Unpolarized gluon PDF for the proton using the twisted mass formulation

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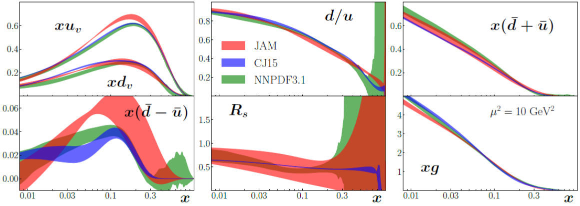
Outline

Motivation

- Gluon contributions to physical quantities play a critical role in hadron structure
- Gluon contributions can be large, eg. gluon momentum fraction $\approx$ 40%
- Dedicated experimental efforts to understand gluonic structure of hadron

[Moffat et al, PRD 104, 016015 (2021)]
[Ball et al, EPJC 77, 663 (2017)]
[Accardi et al, PRD 93, 114017 (2016)]



- Lattice studies of gPDFs can assist in constraining global analysis

gPDFs on the Lattice

- Several challenges in extracting reliable results
  - purely disconnected diagram
  - at least an order of magnitude more statistics than quark counterparts
  - unavoidable mixing with quark singlet PDFs
- *x*-dependence of gluon PDFs even more challenging
- Inverse problem in reconstruction of x-dependence due to limited lattice data

Pseudo-PDF Approach (in a nutshell)

- Matrix elements of **non-local operators** and **momentum-boosted proton states**
  - Several choices for the form of the gluon operator consisting of two field-strength tensors, separated by spatial distance z, and two straight Wilson lines, connecting points $0 \to z$ and $z \to 0$

$$M_{\mu i;\nu j}(P,z) = \langle N(P)|F_{\mu i}(z)W(z,0)F_{\nu j}(0)W(0,z)|N(P)\rangle$$

  - Choice of indices for $F_{\mu\nu}$ not unique
  - This operator avoids finite mixing under renormalization
    - must subtract vacuum expectation value

$$\mathcal{O} = \frac{1}{2}\sum_{i\neq 3} F_{i3}(x+z\hat{z})W(x+z\hat{z},x)F_{i3}(x) - \sum_{i\neq j\neq 3} F_{ij}(x+z\hat{z})W(x+z\hat{z},x)F_{ij}(x)$$

- Matrix elements extracted from ratio of 2pt- and 3pt- functions
- Ground state from plateau fit

$$\frac{C^{3pt}(t,\tau,0,\vec{P})}{C^{2pt}(t,0,\vec{P})} \overset{0<<\tau<<t}{=} \mathcal{M}(\nu,z^2)$$

Pseudo-PDF Approach (in a nutshell)

- $\mathfrak{M}$ - Euclidean, renormalized in ratio scheme, scale $1/z$
  - want: Minkowski, renormalized in $\overline{\mathrm{MS}}$, scale μ
  - we need matching to the light-cone, scheme conversion and evolution
- Form the double ratio (reduced ITD) with zero-momentum and local matrix elements for renormalization and to reduce higher twist contributions [Orginos et al., Phys.Rev.D 96 (2017) 9, 094503]

$$\mathfrak{M}(\nu, z^2) \equiv \left(\frac{M(\nu, z^2)}{M(\nu, 0)|_{z=0}} \right) \Big/ \left(\frac{M(0, z^2)|_{p=0}}{M(0, 0)|_{p=0,z=0}} \right)$$

- Scale evolution and apply matching kernel on ITD
  - neglect mixing with quark singlet
  - normalize with $\langle x \rangle_g$

$$\mathcal{Q}(\nu, z^2, \mu^2) = \mathfrak{M} + \frac{\alpha_s N_c}{2\pi} \int_0^1 du\, \mathfrak{M}(u\nu, z^2) \left\{ \ln\left(\frac{z^2 \mu^2 e^{2\gamma_E}}{4} \right) B(u) + 4\left[\frac{u + \ln(\bar{u})}{\bar{u}} \right]_+ + \frac{2}{3}\left[1 - u^3\right]_+ \right\}$$

- Reconstruct x-dependence (Backus-Gilbert, fitting ansatz, Fourier tansform, etc.)

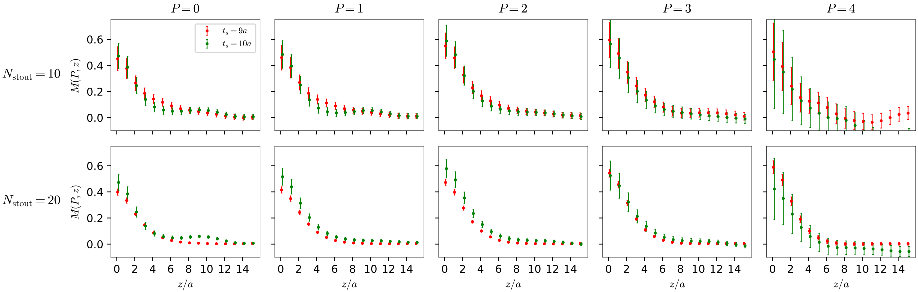
$$\mathcal{Q}(\nu, z^2, \mu^2) = \int_0^1 dx\, cos(x\nu)\, x\, g(x, \mu^2)$$

Lattice Parameters and Statistics

- N_f=2+1+1 ensemble of twisted-mass clover fermions and Iwasaki improved gluons
  - $m_\pi = 260$ MeV
  - $a = 0.09471(39)$ fm
  - $L^3 \times T = 32^3 \times 64$
  - $Lm_\pi = 4$
- Stout smearing ($\omega = 0.129$)
  - field-strength tensor: 10, 20 steps
  - Wilson line: 0, 10 steps
- Momentum smearing (optimized value $\xi = 0.6$) used for $P = 2, 3, 4$ [Bali et al, PRD 93, 094515 (2016)]
- Excited states:
  - Numerical results relatively good up to $t_s = 10a$
- Statistics
  - Average over all 6 spatial directions of Wilson line / momentum ($\pm x$, $\pm y$, $\pm z$)
  - Statistics much higher than quark PDFs

$\|\mathbf{P_3}\|$ $[\frac{2\pi}{L}]$	$\|\mathbf{P_3}\|$ [GeV]	$N_{\rm confs}$	$N_{\rm src}$	N_{dir}	Total statistics
0 to 4	0 - 1.67	1,134	200	6	1,360,800

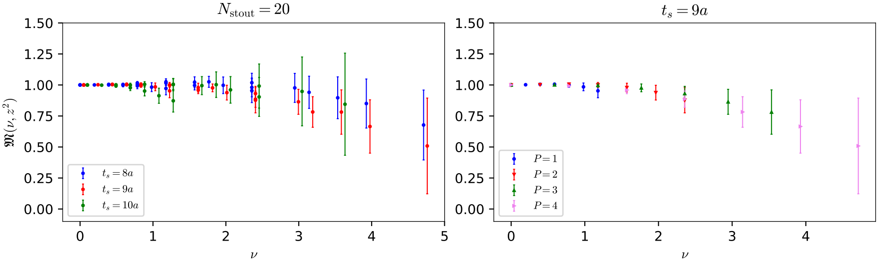
Matrix Elements: Excited States Contamination and Effect of N_{stout}



- Various values of t_s and two stout steps for the gluon operator
- Statistical errors increase with momentum boost and t_s
- MEs have expected behavior (higher boosts decay faster to 0)
- Final results use $N_{\mathrm{stout}} = 20$

Double Ratio (Reduced ITD)

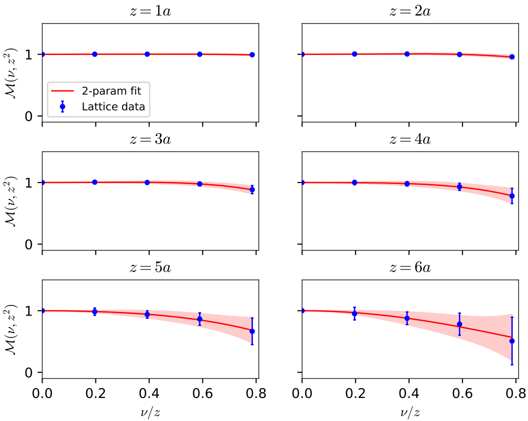
- Double-ratio analysis to narrow down t_s



- Values at different t_s are compatible within uncertainties
- Final results use $t_s = 9a$
  - $z_{max} = 6a = 0.568$ fm motivated by signal
- Lattice data form a smooth function
  - Must interpolate for evolution and matching

Interpolation of Double Ratio

- We interpolate the double-ratio at each z to get a continuous function for the integration
  - interpolation done with linear and 2nd-order polynomial fits

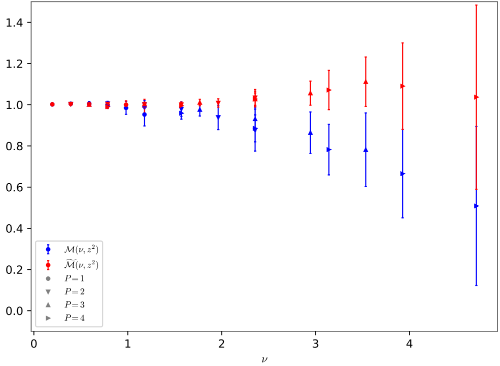


- 2nd-order polynomial fits prove to be the most suitable for evolution and matching
- Choice of fit mostly irrelevant below $z = 4a$

ITD Development

- Apply the evolution kernel to the reduced matrix elements to the scale $\mu = 2$ GeV ahead of final conversion to $\overline{\mathrm{MS}}$ scheme and matching to the light-cone

$$\tilde{\mathfrak{M}}(\nu, z^2, \mu^2) = \mathfrak{M} + \frac{\alpha_s N_c}{2\pi} \int_0^1 du \ln\left(\frac{z^2 \mu^2 e^{2\gamma_E}}{4}\right) B(u) \mathfrak{M}(u\nu, z^2)$$

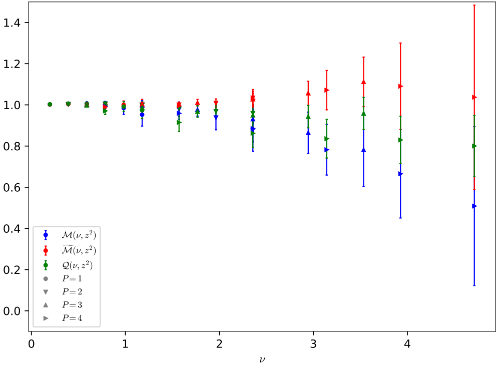


- Data from different (P, z) pairs fall on a universal curve
- We find good agreement up to $z = 6a$

ITD Development

- Apply the matching kernel to convert to $\overline{\mathrm{MS}}$ scheme and match to the light-cone

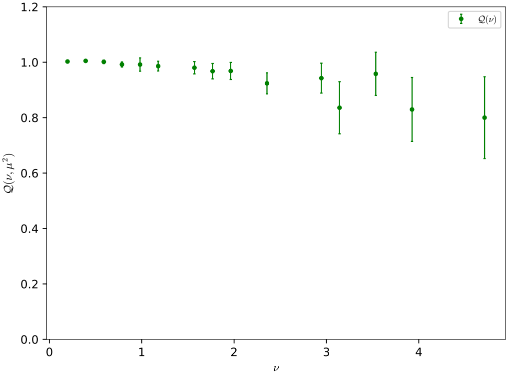
$$\mathcal{Q}(\nu, z^2, \mu^2) = \tilde{\mathfrak{M}}(\nu, z^2, \mu^2) + \frac{\alpha_s N_c}{2\pi} \int_0^1 du\, L(u) \mathfrak{M}(u\nu, z^2)$$



- We continue to find good agreement between common values of Ioffe time from different combinations of momenta and Wilson line lengths
- Matching effects in opposite direction of evolution

ITD Development

- Average over common ν for final pseudo-ITD

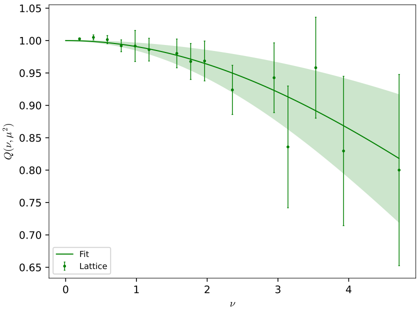


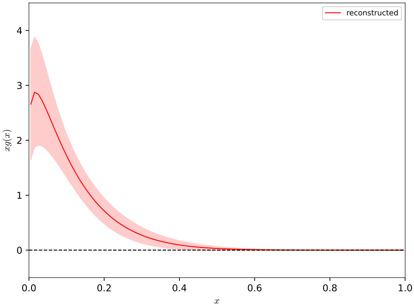
- No information remains regarding initial (P, z) pairs

PDF Reconstruction

- The fit is chosen by the minimization of

$$\chi^2 = \sum_{\nu=0}^{\nu_{max}} \frac{\left(Q(\nu,\mu^2) - \int_0^1 dx \cos(\nu x)\, N\, x^a (1-x)^b\right)^2}{\sigma_Q^2(\nu,\mu^2)}$$

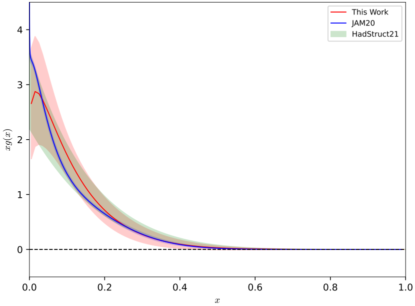




- PDF is normalized using gluon momentum fraction $\langle x\rangle_g^{\overline{\mathrm{MS}}}(\mu{=}2\,\mathrm{GeV}){=}0.427(92)$ [Alexandrou et al, PRD 101, 094513 (2020)]
- Other reconstruction methods (naive Fourier-transform, Backus-Gilbert method) have proven less suitable [Bhat et al, PRD 103, 034510 (2021)]

Comparison with Other Works

- Comparison with lattice results from HadStruc collaboration [Khan et al, PRD 104, 094516 (2021)]
  - $m_\pi = 358$ MeV, $a = 0.094$, $L^3 \times T = 32^3 \times 64$
- JAM20 global analysis [Moffat et al, PRD 104, 016015 (2021)], $\langle x \rangle_g = 0.40(1)$

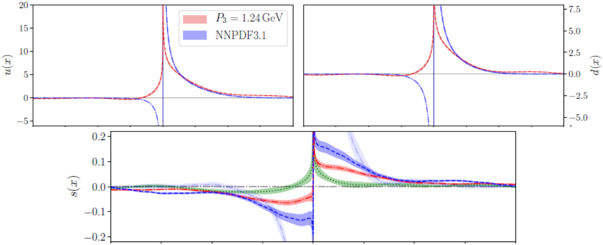


- We find agreement between all results
- This work: $\nu_{max} = 4.71$
- HadStruc: $\nu_{max} = 7.07$

Future Work

- Increasing calculation precision
- Increasing the range of accessed Ioffe times
- Addressing systematic effects
- Investigation of mixing with quark singlet PDFs

[Alexandrou et al, PRD 104, 054503 (2021)]



- Calculations at physical point

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