Large-scale shell model $\beta\beta$ nuclear matrix elements

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Engel et al. Rep. Prog. Phys. 80, 046301, 2017

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$\beta\beta$ DECAY

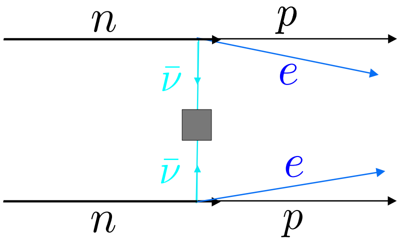
$\beta\beta$ DECAY

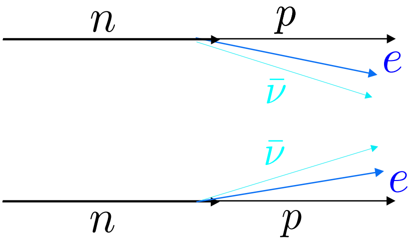
$0\nu\beta\beta : 2n \rightarrow 2p + 2e^-$

- Beyond Standard Model $\rightarrow$ Lepton Number Violation
- Majorana Fermions
- Hypothetical decay

$2\nu\beta\beta : 2n \rightarrow 2p + 2e^- + 2\bar{\nu}_e$

- Standard Model allowed $\rightarrow$ Lepton Number Conservation
- Dirac Fermions
- Already measured

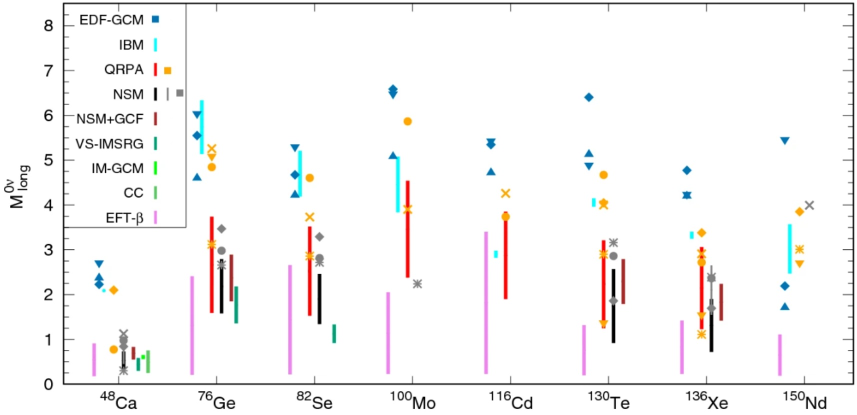




Engel et al. Rep. Prog. Phys. 80, 046301, 2017

$$\left(T_{1/2}^{0\nu}\right)^{-1} \propto G^{0\nu} g_A^4 \left|M^{0\nu}\right|^2 m_{\beta\beta}$$

Gómez-Cadenas, Martín-Albo, Menéndez *et al.*, *Riv. Nuovo Cim.* **46**, 619–692 (2023)



NUCLEAR MATRIX ELEMENTS

NUCLEAR MATRIX ELEMENTS

$$\boxed{M^{\beta\beta} = \langle 0_f^+ | \sum_{a,b} \hat{O}_k \tau_a^- \tau_b^- | 0_i^+ \rangle}$$

Wavefunctions

- Solve the many-body Schrödinger equation
- Phenomenological models:
  - Nuclear shell model (NSM)
  - Quasi-particle random phase approximation (QRPA)
- $|0_{i(f)}^+ \rangle \equiv$ Initial (final) state

Decay Operators

- Chiral effective field theory (χEFT) (LO+NLO+N^{2}LO...)
- $\hat{O}_k \equiv$ Operator (radial/momentum + spin dependence)
  - $k = \mathrm{GT},\ \mathrm{F},\ \mathrm{T}$
- $\tau^- \equiv$ Ladder isospin operator

NUCLEAR SHELL MODEL

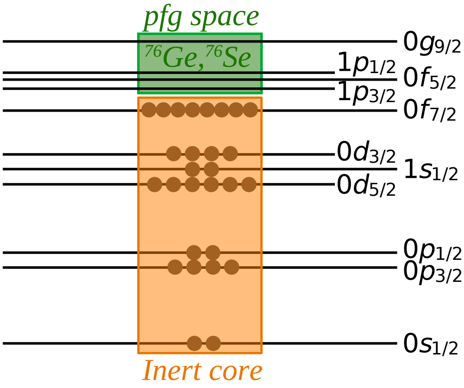
Hamiltonian

- To solve the Schrödinger equation $\rightarrow H_{\text{eff}}|0^+_{\text{GS}}\rangle = E|0^+_{\text{GS}}\rangle$
- $H_{\text{eff}} \equiv$ Phenomenological Hamiltonian. Fitted to experimental data (NN scattering) and renormalized within each valence space
- Good description of nuclear spectroscopy

Orbitals

- Empty orbitals
- Valence Space$\rightarrow H_{\text{eff}}$
- Inert Core

^{76}Ge/^{76}Se:

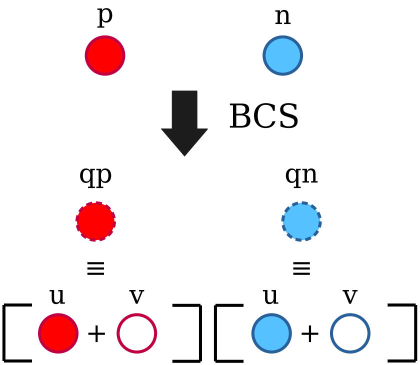


Caurier et al. Rev. Mod. Phys. 77, 427, 2005

QRPA

- $|\mathrm{QRPA}\rangle = |0^+\rangle \rightarrow$ Reference state
- $n = 0 \rightarrow 2$ harmonic oscillator shells above the Fermi level
- Larger valence spaces than NSM but less complex correlations between nucleons
- Parameters:
  - g_{ph} : Gamow-Teller giant resonances
  - g_{pp} : $2\nu\beta\beta$ half-lives

Suhonen, Springer-Verlag, Berlin Heidelberg, 2007



$$|\mathrm{QRPA}\rangle = \sum_{\mathrm{pn}} (\mathrm{a_n qp} + \mathrm{b_n qn})$$

Credit, Jokiniemi

$0\nu\beta\beta$

LO χEFT DIAGRAMS

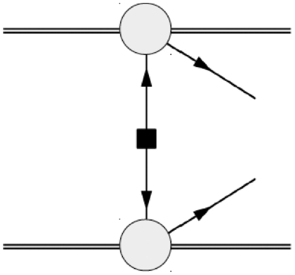
$$\boxed{M^{0\nu} = M^{0\nu}_{\mathrm{L}} + M^{0\nu}_{\mathrm{S}} + M^{0\nu}_{\mathrm{usoft}} + M^{0\nu}_{\mathrm{loops}}}$$

Leading Order (LO)$\rightarrow$already computed

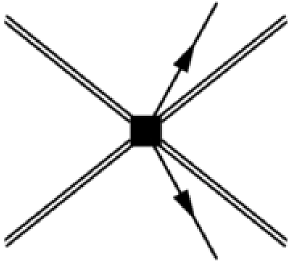
Jokiniemi et al. Phys. Lett. B, 823, 136720, 2021

$q \equiv$ Transferred momentum

- Long-range(L)
  ($q \sim 100$MeV $+q-$dependent N^{2}LO)



- Short-range(S)
  ($q >> 100$ MeV)



Cirigliano et al. Phys. Rev. C 97, 065501, 2018

LO NMEs

$$\boxed{M^{0\nu}_{\rm L} = M^{0\nu}_{\rm L,GT} + M^{0\nu}_{\rm L,F} + M^{0\nu}_{\rm L,T}}$$

Finite Size Corrections:
$q-$dependent N^{2}LO $(g_A^2(q^2),\ g_V^2(q^2))$

$$\boxed{M^{0\nu}_{\rm S} \propto g_\nu^{NN} e^{-q^2/\Lambda^2}}$$

$g_\nu^{NN} \equiv$ Nucleon-Nucleon coupling
$\Lambda \equiv$ Gaussian cutoff

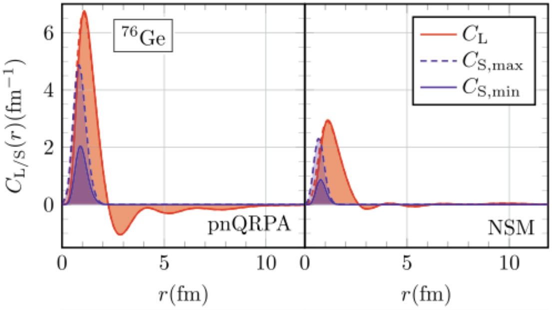
Results

- NSM: $M^{0\nu}_{\rm S}/M^{0\nu}_{\rm L} = +(15-50)\%$
- QRPA: $M^{0\nu}_{\rm S}/M^{0\nu}_{\rm L} = +(30-80)\%$

Jokiniemi et al. Phys. Lett. B 823, 136720, 2021

$$M^{0\nu}_{\rm L/S} = \int_0^\infty C_{\rm L/S}(r)dr$$

- Hard Neutrinos
- Soft Neutrinos

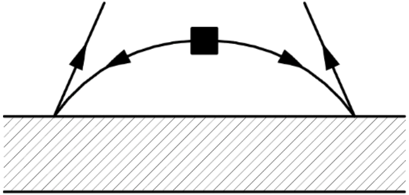


Jokiniemi et al. Phys. Lett. B 823, 136720, 2021

$$\boxed{M^{0\nu} = M_{\mathrm{L}}^{0\nu} + M_{\mathrm{S}}^{0\nu} + M_{\mathrm{usoft}}^{0\nu} + M_{\mathrm{loops}}^{0\nu}}$$

$q \equiv$ Transferred momentum

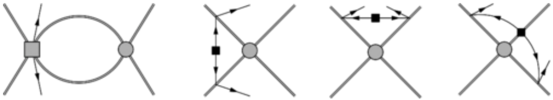
- Ultrasoft(usoft) ($q << 100\text{MeV}$)



Next-to-next leading order(N^2LO)$\rightarrow$New terms

DC, Jokiniemi, Menéndez, Phys. Lett. B 860, 2025

- Loop terms (soft(3)+ultrasoft(1))



Cirigliano et al. Phys. Rev. C 97, 065501, 2018

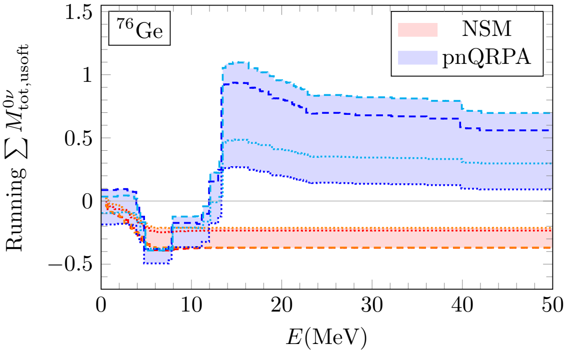
ULTRASOFT NME

- Study of the intermediate states dependence of the total ultrasoft NMEs
- $M^{0\nu}_{\mathrm{usoft}} \propto \langle 0^+_f|\mathrm{GT}|1^+_n\rangle\langle 1^+_n|\mathrm{GT}|0^+_i\rangle \cdot \ln(\mu_{\mathrm{us}})$
- $\mu_{\mathrm{us}} = \frac{m_\pi}{2} \ldots 2m_\pi \equiv$ Ultrasoft scale→Main uncertainty

Results

Different behaviour between models around 10 MeV→Different sign

$$M^{0\nu}_{\mathrm{tot,usoft}} = M^{0\nu}_{\mathrm{usoft}} + M^{0\nu}_{\mathrm{loop,usoft}}$$



Jokiniemi et al. Phys. Lett. B 823, 136720, 2021

CLOSURE VS NON-CLOSURE

Non-closure:

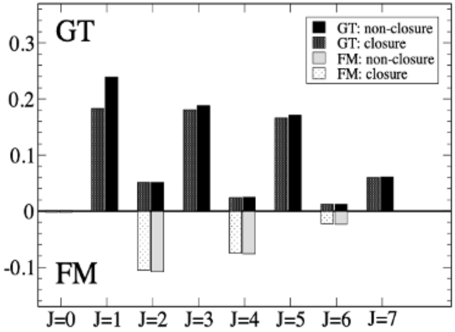
$$M^{0\nu}_{\mathrm{non-cl}} \propto \frac{\langle 0_f^+|J_\mu(x)|J_n^\pi\rangle\langle J_n^\pi|J^\mu(y)|0_n^+\rangle}{q(q+E_n-\frac{1}{2}(E_f+E_i))} \quad \longrightarrow$$

- $q \simeq k_F \simeq 100\mathrm{MeV}$
- $E_n - \frac{1}{2}(E_i + E_f) \to 0$
- $J_\mu J^\mu = h_{\mathrm{GT}} + h_{\mathrm{F}} + h_{\mathrm{T}}$

Closure:

$$M^{0\nu}_{\mathrm{L}} \propto \frac{\langle 0_f^+|J_\mu(x)J^\mu(y)|0_n^+\rangle}{q^2}$$

- Main difference comes from $\mathrm{GT}|1_n^+\rangle$
- Same dependence as $M^{0\nu}_{\mathrm{usoft}}$



Sen'kov, Horoi Phys. Rev. C 88, 064312, 2013

CLOSURE VS NON-CLOSURE

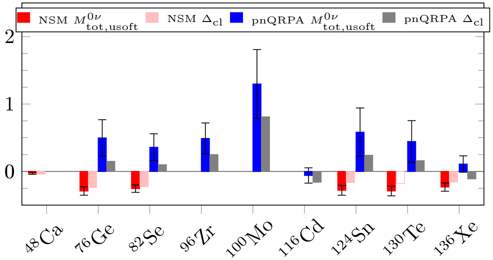
- According to χEFT the ultrasoft term must be the main contribution beyond the closure approximation:
$M^{0\nu}_{\mathrm{non-cl}} - M^{0\nu}_{\mathrm{L}} = \Delta_{\mathrm{cl}} \sim M^{0\nu}_{\mathrm{tot,usoft}}$

Cirigliano et al. Phys. Rev. C 97, 065501, 2018

Results

- Agreement in the sign between Δ_{cl} and $M^{0\nu}_{\mathrm{tot,usoft}}$
- NSM: Good agreement with χEFT
- QRPA: Milder agreement with χEFT

Main uncertainties $\rightarrow$ μ_{us} dependence



DC, Jokiniemi, Menéndez, Soriano, Phys. Lett. B, 860, 2025

LOOP NMEs

$$\boxed{M^{0\nu}_{\text{loops,soft}} = M^{0\nu}_{AA,\text{loops}} + M^{0\nu}_{VV,\text{loops}} + M^{0\nu}_{CT,\text{loops}}}$$

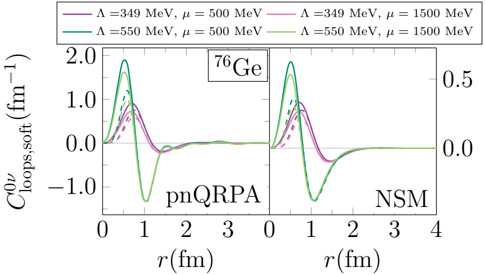
Results

- Important contributions in short-range distances
- Similar behaviour between models
- Main dependence $\rightarrow \Lambda$
  - $\Lambda = 349$ MeV $\Lambda = 550$ MeV
- Most reliable: $\Lambda = 349$ MeV

Uncertainties: H_{eff}, μ, Λ

$\mu \equiv$ Renormalization scale

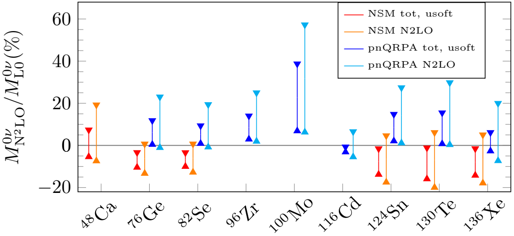
$$M^{0\nu}_{\text{loops,soft}} = \int_0^\infty C^{0\nu}_{\text{loops,soft}}(r)dr$$



DC, Jokiniemi, Menéndez, Soriano, Phys. Lett. B, 860, 2025

TOTAL N^{2}LO NMEs

$$\boxed{M^{0\nu}_{\mathrm{N^2LO}} = M^{0\nu}_{\mathrm{tot,usoft}} + M^{0\nu}_{\mathrm{loops,soft}}}$$



DC, Jokiniemi, Menéndez, Soriano, Phys. Lett. B, 860, 2025

- χEFT expectations $\sim$ (5 - 10)%

NSM

- $\mathrm{N^2LO/LO} \leq$ 20%
- Central values:
  - Total ultrasoft: $-$(5-10)%
  - Total N^{2}LO: $-$(5-10)%

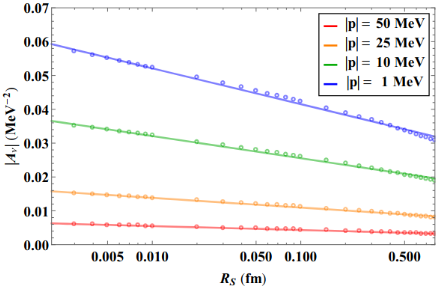
QRPA

- $\mathrm{N^2LO/LO} \leq$ 30%
- Central values:
  - Total ultrasoft: $+$(5-10)%
  - Total N^{2}LO: $+$(10-15)%

g_ν^{NN} EVALUATION

$$\boxed{h_{\rm S}(q^2) = 2g_\nu^{NN} e^{-q^2/2\Lambda^2} \to 2g_\nu^{NN}\delta^{(3)}(r)}$$

- No LNV observation $\to$ no experimental data to fit



- Similar UV divergence than charge-independence breaking
- Connection between EM and the LNV Lagrangians
- $V_{CIB,{\rm S}} = -\frac{e^2}{6}\frac{\mathcal{C}_1+\mathcal{C}_2}{2} T^{(12)}\delta^{(3)}(r)$
  - $T^{(12)} \equiv$ isospin tensor operator
  - $\delta^{(3)}(r) \equiv$ radial delta function
- $g_\nu^{NN} \to \frac{\mathcal{C}_1+\mathcal{C}_2}{2}$
- χEFT potentials: $v_{12}^{CD} = C_0^{\rm IT} T^{(12)}\delta^{(3)}(r)$
- $g_\nu^{NN} \to \frac{\mathcal{C}_1+\mathcal{C}_2}{2} = -\frac{6}{e^2} C_0^{\rm IT}$
- g_ν^{NN} determined by $C_0^{\rm IT}$ and Λ

Cirigliano, Dekens, de Vries et al. Phys. Rev. C 100, 055504 (2019)

NUCLEAR SHELL MODEL SHORT-RANGE NME

- The χEFT potentials use is not consistent with NSM framework
- $v^{\mathrm{IT}}_{\mathrm{NSM}} = \lambda^{\mathrm{IT}}_{\mathrm{coul}} V^{\mathrm{IT}}_{\mathrm{coul}}(r) + \lambda^{\mathrm{IT}}_{\rho} V^{\mathrm{IT}}_{\rho}(r)$
  - $V^{\mathrm{IT}}_{\rho} = e^{-\mu_\rho r}/(\mu_\rho r) \equiv$ rho-exchange
  - $\lambda^{\mathrm{IT}}_{\rho} \equiv$ isotensor strength

$$\boxed{2g^{\mathrm{NN}}_{\nu}\delta^{(3)}(r) \to -\frac{6}{e^2}\lambda^{\mathrm{IT}}_{\rho} V^{\mathrm{IT}}_{\rho}(r)}$$

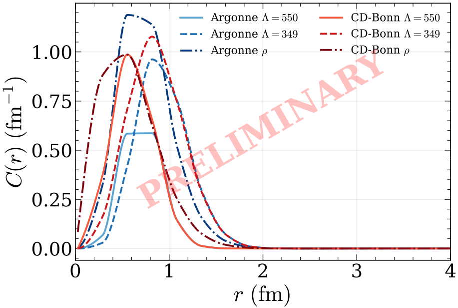
- $\lambda^{\mathrm{IT}}_{\rho} \to$ NSM fit through experimental IMME coefficients

Lam, Smirnova, Caurier Phys. Rev. C 87, 054304 (2013)

- $sd-$shell: USD interaction

Brown et al. Ann. of Phys. 182, 191-236 (1988)

- $\lambda^{\mathrm{IT}}_{\rho,\mathrm{Arg}} = 149.42$ MeV
- $\lambda^{\mathrm{IT}}_{\rho,\mathrm{Bonn}} = 73.52$ MeV
- $^{30}\mathrm{Mg} \to {}^{30}\mathrm{Si}$:
  - $M^{0\nu}_{\mathrm{S},\ \chi\mathrm{EFT}} = 0.29 - 0.85$
  - $M^{0\nu}_{\mathrm{S;NSM}} = 0.79 - 0.89$



HEAVY-ν EXCHANGE: MASS DEPENDENCE

LO:

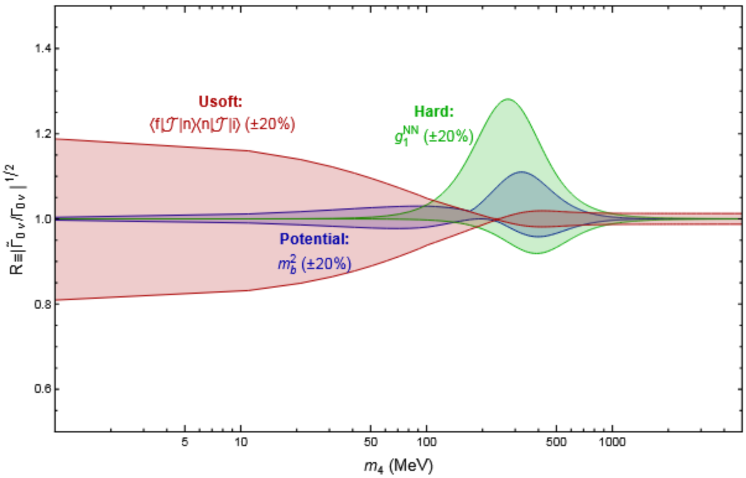
$$M^{0\nu}_{\rm sterile}(m_i) = \begin{cases} M^{0\nu}_{\rm L,<}(m_i) + M^{0\nu}_{\rm S}(m_i) + M^{0\nu}_{\rm usoft}(m_i) & m_i < 100 \text{ MeV}, \\ M^{0\nu}_{\rm L}(m_i) + M^{0\nu}_{\rm S}(m_i) & 100 \text{ MeV} < m_i < 2 \text{ GeV}, \\ M^{0\nu}_{9}(m_i) & 2 \text{ GeV} < m_i \end{cases}$$

- $M^{0\nu}_{\rm L} \propto \frac{1}{q^2+m_i^2}$
- $M^{0\nu}_{\rm L,<}(m_i) = M^{0\nu}_{\rm L}(m_i) - m_i[\frac{d}{dm_i} M^{0\nu}_{\rm L}(m_i)]_{m_i=0}$
  To avoid double-counting (ultrasoft region)
- $M^{0\nu}_{\rm S}(m_i) \propto g^{NN}_{\nu}(m_i)$

Cirigliano, Dekens, Urrutia, J. High Energ. Phys. 2025, 181 (2025)

- $M^{0\nu}_{\rm usoft}(m_i) \propto m_i M^{0\nu}_{\rm usoft}(0) + \mathcal{O}(\Delta E/m_i)$
- $M^{0\nu}_{9}(m_i) \propto 1/m_i \rightarrow g_1^{\pi\pi}, g_1^{\pi N}, g_\nu^{NN}$
  sterile neutrinos integrated out

HEAVY-ν EXCHANGE: ULTRASOFT ENHANCEMENT



- Potential: $M_{\mathrm{L}}^{0\nu}$
- Hard: $M_{\mathrm{S}}^{0\nu}$
- Usoft: $M_{\mathrm{usoft}}^{0\nu}$

Dekens, De Vries, DC et al. J. High Energ. Phys. 2024, 201 (2025)

$$2\nu\beta\beta$$

HALF-LIFE UP TO NLO

$$\boxed{(T^{2\nu}_{1/2})^{-1} = g_A^4 (M^{-1}_{\mathrm{GT}})^2 \Delta_0 [G^{2\nu}_0 + \xi_{31} \tfrac{\Delta_2}{\Delta_0} G^{2\nu}_2 + G^{2\nu}_4 (\xi_{51} \tfrac{\Delta_2}{\Delta_0} + \tfrac{1}{3}\xi^2_{31}) + G^{2\nu}_{22} + G^{2\nu}_M]}$$

$g_A \equiv$ Axial coupling

$|0_i^+\rangle = |0_{gs}^+\rangle$, $|0_f^+\rangle = |0_2^+\rangle$

Leptonic degrees of freedom:

- $G_i^{2\nu} \equiv$ Phase-space factors (PSF)

Nuclear degrees of freedom:

- $M^{-1}_{\mathrm{GT}} \equiv$ Leading order (LO) Nuclear Matrix Element (NME)
- $\xi_{31}, \xi_{51} \equiv$ NME quotients
- $\Delta_0, \Delta_2 \equiv \chi$EFT Next-to-leading order (NLO) NMEs

Taylor expansion: Simkovic et al. Phys. Rev. C 97, 034315, 2018

- $\xi_{i1} = \frac{M^{-i}_{\mathrm{GT}}}{M^{-1}_{\mathrm{GT}}}$
- $M^{-2m-1}_{\mathrm{GT}} \propto \frac{\langle 0_f^+|\mathrm{GT}|1_n^+\rangle\langle 1_n^+|\mathrm{GT}|0_i^+\rangle}{(E_n - \frac{1}{2}(E_i+E_f))^{2m+1}}$
- $G_i^{2\nu}$; $i = 2, 4, 22$

NLO Corrections: el Morabit et al. JHEP, 06, 082, 2025

- $G_M^{2\nu} \equiv$ New weak magnetism contribution (WM)
- Δ_0, $\Delta_2 \equiv$ one-pion exchange terms and WM insertion

PHASE-SPACE FACTORS

Phase space factors integrals:

$$G_i^{2\nu} = \frac{1}{\ln 2}\frac{(G_F V_{ud})^4}{8\pi^7 m_e^2} \times \int_{m_e}^{E_i - E_f - m_e} dE_{e1} \int_{m_e}^{E_i - E_f - E_{e1}} dE_{e2} \times \int_0^{E_i - E_f - E_{e1} - E_{e2}} dE_{\nu 1}\ C_i$$

$$C_i = E_{e1} p_{e1} E_{e2} p_{e2} E_{\nu 1}^2 E_{\nu 2}^2 \times F(E_{e1}, Z) \times F(E_{e2}, Z) \times \mathcal{A}_i .$$

- $G_F \equiv$ Fermi constant
- $V_{ud} \equiv$ CK matrix element
- $m_e \equiv$ electron mass
- $E_e \equiv$ electron energies
- $E_\nu \equiv$ neutrino energies
- $\mathcal{A}_i \equiv$ amplitudes
- $F(E, Z) \equiv$ Fermi function

$$F(E, Z) = g_{-1}^2(E_e, R_A) + f_{+1}^2(E_e, R_A)$$

$g_{-1}(E_e, R_A) \equiv$ upper component of radial w.f.

$f_{+1}(E_e, R_A) \equiv$ lower component of radial w.f.

Wavefunctions are evaluated at the radius of the nucleus $R_A = 1.2A^{1/3}$ fm.

Exact Dirac + Finite Size Coulomb + Screening (Thomas-Fermi)

NUCLEAR MATRIX ELEMENTS

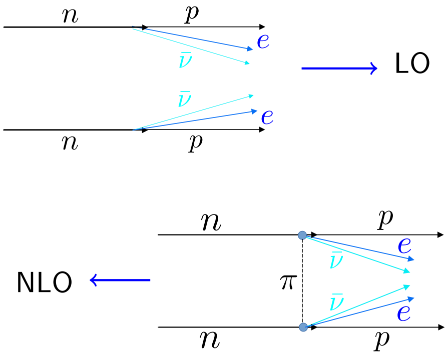
$$M_{\mathrm{GT}}^{-1} = \sum_n \frac{\langle 0_f^+|\mathrm{GT}|1_n^+\rangle\langle 1_n^+|\mathrm{GT}|0_i^+\rangle}{E_n - \frac{1}{2}(E_i + E_f)}$$

Decay operators

- Chiral Effective Field Theory (χEFT) (LO+NLO+N^{2}LO...)
- $2\nu\beta\beta \rightarrow$ Gamow-Teller $\mathrm{GT} = \sum_a \boldsymbol{\sigma}_a \tau_a^-$
- $\tau^- \equiv$ Ladder isospin operator

el Morabit et al. JHEP, 06, 082, 2025

- $E_n \equiv$ Intermediate states energy
- $E_{i(f)} \equiv$ Initial (final) state energy
- Wavefunctions: Nuclear Shell Model (NSM)



QUENCHING

- $2\nu\beta\beta$ $0^+_{\rm gs} \to 0^+_{\rm gs}$ decay has been measured for several nuclei: ^{48}Ca, ^{76}Ge, ^{82}Se, ^{130}Te, ^{136}Xe...
- Theoretical predictions systematically underpredict the experimental $T^{2\nu}_{1/2} \to$ theoretical NME are overestimated
- Fine-tune the NMEs to reproduce the experimental measurements
- $q_i \equiv$ quenching: $M^{2\nu}_{\rm GT} \to q_i^2 M^{2\nu}_{\rm GT}$

q_β

- Fitted to single-β decay strenghts $B(\rm GT)$
- Usually very similar within the same valence space

  Caurier et al. Phys. Lett. B 711 (2012) 62-64
- $q_\beta = 0.74$ ($pf-$shell)

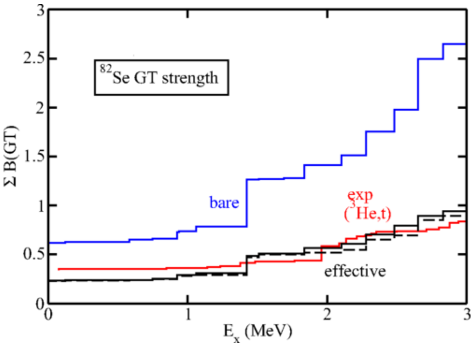
  $q_\beta = 0.6$ ($pfg-$shell)

$q_{\beta\beta}$

- Fitted to reproduce the $T^{2\nu}_{1/2}(0^+_{\rm gs} \to 0^+_{\rm gs})$

GT : BARE VS RENORMALIZED

- $\hat{O}^{\text{bare}}_{\text{GT}} = \boldsymbol{\sigma_a} \cdot \boldsymbol{\sigma_b}$
- Experimental data not reproduced$\rightarrow$ need q_β



Coraggio et al. Phys. Rev. C 100, 014316 (2019)

Effective GT ($\hat{O}^{\text{eff}}_{\text{GT}}$)

- Softened NN Hamiltonian (V_{low-k} approach) $\rightarrow$ Two-body currents less important
- Evolve GT consistently within your valence space $\rightarrow$ Capture many-body correlations Coraggio et al. Phys. Rev. C 100, 014316 (2019)
- We include $q^{\text{eff}}_{\beta\beta} \rightarrow q^{eff}_{\beta\beta} \sim 1$ for ^{76}Ge and ^{82}Se

TAYLOR AND NLO CORRECTIONS

$\beta\beta$ emitters:

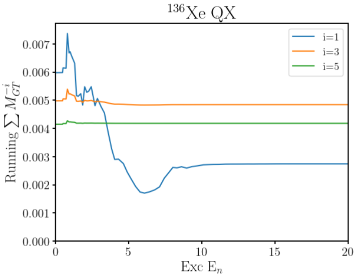
- ^{48}Ca: KB3G and GXPF1A
- ^{76}Ge, ^{82}Se: GCN2850, JUN45, JJ4BB and RG.PROLATE
- ^{124}Sn(GS), ^{130}Te, ^{136}Xe: GCN5082 and QX5082

Taylor Corrections

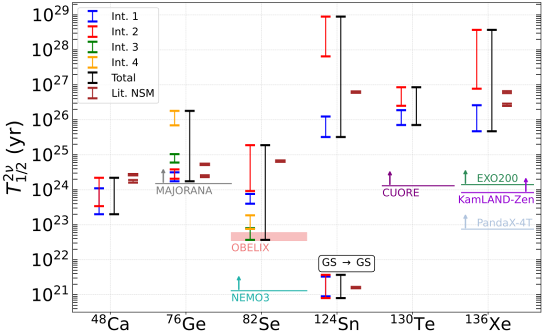
- Typically under a 5% correction
- ^{82}Se and ^{136}Xe: Larger impact reaching 14% $\rightarrow$important cancellations in the leading NME (high-energy excited states)

NLO corrections

- Typically under a 5% correction
- ^{124}Sn and ^{136}Xe (QX): NLO contributions can be comparable to the LO term or even dominant



DECAY TO EXCITED STATES



DC, DF, Menéndez, Benavente, Phys. Lett. B 875 (2026) 140306

- Color bars: Predictions for each Hamiltonian
- **Black bars**: Hamiltonians span
- Brown Bars: Literature results

Uncertainties: quenching range $(q_\beta - q_{\beta\beta})$ + bare/effective operator

Results

- Main uncertainty comes from Hamiltonians: Deformation of initial and final states plays a key role
- ^{76}Ge, ^{82}Se: Experimental evidence close to theoretical predictions

DEFORMATION AND SHAPE COEXISTENCE

Deformation is everywhere:

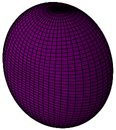
- Magic nuclei are spherical
- Deformation is enhanced mid-shell (most nuclei)

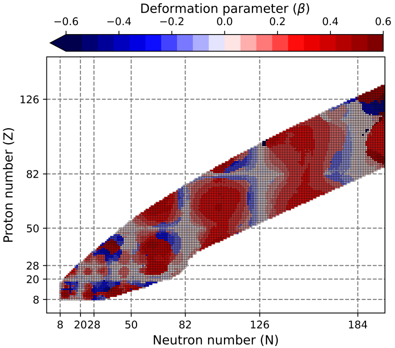
Quadrupole deformation:

- Most extended shape
- Prolate: elongated sphere
- Oblate: flattened sphere

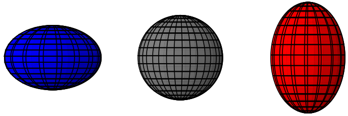
Triaxiality:

- Three disitinct axes
- Present in most nuclei



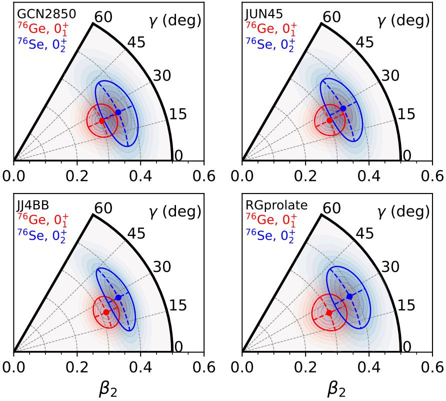


P. Möller, et al. Atom. Data Nucl. Data Tabl. 109-110 (2016)



Shape invariants for different interactions:

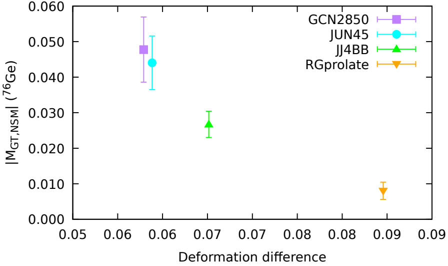
- $\langle Q^2 \rangle = \langle [Q_2 \times Q_2]_0 \rangle \rightarrow \beta$
- $\langle Q^3 \rangle = \langle [[Q_2 \times Q_2]_2 \times Q_2]_0 \rangle \rightarrow \gamma$



DC, DF, Menéndez, Benavente, Phys. Lett. B 875 (2026) 140306

Impact of deformation on $2\nu\beta\beta$:

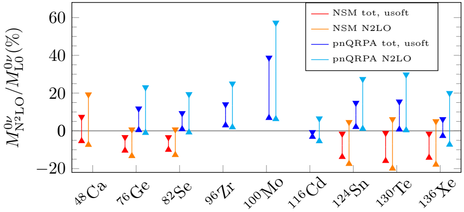
- Deformation differences of $|\Psi_f\rangle$ and $|\Psi_i\rangle$ **reduce** M_{GT}
- δ: polar plane distance
- Correlation: M_{GT} with δ
- Large impact of triaxiality γ



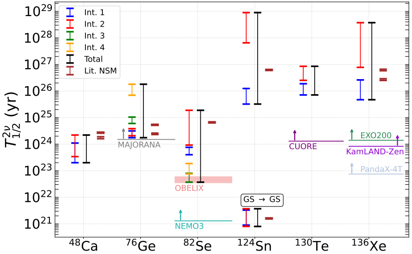
SUMMARY AND OUTLOOK

Summary

- $M^{0\nu}_{\mathrm{tot,usoft}}$ is the main contribution beyond the closure
- $0\nu\beta\beta$ N^{2}LO are non-negligible$\rightarrow$ N^{2}LO/LO$= \pm(5-15)\%$
- $M^{0\nu}_{\mathrm{S,\,NSM}}$ yields similar results to the largest values of $M^{0\nu}_{\mathrm{S,}\,\chi\mathrm{EFT}}$
- $T^{2\nu}_{1/2}$ large uncertainties driven by the Hamiltonian choice $\rightarrow$ Deformation
- Strong correlation between deformation difference and the size of NMEs

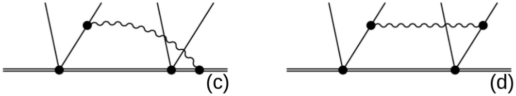


DC, Jokiniemi, Menéndez, Soriano, Phys. Lett. B, 860, 2025

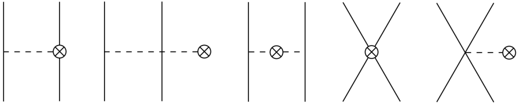


DC, DF, Menéndez, Benavente, Phys. Lett. B 875 (2026) 140306

OUTLOOK



de Vries et al. arXiv:2603.23592v1 (2026)



Philipp Klos, Master's Thesis (2014)

Outlook

- To include explicitly two-body currents
- To use $\chi-$Hamiltonians to be fully consistent with the χEFT corrections (VS-IMSRG)
- To extend the ISB fit to heavier nuclei
- To include radiative and exchange corrections in the $2\nu\beta\beta$ PSFs

de Vries et al. arXiv:2603.23592v1 (2026)

- Given our predictions to hopefully observe an experimental measurement!

Thank you for your attention!

BACKUP SLIDES

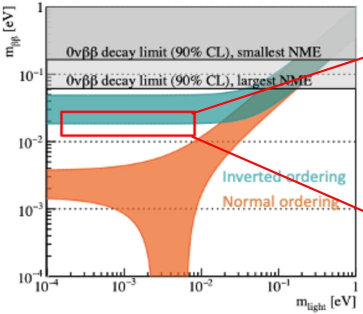
NUCLEAR PHYSICS' ROLE

$$\boxed{(T_{1/2}^{\beta\beta})^{-1} \propto g_A^4 G_0 |M^{\beta\beta}|^2 m_{\beta\beta}^2}$$

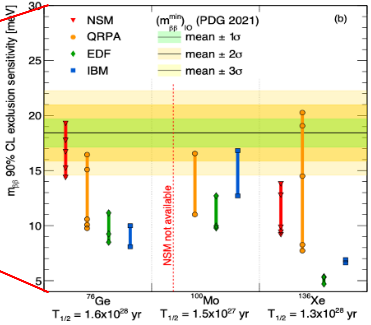
$$m_{\beta\beta} = \begin{cases} 1 & \text{if } 2\nu\beta\beta \\ \sum_{j=\text{light}} U_{ej} m_j & \text{if } 0\nu\beta\beta \end{cases}$$

$M^{\beta\beta} \equiv$ Nuclear matrix elements
$G_0 \equiv$ Phase-space factor (PSF)
$g_A \equiv$ Axial coupling
$m_{\beta\beta} \equiv$ Effective neutrino mass

Next Decade Experiments



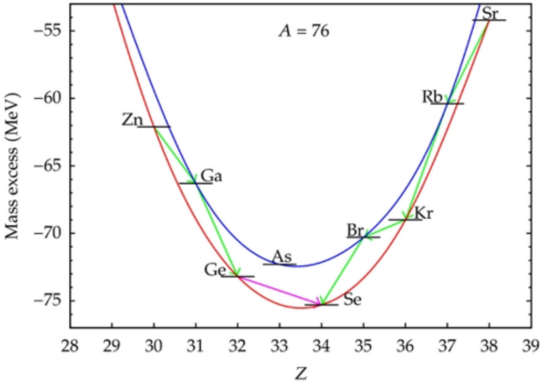
Agostini et al. Rev. Mod. Phys. 95, 025002, 2023



Agostini et al. Phys. Rev. C 104, L042501, 2021

$\beta\beta$ DECAY

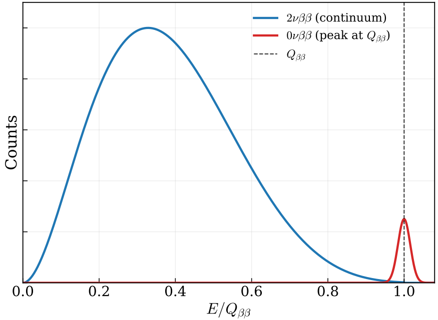
- ^{76}Ge: β decay forbidden
- Energy difference between odd-odd nuclei and even-even nuclei$\rightarrow$ Pairing
- $\beta\beta$ decay allowed:
  $^{76}\text{Ge} \rightarrow {}^{76}\text{Se}$ ($Q_{\beta\beta} > 0$)
- $Q_{\beta\beta} = E_i - E_f - 2m_e$
  - $Q_{\beta\beta} \equiv Q-$value for $\beta\beta$ decay
  - $m_e \equiv$ Electron mass
  - $E_i \equiv$ Initial energy
  - $E_f \equiv$ Final energy



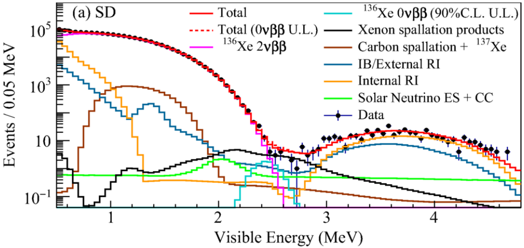
Giuliani, et al. Adv. High Energy Phys, 2012, 857016, 2012.

$\beta\beta$ SPECTRUM

- $0\nu\beta\beta$: Not measured
- $2\nu\beta\beta$: Already measured



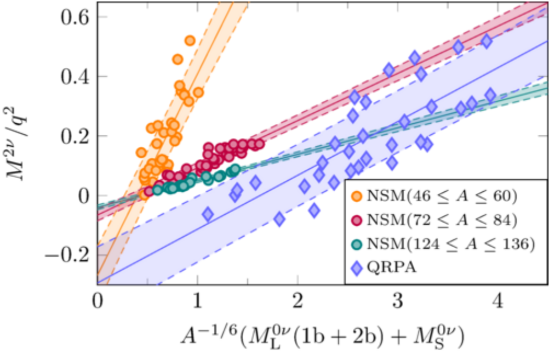
- $2\nu\beta\beta$: irreducible background
- Very important to have a precise knowledge of the $2\nu\beta\beta$ shape



KamLand-Zen Collaboration, Phys. Rev. Lett. 135, 262501 (2025)

EXPERIMENTS

CONNECTION: $0\nu\beta\beta$-$2\nu\beta\beta$



Jokiniemi et al. Phys. Rev. C 107, 044305, 2023

- $M^{2\nu} \equiv$ LO $2\nu\beta\beta$ NME
- $M_L^{0\nu}(1b+2b) + M_S^{0\nu} \equiv$ LO $0\nu\beta\beta$ NMEs + Two-body currents
- $A \equiv$ mass number
- NSM:
  - Systematic calculations of $2\nu\beta\beta$ and $0\nu\beta\beta$ NMEs for different nuclei
  - Good linear correlation depending on the valence space
  - Heavier nuclei: flattest slope
- Reduce $0\nu\beta\beta$ uncertainties by studying $2\nu\beta\beta$

PHENOMENOLOGICAL INTERACTIONS

pf−shell interactions($A = 40 - 80$)

- KB3G: Kuo-Brown interaction Mass dependence and monopole modifications
- GXPF1A: Bonn-C potential Two-body matrix elements from $A = 47 - 66$

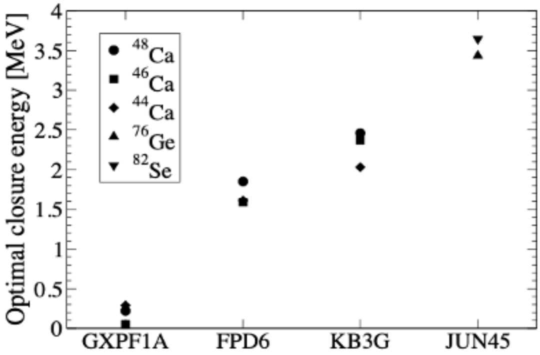
pfg−shell interaction($A = 56 - 100$)

- JUN45: Bonn-C interaction 133 two-body matrix elements, 4 single-particle energies with $A = 63 - 96$
- GCN2850: G-matrix, fit to 300 energy levels
- JJ4BB
- RG.PROLATE

$sdgh$−shell($A = 100 - 140$)

- GCN5082
- QX5082: Bonn-C potential Binding energies of 157 low-lying yrast states from $^{102-132}$Sn

$0\nu\beta\beta$: CLOSURE ENERGIES



Neacsu, Horoi, Phys. Rev. C, 91, 024309, 2015

Nucleus	Interaction	$\langle E_n \rangle$ (MeV)
^{48}Ca	KB3G	2.5
^{48}Ca	GXPF1A	0.23
^{76}Ge	JUN45	3.5
^{82}Se	JUN45	3.6
^{136}Xe	GCN5082	3.7

Sarkar et al. arXiv:2406.13417v1, 2024

HEAVY-ν EXCHANGE: EXPRESSIONS

$$A_\nu^{(9)} = -2\eta(\mu_0, m_i)\frac{m_\pi^2}{m_i^2}[\frac{5}{6}g_1^{\pi\pi}(M_{\mathrm{GT,sd}}^{PP} + M_{\mathrm{T,sd}}^{PP}) + \frac{g_1^{\pi N}}{2}(M_{\mathrm{GT,sd}}^{AP} + M_{\mathrm{T,sd}}^{AP}) - \frac{2}{g_A^2}g_\nu^{NN}M_{\mathrm{F,sd}}]$$

$$A_\nu^{(\mathrm{pot})}(m_i) + A_\nu^{(\mathrm{hard})} = \frac{M_{\mathrm{F}}(m_i)}{g_A^2} - M_{\mathrm{GT}}(m_i) - M_T(m_i) - \frac{2}{g_A^2}g_\nu^{NN}(m_i)m_\pi^2 M_{\mathrm{F,sd}}$$

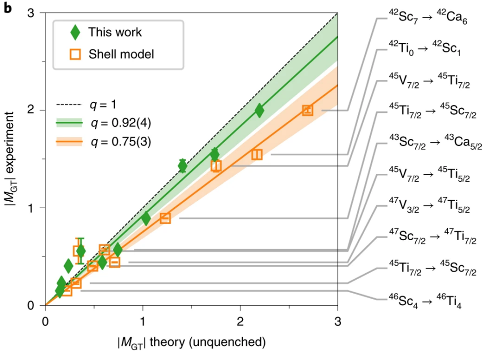
$$A_\nu^{(\mathrm{usoft})} \approx R_A m_i \sum_n N_f N_i + \mathcal{O}(\Delta E/m_i)$$
$$N_k = \langle k|\mathrm{GT}|n\rangle$$

EXPERIMENTAL AND THEORETICAL GT NMEs

- $B(GT) \propto |M_{\mathrm{GT}}^{-1}|^2 \equiv \mathrm{GT}$ strength
- Theory differs from the experimental measurements
- Typically theoretical $B(\mathrm{GT})$ are overestimated
- Two reasons:
  - Many-body correlations $\rightarrow$ wavefunctions too simple
  - Two-body currents (2BC) $\rightarrow$ operator too simple

Shell Model

Ab-initio + 2BC



Gysbers et al. Nature Phys. 15 (2019) 5, 428-431

SHAPE INVARIANTS

Deformation is characterized in the intrinsic frame: (β, γ)

However, measurements are done in lab frame: $(B(E2), Q_s)$

Conversion from lab to intrinsic frame:

$$Q_s(J) \sim Q_{0,s}, \quad J \geq 1/2$$

$$B(E2, J_i \to J_f) \sim Q_{0,t}^2$$

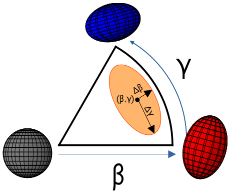
$\to$ Perfect axial rotor assumption: $Q_{0,s} \simeq Q_{0,s}$

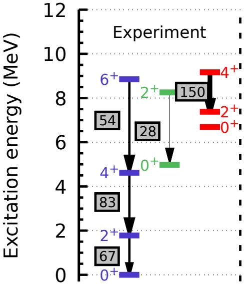
Shape invariants $\langle Q^n \rangle$: model independent Couple Q_2 (tensor) to obtain a scalar:

- $\langle Q^2 \rangle = \langle [Q_2 \times Q_2]_0 \rangle \to \beta$
- $\langle Q^3 \rangle = \langle [[Q_2 \times Q_2]_2 \times Q_2]_0 \rangle \to \gamma$
- All $\langle Q^n \rangle$ up to $\langle Q^6 \rangle$ are needed for fluctuations of (β, γ)

K. Kumar, Phys. Rev. Lett. 28, 249 (1972)

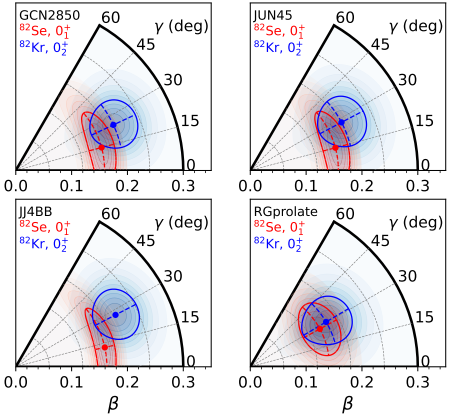
D. Cline, Nuclear Structure 313–326 (1985)





DEFORMATION IMPACT ON $2\nu\beta\beta$ (^{82}Se/Kr)

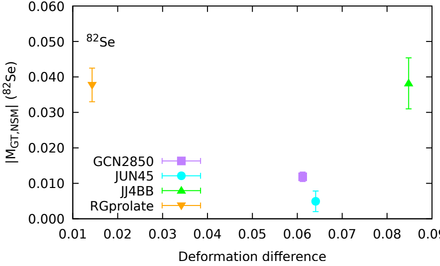
Shape invariants for different interactions:



DC, DF, Menéndez, Benavente, Phys. Lett. B 875 (2026) 140306

Impact of deformation on $2\nu\beta\beta$:

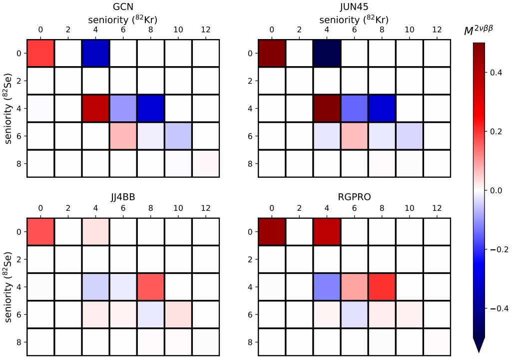
- Deformation differences of $|\Psi_f\rangle$ and $|\Psi_i\rangle$ **reduce** M_{GT}
- δ: polar plane distance
- Correlation: M_{GT} with δ
- JJ4BB: too large M_{GT} for large δ



SENIORITY IMPACT ON $2\nu\beta\beta$

Seniority (s) decomposition of M_{GT}:

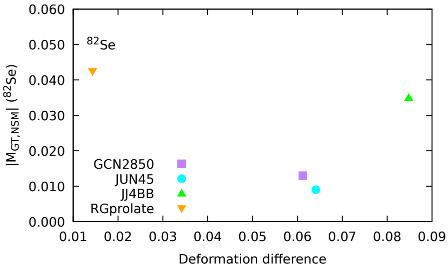
- $s \equiv$ nucleons not coupled to $J = 0$
- $s = 0$ corresponds to sphericity



Positive and negative M_{GT} contributions

Impact of seniority on $2\nu\beta\beta$:

- Large **cancelations** in GCN and JUN45 $\rightarrow$ Smallest NME
- RGprolate: large (4,0) contribution
- JJ4BB barely has cancelations $\rightarrow$ Anomalous $M_{GT}(\delta)$ behaviour



ELECTRONS' WAVEFUNCTIONS

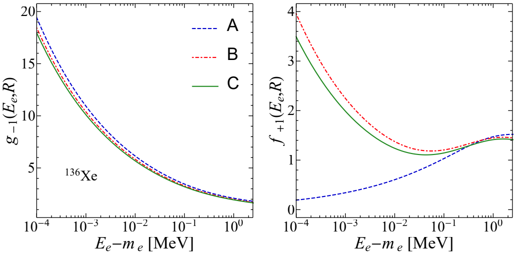
Finite size (FS): uniform sphere

$$V(r) = -\alpha Z_f / r \text{ for } r \geq R_A$$
$$V(r) = -\frac{\alpha Z_f}{2R_A}\left(3 - \frac{r^2}{R_A^2}\right) \text{ for } r < R_A$$

Screening: Thomas-Fermi potential (ϕ)

$$\frac{d^2\phi}{dx^2} = \frac{\phi^{3/2}}{\sqrt{x}},\ \phi(0) = 1,\ \phi(\infty) = 0$$

$x = r/b,\ b = 0.885 a_0 Z_f^{-1/3}$
$a_0 \equiv$Bohr radius
Z_f final nucleus Z



O. Nițescu et al. Universe 7 (2021) 5, 147

$A \equiv$ analytical approximation of $F(E,R)$
$B \equiv$ Exact Dirac point-like wavefunctions
$C \equiv$ Exact Dirac+FS+Screening