Open EFTs for the early and late universe

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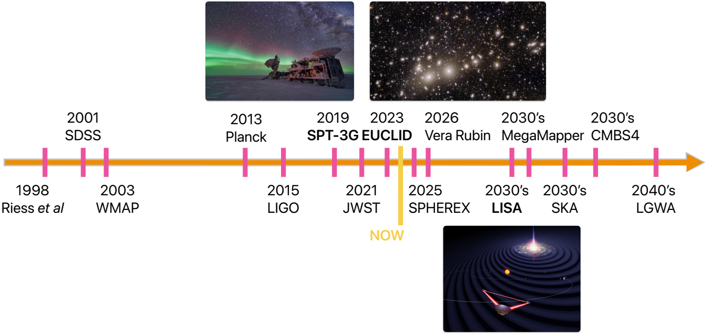
Aspirations

Learn about **fundamental physics**:

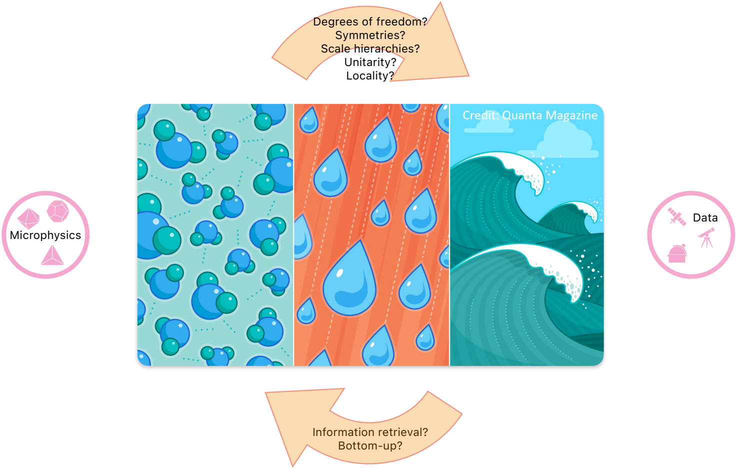
- New degrees of freedom;
- GR at high energy;
- QFT in curved spacetimes;
- Quantum gravity; $\cdots$

A defining moment for **cosmology**:

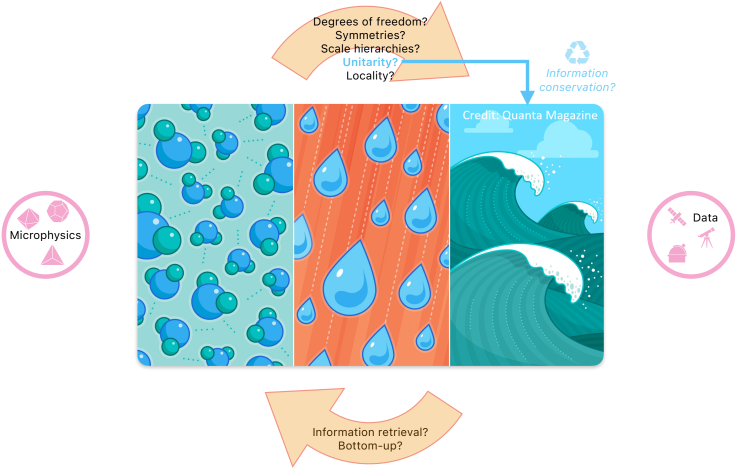


Challenges: i) model degeneracy; ii) enlarged parameter space.

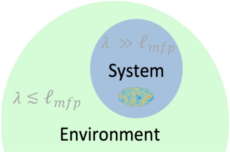
Effective Field Theories



Effective Field Theories



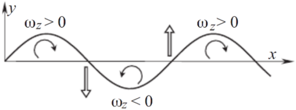
When (non)-unitarity matters



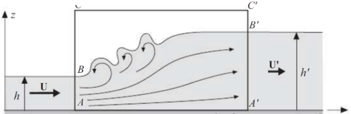
System interacts with environment:
$\Rightarrow$ non-unitary evolution

Dissipation & noise $=$ energy & information losses

Perfect fluid: Wilsonian EFT



Imperfect fluid: non-equilibrium EFT



Credit: [Guyon *et al*, 2012]

What about cosmology?

- **Observable** universe $\neq$ whole; 
- Continuously **evolving**; 
- Always a **medium**; 
- $\exists$ **fluxes** between sectors. 

$\Rightarrow$ effective dynamics often non-unitary [Baumann, Nicolis, Senatore & Zaldarriaga, 2010]

<u>Goal</u>: EFTs accounting for dissipation & noise in cosmology

Combining EFTs and Open Quantum Systems

Langevin equation

$$\mathrm{d}\mathcal{O} = \{H, O\}\mathrm{d}t + \mathrm{d}\xi$$

Martin-Siggia-Rose formalism

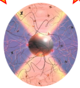
$$P = \int \mathcal{D}x\mathcal{D}\widetilde{x}\, e^{-S[x,\widetilde{x}]}$$

Fokker-Planck equation

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \mathcal{L}_{\mathrm{FP}}[P]$$

Stochastic Schrödinger equation

$$\mathrm{d}\widehat{\mathcal{O}} = -i\left[\widehat{H}, \widehat{\mathcal{O}}\right] + \mathrm{d}\widehat{\xi}$$



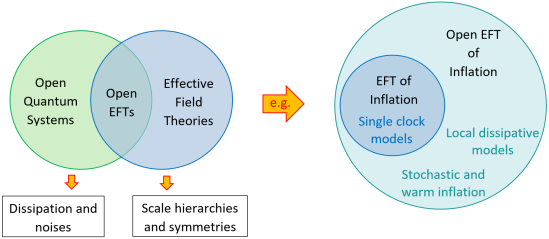
Influence functional

$$\langle\phi_1|\,\widehat{\rho}_{\mathrm{red}}\,|\phi_2\rangle = \int \mathcal{D}\phi_\pm e^{iS_{\mathrm{IF}}[\phi_+,\phi_-]}$$

Master equation

$$\frac{\mathrm{d}\widehat{\rho}_{\mathrm{red}}}{\mathrm{d}t} = \mathcal{L}[\widehat{\rho}_{\mathrm{red}}]$$

Extend the **embedding power** of EFTs:



Outline

1. Open EFTs for gravity
    - Gravitational waves in a medium
    - Dissipative dark sectors
2. Entropy measures
    - In-in formalism
    - Phenomenology

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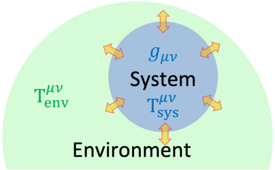
Open gravity

Dissipative theory for a **massless spin** 2 **graviton**: theory of gravity in a medium.



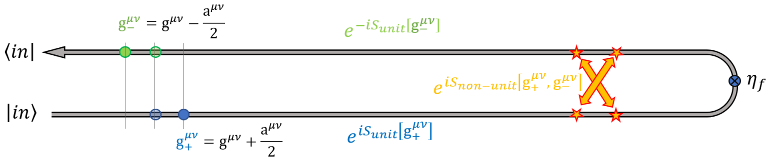
Diffeomorphisms invariance:

$$g^{\mu\nu}_{\pm}(x) \to \frac{\partial(x^\mu + \xi^\mu_\pm)}{\partial x^\alpha}\frac{\partial(x^\nu + \xi^\nu_\pm)}{\partial x^\beta} g^{\alpha\beta}_{\pm}(x)$$

for each branch of SK path integral contour.

Keldysh basis: **retarded** $g_{\mu\nu}$ and **advanced** $a^{\mu\nu}$ metric:

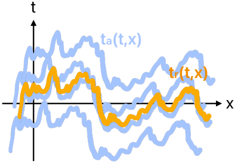
$$g = \frac{g_+ + g_-}{2} = \bar{g} + \delta g, \qquad \text{and} \qquad a = g_+ - g_- = \delta a$$



A tale of two clocks

Fluctuating clocks: $\phi_\pm(t,\mathbf{x})$, i.e.

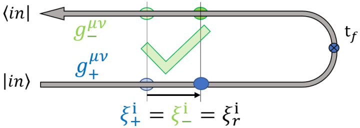
- $\phi_r(t,\mathbf{x}) \Leftrightarrow t_r(t,\mathbf{x})$: average clock;
- $\phi_a(t,\mathbf{x}) \Leftrightarrow t_a(t,\mathbf{x})$: stochasticity.

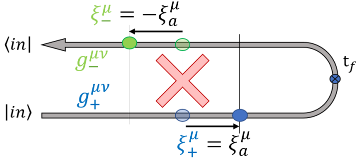


Gauge fixing. Work in *unitary gauges*: $\phi_+(t,\mathbf{x}) = \phi_-(t,\mathbf{x}) = \bar{\phi}(t)$:

$$\phi_r(t,\mathbf{x}) = \bar{\phi}(t) \quad \text{and} \quad \phi_a(t,\mathbf{x}) = 0 \qquad \text{i.e.} \qquad t_r = t \quad \text{and} \quad t_a = 0.$$

Following EFT of Inflation [Cheung *et al.*, 2008] and Dark Energy [Gubitosi, Piazza & Vernizzi, 2013]:





$$\text{4d-diff}_+ \times \text{4d-diff}_- \quad \xrightarrow{\text{open}} \quad \text{4d-diff}_r \quad \xrightarrow{\text{clock}} \quad \text{3d-diff}_r\,.$$

Unitary gauges

Most generic functional invariant under retarded spatial diffeomorphisms.

$$S_{\mathrm{eff}} = \int \mathrm{d}^4 x \sqrt{-g} \sum_{\ell=0} (g^{00}+1)^{\ell} \left[M_{\mu\nu,\ell} a^{\mu\nu} + i N_{\mu\nu\rho\sigma,\ell} a^{\mu\nu} a^{\rho\sigma} + \cdots \right]$$

with $M_{\mu\nu,\ell}$ and $N_{\mu\nu\rho\sigma,\ell}$ rank-2 and 4 cotensors under *retarded spatial diffs*.

$$M_{00,\ell} = \gamma^{tt}_{1,\ell} + \gamma^{tt}_{2,\ell} K + \gamma^{tt}_{3,\ell} K^2 + \gamma^{tt}_{4,\ell} K_{\alpha\beta} K^{\alpha\beta} + \gamma^{tt}_{5,\ell} \nabla^0 K + \gamma^{tt}_{6,\ell} R + \gamma^{tt}_{7,\ell} R^{00} \, ;$$

$$M_{0\mu,\ell} = \gamma^{ts}_{1,\ell} R^0{}_{\mu} + \gamma^{ts}_{2,\ell} \nabla_{\mu} K + \gamma^{ts}_{3,\ell} \nabla_{\beta} K^{\beta}{}_{\mu} \, ;$$

$$\begin{aligned} M_{\mu\nu,\ell} = {} & g_{\mu\nu} \left(\gamma^{ss}_{1,\ell} + \gamma^{ss}_{2,\ell} K + \gamma^{ss}_{3,\ell} K^2 + \gamma^{ss}_{4,\ell} K_{\alpha\beta} K^{\alpha\beta} + \gamma^{ss}_{5,\ell} \nabla^0 K + \gamma^{ss}_{6,\ell} R + \gamma^{ss}_{7,\ell} R^{00} \right) \\ & + \gamma^{ss}_{8,\ell} K_{\mu\nu} + \gamma^{ss}_{9,\ell} \nabla^0 K_{\mu\nu} + \gamma^{ss}_{10,\ell} K_{\mu\alpha} K^{\alpha}{}_{\nu} + \gamma^{ss}_{11,\ell} K K_{\mu\nu} + \gamma^{ss}_{12,\ell} R_{\mu\nu} + \gamma^{ss}_{13,\ell} R_{\mu}{}^{0}{}_{\nu}{}^{0} \\ & + \gamma^{\mathrm{P.O.}}_{1,\ell} \epsilon_{\mu}{}^{\alpha\beta 0} \nabla_{\alpha} K_{\beta\nu} + \gamma^{\mathrm{P.O.}}_{2,\ell} \epsilon_{\mu}{}^{\alpha\beta 0} R_{\alpha\beta}{}^{0}{}_{\nu} . \end{aligned}$$

and similarly for $N_{\mu\nu\rho\sigma,\ell} \Rightarrow$ can be studied **systematically**.

Background evolution

Modified Friedmann equations with α_i functions of EFT coefficients:

$$\text{Open:} \begin{cases} 3M_{\mathrm{Pl}}^2 H^2 = \alpha_1 + \alpha_2 H\,, \\ 2M_{\mathrm{Pl}}^2 \dot{H} = \alpha_3 + \alpha_4 H\,, \end{cases} \quad \text{vs} \quad \text{Closed:} \begin{cases} 3M_{\mathrm{Pl}}^2 H^2 = \rho_\phi\,, \\ 2M_{\mathrm{Pl}}^2 \dot{H} = -(\rho_\phi + P_\phi)\,, \end{cases}$$

1. *Bulk viscosity* [Weinberg, 1971]: $\alpha_4 = 3\zeta$

$$2M_{\mathrm{Pl}}^2 \dot{H} = -\left[\rho_\phi + (P_\phi - 3H\zeta)\right],$$

2. *Brane-world gravity* [Dvali, Gabadadze & Porrati, 2000]: $\alpha_2 = \pm 3M_{\mathrm{Pl}}^2/r_c$

$$3M_{\mathrm{Pl}}^2(H^2 \pm H/r_c) = \rho_\phi,$$

$\Rightarrow$ Modified continuity equation: $\dot{\alpha}_1 + \left(\dot{\alpha}_2 H + \alpha_2 \dot{H}\right) - 3H\left(\alpha_3 + \alpha_4 H\right) = 0$

3. *Interacting dark sector*: $Q = -(\dot{\alpha}_2 H + \alpha_2 \dot{H}) + 3\alpha_4 H^2$

$$\dot{\rho}_\phi + 3H(\rho_\phi + P_\phi) = Q.$$

Primordial gravitational waves

Transverse and traceless (TT) sector: $g_{ij} = a^2(t)\left(\delta_{ij} + h_{ij}\right),\ a^{ij} = a^{-2} h^a_{ij}$:

$$S^{(2)} = \int \mathrm{d}^4 x \sqrt{-g}\, \frac{M_{\mathrm{Pl}}^2}{4c_T^2}\, h^a_{ij} \left\{ \ddot{h}_{ij} - c_T^2 \frac{\nabla^2}{a^2} h_{ij} + \left(\Gamma_T + 3H\right) \dot{h}_{ij} + \frac{\chi_T}{a} \epsilon_{imn} \left(\partial_m \dot{h}_{nj} + 2H \partial_m h_{nj}\right) + i \frac{\beta_T}{M_{\mathrm{Pl}}^2} h^a_{ij} \right\}.$$

1. Speed of propagation c_T^2;
2. Dissipation Γ_T;
3. Birefringence χ_T;
4. Noise β_T.

Effects correlated with **background** and **scalar sector** [Lau, Nishii & Noumi, 2024]:

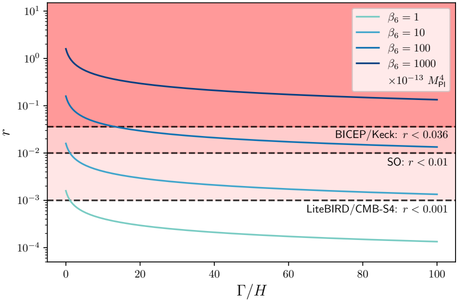
$$S_{\mathrm{eff}} \supset \int \mathrm{d}^4 x \sqrt{-g}\, \gamma^{ss}_{8,\ell} K_{\mu\nu} a^{\mu\nu}.$$

- Background with bulk viscosity: $\zeta = 2\gamma^{ss}_{8,\ell}/3$.
- Dissipation in tensor sector: $\Gamma_T = 2\gamma^{ss}_{8,\ell}/M_{\mathrm{Pl}}^2$.

Tensor-to-scalar ratio

Without birefringence ($\chi_T = 0$): define $\nu_\Gamma \equiv 3/2 + \Gamma_T/(2H)$

$$\Delta_h^2 = \frac{\beta_T}{M_{\mathrm{Pl}}^4} 2^{2\nu_\Gamma} \frac{\Gamma(\nu_\Gamma - 1)\Gamma(\nu_\Gamma)^2}{\Gamma(\nu_\Gamma - \frac{1}{2})\Gamma(2\nu_\Gamma - \frac{1}{2})} \propto \begin{cases} \dfrac{\beta_T}{M_{\mathrm{Pl}}^4}, & \text{for} \quad \Gamma_T \ll H, \\ \dfrac{\beta_T}{M_{\mathrm{Pl}}^4}\sqrt{\dfrac{H}{\Gamma_T}}\left[1 + \mathcal{O}\left(\dfrac{H}{\Gamma_T}\right)\right], & \text{for} \quad \Gamma_T \gg H. \end{cases}$$



⚠ In general, $\beta_T \propto f(\Gamma_T)$:

- Thermal equilibrium: $\beta_T \propto \Gamma_T T$ (FDR);
- What about out-of-equilibrium?

BICEP/Keck bound: $r < 0.036 \quad \Rightarrow \quad \beta_T \lesssim (0.002 M_{\mathrm{Pl}})^4$

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Parametrizing dark sectors

Dark Matter (DM) and Dark Energy (DE) **can interact**

Motivations for interacting dark sectors [Lewis & Chamberlain, 2412.13894]:

- New galaxy survey called DESI claims hints for dynamical DE.
- If DE and DM treated in isolation, most datapoints of DESI **violate the Null-Energy Condition** (NEC):

$$\rho_{\mathrm{DE}} + p_{\mathrm{DE}} \geq 0.$$

- Explaining DESI without violating the NEC may require interacting dark sectors [Khoury, Lin & Trodden, 2503.16415].

Whether or not it becomes more statistically significant,
a good test for parametrizing features in the dark sector.

Principle-based approach

Effective Field Theories (EFTs) embed microphysical models.

EFT of dark energy [Gubitosi, Piazza & Vernizzi, 1210.0201] $\supset$ single-field DE models:

$$S = \int \mathrm{d}^4x \sqrt{-g}\left[\frac{1}{2}f(t)M_*^2 R - \Lambda(t) - c(t)g^{00} + \frac{1}{2}M_2^4(t)(\delta g^{00})^2 - \frac{\bar{M}_1^3(t)}{2}(\delta g^{00})\delta K^\mu{}_\mu - \frac{\bar{M}_2^2(t)}{2}\left(\delta K^\mu{}_\mu\right)^2 - \frac{\bar{M}_3^2(t)}{2}\delta K^\mu{}_\nu \delta K^\nu{}_\mu + \cdots\right]$$

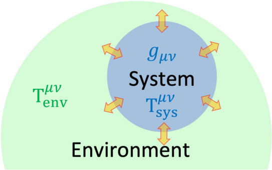
⚠ Shortcomings: **Conservative dynamics** only:
$\Rightarrow$ Cannot describe interacting DM-DE

How do we account for ***sourcing*** *and* ***damping*** *between dark sectors?*

$\Rightarrow$ Open EFT: treat DE as a medium on which gravity & DM propagate.

Matter in a medium

System $=$ {gravity $+$ DM $+$ SM}; **Environment** $=$ {DE $+$ everything else}



{System $+$ environnement}

$$M_{\rm Pl}^2 G_{\mu\nu} = T_{\mu\nu}^{(\rm sys)} + T_{\mu\nu}^{(\rm env)}$$

$$\nabla^\mu T_{\mu\nu}^{(\rm sys)} + \nabla^\mu T_{\mu\nu}^{(\rm env)} = 0$$

Hierarchy of scales: $\lambda^{(\rm sys)} \gg \lambda^{(\rm env)}_{m.f.p.}$: **effective/system-only description**:

$$M_{\rm Pl}^2 G_{\mu\nu} = T_{\mu\nu}^{(\rm sys)} + \langle T_{\mu\nu}^{(\rm env)} \rangle \simeq T_{\mu\nu}^{(\rm sys)} + \text{modif.}$$

$$\nabla^\mu T_{\mu\nu}^{(\rm sys)} = -\nabla^\mu \langle T_{\mu\nu}^{(\rm env)} \rangle \neq 0$$

$\rightarrow$ *An open system approach to gravity* [S.A. Agüí Salcedo, T.C. , L. Dufner & E. Pajer, 2507.03103]

- *Schwinger-Keldysh formalism*: non-equilibrium and open systems;
- *Symmetry-breaking pattern*: 4d-diff$_+ \times$ 4d-diff$_- \xrightarrow{\text{open}}$ 4d-diff$_r \xrightarrow{\text{clock}}$ 3d-diff$_r$.

An open dark fluid (DF)

Dark matter with *bulk viscosity* = dissipative effect from unknown medium:

$$G_{\mu\nu}+\gamma(t)K_{\mu\nu}=\frac{1}{M_{\mathrm{P}l}^2}\left(T_{\mu\nu}^{\mathrm{DF}}+T_{\mu\nu}^{\mathrm{SM}}\right)$$

- **Effective** and **dissipative** theory: DF gain or loose energy from medium:

$$\nabla^\mu T_{\mu\nu}^{\mathrm{SM}}=0 \quad \Rightarrow \quad \nabla^\mu T_{\mu\nu}^{\mathrm{DF}}=\gamma(t)M_{\mathrm{P}l}^2\nabla^\mu K_{\mu\nu}$$

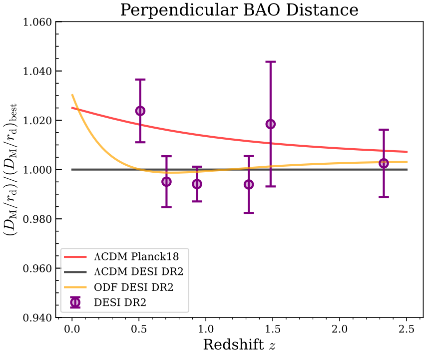
- **No cosmological constant** still **late time acceleration** sourced by $\gamma(t)$.

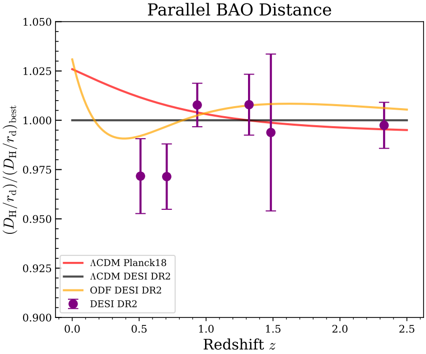
Analysis:

1. *Background*: can reproduce ΛCDM with same number of parameters.
2. *Tensor perturbations*: damped Gravitational Waves (GW) ⇒ testable with GW luminosity distance.
3. *Scalar perturbations*: bulk viscosity = modified clustering ⇒ testable with Weak Lensing and Redshift Space Distortion.

DESI data

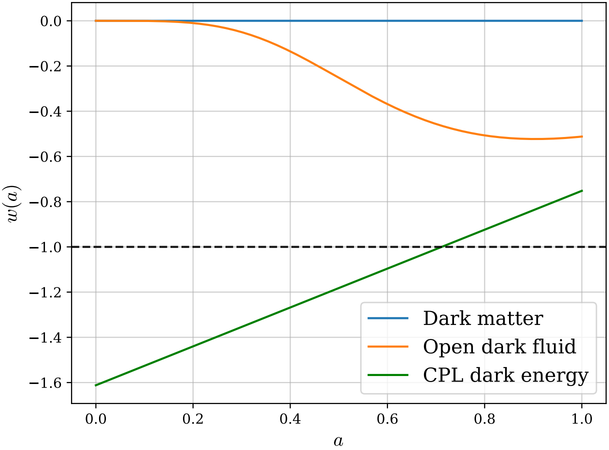
Using $\gamma(a)$ from DESI best-fit:





Reinterpreting DESI results [Kou & Lewis, 2509.16155]

Open dark fluid **never crosses** phantom divide $w = -1$.



[*Ongoing work: scalar perturbations compatible with* ***weak lensing*** *and* ***redshift space distortion****?*]

Gravitational wave cosmology

Transverse and traceless (TT) sector: $g_{ij} = a^2(t)\left(\delta_{ij} + h_{ij}\right),$:

$$\ddot{h}_{ij} + [3H + \gamma(t)]\dot{h}_{ij} - \frac{\nabla^2}{a^2}h_{ij} = 0,$$

$\Rightarrow$ **bulk viscosity** damps gravitational waves.

- Luminosity distances and **damping rates**: [Belgacem, Dirian, Foffa & Maggiore, 1805.08731]

$$\frac{d_L^{\mathrm{gw}}(z)}{d_L^{\mathrm{em}}(z)} = \exp\left[\frac{1}{2}\int_0^z \frac{\mathrm{d}z'}{(1+z')}\frac{\gamma(z')}{H(z')}\right] = \Xi_0 + \frac{1-\Xi_0}{(1+z)^n}\,.$$

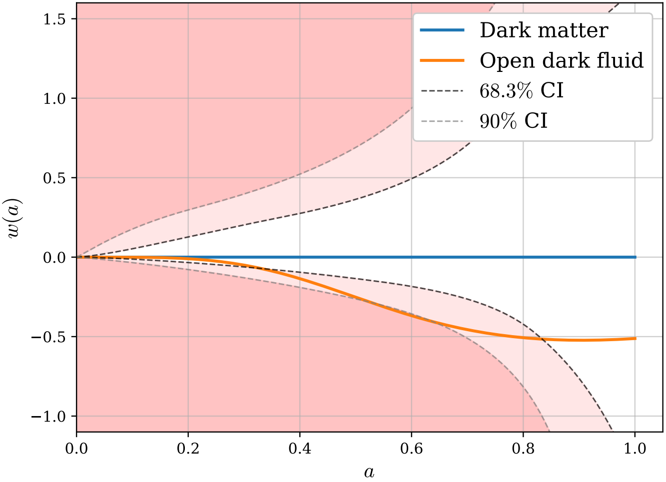
GTWC-4 constraints [LVK, 2509.04348]: $\Xi_0 = 1.2^{+0.8}_{-0.4}$, $n = 2.5^{+1.7}_{-1.1}$.

Constraints on EFT coefficients from GW luminosity distance:

$$\frac{\gamma(z)}{H(z)} = -\frac{2n(1-\Xi_0)}{(1-\Xi_0)+\Xi_0(1+z)^n}.$$

Constraints from GW damping

Bulk viscosity dissipates gravitational waves ⇒ observational constraints!



Summary

Realistic gravitational environments are noisy and dissipative:
*We need **systematic modelling** for **upcoming data**.*

Open gravity:

1. **Dissipation** and **noise** consistent with diff invariance;
2. **Schwinger-Keldysh**: $\text{4d-diff}_{+} \times \text{4d-diff}_{-} \xrightarrow{\text{open}} \text{4d-diff}_{r} \xrightarrow{\text{clock}} \text{3d-diff}_{r}$.

Applications: GW in a medium & dissipative dark sectors

- Tensor modes in early universe;
- GW luminosity distance;
- Open dark fluid;
- Viscosity and clustering.

Outlook: Power counting, renormalization, boostrap, entropy bounds

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Motivations

1. Model interplay visible-hidden sectors using **Open EFTs**;
   - Hidden sector is integrated out;
   - Effective evolution of visible sector.
2. *Goal*: Understand if effective evolution can be approx. as **unitary**;
   - In particle physics, **energy conservation** protects unitarity of the IR;
   - In cosmology, time-translation sym. is **spontaneously broken**.
3. Use **purity** $\gamma \equiv \mathrm{Tr}_{\mathcal{S}}[\widehat{\rho}^2_{\mathrm{red}}]$ as a tracer of non-unitary evolution:
   - When **conserved**: evolution is unitary;
   - When $\gamma \to 0$: **quantum decoherence** takes place.

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In-in formalism

Consider some *observable*

$$\widehat{Q} \equiv \widehat{\mathcal{O}}_1(\eta, \boldsymbol{x}_1)\widehat{\mathcal{O}}_2(\eta, \boldsymbol{x}_2)\cdots\widehat{\mathcal{O}}_n(\eta, \boldsymbol{x}_n).$$

and some *Hamiltonian* evolution controlled by $\widehat{H} = \widehat{H}_0 + g\widehat{H}_{\mathrm{int}}$.

Interaction picture: **state** evolves with $\widetilde{H}_{\mathrm{int}}$, **operators** evolve with $\widetilde{H}_0$

$$\langle\Psi(\eta)|\,\widehat{Q}\,|\Psi(\eta)\rangle = \langle\mathrm{BD}|\left[\bar{\mathcal{T}}e^{ig\int_{-\infty(1-i\epsilon)}^{\eta}\widetilde{H}_{\mathrm{int}}(\eta')\mathrm{d}\eta'}\right]\widetilde{Q}(\eta)\left[\mathcal{T}e^{-ig\int_{-\infty(1+i\epsilon)}^{\eta}\widetilde{H}_{\mathrm{int}}(\eta')\mathrm{d}\eta'}\right]|\mathrm{BD}\rangle$$

In-in expansion: [Weinberg, 2005]

$$\left\langle\widehat{Q}(\eta)\right\rangle^{(n)} = (ig)^n\int_{-\infty}^{\eta}\mathrm{d}\eta_n\int_{-\infty}^{\eta_1}\mathrm{d}\eta_{n-1}\cdots\int_{-\infty}^{\eta_2}\mathrm{d}\eta_1$$

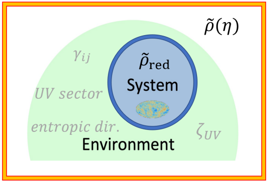
$$\langle\mathrm{BD}|\left[\widetilde{H}_{\mathrm{int}}(\eta_1),\left[\widetilde{H}_{\mathrm{int}}(\eta_2),\cdots\left[\widetilde{H}_{\mathrm{int}}(\eta_n),\widetilde{Q}(\eta)\right]\cdots\right]\right]|\mathrm{BD}\rangle$$

Perturbative purity [TC, Grain, Kaplanek & Vennin, 2406.17856]

Recover previous result from $\langle \widehat{Q}(\eta)\rangle^{(n)} = \mathrm{Tr}[\widetilde{Q}(\eta)\widetilde{\rho}^{(n)}(\eta)]$ with

$$\widetilde{\rho}^{(n)}(\eta) = (-ig)^n \int_{-\infty}^{\eta} \mathrm{d}\eta_1 \int_{-\infty}^{\eta_1} \mathrm{d}\eta_2 \cdots \int_{-\infty}^{\eta_{n-1}} \mathrm{d}\eta_n \left[\widetilde{H}_{\mathrm{int}}(\eta_1), \left[\widetilde{H}_{\mathrm{int}}(\eta_2), \cdots \left[\widetilde{H}_{\mathrm{int}}(\eta_n), |\mathrm{BD}\rangle\langle\mathrm{BD}|\right] \cdots \right]\right]$$

$\Rightarrow$ In-in expansion = statement about the **quantum state** $\widehat{\rho}(\eta)$.



What if we now trace over the environment?

$$\widetilde{\rho}^{(n)}_{\mathrm{red}} = \mathrm{Tr}_{\mathcal{E}}\left[\widetilde{\rho}^{(n)}\right]$$

Compute purity γ order by order:

$$\gamma = \sum_{n=0}^{\infty} g^n \gamma^{(n)} \qquad \text{with} \qquad \gamma^{(n)} = \sum_{m=0}^{n} \mathrm{Tr}_{\mathcal{S}}\left[\widetilde{\rho}^{(m)}_{\mathrm{red}}\,\widetilde{\rho}^{(n-m)}_{\mathrm{red}}\right]$$

Leading order

Consider

$$\widehat{H}_{\rm int}(\eta) = \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3} \widehat{\mathcal{O}}^{\mathcal{S}}_{\boldsymbol{k}}(\eta) \otimes \widehat{\mathcal{O}}^{\mathcal{E}}_{-\boldsymbol{k}}(\eta)$$

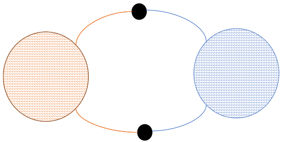
Second-order perturbative purity:

$$\gamma^{(2)} = -2 \prod_{i=1}^{2} \int_{-\infty}^{\eta} \mathrm{d}\eta_i \int \frac{\mathrm{d}^3\boldsymbol{k}_i}{(2\pi)^3} \bar{\mathcal{K}}^{(2)}_{\mathcal{S}}(\boldsymbol{k}_1, \boldsymbol{k}_2, \eta_1, \eta_2) \bar{\mathcal{K}}^{(2)}_{\mathcal{E}}(\boldsymbol{k}_1, \boldsymbol{k}_2, \eta_1, \eta_2)$$

with

$$\bar{\mathcal{K}}^{(2)}_{\mathcal{S}/\mathcal{E}}(\boldsymbol{k}_1, \boldsymbol{k}_2, \eta_1, \eta_2) \equiv \langle \widetilde{\mathcal{O}}^{\mathcal{S}/\mathcal{E}}_{\pm\boldsymbol{k}_1}(\eta_1) \widetilde{\mathcal{O}}^{\mathcal{S}/\mathcal{E}}_{\pm\boldsymbol{k}_2}(\eta_2) \rangle - \langle \widetilde{\mathcal{O}}^{\mathcal{S}/\mathcal{E}}_{\pm\boldsymbol{k}_1}(\eta_1) \rangle \langle \widetilde{\mathcal{O}}^{\mathcal{S}/\mathcal{E}}_{\pm\boldsymbol{k}_2}(\eta_2) \rangle$$

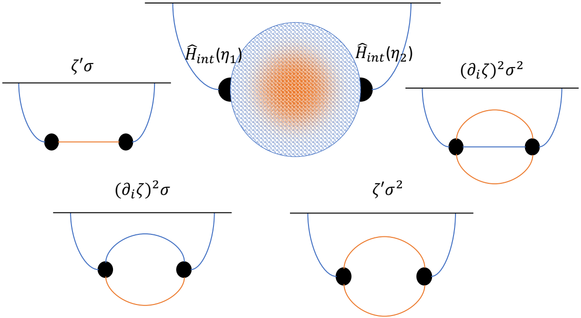
Need *unequal-time* two-point functions of the system and the environment.



Link to observables

Perturbative Gaussianity: If $\mathcal{S} = \{\zeta\}$ and $\widehat{H}_{\rm int} \propto \mathcal{O}(\zeta^2)$ at most:

$$\gamma^{(2)} = -4 \int_{\mathbb{R}^{3+}} \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3} \det{}^{(2)} \begin{pmatrix} \langle \widehat{\zeta}_{\boldsymbol{k}}(\eta)\widehat{\zeta}_{-\boldsymbol{k}}(\eta)\rangle & \langle\{\widehat{\zeta}_{\boldsymbol{k}}(\eta), \widehat{p}_{-\boldsymbol{k}}(\eta)\}\rangle \\ \langle\{\widehat{\zeta}_{\boldsymbol{k}}(\eta), \widehat{p}_{-\boldsymbol{k}}(\eta)\}\rangle & \langle \widehat{p}_{\boldsymbol{k}}(\eta)\widehat{p}_{-\boldsymbol{k}}(\eta)\rangle \end{pmatrix}$$



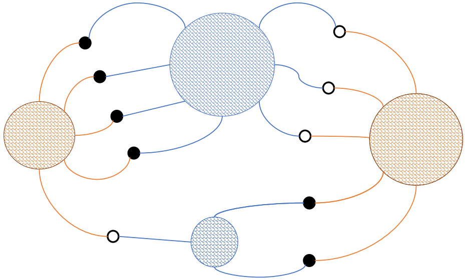
Non-Gaussian statistics (bispectrum, $\cdots$) only enters at higher order.

Higher orders

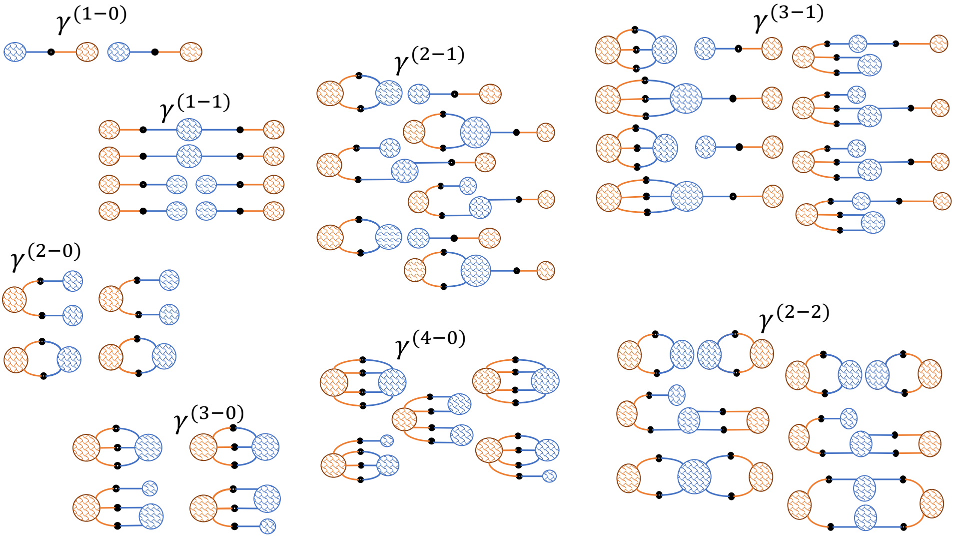
n^{th}-order perturbative purity:

$$\gamma^{(n)} = (-i)^n \sum_{m=0}^{n} \left(\prod_{i=1}^{m} \int_{-\infty}^{\eta} \mathrm{d}\eta_i \int \frac{\mathrm{d}^3 \boldsymbol{k}_i}{(2\pi)^3} \right) \left(\prod_{j=1}^{n-m} \int_{-\infty}^{\eta} \mathrm{d}\bar{\eta}_j \int \frac{\mathrm{d}^3 \bar{\boldsymbol{k}}_j}{(2\pi)^3} \right)$$

$$\sum_{V=\{\circ,\bullet\}} \left[\prod_{\bullet} (-1) \right] \prod_{\mathrm{blob}} \left\langle \widetilde{\mathcal{T}} [\bullet_{\mathcal{S}}] \mathcal{T} [\circ_{\mathcal{S}}] \right\rangle_{\mathcal{S}} \left\langle \widetilde{\mathcal{T}} [\bullet_{\mathcal{E}}] \mathcal{T} [\circ_{\mathcal{E}}] \right\rangle_{\mathcal{E}}$$



Diagrammatic



Summary

1. **In-in formalism**: access the purity $\gamma^{(n)}$ order by order;
   - Same expansion as the one used to compute observables.
2. First deviation to purity controlled by **Gaussian statistics**;
   - Non-Gaussian statistics only enters at higher orders.
3. Develop **diagrammatic** rules to organise higher-order computations
   - Closer and closer to standard in-in computation.

Now have tools to understand
if effective evolution can be approx. as ***unitary***

Outline

1. Open EFTs for gravity
   - Gravitational waves in a medium
   - Dissipative dark sectors
2. Entropy measures
   - In-in formalism
   - Phenomenology

Inflation and alternatives [TC, de Rham G. & Kaplanek, 2401.02832]

Two-field $\{\Phi_1, \Phi_2\}$ **scaling solutions** based on [Tolley & Wesley, 2007]

$$S = \int \mathrm{d}^4 x \sqrt{-g}\left[\frac{M_{\mathrm{Pl}}^2}{2} R - \frac{1}{2} G_{ab} g^{\mu\nu} \partial_\mu \Phi^a \partial_\nu \Phi^b - V(\Phi)\right]$$

Background **FLRW solution** in conformal time $\eta \in]-\infty, 0]$

$$a(\eta) = \left(\frac{\eta}{\eta_0}\right)^{\frac{1}{\epsilon - 1}}$$

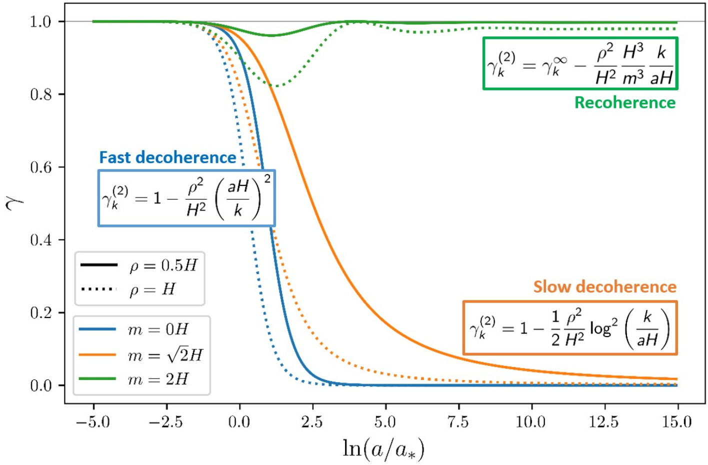
Inhomogeneities decomposed along curvature ζ and isocurvature σ modes:

$$S^{(2)} = \int \mathrm{d}^4 \boldsymbol{x}\, \frac{a^2(\eta)}{2}\left\{\overbrace{2\epsilon\left[\zeta'^2 - (\partial_i \zeta)^2\right]}^{\text{System}} + \overbrace{\sigma'^2 - (\partial_i \sigma)^2 - \frac{m^2(\epsilon)}{\eta^2}\sigma^2}^{\text{Environment}} + \overbrace{\frac{\rho(\epsilon)}{\eta}\zeta'\sigma}^{\text{Interactions}}\right\}$$

Look for **nearly scale-invariant power spectrum** $P_\zeta(k) \propto k^{n_s - 1}$:

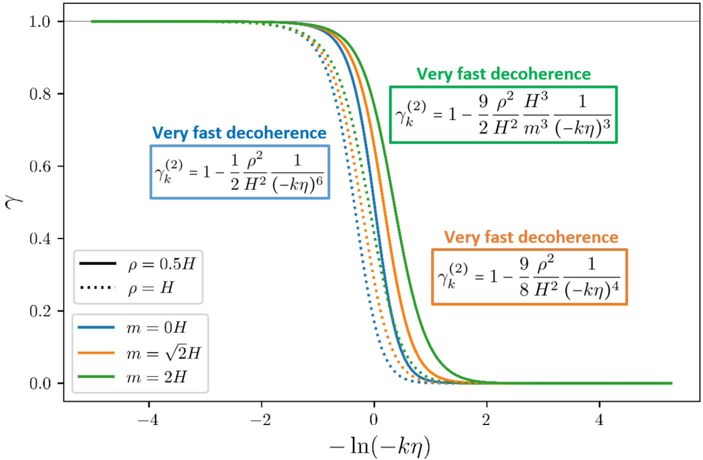
Expanding *(de Sitter):* $\epsilon = 0$; ***Contracting*** *(Ekpyrosis):* $\epsilon = 3/2$.

Decoherence *vs.* recoherence in inflation [TC, Grain & Vennin, 2212.09486]



$$\sigma_l \propto \frac{1}{a^{3/2-\nu}}, \quad \nu \equiv \sqrt{\frac{9}{4} - \frac{m^2}{H^2}} \quad \text{and} \quad \sigma_h \propto \frac{\cos\left(\mu \ln \frac{a}{a_*}\right)}{a^{3/2}}, \quad \mu \equiv \sqrt{\frac{m^2}{H^2} - \frac{9}{4}}$$

Purity loss in Ekpyrosis [TC, de Rham & Kaplanek, 2401.02832]



$$p_{\boldsymbol{k}}^{\text{infl.}}(\eta) = -i\sqrt{\frac{k}{2}}\, e^{-ik\eta} \qquad \Rightarrow \qquad p_{\boldsymbol{k}}^{\text{ekpy.}}(\eta) = -i\sqrt{\frac{k}{2}}\, e^{-ik\eta}\left[1 - \frac{3i}{k\eta} - \frac{3}{(-k\eta)^2}\right]$$

Driven-dissipative cosmologies

	Massless	Conformal	Heavy
Inflation	$(-k\eta)^{-2}$	$1/\log^2(-k\eta)$	$1 - e^{-2\pi \frac{m_\sigma}{H}}$
Ekpyrosis	$(-k\eta)^{-6}$	$(-k\eta)^{-4}$	$(-k\eta)^{-3}$

Table: Late-time scaling of γ_k for $-k\eta \ll 1$

Depending on the **internal** (mass, spin, $\cdots$) and **external parameters** (background dynamics) non-unitary effects can be significant at late-time.

Robustness against non-linearities [TC, Grain, Kaplanek & Vennin, 2406.17856]

<u>NL extensions</u>: EFToI coupled to scalar hidden sector: $\mathcal{L}_{\rm int} \supset \rho \delta g^{00} \sigma$

$$S_{\zeta\sigma} = \int \mathrm{d}^3x \mathrm{d}\eta a^2 \left[\rho a \zeta' \sigma + \frac{1}{\Lambda_1} \zeta_c'^2 \sigma + \frac{c_s^2}{\Lambda_2} \left(\partial_i \zeta\right)^2 \sigma\right]$$

Perturbative Gaussianity:

$$\gamma^{(2)} = -4 \int_{\mathbb{R}^{3+}} \frac{\mathrm{d}^3 \boldsymbol{k}}{(2\pi)^3} \det{}^{(2)} \begin{pmatrix} \langle \widehat{\zeta}_{\boldsymbol{k}}(\eta) \widehat{\zeta}_{-\boldsymbol{k}}(\eta) \rangle & \langle \{ \widehat{\zeta}_{\boldsymbol{k}}(\eta), \widehat{p}_{-\boldsymbol{k}}(\eta) \} \rangle \\ \langle \{ \widehat{\zeta}_{\boldsymbol{k}}(\eta), \widehat{p}_{-\boldsymbol{k}}(\eta) \} \rangle & \langle \widehat{p}_{\boldsymbol{k}}(\eta) \widehat{p}_{-\boldsymbol{k}}(\eta) \rangle \end{pmatrix}$$

$\Rightarrow$ Need to compute **one-loop power spectra**. At late-time $(-k\eta \ll 1)$:

$$\gamma^{(2)}_{\rm NL}(\eta) = \gamma^{(2)}_{\rm lin}(\eta) \left[1 - \frac{3}{8\pi} \frac{1}{c_s^4} \frac{H^4}{(2\pi f_\pi)^4}\right] + \mathcal{O}\left(-k\eta\right).$$

Small loop correction controlled by scale hierarchy $\rho \ll H \ll c_s(2\pi f_\pi)$.

Summary

- **Purity** γ is a tracer of non-unitarity $\Rightarrow$ developped **in-in formalism** to access this quantity.
- Its evolution depends on both the **internal** (mass, spin) and **external** (background) parameters: *light vs. heavy*, *expanding vs. contracting*.

Extensions:

1. Entropy measures from SK path integral;
2. Entropy bounds on EFT coefficients [Duaso Peyo *et al*, 2410.23709], [Cai *et al*, 2507.00850]
3. Link between entropy and chaos: purity and OTOCs [Zhou *et al*, 2504.05237]
4. Constraints from quantum speed limit?
5. NEQ constraints equivalent to CPTP dynamical maps?

Thank you!

Backup

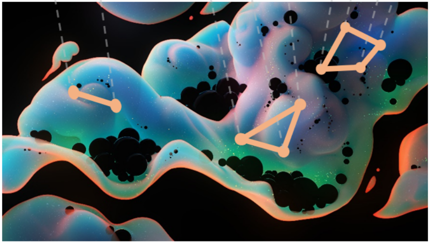
Thomas Colas

Outline

Outline

Cosmological correlators

$$\left\langle \prod_{i=1}^{n} \delta(\boldsymbol{k}_i) \right\rangle$$



$$\left\langle \prod_{i=1}^{n} \hat{\zeta}(\boldsymbol{k}_i, \eta_0) \right\rangle$$

Schwinger-Keldysh formalism

Consider some **observable**

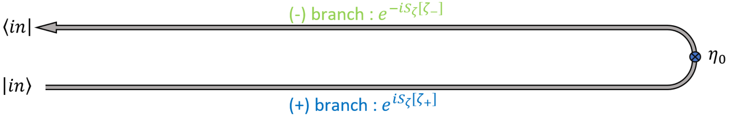
$$\widehat{Q} \equiv \widehat{\zeta}(\boldsymbol{x}_1)\widehat{\zeta}(\boldsymbol{x}_2)\cdots\widehat{\zeta}(\boldsymbol{x}_n)$$

and some unitary **evolution operator** $\widehat{U}(\eta_0, \eta_\mathrm{i})$ so that

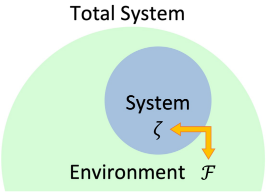
$$|\Psi(\eta_0)\rangle = \widehat{U}(\eta_0, \eta_\mathrm{i})\,|\mathrm{BD}\rangle \quad \text{with} \quad \langle\zeta|\,\widehat{U}(\eta_0, \eta_\mathrm{i})\,|\zeta_1\rangle = \int_{\zeta_1}^{\zeta} \mathcal{D}\left[\Phi\right] e^{iS[\Phi]}.$$

If $S\left[\Phi\right] = S_\zeta\left[\zeta\right]$, see [Weinberg, 2005]:

$$\left\langle \widehat{Q}(\eta_0) \right\rangle = \left\langle \Psi(\eta_0) \middle| \widehat{Q}(\eta_0) \middle| \Psi(\eta_0) \right\rangle = \left\langle \mathrm{BD} \middle| \widehat{U}^\dagger(\eta_0, \eta_\mathrm{i})\widehat{Q}(\eta_0)\widehat{U}(\eta_0, \eta_\mathrm{i}) \middle| \mathrm{BD} \right\rangle$$
$$= \int \mathrm{d}\zeta\mathrm{d}\zeta_1\mathrm{d}\zeta_2\left[\zeta(\boldsymbol{x}_1)\cdots\zeta(\boldsymbol{x}_n)\right] \int_{\zeta_1}^{\zeta} \mathcal{D}\left[\zeta_+\right] \int_{\zeta_2}^{\zeta} \mathcal{D}[\zeta_-] e^{iS_\zeta[\zeta_+] - iS_\zeta[\zeta_-]} \langle\zeta_1|\mathrm{BD}\rangle \langle\mathrm{BD}|\zeta_2\rangle$$



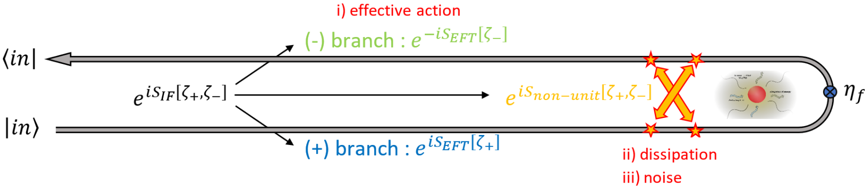
Integrating out an environment



- $S[\Phi] = S_\zeta[\zeta] + S_\mathcal{F}[\mathcal{F}] + S_{\text{int}}[\zeta;\mathcal{F}]$ with $\mathcal{F}$ a **hidden sector**.
- _Goal:_ tracing out $\mathcal{F}$, the environment being **unobservable**.

Effects of the environment captured by the **Influence Functional** (IF):

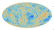
$$\left\langle \widehat{Q}(\eta) \right\rangle = \int \mathrm{d}\zeta \mathrm{d}\zeta_1 \mathrm{d}\zeta_2 \left[\zeta(\boldsymbol{x}_1)\cdots\zeta(\boldsymbol{x}_n)\right] \int_{\zeta_1}^{\zeta} \mathcal{D}[\zeta_+] \int_{\zeta_2}^{\zeta} \mathcal{D}[\zeta_-] e^{iS_\zeta[\zeta_+] - iS_\zeta[\zeta_-] + \boldsymbol{i S_{\mathrm{IF}}[\zeta_+;\zeta_-]}}$$



What are the rules obeyed by $S_{\mathrm{IF}}[\zeta_+;\zeta_-]$*?*

The Open EFT of Inflation [S.A. Agüí Salcedo, T.C. & E. Pajer, 2404.15416]

Early universe: one scalar degree of freedom $\pi(\boldsymbol{x}, t)$:

Observed ⟨  ⟩ $\quad \Leftrightarrow \quad \langle \hat{\pi}^n \rangle(t) = \int \mathrm{d}\pi \pi^n \mathrm{Prob}_\pi(t)\,.$

- EFT of Inflation [Cheung *et al.*, 2008]: most generic wavefunction

$$\mathrm{Prob}_\pi(t) = |\Psi_\pi(t)|^2 = \left| \int_\Omega^\pi \mathcal{D}\pi\, e^{iS_{\mathrm{eff}}[\pi]} \right|^2$$

- **Dissipation** & **noise**: most generic density matrix

$$\mathrm{Prob}_\pi(t) = \rho_{\pi\pi}(t) = \int_\Omega^\pi \mathcal{D}\pi_+ \int_\Omega^\pi \mathcal{D}\pi_-\, e^{iS_{\mathrm{eff}}[\pi_+, \pi_-]}$$

Physical principles restrict $S_{\mathrm{eff}}[\pi_+, \pi_-]$:

1. Unitarity: {Sys. + Env.} closed ⇒ non-equilibrium constraints; [Liu & Glorioso, 2018]
2. Symmetries: in-in coset construction; [Akyuz, Goon & Penco, 2023]
3. Locality: truncatable power counting scheme.

Non-equilibrium constraints [Liu & Glorioso, 2018]

Requiring **Open QFT** originates from a unitary "closed" UV theory:

$$\text{i) } \mathrm{Tr}[\widehat{\rho}] = 1, \qquad \text{ii) } \widehat{\rho}^\dagger = \widehat{\rho} \qquad \text{and} \qquad \text{iii) } \widehat{\rho} \geq 0$$

implies constraints on $S_{\rm eff}\left[\pi_+, \pi_-\right] \equiv S_{\rm unit}[\pi_+] - S_{\rm unit}[\pi_-] + S_{\rm non-unit}[\pi_+, \pi_-]$:

$$\begin{aligned}
&\text{i)} && S_{\rm eff}\left[\pi_+, \pi_+\right] = 0, && S_{\rm eff}\left[\pi_r, \pi_a = 0\right] = 0; \\
&\text{ii)} && S_{\rm eff}\left[\pi_+, \pi_-\right] = -S^*_{\rm eff}\left[\pi_-, \pi_+\right], && S_{\rm eff}\left[\pi_r, \pi_a\right] = -S^*_{\rm eff}\left[\pi_r, -\pi_a\right]; \\
&\text{iii)} && \mathfrak{Im} S_{\rm eff}\left[\pi_+, \pi_-\right] \geq 0, && \mathfrak{Im} S_{\rm eff}\left[\pi_r, \pi_a\right] \geq 0,
\end{aligned}$$

for $\pi_r = (\pi_+ + \pi_-)/2$ and $\pi_a = \pi_+ - \pi_-$. *Consequences*:

1. $S_{\rm eff}\left[\pi_r, \pi_a\right]$ starts **linear** in π_a;
2. **Odd** powers of π_a are purely real; **even** powers of π_a purely imaginary;
3. Bounds on the **noise coefficients**.

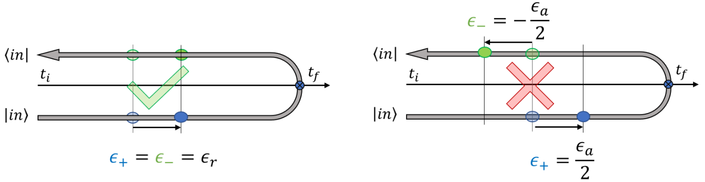
$\Rightarrow$ *Already reduce the scope of available Open EFTs*

Dissipative shift symmetric scalar [Hongo *et al.*, 2018], [Akyuz, Goon & Penco, 2023]

$S_{\rm unit}[\pi_\pm]$ invariant under shift$_+ \times$ shift$_-$:

$$\pi_\pm(t) \to \pi'_\pm(t) = \pi_\pm(t+\epsilon_\pm) + \epsilon_\pm,$$

but $S_{\rm non-unit}[\pi_+;\pi_-]$ **is not**: **SSB** shift$_+ \times$ shift$_-$ $\to$ shift$_r$:



Retarded $\pi_r = (\pi_+ + \pi_-)/2$ and **advanced** $\pi_a = \pi_+ - \pi_-$ fields:

$$\pi_r(t) \to \pi'_r(t) = \pi_r(t+\epsilon_r) + \epsilon_r, \qquad \pi_a(t) \to \pi'_a(t) = \pi_a(t+\epsilon_r).$$

Building blocks:

$$\pi_a, \quad t+\pi_r, \quad \partial_\mu \pi_a, \quad \partial_\mu(t+\pi_r).$$

Effective functional

- *Quadratic order:* $1 \to 5$ EFT param (1 tadpole constraint):

$$S_{\mathrm{eff}}^{(2)} = \int \mathrm{d}^4x\sqrt{-g}\Big\{ \overbrace{\dot{\pi}_r\dot{\pi}_a - c_s^2\partial_i\pi_r\partial^i\pi_a}^{\text{Kinetic term}}$$

$$\underbrace{-\gamma\dot{\pi}_r\pi_a}_{\text{Dissipation}} + i\underbrace{\left[\beta_1\pi_a^2 - (\beta_2-\beta_4)\,\dot{\pi}_a^2 + \beta_2\left(\partial_i\pi_a\right)^2\right]}_{\text{Noise}}\Big\}$$

- *Cubic order:* $1 \to 13$ EFT param: EFToI famous for relating operators at different orders because of **non-linearly realised boosts** [López Nacir *et al.*, 2011].

$$\begin{aligned}
\text{EFToI}: &\qquad \mathcal{L} \supset \left(c_s^2 - 1\right)\left[-2\dot{\pi}_r + \left(\partial_\mu\pi_r\right)^2\right]\dot{\pi}_a \\
\text{Dissipation}: &\qquad \mathcal{L} \supset \gamma\left[-2\dot{\pi}_r + \left(\partial_\mu\pi_r\right)^2\right]\pi_a \\
\text{Noise}: &\qquad \mathcal{L} \supset i\beta_4\left(-\dot{\pi}_a + \partial_\mu\pi_r\partial^\mu\pi_a\right)^2
\end{aligned}$$

Recover and extend EFToI construction.

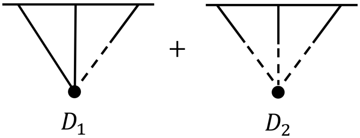
Standard observables

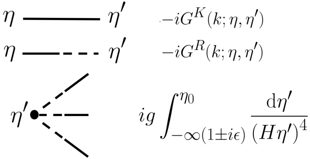
Symmetries ensure existence of nearly scale invariant power spectrum

$$\langle \zeta_{\boldsymbol{k}} \zeta_{\boldsymbol{k}'} \rangle = \frac{H^2}{f_\pi^4} \langle \pi^c_{\boldsymbol{k}} \pi^c_{\boldsymbol{k}'} \rangle \equiv (2\pi)^3 \delta(\boldsymbol{k} + \boldsymbol{k}) \frac{2\pi^2}{k^3} \Delta_\zeta^2(k).$$

$\Rightarrow \Delta_\zeta^2 = 10^{-9}$ obtained by imposing hierarchies of scales.

Bispectrum computed in **perturbation theory** using standard in-in rules.

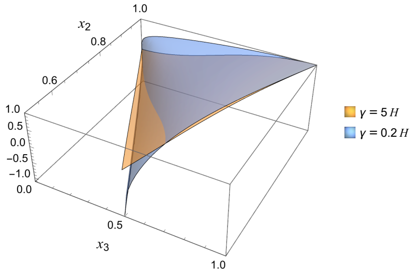




$$\langle \zeta_{\boldsymbol{k}_1} \zeta_{\boldsymbol{k}_2} \zeta_{\boldsymbol{k}_3} \rangle = -\frac{H^3}{f_\pi^6} \langle \pi^c_{\boldsymbol{k}_1} \pi^c_{\boldsymbol{k}_2} \pi^c_{\boldsymbol{k}_3} \rangle \equiv (2\pi)^3 \delta(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3) B(k_1, k_2, k_3).$$

$$S(x_2, x_3) \equiv (x_2 x_3)^2 \frac{B(k_1, x_2 k_1, x_3 k_1)}{B(k_1, k_1, k_1)}, \qquad f_{\mathrm{NL}}(k_1, k_2, k_3) \equiv \frac{5}{6} \frac{B(k_1, k_2, k_3)}{P(k_1)P(k_2) + 2 \text{ perms.}}.$$

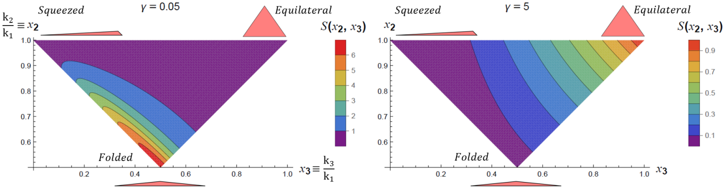
Bispectrum shapes



Main features:

- $\gamma \gg H$: equilateral;
- $\gamma \ll H$: folded;
- Consistency relations;
- Regularized divergence.

Consistent with **flat-space/sub-Hubble** analytic results:

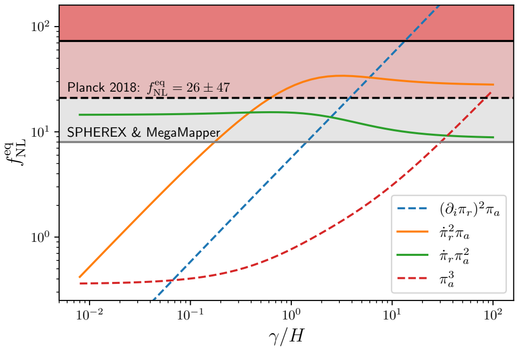


Matching and $f_{\rm NL}$ with [Creminelli *et al.*, 2305.07695]

UV completion: inflaton ϕ + massive scalar field χ with softly-broken $U(1)$:

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}M_{\rm Pl}^2 R - \frac{1}{2}(\partial\phi)^2 - V(\phi) - |\partial\chi|^2 + M^2|\chi|^2 \right.$$
$$\left. -\frac{\partial_\mu\phi}{f}\left(\chi\partial^\mu\chi^* - \chi^*\partial^\mu\chi\right) - \frac{1}{2}m^2\left(\chi^2 + \chi^{*2}\right)\right].$$

$\Rightarrow$ narrow **instability band** in sub-Hubble regime: *local* particle production.

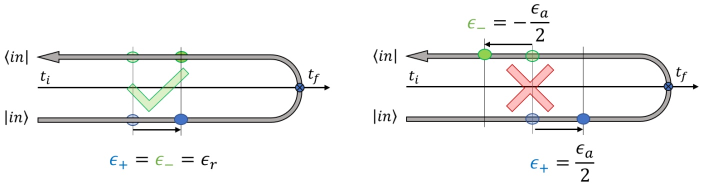


In-in coset construction [Hongo *et al.*, 2018], [Akyuz, Goon & Penco, 2023]

Two **simplifications** from [Cheung *et al.*, 2008]: [regime of validity? ⇒ later]

1. *Decoupling limit*: Mixing $\pi/\delta g$ small as long as $E \sim H \gg E_{\rm mix} \sim \epsilon^{1/2} H$

⇒ **enough** to construct theory of **dissipative shift symmetric scalar**:



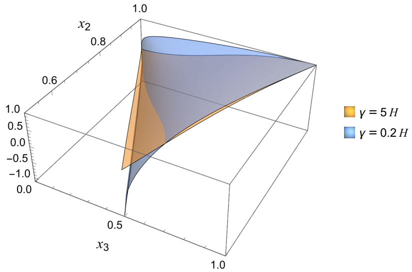
$S_{\rm eff}[\pi_r, \pi_a]$ invariant under *retarded time diffeomorphism*:

$$t \to t + \epsilon_r : \qquad \pi_r \to \pi_r - \epsilon_r, \qquad \pi_a \to \pi_a.$$

<u>*Building blocks:*</u> π_a, $t + \pi_r$, $\partial_\mu \pi_a$, $\partial_\mu (t + \pi_r)$.

2. *Derivative expansion*: locality and truncatable power counting scheme.

Primordial non-Gaussianities

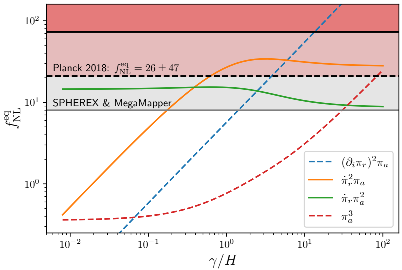


Main features:

- $\gamma \gg H$: equilateral shape;
- $\gamma \ll H$: **new** folded signal;
- Consistency relations hold.

Matching with [Creminelli *et al.*, 2305.07695]

- inflaton ϕ
- massive scalar field χ
- softly-broken $U(1)$



New challenge: dynamical gravity

Details of matching with [Creminelli *et al.*, 2305.07695]

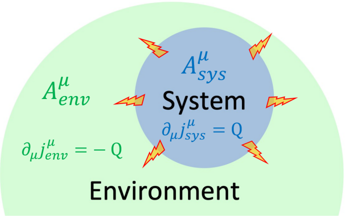
	Parameters:						
UV completion	M	m	f	ρ	ξ	γ	$\nu_{\mathcal{O}}/\nu_{\mathcal{O}^3}$
Open EFT	f_π^2	c_s	γ	γ_2	β_1	β_5	δ_1

Matching:	$f_\pi^2 = \rho f$
$c_s = 1$	$\gamma = \frac{\xi m^4}{\pi M f^2} e^{2\pi\xi}$
$\Gamma = \pi\xi\gamma$	$\beta_1 = \frac{\nu_{\mathcal{O}}}{2\rho f}\frac{m^4}{f^2}$
$\beta_5 = 2\pi\xi\beta_1$	$\delta_1 = \frac{\nu_{\mathcal{O}^3}}{6\sqrt{\rho f}}\frac{m^6}{f^3}$

Outline

Open gauge theories

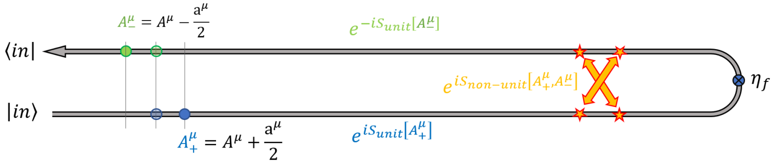
[S.A. Agüí Salcedo, T.C. & E. Pajer, 2412.12299]: *Dissipative theory* for of light in a medium.



- Gauge symmetry: $U(1)_r \times U(1)_a$
- Global symmetry:

$$U(1)_r \times U(1)_a \to U(1)_r$$

Keldysh basis: retarded $A^\mu = \left(A^\mu_+ + A^\mu_-\right)/2$; advanced $a^\mu = A^\mu_+ - A^\mu_-$.



Retarded & advanced gauge transformation

Retarded gauge transformation $\epsilon_+ = \epsilon_- = \epsilon_r$:

$$A^\mu \to A^\mu + \partial^\mu \epsilon_r \,, \qquad\qquad a^\mu \to a^\mu \,.$$

Advanced gauge transformation $\epsilon_+ = -\epsilon_- = \epsilon_a$:

$$A^\mu \to A^\mu \,, \qquad\qquad a^\mu \to a^\mu + \partial^\mu \epsilon_a \,.$$

- Global $U(1)_a$ broken: $B_a^\mu = a^\mu + \partial^\mu \varphi_a$ with $\varphi_a \to \varphi_a - \epsilon_a$.
- Unitary gauge: $\varphi_a = 0$ and **retarded gauge invariance** only.

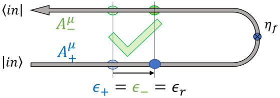
Effective functional constructed out of $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$ and a^μ:

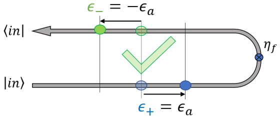
$$S_1 = \int_{\omega,\boldsymbol{k}} \left[a^0 i k_i F^{0i} + a_i \left(\gamma_2 F^{0i} + \gamma_3 i k_j F^{ij} + \gamma_4 \epsilon^i{}_{jl} F^{jl}\right) + a^\mu j_\mu\right]$$

$$S_2 = \int_{\omega,\boldsymbol{k}} i a^\mu N_{\mu\nu} a^\nu , \qquad S_{n\geq 3} = \cdots$$

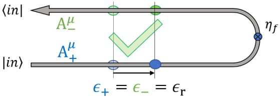
Summary

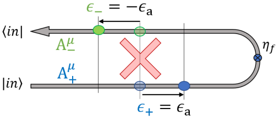
1. **Unitary**: $\Delta S_{\text{eff}}^{\text{adv}} = 0 \quad \Rightarrow \quad \partial^\mu j_\mu = 0$: Maxwell in medium;



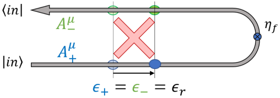


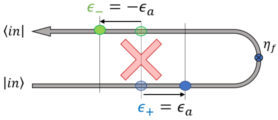
2. **Non-unitary**: $\Delta S_{\text{eff}}^{\text{adv}} \neq 0 \quad \Rightarrow \quad \partial^\mu j_\mu \neq 0$: modified charge conservation;





3. **Conductor**: $\Delta S_{\text{eff}}^{\text{ret}} \neq 0 \quad \Rightarrow \quad$ new d.o.f.: dissipative Proca theory.





Recovering electromagnetism in a medium

From S_{eff}, obtain modified Gauss and Ampère laws:

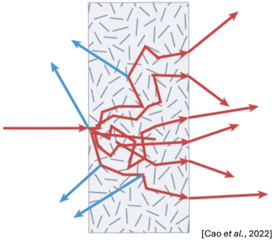
$$\frac{\delta S_{\text{eff}}}{\delta a^0} = 0 \qquad \Rightarrow \qquad \nabla.\mathbf{E} = j_0 + \xi_0,$$
$$\frac{\delta S_{\text{eff}}}{\delta a^i} = 0 \qquad \Rightarrow \qquad \gamma_2\mathbf{E} + \gamma_3\nabla\times\mathbf{B} - 2\gamma_4\mathbf{B} = \mathbf{j} + \xi,$$

and a noise constraint: *modified charge conservation in the system*

$$\partial^\mu(j_\mu + \xi_\mu) = \Gamma(j_0 + \xi_0).$$

Properties:

- Dispersive medium: $n = 1/\sqrt{-\gamma_3}$;
- Dissipative medium: $\gamma_2 = -i\omega + \Gamma$;
- Anisotropic medium: γ_4;
- Random medium: ξ^μ.

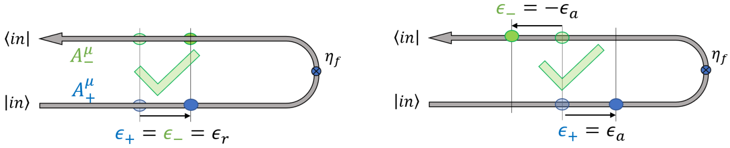


[Cao *et al.*, 2022]

Unitary limit

$$\Delta S_1^{\mathrm{ret}} = 0 \qquad \text{but} \qquad \Delta S_1^{\mathrm{adv}} = \int_{\omega,\boldsymbol{k}} (i\omega + \gamma_2)\epsilon_a i k_i F^{0i}.$$

- For general γ_2, $\Delta S_1^{\mathrm{adv}} \neq 0$;
- When $\gamma_2 = -i\omega$, $\Delta S_1^{\mathrm{adv}} = 0$.



Unitary theory with $S_1[A_+, A_-] = S[A_+] - S[A_-]$ and

$$S[A] = \frac{1}{4}\int \mathrm{d}^4x \left[F^{\mu\nu}F_{\mu\nu} + (c_s^2 - 1)F^{ij}F_{ij} + \theta F^{\mu\nu}\tilde{F}^{\mu\nu}\right]$$

Recover Maxwell in a medium.

Deformed advanced gauge

*When $\gamma_2 \neq -i\omega$, does it exist **physically equivalent** advanced configurations?*

1. Retarded gauge invariance: $M_{\mu\nu} k^\nu = 0$ where $k^\mu = (\omega, \mathbf{k})$.
2. $\exists$ "right kernel" $\Rightarrow$ $\exists$ "left kernel" such that $v^\mu M_{\mu\nu} = 0$.
3. M is non-Hermitian $\Rightarrow$ different left and right kernels: $v^\mu = (i\gamma_2, \boldsymbol{k})$.

Conclusion: retarded gauge invariance generates advanced gauge invariance.

S_1 remains unchanged under

$$A^\mu \to A^\mu + \epsilon_r k^\mu, \qquad a^\mu \to a^\mu + \epsilon_a v^\mu.$$

Not broken but rather **deformed**! [Liu & Glorioso, 2018], [Akyuz, Goon & Penco, 2023].

Advanced gauge redundancy allows us to reduce number of advanced components.

Noise constraint

Add some noise: $N_{\mu\nu}$ positive semi-definite [Kamenev, 2011], [Breuer & Petruccione, 2007]

$$S = \int d^4x \left[a^\mu M_{\mu\nu} A^\nu + i a^\mu N_{\mu\nu} a^\nu\right],$$

Hubbard-Stratonovich trick:

$$\mathcal{Z} = \int [\mathcal{D}A^\mu] \int [\mathcal{D}a^\mu] \int [\mathcal{D}\xi_\mu] \exp\left[\int d^4x \; i\boldsymbol{a^\mu} \left(\boldsymbol{M_{\mu\nu} A^\nu} - j_\nu - \xi_\nu\right) - \frac{1}{4}\xi_\mu (N^{-1})^{\mu\nu} \xi_\nu\right]$$

Advanced gauge symmetry $\boldsymbol{a^\mu \to a^\mu + \epsilon_a v^\mu}$ induces noise constraint:

$$v^\mu (j_\mu + \xi_\mu) = 0.$$

Meaning: $\gamma_2 = \Gamma - i\omega$ leads to

$$\partial^\mu (j_\mu + \xi_\mu) = \Gamma (j_0 + \xi_0).$$

Conservation of the total current $\Leftrightarrow$ ***Non-conservation of the system's current***

Advanced Stueckelberg [Lau, Nishii & Noumi, 2412.21136]

Stueckelberg trick: $X_a \to X_a - \epsilon_a$ s.t. $\mathcal{A}_a^\mu = a^\mu + \partial^\mu X_a \to \mathcal{A}_a^\mu$ invariant

$$\begin{aligned} S_{\text{eff}} &= \int_{\omega,\boldsymbol{k}} \left[\mathcal{A}_{a0} ik_i F^{0i} + \mathcal{A}_{ai}\left(\gamma_2 F^{0i} + \gamma_3 ik_j F^{ij} + \gamma_4 \epsilon^i_{\ jl} F^{jl}\right) - \mathcal{A}_{a\mu}\left(j^\mu + \xi^\mu\right)\right], \\ &= S_{\text{eff}}^{\text{old}} + \int_{\omega,\boldsymbol{k}} X_a \left[(i\omega + \gamma_2) ik_i F^{0i} + \partial_\mu \left(j^\mu + \xi^\mu\right)\right]. \end{aligned}$$

X_a e.o.m. recovers noise constraint: for $\gamma_2 = \Gamma - i\omega$,

$$\frac{\delta S_{\text{eff}}}{\delta X_a} = 0 \quad \Rightarrow \quad \partial^\mu (j_\mu + \xi_\mu) = \Gamma(j_0 + \xi_0).$$

using on-shell relation $\delta S_{\text{eff}}/\delta a_0 = 0$ that is $ik_i F^{0i} = -\left(j^0 + \xi^0\right)$.

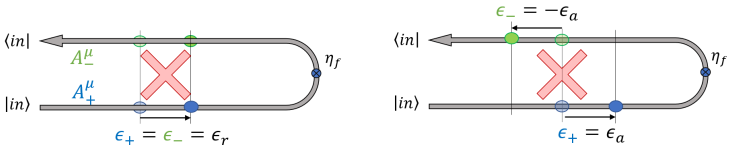
Lessons:

- Manifest advanced gauge invariance at the price of X_a;
- If $\partial^\mu (j_\mu + \xi_\mu) = 0$, only solution is $\gamma_2 = -i\omega$, *i.e.* no dissipation $\Gamma = 0$.

Proca mass term

Adding a mass term breaks **both** retarded and advanced gauge symmetries:

$$S_{\text{eff}} = \int_{\omega,\boldsymbol{k}} ik_i a_0 F^{0i} - i\omega a_i F^{0i} - m^2 a_\mu A^\mu$$



Stueckelberg trick:

- $X_a \to X_a - \epsilon_a$ s.t. $\mathcal{A}^\mu_a = a^\mu + \partial^\mu X_a \to \mathcal{A}^\mu_a$ invariant;
- $X_r \to X_r - \epsilon_r$ s.t. $\mathcal{A}^\mu_r = A^\mu + \partial^\mu X_r \to \mathcal{A}^\mu_r$ invariant

$$S_{\text{eff}} = S_{\text{eff}}^{\text{old}} - m^2 \int \mathrm{d}^4 x \left[\partial_\mu X_a \partial^\mu X_r - a_\mu \partial^\mu X_r - \partial_\mu X_a A^\mu\right]$$

Decoupling: when $E \gg m$, mixing between X and A negligeable.

Breaking retarded gauge invariance triggers new d.o.f.

Dispersion relations

Gauge fixing

- retarded Coulomb gauge: $\partial_i A^i = 0$

$$\exists\ \epsilon_r \quad \text{s.t.} \quad k_i A'^i = 0, \quad \text{where} \quad A'^\mu = A^\mu + \epsilon_r k^\mu .$$

- advanced Coulomb gauge: $\partial_i a^i = 0$

$$\exists\ \epsilon_a \quad \text{s.t.} \quad k_i a'^i = 0, \quad \text{where} \quad a'^\mu = a^\mu + \epsilon_a v^\mu .$$

Eigenvalues of the kinetic matrix: 1 constrained dof, 2 propagating dof

$$\left(k^2\,, i\gamma_2\omega + \gamma_3 k^2 + 2\gamma_4 k, i\gamma_2\omega + \gamma_3 k^2 - 2\gamma_4 k\right).$$

Introduce $\gamma_2 = \Gamma - i\omega$, $\gamma_3 = -c_s^2$:

$$\omega^2 + i\Gamma\omega - c_s^2 k^2 \pm 2\gamma_4 k = 0 \quad \Rightarrow \quad \omega = -i\frac{\Gamma}{2} \pm \sqrt{c_s^2 k^2 - (\Gamma/2)^2 \mp 2\gamma_4 k}\,.$$

Outline

Reintroducing the scalar

Restore covariance by *retarded* and *advanced* time diffeomorphism:

$$S_{\text{eff}}[g_{\mu\nu}, a_{\mu\nu}] \to S_{\text{eff}}[g_{\mu\nu}, a_{\mu\nu}, \pi_r, \pi_a]\,,$$

- **retarded Stückelberg trick**: $t_r \to t_r + \pi_r(x)$:

$$g^{00} \to g^{00} + 2g^{0\mu}\partial_\mu\pi_r + g^{\mu\nu}\partial_\mu\pi_r\partial_\nu\pi_r\,,$$
$$a^{00} \to a^{00} + 2a^{0\mu}\partial_\mu\pi_r + a^{\mu\nu}\partial_\mu\pi_r\partial_\nu\pi_r\,.$$

- **advanced Stückelberg trick**: $t_a \to t_a - \pi_a(x)$:

$$g_{\mu\nu} \to g_{\mu\nu}\,,$$
$$a^{\mu\nu} \to a^{\mu\nu} + 2\nabla^{(\mu}\epsilon_a^{\nu)}\,.$$

Decoupling limit: slow-roll $\epsilon \ll 1$ (**even with dissipation & noise**):

$$\begin{aligned} -a^{00} + (g^{00}/2)g_{\mu\nu}a^{\mu\nu} \quad &\to \quad 2\dot{\pi}_a - 2\partial^\mu\pi_r\partial_\mu\pi_a\,, \\ g_{\mu\nu}a^{\mu\nu} \quad &\to \quad 2\dot{\pi}_a + 6H\pi_a\,, \\ 1 + g^{00} \quad &\to \quad -2\dot{\pi}_r + (\partial_\mu\pi_r)^2\,, \end{aligned}$$

$\Rightarrow$ *recover Open EFToI & more: primordial GWs*

Background analysis

Late-universe: SM = baryons

- Friedmann equation: $3M_{\mathrm{Pl}}^2 H^2 = \rho_{\mathrm{DF}} + \rho_{\mathrm{SM}} = \rho_{\mathrm{tot}}$
- Continuity equations: $\dot{\rho}_{\mathrm{SM}} + 3H\rho_{\mathrm{SM}} = 0,\ \dot{\rho}_{\mathrm{DF}} + 3H\rho_{\mathrm{DF}} = 3\gamma(t)M_{\mathrm{Pl}}^2 H^2$

$$\boxed{\dot{\rho}_{\mathrm{tot}} + 3H\rho_{\mathrm{tot}} = \gamma(t)\rho_{\mathrm{tot}}}$$

Match $\gamma(t)$ to $w_0 w_a$CDM:

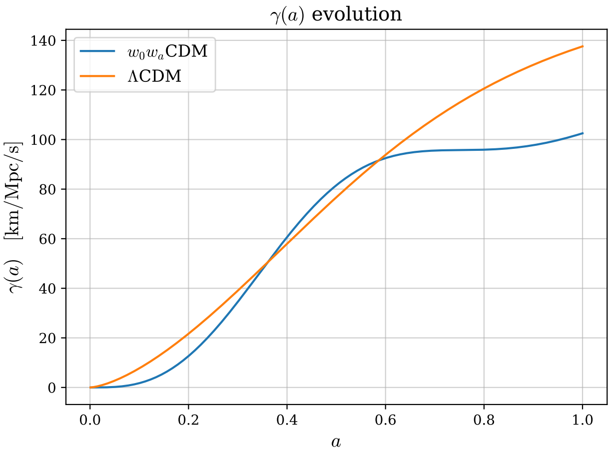
$$\begin{aligned}
\rho_{\mathrm{tot}} \quad &= \quad \frac{\rho_{\mathrm{tot},0}}{a^3}\exp\left[\int_1^a \mathrm{d}\tilde{a}\,\frac{\gamma(\tilde{a})}{\tilde{a}H(\tilde{a})}\right], \\
&\underset{w_0 w_a\mathrm{CDM}}{=} \frac{\rho_{\mathrm{tot},0}}{a^3}\left[\Omega_{\mathrm{m},0} + (1-\Omega_{\mathrm{m},0})a^{-3(w_0+w_a)}e^{3w_a(a-1)}\right].
\end{aligned}$$

Conclusion: $w_0 w_a$CDM $\Leftrightarrow$ open dark fluid for:

$$\gamma(a) = -\frac{3H\left(1-\Omega_{\mathrm{m},0}\right)\left[w_0 + w_a(1-a)\right]}{\left(1-\Omega_{\mathrm{m},0}\right) + \Omega_{\mathrm{m},0}a^{3(w_0+w_a)}e^{3w_a(1-a)}}$$

A guidance for model building?

Deduce $\gamma(a)$ from best-fit DESI DR2 + Planck CMB + DESY5 supernovae



A reinterpretation of the concordance model:
Is there a microphysical model realising this EFT?

A glimpse on what to expect

- Dissipative and stochastic Einstein Equations: $G_{\mu\nu} + \Gamma \mathcal{D}_{\mu\nu} = T_{\mu\nu} + \xi_{\mu\nu}$
- Non-conserved stress-energy tensor: $\nabla_\mu T^{\mu\nu} \neq 0$

Phenomenology:

- **Background**: Viscous cosmology & Interacting dark sectors

$$\dot{\rho}_{DE} + 3H(\rho_{DE} + p_{DE}) = \Gamma \quad \text{and} \quad \dot{\rho}_m + 3H(\rho_m + p_m) = -\Gamma$$

- **Clustering** $\Rightarrow$ redshift space distortion (RSD) and weak lensing (WL)

$$k^2 \langle \psi \rangle = -4\pi G \mu(a,k) a^2 \rho_m \langle \delta \rangle, \qquad k^2 \frac{\langle \psi + \phi \rangle}{2} = -4\pi G \Sigma(a,k) a^2 \rho_m \langle \delta \rangle$$

- **Gravitational waves** $\Rightarrow$ GW production, propagation and dissipation

$$\ddot{h}_{ij} + \Gamma \dot{h}_{ij} + c_T^2 h_{ij} + \chi \epsilon_{ilm} k_m h_{jl} = T_{ij} + \xi_{ij}$$

Rich phenomenology to explore,
eventually ***already constrained from data***.

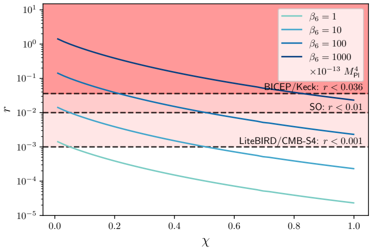
Including birefringence

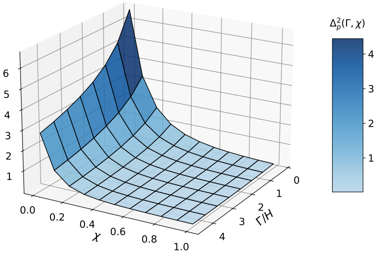
$$\left[\partial_\eta^2 - \left(\frac{2+\frac{\Gamma_T}{H}}{\eta} \mp \chi_T k\right)\partial_\eta + k^2\right] G^R_{\times,+}(k;\eta,\eta') = H^2\eta^2\delta(\eta-\eta').$$

Analytic expression in terms of Appell F_2 function: $\kappa_\pm \equiv \mp \frac{i\chi_T}{\sqrt{1-\chi_T^2}}(\nu_T - 1/2)$

$$\Delta^2_{\times,+} = \frac{\beta_T/M_{\mathrm{Pl}}^4}{2\pi^2\nu_\Gamma^2\chi_T^3} F_2\left(3; \frac{1}{2}+\nu_\Gamma-\kappa_\pm, \frac{1}{2}+\nu_\Gamma+\kappa_\pm; 2\nu_\Gamma+1, 2\nu_\Gamma+1; \mp\sqrt{1-\frac{1}{\chi_T^2}}, \pm\sqrt{1-\frac{1}{\chi_T^2}}\right)$$

Recover **instability band** at low dissipation:





Late-universe: clockless example

Subset that preserves retarded time-diff: **no dynamical scalar**

$$S_{\rm eff} = \int \mathrm{d}^4x\sqrt{-g}\,(f_1 R_{\mu\nu} + f_2 g_{\mu\nu} + f_3 T_{\mu\nu} + \mathrm{h.d.})\, a^{\mu\nu}.$$

$\Rightarrow$ *Very few ingredients!* To lowest order, e.o.m. takes the form:

$$G_{\mu\nu} + \tilde{f}_1 R g_{\mu\nu} = \tilde{f}_2 T_{\mu\nu} + \tilde{f}_3 T g_{\mu\nu}.$$

w.l.o.g., separating **Standard Model**, **dark matter** and **cosmological constant**:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = G_{\rm N}\left[T^{\rm SM}_{\mu\nu} + T^{\rm DM}_{\mu\nu} + (\lambda^{\rm SM} T^{\rm SM} + \lambda^{\rm DM} T^{\rm DM}) g_{\mu\nu}\right].$$

If dark matter $\simeq$ perfect fluid, absorb $\lambda^{\rm DM} T^{\rm DM} g_{\mu\nu}$ into equation of state $w^{\rm DM}$:

$\simeq$ *Gravitational Aether* [Afshordi, 2008] | *Massive Gravity* [Isham, Salam & Strathdee, 1971]

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = G_{\rm N}\left[T^{\rm SM}_{\mu\nu} + T^{\rm DM}_{\mu\nu} + \lambda^{\rm SM} T^{\rm SM} g_{\mu\nu}\right].$$

Blueprint for cosmological analysis [with F. McCarthy, ACT & SO]

- Does not come from an action $\Rightarrow$ **evade [Lovelock, 1971] theorem**;
- Dissipative dark matter:

$$\nabla^\mu T^{\mathrm{SM}}_{\mu\nu} = 0\,, \qquad \nabla^\mu T^{\mathrm{DM}}_{\mu\nu} + \lambda^{\mathrm{SM}} \partial_\nu T^{\mathrm{SM}} = 0\,.$$

- To **UV complete**, need to find sector s.t. $\langle T^{\mathrm{env}}_{\mu\nu} \rangle = \lambda^{\mathsf{SM}} T^{\mathsf{SM}} g_{\mu\nu}$;
- One-parameter extension of Λ-CDM $\Rightarrow$ data-analysis CMB + BAO.

Background $\Rightarrow$ Hubble diagram function of redshift z:

$$\frac{H(z)}{H_0} = \sqrt{\Omega_\Lambda + \Omega_{\mathrm{DM}}(1+z)^{3(1+w_{\mathrm{DM}})} + \Omega_{\mathrm{SM}}(1+z)^{3(1+w_{\mathrm{SM}})}\left[1+\lambda(1-3w_{\mathrm{SM}})\right]}\,.$$

Clustering $\Rightarrow$ redshift space distortion (RSD) and weak lensing (WL):

$$k^2\langle\psi\rangle = -4\pi G\mu(a,k)a^2\rho_m\langle\delta\rangle, \qquad k^2\frac{\langle\psi+\phi\rangle}{2} = -4\pi G\Sigma(a,k)a^2\rho_m\langle\delta\rangle$$

Outline

Benchmarking the perturbative purity

Gaussian systems: purity γ **fully determined** by det $\boldsymbol{\Sigma}$ [Serafini *et al.*, 2003]

$$\boldsymbol{\Sigma} \equiv \begin{pmatrix} \langle \widehat{\zeta}^2 \rangle & \langle \{ \widehat{\zeta}, \widehat{p}_\zeta \} \rangle \\ \langle \{ \widehat{\zeta}, \widehat{p}_\zeta \} \rangle & \langle \widehat{p}_\zeta^2 \rangle \end{pmatrix}$$

with $\widehat{p}_\zeta$ conjugate momentum.

Solve **numerically** and **analytically** in asymptotic regimes (exact).

$\Rightarrow$ Playground to *test pert. purity*. Resummation: $\gamma = 1 - \gamma^{(2)} \to 1/(1 + \gamma^{(2)})$