Massive Continuum QFTs via Qubit Regularized Lattice Gauge Thoeries

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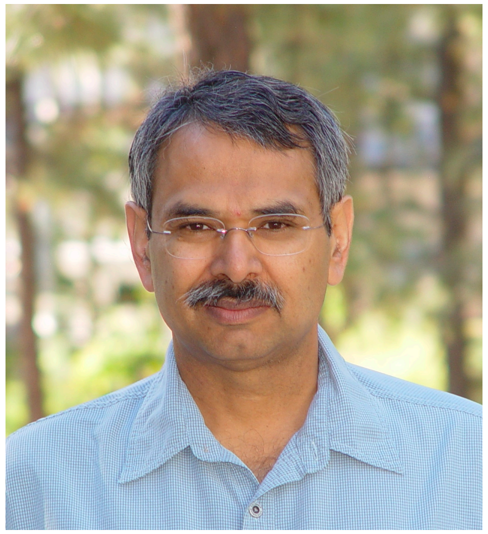
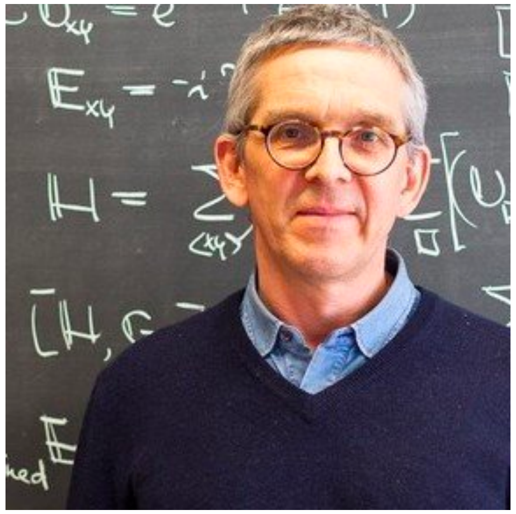

Gupta


Wiese

Outline

Massive Continuum (Euclidean) QFTs

Qubit Regularization of QFTs

Qubit Regularization of SU(N) Gauge Theories

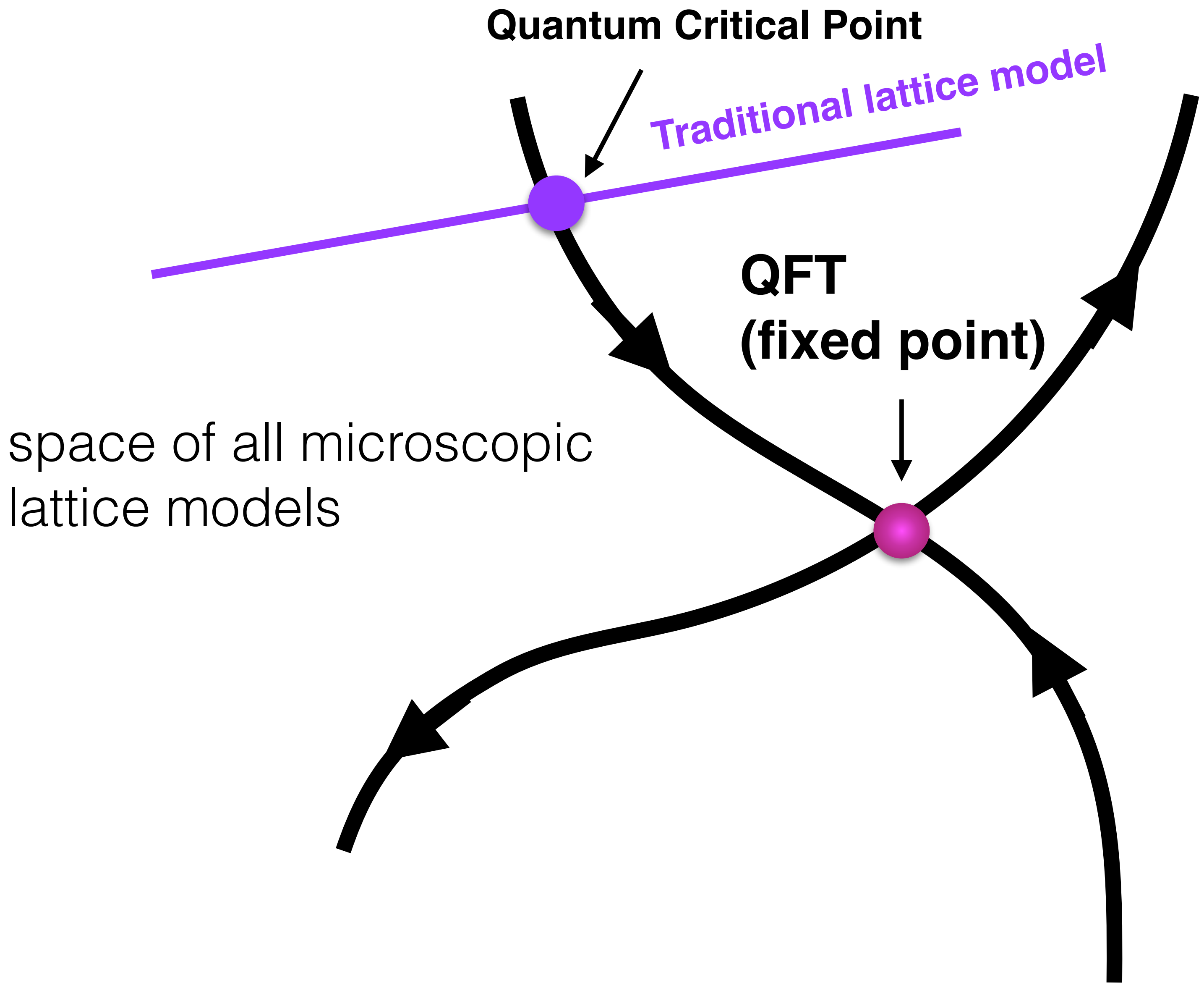
Generalized Dimer Models as Qubit Regularized Gauge Theories

Example 1: Massive continuum QFT via a classical dimer model

Example 2: Massive continuum QFT via a quantum dimer model

Conclusions

Massive Continuum (Euclidean) QFTs



Relevant coupling

$$S_{\text{rel}} = h \int d^d x \, \mathcal{O}_h$$

$$[\mathcal{O}_h] = \delta$$

$$(M_{\text{phys}})^{d-\delta} \sim h, \quad \delta < d$$

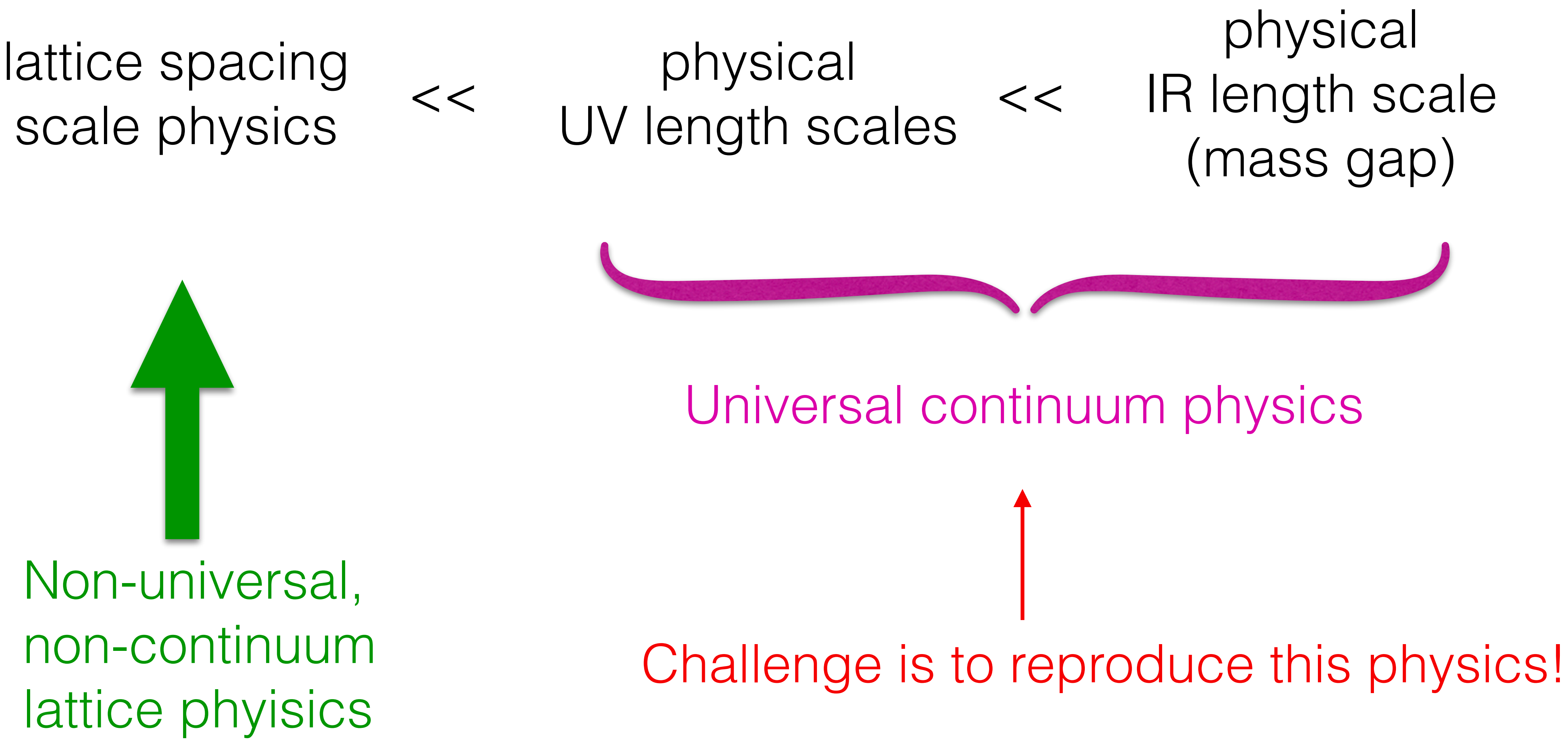
$$(-\log M_{\text{phys}})^p \sim \frac{1}{h}, \quad \delta \approx d$$

Massive QFTs emerge by tuning relevant couplings near a quantum critical point of a lattice field theory

When critical points are "free" often no fine tuning is necessary

The most interesting theories have marginally relevant couplings

Examples: 2d O(N) models, 4d Yang-Mills, …



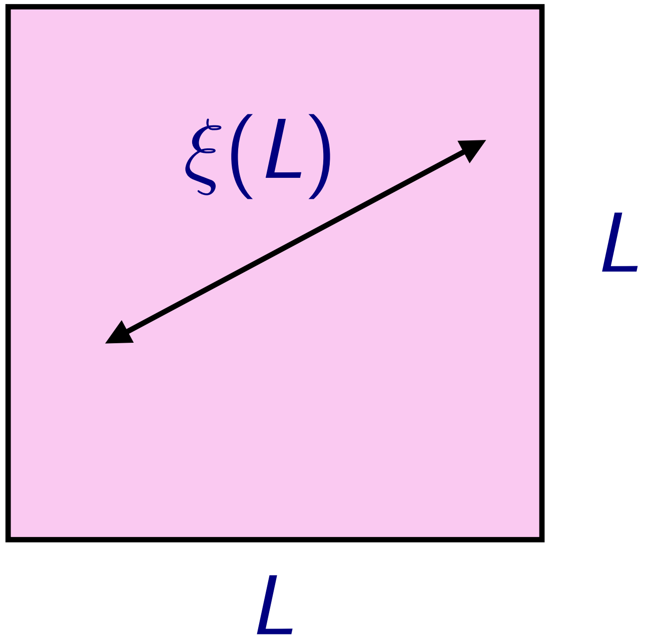
lattice spacing scale physics << physical UV length scales << physical IR length scale (mass gap)

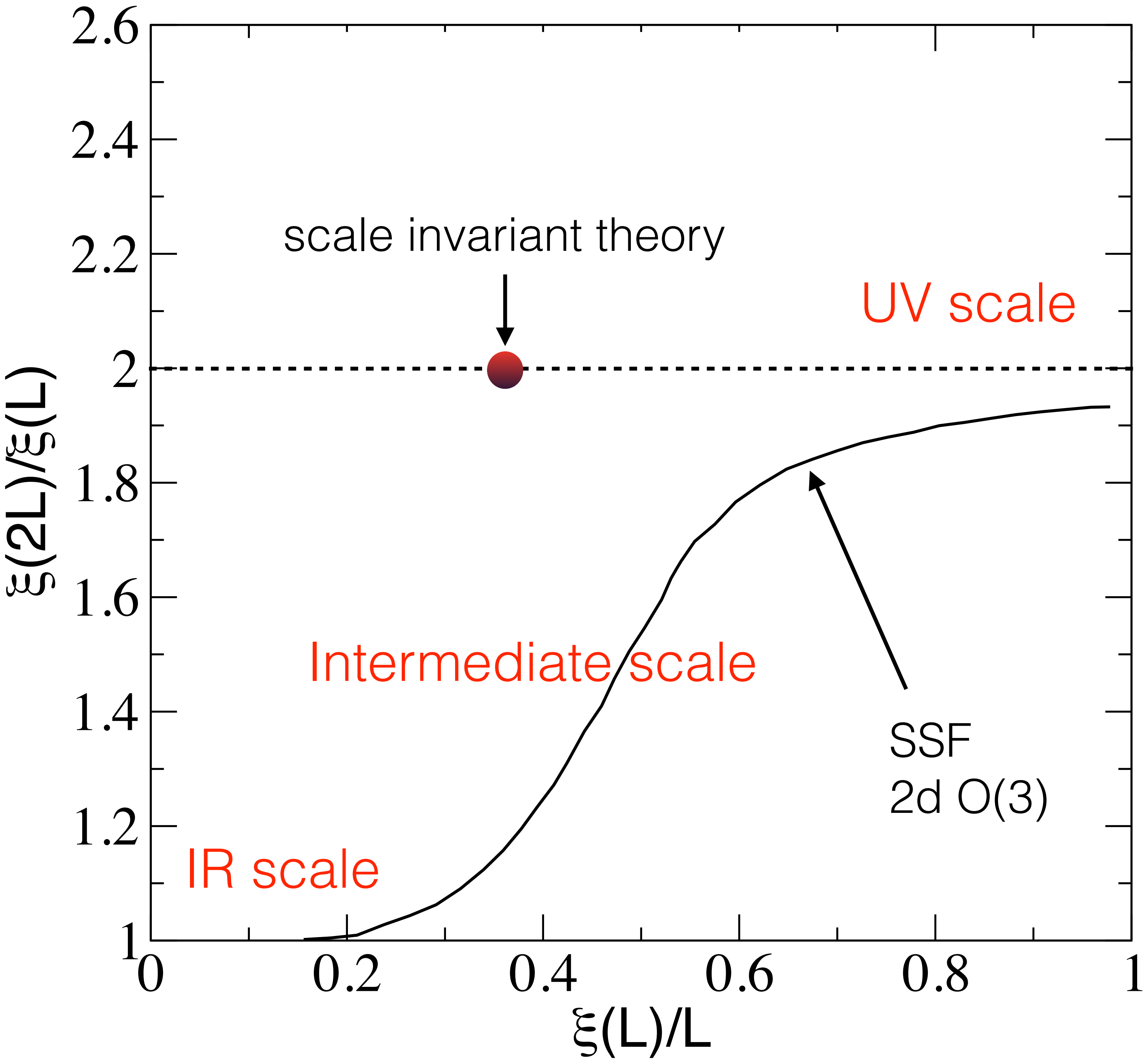
Universal continuum physics

Challenge is to reproduce this physics!

Non-universal, non-continuum lattice phyisics

Universal physics of a massive QFT can be studied via the Step Scaling Function (SSF)





Qubit Regularization of QFTs

A new type of regularization of QFTs where we begin with a Hamiltonian formulation on a lattice with a finite local Hilbert space.

local lattice Hilbert space: $\mathcal{H}_{\text{Full}} \overset{\mathbb{P}_Q}{\Longrightarrow} \mathcal{H}_Q$

On a finite lattice volume, the QFT is replaced by quantum mechanics of a system with a finite Hilbert space!

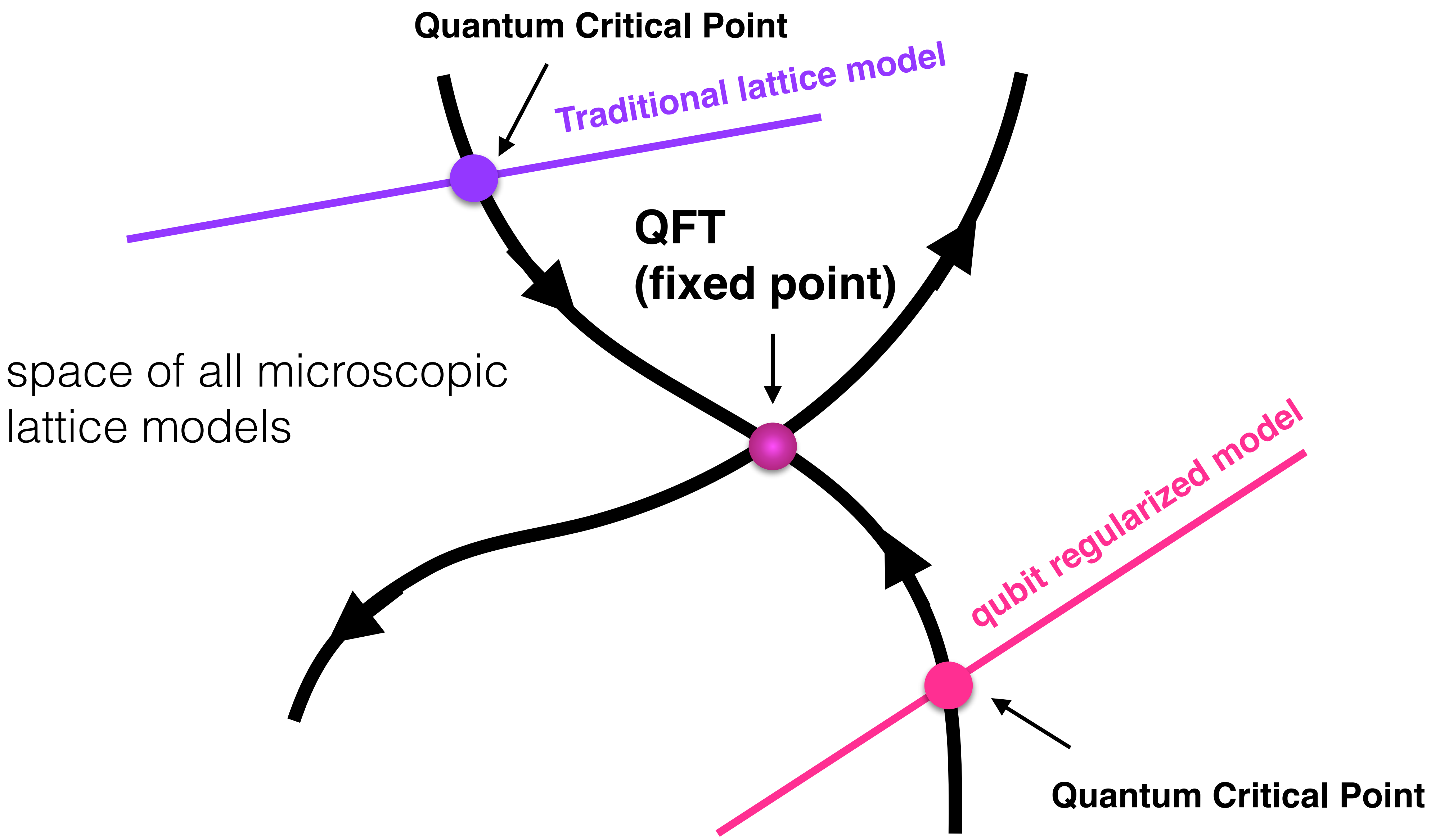
Continuum QFT emerges through a limiting process.

continuum limit $a \to 0$

Thermodynamic limit $L \to \infty$

Traditional Hilbert space limit $Q \to \infty$ ⟵ We may not need this limit!

Qubit Regularization: RG view point



Qubit Regularization unifies HEP and NP with CMP.

Design a Condensed Matter system to study QFT’s of interest

The idea of course is quite well understood in limited cases…

We know of several examples of critical phenomena in the IR can be reproduced by lattice quantum spin-systems.

Classical spin models at second order phase transitions can be recovered by quantum spin models

Assaad, Scalapino, Sandvik, …

The k=1 SU(2) WZW theory can be reproduced by a quantum spin-1/2 chain.

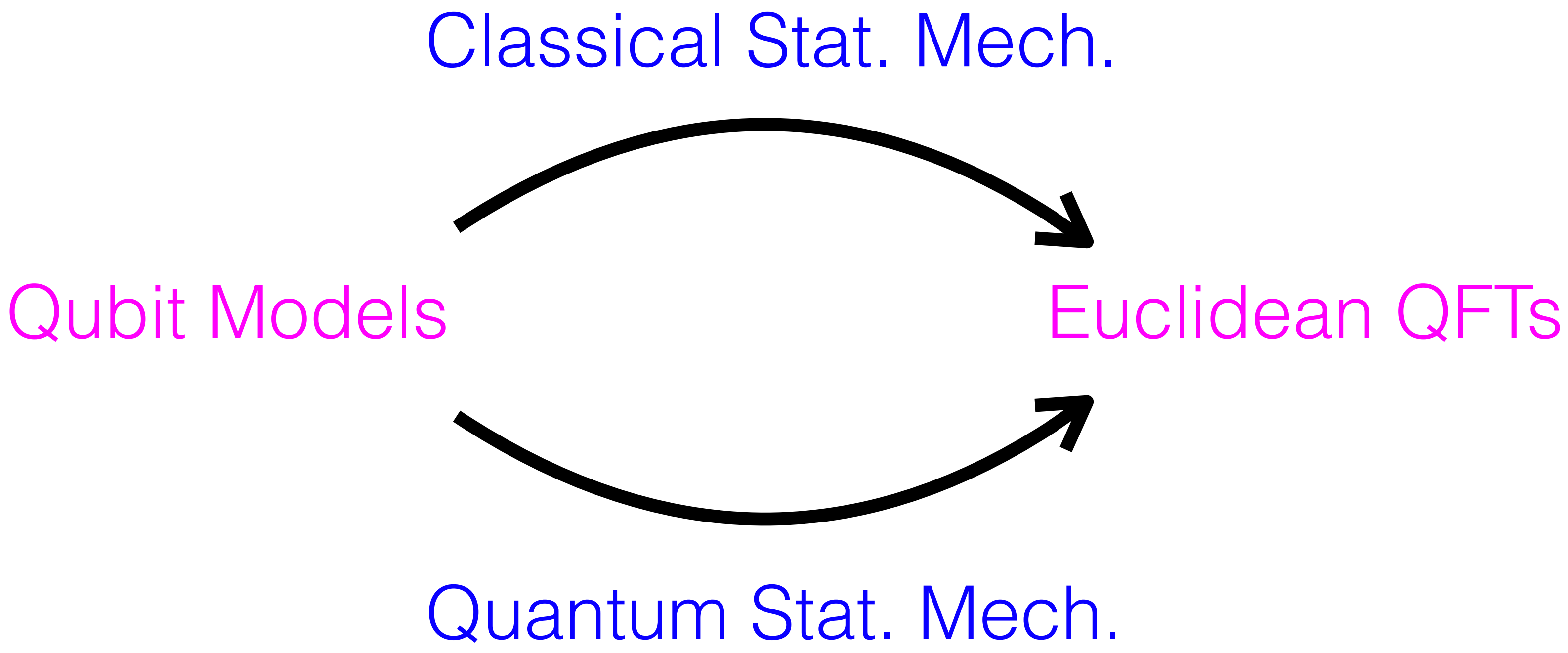
Haldane, Affleck, ….

But can the whole SSF of a massive QFT be reproduced?

What if the UV theory is Gaussian?

Qubit Model (some fixed Q) << physical UV length scales (Gaussian Theory!) << physical IR length scale (mass gap)

Simple examples of QFTs exist where this can indeed be shown!



Classical Stat. Mech.

Qubit Models

Euclidean QFTs

Quantum Stat. Mech.

Example: 2D O(3) Non-linear Sigma Model

Action:

$$S(\phi) = \frac{1}{2G} \int d\tau \; dx \; \left\{ \partial_t \vec{\phi}_x(t) \cdot \partial_t \vec{\phi}_x(t) \; + \; \partial_x \vec{\phi}_x(t) \cdot \partial_x \vec{\phi}_x(t) \right\}$$

$$\vec{\phi}_x(t) \cdot \vec{\phi}_x(t) \; = \; 1$$

Massive QFT (like Yang Mills theory or QCD)

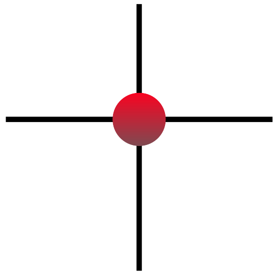
Asymptotically Free

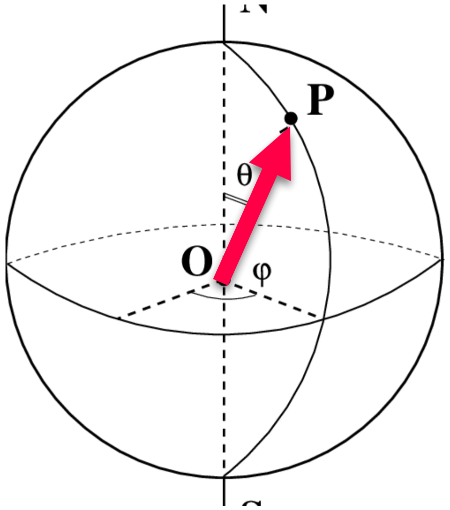
Exactly Solvable!

Studied extensively on the lattice using Lagrangian methods

A 2-qubit model can reproduce this physics!

Local site Hilbert space describes a quantum particle on a surface of a unit sphere





$$\vec{\phi}_x = (\phi^1_x, \phi^2_x, \phi^3_x)$$

$$\vec{\phi}_x \cdot \vec{\phi}_x = 1$$

Basis of the full Hilbert space $\mathcal{H}_{\mathrm{Full}}$:

$$\int d\Omega \; |\theta, \varphi\rangle\langle\theta, \varphi| \; = \; I$$

“position basis”

$$\sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} |\, \ell, m \,\rangle\langle\, \ell, m \,| \; = \; I$$

“momentum basis”

Qubit Regularization: $\mathcal{H}_{\mathrm{Full}} \overset{\mathbb{P}_Q}{\Longrightarrow} \mathcal{H}_Q$

View the local Hilbert space is a direct sum of symmetry representations:

Traditional Hilbert Space: $$\mathcal{H}_{Full} = \bigoplus_{\ell=0,1,2..} V_\ell$$

Introduce the projector $$\mathbb{P}_Q = \sum_{\ell \in Q} \sum_{m=-\ell,\ldots,\ell} |\ell, m\rangle\langle\ell, m|$$

where $Q = \{\ell_1, \ell_2, \ell_3, \ldots\}$

Qubit Regularized Hilbert Space $$\mathcal{H}_Q = \bigoplus_{\ell \in Q} V_\ell$$

Traditional quantum Fields: $\vec{\phi}_x$ (position) $\vec{L}_x$ (angular momentum)

Canonical commutation relations

$$\left.\begin{aligned}[][L^a, L^b] &= i\epsilon_{abc} L^c \\ [L^a, \phi^b] &= i\epsilon_{abc} \phi^c\end{aligned}\right\} \text{symmetry relations} \qquad \left.\begin{aligned}\vec{\phi}_x \cdot \vec{\phi}_x &= 1 \\ [\phi^a, \phi^b] &= 0\end{aligned}\right\} \text{Non-symmtery relations}$$

Qubit regularized quantum fields:

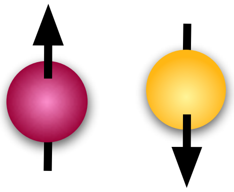
$$\begin{aligned}\vec{\phi}_Q &= \mathbb{P}_Q \, \vec{\phi} \, \mathbb{P}_Q \\ \vec{L}_Q &= \mathbb{P}_Q \, \vec{L} \, \mathbb{P}_Q\end{aligned}$$

Easy to verify:

$$\underbrace{\begin{aligned}[][\, L^a_Q,\ L^b_Q \,] &= i\epsilon_{abc}\, L^c_Q \\ [\, L^a_Q,\ \phi^b_Q \,] &= i\epsilon_{abc}\, \phi^c_Q\end{aligned}}_{\text{Symmetry relations maintained!}} \qquad \left.\begin{aligned}[][\, \phi^a_Q,\ \phi^b_Q \,] &\neq 0 \\ \vec{\phi}_Q \cdot \vec{\phi}_Q &\neq 0\end{aligned}\right\} \text{Non-symmetry relations sacrificed}$$

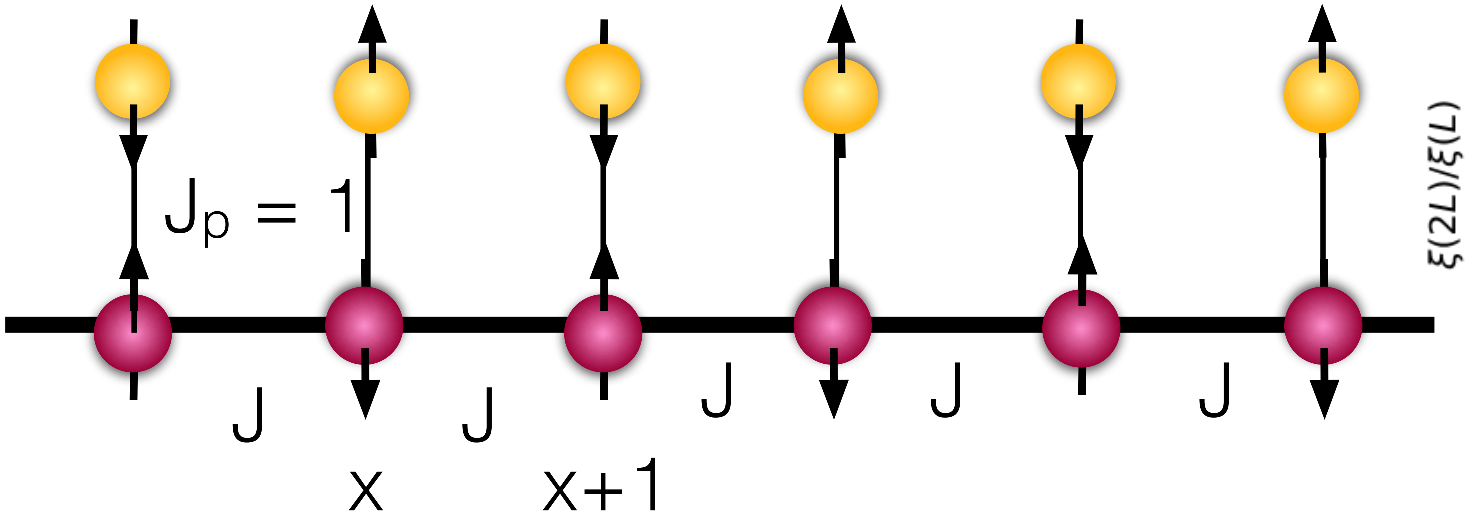
A simple qubit regularization scheme is $Q = \{0, 1\}$

($dim(\mathcal{H}_Q) = 4$) Two qubits per lattice site



Model: Heisenberg-Comb

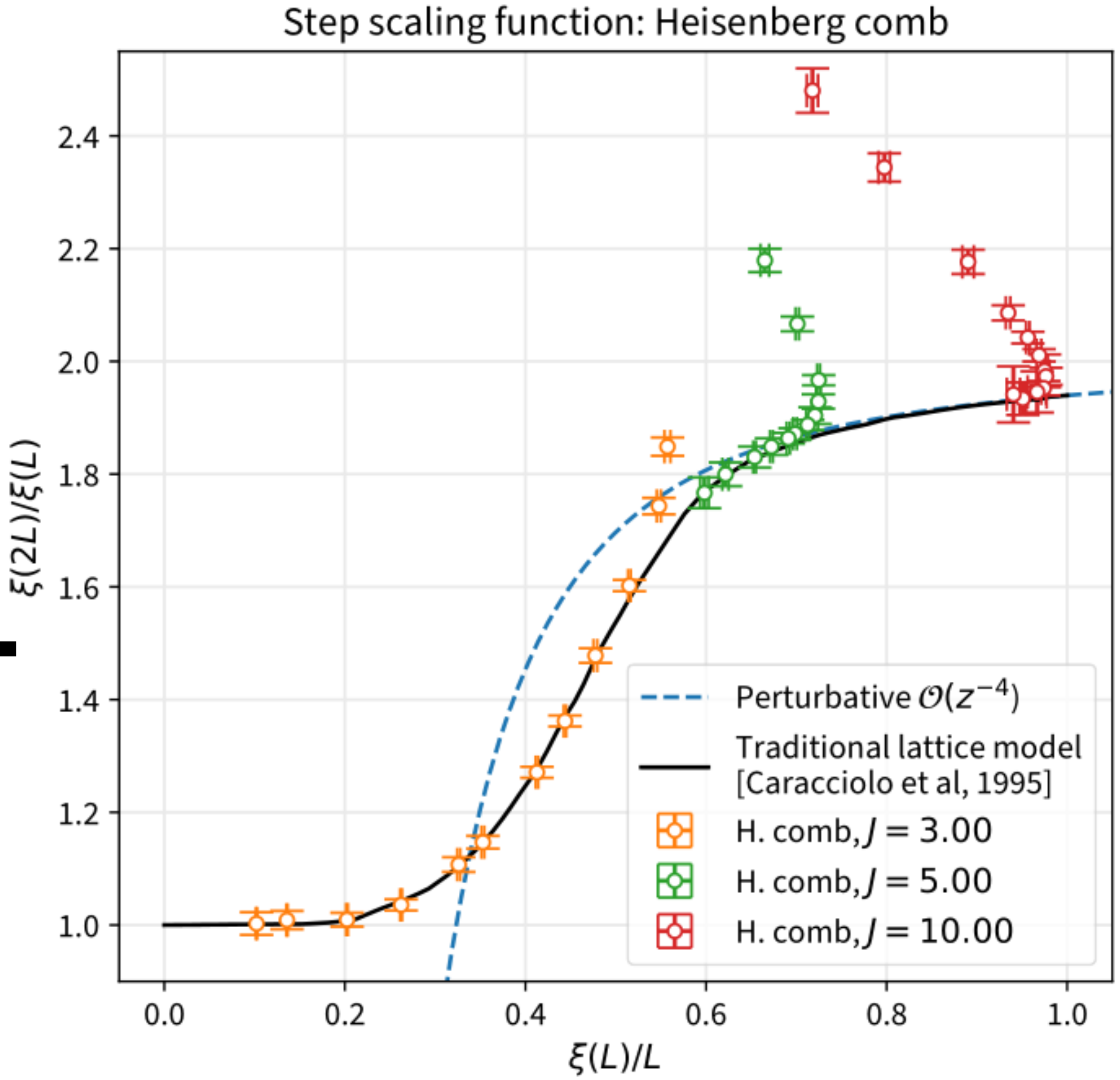
Bhattacharya, Buser, SC, Gupta, Singh
PRL (2021) 2 305



$$H = \sum_x J\, \mathbf{S}_{x,1} \cdot \mathbf{S}_{x+1,1} + \mathbf{S}_{x,1} \cdot \mathbf{S}_{x,2}$$

Quantum Critical Point:

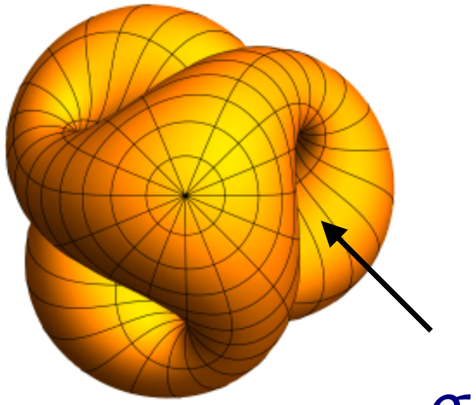
$$J \to \infty$$



Qubit Regularization of SU(N) Gauge Theories

Hanqing Liu, SC Symmetry 14 (2022) 2 305,

Local link Hilbert space describes a quantum particle on the surface of the SU(N) group manifold



$g \in SU(N)$

Basis of the full Hilbert space $\mathcal{H}_{\text{Full}}$:

$$\{|D^{\lambda}_{ij}\rangle,\ \ i,j = 1, 2.., d_{\lambda}\}$$

$$|g\rangle,\ \ \langle g|g'\rangle \ = \ \delta(g - g')$$

$$\langle D^{\lambda}_{ij}|D^{\lambda'}_{kl}\rangle \ = \ \delta_{\lambda\lambda'}\delta_{ik}\delta_{jl}$$

$$\int [dg]\ |g\rangle\langle g| = I$$

$$\sum_{\lambda}\sum_{i,j} |D^{\lambda}_{ij}\rangle\langle D^{\lambda}_{ij}| \ = \ \mathbb{I}$$

"position basis"

λ labels distinct irreps of SU(N)

"momentum basis"

Relating the momentum basis to the position basis

$$|D^{\lambda}_{ij}\rangle \;=\; \int [dg]\,\sqrt{d_\lambda}\; \underset{\substack{\uparrow \\ \text{SU(N) irrep } \lambda}}{D^{\lambda}_{ij}}(g)\;|g\rangle$$

Gauge transformations are implemented by "left" and "right" translations

$$\left.\begin{array}{ll} \text{Left group generators} & L^a \\[2ex] \text{Right group generators} & R^a \end{array}\right\} \quad a \;=\; 1, 2, .., N^2 - 1$$

$$e^{-i\vec{\alpha}\cdot\vec{L}}\;|g\rangle \;=\; |h(\vec{\alpha})\;g\rangle \qquad e^{-i\vec{\alpha}\cdot\vec{R}}\;|g\rangle \;=\; |g\;h^{-1}(\vec{\alpha})\rangle$$

$$e^{-i\vec{\alpha}\cdot\vec{L}}\;|D^{\lambda}_{ij}\rangle \;=\; D^{\lambda}_{ik}(h^{-1}(\vec{\alpha}))\;|D^{\lambda}_{kj}\rangle \quad e^{-i\vec{\alpha}\cdot\vec{R}}\;|D^{\lambda}_{ij}\rangle \;=\; |D^{\lambda}_{ik}\rangle\;D^{\lambda}_{kj}(h(\vec{\alpha}))$$

Each irrep is an invariant subspace under gauge transformations

The subspace of each irrep is given by

$$\{|D^{\lambda}_{ij}\rangle, \;\; i,j = 1,2,..,d_{\lambda}\} \;\; \in \mathcal{H}_{\lambda} \qquad dim(\mathcal{H}_{\lambda}) = (d_{\lambda})^2$$

Inside each irrep, the left and right gauge transformations act on independent spaces of dual irreps, we have

$$\mathcal{H}_{\lambda} = V_{\lambda} \otimes V^{*}_{\lambda}$$

This means the full link Hilbert space is give by

$$\mathcal{H}_{Full} \;\; = \underbrace{\bigoplus_{\lambda}}_{\text{distinct irreps of SU(N)}} \;\; V_{\lambda} \otimes V^{*}_{\lambda} \;\; \longleftarrow \;\; \text{Peter-Weyl Theorem}$$

Qubit Regularization: $\mathcal{H}_{\text{Full}} \overset{\mathbb{P}_Q}{\Longrightarrow} \mathcal{H}_Q$

A projector that preserves the gauge symmetry algebra

$$\mathbb{P}_Q = \sum_{\lambda \in Q} \sum_{i,j} |D^{\lambda}_{ij}\rangle\langle D^{\lambda}_{ij}| \qquad Q = \{\lambda_1, \lambda_2, \ldots\}$$

Qubit Regularized Hilbert Space $\mathcal{H}_Q = \bigoplus_{\lambda \in Q} V_\lambda \otimes V^*_\lambda$

$$dim(\mathcal{H}_Q) = \sum_{\lambda \in Q} (d_\lambda)^2$$

Qubit models depend on the Hilbert space $\mathcal{H}_Q$ and the Hamiltonian.

Quantum Fields on $\mathcal{H}_{\text{Full}}$

$$L^a, R^a, a = 1, 2, .., N^2 - 1 \qquad \text{(momentum fields)}$$

$$\mathcal{U}^{\lambda}_{ij},\ (\mathcal{U}^{\lambda}_{ij})^{\dagger}, i, j = 1, 2, ..., d_{\lambda} \qquad \text{(position fields)}$$

$$\mathcal{U}^{\lambda}_{ij}|g\rangle \ = \ D^{\lambda}_{ij}(g)\ |g\rangle \qquad\qquad (\mathcal{U}^{\lambda}_{ij})^{\dagger}|g\rangle \ = \ [D^{\lambda}_{ij}(g)]^{*}\ |g\rangle$$

matrix representation of the irrep

Action of the link operator mixes irreps

$$\mathcal{U}^{\lambda'}_{ij}\ |D^{\lambda}_{kl}\rangle \ = \ \int [dg]\ \sqrt{d_{\lambda}}\ \underbrace{D^{\lambda}_{kl}(g)\ D^{\lambda}_{ij}(g)}_{\lambda \otimes \lambda'}\ |g\rangle$$

Quantum Fields on $\mathcal{H}_{\mathrm{Q}}$

$$L^a_Q \; = \; \mathbb{P}_Q \, L^a \, \mathbb{P}_Q \qquad \mathcal{U}^{Q,\lambda}_{ij} \; = \; \mathbb{P}_Q \, \mathcal{U}^{\lambda}_{ij} \, \mathbb{P}_Q$$

$$R^a_Q \; = \; \mathbb{P}_Q \, R^a \, \mathbb{P}_Q \qquad (\mathcal{U}^{Q,\lambda}_{ij})^\dagger \; = \; \mathbb{P}_Q \, (\mathcal{U}^{\lambda}_{ij})^\dagger \, \mathbb{P}_Q$$

Gauge Symmetry Algebra

$$[L^a, \; L^b] \; = \; i f^{abc} \, L^c, \quad [R^a, \; R^b] \; = \; i f^{abc} \, R^c, \qquad [L^a, \; R^b] \; = \; 0$$

$$[L^a, \mathcal{U}^{\lambda}_{ij}] \; = \; (T^a_\lambda)_{ik} \, \mathcal{U}^{\lambda}_{kj} \, , \quad [R^a, \; \mathcal{U}^{\lambda}_{ij}] \; = \; - \, \mathcal{U}^{\lambda}_{ik} \, (T^a_\lambda)_{kj}$$

Non-symmetry relations in $\mathcal{H}_{\mathrm{Full}}$

$$[\, \mathcal{U}^{\lambda}_{ij} \; , \; \mathcal{U}^{\lambda}_{kl}] \; = \; 0 \qquad [\, \mathcal{U}^{\lambda}_{ij} \; , \; (\mathcal{U}^{\lambda}_{kl})^\dagger] \; = \; 0 \qquad \mathcal{U}^{\lambda}_{ik} \, (\mathcal{U}^{\lambda}_{kj})^\dagger \; = \; \delta_{ij} \mathbb{I}$$

Qubit gauge fields also satisfy the gauge symmetry algebra
but violate the non symmetry relations!

A simple qubit regularization involves

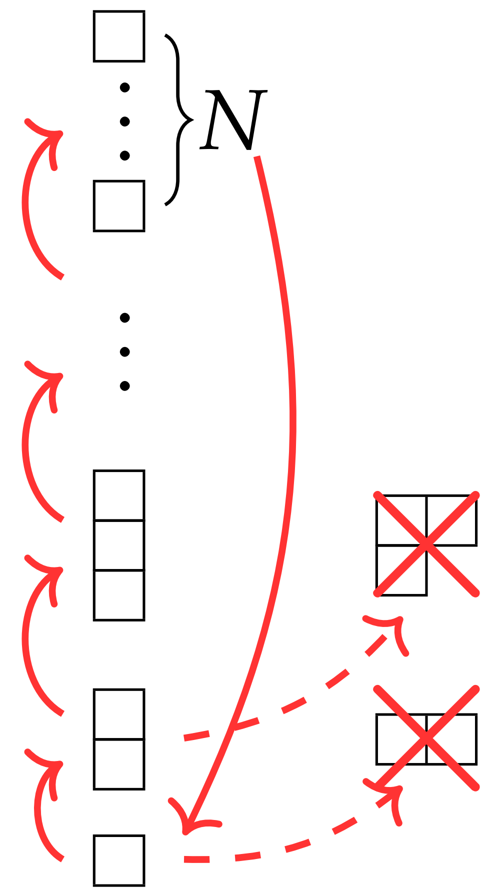
$$Q = \{\underbrace{1, \square, \begin{smallmatrix}\square\\ \square\end{smallmatrix}}_{\text{All anti-symmetric irreps}}, \ldots \}$$

Hanqing Liu, SC Symmetry 14 (2022) 2 305,

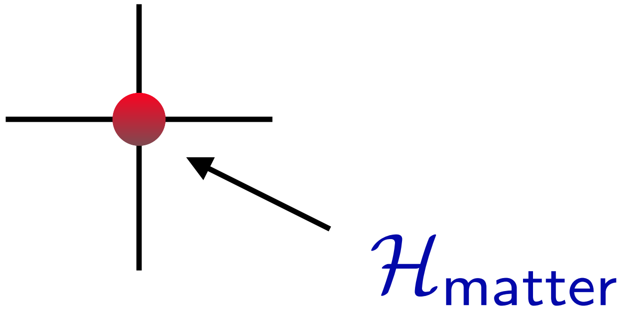
Action of the fundamental link operator is easy

$$\mathcal{U}^f_{ij}\ |D^\lambda_{kl}\rangle \ = \ \int [dg]\ \sqrt{d_\lambda}\ \underbrace{D^\lambda_{kl}(g)\ D^f_{ij}(g)}_{\lambda \otimes f}\ |g\rangle$$

$\mathcal{U}^f$ has the desired cyclic property



Matter can be added through a local Hilbert space on sites.



$\mathcal{H}_{\text{matter}}$ carries a representation of SU(N) that is being gauged.

An example of $\mathcal{H}_{\text{matter}}$ is the N colored of fermionic Fock space

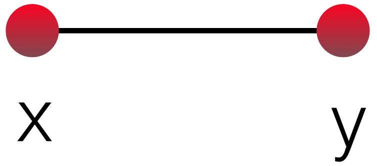
Occupation basis:

$$|n_1, n_2, ..., n_N\rangle = (c_1^\dagger)^{n_1} ... (c_N^\dagger)^{n_N} |0\rangle$$

$$\text{dim}(\mathcal{H}_{\text{matter}}) = 2^N$$

Gauge invariant operators that couple fermions with gauge fields are also easy to construct

$$c_{i,x}^\dagger \; (\mathcal{U}_{ij,xy}^f)^\dagger \; c_{j,y} \qquad\qquad c_{i,y}^\dagger \; (\mathcal{U}_{ij,xy}^f) \; c_{j,x}$$



Generalized Dimer Models

The Hilbert space of a gauge theory is only completely defined after a Gauss law charge is fixed on each lattice site.

For SU(N) gauge theories the Gauss law sectors are defined by fixing local SU(N) irreducible representations, which often turn out to be the trivial representation.

For U(1) and Z2 gauge theories, other Gauss law sectors can also be interesting.

Fixing the Gauss-law forces in the “momentum” basis leads to generalized dimer models.

Gauge Theories $\xrightarrow{\text{“momentum basis”}}$ Generalized Dimer Models

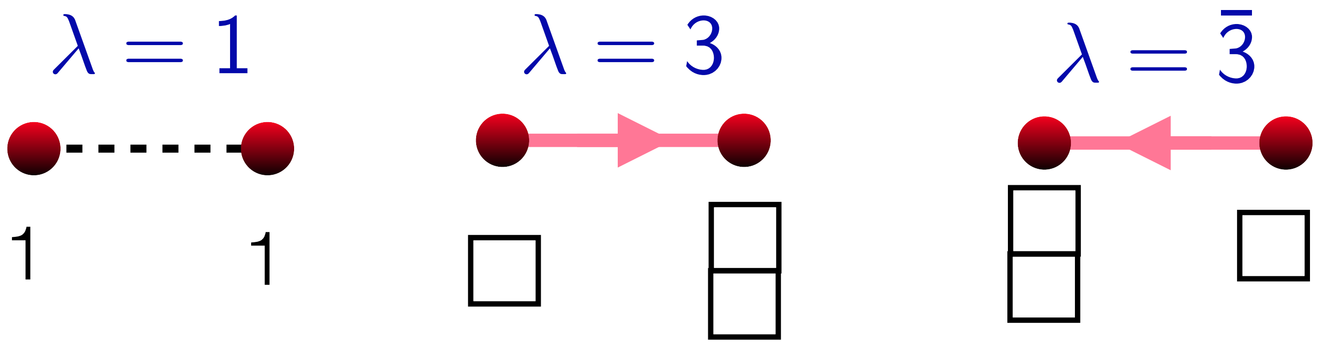
Consider a qubit regularized SU(3)
Hilbert space on a single link:

$$Q = \{1, 3, \bar{3}\}$$

Each irrep on the link has the Hilbert space: $\mathcal{H}_\lambda = V_\lambda \otimes V_\lambda^*$

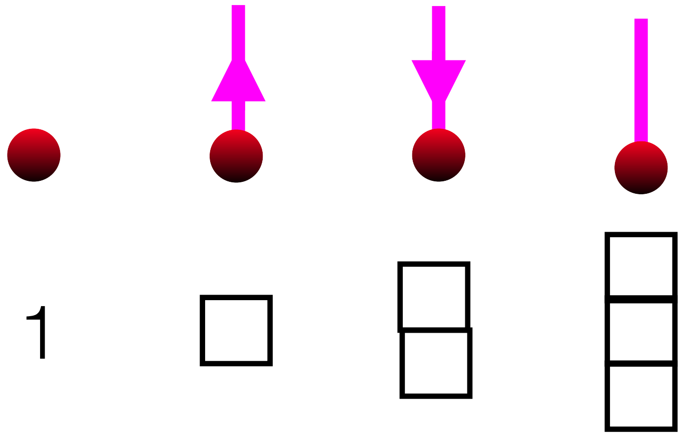
Link Hilbert space

$$\dim(\mathcal{H}_Q) \;=\; 19$$

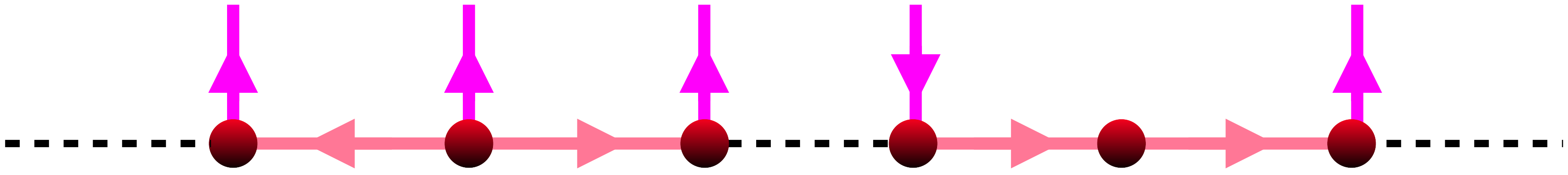


Matter Hilbert space

$$\dim(\mathcal{H}_Q) \;=\; 8$$



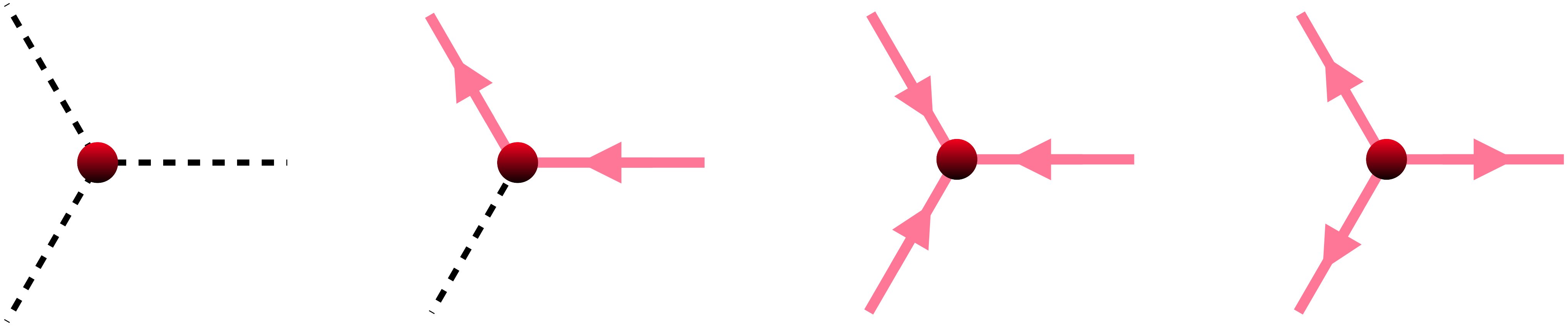
Physical Hilbert space $\mathcal{H}_{\text{Phys}}$ in 1d



Baryon Meson

On a L site lattice $\dim(\mathcal{H}_{\text{Phys}}) = 4^L + 2$ compared to $(8x19)^L$

Physical Hilbert space $\mathcal{H}_{\text{Phys}}$ in 2d without matter is interesting



Site on a honeycomb lattice

These arguments can be generalized to any lattice in any dimension.

Qubit regularized gauge invariant Hamiltonians are easy to construct

$$H = \alpha_1 \sum_{\ell} L^a_{Q,\ell}\ L^a_{Q,\ell} + \alpha_2 \sum_{P} \Big(\mathcal{U}_P^{Q,\lambda} + (\mathcal{U}_P^{Q,\lambda})^\dagger \Big) + \ldots$$

Quantum dimer models.

Diagonal in momentum space

Off-diagonal in momentum space

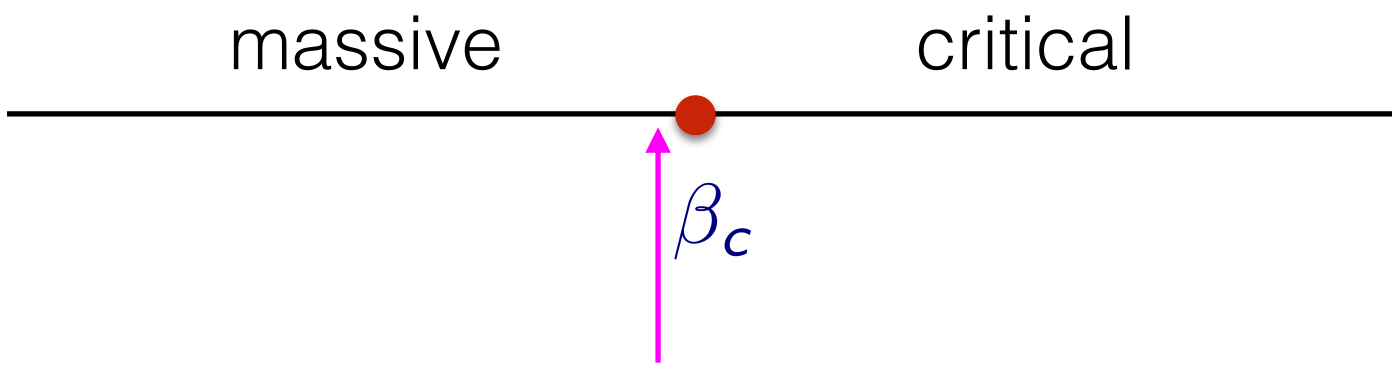
If H contains only diagonal terms, then we get “classical” stat. mech. models, which can also lead to interesting Euclidean QFTs.

Remember! {
Models containing only $\mathcal{U}_P$ and $(\mathcal{U}_P)^\dagger$ lead to
classical lattice gauge theories in $\mathcal{H}_{\mathsf{Full}}$,
since they are diagonal in “position” space.

A massive continuum QFT
via a classical dimer model

It is well known that the classical 2D lattice XY model undergoes a BKT transition from a critical phase to a massive phase.

Action: $S = -\beta \sum_{\langle xy \rangle} \cos(\theta_x - \theta_y)$



massive

critical

β_c

Massive QFT

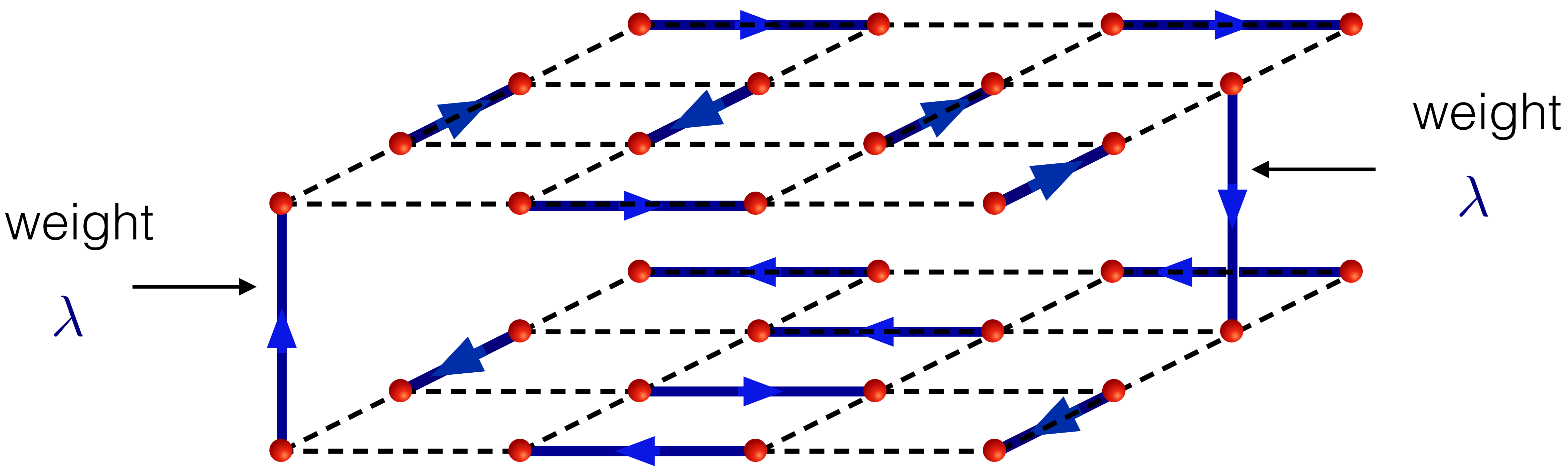
This massive QFT can be obtained using a classical closed packed dimer model without fine tuning.

Desai, Pujari and Damle, PRE 042136 (2021)
Maiti, Banerjee, SC, Marinkovic (in progress)

Each dimer is oriented with constraints on the sites, such that each site can only contain one oriented dimer.

Further the dimer orientations satisfy $Q_x = (-1)^x$

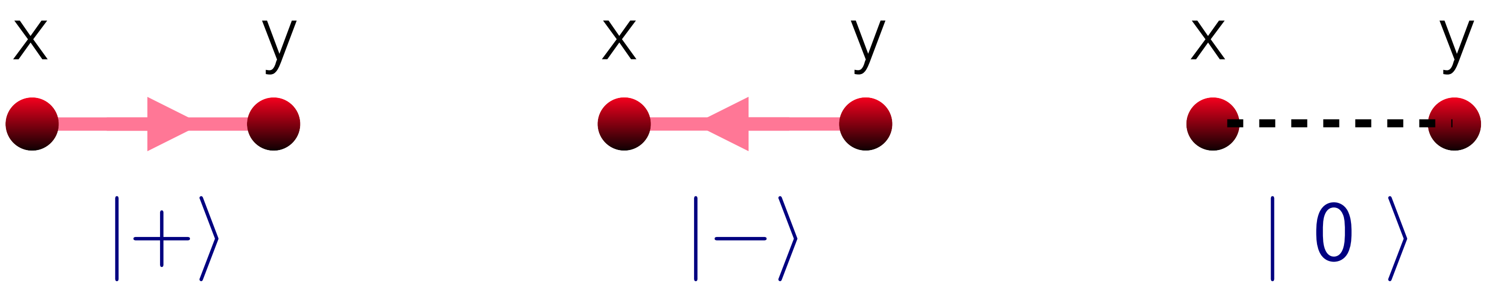
An example of a closed pack dimer configuration



Configuration weight: $W(c) = \lambda^{N_\perp}$

This model can be viewed as a qubit regularized U(1) lattice gauge theory

Consider the U(1) link Hilbert space:



Electric Field Operator:

$$E_{xy}\ |+\rangle\ =\ |+\rangle\,,\quad E_{xy}\ |-\rangle\ =\ -\ |-\rangle\,,\quad E_{xy}\ |\ 0\ \rangle\ =\ 0$$

Gauge charge: $Q_x = (\nabla \cdot E)_x$

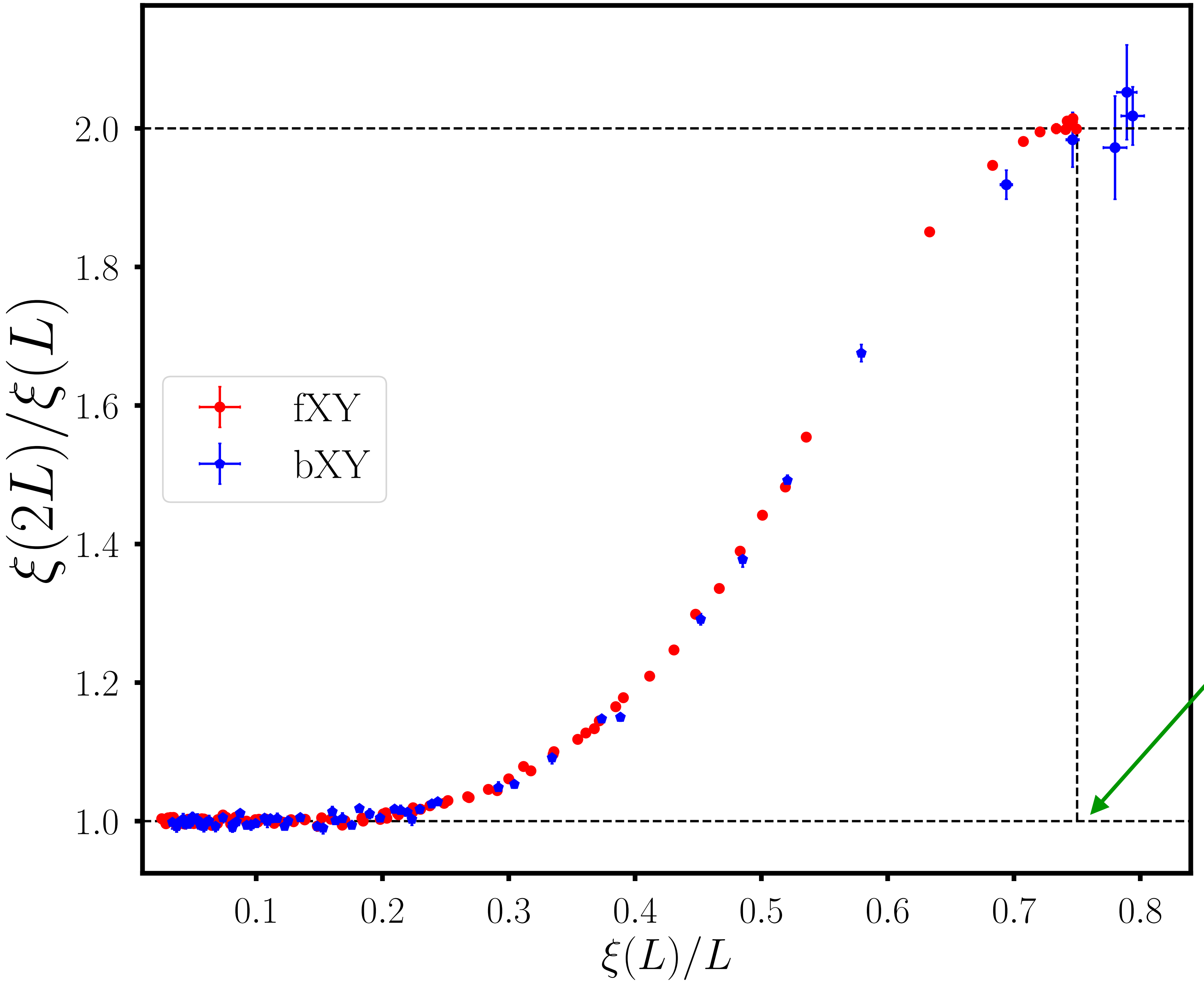
Consider a cubical lattice with dimensions L x L x 2 which is periodic in two dimensions but not in the third dimension.

Fix the Gauss law sector: $Q_x = (-1)^x$

Choose a Hamiltonian that is diagonal in the Electric flux basis and has the appropriate Boltzmann weight for each basis state.

The massive QFT at the BKT transition = $\underbrace{\lambda \to 0 \quad L \to \infty}_{\text{The limits do not commute!}}$ Classical Dimer Model

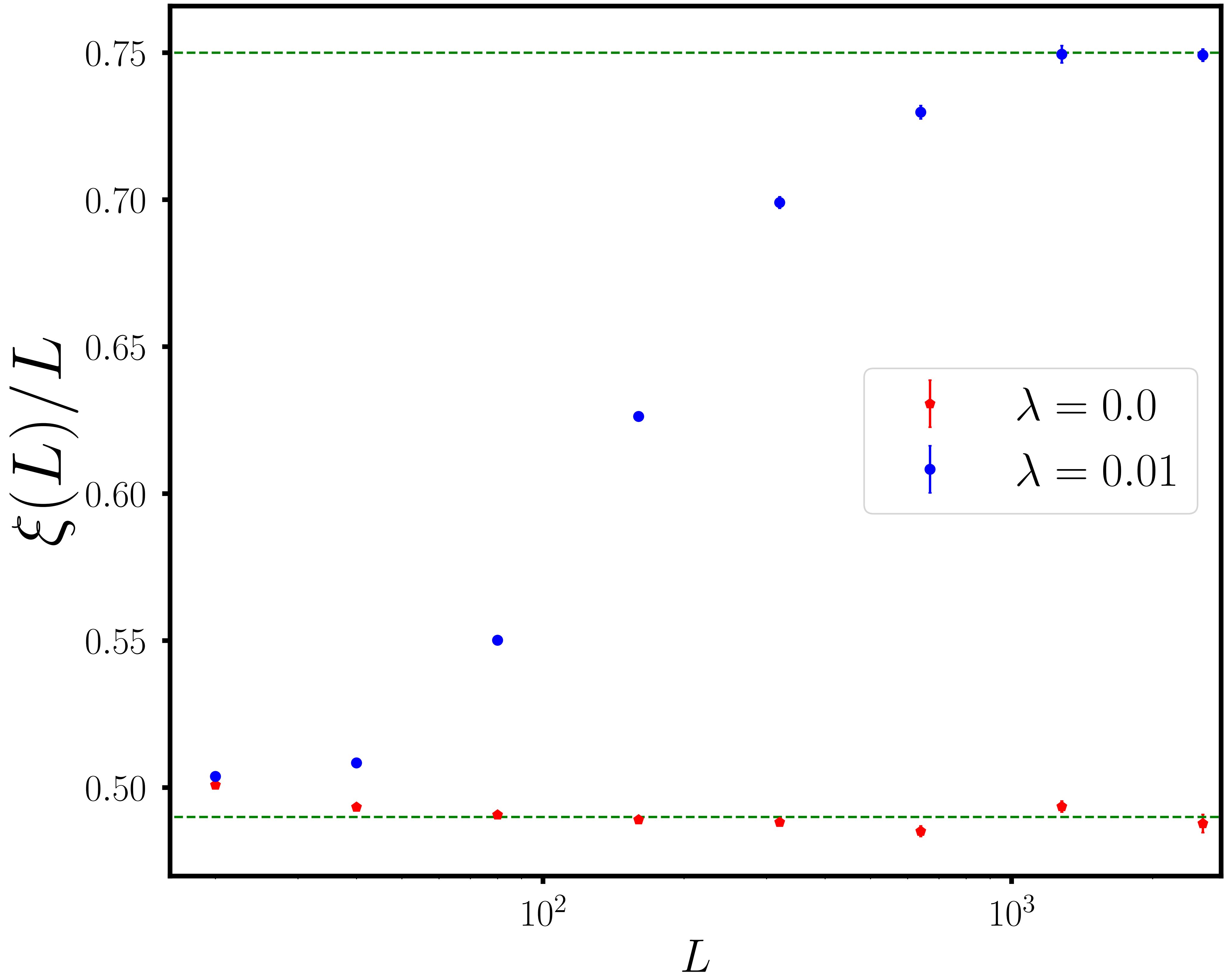
Preliminary Results



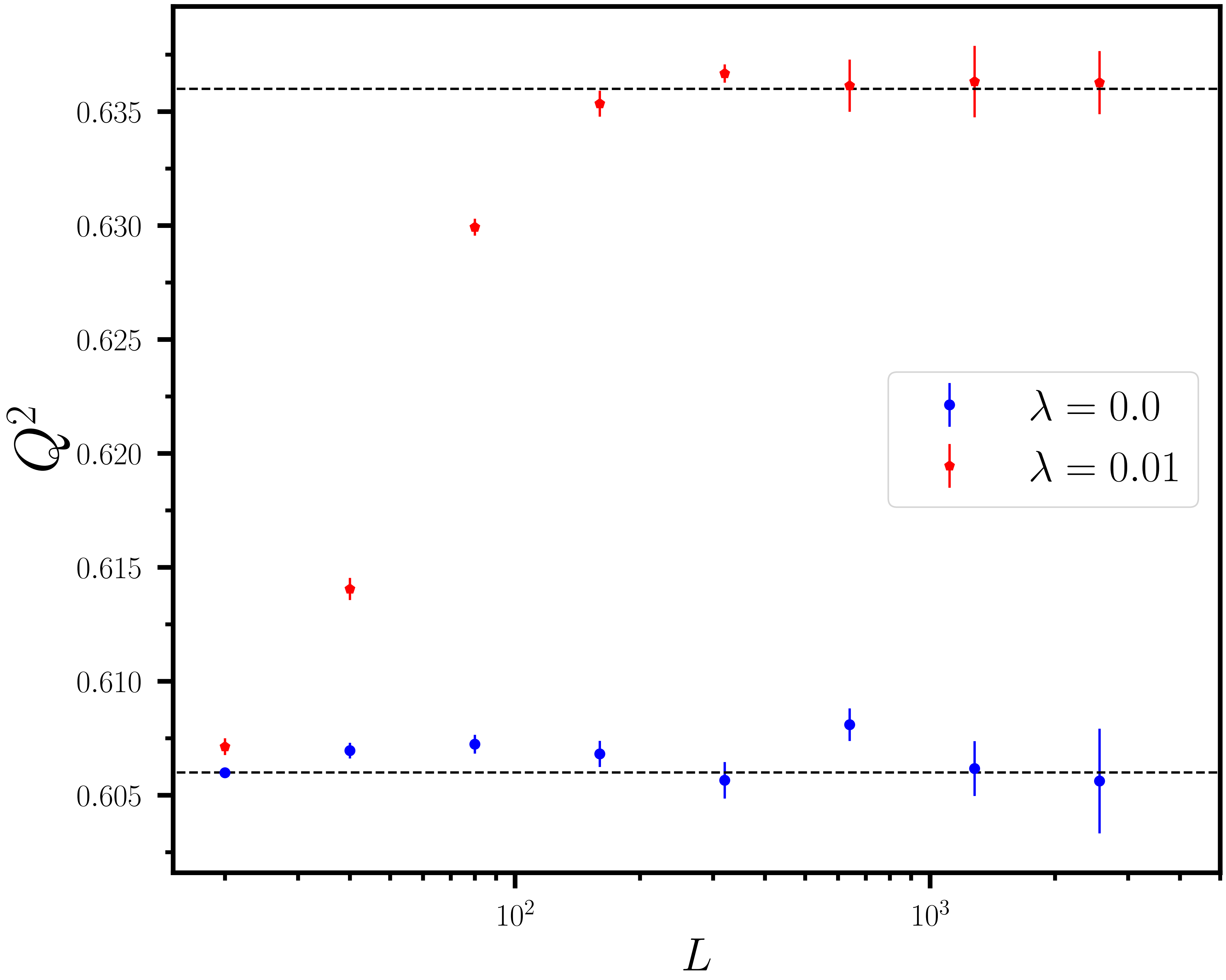
UV fixed point
Hasenbusch, 2008

0.7506912…

Preliminary Results



Preliminary Results



$\frac{2}{\pi}$

famous
BKT result

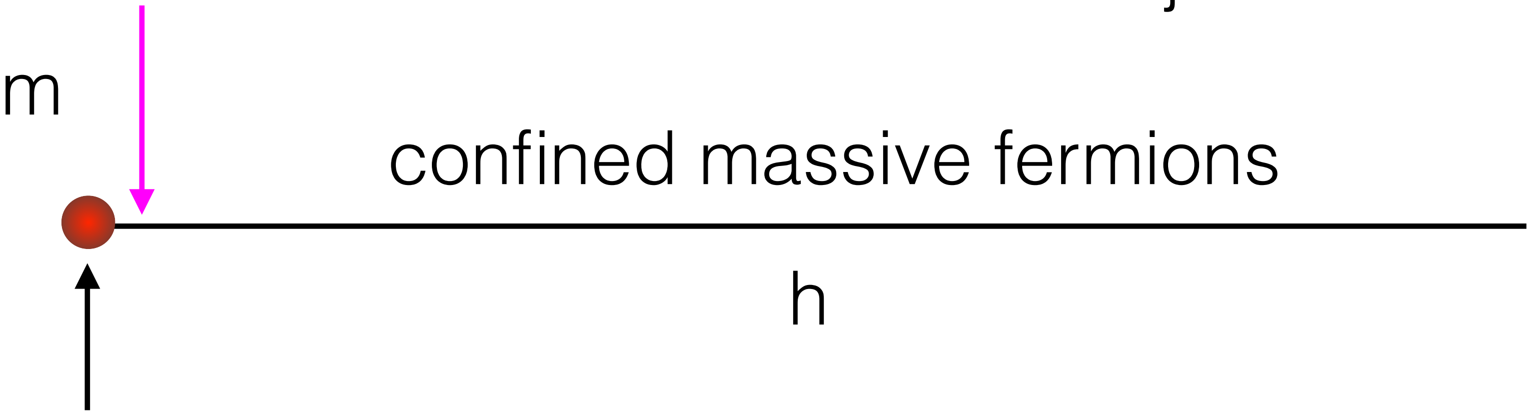
A massive continuum QFT via a quantum dimer model

Hamiltonian (a Z_2 gauge theory)

Frank, Huffman, SC, PLB 806, 135484 (2020)

$$H = \sum_j -\left(c_j^\dagger c_{j+1} + c_{j+1}^\dagger c_j\right)\,\sigma_j^x \;-\; h\,\sigma_j^z$$

gauge fields



Massive continuum QFT

Phase diagram

confined massive fermions

h

h = 0

(deconfined massless fermions)

Define new operators

fermions

$$f_0 = c_0 \qquad f_j = \sigma_0^x\ \sigma_1^x \ldots \sigma_{j-1}^x c_j$$

$$f_0^\dagger = c_0^\dagger \qquad f_j^\dagger = \sigma_0^x\ \sigma_1^x \ldots \sigma_{j-1}^x c_j^\dagger$$

Wilson loop
and its conjugate

$$W_{L-1} = \sigma_0^x\ \sigma_1^x \ldots \sigma_{L-1}^x \qquad E_{L-1} = \sigma_{L-1}^z$$

Local gauge charges

$$Q_j\ =\ \sigma_{j-1}^z \sigma_j^z\ (-1)^{f_j^\dagger f_j}$$

In terms of the new operators and $W_j = 1, \quad j = 0, 1, 2 \ldots L-2$

$$H\ =\ \sum_j -t\ (f_j^\dagger f_{j+1} + f_{j+1}^\dagger f_j)\ W_j\ -h\ E_{L-1} Q_0 (-1)^{n_0} Q_1 (-1)^{n_1} \ldots Q_j (-1)^{n_j}$$

Free fermions

Non-local interaction!

We get free deconfined massless fermions at h = 0!

To study the theory at non-zero values of h we compute three mass scales.

chiral order parameter

$$\phi = \left\langle (-1)^{j+c_j^\dagger c_j} \right\rangle$$

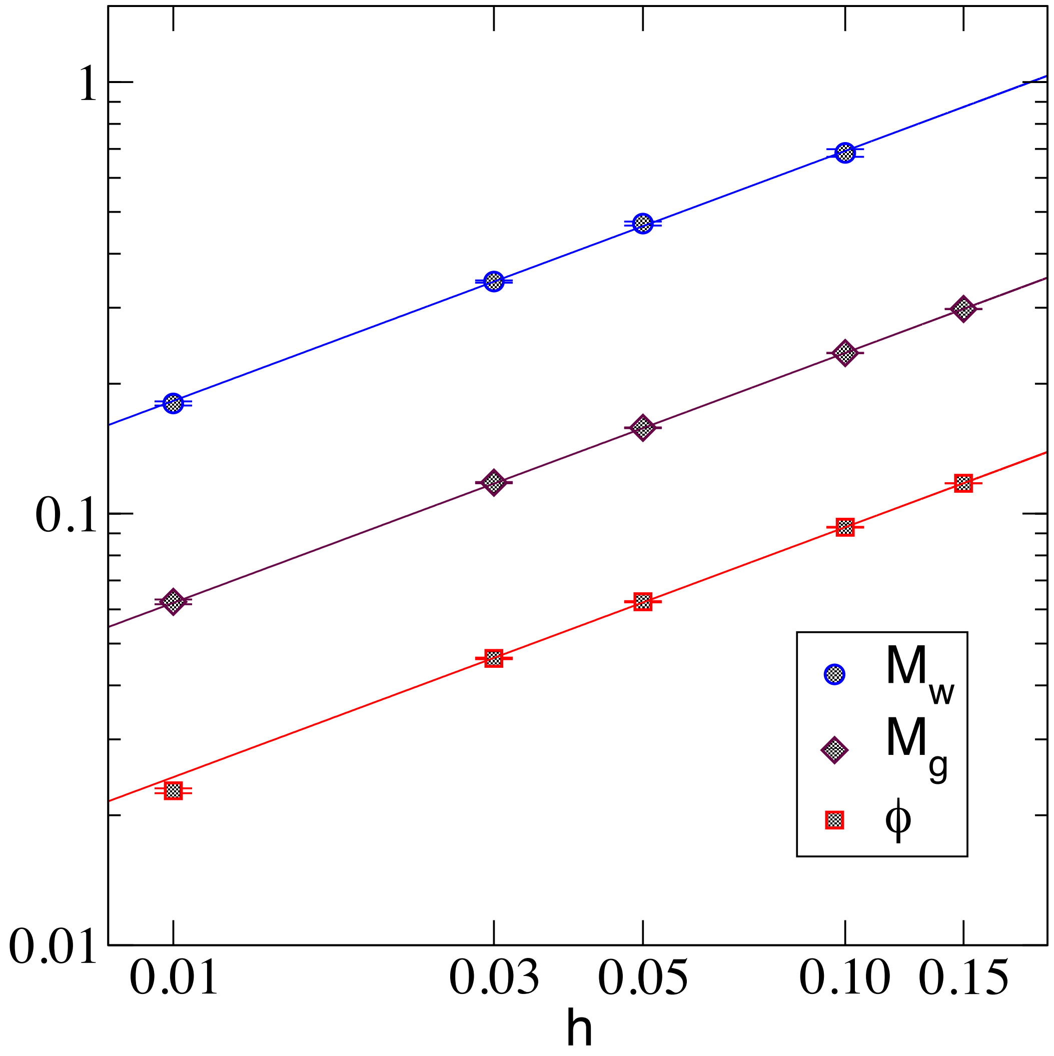
gauge mass

$$M_g = \sqrt{h \,\langle\, \sigma^z \,\rangle}$$

winding mass

$$\langle W^2 \rangle / \beta \sim B e^{-M_w L}$$

Scaling of the three mass scales with h



All three scale as a single power of h!

$$M \sim h^p$$

Fitting the data we found

$$p \approx 0.579$$

Understanding the massive quantum field theory

$$S \;=\; \int d^2x \,\left(\overline{\psi}(x)\sigma_\mu \partial_\mu \psi(x) \;+\; h\,\mathcal{O}_h\right)$$

$$[h] \;=\; \frac{1}{p} \approx 0.73$$

Effect of the gauge interactions!

↑

more relevant than mass!

Borla and Moroz (TU Munich) suggest that bosonization leads to the massive Sine-Gordon field theory with

$$S \;=\; \int d^2x \,\left\{\frac{1}{8\pi}\partial_\mu\phi\,\partial_\mu\phi \;-\; \alpha\; h\; \cos\left(\frac{\phi}{2}\right)\right\}$$

This predicts $p = 0.75$

The Z_2 gauge symmetry affects the periodicity of the angular variable.

Conclusions

Qubit Regularization is a new way to explore QFTs. All QFTs can be qubit regularized in a systematic way.

Qubit regularizations naturally suggest that we reformulate lattice gauge theories in a “momentum” or “representation” basis.

The gauge invariant Hilbert space of gauge theories naturally describes the configuration space of “generalized dimer” models.

Both classical and quantum dimer models have critical points and can lead to massive QFTs, at least in low dimensions.

Exploring similar and other models motivated by SU(N) gauge theories in various dimensions could be exciting.