Lepton (e,μ,τ) - neutron interaction and low-energy scattering

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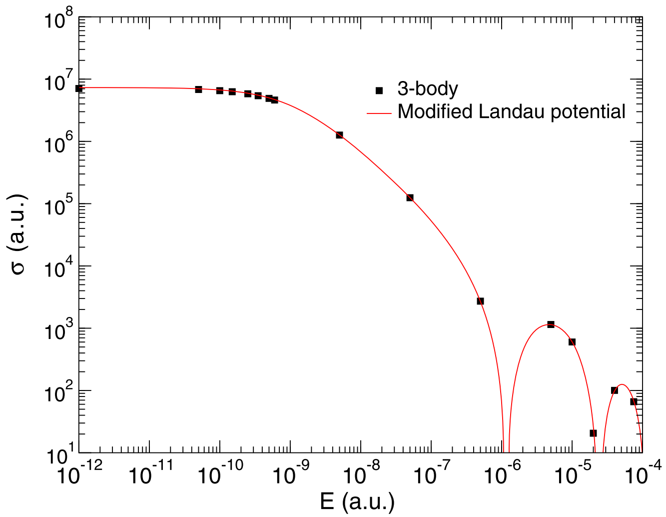
MOTIVATION

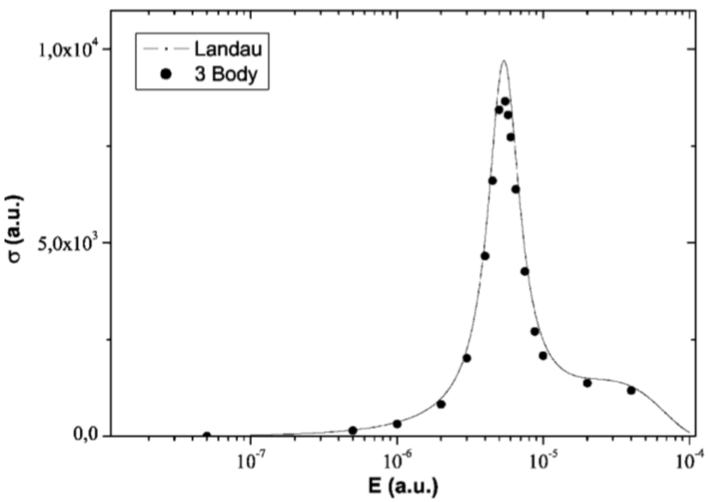
In the nuclear reactions, there are in fact no nuclear targets : targets are atomic
However the effect of the **e⁻**, their interaction with projectile, is (almost) always ignored.

Some years ago **(*)** we realized that the difference between **pp** and **pH** (p-$[pe]_{bound}$) low-energy scattering was huge : the consequences of a single atomic electron were dramatic

While $a_{pp} \approx -0.1$ fm, $a_{pH} = 397$Å ! (not a joke!)

not to talk about a rich series of resonances for L=1,2,3,…





(*) R. Lazauskas, J. Carbonell, Few-Body Syst 31 (2002)125
J. Carbonell, R. Lazauskas, D. Delande, L. Hilico, S.Kilic, Europhys Lett 64 (2003) 316

To go ahead with this study we had several possibilities :

- increase the complexity of the projectile : **p→ ^{2}H → ^{3}H → ^{3}He → ^{3}He**
- increase the number of electrons in the **target : H → 3He → 4He → 6Li**
- both !

In all cases it becomes terribly complicate … and very fast!

So we decided to pursue with an equally simple case : **np** versus **nH**, i.e. **n [pe]**

Not only because it was easier but because, even at zero energy, there are new interesting low-energy processes that occur, e.g. **nH → [pn] + e$^-$**

Since the V_{np} is well known, we needed only a reliable V_{en}

- in configuration space
- to be inserted in a non relativistic Schrodinger equation

We could not find it in the literature, so we decided to built it ourselves

The aim of this talk talk is to present the main properties of the Lepton (e, μ, τ)-Neutron interaction (*)

as well as some low-energy results (mainly limited to S-wave)

(*) J.C. and Tobias Frederico, Phys Rev C 109, 064002 (2024)

Lepton–neutron (Ln) interaction

If neutron was point-like, the **Ln** interaction would be given by the hyperfine Hamiltonian(*)

$$H_{eN} = \frac{\mu_0}{4\pi}\left\{-\frac{8\pi}{3}\mu_e \cdot \mu_N \; \delta(\mathbf{x}) + \left[\mu_e \cdot \mu_N - 3(\mu_e \cdot \hat{\mathbf{x}})(\mu_N \cdot \hat{\mathbf{x}}) - \frac{e}{m_e}\mathbf{L} \cdot \mu_N\right]\frac{1}{r^3}\right\}$$

It is a sum of two terms :

2. The interaction between the Magnetic moments $\mathbf{M_L}$ and $\mathbf{M_n}$

$$V_{M_1 M_2}(\vec{r}) = -\frac{2}{3}\,\vec{M}_1 \cdot \vec{M}_1\;\delta(\vec{r}) \;-\; \frac{3(\vec{M}_1 \cdot \hat{r}_1)(\vec{M}_2 \cdot \hat{r}) - \vec{M}_1 \cdot \vec{M}_1}{4\pi r^3}$$

3. The spin-orbit term

$$H_{LS}^{ln} = \frac{\mu_0}{4\pi}\frac{e}{m_e}\frac{1}{r^3}\mathbf{L} \cdot \vec{M}_n$$

To account for the **n** finite size, these expressions must be integrated over the **n** charge and magnetic densities, contained in the experimentally measured em form factors ($\mathbf{G_E}$ and $\mathbf{G_M}$) The charge distribution gives an addition term (small but relevant)

1. Purely Coulomb **Ln** interaction …despite **n** being neutral.

$$V_{en}^{C}(\vec{r}) = -\frac{e^2}{4\pi\epsilon_0}\int d\vec{r}_n \,\frac{\rho_n(\vec{r}_n)}{\mid \vec{r} + \vec{r}_n \mid} = \alpha(\hbar c)\int d\vec{r}_n \,\frac{\rho_n(\vec{r}_n)}{\mid \vec{r} + \vec{r}_n \mid}$$

(*) See, e.g. Jackson eq 5.73

SOME NOTATIONS

Magnetic moments : **M = μ *σ***

σ = Pauli matrices

μ_l and **q_l** algebrics (+ or -) with (for leptons e^-, μ^-, τ^-) q_L=-e and e>0

$$\vec{M}_l = g_l \frac{q_l \hbar}{2m_l} \vec{S} = \mu_l \vec{\sigma}\,, \qquad \mu_l = -\frac{g_l}{2}\frac{e\hbar}{2m_l} = -1.00116\left(\frac{m_e}{m_l}\right)\mu_B\,, \qquad \mu_B = \frac{e\hbar}{2m_e} = 5.788382\times10^{-5}\ \mathrm{eV\ T^{-1}}$$

$$\vec{M}_n = g_n \frac{e\hbar}{2m_p} \vec{S} = \mu_n \vec{\sigma}\,, \qquad \mu_n = \frac{g_n}{2}\frac{e\hbar}{2m_p} = -1.91304\ \mu_N\,, \qquad \mu_N = \frac{e\hbar}{2m_p} = 3.152451\times10^{-8}\ \mathrm{eV\ T^{-1}}$$

Landé factors **g_l**=+2.00232 **g_n**=-3.82608

NEUTRON FORM FACTORS AND DENSITIES

Neutron charge ($\boldsymbol{\rho}_c$) and magnetic ($\boldsymbol{\rho}_m$) densities are obtained as FT of the Sachs electric ($\mathbf{G}_E$) and magnetic ($\mathbf{G}_M$) form factors in the Breit-frame

$$\rho^n_{c,m}(\vec{r}) = \int \frac{d\vec{q}}{(2\pi)^3} \, G^n_{E,M}(q^2) \, e^{i\vec{q}\cdot\vec{r}} \quad \Longleftrightarrow \quad G^n_{E,M}(q^2) = \int d\vec{r} \, \rho_{c,m}(\vec{r}) \, e^{i\vec{q}\cdot\vec{r}} \, ,$$

$t=q^2=-Q^2$ is the (space-like) momentum transfer

By expanding the plane wave in the r.h.s. and integrating over the angular part, one gets

$$G(q^2) = G(0) - \frac{\langle r^2 \rangle}{6} q^2 + \frac{\langle r^4 \rangle}{120} q^4 + O(q^6).$$

and so

$$\langle r_n^{2k} \rangle_{c,m} = \frac{(-1)^k \, k!}{(2k+1)!} \left[\frac{d^k G_{E,M}}{d(q^2)^k} \right]_{q^2=0},$$

NEUTRON CHARGE DENSITIES

We have considered 3 different parametrizations of the n-charge form factor G_E :

Friar and Negele, Adv Nucl. Phys 8,219(1975)

$$G_E^n(q^2) = \beta_n \frac{q^2}{\left(1+\frac{q^2}{b_n^2}\right)^3}$$

Kelly, Phys Rev C70, 068202(2004)

Atac et al, Nature Communications 12, (2021) (data + LQCD)

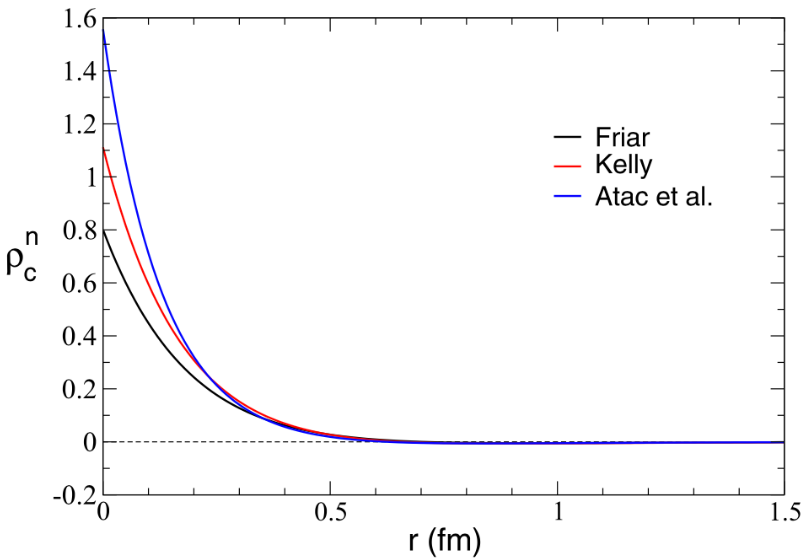
$$G_E^n(q^2) = \frac{A\tau}{1+B\tau} G_D(q^2), \qquad \tau = \frac{q^2}{4m_p^2},$$

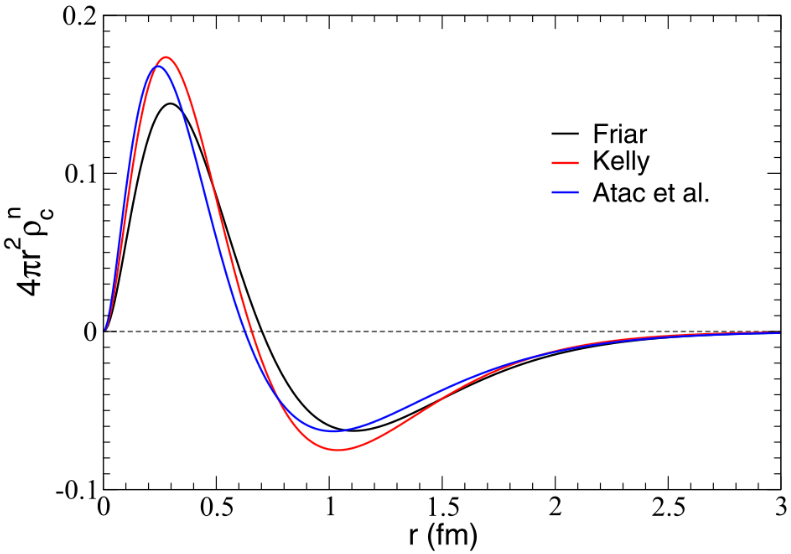
$$G_D(q^2) = \frac{1}{\left(1+\frac{q^2}{b^2}\right)^2}$$

All adjusted to the experimental value

…but differ beyond

$$\langle r_n^2 \rangle = \int d\vec{r}\; r^2 \rho_c^n(\vec{r}) = -\,0.116 \pm 0.002 \text{ fm}^2$$





NEUTRON MAGNETIC DENSITIES

We have considered 2 different parametrizations of the n-magnetic form factor G_M:

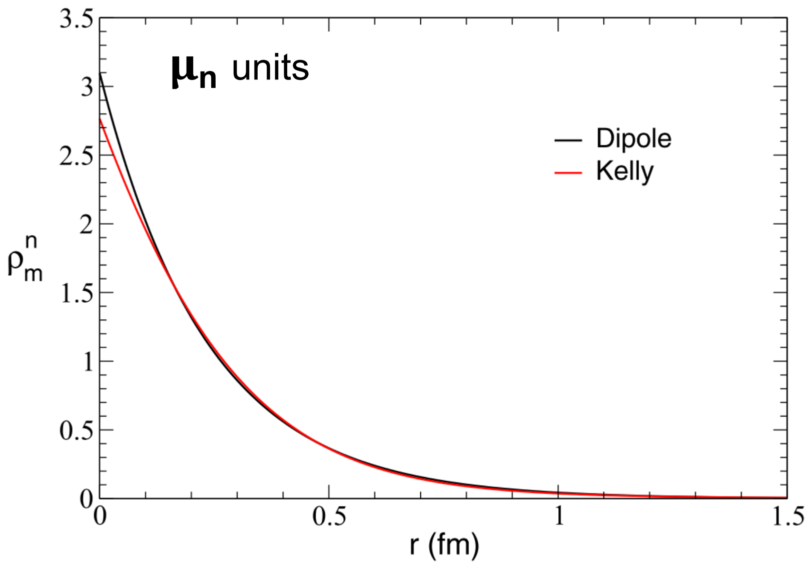
Dipole form, Galster et al Nucl Phys B 32,221(1971)

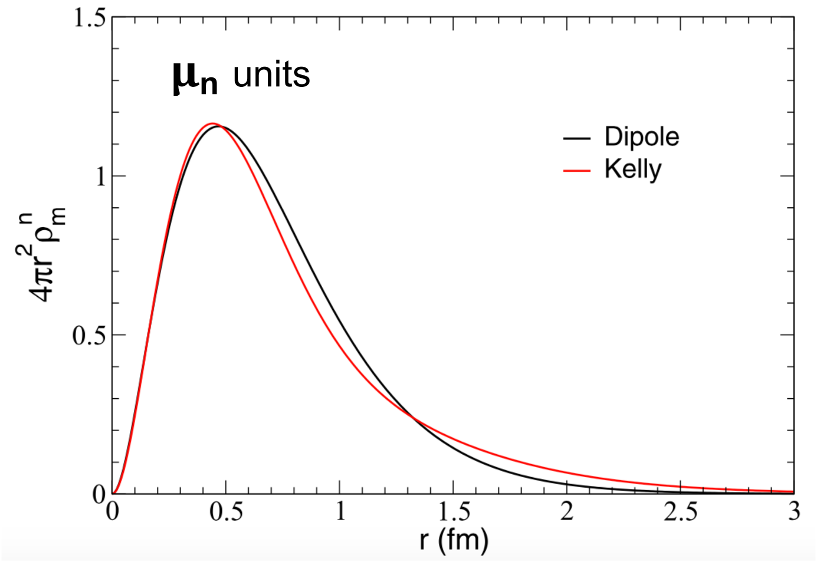
$$G_M^n(q^2) = \frac{\mu_n}{\left(1+\frac{q^2}{b_n^2}\right)^2},$$

Kelly, Phys Rev C70, 068202(2004)

$$G_M^n(Q^2) = \mu_n \frac{1+a_1\tau}{1+b_1\tau+b_2\,\tau^2+b_3\,\tau^3},$$

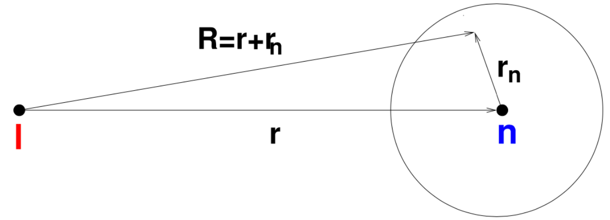
Adjusted to $\int d\vec{r}\;\rho_m^n(\vec{r}) = \mu_n = -1.91$



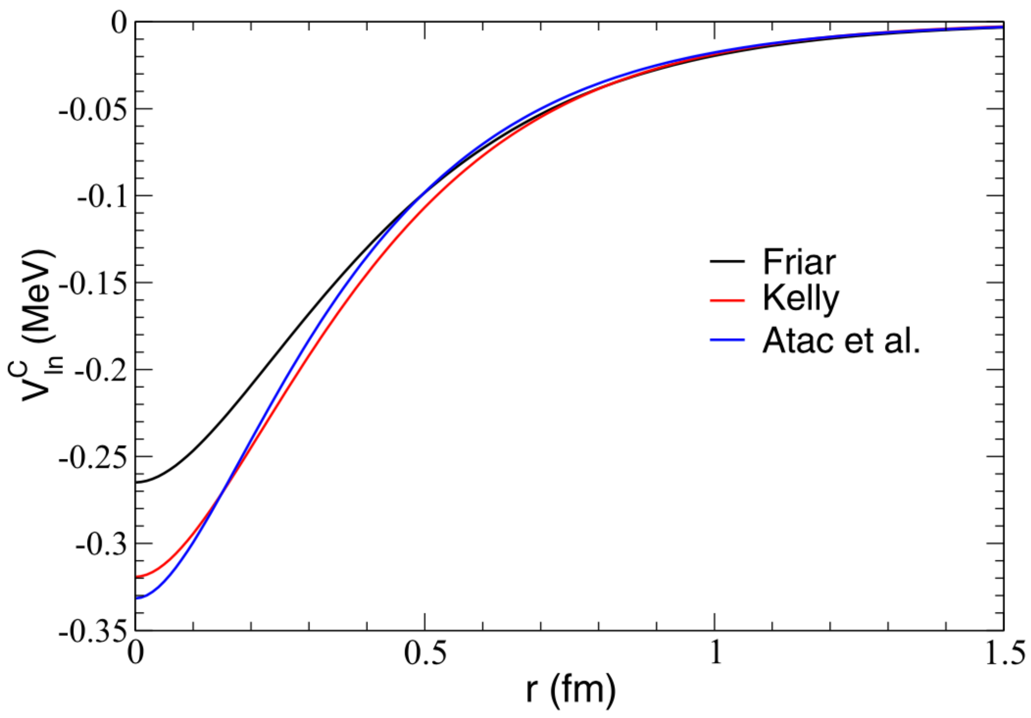


I. L-n INTERACTION : COULOMB TERM

$$V^C_{en}(\vec{r}) = -\frac{e^2}{4\pi\epsilon_0} \int d\vec{r}_n \, \frac{\rho_n(\vec{r}_n)}{|\,\vec{r} + \vec{r}_n\,|} = \alpha(\hbar c) \int d\vec{r}_n \, \frac{\rho_n(\vec{r}_n)}{|\,\vec{r} + \vec{r}_n\,|}$$



Results into a potential well of depth ≃ 0.3 MeV (about the e mass anyway!)
Attractive and the same for the 3 leptons **e-, μ-, τ-** (changing the sign for anti-leptons)



2. Ln INTERACTION : MAGNETIC DIPOLE TERM

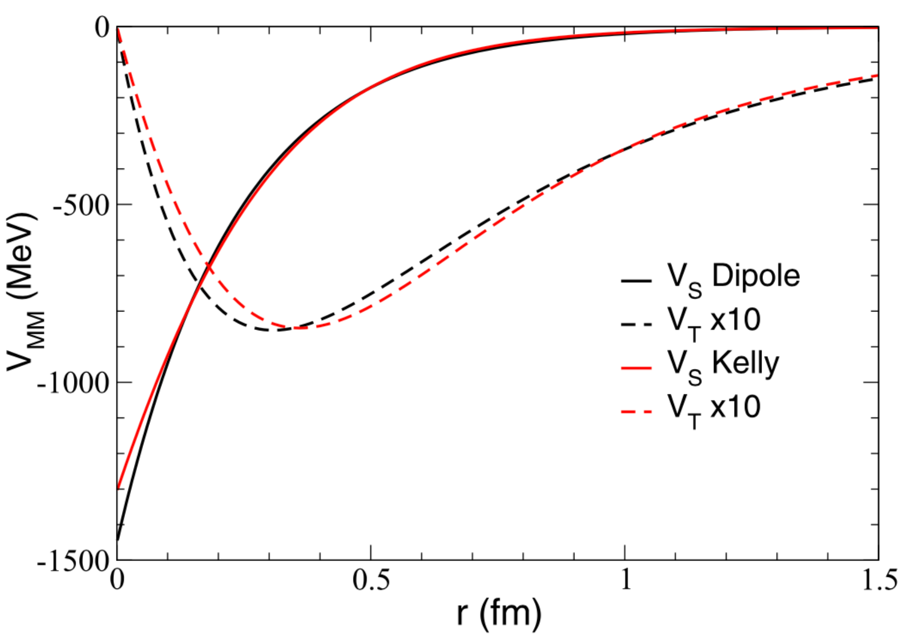
The ponit-like « MM » interaction is written as
$$V_{MM}^{ln}(\vec{r}) = -\frac{\mu_0\mu_l\mu_n}{4\pi}\left[\frac{8\pi}{3}\,\vec{\sigma}_l\cdot\vec{\sigma}_n\,\delta(\vec{r}) \;+\; \frac{\hat{S}_{12}(\hat{r})}{r^3}\right]$$

with the usual tensor operator
$$\hat{S}_{12}(\hat{r}) \equiv 3(\vec{\sigma}_1\cdot\hat{r})(\vec{\sigma}_2\cdot\hat{r}) - \vec{\sigma}_1\cdot\vec{\sigma}_2$$

Integrated over the n magnetic density (quite lengthy !) $\vec{R} = \vec{r} + \vec{r}_n$

$$V_{MM}^{ln}(\vec{r}) = -\frac{\mu_0\mu_l\mu_n}{4\pi}\left[\frac{8\pi}{3}\,\vec{\sigma}_l\cdot\vec{\sigma}_n\int d\vec{r}_n\rho_m^n(\vec{r}_n)\delta(\vec{R}) + \int d\vec{r}_n\,\frac{3(\vec{\sigma}_l\cdot\hat{R})(\vec{\sigma}_n\cdot\hat{R}) - \vec{\sigma}_l\cdot\vec{\sigma}_n}{R^3}\,\rho_m^n(\vec{r}_n)\right]$$

turns into a (huge!) **spin-spin + tensor** potential $V_{MM}^{ln}(x) = V_S(x)(\vec{\sigma}\cdot\vec{\sigma}) + V_T(x)\hat{S}_{12},$



1. When using the **Dipole** FF, the result is analytical and one gets

$$V_S(x) = -\frac{1}{3} C^{ln}_{MM} e^{-x}$$

x=br

$$V_T(x) = -C^{ln}_{MM} \frac{1 - \left(1 + x + \frac{x^2}{2} + \frac{x^3}{6}\right) e^{-x}}{x^3}$$

and for the **en** case $C^{en}_{MM} = 4359.4109 \text{ MeV.}$

The $1/r^3$ of V_T, is naturally regularized at r=0.
It remains asymptotically in the diagonal and coupling term…**with all kind of sorrows !**

With **Kelly** FF it remains anayltical but better to dont show it !

2. The lepton mass m_l appears in the prefactor

$$C^{ln}_{MM} = \mu_0 \frac{\mu_n \mu_l}{4\pi} b_n^3 = -\frac{g_n g_l}{4} \frac{\alpha\ (b\hbar c)^3}{4(m_n c^2)(m_l c^2)}.$$

It is interesting to take as reference V^{en} case and write

$$V^{ln}_{MM}(x) = \left(\frac{m_e}{m_l}\right) V^{en}_{MM}(x).$$

If for **en** case, V_C is negligeable with respect to V_{MM}, the last one scales as $1/m_l$ and these potentials can be comparable for heavier leptons

3. Ln INTERACTION : SPIN-ORBIT TERM

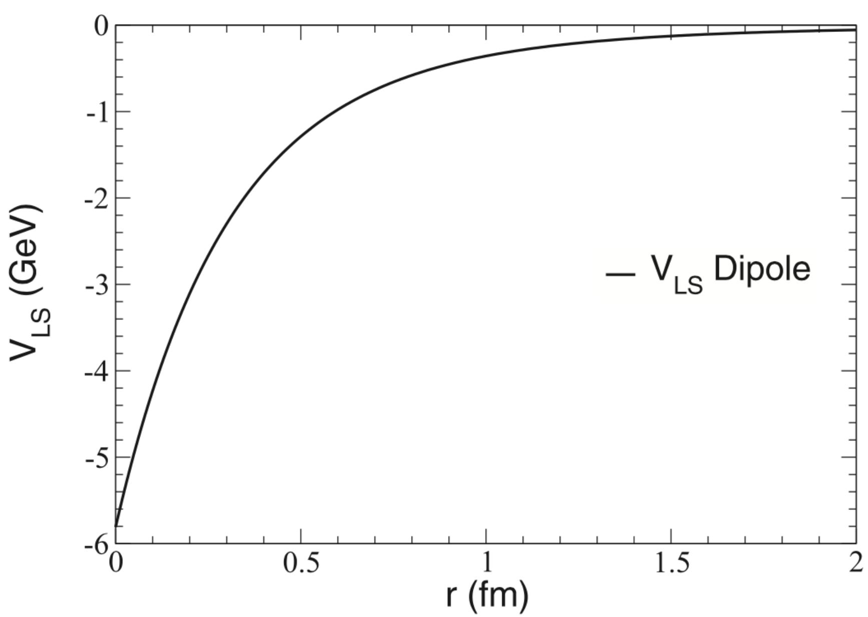
The ponit-like spin-orbit interaction $$H_{LS}^{ln} = \frac{\mu_0}{4\pi}\frac{e}{m_e}\frac{1}{r^3}\mathbf{L}\cdot\vec{M}_n$$

was also Integrated over the n magnetic density

$$H_{LS}^{ln} = -\frac{\mu_0}{4\pi}\frac{e\mu_n}{\mu_{ln}}\,\vec{\sigma}_n\cdot\mathbf{p}\wedge\int d\mathbf{r}_n\,\frac{\mathbf{R}\rho_m^n(\mathbf{r}_n)}{R^3}$$

and turns into $$V_{LS}^{ln} = V_{LS}(x)\;(\vec{L}\cdot\vec{s}_n).$$

For the Dipole FF one gets $$V_{LS}(x) = C_{LS}^{ln}\,\frac{1-\left(1+x+\frac{x^2}{2}\right)e^{-x}}{x^3} \qquad C_{LS}^{ln} = g_n\,\frac{\alpha(b\hbar c)^3}{(\mu_{ln}c^2)(m_pc^2)}$$



The spin-orbit operator $\mathbf{L.s_n}$ is not the usual one (it does not conserve total spin S)

It is convenient to give the matrix elements in the standard |SLJ> basis

- Null for S-waves : $\langle ^1S_0 \mid \vec{L}\cdot\vec{s}_n \mid ^1 S_0\rangle = \langle ^3S_1 \mid \vec{L}\cdot\vec{s}_n \mid ^3 S_1\rangle = 0$

- L>0 triplet « unnnatural parity(*) » states (3P0,3P2,3D1,3D3,….) :

$$\langle ^3L_{L\pm1} \mid \vec{L}\cdot\vec{s}_n \mid ^3 L_{L\pm1}\rangle = \lambda_\pm(L), \qquad \lambda_\pm(L) = \begin{cases} \frac{L}{2} & \text{if} \quad j_n = L+\frac{1}{2} \\ -\frac{L}{2}-\frac{1}{2} & \text{if} \quad j_n = L-\frac{1}{2} \end{cases}.$$

- L>0 « natural(**) parity » states: spin-singlet ($^1P_1$) and spin-triplet ($^3P_1$) are coupled

$$\langle ^{2S+1}L_{J=L} \mid \vec{L}\cdot\vec{s}_n \mid ^{2S'+1} L_{J=L}\rangle = \begin{matrix} \\ S=0 \\ S=1 \end{matrix} \begin{matrix} S=0 & S=1 \\ \left(\begin{matrix} 0 \\ \sqrt{L(L+1)} \end{matrix} \right. & \left. \begin{matrix} \sqrt{L(L+1)} \\ -1 \end{matrix} \right) \end{matrix}$$

…by $1/r^3$ potentials

(*) « unnatural » because L#J
(**) « natural » because L=J

SOME REMARKS

Our expressions for V_C,V_S,V_T,V_{LS} were obtained for arbitrary fermions ($\mathbf{M}_1$,$\boldsymbol{\rho}_1$) ($\mathbf{M}_2$,$\boldsymbol{\rho}_2$)

In the **np** case, by using Friar+Dipole form factor, our results are in agreement with the pionner work **(*)** where the em corrections to S-wave **np** scattering length have been estimated (using AV18)

In the **Λn** case, the main difference with **(*)** is in the 'LS' term
For the **np** they obtained :

$$V_{np}^{LS}(r) = -\frac{\alpha}{2M_n M_R}\,\mu_n\,\frac{F_{LS}}{r^3}\,[\mathbf{L}\cdot\mathbf{S}+\mathbf{L}\cdot\mathbf{A}] \qquad A=\frac{1}{2}(\sigma_n-\sigma_p)$$

and disregarded A …which does not conserve S : standard spin-orbit term.

In **Λn** **<u>this approximation is not justified</u>**… and create some misfortunes (see later)

(*) R.B. Wiringa, V.G.J. Stoks and R. Schiavilla, Phys. Rev. 51 (1995) 38

SUMARY OF Ln INTERACTION

When we put all together :

$$V^{ln}(x) = V_C^{ln}(x) + V_S^{ln}(x)(\vec{\sigma}\cdot\vec{\sigma}) + V_T^{ln}(x)\hat{S}_{12} + V_{LS}^{ln}(x)\ (\vec{L}\cdot\vec{s}_n)$$

The states are labeled by $J^\pi=0^\pm,1^\pm, 2^\pm$… the only conserved QNs

All L>0 states are two-by-two coupled, either by V_T ($^3P_2$-$^3F_2$…) or by V_{LS} ($^1P_1$-$^3P_1$…) with $1/r^3$ potentials, in the diagonal as well as in the coupling terms:

A real cauchemar !!!

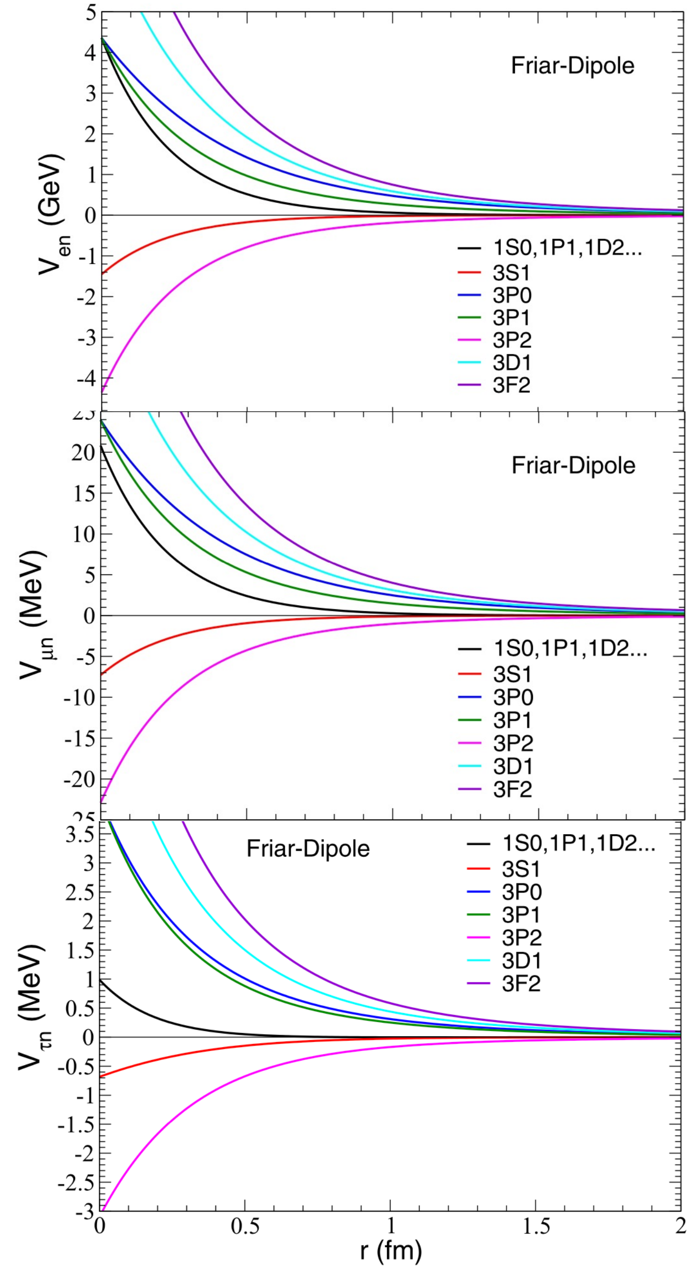
Better to go slowly…and start with S-waves

V_{ln} in some selected Partial Waves

- Very different scales for 3 leptons

- All singlet states are the same

- All V repulsive (except $^3L_{L+1}$)



SOME RESULTS

Ln S-WAVE SCATTERING AND LOW ENERGY PARAMETERS

Scattering length a_0 and effective range r_0 for different choices of G_E / G_M

(no experimental results)

1S0 (all in fm)

$^1S_0$	Friar-Dipole a_0	Friar-Dipole r_0	Kelly-Kelly a_0	Kelly-Kelly r_0	Atac-Kelly a_0	Atac-Kelly r_0
e^-n	2.926×10^{-3}	-149	2.920×10^{-3}	-186	2.920×10^{-3}	-186
μ^-n	2.501×10^{-3}	-170	2.497×10^{-3}	-215	2.501×10^{-3}	-215
τ^-n	1.574×10^{-4}	4814	1.623×10^{-4}	-2145	1.849×10^{-4}	-276
e^+n	-2.949×10^{-3}	150	-2.943×10^{-3}	186	-2.943×10^{-3}	186
μ^+n	-2.518×10^{-3}	171	-2.514×10^{-3}	217	-2.517×10^{-3}	216
τ^+n	-1.577×10^{-4}	-4802	-1.625×10^{-4}	2142	-1.851×10^{-4}	276

SOME REMARKS

Typical sizes: $a_0 \simeq 10^{-3}$ fm but huges (even negative) r_0 !

a_0 quite independent of the FF parametrisation (specially for e and μ)
r_0 quite depedent !

If T=V (perturbative) LEPs will change sign from particle to antiparticle
(Non perturbative effects are of the order of 1%)

Ln S-WAVE SCATTERING AND LOW ENERGY PARAMETERS

3S1 (all in fm)

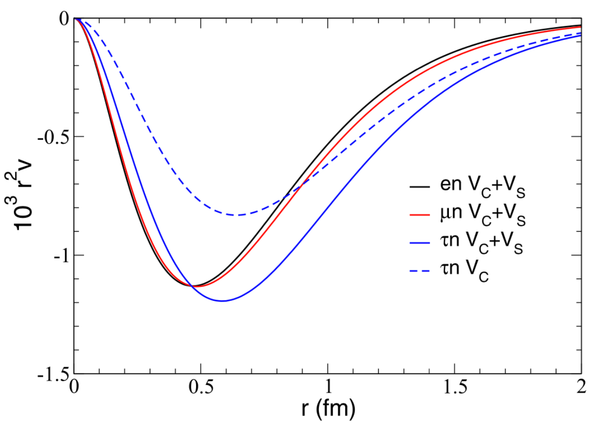
	Friar-Dipole		Kelly-Kelly		Atac-Kelly	
$^3S_1$	a_0	r_0	a_0	r_0	a_0	r_0
e^-n	-0.981×10^{-3}	448	-0.979×10^{-3}	559	-0.979×10^{-3}	559
μ^-n	-1.015×10^{-3}	462	-1.012×10^{-3}	546	-1.009×10^{-3}	557
τ^-n	-1.200×10^{-3}	498	-1.192×10^{-3}	482	-1.117×10^{-3}	535
e^+n	0.979×10^{-3}	-448	0.977×10^{-3}	-559	0.977×10^{-3}	-559
μ^+n	1.012×10^{-3}	-461	1.010×10^{-3}	-545	1.006×10^{-3}	-556
τ^+n	1.200×10^{-3}	-497	1.189×10^{-3}	-481	1.117×10^{-3}	-535

SAME REMARKS AS FOR 1S0

Almost flavour-independent !!! … despite 3 orders of magnitude on $\mathbf{m_l}$'s

$$a_0^B = \int_0^\infty dr\; r^2\; v(r),$$

$$v_S \equiv \frac{2\mu}{\hbar^2} V_S$$



Ln S-WAVE CROSS SECTION

The S-wave phaseshifts $\boldsymbol{\delta}_0$(k) were computed.

In the kinematical domain were the non-relativistic treatement is justified they are well reproduced by the leading term in

$$\delta_0(k) = -k\, a_0 \left[1 + \frac{1}{2} r_0\, a_0\, k^2 + \cdots \right],$$

The zero-energy cross section

$$\sigma_{ln}(0) = \pi \left(|a_s|^2 + 3\, |a_t|^2 \right)$$

provides very close values for the 3 considered leptons

$$\sigma_{en}(0) = 0.358\, \mu b \qquad \sigma_{\mu n}(0) = 0.292\, \mu b \qquad \sigma_{\tau n}(0) = 0.136\, \mu b$$

(no experimental results)

Λn POSSIBLE BOUND STATES

Longstanding debate, in theory as well as in experimental

For : very strong potentials
Against : very small scattering length (negative for attractive chanels)

Most favorable case is the 1S0 **e^+n** and **μ^+n** for which the **V_S** is attractive (5 GeV) but for which $a_0 \simeq -0.003$ fm

When introducing a scaling factor $V_s=s*V$ bound state appears for s=230-270 !!

No any bound state, by far !!!!

COHERENT SCATTERING
and
n – e* « e-bound to heavy Atom » SCATTERING

When very low-energy n scattering by solids, it is pertinent to consider the coherent scattering

$$a_c = \frac{a_s + 3\, a_t}{4}$$

In en S-waves the interaction is dominated by V_S , with **σ.σ**=-3 for S=0 and **σ.σ**=1 for S=1…

If T=V=a (and nothing else in V than V_S !) there would be an exact cancellation between **a_s** and **a_t**…and no zero-energy coherent scattering (a_c=0)!

The coherent scattering is thus provided by :

- The « negligeable » Coulomb potential V_C …

  and/or

- The non perturbative effects (beyond T=V=a)

Numerical results will tell us who is who ….

	Friar-Dipole	Kelly-Kelly	Atac-Kelly
	a_c (fm)	a_c (fm)	a_c (fm)
$e^- n$	-4.50×10^{-6}	-4.42×10^{-6}	-4.43×10^{-6}
$\mu^- n$	-1.36×10^{-4}	-1.35×10^{-4}	-1.32×10^{-4}
$\tau^- n$	-5.07×10^{-4}	-4.88×10^{-4}	-3.76×10^{-4}
	a_c^C (fm)	a_c^C (fm)	a_c^C (fm)
$e^- n$	-7.14×10^{-7}	-7.08×10^{-7}	-6.90×10^{-7}
$\mu^- n$	-1.33×10^{-4}	-1.32×10^{-4}	-1.28×10^{-4}
$\tau^- n$	-8.60×10^{-4}	-8.53×10^{-4}	-8.31×10^{-4}
$e^+ n$	7.14×10^{-7}	7.08×10^{-7}	6.90×10^{-7}

- Coherent en scattering length **(upper half)** are 10^1-10^3 times smaller than 1S0/3S1 separately

- For en and μn cases, great stability w.r. FF

- For **en**, **V_C alone (lower half part) is not enough** : one needs **V_S** and non perturbative !

- Coherent cross sections $\sigma_c = 4\pi \left| \frac{1}{4} \left[f_s(k) + 3 f_t(k) \right] \right|^2$ gives (Atac + Kelly FFs)

$$\sigma_c^{en} = 0.0023\ nb \qquad \sigma_c^{\mu n} = 2.2\ nb \qquad \sigma_c^{\tau n} = 79\ nb$$

(no experimental results) …but who cares about ?

A very different situation occurs when scattering « zero »-energy n's on materials

(the only measured quatity)

Unill now al the scattering fresults were « on flight » scattering

One suposes that the e's are « attached » to atom (a*), which recoils as a whole (M>>m_n)

- The n-Atom reduced mass μ_{nA} is taken equal to m_n=939.565 MeV
- All spin-spin effects are disregarded (averaged to zero, not « compensated »)
- Scattering comes only from V_C !!!

The accepted experimental (*) value is b_{ne}=1.32 +/- 0.03 fm !!!!

Our result a_c^C(ne)=7.14 10^{-7} fm, with $\mu_{nA}=m_n$ turns into $a_c^*(ne^*)$=1.32 10^{-3} fm (Friar)

	Friar-Dipole		Kelly-Kelly		Atac-Kelly	
	a_0	r_0	a_0	r_0	a_0	r_0
e*-n	-0.0013152	+501.48	-0.0013050	+448.75	-0.0012703	518.18
Born	-0.0013152	+501.49	-0.0013050	+448.77	-0.0012703	518.16

This result assumes that only V_C contributes and is givern perturbatively by

$$a_0^B(e^*n) = \frac{(m_n c^2)\alpha}{3(\hbar c)} \langle r^2 \rangle_n \qquad r_0^B(e^*n) = -\frac{1}{5\, a_0^B(ne^*)} \frac{\langle r^4 \rangle_n}{\langle r^2 \rangle_n}.$$

a_0 entirely determined by $<r^2_n>$ and so quite stable wr/FF

r_0 depends on $<r^4_n>$ and is less fixed by FF !!!

(*) Hartmut Abele, Progress in Particle and Nuclear Physics 60 (2008) 1-81

NB.

If **n** would fill the full $\mathbf{V_{ne}}$ interaction (keeping $\boldsymbol{\mu_{nA}=m_n}$):

Totally different results…

	Friar-Dipole	Kelly-Kelly	Atac-Kelly
a_s^*	0.843	0.905	0.905
a_t^*	0.611	0.567	0.567
a_c^*	0.669	0.652	0.652

Enhancement factor m_n/m_e

In particluar there is a n-e* « bound » state in 3S1 (change sign) with B=150 MeV r=0.5 fm !!!

HIGHER ANGULAR MOMENTA (L>0)

Nothing is known in the **L-n** system

For a single channel, short range + C_3/r^3, two key references (*,**)
Results based on the « two-potential formula » (like for Coulomb) :

- L=0 : no a_0 and $\sigma_0(k=0)$ divergent ! (hopefully we are not concerned)

- L>0 : at $k \to 0$ everything is determined by asymptotic coefficient of $\mathbf{V_{ln}}$

$$\beta_3 = \frac{2\mu}{\hbar^2} C_3 \quad \text{with} \quad C_3 = \lim_{r \to \infty} r^3 V(r).$$

It depends on the partial wave $\boldsymbol{\beta_3 = \beta_3(L,S,J)}$ and [Length]

phase shifts
$$\tan \delta_{LSJ}(k) = \frac{k\beta_3}{2L(L+1)} + O(k^2)$$

and so
$$\sigma_{LSJ}(k) = (2J+1)\frac{\pi\beta_3^2}{4L^2(L+1)} + O(k)$$

- Short-range part plays no role
- Non-vanishing contributions at k=0 !!!
- How to extract S-wave contributions from an experiment ?

(*) B. Gao, Phys. Rev. A 59, 2778 (1999)
(**) Tim-Oliver Muller, PRL 110, 260401 (2013)

V_{Ln} asymptotic coefficients for the lowest PW + zero energy cross section (μb)

	en		*μn*		*τn*	
	β_3	$\sigma_L(0)$	β_3	$\sigma_L(0)$	β_3	$\sigma_L(0)$
$^3P_0$	1.76×10^{-2}	0.61	1.70×10^{-2}	0.57	1.38×10^{-2}	0.37
$^3P_1$	0.88×10^{-2}	0.45	0.91×10^{-2}	0.49	1.07×10^{-2}	0.68
$^3P_2$	-5.28×10^{-3}	0.28	-5.34×10^{-3}	0.28	-5.67×10^{-3}	0.32
$^3D_1$	2.06×10^{-2}	0.28	2.03×10^{-2}	0.27	1.86×10^{-2}	0.23
$^3D_2$	0.88×10^{-2}	0.08	0.91×10^{-2}	0.09	1.07×10^{-3}	0.12
$^3D_3$	-1.09×10^{-2}	0.18	-1.10×10^{-2}	0.18	-1.15×10^{-3}	0.20
$^3F_2$	2.58×10^{-2}	0.18	2.56×10^{-2}	0.18	2.43×10^{-2}	0.16
$^3F_3$	0.88×10^{-2}	0.03	0.91×10^{-2}	0.03	1.07×10^{-2}	0.04
$^3F_4$	-1.66×10^{-2}	0.14	-1.67×10^{-2}	0.13	-1.73×10^{-2}	0.15

Remind S-wave (1S0+3S1) values

$$\sigma_{en}(0) = 0.358\,\mu b \qquad \sigma_{\mu n}(0) = 0.292\,\mu b \qquad \sigma_{\tau n}(0) = 0.136\,\mu b$$

Dominated by P+D+F waves

Some misfortunes….

β_3= β_3(L,S,J) is determined by the matrix elements of the LSJ operator

$$V^{ln}(x) = V_C^{ln}(x) + V_S^{ln}(x)(\vec{\sigma}\cdot\vec{\sigma}) + V_T^{ln}(x)\hat{S}_{12} + V_{LS}^{ln}(x)\ (\vec{L}\cdot\vec{s}_n)$$

- Spin-spin is independent of L

- S_{12} tends to a constant when $L\to\infty$

- **$L.s_n$ diverges linearly with L (for some familly of states) !!!!!**

Because of that the sum over L has a logarithmic divergence !!!!!

The origin of that « anomaly » is not clear …. but must be clarified :

- either a consequence of L.s term that may be regularized

- intrinsic property of this operator (disregarded in NNP-waves)

- should be limited to differential cross sections ?

IN CONCLUSION

We have obtained the lepton-neutron (en, μu, τn) potential in configuration space
It is based on the hyperfine (em) interaction integrated over the neutron em densities
It has a central (V_C), spin-spin (V_S) , tensor (V_T) and “spin-orbit” (V_{LS}) terms

The S-waves low-energy scattering parameters were computed and we checked their stability with respect to different form factor parametrisations

The “in medium“ n scattering with electron-bound-to-atom (ne*) was considered
The computed coherent scattering length is compatible with the experimental results.
It is entirely determined by V_C (Coulomb) and it is perturbative

L>0 angular momentum states were considered.
- All of them are 2x2 coupled by tensor or by spin-orbit terms
- The interaction is long range ($1/r^3$) both in the diagonal and in the coupling terms
  No scattering theory is - for the moment - available
- They contribute at zero energy, are dominant, and the PW sum seems to diverge (due to LS !)

First application to nH scattering are coming soon (Next INT Workhsop)

Many thanks for your attention !

…and for this nice workshop

Good luck to everybody for next Tuesday ….!