

Probing the Equation of State Through Heavy-Ion Collisions and Simulations

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Workshop on Dense Nuclear Matter Equation
of State from Heavy-Ion Collisions

Institute for Nuclear Theory

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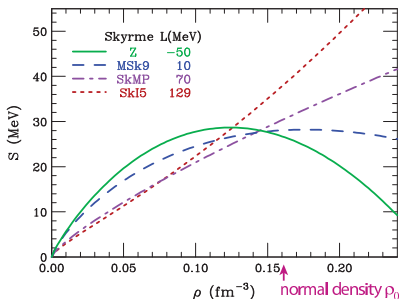
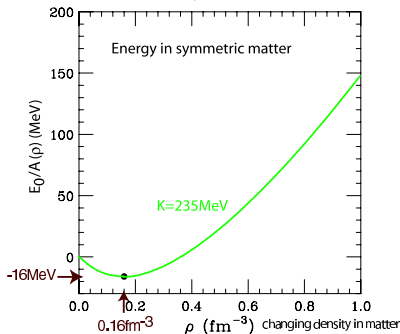
Cold EOS Breakdown

$$\frac{E}{A}(\rho_n, \rho_p) = \frac{E_0}{A}(\rho) + S(\rho) \left(\frac{\rho_n - \rho_p}{\rho} \right)^2 + \mathcal{O}(\dots^4)$$

symmetric matter

(a)symmetry energy

$$\rho = \rho_n + \rho_p$$



$$\frac{E_0}{A}(\rho) = -B + \frac{K}{18} \left(\frac{\rho - \rho_0}{\rho_0} \right)^2 + \dots$$

Known: $B \approx 16\text{MeV}$ $K \sim 235\text{MeV}$

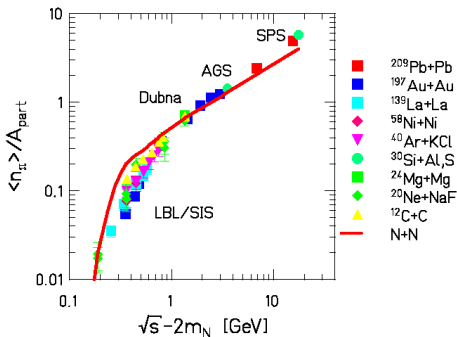
$$S(\rho) = S_0 + \frac{L}{3} \frac{\rho - \rho_0}{\rho_0} + \dots$$

Unknown: $S_0?$ $L?$ 

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Heavy-Ion Collisions

- Equilibrium reached only late in collisions, so transport theory is needed to simulate the collisions and extrapolate to equilibrium
- Collision energy changes densities reached, but also excitation above zero temperature and degrees of freedom



Senger, ProgPartNuclPhys 53(04)1



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Transport Theory

Phase-space characteristics of hadronic degrees of freedom followed in semiclassical transport theory

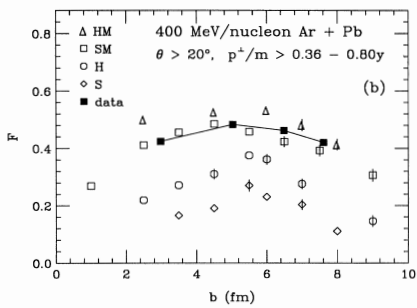
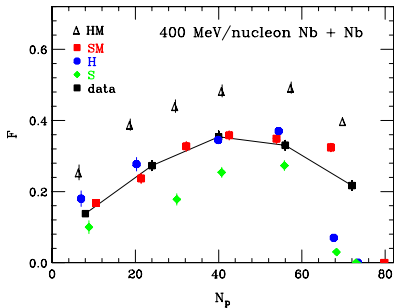
Besides EOS uncertainties include:

- Dependence of mean fields on density, momentum and nonequilibrium features of phase-space distributions
- In-medium interaction rates
- Space-time nonlocalities in collisions → impact on entropy
- Off-shell effects
- Optimal observables for testing individual uncertainties



Example: Mean-Field ρ vs ρ Dependence

Impact of centrality and momentum bracket on flow may be used to resolve these dependencies:



Qiubao Pan & PD PRL70(93)2063



EOS and Flow Anisotropies

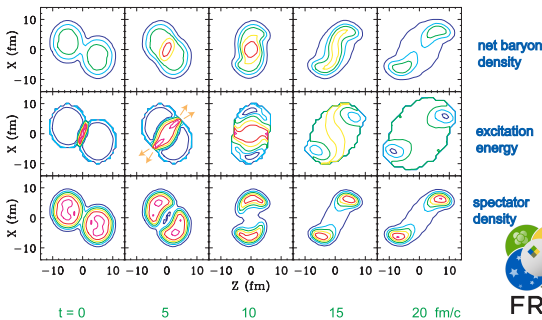
EOS assessed through reaction plane anisotropies characterizing particle collective motion.

Hydro? Euler eq. in $\vec{v} = 0$ frame: $m_N \rho \frac{\partial}{\partial t} \vec{v} = -\vec{\nabla} p$

where p - pressure. From features of v , knowing Δt , we may learn about p in relation to ρ . Δt fixed by spectator motion.

For high p , expansion rapid and much affected by spectators.

For low p , expansion sluggish and completes after spectators gone.



EOS and Flow Anisotropies

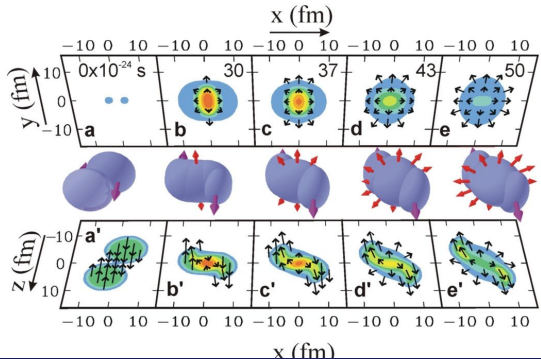
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Sideward Flow Systematics

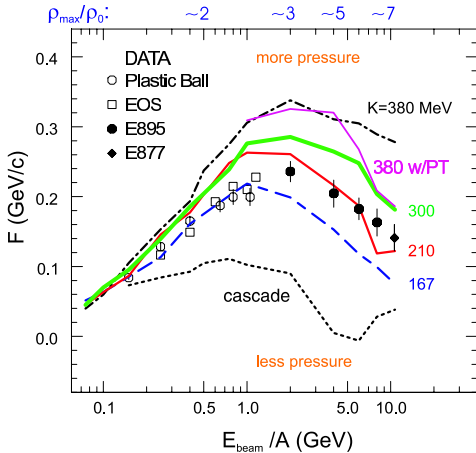
Deflection of forwards and backwards moving particles away from the beam axis, within the reaction plane

Au + Au Flow Excitation Function

Note: K used as a label

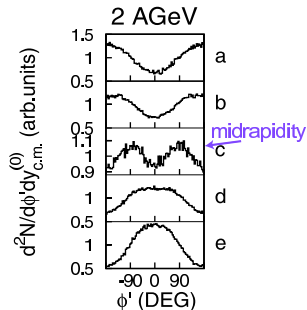
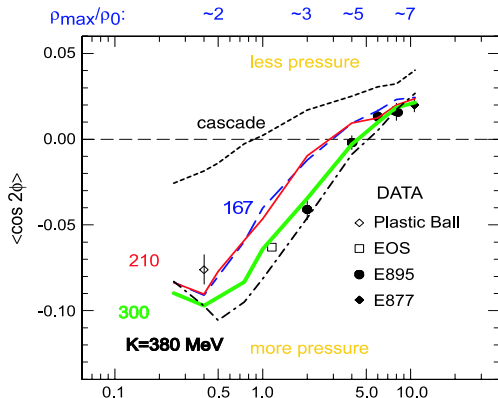
PD, Lacey & Lynch
Science298(02)1592

The sideward-flow observable results from dynamics that spans a density range varying with the incident energy



2nd-Order or Elliptic Flow

Another anisotropy, studied at midrapidity: $v_2 = \langle \cos 2\phi \rangle$, where ϕ is azimuthal angle relative to reaction plane.



Au+Au v_2 Excitation
Function



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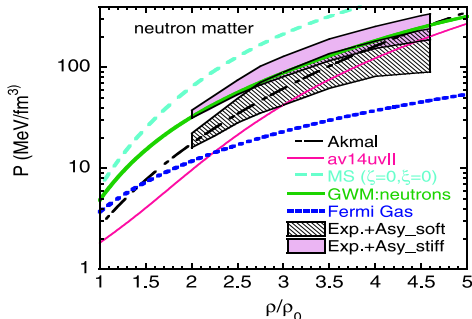
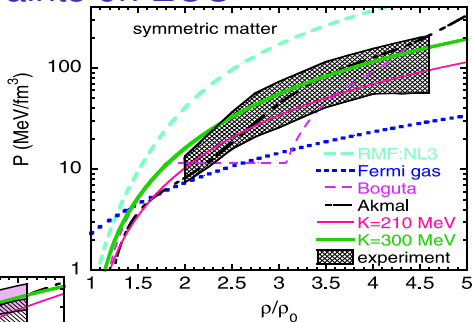
Constraints on EOS

Au+Au flow anisotropies:

$$\rho \simeq (2 - 4.6)\rho_0.$$

No one EOS yields both flows right. Discrepancies: inaccuracy of theory.

Most extreme models for EOS can be eliminated.



PD, Lacey & Lynch
Science298(02)1592

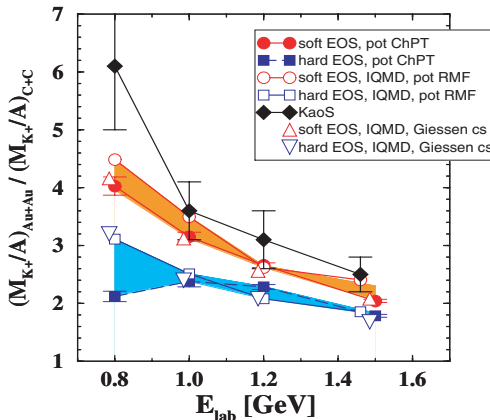
Neutron Matter:

Uncertainty in
symmetry energy



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Subthreshold Kaon Production



Ratio of kaons per participant nucleon in Au+Au collisions to kaons in C+C collisions vs beam energy

filled diamonds: KaoS data

open symbols: theory

Fuchs *et al.*

ProgPartNuclPhys 53(04)113

Kaon yield sensitive to EOS because multiple interactions needed for production, testing density

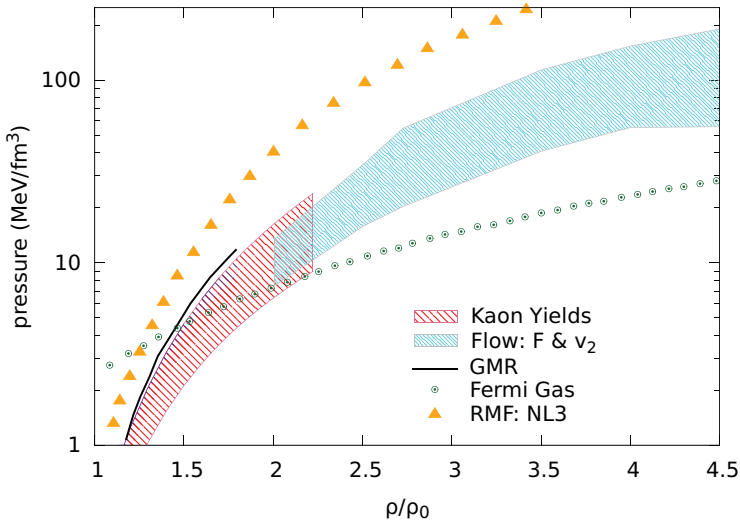
The data suggest a relatively soft EOS

In-medium threshold effects?? (Dan Cozma)



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Combination/Consistency of the Constraints?

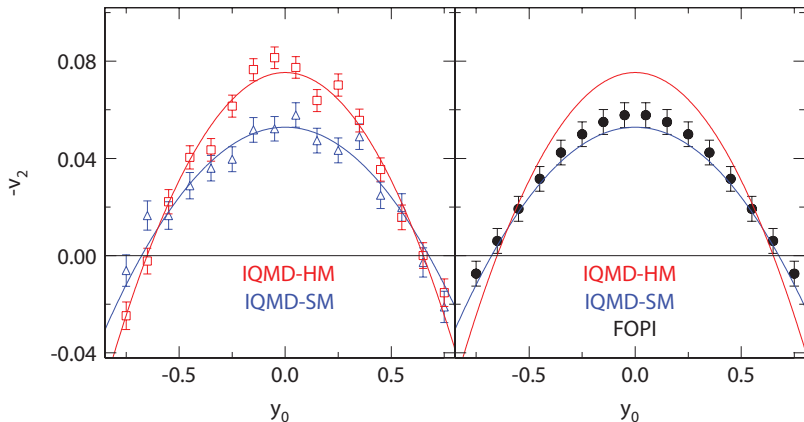


Flow Probing Vicinity of ρ_0

Le Fevre *et al.* NPA945(16)112

Elliptic flow in Au + Au between 0.4 and 1.5 GeV/u

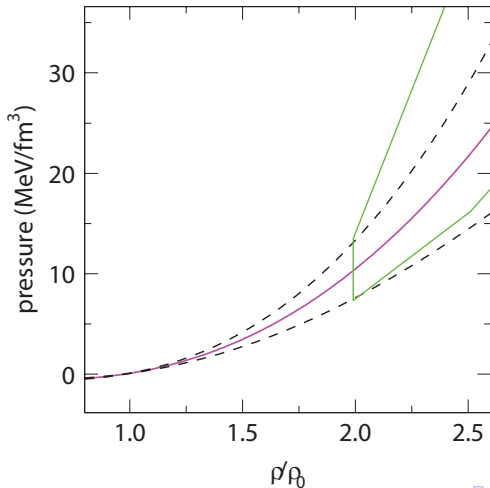
Au+Au 1.2A GeV $0.25 < b_0 < 0.45$ protons



Flow Testing Vicinity of ρ_0

Le Fevre *et al.* NPA945(16)112

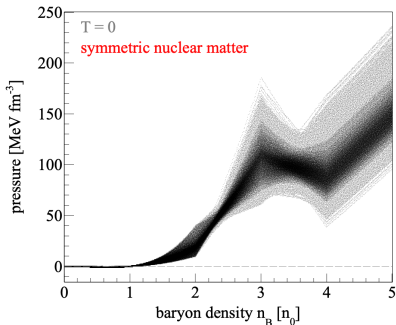
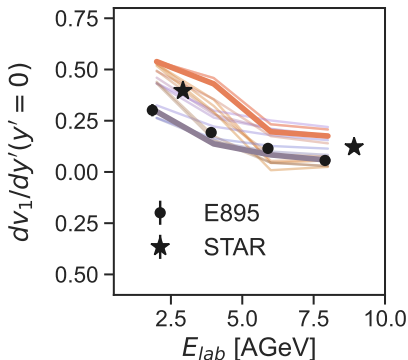
Comparison to higher- ρ inference



Flow in SMASH

Oliinychenko, Sorensen *et al.* arXiv:2208.11996

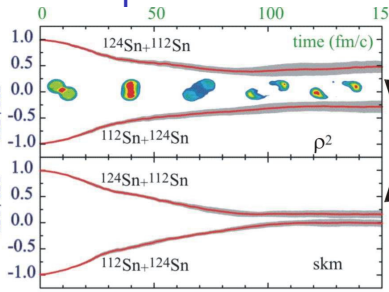
Changing speed of sound in density intervals and comparing to flow data



Isospin Diffusion: Low- ρ Symmetry Energy

$$R = \frac{2\delta - \delta_{124+124} - \delta_{112+112}}{\delta_{124+124} - \delta_{112+112}}$$

normalized asymmetry

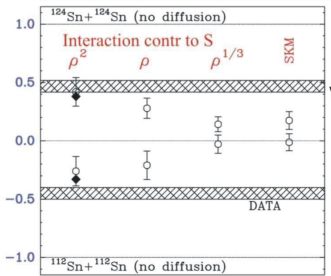


Linear theory:

$$\vec{j}_{np} \simeq -4\nu \vec{\nabla} [S \delta]$$

Tsang *et al.*
PRL92(04)062701

Shi&PD
PRC68(03)064604



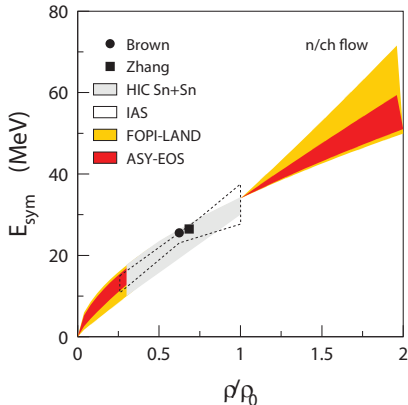
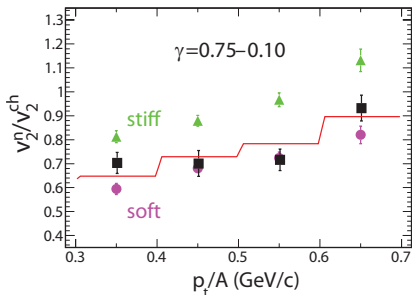
$$\delta = \frac{N - Z}{N + Z}$$



Supranormal Densities: Baryon Differential Flow

Russotto *et al.* PRC94(16)034608

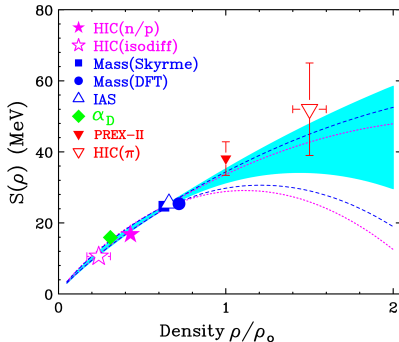
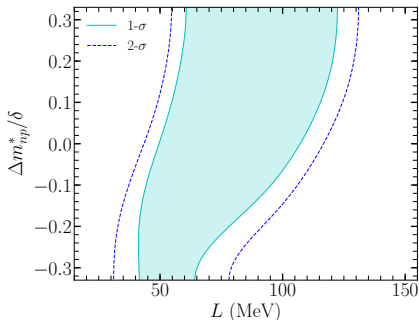
Au + Au @ 400MeV/u, neutron measurements with LAND



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Constraints from Charged Pion Yields

Bao-An Li PRL88(02)192701

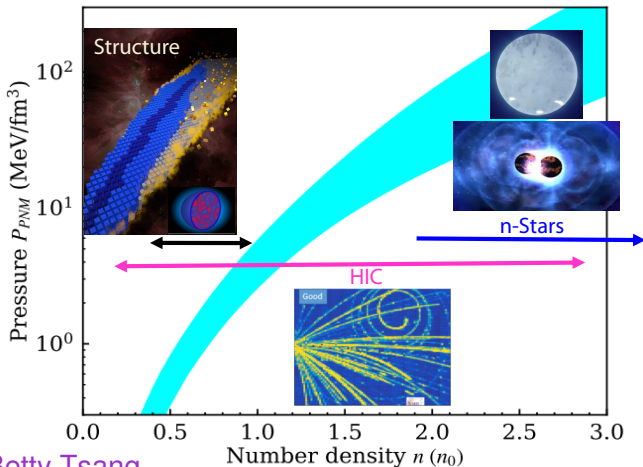


Reisdorf *et al.* NPA781(07)459; Estee *et al.* PRL126(21)162701
Liu *et al.* PRC103(21)014616; Lynch&Tsang PLB830(22)137098



Combined Inferences: HIC + Structure + Astrophysics

Different probed densities



From Betty Tsang

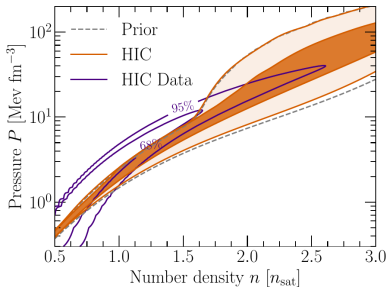
Analysis by Huth et al.

Huth, Pang *et al.* Nature 606(22)276

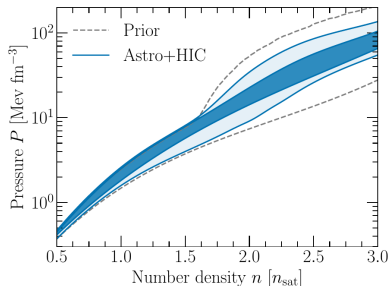
Constraining neutron-star matter with microscopic and macroscopic collisions

Bayesian combinations

HIC experiments:



HIC and Astro combined:

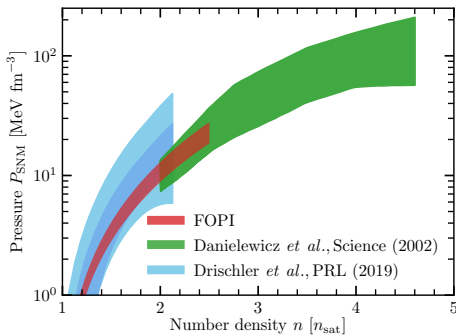


Astrophysical observations narrow constraints above $2\rho_0$

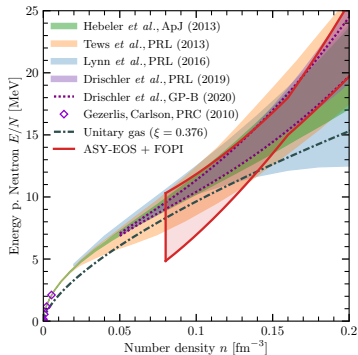


Huth et al: Theory vs Heavy Ions

Symmetric Matter



Neutron Matter



Conclusions

- Heavy-ion collisions allow to dial densities for studying EOS, by changing beam energy
- Window in the energy that addresses the densities around $2\rho_0$, where the collisions are particularly called for, is actually easier from the standpoint of transport than either significantly lower or significantly higher energies
- FRIB400 should deliver a wider range of exotic projectiles, at intensities needed for heavy-ion experiments focussed on EOS, than any other accelerator in the world in the foreseeable future
- More refined observables are needed for more stringent EOS constraints

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