Extracting GPDs from lattice QCD

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OPUS grant No. 2021/43/B/ST2/00497 (2022-2026)

Outline:

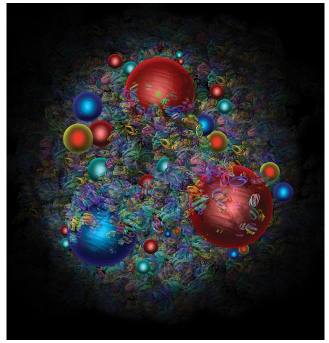
Introduction

GPDs from lattice:
- how to access
- twist-2 GPDs
- frames of reference

Prospects/conclusion

Many thanks to my Collaborators for work presented here:

C. Alexandrou, S. Bhattacharya, M. Constantinou, J. Dodson, X. Gao, K. Hadjiyiannakou, K. Jansen, A. Metz, S. Mukherjee, A. Scapellato, F. Steffens, Y. Zhao



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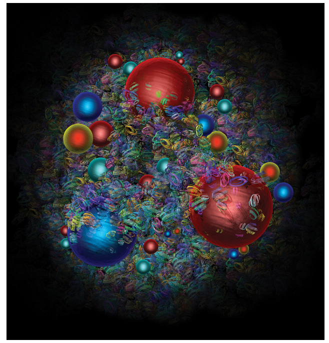
to understand details of 3D nucleon structure.

Particularly important in the context of EIC launch.

Generalized parton distributions (GPDs)



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- related to matrix elements with same incoming/outgoing hadron state,
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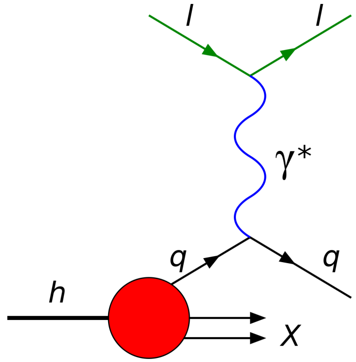

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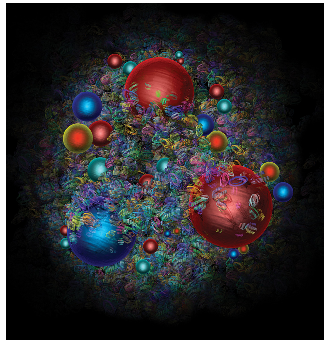

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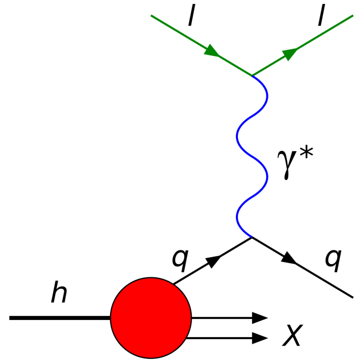

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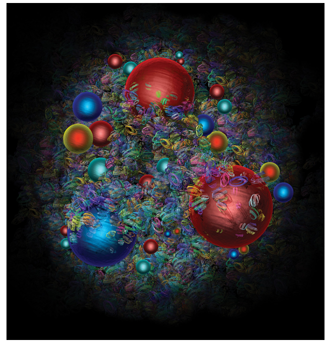

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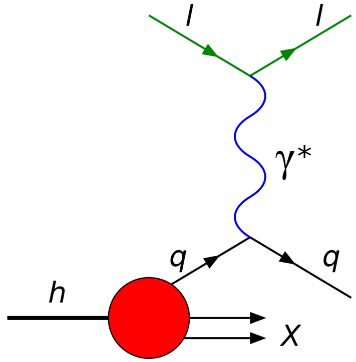

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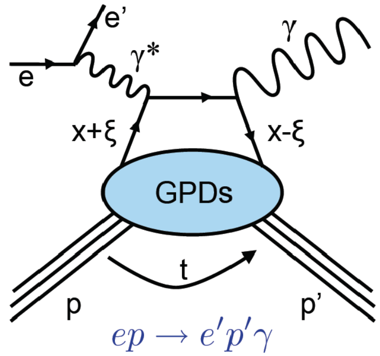
Adding momentum transfer is a natural generalization, leading to **generalized parton distributions** (GPDs):



- experimentally, require exclusive processes like deeply virtual Compton scattering (DVCS) – $ep \longrightarrow e'p'\gamma$,
- reflect spatial distribution of partons in the transverse plane,
- contain information on mechanical properties of hadrons,
- wealth of information on the hadron spin,
- reduce to PDFs in the forward limit, e.g. $H(x,0,0) = q(x)$,
- moments of GPDs are form factors, e.g. $\int dx H(x,\xi,t) = F_1(t)$.

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  1. Set of gauge field configurations on which to measure observables.
     QCD d.o.f.'s put on a **Euclidean** lattice
     ⋆ quarks → sites
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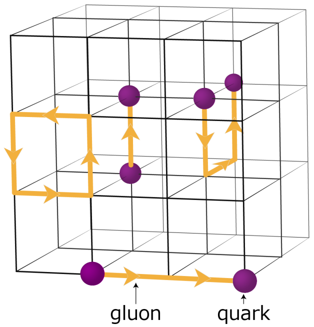
     typical lattice parameters:
     $L/a = [32, 96]$, $a \in [0.04, 0.15]$ fm, $m_\pi \in [135, 500]$ MeV
     ⇒ ∞-dim QCD path integral → $10^8 - 10^9$-dim integral
     Monte Carlo simulations to evaluate the discretized path integral
     feasible, but still require huge computational resources!



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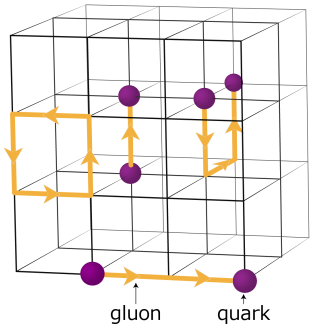

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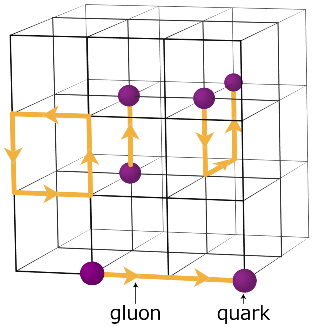

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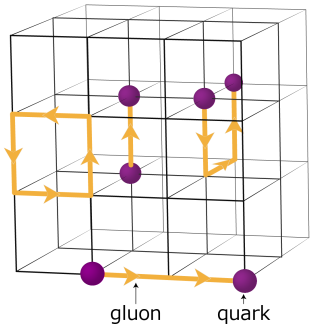

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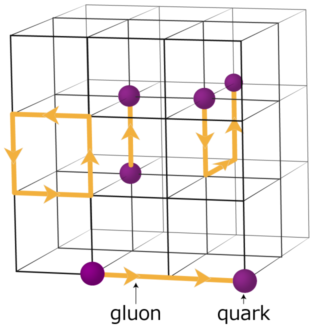

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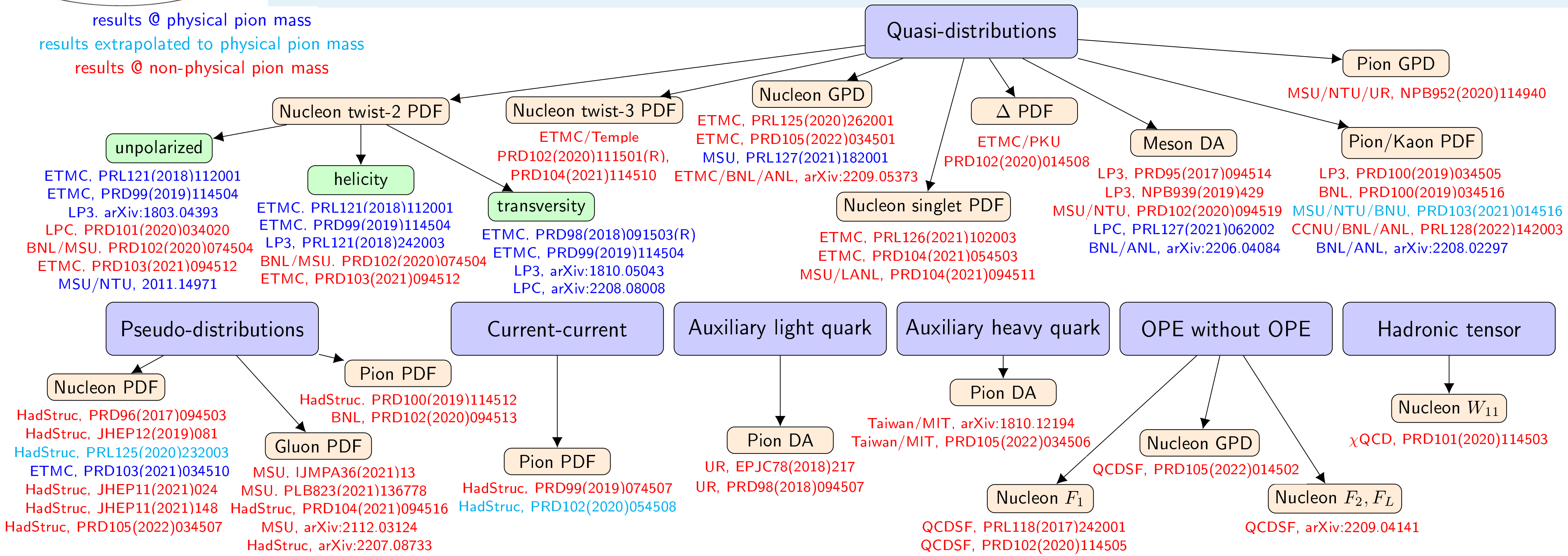
  4. A lot of computing time!
  5. Ingenious analysis techniques, with inputs from perturbation theory.

Lattice PDFs/GPDs: dynamical progress



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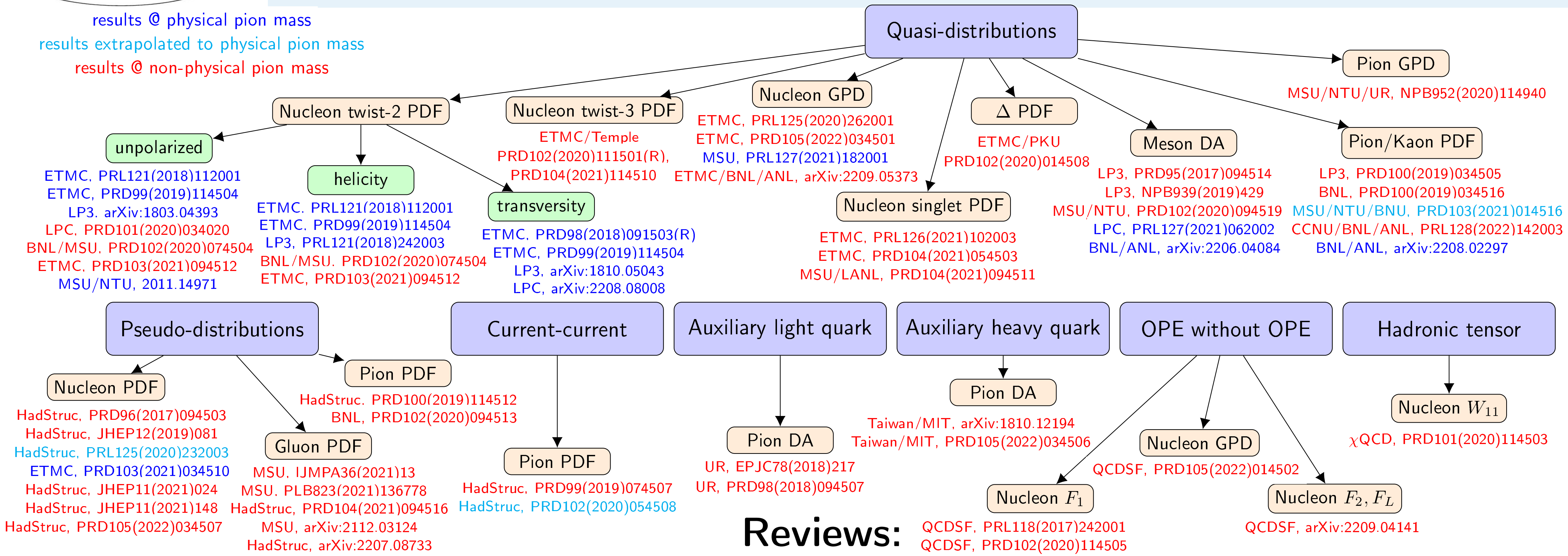

Reviews:

K. Cichy, *Progress in x-dependent partonic distributions from lattice QCD*, plenary talk LATTICE 2021, 2110.07440

K. Cichy, *Overview of lattice calculations of the x-dependence of PDFs, GPDs and TMDs*, plenary talk of Virtual Tribute to Quark Confinement 2021, 2111.04552

K. Cichy, M. Constantinou, *A guide to light-cone PDFs from Lattice QCD: an overview of approaches, techniques and results*, invited review for a special issue of Adv. High Energy Phys. 2019 (2019) 3036904, 1811.07248

M. Constantinou, *The x-dependence of hadronic parton distributions: A review on the progress of lattice QCD* (would-be) plenary talk of LATTICE 2020, EPJA 57 (2021) 77, 2010.02445

X. Ji et al., *Large-Momentum Effective Theory*, Rev. Mod. Phys. 93 (2021) 035005

M. Constantinou et al., *Parton distributions and LQCD calculations: toward 3D structure*, PPNP 121 (2021) 103908

Quasi-GPDs lattice procedure

spatial correlation in a boosted nucleon

$\langle N(\vec{P}')|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N(\vec{P})\rangle$

$\vec{P}' = \vec{P} + \vec{Q}$, $\vec{Q}$ – momentum transfer

lattice computation of bare ME

↓

renormalization
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intermediate RI scheme

↓

reconstruction of x-dependence
z-space$\rightarrow$$x$-space
Backus-Gilbert

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matching to light cone
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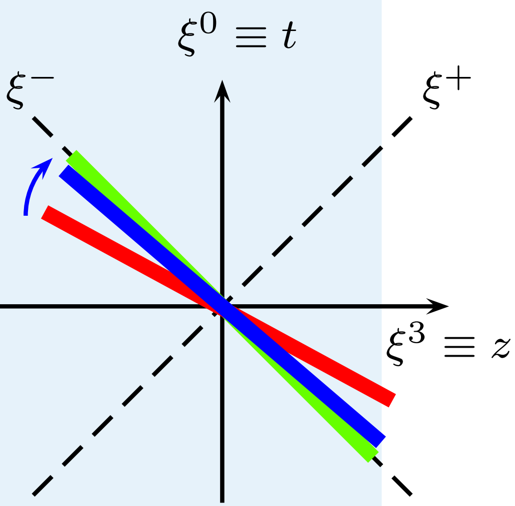

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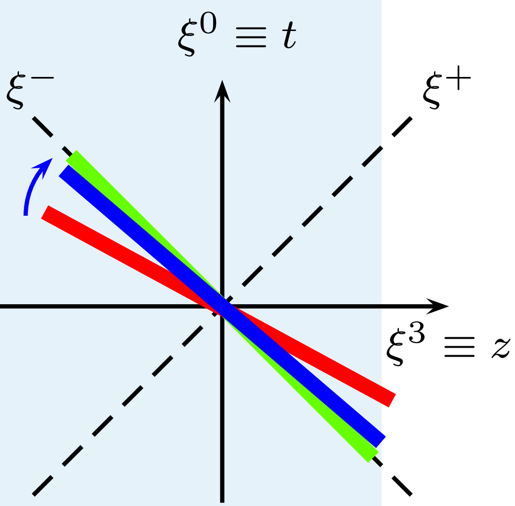

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light-cone GPD

the final desired object!

Setup

Lattice setup:

- fermions: $N_f = 2$ twisted mass fermions + clover term
- gluons: Iwasaki gauge action, $\beta = 1.778$
- gauge field configurations generated by ETMC
- lattice spacing $a \approx 0.093$ fm,
- $32^3 \times 64 \Rightarrow L \approx 3$ fm,
- $m_\pi \approx 260$ MeV.

Twist-2 unpolarized+helicity GPDs ETMC, Phys. Rev. Lett. 125 (2020) 262001
Twist-2 transversity GPDs ETMC, Phys. Rev. D105 (2022) 034501
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Kinematics:

- three nucleon boosts: $P_3 = 0.83, 1.25, 1.67$ GeV,
- momentum transfers: $-t = 0, 0.69, 1.02$ GeV^2,
- skewness: $\xi = 0, 1/3$.

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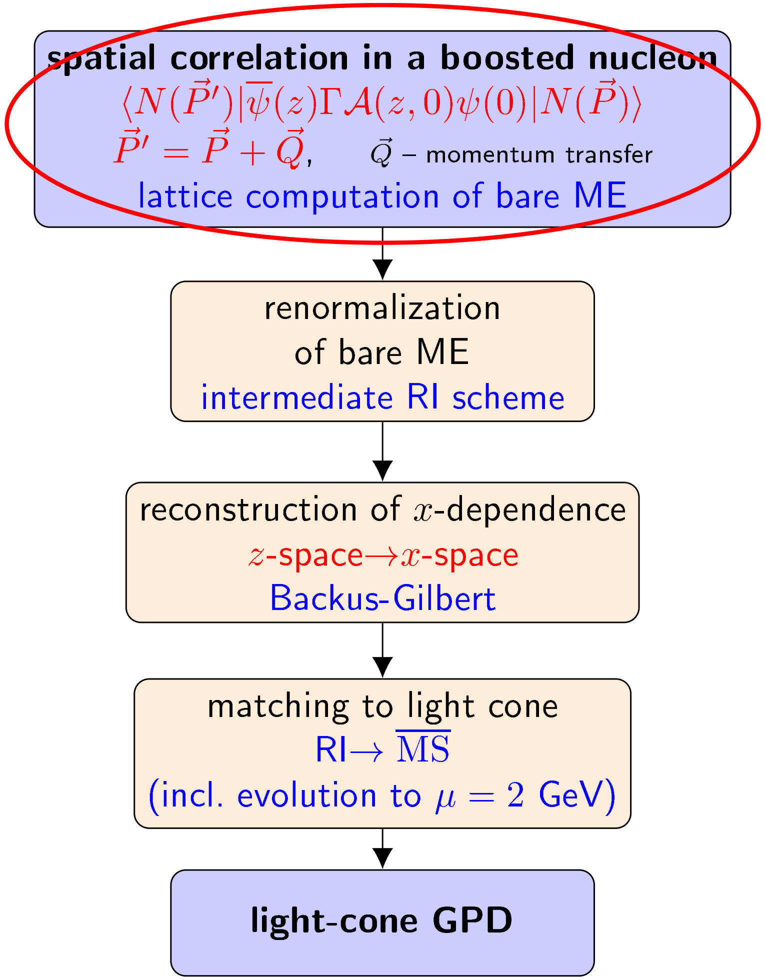
Below for the unpolarized Dirac insertion (for unpolarized GPDs)



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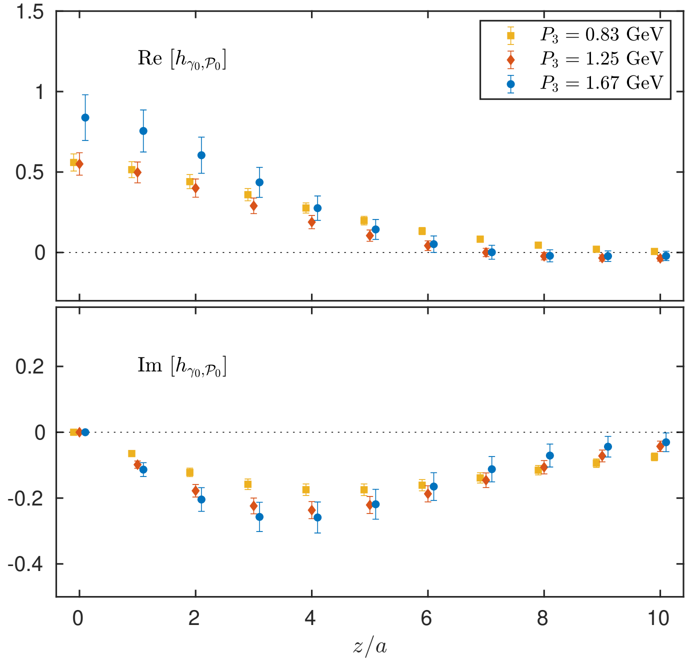
light-cone GPD

unpolarized projector



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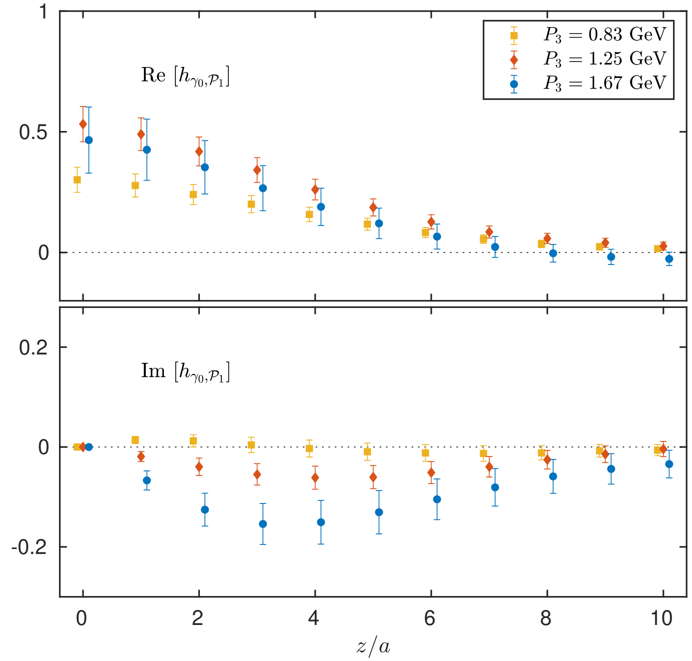

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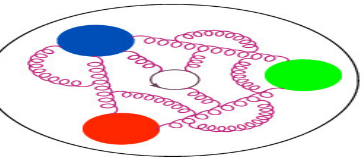
ETMC, Phys. Rev. Lett. 125 (2020) 262001



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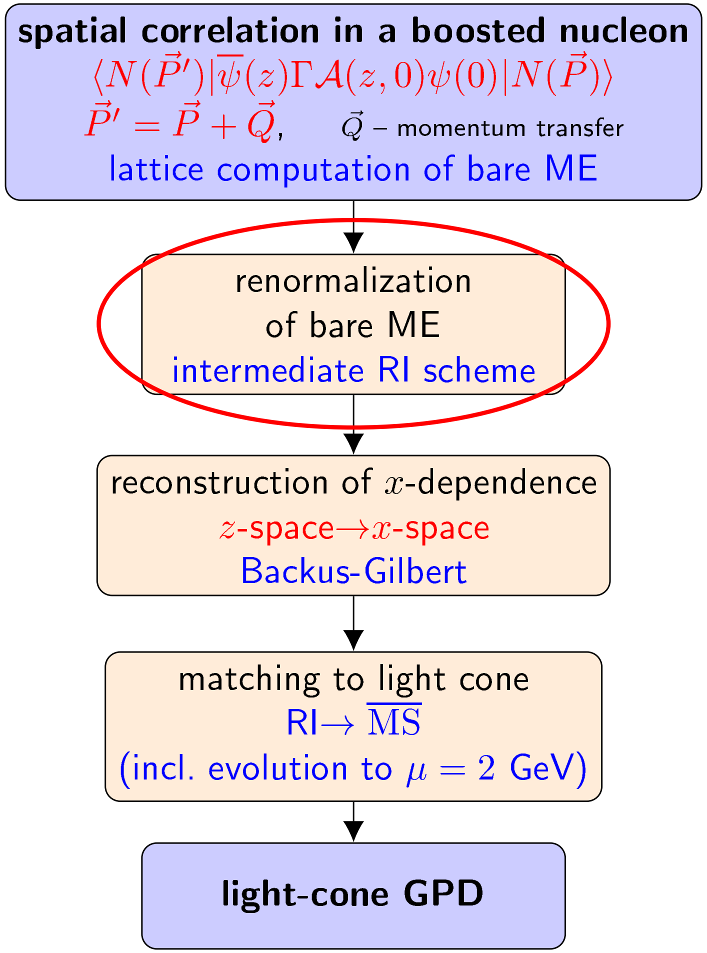

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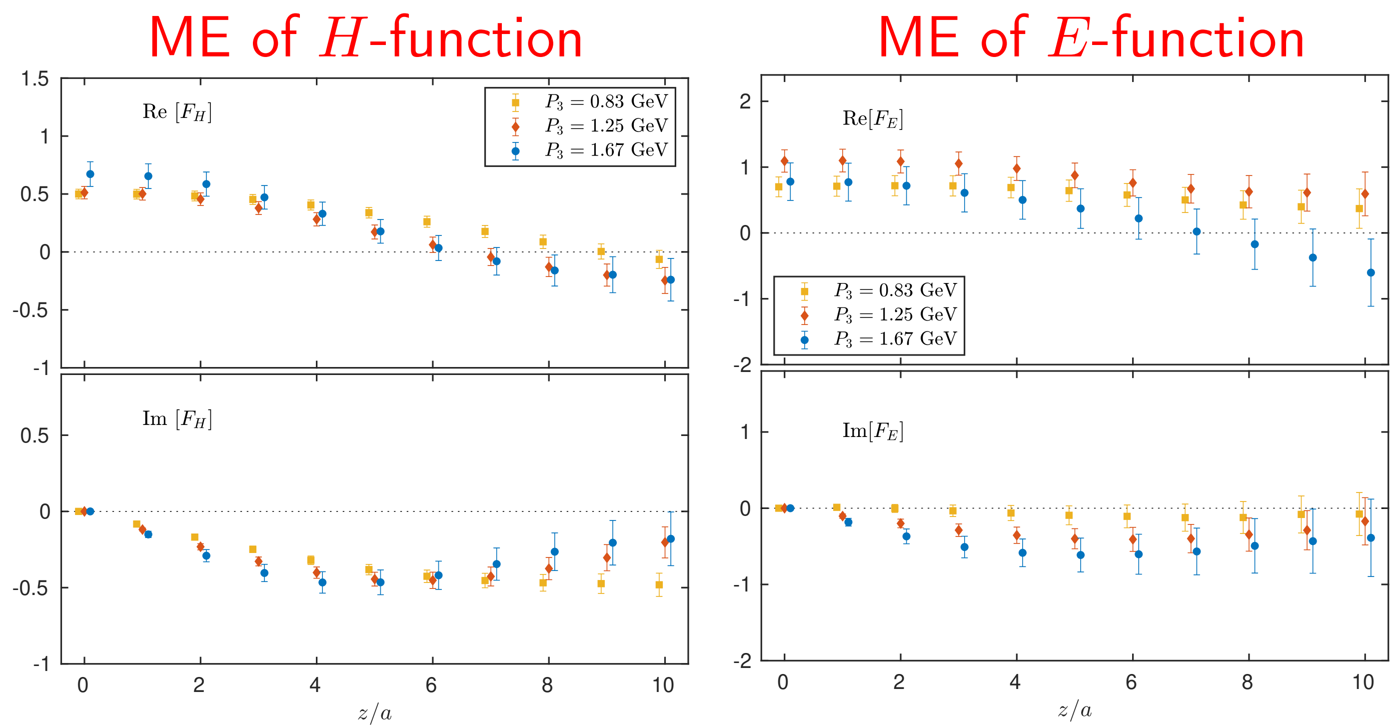
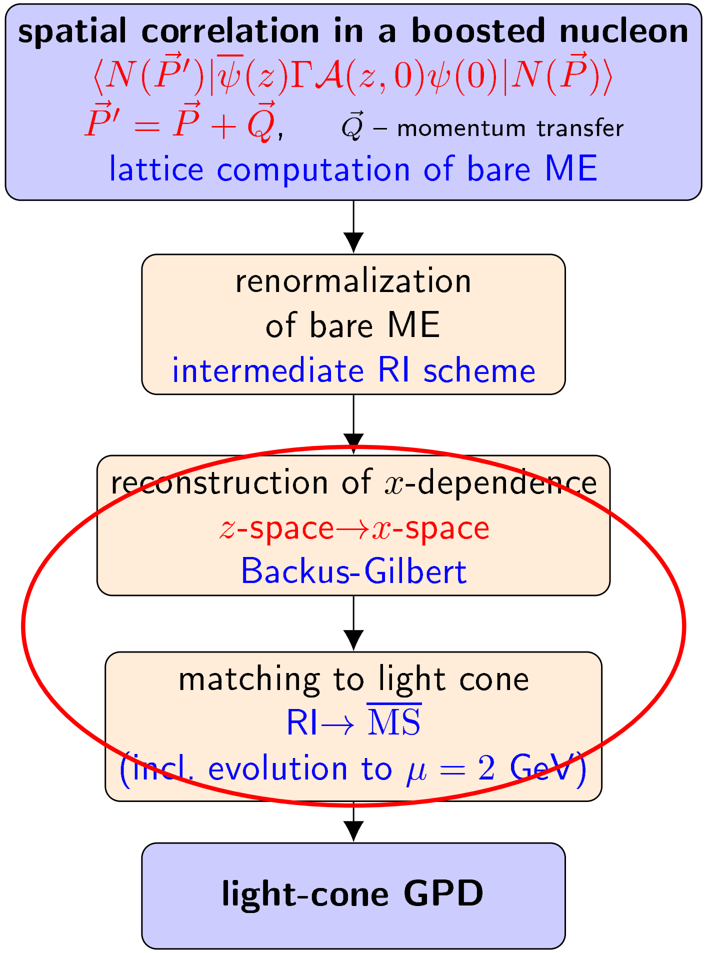
light-cone GPD



Three nucleon boosts: $P_3 = 0.83, 1.25, 1.67$ GeV
Momentum transfer: $-t = 0.69$ GeV^2
Zero skewness: $\xi = 0$
ETMC, Phys. Rev. Lett. 125 (2020) 262001

Light-cone distributions

Reconstruction of x-dependence and matching to light cone.
Unpolarized Dirac insertion (for unpolarized GPDs)



Reconstruction of x-dependence and matching to light cone.
Unpolarized Dirac insertion (for unpolarized GPDs)

spatial correlation in a boosted nucleon
$\langle N(\vec{P}')|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N(\vec{P})\rangle$
$\vec{P}' = \vec{P} + \vec{Q}$, $\vec{Q}$ – momentum transfer
lattice computation of bare ME

↓

renormalization
of bare ME
intermediate RI scheme

↓

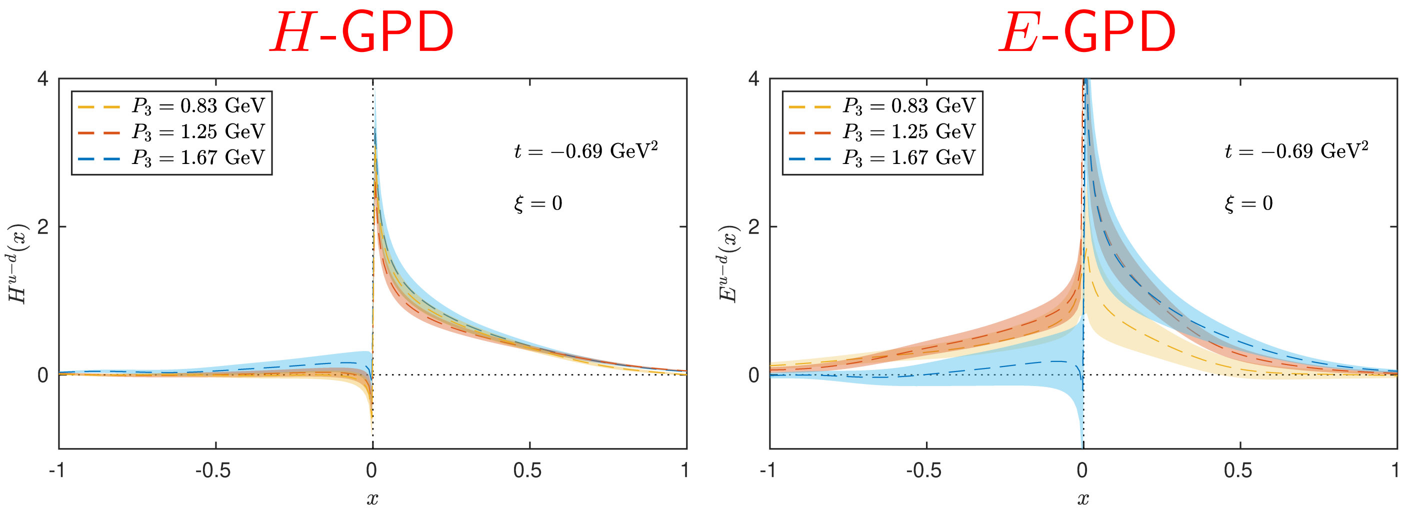
reconstruction of x-dependence
z-space$\rightarrow$$x$-space
Backus-Gilbert

↓

matching to light cone
RI$\rightarrow$ $\overline{\text{MS}}$
(incl. evolution to $\mu = 2$ GeV)

↓

light-cone GPD



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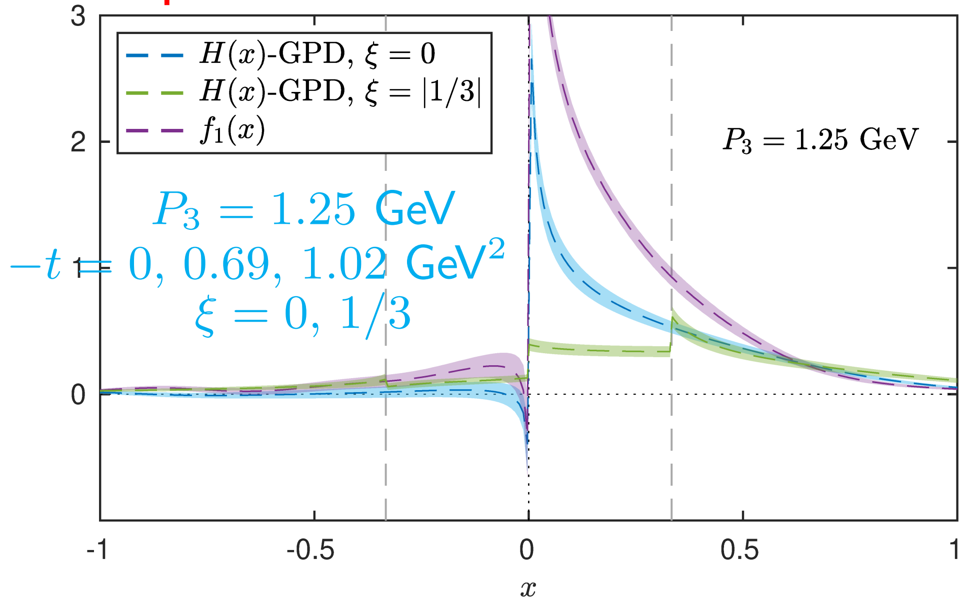
ETMC, Phys. Rev. Lett. 125 (2020) 262001

Comparison of PDFs and H-GPDs

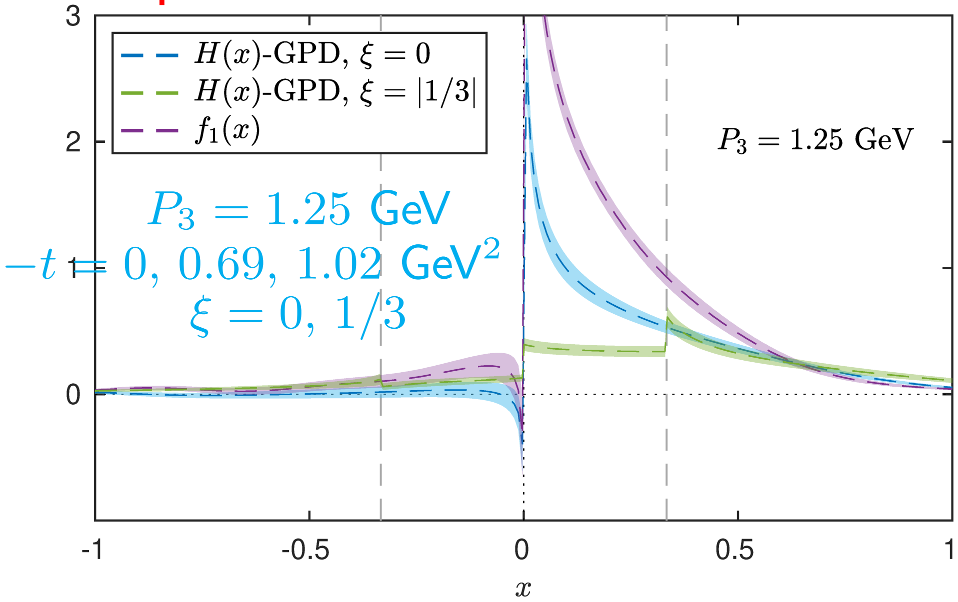
unpolarized

ETMC, Phys. Rev. Lett. 125 (2020) 262001



unpolarized

ETMC, Phys. Rev. Lett. 125 (2020) 262001

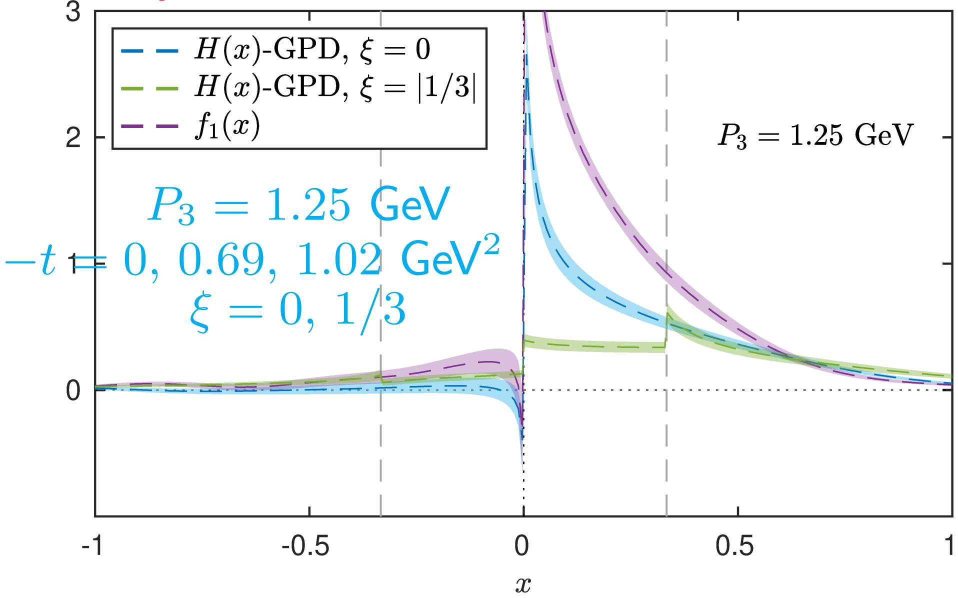


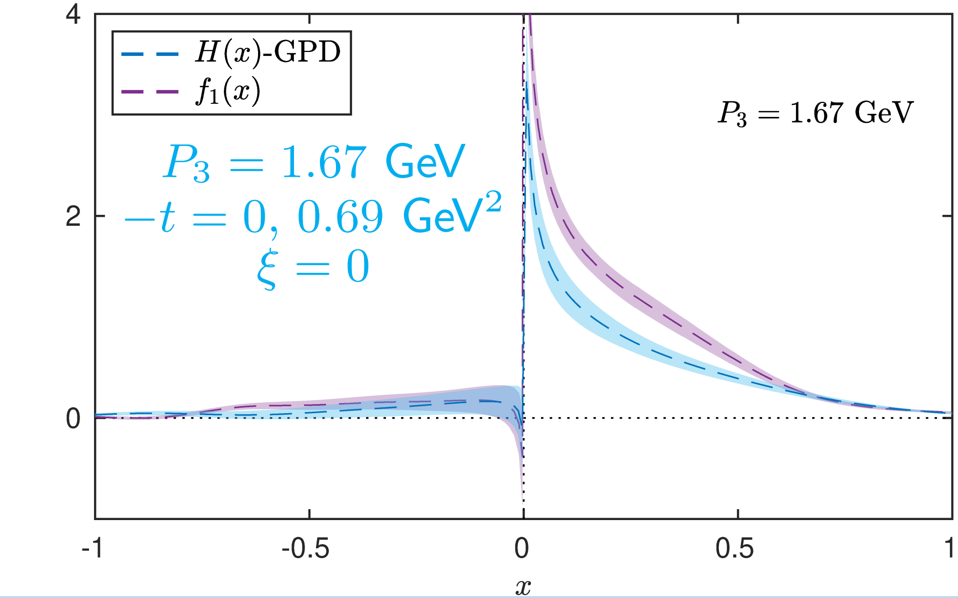
Important insights from models:

S. Bhattacharya, C. Cocuzza, A. Metz
Phys. Lett. B788 (2019) 453
Phys. Rev. D102 (2020) 054201

unpolarized

ETMC, Phys. Rev. Lett. 125 (2020) 262001



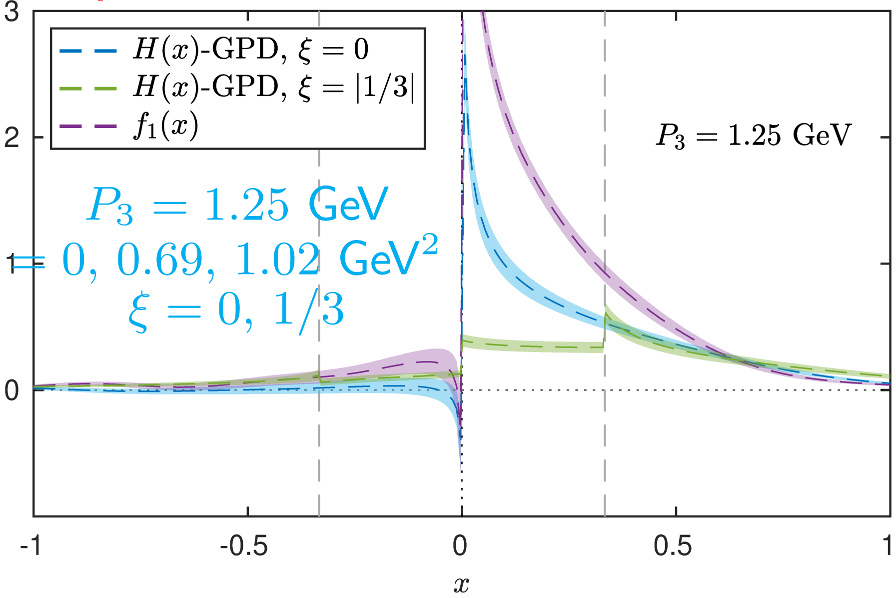


Comparison of PDFs and H-GPDs

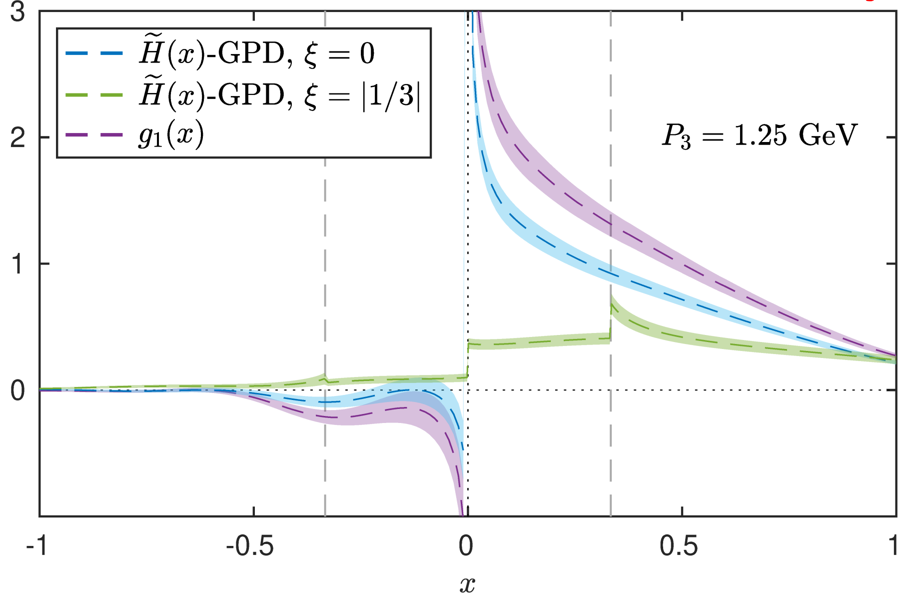
unpolarized

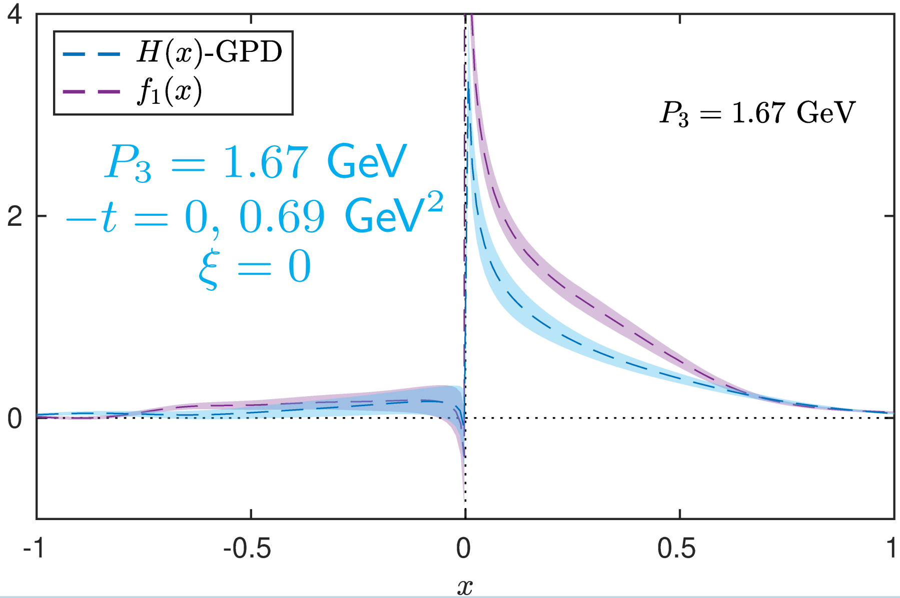
ETMC, Phys. Rev. Lett. 125 (2020) 262001

helicity



$P_3 = 1.25$ GeV
$-t = 0,\ 0.69,\ 1.02$ GeV^2
$\xi = 0,\ 1/3$





$P_3 = 1.67$ GeV
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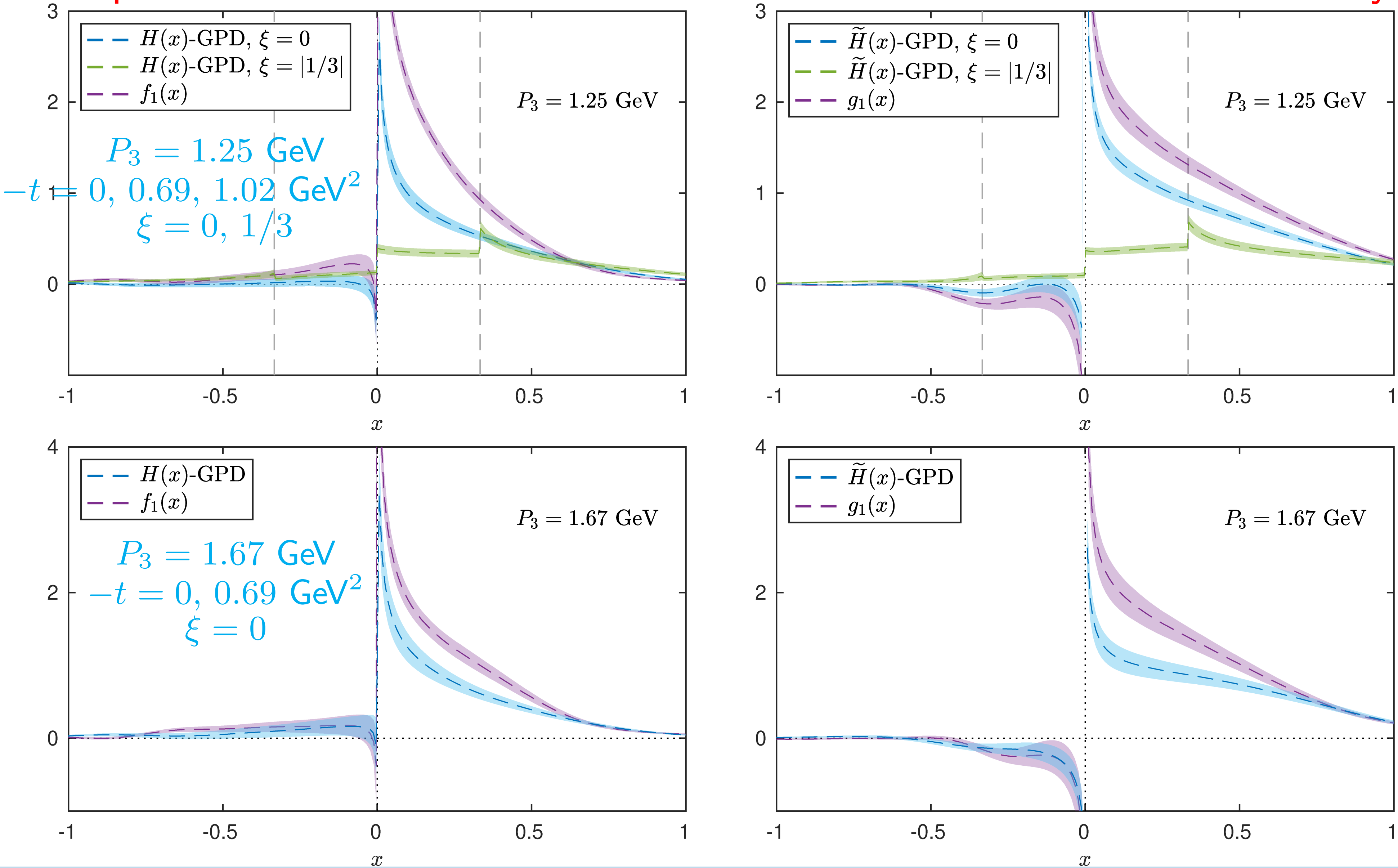
$\xi = 0$

Comparison of PDFs and H-GPDs

unpolarized

ETMC, Phys. Rev. Lett. 125 (2020) 262001

helicity



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Main theoretical tool: See Shohini's talk later today! S. Bhattacharya et al., arXiv:2209.05373
Lorentz-covariant parametrization of matrix elements (e.g. vector case):

$$F^{\mu}(z, P, \Delta) = \bar{u}(p', \lambda')\left[\frac{P^{\mu}}{m}A_1 + mz^{\mu}A_2 + \frac{\Delta^{\mu}}{m}A_3 + im\sigma^{\mu z}A_4 + \frac{i\sigma^{\mu\Delta}}{m}A_5 + \frac{P^{\mu}i\sigma^{z\Delta}}{m}A_6 + \frac{z^{\mu}i\sigma^{z\Delta}}{m}A_7 + \frac{\Delta^{\mu}i\sigma^{z\Delta}}{m}A_8\right]u(p, \lambda),$$

(inspired by: S. Meissner, A. Metz, M. Schlegel, JHEP08(2009)056).

- most general parametrization in terms of 8 linearly-independent Lorentz structures,
- 8 Lorentz-invariant amplitudes $A_i(z \cdot P, z \cdot \Delta, \Delta^2, z^2)$.

Example

S. Bhattacharya et al., arXiv:2209.05373

The relation between lattice-calculated matrix elements and the Lorentz-invariant amplitudes A_i is different in the symmetric and the non-symmetric frame.

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For example: (γ_0 insertion, unpolarized projector)

symmetric frame:

$$\Pi_0^s(\Gamma_0) = C\left(\frac{E\left(E(E+m)-P_3^2\right)}{2m^3}A_1 + \frac{(E+m)\left(-E^2+m^2+P_3^2\right)}{m^3}A_5 + \frac{EP_3\left(-E^2+m^2+P_3^2\right)z}{m^3}A_6\right),$$

asymmetric frame:

$$\begin{aligned}\Pi_0^a(\Gamma_0) \;=\; & C\Bigg(-\frac{(E_f+E_i)(E_f-E_i-2m)(E_f+m)}{8m^3}A_1 - \frac{(E_f-E_i-2m)(E_f+m)(E_f-E_i)}{4m^3}A_3 + \frac{(E_i-E_f)P_3z}{4m}A_4 \\ + \; & \frac{(E_f+E_i)(E_f+m)(E_f-E_i)}{4m^3}A_5 + \frac{E_f(E_f+E_i)P_3(E_f-E_i)z}{4m^3}A_6 + \frac{E_fP_3(E_f-E_i)^2z}{2m^3}A_8\Bigg).\end{aligned}$$

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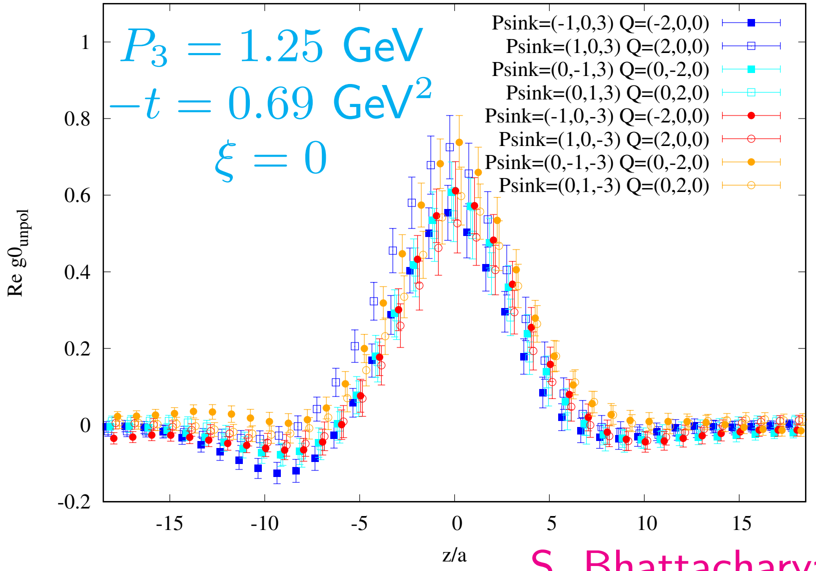
Thus,

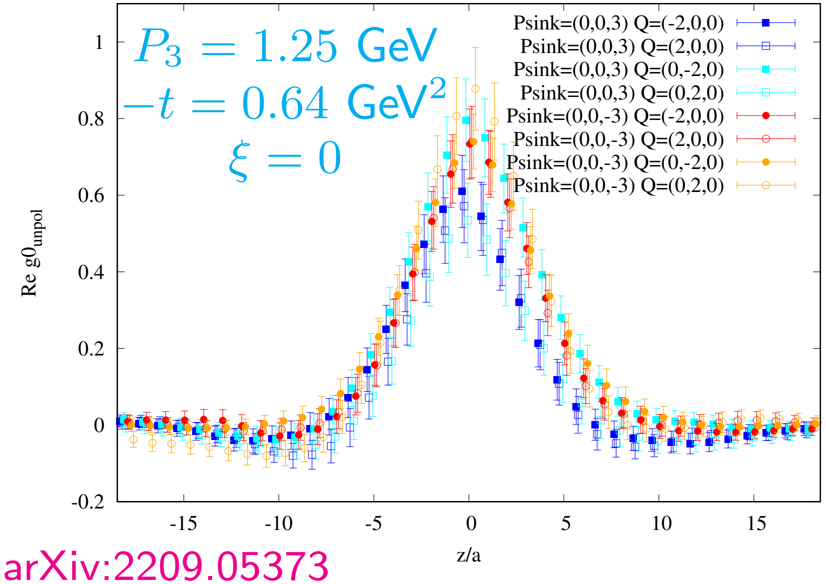
- matrix elements $\Pi_\mu(\Gamma_\nu)$ are frame-dependent,
- but the amplitudes A_i are frame-invariant.

Bare matrix elements of $\Pi_0(\Gamma_0)$

symmetric frame / non-symmetric frame

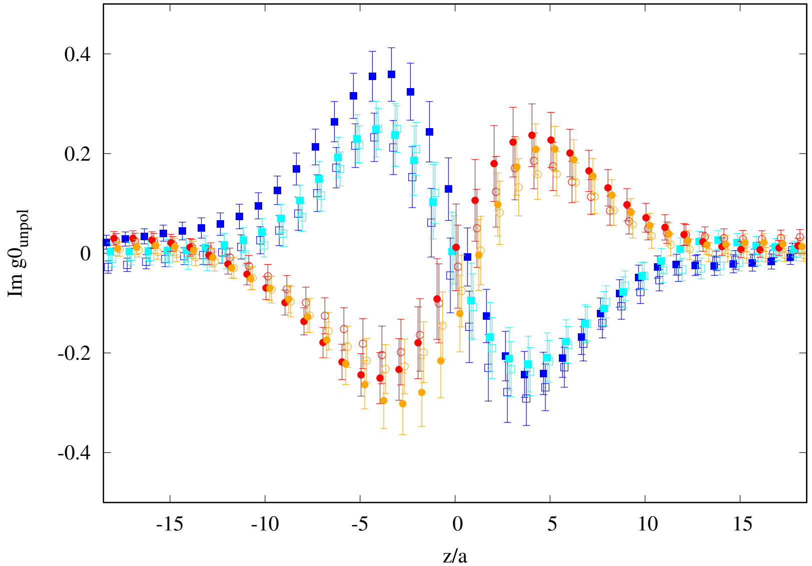
Re

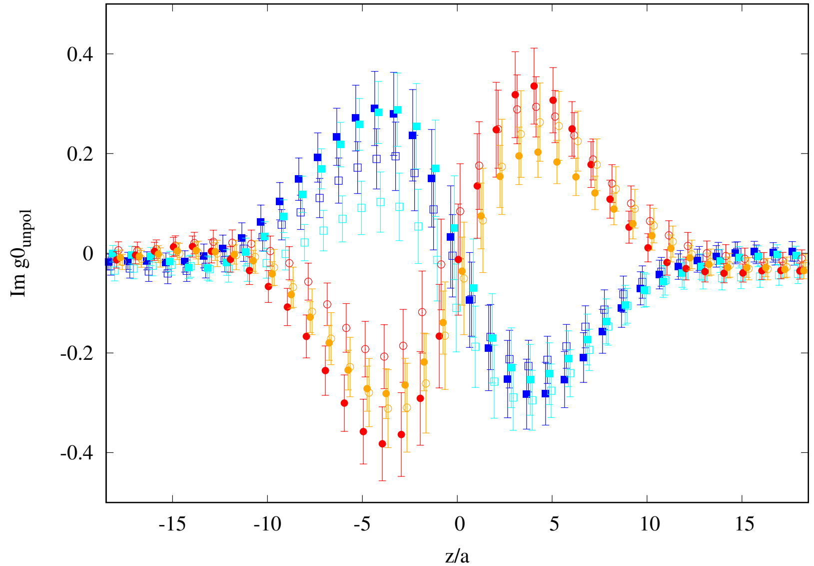




S. Bhattacharya et al., arXiv:2209.05373

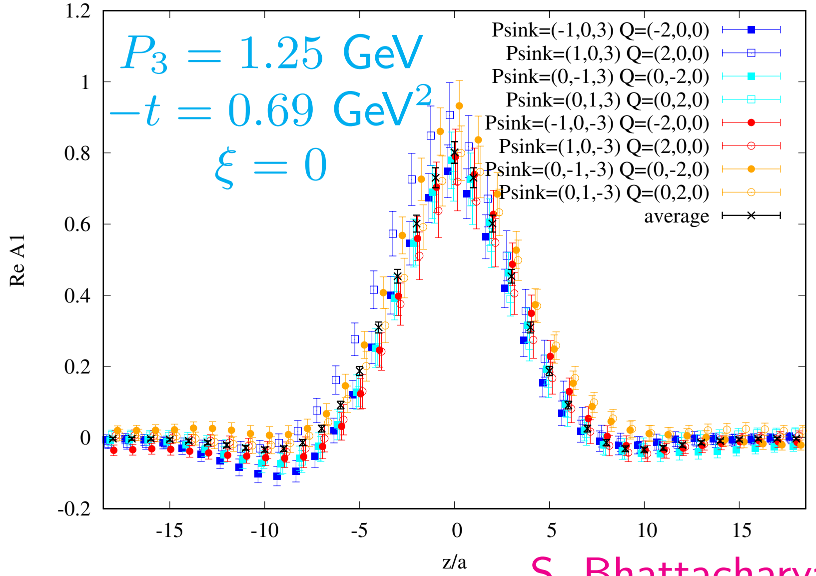
Im



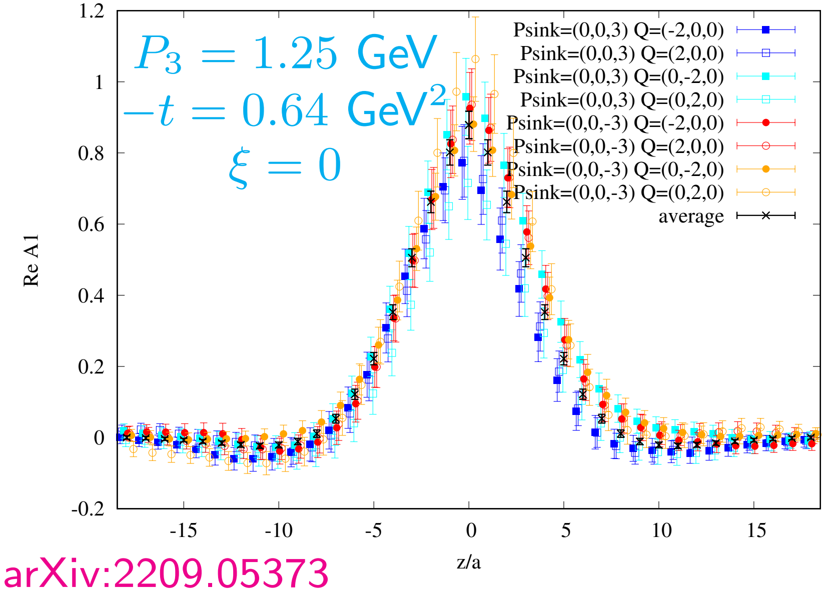


Example amplitude A_1

symmetric frame



non-symmetric frame



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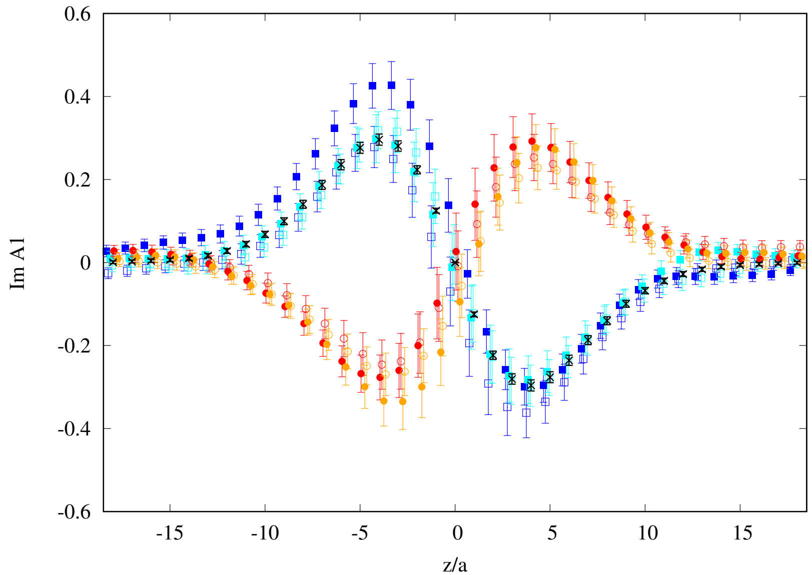
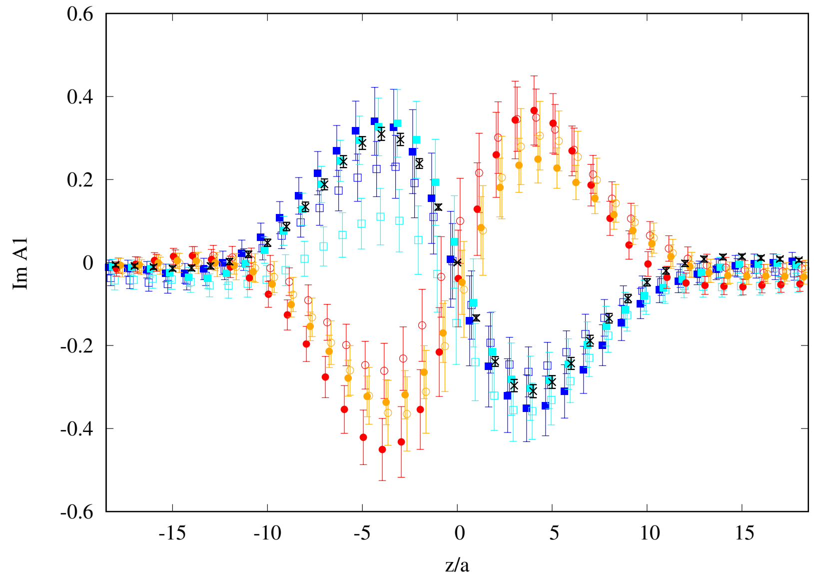
S. Bhattacharya et al., arXiv:2209.05373





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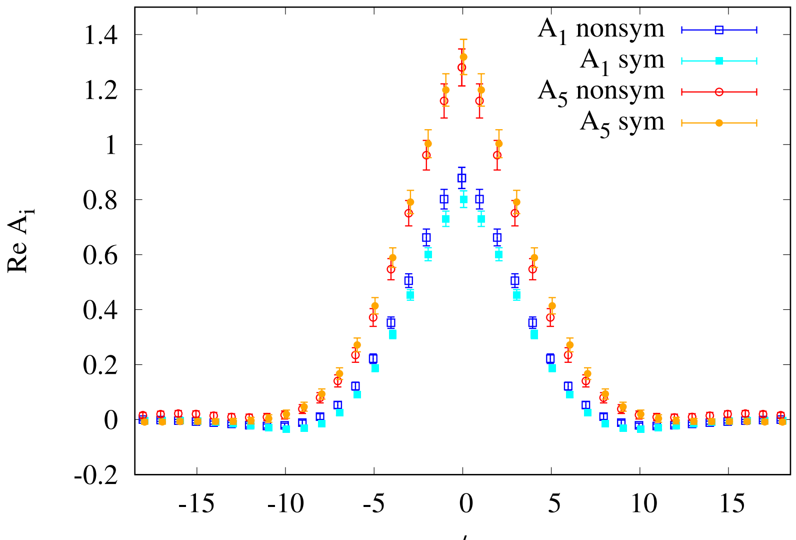
Comparison of amplitudes between frames

A_1, A_5 (leading ones) A_2, A_3, A_4, A_6, A_7, A_8 (subleading ones)



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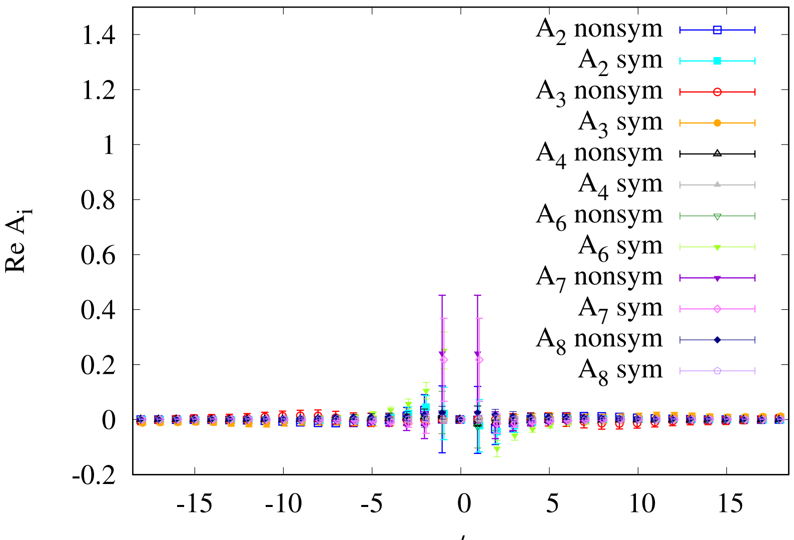

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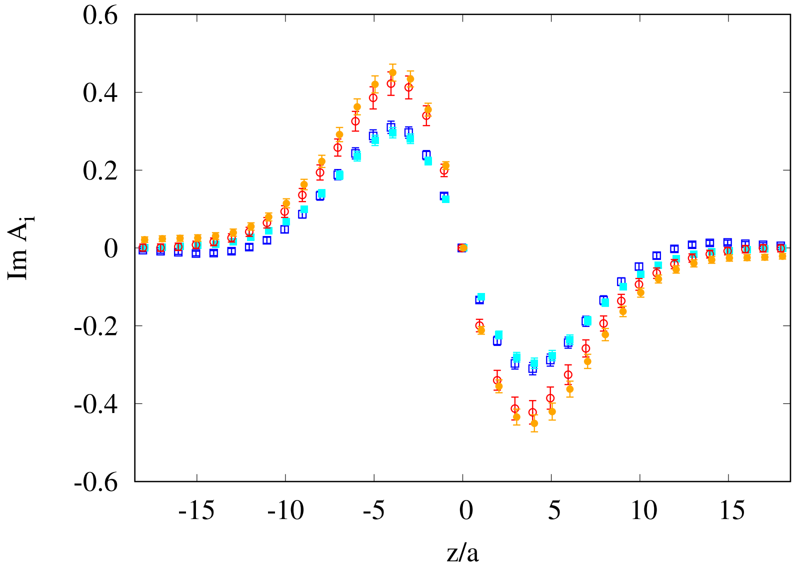

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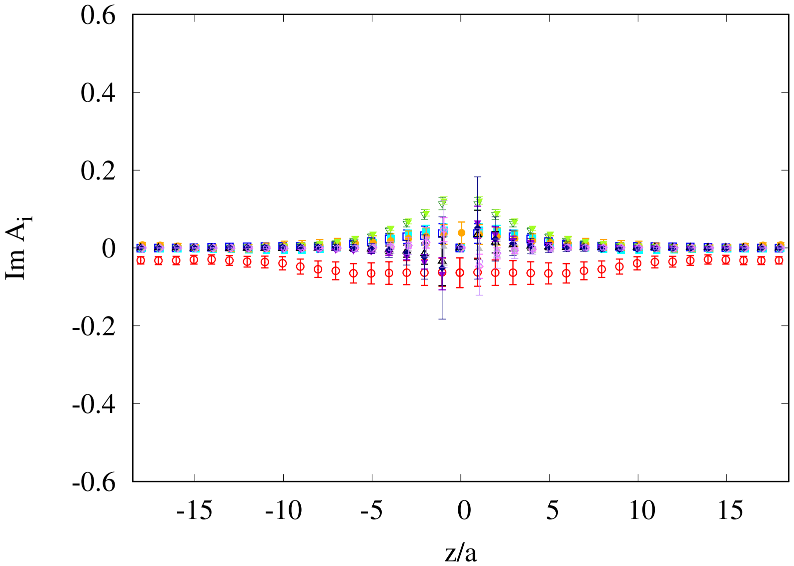

H and E GPDs – standard definition

The standard definition of H and E GPDs:

S. Bhattacharya et al., arXiv:2209.05373

See Shohini's talk later today!

$$F^0(z,P,\Delta) = \bar{u}(p',\lambda')\left[\gamma^0 F_{H^{(0)}}(z,P,\Delta) + \frac{i\sigma^{0\mu}\Delta_\mu}{2m} F_{E^{(0)}}(z,P,\Delta)\right] u(p,\lambda)\,.$$

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Thus-defined GPDs are obviously frame-dependent! In terms of A_i's ($\xi = 0$ case):

symmetric frame:

$$F_{H^{(0)}} = A_1 + \frac{z(Q_1^2+Q_2^2)}{2P_3} A_6\,,$$

$$F_{E^{(0)}} = -A_1 - \frac{m^2 z}{P_3} A_4 + 2A_5 - \frac{z\left(4E^2+Q_1^2+Q_2^2\right)}{2P_3} A_6\,.$$

asymmetric frame:

$$F_{H^{(0)}} = A_1 + \frac{Q_0}{P_0} A_3 + \frac{m^2 z Q_0}{2P_0 P_3} A_4 + \frac{z(Q_0^2+Q_\perp^2)}{2P_3} A_6 + \frac{z(Q_0^3+Q_0 Q_\perp^2)}{2P_0 P_3} A_8\,,$$

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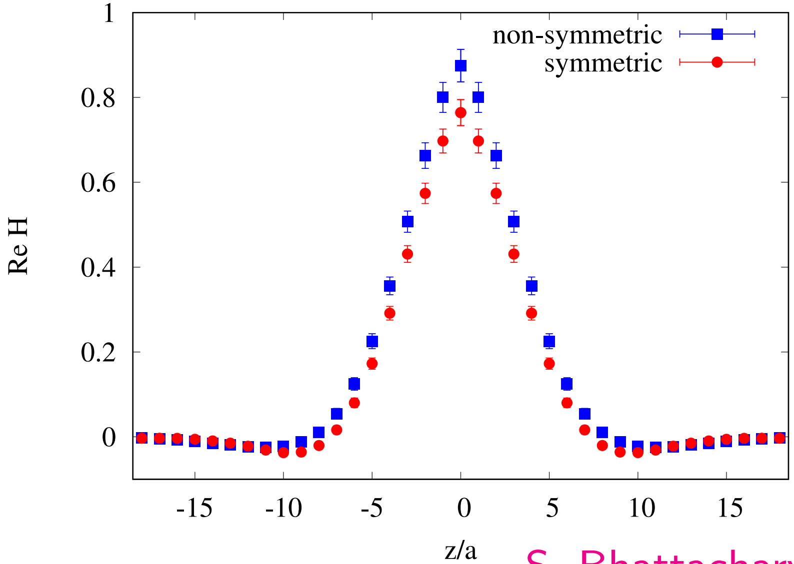
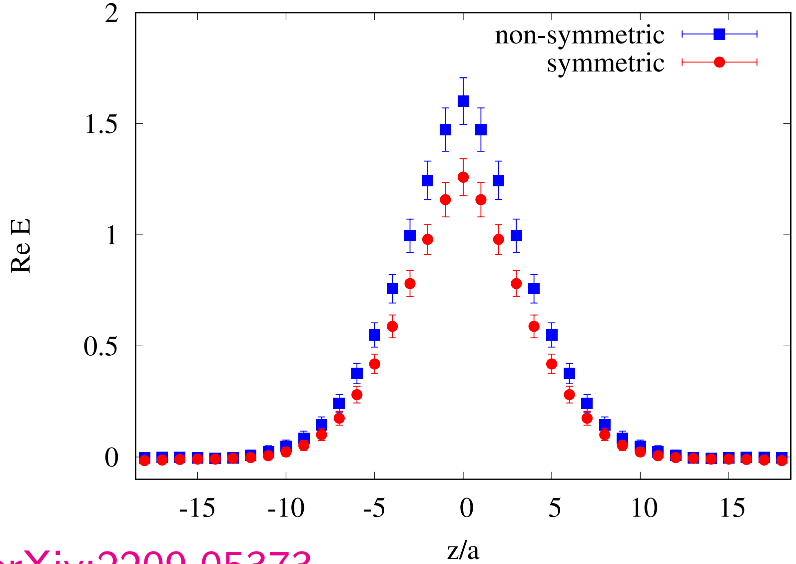
Note: the standard definition is frame-dependent, but still valid in the sense of approaching the correct GPDs in the light-cone limit.

H and E GPDs – standard definition

H-GPD



E-GPD



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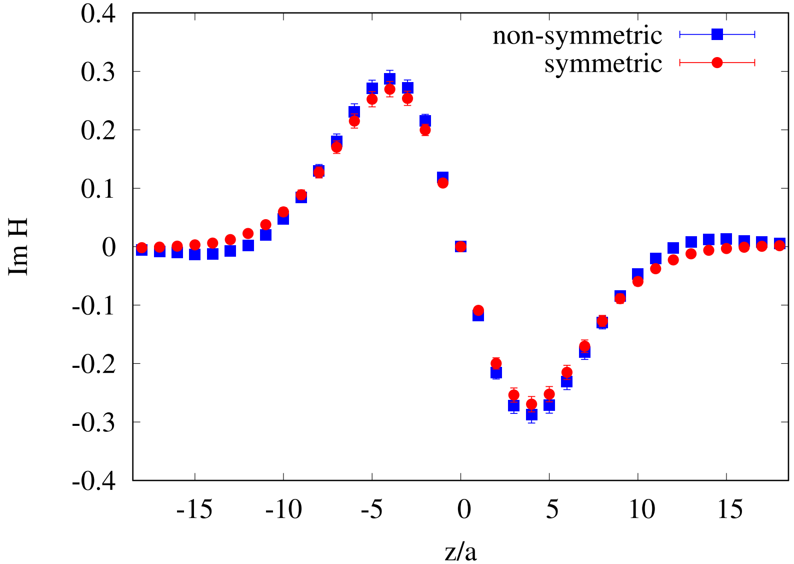
S. Bhattacharya et al., arXiv:2209.05373



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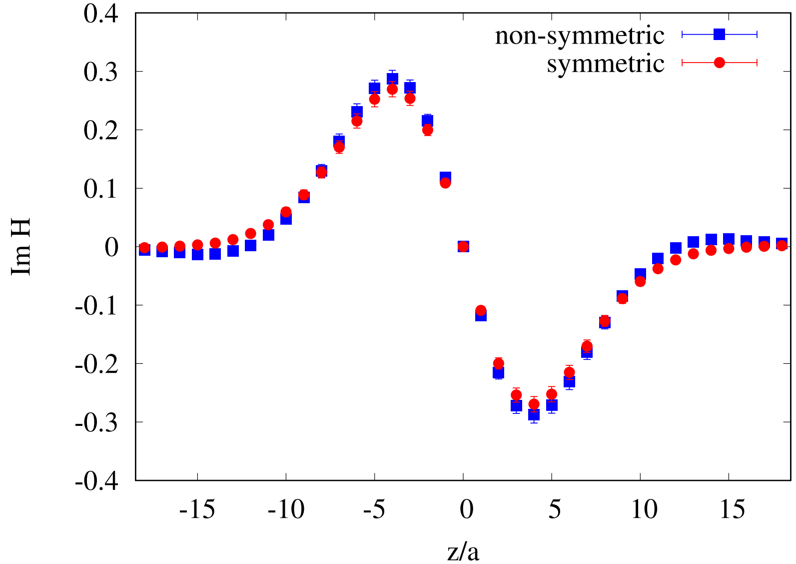

The definition of H and E GPDs can be made Lorentz-invariant in the following way:

S. Bhattacharya et al., arXiv:2209.05373

See Shohini's talk later today!

$$F_H = A_1 + \frac{\Delta \cdot z}{P \cdot z} A_3 \,,$$

$$F_E = -A_1 - \frac{\Delta \cdot z}{P \cdot z} A_3 + 2A_5 + 2P \cdot z A_6 + 2\Delta \cdot z A_8 \,.$$

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At zero-skewness:

$$F_H = A_1 \,,$$

$$F_E = -A_1 + 2A_5 + 2zP_3 A_6 \,.$$

With respect to the standard definition, removed/reduced contribution from A_3, A_4, A_6, A_8.

H and E GPDs – Lorentz-invariant definition

The definition of H and E GPDs can be made Lorentz-invariant in the following way:

S. Bhattacharya et al., arXiv:2209.05373

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See Shohini's talk later today!

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With respect to the standard definition, removed/reduced contribution from A_3, A_4, A_6, A_8.

In terms of matrix elements:

- standard definition – only $\Pi_0(\Gamma_0)$, $\Pi_0(\Gamma_{1/2})$,
- Lorentz-invariant definition – additionally:
  - ⋆ symmetric: $\Pi_{1/2}(\Gamma_3)$,
  - ⋆ non-symmetric: $\Pi_{1/2}(\Gamma_3)$, $\Pi_{1/2}(\Gamma_0)$, $\Pi_1(\Gamma_2)$, $\Pi_2(\Gamma_1)$.

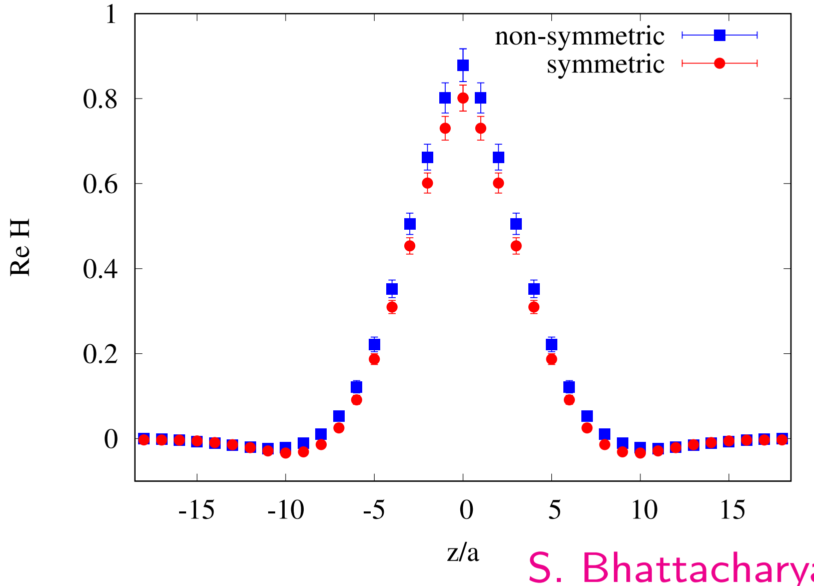
Thus, adding info from additional MEs potentially improves convergence (to be investigated).

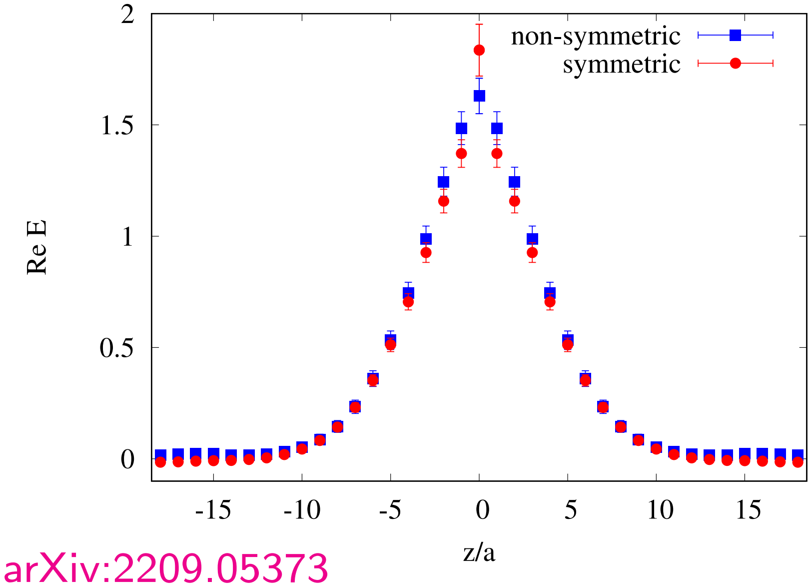
H and E GPDs – Lorentz-invariant definition

H-GPD

E-GPD

Re





S. Bhattacharya et al., arXiv:2209.05373

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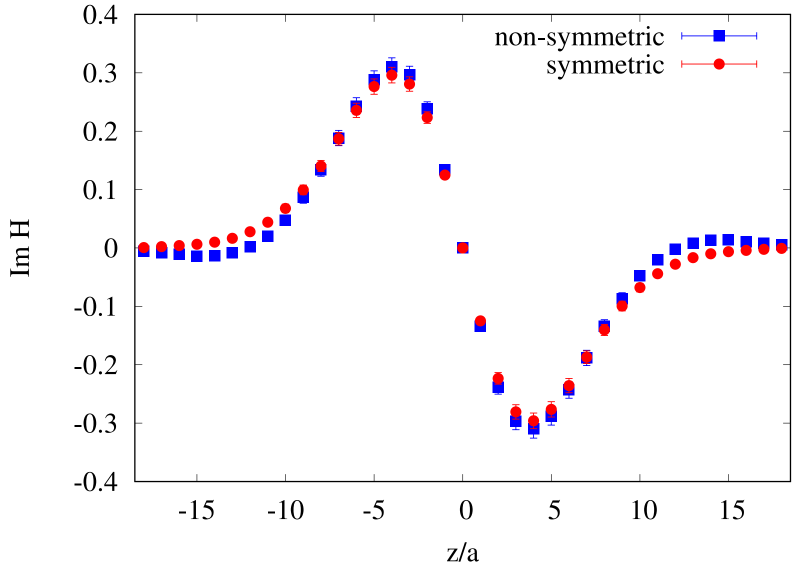
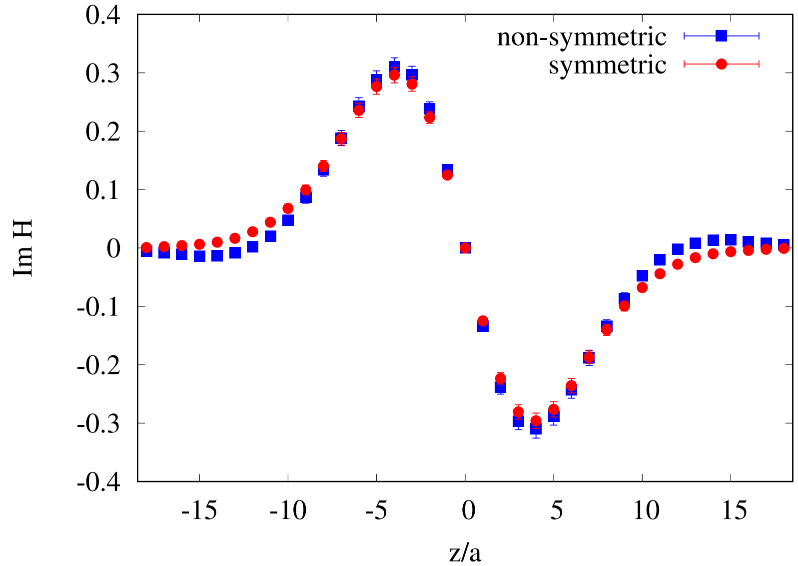
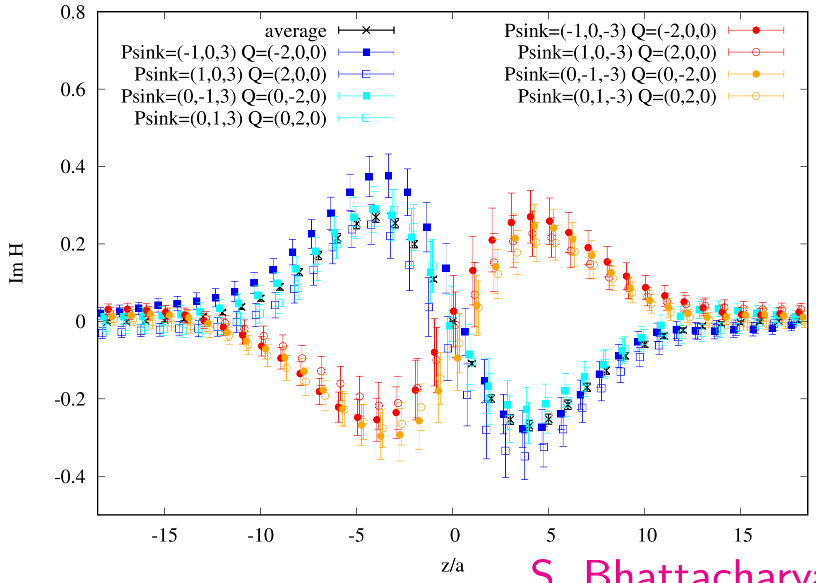


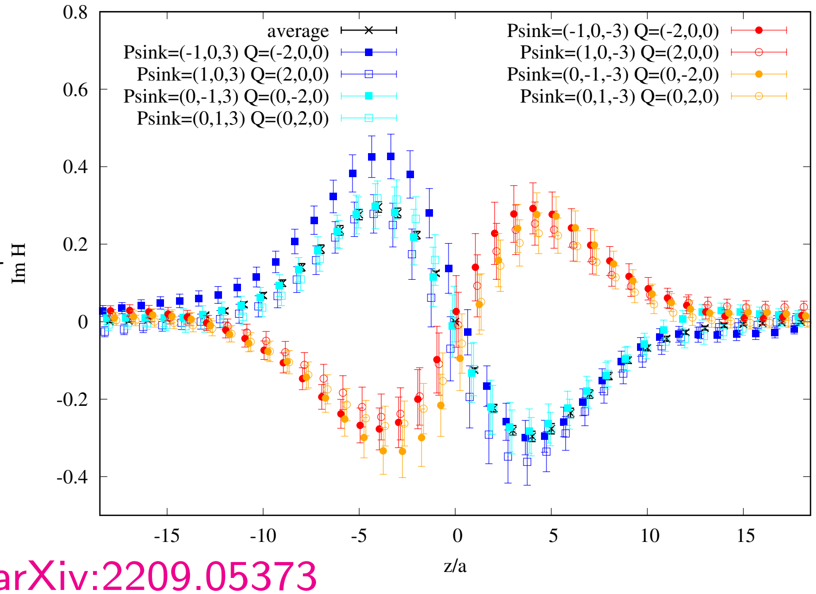


H and E GPDs – signal improvement

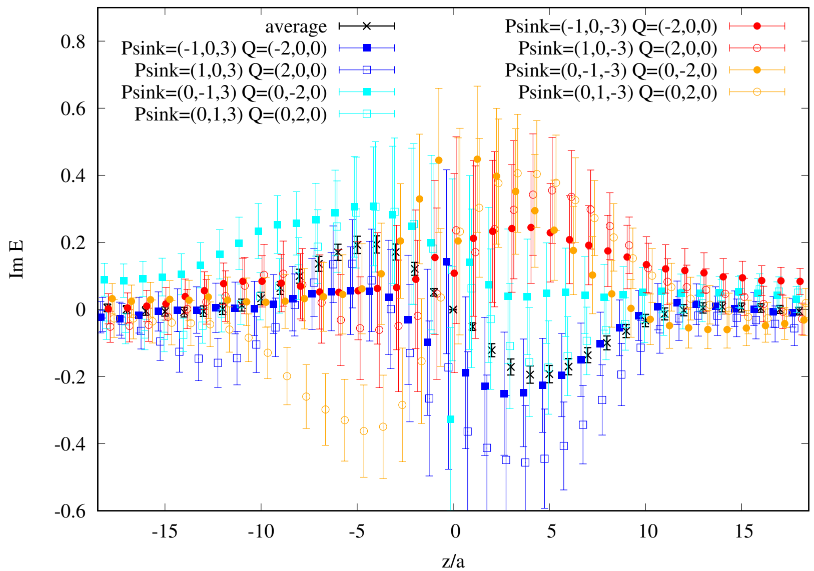
standard | Lorentz-invariant



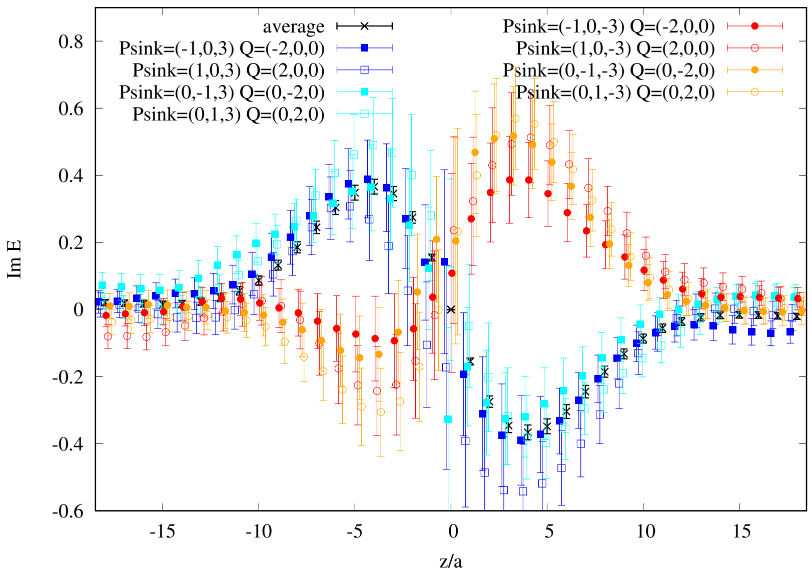
Im H



S. Bhattacharya et al., arXiv:2209.05373

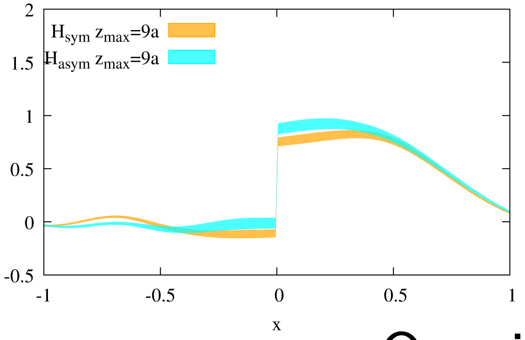


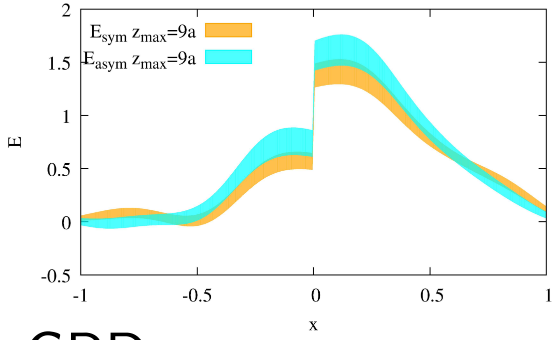
Im E

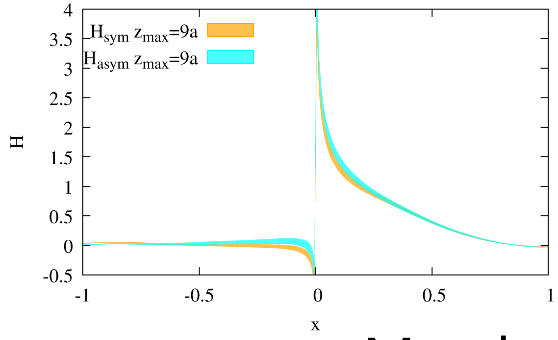


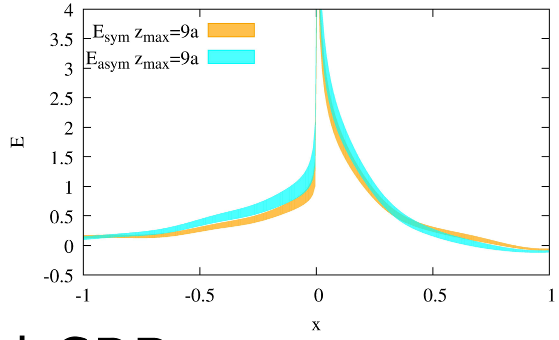
Quasi- and matched H and E GPDs

STANDARD DEFINITION







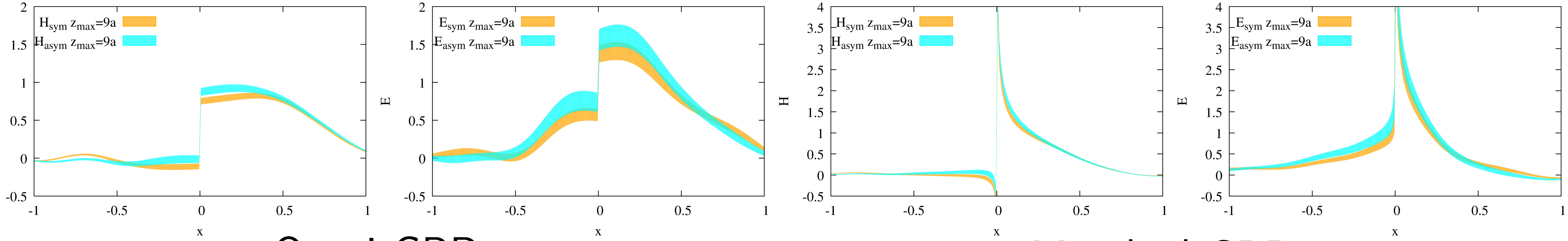


Quasi-GPDs S. Bhattacharya et al., arXiv:2209.05373 Matched GPDs

H-GPD E-GPD H-GPD E-GPD

Quasi- and matched H and E GPDs

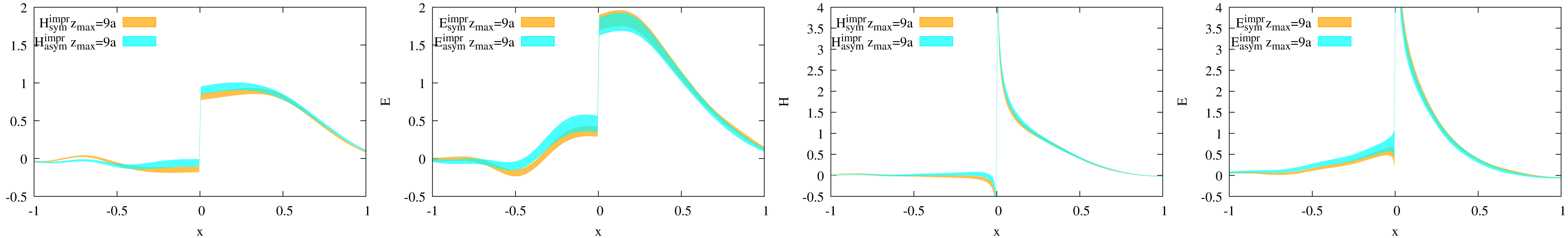
STANDARD DEFINITION



Quasi-GPDs S. Bhattacharya et al., arXiv:2209.05373 Matched GPDs

H-GPD E-GPD H-GPD E-GPD

LORENTZ-INVARIANT DEFINITION



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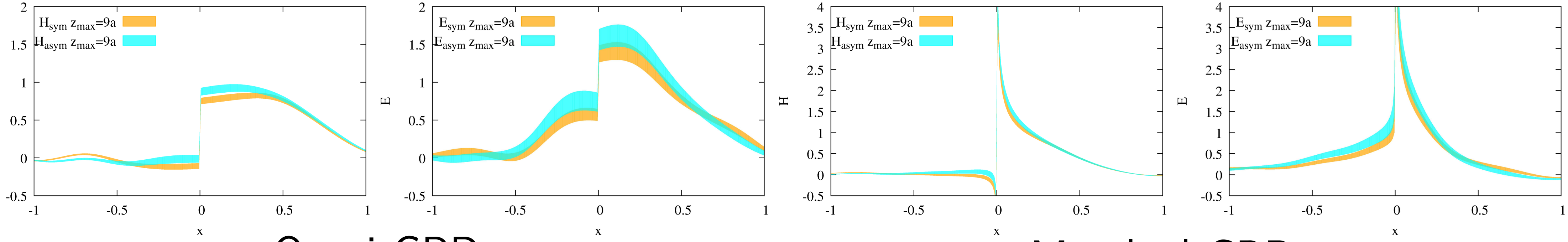

Quasi-GPDs S. Bhattacharya et al., arXiv:2209.05373 Matched GPDs

H-GPD E-GPD H-GPD E-GPD

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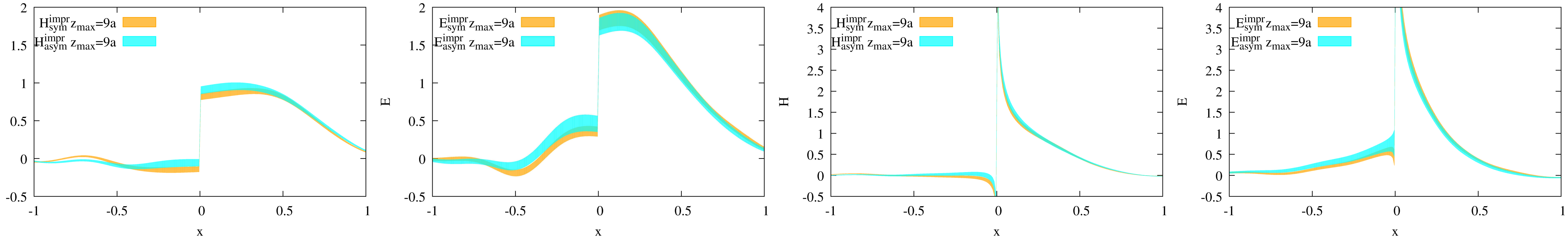

Main conclusions:

– GPDs can be computed in non-symmetric frames, reducing the computational cost

– GPDs can be made frame-independent (Lorentz-invariant definition) – potentially better convergence

Overall, it gives much better perspectives for lattice GPDs!

Transversity GPDs:
4 GPDs: H_T, E_T, $\tilde{H}_T$, $\tilde{E}_T$

spatial correlation in a boosted nucleon
$\langle N(\vec{P}')|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N(\vec{P})\rangle$
$\vec{P}' = \vec{P} + \vec{Q}$, $\vec{Q}$ – momentum transfer
lattice computation of bare ME

↓

renormalization
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reconstruction of x-dependence
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↓

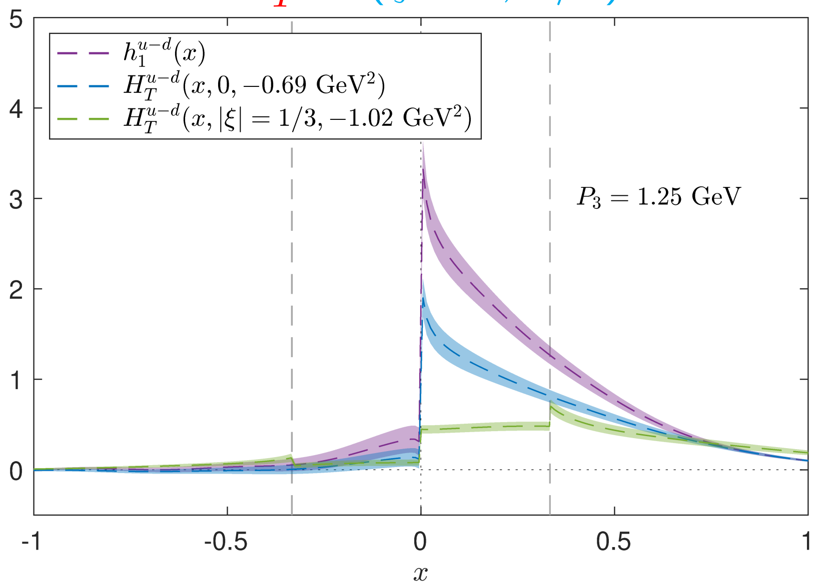
matching to light cone
RI→ $\overline{\text{MS}}$
(incl. evolution to $\mu = 2$ GeV)

↓

light-cone GPD

ETMC, Phys. Rev. D105 (2022) 034501

H_T^{u-d} $(\xi = 0,\ 1/3)$



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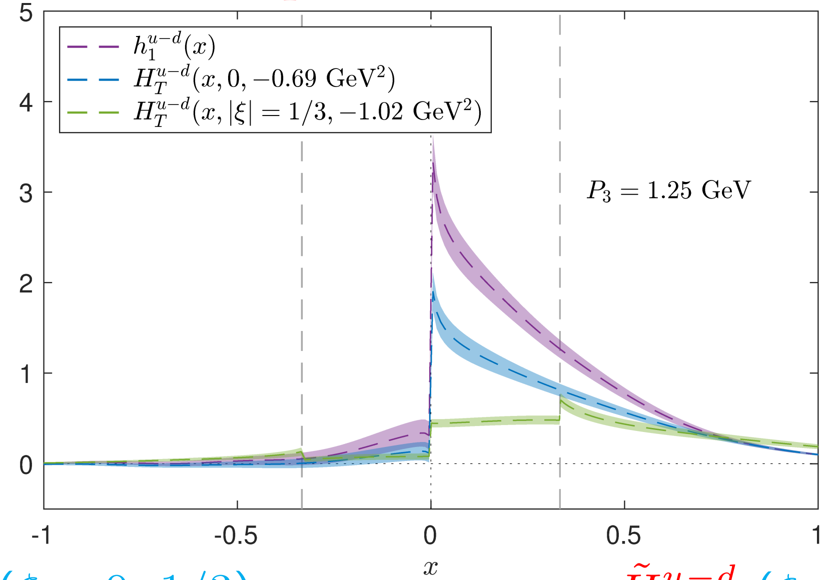
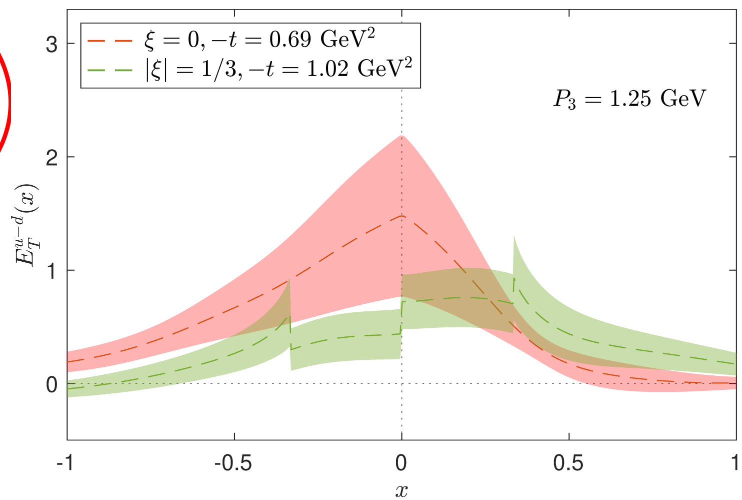
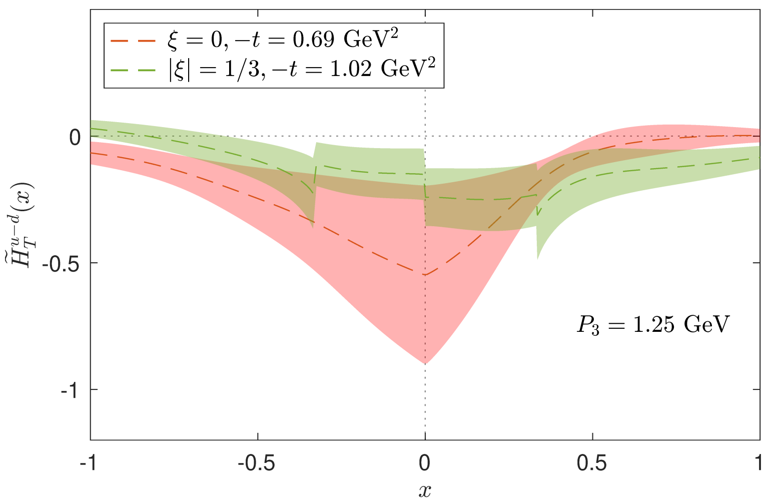


E_T^{u-d} $(\xi = 0,\ 1/3)$



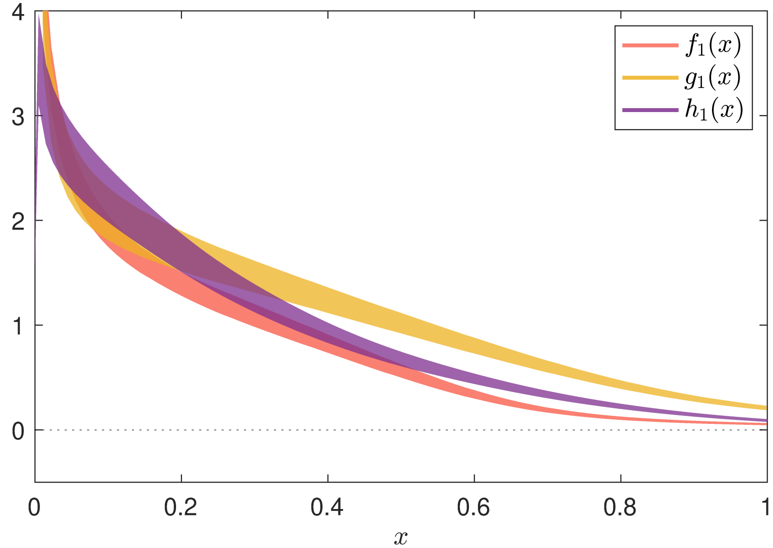
$\tilde{H}_T^{u-d}$ $(\xi = 0,\ 1/3)$



Comparison of different types of PDFs/GPDs

ETMC, Phys. Rev. Lett. 125 (2020) 262001

ETMC, Phys. Rev. D105 (2022) 034501



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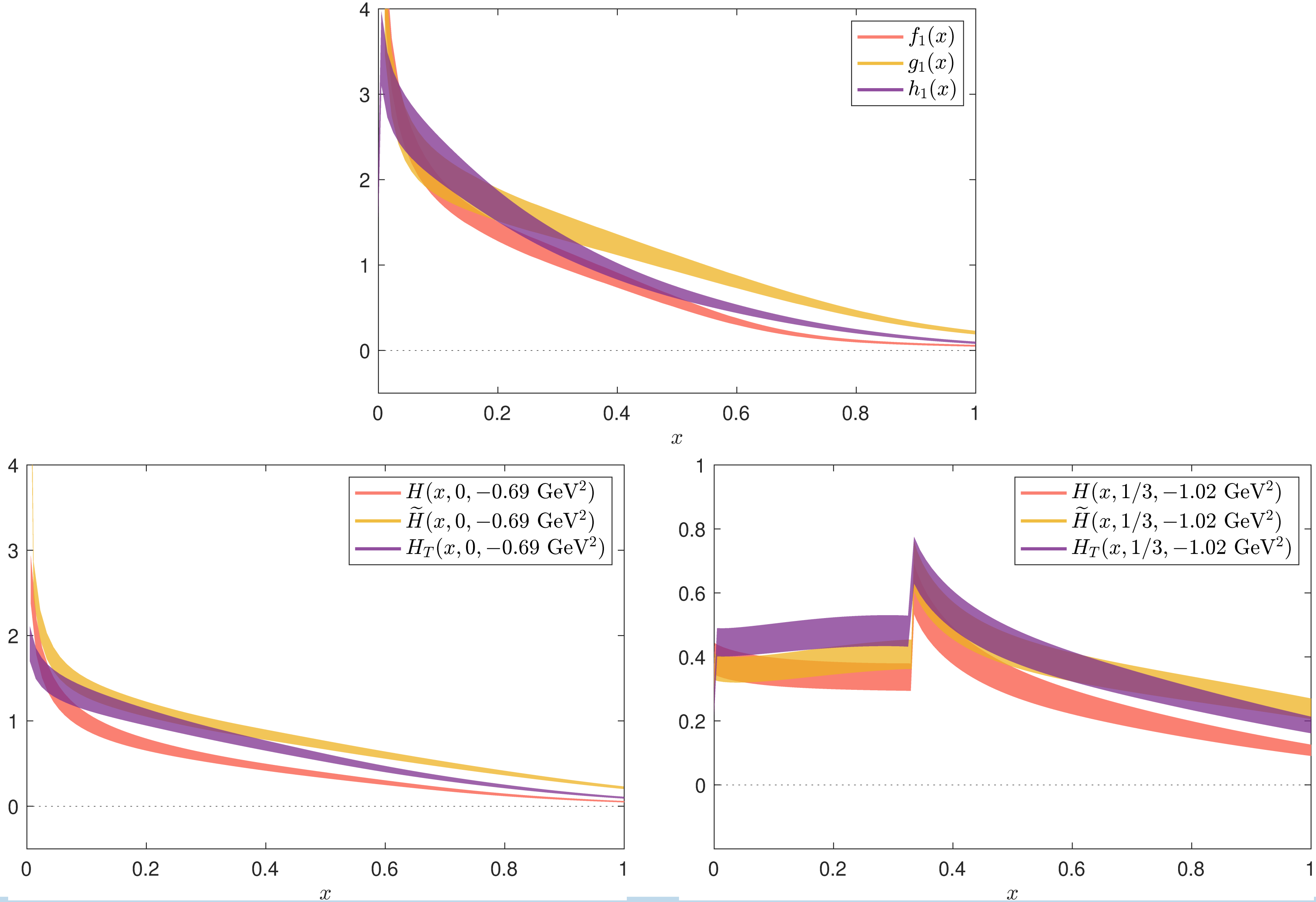
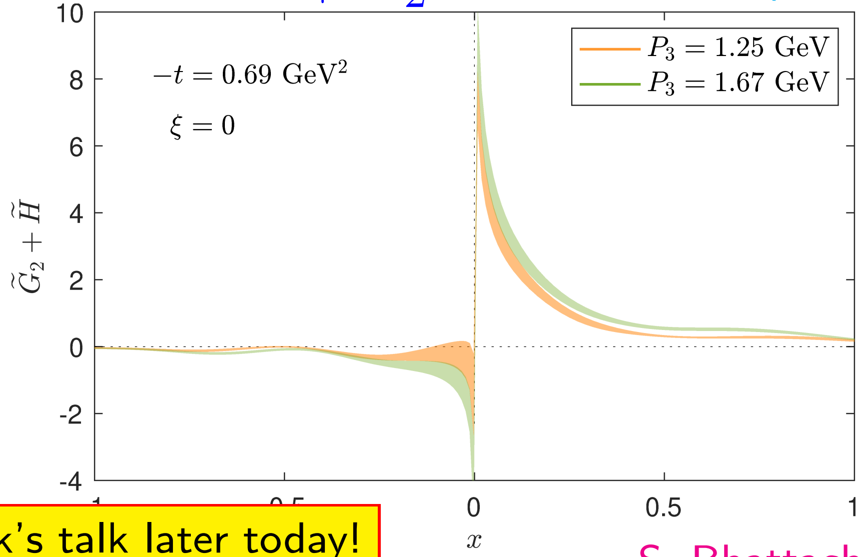
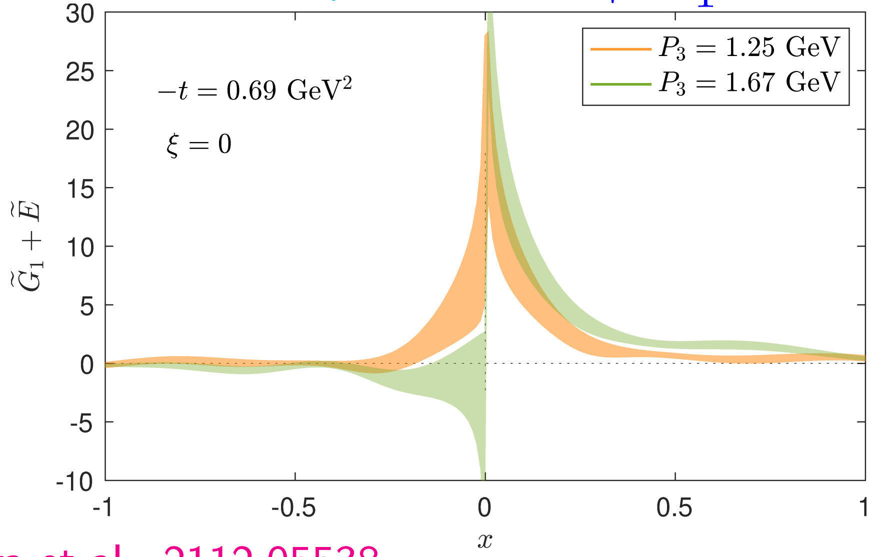


First exploration of twist-3 GPDs

$\widetilde{H}+\widetilde{G}_2$ $(\xi=0, -t=0.69$ GeV$^2)$ $\widetilde{E}+\widetilde{G}_1$

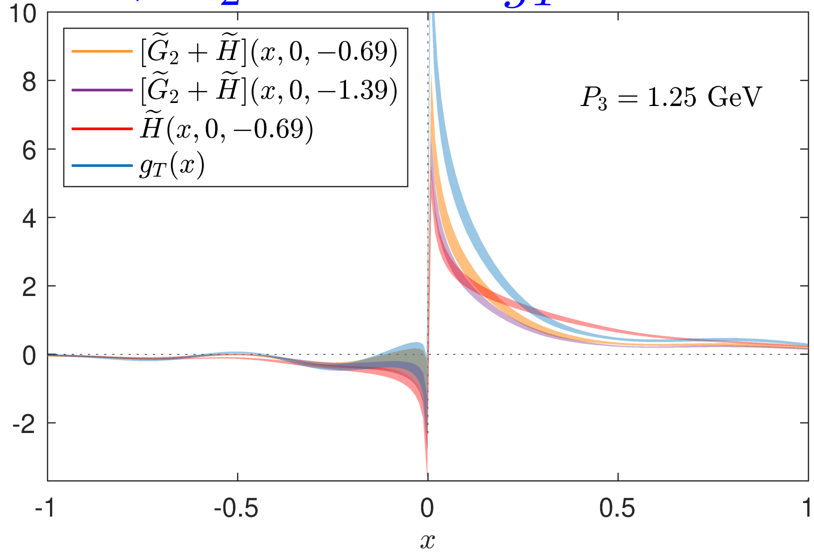


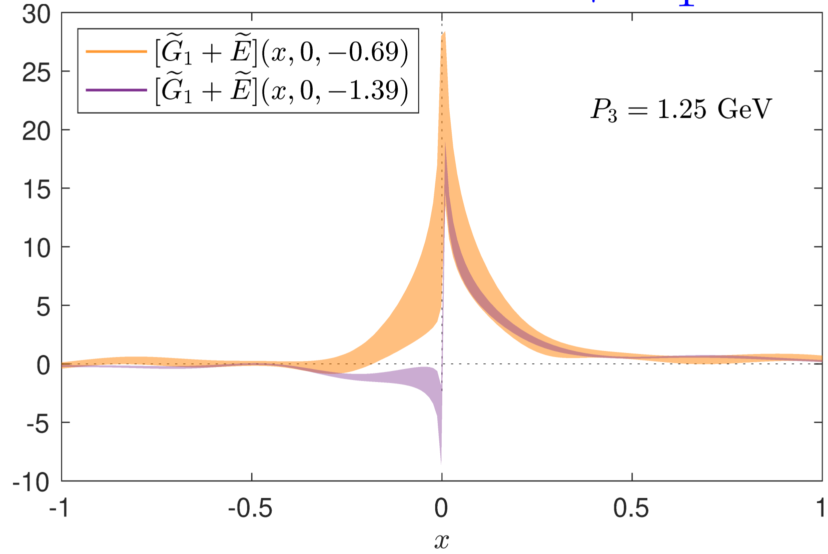


See Jack's talk later today!

S. Bhattacharya et al., 2112.05538

$\widetilde{H}+\widetilde{G}_2$ vs. $\widetilde{H}$ vs. g_T $(\xi=0, -t=0.69, 1.39$ GeV$^2)$ $\widetilde{E}+\widetilde{G}_1$





Conclusions and prospects

- **Huge progress in lattice calculations of GPDs!**
- Recent breakthrough:
  - ⋆ computationally more efficient calculations in non-symmetric frames,
  - ⋆ with, potentially, faster convergence to the light-cone (to be investigated).
- Overall very encouraging results!
- Still several challenges to overcome (control of systematics).
- Obviously, GPDs much more challenging than PDFs.
- Expect slow, but consistent progress and complementary role to pheno.

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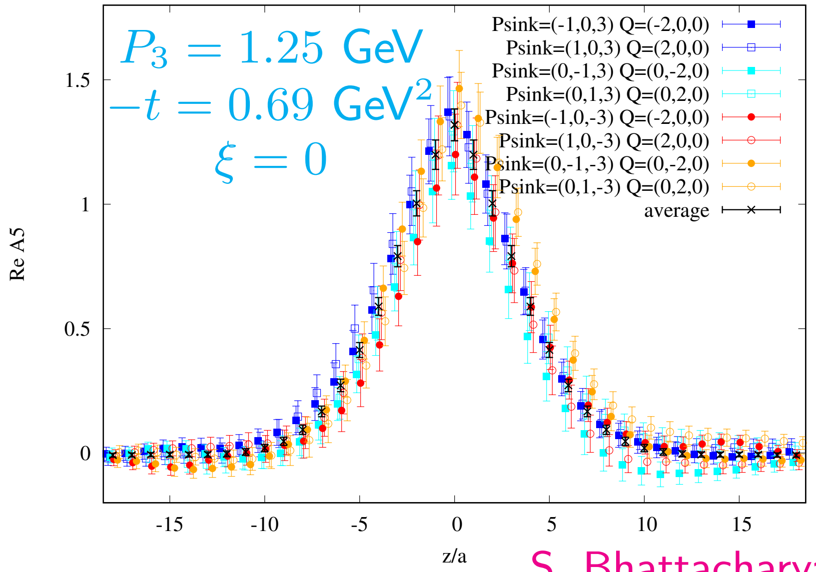
Thank you for your attention!

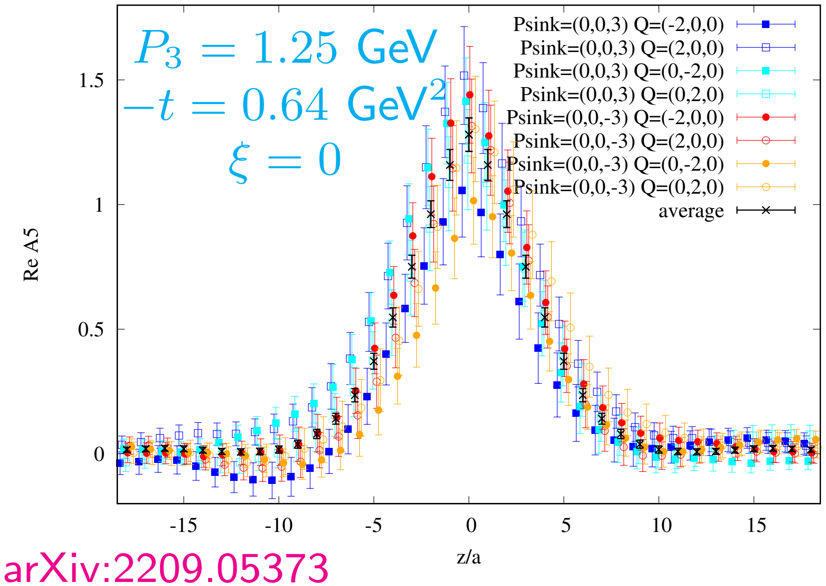
Backup slides

Example amplitude A_5

symmetric frame | non-symmetric frame

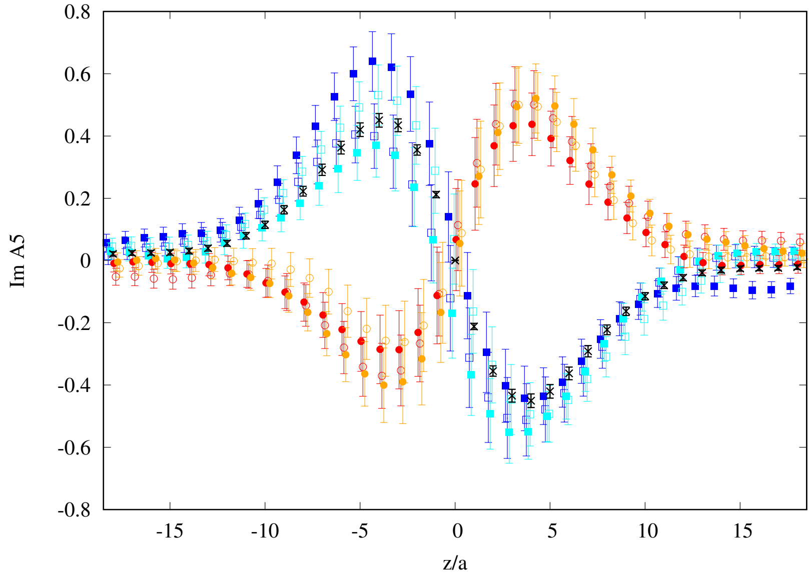
Re

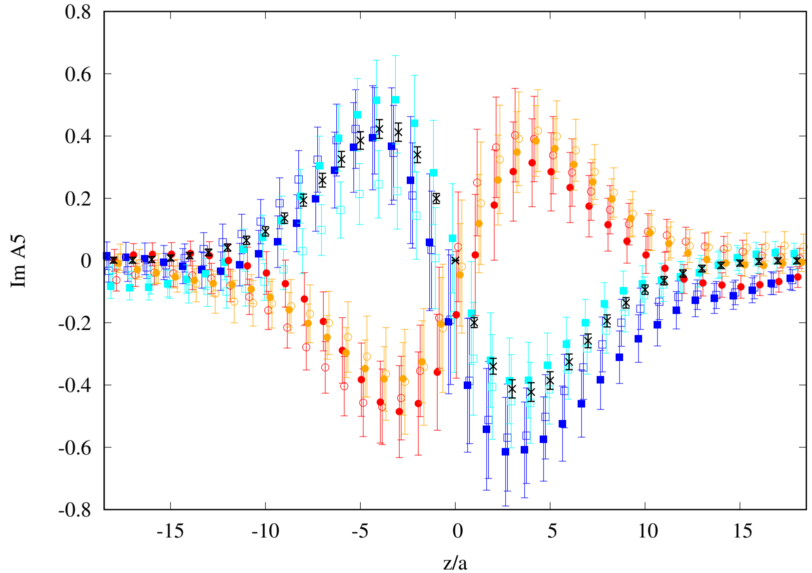




S. Bhattacharya et al., arXiv:2209.05373

Im

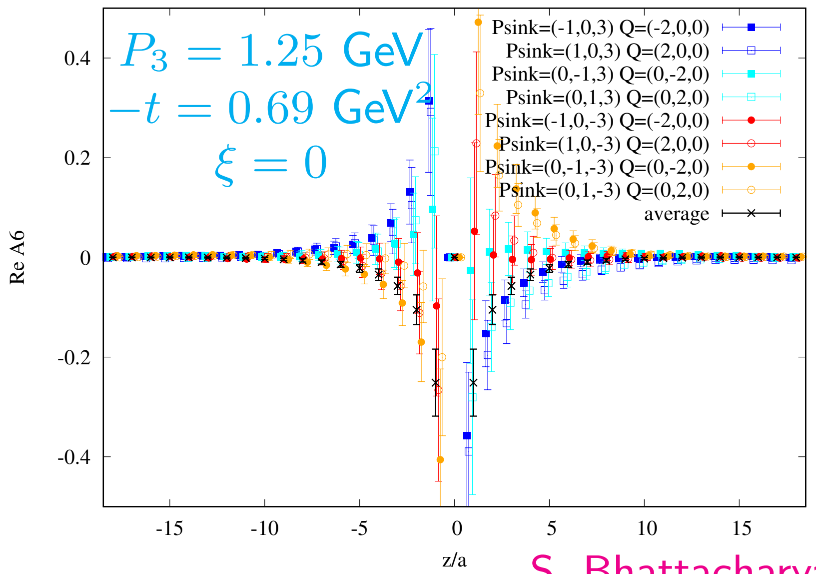


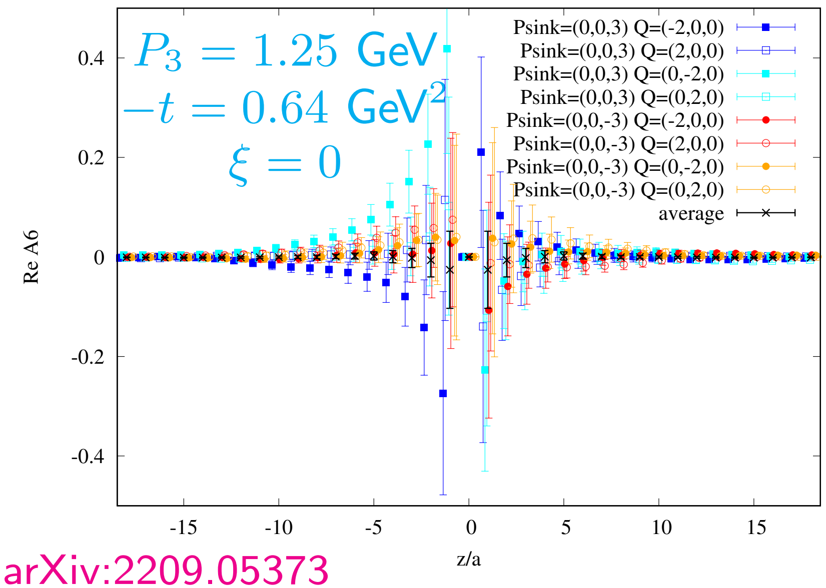


Example amplitude A_6

symmetric frame | **non-symmetric frame**

Re





S. Bhattacharya et al., arXiv:2209.05373

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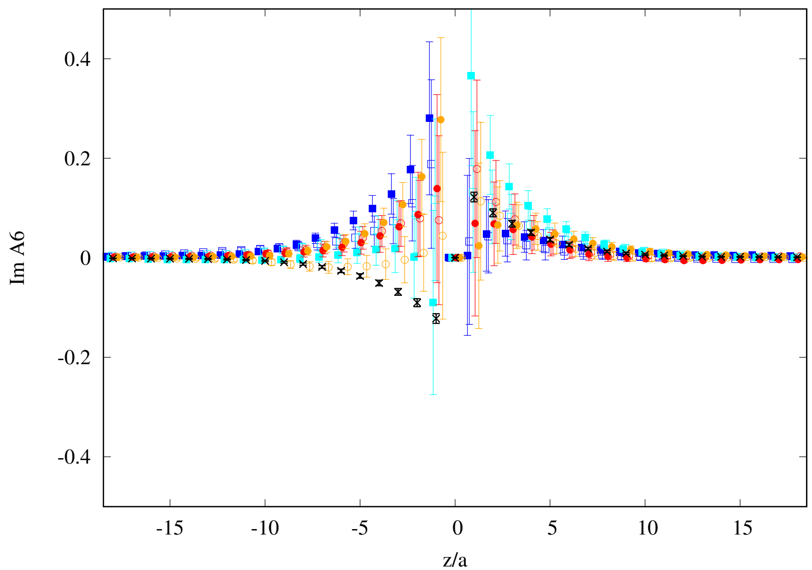

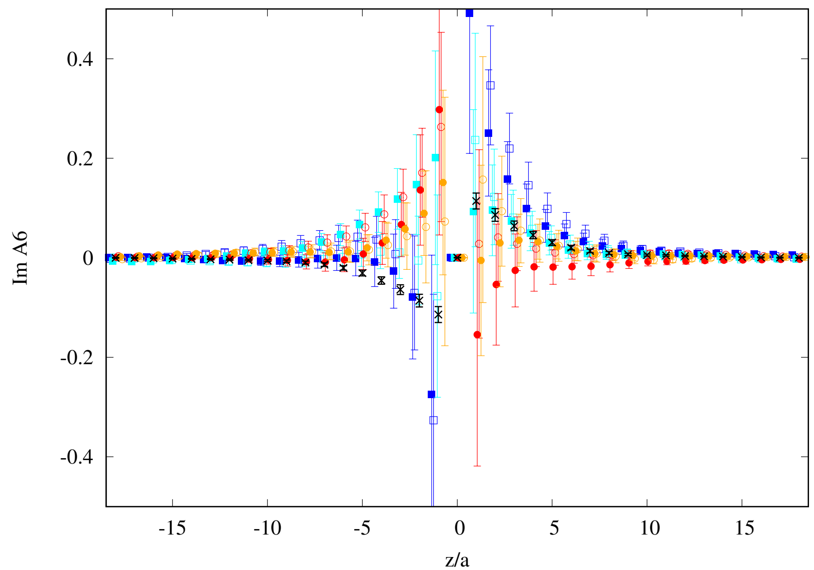

Transversity GPDs

Transversity GPDs: ETMC, Phys. Rev. D105 (2022) 034501

4 GPDs: H_T, E_T, $\tilde{H}_T$, $\tilde{E}_T$

spatial correlation in a boosted nucleon
$\langle N(\vec{P}')|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N(\vec{P})\rangle$
$\vec{P}' = \vec{P} + \vec{Q}$, $\vec{Q}$ – momentum transfer
lattice computation of bare ME

↓

renormalization
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intermediate RI scheme

↓

reconstruction of x-dependence
z-space→x-space
Backus-Gilbert

↓

matching to light cone
RI→ $\overline{\text{MS}}$
(incl. evolution to $\mu = 2$ GeV)

↓

light-cone GPD

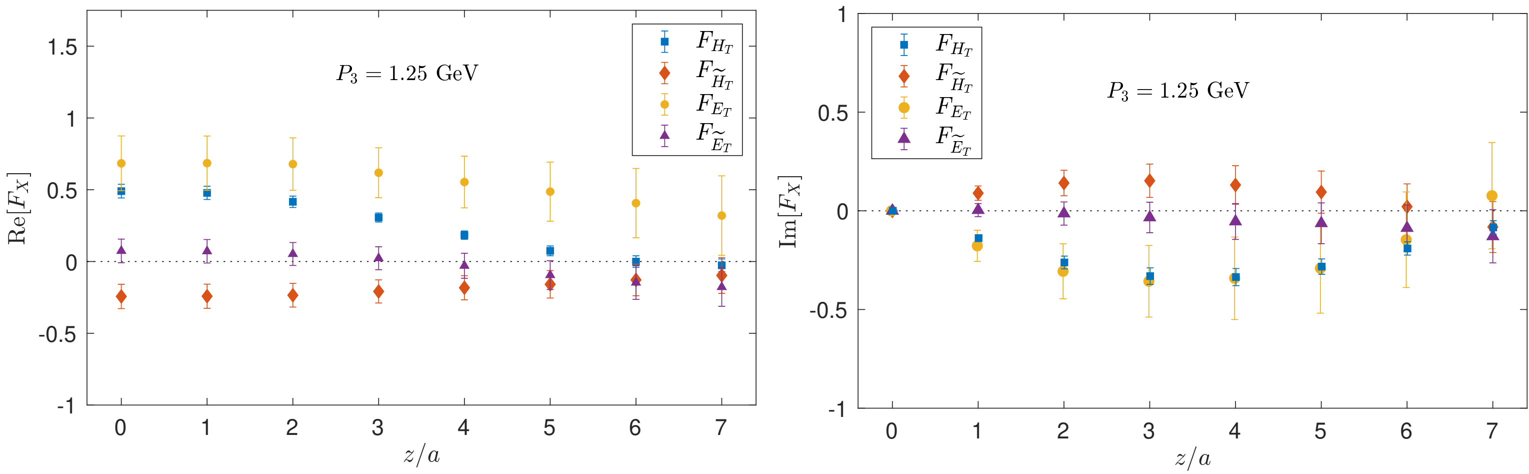
Three nucleon boosts ($\xi = 0$): $P_3 = 0.83, 1.25, 1.67$ GeV
Nucleon boost ($\xi \neq 0$): $P_3 = 1.25$ GeV

Momentum transfer ($\xi = 0$): $-t = 0.69$ GeV^2
Momentum transfer ($\xi \neq 0$): $-t = 1.02$ GeV^2

Renormalized ME

Real part — Imaginary part

$\xi = 1/3$



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ETMC, Phys. Rev. D105 (2022) 034501

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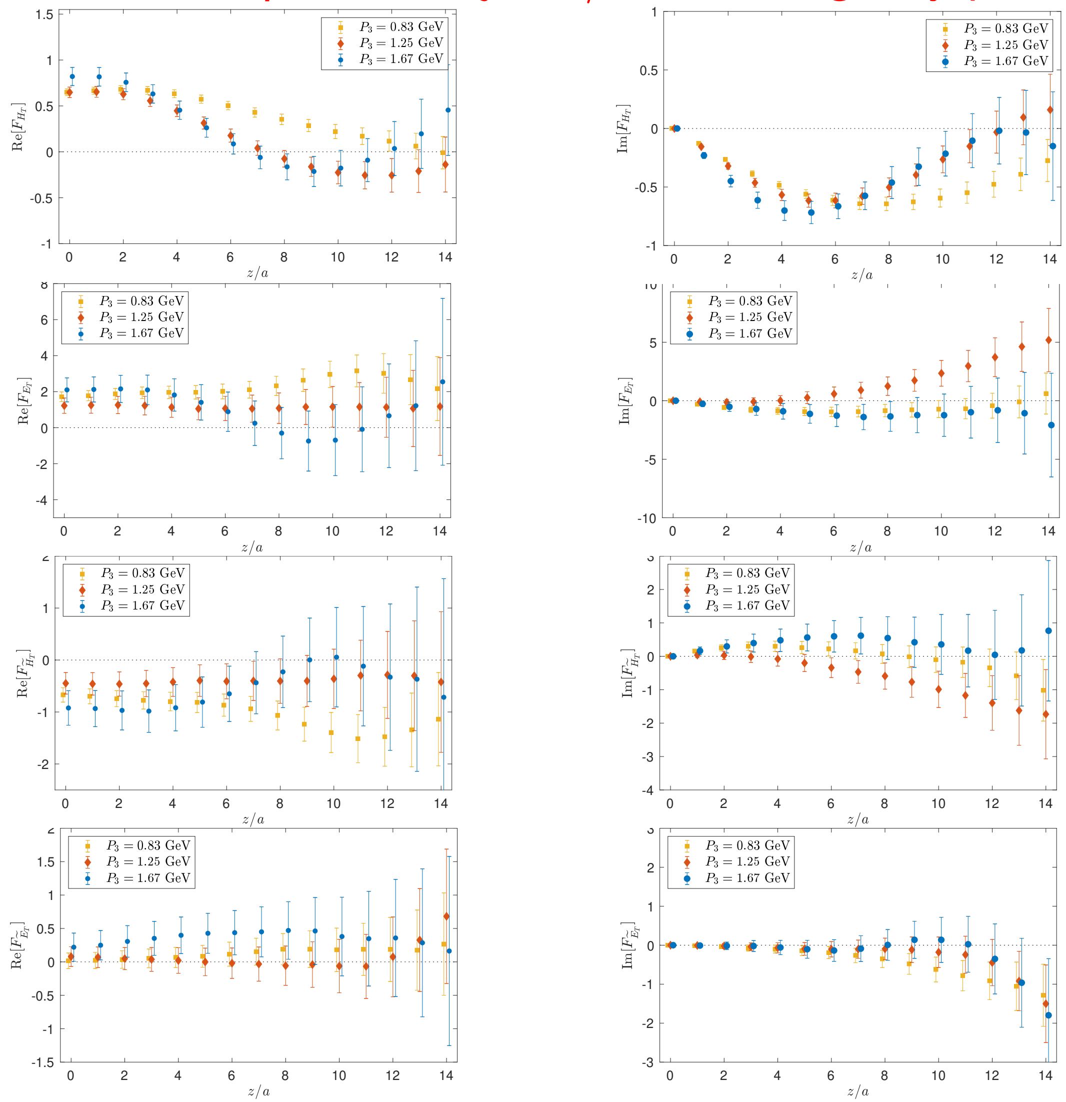
Real part $\xi = 1/3$ Imaginary part



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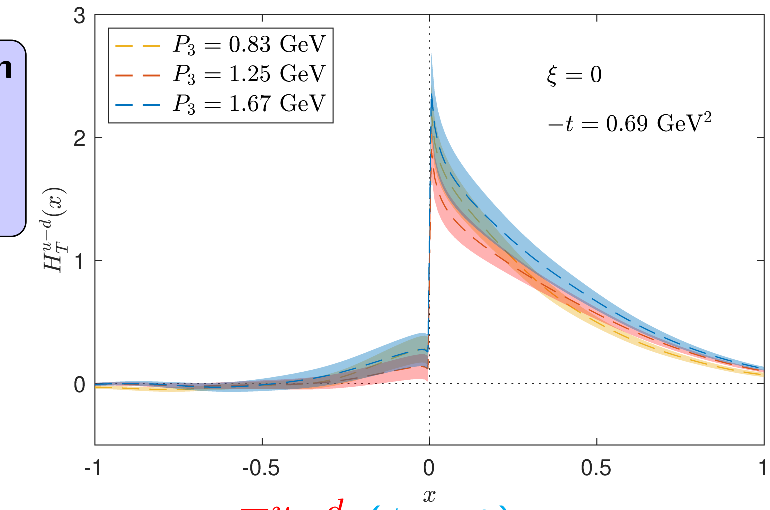
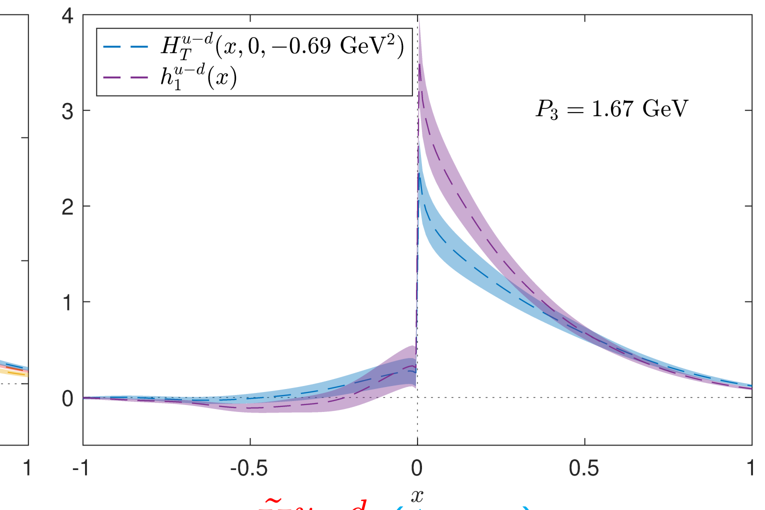
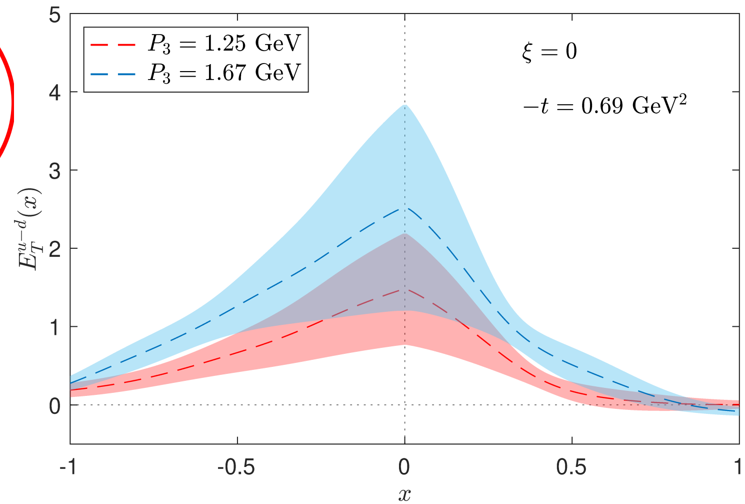
H_T^{u-d} $(\xi = 0)$

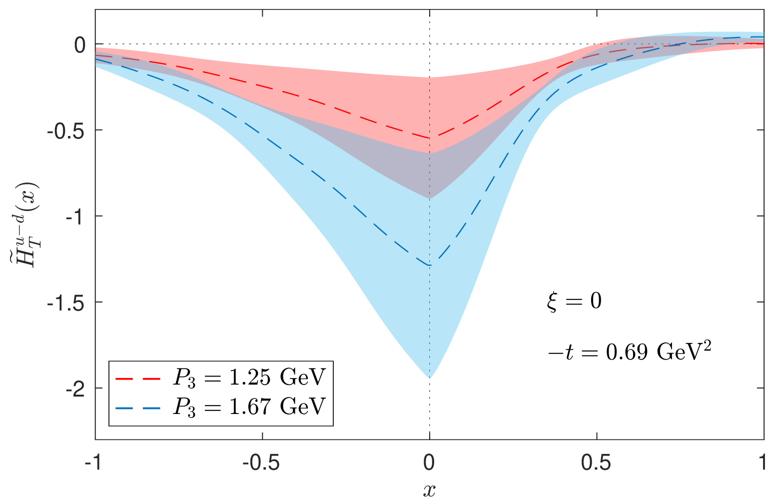




E_T^{u-d} $(\xi = 0)$



$\tilde{H}_T^{u-d}$ $(\xi = 0)$



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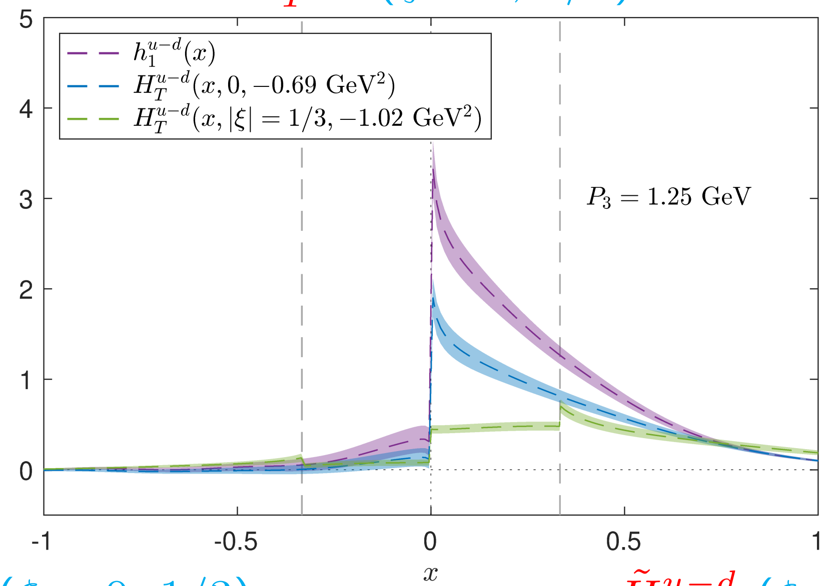
ETMC, Phys. Rev. D105 (2022) 034501

H_T^{u-d} ($\xi = 0,\ 1/3$)



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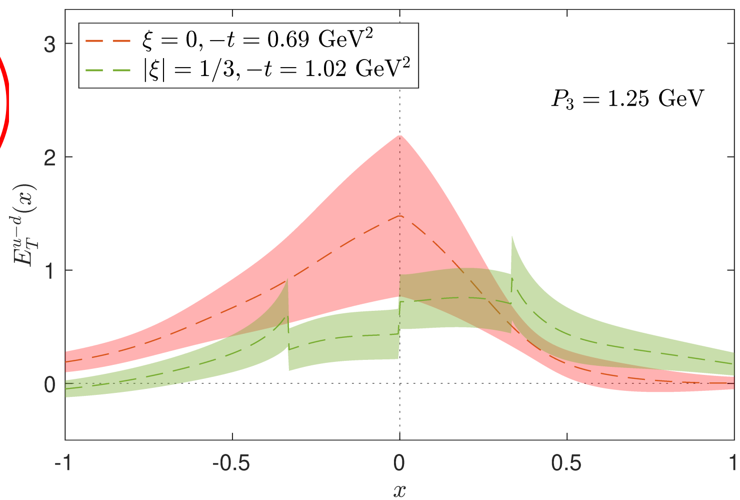

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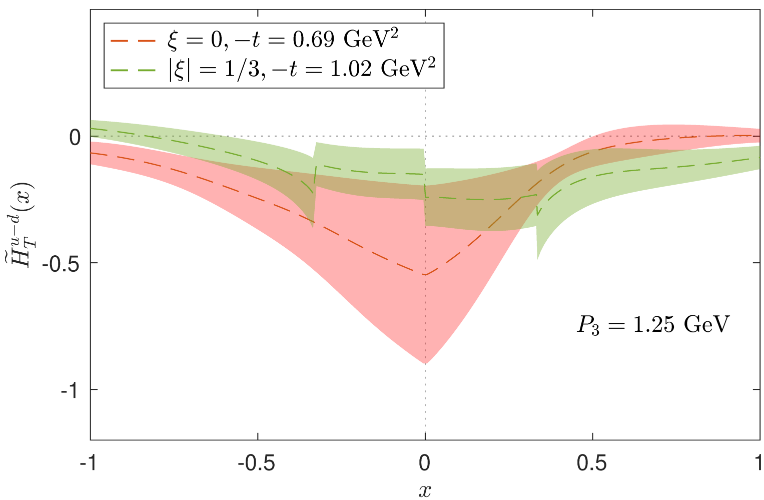

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ETMC, Phys. Rev. D105 (2022) 034501

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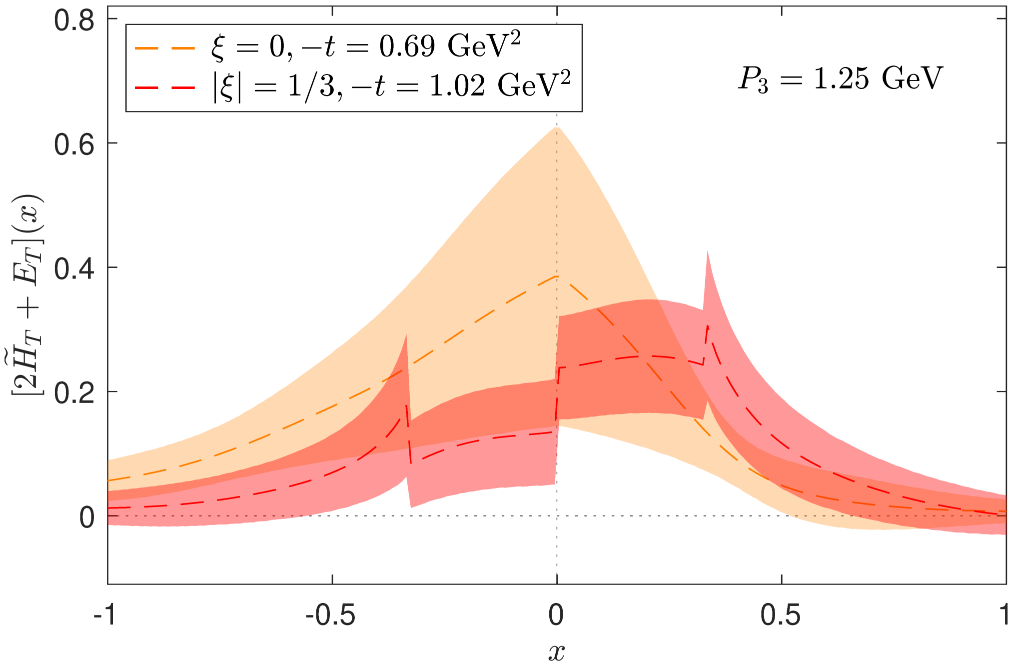
↓

light-cone GPD

More fundamental quantity: $E_T + 2\tilde{H}_T$

– related to the transverse spin structure of the proton
– physically interpreted as lateral deformation in the distribution of transversely polarized quarks in an unpolarized proton
– lowest Mellin moment in the forward limit: transverse spin-flavor dipole moment in an unpolarized target (k_T)
– second moment related to the transverse-spin quark angular momentum in an unpolarized proton



Moments of transversity GPDs

$n = 0$ Mellin moments:

$$\begin{aligned}
\int_{-1}^{1} dx\, H_T(x,\xi,t) &= \int_{-\infty}^{\infty} dx\, H_{Tq}(x,\xi,t,P_3) = A_{T10}(t)\,, \\
\int_{-1}^{1} dx\, E_T(x,\xi,t) &= \int_{-\infty}^{\infty} dx\, E_{Tq}(x,\xi,t,P_3) = B_{T10}(t)\,, \\
\int_{-1}^{1} dx\, \widetilde{H}_T(x,\xi,t) &= \int_{-\infty}^{\infty} dx\, \widetilde{H}_{Tq}(x,\xi,t,P_3) = \widetilde{A}_{T10}(t)\,, \\
\int_{-1}^{1} dx\, \widetilde{E}_T(x,\xi,t) &= \int_{-\infty}^{\infty} dx\, \widetilde{E}_{Tq}(x,\xi,t,P_3) = 0\,,
\end{aligned} \quad (1)$$

- lowest moments of GPDs skewness-independent,
- lowest moments of quasi-GPDs boost-independent.

$n = 1$ Mellin moments (related to GFF of one-derivative tensor operator):

$$\begin{aligned}
\int_{-1}^{1} dx\, x\, H_T(x,\xi,t) &= A_{T20}(t)\,, \\
\int_{-1}^{1} dx\, x\, E_T(x,\xi,t) &= B_{T20}(t)\,, \\
\int_{-1}^{1} dx\, x\, \widetilde{H}_T(x,\xi,t) &= \widetilde{A}_{T20}(t)\,, \quad (3) \\
\int_{-1}^{1} dx\, x\, \widetilde{E}_T(x,\xi,t) &= 2\xi \widetilde{B}_{T21}(t)\,, \quad (2)
\end{aligned}$$

- skewness-dependence only in for $\widetilde{E}_T$ (only ξ-odd GPD).

Moments of transversity GPDs

Moments of	$H_T(x,\xi=0,t=-0.69\,\text{GeV}^2)$			$H_T(x,\xi=1/3,t=-1.02\,\text{GeV}^2)$
	$P_3 = 0.83$ GeV	$P_3 = 1.25$ GeV	$P_3 = 1.67$ GeV	$P_3 = 1.25$ GeV
H_{Tq}	0.65(4)	0.64(6)	0.81(10)	0.49(5)
H_T	0.69(4)	0.67(6)	0.84(10)	0.45(4)
xH_T	0.20(2)	0.21(2)	0.24(3)	0.15(2)
$A_{T10}\ (z=0)$	0.65(4)	0.65(6)	0.82(10)	0.49(5)

Mellin moments P_3-independent, preserved by matching, suppressed with increasing $-t$.

Moments of	$E_T(x,\xi=0,t=-0.69\,\text{GeV}^2)$			$H_T(x,\xi=1/3,t=-1.02\,\text{GeV}^2)$
	$P_3 = 0.83$ GeV	$P_3 = 1.25$ GeV	$P_3 = 1.67$ GeV	$P_3 = 1.25$ GeV
E_{Tq}		1.20(42)	2.05(65)	0.67(19)
E_T		1.15(43)	2.10(67)	0.73(19)
xE_T		0.06(4)	0.13(5)	0.11(11)
$B_{T10}\ (z=0)$	1.71(28)	1.22(43)	2.10(67)	0.68(19)

Moments of	$\widetilde{H}_T(x,\xi=0,t=-0.69\,\text{GeV}^2)$			$\widetilde{H}_T(x,\xi=1/3,t=-1.02\,\text{GeV}^2)$
	$P_3 = 0.83$ GeV	$P_3 = 1.25$ GeV	$P_3 = 1.67$ GeV	$P_3 = 1.25$ GeV
$\widetilde{H}_{Tq}$		-0.44(20)	-0.90(32)	-0.26(9)
$\widetilde{H}_T$		-0.42(21)	-0.92(33)	-0.27(9)
$x\widetilde{H}_T$		-0.17(8)	-0.30(10)	-0.05(5)
$\widetilde{A}_{T10}\ (z=0)$	-0.67(14)	-0.45(21)	-0.92(33)	-0.24(8)

Similar conclusions (but very large errors).