The Equation of State of Symmetric Nuclear Matter from Intermediate Energy Heavy-Ion Data

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“Dense Nuclear Matter Equation of State from Heavy-Ion Collisions”
5-9 December 2022, INT, USA
Overview

Motivation

Model Details
   dcQMD – interaction parametrization
   Medium modification of cross-section
   Threshold effects
   Initial/final state treatment

Study of EoS of SNM
   Stopping observables
   Transverse flow
   Elliptical flow

Perspectives
   Prerequisites for HIC above 1.0 GeV/nucleon
   Improving reaction dynamics for QMD models

Summary & Conclusions
**Motivation**

- latest version of the dcQMD model used to study symmetry energy with pion production
- empirical effective isoscalar mass $m^*=0.70$
- compressibility modulus close to world average $K_0=245$ MeV
- in-medium modification factor adjusted to qualitatively describe nucleonic observables

J. Estee et al., PRL 126, 162701 (2021)  
D. Cozma et al., EPJA 57, 309 (2021)

possible source of discrepancy at medium $p_T$ values: somewhat unrealistic dynamics

Experimental data: (FOPI) NPA 876, 1 (2012) similar quality of description of exp. data for p,d,t,α in the energy range 150-800 AMeV
Model Details

dcQMD transport model: newest version EPJA 57, 309 (2021)

an upgraded version of TuQMD, see H. Wolter et al.

Prog.Part.Nucl.Phys. 125, 103962 (2022)
Interaction (nucleonic d.o.f.)

momentum dependent potential \textbf{MDI2} - generalization of MDI ofDas, Das Gupta, Gale, Li PRC 67, 034611 (2003)

\[
\frac{E}{N}(\rho, \beta, x, y) = \frac{1}{2} A_1 u + \frac{1}{2} A_2(x, y) u \beta^2 + \frac{B u^\sigma}{\sigma + 1} (1 - x \beta^2) + \frac{D u^2}{3} (1 - y \beta^2)
\]

\[
\frac{1}{u \rho_0^2} \sum_{\tau, \tau'} C_{\tau \tau'} \int \int d^3 p d^3 p' \frac{f_\tau(p, p') f_{\tau'}(p, p')}{1 + (\vec{p} - \vec{p}')^2 / \Lambda^2}
\]

\[
A_2(x, y) = A_2^0 + \frac{2 x B}{\sigma + 1} u^{-1} + \frac{2 y D}{3} u
\]

momentum dependent part: similar with that of J. Xu et al. PRC 91, 014611 (2015)
(see also C. Hartnack, J. Aichelin PRC 49, 2801 (1994))
used previously to test model dependence: flow ratio PRC 88, 44912 (2013)
pion multiplicity ratio PLB 753, 166 (2016)
independent part: extra term (\textit{vary} L vs. K_{sym} and also J_0 vs. K independently)

Fit:
\begin{align*}
U_\infty, K_0, m^* \text{- isoscalar} \\
S(\bar{\nu}), L, K_{sym}, \delta m_{isv} \text{- isovector}
\end{align*}

\[C_I - C_u\]

<table>
<thead>
<tr>
<th>Input</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_0$ [fm$^{-3}$]</td>
<td>0.16</td>
</tr>
<tr>
<td>$m^*_u / m$</td>
<td>0.70</td>
</tr>
<tr>
<td>$\delta_{u-p}(\rho_0, \beta = 0.5)$</td>
<td>0.165</td>
</tr>
<tr>
<td>$K_0$ [MeV]</td>
<td>245.0</td>
</tr>
<tr>
<td>$J_0$ [MeV]</td>
<td>-350.0</td>
</tr>
<tr>
<td>$\tilde{\rho}$ [fm$^{-3}$]</td>
<td>0.10</td>
</tr>
<tr>
<td>$S(\tilde{\rho})$ [MeV]</td>
<td>25.4</td>
</tr>
</tbody>
</table>

from J. Xu et al. PRC 91, 014611 (2015)
Collision Term

**Elastic baryon-baryon collisions**

below pion production threshold: Li-Machleidt

- Li, Machleidt PRC 48, 1702 (1993),
- Li, Machleidt PRC 49, 566 (1994)

above pion production threshold: Cugnon

**Inelastic baryon-baryon, meson-baryon collisions** (related only to pion production)

- collision criterion based on effective masses determined using EoM (consistency with the $dt \rightarrow 0$ fm/c limit)
- in-medium modification of elastic cross-sections

$$\sigma^{med} = f(\rho, \delta) \sigma^{vac}_{mod}$$

$$f(\rho, \delta) = \exp\left[ \alpha \rho / \rho_0 + \beta_1 \delta \rho / \rho_0 + \beta_2 (\tau_1 + \tau_2) \delta \rho / \rho_0 \right]$$

- in-medium modification factor

- flux and phase-space factors computed using effective masses

B.A. Li et al. PRC72, 064611 (2005)

- in-medium modification of elastic cross-sections

$$\sigma^{vac}_{mod} = f(\rho, \delta)$$

- accounts for medium modifications of transition matrix due to departure from the quasi-particle picture

C. Fuchs et al. PRC 64, 024003 (2001)

$$\beta_1 = 0 \text{ and } \beta_2 = 0 \text{ in this study}$$

**Inelastic baryon-baryon, meson-baryon collisions**

- two step process:
  - resonance excitation in baryon-baryon collisions
  - resonance decay:
    - resonance model (all 4* resonances below 2 GeV)
      - charge exchange reactions: NR->NR'

J. Weil et al, PRC 94, 054905 (2016)

- inelastic channels: mass scaling formula

$$\sigma_{N\Delta}(\rho, \beta, p) = \sigma^{vac}_{N\Delta}(p) \frac{\mu_{ini}(\rho, p)}{\mu_{ini}^{vac}(p)} \frac{\mu_{fin}(\rho, p)}{\mu_{fin}^{vac}(p)}$$


See also Larionov et al., NPA 728, 135 (2003)
Threshold Effects (dcQMD)

- direct consequence of imposing (total) energy conservation in the medium

\[
\sqrt{p_1^2 + m_1^2 + U(p_1)} + \sqrt{p_2^2 + m_2^2 + U(p_2)} = \sqrt{p_1'^2 + m_1'^2 + U(p_1')} + \sqrt{p_2'^2 + m_2'^2 + U(p_2')}
\]

- rarely considered in transport models below 1 AGeV, with a few exceptions:
  RBUU: G. Ferini et al. PRL 97, 202301 (2006), RVUU: T. Song, C.M. Ko PRC 91, 014901 (2015);
  χBUU: Z. Zhang et al, PRC 98, 054614 (2018)

- required for thermodynamical consistency of the model

- reactions: \(NN \leftrightarrow NR, R \leftrightarrow N\pi\) (\(R \leftrightarrow N\pi\pi\) not corrected)

- assumptions (dcQMD): - two-body collisions are part of N-body one
  - in-medium two-body collisions modeled as a succession of bare (vacuum-like) collisions followed/preceded by energy exchanges with the fireball, while momentum is conserved
  - reaction with highest probability: corresponds to the one which included the bare collision of highest probability

Example: \(NN \rightarrow N\Delta\)

\[
\sigma_{NN \rightarrow N\Delta}^{(med)}(s^*) = \frac{\mu_{(ini)*}}{\mu_{(ini)}} \cdot \frac{\mu_{(fin)*}}{\mu_{(fin)}} \cdot \sigma_{NN \rightarrow N\Delta}^{(vac)}(s^*)
\]

\[s^* = \text{Max}\{s_{ini}^*, s_{fin}^*\}\]

Introduced in TuQMD/dcQMD in DC, PLB 753, 166 (2016)
**Initial/Final State**

**Initial state density profile of nuclei**

- nuclei initialized with realistic charge radii and neutron skins
- larger $L_N^2$ leads to stronger tails and consequently lower reduced impact parameter (flow at projectile/target rapidities affected most visibly)

*this study:* $L_N^2 = 5.0 \text{ fm}^2$

**Minimum spanning tree (MST) algorithm**

all clusters with $A \leq 15$, 23 additional $A > 15$ (B,C,N,O)

**Stable** : lifetime > 1ms
**Unstable** : decay into stable using known decay channels

Au+Au @ 400 AmeV $b < 2.0 \text{ fm}$

*this study:* $\delta r = 4.0 \text{ fm}$, $\delta p = 0.2 \text{ GeV/c}$
Study EoS of SMN

stopping and flow observables for protons and light clusters in AuAu collisions of impact energy 0.15-0.80 GeV/nucleon (FOPI Coll)
Rapidy Spectra

- used in the past to fix in-medium modification factor of elastic cross-sections
  
  see e.g. P. Danielewicz et al., Science 298, 1592 (2002)

- varxz (H) and constrained transverse CI rapidity spectra for AuAu used in this study
  
  FOPI exp data: W. Reisdorf et al. NPA 848, 366 (2010)

ratio of transverse-to-longitudinal variances of rapidy spectra

weaker correlation between $\alpha$ and $K_0$ also evidenced
Rapidity Spectra

- sizable sensitivity of varxz observables to EoS has been previously evidenced
  W. Reisdorf et al. NPA 848, 366 (2010)

- current model reproduces impact energy dependence of varxz observables
  EPJA 57, 309 (2021)

$K_0 = 218 \pm 40$ MeV

- different $(\alpha, m^*)$ values depending on choice of observable(s)
- medium modification factor of cross-sections at $\rho_0$ and $p_F$ is similar ($\sim 0.65$)
Proton Transverse Flow

Experimental data set: proton rapidity dependent $v_1$

- $u \tau_0 > 0.8 - 150, 250 \text{ AuAu } 0.25 < b_0 < 0.45$
- $u \tau_0 > 0.4 - 400, 600, 800 \text{ AuAu } 0.25 < b_0 < 0.45$

68% CL Result

$\alpha = 0.040 - 0.051 + 0.067$

$m^* = 0.936 - 0.032 + 0.022$

$K_0 = 200 - 22 + 31 \text{ MeV}$

stronger in-medium modification factor at lower impact energies possibly connected to an insufficient Pauli blocking of final state of two-body collisions

W. Reisdorf et al., NPA 876, 1 (2012)
Proton Elliptic Flow

Experimental data set: proton rapidity dependent $v_2$
$ut0=p_T/p_p > 0.8$ – 150-800 AuAu $0.25<b_0<0.45$

W. Reisdorf et al., NPA 876, 1 (2012)

68% CL Result
$\alpha=0.130-0.063+0.090$
$m^*=0.751-0.036+0.030$
$K_0=248-18+18$ MeV

IQMD protons $v_{2n}$: $K_0=232\pm30$ MeV (light cluster soften the reported combined result)

A. Le Fevre et al., NPA 945, 112 (2016)
**$P_T$ dependent Elliptic Flow**

**Experimental data set:** $p,d,t$ transverse momentum dependent $v_2$

$|y|<0.4 - 150-800$ AuAu $0.25<b_0<0.45$

W. Reisdorf et al., NPA 876, 1 (2012)

**68% CL Result**

$\alpha = 0.192 - 0.048 + 0.049$

$m^* = 0.621 - 0.008 + 0.008$

$K_0 = 250 - 8 + 8$ MeV

**PRELIMINARY**

Model dependence due to coalescence afterburner not accounted!

**IQMD full result $v_{2n}$:** $K_0 = 190 \pm 30$ MeV  (light clusters: $p,d,t,\alpha$)

A. Le Fevre et al., NPA 945, 112 (2016)
Elliptic Flow (Combined Result)

Experimental data set:  

- $p$ rapidity dependent $v_2$
- $p,d,t$ transverse momentum dependent $v_2$

Inclusion of $v_1$ data leads to a sub-optimal fit ($\chi^2$/dof~2)

68% CL Result

$\alpha=0.287 \pm 0.036$
$m^*=0.624 \pm 0.009$
$K_0=236 \pm 6$ MeV

Microscopic calculations:

1. A. Ekström et al., PRC 91, 051301 (2015)
2. C. Drischler et al., PRC 102, 054315 (2020)
3. A. Carbone, PRR 2, 023227 (2020)

IQMD result:

A. Le Fevre et al., NPA 945, 112 (2016)
Perspectives
Pion production in AuAu at 1.23 AGeV

J. Adamczewski-Musch et al. (HADES), EPJA 56, 259 (2020)

Note: HADES pion total yield are larger than FOPI yields at 1.2 GeV by about 30%

very preliminary calculation addressing the feasibility of studying the symmetry energy

-alternative approach to a systematic description of HADES rapidity and transverse mass spectra: K. Godbey et al. PLB 829, 137134 (2022)
Sensitivity to Resonance Meanfield

chosen impact energy: 1.0 GeV/nucleon (AuAu)
about 20% of nucleons excited into $\Delta(1232)$ at the highest density for central collisions

compressibility modulus: extracted from $v^2(y)$
impact of $\Delta(1232)$ potential on proton $v^2(y)$: ~ 15% (1.5$\sigma$)
 omission equivalent to $\delta K_0 = -30$ MeV

impact on $v^2_n/v^2_Y$: ~5% equivalent to $\delta L = 10$ MeV
Momentum dependence

elliptic flow constraint for effective mass: \( m^* = 0.624 \pm 0.009 \)

possible reasons: compensate for unrealistic density evolution; high density dependence of optical potential deviates from the assumed linear

for a comparison to BUU models, see M. Colonna et al. (TMEP Coll), PRC 104, 024603 (2021)
Proposed Approximation

- zero range two-body + zero range density dependent two-body interaction operators

\[ V = \sum_{j>i} t_1 \delta(\vec{R}_i - \vec{R}_j) + \sum_{j>i} t_2 \rho^{y-1}(\vec{R}_i - \vec{R}_j) \frac{1}{2} \delta(\vec{R}_i - \vec{R}_j) \]

- perform Weyl transform and use the Ansatz for the total wave function of the system as a product of Gaussian wave-packets

\[ V = t'_1 \sum_i \rho_{\text{int}}(\vec{r}_i) + \frac{t_2}{(\pi L^2)^{3/2}} \sum_{j>i} \frac{e^{-L^2 \rho^{y-1}(\vec{r}_i - \vec{r}_j)^2}}{L^2} I(\gamma; L^2; \vec{r}_i + \vec{r}_j) \]

\[ \rho_{\text{int}}(\vec{r}_i) = \frac{1}{(\pi L^2)^{3/2}} \sum_{j\neq i} \frac{e^{-L^2 \rho^{y-1}(\vec{r}_i - \vec{r}_j)^2}}{L^2} \]

\[ I(\gamma; L^2; \vec{r}_0) = \frac{1}{(\pi L^2/4)^{3/2}} \int d^3r \rho^{y-1}(\vec{r}) e^{-4(\vec{r} - \vec{r}_0)^2/L^2} \]

- the expression \( I(\ldots) \) can be evaluated analytically for \( \gamma = 2 \) (besides the trivial case \( \gamma = 1 \))

\[ I(\gamma = 2; L^2; \vec{r}_0) = \frac{1}{(\pi L^2)^{3/2}} \sum_k \frac{e^{-L^2 \rho^{y-1}(\vec{r}_i)^2}}{\alpha L^2} = \rho_{b}^{y-1}(\vec{r}_0; \alpha L^2) \]

make the approximation:

\[ I(\gamma; L^2; \vec{r}_0) \approx \rho_{b}^{y-1}(\vec{r}_0; \alpha L^2) \]

\[ V = t'_1 \sum_i \rho_{\text{int}}(\vec{r}_i) + \frac{t_2}{(\pi L^2)^{3/2}} \sum_{j>i} \frac{e^{-L^2 \rho^{y-1}(\vec{r}_i - \vec{r}_j)^2}}{L^2} \rho_{b}^{y-1}(\frac{\vec{r}_i + \vec{r}_j}{2}; \alpha L^2) \]

\[ \alpha = 3/4 \] good approximation

N^2 approximation QMD: the above

N approximation QMD:
\[ \rho_{b}^{y-1}(\frac{\vec{r}_i + \vec{r}_j}{2}; \alpha L^2) \approx \frac{1}{2} [\rho_{\text{int}}^{y-1}(\vec{r}_i; \alpha L^2) + \rho_{\text{int}}^{y-1}(\vec{r}_j; \alpha L^2)] \]

traditional QMD:
\[ \rho_{b}^{y-1}(\frac{\vec{r}_i + \vec{r}_j}{2}; \alpha L^2) \approx \frac{1}{2} [\rho_{\text{int}}^{y-1}(\vec{r}_i) + \rho_{\text{int}}^{y-1}(\vec{r}_j)] \]
Required/Desired Future Developments

**Theoretical side:**
- improve time evolution of the reaction using more accurate approximations for 3-body terms of the interaction

  - does description of experimental data require a density dependence of the optical potential that deviates from the assumed linear one?

  - improve model to be used for a robust & accurate study of the EoS using reactions above 1 GeV/nucleon (relevance of nucleonic resonances for the evolution of the system)

- relativistic dynamics

**Experimental side:**

- coalescence invariant (H+He at minimum) observables to avoid model dependence on determining final state spectra

- understanding of the observed discrepancy between FOPI and HADES pion multiplicities in AuAu collisions at 1.2 GeV/nucleon (of utmost importance for extracting accurate Information on the EoS in the vicinity of $2\rho_0$ and above)
Summary & Conclusions

- **Study of SNM EoS using nucleonic observables** in AuAu collision of intermediate impact energy (0.15-0.80 GeV/nucleon)

- **Transverse rapidity spectra**: mostly constrain the in-medium modification factor of elastic NN cross-sections

- **Rapidity dependent transverse flow**: moderately accurate value for $K_0$, points towards stronger in-medium modification of $c_s$ at low impact energy

- **Rapidity dependent elliptic flow**: moderately accurate value for $K_0$, compatible with other similar studies (IQMD+FOPI)

- **Transverse momentum dependent elliptic flow**: very accurate constraint for $K_0$, some tension with momentum dependence of the empirical optical pot.

- **Probing higher densities**: impact of nucleonic resonances has to be accounted accurately

**Perspectives:**

- Improve description of reaction dynamics using more accurate estimation of the 3-body term

- Extend the model to accurately study EoS above $2\rho_0$ using HIC of impact energy above 1.0 GeV/nucleon (multi-pion decay channels of resonances)

**68% CL Result**

- $\alpha = 0.287 \pm 0.036$
- $m^* = 0.624 \pm 0.009$
- $K_0 = 236 \pm 6$ MeV
Stopping: Theory vs. Experiment

10 systems (FOPI experiment, see W. Reisdorf et al. NPA 848, 366 (2010))

Best fit achieved for: \( \alpha = 0.33, \beta = -0.60 \)
\( m^* = 0.705, \Delta m^*_{np} = 0.250 \delta \)

Quality of the fit: \( \chi^2_{\text{min}} / \text{d.o.f.} = 4.2 \)

Fitting each specie (p, d or t) separately/alone does improve the quality of the fit
Correlations between parameters

- No statistically significant correlation between isovector parameters

- Contours – limits of allowed parameter space at 1σ CL

- Medium modification factor of cross-sections at saturation varies from 0.65 to 0.90 for the shown range of isoscalar effective mass

- Flows – $\alpha$ and $m^*$ are correlated (rather than anti-correlated as observed for varxz)

- No statistically significant correlation between isovector parameters
Stopping

Model dependence

System size dependence

light cluster experimental stopping underestimated: deuterons (moderately), tritons (severely)

Experimental data: W. Reisdorf et al. (FOPI) NPA 848, 366 (2010)