$0\nu\beta\beta$ decay matrix elements with effective Hamiltonians and operators from ChEFT

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Recommended Values for $0\nu\beta\beta$ *Decay Nuclear Matrix Elements*

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  - S. Pastore

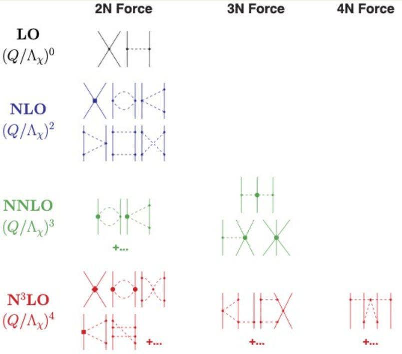
EFT and nuclear structure

- The introduction of effective field theory (EFT) represents, nowadays, a major turning point in studying the nuclear many-body problem
- Starting from the symmetries of QCD, nuclear Hamiltonians and electroweak currents can be constructed consistently through chiral perturbation theory (ChPT)
- This framework should enforce the predictive power of nuclear structure calculations

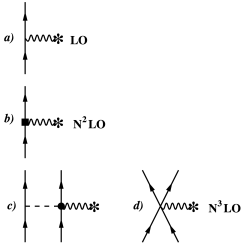
EFT and electroweak decays: GT transitions

- The EFT approach to the derivation of electroweak operator currents has been successfully applied to the study of GT transitions
- Both nuclear potentials and GT operators may be consistently constructed, and three-body forces, as well as two-body GT currents (2BC), show up on equal footing with two-body forces and one-body decay components, respectively, within the ChPT expansion

Nuclear Hamiltonian



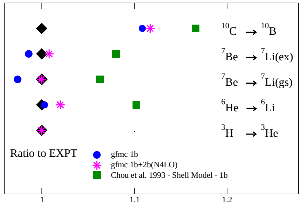
Two-body currents



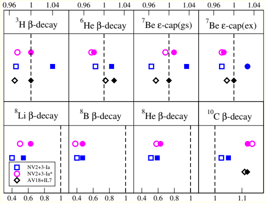
The contribution of 2BC has proved to be crucial to overcome the "quenching" problem of calculated GT transitions

EFT and β-decay: *ab initio* calculations

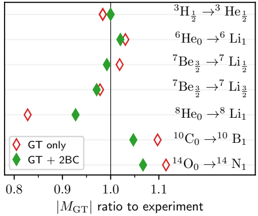
β-decay processes of light- and medium-mass nuclei have been calculated with *ab initio* methods and including contributions from 2BC, reproducing satisfactorily GT nuclear matrix elements

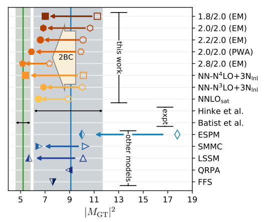


S. Pastore et al., Phys. Rev. C **97** *022501(R) (2018)*



G. B. King et al., Phys. Rev. C **102** *025501 (2020)*





P. Gysbers et al., Nat. Phys. **15** *428 (2019)*

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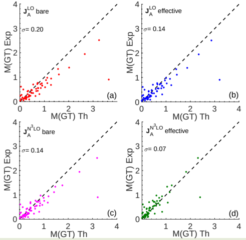
EFT and β-decay: nuclear shell model

ChEFT Hamiltonians and GT decay-operators have been also successfully employed within realistic shell model (RSM), which allows to construct SM effective Hamiltonians (H_{eff}) and transition operators (Θ_{eff}) starting from realistic nuclear forces.



GT matrix elements of 60 experimental decays of 43 $0f1p$-shell nuclei, only yrast states are considered

(a) bare $\mathbf{J}_A$ at LO in ChPT (namely the GT operator $g_A \boldsymbol{\sigma} \cdot \boldsymbol{\tau}$);

(b) effective $\mathbf{J}_A$ at LO in ChPT;

(c) bare $\mathbf{J}_A$ at N^{3}LO in ChPT (namely includy 2BC contributions too);

(d) effective $\mathbf{J}_A$ at N^{3}LO in ChPT.

Two-neutrino double-β decay ($2\nu\beta\beta$) nuclear matrix elements

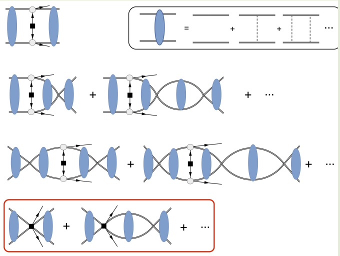
$2\nu\beta\beta$ decay	(a)	(b)	(c)	(d)	Expt
^{48}Ca$\rightarrow$^{48}Ti	0.057	0.048	0.033	0.019	0.042 ± 0.004
^{76}Ge$\rightarrow$^{76}Se	0.211	0.153	0.160	0.118	0.129 ± 0.004
^{82}Se$\rightarrow$^{82}Kr	0.173	0.123	0.136	0.095	0.103 ± 0.001

L. C., N. Itaco, G. De Gregorio, A. Gargano, Z. H. Cheng, Y. Z. Ma, F. R. Xu, and M. Viviani, Phys. Rev. C **109**, 014301 (2024)

EFT and $0\nu\beta\beta$ decay

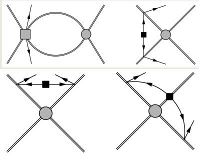
In recent years, EFT has been also applied to construct electroweak currents for the $0\nu\beta\beta$ decay, performing a ChPT expansion of the two-body current up to N^{2}LO

Leading order



V. Cirigliano, W. Dekens, J. de Vries, M. Hoferichter, and E. Mereghetti, Phys. Rev. Lett. **126***, 172002 (2021)*

Next-to-next-to-leading order



V. Cirigliano, W. Dekens, E. Mereghetti, and A. Walker-Loud, Phys. Rev. C **97***, 065501 (2018)*

The novelty of EFT derivation of $0\nu\beta\beta$ operator currents is the emergence at LO of a contact operator, to guarantee the renormalizability of the process

This short-range component is characterized only by a new LEC – $g_\nu^{\rm NN}$ –, that has to be renormalized by evaluating the $nn \rightarrow ppe^- e^-$ amplitude

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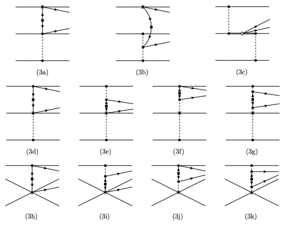
EFT and $0\nu\beta\beta$ decay

At N^{3}LO, also three-body contributions to the $0\nu\beta\beta$ decay operator (3BC) occur within the ChPT expansion

Three-body currents



G. Chambers-Wall, J. Lieffers, G. B. King, E. Mereghetti, S. Pastore, M. Piarulli, and R. B. Wiringa, Phys. Rev. C **113***, 025502 (2026)*

Then, a thorough investigation of the perturbative properties of the ChPT expansion of the $0\nu\beta\beta$ operator, for decays that are of current experimental interst, is a relevant topic

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EFT and $0\nu\beta\beta$ decay

Castillo, Jokiniemi, Soriano, and Menéndez have already performed a study of the ChPT expansion up to N^{2}LO, within the framework of the pnQRPA and nuclear shell model (ISM)

Table 1
Central values and ranges of all NMEs up to N^{2}LO: LO long range (second and sixth columns), LO short range (third and seventh), N^{2}LO total ultrasoft (fourth and eighth) and N^{2}LO soft loops (fifth and ninth). The NME ranges cover two interactions for the NSM or a range of g_{pp} values for the pnQRPA, as well as various couplings $g_\nu^{\rm NN}$, Gaussian regulators $\Lambda = 349 \cdots 550$ MeV, CD-Bonn and Argonne SRCs, the renormalization scale range $\mu = 500 \cdots 1500$ MeV, and the ultrasoft scale $\mu_{\rm us} = m_\pi/2 \cdots 2m_\pi$ (see text for details).

Nucleus	pnQRPA				NSM			
	LO		N^{2}LO		LO		N^{2}LO	
	L	S	tot usoft	loop soft	L	S	tot usoft	loop soft
^{48}Ca					0.92(14)	0.43(20)	0.01(6)	0.05(7)
^{76}Ge	5.23(58)	2.65(116)	0.39(31)	0.28(41)	3.57(25)	0.97(48)	-0.29(9)	0.05(16)
^{82}Se	4.56(36)	2.26(99)	0.30(19)	0.21(34)	3.38(20)	0.91(43)	-0.27(8)	0.05(15)
^{96}Zr	4.28(24)	2.22(98)	0.49(23)	0.25(33)				
^{100}Mo	3.44(73)	2.96(130)	1.15(52)	0.38(43)				
^{116}Cd	4.85(38)	1.95(85)	-0.12(3)	0.15(28)				
^{124}Sn	5.20(32)	2.99(130)	0.59(35)	0.37(46)	2.79(63)	1.06(52)	-0.23(13)	0.06(16)
^{130}Te	3.70(34)	2.12(94)	0.46(38)	0.31(34)	2.68(79)	1.07(50)	-0.23(15)	0.06(16)
^{136}Xe	2.99(28)	1.36(60)	0.06(14)	0.16(32)	2.26(53)	0.86(41)	-0.19(11)	0.05(13)

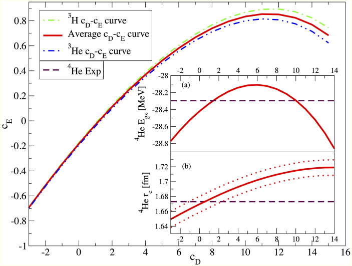
D. Castillo, L. Jokiniemi, P. Soriano, and J. Menéndez, Phys. Lett. B **860**, *139181 (2025)*

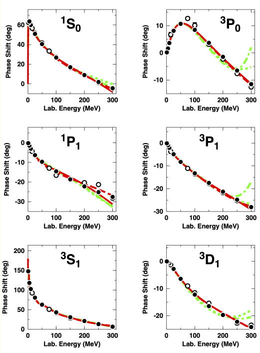
The study has been carried out by using phenomenological Hamiltonians, and varying the LECs associated at each vertex.

The general conclusion is that corrections at N^{2}LO are about one order of magnitude smaller than LO

Shell model and EFT

- Nuclear Hamiltonian: Entem-Machleidt N^{3}LO two-body potential plus N^{2}LO three-body potential





- $0\nu\beta\beta$ operator currents for the two-body component have been expanded up to N^{2}LO, and included also contributions of the 3BC at N^{3}LO, all LECs are consistent with the 2NF and 3NF potentials
- $H_{\rm eff}$ calculated at 3rd order in perturbation theory
- Effective operators are consistently derived using MBPT

The effective operators for decay amplitudes

- Ψ_α eigenstates of the full Hamiltonian H with eigenvalues E_α
- Φ_α eigenvectors obtained diagonalizing $H_{\rm eff}$ in the model space P and corresponding to the same eigenvalues E_α

$$\Rightarrow |\Phi_\alpha\rangle = P\,|\Psi_\alpha\rangle$$

Obviously, for any decay-operator Θ:

$$\langle\Phi_\alpha|\Theta|\Phi_\beta\rangle \neq \langle\Psi_\alpha|\Theta|\Psi_\beta\rangle$$

We then require an effective operator $\Theta_{\rm eff}$ defined as follows

$$\Theta_{\rm eff} = \sum_{\alpha\beta} |\Phi_\alpha\rangle\,\langle\Psi_\alpha|\Theta|\Psi_\beta\rangle\,\langle\Phi_\beta|$$

Consequently, the matrix elements of $\Theta_{\rm eff}$ are

$$\langle\Phi_\alpha|\Theta_{\rm eff}|\Phi_\beta\rangle = \langle\Psi_\alpha|\Theta|\Psi_\beta\rangle$$

This means that the parameters characterizing $\Theta_{\rm eff}$ are renormalized with respect to $\Theta \Rightarrow g_A^{\rm eff} = q \cdot g_A \neq g_A$

The effective shell-model Hamiltonian

We start from the many-body Hamiltonian H defined in the full Hilbert space:

$$H = H_0 + H_1 = \sum_{i=1}^{A}(T_i + U_i) + \sum_{i<j}(V_{ij}^{NN} - U_i)$$

$$\left(\begin{array}{c|c} PHP & PHQ \\ \hline QHP & QHQ \end{array}\right) \quad \begin{array}{c} \mathcal{H} = \Omega^{-1} H \Omega \\ \Longrightarrow \\ Q\mathcal{H}P = 0 \end{array} \quad \left(\begin{array}{c|c} P\mathcal{H}P & P\mathcal{H}Q \\ \hline 0 & Q\mathcal{H}Q \end{array}\right)$$

$$H_{\mathrm{eff}} = P\mathcal{H}P$$

Suzuki & Lee $\Rightarrow \Omega = e^{\omega}$ with $\omega = \left(\begin{array}{c|c} 0 & 0 \\ \hline Q\omega P & 0 \end{array}\right)$

$$H_1^{\mathrm{eff}}(\omega) = PH_1P + PH_1Q\frac{1}{\epsilon - QHQ}QH_1P - PH_1Q\frac{1}{\epsilon - QHQ}\omega H_1^{\mathrm{eff}}(\omega)$$

The perturbative approach to the shell-model $H^{\rm eff}$

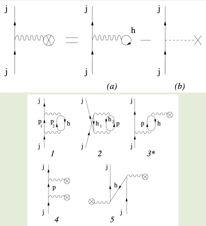
The $\hat{Q}$-box vertex function

$$\hat{Q}(\epsilon) = PH_1P + PH_1Q\frac{1}{\epsilon - QHQ}QH_1P$$

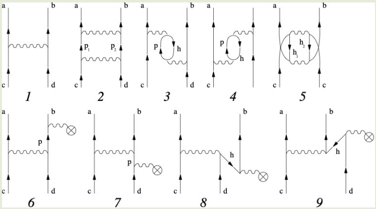
Exact calculation of the $\hat{Q}$-box is computationally prohibitive for many-body system $\Rightarrow$ we perform a perturbative expansion

$$\frac{1}{\epsilon - QHQ} = \sum_{n=0}^{\infty}\frac{(QH_1Q)^n}{(\epsilon - QH_0Q)^{n+1}}$$

$\hat{Q}$-box: 1st- & 2nd-order 1-b diagrams



$\hat{Q}$-box: 1st- & 2nd-order 2-b diagrams



The effective SM operators for decay amplitudes

Any shell-model effective operator may be derived consistently with the $\hat{Q}$-box-plus-folded-diagram approach to $H_{\rm eff}$

It has been demonstrated that, for any bare operator Θ, a non-Hermitian effective operator $\Theta_{\rm eff}$ can be written in the following form:

$$\begin{aligned}\Theta_{\rm eff} \quad &= \quad (P + \hat{Q}_1 + \hat{Q}_1 \hat{Q}_1 + \hat{Q}_2 \hat{Q} + \hat{Q} \hat{Q}_2 + \cdots)(\chi_0 + \\ &\quad +\chi_1 + \chi_2 + \cdots) \ ,\end{aligned}$$

where

$$\hat{Q}_m = \frac{1}{m!} \frac{d^m \hat{Q}(\epsilon)}{d\epsilon^m} \bigg|_{\epsilon=\epsilon_0} \ ,$$

ϵ_0 being the model-space eigenvalue of the unperturbed Hamiltonian H_0

K. Suzuki and R. Okamoto, Prog. Theor. Phys. **93** *, 905 (1995)*

The effective SM operators for decay amplitudes

The χ_n operators are defined in terms of the vertex function $\hat{\Theta}$ as:

$$
\begin{aligned}
\chi_0 &= (\hat{\Theta}_0 + h.c.) + \Theta_{00} \quad , \\
\chi_1 &= (\hat{\Theta}_1 \hat{Q} + h.c.) + (\hat{\Theta}_{01} \hat{Q} + h.c.) \quad , \\
\chi_2 &= (\hat{\Theta}_1 \hat{Q}_1 \hat{Q} + h.c.) + (\hat{\Theta}_2 \hat{Q}\hat{Q} + h.c.) + \\
&\quad (\hat{\Theta}_{02} \hat{Q}\hat{Q} + h.c.) + \hat{Q}\hat{\Theta}_{11} \hat{Q} \quad , \\
&\quad \ldots
\end{aligned}
$$

and

$$\hat{\Theta}(\epsilon) = P\Theta P + P\Theta Q \frac{1}{\epsilon - QHQ} QH_1 P$$

$$
\begin{aligned}
\hat{\Theta}(\epsilon_1; \epsilon_2) = PH_1 Q \frac{1}{\epsilon_1 - QHQ} \times \\
Q\Theta Q \frac{1}{\epsilon_2 - QHQ} QH_1 P
\end{aligned}
$$

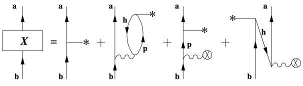
$$\hat{\Theta}_m = \frac{1}{m!} \frac{d^m \hat{\Theta}(\epsilon)}{d\epsilon^m}\bigg|_{\epsilon=\epsilon_0}$$

$$\hat{\Theta}_{nm} = \frac{1}{n!m!} \frac{d^n}{d\epsilon_1^n} \frac{d^m}{d\epsilon_2^m} \hat{\Theta}(\epsilon_1; \epsilon_2)\bigg|_{\epsilon_{1,2}=\epsilon_0}$$

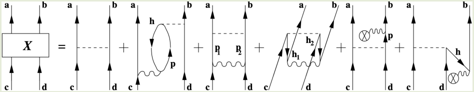
The effective SM operators for decay amplitudes

The $\hat{\Theta}$-box is then calculated perturbatively, here are diagrams up to 2nd order of the effective decay operator Θ_{eff} expansion:

One-body operator



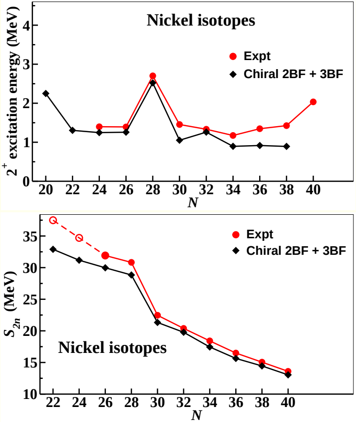
Two-body operator



- *L. C., L. De Angelis, T. Fukui, A. Gargano, and N. Itaco, Phys. Rev. C* **95**,
- *L. C., L. De Angelis, T. Fukui, A. Gargano, N. Itaco, and F. Nowacki, Phys. Rev. C* **100**, *014316 (2019).*
- *L. C., A. Gargano, N. Itaco, R. Mancino, and F. Nowacki, Phys. Rev. C* **101**, *044315 (2020).*
- *L. C., N. Itaco, G. De Gregorio, A. Gargano, R. Mancino, and F. Nowacki, Phys. Rev. C* **105**, *034312 (2022).*

$0f1p$-shell nuclei

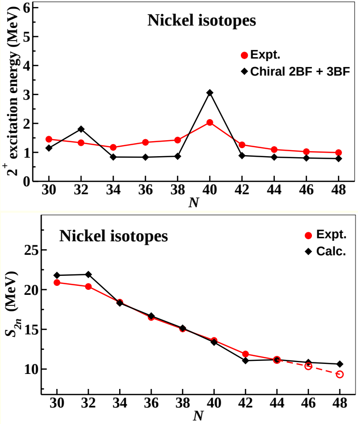
- Model space spanned by 4 proton and neutron orbitals $0f_{7/2}, 0f_{5/2}, 1p_{3/2}, 1p_{1/2}$
- Effects of induced 3-body forces have been included
- Single-particle energies and residual two-body interaction are derived from the theory. No empirical input



Y. Z. Ma, L. C., L. De Angelis, T. Fukui, A. Gargano, N. Itaco, and F. R. Xu, Phys. Rev. C **100**, *034324 (2019)*

$0f1p0g$-shell nuclei

- Model space spanned by 4 proton and neutron orbitals $0f_{5/2}, 1p_{3/2}, 1p_{1/2}, 0g_{9/2}$
- Effects of induced 3-body forces have been included
- Single-particle energies and residual two-body interaction are derived from the theory. No empirical input



L. C., N. Itaco, G. De Gregorio, A. Gargano, Z. H. Cheng, Y. Z. Ma, F. R. Xu, and M. Viviani, Phys. Rev. C **109**, *014301 (2024)*

Shell-model calculations and results

Nuclear model $\Rightarrow$ Accurate reproduction of experimental data

$\Downarrow$

Predictive power

ISM calculations, starting from ChPT two- and three-body potentials and two-body meson-exchange currents, for ^{48}Ca, ^{76}Ge, ^{82}Se

$\Downarrow$

Check our approach calculating spectroscopic and spin-isospin observables ($2\nu\beta\beta$-decay nuclear matrix elements $M^{2\nu}$s), and validating the results with experiment

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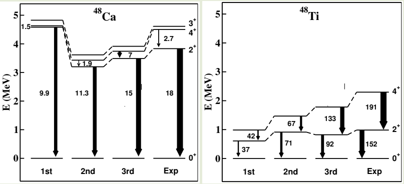
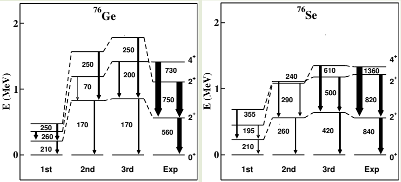
Spectroscopy: theory vs experiment

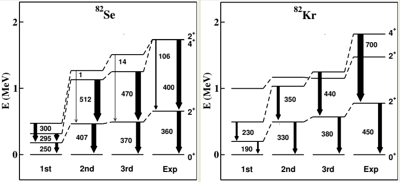
$0f1p$-shell nuclei



$0f_{5/2}1p0g_{9/2}$-shell nuclei



$0f_{5/2}1p0g_{9/2}$-shell nuclei



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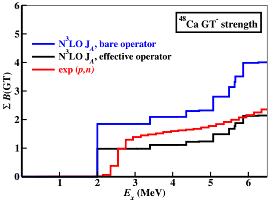
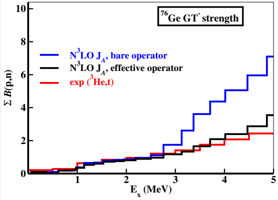
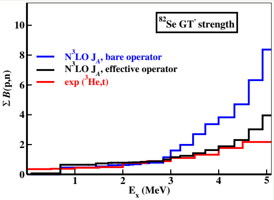
Gamow-Teller observables: theory vs experiment

Two-neutrino double-β decay ($2\nu\beta\beta$) nuclear matrix elements

$2\nu\beta\beta$ decay	1st order	2nd order	3rd order	Expt
$^{48}Ca \rightarrow {}^{48}Ti$	0.055	0.014	0.019	0.042 ± 0.004
$^{76}Ge \rightarrow {}^{76}Se$	0.022	0.056	0.118	0.129 ± 0.004
$^{82}Se \rightarrow {}^{82}Kr$	0.042	0.099	0.095	0.103 ± 0.001







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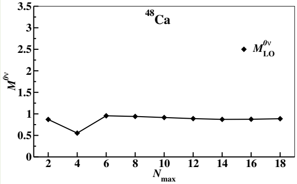
Calculation of $0\nu\beta\beta$ nuclear matrix elments $M^{0\nu}$s

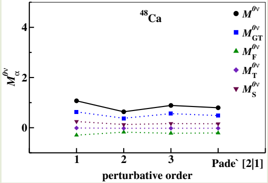
- The calculations of $M^{0\nu}$shas been performed employing SM effective Hamiltonians at third order in many-body perturbation theory
- The contribution from the LO short-range component has been calculated employing a value that is estimated by considering the coefficients of the charge-independence breaking (CIB) contact interaction
- For the Entem-Machleidt N^3LO potential this leads to a value of the LEC $g_\nu^{\mathrm{NN}} = -0.47$ fm^2, when employing a local regulator with a cutoff $\Lambda = 500$ MeV
- All other contributions up to N^2LO – as well as those coming from 3BC – are renormalized with LECs that are consistent with the nuclear Hamiltonian

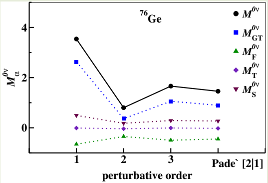
L. C., G. De Gregorio, S. L. Lyu, and N. Itaco, arXiv:2606.25486[nucl-th] (2026)

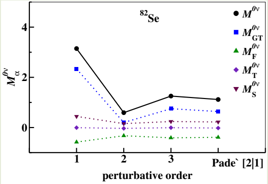
Perturbative behavior of the effective $0\nu\beta\beta$ operator



The $0\nu\beta\beta$ SM effective operator converges quite rapidly in terms of the number of intermediate states of the many-body perturbative expansion of the $\hat{\Theta}$ box
The calculated $M^{0\nu}$s substantially converge from a number of oscillator quanta $N_{max} = 14$ on

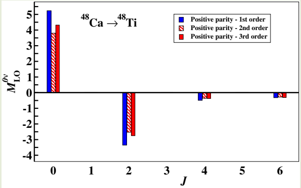




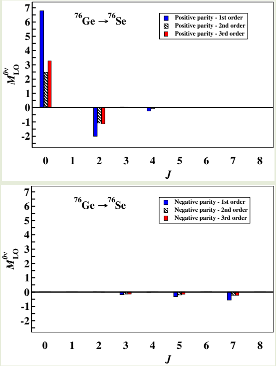


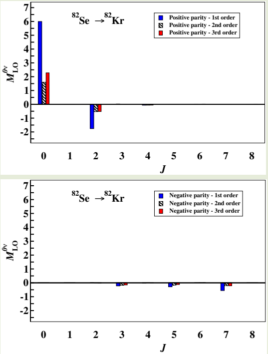
The order-by-order perturbative behavior is driven by the GT component, since LO contact term is rather flat between 2nd and 3rd order, and both Fermi and tensor matrix elements are weakly affected by the renormalization procedure

Perturbative behavior of the effective $0\nu\beta\beta$ operator



A more refined analysis of the order-by-order perturbative behavior may be obtained performing a decomposition of $M^{0\nu}$s in terms of the contributions from the decaying pair of neutrons coupled to a given angular momentum and parity J^{π}





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Perturbative behavior of the ChPT expansion $0\nu\beta\beta$ two-body currents

Summary of the results with LO two-body currents:

Decay	$M^{0\nu}_{3rd}$ (1)	$M^{0\nu}_{Padé}$ (2)	$\Delta = \|(2)-(1)\|$
$^{48}Ca \rightarrow {}^{48}Ti$	0.9 (0.89)	0.8 (0.80)	0.1 (0.09)
$^{76}Ge \rightarrow {}^{76}Se$	1.7 (1.66)	1.5 (1.46)	0.2 (0.20)
$^{82}Se \rightarrow {}^{82}Kr$	1.3 (1.25)	1.1 (1.12)	0.1 (0.13)

Summary of the results with N^2LO two-body currents:

Decay	$M^{0\nu}_{3rd}$ (1)	$M^{0\nu}_{Padé}$ (2)	$\Delta = \|(2)-(1)\|$
$^{48}Ca \rightarrow {}^{48}Ti$	0.8 (0.84)	0.8 (0.75)	0.1 (0.09)
$^{76}Ge \rightarrow {}^{76}Se$	1.8 (1.80)	1.6 (1.59)	0.2 (0.21)
$^{82}Se \rightarrow {}^{82}Kr$	1.4 (1.40)	1.3 (1.26)	0.1 (0.14)

- To estimate the uncertainties associated with the calculated $M^{0\nu}$s, we have considered the Padé approximant [2|1] which accounts for the perturbative behavior up to third-order
- In the last column, we have reported the absolute value $\Delta = |M^{0\nu}_{3rd} - M^{0\nu}_{Padé}|$ that we propose as the estimate of the uncertainties associated to our calculated $M^{0\nu}$s
- The $M^{0\nu}$s at LO and N^2LO differ at most 10%, the convergence of the ChPT expansion is excellent

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Comparison with recent results

- It is worth to compare our calculated $M^{0\nu}\mathrm{s}$ with those of recent calculations
- We consider here those with ISM in *D. Castillo et al, L. Jokiniemi, P. Soriano, Phys. Lett. B* **860**, *139181 (2025)* , labelled with (I), our $M^{0\nu}\mathrm{s}$ with (II) but only with the effective operator at 1st order
- Only LO contributions are taken into account

Decay	$M^{0\nu}_{\mathrm{L}}$ (I)	$M^{0\nu}_{\mathrm{L}}$ (II)	$M^{0\nu}\mathrm{s}$ (I)	$M^{0\nu}\mathrm{s}$ (II)
$^{48}\mathrm{Ca} \rightarrow {}^{48}\mathrm{Ti}$	0.92 (0.14)	0.83	0.43 (0.20)	0.25
$^{76}\mathrm{Ge} \rightarrow {}^{76}\mathrm{Se}$	3.57 (0.25)	3.05	0.97 (0.48)	0.49
$^{82}\mathrm{Se} \rightarrow {}^{82}\mathrm{Kr}$	3.38 (0.20)	2.70	0.91 (0.43)	0.44

- In *A. Belley et al., Phys. Rev. Lett.* **132**, *182502 (2024)* $^{76}\mathrm{Ge}$ $M^{0\nu}$ was calculated with recently developed many-body emulators, their numerical result being $M^{0\nu} = 2.60^{+1.28}_{-1.36}$, to be compared with our result $M^{0\nu} = 1.6(0.2)$, which is smaller but consistent within theoretical uncertainties

Role of $0\nu\beta\beta$ three-body currents

- We have also evaluated the impact of 3BC
- The three-body matrix elements have been calculated at N^3LO in ChPT expansion, that is the lowest order where 3BC start to occur
- Since at present SM codes are not able to manage three-body components of decay operator, we have performed a normal ordering decomposition of the 3BC matrix elements in the HO basis, with the isotope of interest as reference state, and retained only the two-body component

Decay	κ_V	c_1	c_3	c_4	c_D	$M^{0\nu}_{3BC}$
$^{48}Ca \rightarrow {}^{48}Ti$	$-7 * 10^{-5}$	$-1 * 10^{-5}$	$4 * 10^{-5}$	$-7.3 * 10^{-4}$	$1.8 * 10^{-3}$	$1.1 * 10^{-3}$
$^{76}Ge \rightarrow {}^{76}Se$	$2.2 * 10^{-4}$	$1.8 * 10^{-4}$	$-7.5 * 10^{-3}$	$-1.8 * 10^{-3}$	$4.3 * 10^{-3}$	$-4.7 * 10^{-3}$
$^{82}Se \rightarrow {}^{82}Kr$	$1.5 * 10^{-4}$	$1.8 * 10^{-4}$	$-7.5 * 10^{-3}$	$-2.1 * 10^{-3}$	$3.4 * 10^{-3}$	$-5.8 * 10^{-3}$

- As can be observed, the contribution of 3BC to $M^{0\nu}$s is smaller than 1%, a result that is consistent with the impact of N^2LO contributions from the ChPT expansion of the $0\nu\beta\beta$ decay operator

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Nuclear matrix element in light nuclei : p-shell

- RSM + E&M N3LO+ (Non-local)

Decay	$M^{\rm N3LO}_{0\nu,3}$ (10^{-3})					
	κ_v	c_1	c_3	c_4	c_D	tot 3b
^{6}He $\to$ ^{6}Be	0.8	-0.1	-1.9	1.7	-0.8	-0.3
^{8}He $\to$ ^{8}Be	0.3	-0.1	-0.4	0.9	-8.4	-7.7
^{12}Be $\to$ ^{12}C	-0.5	-0.1	1.2	-3.1	-6.1	-8.6

c_1	-0.81
c_3	-3.2
c_4	5.4
c_D	-1
c_E	-0.34

- VMC + Norfolk Ia* Δ – full (Local)

Decay	$M^{\rm N3LO}_{\nu,3}$ (10^{-3})(Ia$*$)					
	$\kappa_v(J=0)$	c_1	c_3	c_4	c_D	tot
^{6}He $\to$ ^{6}Be	3.28	0.59	6.97	-6.63	-4.99	-0.78
^{8}He $\to$ ^{8}Be	-0.50	0.16	-15.76	20.09	4.56	8.55
^{12}Be $\to$ ^{12}C	0.64	1.08	-36.70	38.60	4.40	8.02

c_1	-0.57
c_3	-0.79
c_4	1.33
c_D	-0.635
c_E	-0.09

Preliminary

- most contributions arise from the c_3, c_4 and c_D terms
- the c_3, c_4 contributions largely cancel each other
- significant differences in the c_3, c_4 terms between the two approaches are driven by interactions

Nuclear matrix element in light nuclei : p-shell

- RSM

LO : Fermi + GT + Tensor

Decay	$M_{0\nu}^{\mathrm{LO}}$	$M_{0\nu,3}^{\mathrm{N3LO}}$ (10^{-3})	$M_{0\nu,3}^{\mathrm{N3LO}}/M_{0\nu}^{\mathrm{LO}}$
$^{6}\mathrm{He} \to {}^{6}\mathrm{Be}$	-5.29	-0.3	0.01%
$^{8}\mathrm{He} \to {}^{8}\mathrm{Be}$	-0.25	-7.7	3%
$^{12}\mathrm{Be} \to {}^{12}\mathrm{C}$	-1.59	-8.6	0.5%

- VMC

Decay	$M_{\nu,2}^{\mathrm{LO}}$	$M_{h_A,3}^{\mathrm{N2LO}}$ (10^{-3})	$M_{\nu,3}^{\mathrm{N3LO}}$ (10^{-3})	$M_{\nu,3}^{\mathrm{N3LO}}/M_{\nu,2}^{\mathrm{LO}}$
$^{6}\mathrm{He} \to {}^{6}\mathrm{Be}$	7.57	18.8	-0.78	-0.01%
$^{8}\mathrm{He} \to {}^{8}\mathrm{Be}$	0.29	-36.8	8.55	2.9%
$^{12}\mathrm{Be} \to {}^{12}\mathrm{C}$	1.42	-94.9	8.02	0.6%

Preliminary

- contribution from N3LO 3N potential is small
- VMC calculation shows large contribution from N2LO $\Delta -$ full potentials
- Size roughly in agreement with power counting

Concluding remarks

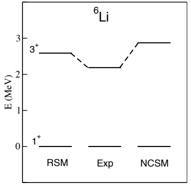
- ChEFT is now a major and well-established tool to answer to the question marks about the nature of the nuclear force and the structure of decay operators
- The explanation of the "quenching puzzle" has been successfully achieved, and now the focus can be spotted in providing reliable $0\nu\beta\beta$ nuclear matrix elements with an enhanced predictive power
- We plan to expand soon our study by:
  - performing calculations for ^{100}Mo, whose decay is currently of experimental interest, and represents a real challenge for microscopic nuclear structure;
  - starting calculations from different ChPT Hamiltonians, and investigate the dependence of the calculated $M^{0\nu}$s on the choice of the chosen LECs

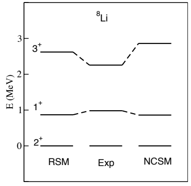
Thanks a lot for your attention!

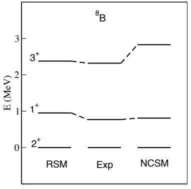
Backup slides

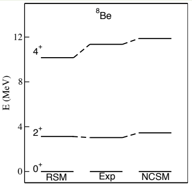
Benchmark calculation - low-lying excitation spectra with H_{eff}^{2b}

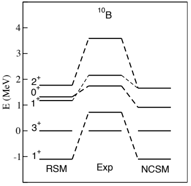
Results with two-body-force Hamiltonian H_{eff}^{2b}

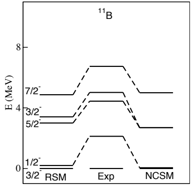


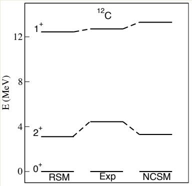


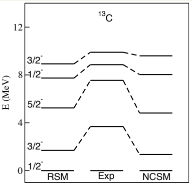






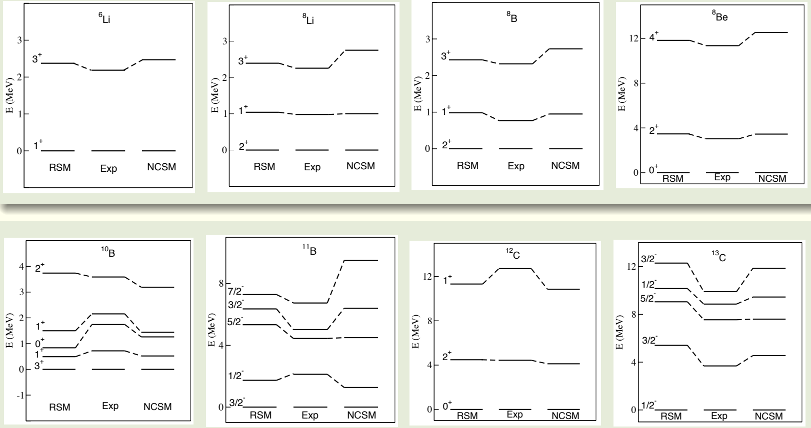






Benchmark calculation - low-lying excitation spectra with H_{eff}^{2b+3b}

Results with two-body-force Hamiltonian H_{eff}^{2b+3b}



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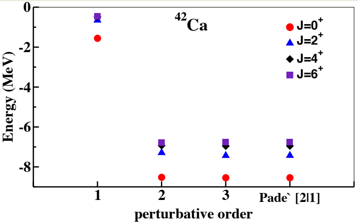
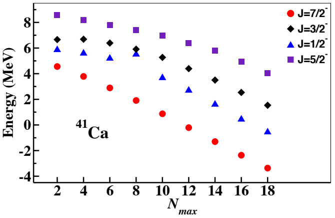
Perturbative properties

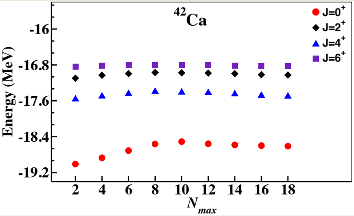
Order-by-order convergence



Intermediate-state convergence



Intermediate-state convergence



Y. Z. Ma, L. C., L. De Angelis, T. Fukui, A. Gargano, N. Itaco, and F. R. Xu, Phys. Rev. C **100**, *034324 (2019)*

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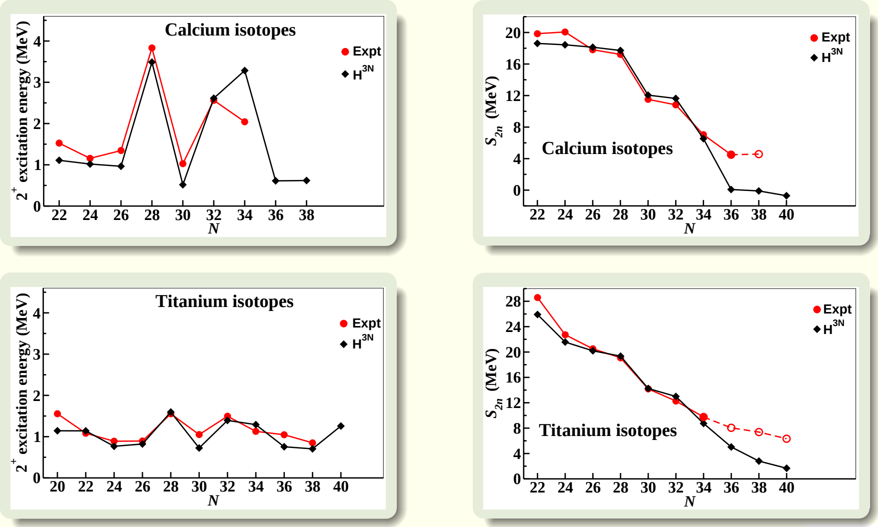
Shell-evolution properties



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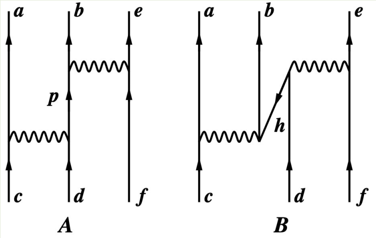
Induced three-body forces

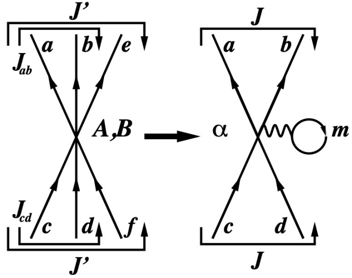


For many-valence nucleon systems (≥ 3) $H_{\rm eff}$ has to include the induced many-body components

Namely, at least three-body diagrams needs to be included in the perturbative expansion of the vertex function $\hat{Q}$ box

Shell model codes, at present, cannot manage three-body components of the shell-model Hamiltonian in large model spaces

We then resort to normal-ordering approximation, this means that TBME are different for each nuclear system



The structure of $M^{0\nu}$ at LO: long-range component

Within the light-neutrino exchange, the matrix elements $M^{0\nu}_{\alpha}$ at LO is expressed as the sum of a long- and short-range components:

$$M^{0\nu} = M^{0\nu}_{\mathrm{L}} + M^{0\nu}_{\mathrm{S}}$$

In turn, the formal expression of the LO long-range component $M^{0\nu}_{\mathrm{L}}$ is expressed in terms of

- the 2-body transition density matrix elements between the parent (i) and grand-daughter (f) nuclei;
- the matrix elements of the Gamow-Teller (GT), Fermi (F), and tensor (T) decay-operators:

$$M^{0\nu}_{\mathrm{L}\alpha} = \sum_{j_n j_{n'} j_p j_{p'}} \langle f | a^{\dagger}_{p} a_n a^{\dagger}_{p'} a_{n'} | i \rangle \times \left\langle j_p j_{p'} \mid \Theta_{\mathrm{L}\alpha} \mid j_n j_{n'} \right\rangle$$

with $\alpha = (GT,\ F,\ T)$

$$\begin{aligned}
\Theta^{GT} &= \vec{\sigma}_1 \cdot \vec{\sigma}_2 H_{GT}(r) \\
\Theta^{F} &= H_F(r) \\
\Theta^{T} &= [3\,(\vec{\sigma}_1 \cdot \hat{r})\,(\vec{\sigma}_1 \cdot \hat{r}) \\
&\quad - \vec{\sigma}_1 \cdot \vec{\sigma}_2]\, H_T(r)
\end{aligned}$$

The neutrino potentials H_{α} are defined as:

$$H_{\alpha}(r) = \frac{2R}{\pi} \int_0^{\infty} \frac{j_{n_{\alpha}}(qr) h_{\alpha}(q^2) q dq}{q}$$

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The structure of $M^{0\nu}$ at LO: short-range component

Within the framework of ChEFT, there is the need to introduce a LO short-range operator, which is missing in standard calculations of $M^{0\nu}$s, to renormalize the operator and make it independent of the ultraviolet regulator

V. Cirigliano et al., Phys. Rev. Lett. **120** *202001 (2020)*

$$M^{0\nu}_{\mathrm{S}} = \sum_{j_n j_{n'} j_p j_{p'}} \langle f | a^{\dagger}_{p} a_{n} a^{\dagger}_{p'} a_{n'} | i \rangle \langle j_p j_{p'} \mid \Theta_{\mathrm{S}} \mid j_n j_{n'} \rangle$$

where Θ_{S} is:

$$\Theta_{\mathrm{S}} = \tau^{-}_{1} \tau^{-}_{2} H_{\mathrm{S}}(r)$$

and the neutrino potential is:

$$H_{\mathrm{S}}(r) = \frac{2R}{\pi} \int_{0}^{\infty} j_0(qr) h_{\mathrm{S}}(q^2) q^2 dq$$

The contact term is regularized with a Gaussian regulator and a cutoff Λ:

$$h_{\mathrm{S}}(q^2) = -2(g^{\mathrm{NN}}_{\nu}/g^2_A) e^{-q^2/(2\Lambda^2)}$$

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