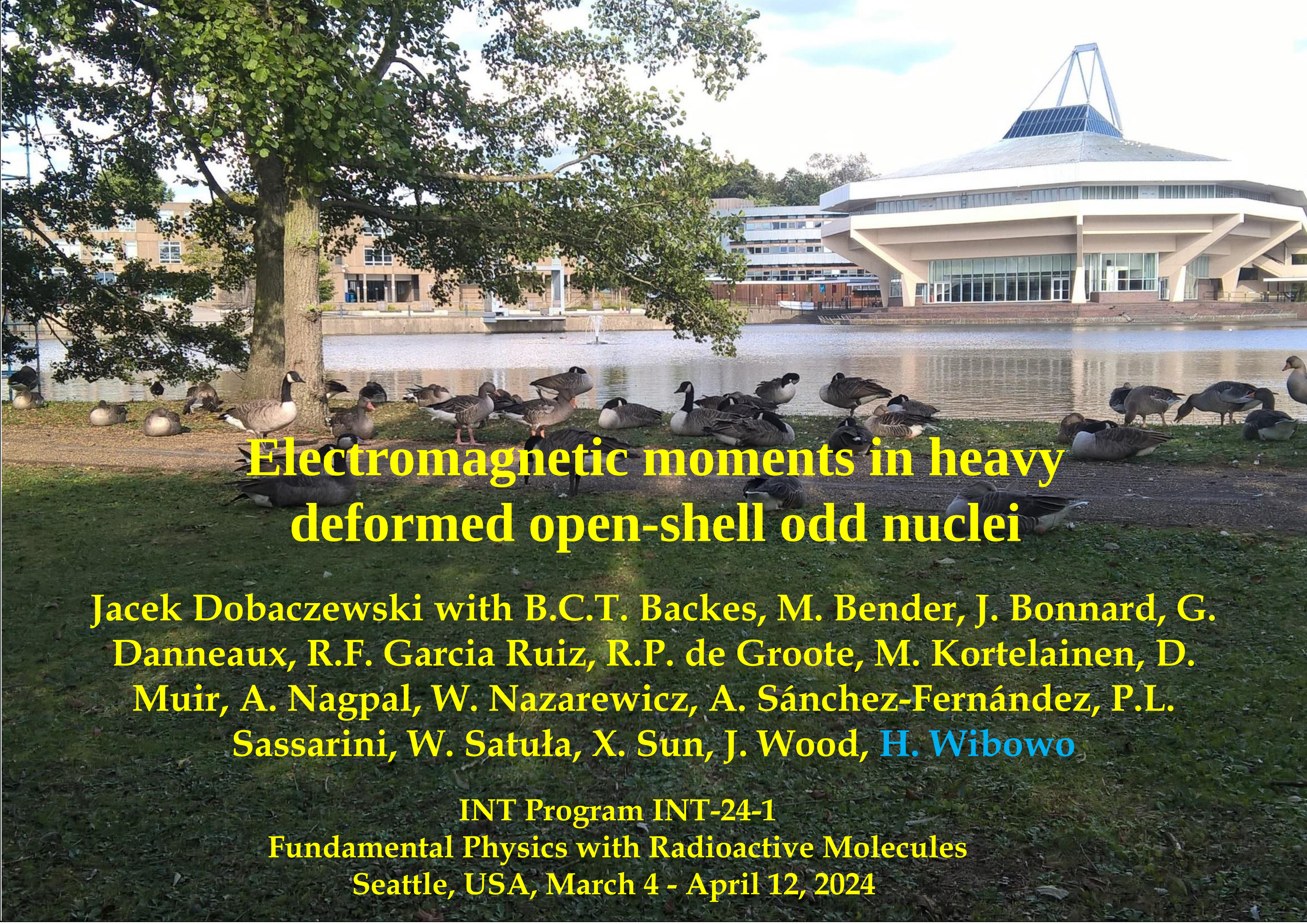

Electromagnetic moments in heavy deformed open-shell odd nuclei

Jacek Dobaczewski with B.C.T. Backes, M. Bender, J. Bonnard, G. Danneaux, R.F. Garcia Ruiz, R.P. de Groote, M. Kortelainen, D. Muir, A. Nagpal, W. Nazarewicz, A. Sánchez-Fernández, P.L. Sassarini, W. Satuła, X. Sun, J. Wood, H. Wibowo

INT Program INT-24-1
Fundamental Physics with Radioactive Molecules
Seattle, USA, March 4 - April 12, 2024

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The bottom line

The nuclear electromagnetic moments are all about:

1. **Polarization**
2. **Self-consistency**
3. **Symmetry restoration**

Outline

1. Methodology*
2. Gd-Pb*
3. Dy-Os
4. Sn-Gd
5. In*
6. Ag*
7. Sn*
8. Sb
9. What next?
10. Two-body currents
11. K-mixing
12. Octupole vibration ^{171}Yb

*published

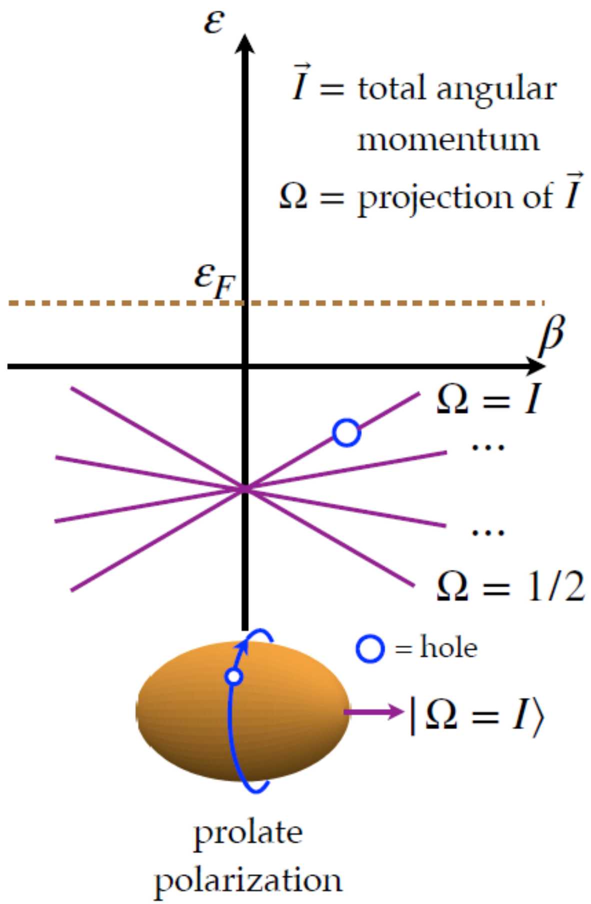
Herlik Wibowo

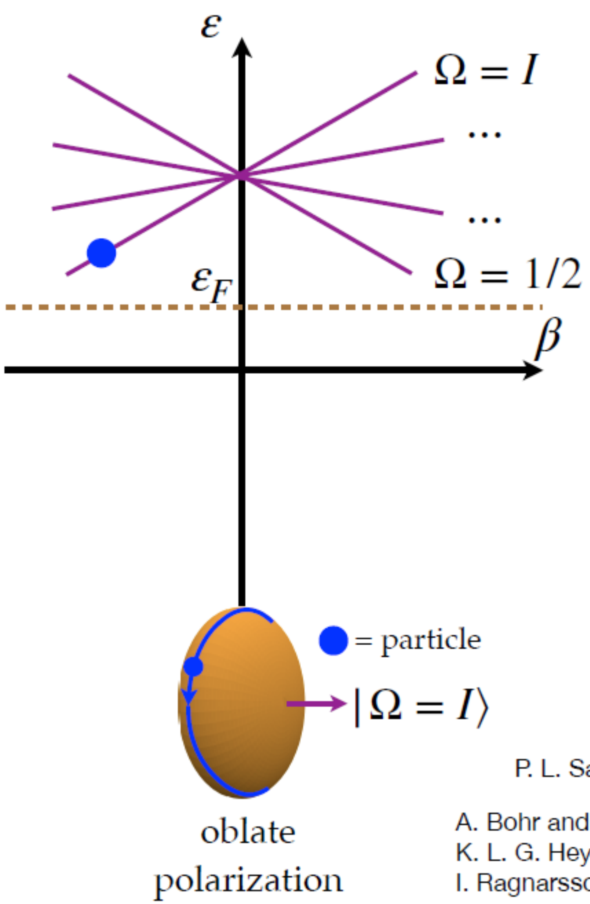
Collaborators: D. Muir, A. Sánchez-Fernández, X. Sun, and J. Dobaczewski

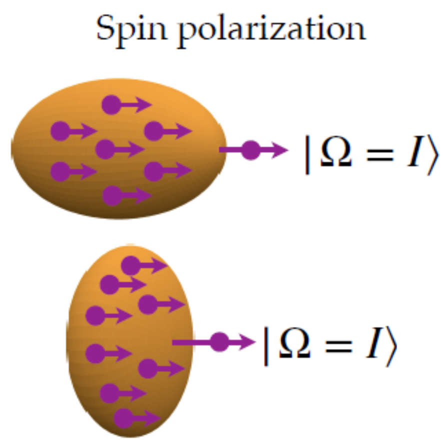
UK Nuclear Physics Conference 2023 at the University of York
April 4-6, 2023

2

Shape and spin core polarizations







Landau parameter g_0' ($g_0' = 1.7$)

$$g_0' = N_0 \left(2C_1^s + 2C_1^T (3\pi^2 \rho_0/2)^{2/3}\right)$$

$$\frac{1}{N_0} \approx 150 \frac{m}{m^*}\ \mathrm{MeV} \cdot \mathrm{fm}^3$$

P. L. Sassarini et al., J. Phys. G: Nucl. Part. Phys. **49**, 11LT01 (2022)

A. Bohr and B. R. Mottelson, *Nuclear Structure* Vol. 1
K. L. G. Heyde, *The Nuclear Shell Model*
I. Ragnarsson and S. G. Nilsson, *Shapes and Shells in Nuclear Structure*

Herlik Wibowo

Collaborators: D. Muir, A. Sánchez-Fernández, X. Sun, and J. Dobaczewski

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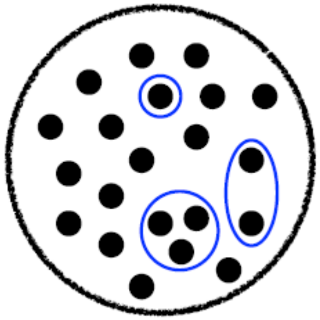
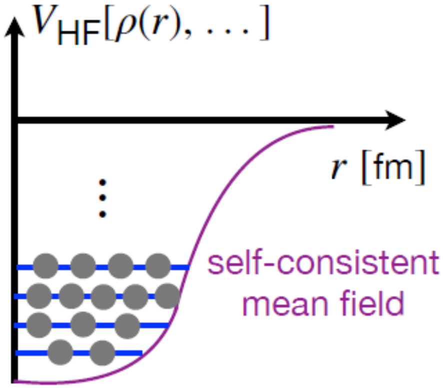
April 4-6, 2023

1

Nuclear density functional theory





Trial s.p. wave-function $\{ | \varphi_i^{\mathrm{trial}} \rangle \}$

↓

Calculate the densities $\{\rho, \ldots\}$

↓

Calculate the mean field $V_{\mathrm{HF}}[\rho, \ldots]$

↓

Solve the HF equation

$$\left[\frac{p^2}{2m} + V_{\mathrm{HF}}[\rho, \ldots] \right] |\varphi_i\rangle = \varepsilon_i |\varphi_i\rangle$$

↓

New s.p. wave-function $\{ | \varphi_i^{\mathrm{new}} \rangle \}$ (→ back to Calculate the densities)

Energy density functional

$$\mathscr{E}[\rho(\mathrm{r}), \mathrm{s}(\mathrm{r}), \tau(\mathrm{r}), \mathrm{T}(\mathrm{r}), \mathrm{j}(\mathrm{r}), \overleftrightarrow{J}(\mathrm{r})]$$

Coupling constants

T-even : $C_t^\rho, C_t^{\Delta\rho}, C_t^\tau, C_t^J, C_t^{\nabla J}$

T-odd : $C_t^s, C_t^{\Delta s}, C_t^T, C_t^j, C_t^{\nabla j}$

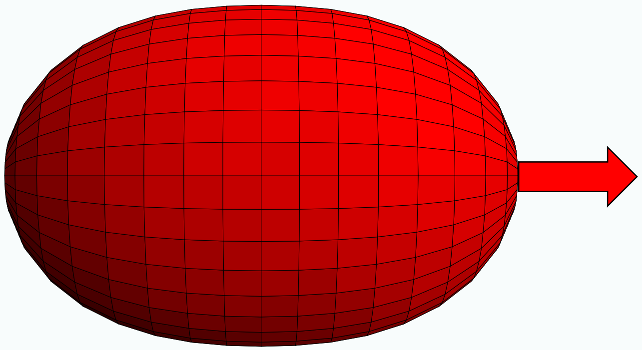
Parametrization: UNEDF1

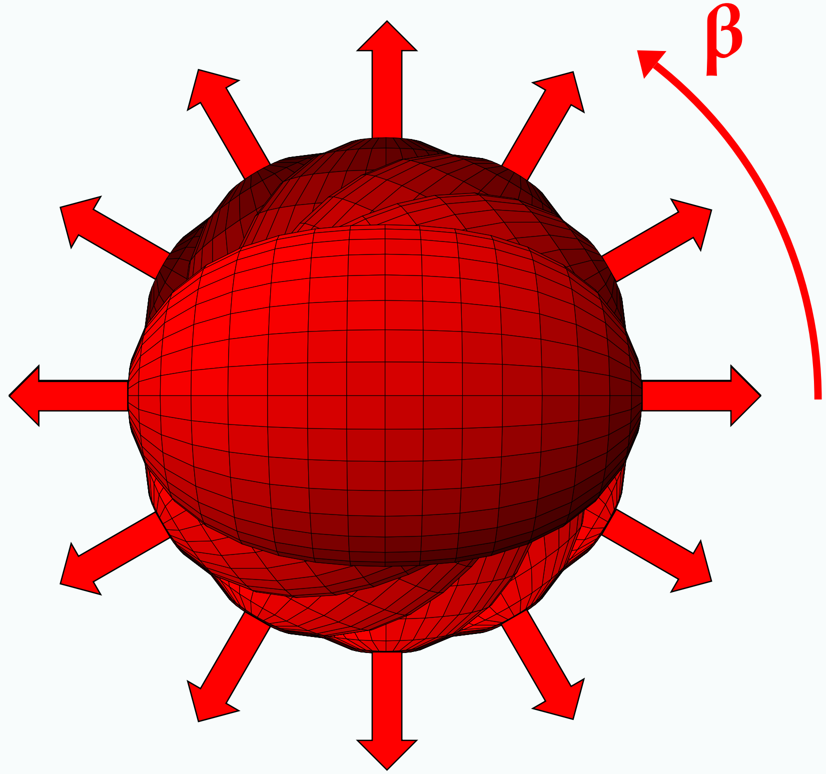
→ Hartree-Fock (HF) equation

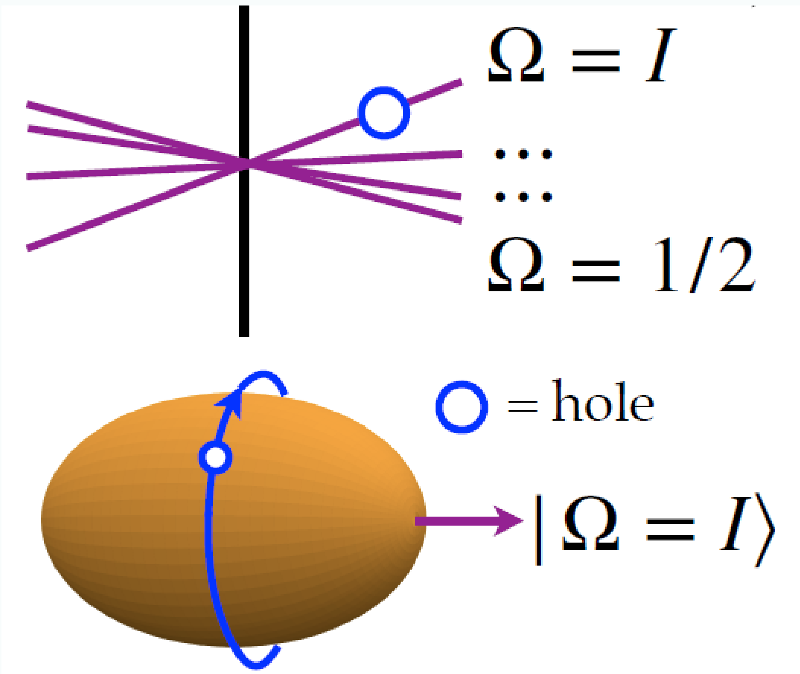
M. Kortelainen et al., Phys. Rev. C **85**, 024304 (2012)

Time-odd spin alignment & symmetry restoration

“Intrinsic”
Symmetry broken







“Laboratory”
Symmetry restored

$$|IM\rangle = \mathcal{N}_I \int_{\beta=0}^{\pi} \mathrm{d}\beta \, d^I_{M\Omega}(\beta) |\Omega, \beta\rangle$$

J. A. Sheikh et al., J. Phys. G48, 123001 (2021)

Nuclear quadrupole & dipole moments

Spectroscopic electric quadrupole Q and magnetic dipole μ moments are :

$$Q = \sqrt{\frac{16\pi}{5}}\langle II\,|\,\hat{Q}_{20}\,|\,II\rangle \quad \text{and} \quad \mu = \sqrt{\frac{4\pi}{3}}\langle II\,|\,\hat{M}_{10}\,|\,II\rangle\,.$$

P. Ring and P. Schuck, *The Nuclear Many-Body Problem*

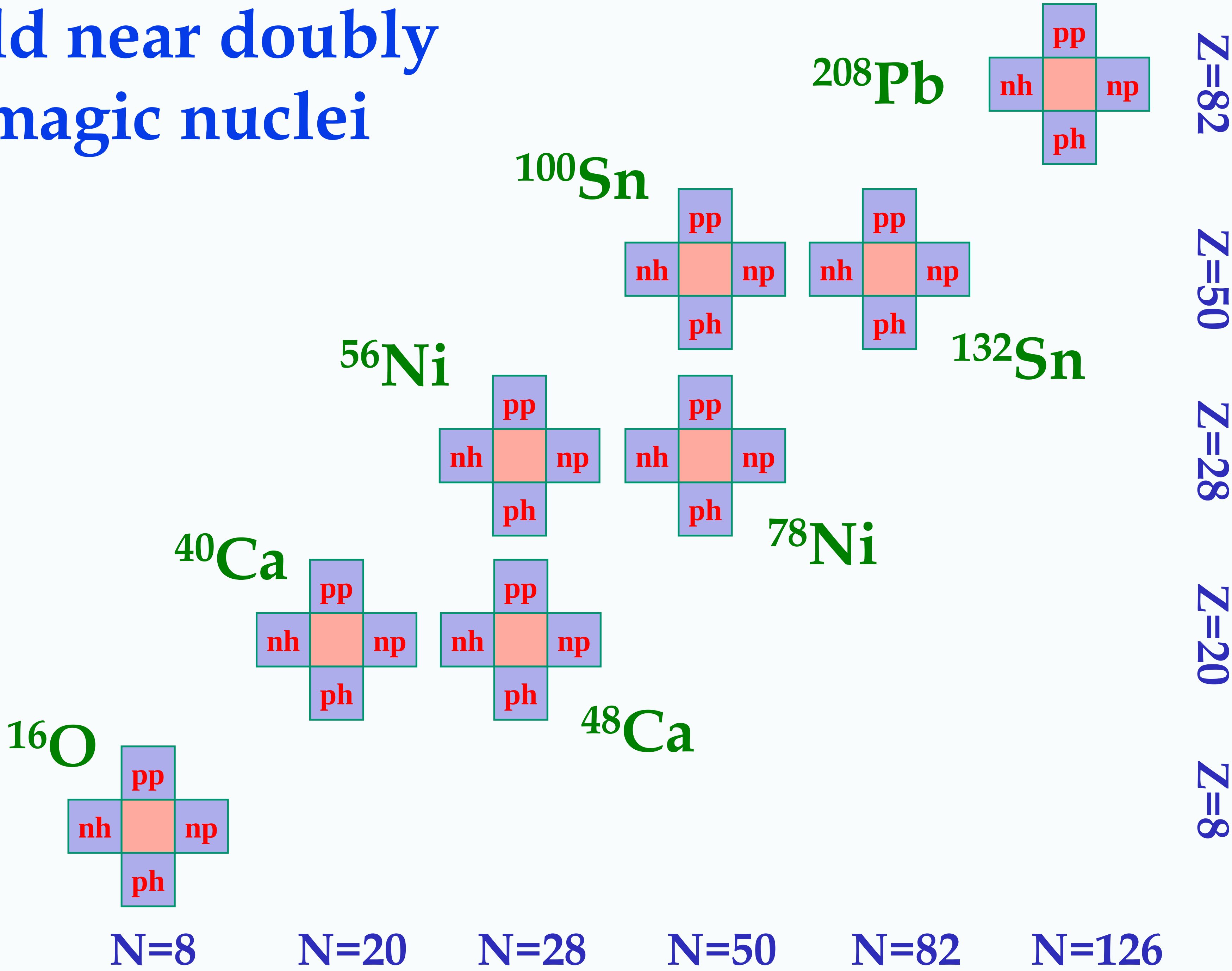
$$\hat{Q}_{20} = \sqrt{\frac{5}{16\pi}}e\sum_{i=1}^{A}\left(\frac{1}{2}-t_3^{(i)}\right)\left\{3z_i^2 - r_i^2\right\};\quad \hat{M}_{10} = \sqrt{\frac{3}{4\pi}}\mu_N\sum_{i=1}^{A}\left\{g_s^{(i)}s_{zi}+g_\ell^{(i)}\ell_{zi}\right\};$$

$$g_s^{(i)} = g_p(g_n) = 5.59(-3.83)$$
$$g_\ell^{(i)} = 1(0)$$

Intrinsic moments = moments of the symmetry-broken state
Spectroscopic moments = moments of the symmetry-restored state

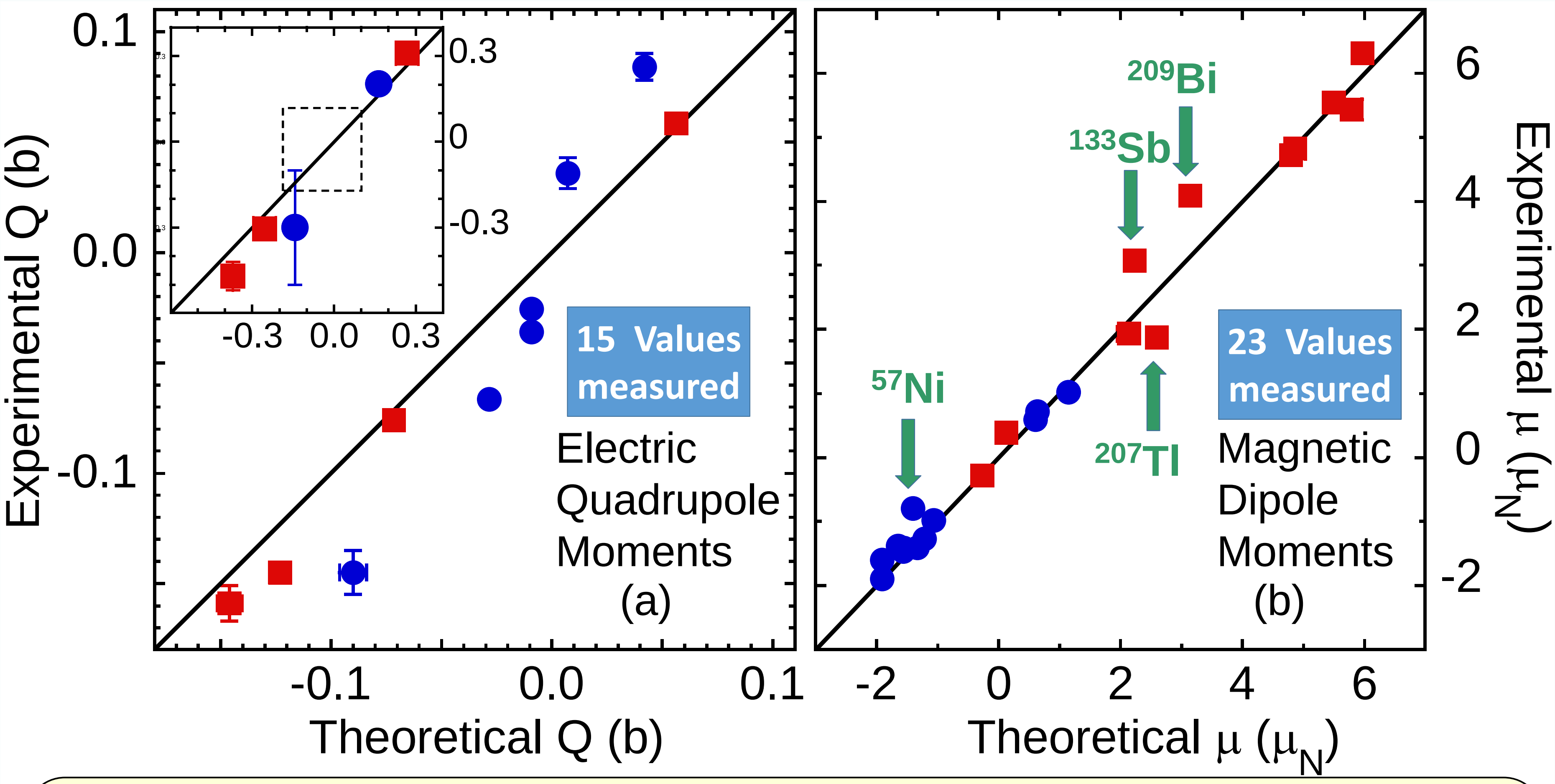
Spectroscopic moments = moments measured experimentally

Odd near doubly magic nuclei



Quadrupole & dipole moments

P. Sassarini *et al.*, J. Phys. G49 (2022) 11LT01



- **Proton-odd (squares) & neutron-odd (circles) nuclei**
- **Average of UNEDF1, SLy4, SkO', D1S, N3LO functionals**
- **RMS deviations much smaller than the residuals**

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Effective spin g-factor? Who ordered that?

P. Sassarini *et al.*, J. Phys. G49 (2022) 11LT01

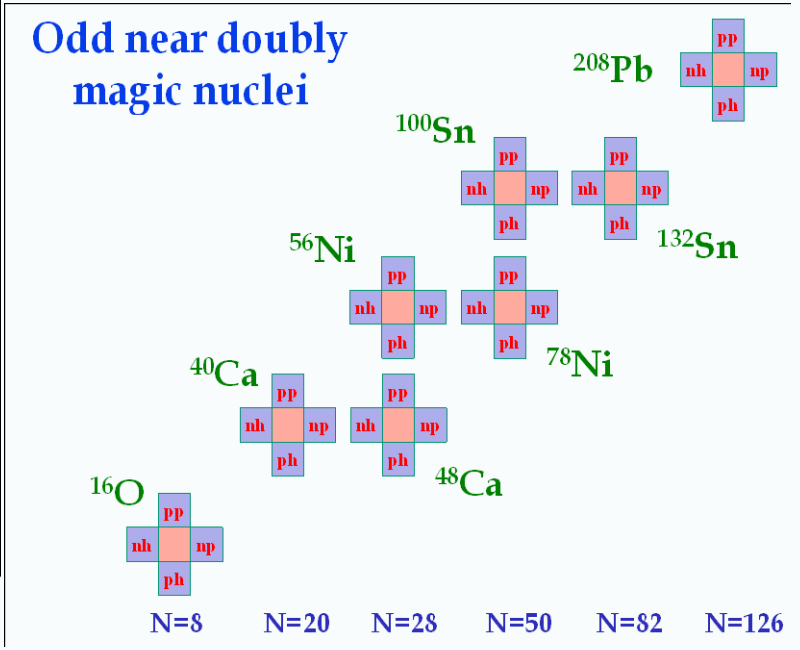
$$g_s^{(i)} = g_p(g_n) = 5.59(-3.83) \times g^{\mathrm{eff}}\ ???$$

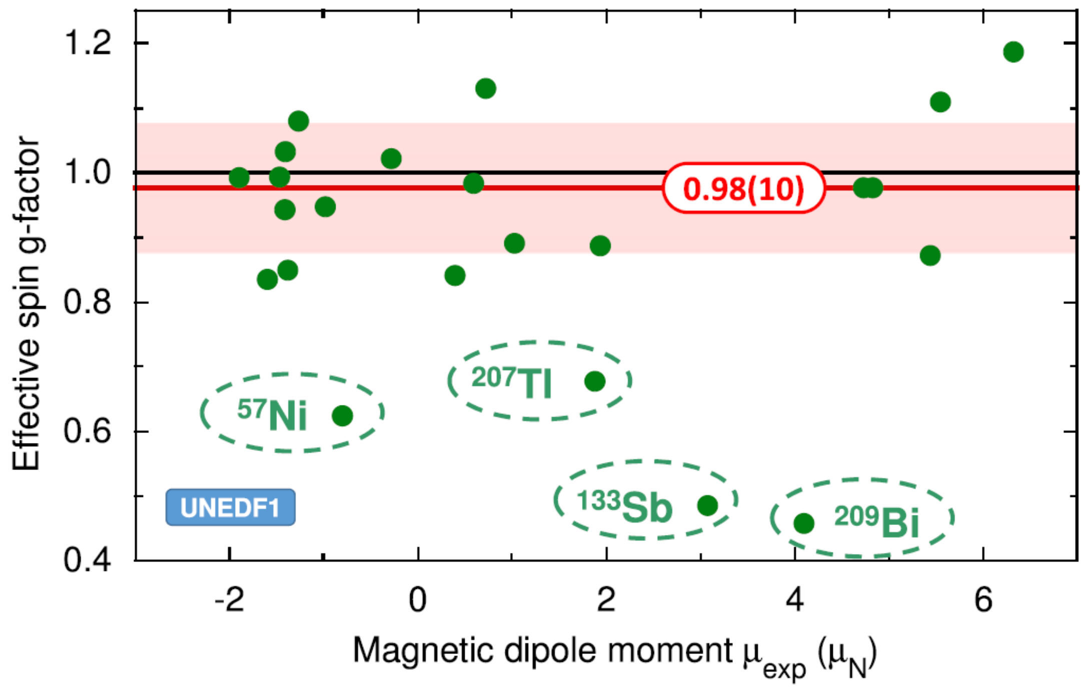
$$g_\ell^{(i)} = 1(0)$$

Landau parameter g_0' ($g_0' = 1.7$)

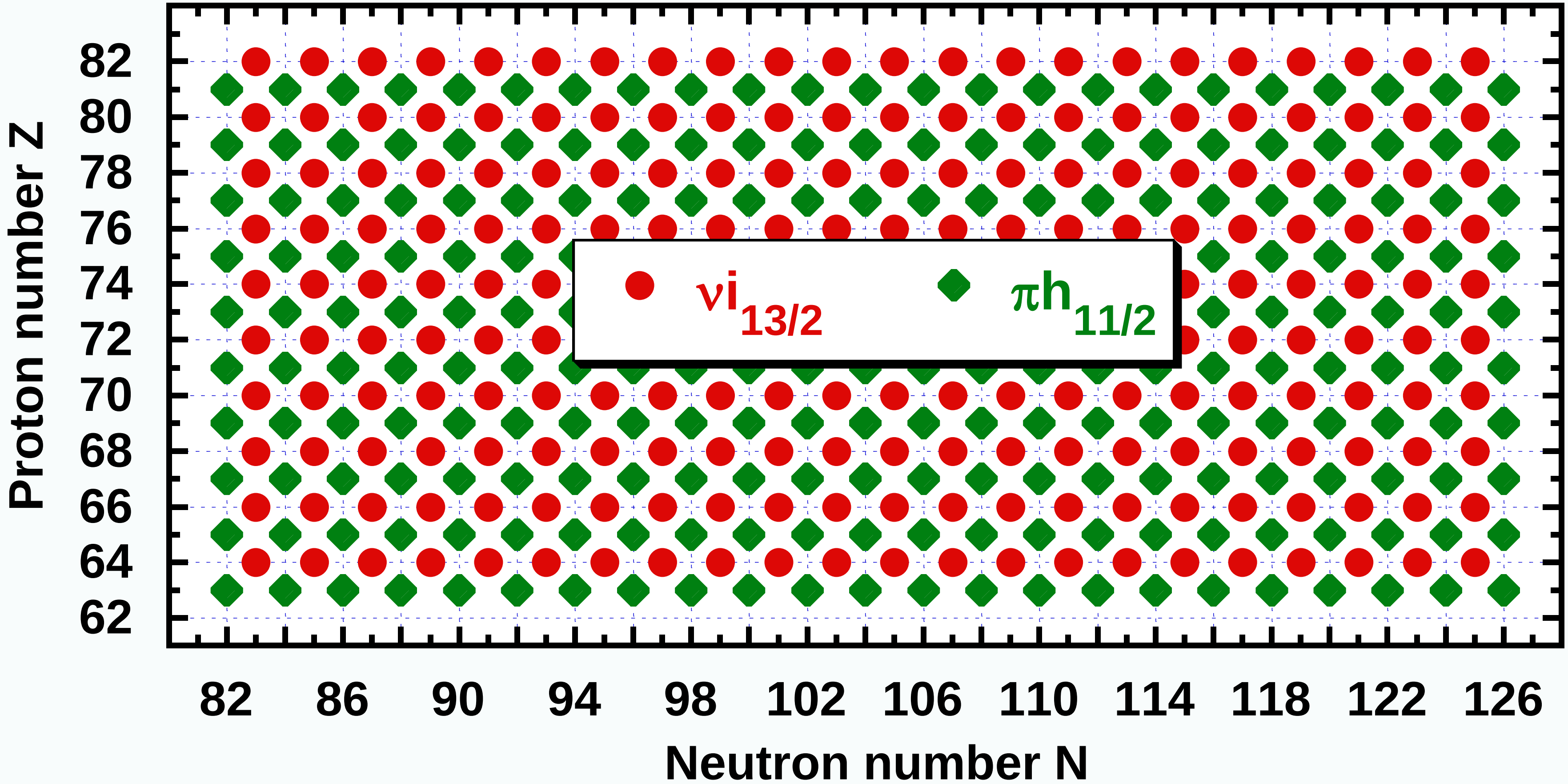
$$g_0' = N_0\left(2C_1^s + 2C_1^T(3\pi^2\rho_0/2)^{2/3}\right)$$

$$\frac{1}{N_0} \approx 150\frac{m}{m^*}\ \mathrm{MeV}\cdot\mathrm{fm}^3$$





The first systematic nuclear-DFT analysis of the electromagnetic moments in heavy deformed open-shell odd nuclei



Blocked quasiparticles were tagged by the neutron $i_{13/2}$ (Ω=+13/2) or proton $h_{11/2}$ (Ω=+11/2) single-particle orbitals

J. Bonnard *et al.*, Phys. Lett. B 843 (2023) 138014

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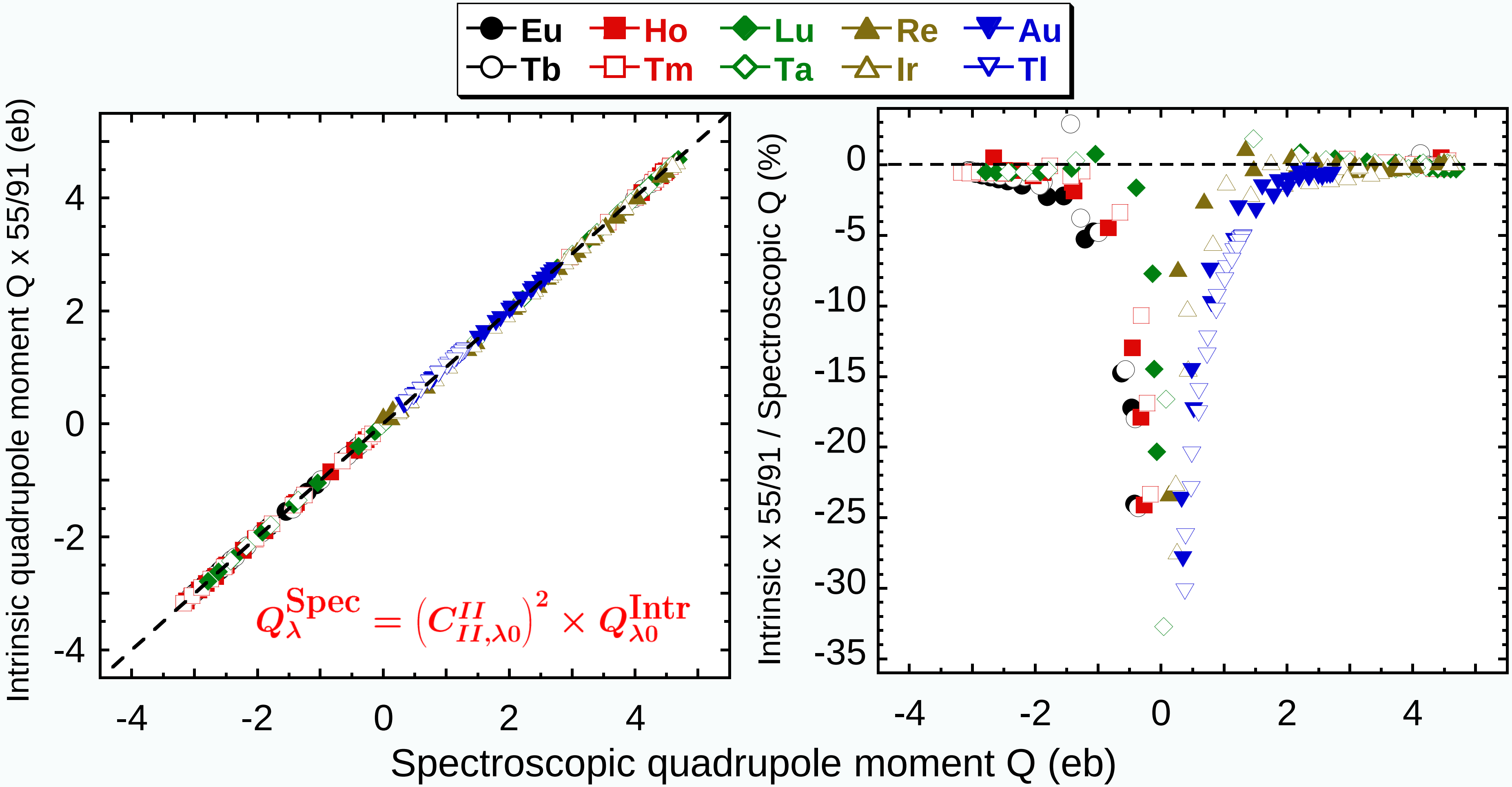
Heavy deformed π11/2⁻ odd-Z nuclei



J. Bonnard *et al.*, Phys. Lett. B 843 (2023) 138014

Conclusion:

Spectroscopic electric quadrupole moments can be inferred from the intrinsic ones at ~5% precision only at |Q|>1b)

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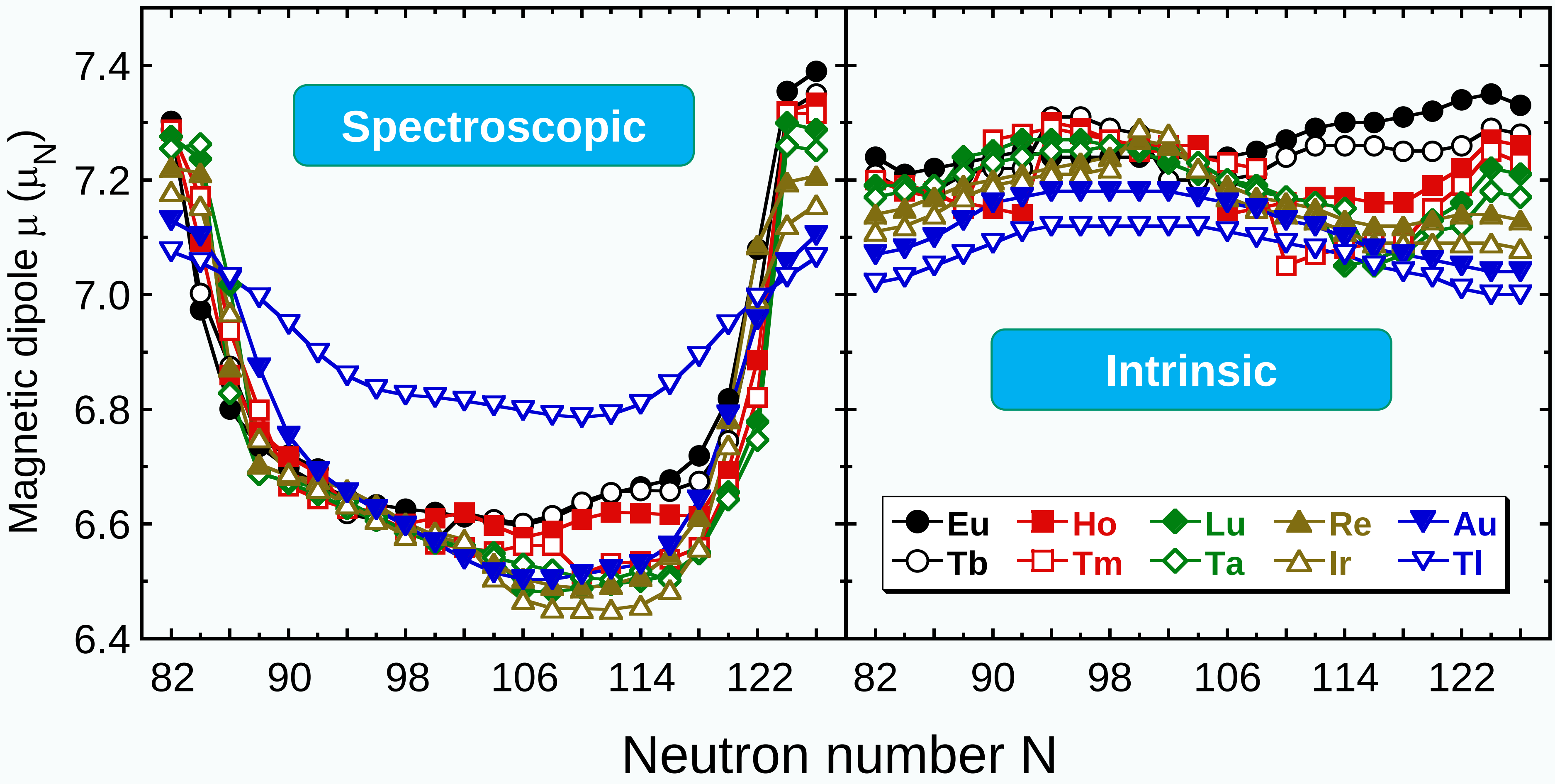
Heavy deformed π11/2⁻ odd-Z nuclei

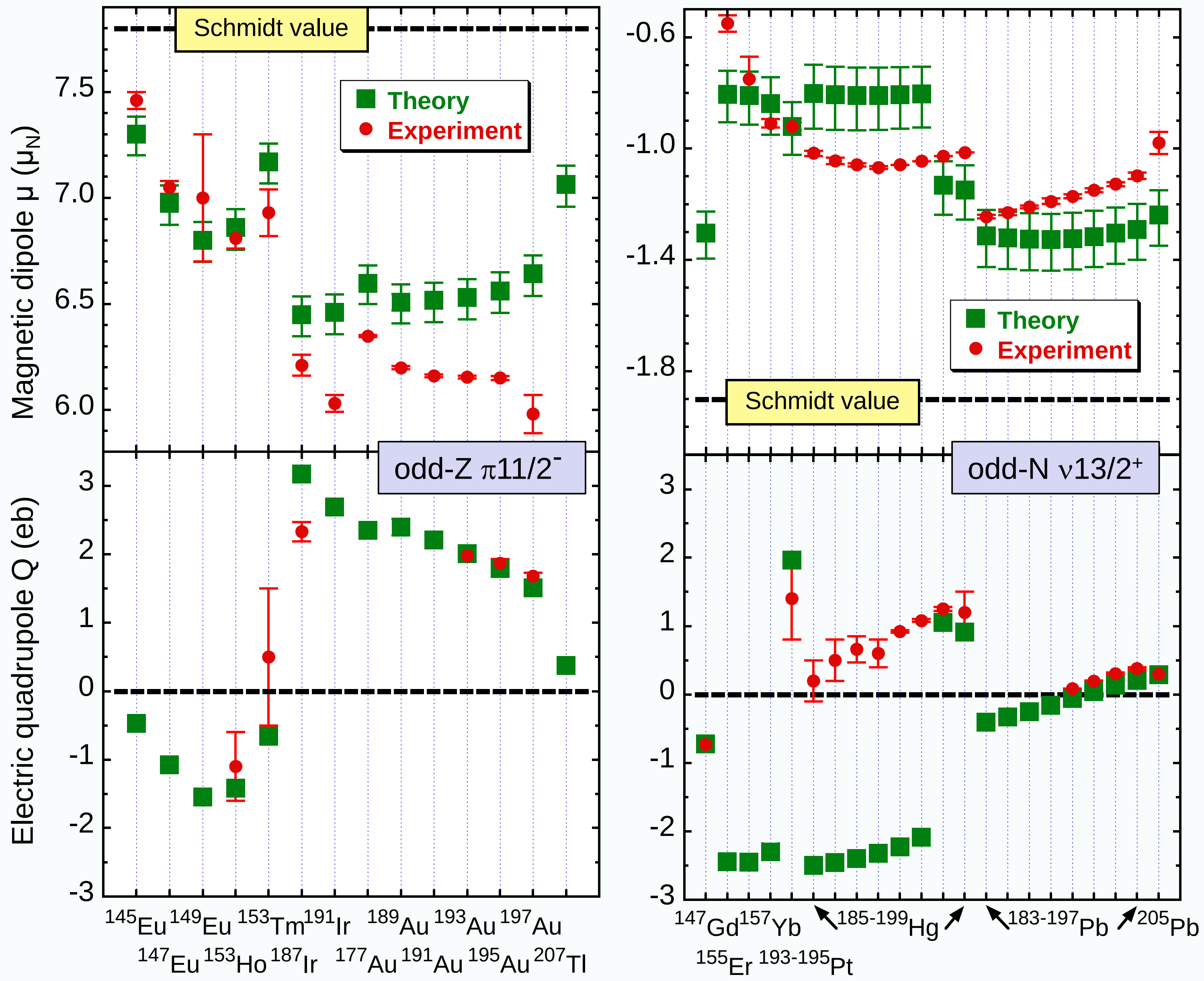


J. Bonnard *et al.*, Phys. Lett. B 843 (2023) 138014

Conclusion:

Spectroscopic magnetic dipole moments cannot be inferred from the intrinsic ones

Spectroscopic moments: theory vs. experiment



J. Bonnard *et al.*, Phys. Lett. B 843 (2023) 138014

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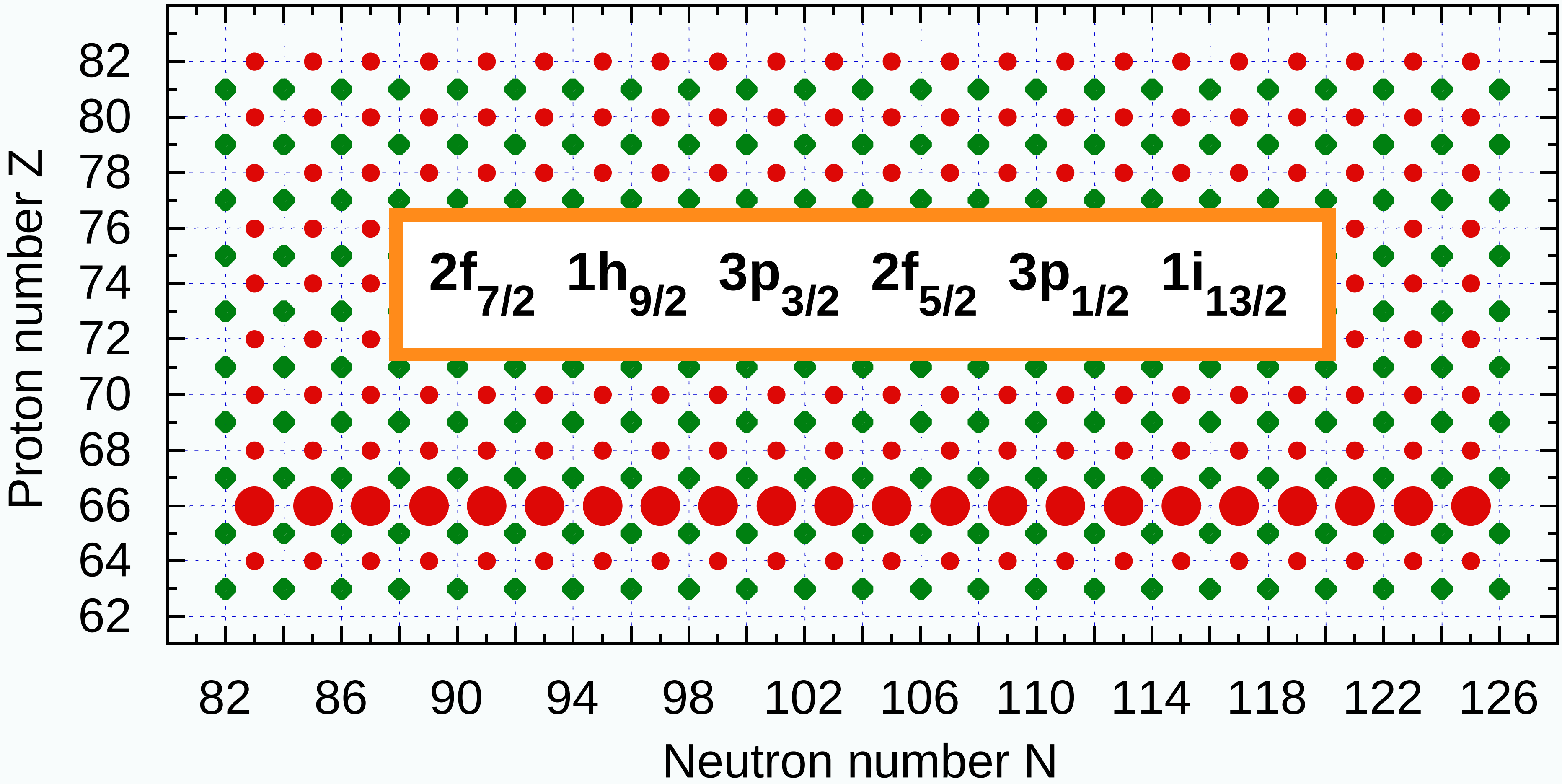
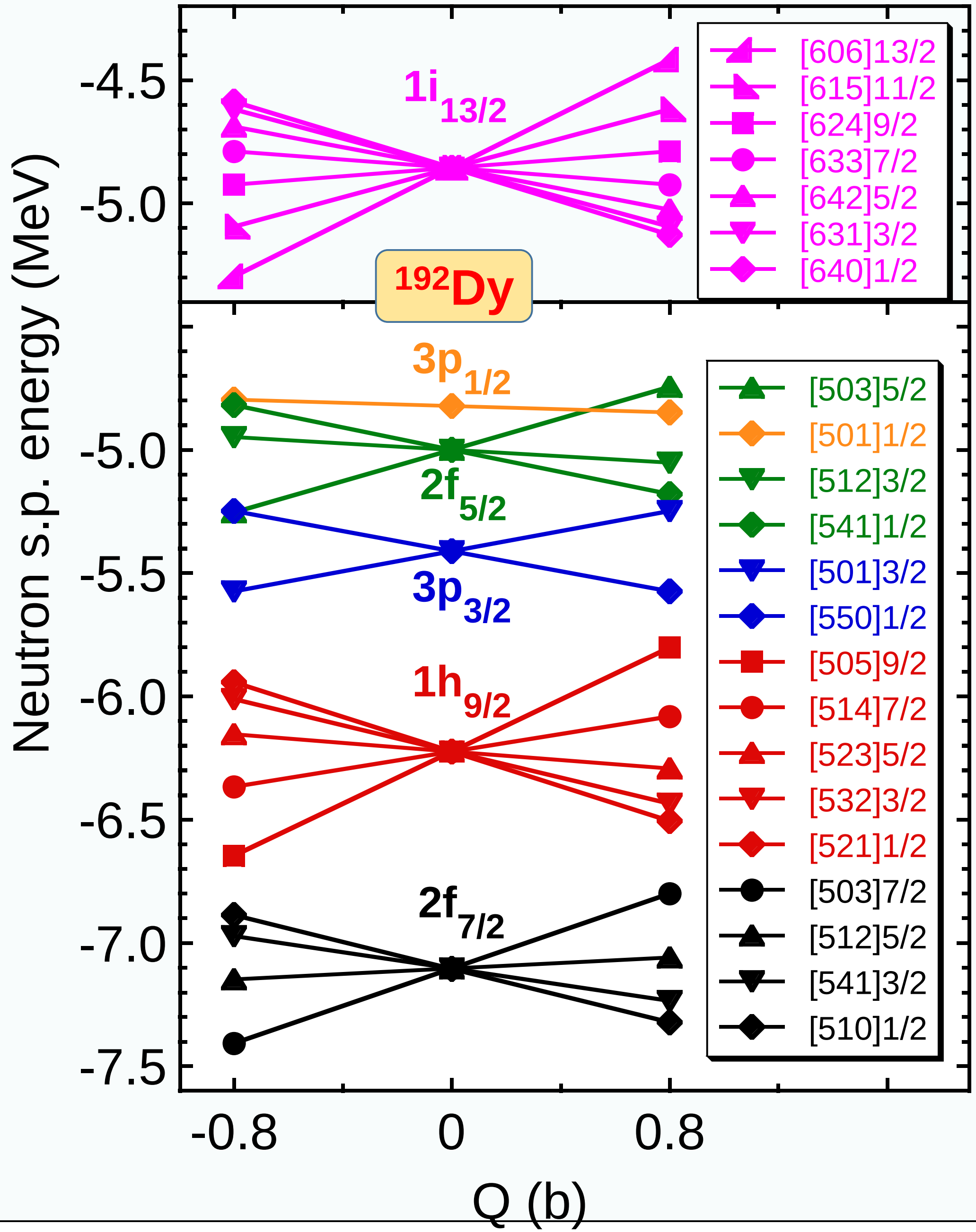
The first systematic nuclear-DFT analysis of the electromagnetic moments in excited quasiparticle states



J. Dobaczewski *et al.*, to be published

How to calculate odd nuclei in nuclear DFT?



without pairing

A even, $p > A$, $h \leq A$

$$|\Psi\rangle^{\text{even}}_{\text{HF}} = a^+_A \ldots a^+_2 a^+_1 |0\rangle$$

$$|\Psi\rangle^{\text{odd}}_{\text{HF}} = \begin{cases} a^+_p |\Psi\rangle^{\text{even}}_{\text{HF}} \\ a_h |\Psi\rangle^{\text{even}}_{\text{HF}} \end{cases}$$

with pairing

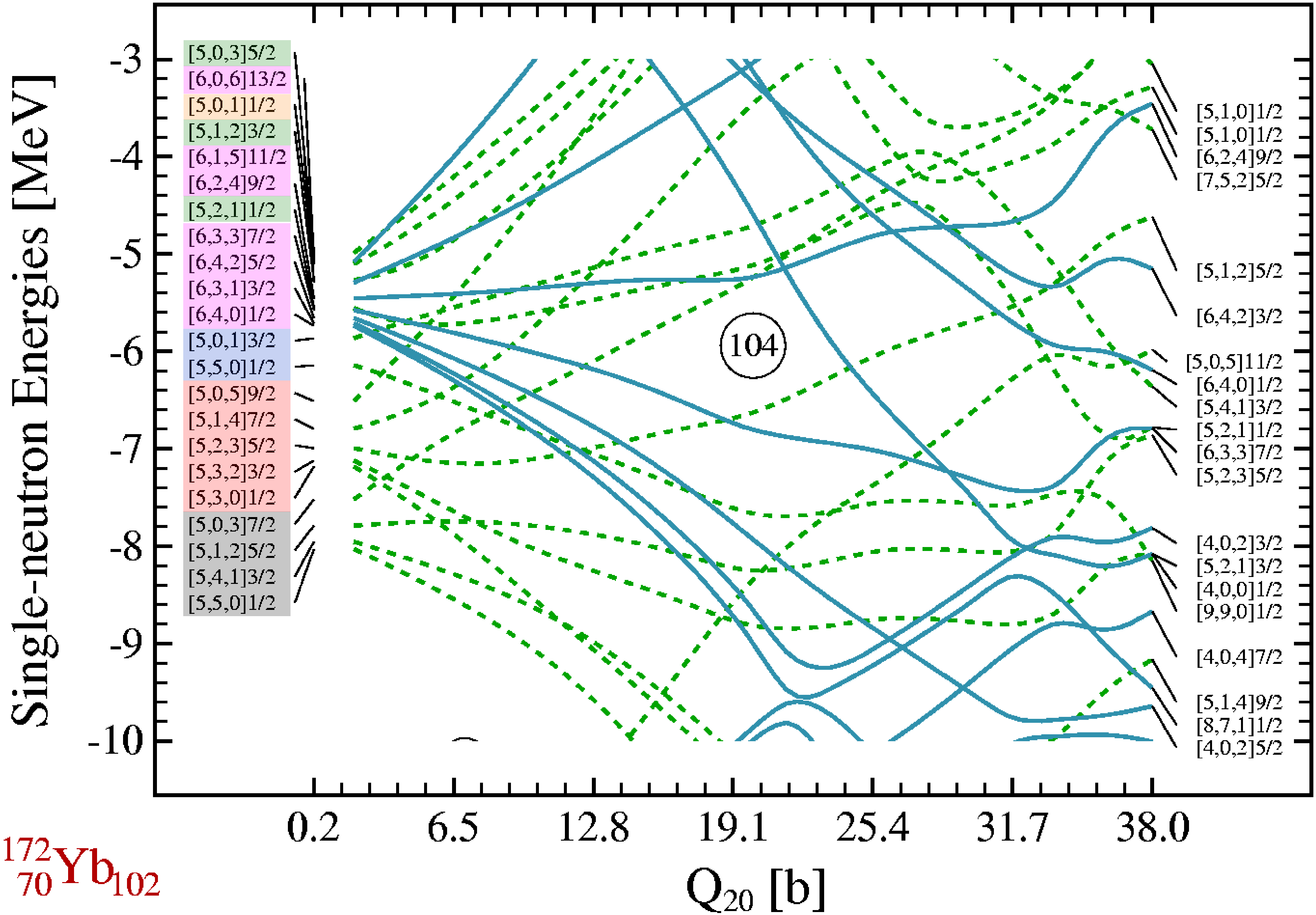
$$|\Psi\rangle^{\text{even}}_{\text{HFB}} = \prod_{\mu>0} \left(u_\mu + v_\mu a^+_{\bar{\mu}} a^+_\mu\right) |0\rangle$$

$$|\Psi\rangle^{\text{odd}}_{\text{HFB}} = \beta^+_\nu |\Psi\rangle^{\text{even}}_{\text{HFB}}$$

$$= a^+_\nu \prod_{\nu\neq\mu>0} \left(u_\mu + v_\mu a^+_{\bar{\mu}} a^+_\mu\right) |0\rangle$$

tagging quasiparticle states

$$\max_\mu \left\{ \langle \varphi_\nu | \phi^{\text{upper}}_\mu \rangle, \langle \varphi_\nu | \phi^{\text{lower}}_\mu \rangle \right\}$$



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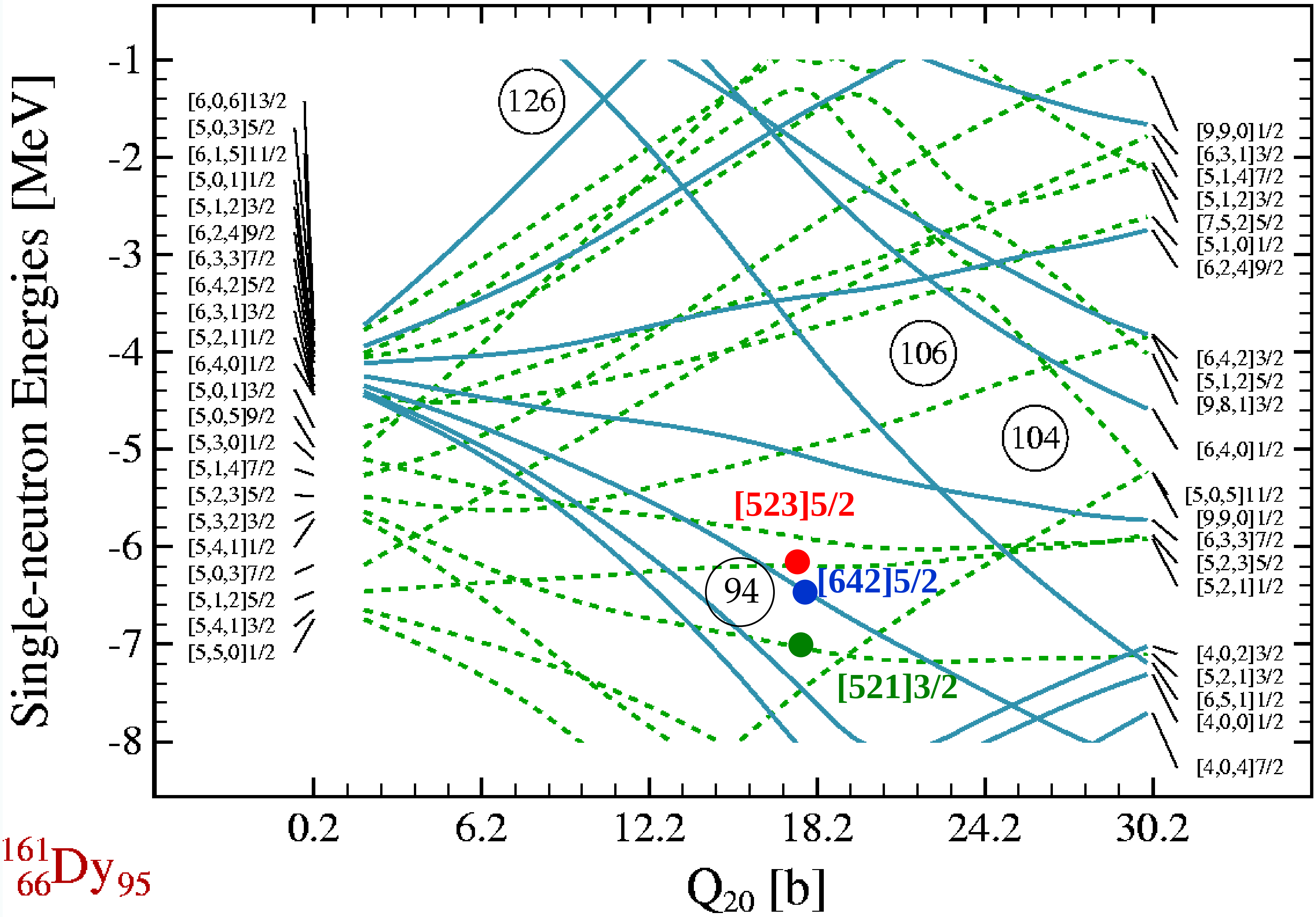

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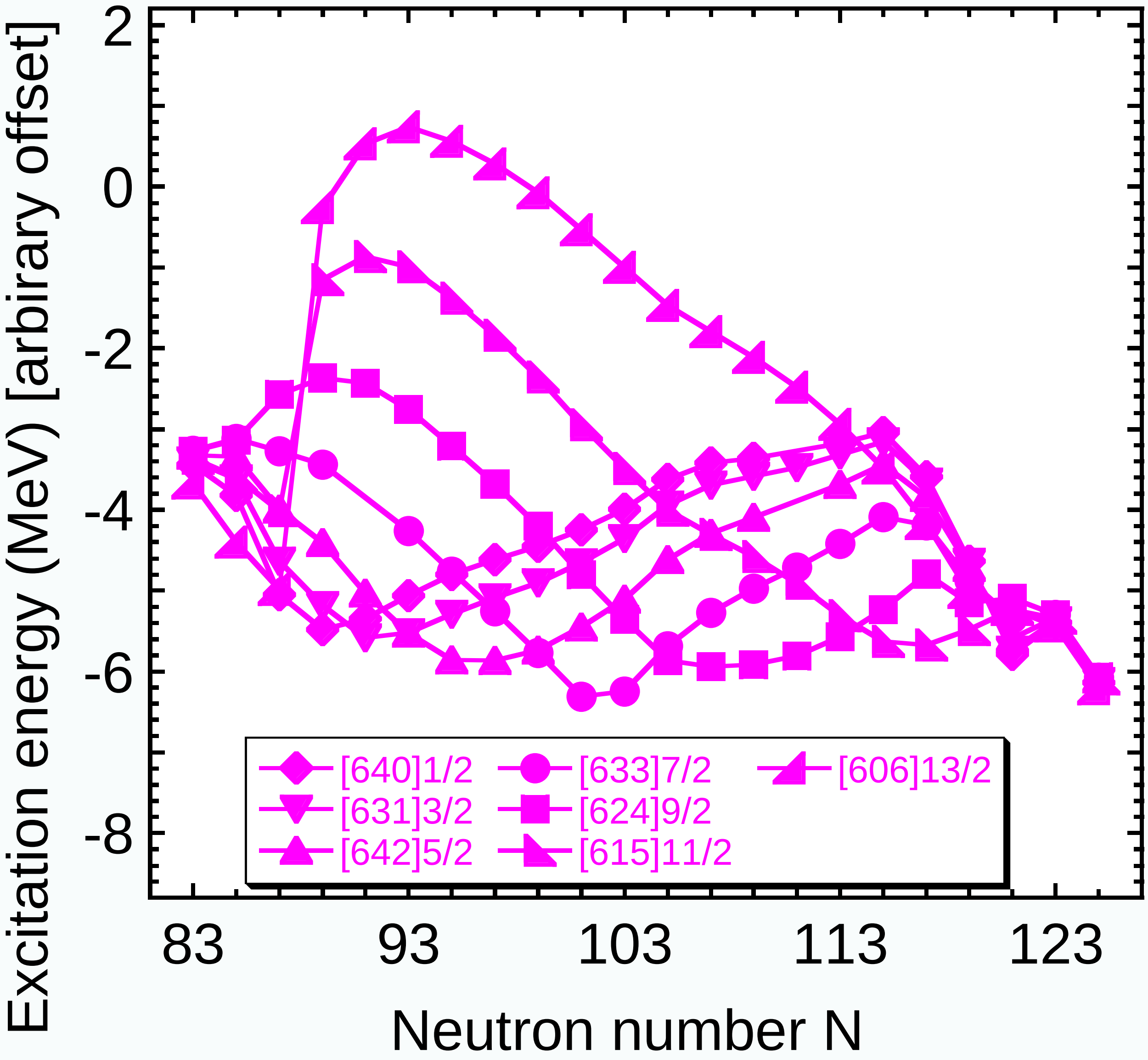
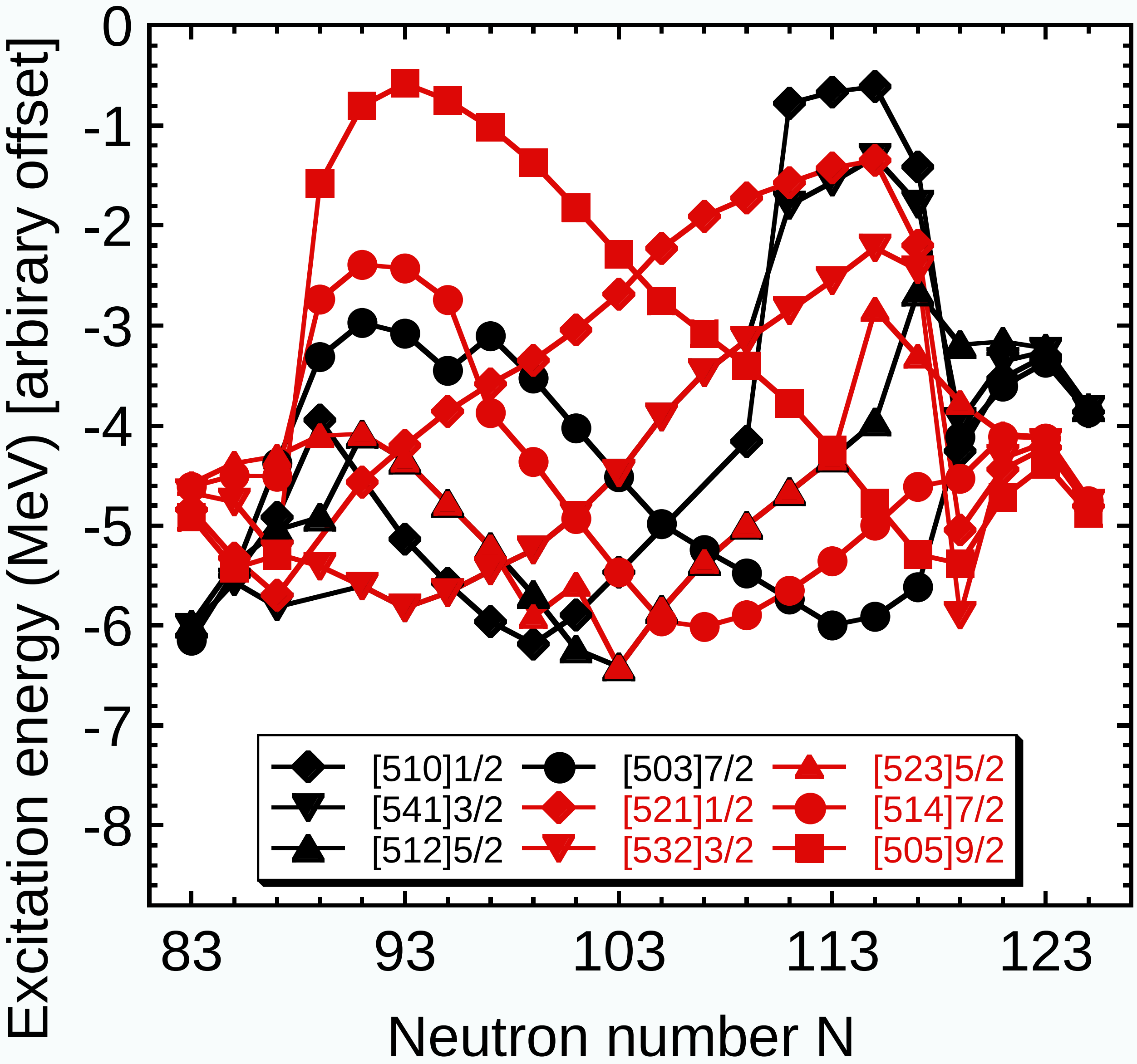
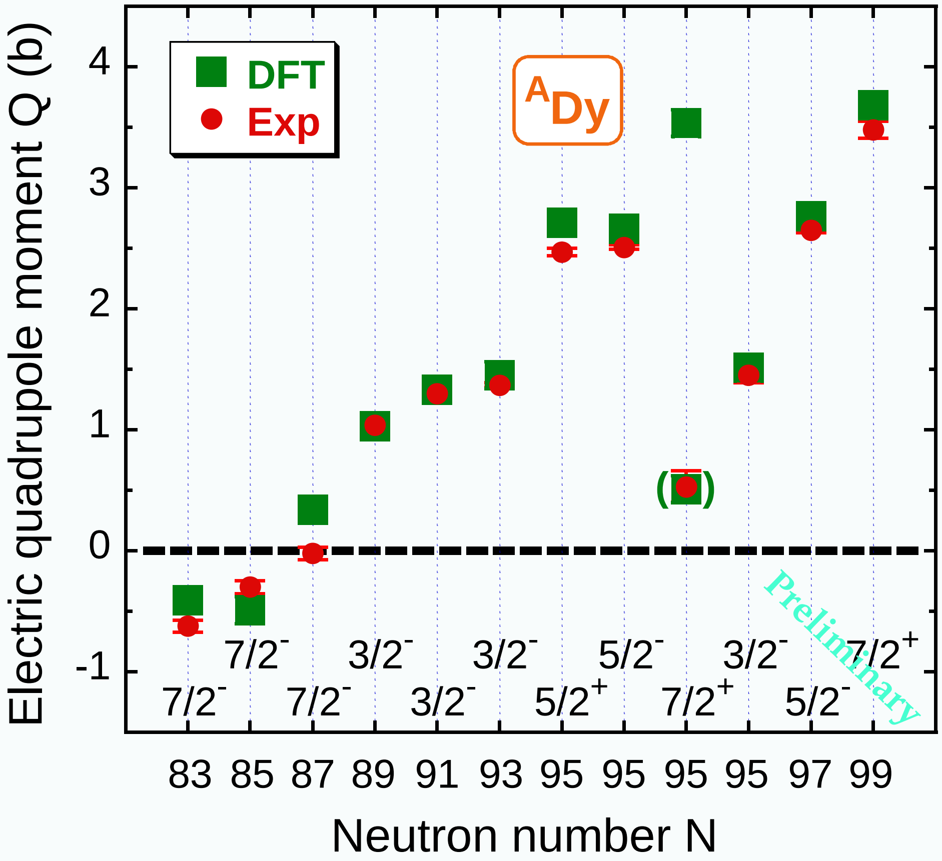
Excitation energies of odd dysprosium isotopes

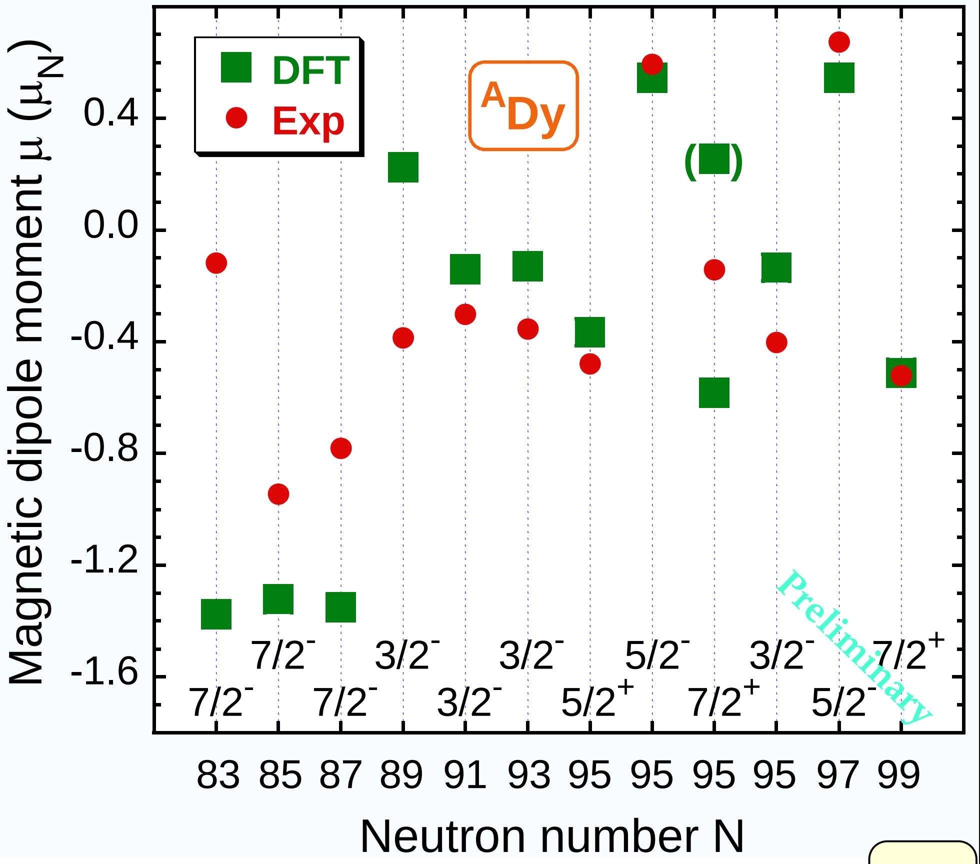


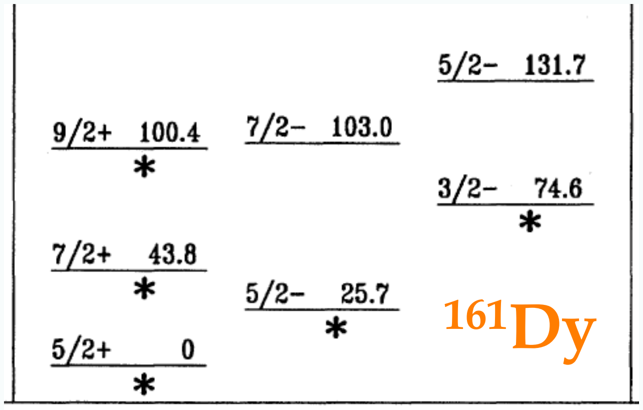
Excitation energies of odd dysprosium isotopes



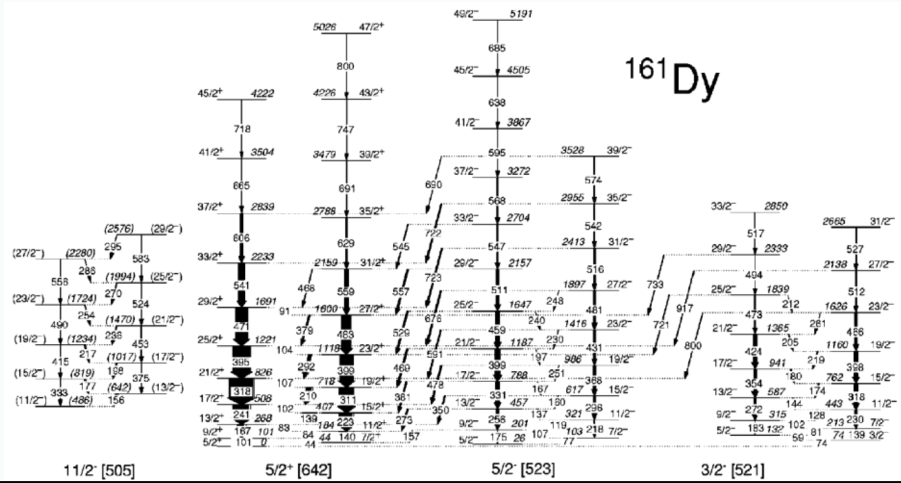
Electromagnetic moments of odd dysprosium isotopes







S. J. Margraf *et al.*, Phys. Rev. C52, 2429 (1995)



A. Jungclaus *et al.*, Phys. Rev. C67, 034302 (2003)

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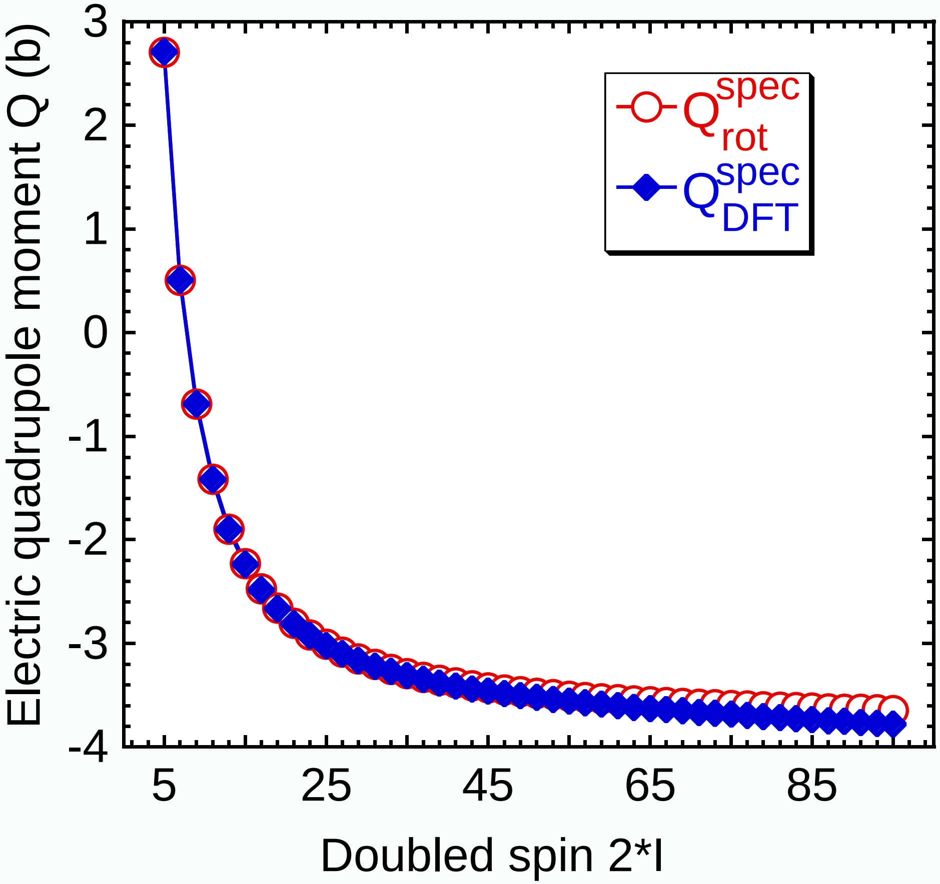
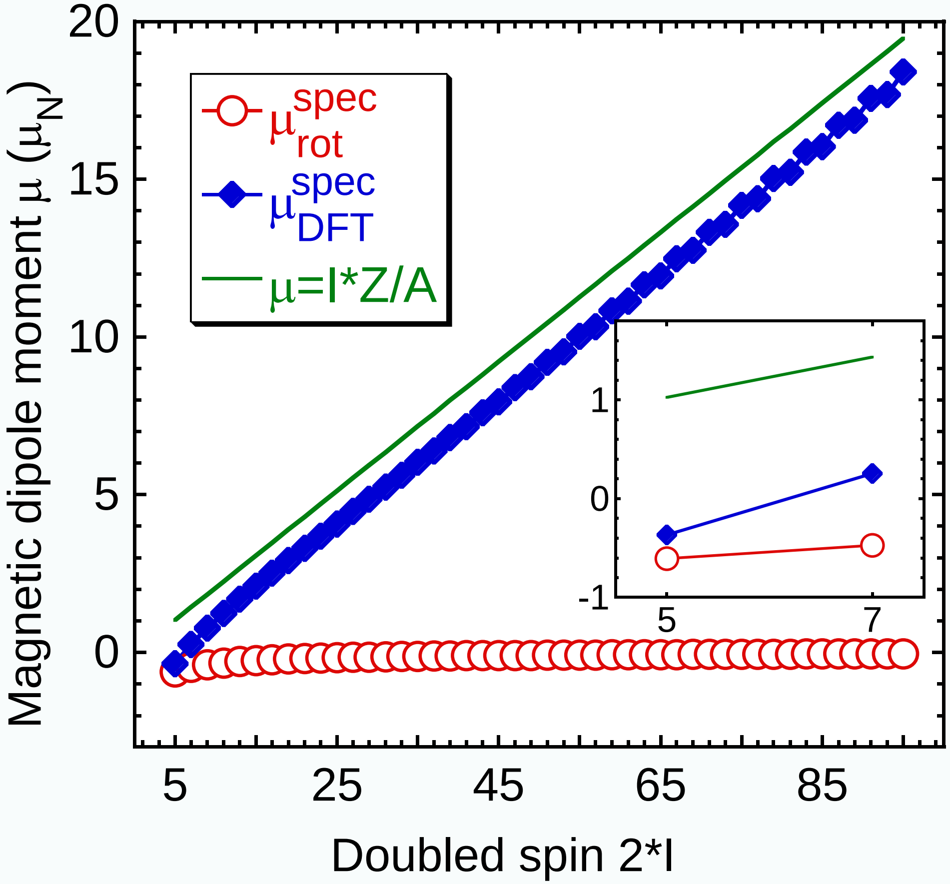
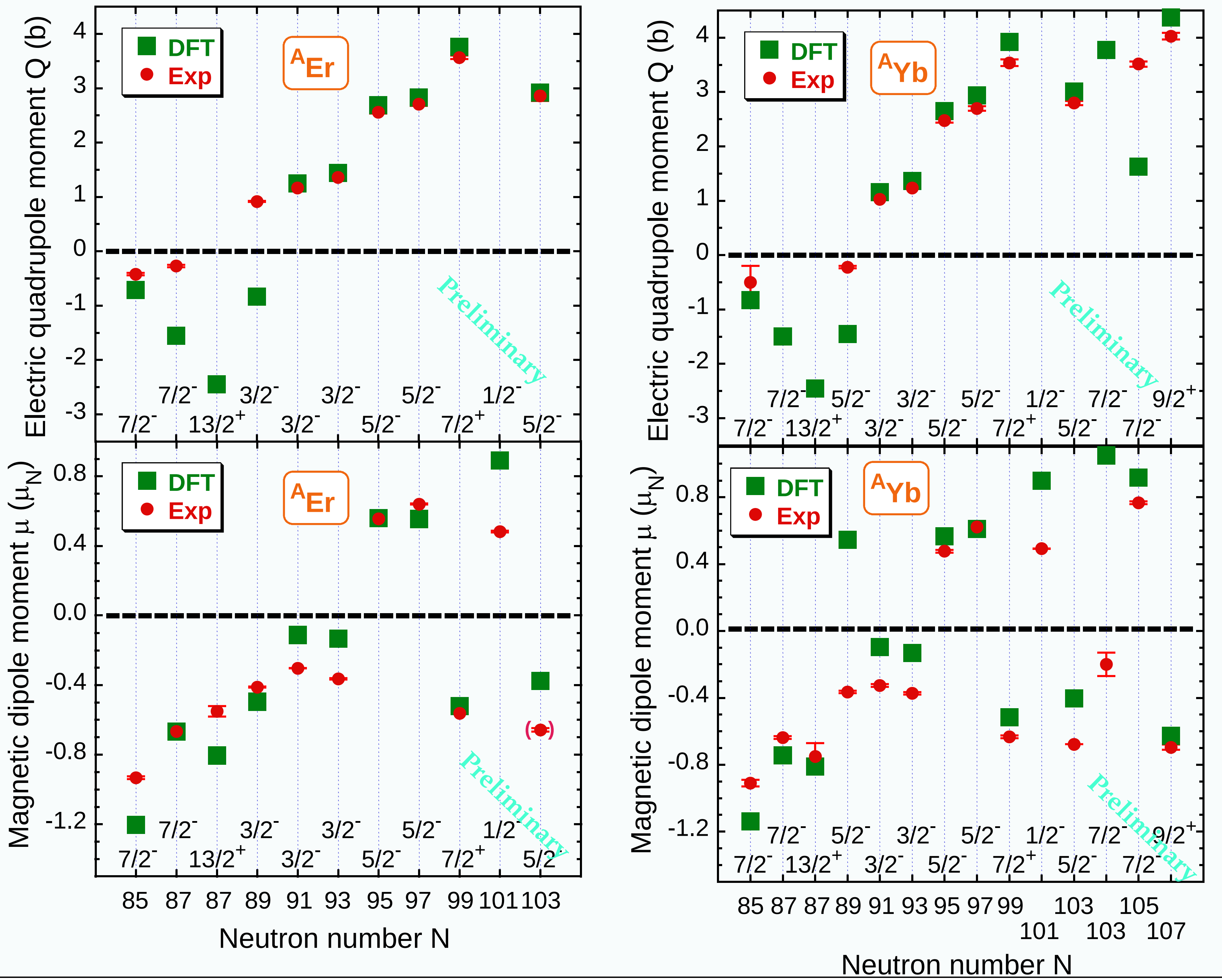
Electromagnetic moments – the rigid-rotor approximation

^{161}Dy 5/2$^+$ UNEDF1, g'_0=1.7

$$Q^{spec}_{rot} = Q^{intr}_{20} \times C^{II}_{II,20} \times C^{IK}_{IK,20} \qquad \mu^{spec}_{rot} = \mu^{intr}_{z} \times C^{II}_{II,10} \times C^{IK}_{IK,10}$$

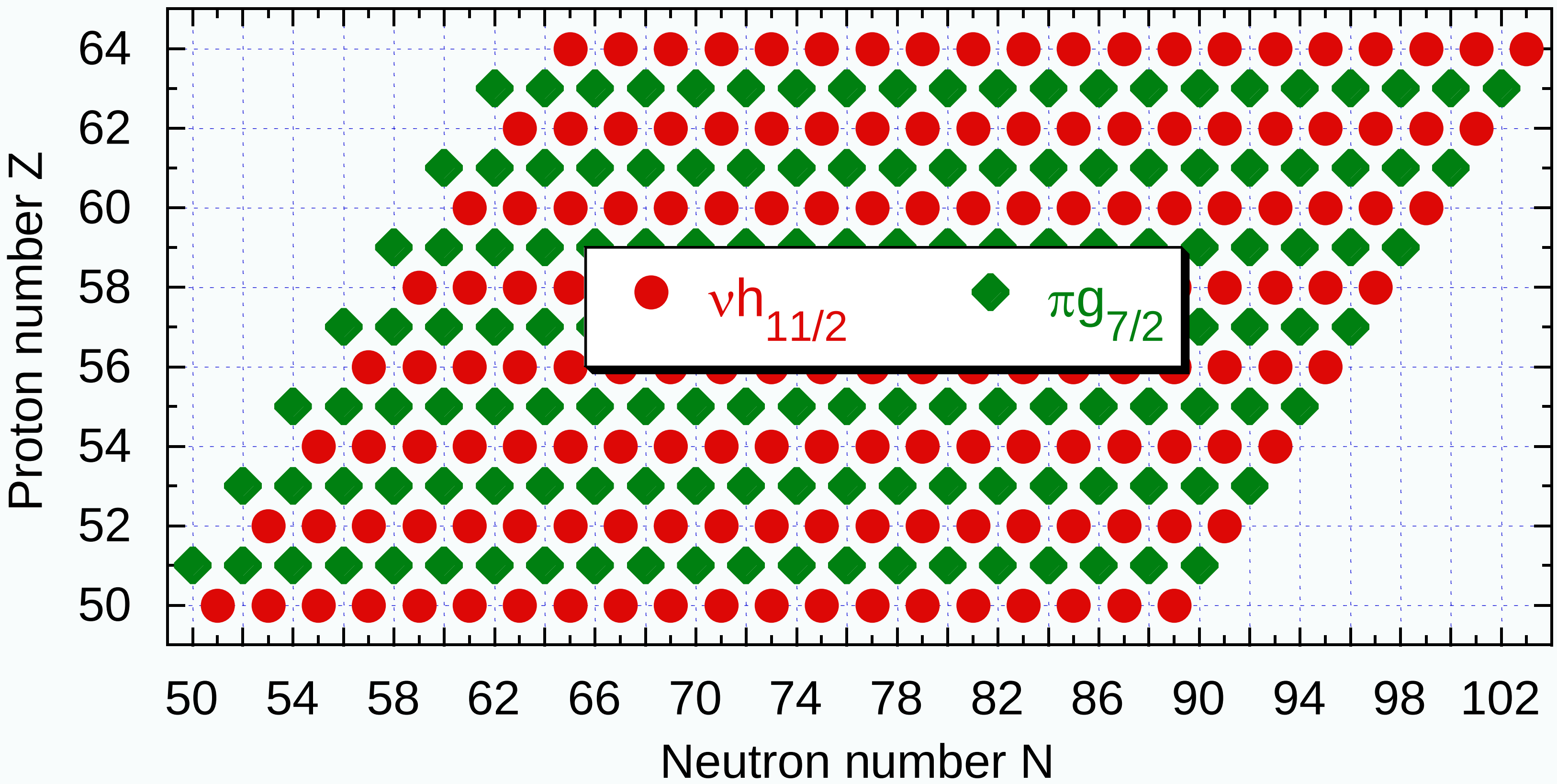






Jacek Dobaczewski

Nuclear-DFT analysis of electromagnetic moments between the Sn and Gd isotopes



H. Wibowo *et al.*, to be published

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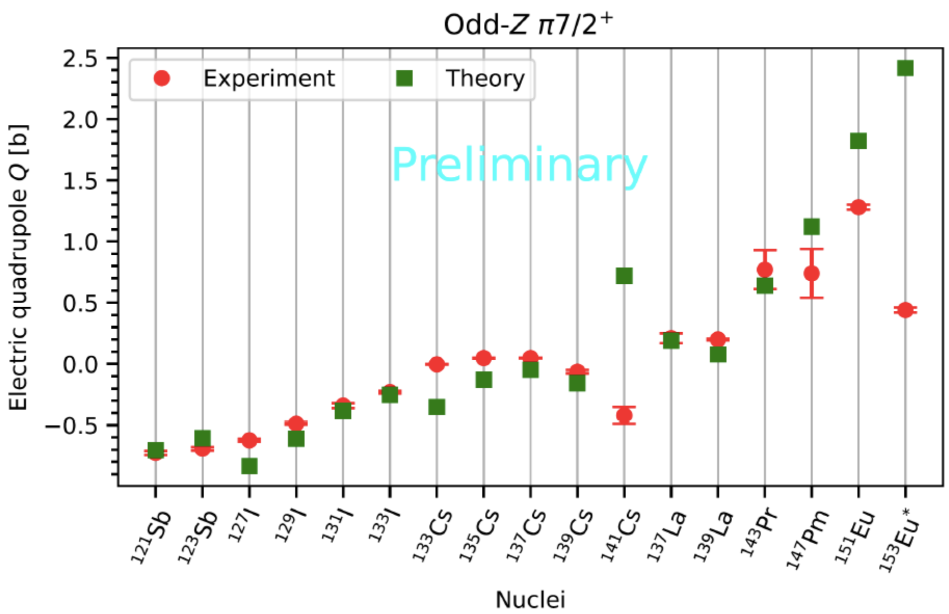
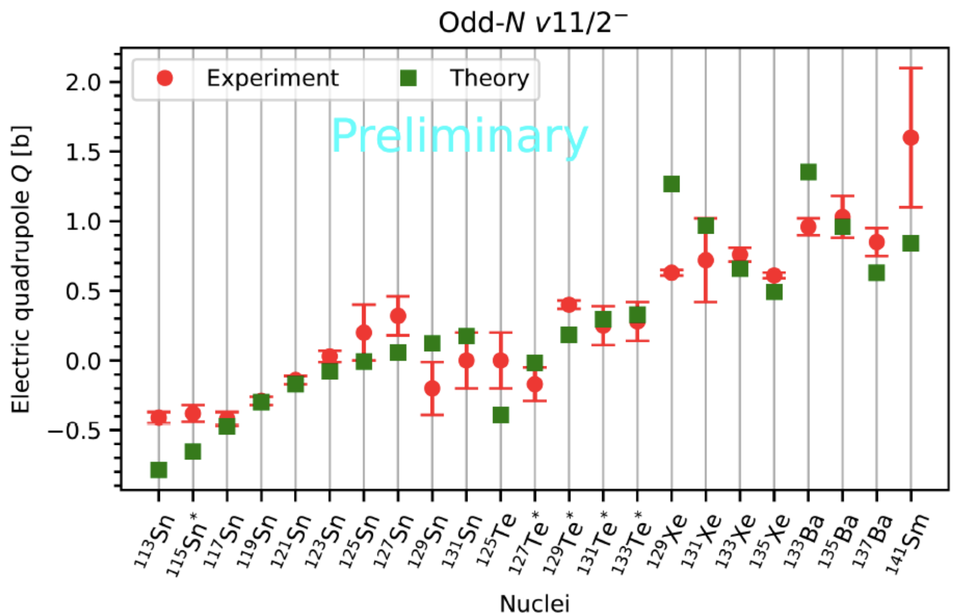
Quadrupole moments: theory vs. experiment



N. J. Stone, *Table of nuclear magnetic dipole and electric quadrupole moments* (2014), INDC, report INDC(NDS)-0658

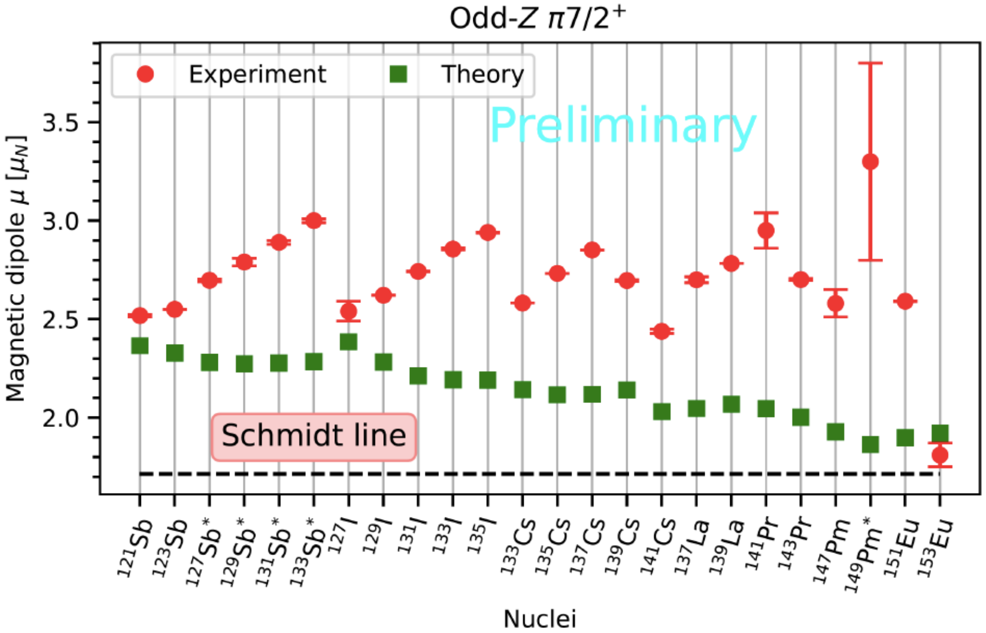
N. J. Stone, *Table of nuclear electric quadrupole moments, ADNDT 111-112*, 1 (2016)



Picture: courtesy H. Wibowo

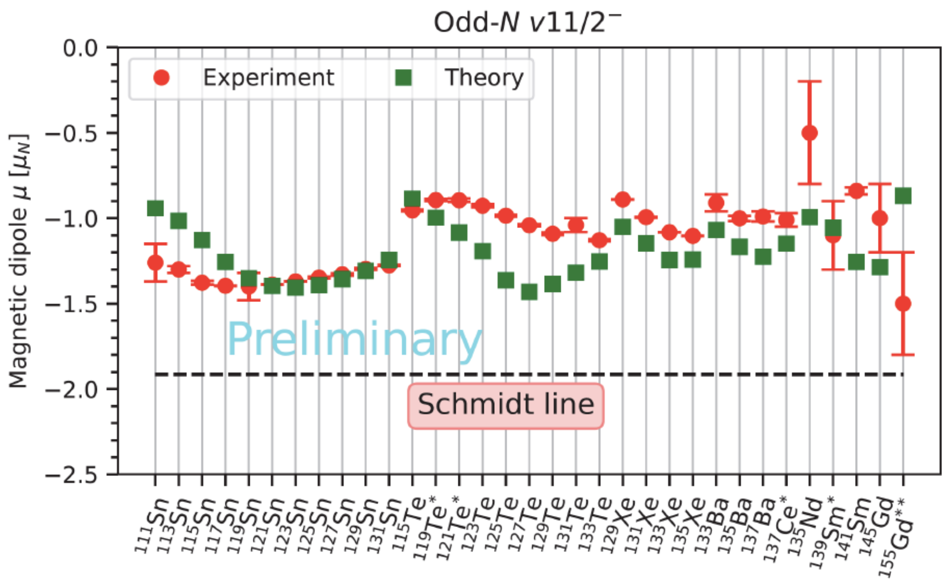
H. Wibowo *et al.*, to be published

Magnetic dipole moments: theory vs. experiment



N. J. Stone, *Table of nuclear magnetic dipole and electric quadrupole moments* (2014), INDC, report INDC(NDS)-0658

Schmidt lines represent the value of magnetic dipole moment of an odd-mass nucleus which is completely determined by the ℓ and j values of the unpaired nucleon (single-particle model).



Picture: courtesy H. Wibowo

H. Wibowo *et al.*, to be published

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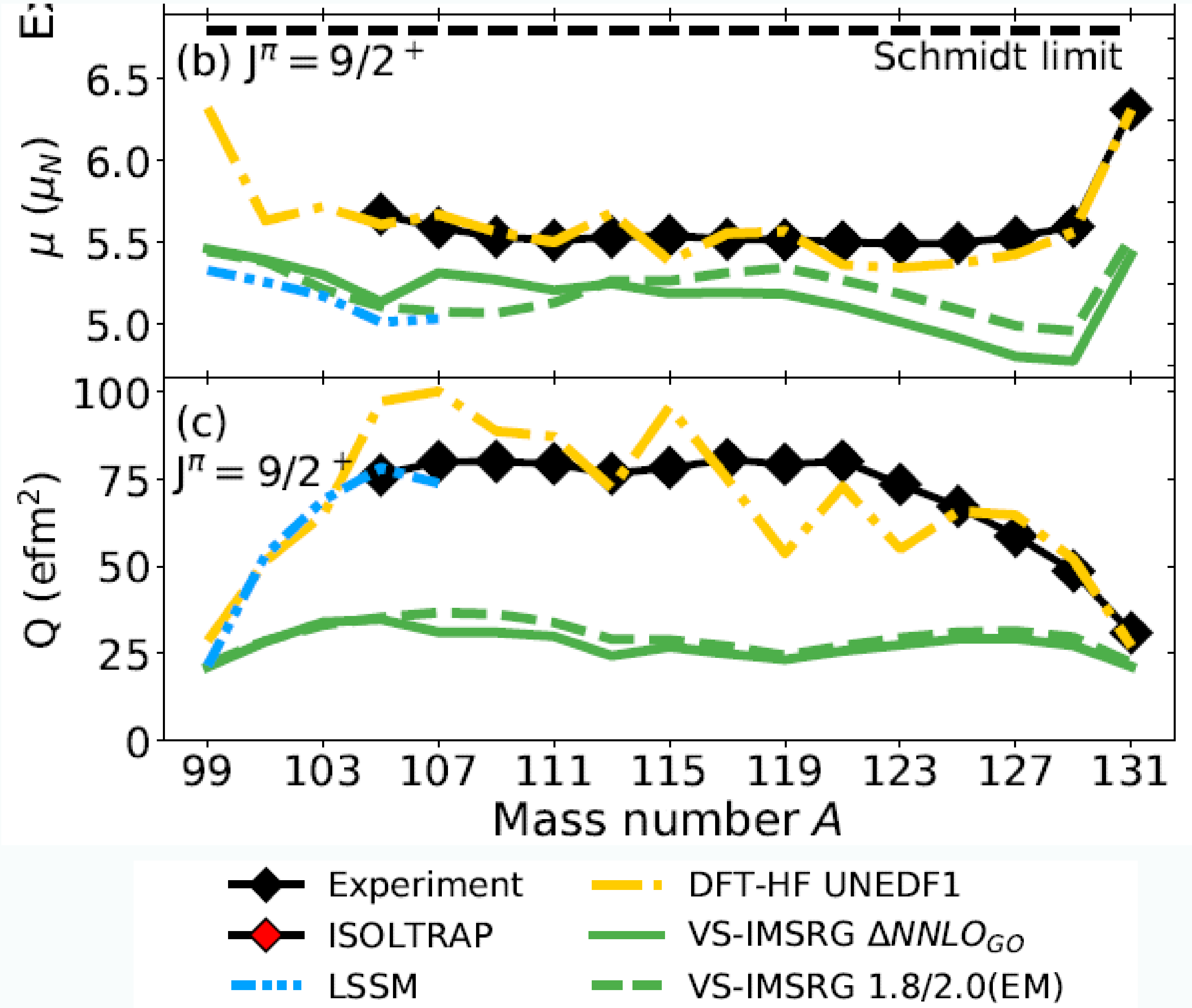
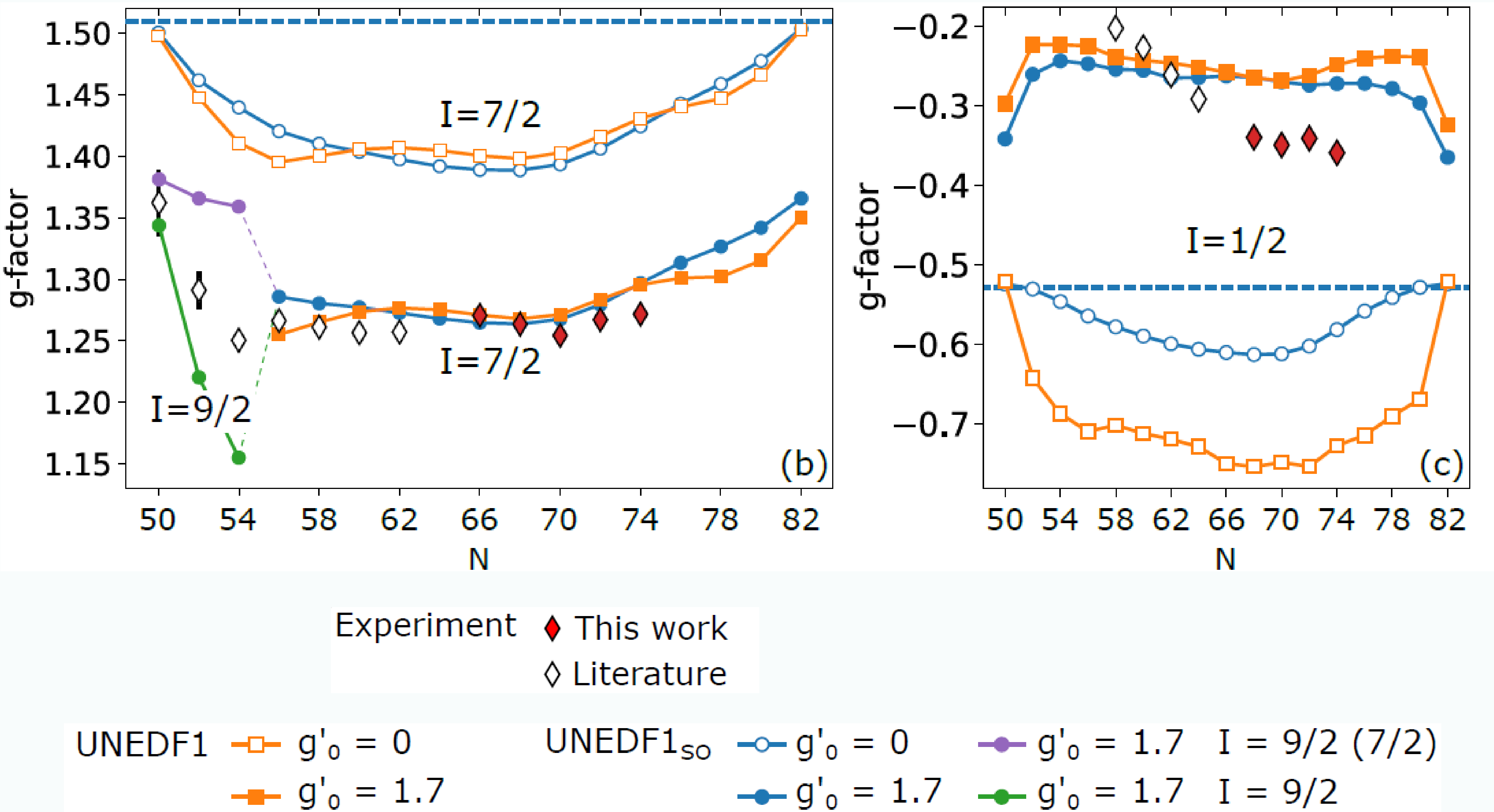
Moments of the 9/2 states in In



A.R. Vernon *et al.*, Nature 607, 260 (2022)

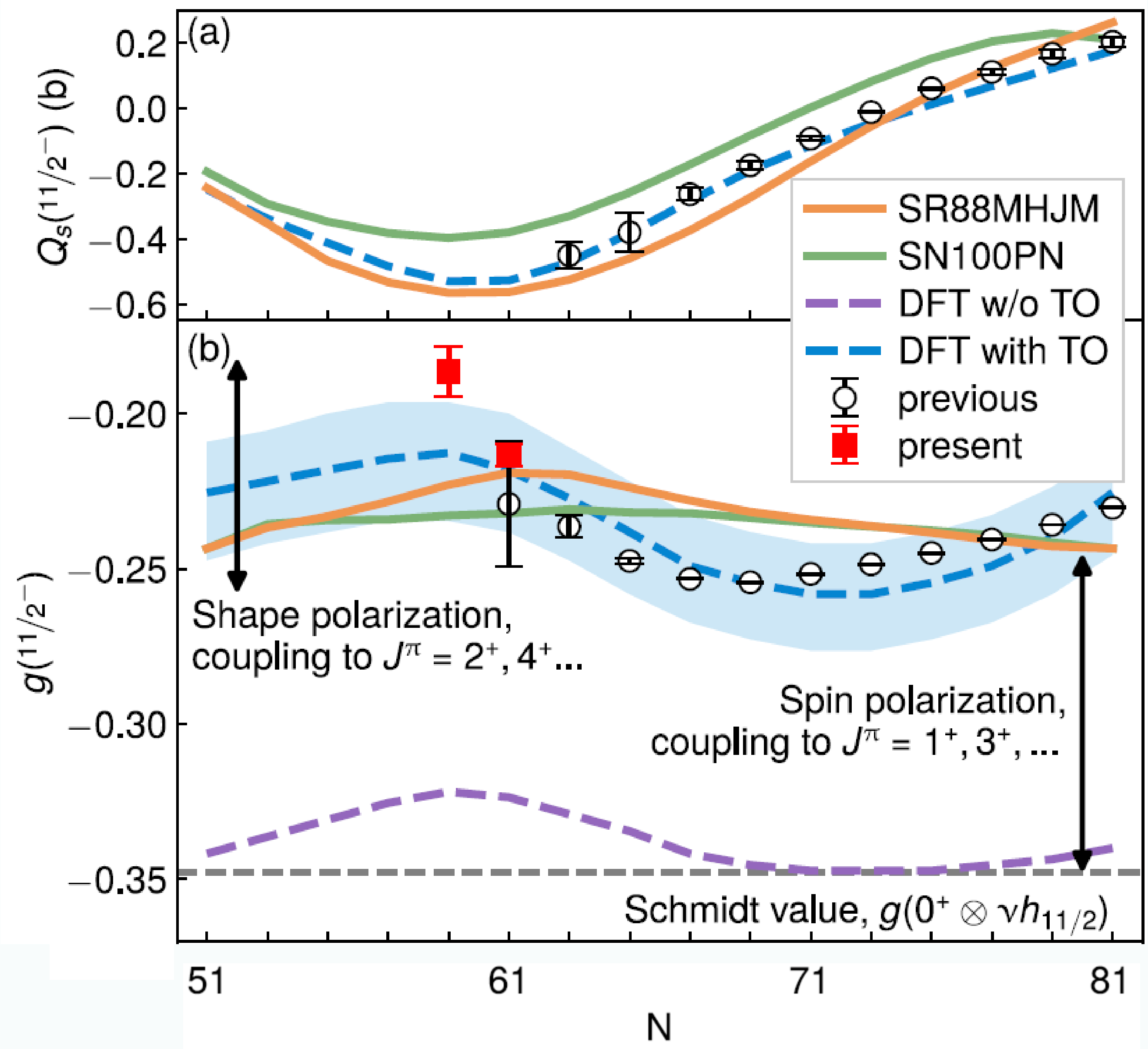
L. Nies *et al.*, Phys. Rev. Lett. 131, 022502 (2023)

Moments of the 1/2, 7/2 & 9/2 states in Ag



R. P. de Groote *et al.*, submitted to Phys. Lett. B

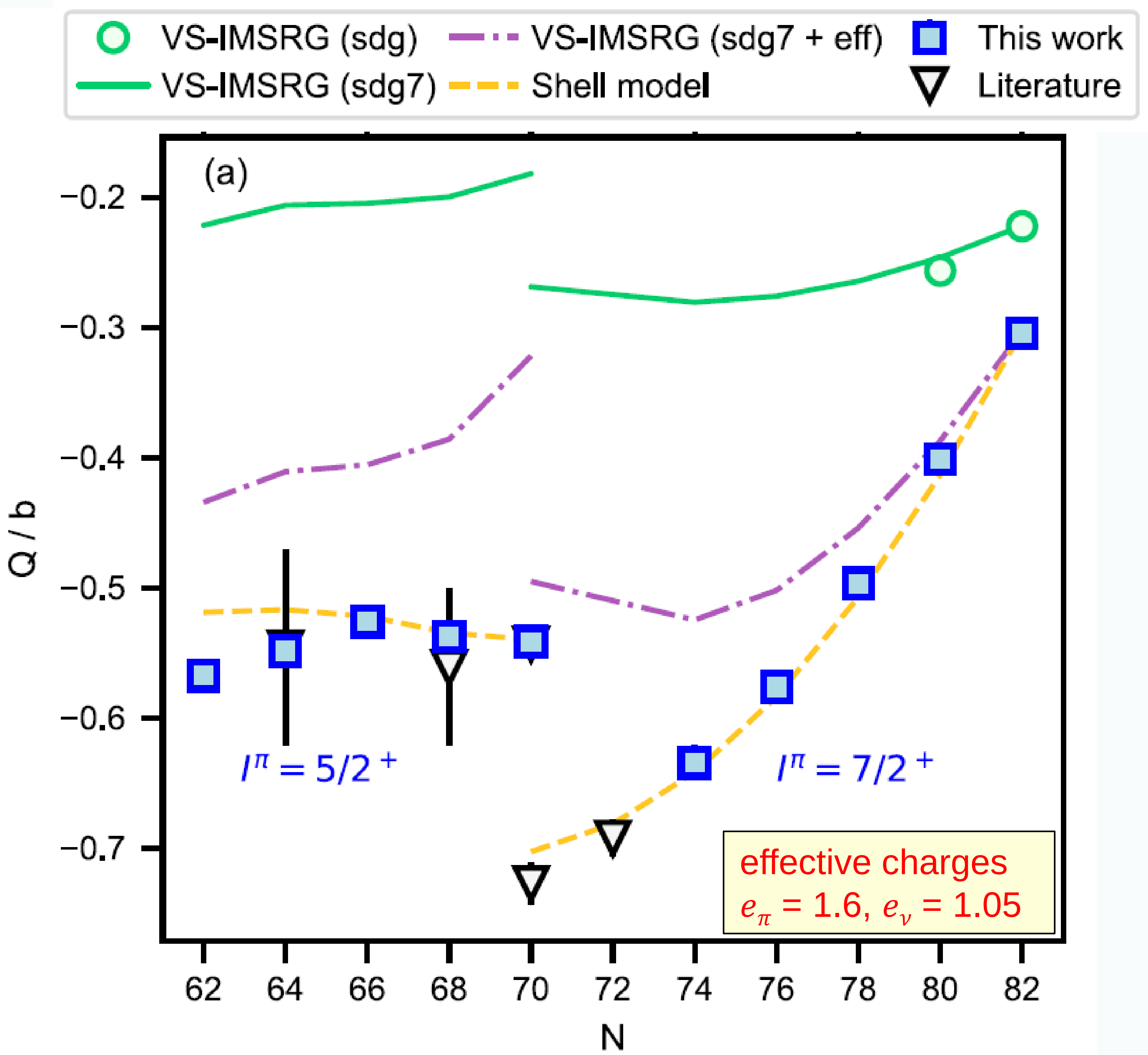
Moments of the $\nu h_{11/2}$ isomers in Sn



g = μ/I

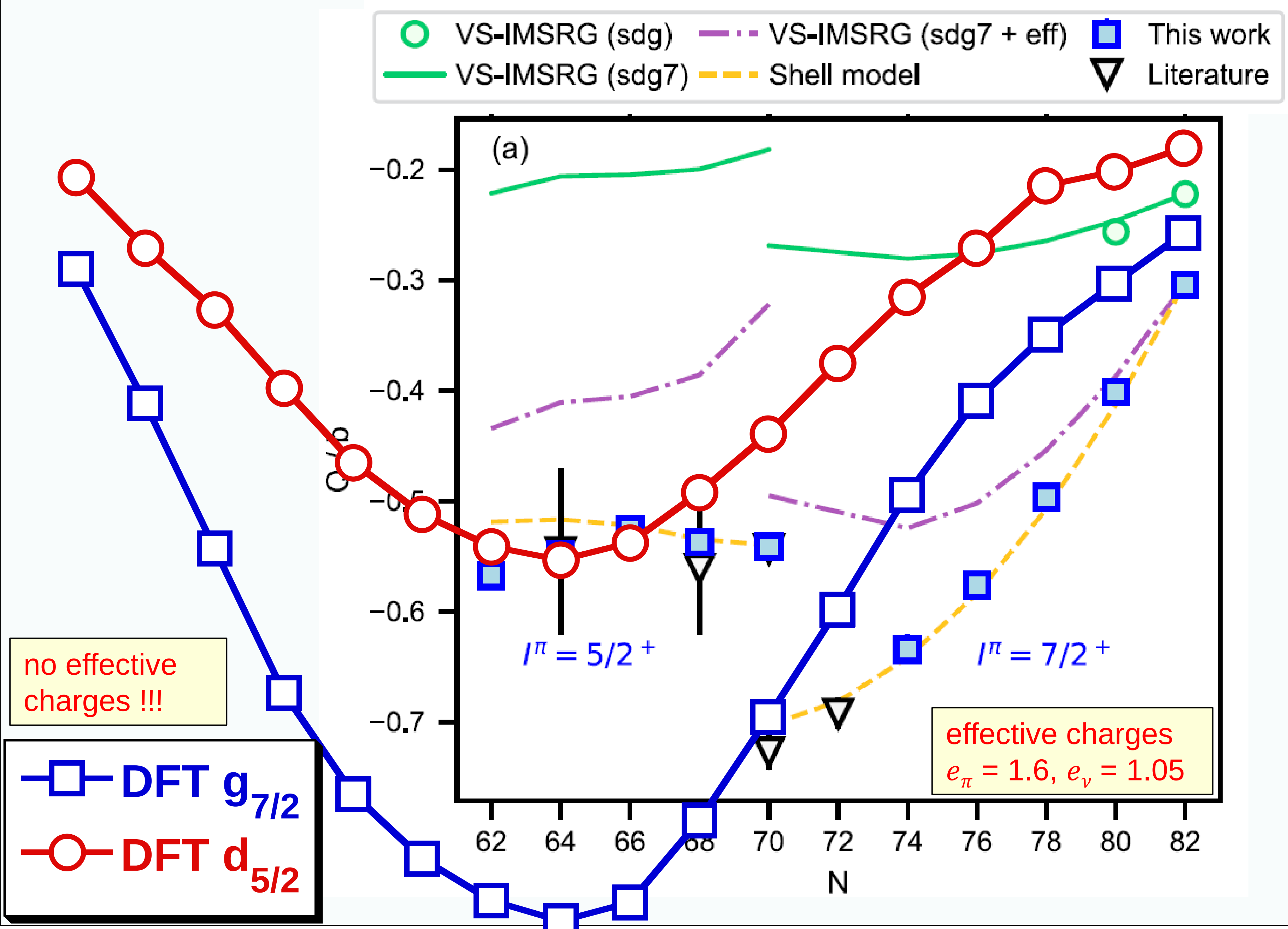
T.J. Gray *et al.*, Phys. Lett. B 847 (2023) 138268

Quadrupole moments in Sb



S. Lechner *et al.*, Phys. Lett. B 847 (2023) 138278

Quadrupole moments in Sb



S. Lechner *et al.*, Phys. Lett. B 847 (2023) 138278

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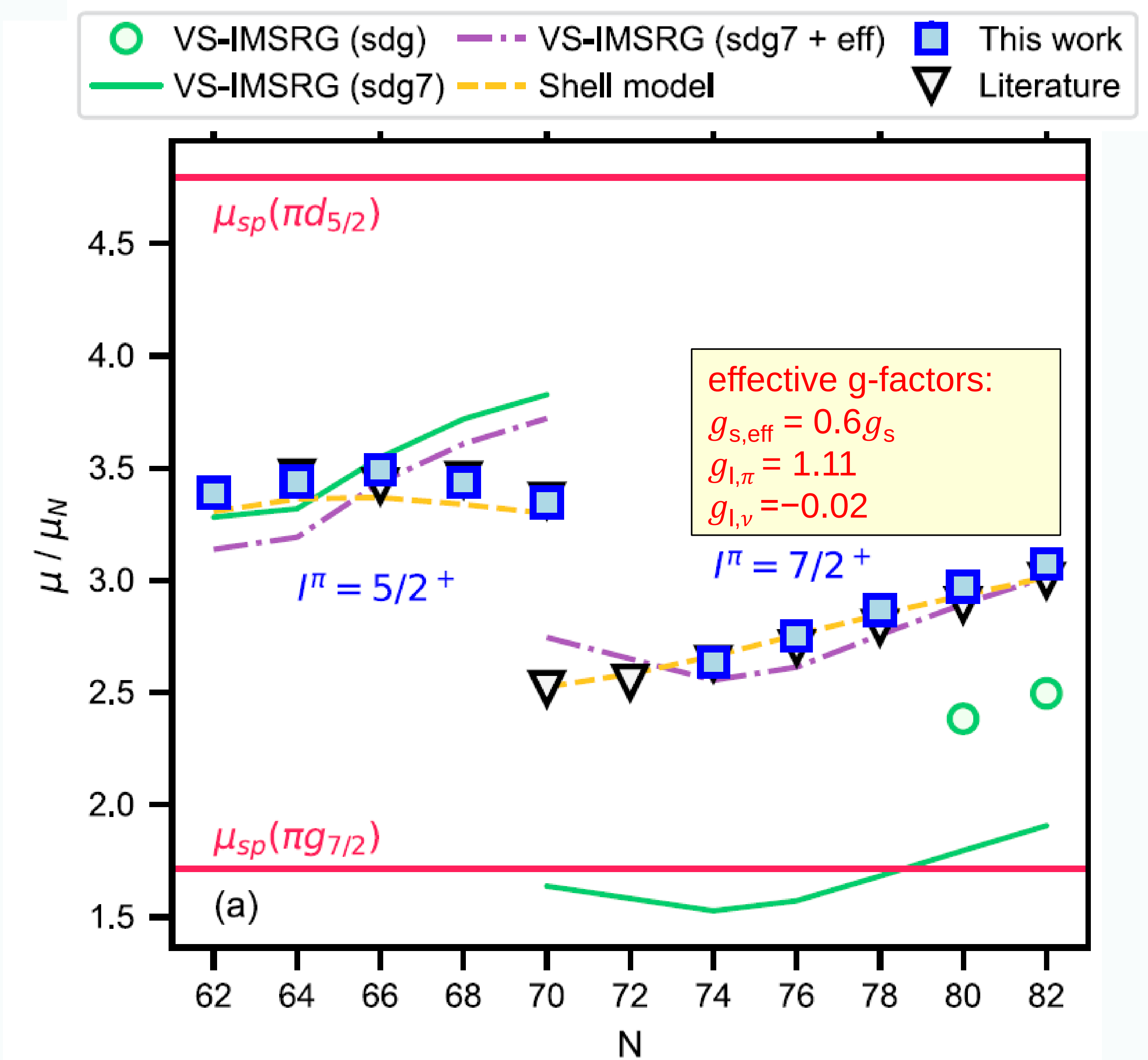
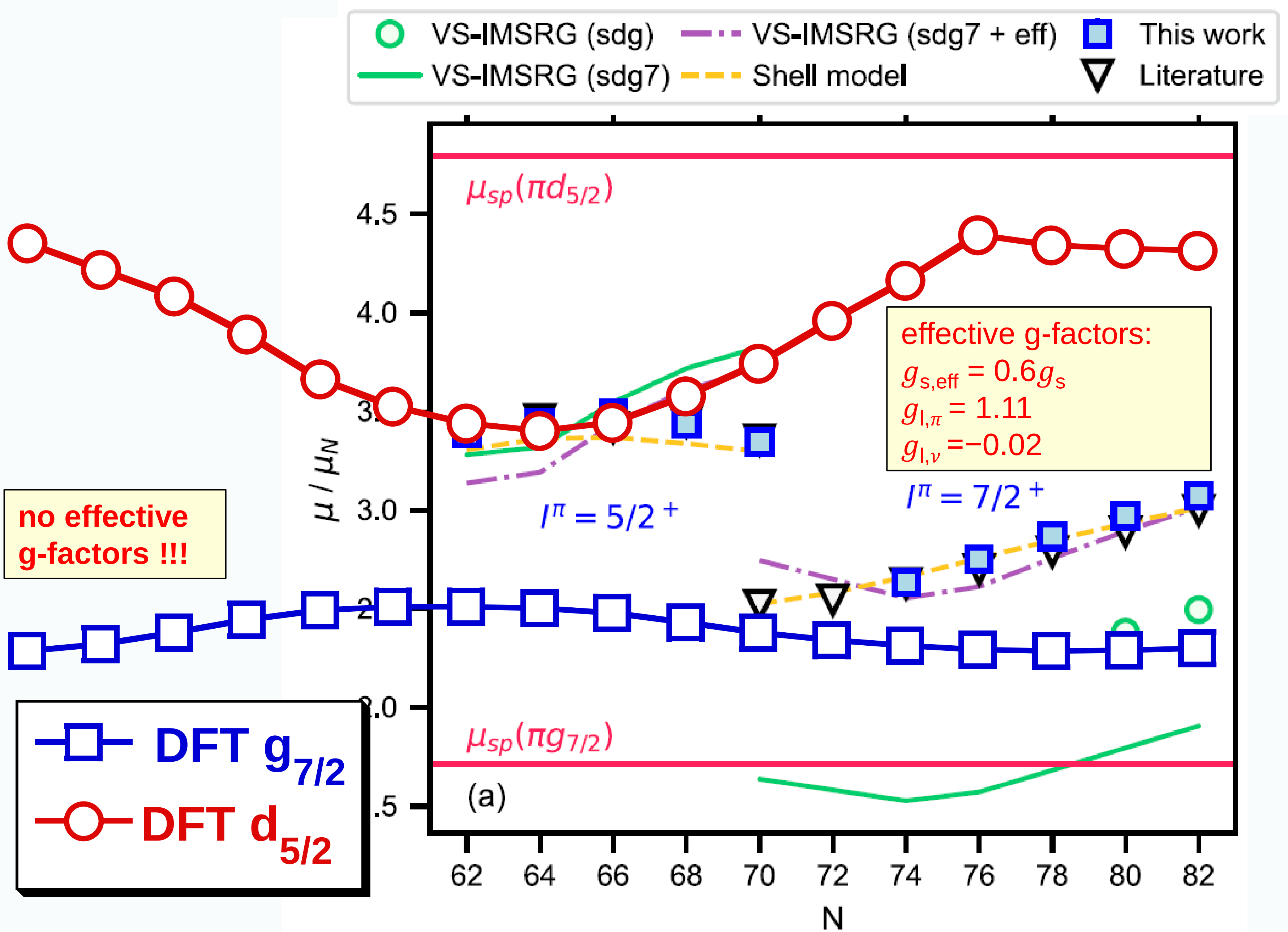
Magnetic dipole moments in Sb



S. Lechner *et al.*, Phys. Lett. B 847 (2023) 138278

Magnetic dipole moments in Sb



S. Lechner *et al.*, Phys. Lett. B 847 (2023) 138278

What’s next to consider	
Segré chart of electromagnetic moments	
Electromagnetic moments of odd-odd nuclei	★
More advanced functionals	★
Octupole deformation	
Triaxiality	★
Configuration interaction	
K-mixing	★★
Quadrupole/octupole collectivity	
Two-body meson-exchange currents	★★

Two-body-current corrections to magnetic moments

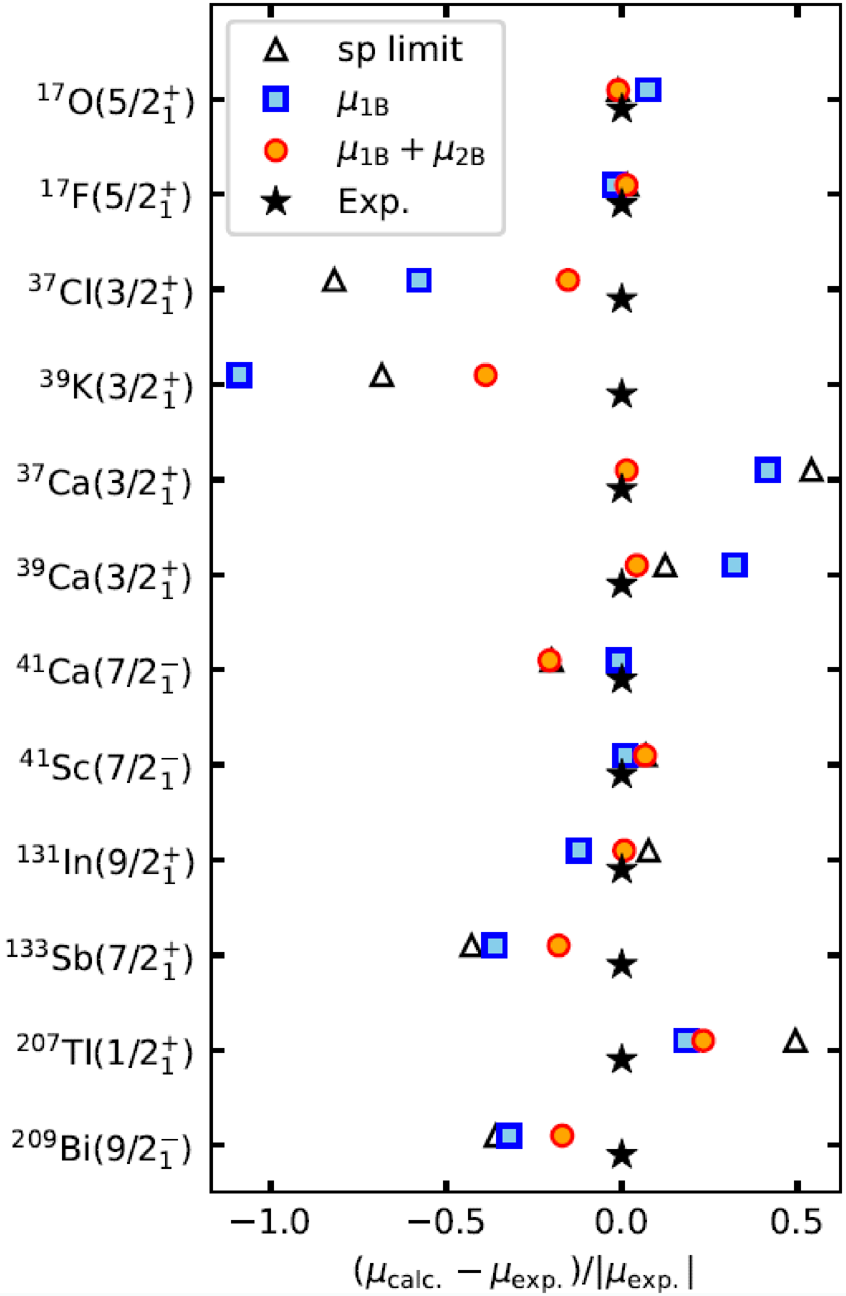


FIG. 1. Magnetic dipole moments of near doubly magic nuclei from $A = 17-209$ computed with the VS-IMSRG(2) relative to the experimental values. Results are shown at the one-body level, μ_{1B} (blue squares), and including 2BC, $\mu_{1B} + \mu_{2B}$ (red circles) based on the 1.8/2.0 (EM) NN+3N interactions. The experimental dipole moments (stars) are taken from Ref. [21, 35]. In addition, we show the simple single-particle (sp) limit (without many-body correlations and without 2BC).

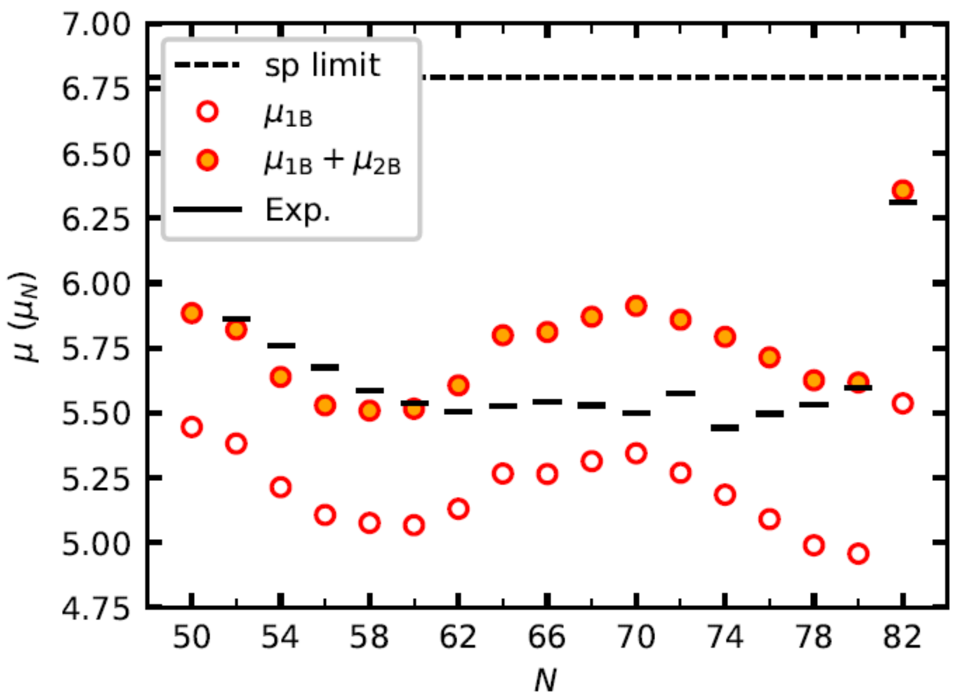


FIG. 4. Magnetic dipole moments of the $9/2^+$ ground state for the odd-mass indium isotopes isotopes computed with the VS-IMSRG(2) including 2BC, in comparison to experiment [21, 52].

T. Miyagi *et al.*, arXiv:2311.14383

NLO magnetic moment operators

(slide by Herlik Wigowo)

The NLO intrinsic and Sachs contributions to the magnetic moment operator are given by

$$\hat{\boldsymbol{\mu}}_{2\mathrm{b}}^{\mathrm{NLO,\ int}}(\mathbf{r}) = -\frac{g_A^2 m_\pi}{8\pi F_\pi^2}\left(\hat{\boldsymbol{\tau}}_1 \times \hat{\boldsymbol{\tau}}_2\right)_z \left\{\left(1+\frac{1}{m_\pi r}\right)\left[\left(\hat{\boldsymbol{\sigma}}_1 \times \hat{\boldsymbol{\sigma}}_2\right)\cdot\hat{\mathbf{r}}\right]\hat{\mathbf{r}} - \left(\hat{\boldsymbol{\sigma}}_1 \times \hat{\boldsymbol{\sigma}}_2\right)\right\} e^{-m_\pi r}$$

and

$$\hat{\boldsymbol{\mu}}_{2\mathrm{b}}^{\mathrm{NLO,\ Sachs}}(\mathbf{r}) = -\frac{1}{2}\left(\hat{\boldsymbol{\tau}}_1 \times \hat{\boldsymbol{\tau}}_2\right)_z V_{1\pi}(r)\mathbf{R}_{\mathrm{NN}} \times \mathbf{r},$$

relative coordinate

COM

respectively, where

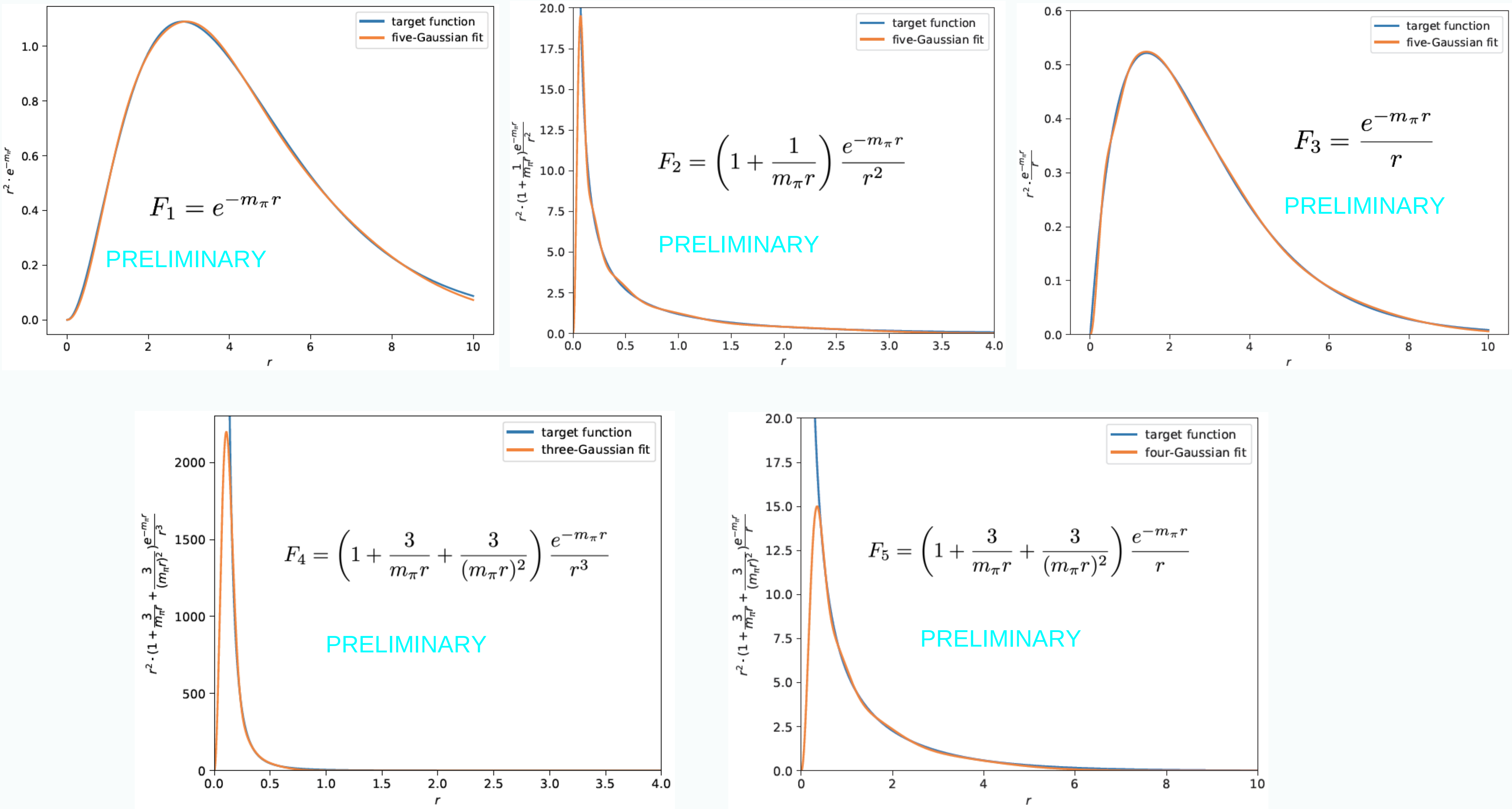
$$V_{1\pi}(r) = \frac{m_\pi^2 g_A^2}{12\pi F_\pi^2}\left(\hat{\boldsymbol{\tau}}_1 \cdot \hat{\boldsymbol{\tau}}_2\right)\left[\hat{S}_{12}\left(1+\frac{3}{m_\pi r}+\frac{3}{(m_\pi r)^2}\right)+\hat{\boldsymbol{\sigma}}_1\cdot\hat{\boldsymbol{\sigma}}_2\right]\frac{e^{-m_\pi r}}{r}$$

Tensor operator: $\hat{S}_{12} = 3\left(\hat{\mathbf{r}}\cdot\hat{\boldsymbol{\sigma}}_1\right)\left(\hat{\mathbf{r}}\cdot\hat{\boldsymbol{\sigma}}_2\right) - \hat{\boldsymbol{\sigma}}_1\cdot\hat{\boldsymbol{\sigma}}_2.$

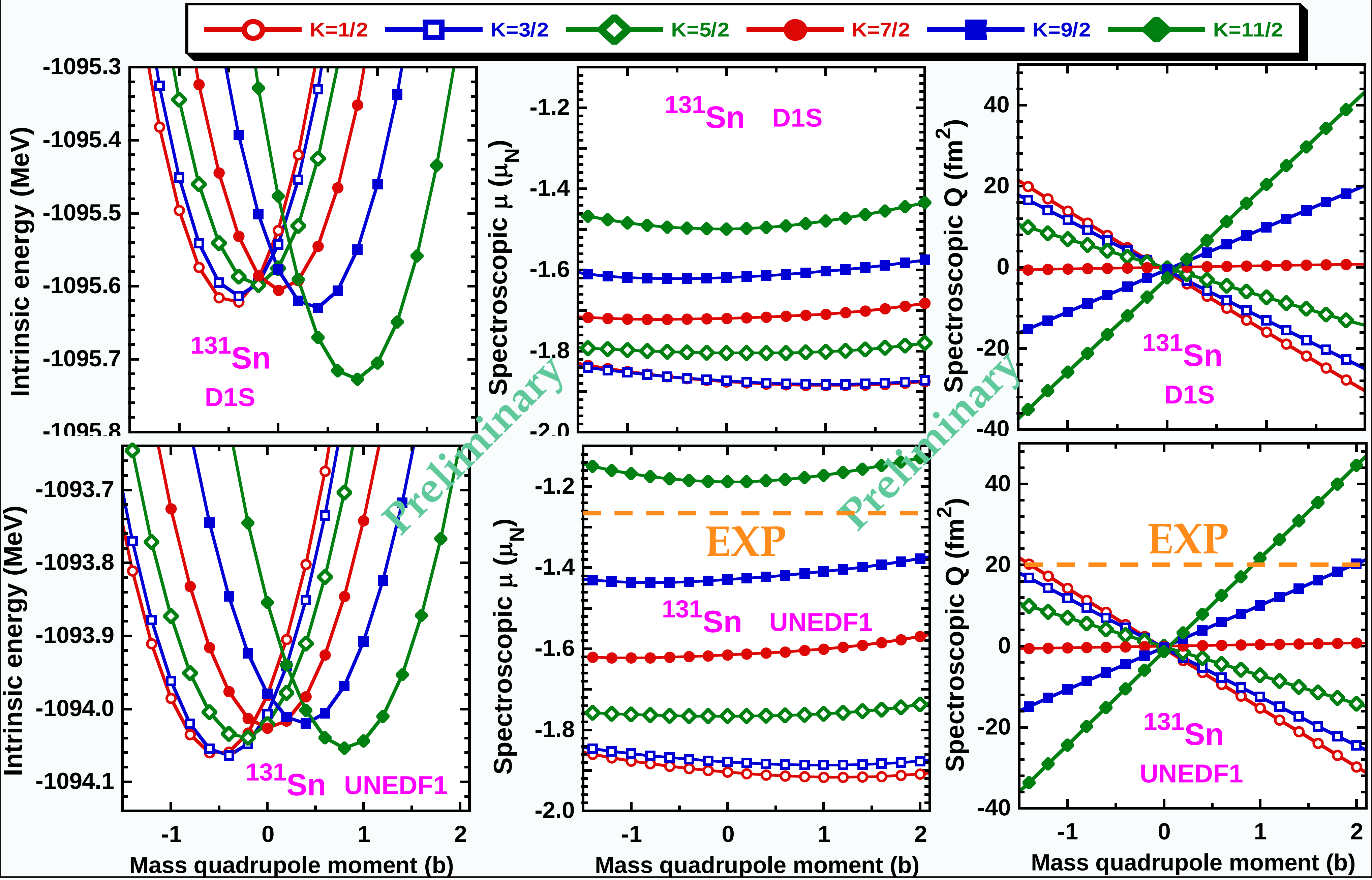
R. Seutin, et.al, PRC 108, 054005 (2023)

NLO magnetic moment operators

(slide by Herlik Wigowo)



K-mixing



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Energies of the K-mixed states

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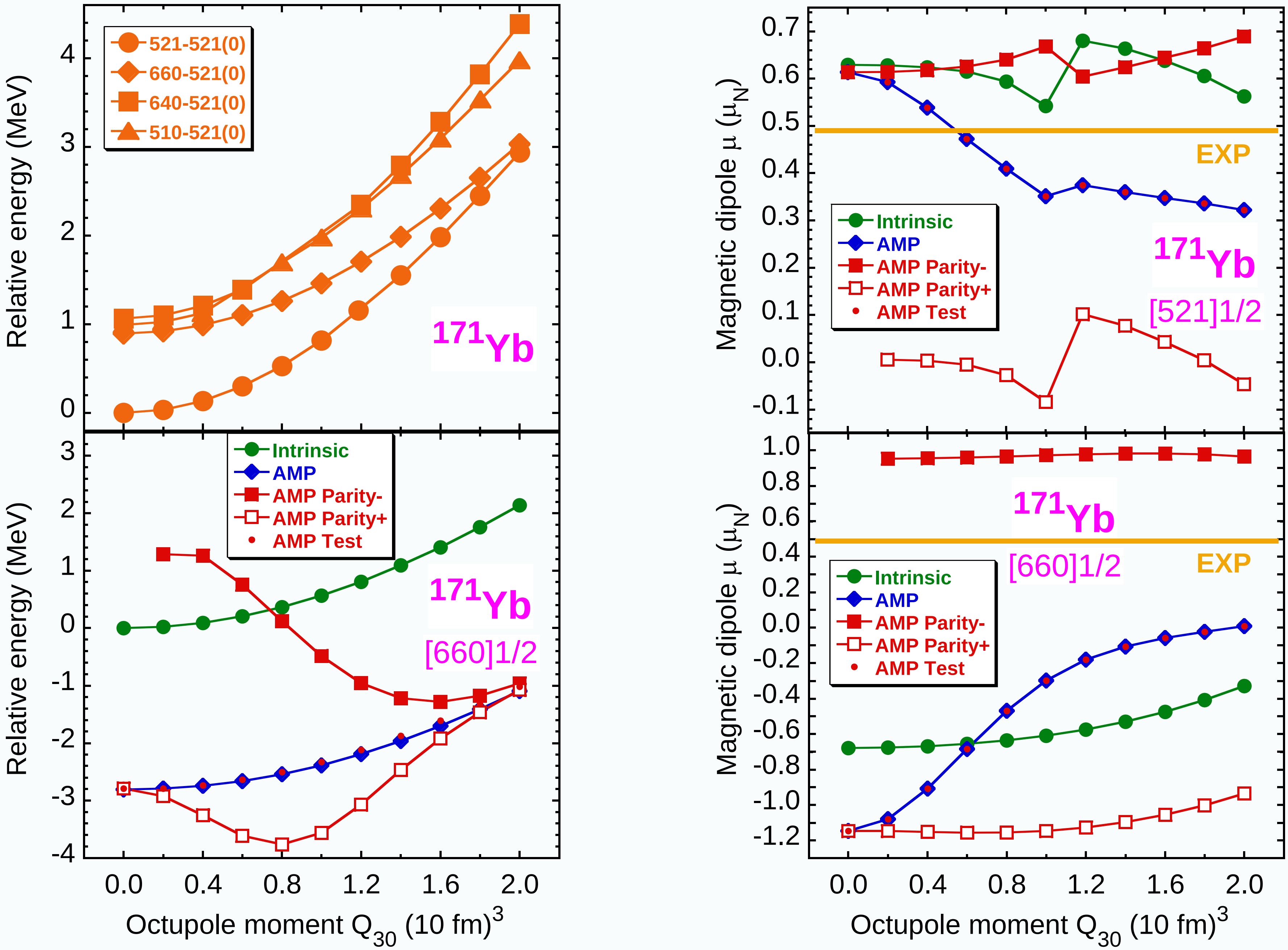
**************************************************************************
*                                                                        *
*                RESULTS OF THE MULTI-REFERENCE CALCULATION              *
*                                                                        *
**************************************************************************
*  SPIN  N   EIG_OVERLAP   EIG_ENERGY  |                                 *
*  ----  -   -----------   ----------  |                                 *
*  11/2  1   5.979982E+00 -1092.439526 |                                 *
*  11/2  2   1.575148E-02 -1083.008302 |                                 *
*  11/2  3   2.225877E-03 -1080.289292 |                                 *
*  11/2  4   1.132094E-03 -1078.423819 |                                 *
*  11/2  5   6.412910E-04 -1069.869511 |                                 *
*  11/2  6   2.674966E-04 -1067.359363 |                                 *
**************************************************************************

E_intrinsic(11/2) = -1092.055162
E_projected(11/2) = -1092.310241
```

Preliminary

Octupole deformation – a case study



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Thank you

Basic definitions

The electric and magnetic moments are defined as

$$Q_{\lambda\mu} = \langle\Psi|\hat{Q}_{\lambda\mu}|\Psi\rangle = \int q_{\lambda\mu}(\vec{r})\, d^3\vec{r},$$

$$M_{\lambda\mu} = \langle\Psi|\hat{M}_{\lambda\mu}|\Psi\rangle = \int m_{\lambda\mu}(\vec{r})\, d^3\vec{r},$$

where $|\Psi\rangle$ is a many-body state, and $q_{\lambda\mu}(\vec{r})$ and $m_{\lambda\mu}(\vec{r})$ are the corresponding electric and magnetic-moment densities:

$$q_{\lambda\mu}(\vec{r}) = e\rho(\vec{r})Q_{\lambda\mu}(\vec{r}),$$

$$m_{\lambda\mu}(\vec{r}) = \mu_N\left[g_s\vec{s}(\vec{r}) + \tfrac{2}{\lambda+1}g_l\left(\vec{r}\times\vec{j}(\vec{r})\right)\right]\cdot\vec{\nabla}Q_{\lambda\mu}(\vec{r}),$$

and e, g_s, and g_l are the elementary charge, and the spin and orbital gyromagnetic factors, respectively. The multipole functions (solid harmonics) have the standard form: $Q_{\lambda\mu}(\vec{r}) = r^\lambda Y_{\lambda\mu}(\theta,\phi)$.

Schmidt limits

The magnetic operator $\bar{\mu}$ is a one-body operator and the magnetic dipole moment μ is the expectation value of $\bar{\mu}_z$. The M1 operator acting on a composed state $|\mathrm{Im}\rangle$ can then be written as the sum of single particle M1 operators $\bar{\mu}_z(j)$ acting each on an individual valence nucleon with total momentum j:

$\mu = g_L\, \mathbf{L} + g_s\, \mathbf{s}$

$$\mu(I) \equiv \left\langle I(j_1, j_2, \ldots, j_n), m = I \left| \sum_{i=1}^{n} \bar{\mu}_z(i) \right| I(j_1, j_2, \ldots, j_n), m = I \right\rangle \tag{2.1}$$

The single particle magnetic moment $\mu(j)$ for a valence nucleon around a doubly magic core is uniquely defined by the quantum numbers l and j of the occupied single particle orbit [22]:

$$\text{for an odd proton: } \begin{cases} \mu = j - \frac{1}{2} + \mu_\mathrm{p} & \text{for } j = l + \frac{1}{2} \\ \mu = \dfrac{j}{j+1}\left(j + \dfrac{3}{2} - \mu_\mathrm{p}\right) & \text{for } j = l - \frac{1}{2} \end{cases} \tag{2.2}$$

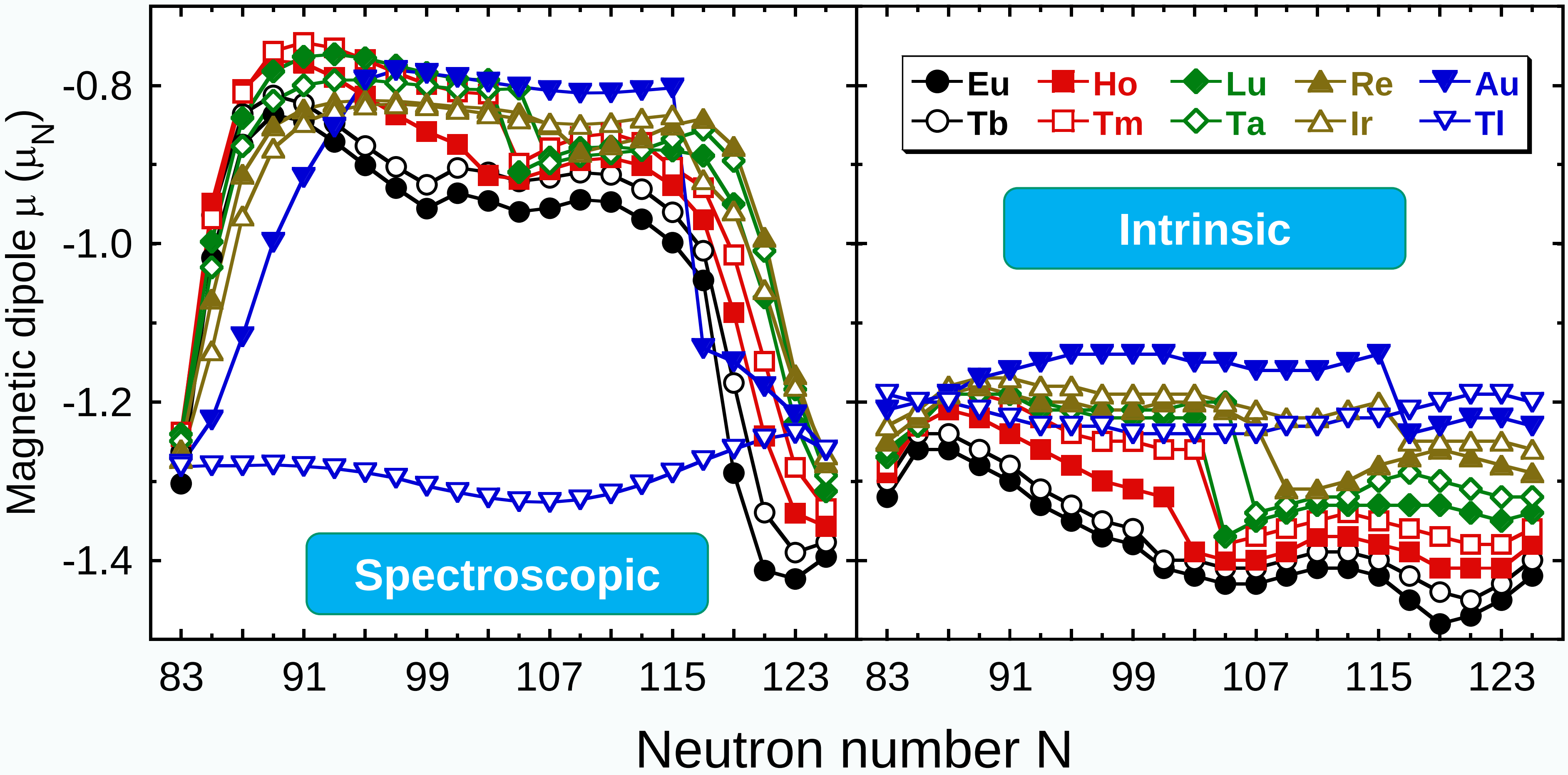
$$\text{for an odd neutron: } \begin{cases} \mu = \mu_\mathrm{n} & \text{for } j = l + \frac{1}{2} \\ \mu = -\dfrac{j}{j+1}\mu_\mathrm{n} & \text{for } j = l - \frac{1}{2} \end{cases} \tag{2.3}$$

Schmidt limits

These single particle moments calculated using the free proton and free neutron moments ($\mu_\mathrm{p} = +2.793$, $\mu_\mathrm{n} = -1.913$) are called the Schmidt moments. In a nucleus, the magnetic

Gerda Neyens, Rep. Prog. Phys. 66 (2003) 633–689

Heavy deformed ν13/2^{+} odd-N nuclei

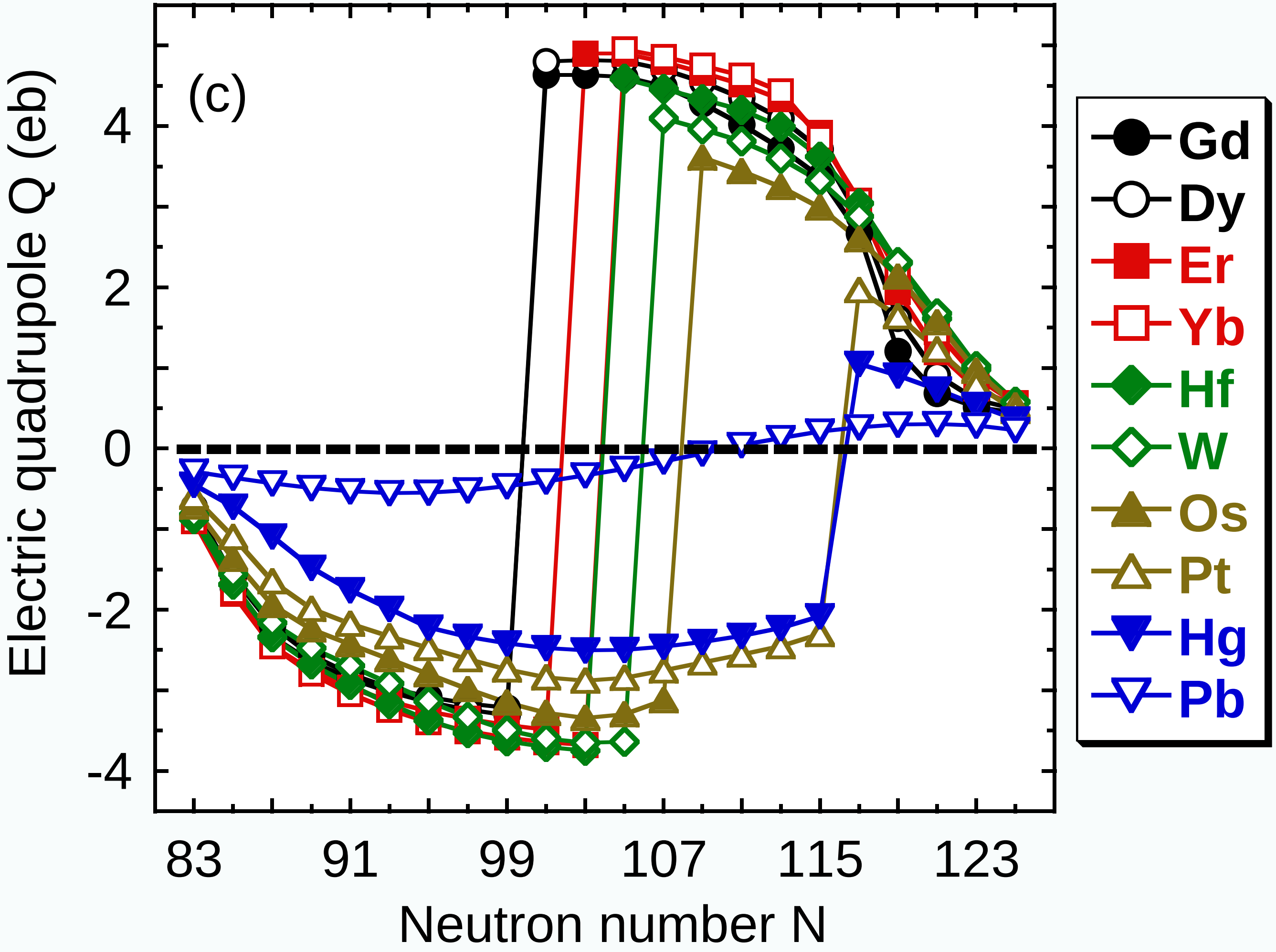


J. Bonnard *et al.*, Phys. Lett. B 843 (2023) 138014

Conclusion:

Spectroscopic magnetic dipole moments cannot be inferred from the intrinsic ones

Heavy deformed ν13/2^{+} odd-N nuclei

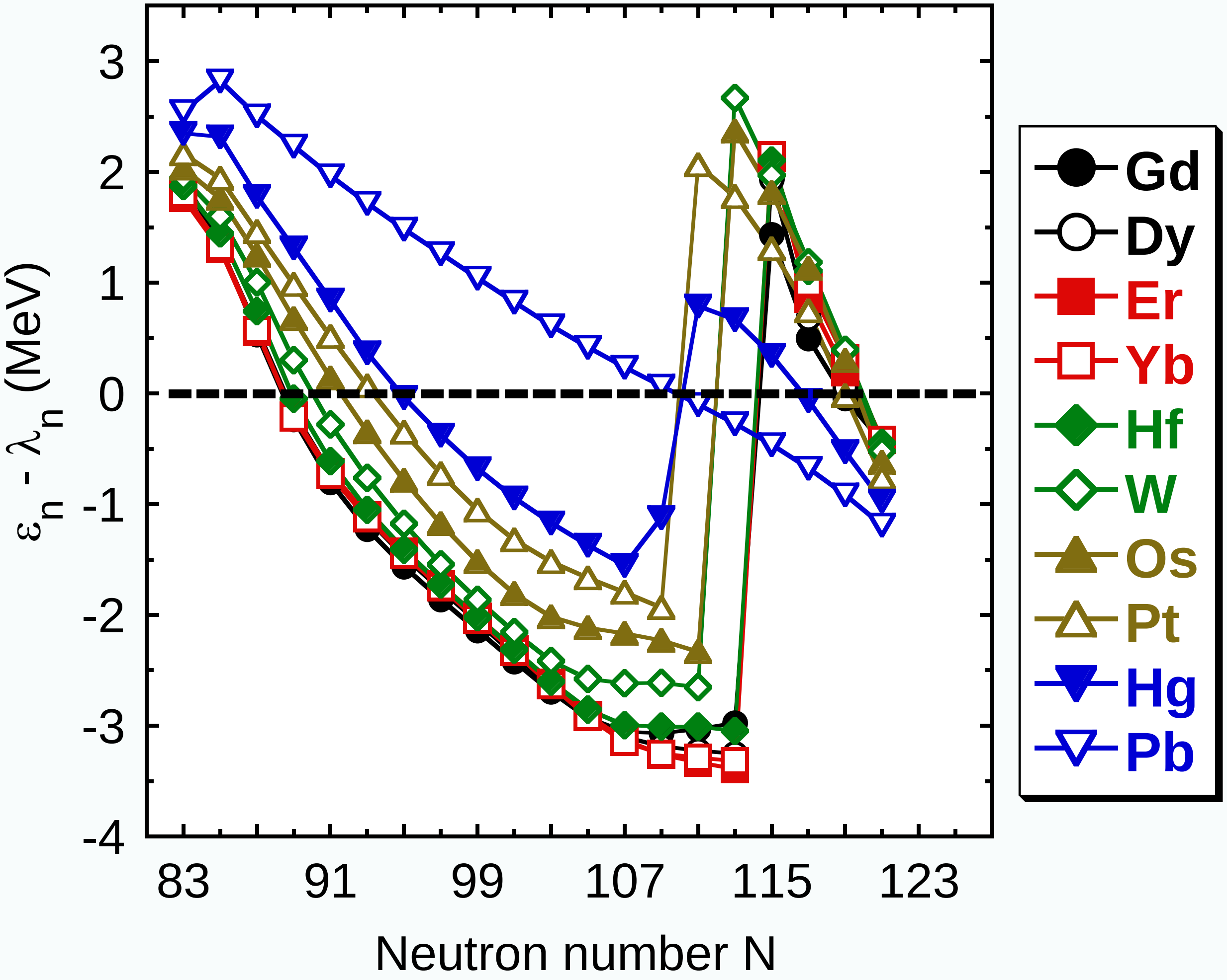


Conclusion:

Rules of oblate and prolate polarizations do extend from the magicity towards the open shell systems.

J. Bonnard *et al.*, Phys. Lett. B 843 (2023) 138014

Heavy deformed ν13/2$^+$ odd-N nuclei



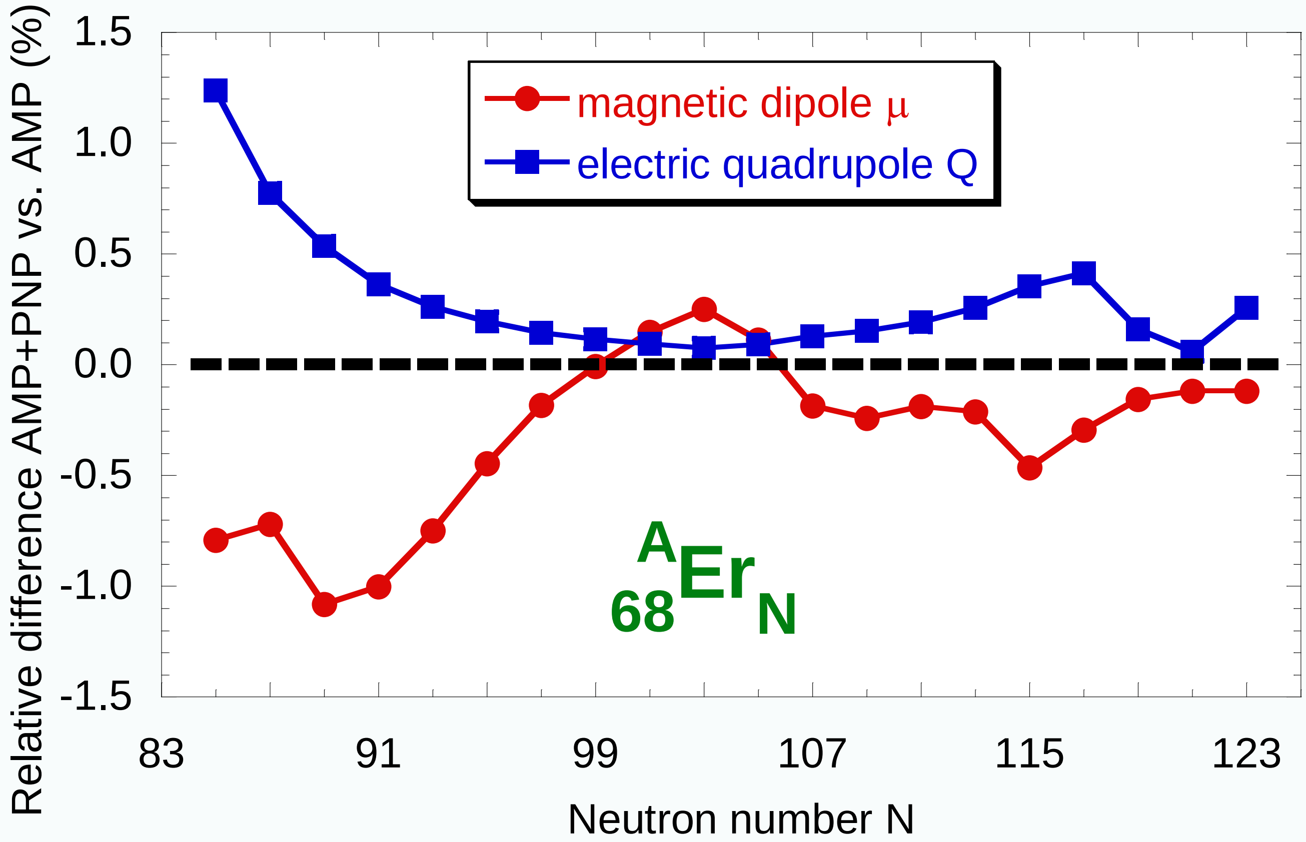
J. Bonnard *et al.*, Phys. Lett. B 843 (2023) 138014

Conclusion:

Rules of particle and hole polarizations do not extend from the magicity towards the open shell systems.

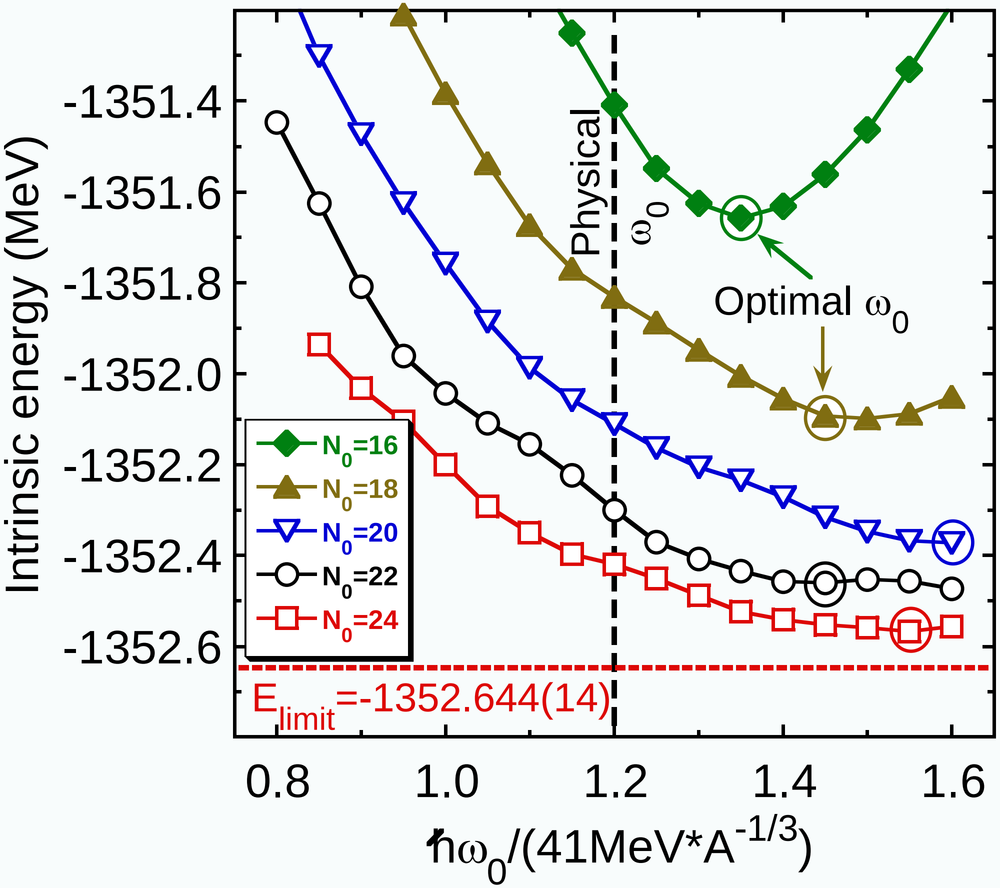
Insignificant impact of the PNP

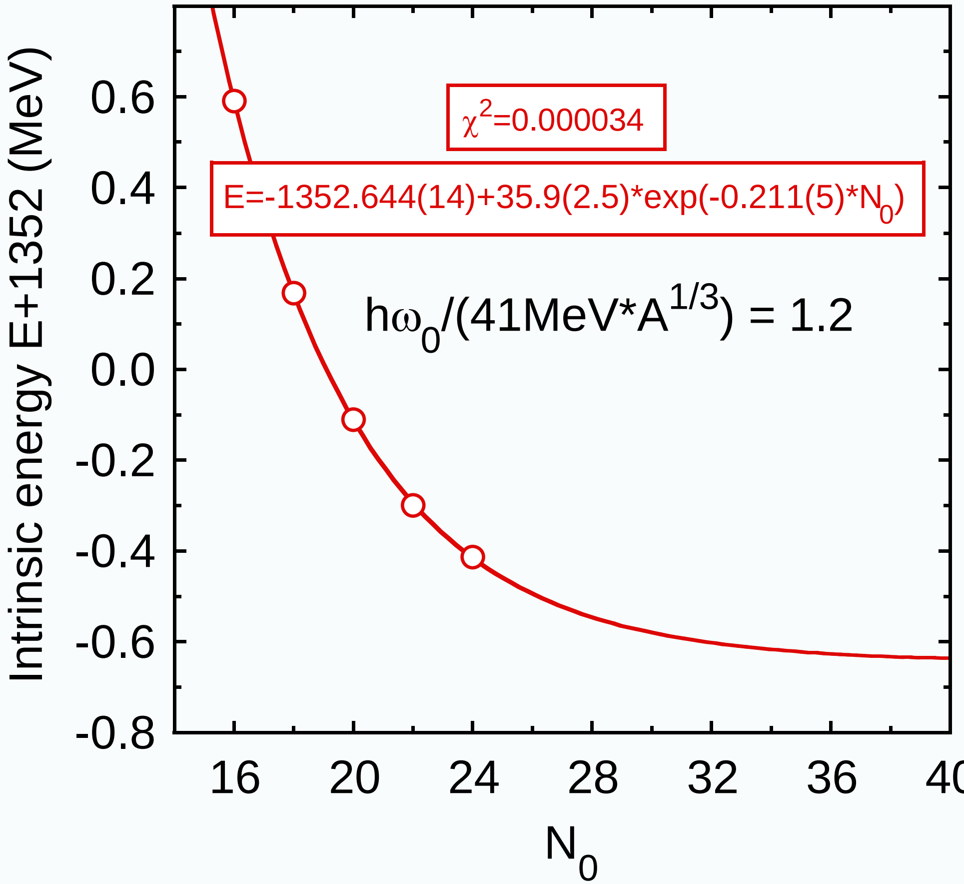
UNEDF1, g'_0=1.7



Convergence of the total HFB intrinsic energy

^{167}Ho 11/2$^-$, UNEDF1, g'_0=1.7

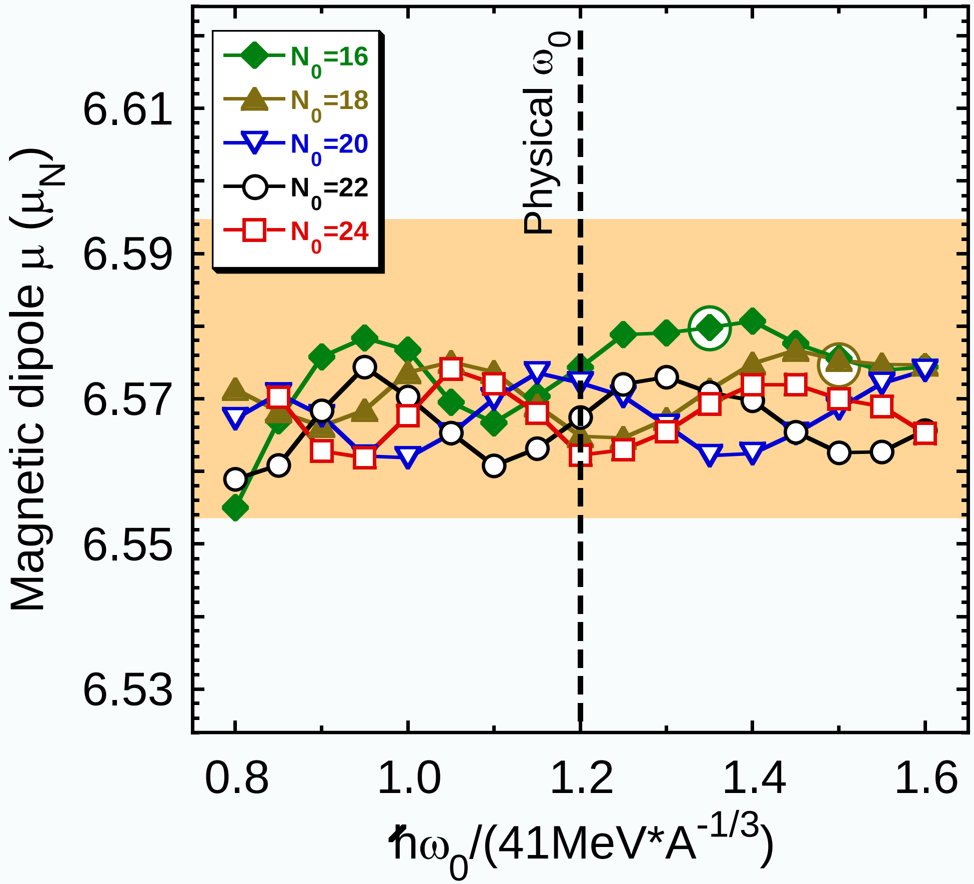


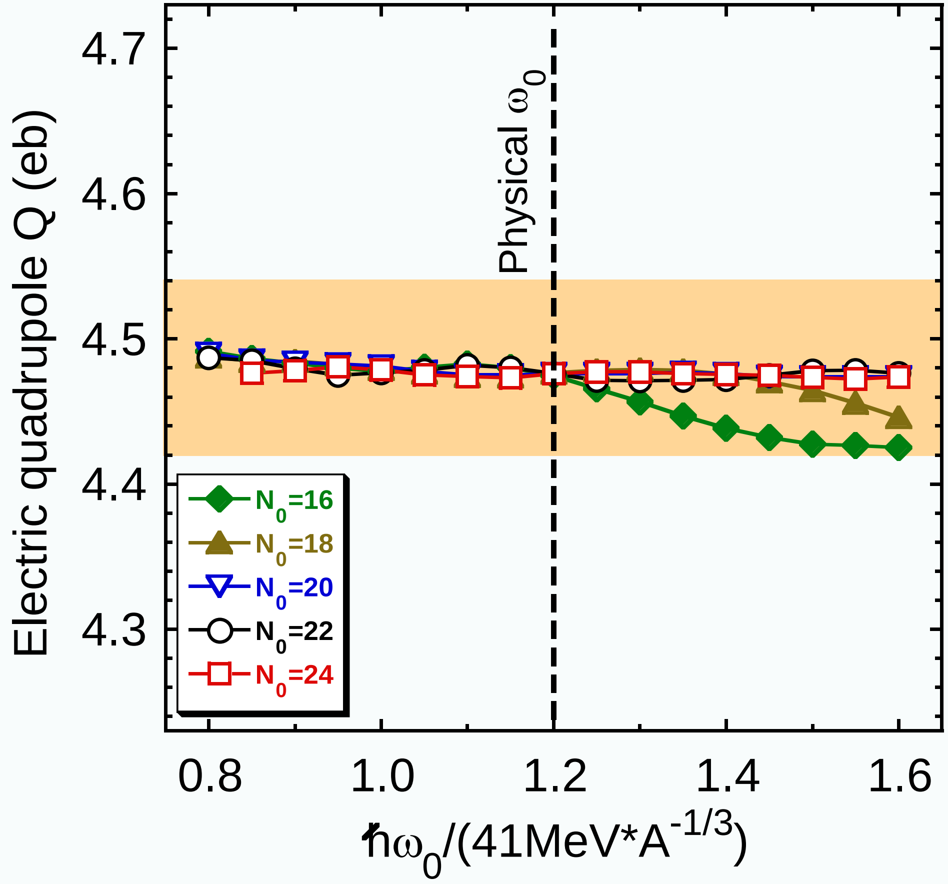


E_{exp}=-1357.77346

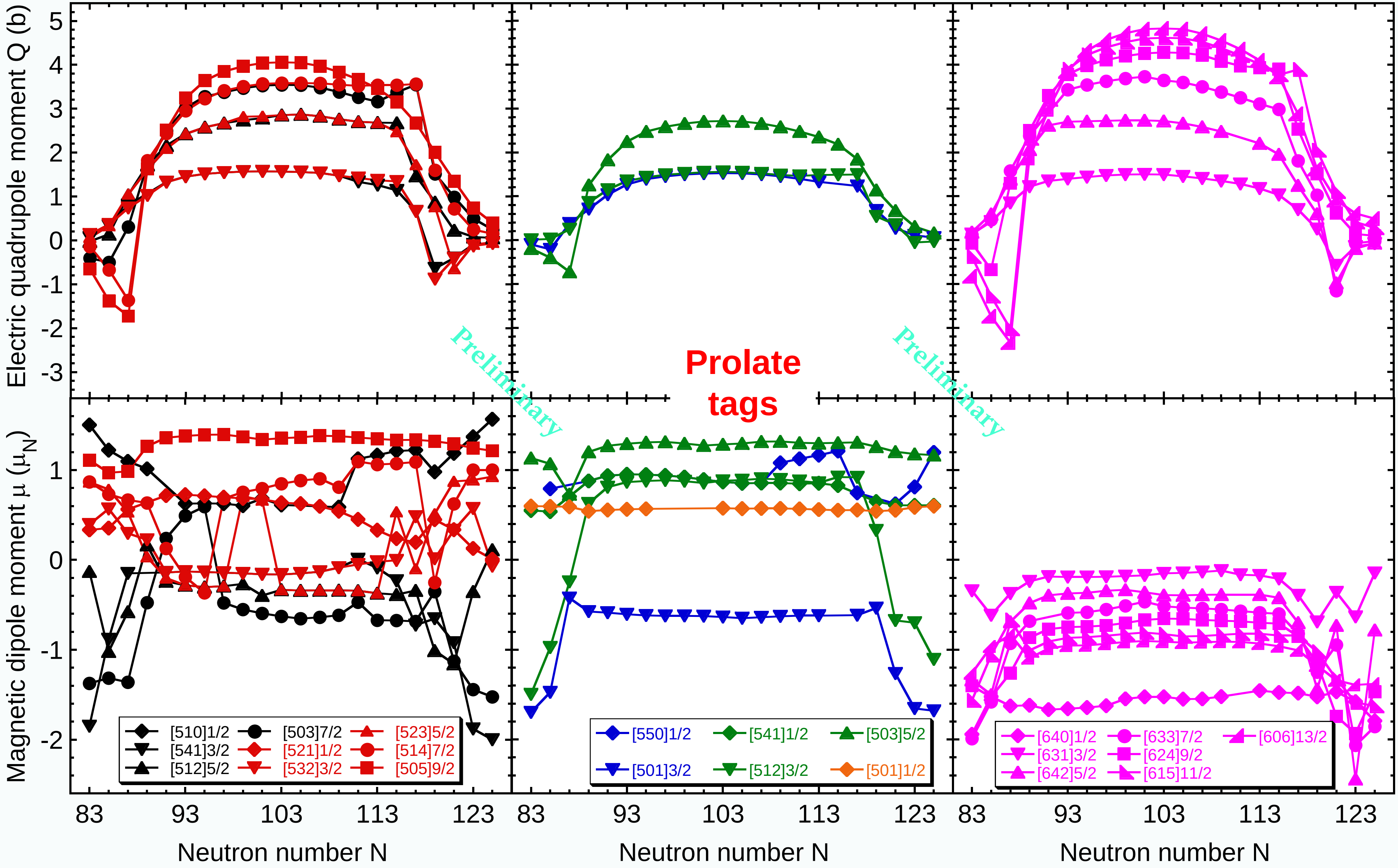
Convergence of the spectroscopic moments

^{167}Ho 11/2$^-$, UNEDF1, g'_0=1.7





Electromagnetic moments of odd dysprosium isotopes



Jacek Dobaczewski