Probing the equation of state of nuclear matter with heavy ion collisions

Zbigniew Chajęcki

for the HiRA Collaboration

WESTERN MICHIGAN
UNIVERSITY

Anatomy of Neutron Stars

Neutron stars are a rich playground for the EoS and symmetry energy

- Inner crust: $\rho < \rho_0$
  - Mix of neutron gas and lattice of finite nuclei
  - $S(\rho)$ determines the thickness of the crust, frequency of star quakes
- Boundary region → pasta
- Outer core: $\rho > \rho_0$
  - Neutron-rich nuclear matter
  - Determines radii, moments of inertia
- Inner core → ????

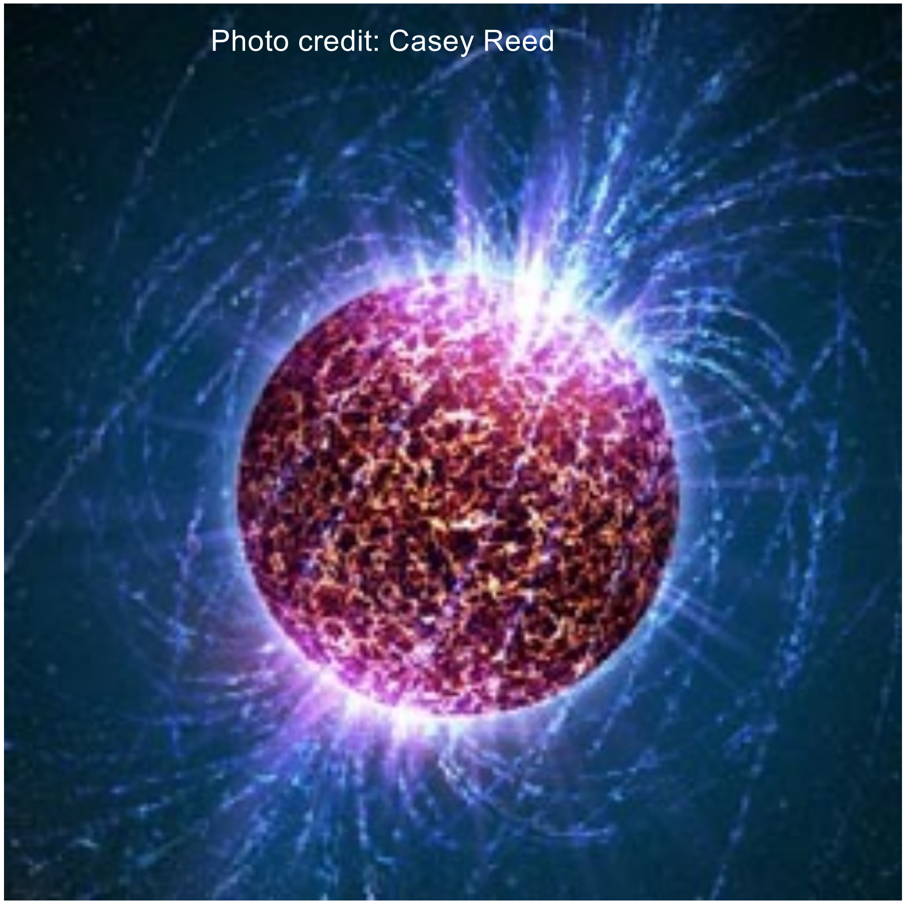


Photo credit: Casey Reed

Need to know the symmetry energy $S(\rho)$ at both high and low density to understand neutron stars. Laboratory measurements allow us to probe the nuclear matter at various densities.

EOS: Symmetry energy

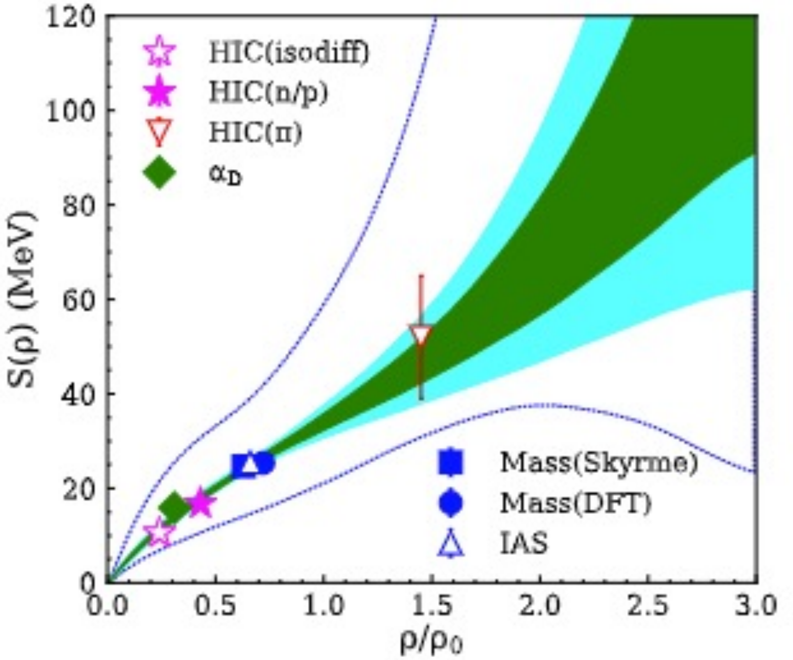
To probe the EOS of asymmetric nuclear matter (neutron stars) we need to be able to study the nuclear matter at **various densities** and **asymmetries**

$$E/A\,(\rho,\delta) = E/A\,(\rho,0) + \delta^2 \cdot S(\rho)$$

$$\delta = (\rho_n - \rho_p)/(\rho_n + \rho_p) = (N-Z)/A$$

The only way we can do it in the laboratory is via heavy ion collisions:

- nucleus – nucleus collisions at higher densities ⇒ require higher beam energies and new detectors
- Increase sensitivity to symmetry energy by increasing δ (asymmetry between the # of neutrons and protons)
- To obtain S(ρ), P(ρ), .. we need theory.

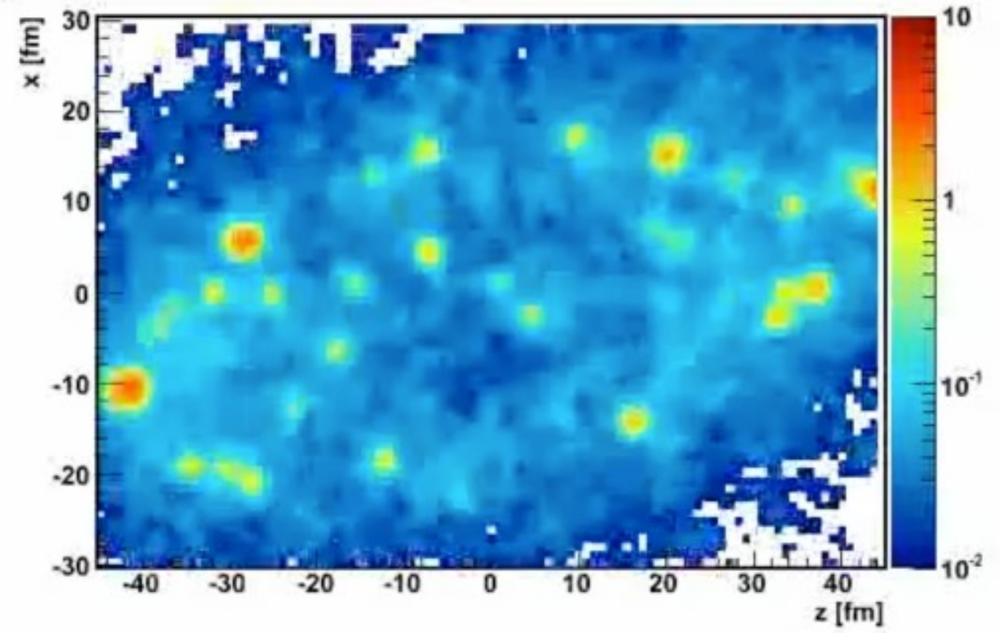


See T. Tsang's talk

Modeling heavy ion collisions

Our tool: Transport models

- BUU – Boltzmann-Uehling-Uhlenbeck
- QMD – Quantum Molecular Dynamics
- AMD – Antisymmetrized Molecular Dynamics
- Simulates the time-dependent evolution of the collision



Danielewicz, Acta. Phys. Pol. B 33, 45 (2002)

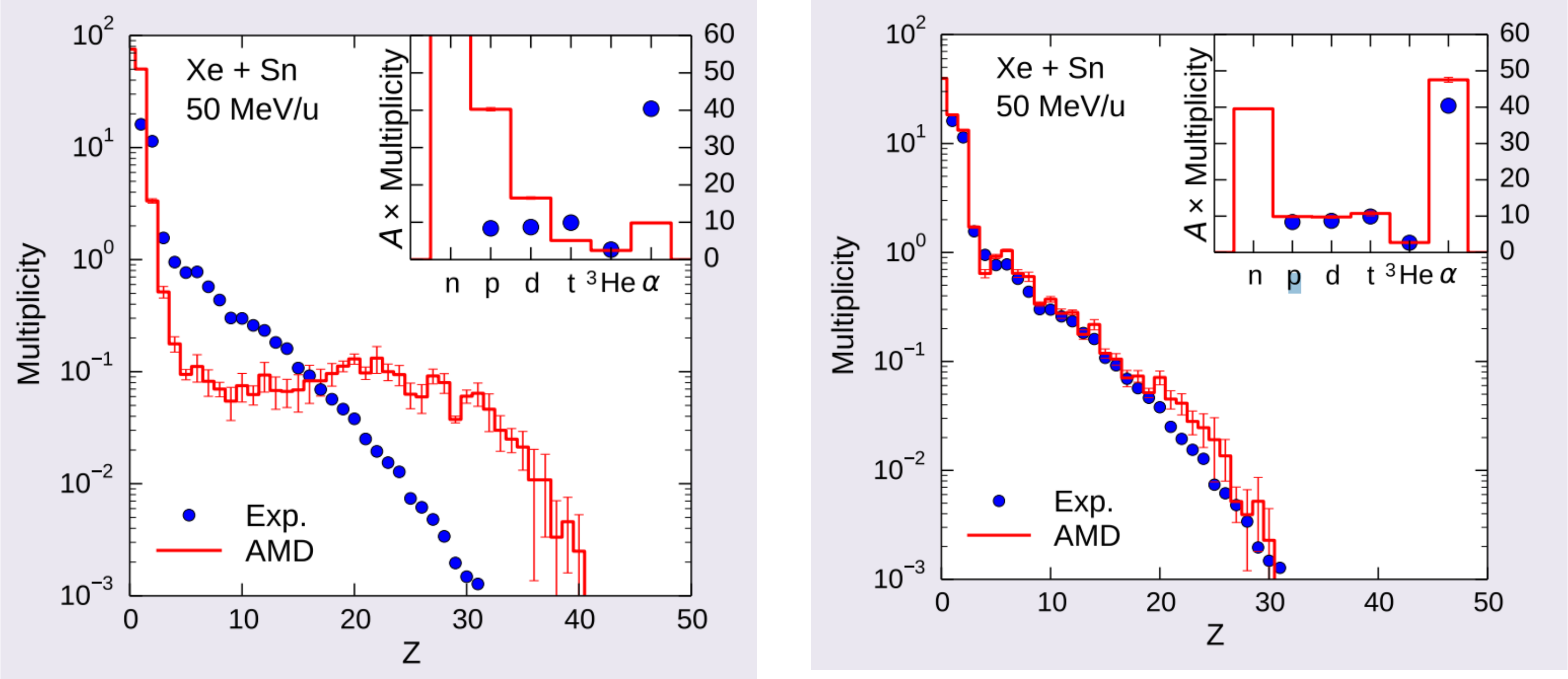
Danielewicz, Bertsch, NPA533 (1991) 712

Main ingredients

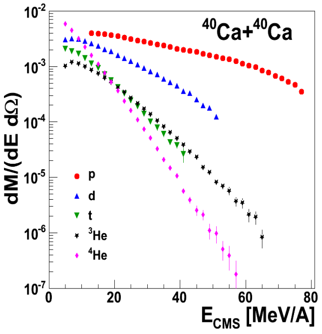
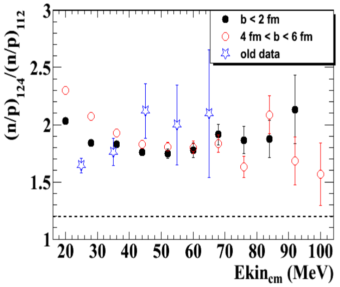
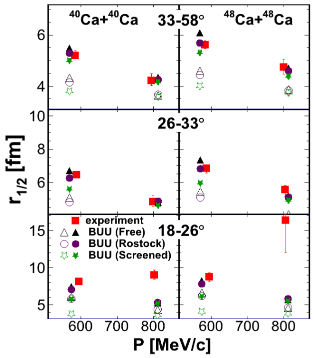
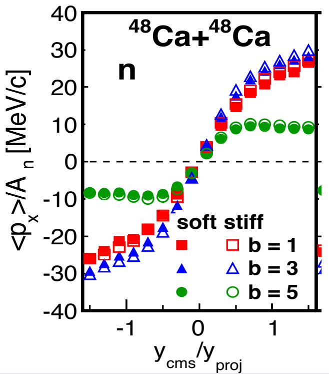
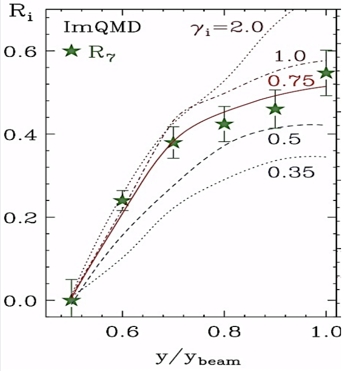
- Nucleons in mean-field
- **Symmetry energy**
- Momentum (in-)dependent nuclear interaction
- ***effective mass***
- **In-medium cross section**
- **Cluster production**

Importance of clusters in AMD

Multiplicity in heavy ion collisions
Without (left) and with (right) dynamical cluster production



What we hope to learn?

Observables	**Spectra**	**(Double-) ratios**	**Femtoscopy**	**Flow**	**Isospin diffusion**
Transport model ingredients					
Symmetry energy		✓		✓	✓
Effective mass		✓		?	?
Cross section	✓	✓	✓	✓	✓
Cluster production	✓	✓	✓	✓	✓

Our approach: Use different isotopes
(fix Z of your initial system and vary N)

Momentum Dependence of Symmetry Energy Potential

Effective mass

- The effective mass describes how the potential energy depends on the momentum

$$m^* = \frac{m}{1 + \frac{m}{p}\frac{\partial V}{\partial p}}$$

- At saturation density this reduction is ~70% from the free nucleon mass
- In asymmetric matter the potentials that neutrons and protons feel are expected to be different → effective-mass splitting

$$\Delta m^*_{np} = \frac{m^*_n - m^*_p}{m_N}$$

Constraints on the effective mass

- Optical model energy dependence: constrains m* for $\rho \leq \rho_0$
- Elliptical flow: constrains m^*_s at $\sim 2\rho_0$
- ISGQR and IVGQR: constrains m^*_s and m^*_v at ρ_0 $(m_n^* - m_p^*) \propto (m_s^* - m_v^*)$
- **n/p spectral ratio** in heavy ions
  - Constrains Δm^*_{np} both above and below saturation density

Thermal properties of proto-neutron stars are sensitive to effective mass m^* which changes the temperature

- Changes neutrino spectrum
- Changes neutrino mean free path (& cross section)
- Changes neutron flux from (ν,n) reactions (r-process)

- m^*_{np} also affects equilibrium ratio of n and p in Big Bang Nucleosynthesis, neutrino opacity in supernovae and neutron stars, and level densities

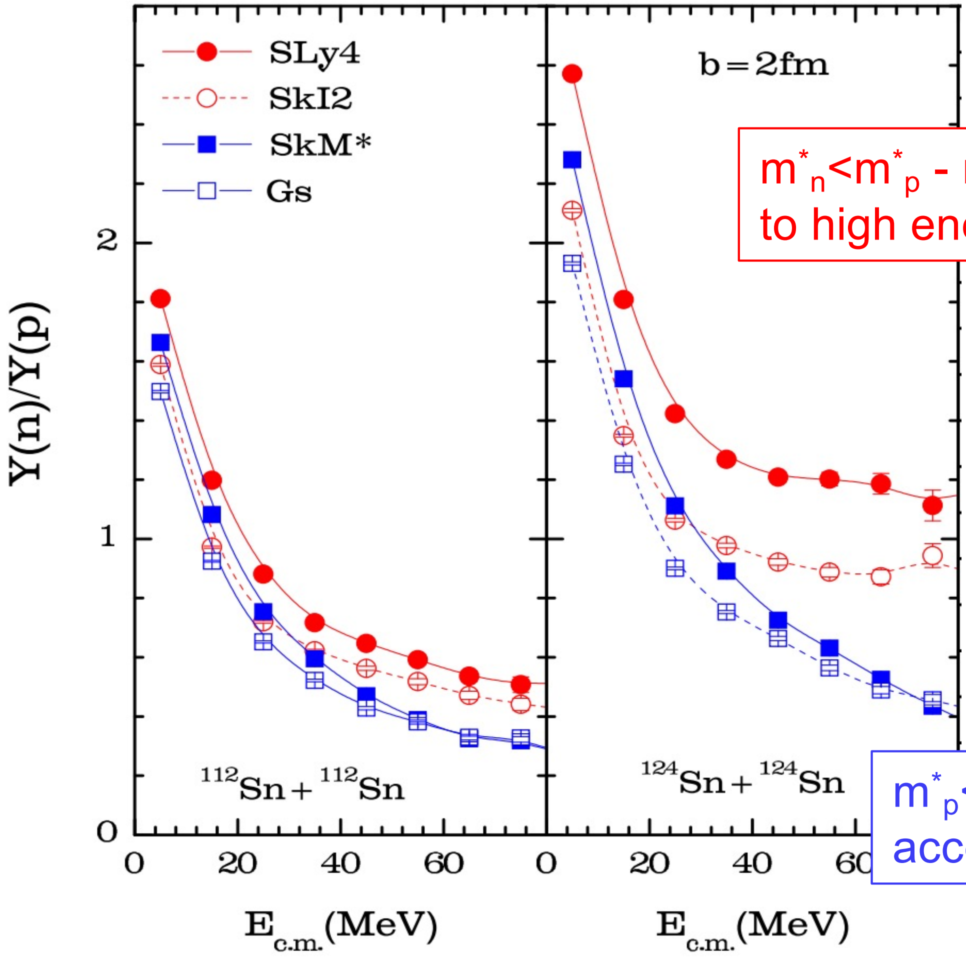
Effective mass in simulations

$m^*_p > m^*_n$

$m^*_n > m^*_p$

○ = stiff AsyEOS

● = soft AsyEOS



$m^*_n < m^*_p$ - more neutrons accelerated to high energies

$m^*_p < m^*_n$ - more protons accelerated to high energies

ImQMD, Zhang *et al*. PLB **732**, 186 (2014)

$$U_{sym}\left(\rho(\vec{r}),\vec{p},\delta\right) = \left(\frac{1}{2m^*} - \frac{1}{2m}\right)p^2 + U_{sym}\left(\rho(\vec{r}),0,\delta\right)$$

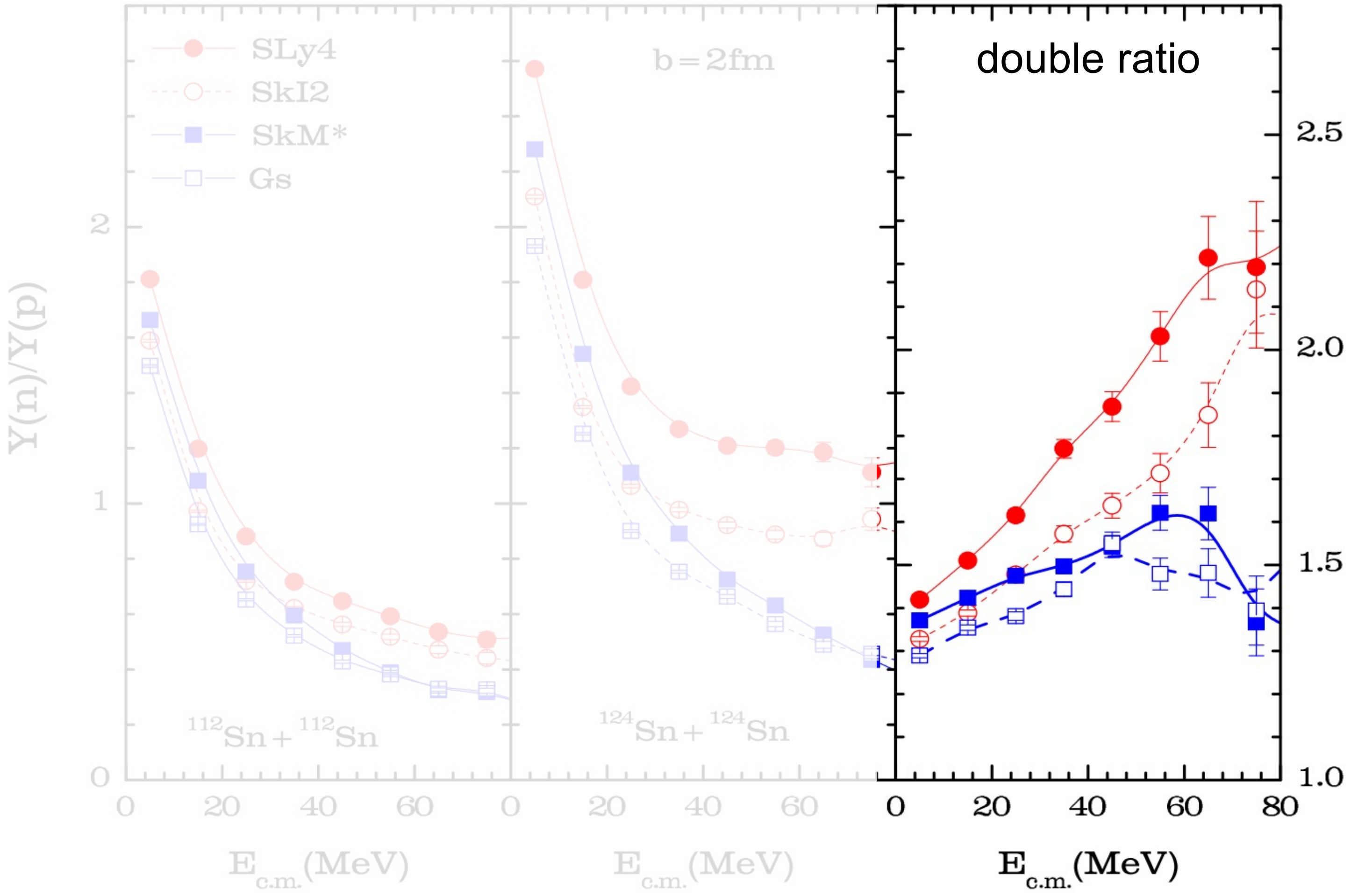
acceleration:

$$\boxed{a = \frac{dv}{dt} = \frac{\nabla U}{m^*}}$$

Effective mass in simulations

$m^*_p > m^*_n$ ○ = stiff AsyEOS

$m^*_n > m^*_p$ ● = soft AsyEOS



ImQMD, Zhang *et al*. PLB **732**, 186 (2014)

$$DR(n/p) = \frac{Y(n)/Y(p)_{sys1}}{Y(n)/Y(p)_{sys2}}$$

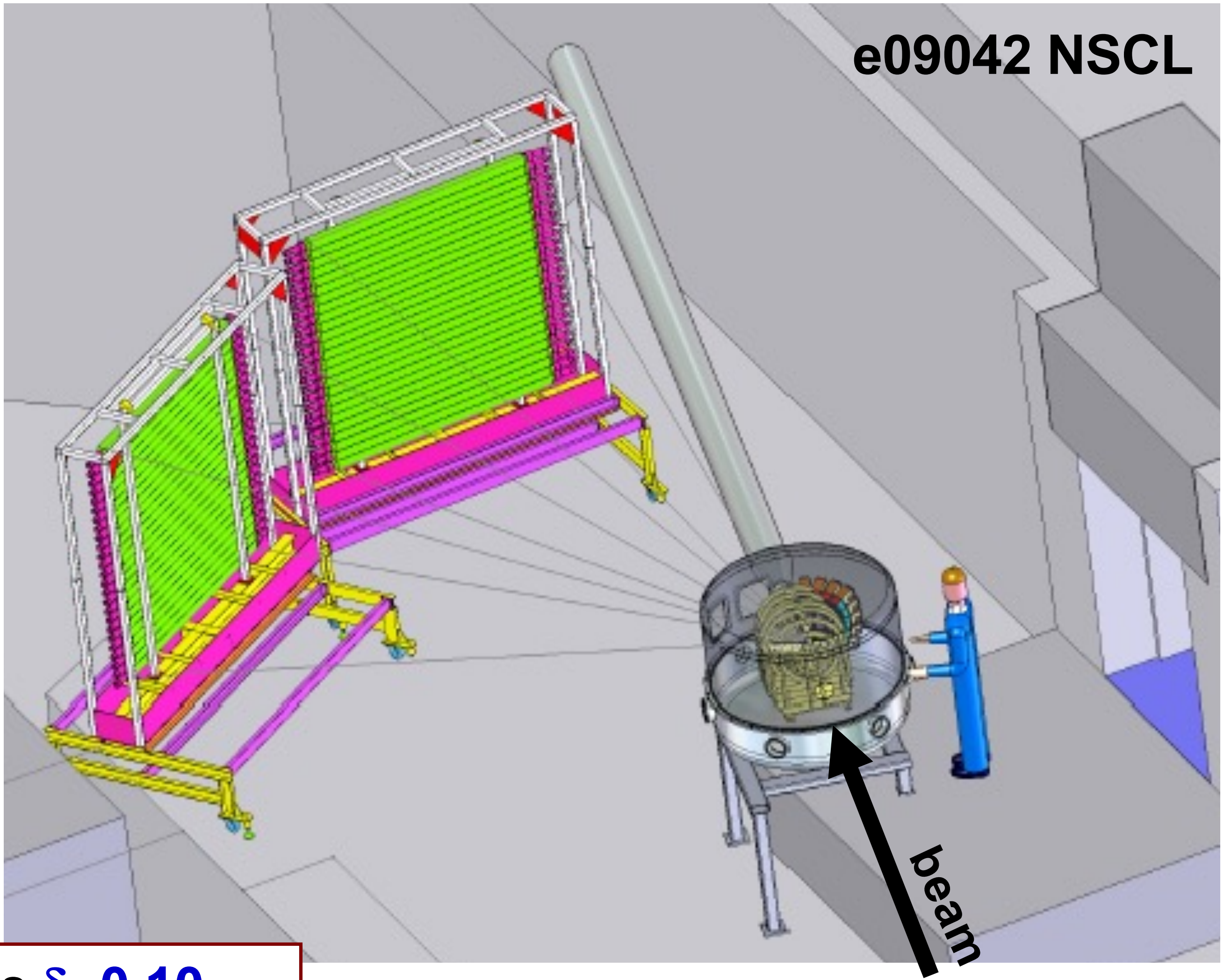
Divide (coalescence) n/p ratios for two reactions

- Minimize systematic uncertainties in detection efficiencies of neutrons and charged particles
- Reduces effects from the Coulomb force

Experiment

^{112}Sn+^{112}Sn and ^{124}Sn+^{124}Sn at 50 and 120 MeV/A

measure emitted neutrons, protons and light clusters from central collisions



$$E/A(\rho,\delta) = E/A(\rho,0) + \delta^2 \cdot S(\rho)$$

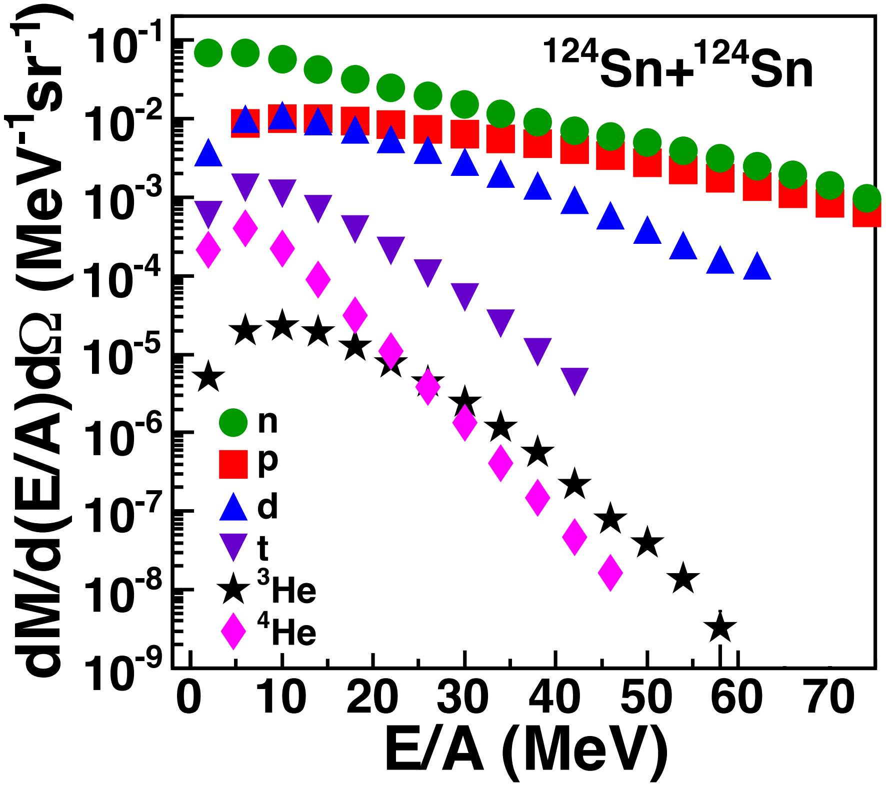
$$\delta = (\rho_n - \rho_p)/(\rho_n + \rho_p) = (N-Z)/A$$

^{112}**Sn**: **50** protons, **62** neutrons **δ~0.10**

^{124}**Sn**: **50** protons, **74** neutrons **δ~0.19**

D. D. S. Coupland *et al*, PRC **94**, 011601 (R) (2016)

Constraining Δm^*_{np} in experiment



Coalescence spectra

$$\frac{dM_{n,coal}}{d\Omega d(E/A)} = \sum_i N_i \frac{dM(N,Z)}{d\Omega d(E/A)}$$

$$\frac{dM_{p,coal}}{d\Omega d(E/A)} = \sum_i Z_i \frac{dM(N,Z)}{d\Omega d(E/A)}$$

D. D. S. Coupland *et al*, PRC **94**, 011601 (R) (2016)

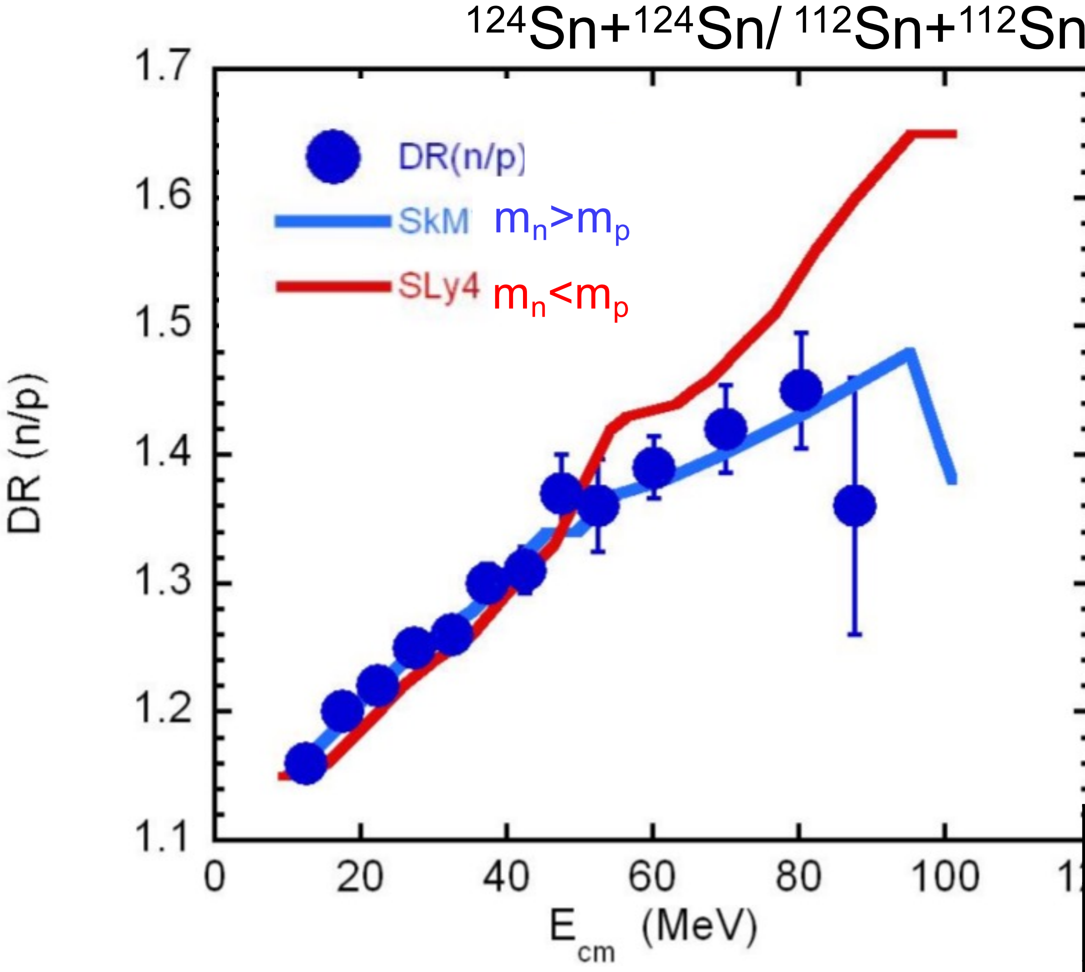
Current approach:

- Look at the coalesce spectra
  - So we can compare to transport models*

Constraining Δm^*_{np} in experiment

$$DR(n/p) = \frac{Y(n)/Y(p)_{sys1}}{Y(n)/Y(p)_{sys2}}$$

^{124}Sn+^{124}Sn/ ^{112}Sn+^{112}Sn
@ 120 MeV/u



D. D. S. Coupland *et al*, PRC **94**, 011601 (R) (2016)

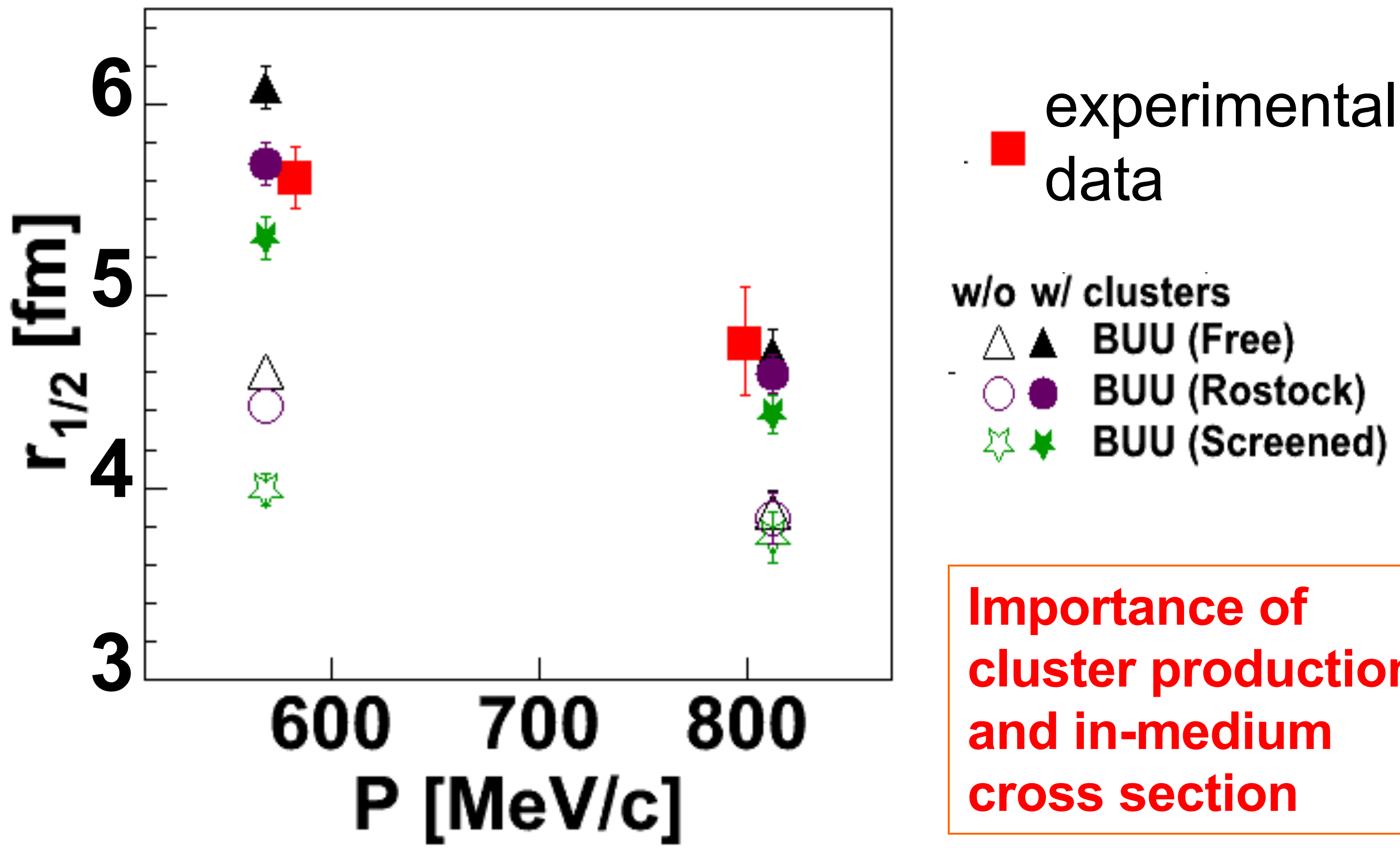
Current approach:

- Look at the coalesce spectra
    - So we can compare to transport models*
- Divide (coalescence) n/p ratios for two reactions
    - Minimize systematic uncertainties in detection efficiencies of neutrons and charged particles
    - Reduces effects from the Coulomb force
    - "removes" the uncertainties in the model due to cluster production

Skyrme	S_0(MeV)	L (MeV)	m_n^*/m_n	m_p^*/m_p
SLy4	32	46	0.68	0.71
SkM*	30	46	0.82	0.76

Cluster production and in-medium x-section

Proton femtoscopy in $^{48}Ca+^{48}Ca$ @ 80 AMeV

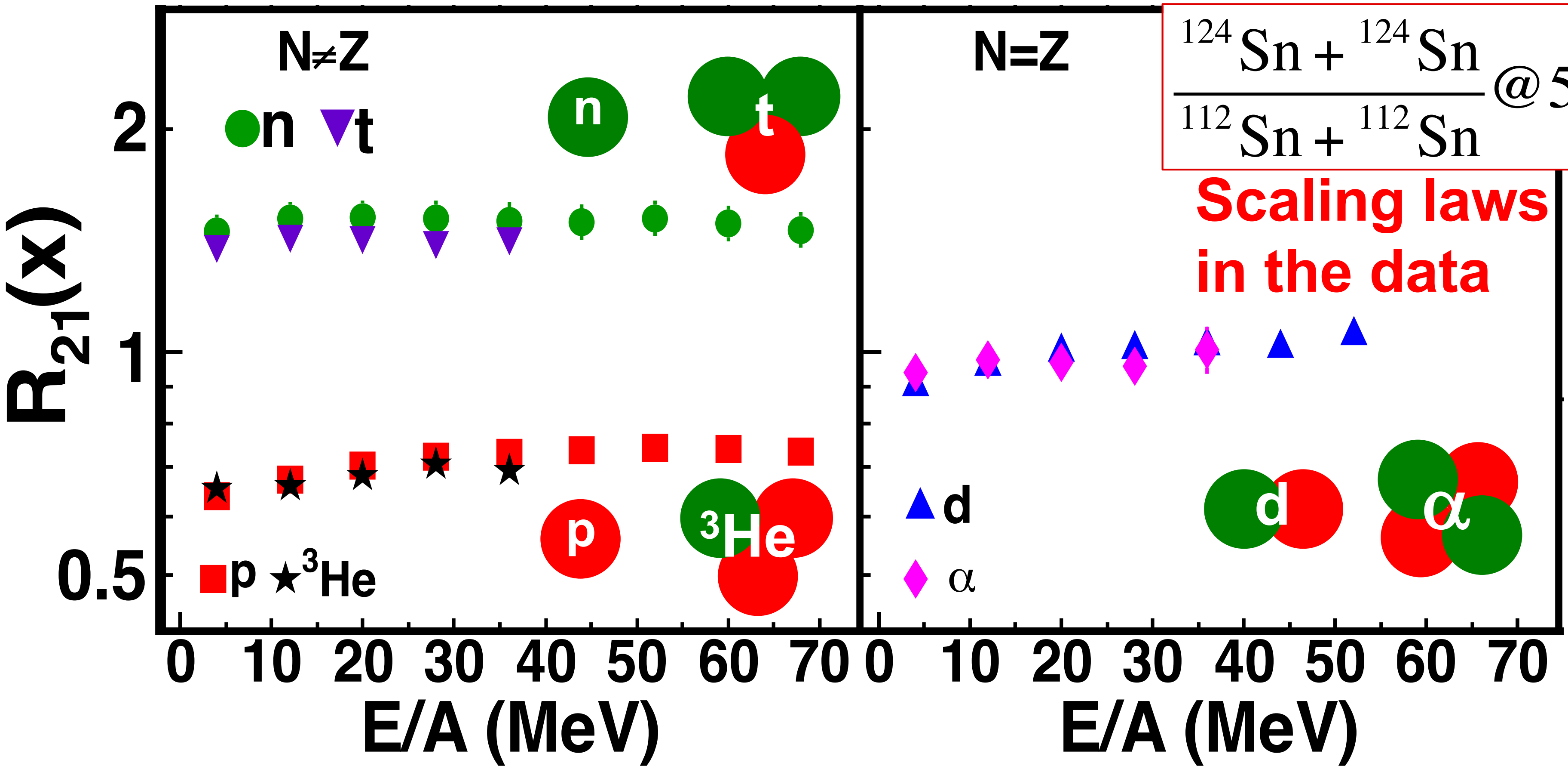


Importance of cluster production and in-medium cross section

Phys.Rev. C85 (2012) 014606

What can we learn from the data?

What can we learn from data?



$$\frac{{}^{124}\mathrm{Sn}+{}^{124}\mathrm{Sn}}{{}^{112}\mathrm{Sn}+{}^{112}\mathrm{Sn}}\ @50\mathrm{MeV}$$

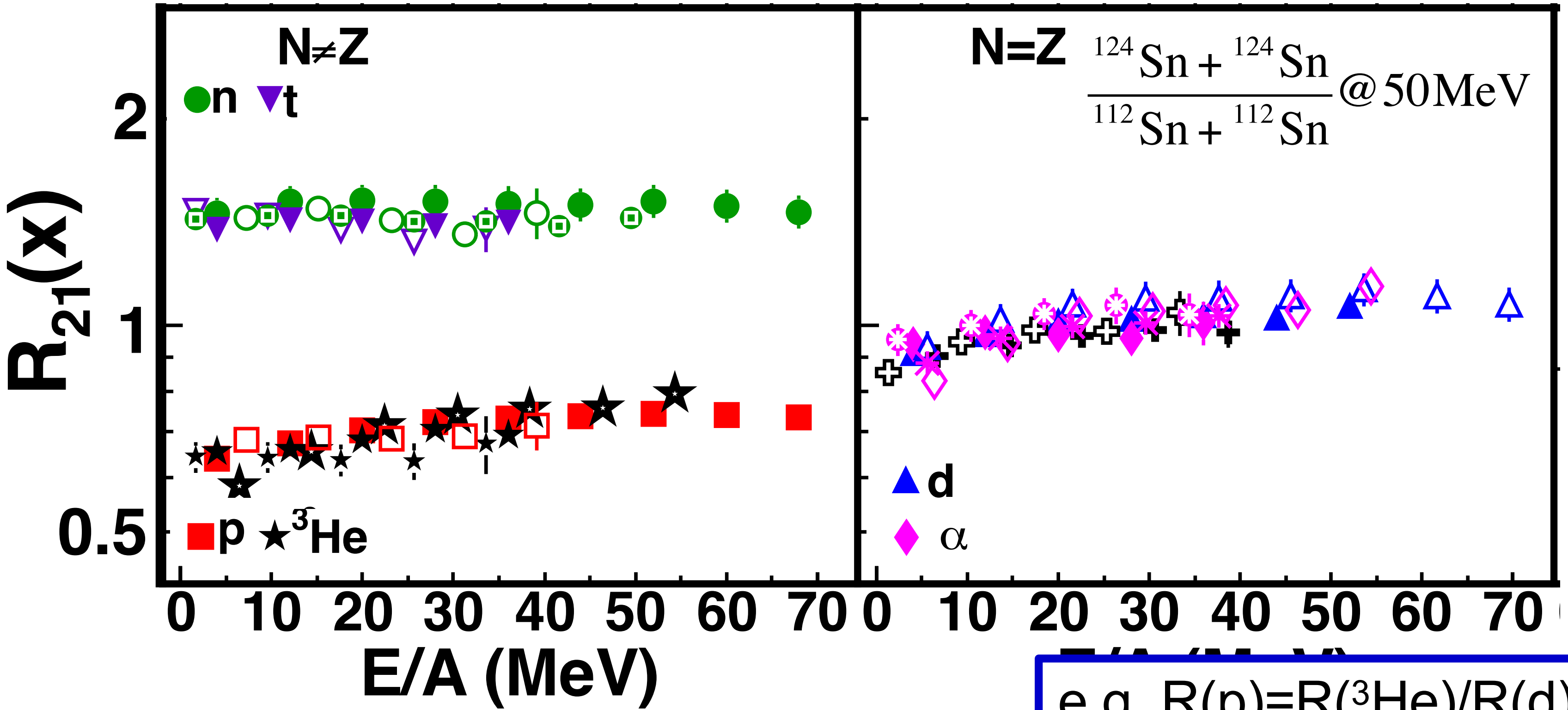
Scaling laws in the data

ArXiv:1402.5216

$$R_{21}(N,Z)=\frac{dM_2(N,Z)}{dM_1(N,Z)}$$

as a function of E_{CM}

Single particle ratios



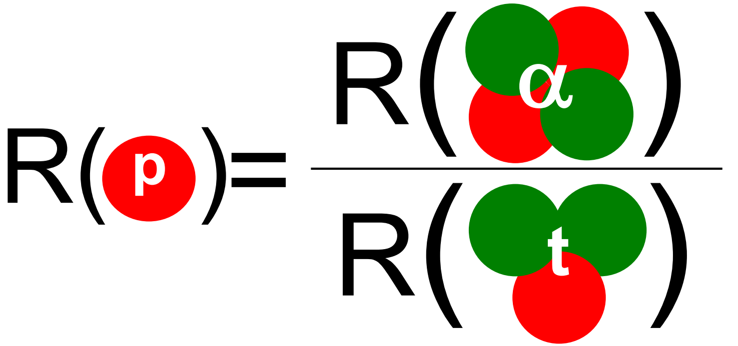
$$\boxed{R_{21}(N,Z) = \exp\left[\left(N\Delta\mu_n + Z\Delta\mu_p\right)/T\right]}$$

Useful relations:

$$R_{21}(N_1+N_2,Z_1+Z_2) = R_{21}(N_1,Z_1)\cdot R_{21}(N_2,Z_2)$$

$$R_{21}(N_1-N_2,Z_1-Z_2) = R_{21}(N_1,Z_1)/R_{21}(N_2,Z_2)$$

e.g. R(p)=R(³He)/R(d) and



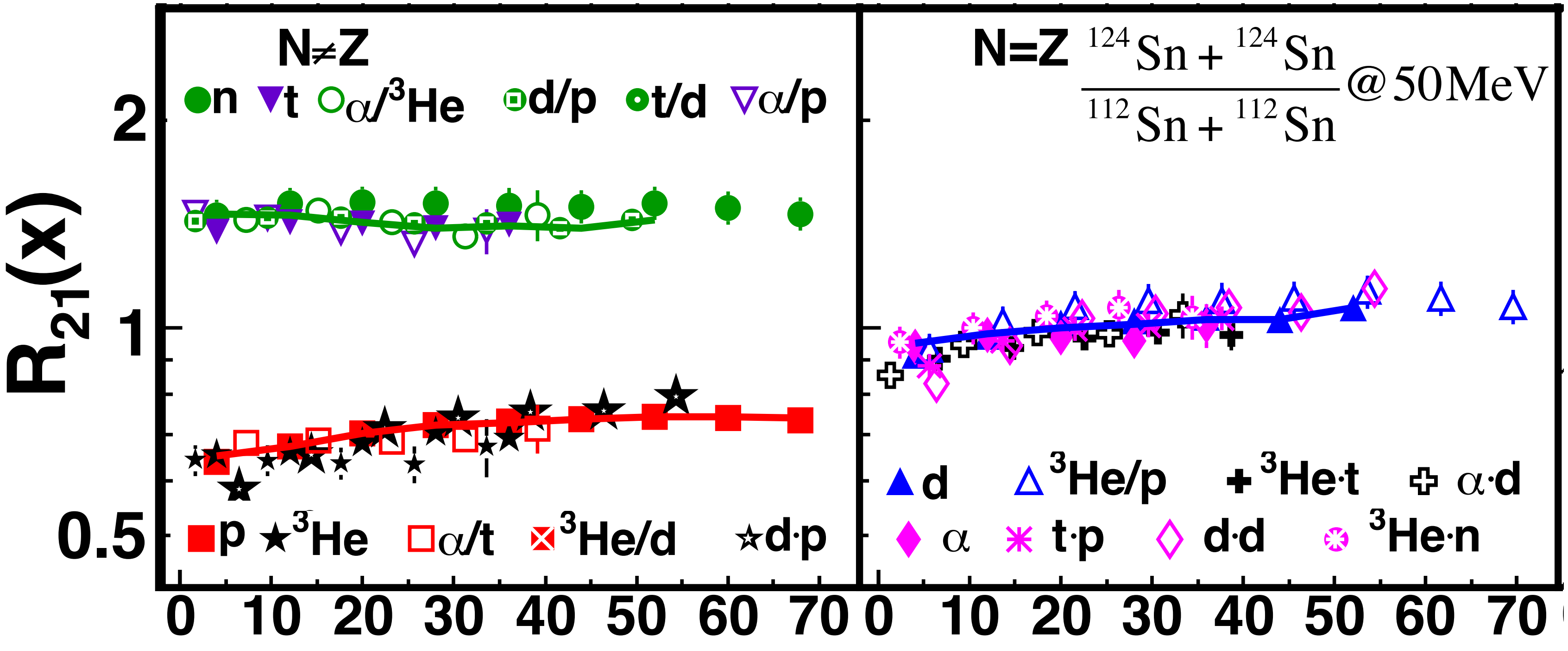
$$R(p) = \frac{R(\alpha)}{R(t)}$$

R(t)=R(d)R(n)

R(α)=R(d)R(d)

Works remarkably well!

Chemical potential scaling



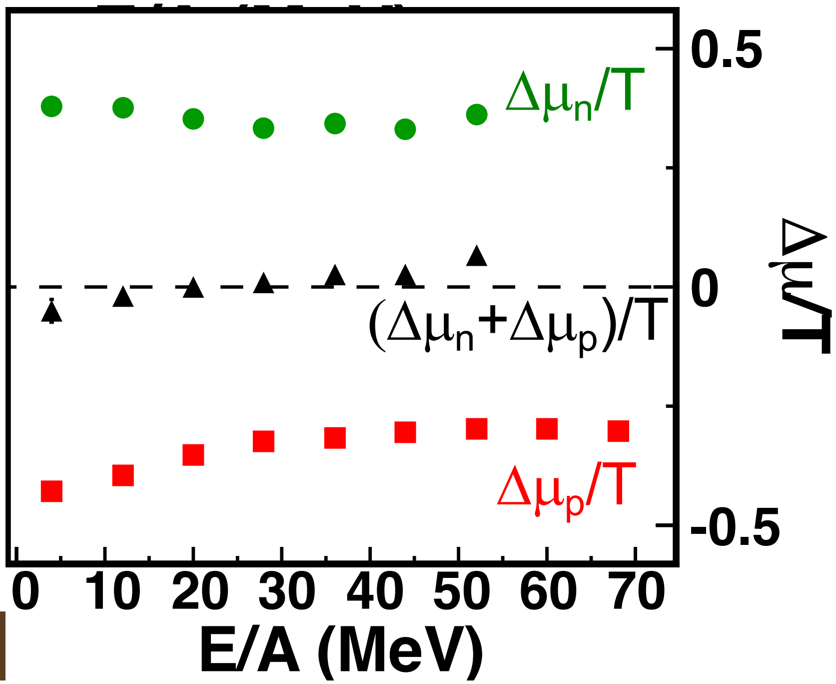
$$R_{21}(N,Z)=\exp\left[\left(N\Delta\mu_n+Z\Delta\mu_p\right)/T\right]$$

e.g.

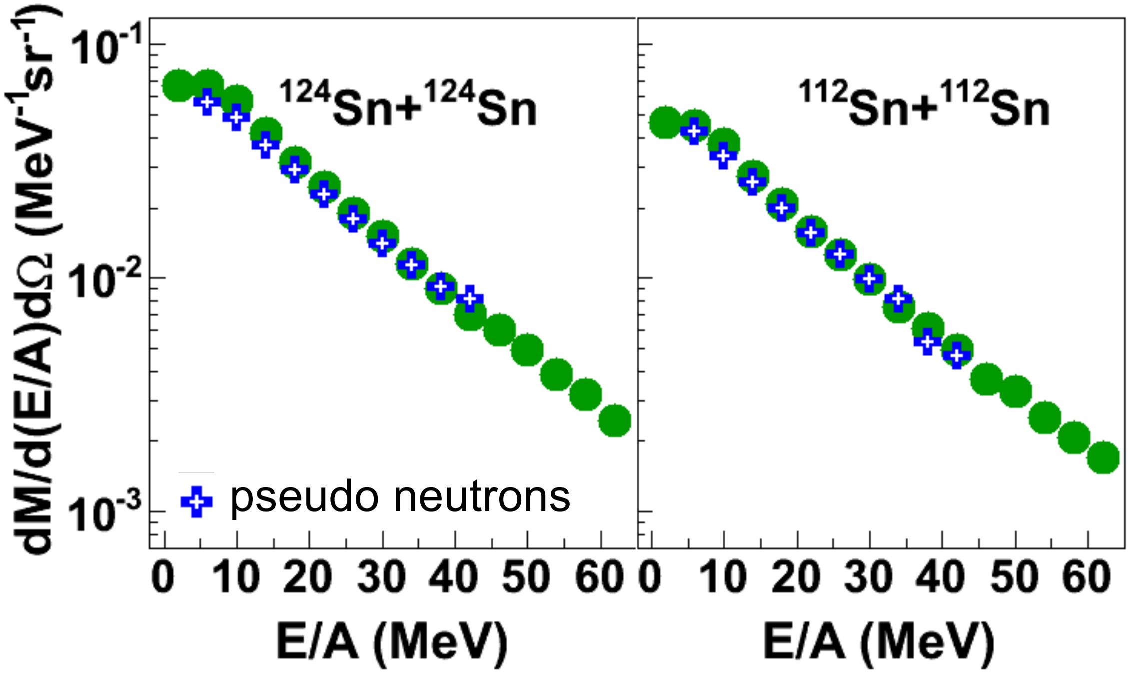
$$R_{21}(p)=\exp\left[\Delta\mu_p/T\right]$$

$$R_{21}(^3He)=\exp\left[\Delta\mu_p/T+\cancelto{\approx 0}{\left(\Delta\mu_n+\Delta\mu_p\right)/T}\right]$$

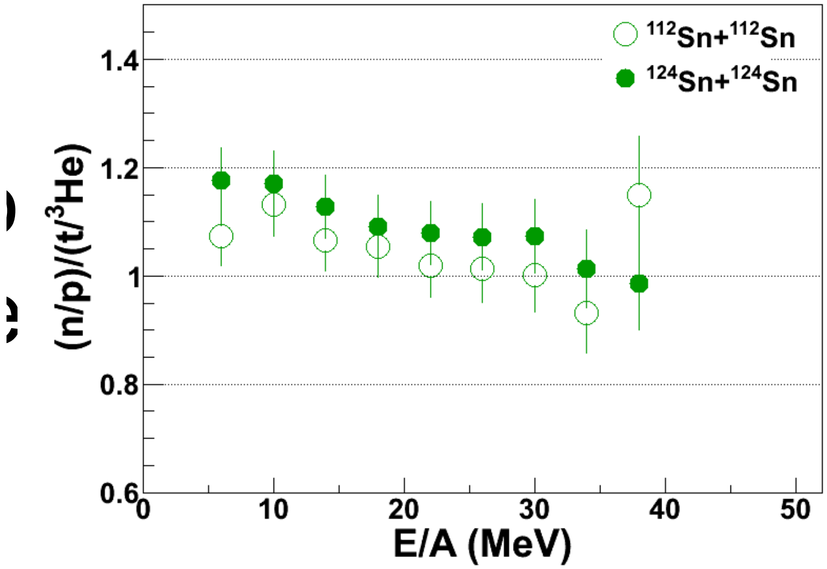
$$=R_{21}(p)$$



"pseudo" neutron spectra



neutrons / pseudo neutrons



$$\frac{Y(t)}{Y(^3He)} = \frac{\exp\left[\left(2\mu_n + \mu_p\right)/T\right]}{\exp\left[\left(\mu_n + 2\mu_p\right)/T\right]}$$

$$\Rightarrow \boxed{Y(n) = \frac{Y(t)}{Y(^3He)} Y(p)}$$

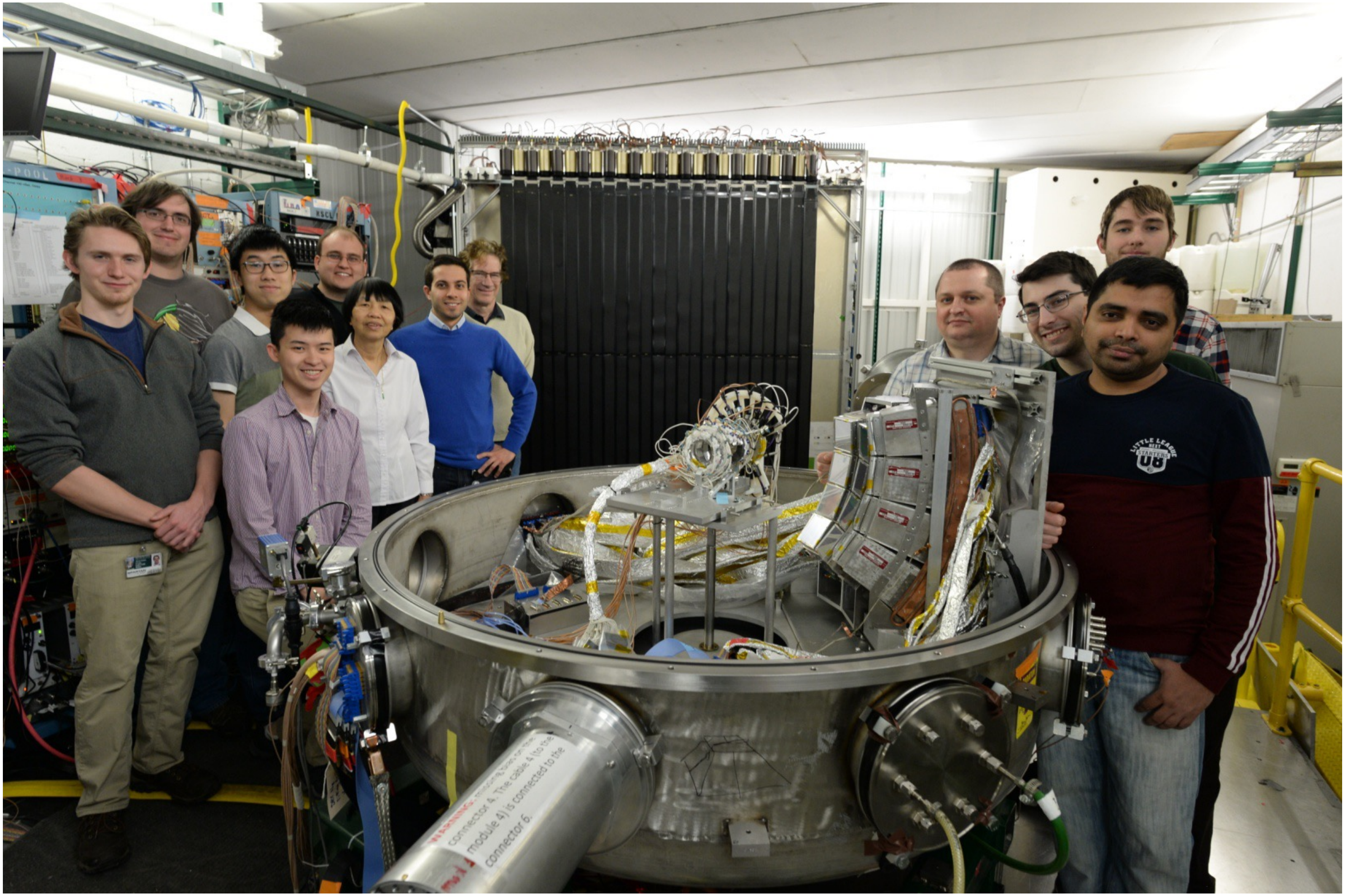
Pseudo neutron yields

Calculating pseudo neutron spectra from charge-particle spectra introduces 10% uncertainties (within systematics of measuring neutrons)

Summary

- Heavy ion collisions provide a unique probe to study the physics of neutron stars in the laboratory
- Analysis suggests that $m^*_n > m^*_p$.
- Recent experiment at NSCL will allow us further constraints of symmetry energy and effective mass at various densities of nuclear matter and to *validate* theoretical models
- Further simulations and theoretical calculations needed to precisely determine Δm^*_{np} and other components of EOS
- We should take full advantage of unique opportunities at new facilities (FRIB, FAIR, RIKEN, ..) to study reactions and observables relevant to EOS despite the current issues with transport models

Thank you



Workshop questions

- Can we reconcile data from current and previous experiment?
  - Once we know the energy range that shows the chemical potential scaling we can ”extract” neutrons from charge particle ratios
- What other observables could enable the extraction of the EOS?
  - Ultimately, we need a multidimensional analysis (spectra, ratios, flow, femtoscopy) and models should be consistent with them over a range of energies
- What improvements on the constraints on the EOS can we expect from future heavy-ion experiments?
  - Improved constraints on S0 and P above ρ/ρ_0=1 (at FRBI) and ρ/ρ_0=2 (at FRIB400)
- What development is necessary for transport codes to address the above questions?
  - Better handling of the cluster production and their dynamics at low energies