The Equation of State of Symmetric Nuclear Matter from Intermediate Energy Heavy-Ion Data

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“Dense Nuclear Matter Equation of State from Heavy-Ion Collisions”
5-9 December 2022, INT, USA

Overview

Motivation

Model Details

- dcQMD – interaction parametrization
- Medium modification of cross-section
- Threshold effects
- Initial/final state treatment

Study of EoS of SNM

- Stopping observables
- Transverse flow
- Elliptical flow

Perspectives

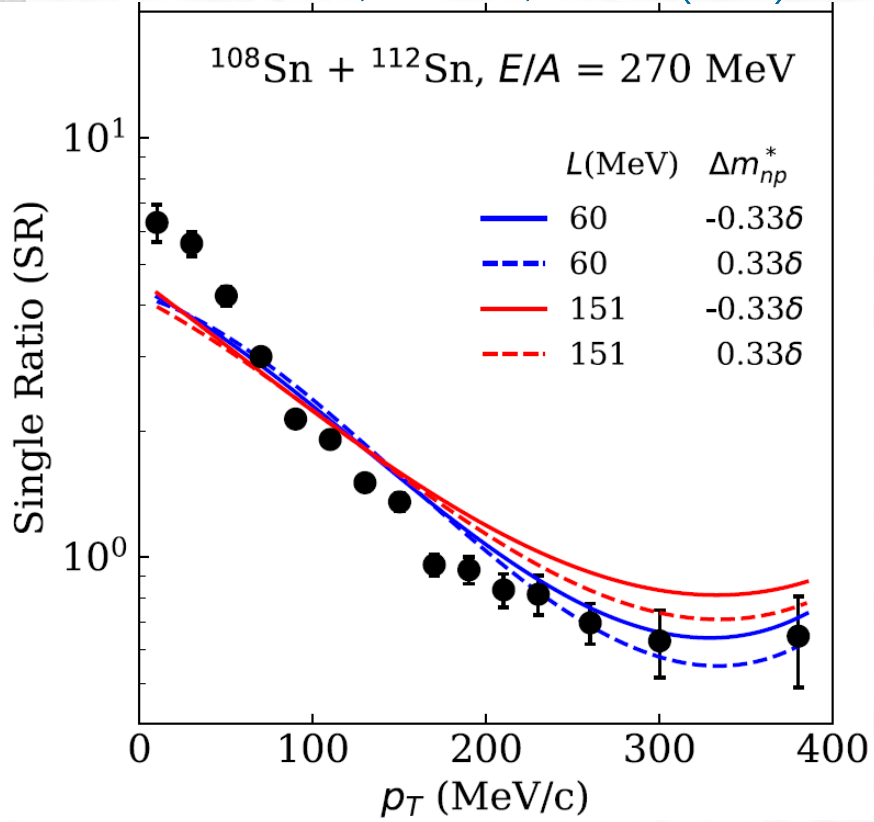
- Prerequisites for HIC above 1.0 GeV/nucleon
- Improving reaction dynamics for QMD models

Summary & Conclusions

Motivation

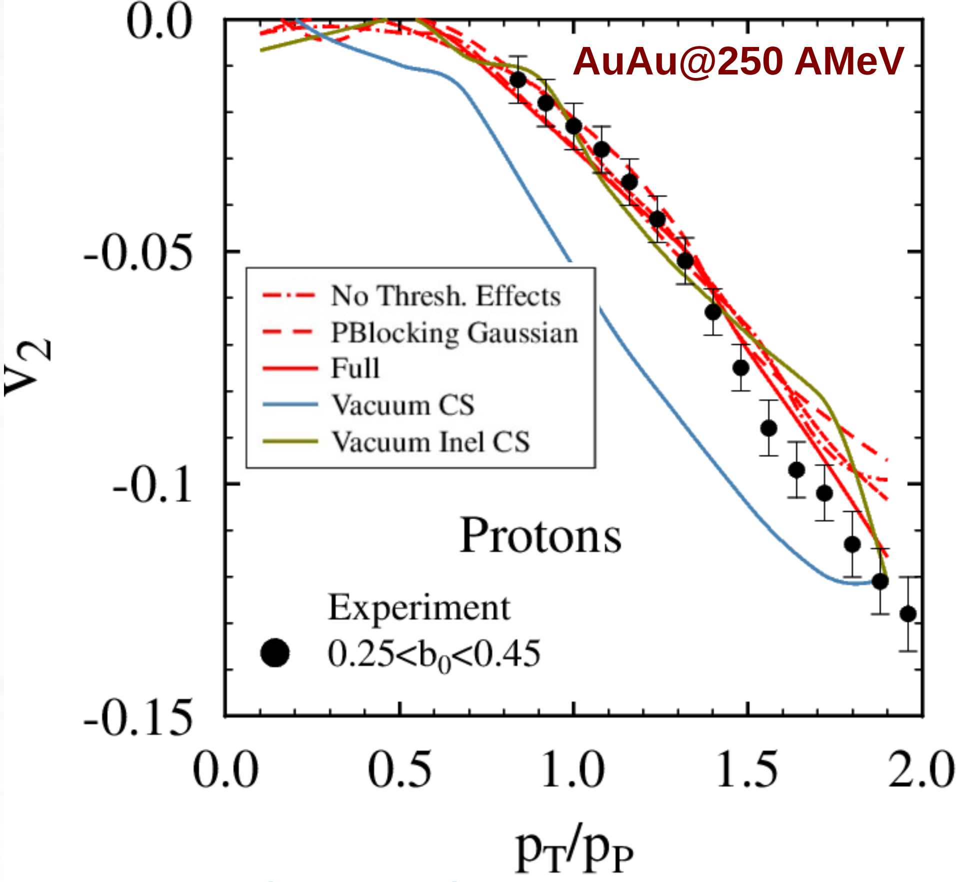
- latest version of the dcQMD model used to study symmetry energy with pion production
- empirical effective isoscalar mass m*=0.70
- compressibility modulus close to world average K_0=245 MeV
- in-medium modification factor adjusted to qualitatively describe nucleonic observables

J.Estee et al., PRL 126, 162701 (2021)



possible source of discrepancy at medium p_T values: somewhat unrealistic dynamics

D. Cozma et al.,EPJA 57, 309 (2021)



Experimental data: (FOPI) NPA 876, 1 (2012)

similar quality of description of exp. data for p,d,t,α in the energy range 150-800 AMeV

Model Details

dcQMD transport model: newest version EPJA 57, 309 (2021)

an upgraded version of TuQMD, see H. Wolter et al.

Prog.Part.Nucl.Phys. 125, 103962 (2022)

Interaction (nucleonic d.o.f.)

momentum dependent potential **MDI2** -generalization of MDI of

Das, Das Gupta, Gale, Li PRC67, 034611 (2003)

$$\frac{E}{N}(\rho,\beta,x,y)=\frac{1}{2}A_1 u+\frac{1}{2}A_2(x,y)u\beta^2+\frac{Bu^\sigma}{\sigma+1}(1-x\beta^2)+\frac{Du^2}{3}(1-y\beta^2)$$

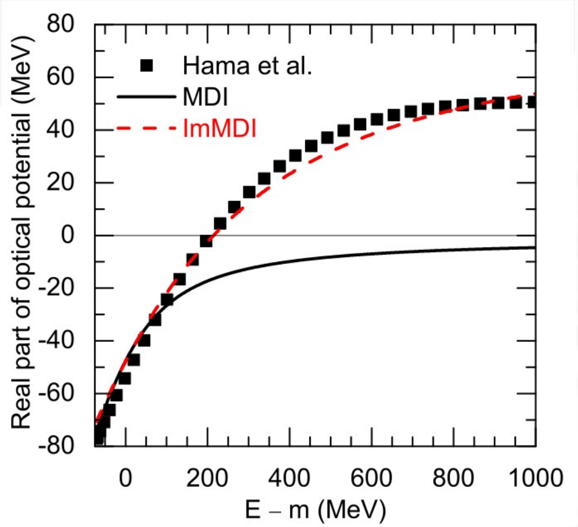
$$+\frac{1}{u\rho_0^2}\sum_{\tau,\tau'}C_{\tau\tau'}\int\int d^3p\,d^3p'\frac{f_\tau(p,p')f_{\tau'}(p,p')}{1+(\vec{p}-\vec{p}\,')^2/\Lambda^2}$$

$$A_2(x,y)=A_2^0+\frac{2xB}{\sigma+1}\bar{u}^{\sigma-1}+\frac{2yD}{3}\bar{u} \qquad u=\frac{\rho}{\rho_0}$$

Fit:
U_∞,K,J_0,m* -isoscalar
S(ũ),L,K_{sym},δm_{isv} -isovector

$C_l - C_u$

momentum dependent part: similar with that of J. Xu et al. PRC 91, 014611 (2015)
(see also C. Hartnack, J. Aichelin PRC 49, 2801 (1994))
used previously to test model dependence: flow ratio PRC 88, 44912 (2013)
pion multiplicity ratio PLB 753, 166 (2016)
independent part: extra term (vary L vs. K_{sym} and also J_0 vs. K independently)



from J. Xu et al. PRC 91, 014611 (2015)

Input		Parameters	
ρ_0 [fm^{-3}]	0.16	Λ [MeV]	708.001
E_B [MeV]	-16.0	C_l [MeV]	-13.183
m_s^*/m	0.70	C_u [MeV]	-140.405
δ_{n-p}^* $(\rho_0, \beta=0.5)$	0.165	B [MeV]	137.305
K_0 [MeV]	245.0	σ	1.2516
J_0 [MeV]	-350.0	$\tilde{A}_l$ [MeV]	-130.495
$\tilde{\rho}$ [fm^{-3}]	0.10	$\tilde{A}_u$ [MeV]	-8.828
S($\tilde{\rho}$) [MeV]	25.4	D [MeV]	7.357

Collision Term

Elastic baryon-baryon collisions
below pion production threshold: Li-Machleidt

Li, Machleidt PRC 48, 1702 (1993),
Li, Machleidt PRC 49, 566 (1994)

above pion production threshold: Cugnon

in-medium modification factor

- collision criterion based on effective masses determined using EoM (consistency with the dt → 0 fm/c limit)
- in-medium modification of elastic cross-sections

$$\sigma^{med} = f(\rho,\delta,)\,\sigma^{vac}_{mod}$$

$$f(\rho,\delta) = \exp[\alpha\rho/\rho_0 + \beta_1\delta\rho/\rho_0 + \beta_2(\tau_1+\tau_2)\delta\rho/\rho_0]$$

σ^{vac}_{mod} – flux and phase-space factors computed using effective masses

B.A. Li et al. PRC72, 064611 (2005)

f(ρ,δ) – accounts for medium modifications of transition matrix due to departure from the quasi-particle picture

C. Fuchs et al. PRC 64, 024003 (2001)

$\beta_1=0$ and $\beta_2=0$ in this study

Inelastic baryon-baryon, meson-baryon collisions (related only to pion production)

two step process:

- resonance excitation in baryon-baryon collisions
  parametrization of the OBE model of
  S.Huber et al., NPA 573, 587 (1994)
- resonance decay:
  Breit-Wigner shape of the resonance spectral function J. Weil et al, PRC 94, 054905 (2016)
- charge exchange reactions: NR->NR'

pion absorption:

-resonance model (all 4* resonances below 2 GeV)
K. Shekhter, PRC 68, 014904 (2003)

inelastic channels: mass scaling formula

$$\sigma_{N\Delta}(\rho,\beta,p) = \sigma^{vac}_{N\Delta}(p)\,\frac{\mu_{ini}(\rho,p)}{\mu^{vac}_{ini}(p)}\,\frac{\mu_{fin}(\rho,p)}{\mu^{vac}_{fin}(p)}$$

See also Larionov et al., NPA 728, 135 (2003)

Threshold Effects (dcQMD)

- direct consequence of imposing (total) energy conservation in the medium

$$\sqrt{p_1^2+m_1^2}+U(p_1)+\sqrt{p_2^2+m_2^2}+U(p_2)=\sqrt{p'^2_1+m'^2_1}+U(p'_1)+\sqrt{p'^2_2+m'^2_2}+U(p'_2)$$

- rarely considered in transport models below 1 AGeV, with a few exceptions:

RBUU: G. Ferini et al. PRL 97, 202301 (2006), RVUU: T. Song, C.M. Ko PRC 91, 014901 (2015);
χBUU: Z. Zhang et al, PRC 98, 054614 (2018)

- required for thermodynamical consistency of the model

Z.Zhang et al, PRC 97, 014610 (2018)

- reactions: NN↔NR, R↔Nπ (R↔Nππ not corrected)

- assumptions (dcQMD): - two-body collisions are part of N-body one
  - in-medium two-body collisions modeled as a succession of bare (vacuum-like) collisions followed/preceded by energy exchanges with the fireball, while momentum is conserved
  - reaction with highest probability: corresponds to the one which included the bare collision of highest probability

Example: NN->NΔ

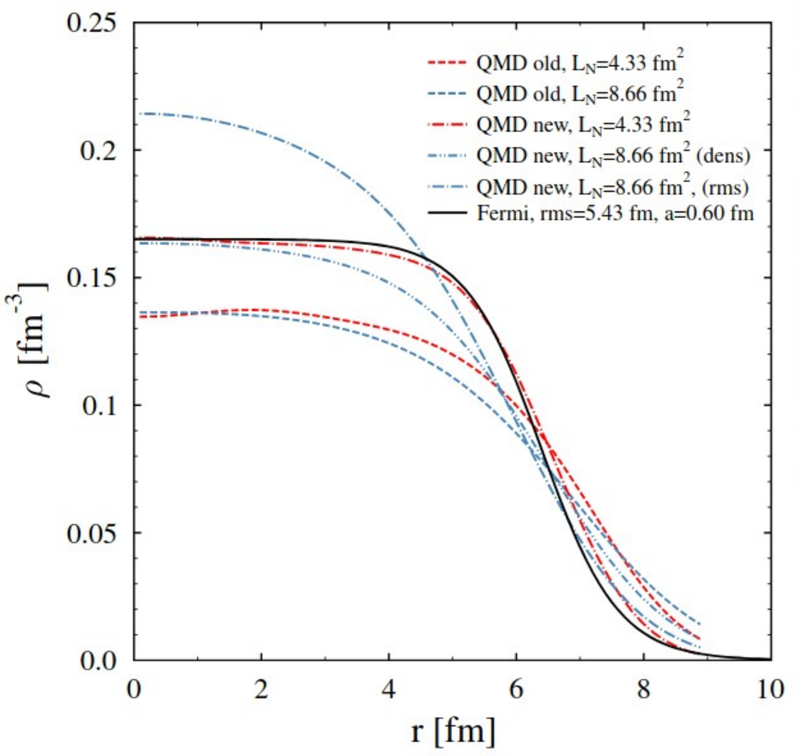
$$\sigma^{(med)}_{NN\to N\Delta}(s^*)=\frac{\mu^{(ini)*}}{\mu^{(ini)}}\,\frac{\mu^{(fin)*}}{\mu^{(fin)}}\,\sigma^{(vac)}_{NN\to N\Delta}(s^*)$$

$s^*=Max\{s^{ini},s^{fin}\}$

Introduced in TuQMD/dcQMD in DC, PLB 753, 166 (2016)

Initial/Final State

Initial state density profile of nuclei



- nuclei initialized with realistic charge radii and neutron skins
- larger L^2_N leads to stronger tails and consequently lower reduced impact parameter (flow at projectile/target rapidities affected most visibly)

this study: L^2_N=5.0 fm^2

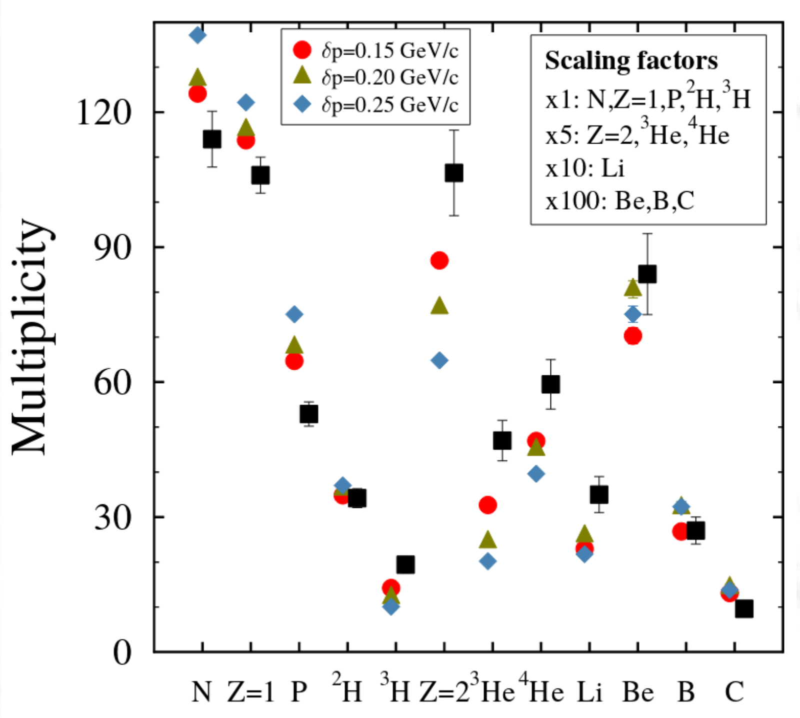
Minimum spanning tree (MST) algorithm

all clusters with A≤15, 23 additional A>15 (B,C,N,O)

Stable : lifetime > 1ms

Unstable : decay into stable using known decay channels

Au+Au @ 400 AmeV b< 2.0 fm



this study: δr=4.0 fm, δp=0.2 GeV/c

Study EoS of SMN

stopping and flow observables for protons and light clusters
in AuAu collisions of impact energy 0.15-0.80 GeV/nucleon (FOPI Coll)

Rapidy Spectra

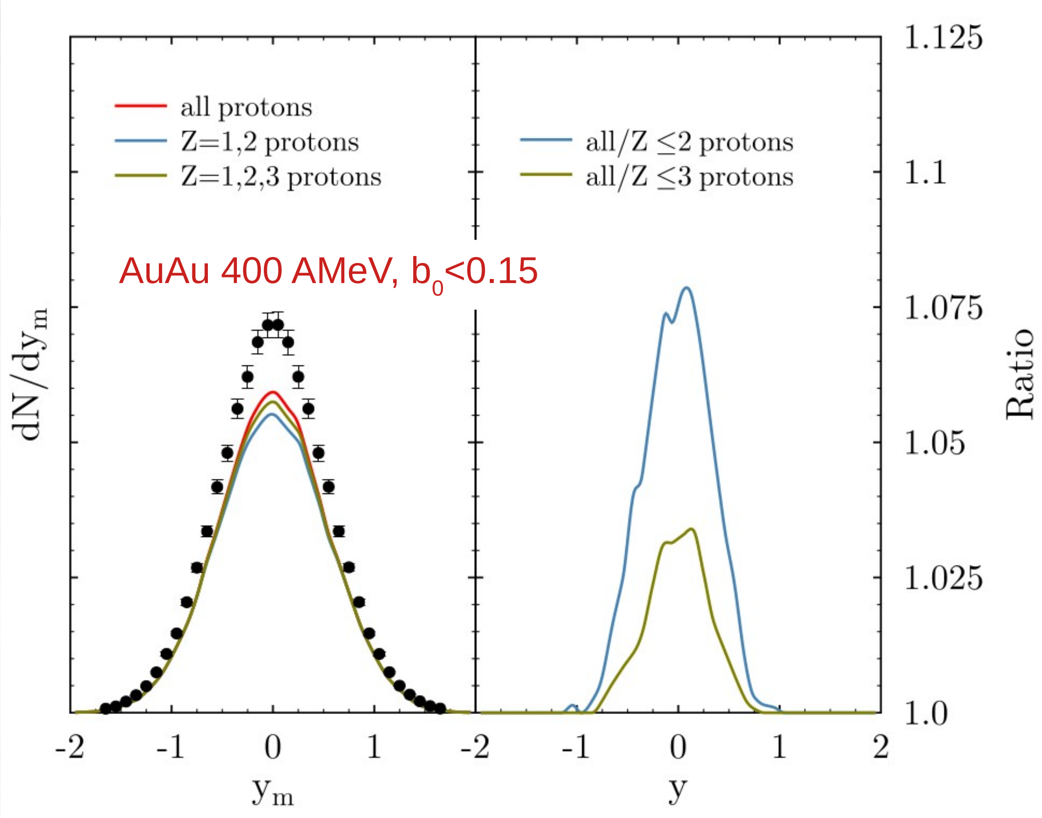
- used in the past to fix in-medium modification factor of elastic cross-sections

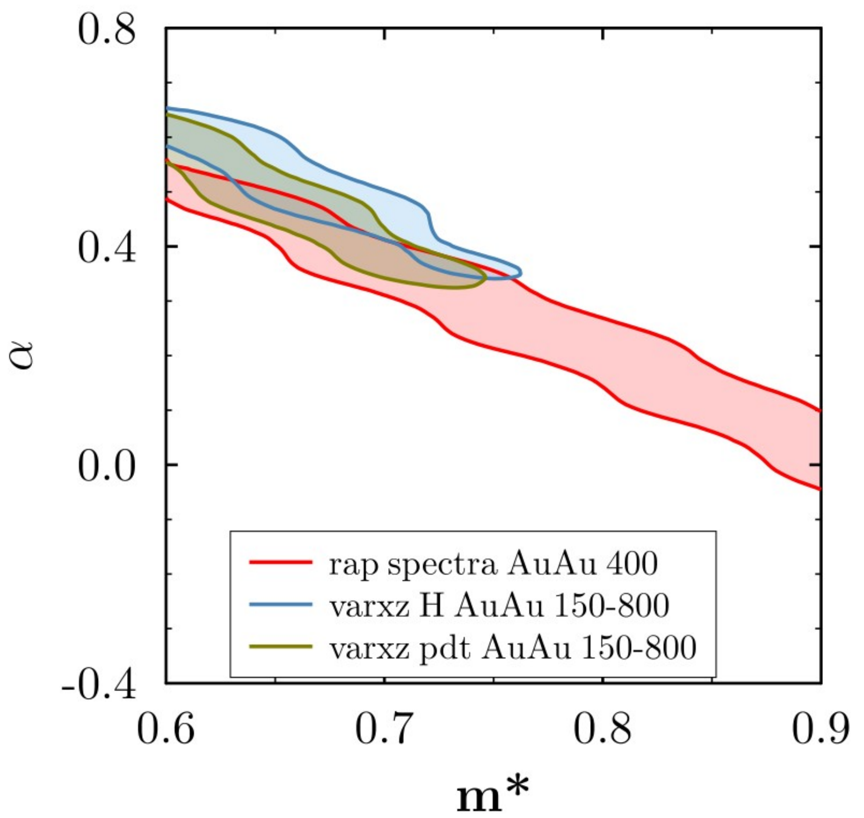
see e.g. P. Danielewicz et al., Science 298, 1592 (2002)

- varxz (H) and constrained transverse CI rapidity spectra for AuAu used in this study

FOPI exp data: W. Reisdorf et al. NPA 848, 366 (2010)

ratio of transverse-to-longitudinal variances of rapidy spectra



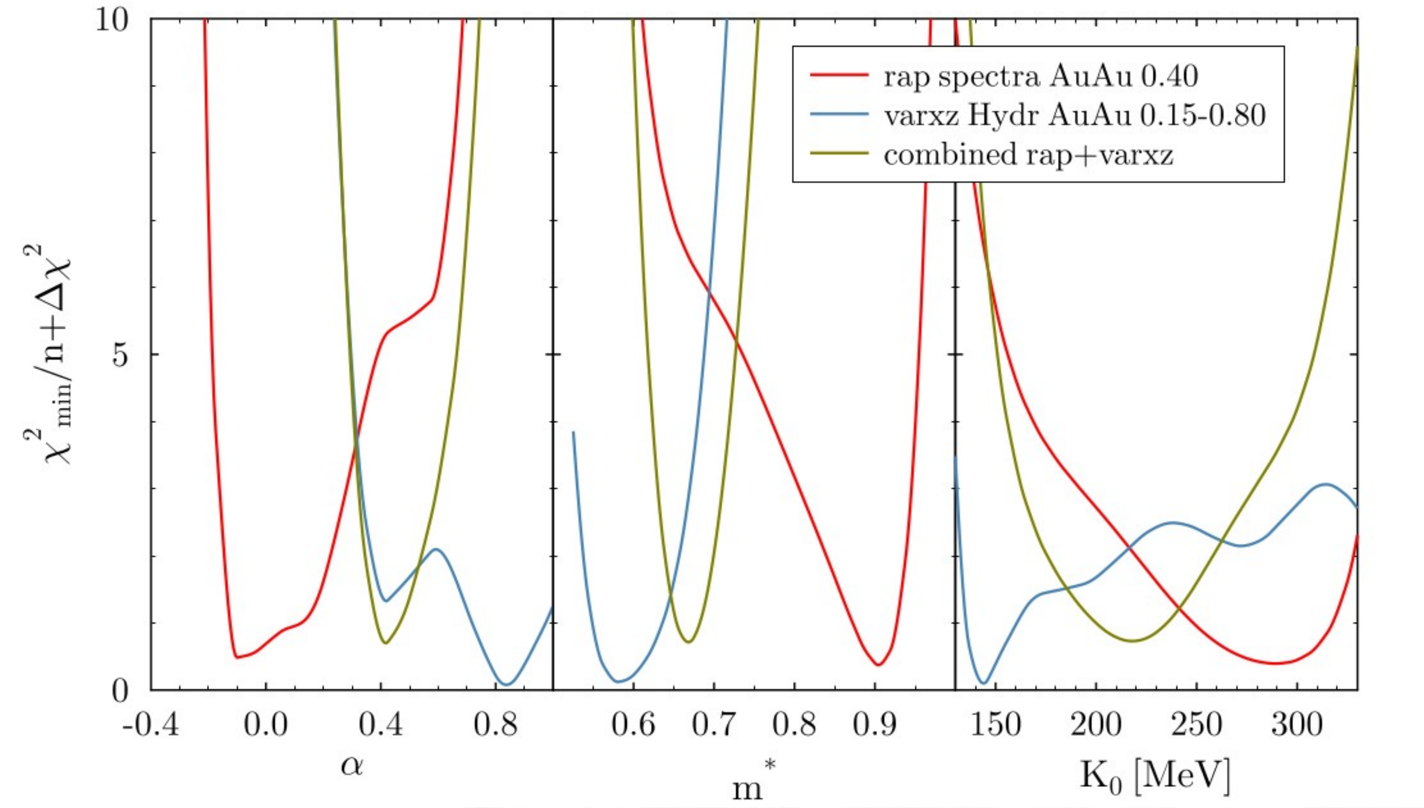


weaker correlation between α and K_0 also evidenced

Rapidity Spectra

- sizable sensitivity of varxz observables to EoS has been previously evidenced

W. Reisdorf et al. NPA 848, 366 (2010)



K_0=218±40 MeV

- current model reproduces impact energy dependence of varxz observables

EPJA 57, 309 (2021)

- different (α,m*) values depending on choice of observable(s)
- medium modification factor of cross-sections at ρ_0 and p_F is similar (~0.65)

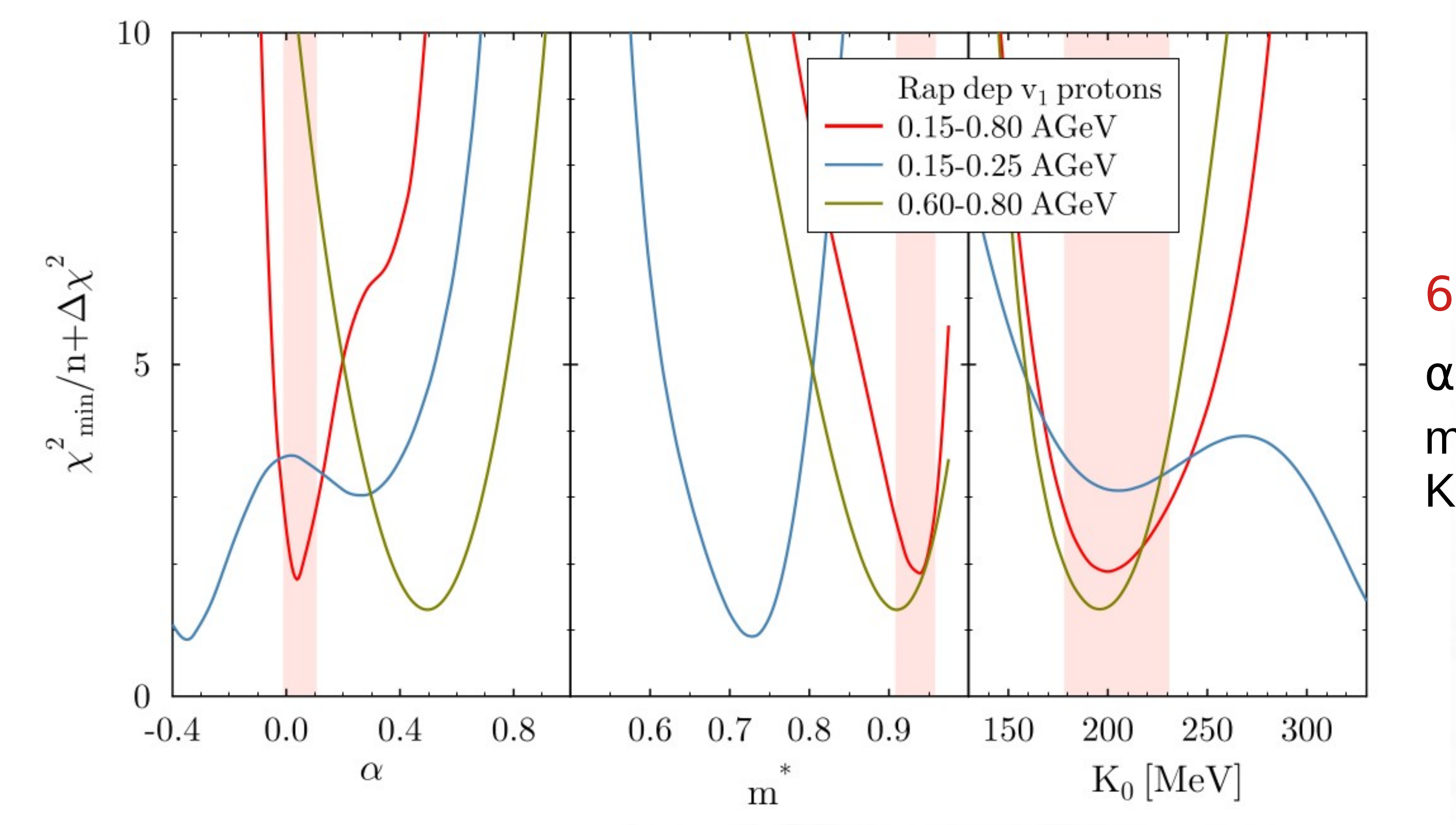
Proton Transverse Flow

Experimental data set: proton rapidity dependent v_1

ut0 > 0.8 – 150, 250 AuAu $0.25<b_0<0.45$

ut0 > 0.4 – 400, 600, 800 AuAu $0.25<b_0<0.45$

W. Reisdorf et al.,
NPA 876, 1 (2012)



68% CL Result

$\alpha=0.040-0.051+0.067$

$m^*=0.936-0.032+0.022$

$K_0=200-22+31$ MeV

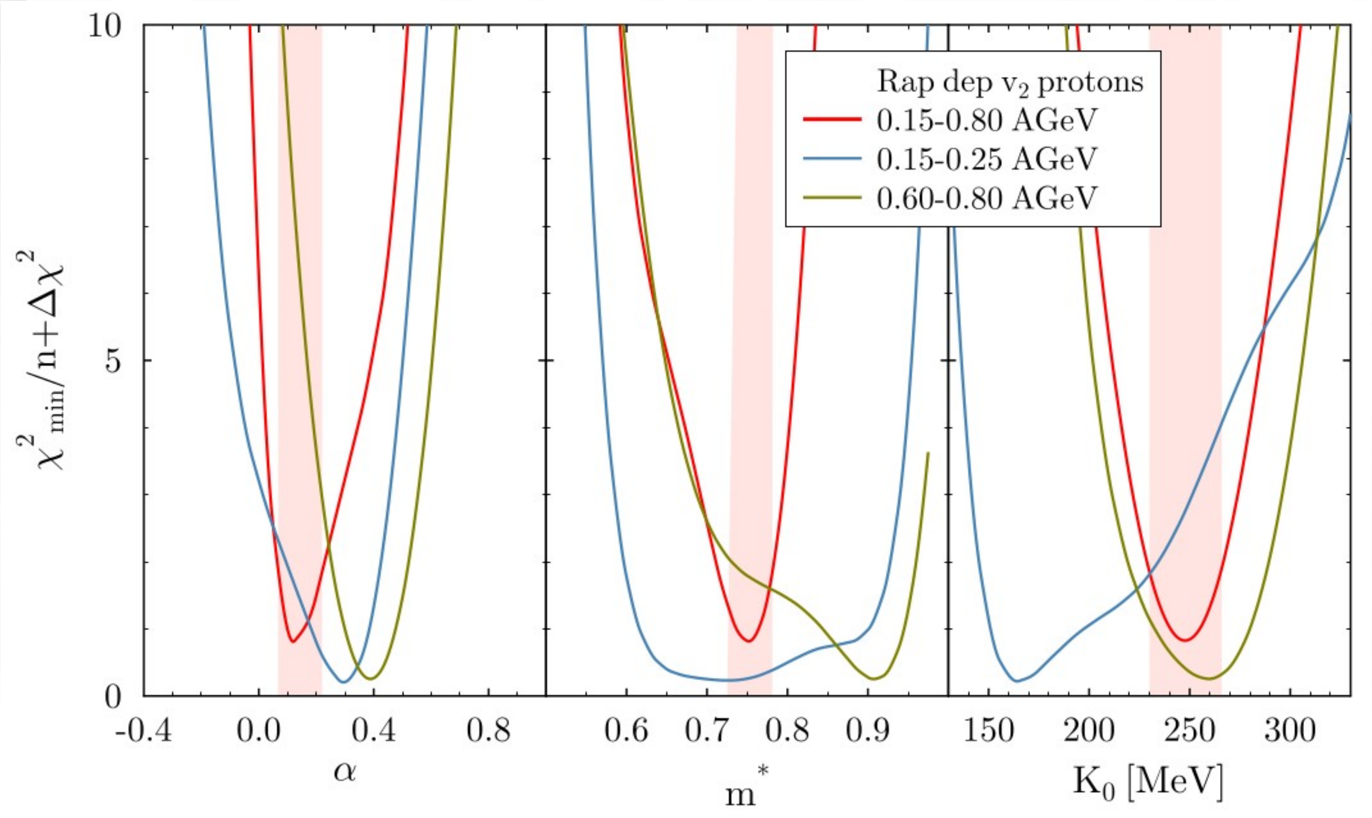
stronger in-medium modification factor at lower impact energies possibly connected to an insufficient Pauli blocking of final state of two-body collisions

Proton Elliptic Flow

Experimental data set: proton rapidity dependent v_2

$ut0=p_T/p_P > 0.8$ – 150-800 AuAu $0.25<b_0<0.45$

W. Reisdorf et al.,
NPA 876, 1 (2012)



68% CL Result

$\alpha=0.130-0.063+0.090$

$m^*=0.751-0.036+0.030$

$K_0=248-18+18$ MeV

$v_2=v_{20}+v_{22}y_0^2$

$v_{2n}=|v_{20}|+|v_{22}|$

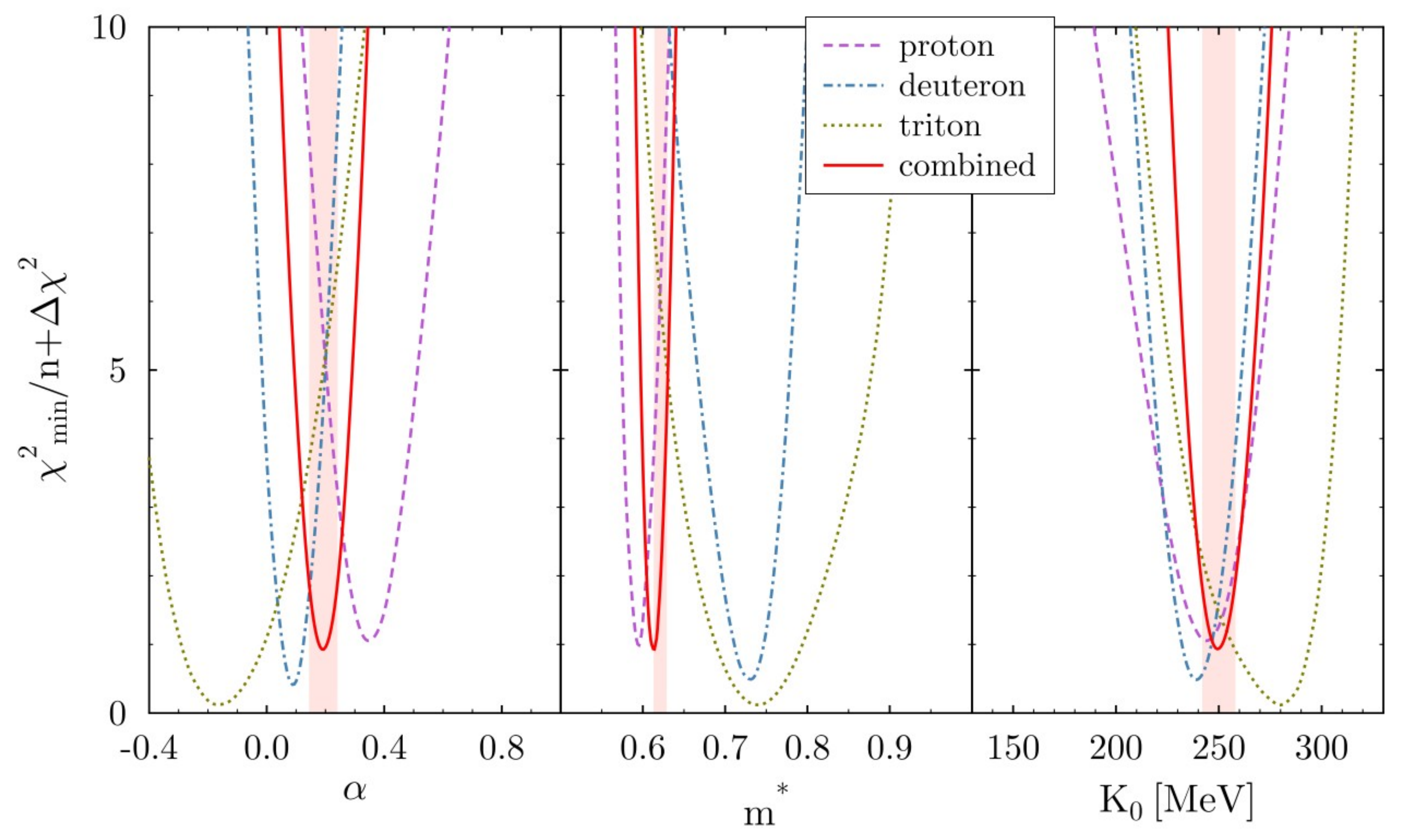
IQMD protons v_{2n}: $K_0=232\pm30$ MeV (light cluster soften the reported combined result)

A.Le Fevre et al., NPA 945, 112 (2016)

P_T dependent Elliptic Flow

Experimental data set: p,d,t transverse momentum dependent v_2
$|y|<0.4$ – 150-800 AuAu $0.25<b_0<0.45$

W. Reisdorf et al.,
NPA 876, 1 (2012)



PRELIMINARY

68% CL Result

α=0.192-0.048+0.049

m*=0.621-0.008+0.008

K_0=250-8+8 MeV

model dependence
due to coalescence
afterburner not accounted !

IQMD full result v_{2n}: K_0=190±30 MeV (light clusters: p,d,t,α)

A.Le Fevre et al., NPA 945, 112 (2016)

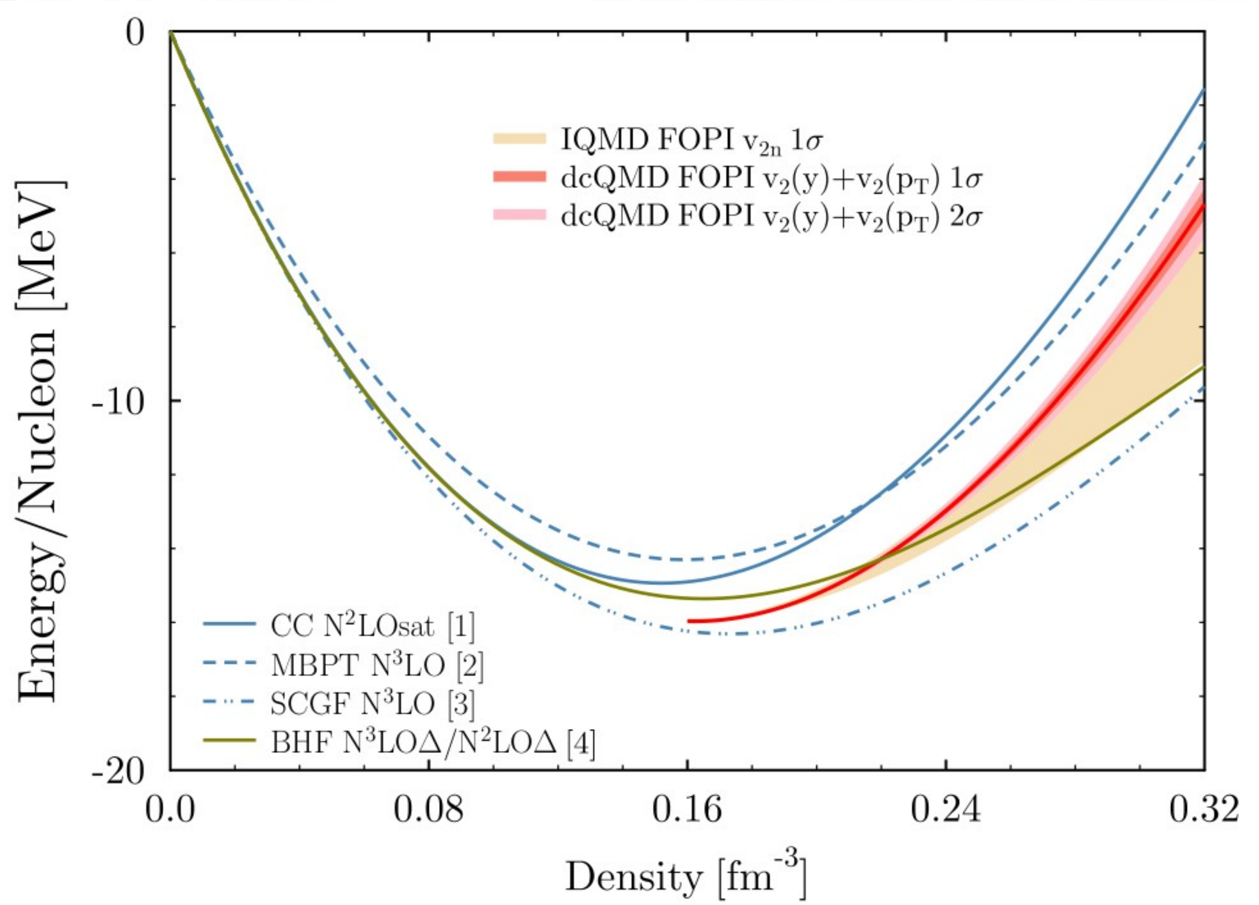
Elliptic Flow (Combined Result)

Experimental data set: p rapidity dependent v_2
p,d,t transverse momentum dependent v_2

inclusion of v_1 data leads to a sub-optimal fit (χ^2/dof~2)

68% CL Result

α=0.287 ± 0.036
m*=0.624 ± 0.009
K_0=236 ± 6 MeV



IQMD result:

A.Le Fevre et al., NPA 945, 112 (2016)

Microscopic calculations:

1. A. Ekstrom et al., PRC 91, 051301 (2015)
2. C. Drischler et al., PRC 102, 054315 (2020)
3. A. Carbone, PRR 2, 023227 (2020)
4. D. Logoteta, PRC 94, 064001 (2016)

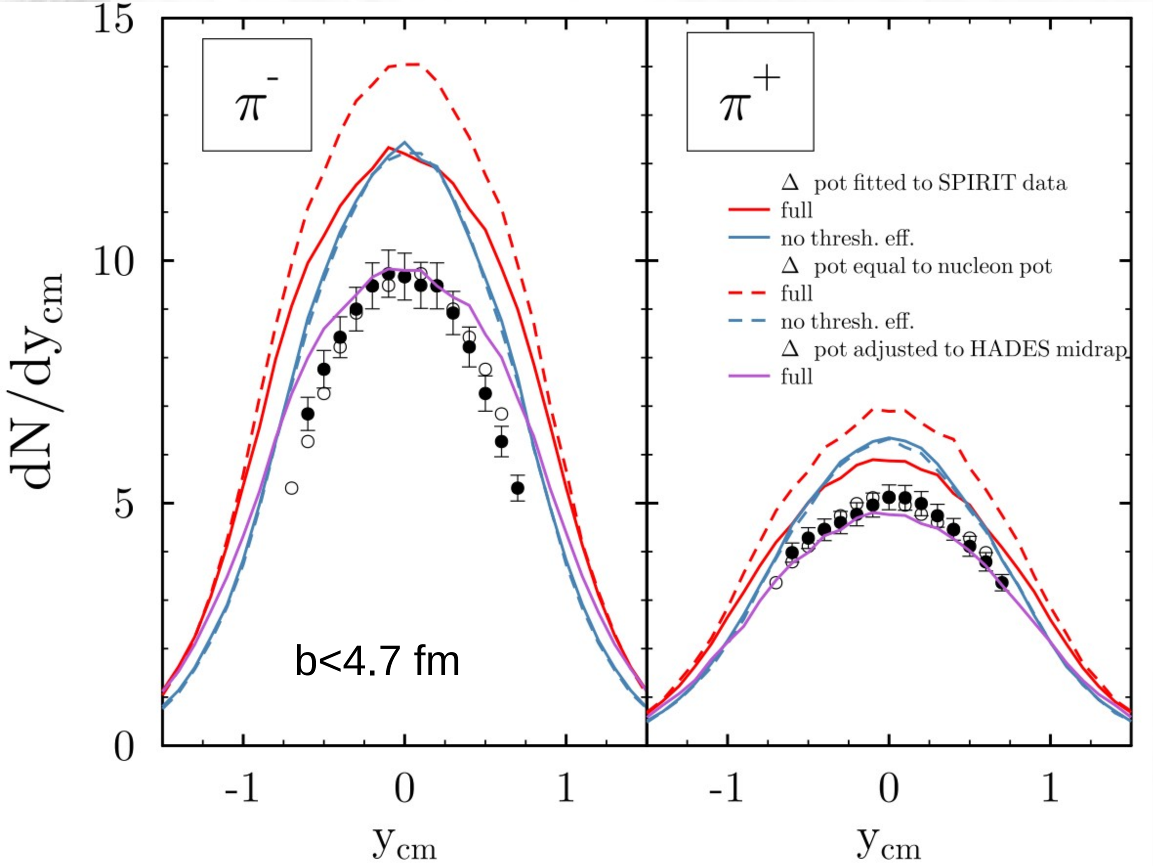
Perspectives

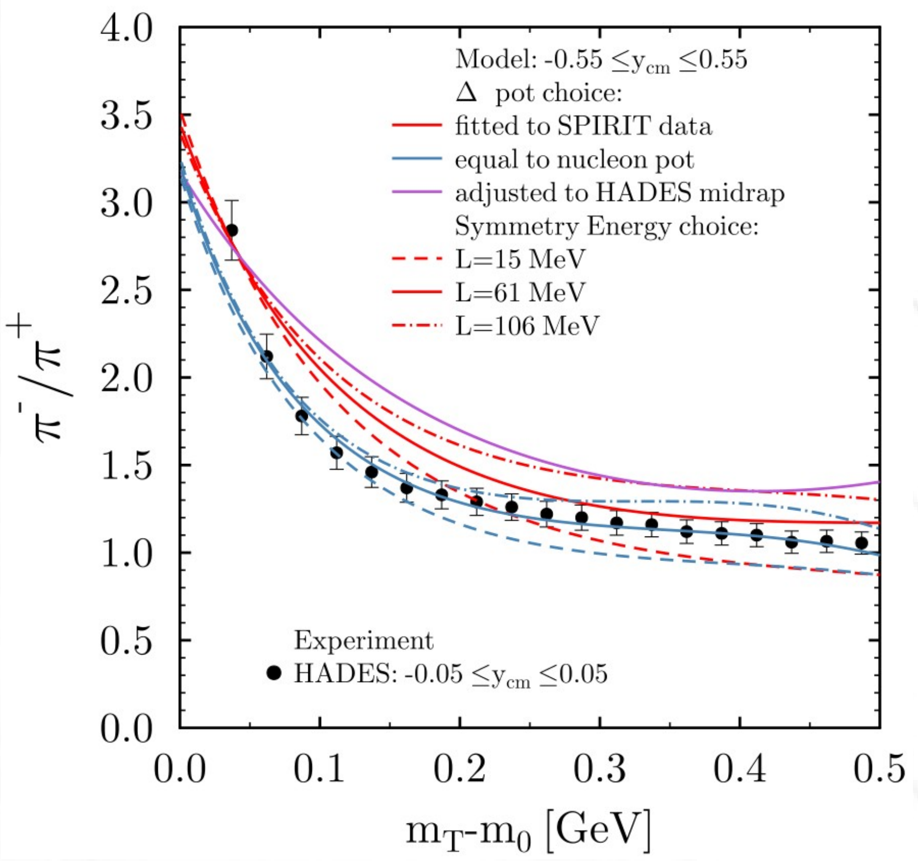
Pion production in AuAu at 1.23 AGeV

J. Adamczewski-Musch et al. (HADES), EPJA 56, 259 (2020)

Note: HADES pion total yield are larger then FOPI yields at 1.2 GeV by about 30%

very preliminary calculation addressing the feasibility of studying the symmetry energy

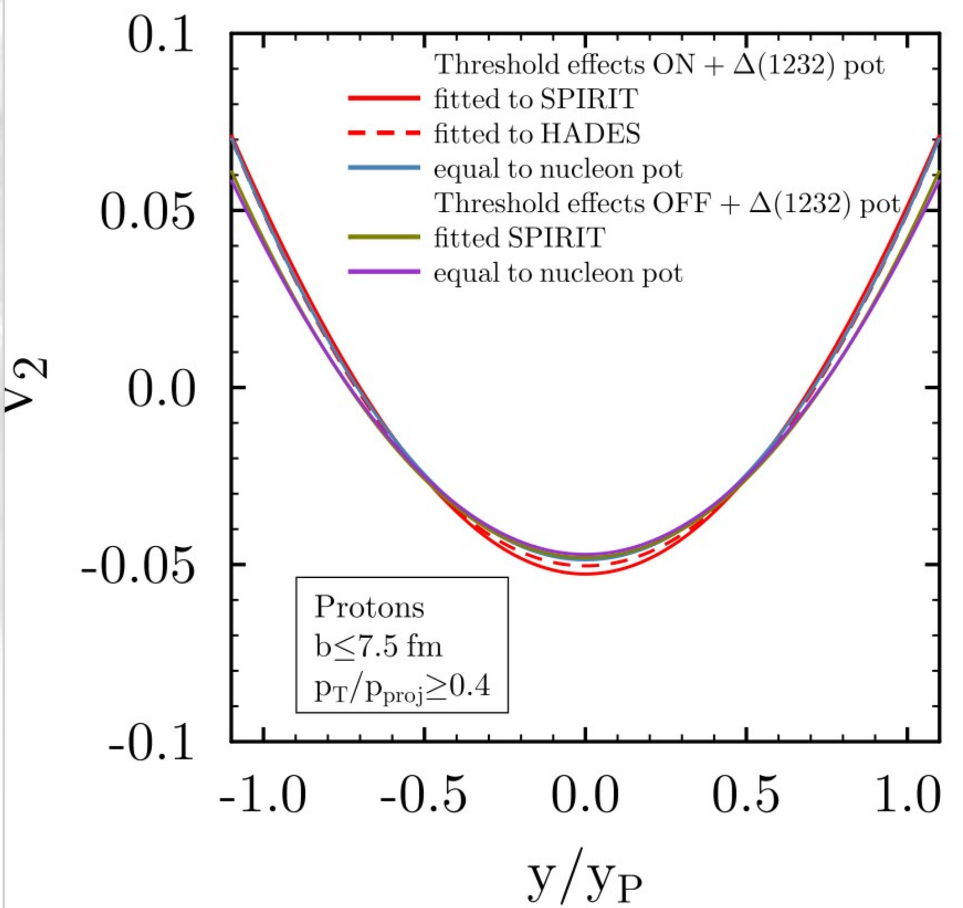


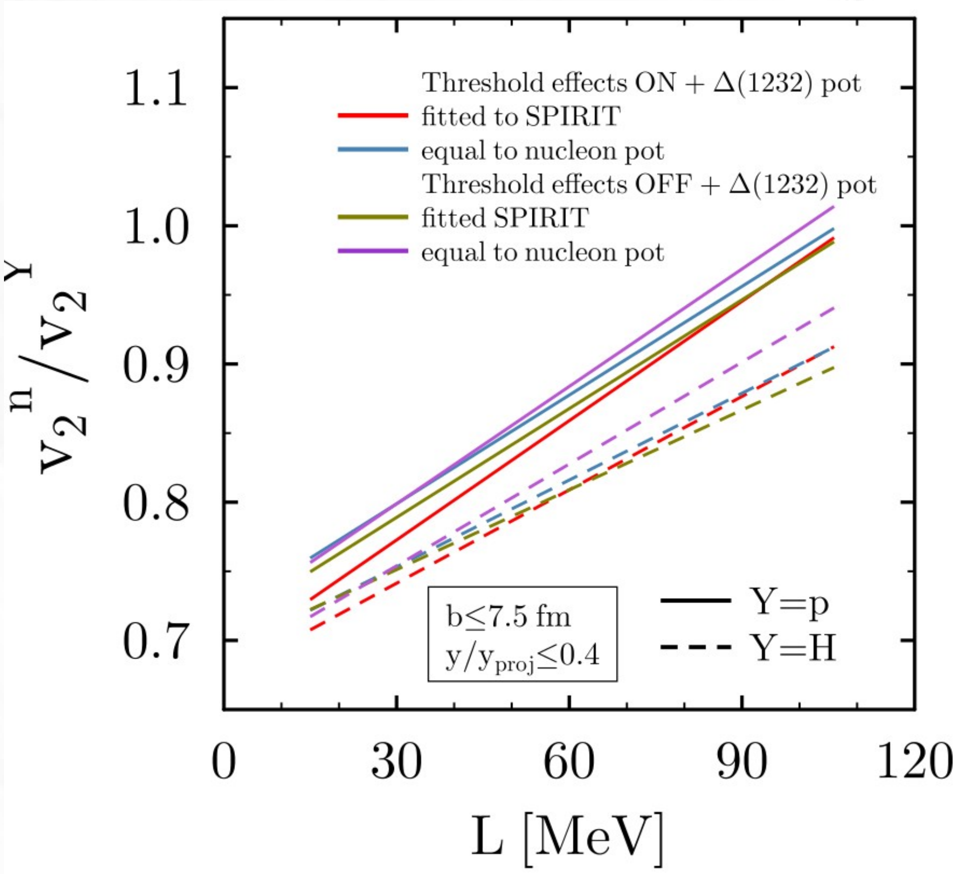


-alternative approach to a systematic description of HADES rapidity and transverse mass spectra: K. Godbey et al. PLB 829, 137134 (2022)

Sensitivity to Resonance Meanfield

chosen impact energy: 1.0 GeV/nucleon (AuAu)
about 20% of nucleons excited into Δ(1232) at the highest density for central collisions





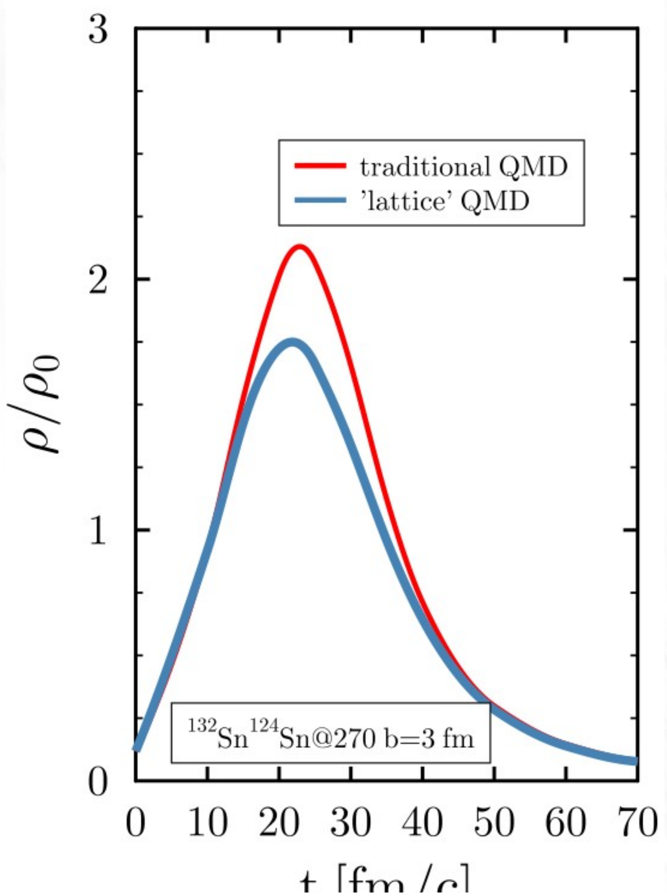
compressibility modulus: extracted from $v_2(y)$
impact of Δ(1232) potential on proton $v_2(y)$: ~ 15% (1.5σ)
omission equivalent to δK_0=-30 MeV

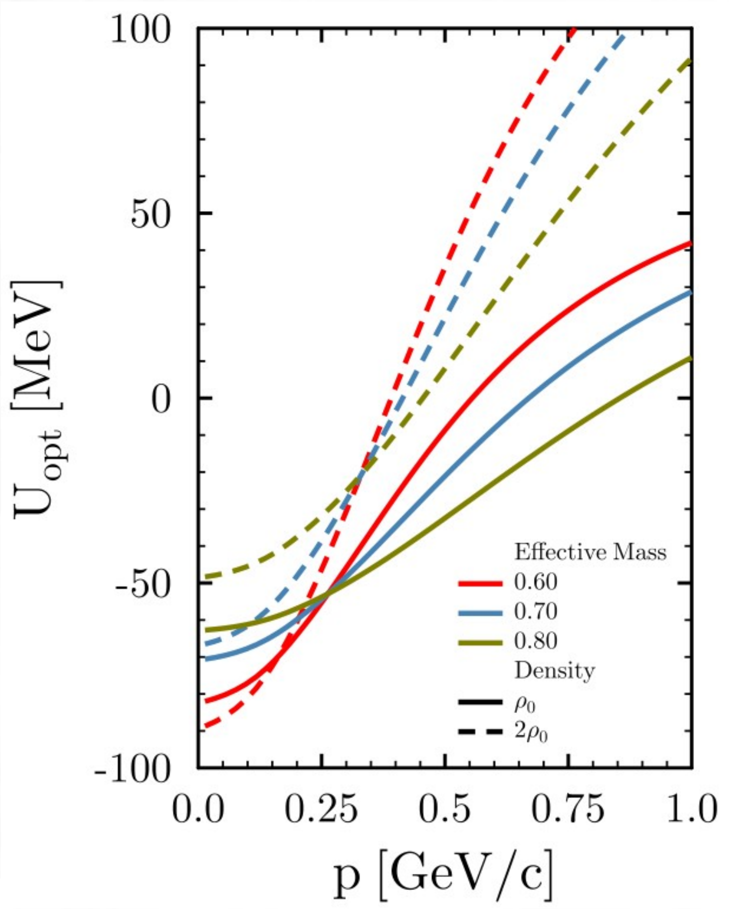
impact on v_2^n/v_2^Y: ~5% equivalent to δL=10 MeV

Momentum dependence

elliptic flow constraint for effective mass: m*=0.624 ± 0.009

possible reasons: compensate for unrealistic density evolution; high density dependence of optical potential deviates from the assumed linear



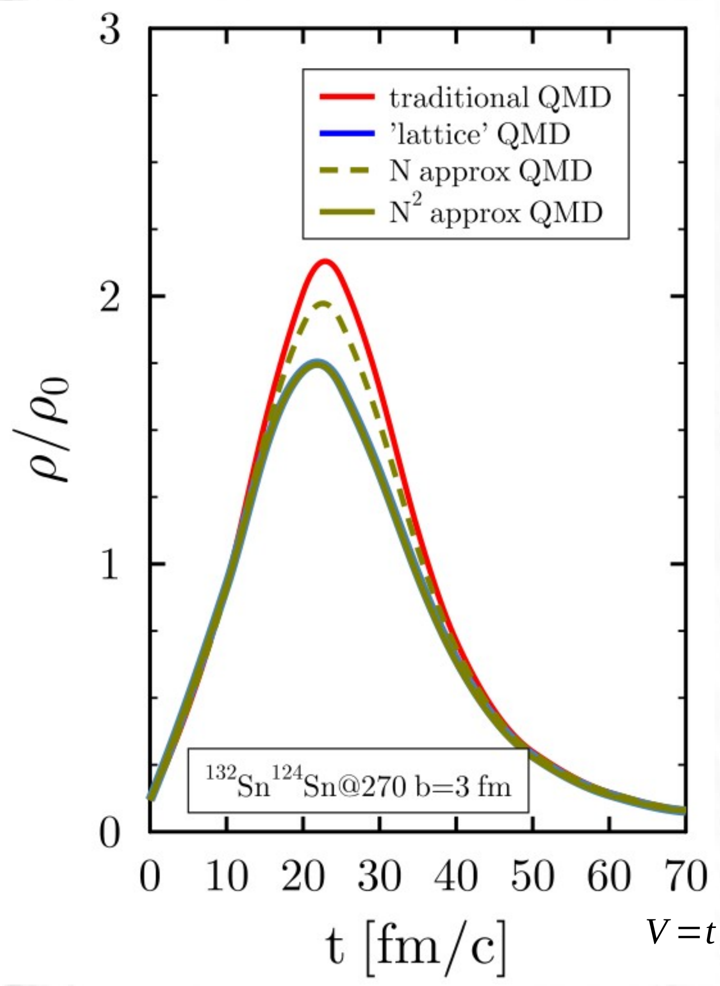


$$\frac{\beta}{\eta+1}\int \frac{\rho^\eta}{\rho_0^\eta}\rho d^3\mathbf{r} = \frac{\beta}{\eta+1}\sum_{i=1}^{N}\left\langle \frac{\rho^\eta}{\rho_0^\eta}\right\rangle_i \approx \frac{\beta}{\eta+1}\sum_{i=1}^{N}\left\langle \frac{\rho}{\rho_0}\right\rangle_i^\eta$$

Y. Yang et al., PRC 104, 024605 (2021)

for a comparison to BUU models, see M. Colonna et al. (TMEP Coll), PRC 104, 024603 (2021)

Proposed Approximation



- zero range two-body+ zero range density dependent two-body interaction operators

$$V=\sum_{j>i} t_1\,\delta(\vec{R}_i-\vec{R}_j)+\sum_{j>i} t_2\,\rho^{\gamma-1}\left(\frac{\vec{R}_i-\vec{R}_j}{2}\right)\delta(\vec{R}_i-\vec{R}_j)$$

- perform Weyl transform and use the Ansatz for the total wave function of the system as a product of Gaussian wave-packets

$$V=t'_1\sum_i \rho_{\text{int}}(\vec{r}_i)+\frac{t_2}{(\pi L^2)^{3/2}}\sum_{j>i} e^{\frac{-(\vec{r}_i-\vec{r}_j)^2}{L^2}}\, I\left(\gamma;L^2;\frac{\vec{r}_i+\vec{r}_j}{2}\right)$$

$$\rho_{\text{int}}(\vec{r}_i)=\frac{1}{(\pi L^2)^{3/2}}\sum_{j\neq i} e^{\frac{-(\vec{r}_i-\vec{r}_j)^2}{L^2}}$$

$$I(\gamma;L^2;\vec{r}_0)=\frac{1}{(\pi L^2/4)^{3/2}}\int d^3r\,\rho^{\gamma-1}(\vec{r})\,e^{-4(\vec{r}-\vec{r}_0)^2/L^2}$$

- the expression I(…) can be evaluated analytically for γ=2 (besides the trivial case γ=1)

$$I(\gamma=2;L^2;\vec{r}_0)=\frac{1}{(3/4\,\pi L^2)^{3/2}}\sum_k e^{\frac{-(\vec{r}_k-\vec{r}_0)^2}{3/4\,L^2}}=\rho_{3b}(\vec{r}_0;\alpha L^2)$$

make the approximation: $I(\gamma;L^2;\vec{r}_0)=\rho_{3b}^{\gamma-1}(\vec{r}_0;\alpha L^2)$

$$V=t'_1\sum_i \rho_{\text{int}}(\vec{r}_i)+\frac{t_2}{(\pi L^2)^{3/2}}\sum_{j>i} e^{\frac{-(\vec{r}_i-\vec{r}_j)^2}{L^2}}\,\rho_{3b}^{\gamma-1}\left(\frac{\vec{r}_i+\vec{r}_j}{2};\alpha L^2\right)$$

α=3/4 good approximation

$N^2\,approximation\,QMD: the\,above$

$N\,approximation\,QMD: \rho_{3b}^{\gamma-1}\left(\frac{\vec{r}_i+\vec{r}_j}{2};\alpha L^2\right)\approx\frac{1}{2}\left[\rho_{3b}^{\gamma-1}(\vec{r}_i;\alpha L^2)+\rho_{3b}^{\gamma-1}(\vec{r}_j;\alpha L^2)\right]$

$traditional\,QMD:$ $\rho_{3b}^{\gamma-1}\left(\frac{\vec{r}_i+\vec{r}_j}{2};\alpha L^2\right)\approx\frac{1}{2}\left[\rho_{\text{int}}^{\gamma-1}(\vec{r}_i)+\rho_{\text{int}}^{\gamma-1}(\vec{r}_j)\right]$

Required/Desired Future Developments

Theoretical side: - improve time evolution of the reaction using more accurate approximations for 3-body terms of the interaction

- does description of experimental data require a density dependence of the optical potential that deviates from the assumed linear one ?

- improve model to be used for a robust & accurate study of the EoS using reactions above 1 GeV/nucleon (relevance of nucleonic resonances for the evolution of the system)

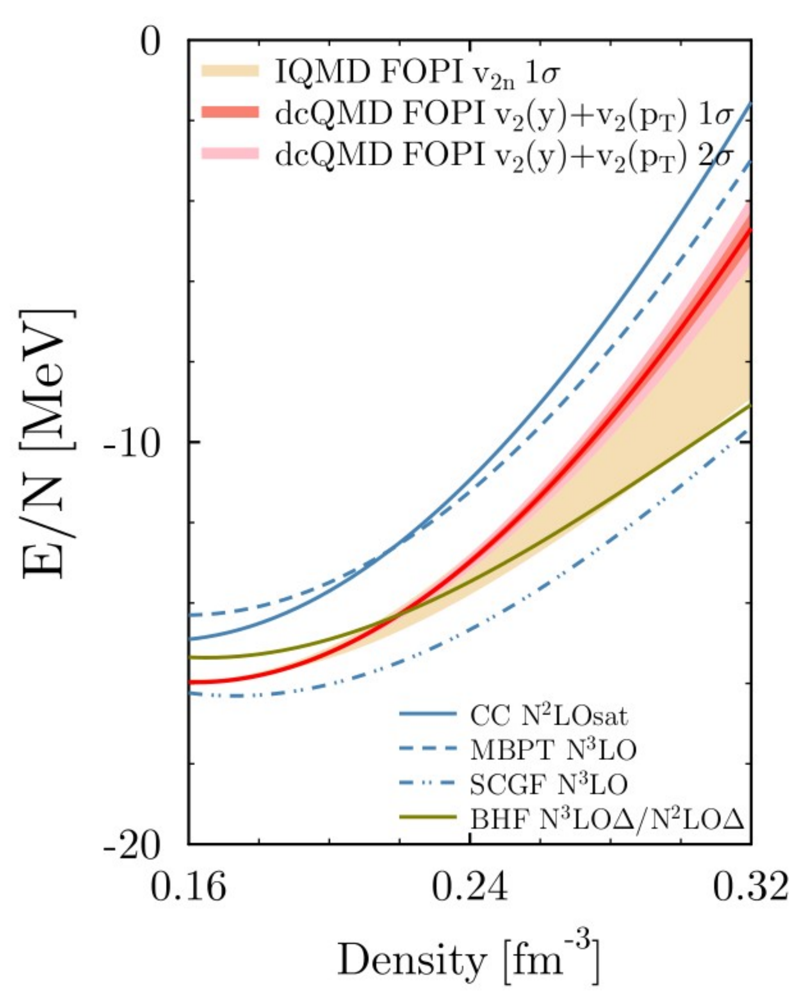
- relativistic dynamics

Experimental side:

- coalescence invariant (H+He at minimum) observables to avoid model dependence on determining final state spectra

- understanding of the observed discrepancy between FOPI and HADES pion multiplicities in AuAu collisions at 1.2 GeV/nucleon (of utermost importance for extracting accurate Information on the EoS in the vicinity of $2\rho_0$ and above)

Summary & Conclusions



- **Study of SNM EoS using nucleonic observables** in AuAu collision of intermediate impact energy (0.15-0.80 GeV/nucleon)
- transverse rapidity spectra: mostly constrain the in-medium modification factor of elastic NN cross-sections
- rapidity dependent transverse flow: moderately accurate value for K_0, points towards stronger in-medium modification of cs at low impact energy
- rapidity dependent elliptic flow: moderately accurate value for K_0, compatible with other similar studies (IQMD+FOPI)
- transverse momentum dependent elliptic flow: very accurate constraint for K_0, some tension with momentum dependence of the empirical optical pot.
- probing higher densities: impact of nucleonic resonances has to be accounted accurately

68% CL Result

$\alpha=0.287 \pm 0.036$

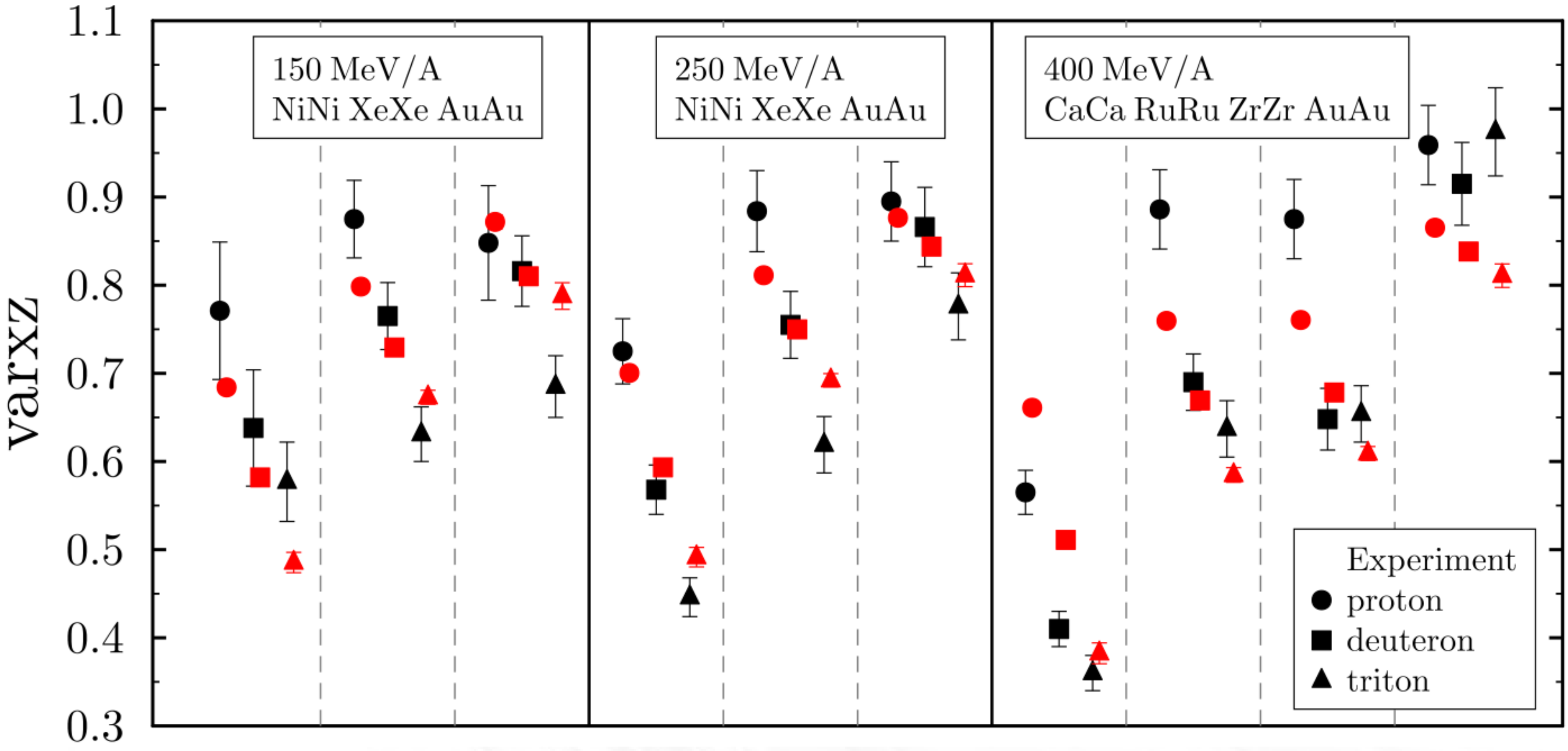
$m^*=0.624 \pm 0.009$

$K_0=236 \pm 6$ MeV

Perspectives: - improve description of reaction dynamics using more accurate estimation of the 3-body term
- extend the model to accurately study EoS above $2\rho_0$ using HIC of impact energy above 1.0 GeV/nucleon (multi-pion decay channels of resonances)

Stopping: Theory vs. Experiment

10 systems (FOPI experiment, see W.Reisdorf et al. NPA 848, 366 (2010)

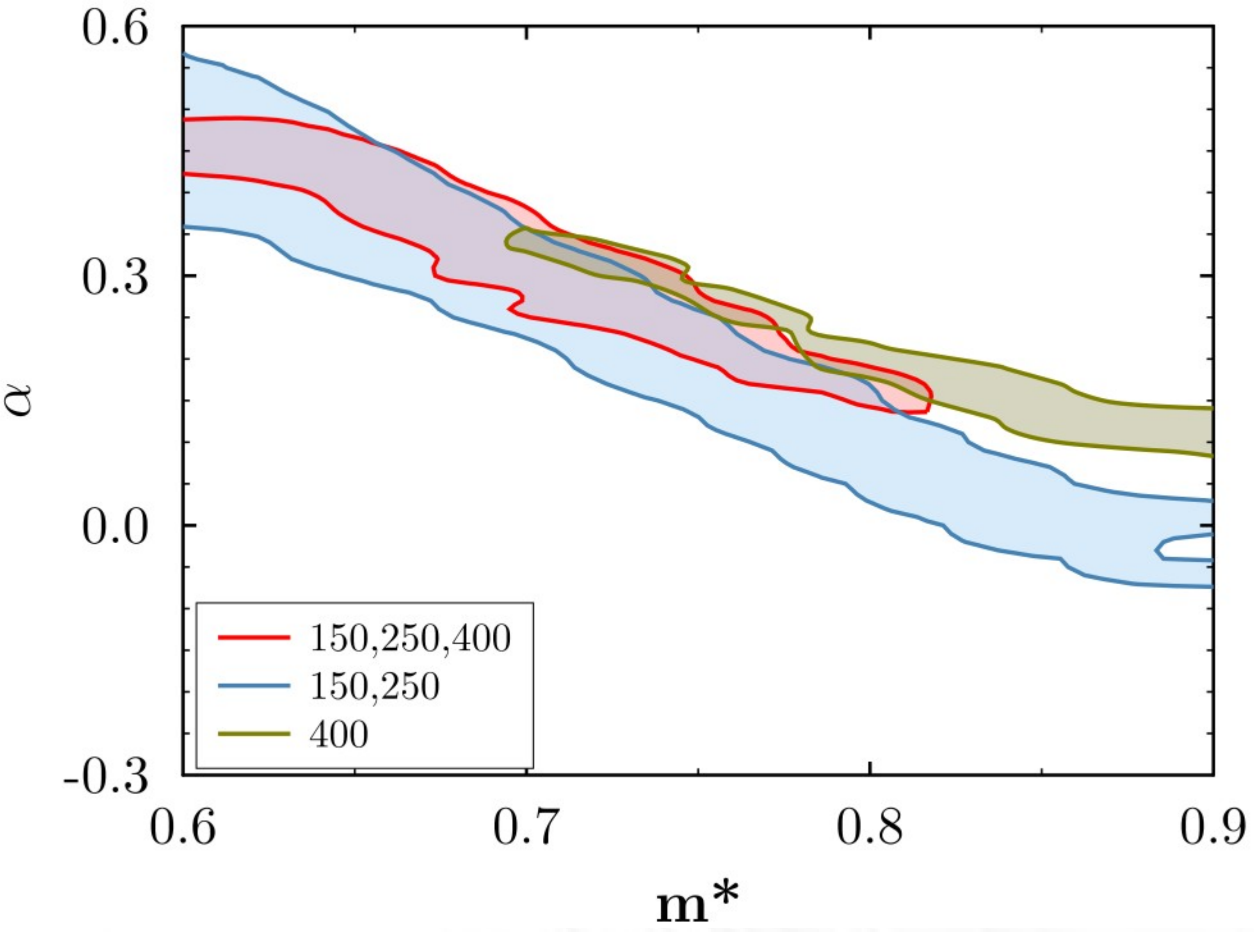


Best fit achieved for: α=0.33, β=-0.60 $m^*=0.705$, $\Delta m^*_{np}=0.250\,\delta$

Quality of the fit: $\chi^2_{min}/\mathrm{d.o.f.}=4.2$

Fitting each specie (p, d or t) separately/alone does improve the quality of the fit

Correlations between parameters



Contours – limits of allowed parameter space at 1σ CL

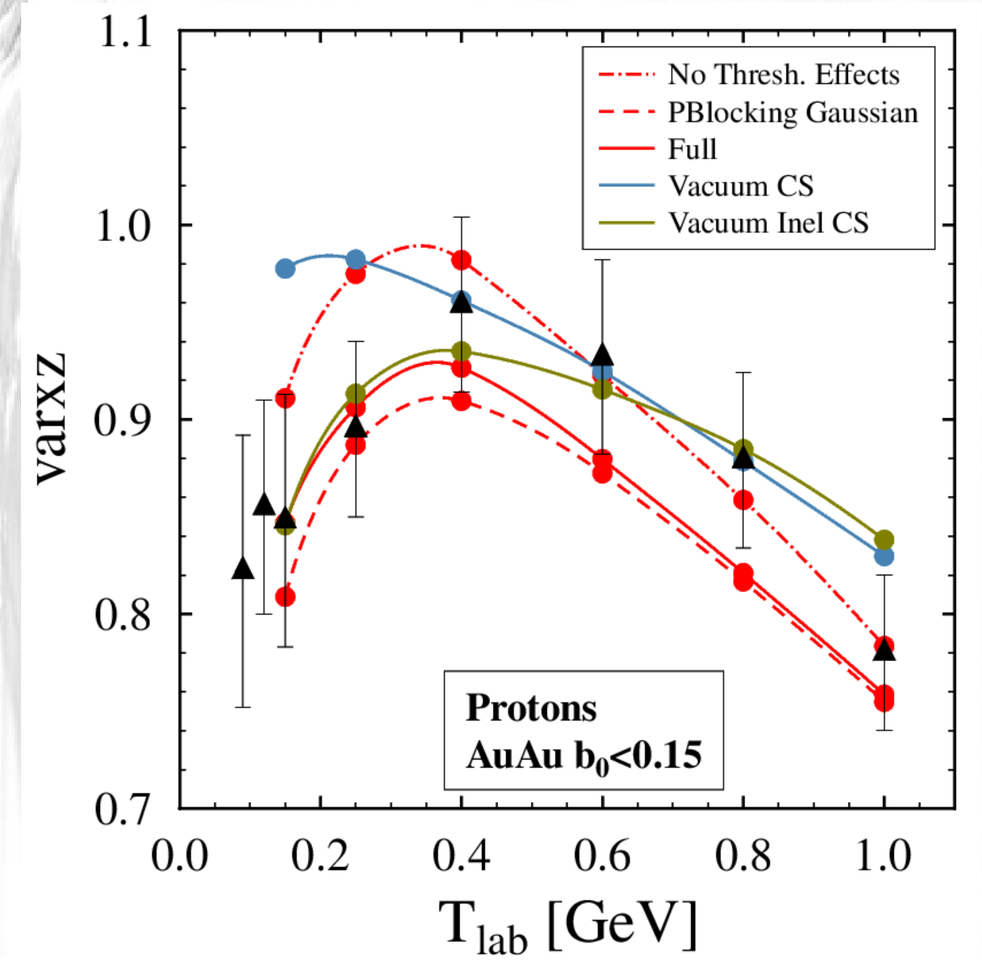
Medium modification factor of cross-sections at saturation varies from 0.65 to 0.90 for the shown range of isoscalar effective mass

Flows – α and m* are correlated (rather than anti-correlated as observed for varxz)

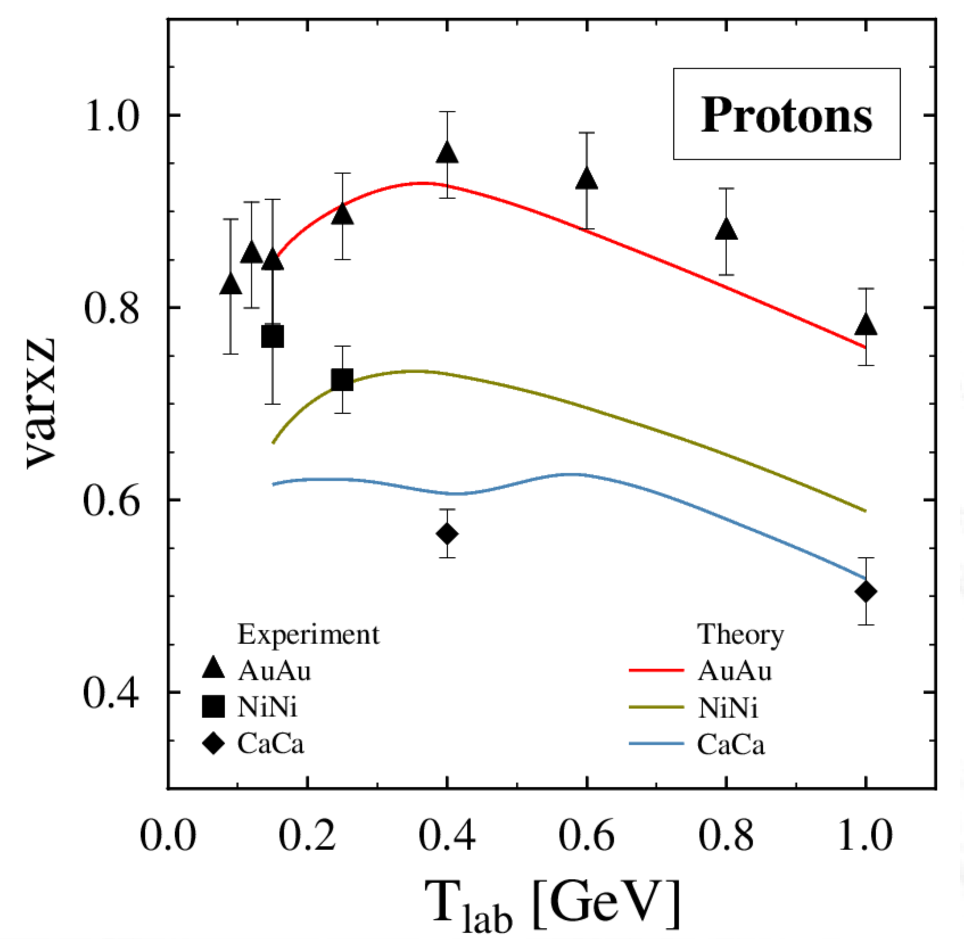
no statistically significant correlation between isovector parameters

Stopping

Model dependence



System size dependence



light cluster experimental stopping underestimated: deuterons (moderately), tritons (severely)

Experimental data: W. Reisdorf et al. (FOPI) NPA 848, 366 (2010)