Dispersive analysis of the $\gamma\gamma \rightarrow D\bar{D}$ data and the confirmation of the $D\bar{D}$ bound state

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in coll. with Oleksandra Deineka, Marc Vanderhaeghen

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New states



Date of arXiv submission

+ data from Babar, Belle, COMPASS, ...

What are we looking for?



PDG 2021

Everything is fine with $\chi_{c2}(2P)$



What are we looking for?

PDG 2021

is X(3915) a $\chi_{c0}(2P)$?

[Belle 2005] $B \rightarrow J/\psi \omega K$: X(3915): later confirmed by [BaBar 2008, 2010]

[Belle 2010] $\gamma \gamma \rightarrow X(3915) \rightarrow J/\psi \omega$ [BaBar 2012] spin-parity analysis : $J^{PC} = 0^{++}$ (assuming helicity-2 dominance of tensor resonance)

Problems: [Brambilla et al. 2011, Olsen 2015, ...]

- No decay mode to DD (S-wave) was observed
- The $X(3915) \rightarrow J/\psi\omega$ decay should be OZI suppressed
- Narrow, width of ~20 MeV
- Small mass splitting with $\chi_{c2}(3930)$
- Might actually be the same state as $\chi_{c2}(3930)$ [Zhou et al. 2015]

[LHCb 2021] $B^+ \rightarrow D^+ D^- K^+$

found narrow $J^{PC} = 0^{++}$ resonance around ~3.92 GeV (amplitude analysis)

$X(3915) ? \chi_{c0}(3915)$	$\chi_{c0}(3915)$	$I^G(J^{PC}) = 0^+(0^{++})$	PDG 2022
	$\chi_{c0}(3915)$ MASS		$3921.7\pm1.8~{ m MeV}$
	$\chi_{c0}(3915)$ WIDTH		$18.8\pm3.5~{ m MeV}$

Another possibility for $\chi_{c0}(2P)$?

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[Guo Meißner 2010]

two Breit-Wigner functions: mass and width of $\chi_{c2}(3930)$ is fixed, and $\chi_{c0}(2P)$ is fitted

$$B_{L}(s) = \left(\frac{p(s)}{p(m_{R}^{2})}\right)^{2L+1} \frac{m_{R}}{\sqrt{s}} \frac{F_{L}^{2}(s)}{(s-m_{R}^{2})^{2} + m_{R}^{2} \Gamma^{2}(s)}, \quad \Gamma(s) = \Gamma_{R} \left(\frac{p(s)}{p(m_{R}^{2})}\right)^{2L+1} \frac{m_{R}}{\sqrt{s}} F_{L}^{2}(s)$$
$$M_{\chi_{c0}(2P)} = 3837.6 \pm 11.5 \,\text{MeV}, \quad \Gamma_{\chi_{c0}(2P)} = 221 \pm 19 \,\text{MeV}$$

is X(3860) a $\chi_{c0}(2\overline{P})?$

Events / $(\pi/5)$ 20 [Guo Meißner 2010] Breit-Wigner analysis of $\gamma \gamma \rightarrow D\bar{D}$: found $\chi_{c0}(3860)$ \bigcirc 18 16 [Wang et al. 2021, Gemerman et al. 2007] Unitary approach $(B_{\pm}^{14}E)$ found that $\gamma\gamma \rightarrow D\bar{D}$ & 14 0 12 $e^+e^- \rightarrow J/\psi D\bar{D}$ data do not contradict dynamics with the encoded **bound state**: 10 no broad $\chi_{c0}(3860)$ is needed 8 6 4 2 2 0___3 0_1

-0.5

0

0.5

 $\cos \theta_{x}$

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3) $D_s \bar{D}_s$ quasi-bound state (i.e. bound state if $D\bar{D}$ is off) which might be $\chi_{c0}(3915)$ or X(3960)

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In order to figure out what is going on with 0^{++} we employ a data driven dispersive analysis of $\gamma\gamma \rightarrow D\bar{D}$ and $e^+e^- \rightarrow J/\psi D\bar{D}$

Formalism

• Full **p.w. dispersion relation** (causality, crossing, unitarity)

$$t_{ab}(s) = \int_{-\infty}^{s_L} \frac{ds'}{\pi} \frac{\text{Im } t_{ab}(s')}{s' - s} + \int_{s_{th}}^{\infty} \frac{ds'}{\pi} \frac{\text{Im } t_{ab}(s')}{s' - s}$$

$$\lim t_{ab}(s) = \sum_{c} t_{ac}(s) \rho_{c}(s) t_{cb}^{*}(s) - \frac{1}{2\rho_{1}} \le \operatorname{Re} t_{11}(s) \le \frac{1}{2\rho_{1}}, \quad 0 < \operatorname{Im} t_{11}(s) \le \frac{1}{\rho_{1}}, \quad \dots$$

• Assuming $t(\infty) \rightarrow const$ we subtract the dispersion relation once

$$t_{ab}(s) = t_{ab}(0) + \frac{s}{\pi} \int_{-\infty}^{s_L} \frac{ds'}{s'} \frac{\operatorname{Im} t_{ab}(s')}{s' - s} + \frac{s}{\pi} \sum_{c} \int_{s_{th}}^{\infty} \frac{ds'}{s'} \frac{t_{ac}(s') \rho_c(s') t_{cb}^*(s')}{s' - s}$$

$$\underbrace{U_{ab}(s)} \qquad \text{(asymptotically bounded unknown function)}$$

• Once-subtracted p.w. dispersion relation

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$$\underbrace{U_{ab}(s)}$$

can be solved using N/D method with input from $U_{ab}(s)$ above threshold

Chew, Mandelstam (1960) Luming (1964) Johnson, Warnock (1981)

$$t_{ab}(s) = \sum_{c} D_{ac}^{-1} N_{cb}(s)$$

$$N_{ab}(s) = U_{ab}(s) + \frac{s}{\pi} \sum_{c} \int_{s_{th}}^{\infty} \frac{ds'}{s'} \frac{N_{ac}(s') \rho_{c}(s') (U_{cb}(s') - U_{cb}(s))}{s' - s}$$

$$D_{ab}(s) = \delta_{ab} - \frac{s}{\pi} \int_{s_{th}}^{\infty} \frac{ds'}{s'} \frac{N_{ab}(s') \rho_{b}(s')}{s' - s}$$
the obtained N/D solution

the obtained N/D solution can be checked that it **fulfils** the p.w. dispersion relation

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Bound state case

$$\det(D_{ab}(s_B)) = 0$$

$$t_{ab}(s) = U_{ab}(s) + \frac{s}{s_B} \frac{g_a g_b}{s_B - s} + \frac{s}{\pi} \sum_c \int_{s_{th}}^{\infty} \frac{ds'}{s'} \frac{t_{ac}(s') \rho_c(s') t_{cb}^*(s')}{s' - s}$$

• I = 0 has $\{\gamma\gamma, \pi\pi, K\bar{K}, \dots, D\bar{D}, \dots\}$ channels, but the coupling of charmed $\{D\bar{D}, \dots\}$ with uncharmed $\{\pi\pi, K\bar{K}, \dots\}$ are strongly suppressed: separately focus on $\{\gamma\gamma, D\bar{D}, \dots\}$ and $\{\gamma\gamma, \pi\pi, K\bar{K}, \dots\}$

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- Neglect $\gamma\gamma$ intermediate states in the unitary relation => coupled-channel { $\gamma\gamma$, $D\bar{D}$,...} equations reduce to the **hadronic part** and **photon-fusion part** Focus on energies $4m_D^2 < \sqrt{s} < 4.0 \text{ GeV}$ ($\approx 4m_{D_s}^2$)

$$t_{DD}(s) = U_{DD}(s) + \frac{s}{\pi} \int_{4m_D^2}^{\infty} \frac{ds'}{s'} \frac{t_{DD}(s') \rho_D(s') t_{DD}^*(s')}{s' - s} = \frac{N_{DD}(s)}{D_{DD}(s)}$$
$$t_{\gamma\gamma,DD}(s) = U_{\gamma\gamma,DD}(s) + D_{DD}^{-1}(s) \left(-\frac{s}{\pi} \int_{4m_D^2}^{\infty} \frac{ds'}{s'} \frac{\operatorname{Im} D_{DD}(s') U_{\gamma\gamma,DD}(s')}{s' - s} \right)$$

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$$U_{DD}(s) \simeq \sum_{n=0}^{\infty} C_{p,c}^{n}(s)$$

$$U_{DD}(s) \simeq Born$$

$$U_{\gamma\gamma,DD}(s) \approx Born$$

$$t_{\gamma\gamma,DD}(s = 0) = Born$$

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How good is Born left-hand cut for $\{\gamma\gamma, \pi\pi, K\bar{K}\}$?

Analysis of $\gamma \gamma \to D^+ D^-$ and $\gamma \gamma \to D^0 \overline{D}^0$ data

Analysis of $\gamma \gamma \rightarrow D^+ D^-$ and $\gamma \gamma \rightarrow D^0 \overline{D}^0$ data

S-wave: I = 0 Born with dispersive rescattering, I = 1 only Born D-wave: $\chi_{c2}(3930)$ is a Breit-Wigner hel-2 with PDG mass/width \rightarrow 1 normalisation ratio (S/D wave)

 $\sigma_c(s), \sigma_n(s)$

fix hadronic part

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 $\sigma_c(s), \sigma_n(s)$

 $\sigma_c(s) + \sigma_n(s)$

1) Consider again { $\pi\pi, K\bar{K}$ } coupled channel system $\sqrt{s_{f_0}} = 993 - i\,21$ MeV

2) Switch off $\pi\pi$ channel $\implies f_0(980)$ becomes a $K\bar{K}$ bound state with $\sqrt{s_B} = 961$ MeV

1) Consider again { $\pi\pi, K\bar{K}$ } coupled channel system $\sqrt{s_{f_0}} = 993 - i\,21$ MeV

Suppression of K^+K^- channel compared to $K^0\bar{K}^0$ is also seen in the full amplitude analysis of [Dai Pennington 2014]

future BESIII data

Prediction for $\gamma\gamma \rightarrow \pi\pi$ cross section for unphysical m_{π}

in the case of a **bound state**: charged channel is suppressed compared to neutral

$\gamma \gamma \rightarrow D\bar{D}$ angular distribution

Angular distribution is mainly D-wave: however, one cannot exclude an additional small S-wave contribution from $\chi_{c0}(3915)$ $e^+e^- \rightarrow J/\psi D\bar{D}$ data

 $e^+e^- \rightarrow J/\psi D\bar{D}$ data

\square Dispersive analysis of the $\gamma\gamma \rightarrow D^+D^-$, $D^0\bar{D}^0$ data, consistency check with the $e^+e^- \rightarrow J/\psi D\bar{D}$ data

 \mathbf{V} No broad resonance corresponding to *X*(3860) found

 \mathbf{V} Bound state below the $D\bar{D}$ threshold, $\sim D\bar{D}$ molecule

Δ More data is needed: Belle II $\gamma\gamma \rightarrow D\bar{D}$, BESIII decay $\psi(3770) \rightarrow D\bar{D}\gamma$, PANDA...

Dispersion theory can be applied to many other exciting processes

 \square Light: $\gamma \gamma \rightarrow \pi^0 \eta$, $K\bar{K}$; $(g-2)_{\mu}$ HLBL contributions

 $\Box \{J/\psi J/\psi, J/\psi \psi(2S)\}$ scattering LHCb

Thank you!

Extra slides

How good is Born left-hand cut for $\{\gamma\gamma, \pi\pi, K\bar{K}\}$?

Input: experimental data/Roy analysis + threshold parameters NNLO + Adler zero NLO

	Our results		Roy-like analyses	
	pole position, MeV	couplings, GeV	pole position, MeV	couplings, GeV
$\sigma/f_0(500)$	$458(10)^{+7}_{-15} - i256(9)^{+5}_{-8}$	$\gamma\gamma: 5.6(1)(1)\cdot 10^{-3}$	$449_{-16}^{+22} - i275(15)$	$\gamma\gamma: 6.1(7)\cdot 10^{-3}$
		$\pi\pi: 3.33(8)^{+0.12}_{-0.20}$		$\pi\pi: 3.45^{+0.25}_{-0.29}$
		$K\bar{K}: 2.11(17)^{+0.27}_{-0.11}$		$K\bar{K}:-$
$f_0(980) \qquad 993(2)^{+2}_{-1} - i22$		$\gamma\gamma: 4.0(8)^{+0.3}_{-1.1} \cdot 10^{-3}$	996 ⁺⁷ ₋₁₄ - $i 25^{+11}_{-6}$	$\gamma\gamma: 3.8(1.4)\cdot 10^{-3}$
	$993(2)_{-1}^{+2} - i21(3)_{-4}^{+2}$	$\pi\pi: 1.93(15)^{+0.07}_{-0.12}$		$\pi\pi: 2.3(2)$
		$K\bar{K}: 5.31(24)^{+0.04}_{-0.24}$		$K\bar{K}:-$

I.D, Deineka, Vanderhaeghen (2020)

Caprini et al. (2006), Garcia-Martin et al. (2011) Moussallam (2011), Dai Pennington (2016)

Convergence of conformal expansion

Good convergence with 3 parameters in conformal mapping expansion \implies there is no need for more parameters

Analysis of $\gamma \gamma \rightarrow D^+ D^-$ and $\gamma \gamma \rightarrow D^0 \overline{D}^0$ data

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