INT program on
Nuclear Hamiltonians for Advancing Nuclear Physics and Beyond
May 14 2026

Chiral EFT & Power Counting:

From electroweak and BSM physics to 3N forces

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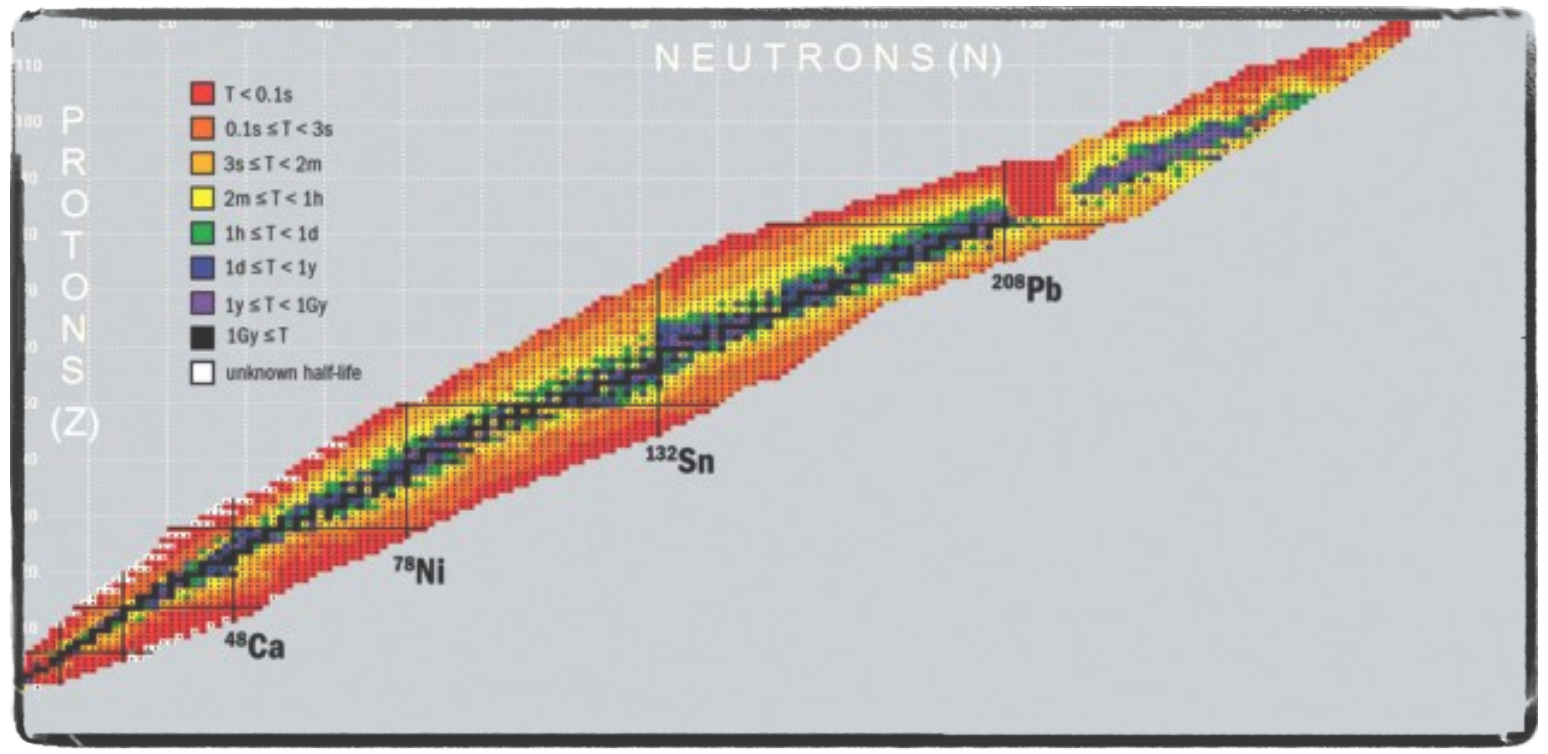
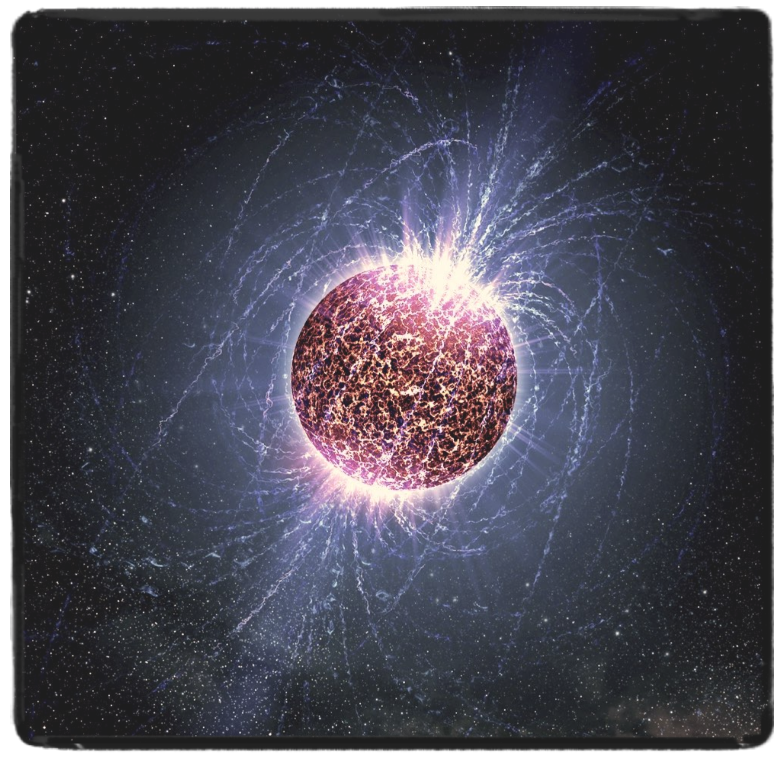
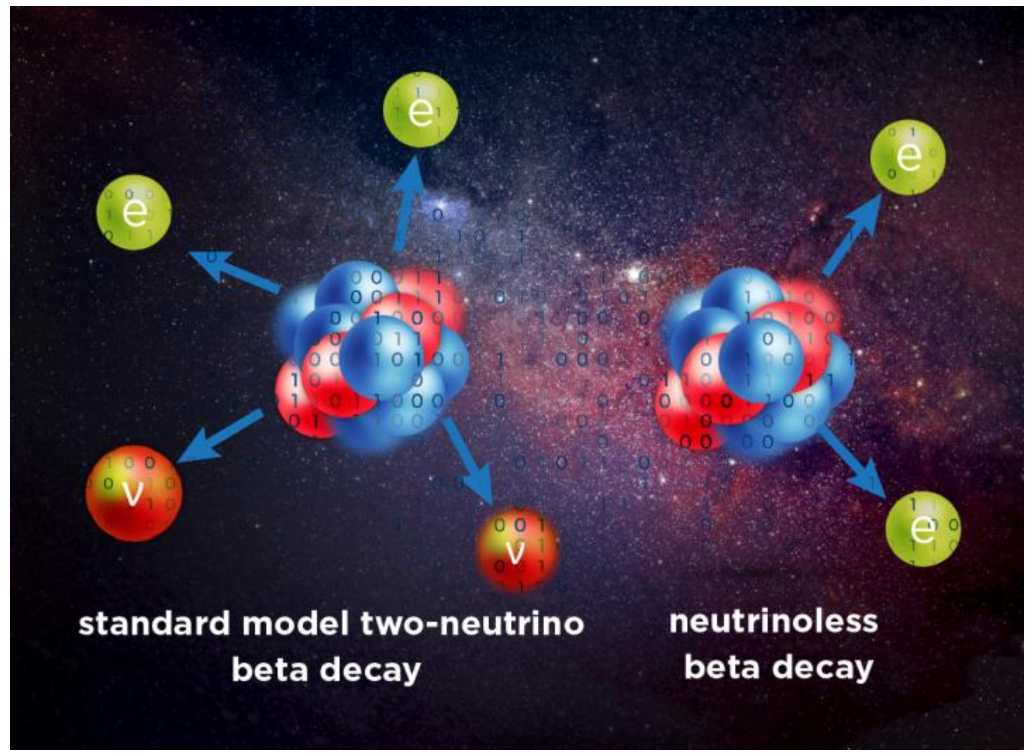
Nuclear Hamiltonians

- Nuclear Hamiltonians influence a vast range of phenomena
    - Properties of nuclei
    - Dense matter
    - Nuclei as probes of physics beyond the Standard Model
- Effective Field Theory (EFT) has emerged as a valuable tool to relate nuclear Hamiltonians to QCD and (B)SM physics







Outline

- Introduction:
  - Chiral EFT and the role of (non-perturbative) renormalization in power counting

- Three examples:
  - Searching for new physics: neutrinoless double beta decay
  - Precision tests of the Standard Model: superallowed nuclear β decays
  - Three nucleon interactions

Special thanks to collaborators:
M. Dawid, W. Dekens, J. de Vries, S. Gandolfi, M. Hoferichter,
E. Mereghetti, S. Pastore, M. Piarulli, S. Reddy , B. van Kolck

Apologies to everyone else for sloppy and incomplete references

Chiral EFT

Weinberg, van Kolck, Ordonez, Kaplan, Savage, Wise, Meissner, Epelbaum, Krebs, Bernard, Hammer, Bedaque, Beane, ...

- Expand amplitudes in $\frac{Q}{\Lambda_\chi}$

← low scale: $p, m_\pi, \ldots$

← high scale (EFT breakdown): $M_{\mathrm{QCD}}, 4\pi F_\pi, m_N, \ldots$

$$\mathscr{L}_{\mathrm{eff}} = \sum_i c_i(\Lambda, \Lambda_\chi)\, O_i(\Lambda)$$

O_i ordered by Δ = #derivatives + 2 #m_q

c_i: Low Energy Constants (LECs)

Λ: regulator scale or subtraction point or dim-reg scale

Power counting: rules that determine which diagrams and vertices from $\mathscr{L}_{\mathrm{eff}}$ contribute to amplitudes at given order in Q/Λ_χ

$$\Big\downarrow$$

$$T(Q) = \sum_\nu \left(\frac{Q}{\Lambda_\chi}\right)^\nu F_\nu\left(\frac{Q}{\Lambda}, c_i(\Lambda, \Lambda_\chi)\right)$$

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$$\Big\downarrow$$

A physical amplitude does not depend on Λ →

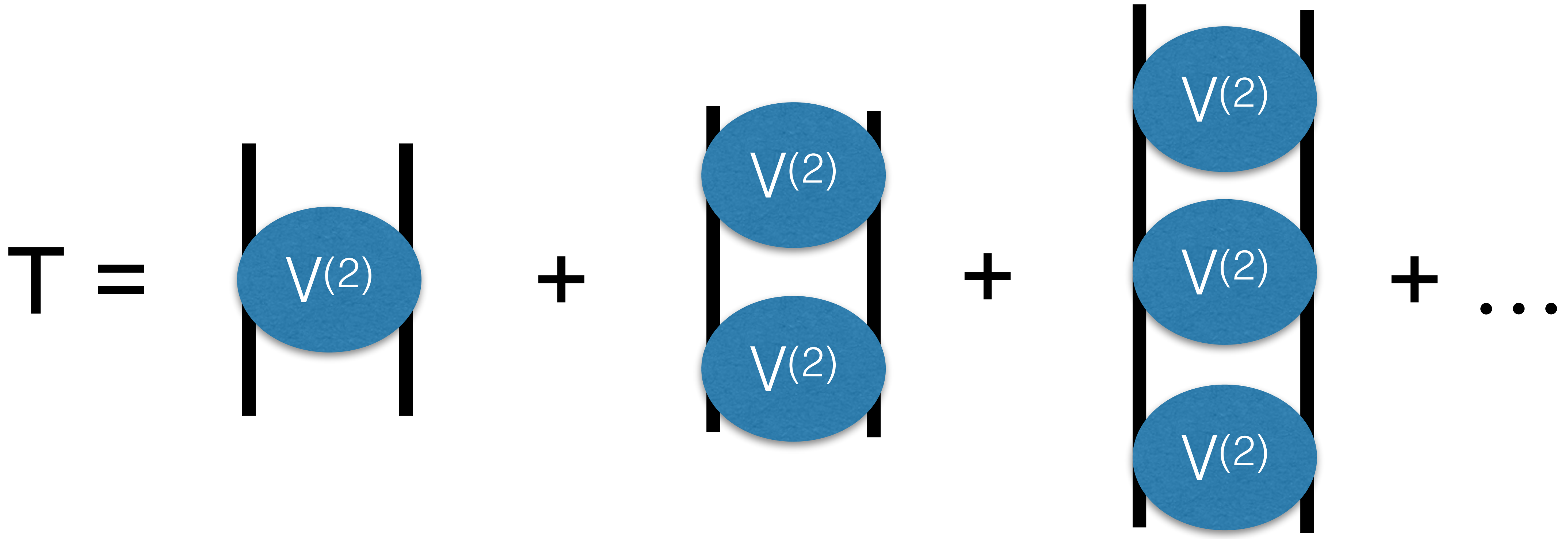
$$T(Q) = \sum_\nu \left(\frac{Q}{\Lambda_\chi}\right)^\nu F_\nu\left(\frac{Q}{\Lambda}, c_i(\Lambda, \Lambda_\chi)\right)$$

- Renormalization: absorb UV divergences in effective couplings, $c_i(\Lambda, \Lambda_\chi)$, in such a way that observables do not depend on the regulator Λ or subtraction point up to higher order ('RG-invariance')

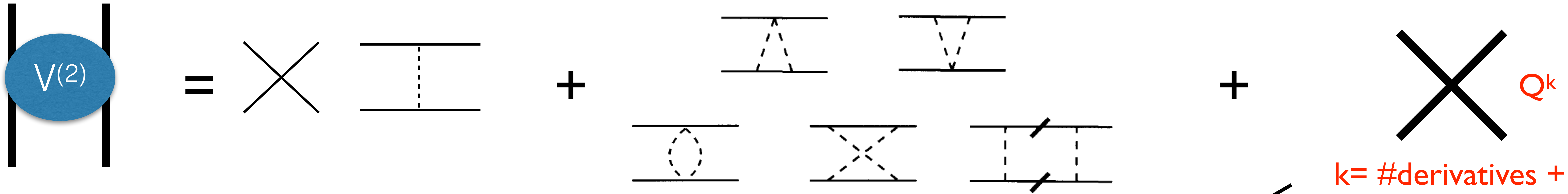
Potentials and amplitudes

Weinberg ’91

- Weinberg’s observation: IR-enhancements in A-nucleon reducible diagrams require resummation



1. Power-count & renormalize the potential (irreducible diags)
2. Iterate it in the Schroedinger equation



k= #derivatives + 2 #m$_q$

$$\Lambda_{NN} = \frac{16\pi F_\pi^2}{g_A^2 m_N} \sim 3F_\pi$$

$$\Lambda_\chi \sim 4\pi F_\pi$$

$$\boxed{C_k \sim \frac{4\pi}{m_N \Lambda_{NN}} \frac{1}{\Lambda_\chi^k}}$$

LECs of contact interactions scale in the same way as the (pion) loops for which they absorb divergences (Naive Dimensional Analysis, NDA)

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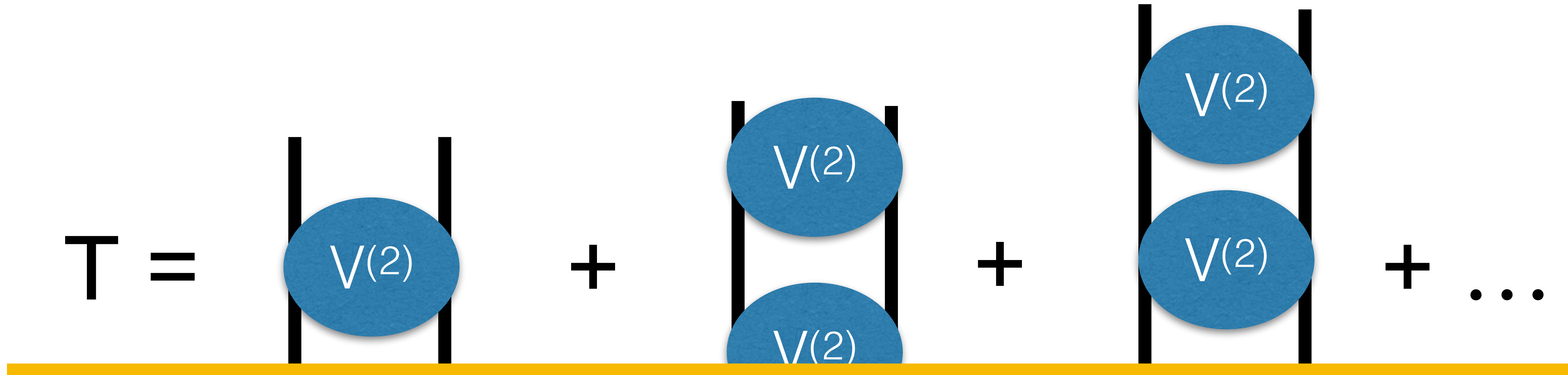

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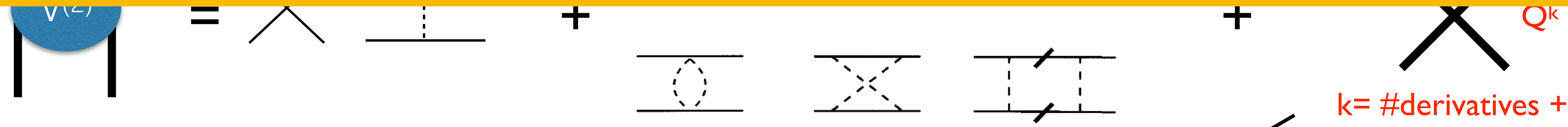
UV divergences in the iteration of the potentials (reducible loops) can upset NDA scaling for the short-range C_k

and require new contact terms to ensure RG-invariance at a given order



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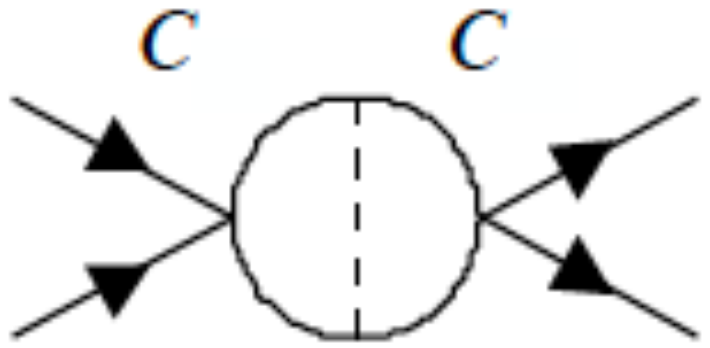
Renormalized amplitudes

Kaplan-Savage-Wise ’96, Mehen-Stewart ’99, Beane et al’ 02, Nogga-Timmermans-van Kolck ’05 , Long-Yang ’11,’12, , …

- No problem if at a given order one can reabsorb the ‘iteration divergences’ in the potential at that order, but there are cases in which this doesn’t work, e.g.:



  - $^1S_0$ channel: iteration of the LO contact and one-pion exchange. Need enhanced contact term ~ m_π^2

$$C \sim \frac{4\pi}{m_N \Lambda_{NN}} \sim \frac{1}{F_\pi^2}$$

$$D_2 \sim \frac{4\pi}{m_N \Lambda_{NN}} \frac{1}{(4\pi F_\pi)^2}$$

$$\mathscr{L} = -C\,\bar{N}N\bar{N}N - m_\pi^2 D_2\,\bar{N}N\bar{N}N$$

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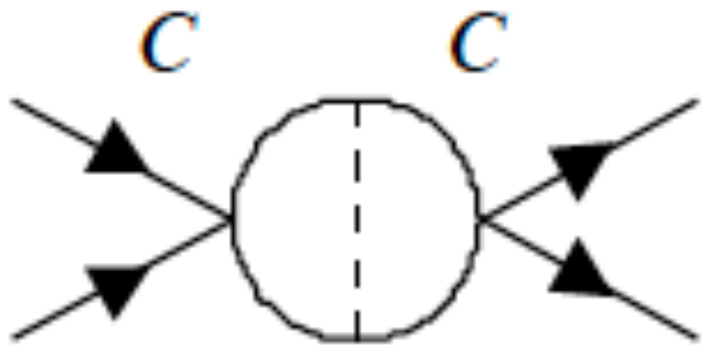

$$C \sim \frac{4\pi}{m_N \Lambda_{NN}} \sim \frac{1}{F_\pi^2}$$

$$D_2 \sim \frac{4\pi}{m_N \Lambda_{NN}} \frac{1}{(\cancel{4}\pi F_\pi)^2}$$

$$\frac{1}{(4\pi F_\pi^2)} \to \frac{1}{\Lambda_{NN}^2}$$

$$\mathscr{L} = -C\, \bar{N}N\bar{N}N - m_\pi^2 D_2\, \bar{N}N\bar{N}N$$

$$\sim \frac{g_A^2 m_\pi^2 m_N^2}{16\pi^2 F_\pi^2} C^2 \ \log \Lambda$$

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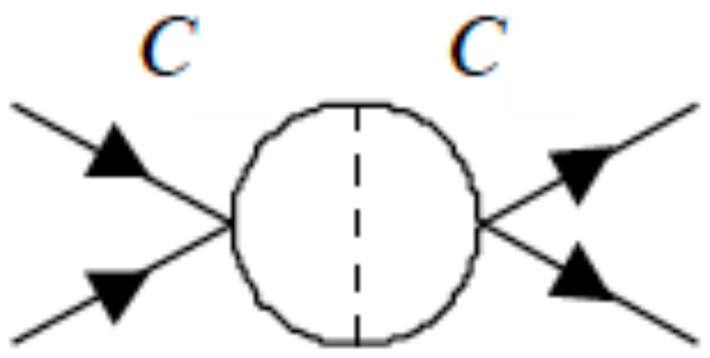
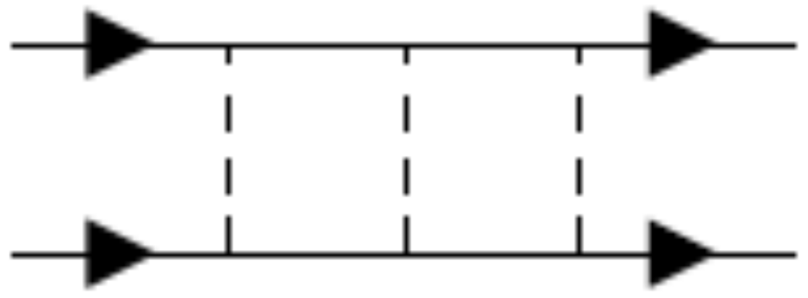


  - Spin triplet waves with attractive 1π-exchange tensor potential (singular potential ~$-1/r^3$) : $^3P_0$, … Need *derivative* contact terms at leading order



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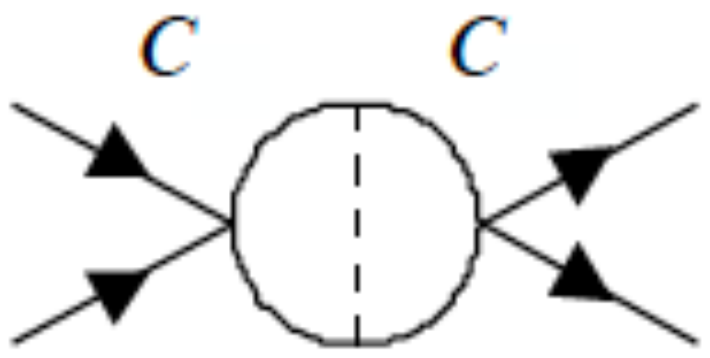

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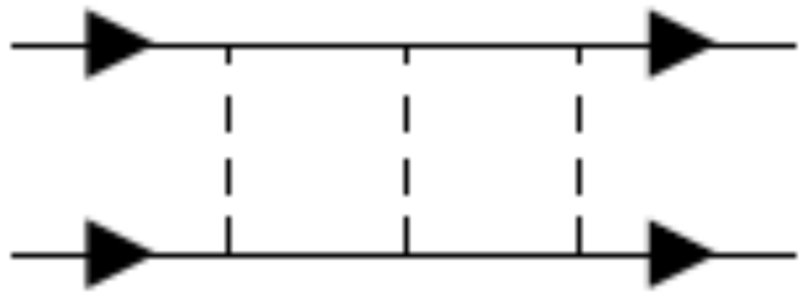

- *'Renormalized' chiral EFT**:* in addition to NDA, use RGEs scaling as guiding principle for power counting — point of view adopted by a fraction of the community

** Several proposals in the literature, see for example talks at INT-25-92W (https://www.int.washington.edu/programs-and-workshops/25-92w)

Why bother with renormalization?

- Limitations of Weinberg's scheme are clearly visible in precision tests of the SM and BSM searches with nuclei:

Neutrinoless double beta decay

$$T_{1/2}^{-1}(0\nu\beta\beta) = \tilde{k}\,|m_{\beta\beta}|^2\,|M(\Lambda)|^2\,\Phi_{PS}$$

Superallowed beta decays

$$T_{1/2}^{-1}(\beta) = kG_F^2\,|V_{ud}|^2\,m_e^5\left[1+\frac{\alpha}{\pi}\Delta(\Lambda)\right]\Phi_{PS}$$

In these cases even a lower order calculation with robust error estimate (even at the 20-30% level) is extremely valuable

Λ-independent experimental result

Want to extract (or bound) these parameters from experiment with precision and accuracy

Nuclear matrix element should not introduce spurious Λ dependence

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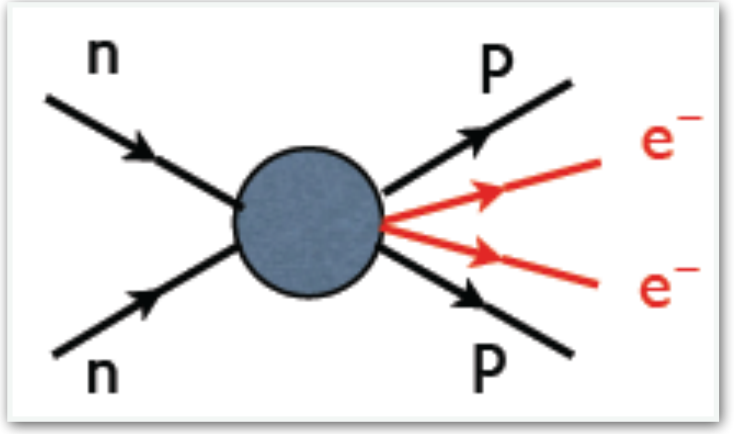
- Similar issues with CP-violating interactions*, NN EW currents**, WIMP-nucleus scattering, **3N forces**
- Next: will discuss **0νββ decay**, **β decay**, and **3NF** examples. In some cases data can provide guidance

* de Vries, Gnech, Shain 2020, ** Phillips, Pavon-Valderrama 2015

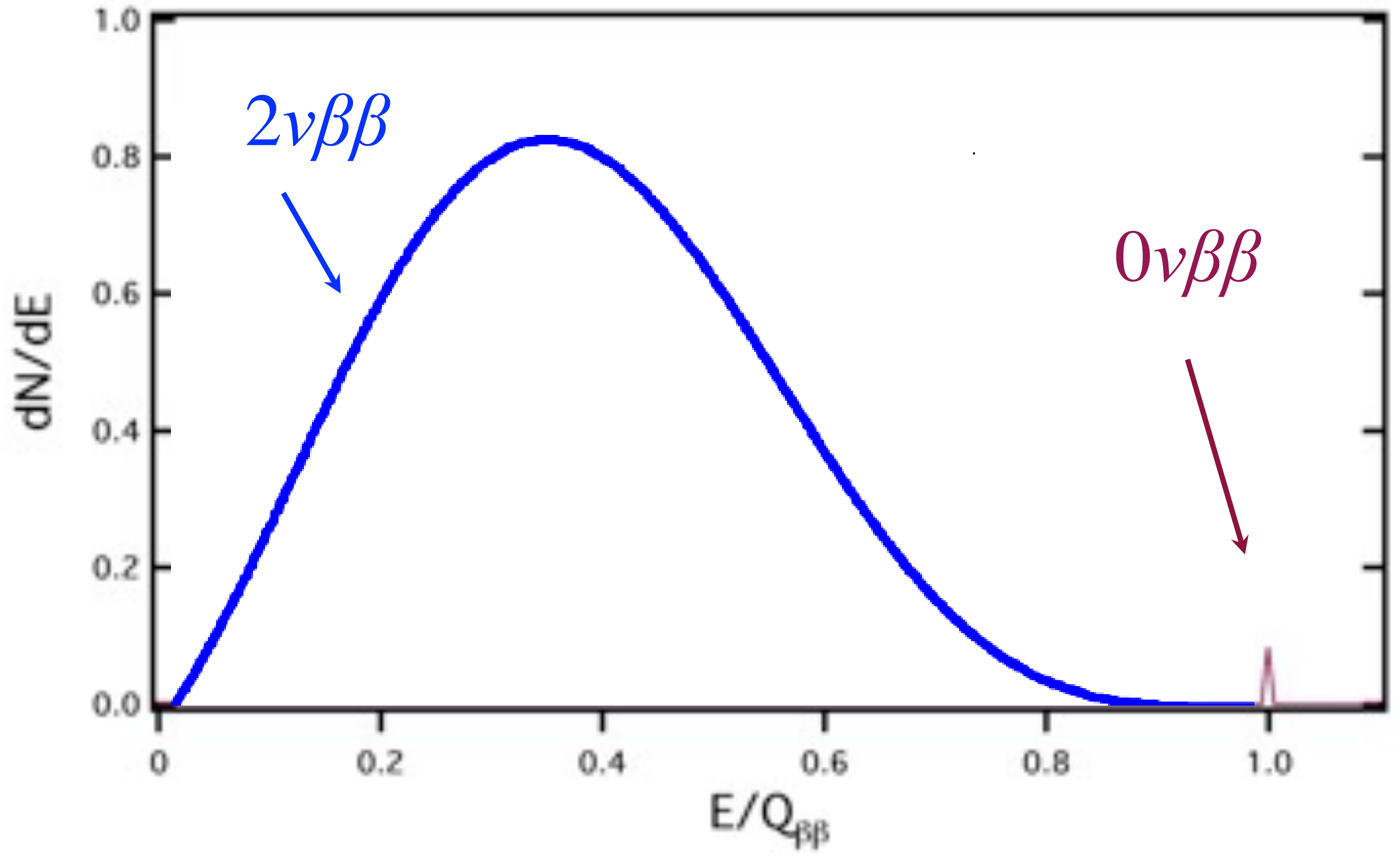
Topic 1: Neutrinoless double beta decay

$$(N,Z) \rightarrow (N-2,Z+2) + e^- + e^-$$

$$\boxed{T_{1/2} > \# 10^{25}\mathrm{yr}}$$



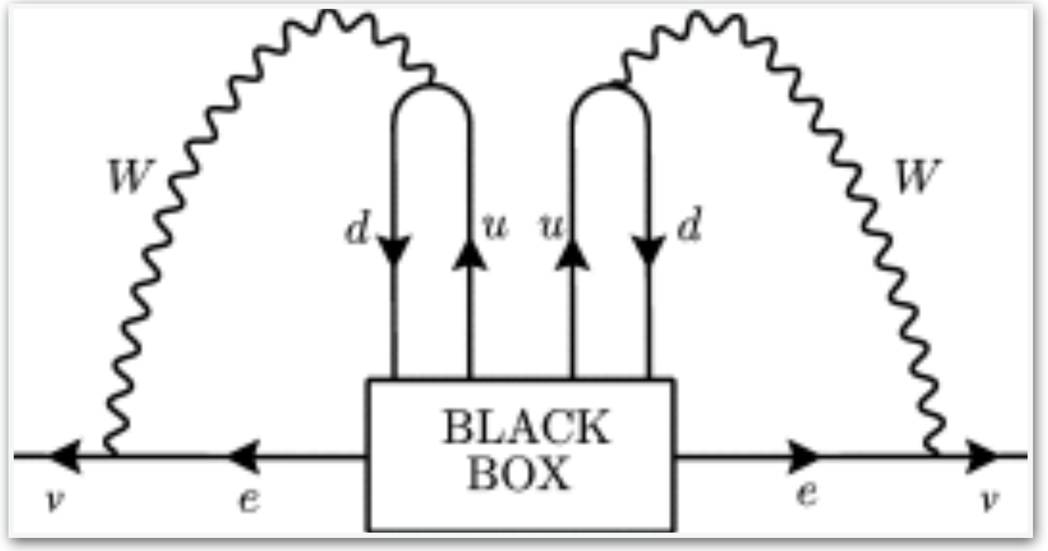
ΔL=2



Potentially observable in certain even-even nuclei (^{48}Ca, ^{76}Ge, ^{136}Xe, …) for which single beta decay is energetically forbidden

- Observation would have far-reaching implications
  - Demonstrate that neutrinos are Majorana fermions
  - Establish LNV, key ingredient to generate baryon asymmetry via leptogenesis

Shechter-Valle 1982



Fukujita-Yanagida 1987

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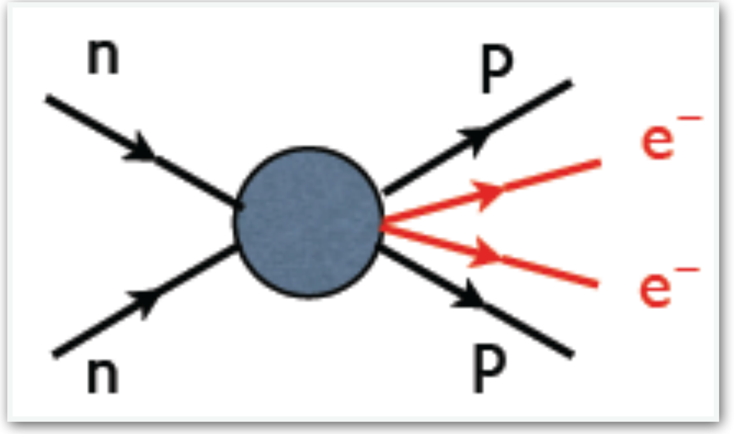

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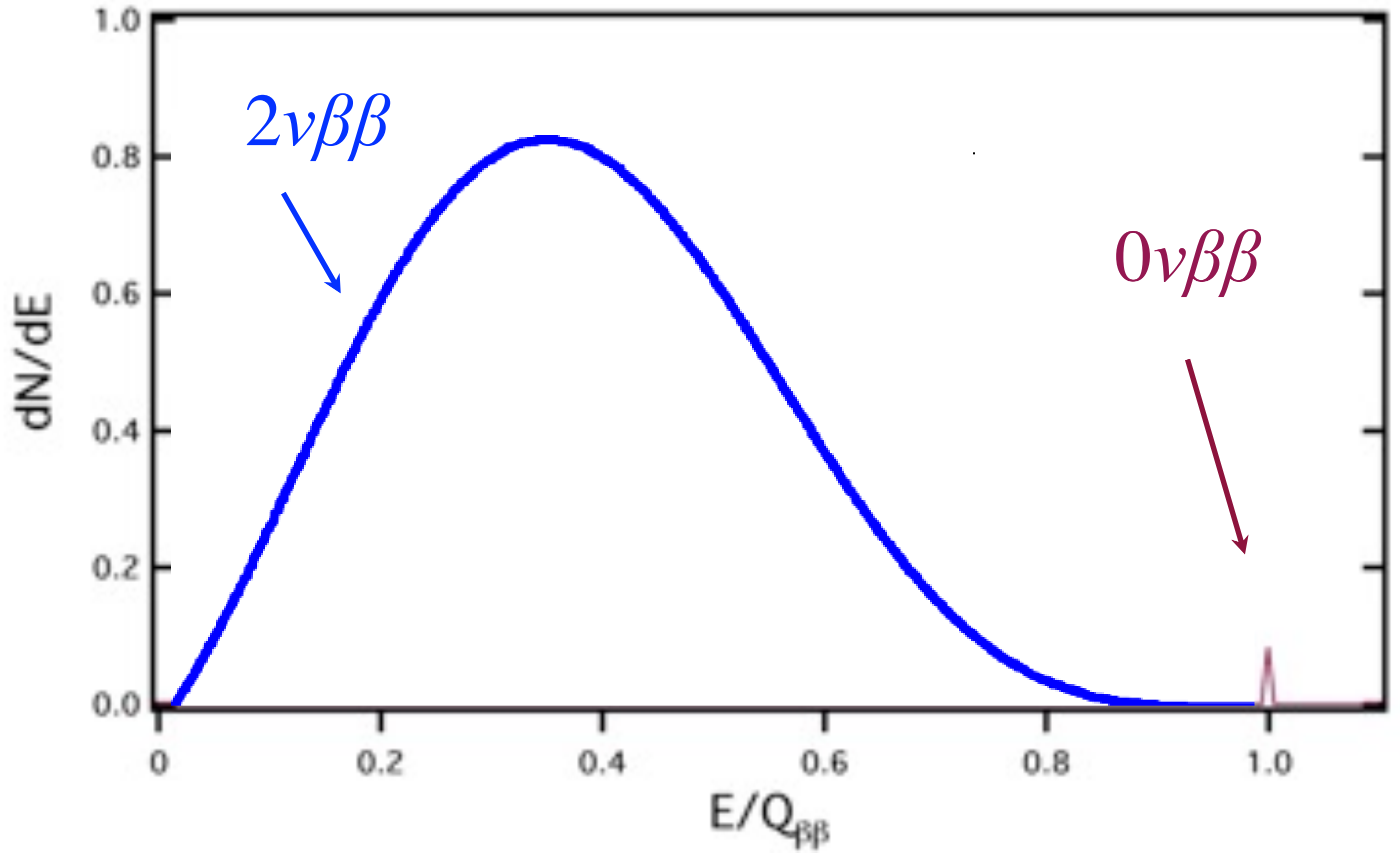

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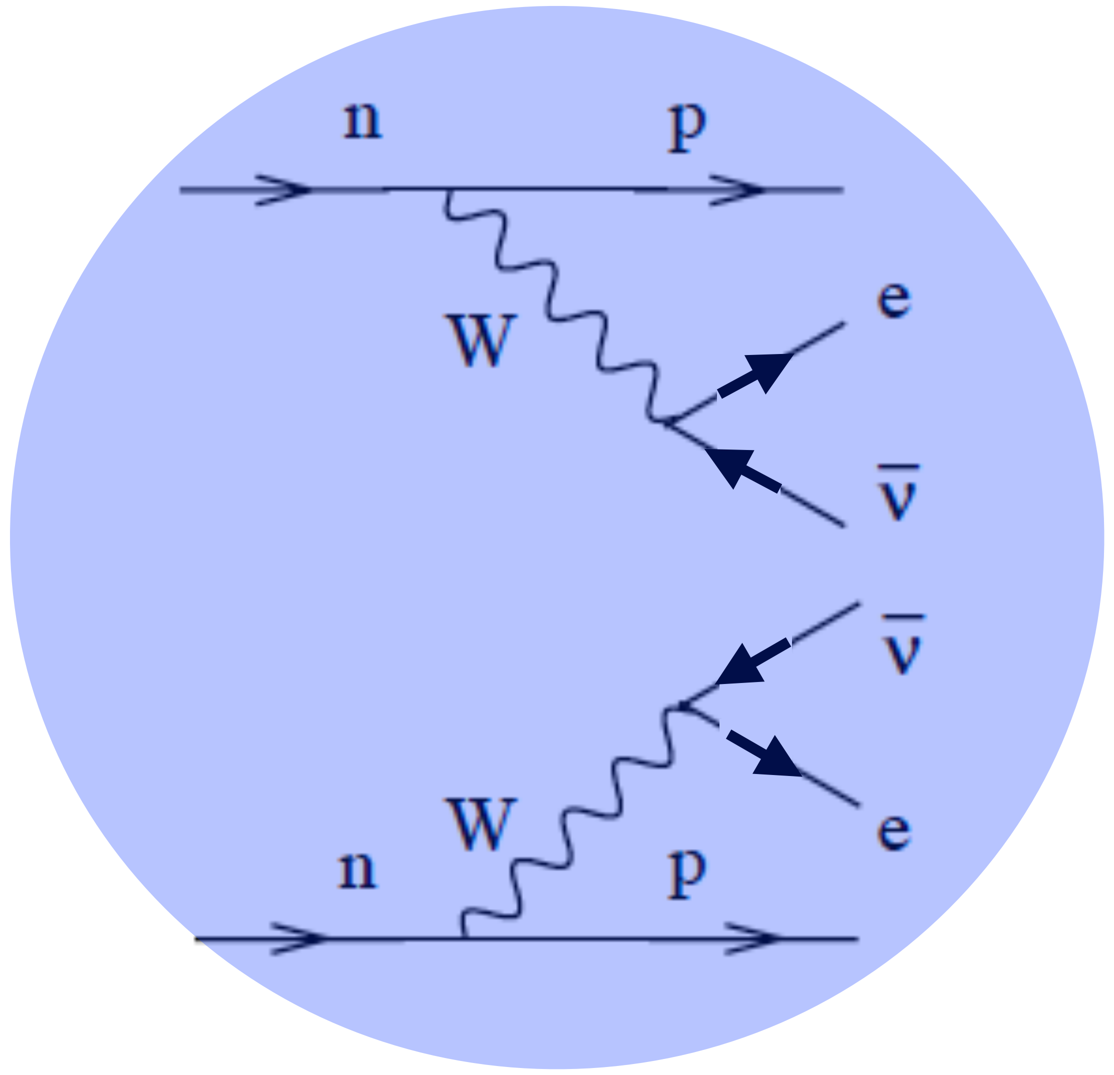
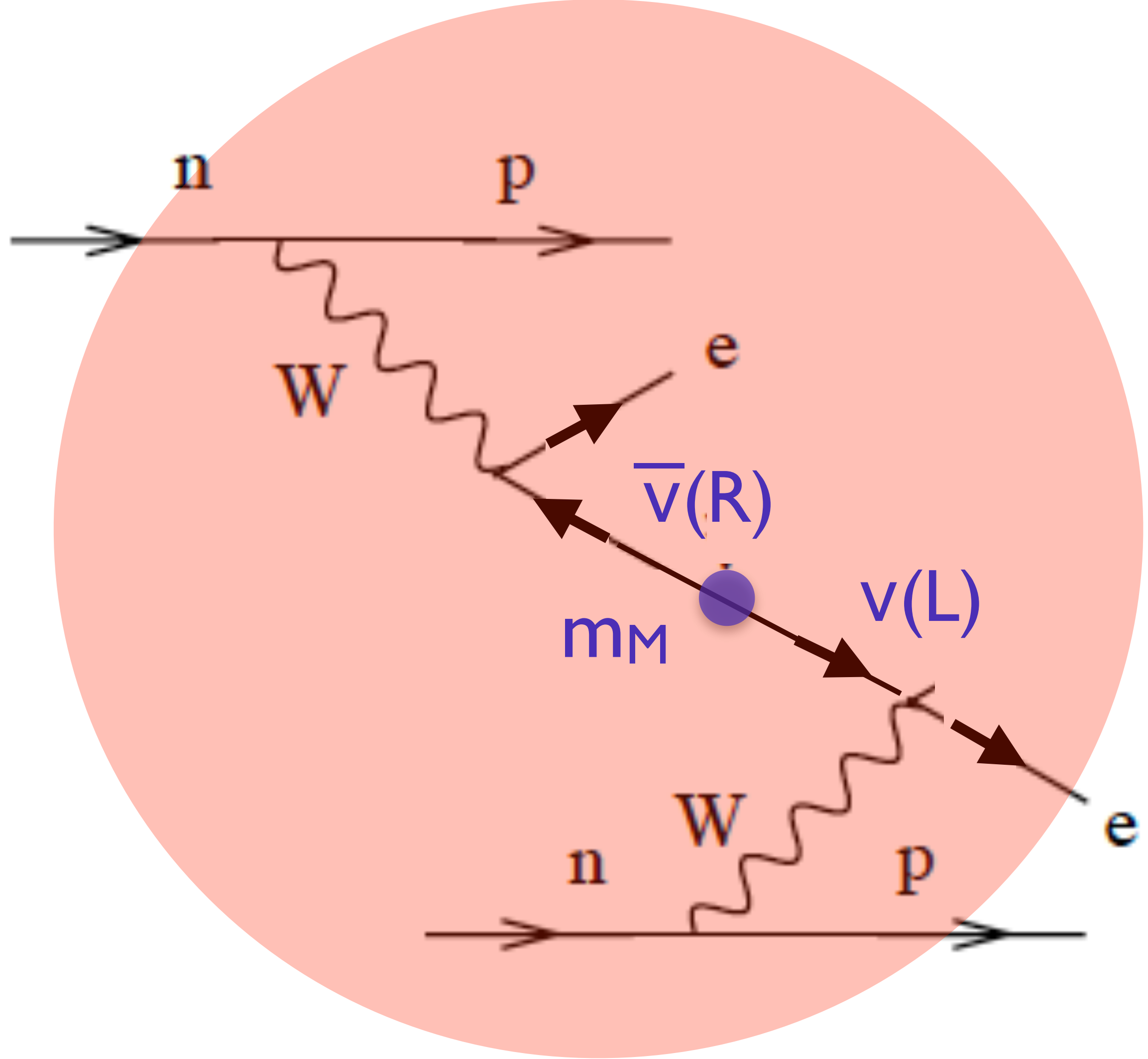


High-scale seesaw →
light neutrino Majorana mass term

(But LNV could come from other source)

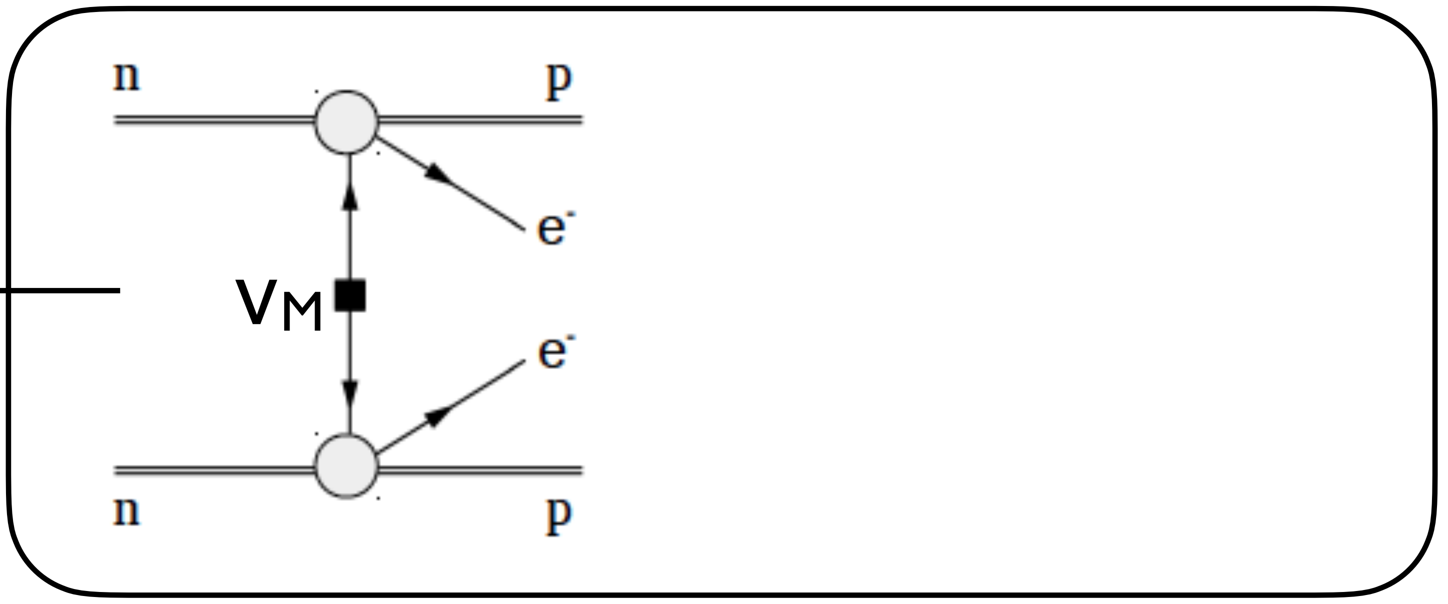


Leading order transition operator

VC, W. Dekens, E. Mereghetti, A. Walker-Loud, 1710.01729
VC, W. Dekens, J. de Vries, M. Graesser, E. Mereghetti, S. Pastore, U. van Kolck 1802.10097

- To leading order (LO) in Q/Λ_χ the nn → pp transition operator has *two contributions*

'Usual' ν_M exchange ~ $1/k_F^2 \sim 1/Q^2$
Coulomb-like long-range potential



Potential neutrino exchange

$$V_{\nu,0}^{(a,b)} = \tau^{(a)+}\tau^{(b)+} \boxed{\frac{1}{\mathbf{q}^2}} \left[1 + 2g_A^2 + \frac{g_A^2 m_\pi^4}{(\mathbf{q}^2 + m_\pi^2)^2}\right]$$

Only hadronic input: g_A

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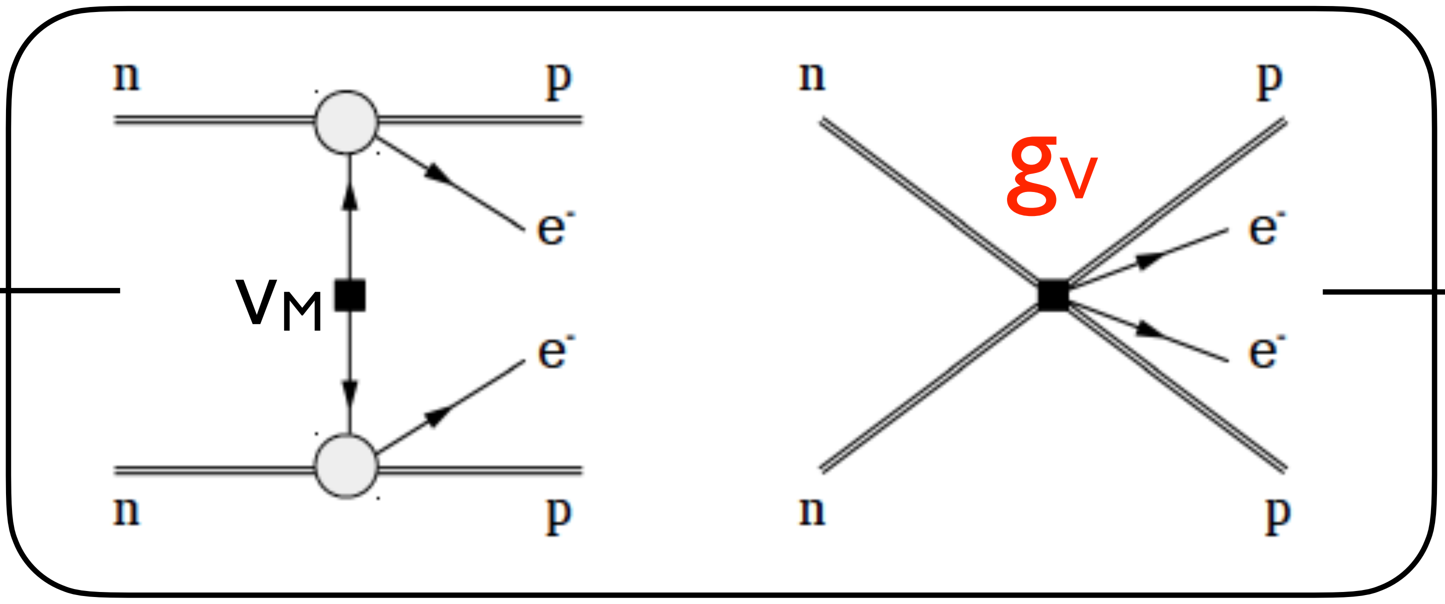
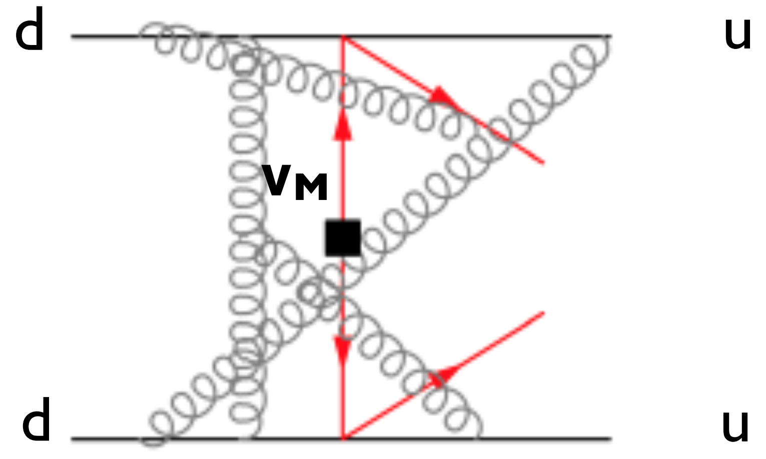


‘New’: short-range potential with
coupling g_ν ~$1/Q^2$



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Only
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Hard
neutrino
exchange

$$V_{\nu,CT}^{(a,b)} = -2\, g_\nu\, \tau^{(a)+}\tau^{(b)+}$$

g_ν ~ $1/Q^2$ >> $1/\Lambda_\chi^2$ ~$1/(4\pi F_\pi)^2$
(Much larger than estimate from
Naive Dimensional Analysis)

Leading order transition operator

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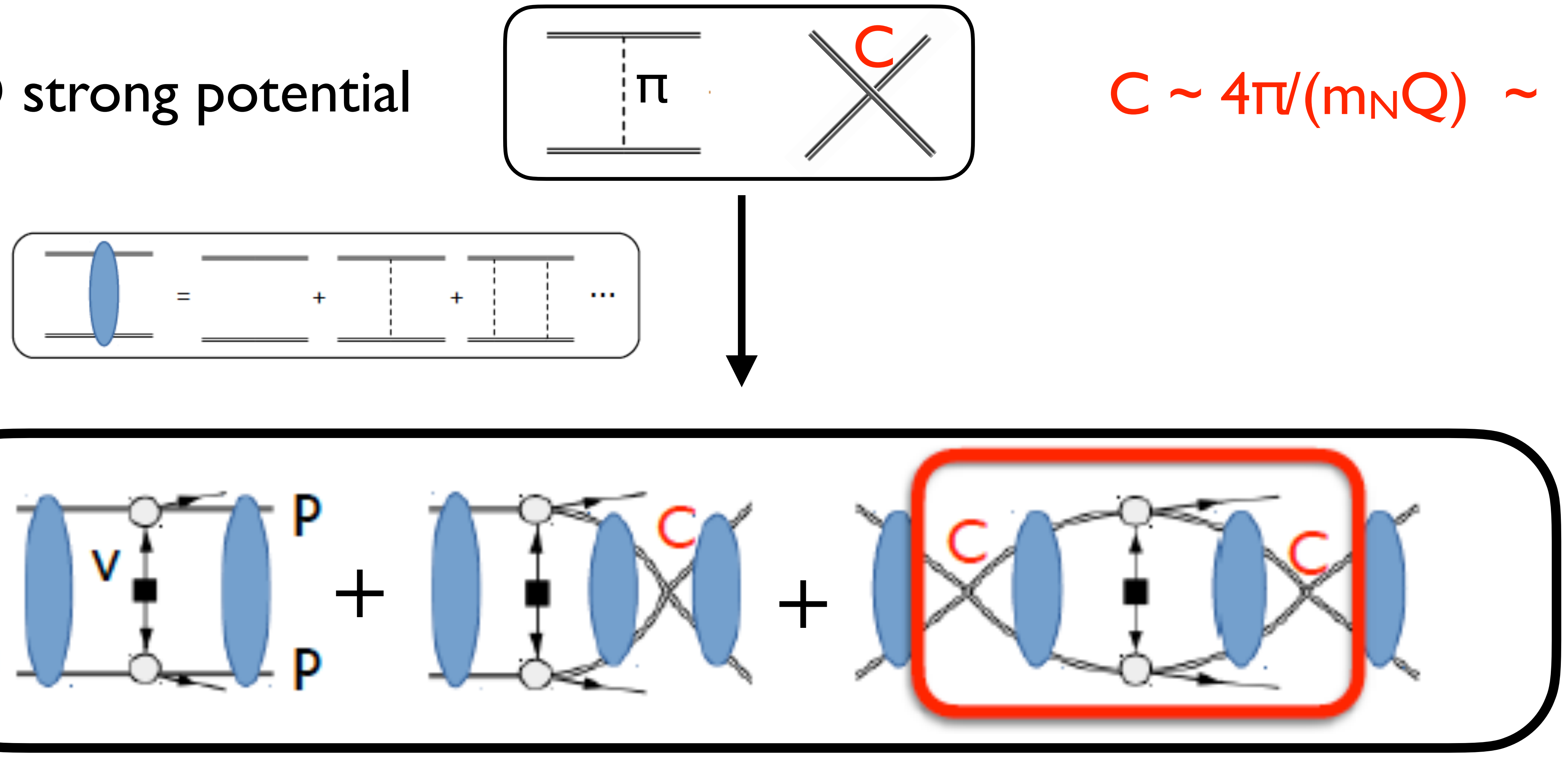
- Contact term is required at LO to renormalize of the $^1S_0$ nn→pp amplitude in presence of strong interactions

LO strong potential



$C \sim 4\pi/(m_N Q) \sim 1/F_\pi^2$

$+\ldots$

UV divergence

$$\sim \frac{1}{2}(1+2g_A^2)\left(\frac{m_N C}{4\pi}\right)^2\left(\frac{1}{4-d}+\log\mu^2\right)$$

Use dim-reg to illustrate the point

$(m_N C/4\pi)^2 \sim 1/Q^2 \sim 1/F_\pi^2$

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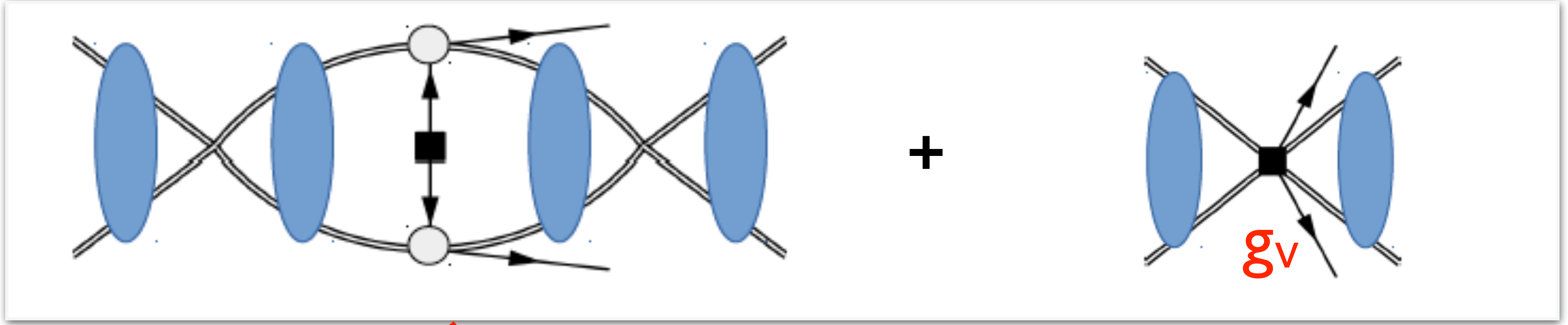
- Renormalization group implies that the coupling flows to $g_\nu \sim 1/Q^2 \gg 1/(4\pi F_\pi)^2$

The sum is finite
and scale
independent



$$\boxed{\sim \frac{1}{2}(1+2g_A^2)\left(\frac{m_N C}{4\pi}\right)^2\left(\frac{1}{4-d}+\log\mu^2\right)}$$

This holds in any regularization

- Same conclusion obtained by solving the Schrodinger equation
  - Example: use smeared delta function to regulate short range strong potential: $C \to C\,(R_S)$
  - Compute amplitude

$$\delta^{(3)}(\mathbf{r}) \to \frac{1}{\pi^{3/2} R_S^3} e^{-\frac{r^2}{R_S^2}}$$

$$\mathcal{A}_\nu = \int d^3\mathbf{r}\ \psi^-_{\mathbf{p}'}(\mathbf{r})\, V_{\nu,0}(\mathbf{r})\, \psi^+_{\mathbf{p}}(\mathbf{r})$$

Scattering states "fully correlated" according to the leading order strong potential in the $^1S_0$ channel

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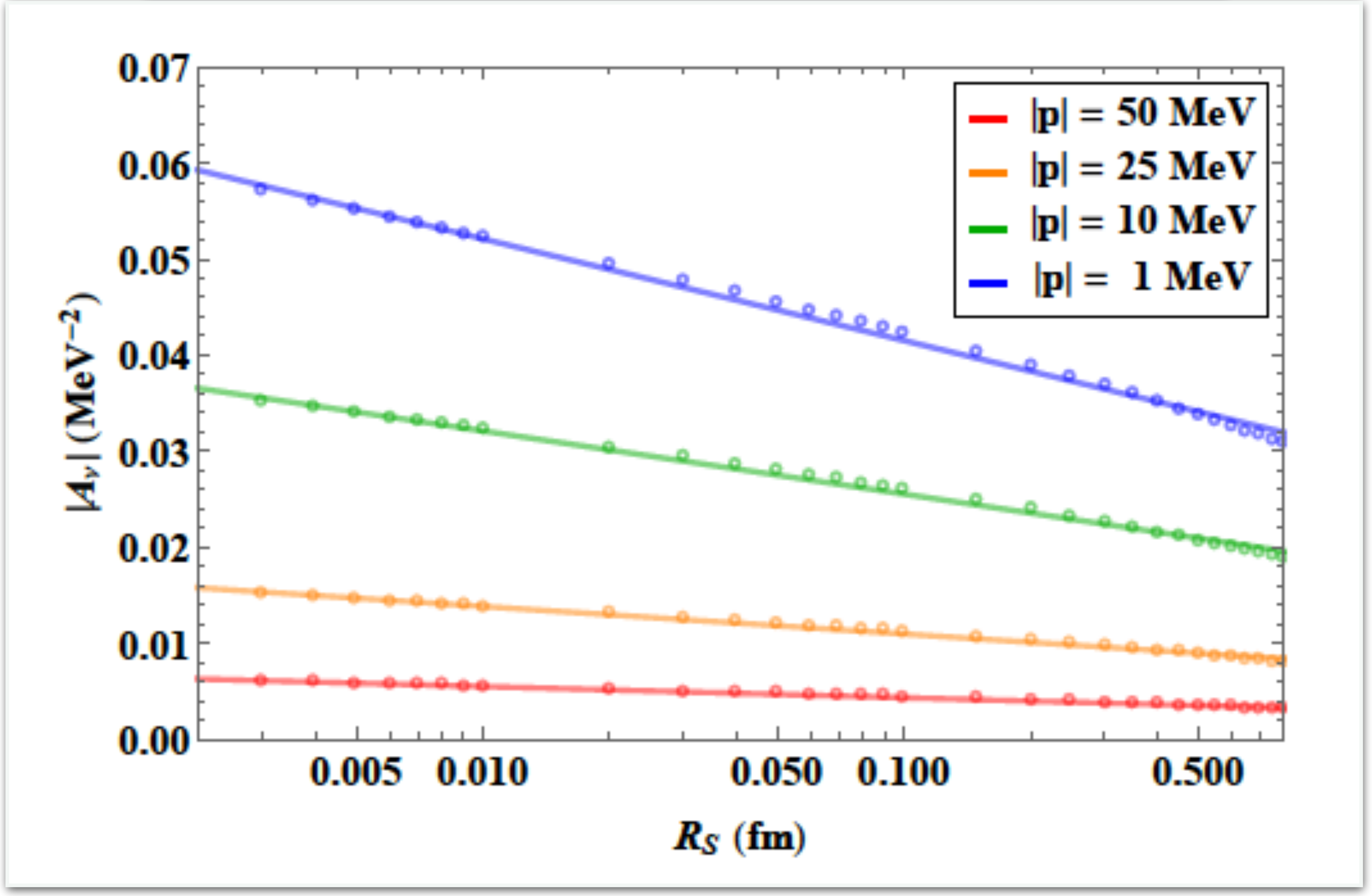


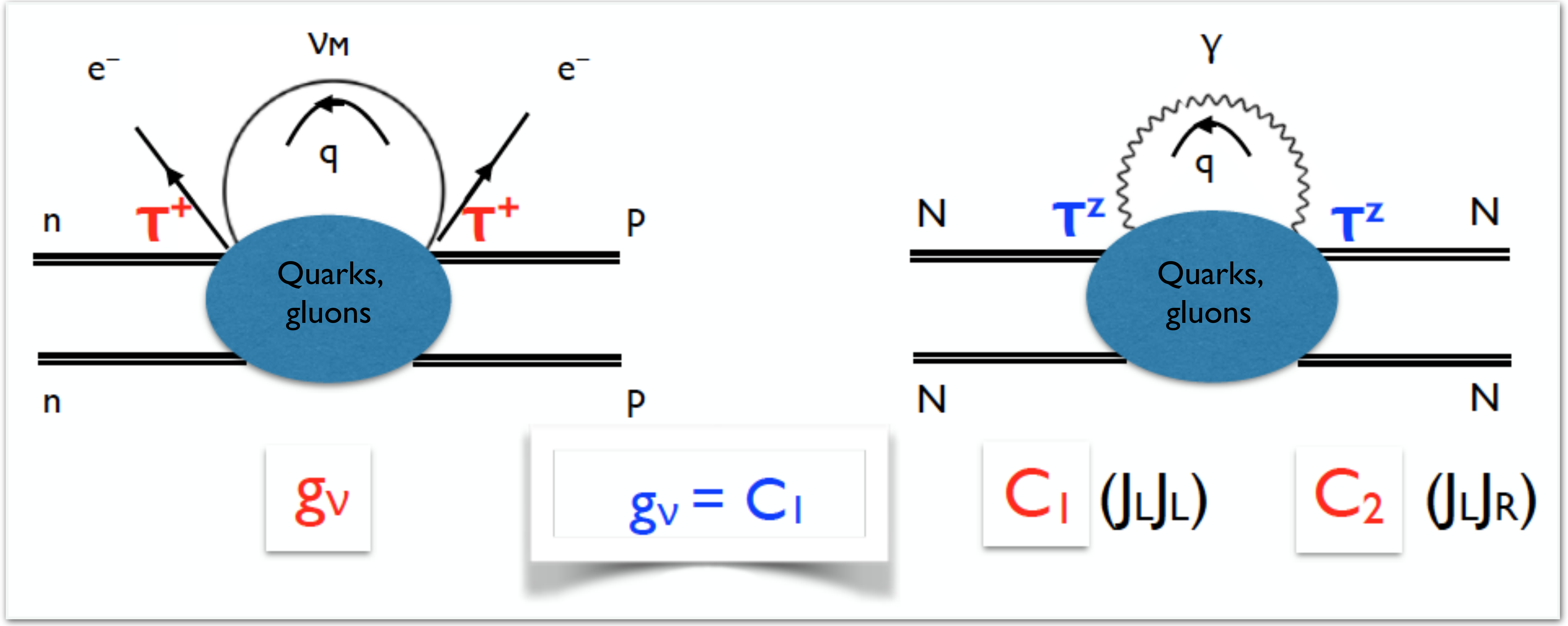
- Logarithmic dependence on R_S ⇒

  need LO contact term $g_\nu \sim 1/F_\pi^2 \log R_S$ to obtain physical, regulator-independent result

- Similar story with Lippmann-Schwinger equation with momentum cutoff

Connection with electromagnetism (and data)

- Isospin symmetry relates g_ν to one of two I=2 e.m. couplings (hard γ's versus hard ν's)

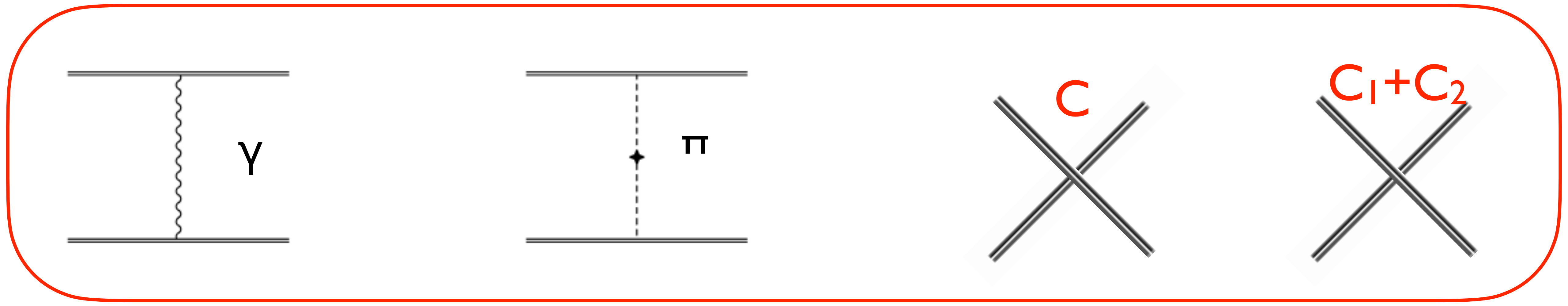


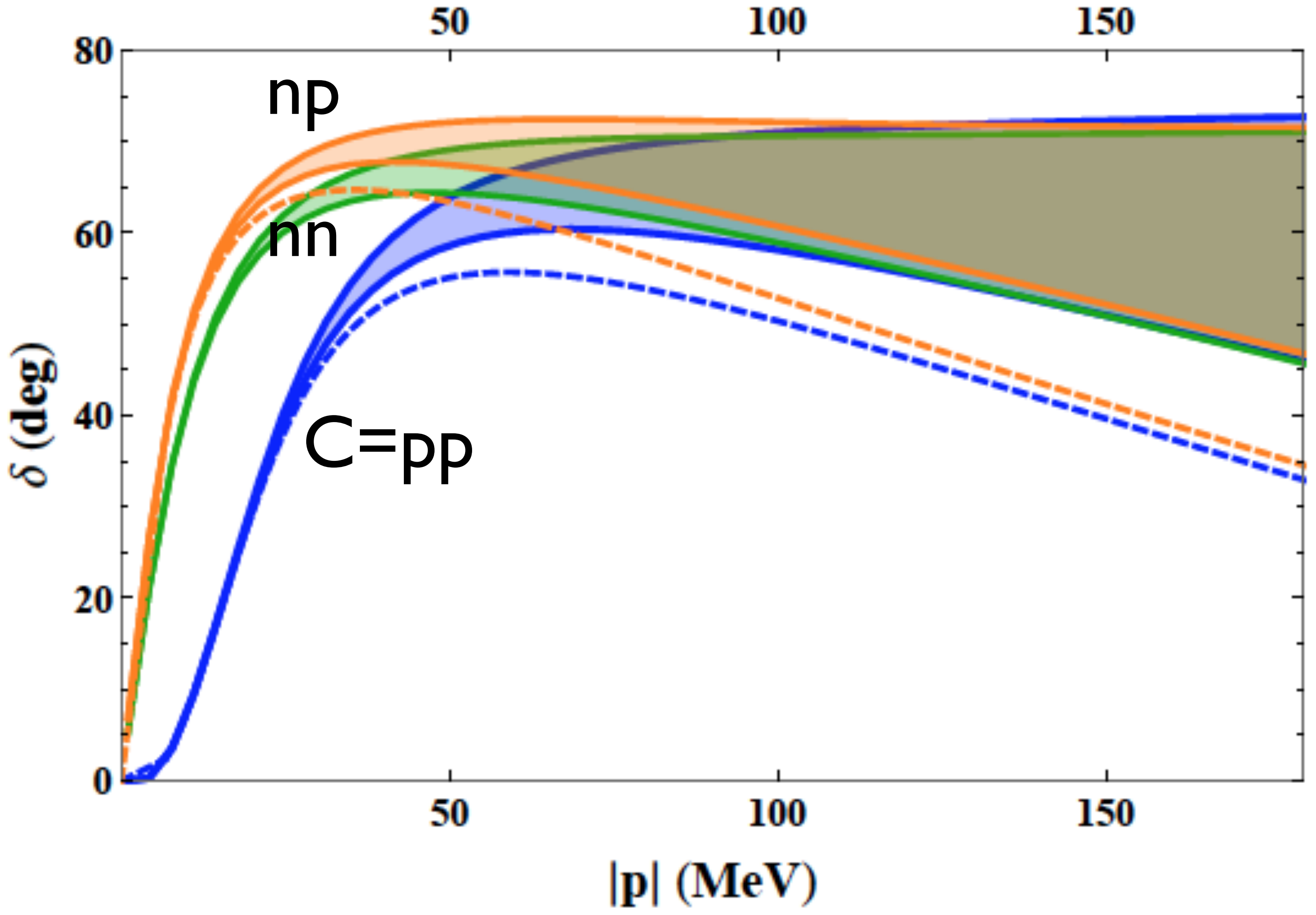
- NN scattering data at low energy ($a_{nn}+a_{pp}-2a_{np}$) determine C_1+C_2, confirming LO scaling!

Connection with data — some details

$$a_{np} = -23.7 \pm 0.02 \text{ fm}\,, \qquad a_{nn} = -18.90 \pm 0.40 \text{ fm}\,, \qquad a_C = -7.804 \pm 0.005 \text{ fm}\,.$$

- $C_1 + C_2$ controls CIB combination of $^1S_0$ scattering lengths $a_{nn} + a_C - 2\,a_{np}$





- Fit to data, including LO strong, Coulomb potential, pion EM mass splitting, and contact terms confirms the scaling $C_1 + C_2 \gg 1/(4\pi F_\pi)^2$

$$\boxed{\frac{\mathcal{C}_1 + \mathcal{C}_2}{2} \equiv \left(\frac{m_N C}{4\pi}\right)^2 \left(2.5 - 1.8\,\ln(m_\pi/\mu)\right)}$$

Dim-reg with Minimal Subtraction

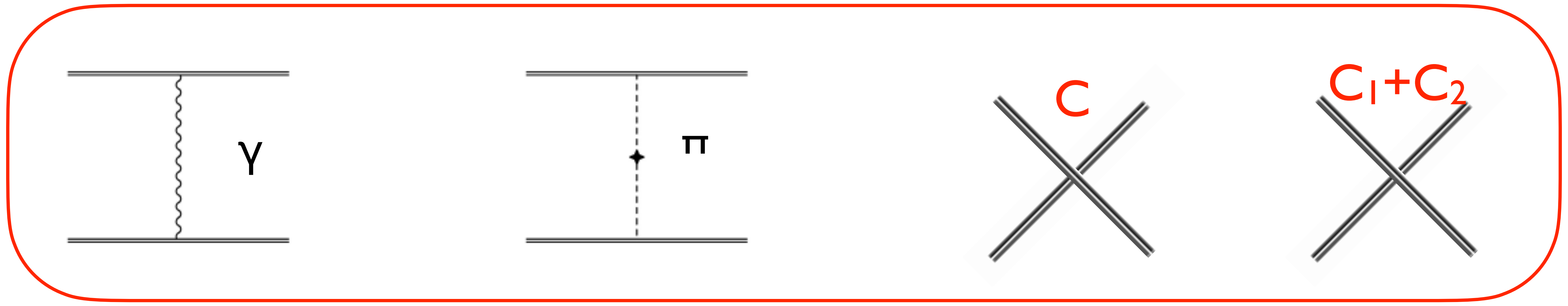
$$C = -\frac{1}{\tilde{\Lambda}^2}$$

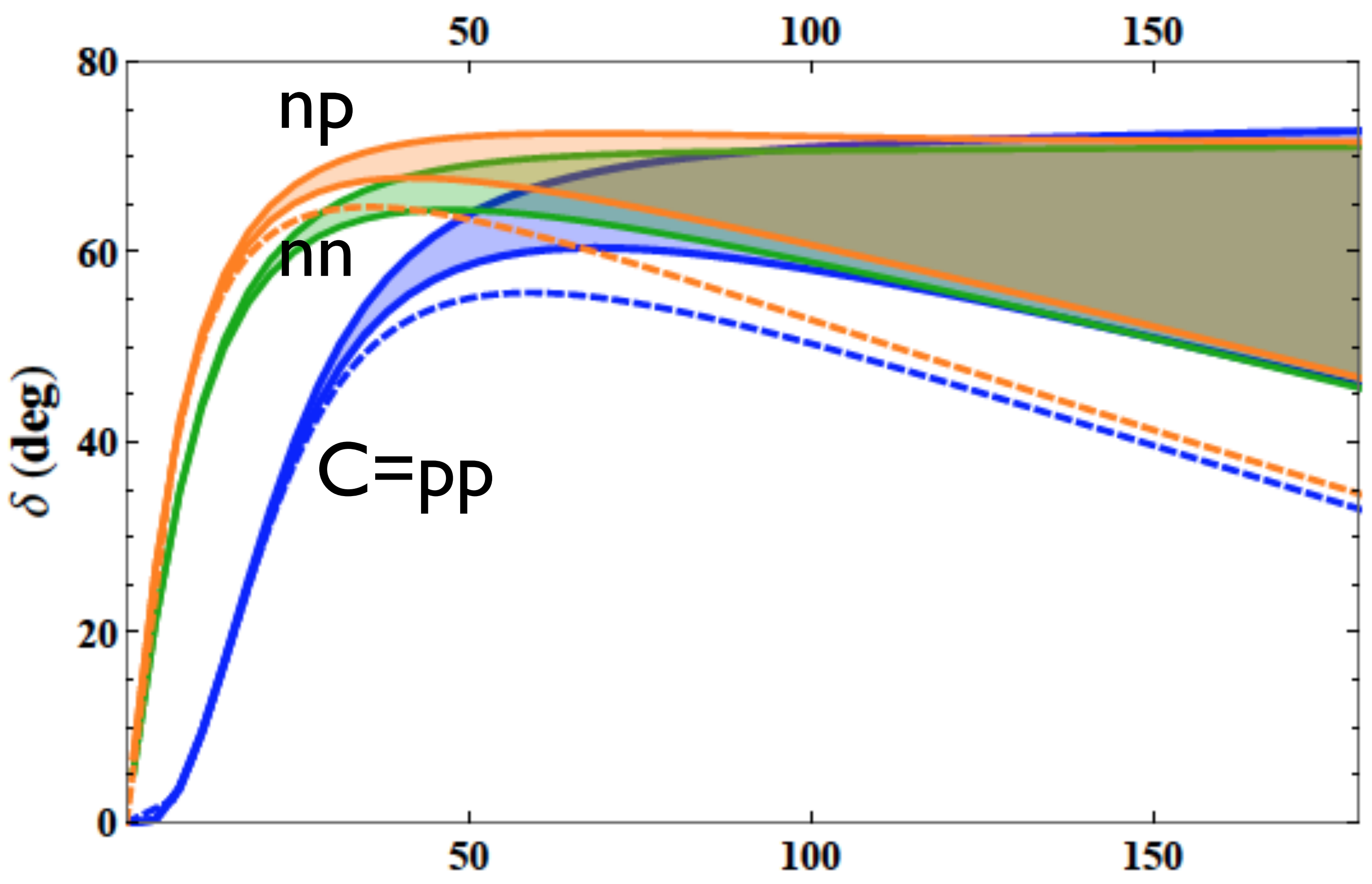
$$\tilde{\Lambda}(\mu = m_\pi) = O(100 \text{ MeV})$$

Connection with data — some details

$$a_{np} = -23.7 \pm 0.02 \text{ fm}\,, \qquad a_{nn} = -18.90 \pm 0.40 \text{ fm}\,, \qquad a_C = -7.804 \pm 0.005 \text{ fm}\,.$$

- $C_1 + C_2$ controls CIB combination of $^1S_0$ scattering lengths $a_{nn} + a_C - 2\, a_{np}$





- Fit to data, including LO strong, Coulomb potential, pion EM mass splitting, and contact terms confirms the scaling $C_1 + C_2 \gg 1/(4\pi F_\pi)^2$

In this case, the RG-based EFT counting survives comparison with data!

The analog of $e^2(C_1+C_2)$ is included in all high-quality potentials (AV18, CD-Bonn, chiral, …)

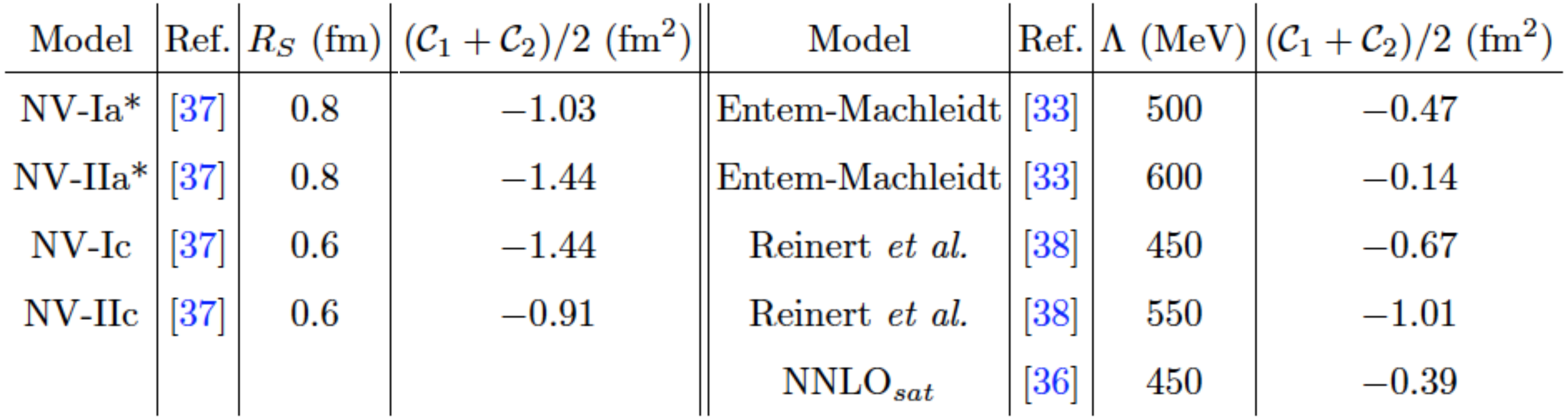
MeV)

C_1+C_2 in commonly used NN potentials

1907.11254

$$V_{\mathrm{CIB,S}} = -\frac{e^2}{6}\frac{\mathcal{C}_1+\mathcal{C}_2}{2}\,T^{(12)}\,\delta^{(3)}_{R_S}(\mathbf{r})$$

$$T^{(12)} = 3\,\tau_3^{(1)}\tau_3^{(2)} - \vec{\tau}^{(1)}\cdot\vec{\tau}^{(2)}$$



Model	Ref.	R_S (fm)	$(\mathcal{C}_1+\mathcal{C}_2)/2$ (fm^2)	Model	Ref.	Λ (MeV)	$(\mathcal{C}_1+\mathcal{C}_2)/2$ (fm^2)
NV-Ia*	[37]	0.8	-1.03	Entem-Machleidt	[33]	500	-0.47
NV-IIa*	[37]	0.8	-1.44	Entem-Machleidt	[33]	600	-0.14
NV-Ic	[37]	0.6	-1.44	Reinert *et al.*	[38]	450	-0.67
NV-IIc	[37]	0.6	-0.91	Reinert *et al.*	[38]	550	-1.01
				NNLO$_{sat}$	[36]	450	-0.39

- The above fits include higher order chiral terms
- Our LO fit gives $(C_1+C_2)/2 = 0.71$ fm^2 @ $R_S = 0.8$ fm
- All seem to confirm violation of Weinberg counting scaling $(C_1+C_2)/2 \sim 0.04$ fm^2

Impact of g_V and future directions

- Several approaches to determine g_V
  - Large-N_C arguments point to $g_V \sim (C_1+C_2)/2$

Richardson, Shindler, Pastore, Springer, 2102.02814

  - Lattice QCD — gearing up

Tuo et al. 1909.13525;
Detmold, Murphy 2004.07404

Davoudi, Kadam, 2012.02083
Davoudi et al, 2402.09362

  - Dispersive approach, estimated g_V with ~30% uncertainty (validated with NN CIB data)

VC, Dekens, deVries, Hoferichter, Mereghetti, 2012.11602, 2102.03371, Van Groffier 2024

- Contact term fit to synthetic data and used in ab-initio calculations for

^{48}Ca [1], ^{130}Te [2], ^{136}Xe, [2], ^{76}Ge [3]

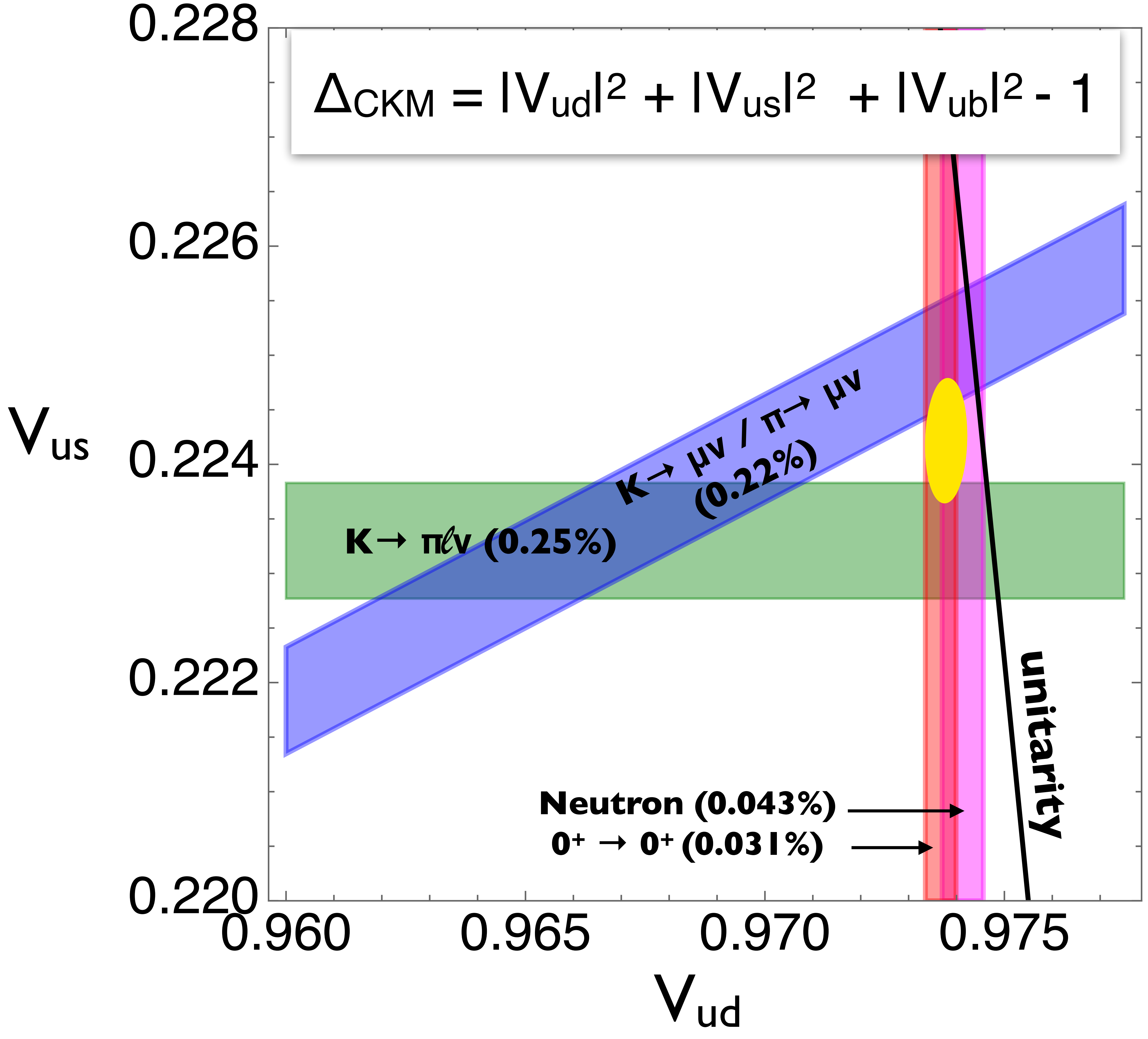
[2] Wirth, Yao, Hergert, 2105.05415 [3] Belley et al, 2307.15156 [4] Belley et al, 2308.15634

Enhances matrix elements by ~40% [Ca, Ge] and >50% [Te, Xe] —
good news for phenomenology, while we wait for Lattice QCD results

- Corrections up to N2LO in chiral EFT known

Topic 2: nuclear β decays and CKM unitarity

VC-Crivellin-Hoferichter-Moulson 2208.11707
[and references therein]

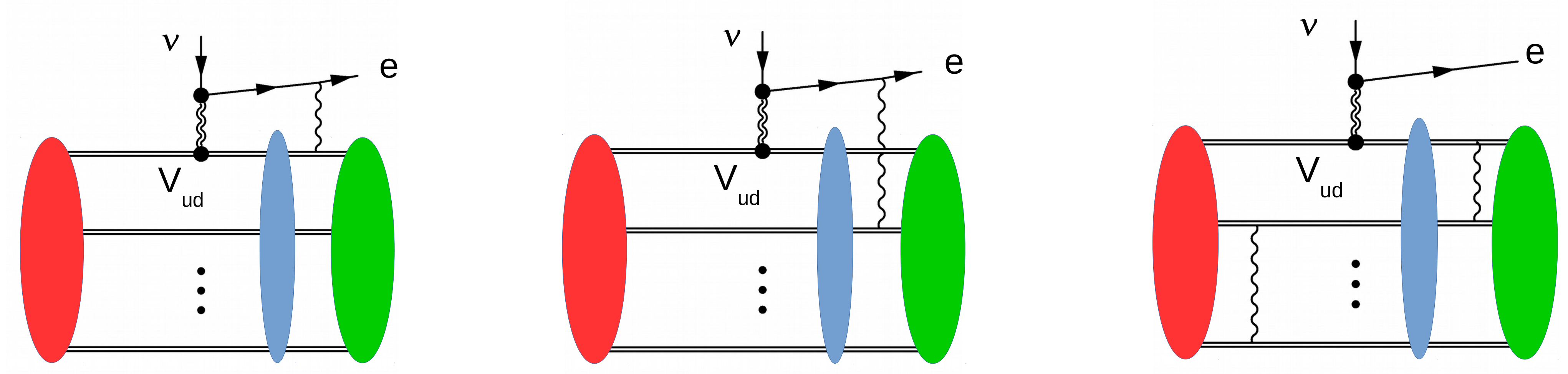


$$\Delta_{CKM} = |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1$$

At face value ~ 3σ deviation from unitarity

- Most precise determination of V_{ud} currently from nuclear decay
- Required radiative corrections under scrutiny
- Several approaches are currently pursued (EFT, dispersive, …)

Seng-Gorchtein 2022,
Gennari et al. 2024,
…



EFT for multi-nucleon weak decays

VC, W. Dekens,, J.de Vries, S. Gandolfi, M. Hoferichter, E, Mereghetti, 2405.18469, 2405.18464

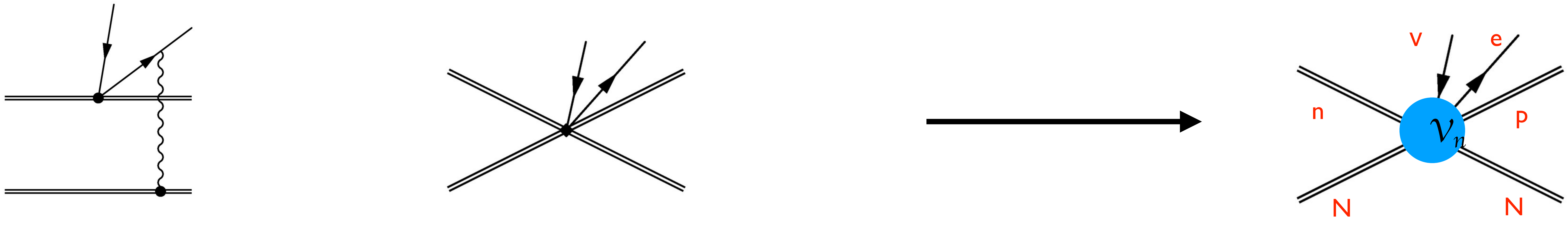
- Chiral EFT (NN, NNN, …) with dynamical leptons and photons

- Hard photons leave behind local multi-nucleon electroweak operators

$$\mathcal{L}_W^{2\mathrm{b}} = -\sqrt{2}e^2 G_F V_{ud}\, \bar{e}_L \gamma_0 \nu_L \times$$

$$N^\dagger \tau^+ N \left(e^2 g_{V1}^{NN}\, N^\dagger N + e^2 g_{V2}^{NN}\, N^\dagger \tau^3 N\right)$$

- ‘Integrate out' soft & potential photons (and π’s) → obtain EW n-body transition operators (‘potentials’)



$$\mathcal{V}_{\mathrm{mag}} \sim \frac{e^2}{m_N \mathbf{q}^2}$$

$$\mathcal{V}_{\mathrm{contact}} \sim e^2\, g_{V_1,V_2}^{NN}$$

$$H_{EW} \supset \sqrt{2}G_F V_{ud}\, \bar{e}_L \gamma_0 \nu_L \times \sum_n c_n \mathcal{V}_n$$

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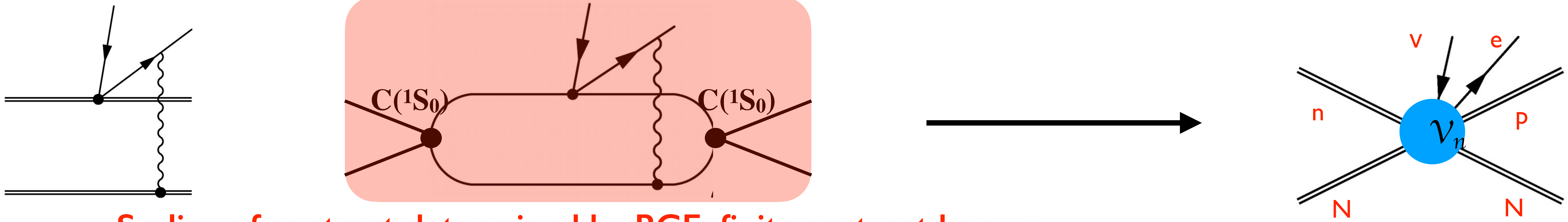

Scaling of contract determined by RGE, finite part not known

$$\mathcal{V}_{\mathrm{mag}} \sim \frac{e^2}{m_N \mathbf{q}^2} \qquad \mathcal{V}_{\mathrm{contact}} \sim e^2\, g_{V_1,V_2}^{NN} \sim e^2 \frac{1}{\Lambda_\chi F_\pi^2}$$

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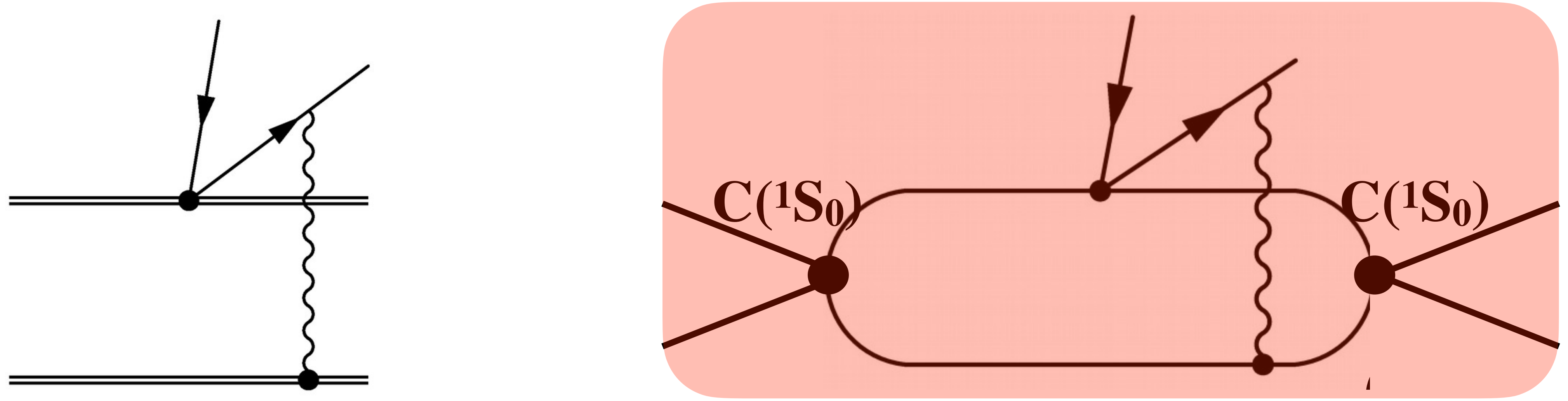

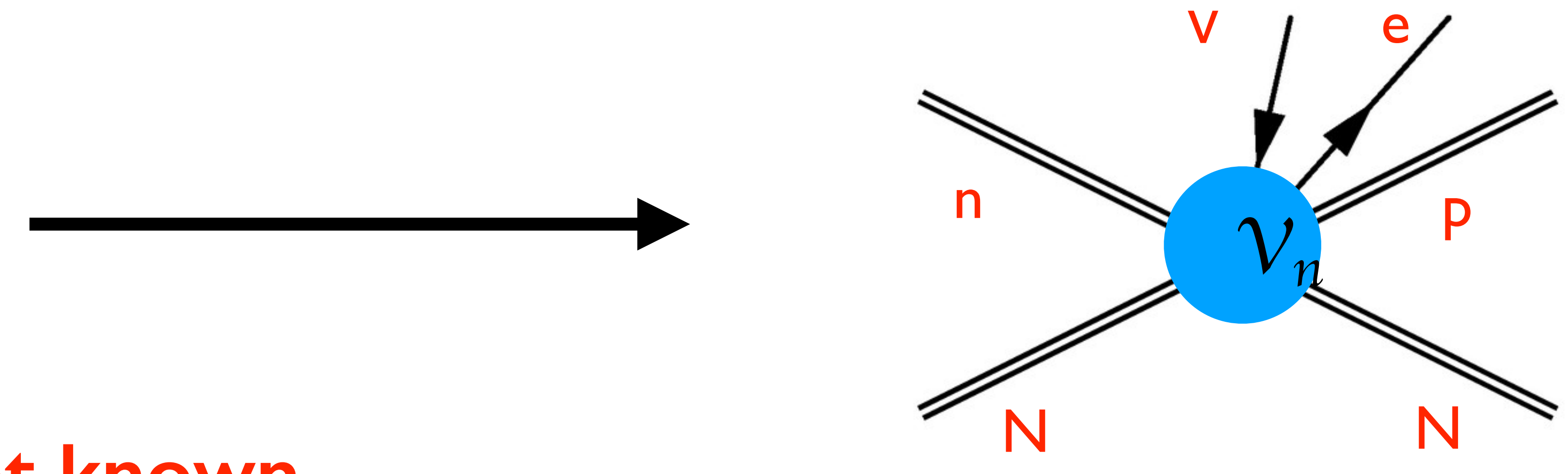

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Scaling relative to LO:

α (Q/Λχ)

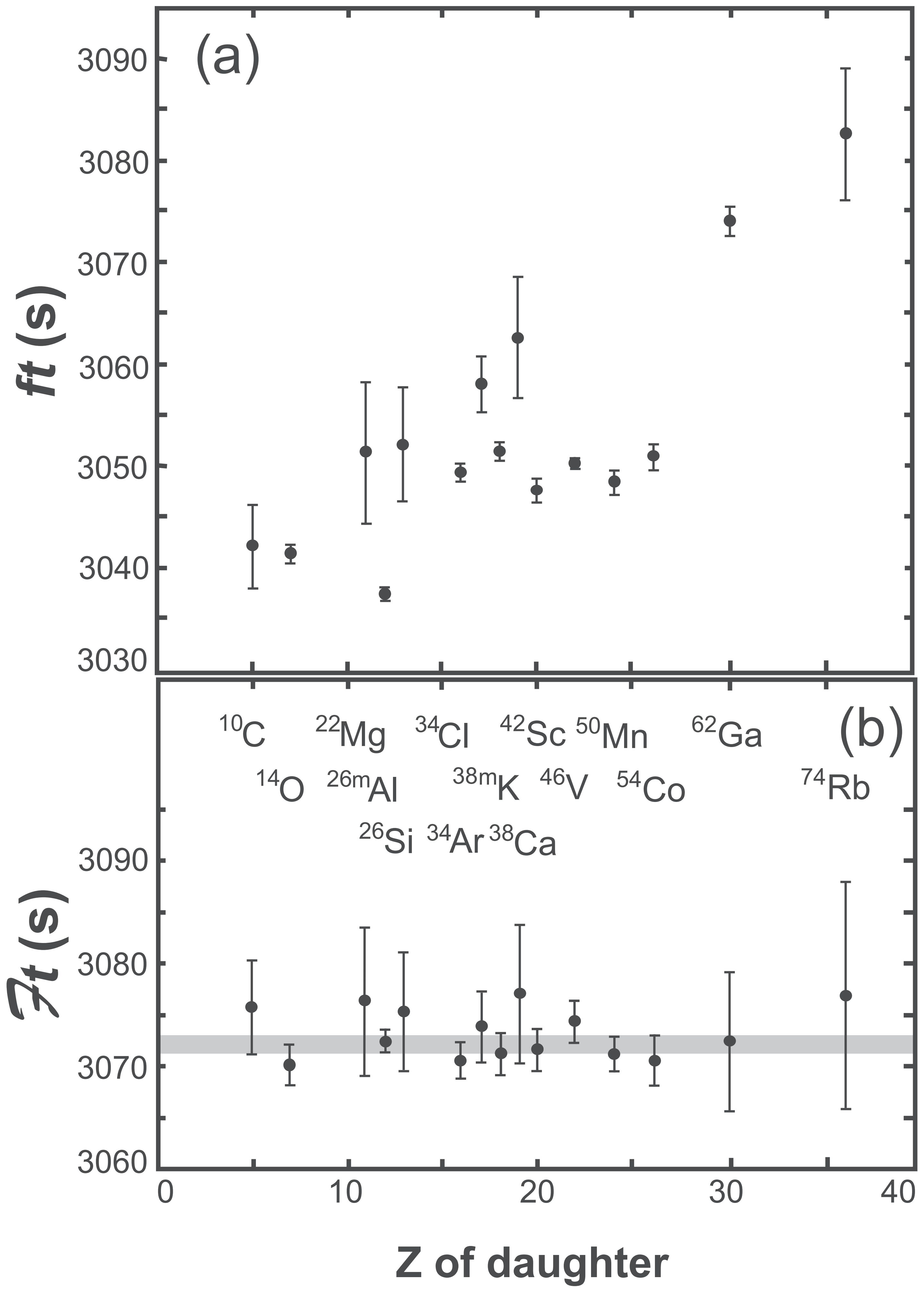
α (Q/Λχ)

α (Q/Λχ) — crucial to claim 0.03% uncertainty

Path forward in the EFT approach

** S. Novario, G. Chambers-Wall, VC, W. Dekens,, J.de Vries, S. Gandolfi, M. Hoferichter, E, Mereghetti, in progress

Hardy-Towner, PRC 2020

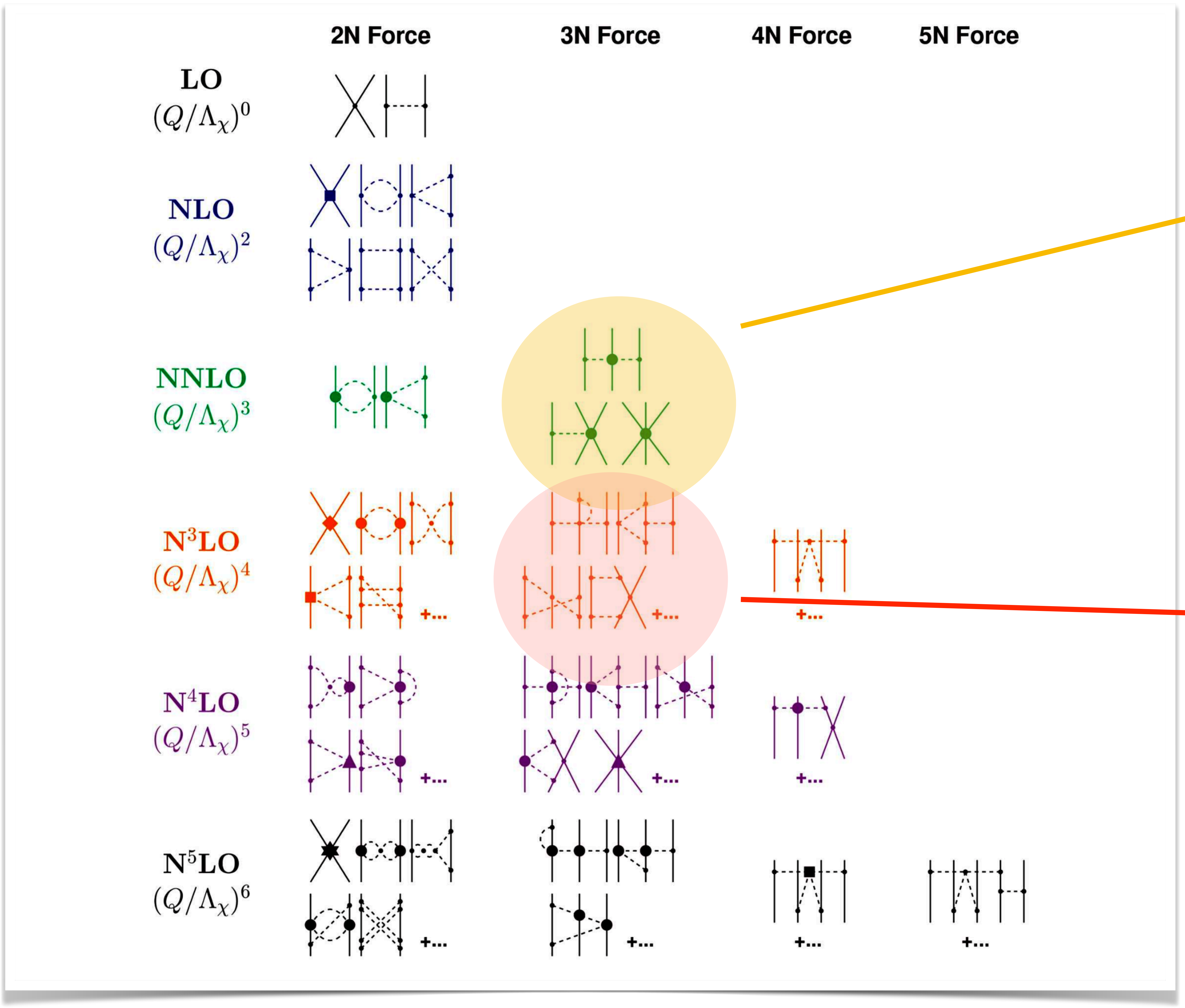


- Chiral EFT analysis has (temporarily) increased the uncertainty. But in the long run it will allow for smaller errors and robust uncertainty quantification

- Paths forward to determine the LECs:
  - Data to the rescue? Fit the two LECs (along with V_{ud} and possibly BSM effective couplings) once NME calculations for several isotopes become available **
  - Theory: match EFT to full QCD + EW theory in the two-nucleon sector, using dispersive analysis, Lattice QCD

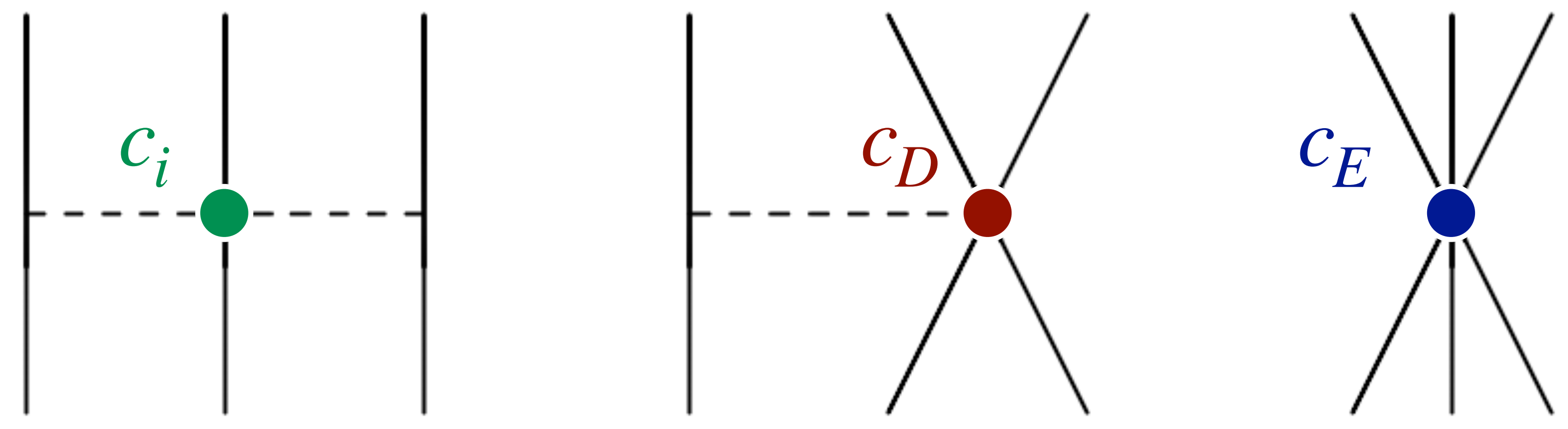
Topic 3: three nucleon interactions

- Hierarchy of nuclear forces in Weinberg counting



Entem, Machleidt, Y. Nosyk, 1703.05454

Van Kolck '94; Epelbaum Nogga, Glöckle Hamada, Meissner, '02



- c_i from πN scattering Hoferichter et a 2015-16
- $c_{D,E}$ from Nd scattering, light nuclei, tritium β decay

- Tree diagrams with relativistic corrections
- 1-loop diagrams with LO vertices
- No new LECs

Bernard, Epelbaum, Krebs, Meissner '08,'11; Ishikawa, Robilotta '07,

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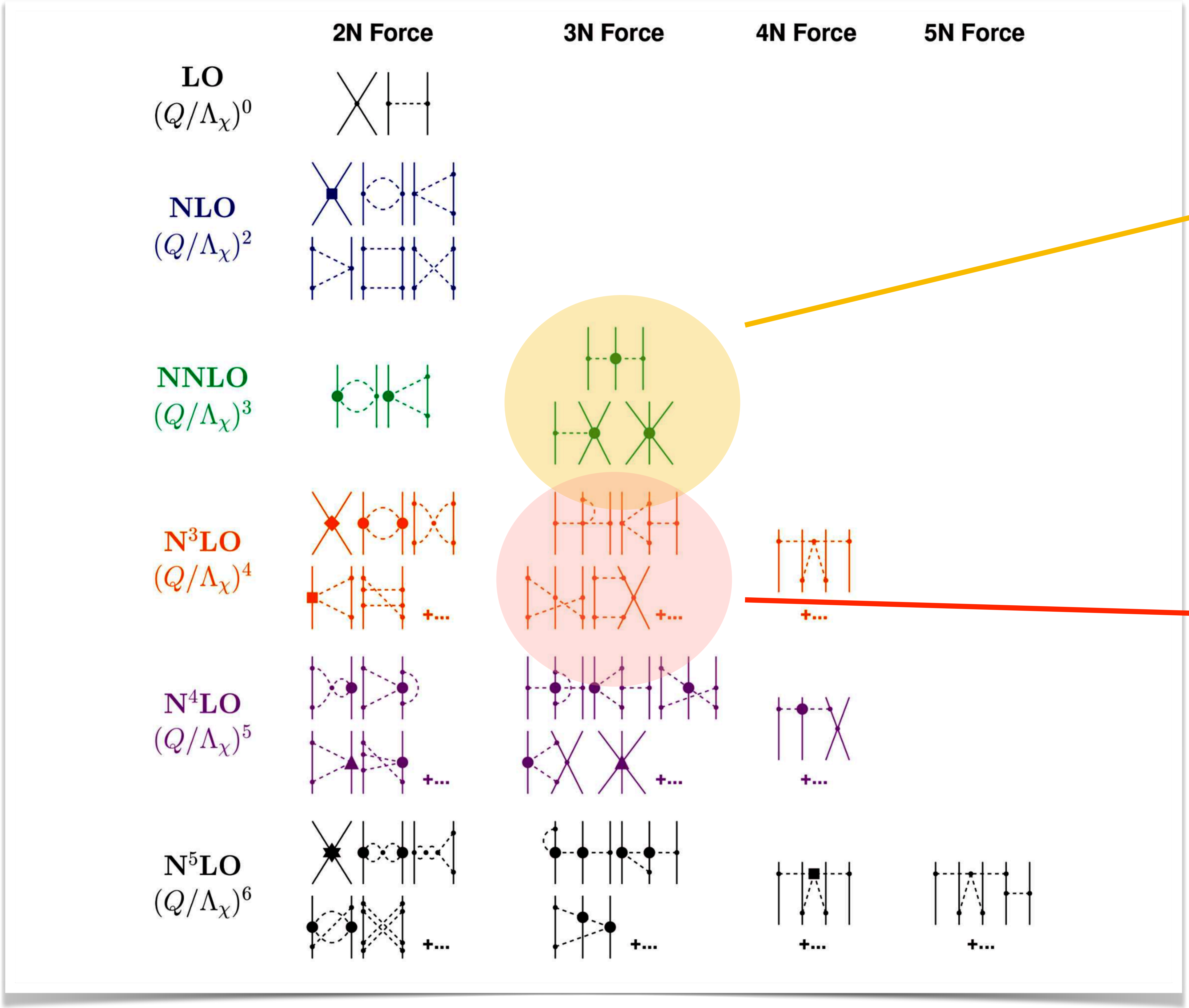

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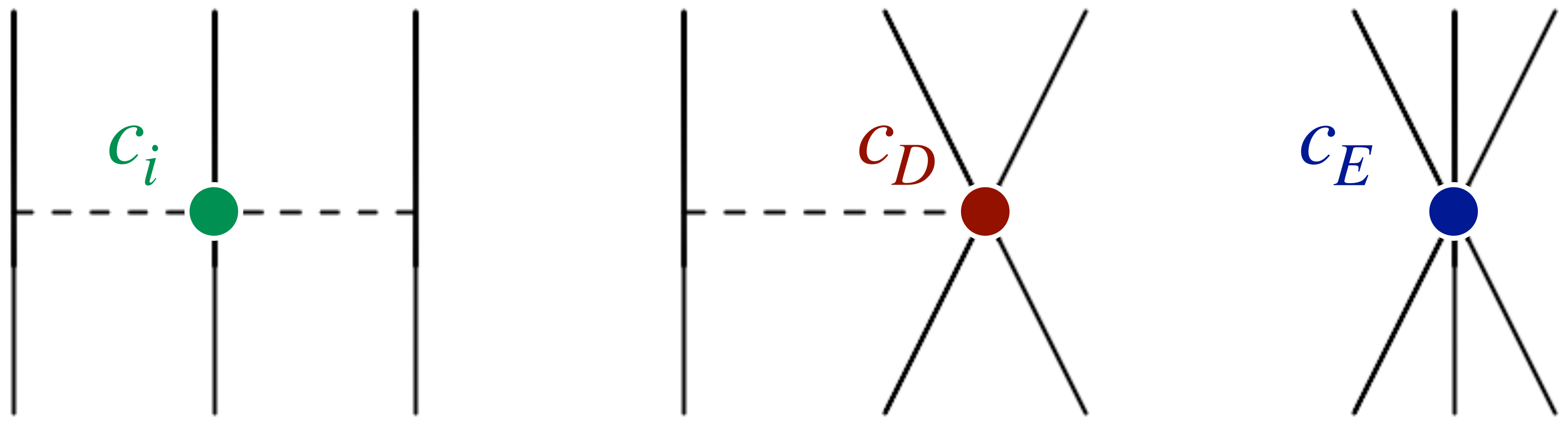

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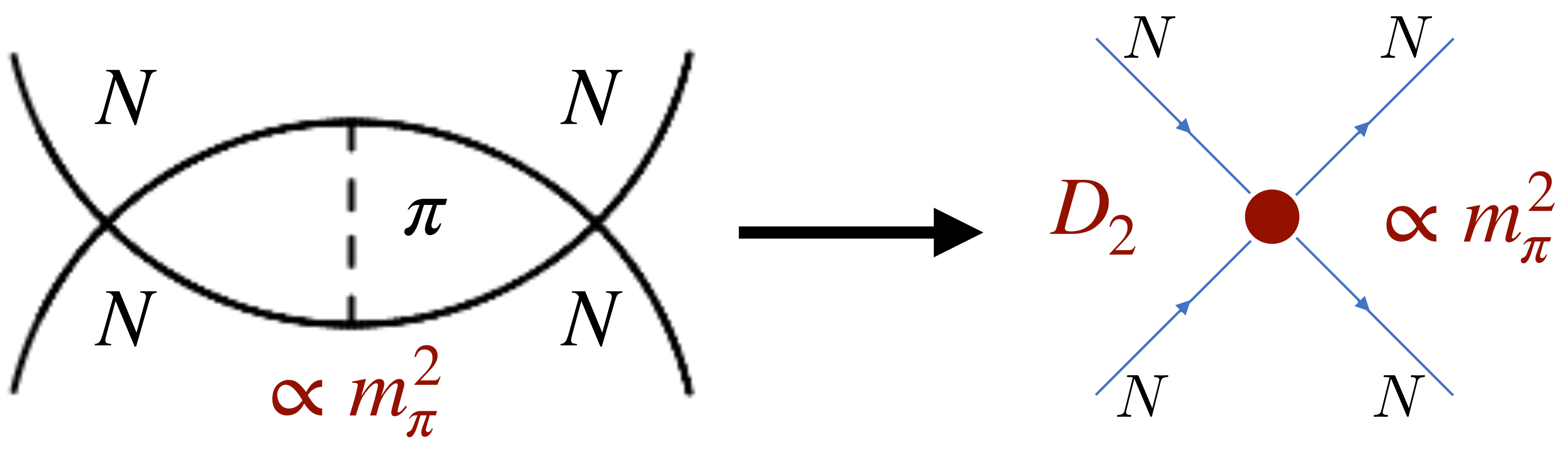
Expect modifications in 'renormalized' approach,
due to different scaling of contact interactions

RG-based power counting: D$_2$ case

VC, M. Dawid, W. Dekens, S. Reddy, 2411.00097

- We have already seen an operator with enhanced scaling due to RG — the chiral-symmetry breaking D$_2$



$$\frac{d}{d\ln\mu}\left[\frac{D_2}{C^2}\right] = \frac{g_A^2 m_N^2}{64\pi^2 F_\pi^2} \equiv \frac{m_N}{4\pi}\frac{1}{\Lambda_{NN}} \xrightarrow[\Lambda_{NN} = \frac{16\pi F_\pi^2}{g_A^2 m_N} \sim 3F_\pi]{C \sim \frac{4\pi}{m_N \Lambda_{NN}}} D_2 \sim \frac{4\pi}{m_N \Lambda_{NN}}\frac{1}{\Lambda_\chi^2} \to \frac{4\pi}{m_N \Lambda_{NN}}\frac{1}{\Lambda_{NN}^2}$$

Enhancement by ~2 chiral orders

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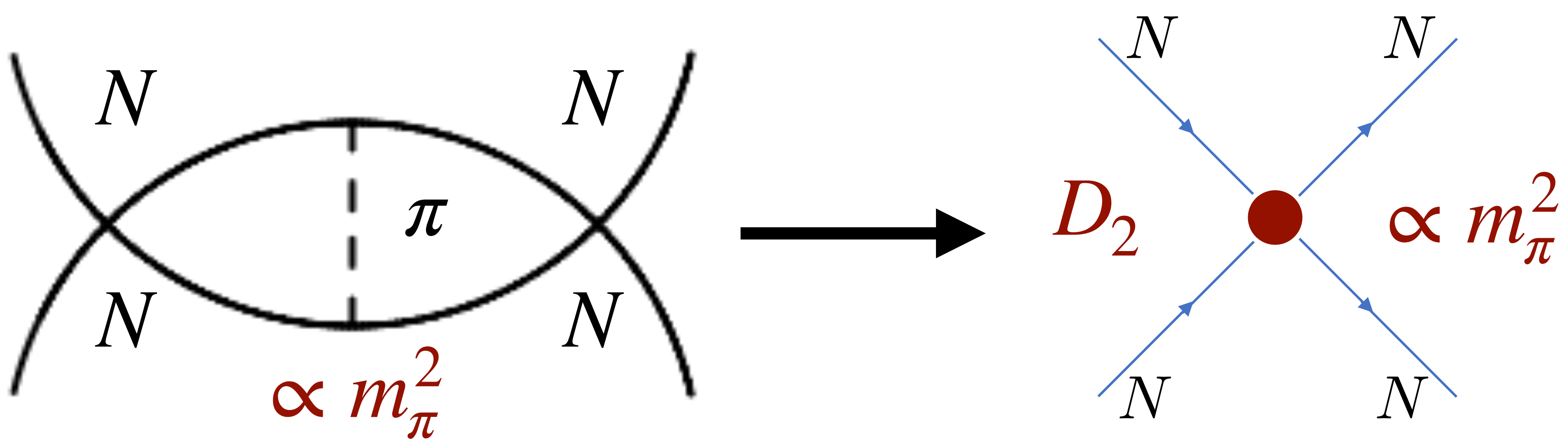

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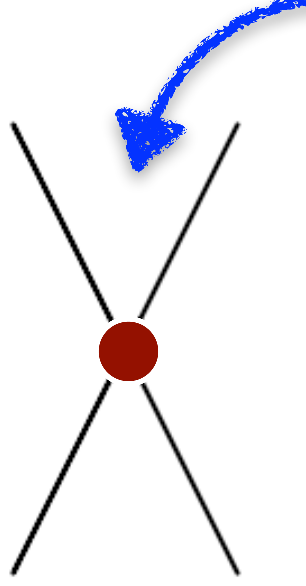
Enhancement by ~2 chiral orders

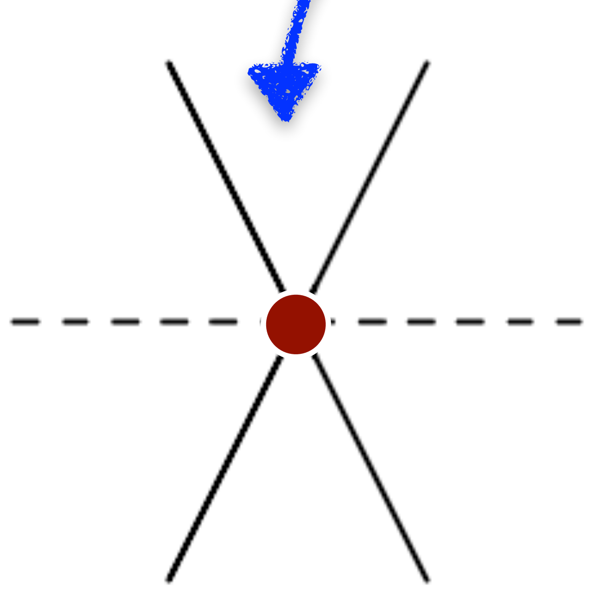
- In chiral covariant form

$$\boxed{\mathscr{L} = D_2 \bar{N}N\bar{N}N < \chi_+ > = D_2 \bar{N}N\bar{N}N m_\pi^2 \left(1 - \frac{1}{2F_\pi}\pi^a\pi^b\delta^{ab} + \mathcal{O}\left(\frac{\pi^4}{F_\pi^4}\right)\right)}$$

Can be absorbed in

$C' = C + m_\pi^2 D_2$





Leads to 3-nucleon force

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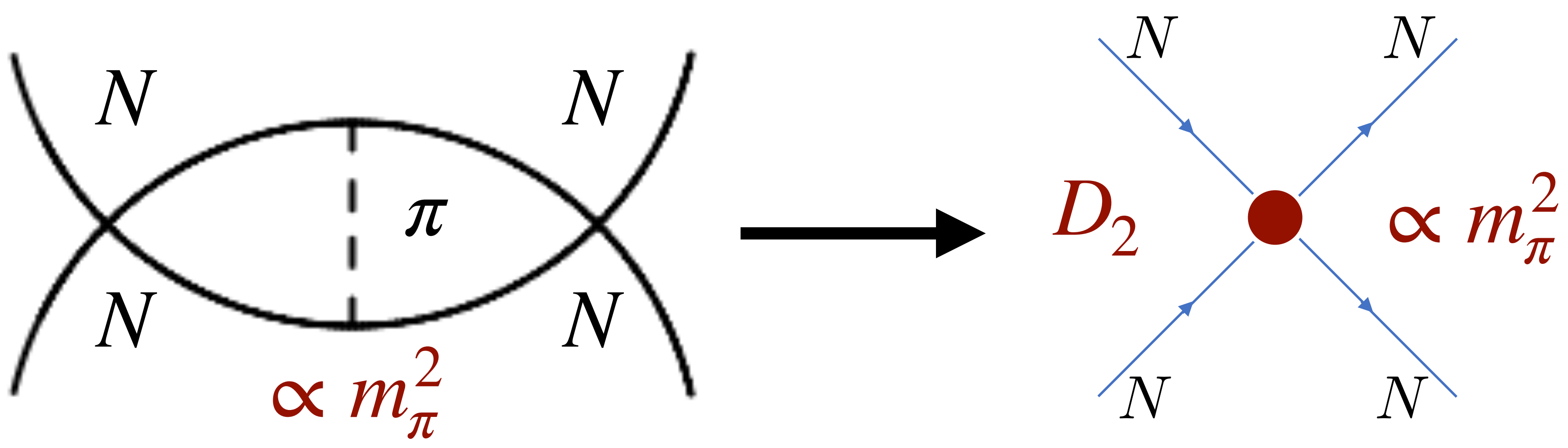

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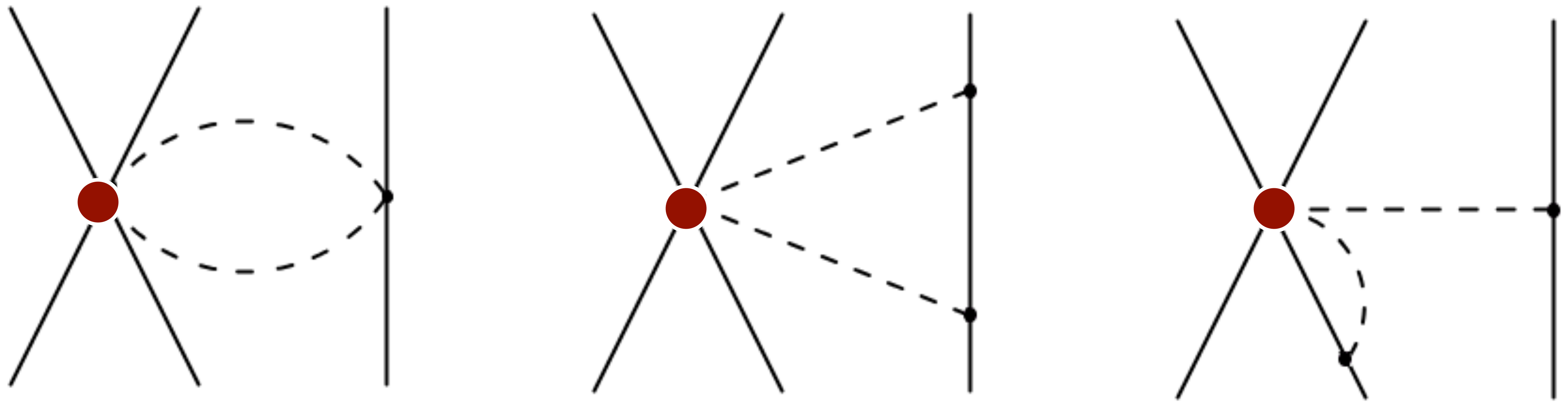
- 3NF generated at one-loop

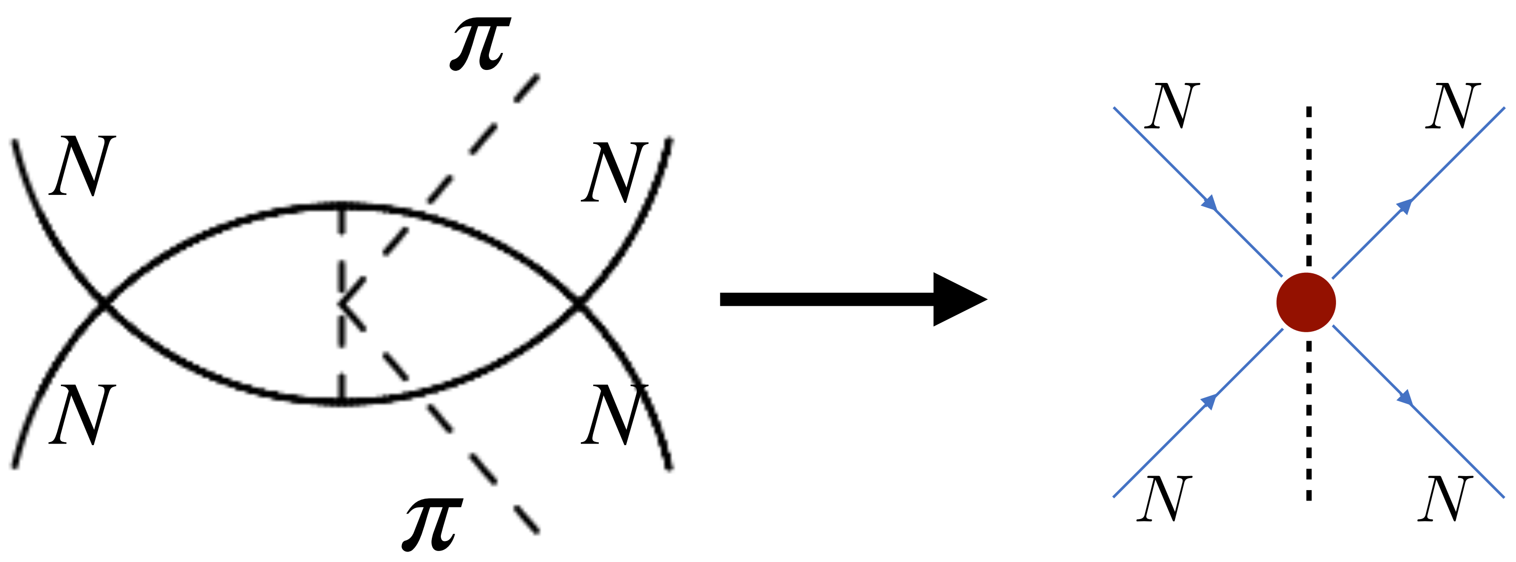


RG-based power counting: F_2, E_2

VC, M. Dawid, W. Dekens, S. Reddy, 2411.00097

- Similar RG-enhanced scaling in derivative operators

Borasoy, Griesshammer '01, '03



$$\mathscr{L} = \frac{1}{4}\left[E_2\langle(v\cdot u)^2\rangle + F_2\langle u\cdot u - (v\cdot u)^2\rangle\right]\ (N^T P_i N)^\dagger\,(N^T P_i N)$$

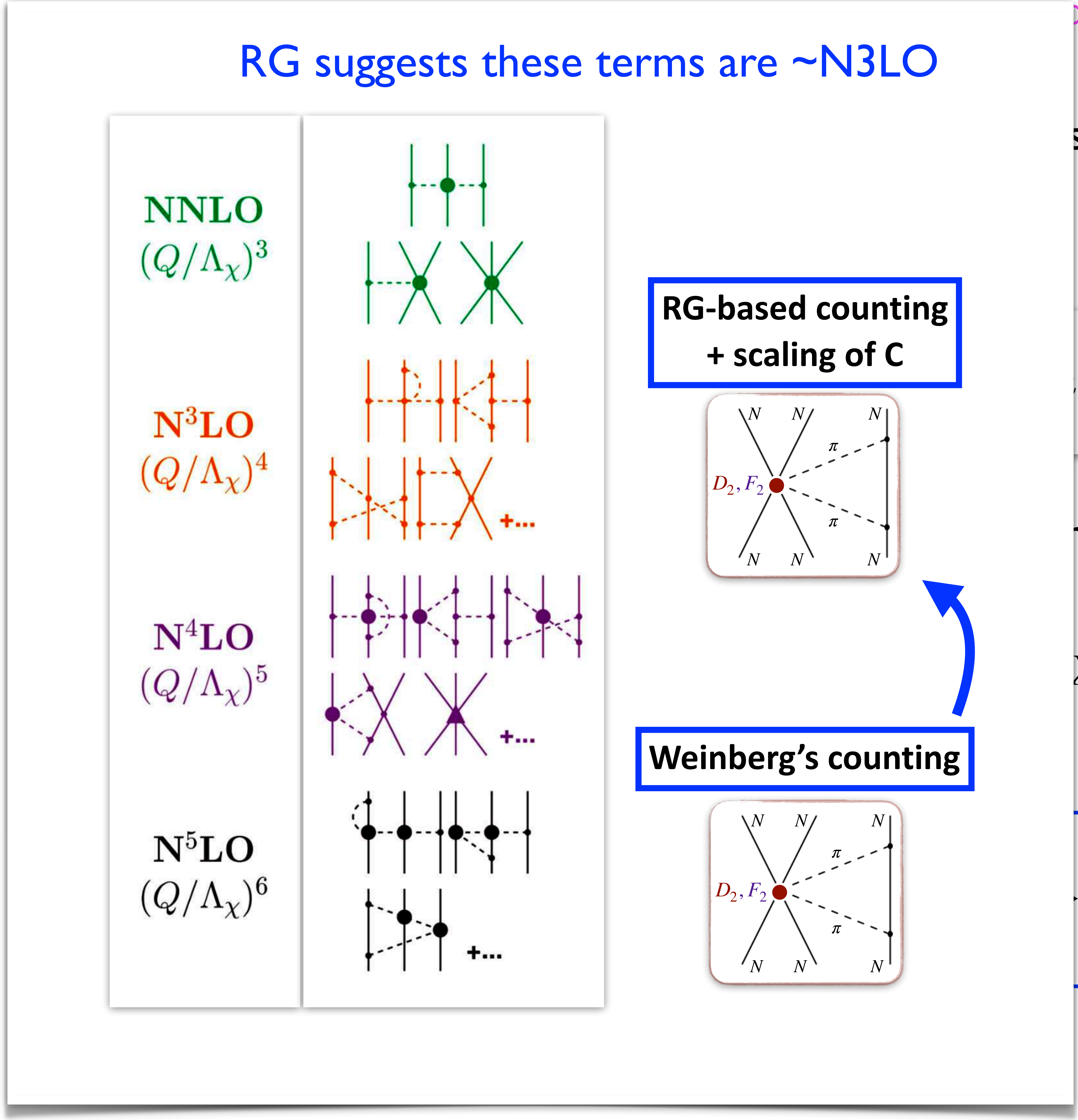
- RGEs similar to D_2 case and same one-loop diagrams generate 3N force

$$\frac{d}{d\ln\mu}\left[\frac{X}{C^2}\right] = \gamma_X\left(\frac{m_N}{4\pi f_\pi}\right)^2, \qquad X \in \{D_2, E_2, F_2\}$$

$$\boxed{\begin{aligned}\gamma_{D_2} &= g_A^2/4\\ \gamma_{E_2} &= -(1+g_A^2)/3\\ \gamma_{F_2} &= -g_A^2/3\end{aligned}}$$

$$\boxed{X \sim \frac{4\pi}{m_N\Lambda_{NN}}\,\frac{1}{\Lambda_\chi^2}\bigg|_{NDA} \rightarrow \frac{m_N}{4\pi\Lambda_{NN}}C^2 \sim \frac{4\pi}{m_N\Lambda_{NN}}\,\frac{1}{\Lambda_{NN}^2}}$$

RG-based power counting: F_2, E_2



RG suggests these terms are ~N3LO

NNLO
$(Q/\Lambda_\chi)^3$

N^{3}LO
$(Q/\Lambda_\chi)^4$

N^{4}LO
$(Q/\Lambda_\chi)^5$

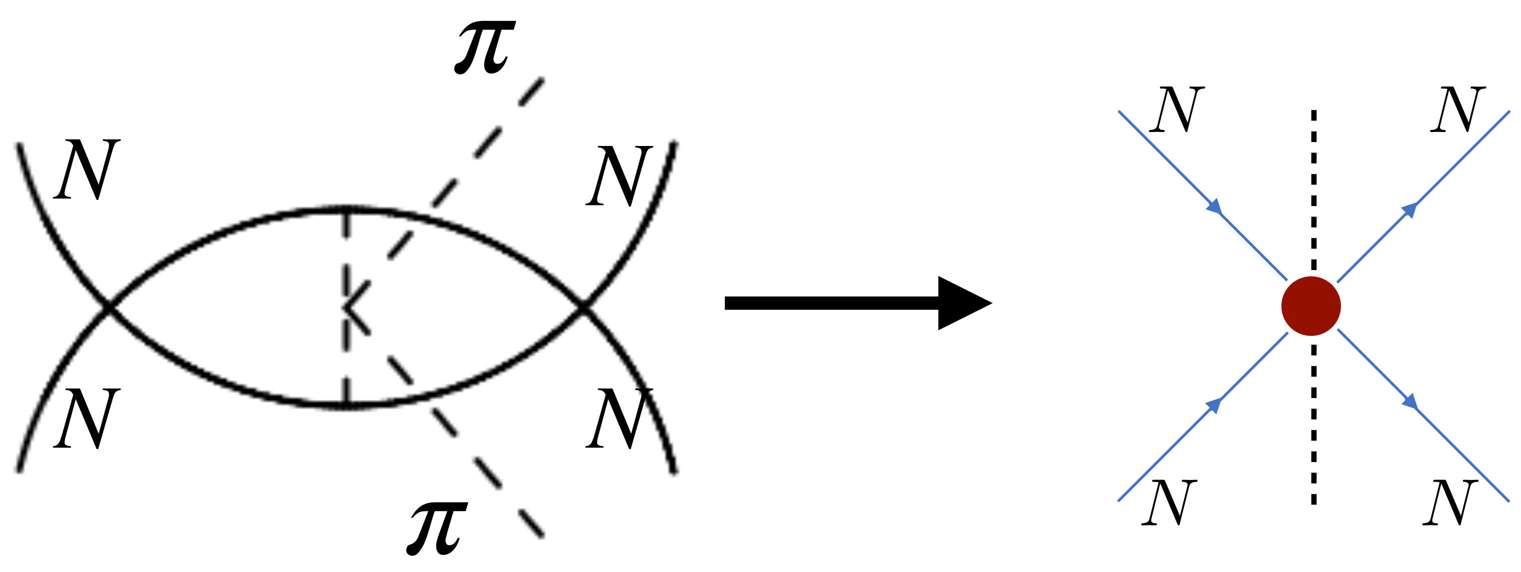
N^{5}LO
$(Q/\Lambda_\chi)^6$

RG-based counting + scaling of C

Weinberg's counting

Dekens, S. Reddy, 2411.00097

s



$$-(v\cdot u)^2\rangle\big]\ (N^TP_iN)^\dagger\,(N^TP_iN)$$

ns generate 3N force

$X \in \{D_2, E_2, F_2\}$

$$\boxed{\begin{aligned}\gamma_{D_2} &= g_A^2/4\\ \gamma_{E_2} &= -(1+g_A^2)/3\\ \gamma_{F_2} &= -g_A^2/3\end{aligned}}$$

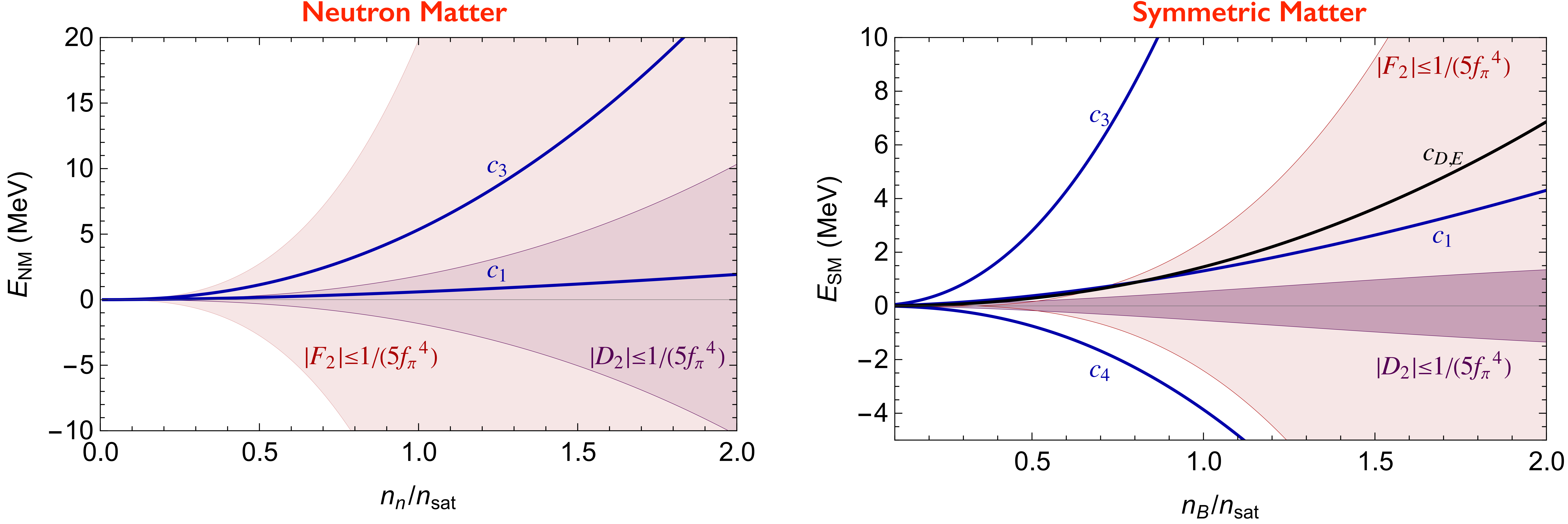
$$\frac{m_N}{4\pi\Lambda_{NN}}\,C^2 \sim \frac{4\pi}{m_N\Lambda_{NN}}\ \frac{1}{\Lambda_{NN}^2}$$

Explored impact on nuclear matter assuming

$$C \sim \frac{1}{F_\pi^2} \ \rightarrow\ |X| \le \frac{1}{5F_\pi^4}$$

Energy per particle in dense matter

VC, M. Dawid, W. Dekens, S. Reddy, 2411.00097



- This rough estimate (Fermi gas) shows sizable effect compared to standard N2LO 3NF (RG scaling and ‘π’ enhancement of loop)
- Caveats: (i) scheme and regulator dependence: using dispersive regulators with Λ= 500 MeV decreases result to ~1/3 of this; (ii) missing the standard N3LO 3N interaction

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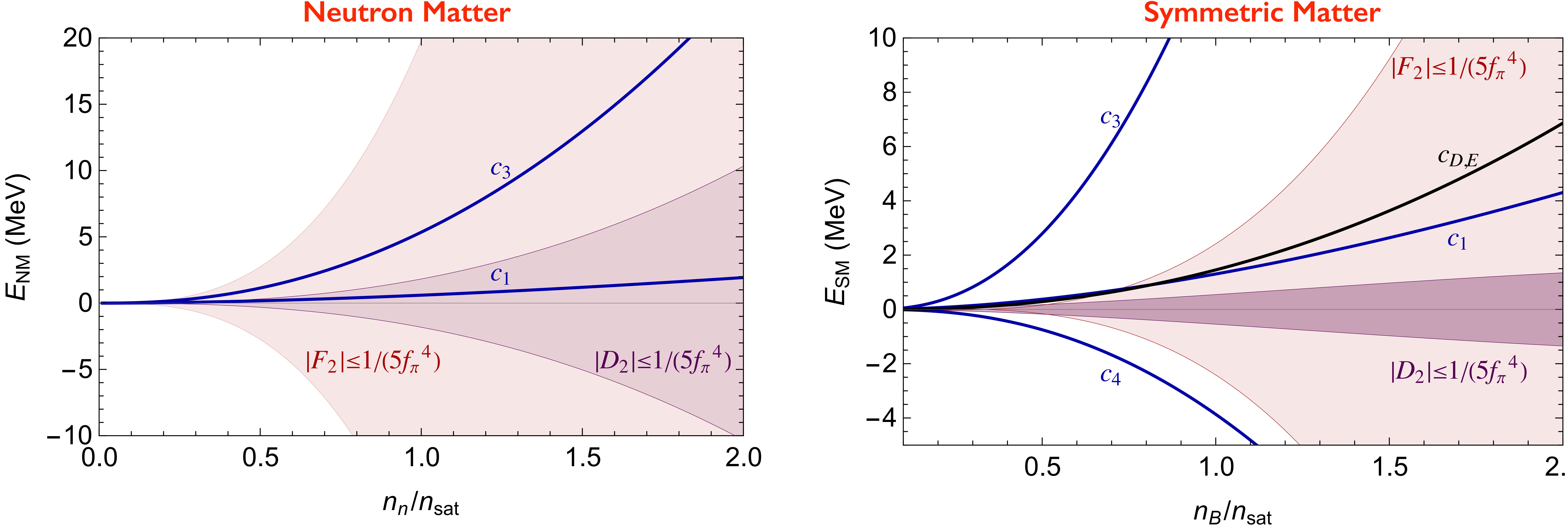

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Different conclusion reached in Epelbaum-Gasparyan-Gegelia-Hog-Krebs 2512.14117
(In part due to different regulator and different input for C)

Current and future directions

- Lattice QCD data on m_q dependence of NN amplitude will shed light on D_2
- It will be interesting to confront the RG-suggested ordering scheme with data in low- and medium-mass nuclei (i.e. simultaneous determination of c_D, c_E, D_2, F_2)
- Recent / ongoing activity
    - Light and medium nuclei
    - Properties of dense matter
    - Impact on neutron matter EOS and neutron star observables

Vernik, Hebeler, Schwenk 2512.20454
Chambers-Wall, Dawid, Pastore (in progress)
Navratil, Palkanoglu (in progress)

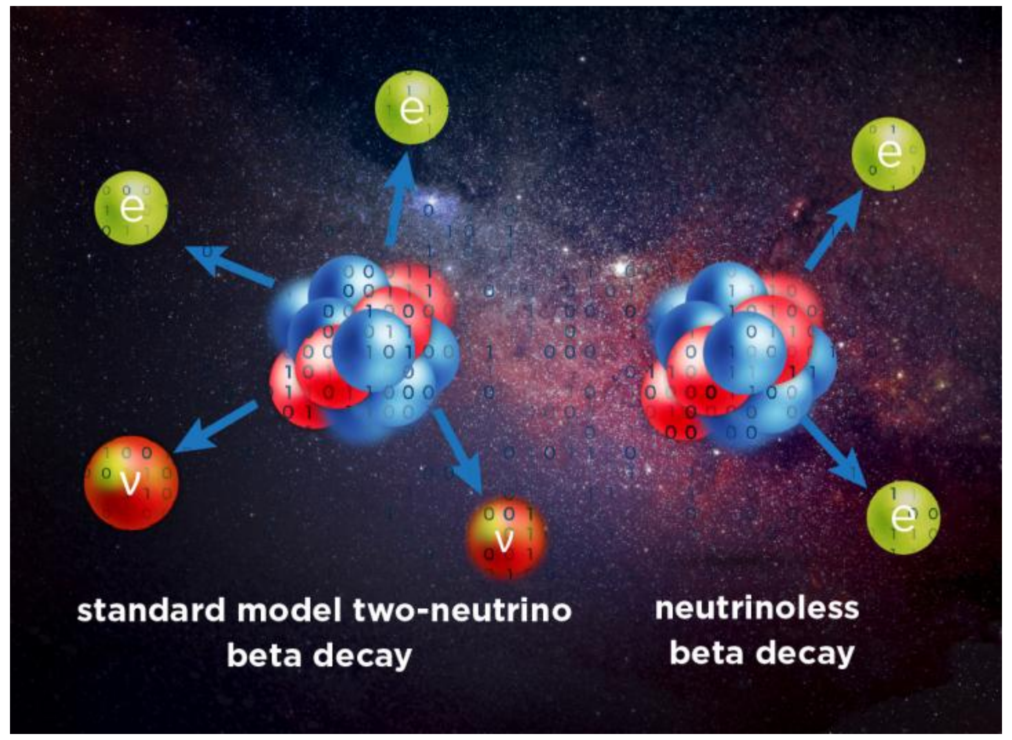
Dawid, Dekens, Drischler, Kumamoto, Reddy (in progress)

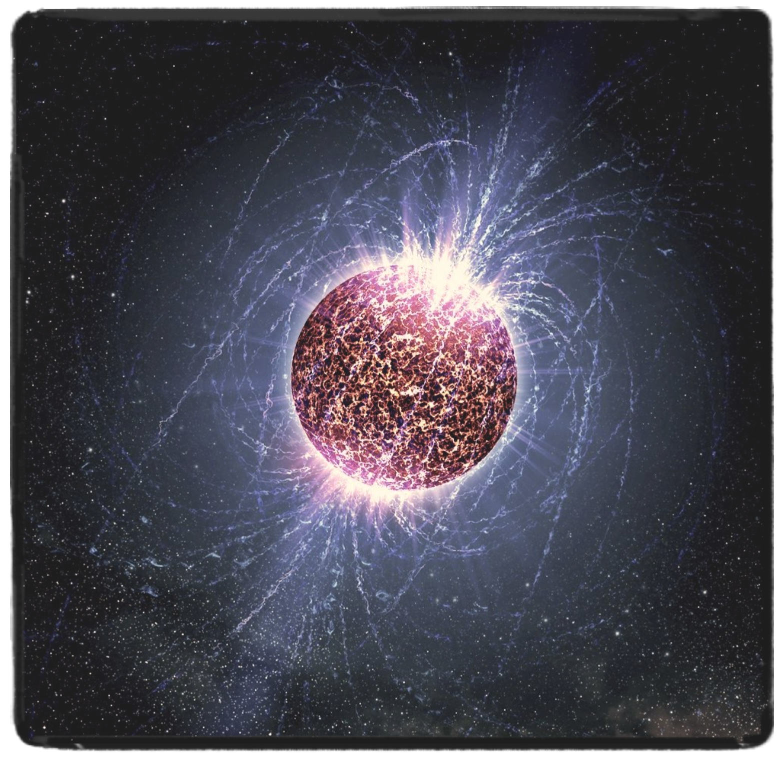
Armstrong, VC, Curry, Dawid, Dekens, King, Reddy, Tews (in progress)

- Future: other implications of RG-based counting in 3N sector?

Conclusions

- Discussion about power counting for nuclear EFT has implications beyond nuclear phenomenology, e.g.
    - EW and BSM physics
    - Astrophysics





- Discussed few examples:
    - Neutrinoless double beta decay
    - Superallowed beta decays and CKM unitarity
    - Three-nucleon interactions and impact on neutron matter

→

In these case, a lower order calculation with robust error estimate (even at the 20-30% level) is extremely valuable

Appendix

NN couplings

- Two I=2 operators involving four nucleons

(See also Walzl-Meißner-Epelbaum nucl-th/0010109)

EM case

$\mathcal{Q}_L = \frac{\tau^z}{2}, \mathcal{Q}_R = \frac{\tau^z}{2}$

$$\frac{e^2 C_1}{4}\left(\bar{N}\mathcal{Q}_L N \bar{N}\mathcal{Q}_L N - \frac{\mathrm{Tr}[\mathcal{Q}_L^2]}{6}\bar{N}\boldsymbol{\tau}N\cdot\bar{N}\boldsymbol{\tau}N + L\to R\right)$$

$$\frac{e^2 C_2}{4}\left(\bar{N}\mathcal{Q}_L N \bar{N}\mathcal{Q}_R N - \frac{\mathrm{Tr}[\mathcal{Q}_L\mathcal{Q}_R]}{6}\bar{N}\boldsymbol{\tau}N\cdot\bar{N}\boldsymbol{\tau}N + L\to R\right)$$

$$\mathcal{Q}_L = u^\dagger Q_L u$$

$$\mathcal{Q}_R = u Q_R u^\dagger$$

$$u = 1 + \frac{i\boldsymbol{\pi}\cdot\boldsymbol{\tau}}{2F_\pi} + \dots$$

ΔL=2 case

$\mathcal{Q}_L = \tau^+, \mathcal{Q}_R = 0$

$$8G_F^2 V_{ud}^2 m_{\beta\beta}\bar{e}_L e_L^c\ \frac{g_\nu}{4}\left(\bar{N}\mathcal{Q}_L N \bar{N}\mathcal{Q}_L N - \frac{\mathrm{Tr}[\mathcal{Q}_L^2]}{6}\bar{N}\boldsymbol{\tau}N\cdot\bar{N}\boldsymbol{\tau}N + L\to R\right)$$

- Chiral symmetry ⇒ $g_\nu = C_1$

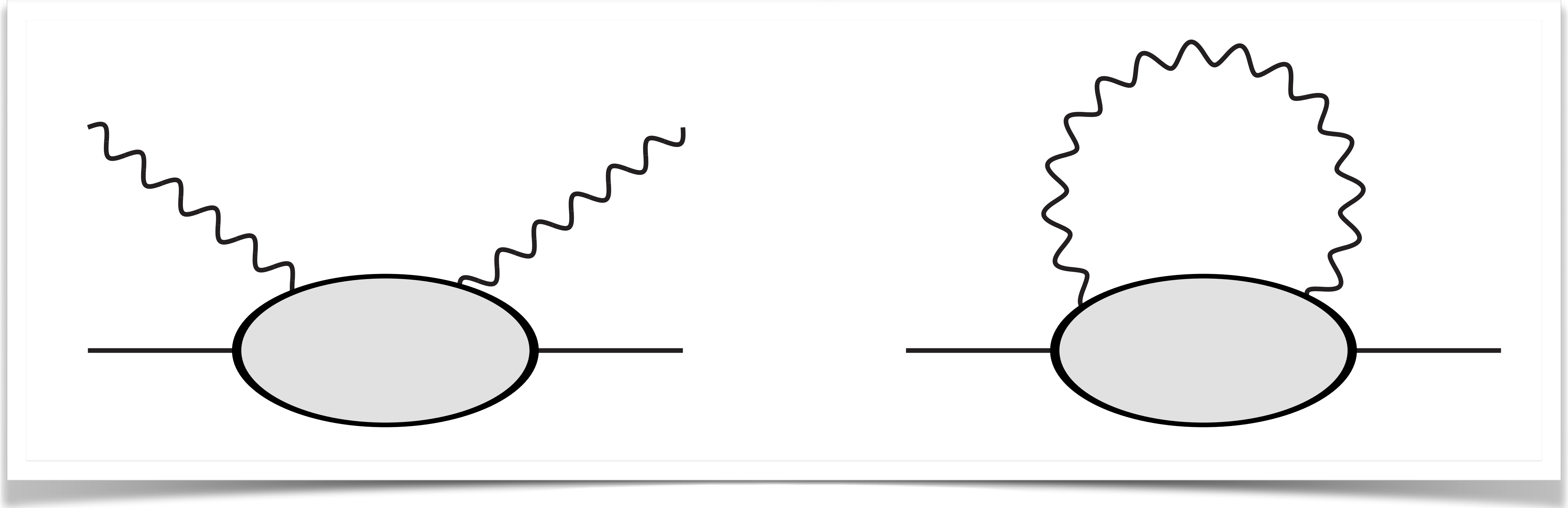
- Can we get C_1 from experiment ?

Estimating the contact term

VC, Dekens, deVries, Hoferichter, Mereghetti, 2012.11602, 2102.03371

Forward Compton amplitude

Self-energy ~ mass

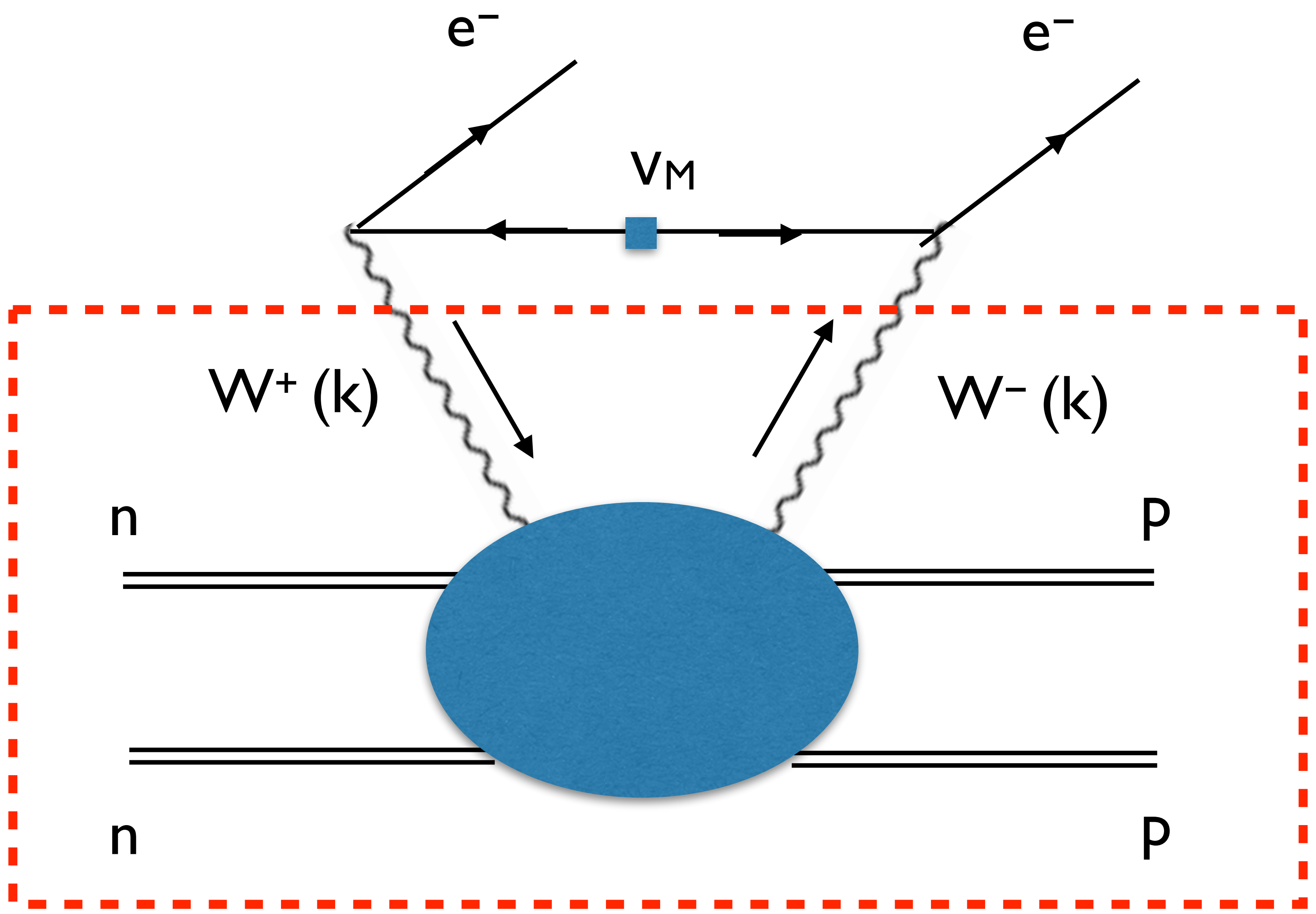


Cottingham (1963) approach to electromagnetic contributions to hadron masses

Estimating the contact term

VC, Dekens, deVries, Hoferichter, Mereghetti, 2012.11602, 2102.03371

nn → pp amplitude controlled by a forward "Compton" amplitude

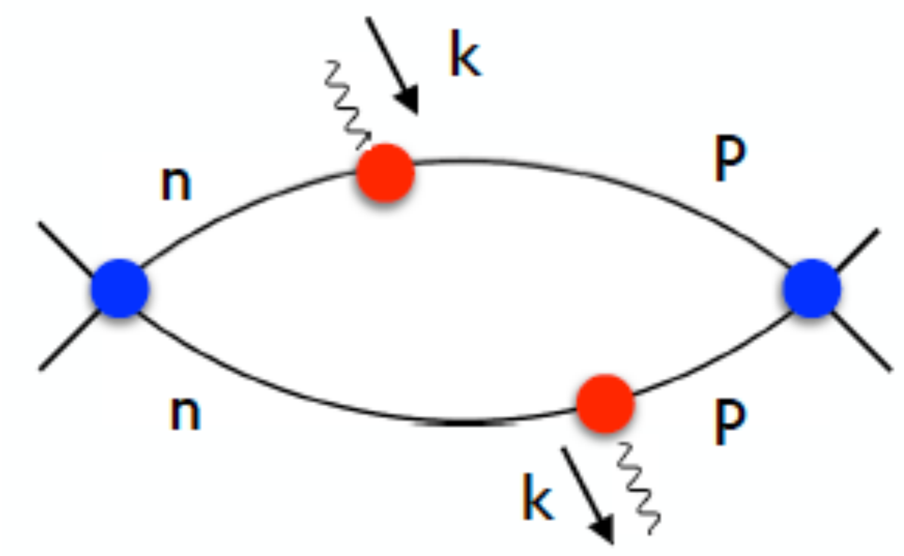


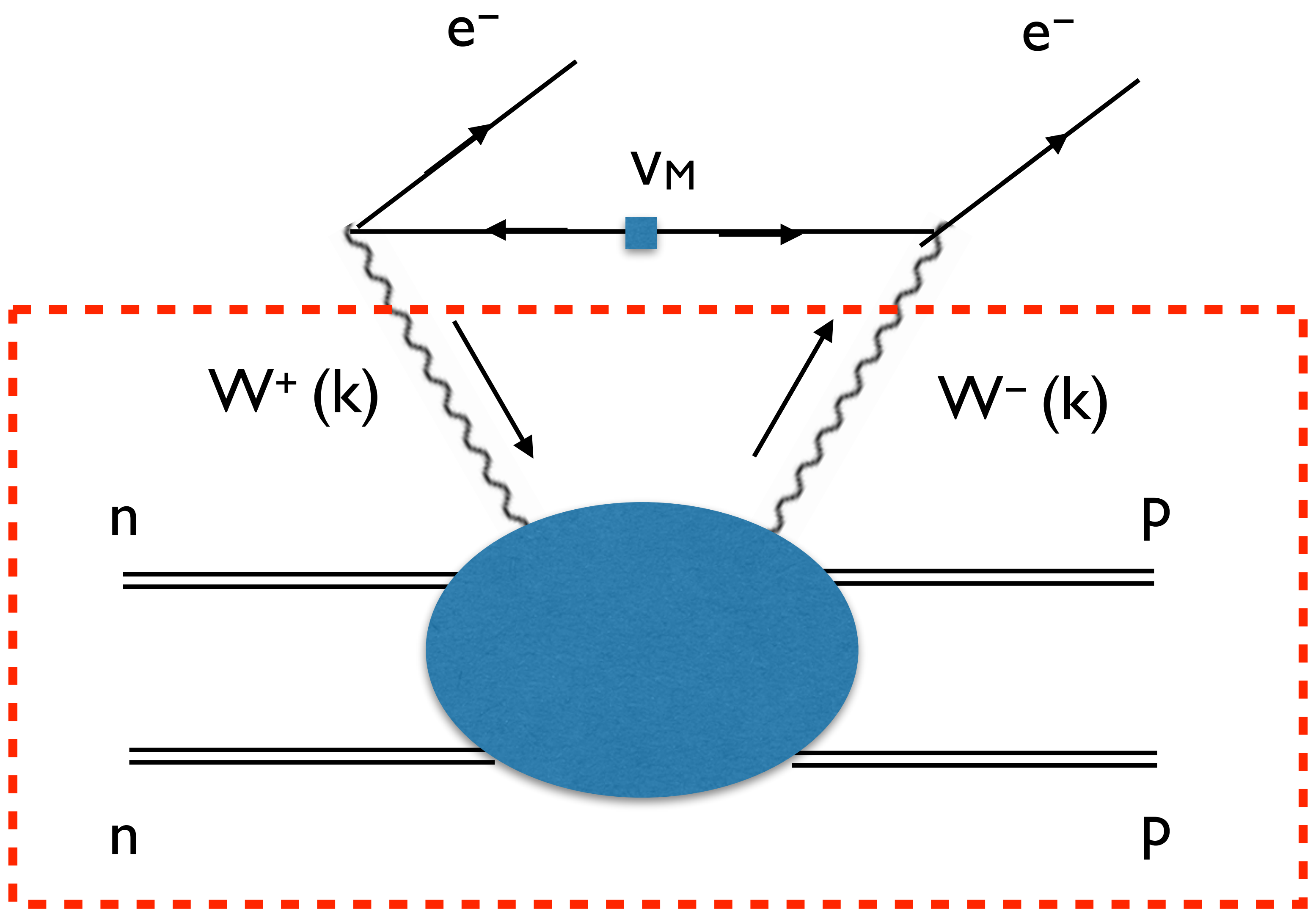
Estimating the contact term

VC, Dekens, deVries, Hoferichter, Mereghetti, 2012.11602, 2102.03371

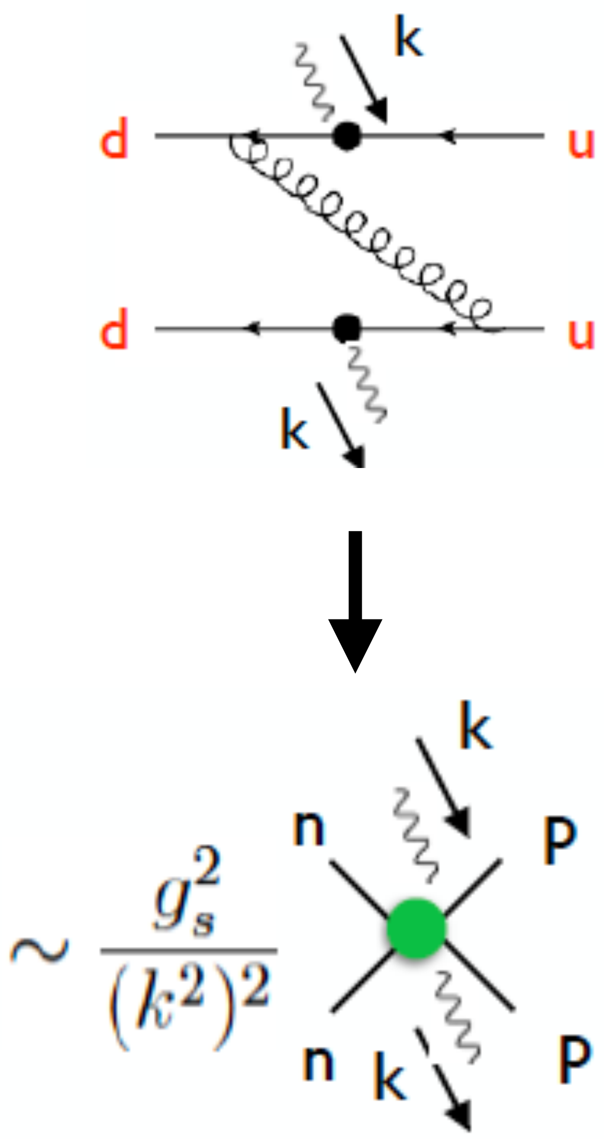
nn → pp amplitude controlled by a forward "Compton" amplitude

Low k: chiral EFT to NLO





High k: QCD OPE



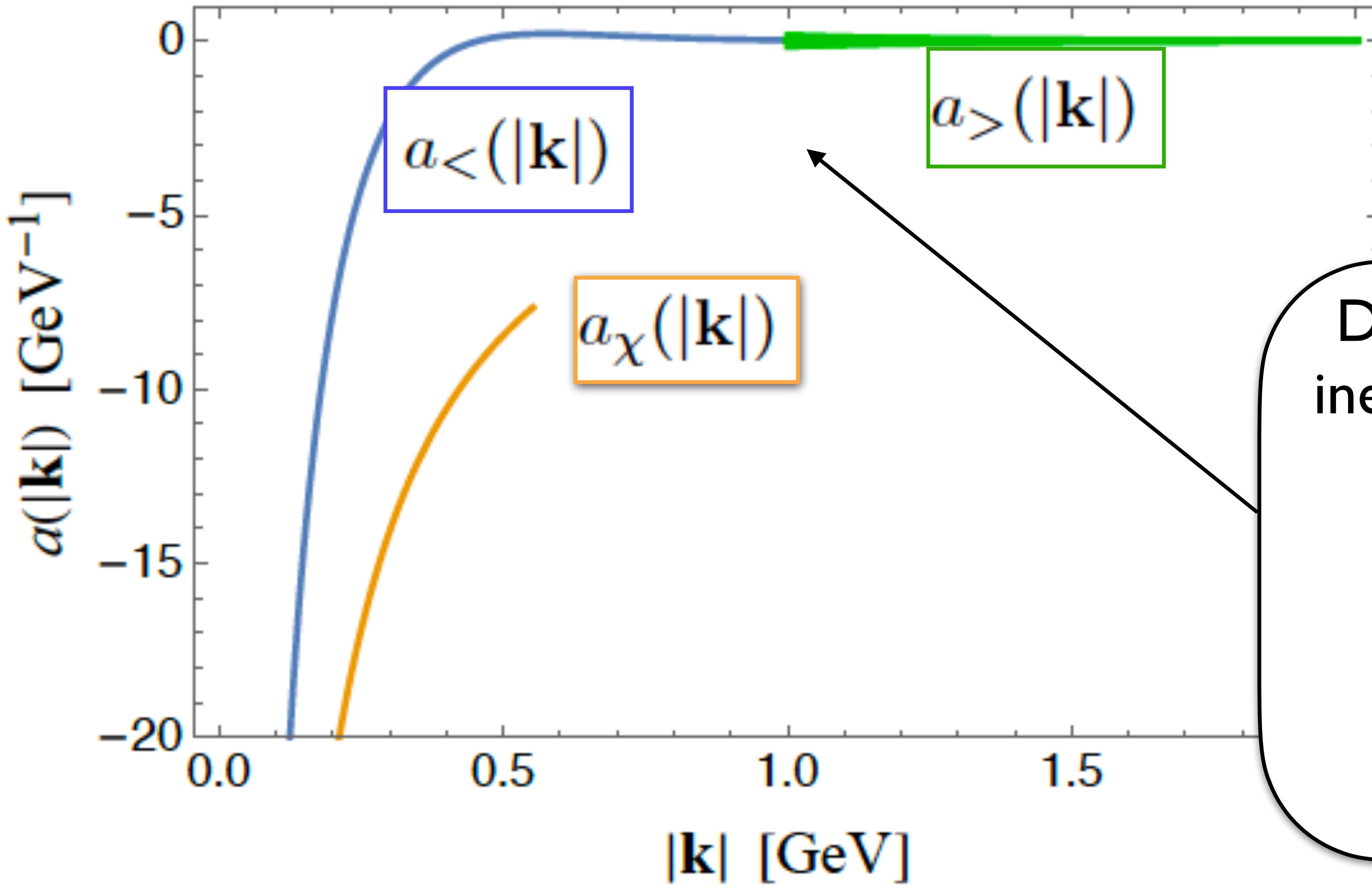
Intermediate k: resonance contributions
in and ●, ●
πNN intermediate state, …

Estimating the contact term

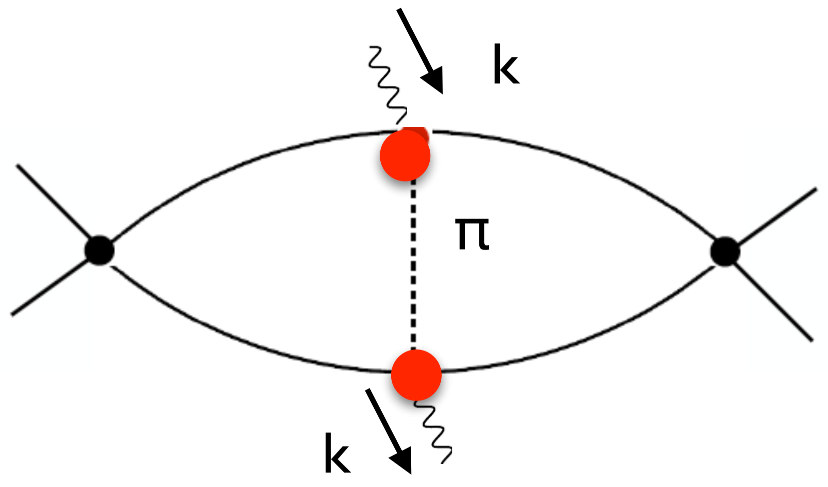
VC, Dekens, deVries, Hoferichter, Mereghetti, 2012.11602, 2102.03371

Determined g_V with ~30% uncertainty (validated with ΔI=2 NN electromagnetic coupling)

$$\mathcal{A}_\nu \propto \int_0^\Lambda d|\mathbf{k}|\; a_<(|\mathbf{k}|) + \int_\Lambda^\infty d|\mathbf{k}|\; a_>(|\mathbf{k}|)$$

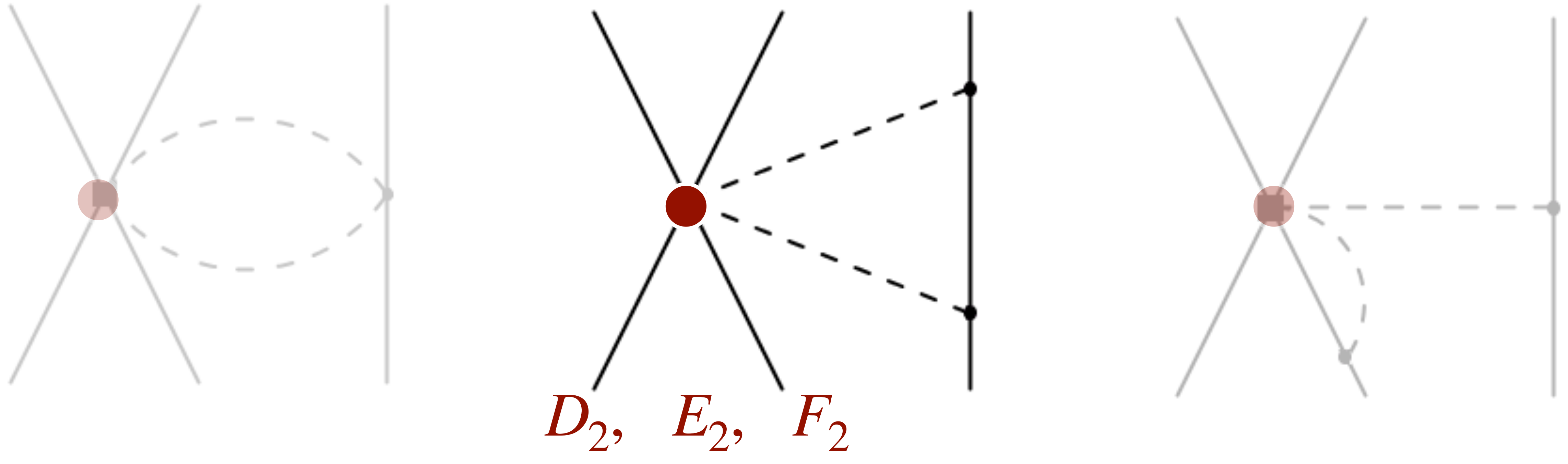


Dominant uncertainty from inelastic channels (NNπ , …):



Explicit form of the 3N potential

Courtesy: Wouter Dekens



Three-nucleon potential from $m_\pi^2 D_2$

$$V(\vec{q}_1, \vec{q}_2, \vec{q}_3) = \frac{9 g_A^2 D_2 m_\pi^3}{512 \pi f_\pi^4} F\left(\frac{\vec{q}_3^{\,2}}{4 m_\pi^2}\right) \left(1^{(i)} 1^{(j)} - \vec{\sigma}^{(i)} \cdot \vec{\sigma}^{(j)}\right) \quad F(b) = \frac{2}{3}\left(1 + \left(\frac{1}{2\sqrt{b}} + \sqrt{b}\right) \tan^{-1}(\sqrt{b})\right) .$$

Contributions from $\sim E_\pi^2 E_2$

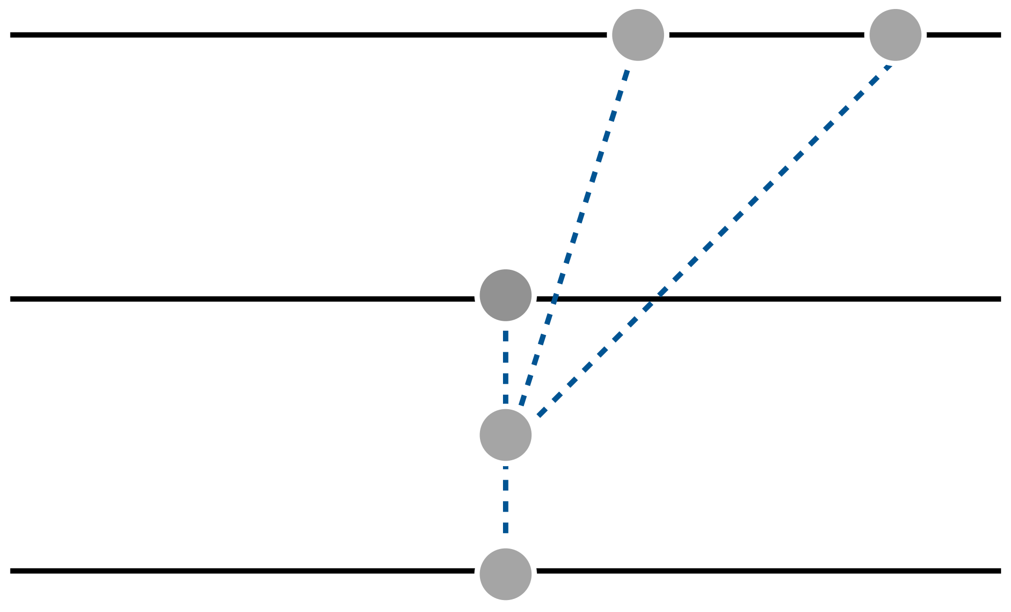
- E_2 induces same structure
  - With $m_\pi^2 \to (\vec{q}^{\,2}/m_N)^2$

Contributions from $\sim \vec{q}^{\,2} F_2$

- F_2 induces same structure as D_2
  - With additional factors of $\vec{q}^{\,2}$

More on 3N interactions

Courtesy: Wouter Dekens



N3LO potential

Bernard, Epelbaum, Krebs, Meissner ’08,’11; Ishikawa, Robilotta ’07

More on 3N interactions

Courtesy: Wouter Dekens

N3LO potential
+ iterations

- Part of the ‘conventional’ N3LO potential is connected to D_2, E_2, F_2
  - Generates the divergent diagrams that induce D_2, E_2, F_2
  - Need to be considered simultaneously for a consistent calculation

Bernard, Epelbaum, Krebs, Meissner ’08,’11; Ishikawa, Robilotta ’07