A fluid-dynamic approach to heavy quarks in the quark-gluon plasma

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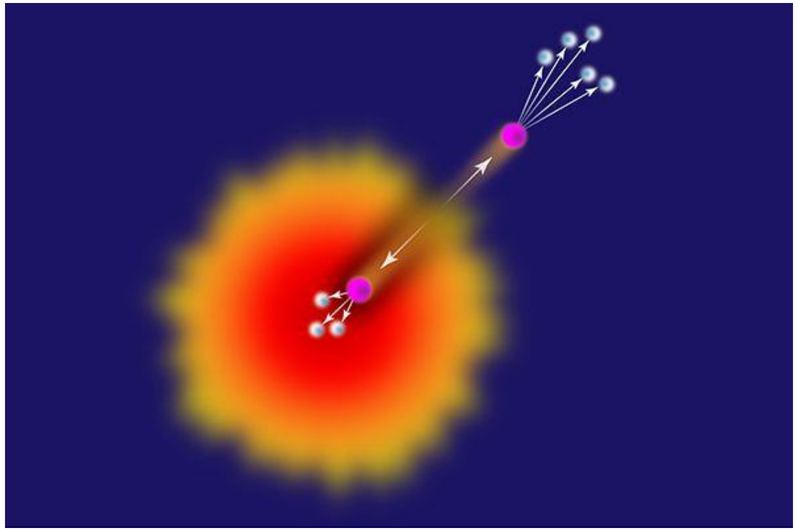
Heavy Flavor Production in Heavy-Ion and Elementary Collisions

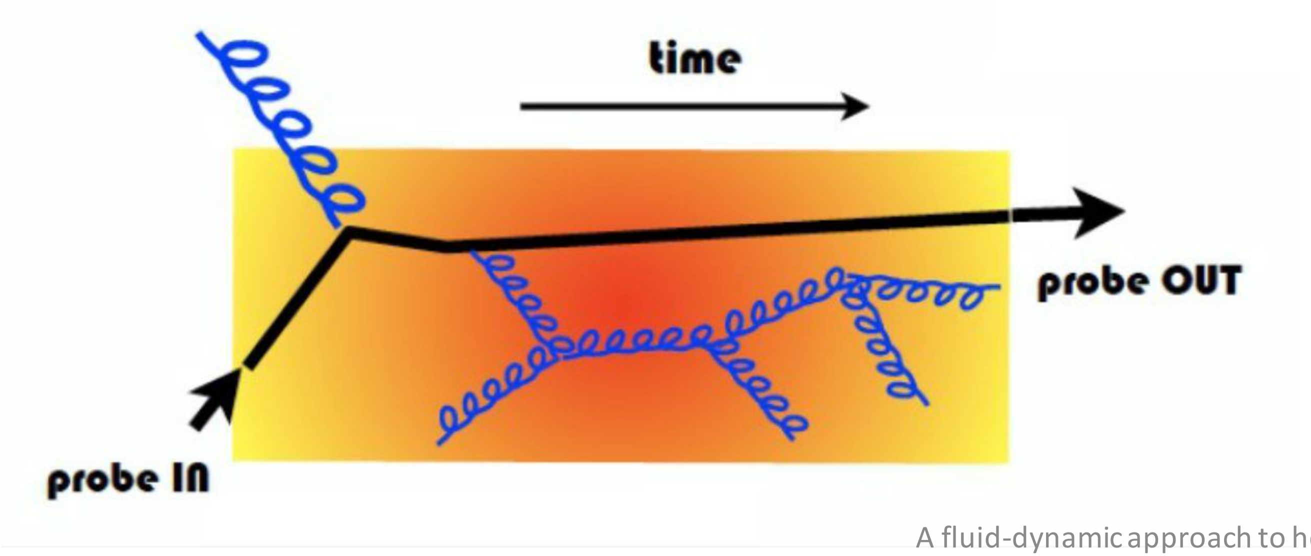
Heavy quarks as hard probes

Heavy quarks in heavy-ion collisions

- Are produced via hard scatterings at the beginning of the collision: they go through all the stages of the expanding fireball

- They interact with light quarks and gluons in the QGP via elastic and radiative processes → they can lead to a better understanding of the parton-medium interaction





- They help us in the **characterization of the QGP**

- In the low p_T region they provide a window to study **equilibration processes**

Thermalization

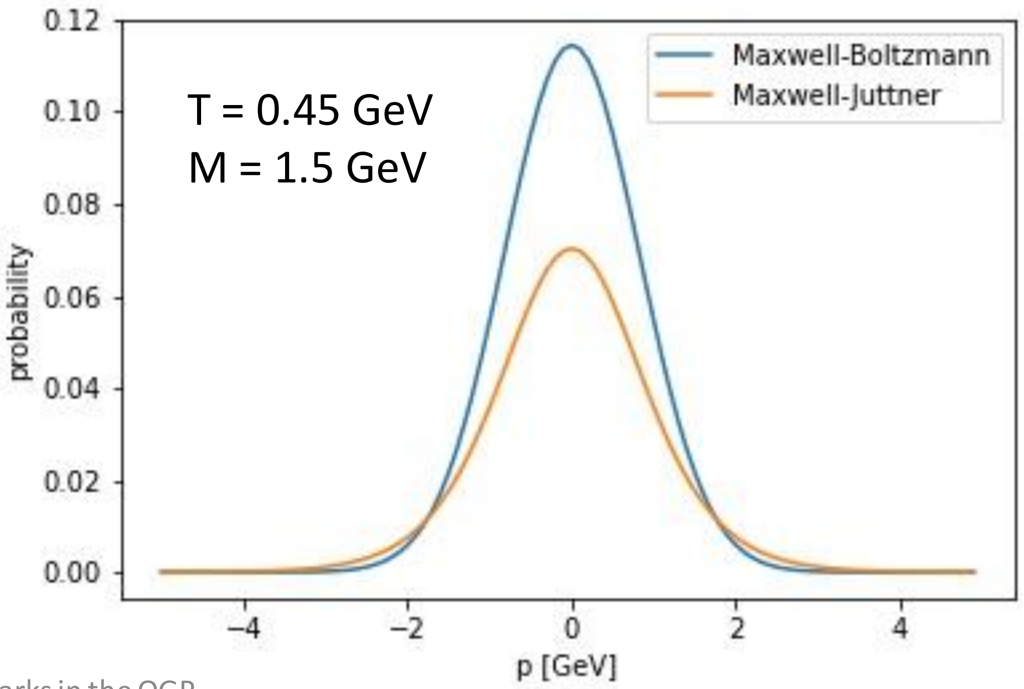
If particles have enough time to interact with each other they will eventually relax to (at least local) thermal equilibrium.

- **chemical equilibrium:**

the particle multiplicity is given by a thermal distribution at chemical potential $\mu = 0$
For HQs: initial hard production very far from chemical equilibrium! **Fugacity factor** needed.

- **kinetic equilibrium:**

the momentum distribution of the particles approaches a Maxwell-Boltzmann (Maxwell-Juttner) distribution
For HQs: possibly get quite close to local kinetic equilibrium within the lifetime of the fireball

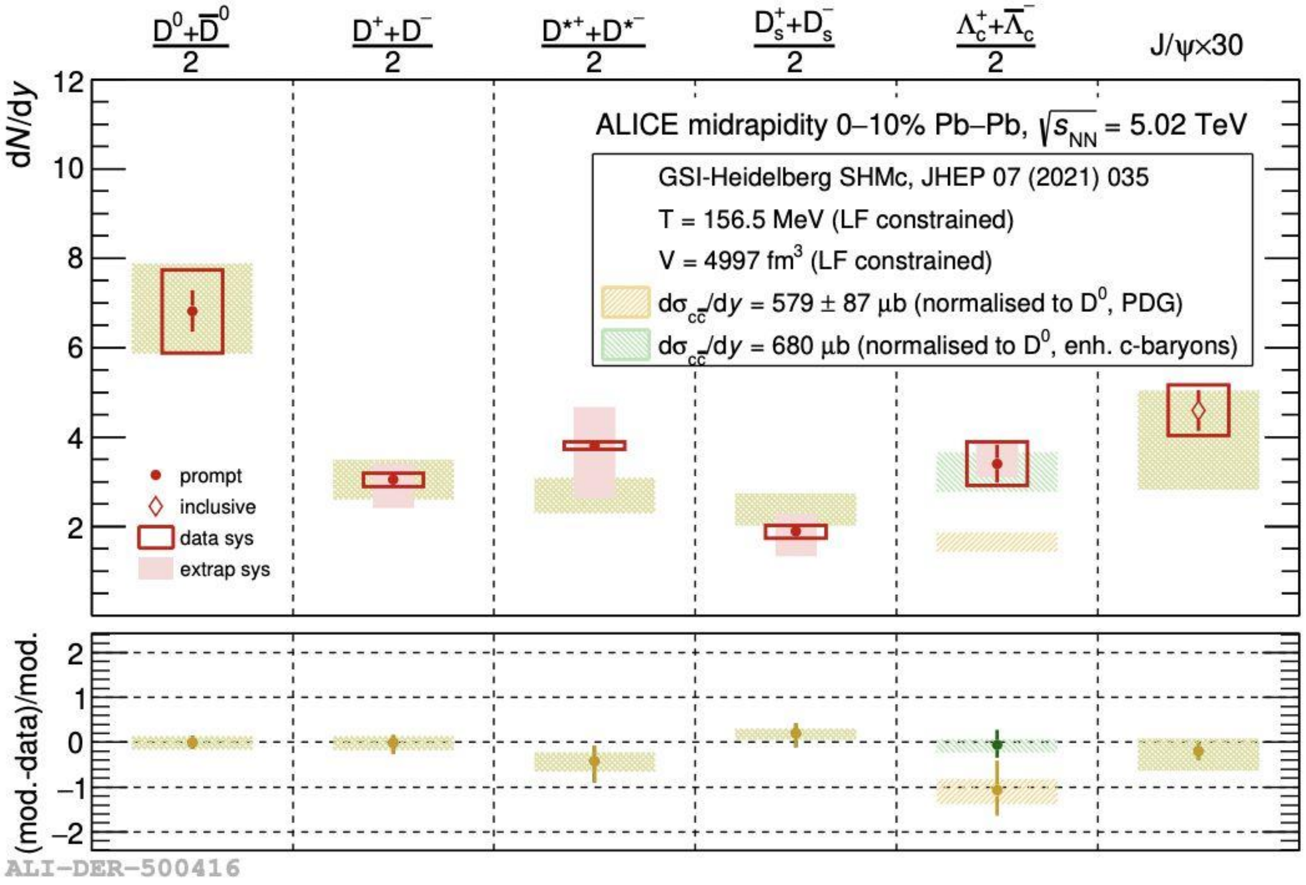


Statistical hadronization model for charm

An extension to the SHM was proposed to include charmed hadrons. The assumption of their production from a thermal source at the freeze-out temperature of the QGP reproduces well the experimental data!

Only one parameter - the fugacity factor for charm ($\sim$30) - is needed to reproduce multiplicities of charmed hadrons.

First signs of **full thermalization** of charm quarks!



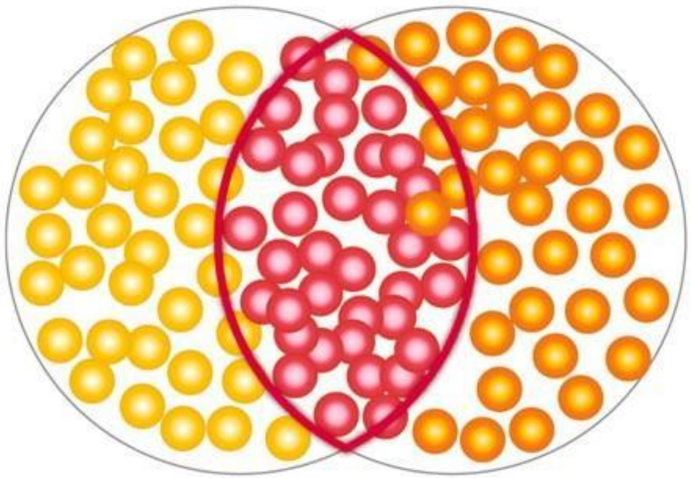
JHEP 07 (2021) 035 Andronic et al.

Hydrodynamics in the light sector

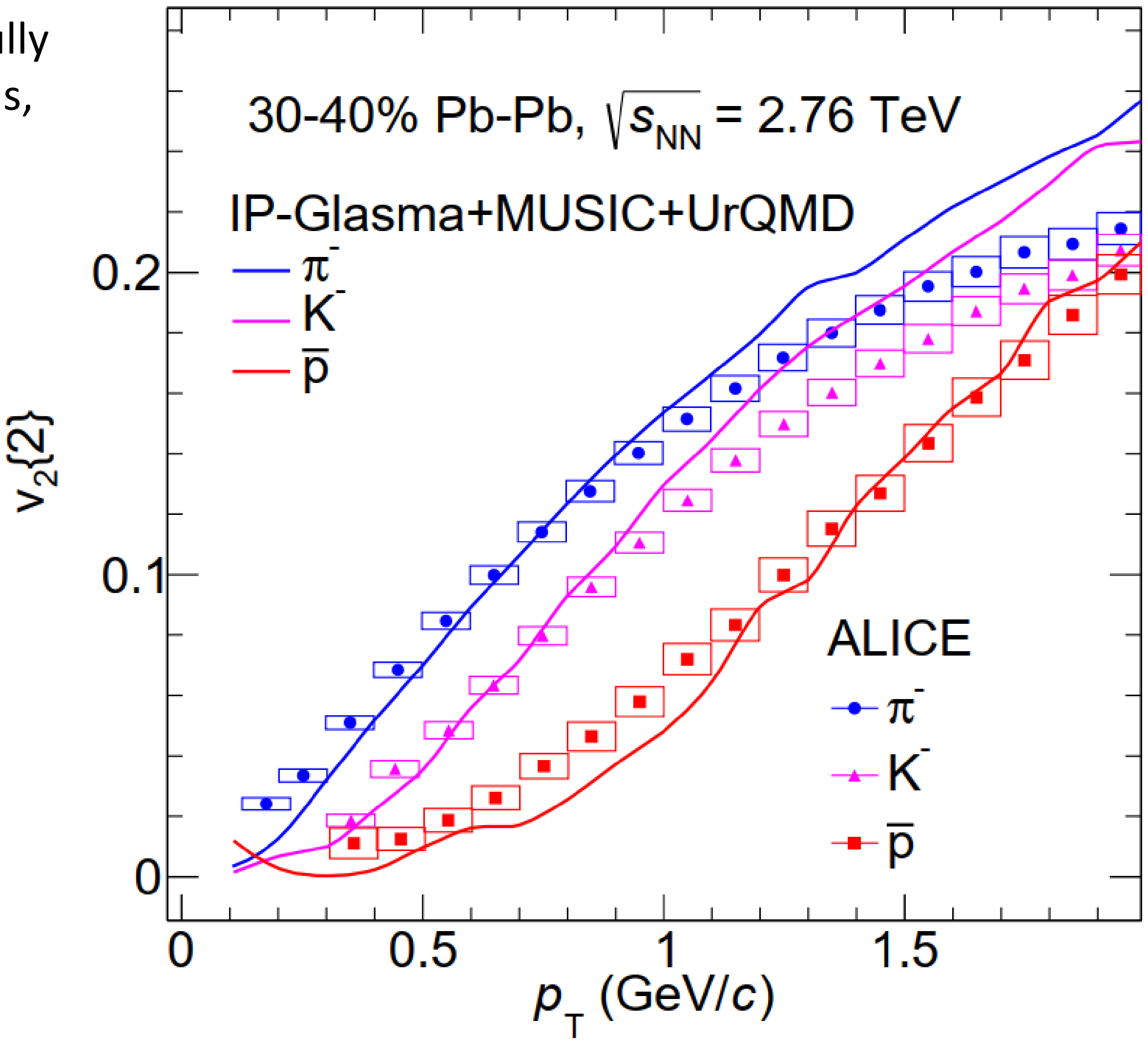
Relativistic viscous hydrodynamics: powerful tool in order to capture the features of the collective motion of the medium. It successfully describes light-flavour observables such as elliptic flow of protons, pions and kaons.

Elliptic flow v_2 is an important probe of collectivity in the system created in HICs: it is a response of the dense system to the initial conditions → sensitive to the early and strongly interacting phase of the evolution.

$$E\frac{d^3N}{d^3p} = E\frac{d^2N}{2\pi p_T dp_T dy}\left(1+\sum_{n=1}^{\infty} 2v_n \cos[n(\phi-\psi_{\rm RP})]\right)$$



NPA 979 (2018) 251-264 Dubla et al.



Further hints of thermalization

PLB 813 (2021) 136054 ALICE coll.

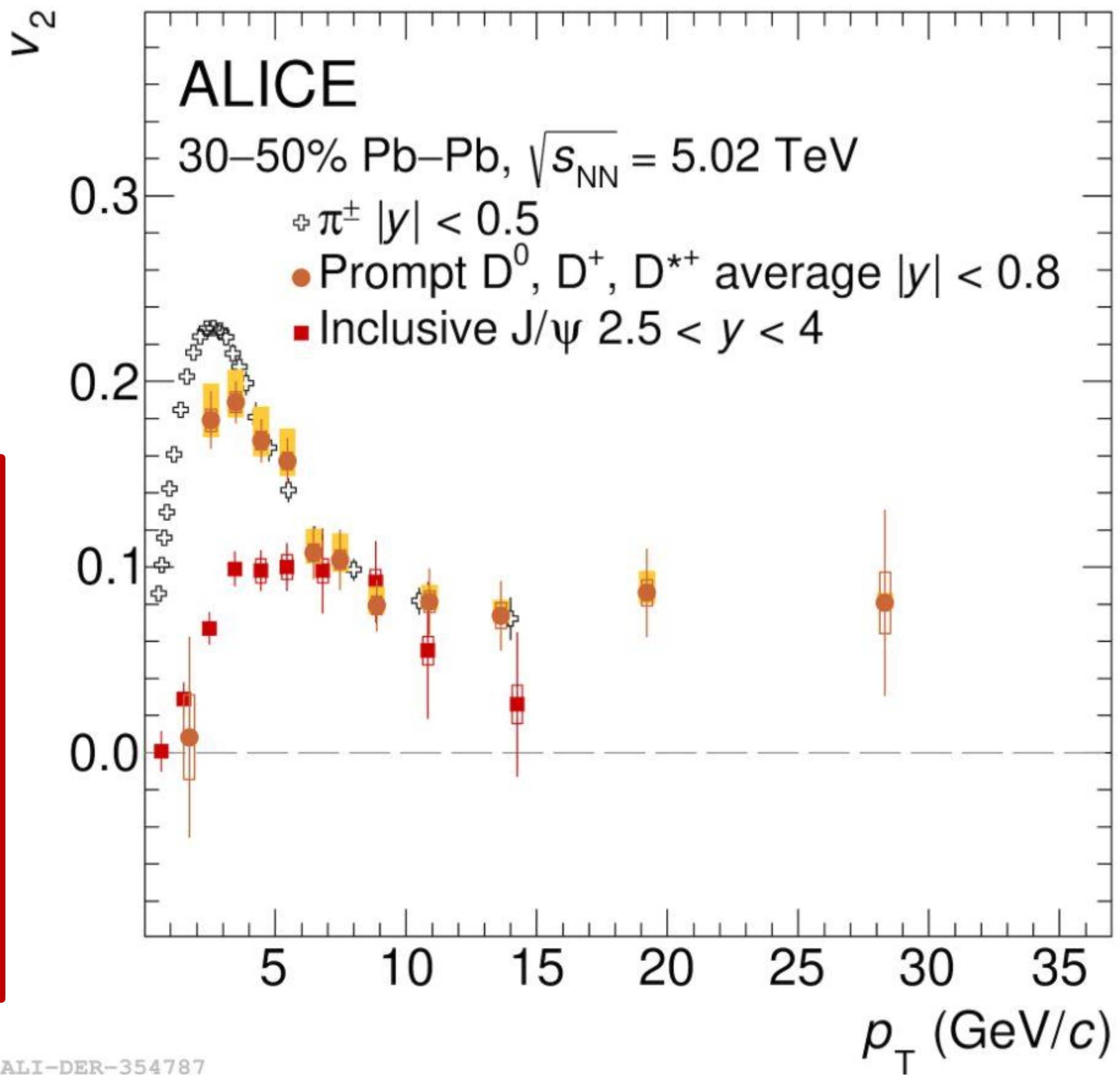
Significant measurements of J/Ψ and D mesons elliptic flow

- How strong are charm quarks interacting with the partons in the QGP?
- Do they interact long enough with the medium to be considered part of the medium itself?

Driven by the experimental data, we propose a new way of studying the heavy-quark dynamics in the QGP.

We assume heavy quarks interacted long enough with the medium to approach local kinetic equilibrium

→ we treat them with a **fluid-dynamic approach**!



Hydrodynamics with conserved currents

The equations describing the QGP as a relativistic viscous fluid can be written as

$$\nabla_\mu T^\mu{}_\nu = 0 \qquad T^{\mu\nu} = \epsilon u^\mu u^\nu + (P+\Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}$$

Energy-momentum conservation

$$\tau_\Pi \dot{\Pi} + \Pi = -\zeta\theta + \mathcal{J} + \mathcal{K} + \mathcal{R}$$

$$\tau_\pi \dot{\pi}^{\langle\mu\nu\rangle} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} + \mathcal{J}^{\mu\nu} + \mathcal{K}^{\mu\nu} + \mathcal{R}^{\mu\nu}$$

Equations of motion for dissipative quantities

Additionally, one can consider other conserved currents e.g. baryon number, strangeness etc.

$$\nabla_\mu N^\mu = 0 \qquad N^\mu = n u^\mu + \nu^\mu$$

Number conservation

$$\tau_n \dot{\nu}^\mu + \nu^\mu = \kappa_n \nabla^\mu\left(\frac{\mu_Q}{T}\right) + J^\mu + K^\mu$$

Equation of motion for diffusion current

To close the system of equation, one needs to provide an **Equation of State**.

The QGP is characterized by the **transport coefficients** ζ, η, κ_n .
The presence of **relaxation times** ensures the causality of the equations.

PRD 85 (2012) 114047 Denicol et al.
PRD 101 (2020) 076007 Fotakis et al.

Our fluid-dynamic framework: Fluid*u*M

Fluid dynamics of heavy-ion collisions with Mode expansion

- Bjorken coordinates r, τ, φ, η
- Background-fluctuation splitting

$$\Phi(r,\tau,\varphi,\eta) = \Phi_0(r,\tau) + \Phi_1(r,\tau,\varphi,\eta)$$

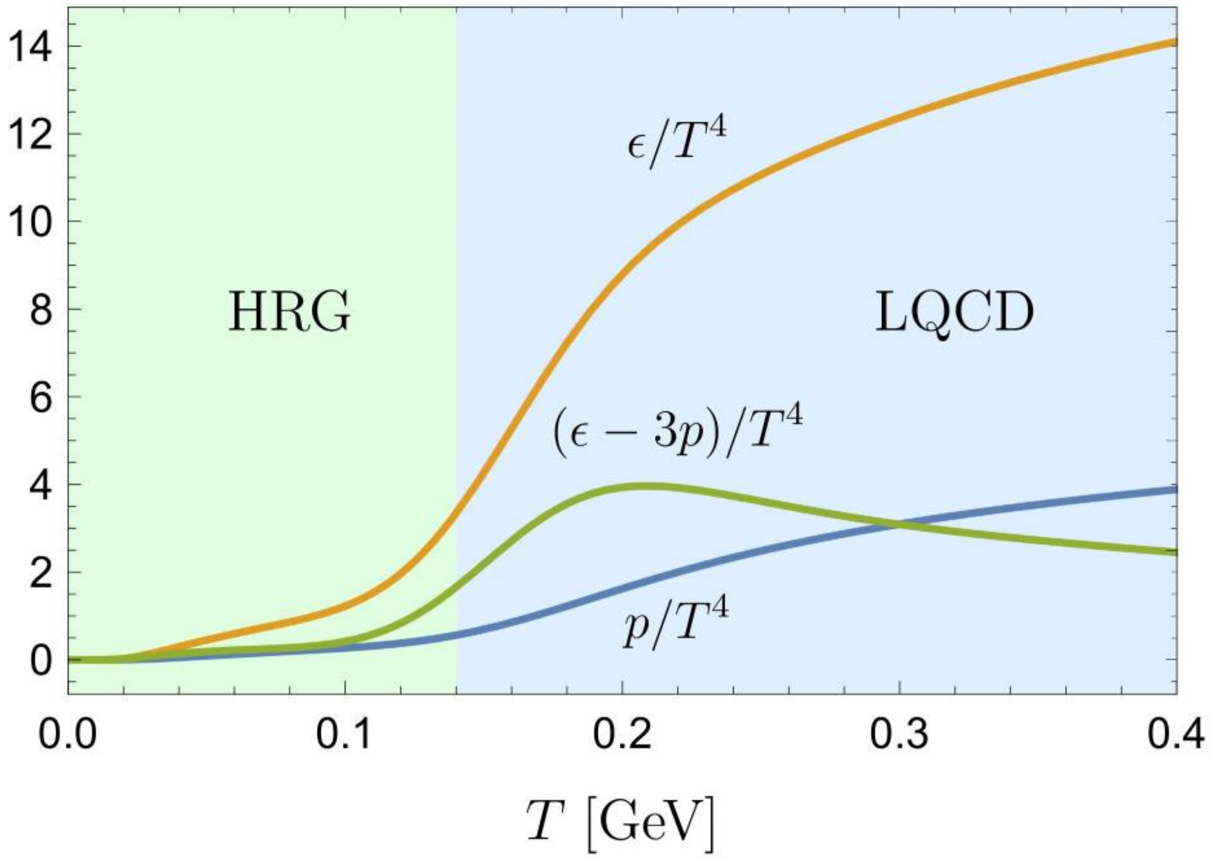
- Initial conditions from Trento
  PRC 92 (2015) 011901 Moreland et al.

- Resonance decays with FastReso
  Eur. Phys. J. C (2019) 79: 284 Mazeliauskas et al.

- Includes partial chemical equilibrium $T_{chem} \neq T_{kin}$

- From the background we can compute spectra and mean transverse momentum
- From the perturbations we can compute flow coefficients

PRC 100, 014905 (2019) Floerchinger et al.

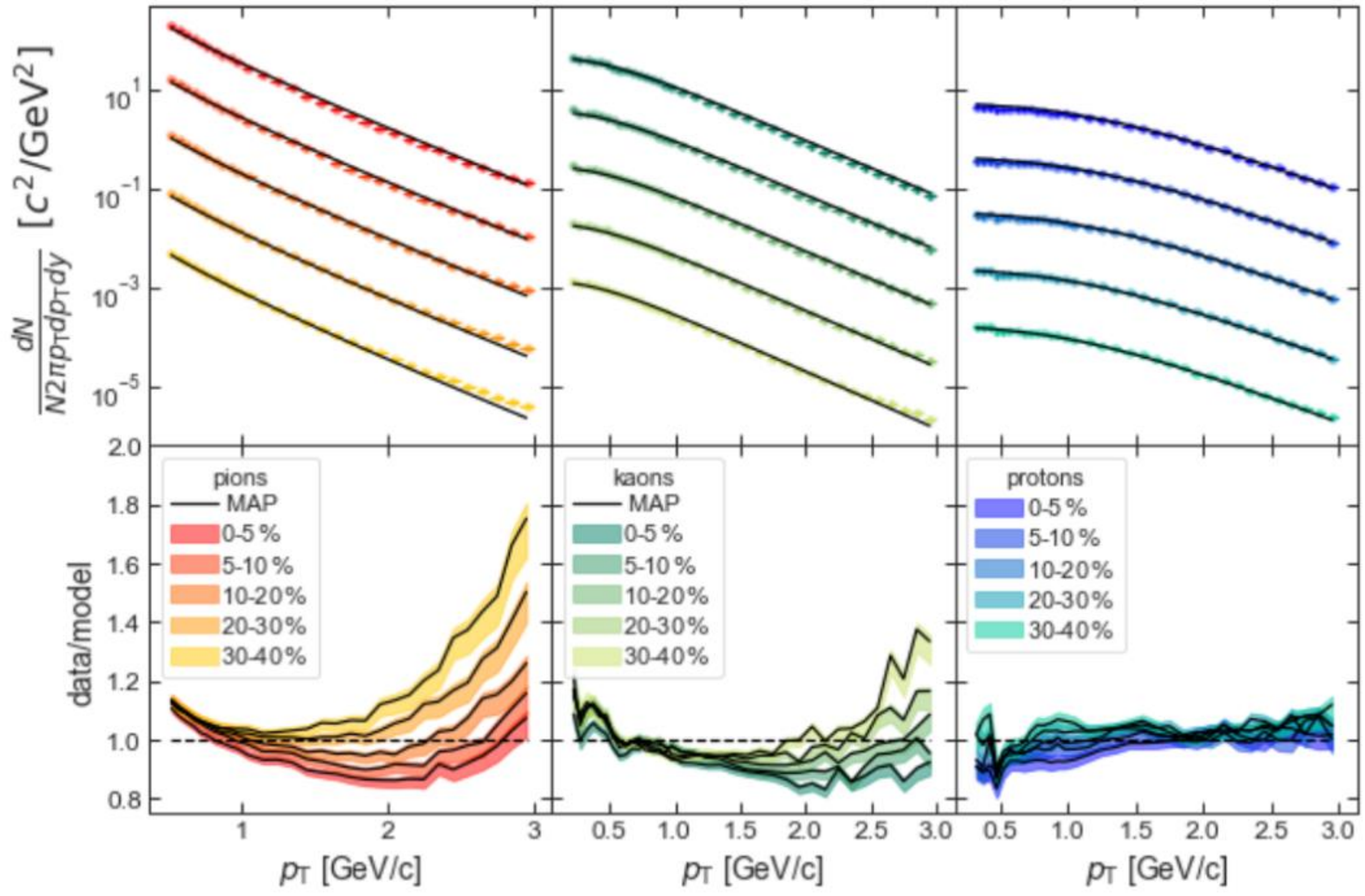


Fluid*u*M: background results

Y. Seemann, master thesis
JHEP 06 (2020) 044 10.1007 Devetak et al.

The spectra can be reproduced fairly well by our simulation.

We aim now at extending the framework to include a HQ conserved current.



Extending the framework: HQ conserved current

We write down a current associated to the conservation of QQbar pairs in the medium

$$N^\mu = n\, u^\mu + \nu^\mu$$

HQ density

HQ diffusion current

$$\partial_\mu N^\mu = 0$$ + equation of motion for the diffusion current

We need to determine the equation of motion for the diffusion current. We do it in a **kinetic theory** setup.

HQ distribution function

Heavy quarks are initially produced **out of kinetic equilibrium**: they cannot be described by a thermal Boltzmann distribution at the same temperature of the QGP

$$f(E, x) \sim e^{-E/T(x)}$$

HQ distribution function

Heavy quarks are initially produced **out of kinetic equilibrium**: they cannot be described by a thermal Boltzmann distribution at the same temperature of the QGP

$$f(E,x) \sim e^{-E/T(x)} e^{\mu(x)/T(x)} + \delta f(E,x)$$

Fugacity: needed to match the number of QQbar pairs

Out-of-equilibrium component

The **diffusion current** is given by the first moment of the out-of-equilibrium distribution function and the **density** is given by integrating the equilibrium part, in the Landau frame. .

PRD 85 (2012) 114047 Denicol et al.

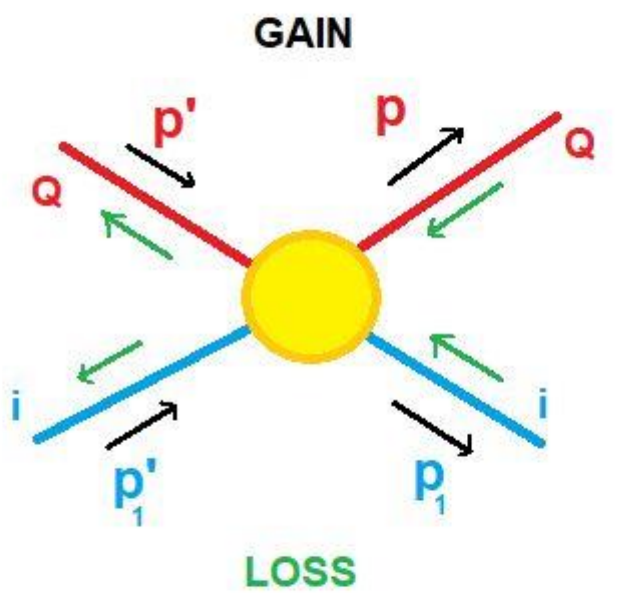
Boltzmann equation

We start with the relativistic Boltzmann equation, which studies how the distribution function of the heavy quark varies in time

$$p^\mu \partial_\mu f(\mathbf{p},\mathbf{x},\mathrm{t}) = C[f(\mathbf{p},\mathbf{x},\mathrm{t})]$$

where the **collision integral** reads

$$C[f(\mathbf{p},\mathbf{x},\mathrm{t})] = \int dP_1 dP_1' dP' W_{p',p_1' \to p,p_1} \left(f(\mathbf{p}') f_i(\mathbf{p_1'}) - f(\mathbf{p}) f_i(\mathbf{p_1}) \right)$$



Heavy quarks

- treated like Brownian particles
- asymptotically reach thermal equilibrium

Great effort was put to determine the value of the **transport coefficients** that encode the interaction between the heavy quark and the partons from the medium.

Equation of motion for the HQ diffusion current

We derive hydrodynamic equations of motion from kinetic theory (Boltzmann) in a **Fokker-Planck approximation**

$$p^\mu \partial_\mu f(\mathbf{p},\mathbf{x},t) = \frac{\partial}{\partial p^i}\left[A(p)p^i f(\mathbf{p},\mathbf{x},t) - g^{ij}\frac{\partial}{\partial p^j}D(p)f(\mathbf{p},\mathbf{x},t)\right]$$

Drag coefficient - inverse relaxation time

Momentum-diffusion coefficient

Spatial diffusion coefficient

$$D_s = \lim_{k\to 0}\frac{T}{MA(k)}$$

Einstein Fluctuation Dissipation

$$A = \eta = \frac{D}{ET} = \frac{\kappa}{2ET}$$

where η and κ are the Langevin coefficients.

Equation of motion for the HQ diffusion current

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By integrating the first moment of the equation

$$\int dP p^\nu p^\mu \partial_\mu f(\mathbf{p},\mathbf{x},\mathrm{t}) = \int dP p^\nu \frac{\partial}{\partial p^i}\left[A(p)p^i f(\mathbf{p},\mathbf{x},\mathrm{t}) - g^{ij}\frac{\partial}{\partial p^j}D(p)f(\mathbf{p},\mathbf{x},\mathrm{t})\right]$$

We obtain a relaxation-type equation for the diffusion current

$$\tau_n \partial_t \nu^i + \nu^i = \kappa_n \nabla^i \left(\frac{\mu}{T}\right)$$

Relaxation time

HQ diffusion coefficient

$$\tau_n = \frac{D_s I_{31}}{T P_o}$$

$$\kappa_n = \frac{T^2}{D} n = D_s n$$

Fluid-dynamic transport coefficients

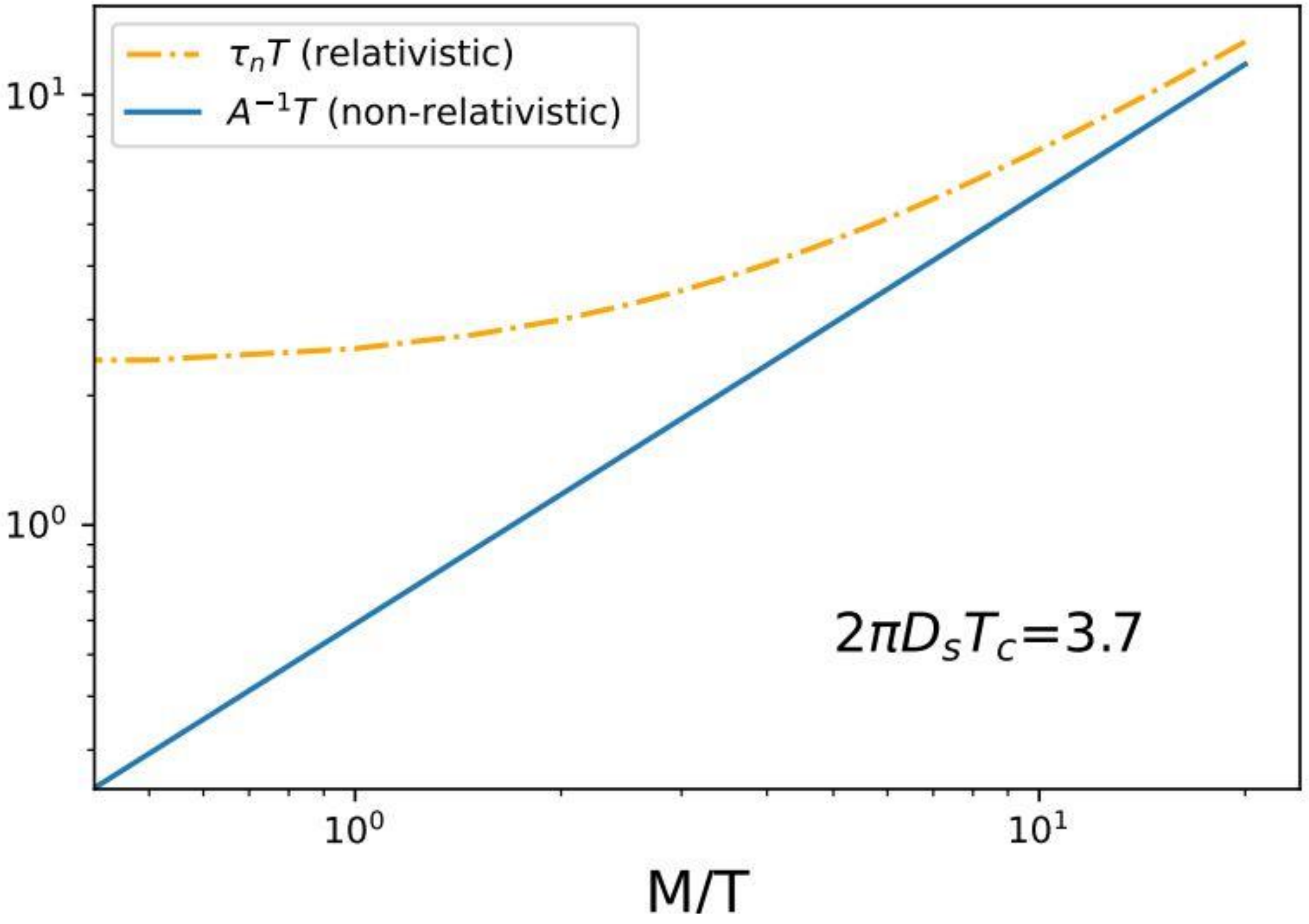
$$\tau_n = \frac{D_s I_{31}}{T P_o}$$
$$\kappa_n = \frac{T^2}{D} n = D_s n$$

$$I_{31} = \frac{1}{3}\int dP p^0 p^2 f_0(p)$$

$$p^0 \sim M \qquad I_{31} \sim M P_0$$

$$\tau_n \sim \frac{D_s M P_0}{T P_0} = D_s \frac{M}{T}$$

$$\tau_n T \sim (2\pi D_s T)\frac{1}{2\pi}\frac{M}{T} = A^{-1} T$$

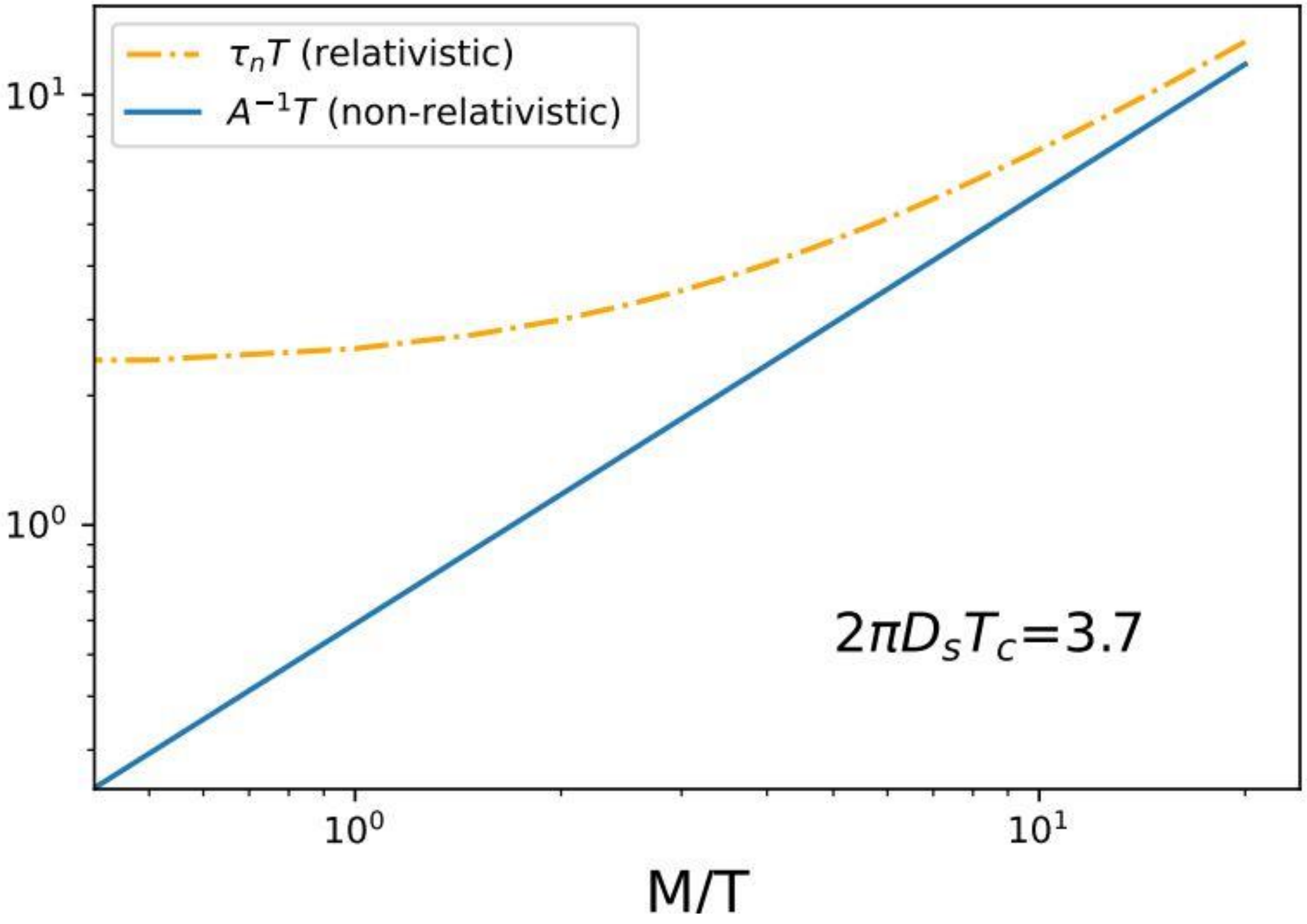


Important check: the hydrodynamic **relaxation time** is consistent with the relaxation time found within the Fokker-Planck approach in the non-relativistic limit.

Fluid-dynamic transport coefficients

$$\tau_n = \frac{D_s I_{31}}{T P_o}$$
$$\kappa_n = \frac{T^2}{D} n = D_s n$$

The relation between **spatial diffusion coefficient** and **momentum-diffusion coefficient** - usually found in a non-relativistic setup - arises naturally also in a relativistic framework.

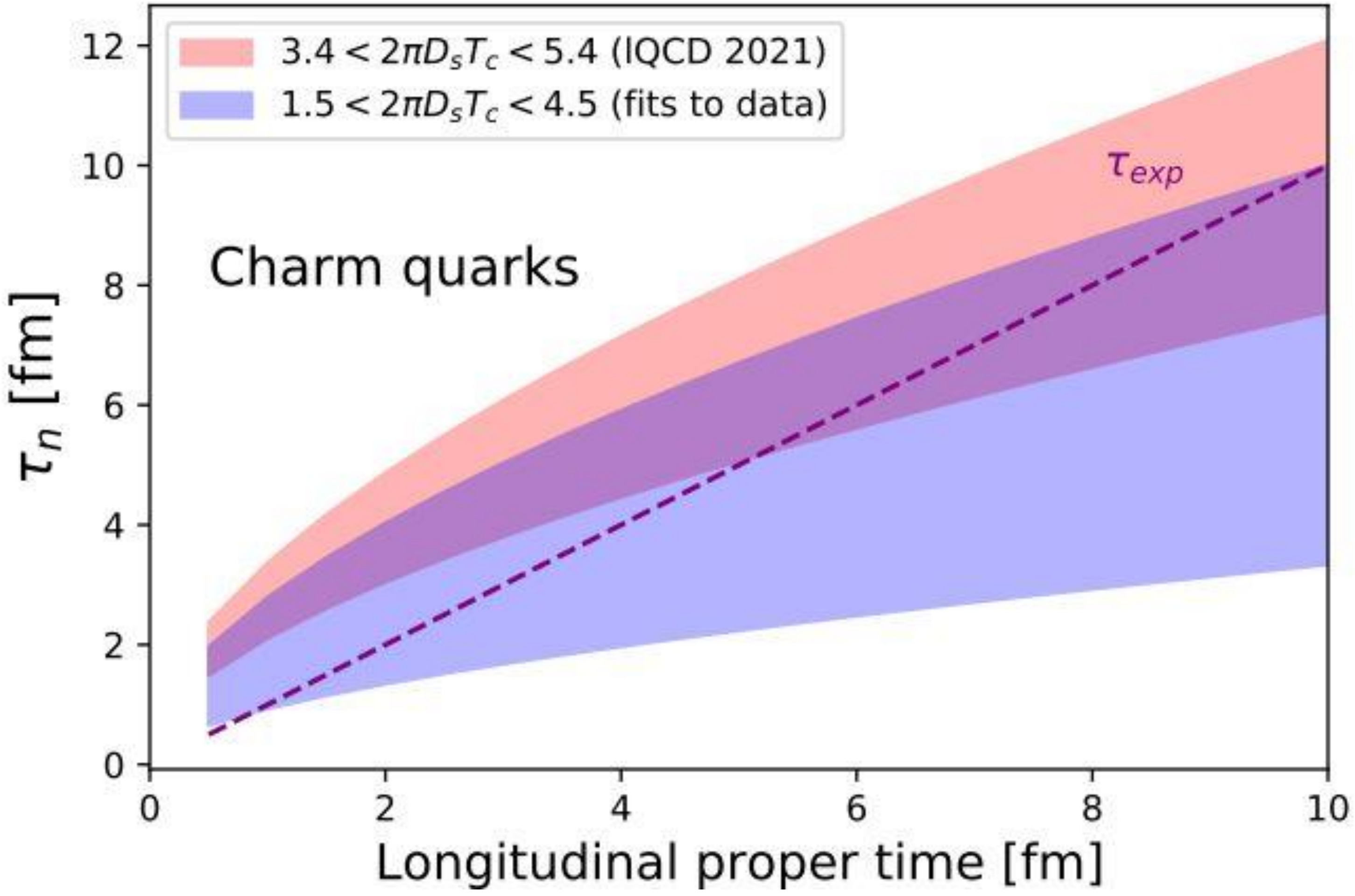


Results for charm quarks

This work, PRD 106 (2022) 034021

We compare the typical expansion time of the QGP undergoing a **Bjorken flow** with the relaxation time obtained for charm.

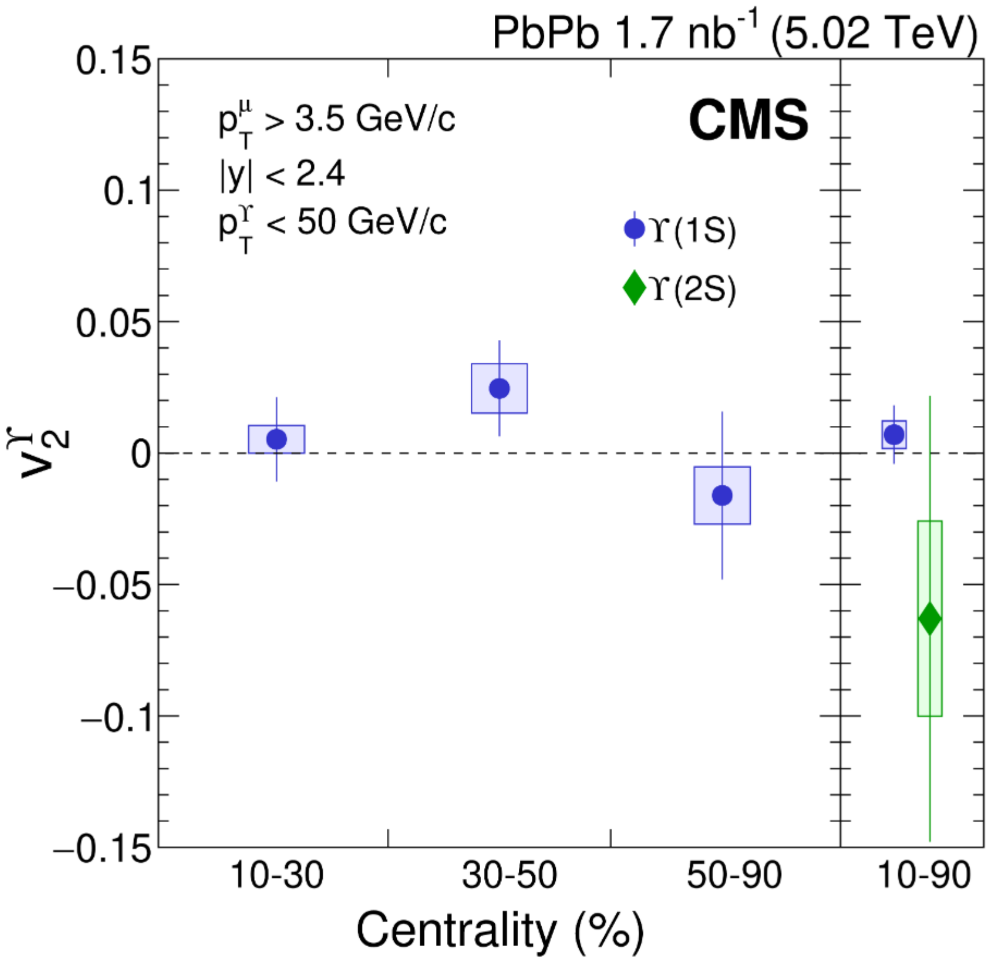
Remarkably, the relaxation time for charm becomes much smaller than the expansion time of the fluid very quickly: **a fluid-dynamic description for charm looks feasible**.



lQCD 2021: PRD 103 (2021) 014511 Altenkort et al.
ALICE fits to data: JHEP 01 (2022) 174 ALICE coll.

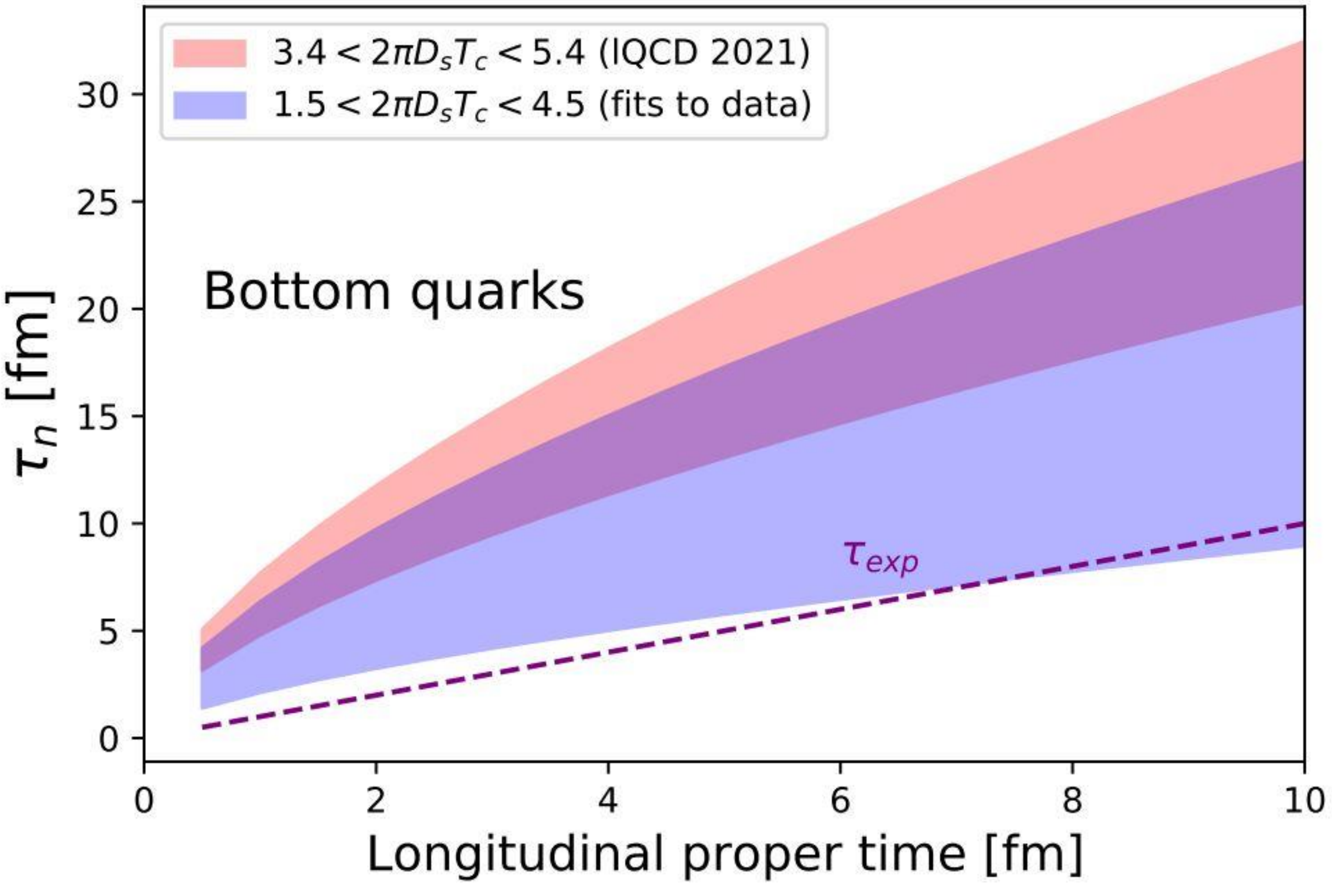
Results for bottom quarks

- From new data at low p_T, can we see partial thermalization of beauty quarks in the QGP?
- Can we constrain estimates for the transport coefficients by systematic comparison with data?



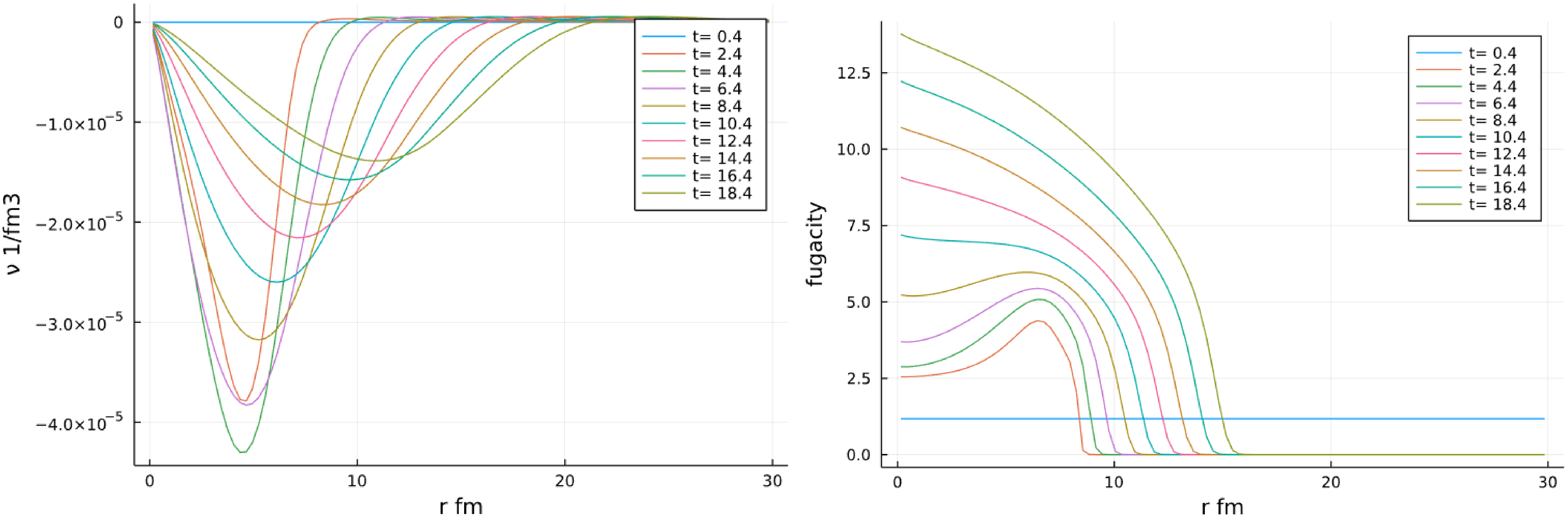
PLB 819 (2021) 136385 CMS coll.

This work, PRD 106 (2022) 034021



Preliminary results for charm in Fluid*u*M

- Very small diffusion current → it would be interesting to study different initial conditions for it
- A sizeable fugacity indicates that HQs are quite far from kinetic equilibrium (if chemical equilibrium, μ should tend to 0)
- **In the near future: charmed hadrons spectra and flow coefficients**



A challenging problem: initial conditions

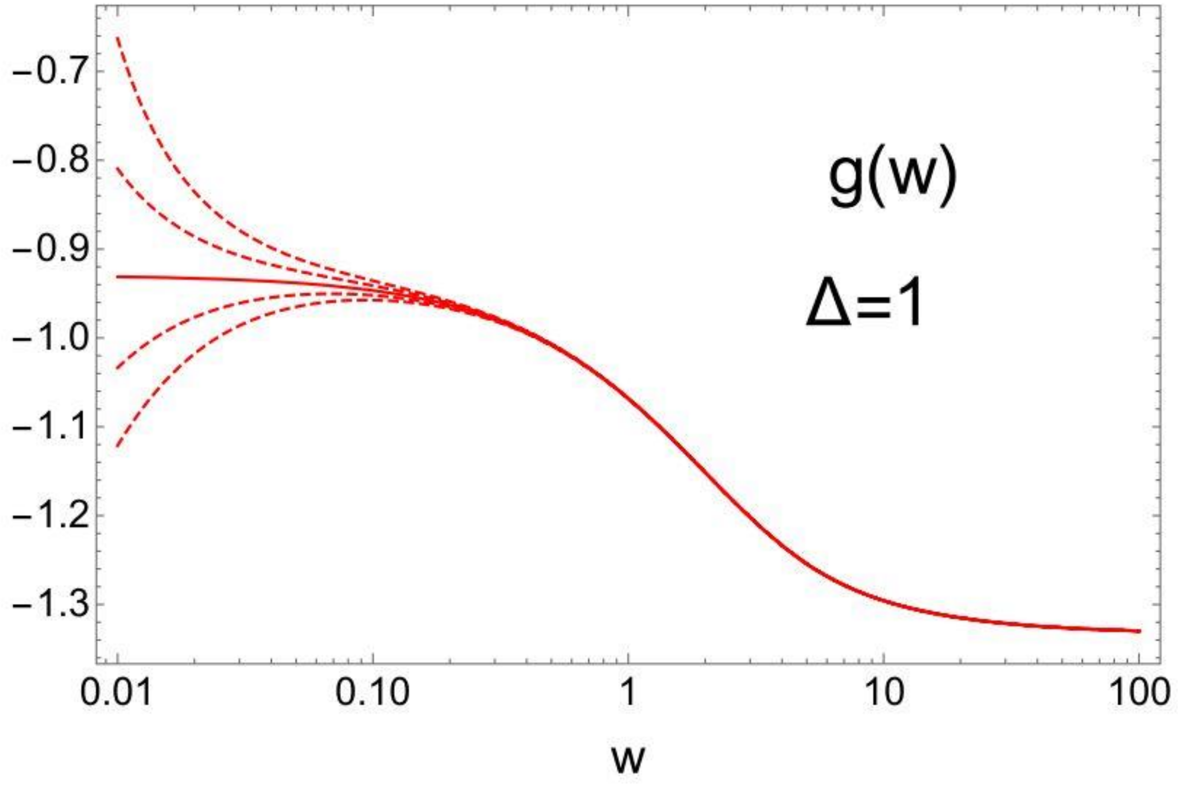
- What's the connection between the initial spatial and momentum distribution?
- Under which conditions fluid-dynamics and kinetic theory coincide for heavy quarks?

- Is there an **attractor solution** for the hydrodynamic behaviour of HQs?

IF YES: do we lose the information on the initial state?

IF NO: can we infer something on the initial conditions?
Can we observe a preferred phase-space trajectory for heavy quarks in the QGP?

PLB 820 (2021) 136478 Blaizot et al.



Measure of pressure anisotropy under Bjorken flow for different initial conditions

Relativistic vs non-relativistic

Non-relativistic diffusion equation: non-causal, no relaxation time

$$\frac{\partial n}{\partial t} = Ds\,\frac{\partial^2 n}{\partial x^2}$$

Relativistic diffusion equation: causal, relaxation time

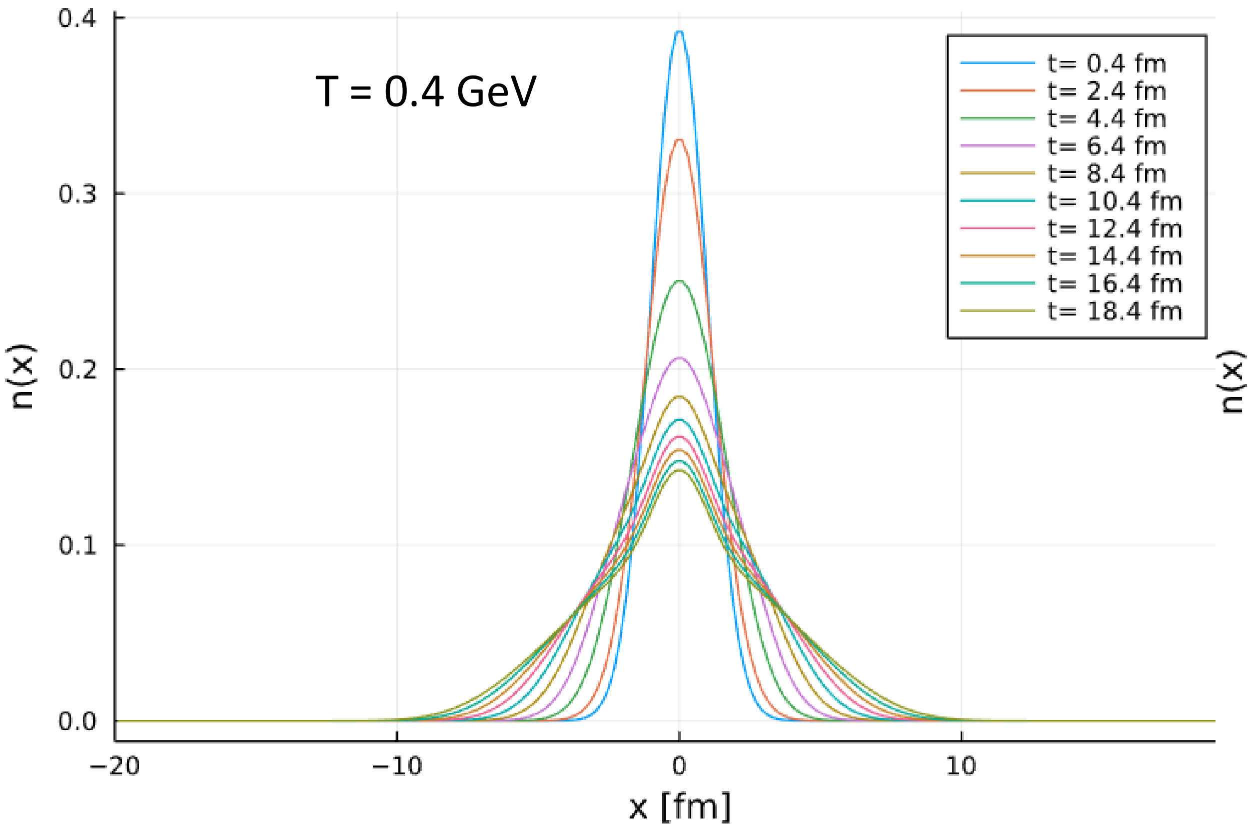
$$\tau_n \frac{\partial^2 n}{\partial t^2} + \frac{\partial n}{\partial t} = Ds\,\frac{\partial^2 n}{\partial x^2}$$

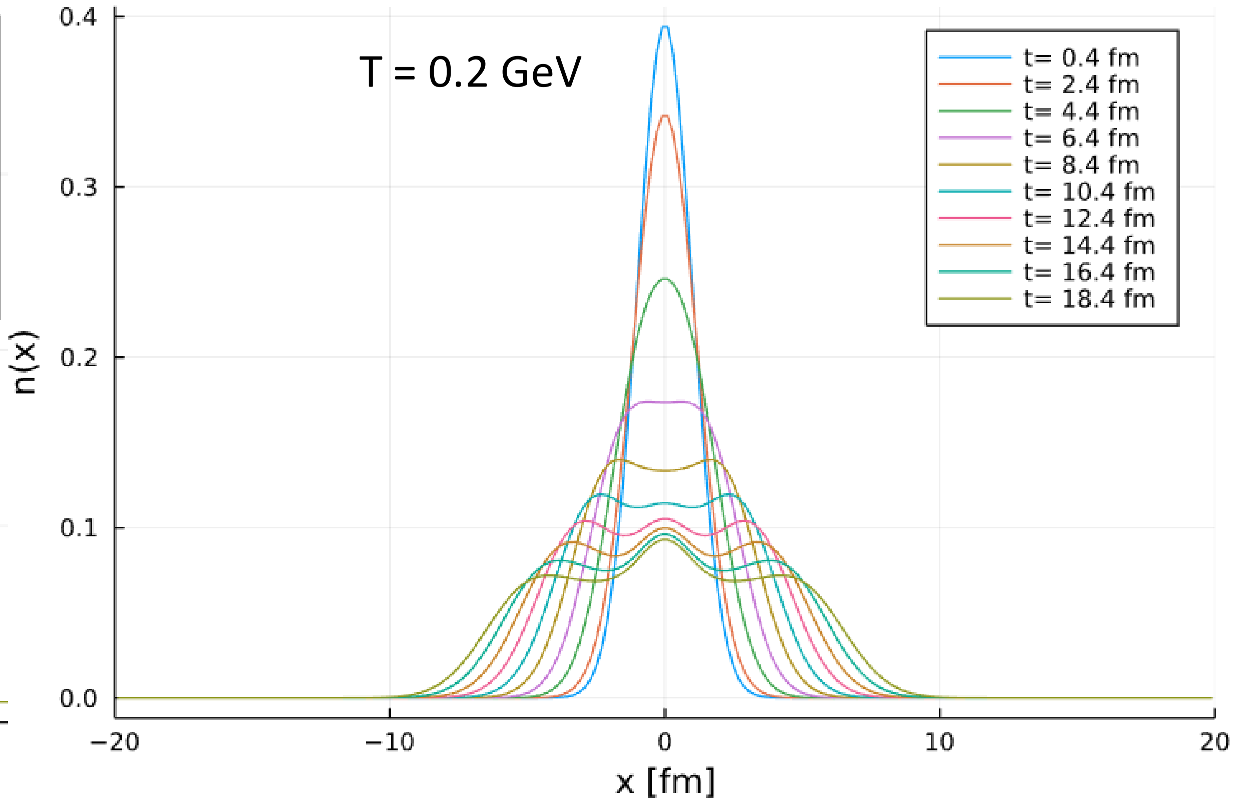
If there was no first-order time derivative: wave equation
Depending on the transport coefficients, the wave behaviour or the non-relativistic behaviour will be predominant.

For very large times >> τ_n , at least in a constant temperature case, we expect the two formulations to coincide.

Relativistic behaviour at different T

$$\tau_n \frac{\partial^2 n}{\partial t^2} + \frac{\partial n}{\partial t} = Ds \frac{\partial^2 n}{\partial x^2}$$





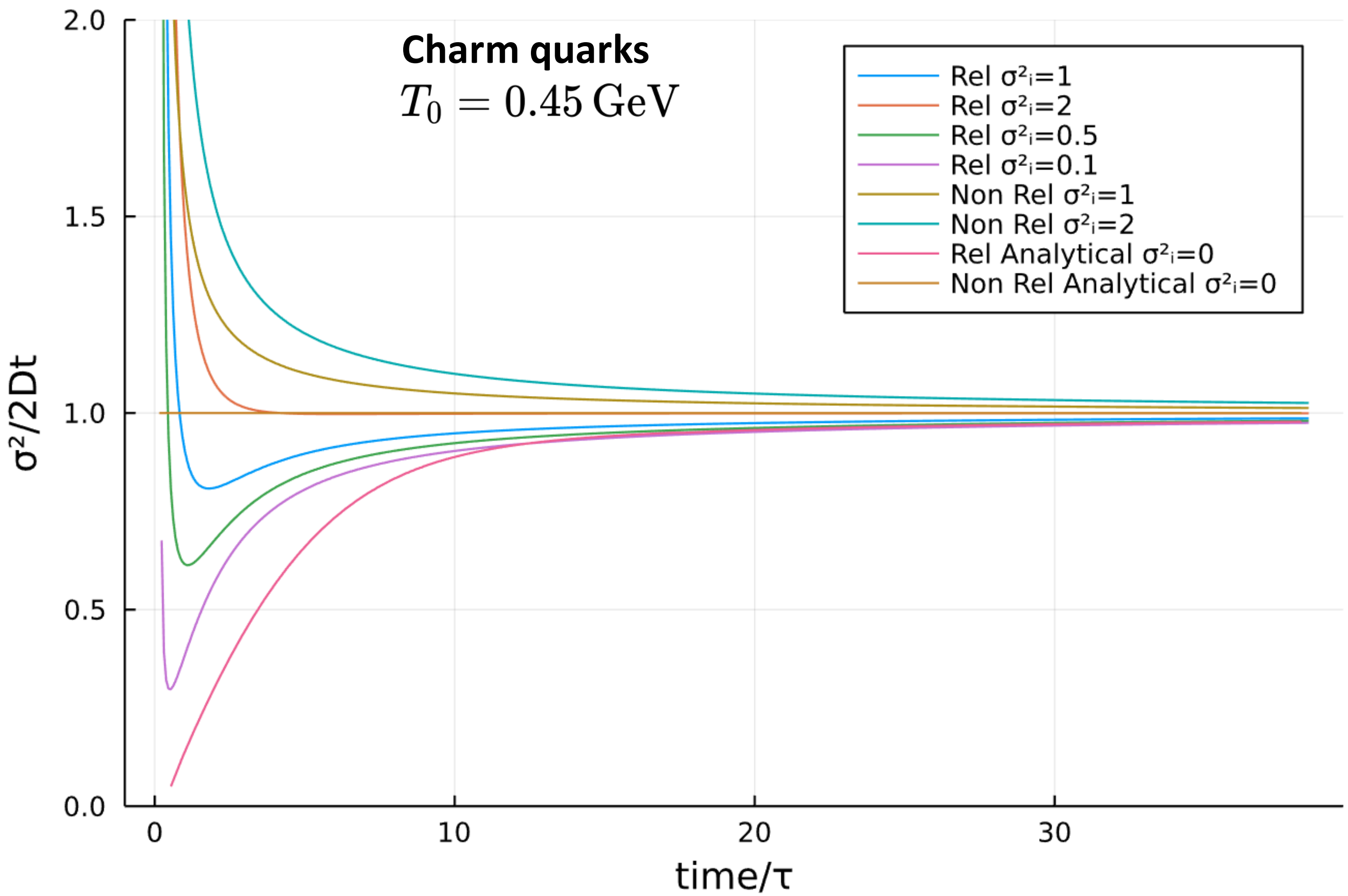
Equation solved numerically at static temperature for charm quarks.

Searching for attractors: static fireball

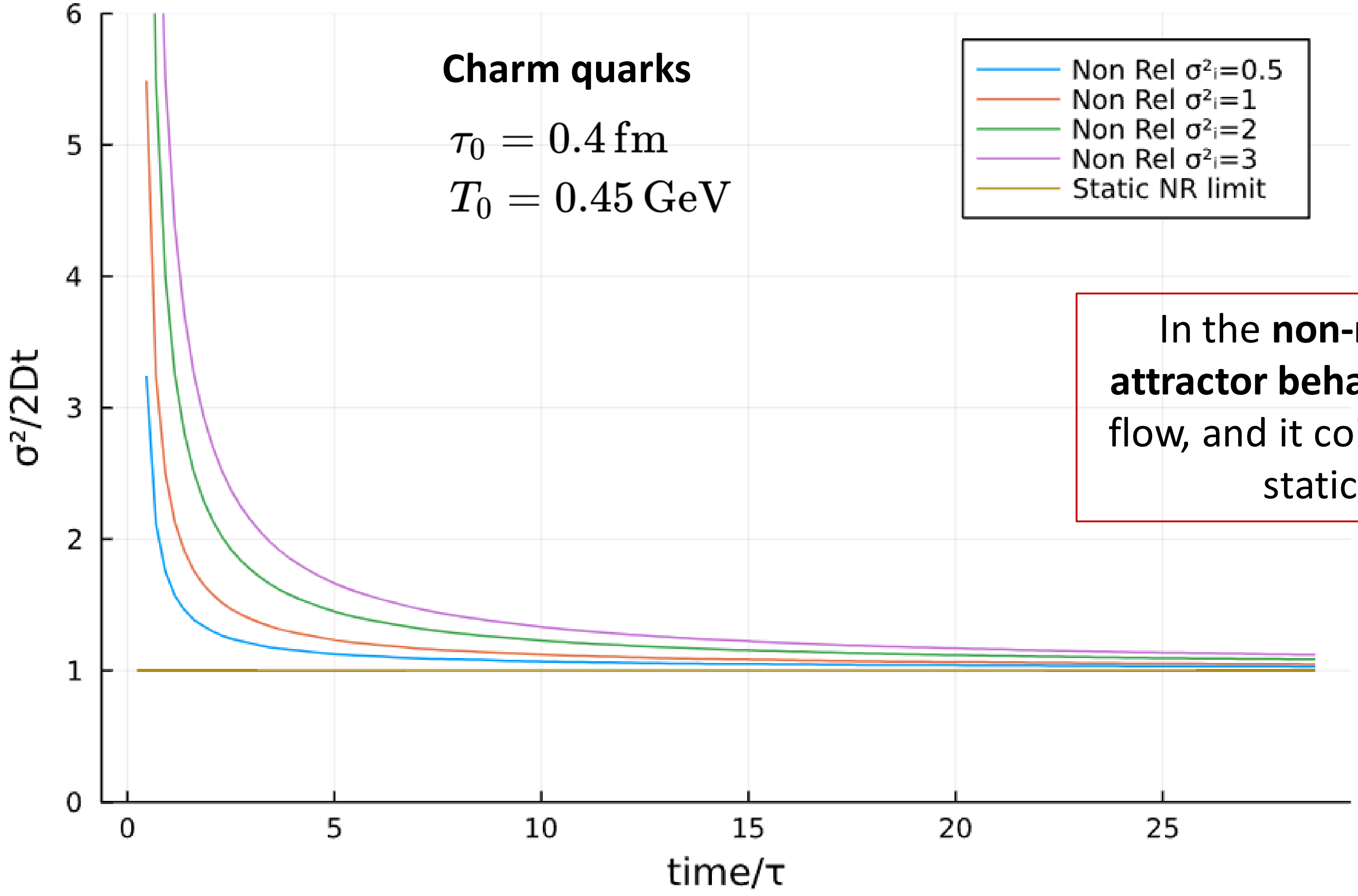
We study the **variance** of the HQ density as a function of dimensionless time t divided by the HQ relaxation time in 1D.

We normalize the variance by 2 Ds t because this quantity is 1 in the non-relativistic static (=constant temperature) case.

We initialize the HQ density with different initial variance.

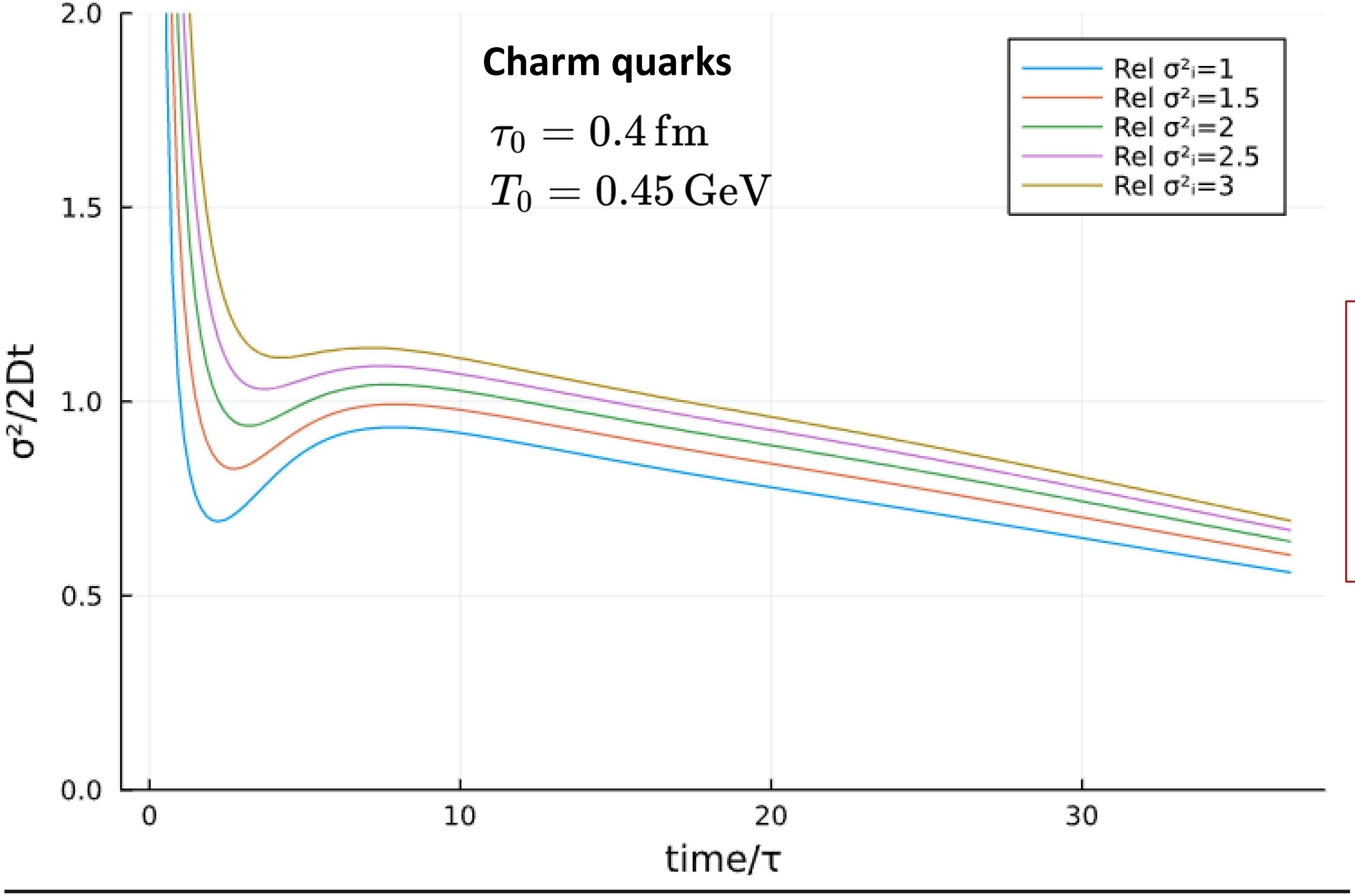


Searching for attractors: expanding QGP



In the **non-relativistic** case, the **attractor behaviour** survives Bjorken flow, and it coincides with the one at static temperature.

Searching for attractors: expanding QGP



In the **relativistic case** the attractor behaviour seems to exist but it doesn't coincide with the non-relativistic one.

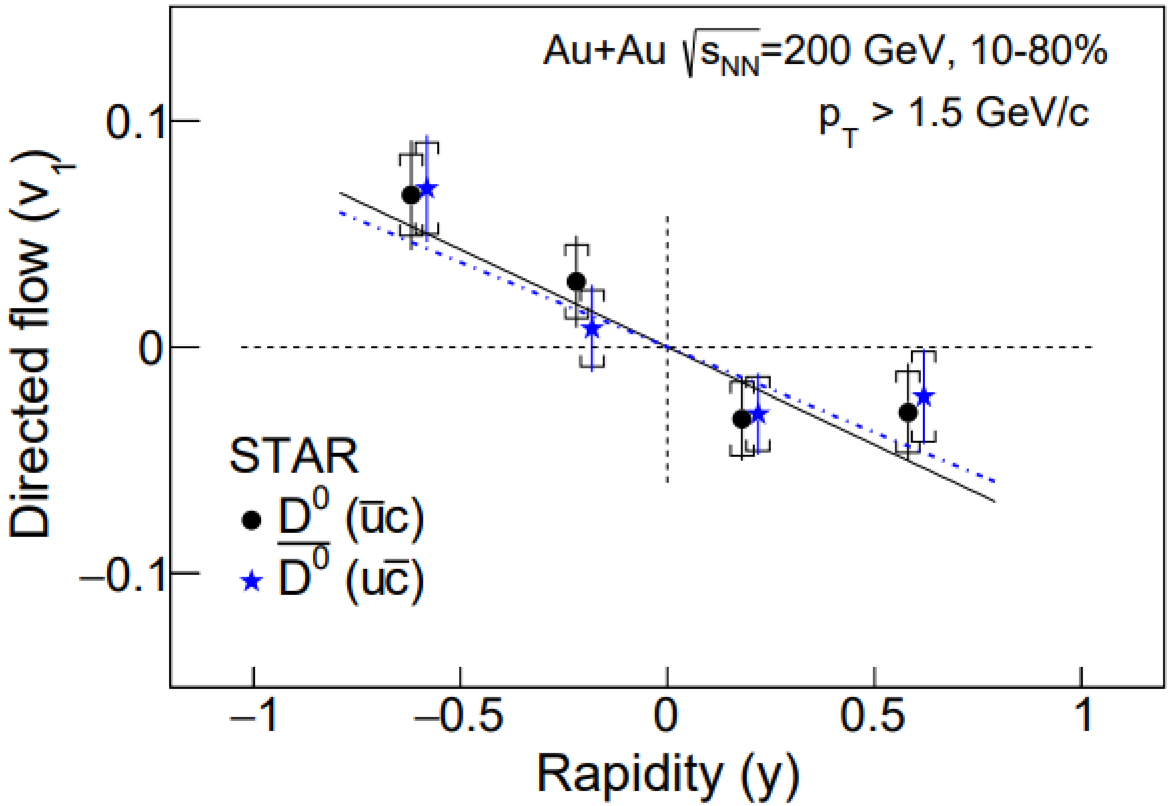
Summary

- A fluid-dynamic description of heavy-quark diffusion seems feasible for charm quarks
- There seems to exist an attractor solution under Bjorken flow

Outlook

- Study charmed hadron spectra and flow coefficients with Fluid*u*M
- Study attractor behaviour under realistic hydro simulation of the QGP
- Study directed flow of charm quarks
- Couple HQ diffusion with magnetic fields
- Study multiple conserved charges

PRL 123, 162301 (2019) STAR coll.



Back up

Charm in Fluid*u*M: causality

In order to have causal EOM for the diffusion current, the relation

$$\tau_n > \frac{\kappa_n}{\chi_n}\left(T\frac{\alpha}{\beta} + \mu\right)\left(\frac{n}{\epsilon+P}\right)^2$$

must hold, where we defined

$$\alpha = T\partial_T^2 p + \mu\partial_T\partial_\mu p \qquad \beta = T\partial_T\partial_\mu p + \mu\partial_\mu^2 p$$

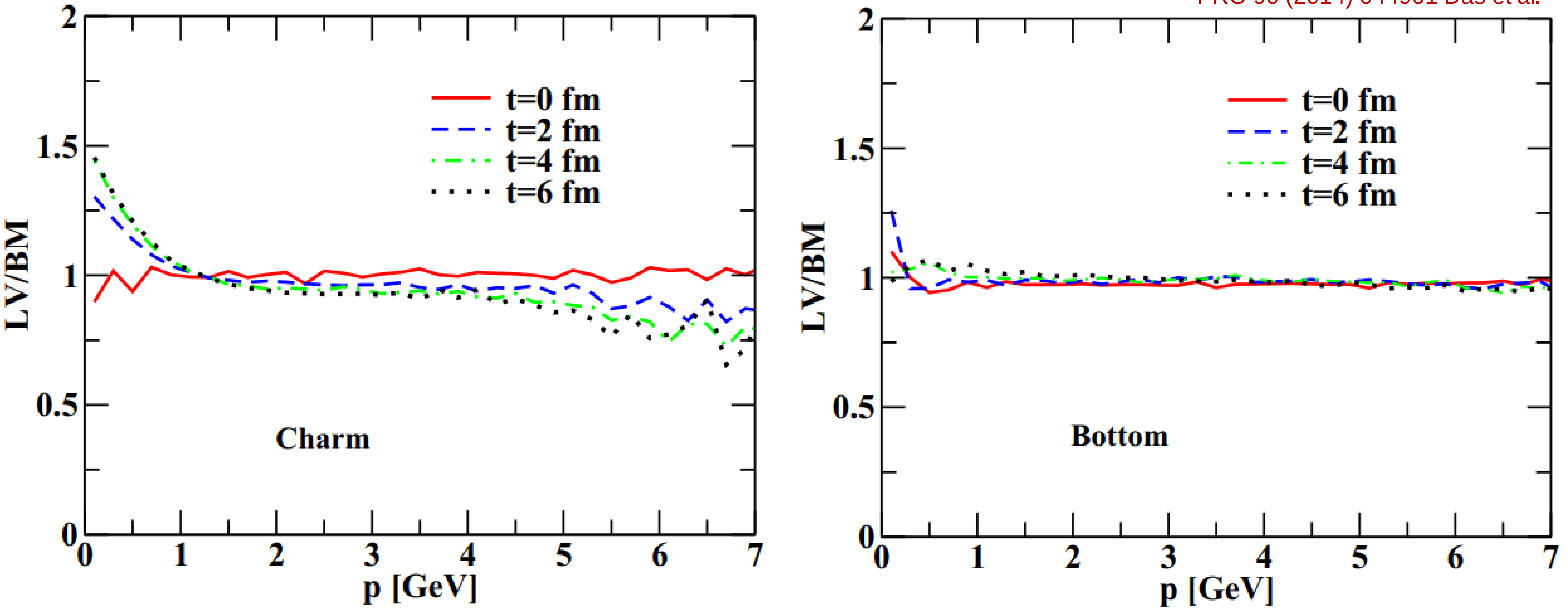
And

$$\chi_n \equiv \partial n/\partial\mu$$

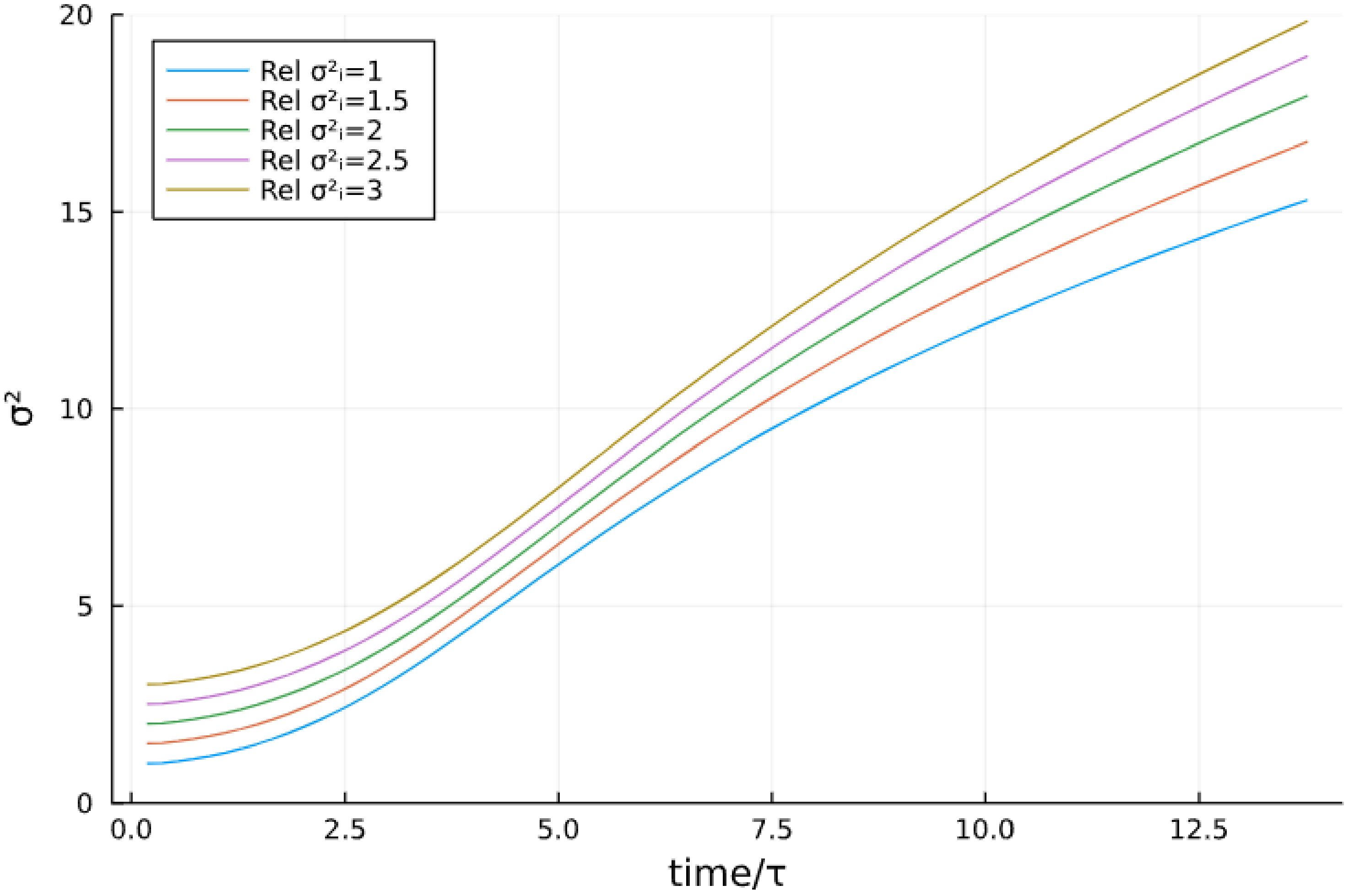
is the isothermal susceptibility.

Boltzmann vs Langevin/Fokker-Planck

PRC 90 (2014) 044901 Das et al.



Searching for attractors: expanding QGP



Charm quarks

$$\tau_0 = 0.4\,\mathrm{fm}$$
$$T_0 = 0.45\,\mathrm{GeV}$$

In the **relativistic case** the attractor behaviour seems to exist but it doesn't coincide with the non-relativistic one.

Study the equation with Bjorken flow

To implement Bjorken flow, I write the equation in Bjorken coordinates

$$\tau_n \frac{\partial^2 n}{\partial \tau^2} + \left(\frac{\tau_n}{\tau} + 1\right)\frac{\partial n}{\partial \tau} + \frac{n}{\tau} - \tau_n \frac{n}{\tau^2} = D_s \frac{\partial^2 n}{\partial r^2} + D_s \frac{1}{r}\frac{\partial n}{\partial r}$$

and then use time-dependent transport coefficients.

If $\tau_n < \tau$ this term is **positive** → density decreases
If $\tau_n > \tau$ this term is **negative** → density increases

Depending on the value of τ_0 and T_0 I will see different behaviours at the early stages of the evolution.

E.g. in our case for charm quarks

$\tau_0 = 0.4\,\mathrm{fm}$

$T_0 = 0.45\,\mathrm{GeV}$

$\tau_n \sim 1.7\,\mathrm{fm}$

The density will increase at the beginning of the evolution.