

# Designing Topological Quantum Matter

Claudio Chamon



# Collaborators

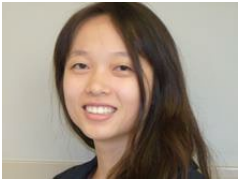
2020: PRL  
2021: 2x PRB, 2x PRX Quantum  
2022: 3x SciPost submissions  
2023: arXiv



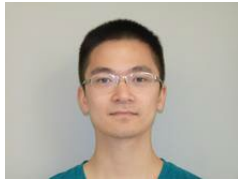
Dmitry Green



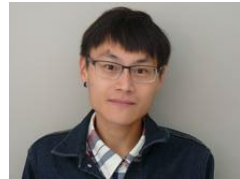
Zhi-Cheng Yang



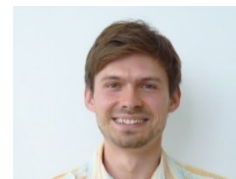
Shiyu Zhou



Elliot Yu



Kai-Hsin Wu



Aleksey Khudorozhkov



Guilherme Delfino



Garry Goldstein

@ MIT LL

@ ColdQuanta

@ Univ. of Cambridge



Anders Sandvik



Andrei Ruckenstein



Andrew Kerman



Edward Dahl



Claudio Castelnovo



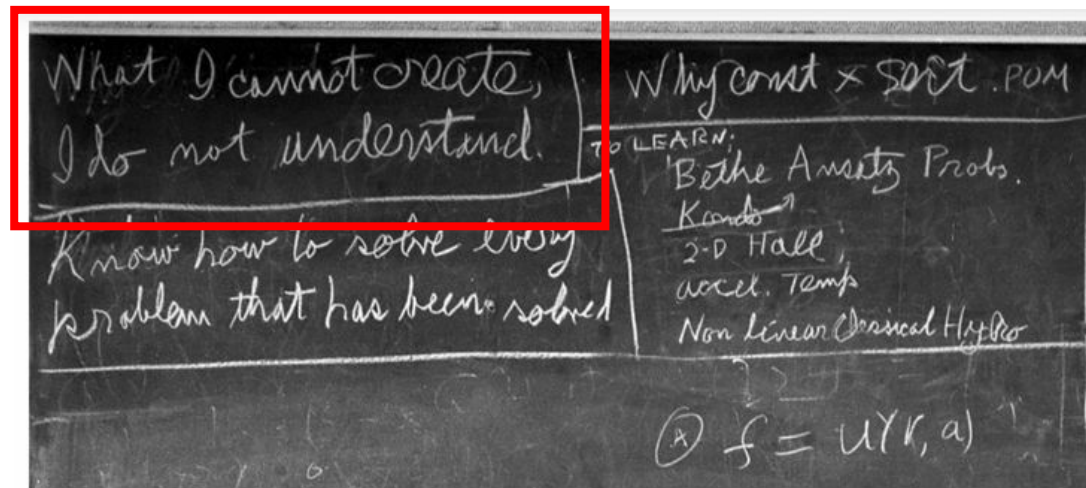
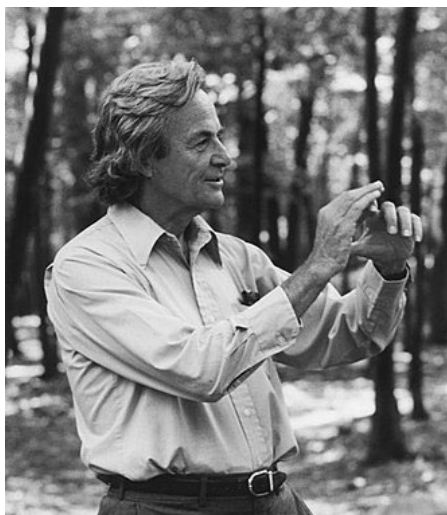
Maria Zelenayova



Oliver Hart

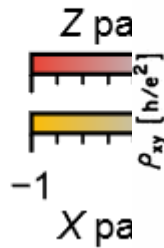
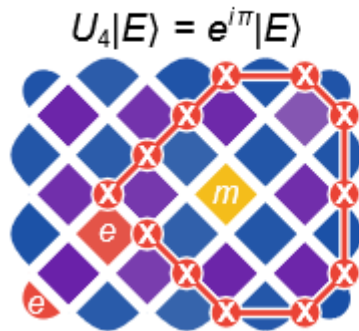
# Motivation

Build topological phases (e.g., toric code or  $\mathbb{Z}_2$  gauge models) with **physical** interactions (2-spin interactions or Josephson couplings)



# Motivation

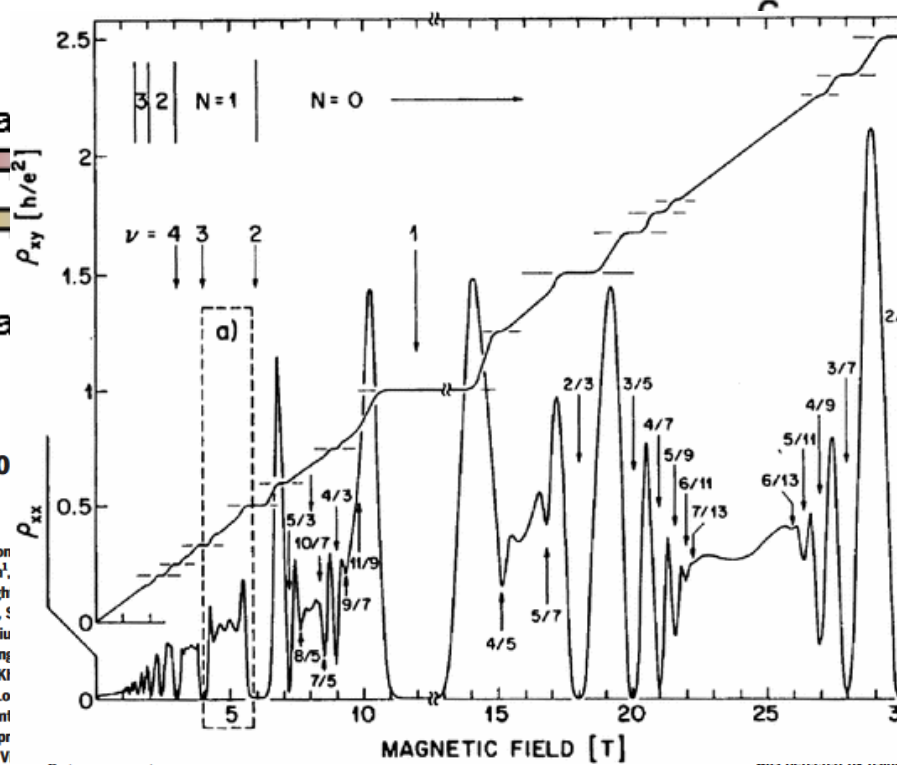
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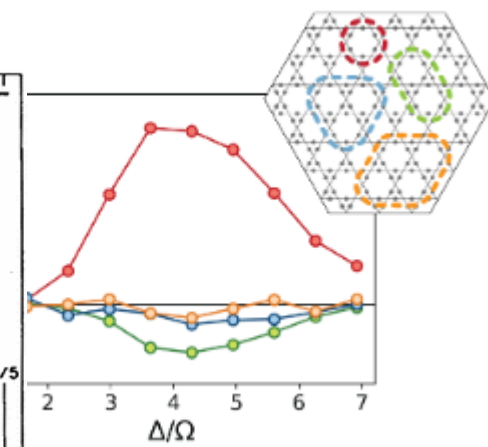
## TOPOLOGICAL MATTER

### Realizing topologically ordered states on a quantum processor

K. J. Satzinger<sup>1\*</sup>, Y.-J. Liu<sup>2,3</sup>, A. Smith<sup>2,4,5</sup>, C. Knapp<sup>6,7</sup>, M. Newman<sup>1</sup>, C. Jon X. Mi<sup>1</sup>, A. Dunsworth<sup>1</sup>, C. Gidney<sup>1</sup>, I. Aleiner<sup>1</sup>, F. Arute<sup>1</sup>, K. Arya<sup>1</sup>, J. Atalaya<sup>1</sup>, J. C. Bardin<sup>1,8</sup>, R. Barends<sup>1</sup>, J. Basso<sup>1</sup>, A. Bengtsson<sup>1</sup>, A. Bilmes<sup>1</sup>, M. Brought D. A. Buell<sup>1</sup>, B. Burkett<sup>1</sup>, N. Bushnell<sup>1</sup>, B. Chiaro<sup>1</sup>, R. Collins<sup>1</sup>, W. Courtney<sup>1</sup>, D. Eppens<sup>1</sup>, C. Erickson<sup>1</sup>, L. Faoro<sup>9</sup>, E. Farhi<sup>1</sup>, A. G. Fowler<sup>1</sup>, B. Foxen<sup>1</sup>, M. G. J. A. Gross<sup>1</sup>, M. P. Harrigan<sup>1</sup>, S. D. Harrington<sup>1</sup>, J. Hilton<sup>1</sup>, S. Hong<sup>1</sup>, T. Huang L. B. Ioffe<sup>1</sup>, S. V. Isakov<sup>1</sup>, E. Jeffrey<sup>1</sup>, Z. Jiang<sup>1</sup>, D. Kafri<sup>1</sup>, K. Kechedzhii<sup>1</sup>, T. Ki P. V. Klimov<sup>1</sup>, A. N. Korotkov<sup>1,11</sup>, F. Kostritsa<sup>1</sup>, D. Landhuis<sup>1</sup>, P. Laptev<sup>1</sup>, A. Lo O. Martin<sup>1</sup>, J. R. McClean<sup>1</sup>, M. McEwen<sup>1,12</sup>, K. C. Miao<sup>1</sup>, M. Mohseni<sup>1</sup>, S. Mont J. Mutus<sup>1</sup>, O. Naaman<sup>1</sup>, M. Neeley<sup>1</sup>, C. Neill<sup>1</sup>, M. Y. Niu<sup>1</sup>, T. E. O'Brien<sup>1</sup>, A. Opr A. Petukhov<sup>1</sup>, N. C. Rubin<sup>1</sup>, D. Sank<sup>1</sup>, V. Shvarts<sup>1</sup>, D. Strain<sup>1</sup>, M. Szalay<sup>1</sup>, B. V Z. Yao<sup>1</sup>, P. Yeh<sup>1</sup>, J. Yoo<sup>1</sup>, A. Zalcman<sup>1</sup>, H. Neven<sup>1</sup>, S. Boixo<sup>1</sup>, A. Megrant<sup>1</sup>, Y. Chen<sup>1</sup>, J. Kelly<sup>1</sup>, V. Smelyanskiy<sup>1</sup>, A. Kitaev<sup>1,6,7</sup>, M. Knap<sup>2,3,13</sup>, F. Pollmann<sup>2,3\*</sup>, P. Roushan<sup>1\*</sup>



quantum Hall effect



## TOPOLOGICAL MATTER

### Realizing topological spin liquids on a programmable quantum processor

A. Keesling<sup>1,2</sup>, S. Ebadi<sup>1</sup>, T. T. Wang<sup>1</sup>, D. Bluvstein<sup>1</sup>, R. Verresen<sup>1</sup>, K. R. Samajdar<sup>1</sup>, A. Omran<sup>1,2</sup>, S. Sachdev<sup>1,5</sup>, A. Vishwanath<sup>1\*</sup>, M. D. Lukin<sup>1\*</sup>

Topological phases of matter with topological order, have been a major focus in physics for decades. Such phases feature long-range quantum entanglement that can be used to realize robust quantum computation. We used a 219-atom programmable quantum processor to realize quantum spin liquid states. In our approach, arrays of atoms were placed on a diamond lattice, and evolution under Rydberg blockade created frustrated quantum states that host a quantum spin liquid phase of the paradigmatic toric code type. We observed topological string operators that provide direct signatures of topological order and quantum correlations. Our observations enable the controlled experimental exploration of topological matter and protected quantum information processing.

# Motivation

Build topological phases (e.g., toric code or  $\mathbb{Z}_2$  gauge models) with *physical* interactions (2-spin interactions or Josephson couplings)

Our goal is to build static Hamiltonians  
hosting topological ground states!!!

Ground state is a quiet place

# Motivation

Build topological phases (e.g., toric code or  $\mathbb{Z}_2$  gauge models) with *physical* interactions (2-spin interactions or Josephson couplings)

Ioffe and Feigel'man, PRB 2002

Ioffe, Feigel'man, Ioselevich, Ivanov, Troyer, and Blatter, Nature 2002

Douçot, Feigel'man, and Ioffe, PRL 2003, PRB 2005

Gladchenko, Olaya, Dupont-Ferrier, Douçot, Ioffe, and Gershenson, Nat. Phys. 2009

Douçot and Ioffe, Rep. Prog. Phys. 2012

Jordan and Farhi, Sci. Adv. 2016

J. D. Biamonte, PRA 2008

Bravyi, DiVincenzo, Loss, and Terhal, PRL 2008

Leib, Zoller, and Lechner, Quant. Sci. and Tech. 2016

Subas and Jarzynski, PRA 2016

Chancellor, Zohren, and Warburton, Quant. Info. 2017

Gaps are perturbative: how can we try to increase these gaps?

# Motivation

Build topological phases (e.g., toric code or  $\mathbb{Z}_2$  gauge models) with ***physical*** interactions (2-spin interactions or Josephson couplings)

**“The definition of insanity is doing the same thing over and over again and expecting different results.”**

Albert Einstein often gets the credit for this saying, but you probably won't be surprised to learn that he never actually said it. This misattributed quotation has been well documented: it appears to have originated around 1980 in literature published by Narcotics Anonymous (Becker; “Insanity”).

Becker, Michael. “Einstein Probably Didn't Say That Famous Quote about Insanity.” *Becker's Online Journal*, 13 Nov. 2012, [www.news.hypercrit.net/2012/11/13/einstein-on-insanity](http://www.news.hypercrit.net/2012/11/13/einstein-on-insanity).

<https://style.mla.org/five-commonly-misattributed-quotations/>

Design **exact** gauge symmetries

# Combinatorial gauge symmetry

EXACT

**NOT** emergent



# Combinatorial gauge symmetry

What is it?

# What symmetries preserve commutation relations for $n$ spins?

Compare with the case of  $n$  fermions or bosons

$$\psi_i \rightarrow \tilde{\psi}_i = \sum_j U_{ij} \psi_j$$

$$\{\psi_i, \psi_j^\dagger\} = \{\tilde{\psi}_i, \tilde{\psi}_j^\dagger\}$$

$$\phi_i \rightarrow \tilde{\phi}_i = \sum_j U_{ij} \phi_j$$

$$[\phi_i, \phi_j^\dagger] = [\tilde{\phi}_i, \tilde{\phi}_j^\dagger]$$

# What symmetries preserve commutation relations for $n$ spins?

Eg.: 2 fermions

$$\tilde{\psi}_1 = \frac{1}{\sqrt{2}} (\psi_1 + \psi_2)$$

$$\tilde{\psi}_2 = \frac{1}{\sqrt{2}} (\psi_1 - \psi_2)$$

All anti-commutation relations are preserved

# What symmetries preserve commutation relations for $n$ spins?

Now say we try this (please don't) for spins

$$\tilde{\sigma}_1^a = \frac{1}{\sqrt{2}} (\sigma_1^a + \sigma_2^a)$$

$$\tilde{\sigma}_2^a = \frac{1}{\sqrt{2}} (\sigma_1^a - \sigma_2^a)$$

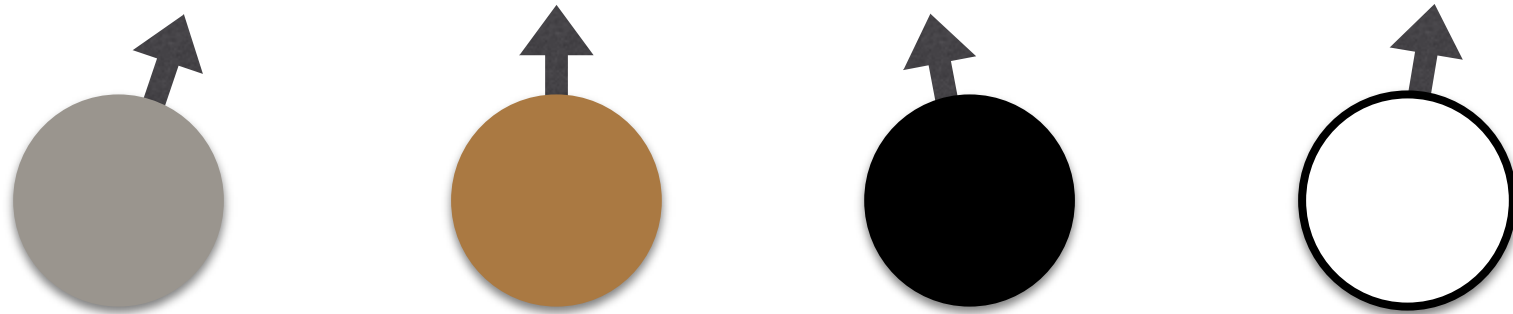
commutation and  
anti-commutation  
relations are messed up

## Which transformations are allowed?

$$\sigma_i^a \rightarrow U \sigma_i^a U^\dagger \quad U \in \text{SU}(2^n)$$

$$\sigma_i^a \rightarrow \sum_j \sum_b R_{ij}^{ab} \sigma_j^b + \sum_{jk} \sum_{bc} \Lambda_{l,j \neq k}^{abc} \sigma_j^b \sigma_k^c + \dots$$

$$U \in \text{SU}(2) \otimes \text{SU}(2) \otimes \dots \otimes \text{SU}(2)$$

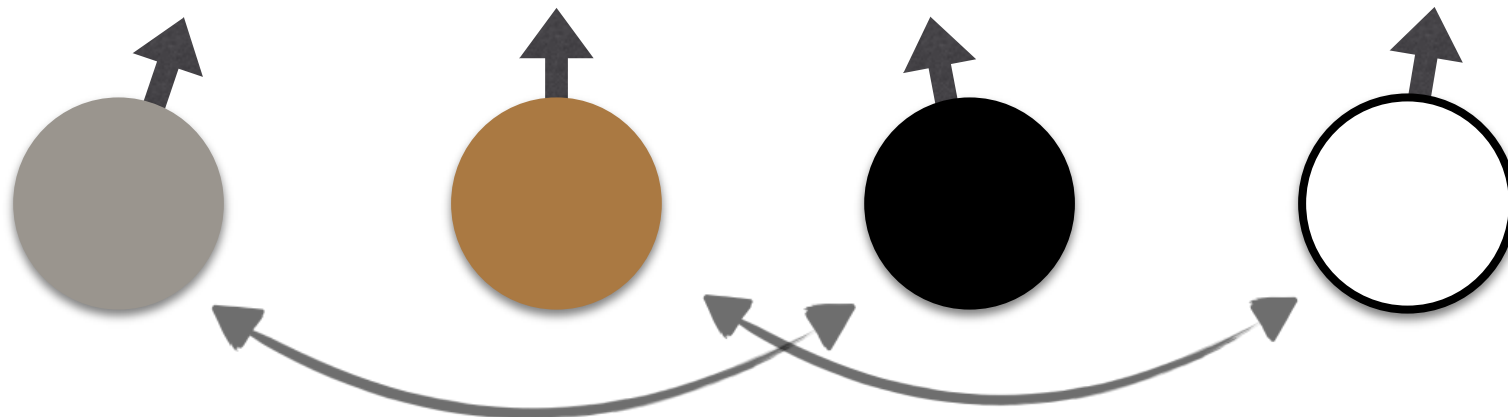


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$$U \in \text{SU}(2) \otimes \text{SU}(2) \otimes \dots \otimes \text{SU}(2)$$



Single-spin rotations  
+ permutations

# Monomial transformations

Eg.: 4 spins

$$\begin{pmatrix} \vec{\sigma}_1 \\ \vec{\sigma}_2 \\ \vec{\sigma}_3 \\ \vec{\sigma}_4 \end{pmatrix} \rightarrow \begin{pmatrix} 0 & 0 & g_1 & 0 \\ 0 & 0 & 0 & g_2 \\ g_3 & 0 & 0 & 0 \\ 0 & g_4 & 0 & 0 \end{pmatrix} \begin{pmatrix} \vec{\sigma}_1 \\ \vec{\sigma}_2 \\ \vec{\sigma}_3 \\ \vec{\sigma}_4 \end{pmatrix}$$

$$g_i \in SO(3)$$

Spin commutation relations are all preserved

# Monomial transformations

$$\begin{pmatrix} \vec{\sigma}_1 \\ \vec{\sigma}_2 \\ \vec{\sigma}_3 \\ \vec{\sigma}_4 \end{pmatrix} \rightarrow \begin{pmatrix} 0 & 0 & g_1 & 0 \\ 0 & 0 & 0 & g_2 \\ g_3 & 0 & 0 & 0 \\ 0 & g_4 & 0 & 0 \end{pmatrix} \begin{pmatrix} \vec{\sigma}_1 \\ \vec{\sigma}_2 \\ \vec{\sigma}_3 \\ \vec{\sigma}_4 \end{pmatrix}$$

$GP$   $\bowtie$  semi-direct or product of two groups

$G$  rotations  
 $P$  permutations

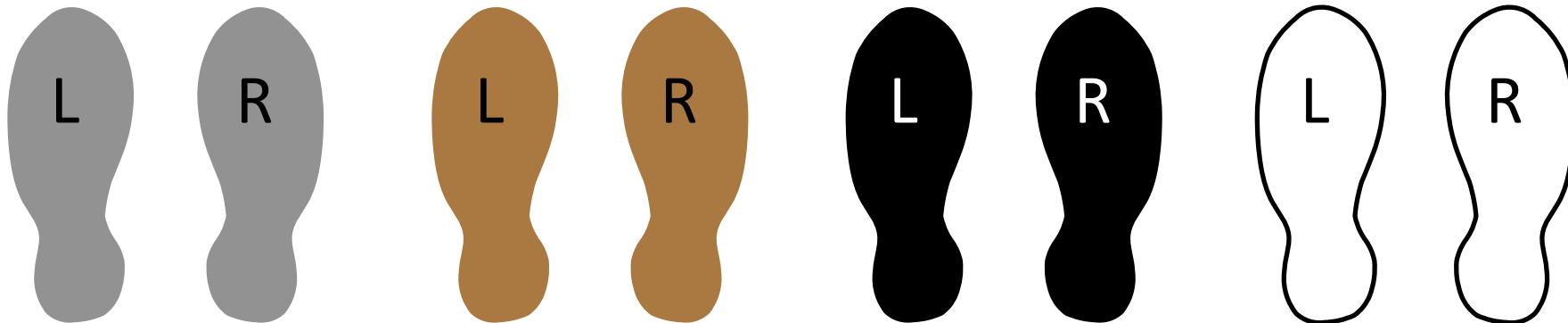


# Monomial transformations

$$\begin{pmatrix} \vec{\sigma}_1 \\ \vec{\sigma}_2 \\ \vec{\sigma}_3 \\ \vec{\sigma}_4 \end{pmatrix} \rightarrow \begin{pmatrix} 0 & 0 & g_1 & 0 \\ 0 & 0 & 0 & g_2 \\ g_3 & 0 & 0 & 0 \\ 0 & g_4 & 0 & 0 \end{pmatrix} \begin{pmatrix} \vec{\sigma}_1 \\ \vec{\sigma}_2 \\ \vec{\sigma}_3 \\ \vec{\sigma}_4 \end{pmatrix}$$

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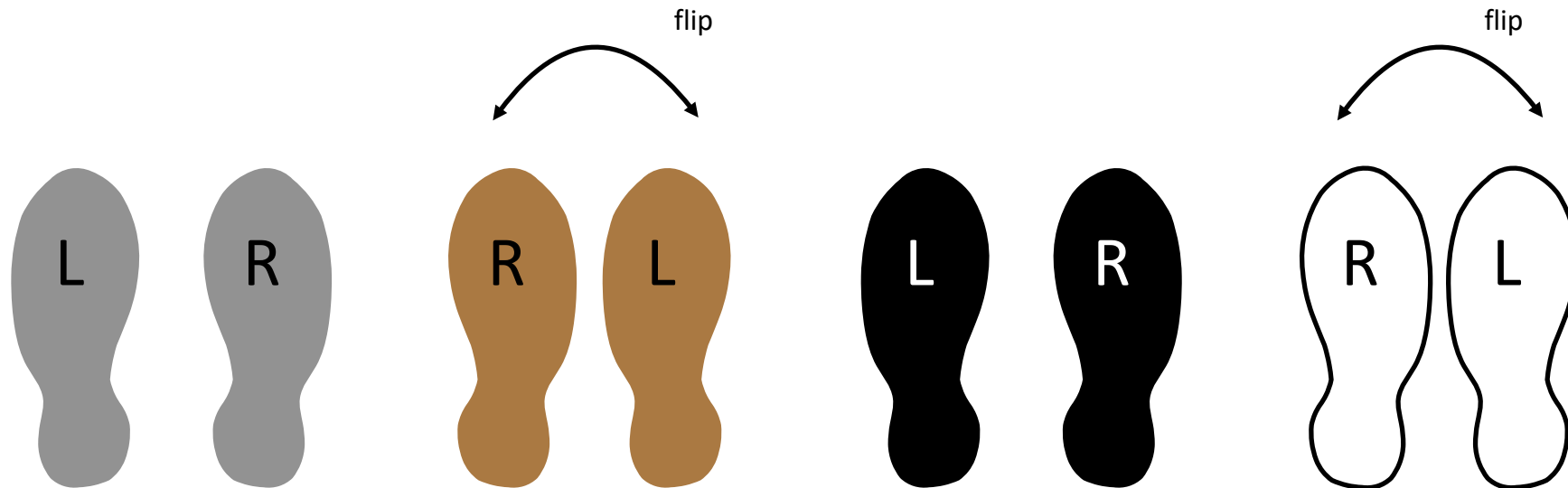


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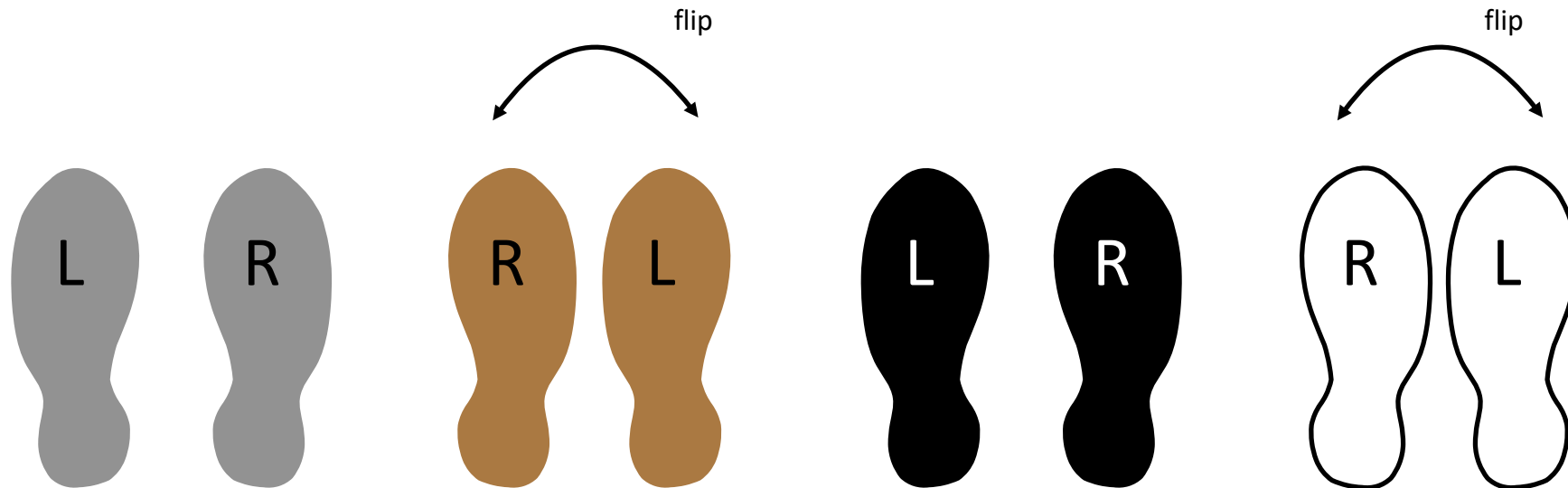


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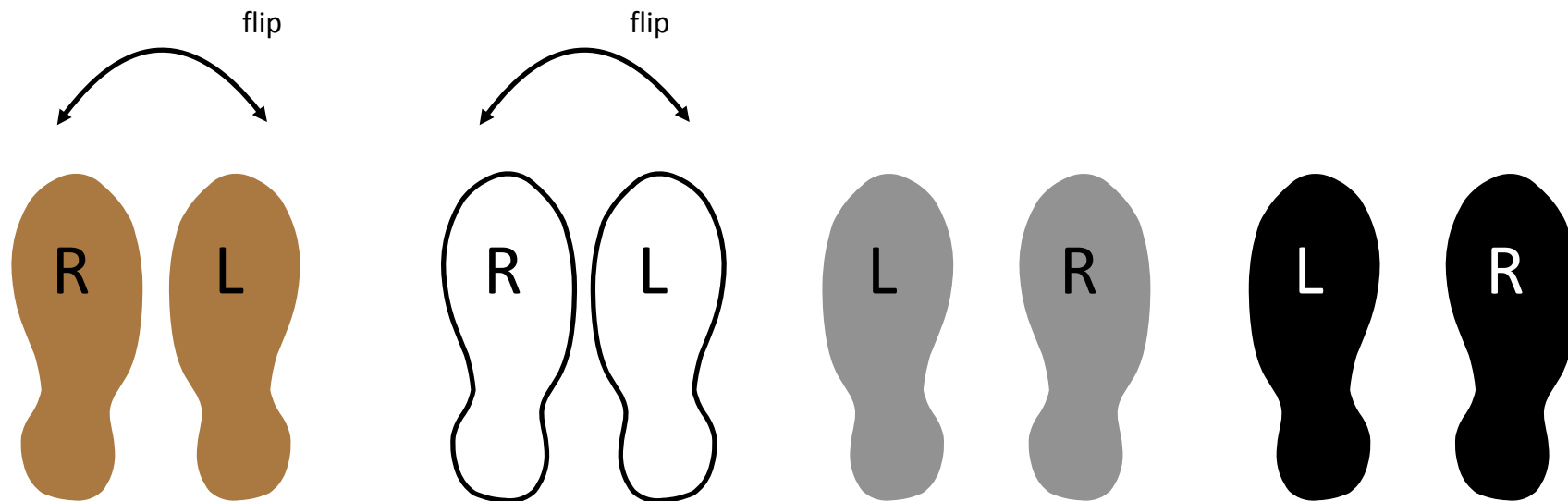


# Monomial transformations

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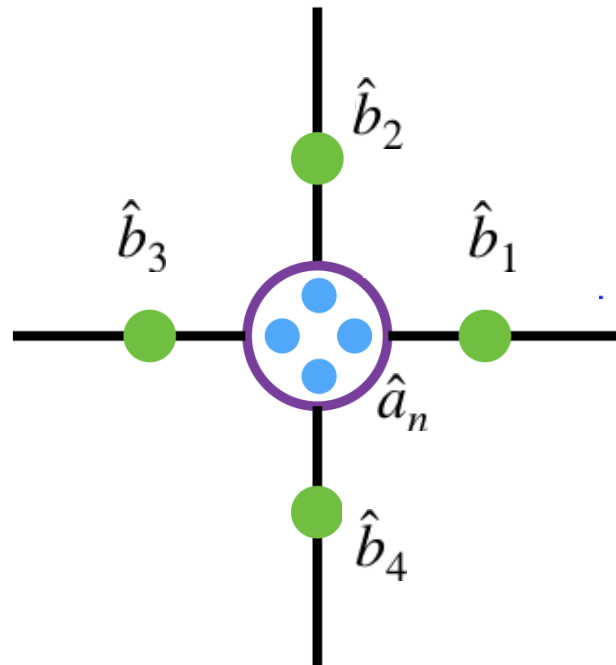
$GP$   $\bowtie$  semi-direct or product of two groups

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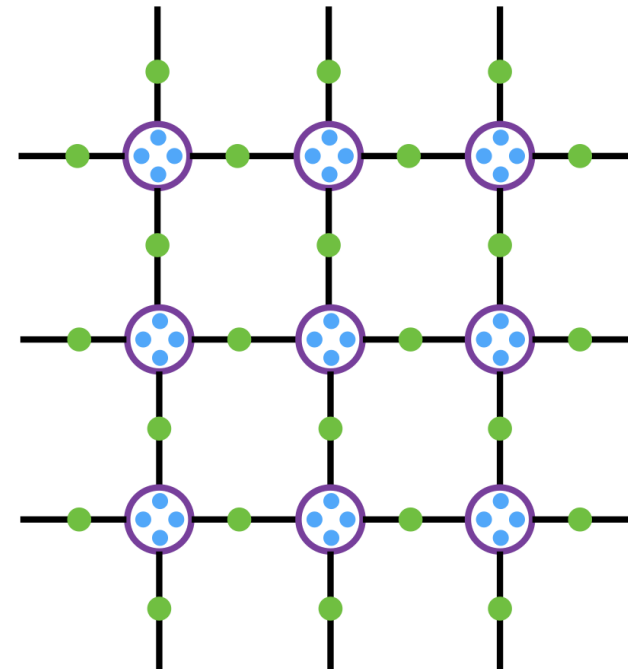


# Combinatorial gauge symmetry on lattice

- “Matter” fields  $\hat{a}_n$  at each vertex (  $i = 1 \dots 4$  )
- “Gauge” fields  $\hat{b}_i$  shared by vertices (  $n = 1 \dots 4$  )



star  $s$



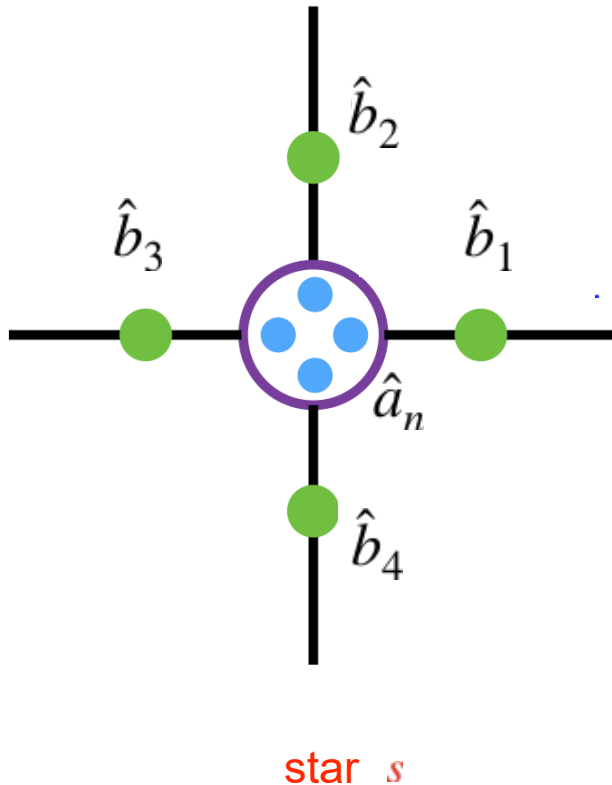
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- “Gauge” fields  $\hat{b}_i$  shared by vertices (  $n = 1 \dots 4$  )

e.g.

$$\hat{a}_m = \mu_m^z$$

$$\hat{b}_i = \sigma_i^z$$



Operators transform as:

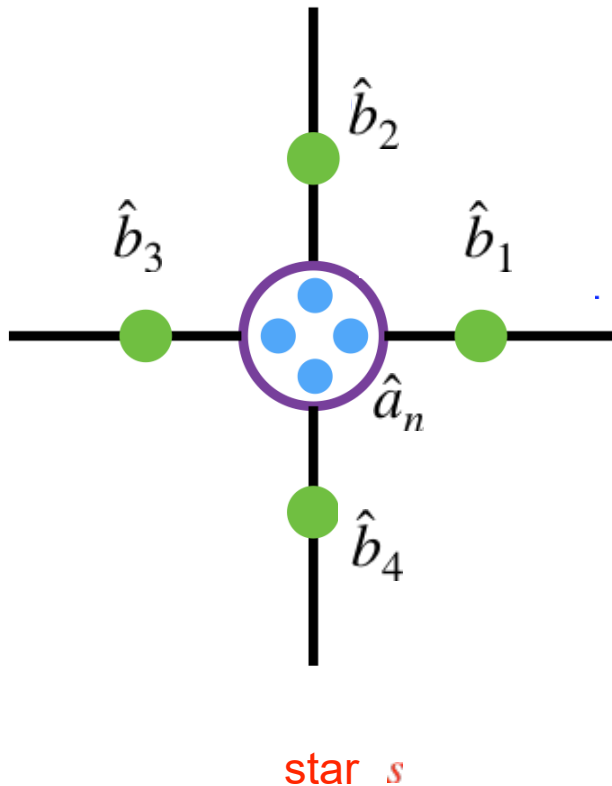
$$\hat{a}_n \rightarrow \sum_m \hat{a}_m (L^{-1})_{mn} \quad \text{and} \quad \hat{b}_i \rightarrow \sum_j R_{ij} \hat{b}_j$$

$\hat{a}$ ,  $\hat{b}$  can be spins, phases operators, etc.

$L, R$  are monomial matrices

# Combinatorial gauge symmetry on lattice

- “Matter” fields  $\hat{a}_n$  at each vertex (  $i = 1 \dots 4$  )
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Hamiltonian

$$H_J = -J \sum_s \sum_{n,i \in s} W_{ni} \left( \hat{a}_n^\dagger \hat{b}_i + \hat{b}_i^\dagger \hat{a}_n \right)$$

$$W = \begin{pmatrix} -1 & +1 & +1 & +1 \\ +1 & -1 & +1 & +1 \\ +1 & +1 & -1 & +1 \\ +1 & +1 & +1 & -1 \end{pmatrix}$$

4x4 Hadamard

Symmetry (Automorphism)

$$L^{-1} W R = W$$

# Mathematically: Hadamard automorphism

$$L^{-1} W R = W$$

Hadamard  
automorphism

$$\begin{pmatrix} 0 & +1 & 0 & 0 \\ +1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

Monomial matrix

$$\hat{a}_n = \sum_{m=1}^4 \hat{a}_m (L^{-1})_{m,n}$$

$$\begin{pmatrix} -1 & +1 & +1 & +1 \\ +1 & -1 & +1 & +1 \\ +1 & +1 & -1 & +1 \\ +1 & +1 & +1 & -1 \end{pmatrix}$$

$$\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & +1 & 0 \\ 0 & 0 & 0 & +1 \end{pmatrix}$$

Monomial matrix

$$\hat{b}_i = \sum_{j=1}^4 R_{i,j} \hat{b}_j$$

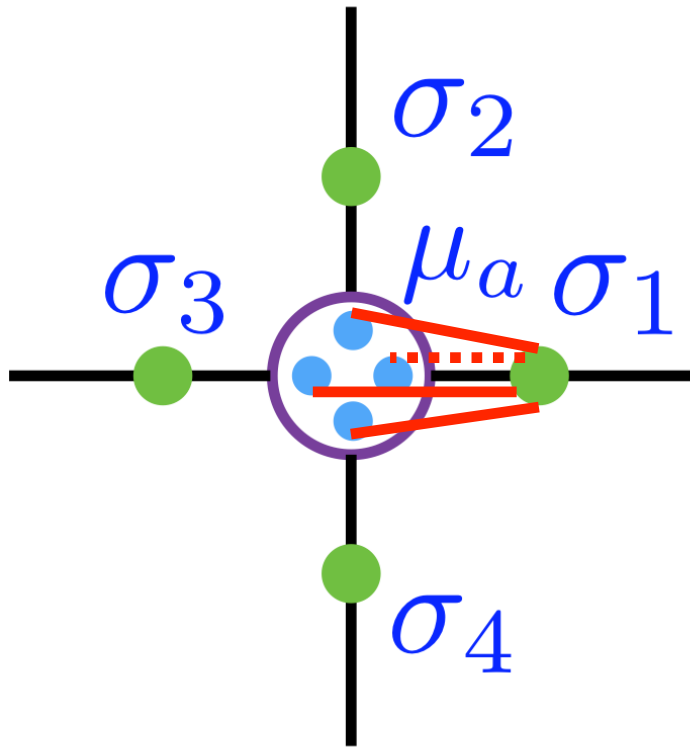
$$R \Rightarrow L \quad L = W R W^{-1}$$





# E.g.: Spin model with gauge-matter spin-spin interaction

- ■ ■ ■ Anti-ferromagnetic
- Ferromagnetic

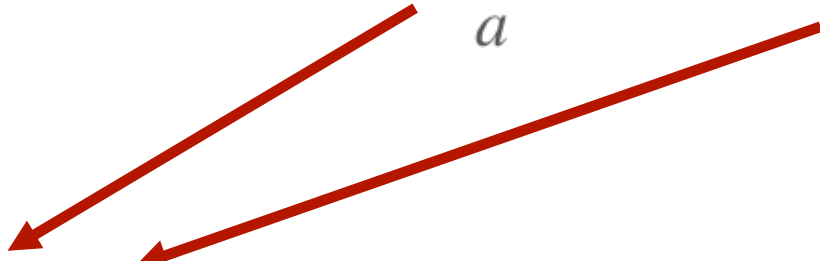


$$H_0 = -J \sum_{a=1}^4 \sum_{i=1}^4 W_{ai} \sigma_i^z \mu_a^z$$

$$W = \begin{pmatrix} -1 & +1 & +1 & +1 \\ +1 & -1 & +1 & +1 \\ +1 & +1 & -1 & +1 \\ +1 & +1 & +1 & -1 \end{pmatrix}$$

4 x 4 Hadamard matrix

Invariance for *all*  $J, \Gamma, \tilde{\Gamma}$

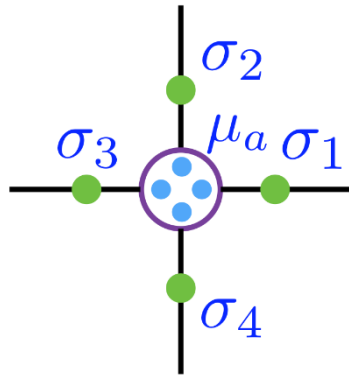
$$H = -J \sum_s \sum_{a, i \in s} W_{ai} \mu_a^z \sigma_i^z - \Gamma \sum_a \mu_a^x - \tilde{\Gamma} \sum_i \sigma_i^x$$


Transverse fields: invariant under spin flips and permutations

Monomial transformations preserve spin algebra

Warning: would lead to small gaps (sanity check!)

Simple limit: single star  $\Gamma \gg J$



$\mu$  in effective field of  $\sigma$

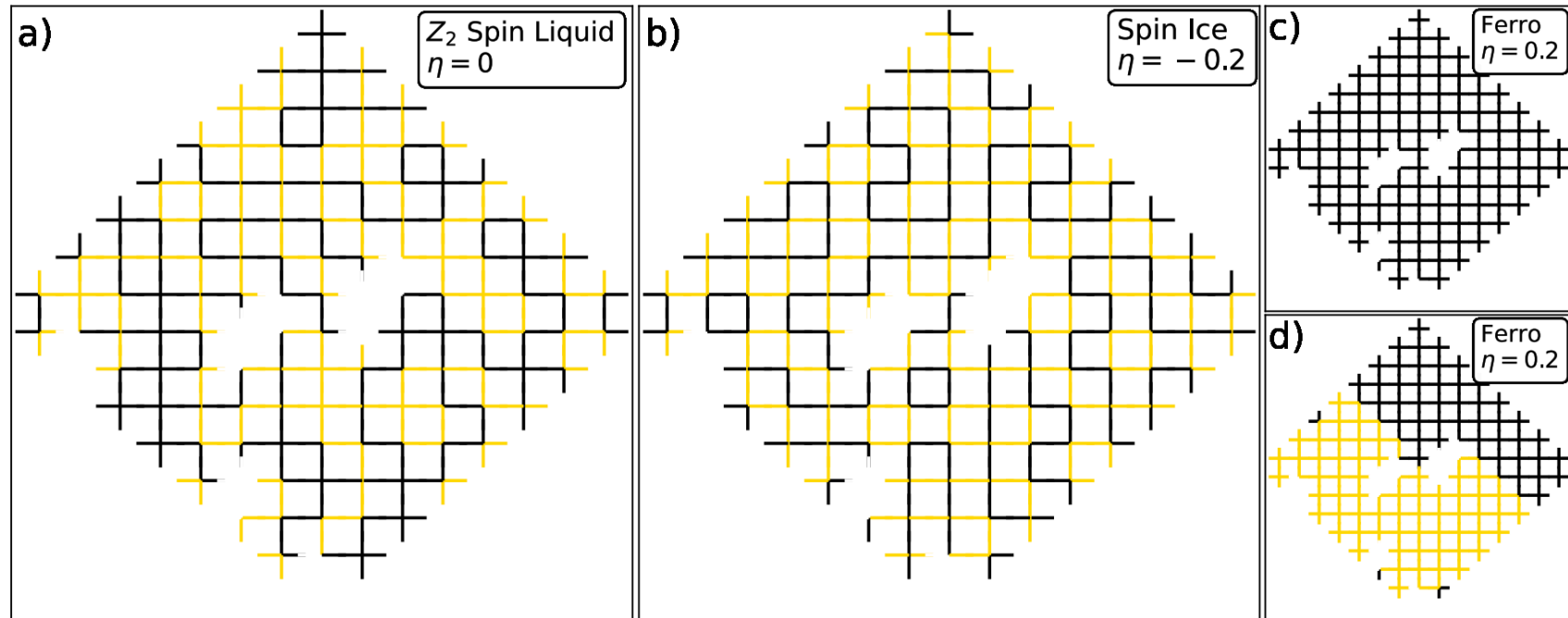
$$H = -J \sum_{a=1}^4 \left( \sum_{i=1}^4 W_{ai} \sigma_i^z \right) \mu_a^z - \Gamma \sum_{a=1}^4 \mu_a^x$$

$\underbrace{\hspace{15em}}_{B_z \mu^z}$ 
 $\underbrace{\hspace{15em}}_{B_x \mu^x}$

$$E \sim - \sum \sqrt{(W\sigma^z)^2 + \Gamma^2} \sim \boxed{\text{const} - \sigma_1^z \sigma_2^z \sigma_3^z \sigma_4^z}$$

# Realization in D-Wave DW-2000Q for spins (classical limit only)

First\* experimental 8-vertex model (classical  $\mathbb{Z}_2$  spin liquid)

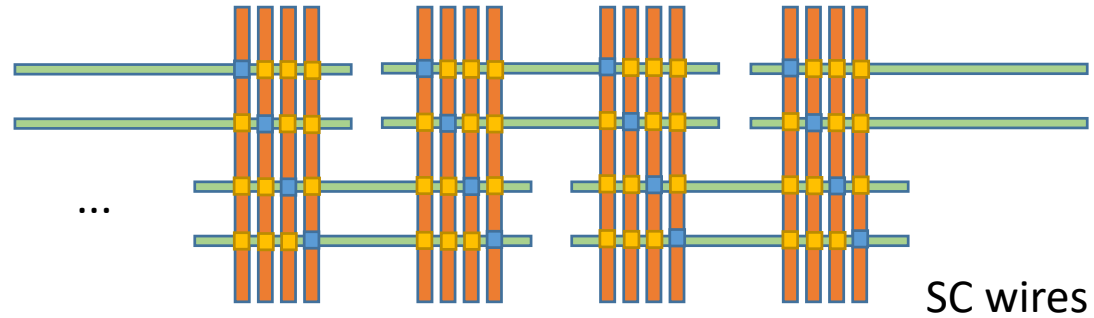


\* As far as we know

Zhou, Green, Dahl, Chamon, Phys. Rev. B (2021)

How to get large (non-perturbative) gaps,  
back to the program

# SC wire array



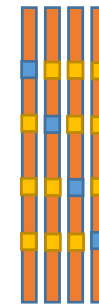
$\pi$  junction



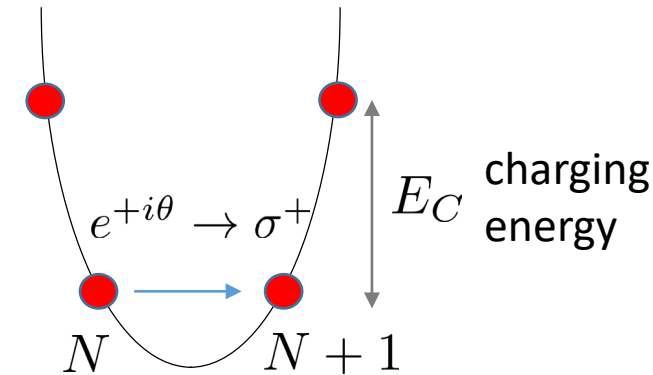
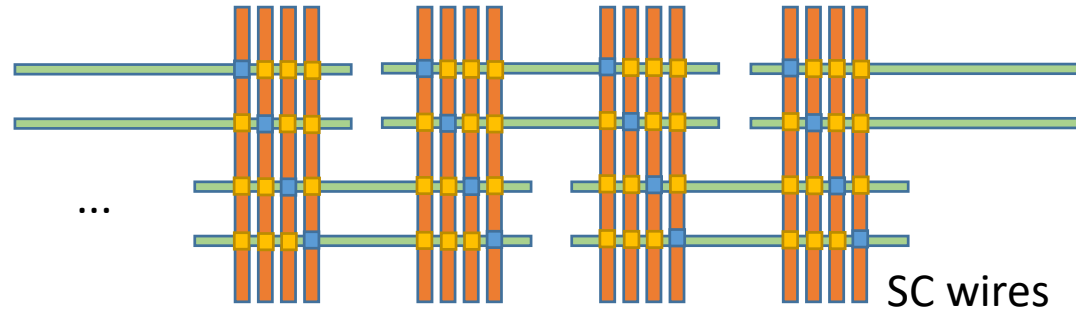
regular junction

$$H_J = -J \sum_{ia} W_{ia} e^{i\phi_i} e^{-i\theta_a} + H.c.$$

$$W = \begin{pmatrix} - & + & + & + \\ + & - & + & + \\ + & + & - & + \\ + & + & + & - \end{pmatrix}$$



# SC wire array



$\pi$  junction



regular junction

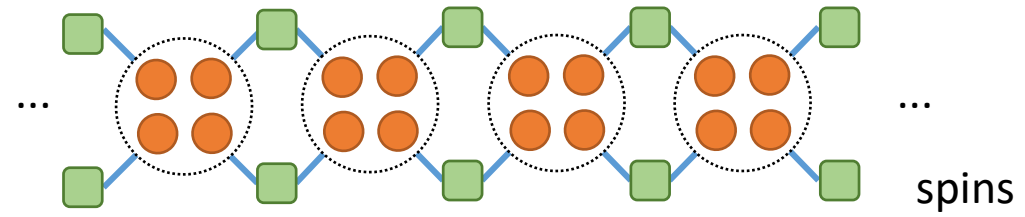
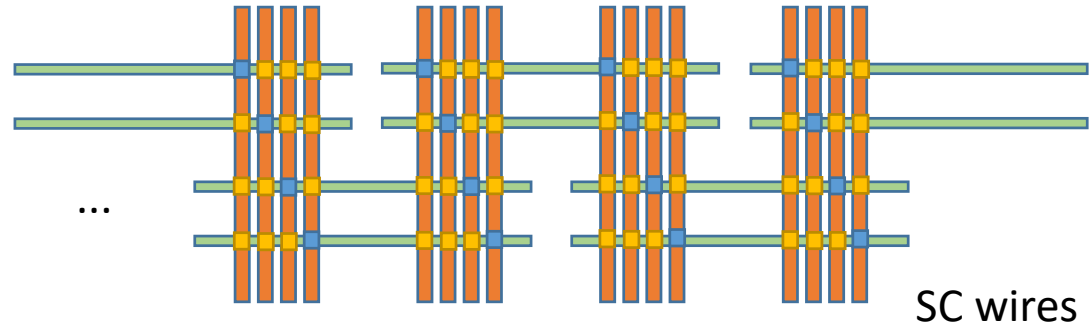
Small capacitance limit (charge degenerate point)

$$H_J = -J \sum_{ia} W_{ia} \mu_i^+ \sigma_a^- + H.c.$$

**WXY** model

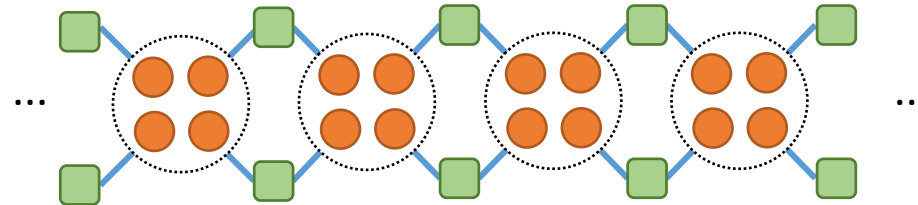


# WXY model

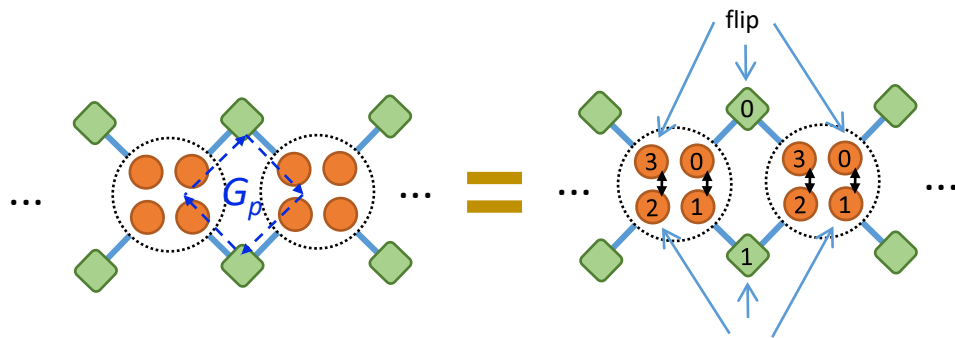


$$H_J = -J \sum_{ia} W_{ia} \mu_i^+ \sigma_a^- + H.c.$$

# WXY model symmetries



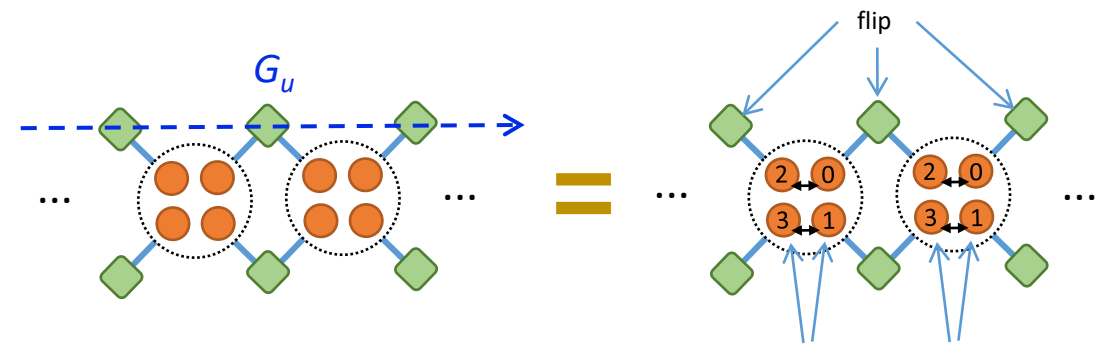
Local symmetries



plaquette operators

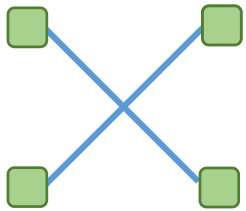
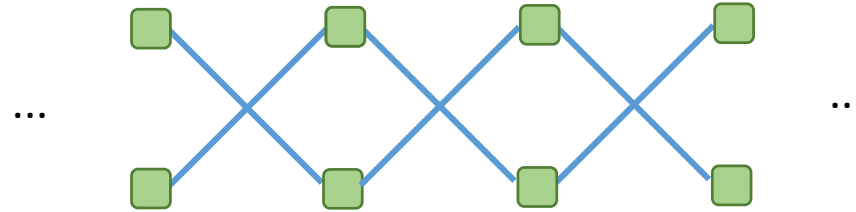
$$[H, G_p] = 0$$

Nonlocal symmetries



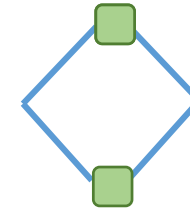
$$[H, G_u] = 0$$

# Toric code strip



$$A_s = \sigma_1^x \sigma_2^x \sigma_3^x \sigma_4^x$$

star operators



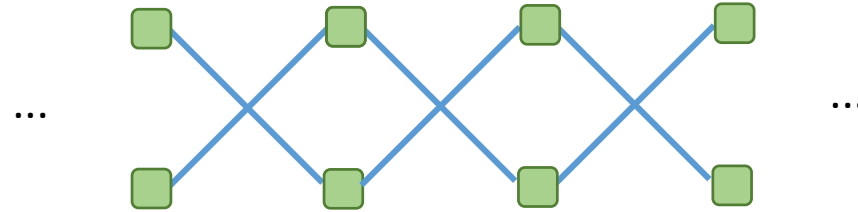
$$B_p = \sigma_1^z \sigma_2^z$$

plaquette operators

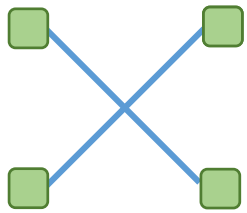
$$H_{\text{toric-ladder}} = -\lambda_A \sum_s A_s - \lambda_B \sum_p B_p$$

# U(1) toric code strip

same symmetries of  
WXY model

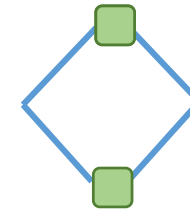


$$H_{\text{toric-ladder}} = -\lambda_A \sum_s A_s$$



star operators

$$A_s = \sigma_1^+ \sigma_2^+ \sigma_3^- \sigma_4^- + 5 \text{ terms}$$



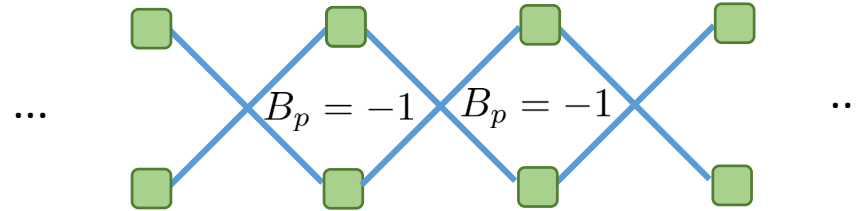
plaquette operators

$$B_p = \sigma_1^z \sigma_2^z$$

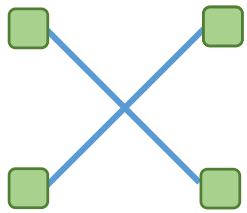
$$B_p = \mp 1$$

# U(1) toric code strip

same symmetries of  
WXY model



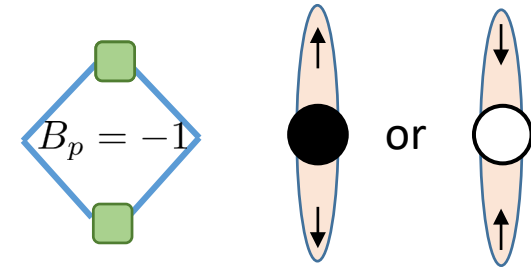
$$H_{\text{toric-ladder}} = -\lambda_A \sum_s A_s$$



star operators

$$A_s = \sigma_1^+ \sigma_2^+ \sigma_3^- \sigma_4^-$$

+ 5 terms



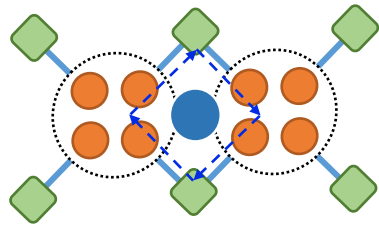
$$B_p = \sigma_1^z \sigma_2^z$$

Absence of visons:  
Model maps onto  
p-wave SC chain

$$H_- = -\lambda_A \sum_p \left[ c_{p+1}^\dagger c_p + c_p^\dagger c_{p+1} + c_{p+1}^\dagger c_p + c_p c_{p+1} \right]$$

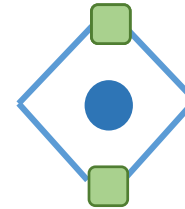
# WXY ladder / toric ladder side-by-side

visons



plaquette operators

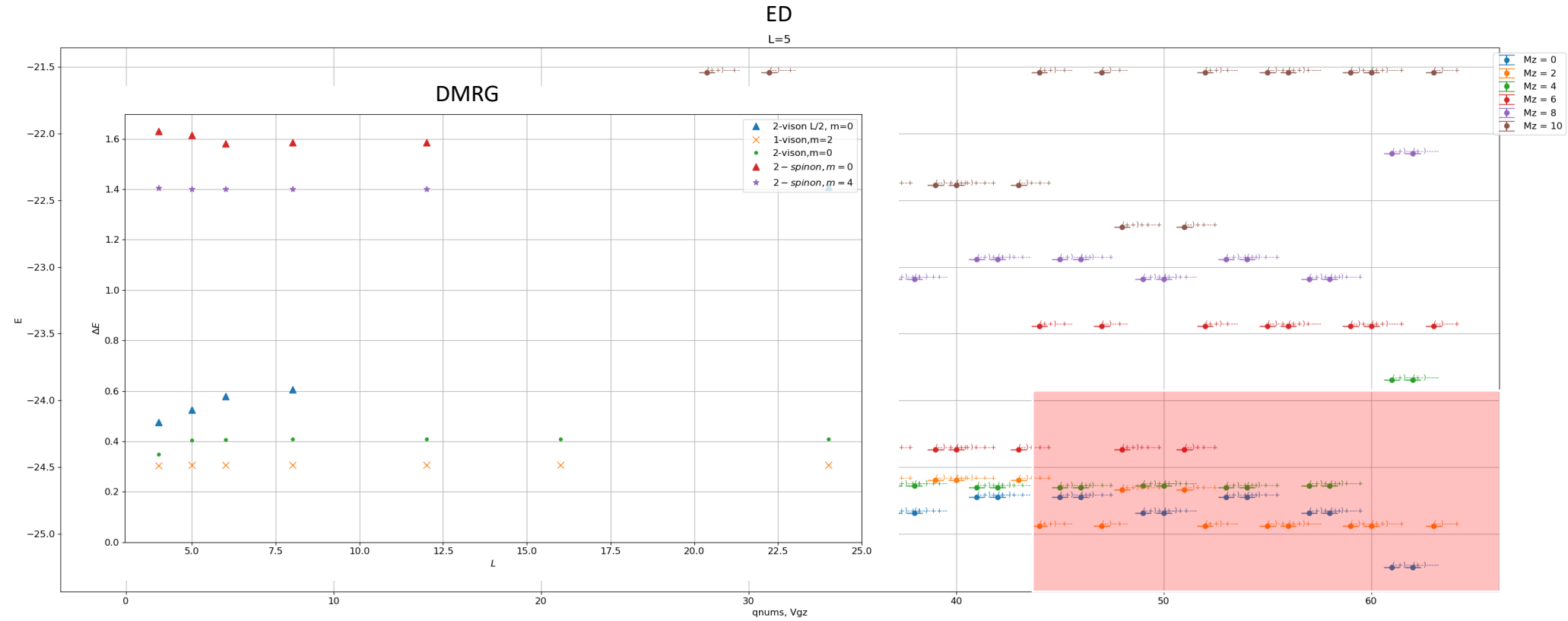
$$G_p = \mp 1$$



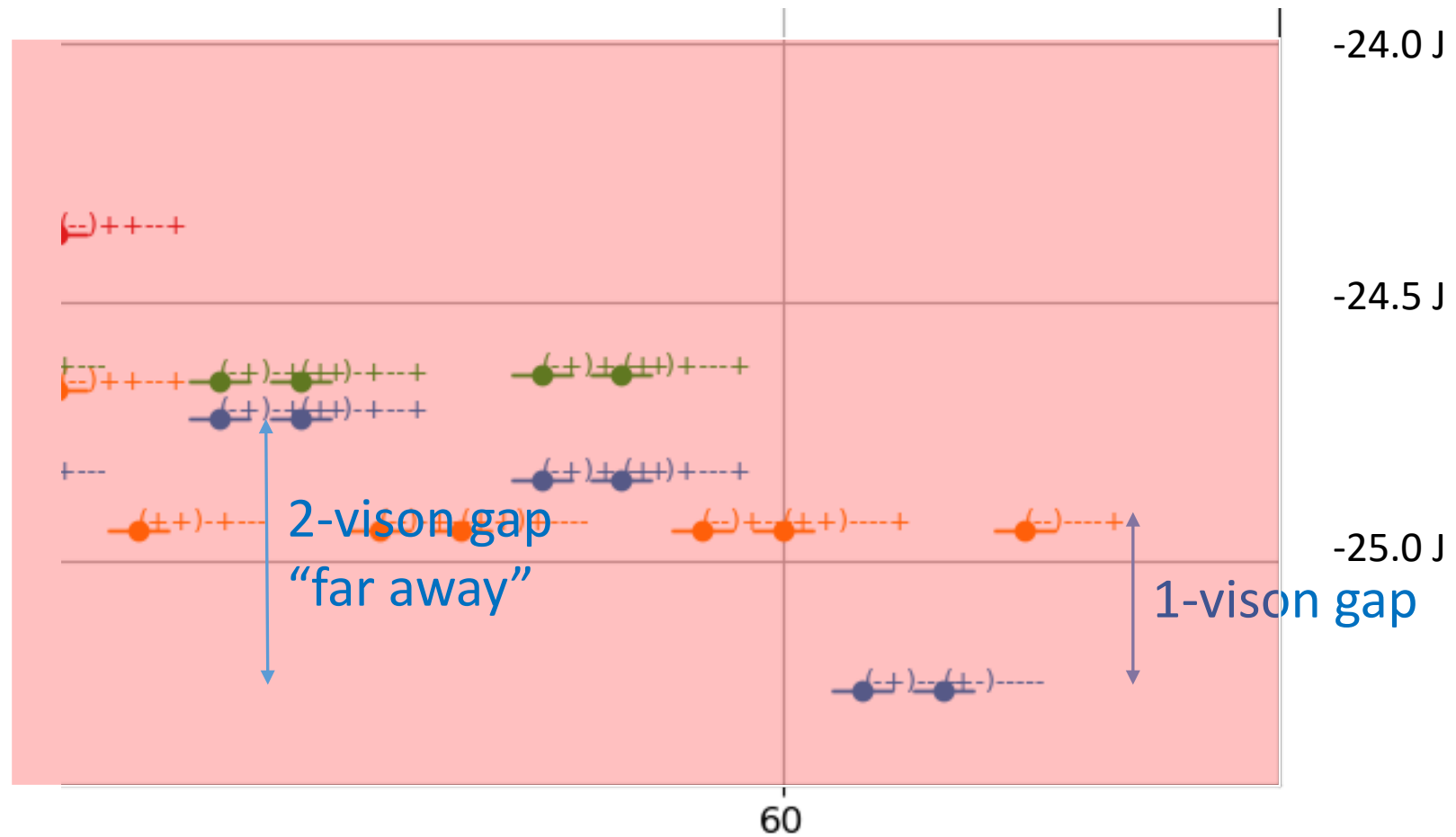
plaquette operators

$$B_p = \pm 1$$

# WXY ladder spectrum (preliminary data)

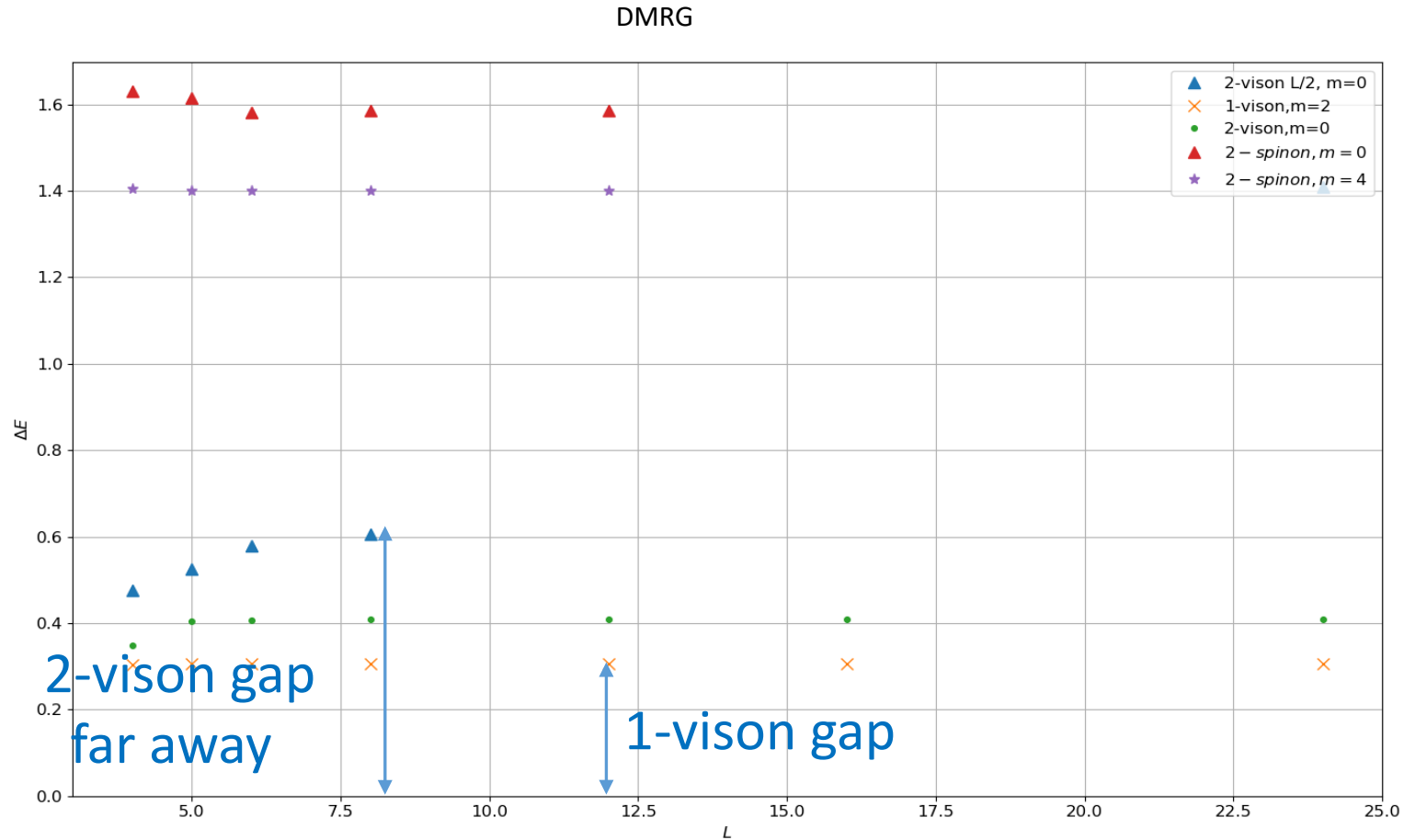


# WXY ladder spectrum (preliminary data)

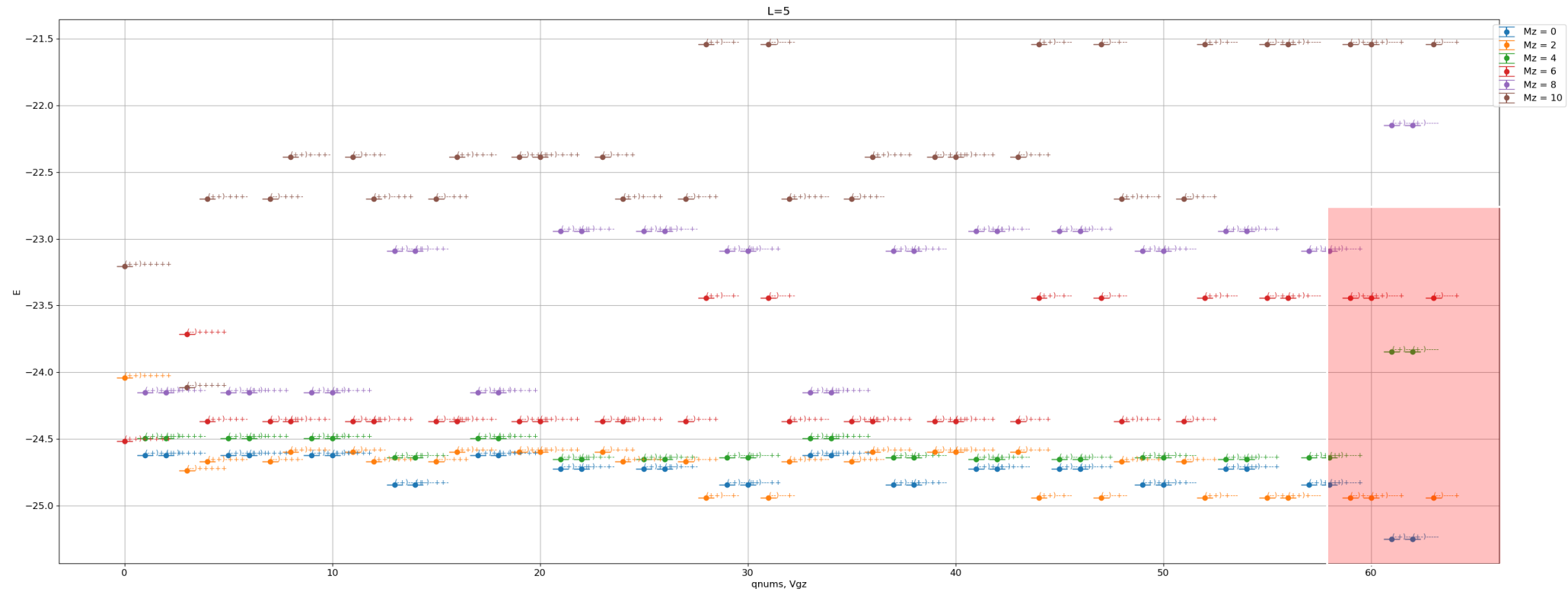




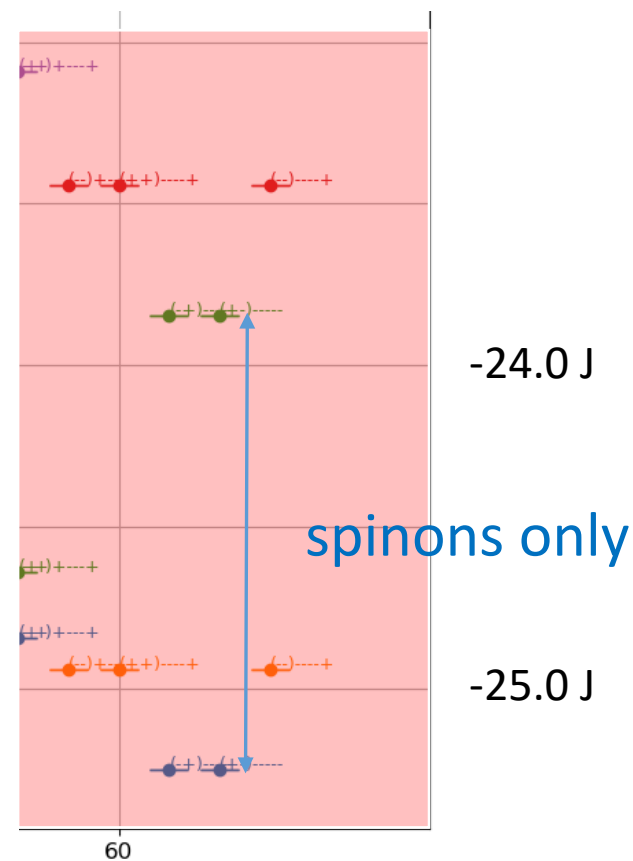
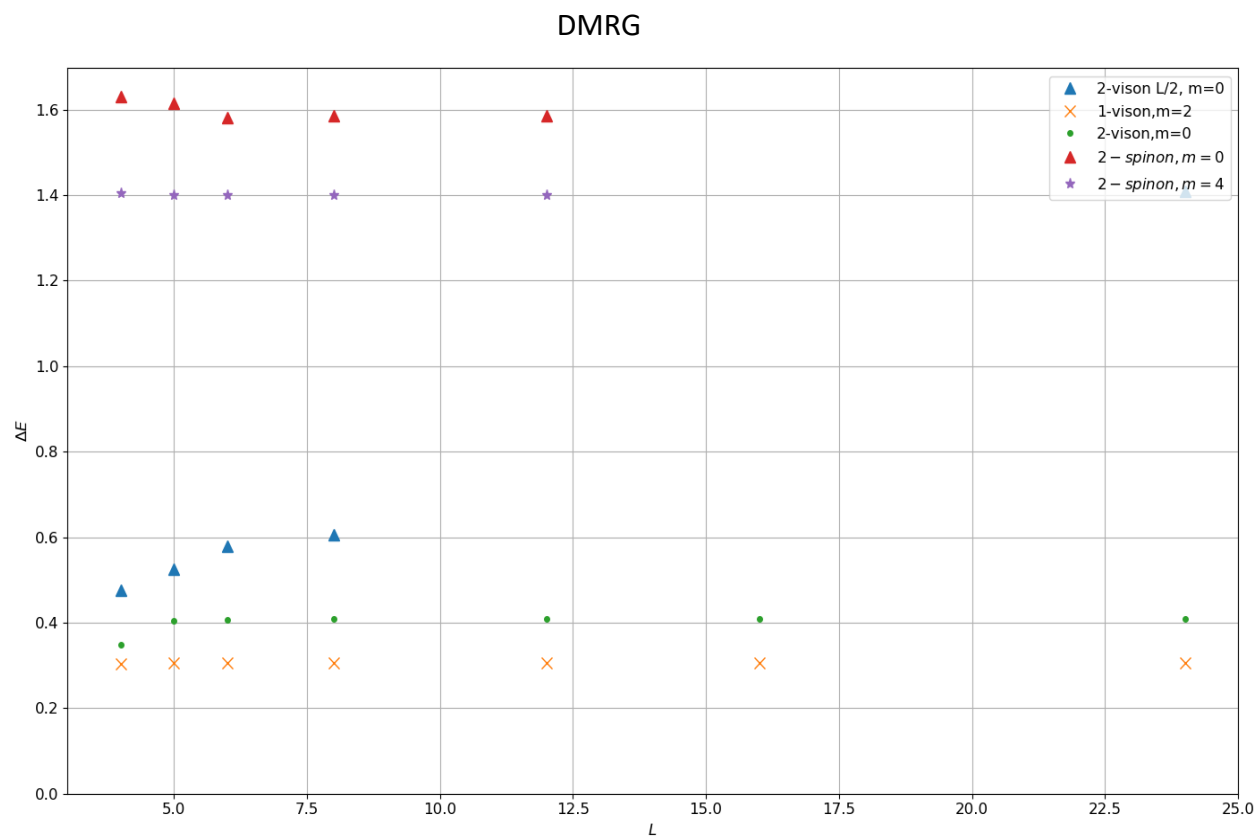
# WXY ladder spectrum (preliminary data)



# WXY ladder spectrum (preliminary data)



# WXY ladder spectrum (preliminary data)



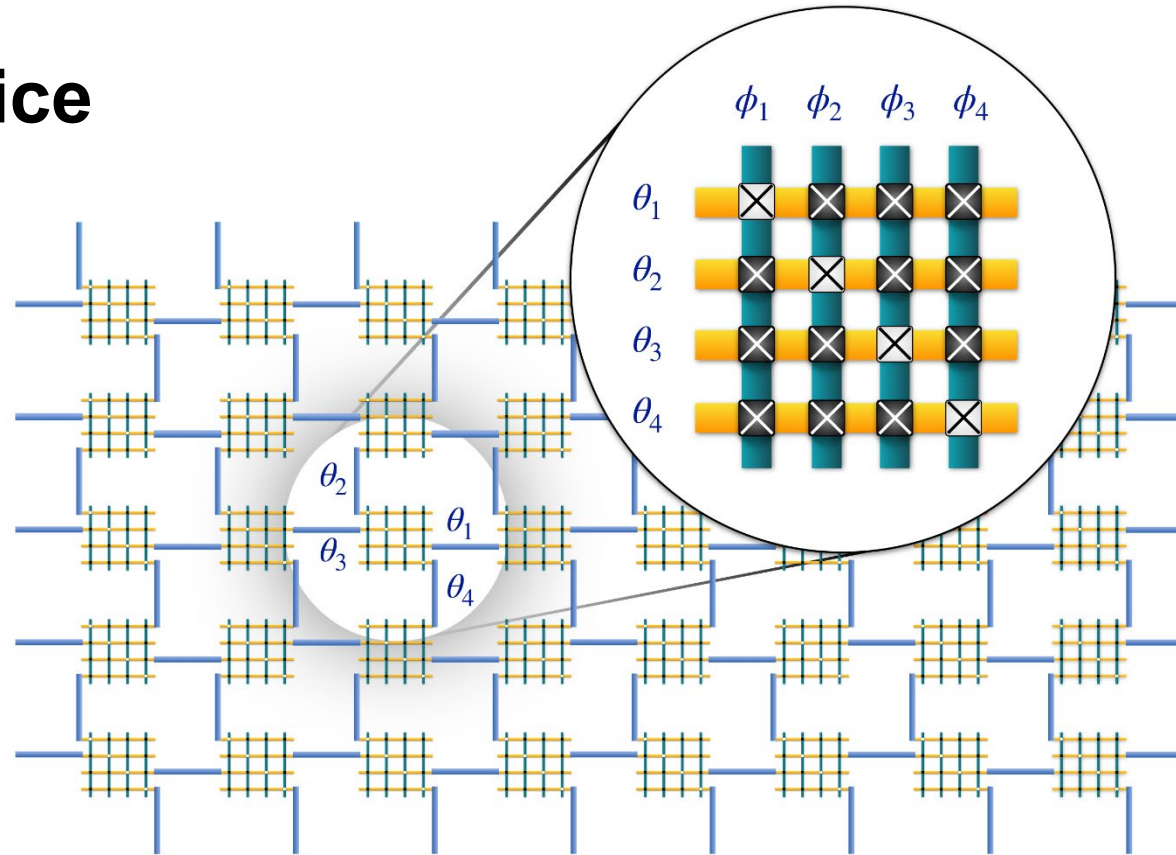
# WXY ladder spectrum (preliminary data)

vison and spinon gaps

$$\Delta_v \sim \Delta_s \sim \mathcal{O}(J)$$

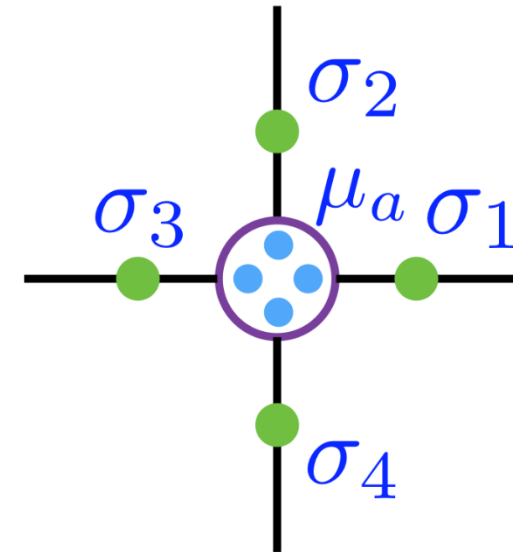
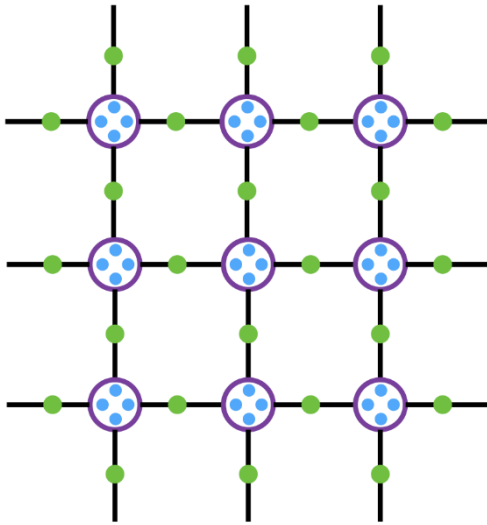
# 2D version

## Lattice



# WXY model

$$H_J = -J \sum_s \sum_{ia \in s} W_{ia} \mu_i^+ \sigma_a^- + H.c.$$



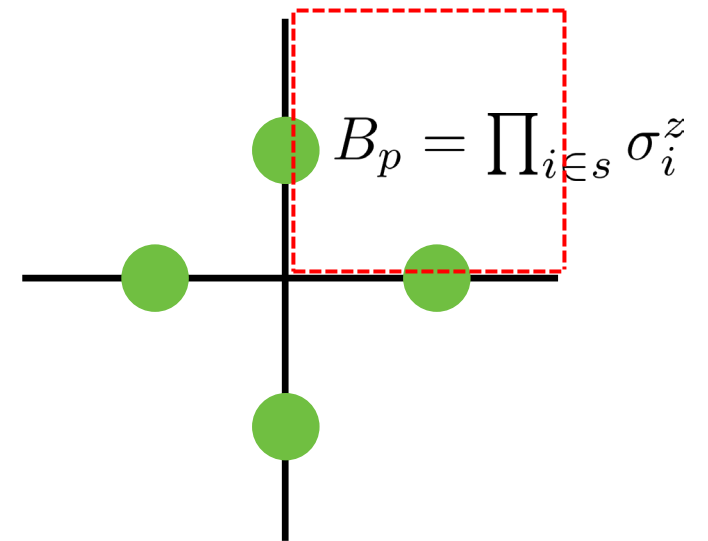
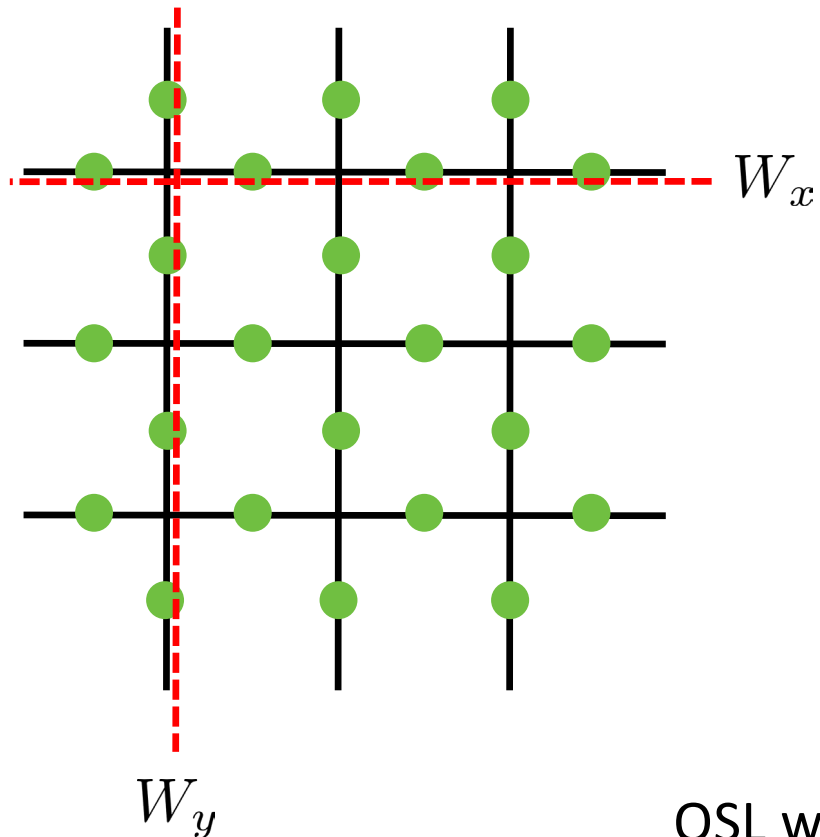
QSL with gap of order  $J$  ?

**U(1) toric code – YES!**

UV/IR mixing, strange topological degeneracies,  
Hilbert space fragmentation, possibly non-Abelian

# U(1) symmetry-enhanced toric

$$H_J = -J \sum_s \mathcal{A}_s - \lambda \sum_p B_p$$

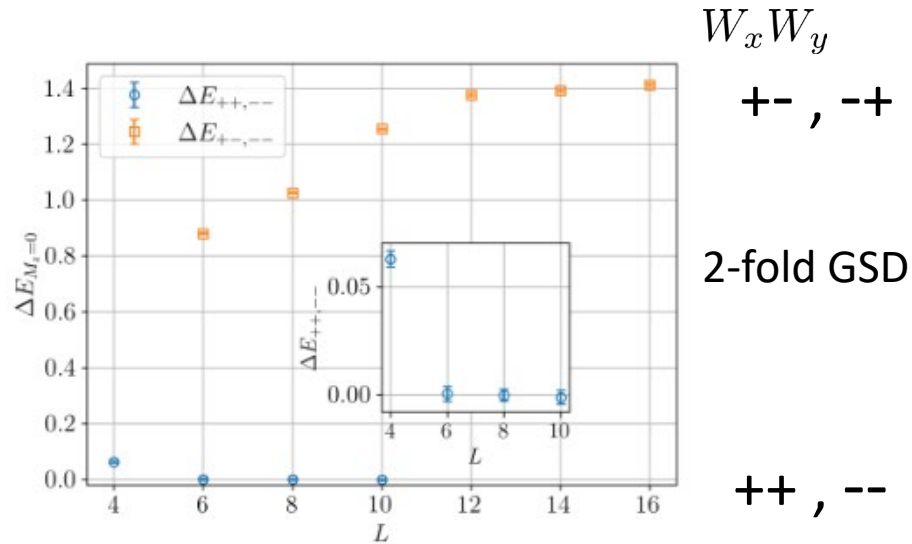


$$\mathcal{A}_s = \sigma_1^+ \sigma_2^+ \sigma_3^- \sigma_4^- + 5 \text{ terms}$$

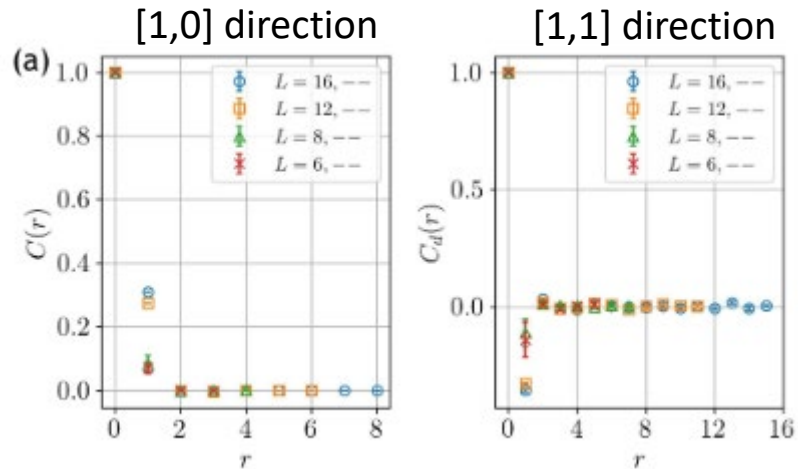
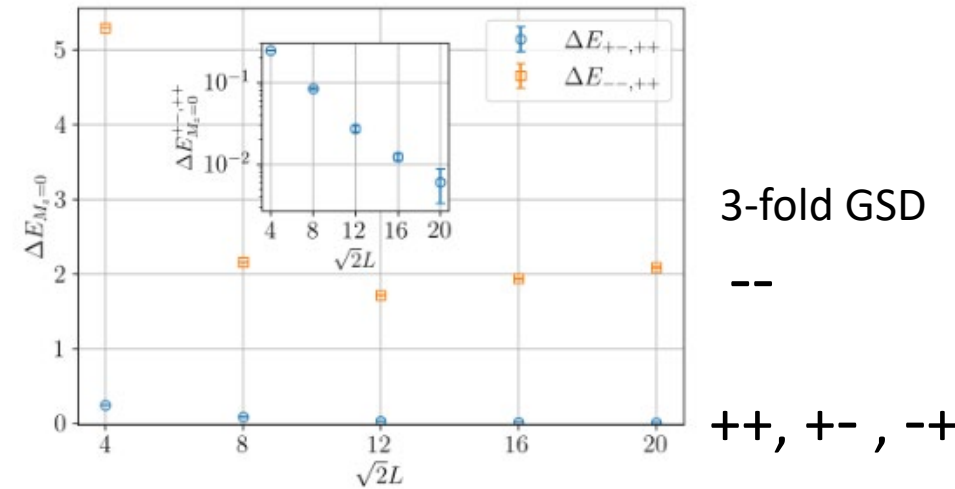
QSL with gap of order  $J$  !

# U(1) symmetry-enhanced toric

0° compactification



45° compactification



Gapped spin-liquid

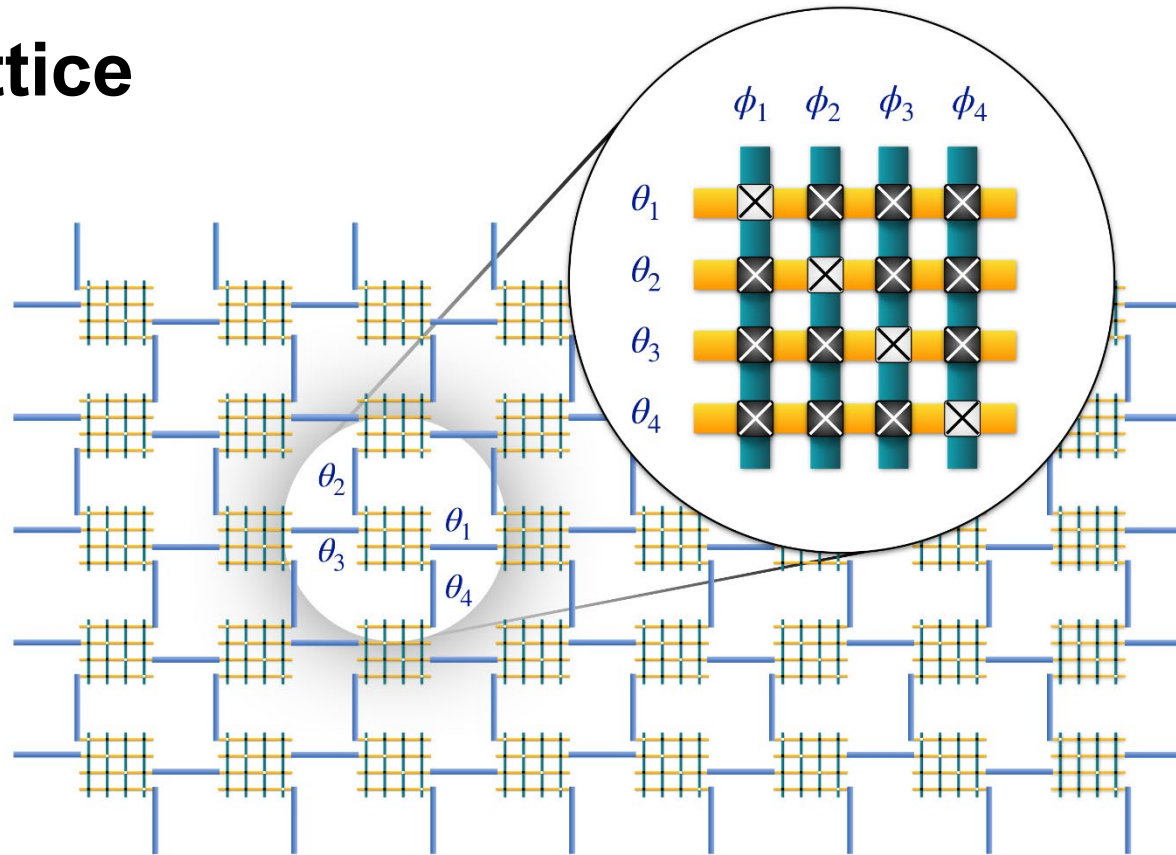
Spin-spin correlation

UV/IR mixing, strange topological degeneracies,  
Hilbert space fragmentation, possibly non-Abelian



# Motivated by the SC wire array!

## Lattice



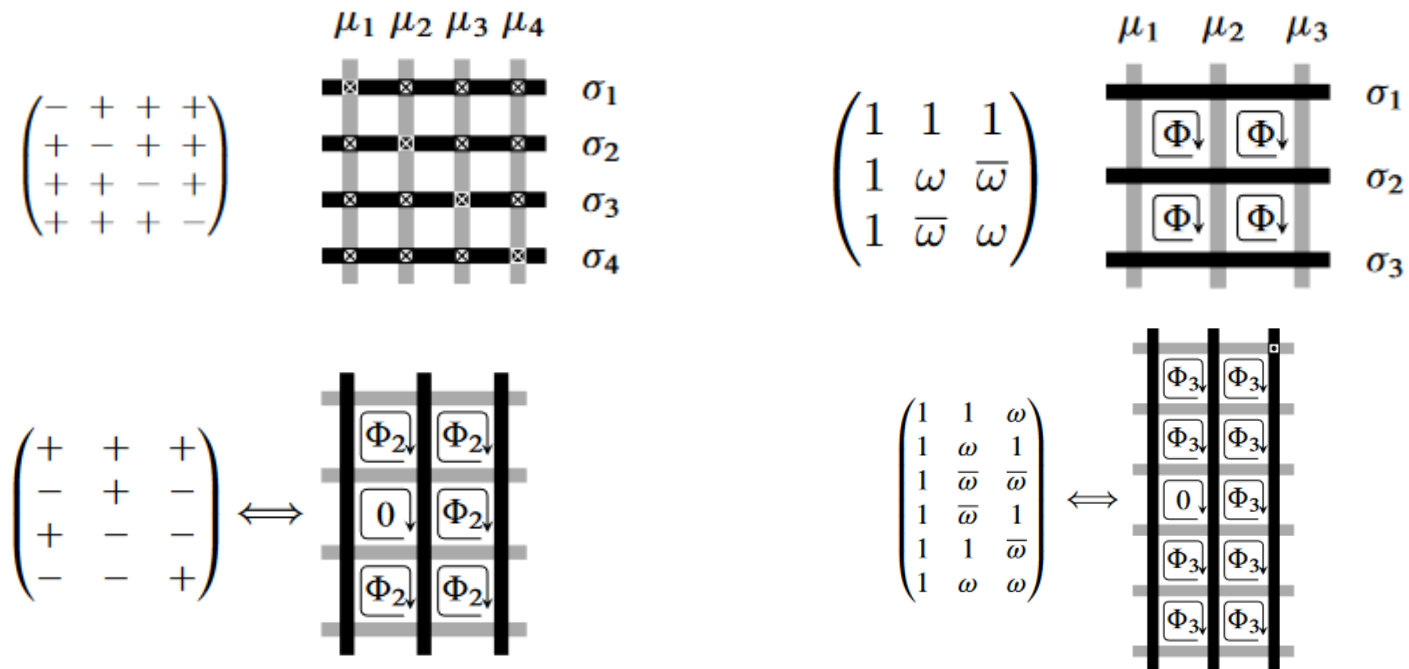
# Abelian combinatorial gauge symmetry

Generalized framework for all Abelian groups and lattice connectivities

arXiv:2212.03880

Yu, Goldstein, Green, Ruckenstein, and Chamon

W matrices translate into “waffle” arrays



E.g. application

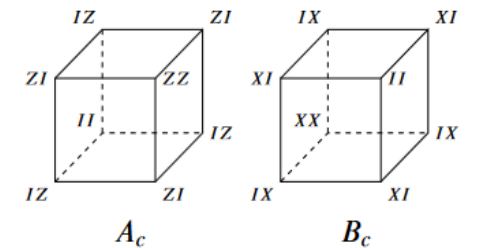
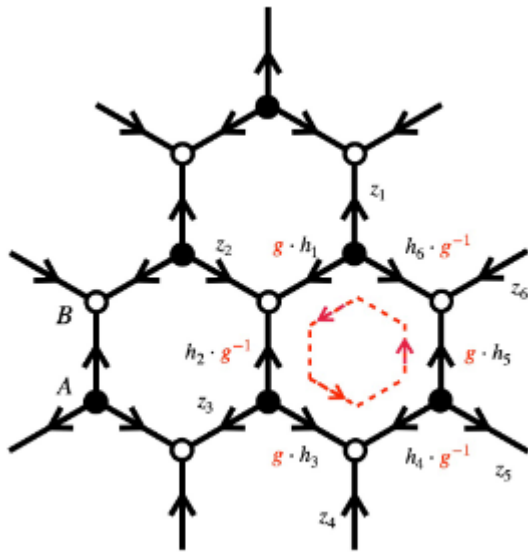


FIG. 3. Illustration of the two types of operators in the Hamiltonian of the Haah's code.

# Non-Abelian combinatorial gauge symmetry

arXiv:2209.14333  
Green and Chamon



Quaternion group

$$\begin{aligned}
 v(+1) &= [+++ ] & v(-1) &= [--- ] \\
 v(+i) &= [+ - + -] & v(-i) &= [- + - +] \\
 v(+j) &= [ + + - -] & v(-j) &= [- - + +] \\
 v(+k) &= [- + + -] & v(-k) &= [+ - - +]
 \end{aligned}$$

$$W = \frac{1}{4} \begin{bmatrix} v(f_1) & v(h_1) & v((f_1 h_1)^{-1}) \\ v(f_2) & v(h_2) & v((f_2 h_2)^{-1}) \\ \vdots & \vdots & \vdots \\ v(f_{64}) & v(h_{64}) & v((f_{64} h_{64})^{-1}) \end{bmatrix}$$

64 × 12 matrix

lots of SC wires and junctions!

General (discrete) non-Abelian groups:  
Yu, Green and Chamon, in preparation

# Summary

- Framework for constructing systems with **exact** (not emergent) local **Abelian** and **non-Abelian gauge symmetries** using **physical** interactions
- Proposed a 2-leg ladder SC wire array with **non-perturbative spinon/vison gap**
- Presented a U(1)-symmetry enhanced toric code with unusual topological features

# Collaborators

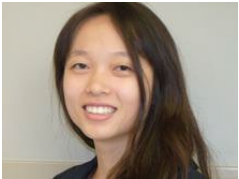
2020: PRL  
2021: 2x PRB, 2x PRX Quantum  
2022: 3x SciPost submissions  
2023: arXiv



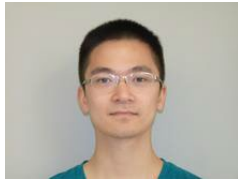
Dmitry Green



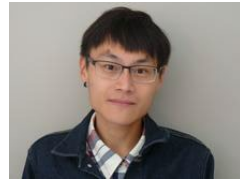
Zhi-Cheng Yang



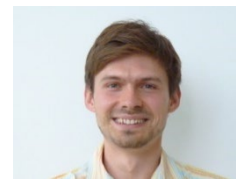
Shiyu Zhou



Elliot Yu



Kai-Hsin Wu



Aleksey Khudorozhkov



Guilherme Delfino



Garry Goldstein

@ MIT LL

@ ColdQuanta

@ Univ. of Cambridge



Anders Sandvik



Andrei Ruckenstein



Andrew Kerman



Edward Dahl



Claudio Castelnovo



Maria Zelenayova



Oliver Hart