Factorization and Resummation of QED radiative corrections for neutron beta decay

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Phys. Rev. D 112 (2025) 11, 113006
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Motivation

- Traditional RC to neutron beta decay:

$$\delta\Gamma_n = 1 + 4.6\alpha + 16\alpha^2 + 35\alpha^3 + \cdots$$

Nucl. Phys. A 377, 474 (1982), Wilkinson

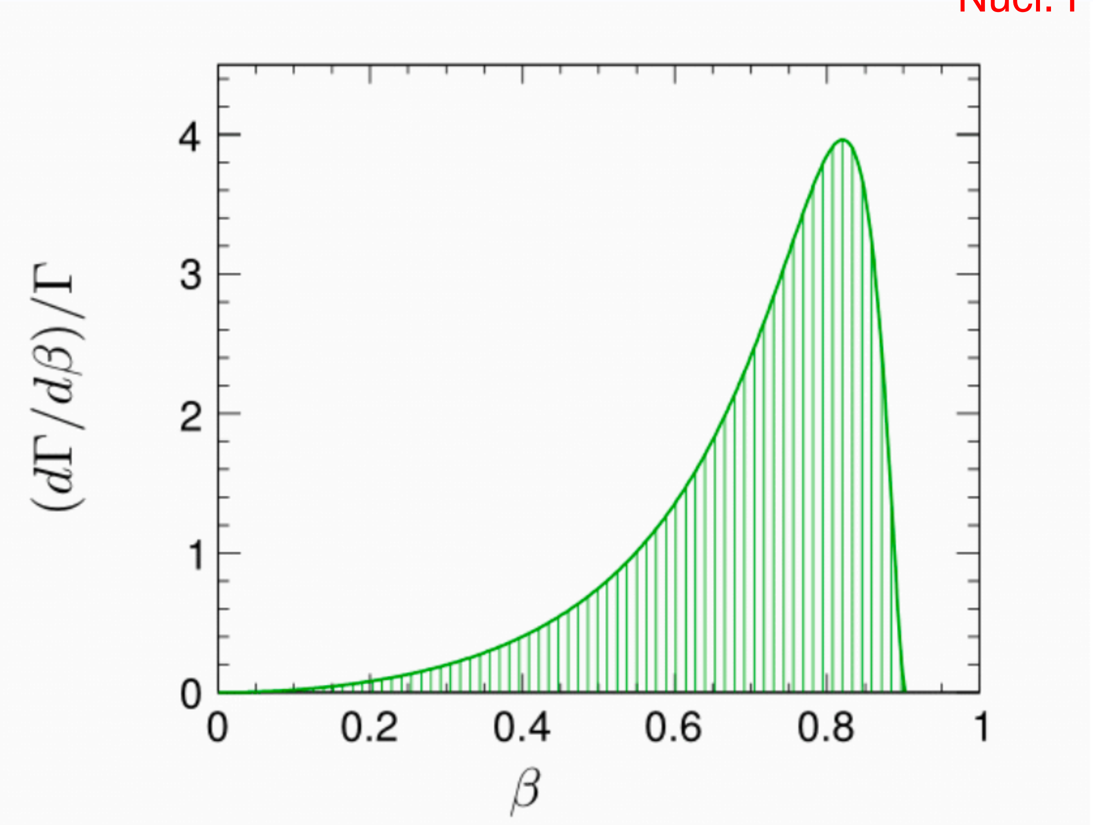
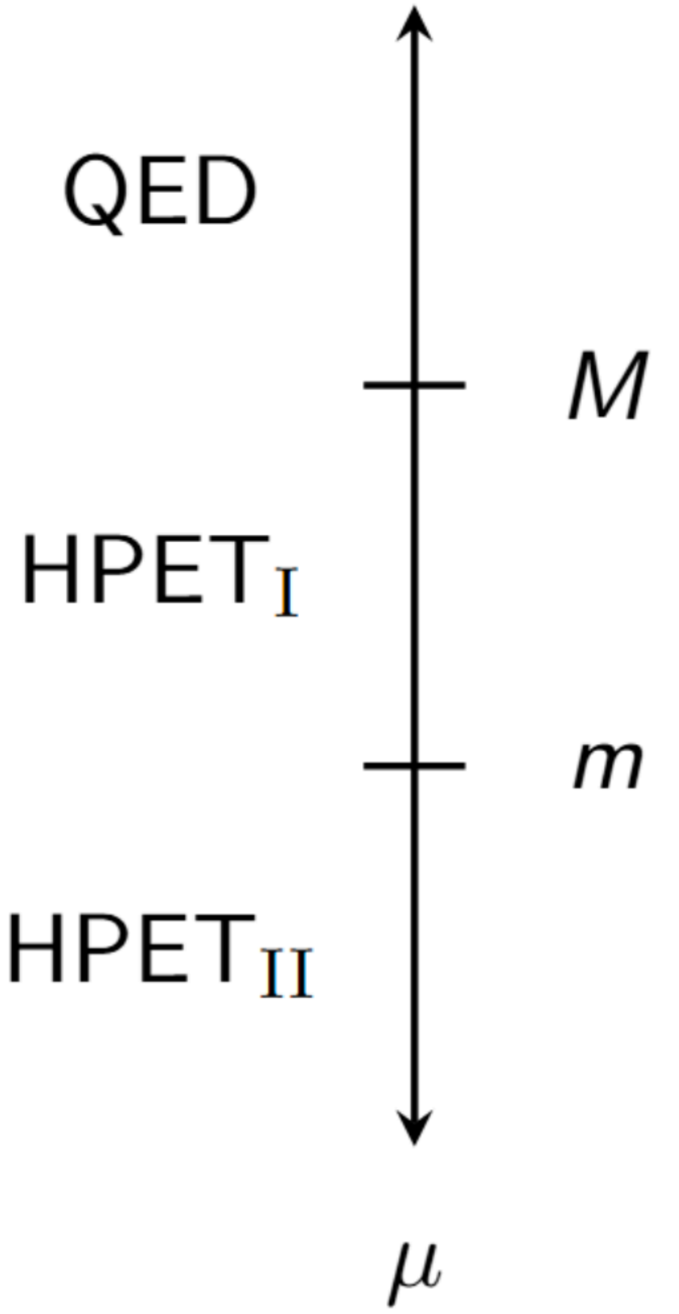


Fig: **tree level neutron beta decay rate as a function of electron speed β**

- Electron velocity is not small, i.e., Fermi function (valid at small β) doesn't apply.

Z. Phys. 88 (1934) 161-177, Fermi

HPEFT Framework & Factorization



QED

M

HPET_{I}

m

HPET_{II}

μ

$$\mathscr{M} \sim \mathscr{M}_{\text{UV}}(\Lambda_{\text{UV}}/\mu_{\text{UV}})\mathscr{M}_H(p/\mu_{\text{UV}}, p/\mu)\mathscr{M}_S(\lambda/\mu)$$

Phys. Rev. Lett. 133 (2024) 2, 021803, Hill, Plestid

- Λ_{UV} is the hadronic scale ($\sim 100\,\text{MeV}$), p is the kinematic scale of neutron beta decay ($p \sim \Delta \sim m \sim \text{MeV}$), and λ is the soft scale ($\sim \varepsilon_\gamma \ll p$)
- μ_{UV} and μ are factorization scales separating UV, hard and soft regions; $\mathscr{M}$ is μ_{UV} and μ independent
- $\mathscr{M}_{\text{UV}}$ related to $-\sqrt{2}\,G_F\,V_{ud}\,g_{V,A}$, encoding hadronic structure
- $\mathscr{M}_S$ exponentiates for small soft cutoff, i.e., is known to all orders from a one-loop calculation
- $\mathscr{M}_H$ extractable from calculations in the literature

The Challenge of π Enhancement

Phys. Rev. D 109 (2024) 11, 113007, Borah, Hill, Plestid

JHEP 07 (2024) 216, Plestid

- Hard amplitude has the structure

$$\mathscr{M}_H(w,\mu^2) = \mathscr{A}_H(w,\mu^2) + \frac{\not{v}}{w}\mathscr{B}_H(w,\mu^2)$$

  - Time-like amplitude contains factors $\sim \pi^2$ even in regime without kinematic enhancements (w is order 1), at one-loop order

$$\mathscr{A}_H(w) - \mathscr{A}_H(-w) = \frac{\alpha}{2\pi}\left[\frac{i\pi w}{\sqrt{w^2-1}}\left(\log\left(\frac{-4\mathbf{p}^2 - i0}{\mu^2}\right) - 1\right)\right]$$

$$\mathscr{B}_H(w) - \mathscr{B}_H(-w) = \frac{\alpha}{2\pi}\left[\frac{i\pi w}{\sqrt{w^2-1}}\right]$$

- Such large π can be eliminated by evaluating $\mathscr{M}_H$ at $\mu^2 = -4\mathbf{p}^2$

Factorization of Hard Function

- Neglecting contributions suppressed by powers of m/E, hard function can be decomposed

$$\mathscr{M}_H(w) \approx \mathscr{A}_H(w) \approx F_R(w, m) F_J(m) F_H(E)$$

- The amplitude factorizes into a collinear or “jet” function F_J depending only on the mass scale m, and a “hard” function F_H depending only on the energy scale E. "Remainder" function F_R, which converts between $n_e = 1$ and $n_e = 0$ dynamical electrons in the low-energy effective theory

- $F_H(-E, \mu) = F_H(E, -\mu)$ computed for space-like kinematics; need to resum from $\hat{\mu}(=-\mu - i0)$ to μ

$$\log \frac{\mu_*^2}{\hat{\mu}_*^2} = \int_{\alpha(\hat{\mu}_*^2)}^{\alpha(\mu_*^2)} \frac{\mathrm{d}\alpha}{\beta(\alpha)} = 2\mathrm{i}\pi = X_* \qquad \text{where } \hat{\mu}_* = -\mu_*,\ \mu_* = 2E + \mathrm{i}0$$

- Large logarithms($\sim \pi$) spoil naive power counting and are assigned with power counting $|X_*| \sim \alpha^{-\frac{1}{4}}$

Resummation of Hard Function

- Factors $\alpha^3 X_*^4$ numerically relevant at $\mathcal{O}\left(\alpha^2\right)$
- Renormalization group evolution of F_H is given by

$$\frac{\mathrm{d}\log F_H(E,\mu)}{\mathrm{d}\log\mu} = -\gamma_{\mathrm{UV}} + \gamma^h(\bar{\alpha}) + \gamma^\psi(\bar{\alpha}) + \gamma_{\mathrm{cusp}}(\bar{\alpha})\log\frac{-2E}{\mu}$$

where the terms are the UV, heavy particle, light particle, and massive cusp anomalous dimensions, respectively.

- Solve the renormalization group running including π-enhanced term of $\mathcal{O}\left(\alpha^3\right)$

$$\left|\frac{F_H(E,\mu=2E)}{F_H(-E,\mu=2E)}\right|^2 = \left|\frac{F_H(E,\mu=2E)}{F_H(E,-\mu=-2E)}\right|^2 = \exp\left[-X_*^2\frac{\bar{\alpha}}{4\pi} + \frac{32}{9}n_e X_*^2\left(\frac{\bar{\alpha}}{4\pi}\right)^2 - \frac{8}{27}n_e^2 X_*^4\left(\frac{\bar{\alpha}}{4\pi}\right)^3 + \ldots\right],$$

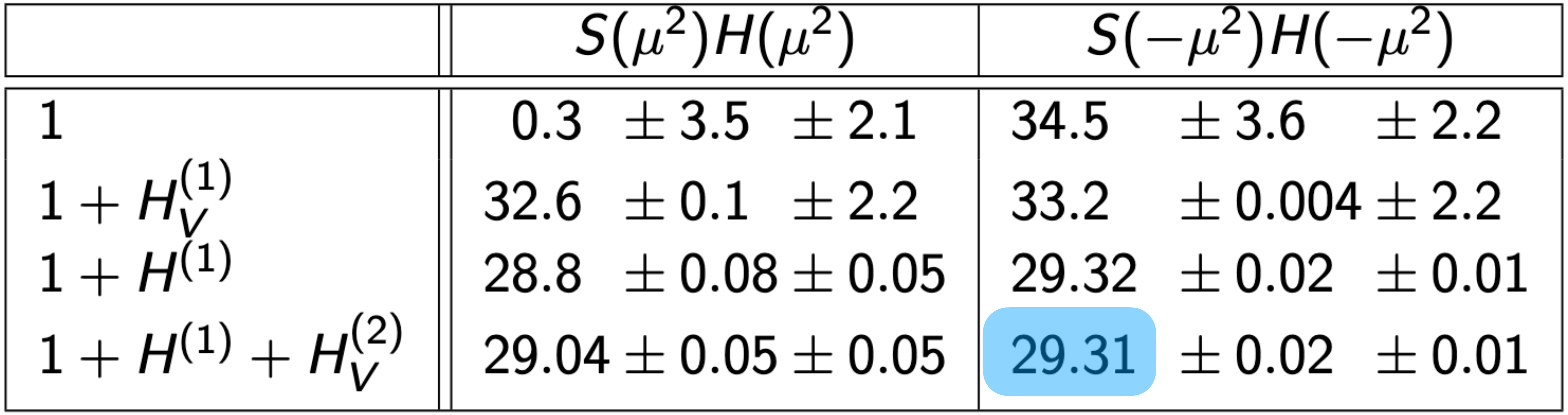
- X_*^2 slows the convergence of perturbation theory

$$|F_H(\hat{\mu})|^2 = 1 - 3.1\left(\frac{\bar{\alpha}}{\pi}\right) + (5.4 + 3.6n_e)\left(\frac{\bar{\alpha}}{\pi}\right)^2 + \ldots\ ,$$

$$|F_H(\mu)|^2 = 1 + 6.8\left(\frac{\bar{\alpha}}{\pi}\right) + (23.9 - 5.1n_e)\left(\frac{\bar{\alpha}}{\pi}\right)^2 + \ldots\ ,$$

Results

- Elimination of π enhancements in H leads to better convergence



	$S(\mu^2)H(\mu^2)$			$S(-\mu^2)H(-\mu^2)$		
1	0.3	± 3.5	± 2.1	34.5	± 3.6	± 2.2
$1 + H_V^{(1)}$	32.6	± 0.1	± 2.2	33.2	± 0.004	± 2.2
$1 + H^{(1)}$	28.8	± 0.08	± 0.05	29.32	± 0.02	± 0.01
$1 + H^{(1)} + H_V^{(2)}$	29.04	± 0.05	± 0.05	29.31	± 0.02	± 0.01

Table: long-distance radiative correction to Γ_n in units of 10^{-3}. Columns computed at time-like (left) and space-like (right) renormalization scale. Central values evaluated at $\mu^2 = m\Delta$, $\Lambda_\gamma = \Delta$, and $\mu_{\mathrm{UV}} = \Delta$ (where Λ_γ parametrizes uncertainty due to imposing cancelation of ε_γ in $H^{(2)}$); errors denote scale variation $\mu = m/2..2\Delta$, and $\Lambda_\gamma = \Delta/2..2\Delta$.

- Neglected power corrections ($[H_V^{(2)}]_i$, for $i \geq 1$) shifts the central value of the outer corrections $29.31 \rightarrow 29.18$; we assign an uncertainty of half this shift

$$\delta_{R,\mathrm{static}} = (29.18 \pm 0.07 \pm 0.01 \pm 0.02) \times 10^{-3}$$

- Other uncertainties are from neglected real radiation at two-loop order, and from perturbative corrections at three-loop order

Thank You