Heavy quark diffusion coefficient from lattice QCD

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Based on: D. Banerjee, S. Datta, M. Laine, JHEP 08('22) 128 (2204.14075),
D. Banerjee, S. Datta, R. Gavai, P. Majumdar, 2206.14571

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Heavy-light mesons in deconfined plasma

- R_{AA} and v_2 of heavy mesons, in particular D mesons indicate thermalization of charm: small $t_{\rm kin}$

  ALICE, JHEP 01 (2022) 174

  Dong, Lee & Rapp, Ann. Rev. Nucl. Part. Sc. 69 (2019) 417

- For thermal heavy quark, $M \gg T, \quad p \gtrsim \sqrt{MT}$
- Even in hard collisions momentum transfer $\sim T$:
  Takes $\mathcal{O}(M/T)$ hard collisions to change momentum by $\mathcal{O}(1)$
  Thermal scattering: uncorrelated momentum kicks.
- A Langevin formalism can be written.

$$\frac{dp_i}{dt} = \xi_i(t) - a(p)p_i, \qquad \langle \xi_i(t)\,\xi_j(t')\rangle = b_{ij}(p)\,\delta(t-t')$$

- Leads to the Fokker-Planck equation

$$\frac{\partial f_Q(p,t)}{\partial t} = -\frac{\partial}{\partial p_i}\left[p_i\, a(p)\, f_Q(p,t)\right] + \frac{\partial^2}{\partial p_i\,\partial p_j}\left[b_{ij}(p)\, f_Q(p,t)\right]$$

Svetitsky '88; Mustafa, Pal, Srivastava, '97; Moore & Teaney '05; Rapp & van Hees '05;

Diffusion coefficient

- At small p, the **drag** and **momentum diffusion coefficients**

$$a(p) \to \eta_D, \quad B_{ij}(p) \to \kappa$$

- The thermal distribution is the stable fixed point of FP equation

$$\to \eta_D = \frac{\kappa}{2\,M\,T}$$

- Langevin equation leads to diffusion in position space,

$$\langle x^2(t) \rangle = 6 D_s t, \qquad D_s = \frac{2T^2}{\kappa}$$

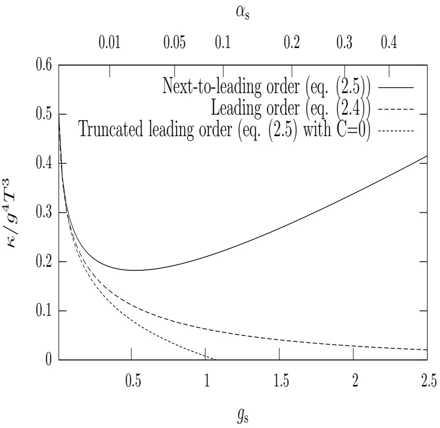
- One can calculate κ from the $2 \to 2$ collision processes.

  Svetitsky '88, Moore & Teaney 2005, ...

- Leads to a value of κ substantially smaller than required to interpret the R_{AA} and v_2 results.

κ in kinetic theory

NLO corrections very large.



Caron-Huot & Moore, PRL 100 (2008) 052301

Calculation of κ

- For phenomenological studies, one often tweaks the LO kinetic theory calculation to get κ in the right ballpark.

  See, e.g., Cao et al., PRC 99 (2019) 054907

- In field theory **Kubo relation** connects spatial diffusion to spectral function of number current $J_i = Q \gamma_i Q$.

  Kapusta & Gale's book

- Very difficult to extract D_s: the diffusion part leads to a near-flat contribution to the correlator.

  Umeda, 2007; Teaney & Petreczky, 2007

- Moments of correlator can be looked at to enhance the diffusive part.

  H-T. Ding, et al., PRD 104 (2021) 114508

Momentum diffusion coefficient and force-force correlator

- An alternate approach, more directly associated with the Langevin formalism, has been more promising.
- The momentum diffusion coefficient,

$$\kappa = \frac{1}{3\chi} \lim_{\omega \to 0} \int_{-\infty}^{\infty} dt\, e^{i\omega t} \int d^3x \left\langle \frac{1}{2} \{F_i(t,x)\,,\, F_i(0,0)\} \right\rangle$$

J. Casalderrey-Solana & D. Teaney, PRD 74 (2006) 085012;
S. Caron-Huot, M. Laine & G. Moore, JHEP 04 (2009) 53;
A. Bouttefeux & M. Laine, JHEP 12 (2020) 150

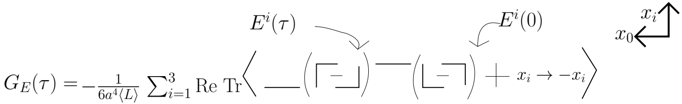
- The force term can be expanded in a series in $1/M$:

$$F^i = M\frac{dJ^i}{dt} = \phi^\dagger \left\{ -gE^i + \frac{[D^i, D^2 + g\sigma\cdot B]}{2M} + \frac{g\,[D_0, \sigma\times E]^i}{4M} + \ldots \right\} \phi$$

- Nonperturbatively, we can calculate the Matsubara correlator. This can be connected to the above correlator using standard relations.

$m_Q \to \infty$ limit

- In the static limit, we get the electric field correlator.
- Discretizing $gE_i = [D_i, D_0]$, one gets the correlator



- Only a finite renormalization $Z_E(a)$ required to connect the lattice discretized E to continuum E. No other renormalization factor required.
- $Z_E(a)$ has been evaluated in perturbation theory.

C. Christensen & M. Laine, PLB 755 (2016) 316.

- One can connect the correlator to the spectral function

$$G_E(\tau) = \int_0^\infty \frac{d\omega}{\pi} \rho_{EE}(\omega) \frac{\cosh \omega(\tau - 1/2T)}{\sinh \omega/2T}$$

κ_E from lattice

- Then κ can be extracted from the infrared behavior of $\rho_{EE}(\omega)$:

$$\rho_{IR} \underset{\omega \to 0}{\approx} \frac{\kappa\,\omega}{2T}$$

- The EE correlator has been investigated for a gluonic plasma by multiple groups, and κ_E extracted.

  Banerjee, Datta, Gavai, Majumdar, PRD 85 (2012) 014510.
  Francis, Kaczmarek, Laine, Neuhaus, Ohno, PRD 92 (2015) 116003.
  Brambilla, Leino, Petreczky, Vairo, PRD 102 (2020) 074503.
  Altenkort, Eller, Kaczmarek, Mazur, Moore, Shu, PRD 103 (2021) 014511.
  Brambilla, Leino, Mayer-Steudte, Petreczky, arXiv:2206.02861.
  Banerjee, Datta, Gavai, Majumdar, arXiv:2206.15471.
- We have used the multilevel method to enhance the signal, and used the perturbative renormalization.
- Besides, Gradient flow has been used by others for both opertor renormalization and signal enhancement.
- Structure of spectral function not complicated.
- Techniques used: modelling of the ultraviolet, Brackus-Gilbert inversion, Bayesian techniques (MEM), ...

$1/m_Q$ correction

- The $1/m_Q$ correction has only recently been estimated.
- Under certain assumptions, (Bouttefeux & Laine, JHEP 12 ('20) 150)

$$\kappa_Q \approx \kappa_E + \frac{2}{3} \langle v^2 \rangle \, c_B(\mu) \, \kappa_B \,, \qquad \langle \gamma v^2 \rangle = \frac{3T}{M_{\rm kin}}$$

  where κ_B is the equivalent of κ_E from the $B - B$ correlator.
- The BB correlator has an anomalous dimension. This cancels with the scale dependence of c_B, to give a physical result.
- This becomes equivalent to evaluating $Z_B(\mu = 19.2\,T)$.

  M. Laine, JHEP 06 ('21) 139

- We used the clover discretization of the B field.
- For this discretization, the matching to the $\overline{MS}$ operator can be done nonperturbatively using the RGI matching constant ϕ_{RGI} calculated by the ALPHA collaboration.

  Guazzini, Meyer & Sommer, JHEP 10 (2007) 081.

$1/m_Q$ correction

- Then we get to $Z_{\overline{MS}}(\mu = 19.2T)$ by integrating the RGE:

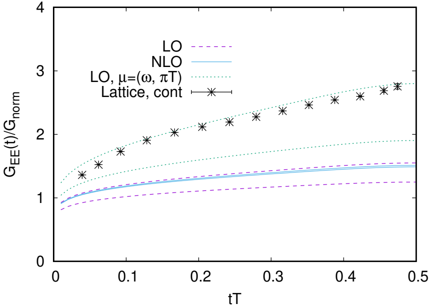
$$\mu \frac{d}{d\mu} \left[\frac{\phi_{RGI}}{\phi_{\overline{MS}}(\mu)} \right] = -\gamma(g) \left[\frac{\phi_{RGI}}{\phi_{\overline{MS}}(\mu)} \right]$$

Banerjee, Datta & Laine, JHEP 08 ('22) 128

- $\langle v^2 \rangle$ is obtained from the constant part of $\frac{\langle J^i J^i \rangle}{\langle J^0 J^0 \rangle}$.
- We evaluate them from the NLO calculation of the vector correlator.

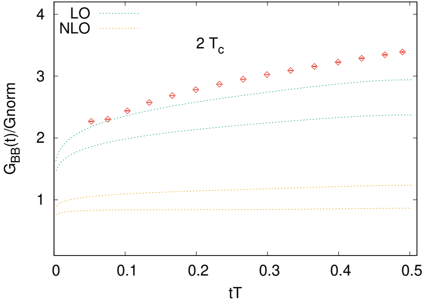
Burnier & Laine, JHEP 11 ('12) 086

Perturbative result



EE correlator at 3 T_c vs PT

Banerjee, Datta, Gavai & Majumdar, arXiv:2206.15471
Pert.: Burnier, Laine, Langelage, MEHER, JHEP 08('10) 094



BB correlator at 2 T_c vs PT

$\mu = (\omega^{1-\gamma_0/b_0} (\pi T)^{\gamma_0/b_0}, \ \pi T)$

Banerjee, Datta & Laine, 2204.14075

Both figures normalized by corresponding LO shape: $G_{\rm norm} = \frac{G_{LO}(\tau)}{g_0^2 \, C_f}$

$\rho(\omega)$ and κ_E

- $\rho_{IR}(\omega) \equiv \frac{\kappa\,\omega}{2T}$, $\qquad \rho_{UV}(\omega) \equiv \frac{g^2(\mu) C_f \omega^3}{6\pi}$, $\qquad \mu \equiv (\omega,\, \pi T)$
- Model $\rho(\omega)$ as

  $\rho_1(\omega) \equiv \max\left(\rho_{IR}(\omega),\; c\rho_{UV}(\omega)\right)$
- Physically better motivated: Francis et al. '15

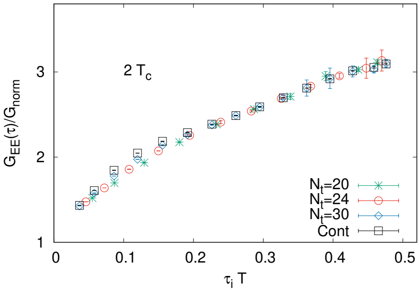
  $\rho_2(\omega) \equiv \sqrt{(\rho_{IR}(\omega))^2 + (c\rho_{UV}(\omega))^2}$
- Also tried adding a Fourier mode (Francis et al. '15)

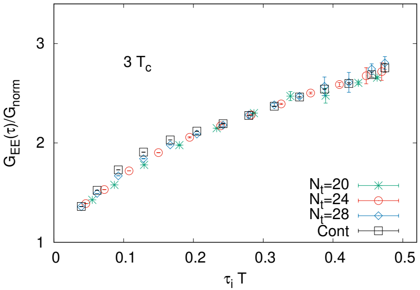
$$\rho_{1f,2f}(\omega) \equiv (1 + d \sin \pi y)\; \rho_{1,2}^{c=1}(\omega), \qquad y = \frac{\log\left(1 + \frac{\omega}{\pi T}\right)}{1 + \log\left(1 + \frac{\omega}{\pi T}\right)}$$

- At each temperature, calculate the correlator at three lattice spacings and take continuum limit.
- took a superset of the values obtained from different forms.
- Also looked at the fits for individual lattices.

EE correlator

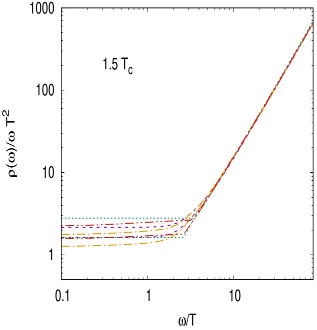
For gluonic plasma, careful continuum limit of the correlator taken and κ_E calculated.

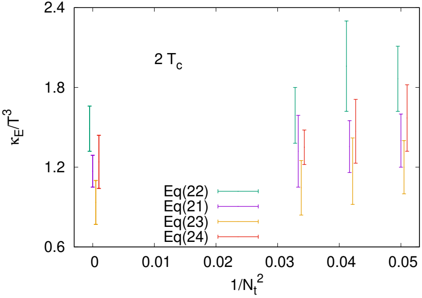




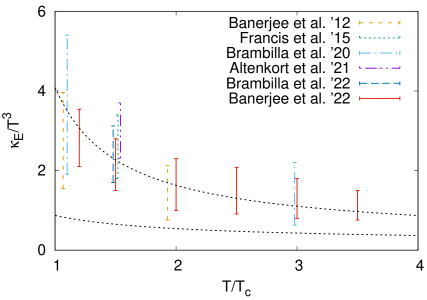
Banerjee, Datta, Gavai, Majumdar, arXiv:2206.14571

κ_E


1000
100
10
1
0.1
1
10
1.5 T_c
ρ(ω)/ω T^2
ω/T

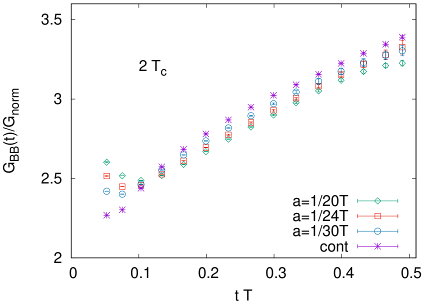

2.4
1.8
1.2
0.6
0
0.01
0.02
0.03
0.04
0.05
2 T_c
κ_E/T^3
1/N_t^2
Eq(22)
Eq(21)
Eq(23)
Eq(24)

κ_E from lattice and NLO PT



NLO pert. theory from Caron-Huot & Moore, PRL100,052301.

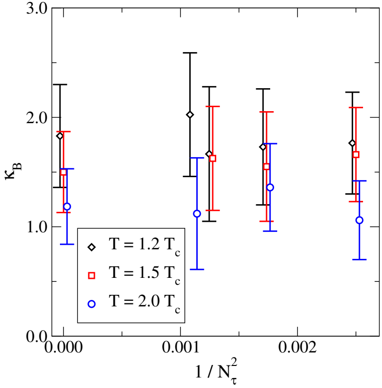
BB correlator from lattice

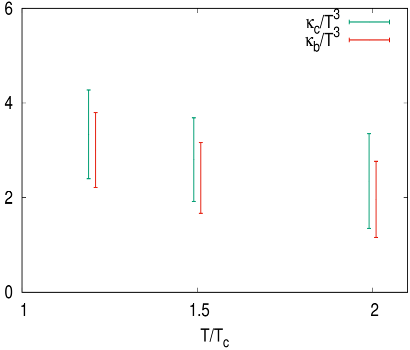


Banerjee, Datta, Laine, JHEP 08 ('22) 128

Unlike the EE correlator, the BB correlator has anomalous dimension, and renormalization is nontrivial.
The renormalization factor we get is large, $Z^2 \approx 3$ for much of our data set.

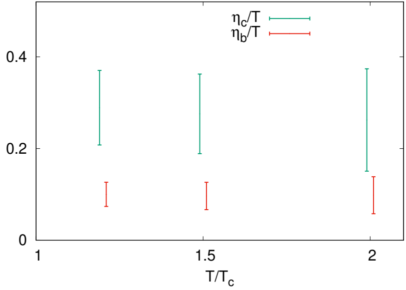
κ_B from BB correlator





Banerjee, Datta, Laine, JHEP08('22)128 (2204.14075)
(see also Brambilla, et al., arXiv:2206.02861)

$\eta_{c,b}$



Indicates $\tau_b \approx 3\tau_c \gtrsim 10 fm$
Also $2\pi D_s T_c \sim 2.4 - 5.8$ for charm and $\sim 2.8 - 6.5$ for bottom.

Banerjee, Datta, Laine, JHEP 08 (2022) 128 (2204.14075)

Summary

- For low p_T heavy quark interactions with QGP can be studied using a Fokker-Planck equation.
- Lattice QCD provides an important ingredient: QCD estimate of the momentum diffusion coefficient.
- Current results are for quenched QCD. However, they give confidence in Langevin description of heavy quark interactions with plasma.
- Results for κ_b and κ_c are available for the gluonic plasma.
- Relaxation time short for charm but larger for the bottom, indicating incomplete thermalization for bottom.
- First studies in full QCD: substantial unquenching effect.

Altenkort's talk