Probing the effect of Nuclear Shape Fluctuations in Heavy-Ion Collisions using Glauber Model

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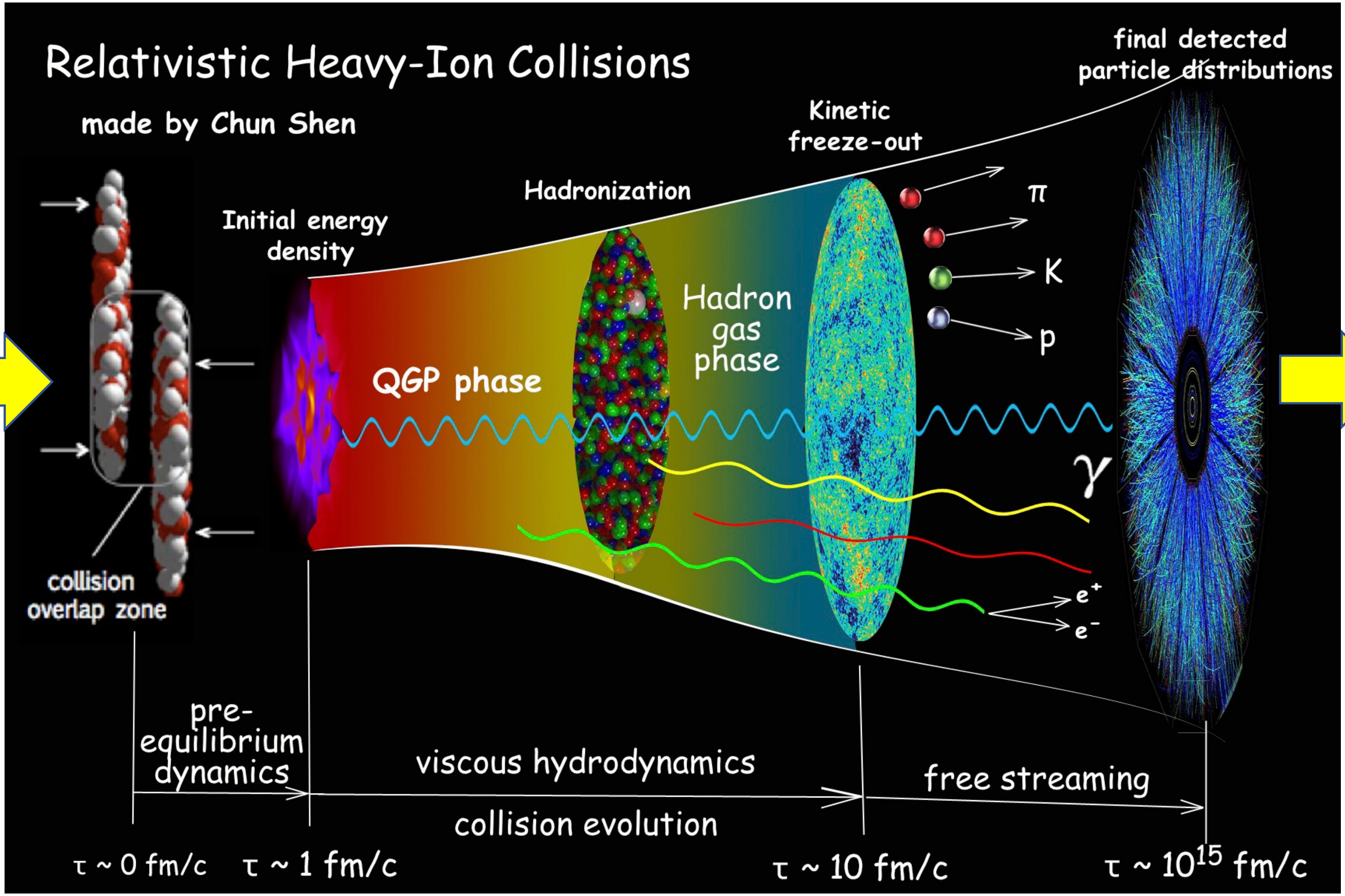
Intersection of nuclear structure and high-energy nuclear collisions

Outline of the Presentation:

- Introduction
- Liquid Drop Model Estimates
- Glauber Model
- Results
  - Effect of γ fluctuations
  - Effect of β fluctuations
- Summary

Stages of Relativistic Heavy-Ion collisions

Initial Stage inputs



Final Stage outputs

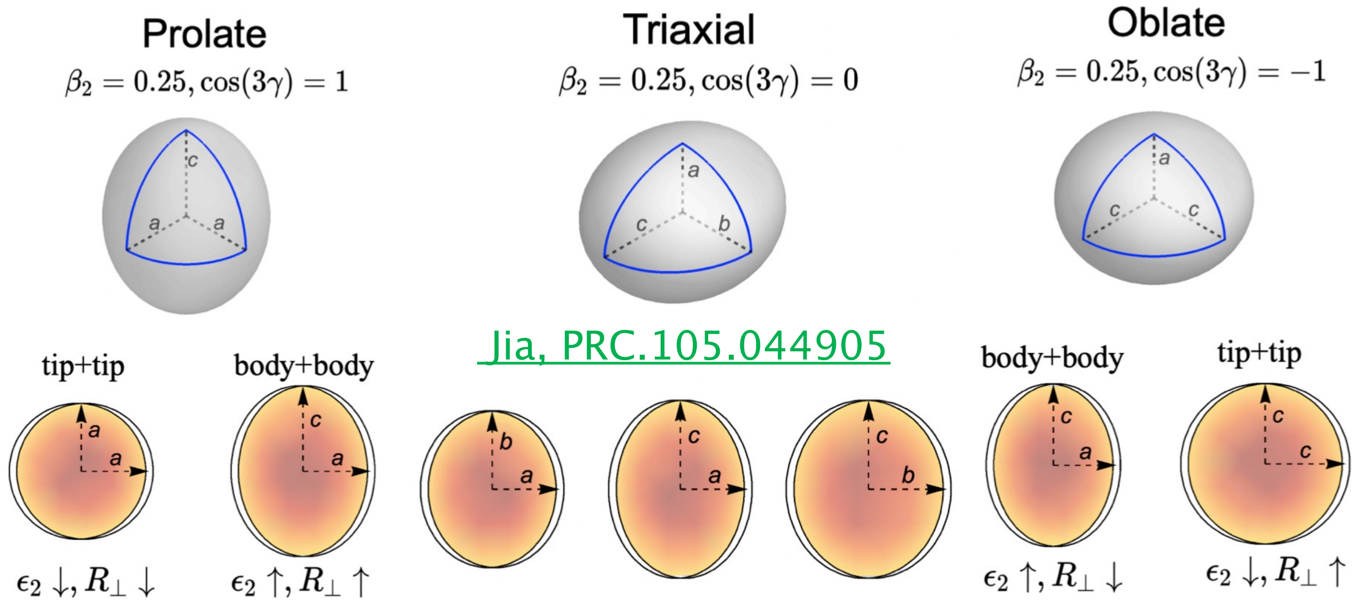
Withheld due to lack of understanding of Initial-State!!

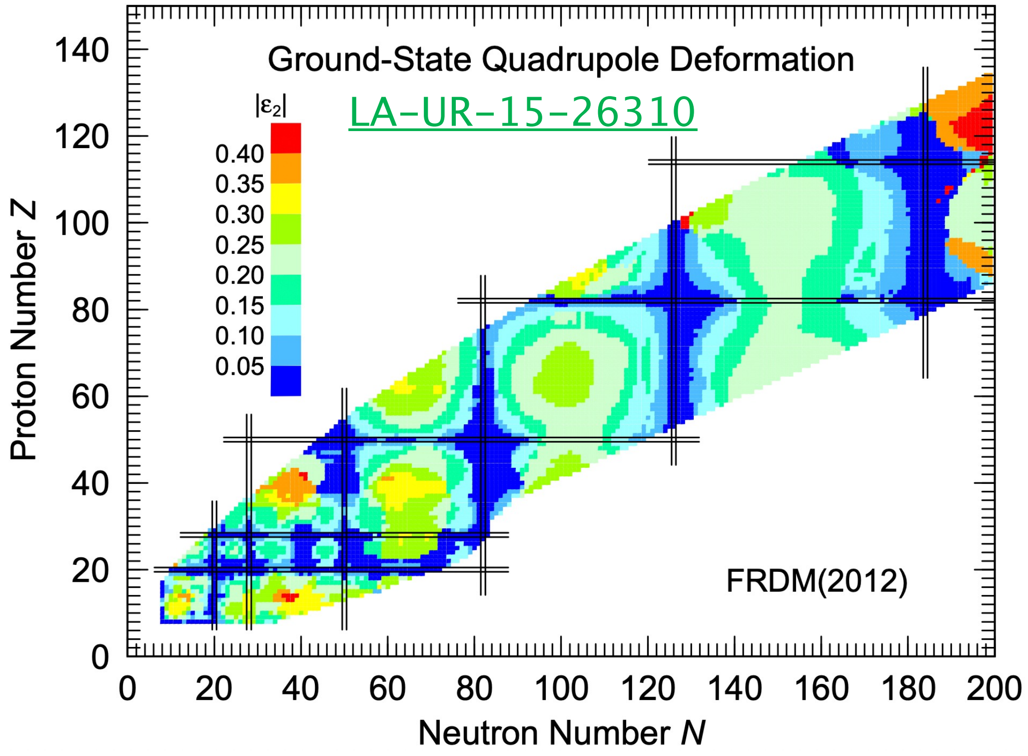
Connection to nuclear geometry

Nuclear geometry is parametrized by Woods-Saxon distribution

$$\rho(r) = \frac{\rho_0}{[1 + exp(r - R(\theta, \phi))/a]}$$

$$R(\theta, \phi) = R_0(1 + \underbrace{\beta(cos\gamma Y_{20}(\theta, \phi) + sin\gamma Y_{22}(\theta, \phi))}_{\text{Quadrupole Deformations}})$$

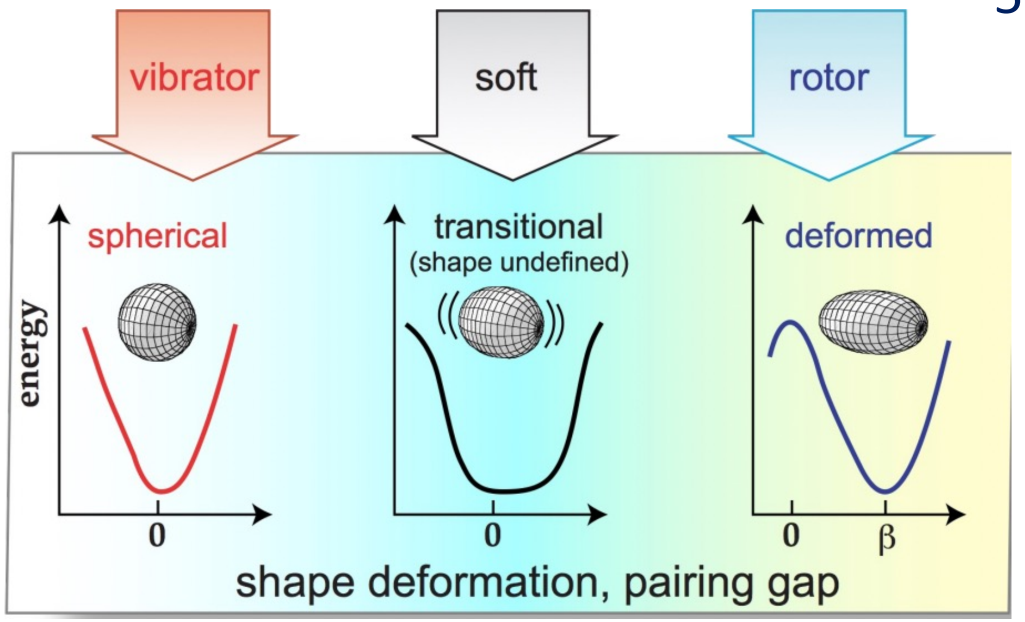




- Nuclear geometric deformation impacts shape and size of overlap area in initial state.
- Deformation effects propagate via viscous hydrodynamics to affect the Final state observables.

Nuclear Shape Fluctuations

- Shallow minimums in potential energy surface allow nuclei to change shape for small energy fluctuations.



Witek, Week 1

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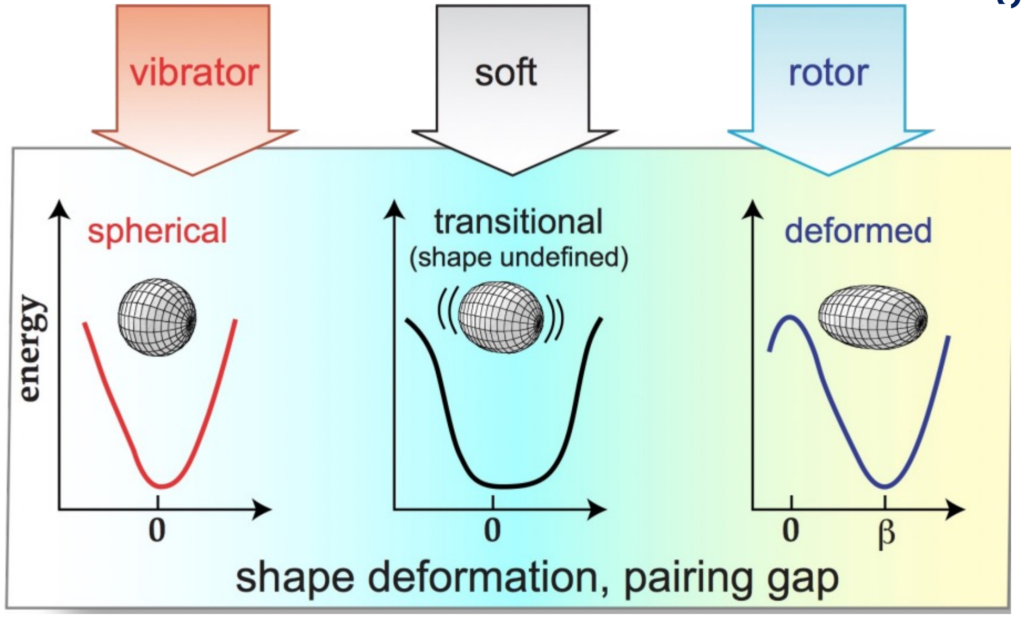
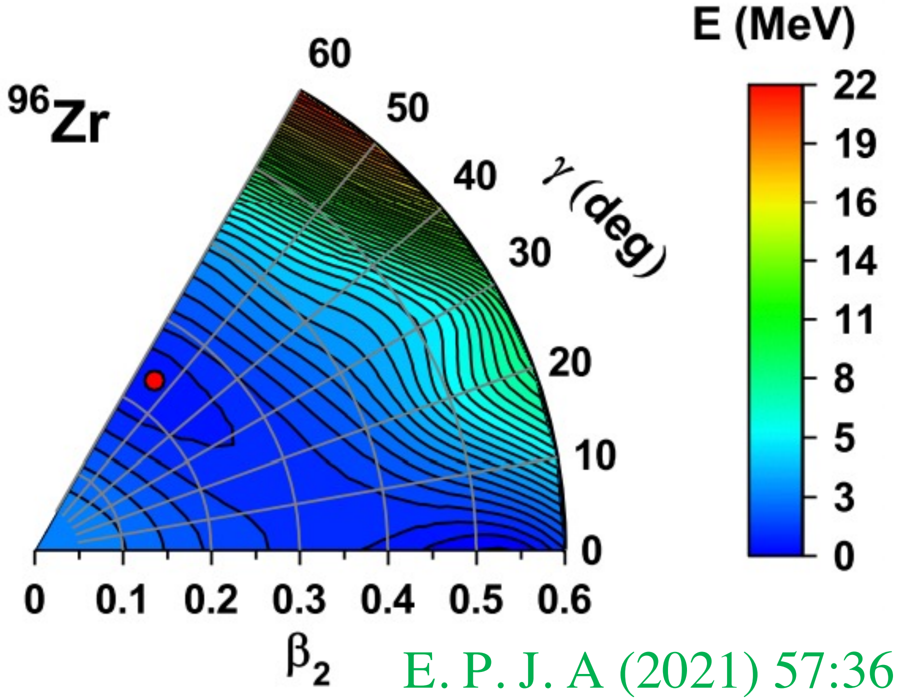


Witek, Week 1



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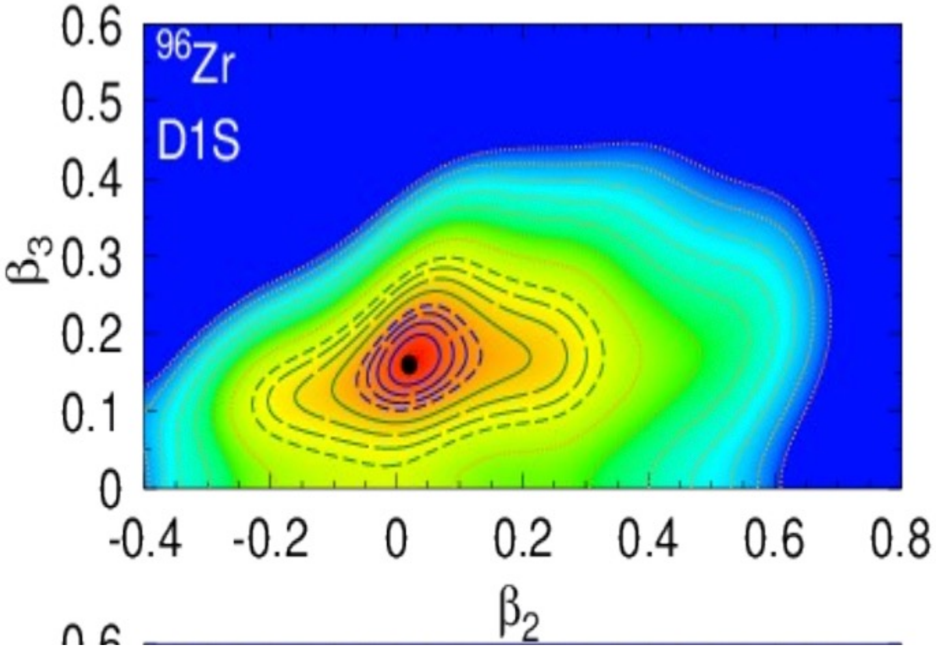
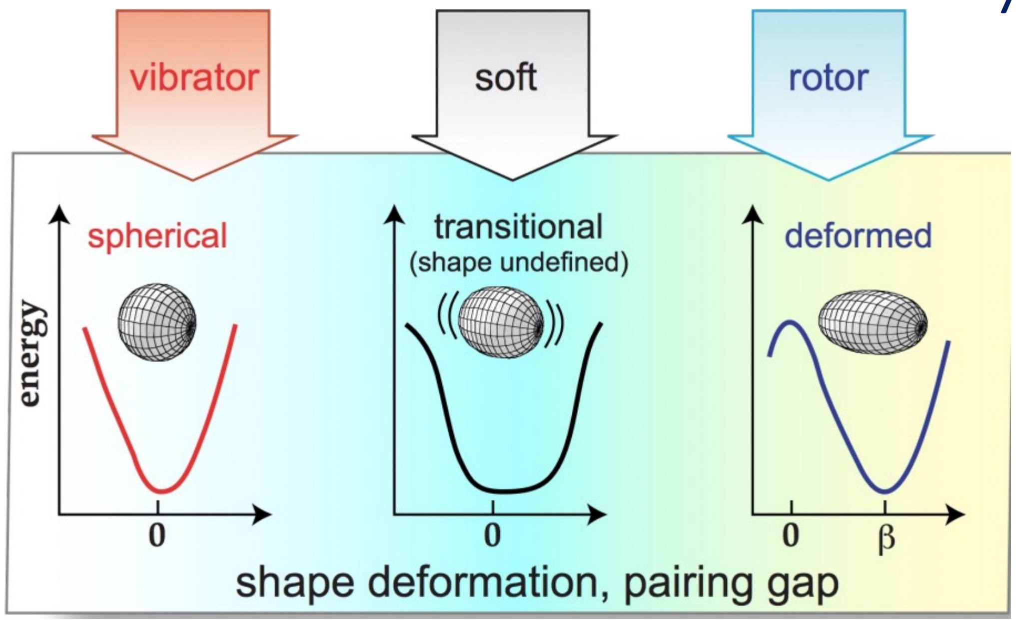


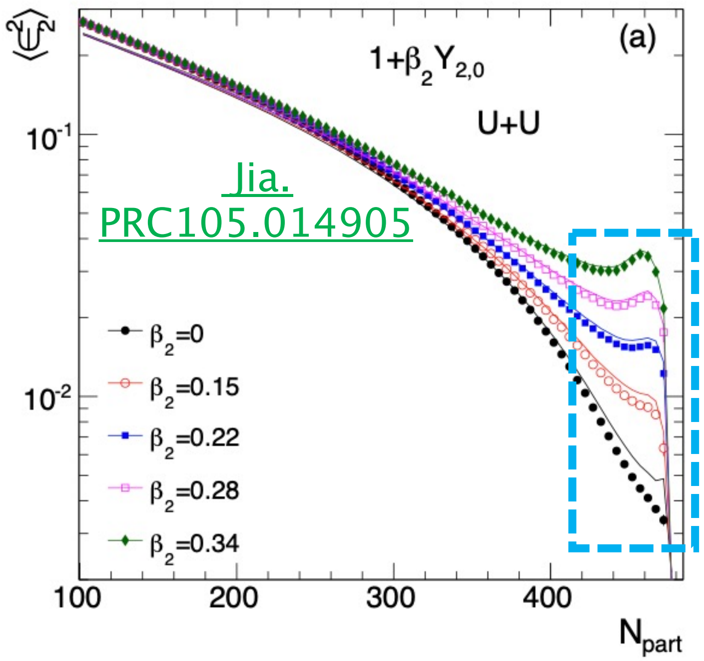
R. Luis, Week 3

Nuclear Shape Fluctuations

- ➢ Shallow minimums in potential energy surface allow nuclei to change shape for small energy fluctuations.

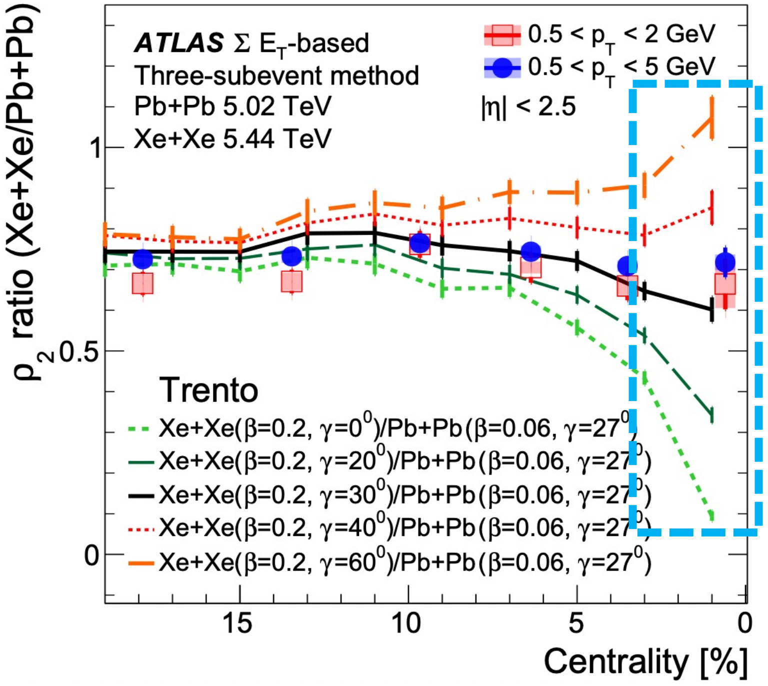


Witek, Week 1



Jia. PRC105.014905

$$\langle \varepsilon_2^2 \rangle = a + b\beta^2$$



$$\langle v_2^2 \delta p_T \rangle \sim a_3 - b_3 \ cos(3\gamma)\beta^3$$

ATLAS. arXiv:2205.00039

- ➢ How does fluctuations in β/γ arising from shape fluctuation manifest in heavy ion collisions?

Liquid Drop Model Estimates:

- Assumptions:
  1. Liquid Drop Model implying each nucleus has a sharp boundary.
  2. The density of nuclear matter is uniform inside the nucleus.
  3. The impact parameter for the colliding system is 0.

$$\rho_{NP}(r) = \begin{cases} \rho_0 & r \leq R(\theta,\phi) \\ 0 & r > R(\theta,\phi) \end{cases}$$

Performing a first order analysis gives:

$$\frac{\delta d_\perp}{d_\perp} \approx \delta_d + p_0(\Omega_p,\gamma_p)\beta_p + p_0(\Omega_t,\gamma_t)\beta_t, \qquad \boldsymbol{\epsilon}_2 \approx \boldsymbol{\epsilon}_0 + \mathbf{p}_2(\Omega_p,\gamma_p)\beta_p + \mathbf{p}_2(\Omega_t,\gamma_t)\beta_t$$

$$d_\perp = \sqrt{N_{part}/\langle r_\perp^2\rangle}\,,\ \frac{\delta[p_\mathrm{T}]}{[p_\mathrm{T}]} \propto \frac{\delta d_\perp}{d_\perp} \qquad \boldsymbol{\epsilon}_2 = -\frac{\langle r_\perp^2 e^{i2\phi}\rangle}{\langle r_\perp^2\rangle}, v_2 \propto \varepsilon_2$$

Averaging over multiple events gives:

$$\langle Obs\rangle = \int\int P(\beta_P,\gamma_P)P(\beta_T,\gamma_T) \int\int Obs(\beta_{2P},\gamma_P,\Omega_P,\beta_{2T},\gamma_T,\Omega_T)\ \frac{d\Omega_P}{8\pi^2}\frac{d\Omega_T}{8\pi^2} d\beta_P d\beta_T d\gamma_P d\gamma_T$$

Cumulant	Formula
$\left\langle(\delta d_\perp/d_\perp)^2\right\rangle$	$\frac{1}{32\pi}\left\langle\beta^2\right\rangle$
$\left\langle(\delta d_\perp/d_\perp)^3\right\rangle$	$\frac{\sqrt{5}}{896\pi^{3/2}}\left\langle\cos(3\gamma)\beta^3\right\rangle$
$\left\langle(\delta d_\perp/d_\perp)^4\right\rangle-3\left\langle(\delta d_\perp/d_\perp)^2\right\rangle^2$	$-\frac{3}{14336\pi^2}\left(7\left\langle\beta^2\right\rangle^2-5\left\langle\beta^4\right\rangle\right)$
$\left\langle\varepsilon_2^2\right\rangle$	$\frac{3}{4\pi}\left\langle\beta^2\right\rangle$
$\left\langle\varepsilon_2^4\right\rangle$	$\frac{9}{112\pi^2}\left(5\left\langle\beta^4\right\rangle+7\left\langle\beta^2\right\rangle^2\right)$
$\left\langle\varepsilon_2^4\right\rangle-2\left\langle\varepsilon_2^2\right\rangle^2$	$-\frac{9}{112\pi^2}\left(7\left\langle\beta^2\right\rangle^2-5\left\langle\beta^4\right\rangle\right)$
$\left(\left\langle\varepsilon_2^6\right\rangle-9\left\langle\varepsilon_2^4\right\rangle\left\langle\varepsilon_2^2\right\rangle+12\left\langle\varepsilon_2^2\right\rangle^3\right)/4$	$\frac{81}{256\pi^3}\left[\left\langle\beta^2\right\rangle^3-\frac{45}{14}\left\langle\beta^4\right\rangle\left\langle\beta^2\right\rangle-\frac{1175}{6006}\left\langle\beta^6\right\rangle+\frac{25}{3003}\left\langle\cos(6\gamma)\beta^6\right\rangle\right]$
$\left\langle\varepsilon_2^2\left(\delta d_\perp/d_\perp\right)\right\rangle$	$-\frac{3\sqrt{5}}{112\pi^{3/2}}\left\langle\cos(3\gamma)\beta^3\right\rangle$
$\left\langle\varepsilon_2^2\left(\delta d_\perp/d_\perp\right)^2\right\rangle-\left\langle\varepsilon_2^2\right\rangle\left\langle(\delta d_\perp/d_\perp)^2\right\rangle$	$-\frac{3}{1792\pi^2}\left(7\left\langle\beta^2\right\rangle^2-5\left\langle\beta^4\right\rangle\right)$
$\left\langle\epsilon_2^2\epsilon_4^*\right\rangle$	$\frac{45}{56\pi^2}\left\langle\beta^4\right\rangle$

Table 1: The leading-order results of various cumulants of $\boldsymbol{\epsilon_2}$ and $\frac{\delta d_\perp}{d_\perp}$ calculated under the Liquid Drop model

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$\left\langle \varepsilon_2^2 (\delta d_\perp/d_\perp) \right\rangle$	$-\frac{3\sqrt{5}}{112\pi^{3/2}}\left\langle \cos(3\gamma)\beta^3 \right\rangle$
$\left\langle \varepsilon_2^2 (\delta d_\perp/d_\perp)^2 \right\rangle - \left\langle \varepsilon_2^2 \right\rangle\left\langle (\delta d_\perp/d_\perp)^2 \right\rangle$	$-\frac{3}{1792\pi^2}\left(7\left\langle \beta^2 \right\rangle^2 - 5\left\langle \beta^4 \right\rangle\right)$
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Glauber Model:

- Nucleons inside the nuclei are distributed according to the deformed Wood-Saxon distribution.

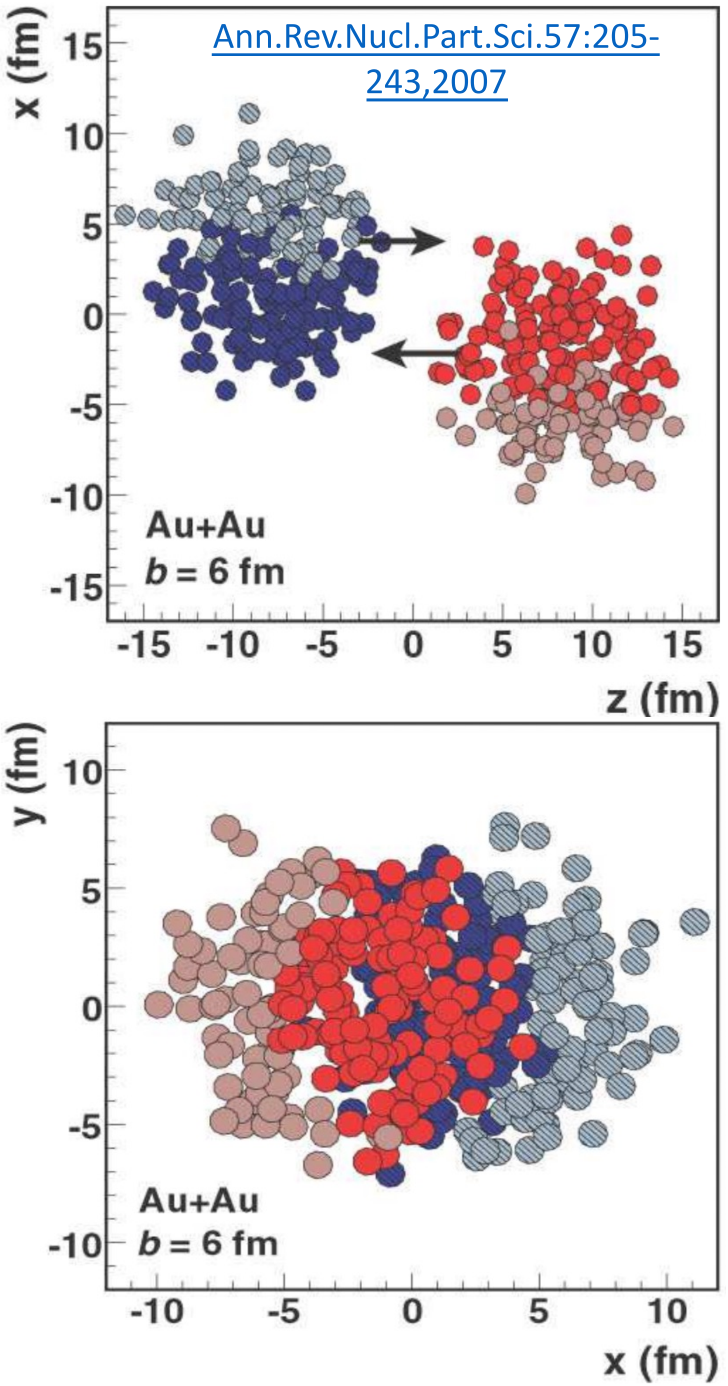
$$\rho_{WS}(r) = \frac{\rho_0}{[1 + \exp(r - R(\theta,\phi))/a]}$$
$$R(\theta,\phi) = R_0\left(1 + \beta\left(\cos\gamma Y_{20}(\theta,\phi) + \sin\gamma Y_{22}(\theta,\phi)\right)\right)$$

- Random impact parameter `b' is sampled along the x-axis from the distribution $\frac{dN}{db} \propto b$ between the range $[b_{min}, b_{max}]$.
- The nucleus-nucleus collision is assumed to be a collection of multiple independent binary nucleon-nucleon collisions.
- A nucleon-nucleon collision is assumed to have occurred if:

$$d_\perp \leq \sqrt{\sigma_{\rm inel}^{\rm NN}/\pi}$$

σ_{inel}^{NN}: inelastic nucleon-nucleon cross-section
$d_\perp$: Distance in x-y plane between the nucleon

Participating nucleons are identified and event quantities are computed.



Results:

- Effect of γ fluctuations:
  - cov $\equiv \left\langle \varepsilon_2^2 \frac{\delta d_1}{d_\perp} \right\rangle$ and $C_d\{3\} \equiv \left\langle \left(\frac{\delta d_\perp}{d_\perp}\right)^3 \right\rangle$

- Effects of β fluctuations:
  - $\langle \varepsilon_2^2 \rangle$ and $C_d\{2\} \equiv \left\langle \left(\frac{\delta d_\perp}{d_\perp}\right)^2 \right\rangle$
  - $c_{2,\varepsilon}\{4\} \equiv \langle \varepsilon_2^4 \rangle - 2\langle \varepsilon_2^2 \rangle^2$

Part I – Impact of γ fluctuations:

- Nuclear shape has a three-fold symmetry under the triaxial parameter γ. Hence any observable $\langle O \rangle$ can be parameterized as:

$$\langle O \rangle = a_0 + \sum_{n=1}^{\infty} \left[a_n \cos(3n\bar{\gamma}) + b_n \sin(3n\bar{\gamma}) \right] e^{-\frac{n^2 \sigma_\gamma^2}{2}}$$

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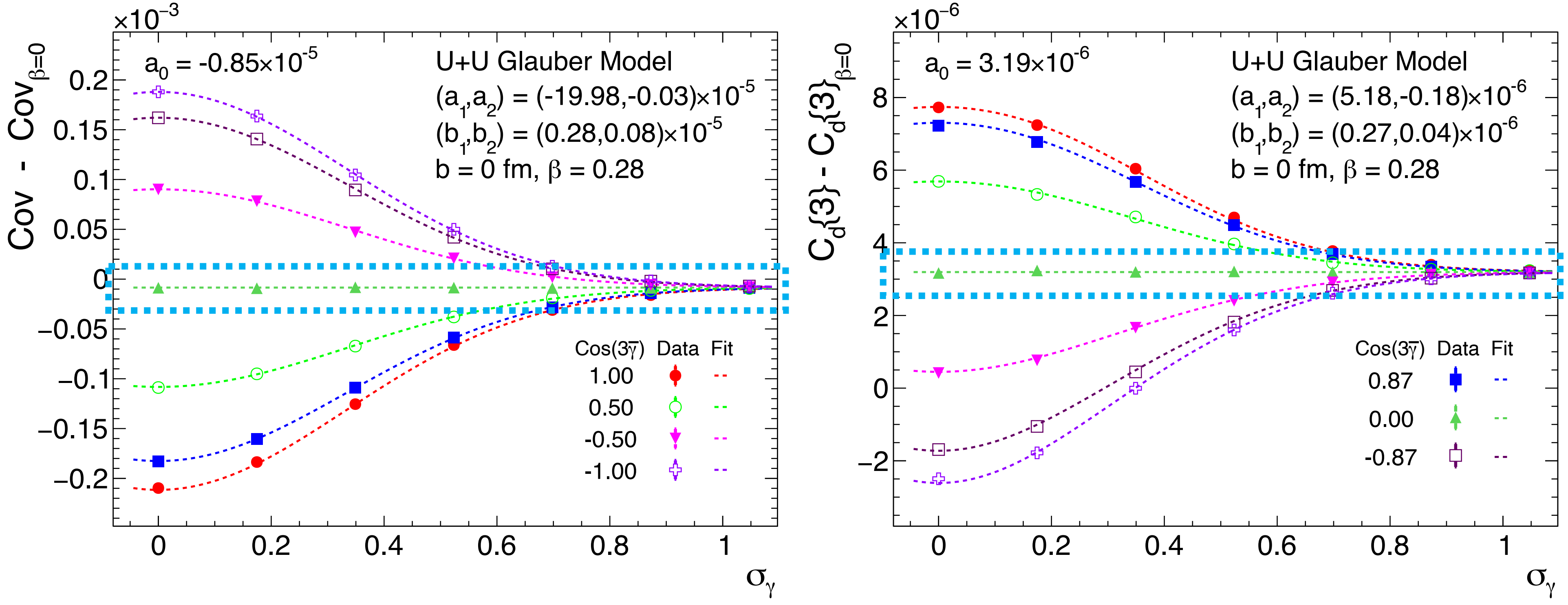
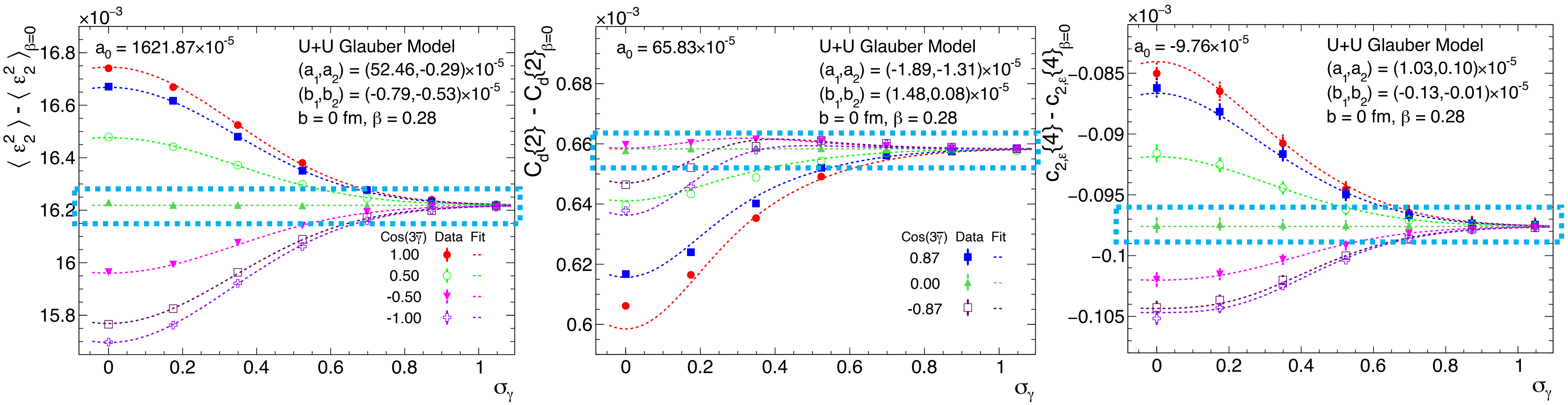
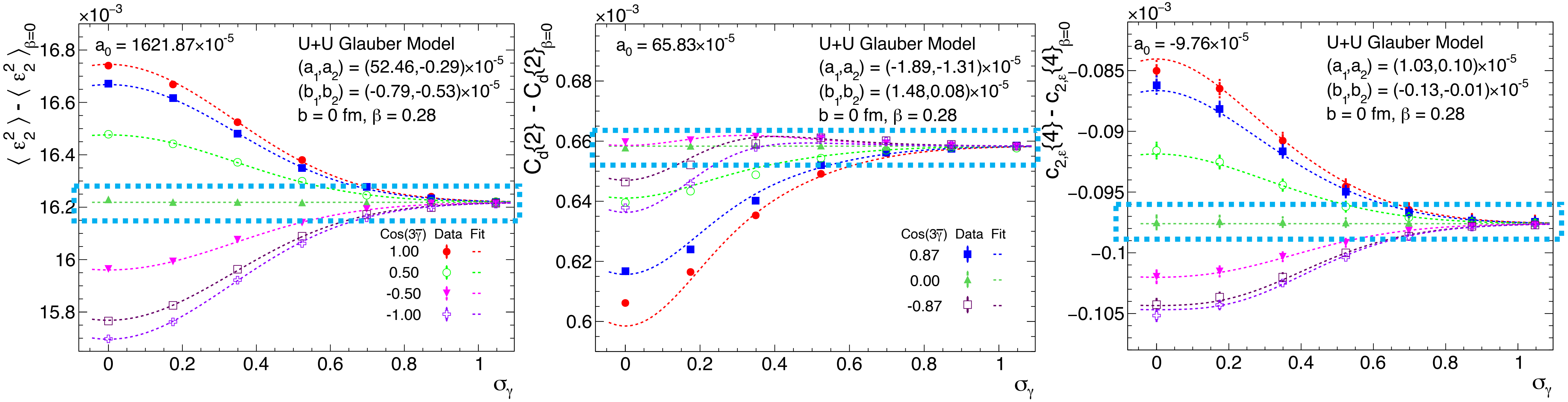




a_0 = 1621.87×10^{-5}
U+U Glauber Model
(a_1,a_2) = (52.46,-0.29)×10^{-5}
(b_1,b_2) = (-0.79,-0.53)×10^{-5}
b = 0 fm, β = 0.28
Cos(3γ̄) Data Fit
1.00
0.50
-0.50
-1.00
$\langle \varepsilon_2^2 \rangle - \langle \varepsilon_2^2 \rangle_{\beta=0}$
σ_γ
a_0 = 65.83×10^{-5}
U+U Glauber Model
(a_1,a_2) = (-1.89,-1.31)×10^{-5}
(b_1,b_2) = (1.48,0.08)×10^{-5}
b = 0 fm, β = 0.28
Cos(3γ̄) Data Fit
0.87
0.00
-0.87
$C_d\{2\} - C_d\{2\}_{\beta=0}$
σ_γ
a_0 = -9.76×10^{-5}
U+U Glauber Model
(a_1,a_2) = (1.03,0.10)×10^{-5}
(b_1,b_2) = (-0.13,-0.01)×10^{-5}
b = 0 fm, β = 0.28
$c_{2,\varepsilon}\{4\} - c_{2,\varepsilon}\{4\}_{\beta=0}$
σ_γ



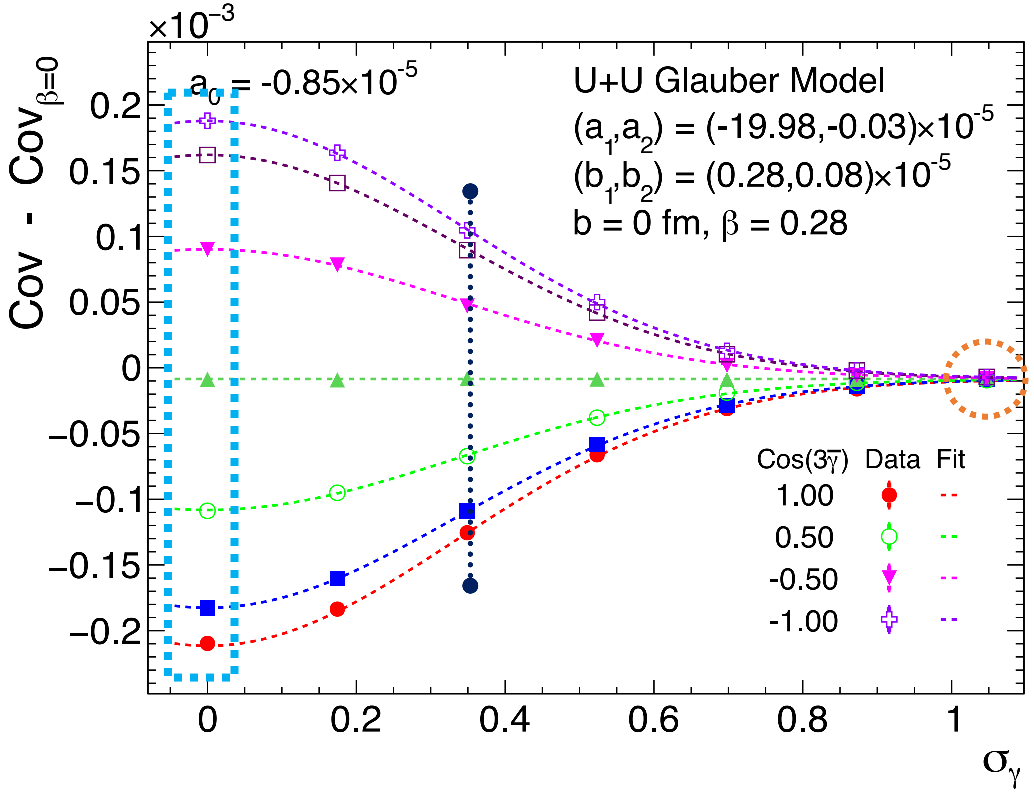
- Imposing the condition that a triaxial nucleus ($\bar{\gamma}$ = π/6) is unaffected by the variance of the parent distribution (σ_γ)

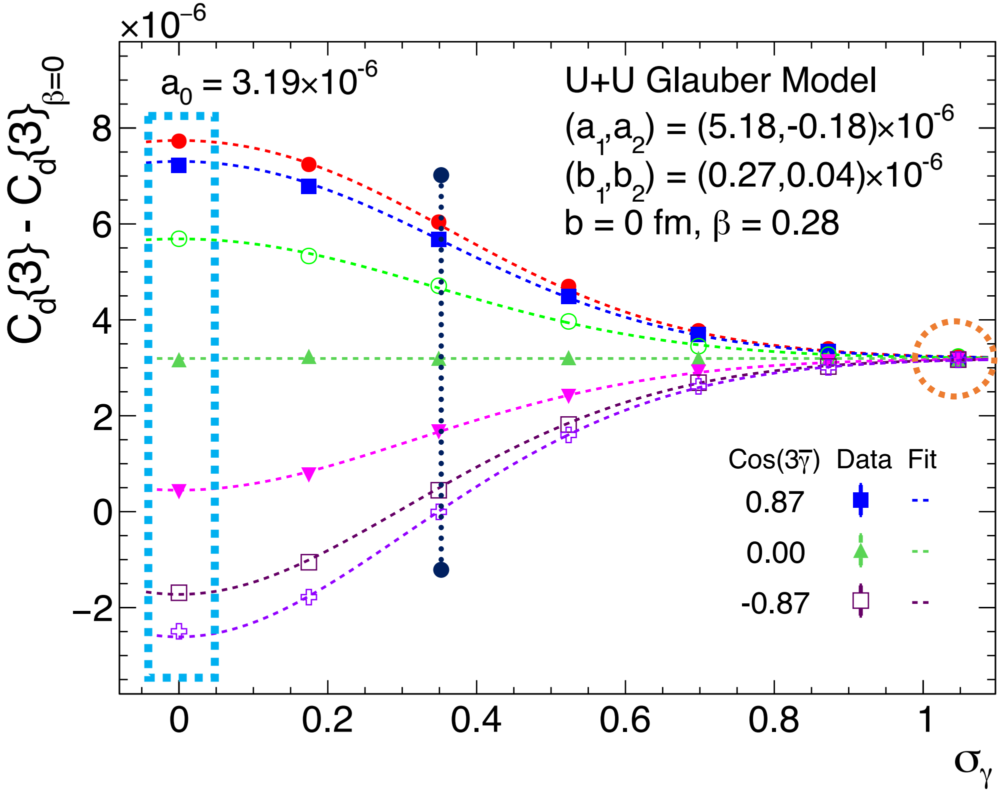
$$\langle O \rangle = a_0 + \sum_{n=1}^{\infty}\left[a_n\left(\cos(3n\bar{\gamma}) - \cos\left(3n\frac{\pi}{6}\right)\right) + b_n\left(\sin(3n\bar{\gamma}) - \sin\left(3n\frac{\pi}{6}\right)\right)\right]e^{-\frac{n^2\sigma_\gamma^2}{2}}$$

- Restricting ourselves to leading and sub-leading terms

$$\langle O \rangle - \langle O \rangle_{\beta=0} = a_0 + \left(a_1\cos(3\bar{\gamma}) + b_1[\sin(3\bar{\gamma}) - 1]\right)e^{-\frac{9\sigma_\gamma^2}{2}} + \left(a_2[\cos(6\bar{\gamma}) + 1] + b_2\sin(6\bar{\gamma})\right)e^{-\frac{36\sigma_\gamma^2}{2}}$$

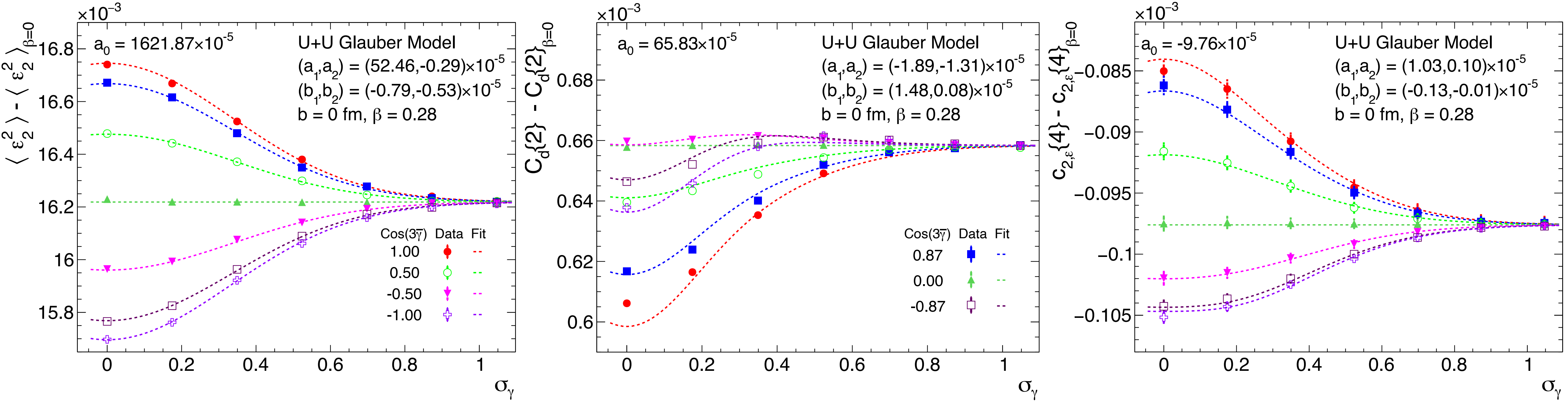
Impact on $\left\langle \varepsilon_2^2 \frac{\delta d_1}{d_\perp} \right\rangle$ and $\left\langle \left(\frac{\delta d_\perp}{d_\perp} \right)^3 \right\rangle$





- Observables can largely be described by their leading order fits.
- The signature of triaxiality is greatly reduced for γ - soft nuclei. A twenty-degree fluctuation in triaxiality roughly reduces the signal by 50%.
- Nuclei that fluctuate uniformly between prolate and oblate shapes become indistinguishable from a rigid triaxial nuclei.

Impact on $\langle\varepsilon_2^2\rangle$, $\left\langle\left(\frac{\delta d_\perp}{d_\perp}\right)^2\right\rangle$ and $\langle\varepsilon_2^4\rangle - 2\langle\varepsilon_2^2\rangle^2$



- The fitting function describes each observable quite decently.
- $\langle\varepsilon_2^2\rangle$ and $\langle\varepsilon_2^4\rangle - 2\langle\varepsilon_2^2\rangle^2$ are majorly described by leading order terms. A visible asymmetry about the triaxial nuclei is observed which is accounted for by the sub-leading term.
- $\left\langle\left(\frac{\delta d_\perp}{d_\perp}\right)^2\right\rangle$ requires inclusion of all the terms with comparable magnitudes.

Part II – Impact of β fluctuations:

- In the most general case, $\frac{\delta d_\perp}{d_\perp}$, $\boldsymbol{\epsilon_2}$ can have the following form at the event level.

$$\frac{\delta d_\perp}{d_\perp} = \delta_d + \sum_{i=1}^{\infty} p_{0,i}(\Omega_p, \gamma_p)\beta_p^i + p_{0,i}(\Omega_t, \gamma_t)\beta_t^i,$$

$$\boldsymbol{\epsilon_2} = \boldsymbol{\epsilon_0} + \sum_{i=1}^{\infty} \boldsymbol{p_{2,i}}(\Omega_p, \gamma_p)\beta_p^i + \boldsymbol{p_{2,i}}(\Omega_t, \gamma_t)\beta_t^i$$

- Then any cumulant of the form $\langle O_{2\alpha+\nu}\rangle \equiv \left\langle \boldsymbol{\epsilon_2^{2\alpha}} \left(\frac{\delta d_\perp}{d_\perp}\right)^\nu \right\rangle$ will assume the form:

$$\langle O_{2\alpha+\nu}\rangle - \langle O_{2\alpha+\nu}\rangle_{\beta=0} = \sum_{i=2\alpha+\nu}^{\infty} d_i \left\langle \beta^i \right\rangle + \sum_{\substack{j=4 \\ 2\alpha+\nu\geq 4}}^{\infty} \sum_{k=2}^{k\leq j/2} e_{jk} \left\langle \beta^k \right\rangle \left\langle \beta^{j-k} \right\rangle$$

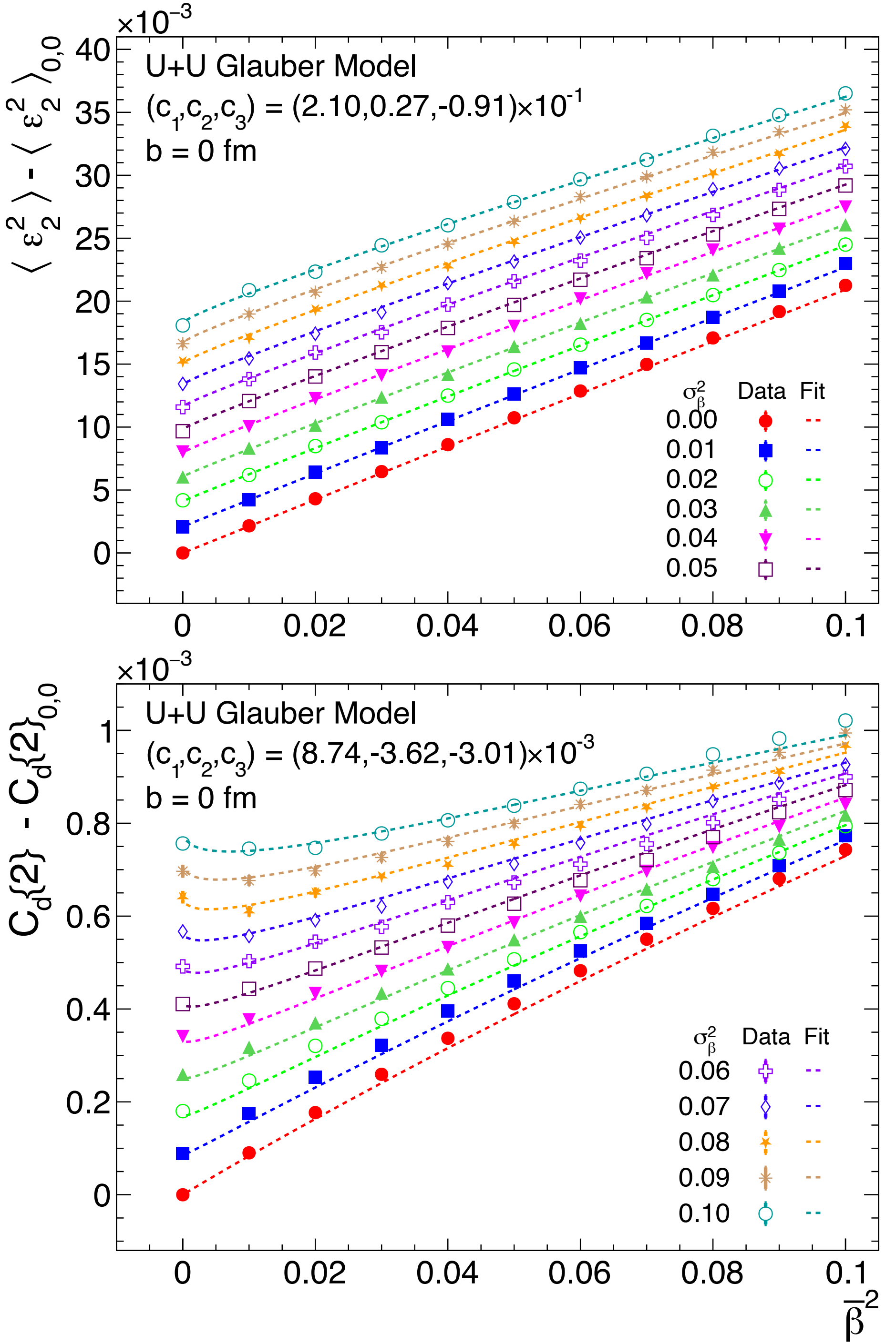
- In the analysis, we will try to constrain the impact of β fluctuations using the minimum possible number of higher order corrections.

Impact on $\langle\varepsilon_2^2\rangle$ and $\left\langle\left(\frac{\delta d_\perp}{d_\perp}\right)^2\right\rangle$

- Approximately linear dependencies on $\bar{\beta}$ are observed for both observables.

Slopes of the data points also vary with σ_β. To describe this feature, we include two higher-order terms

$$\langle\varepsilon_2^2\rangle - \langle\varepsilon_2^2\rangle_{\beta=0} \text{ or } \left\langle\left(\frac{\delta d_\perp}{d_\perp}\right)^2\right\rangle - \left\langle\left(\frac{\delta d_\perp}{d_\perp}\right)^2\right\rangle_{\beta=0}$$
$$= c_1\langle\beta^2\rangle + c_2\langle\beta^3\rangle + c_3\langle\beta^4\rangle$$



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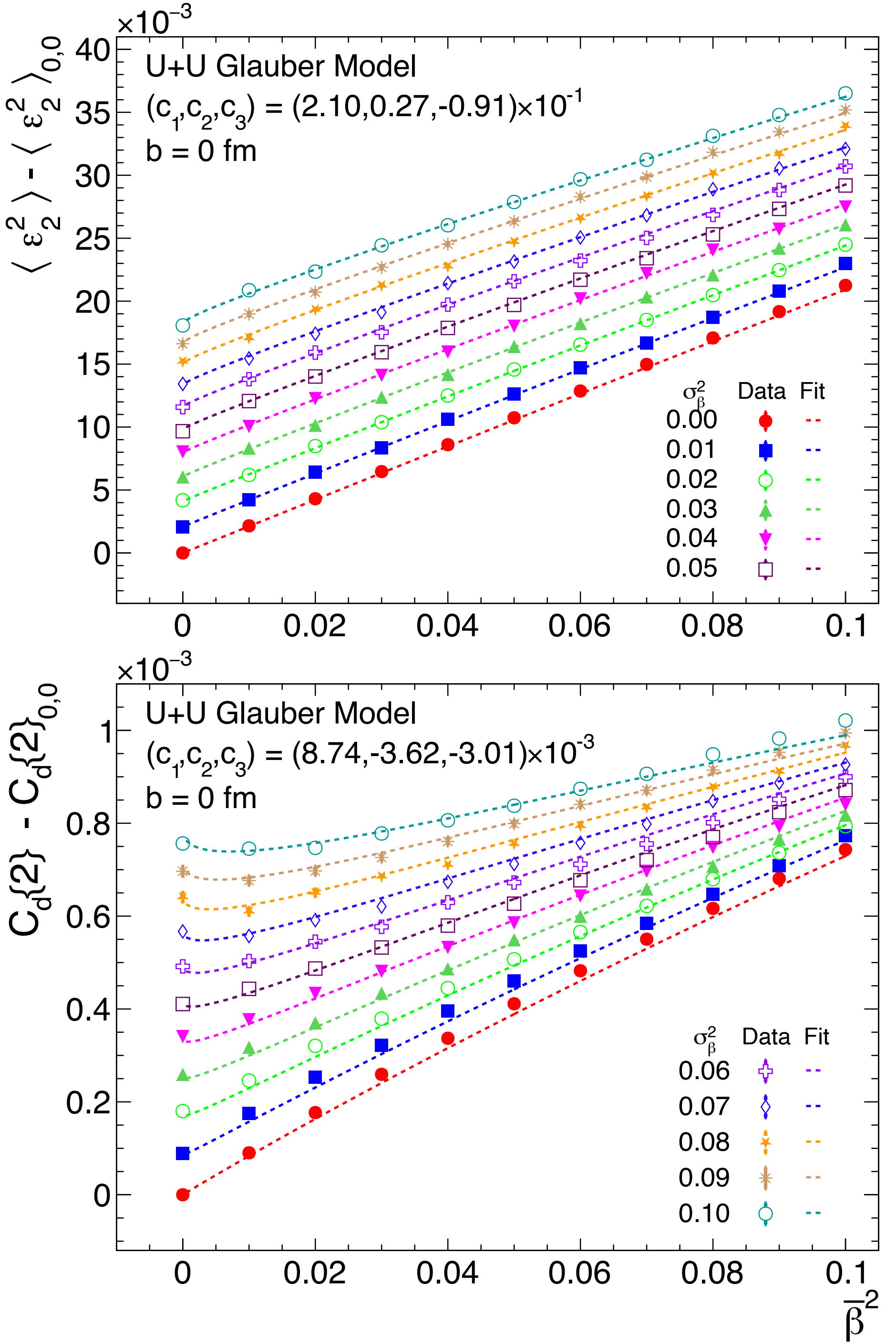
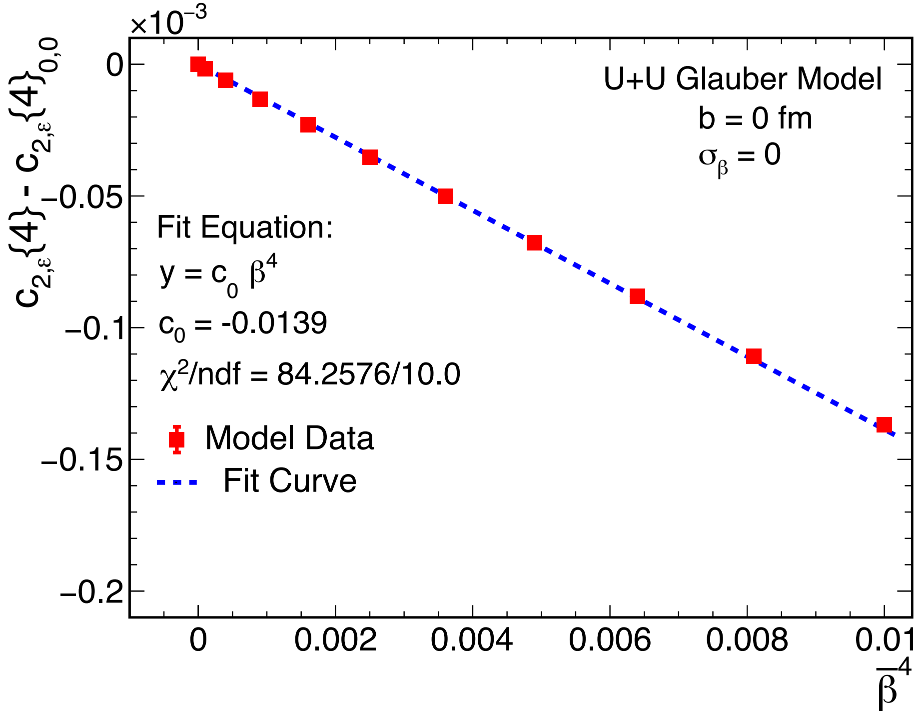
- $\langle \boldsymbol{\varepsilon_2^2} \rangle$ has its major contribution from the leading order $\langle \beta^2 \rangle$ with a small correction from $\langle \beta^4 \rangle$.
- $\left\langle \left(\frac{\boldsymbol{\delta d_\perp}}{\boldsymbol{d_\perp}}\right)^2 \right\rangle$ requires contribution from all the terms.

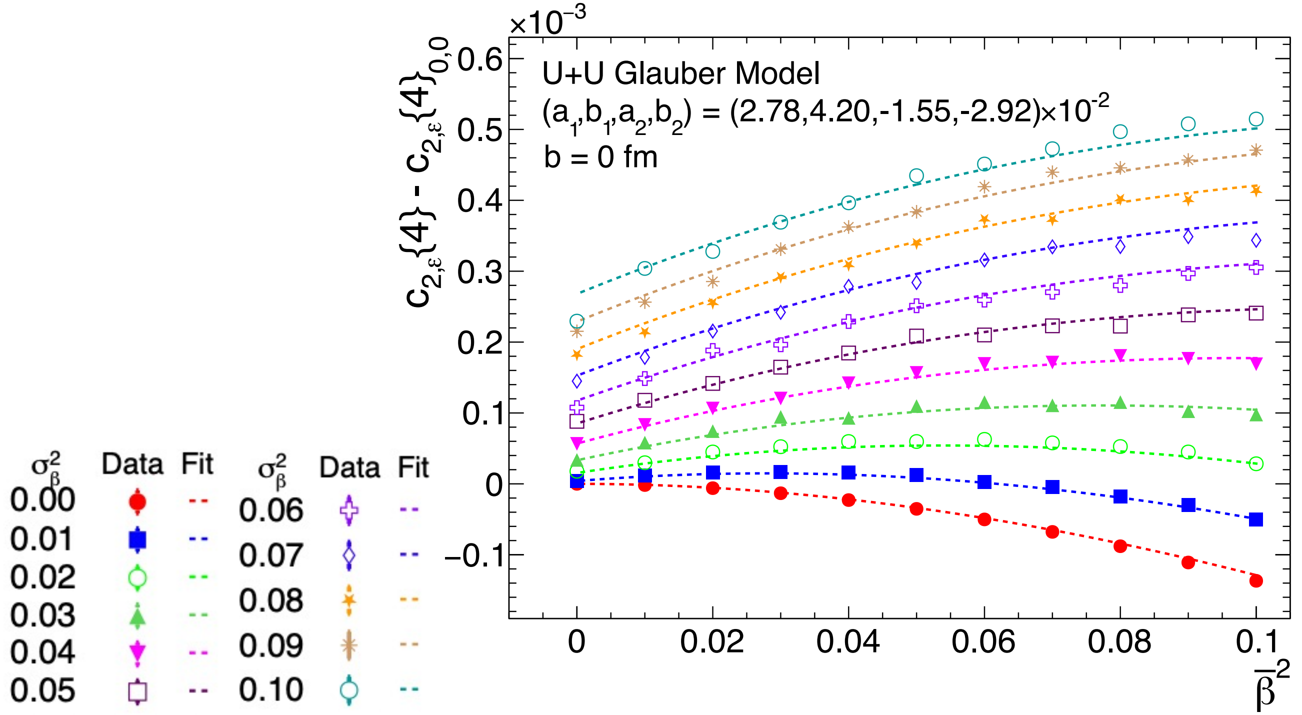


Impact on $c_{2,\varepsilon}\{4\} = \langle\varepsilon_2^4\rangle - 2\langle\varepsilon_2^2\rangle^2$

Guided by the fits for $\langle\boldsymbol{\varepsilon_2^2}\rangle$:

$$c_{2,\varepsilon}\{4\} - c_{2,\varepsilon}\{4\}_{\beta=0} =$$
$$a_1\langle\beta^4\rangle - b_1\langle\beta^2\rangle^2 + a_2\langle\beta^6\rangle - b_2\langle\beta^2\rangle\langle\beta^4\rangle$$





Impact on $c_{2,\varepsilon}\{4\} = \langle\varepsilon_2^4\rangle - 2\langle\varepsilon_2^2\rangle^2$

Guided by the fits for $\langle\boldsymbol{\varepsilon_2^2}\rangle$:

$$c_{2,\varepsilon}\{4\} - c_{2,\varepsilon}\{4\}_{\beta=0} =$$
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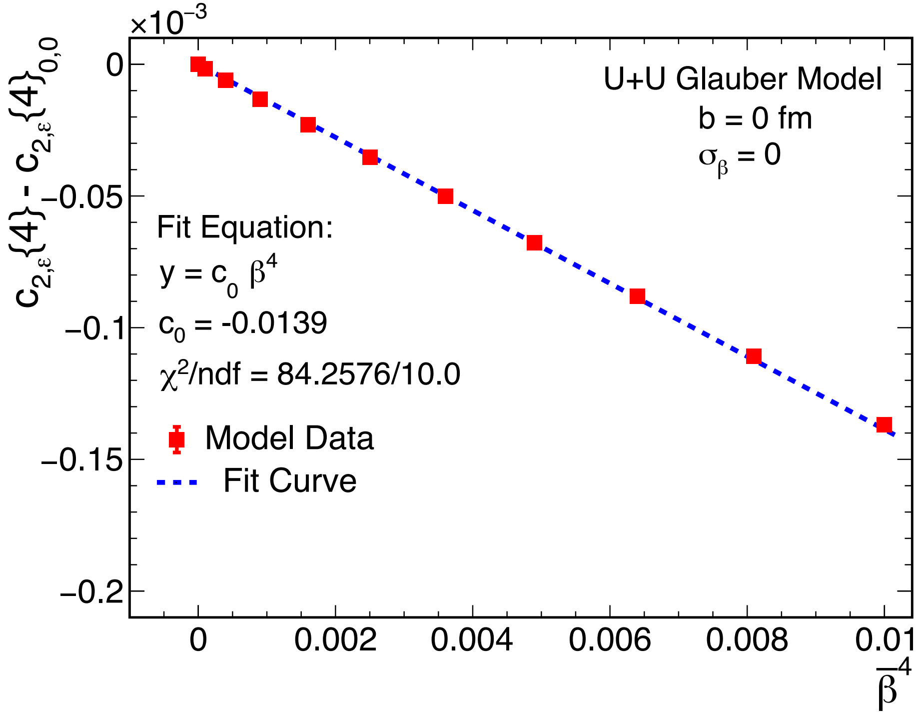
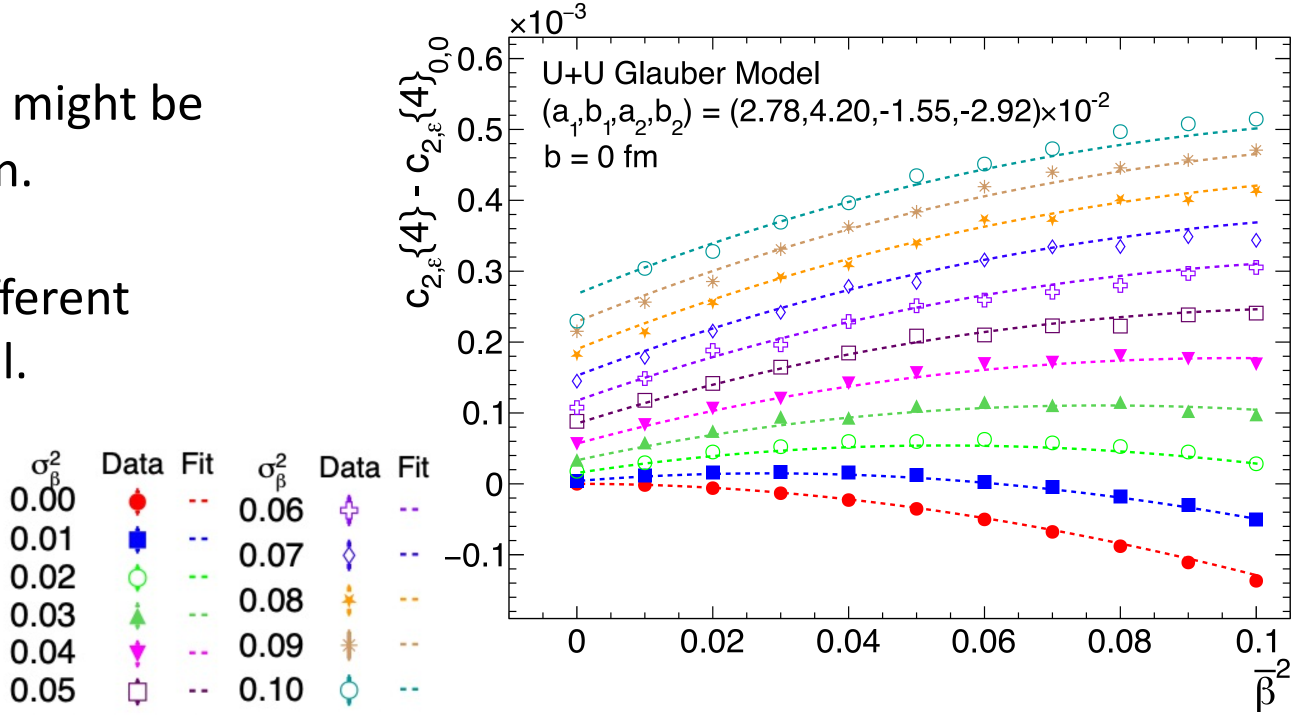




- A large negative $c_{2,\varepsilon}\{4\}$ value in the central collisions might be an indication of a large static quadrupole deformation.
- We also observe that $b_1 \approx 1.5a_1$ which is slightly different from ratio (1.4) calculated from the liquid drop model.

Summary

- Impact of the fluctuations of nuclear quadrupole deformation on various initial state heavy ion observables under the framework of a Monte Carlo Glauber model was studied.
- Triaxiality γ has a strong impact on three-particle correlators, but the impact diminishes for larger σ_γ. When σ_γ is large, the observables fail to distinguish between prolate deformation and oblate deformation.
- Quadrupole fluctuations had a significant impact on all the observables.
    - $\langle \boldsymbol{\varepsilon_2^2} \rangle$ and $\left\langle \left(\frac{\boldsymbol{\delta d_\perp}}{\boldsymbol{d_\perp}} \right)^{\mathbf{2}} \right\rangle$ are proportional to $\langle \beta^2 \rangle$ up to leading order. $\left\langle \left(\frac{\boldsymbol{\delta d_\perp}}{\boldsymbol{d_\perp}} \right)^{\mathbf{2}} \right\rangle$ also has a significant contribution from $\langle \beta^3 \rangle$.
    - $c_{2,\varepsilon}\{4\}$ is found to be negative for static deformation and quickly becomes positive under the influence of a small fluctuation. It requires both leading ($\langle \beta^4 \rangle, \langle \beta^2 \rangle^2$) and sub-leading orders ($\langle \beta^6 \rangle, \langle \beta^4 \rangle \langle \beta^2 \rangle$) to achieve a good fit.

Thank You For Your Attention

Backup Slides

Initial state observables:

- The second order eccentricity of the overlap region is usually quantified by:

$$\epsilon_{\mathbf{2}} \equiv \varepsilon_2 e^{i2\Phi_2} = -\frac{\langle r_\perp^2 e^{i2\phi}\rangle}{\langle r_\perp^2\rangle}$$ average over all participating nuclei

- The inverse transverse size of the overlapping region is given as:

$$d_\perp = \sqrt{N_{\text{part}}/S_\perp}$$

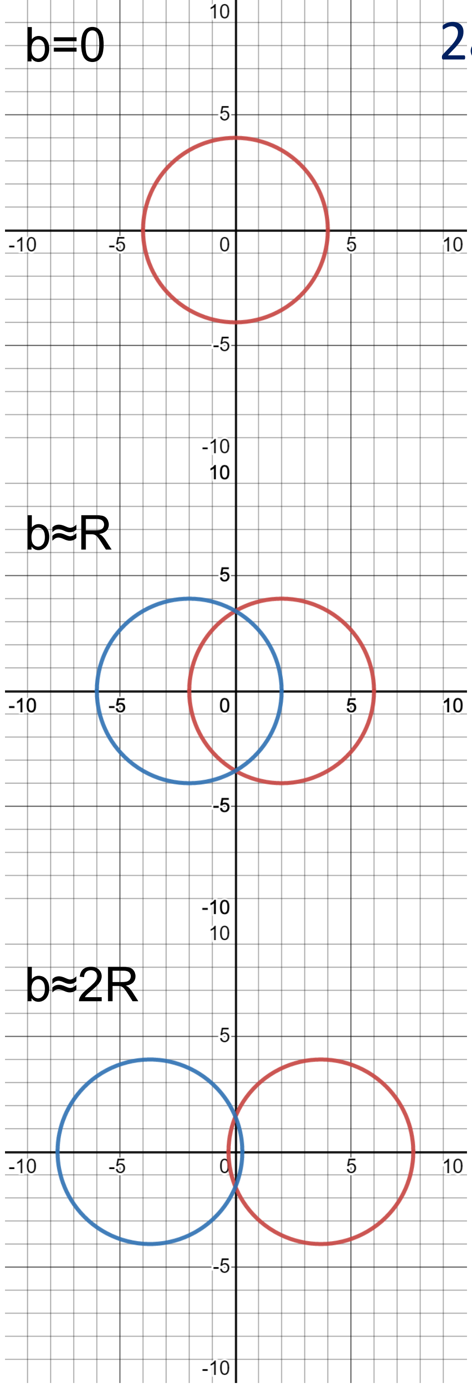
N_{part} is number of participating nuclei

$S_\perp$ is transverse area computed $\pi\sqrt{\langle x^2\rangle\langle y^2\rangle}$

- Useful owing to the relation:

$$v_2 = k_2\varepsilon_2, \quad \frac{\delta[p_\mathrm{T}]}{[p_\mathrm{T}]} = k_0\frac{\delta d_\perp}{d_\perp} = -k_0\frac{\delta R_\perp}{R_\perp} = -k_0\frac{1}{2}\frac{\delta S_\perp}{S_\perp}$$

The following five cumulants of $\epsilon_{\mathbf{2}}$ and $d_\perp$ act as our initial state observables: $\langle\varepsilon_2^2\rangle$ (elliptic deformation), $\left\langle\varepsilon_2^2\frac{\delta d_\perp}{d_\perp}\right\rangle$, $\left\langle\left(\frac{\delta d_\perp}{d_\perp}\right)^2\right\rangle$), $\left\langle\left(\frac{\boldsymbol{\delta d_\perp}}{\boldsymbol{d_\perp}}\right)^{\mathbf{3}}\right\rangle$ (variance and skewness in inverse transverse size respectively) and $\langle\varepsilon_2^4\rangle - 2\langle\varepsilon_2^2\rangle^2$ (fourth order cumulant of elliptic eccentricity).



Liquid Drop Model Estimates:

- Assumptions:
  1. Liquid Drop Model implying each nucleus has a sharp boundary.
  2. The density of nuclear matter is uniform inside the nucleus.
  3. The impact parameter for the colliding system is 0.
  4. Both target and projectile nuclei have the same orientation i.e. they are aligned.
  5. In a given collision, the deformation parameters for both nuclei are exactly equal.

$$\rho_{NP}(r) = \begin{cases} \rho_0 & r \le R(\theta,\phi) \\ 0 & r > R(\theta,\phi) \end{cases}$$

Performing a first order analysis gives:

$$\frac{\delta d_\perp}{d_\perp}(\beta_2,\gamma,\Omega) = \sqrt{\frac{5}{16\pi}}\beta_2\left(\cos\gamma\, D^2_{0,0}(\Omega) + \frac{\sin\gamma}{\sqrt{2}}\left[D^2_{0,2}(\Omega) + D^2_{0,-2}(\Omega)\right]\right)$$

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$$\boldsymbol{\epsilon}_2(\beta_2,\gamma,\Omega) = -\sqrt{\frac{15}{2\pi}}\beta_2\left(\cos\gamma\, D^2_{2,0}(\Omega) + \frac{\sin\gamma}{\sqrt{2}}\left[D^2_{2,2}(\Omega) + D^2_{2,-2}(\Omega)\right]\right)$$

- (β_2,γ) are deformation parameters
- Ω is the set of Euler angles defining the intrinsic orientation of the nucleus
- $D^m_{n,o}$ are Wigner-Matrix elements

Relaxing conditions 3 and 4 leads to:

$$\frac{\delta d_\perp}{d_\perp}^{evt} \equiv \frac{\delta d_\perp}{d_\perp}^{evt}(\beta_{2P},\gamma_P,\Omega_P,\beta_{2T},\gamma_T,\Omega_T) = \frac{1}{2}\left(\frac{\delta d_\perp}{d_\perp}(\beta_{2P},\gamma_P,\Omega_P) + \frac{\delta d_\perp}{d_\perp}(\beta_{2T},\gamma_T,\Omega_T)\right)$$

$$\boldsymbol{\epsilon_2}^{evt} \equiv \boldsymbol{\epsilon_2}^{evt}(\beta_{2P},\gamma_P,\Omega_P,\beta_{2T},\gamma_T,\Omega_T) = \frac{1}{2}\left(\boldsymbol{\epsilon_2}(\beta_{2P},\gamma_P,\Omega_P) + \boldsymbol{\epsilon_2}(\beta_{2T},\gamma_T,\Omega_T)\right)$$

- Subscript P and T are for Projectile and Target nuclei respectively.

Finally, we calculate the averaged quantities over multiple events.

$$< Obs > = \int\int P(\beta_{2P})P(\beta_{2T}) \int\int Obs(\beta_{2P},\gamma_P,\Omega_P,\beta_{2T},\gamma_T,\Omega_T)\ \frac{d\Omega_P}{8\pi^2}\frac{d\Omega_T}{8\pi^2}\ d\beta_{2P}d\beta_{2T}$$

- Here Obs is a proxy for the observable one is calculating the event averages of.

Setting up the Glauber Model:

- A symmetric collision system of ^{238}U - ^{238}U is chosen.
- The radius R_0 is set at 6.81 fm and the skin depth parameter a_0 is set to 0.55 fm.
- The nucleon-nucleon inelastic cross section σ_{inel}^{NN} is chosen to be 42.1 mb.
- Ultra-central collisions (b=0) are simulated by setting b_{min} =b_{max} = 0. These collisions are chosen as the impact of deformation parameters on chosen initial state observables is maximum for UCC collisions.
- Deformation parameters sampled independently from a Gaussian distribution.

$$\rho(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$x \equiv \beta, \gamma$$
$$\mu = \bar{\beta}, \bar{\gamma}$$
$$\sigma \equiv \sigma_\beta, \sigma_\gamma$$

✓ For the study of β fluctuations:

- 11 $\overline{\beta^2}$ values chosen : 0,0.01,0.02…..0.1.
- 11 σ_β^2 values chosen : 0,0.01,0.02…..0.1.
- Default γ=0^0

✓ For the study of γ fluctuations:

- 7 $cos(3\bar{\gamma})$ values chosen : -1,- 0.87, -0.5, 0, 0.5, 0.87, 1
- 7 σ_γ values chosen : 0, π/18, 2π/18, … , 6π/18
- Default β= 0.28

Handling generalized probability distribution:

$$\langle \cos(n\gamma) \rangle = \frac{1}{2}\left(\left\langle e^{in\bar{\gamma}} \right\rangle + \left\langle e^{-in\bar{\gamma}} \right\rangle\right) = \frac{1}{2}\left(\exp\left(\sum_{m=1}^{\infty} \kappa_{m,\gamma} \frac{(in)^m}{m!}\right) + \exp\left(\sum_{m=1}^{\infty} \kappa_{m,\gamma} \frac{(-in)^m}{m!}\right)\right)$$

$$= \exp\left(\sum_{m=1}^{\infty} \kappa_{2m,\gamma} \frac{(-1)^m (n)^{2m}}{2m!}\right)\left[\cos\left(\sum_{m=1}^{\infty} \kappa_{2m+1,\gamma} \frac{(-1)^m (n)^{2m+1}}{(2m+1)!} + n\bar{\gamma}\right)\right]$$

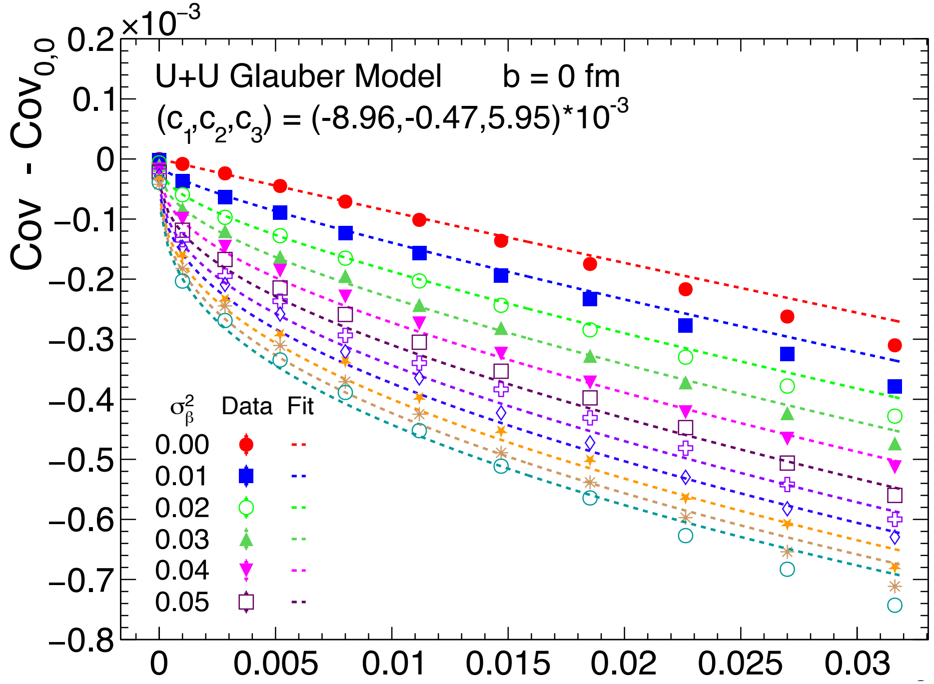
$$\approx e^{-\frac{n^2\sigma_\gamma^2}{2} + \frac{n^4 k_{4,\gamma}}{24}} \cos\left(n\bar{\gamma} + \frac{n^3}{6} k_{3,\gamma}\right) \approx e^{-\frac{n^2\sigma_\gamma^2}{2}} \left[\cos(n\bar{\gamma}) + \sin(n\bar{\gamma}) \frac{n^3}{6} k_{3,\gamma}\right] (1 + \frac{n^4}{24} k_{4,\gamma}).$$

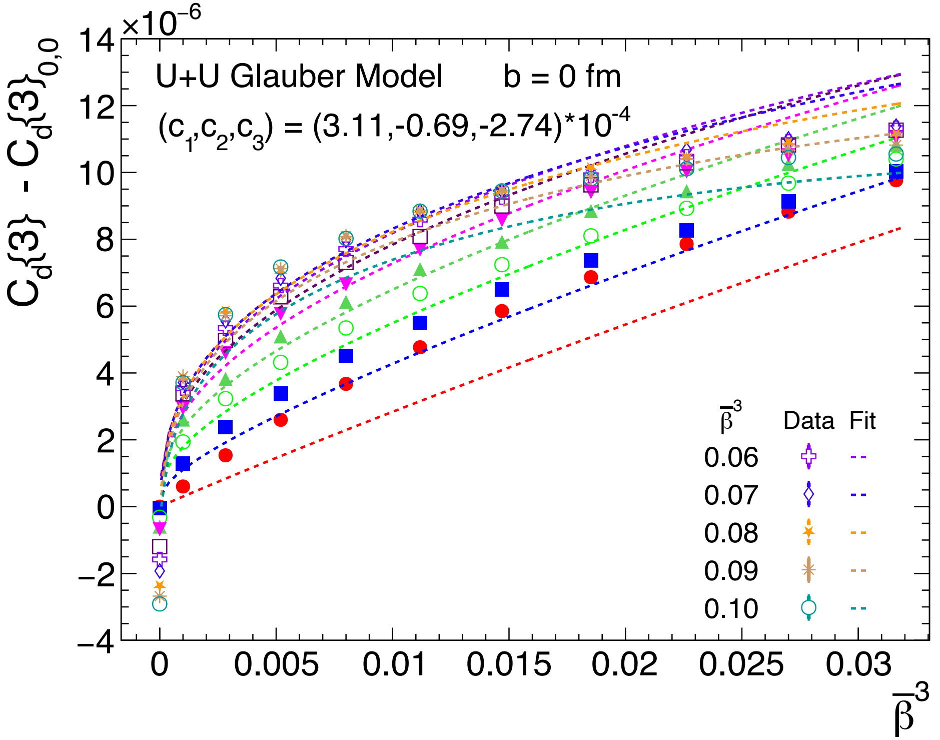
Impact on $\left\langle \varepsilon_2^2 \frac{\delta d_1}{d_\perp} \right\rangle$ and $\left\langle \left(\frac{\delta d_\perp}{d_\perp}\right)^3 \right\rangle$

- Both observables show a definite curve for non-zero σ_β suggesting significant higher order corrections. To account for this behavior two higher order corrections are taken into account.

$$\left\langle \varepsilon_2^2 \frac{\delta d_\perp}{d_\perp} \right\rangle - \left\langle \varepsilon_2^2 \frac{\delta d_\perp}{d_\perp} \right\rangle_{\beta=0} \text{ or } \left\langle \left(\frac{\delta d_\perp}{d_\perp}\right)^3 \right\rangle - \left\langle \left(\frac{\delta d_\perp}{d_\perp}\right)^3 \right\rangle_{\beta=0}$$
$$= c_1\langle \beta^3 \rangle + c_2\langle \beta^4 \rangle + c_3\langle \beta^5 \rangle$$

- Primarily, $\langle \beta^5 \rangle$ component contributes to the overall impact on β fluctuations on both these observables.
- Fit function is not sufficient to describe $\left\langle \left(\frac{\delta d_\perp}{d_\perp}\right)^3 \right\rangle$ in large $\bar{\beta}$ and σ_β.





A possible way of constraining $\bar{\beta}$ and σ_β

- Experimentally, access to both $\langle v_2^2 \rangle \propto \langle \beta^2 \rangle$ and $\langle v_2^4 \rangle \propto \langle \beta^4 \rangle$ is available.

- Can a combination of $\langle \beta^4 \rangle$ and $\langle \beta^2 \rangle$ isolate the value of $\bar{\beta}$? Yes!! But only approximately.

- Consider the following general linear combination

$$f(\bar{\beta}, \sigma_\beta; k) = \langle \epsilon_2^4 \rangle - k\langle \epsilon_2^2 \rangle^2$$

- At k=2.541, $f(\bar{\beta}, \sigma_\beta; k)$ - $f(\bar{0}, 0; k) \propto \overline{\beta^4}$. Determines $\bar{\beta}$ with a precision of 7% in the Glauber model.