Compton amplitude and structure function calculations of the nucleon from a lattice QCD perspective

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EW and BSM Physics at the EIC
12-16 February 2024

CSSM/QCDSF/UKQCD Collaborations

Granada, Lattice 2017

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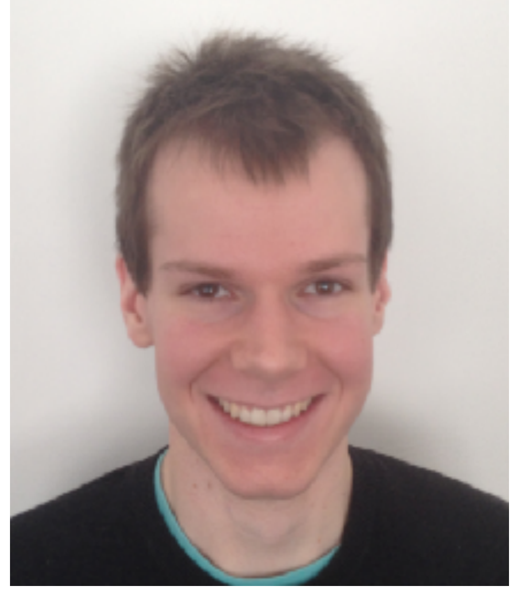

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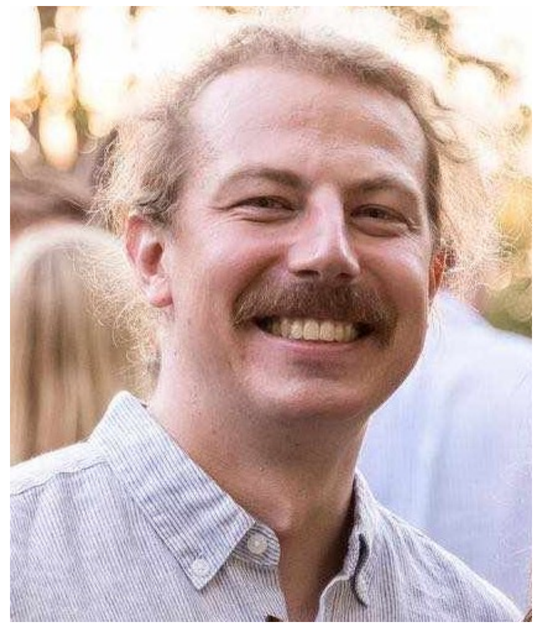


Joshua Crawford
U.Adelaide
PhD ongoing

Motivation

- <u>Nucleon structure</u> (leading twist)
  - Parton distribution functions from first principles
  - Understanding the behaviour in the high- and low-x regions

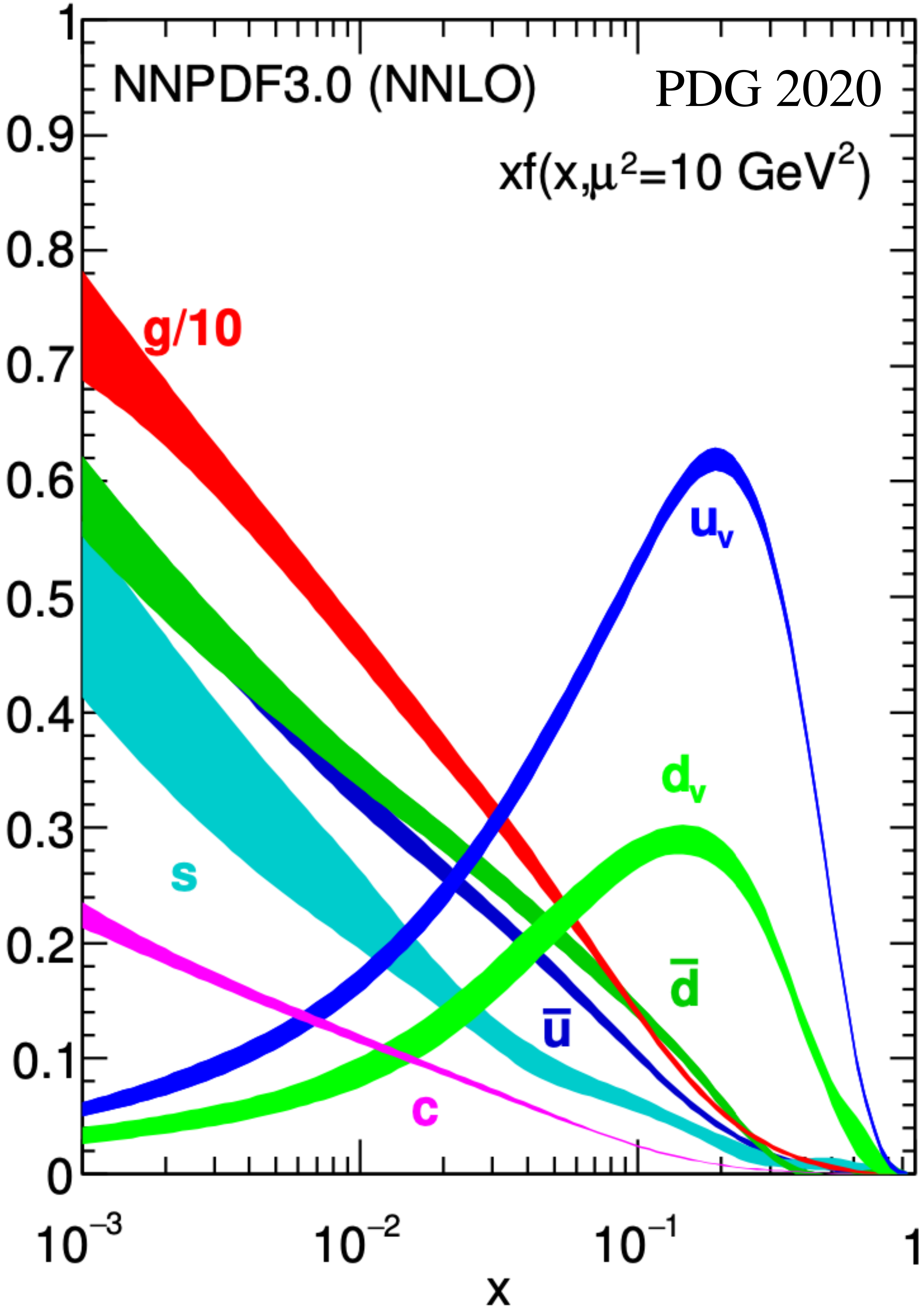
- Parton model

$$F_2 \propto (q + \bar{q})$$

$$F_3^{\gamma Z} \propto (q - \bar{q})$$

$$F_2^{W-} \propto u + \bar{d} + \bar{s} + c\ldots$$

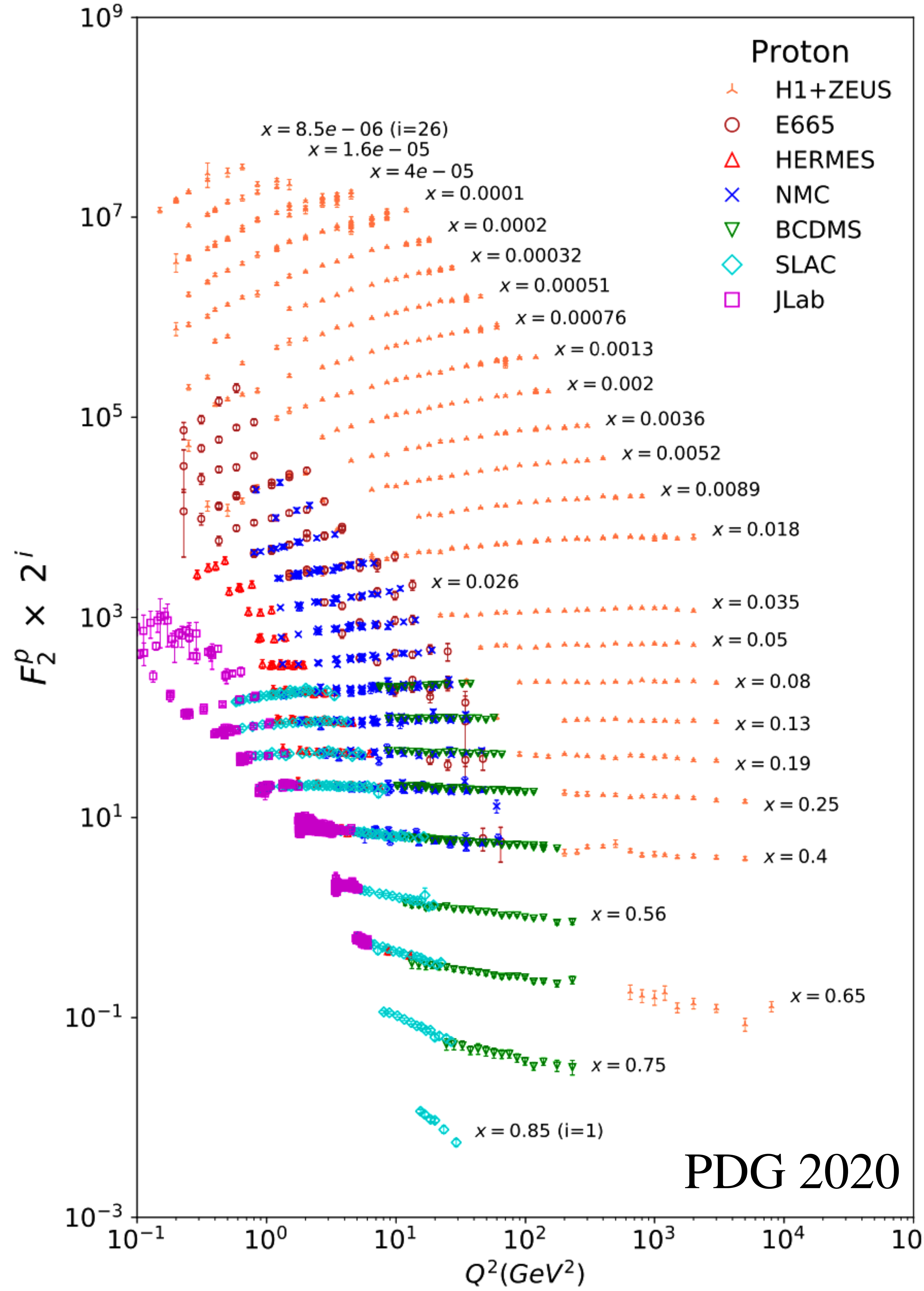
$$F_3^{W-} \propto u - \bar{d} - \bar{s} + c\ldots$$



Motivation

- Scaling
  - Q^2 cuts of global QCD analyses

- Power corrections / Higher twist effects
  - Target mass corrections
  - Twist-4 contributions



PDG 2020

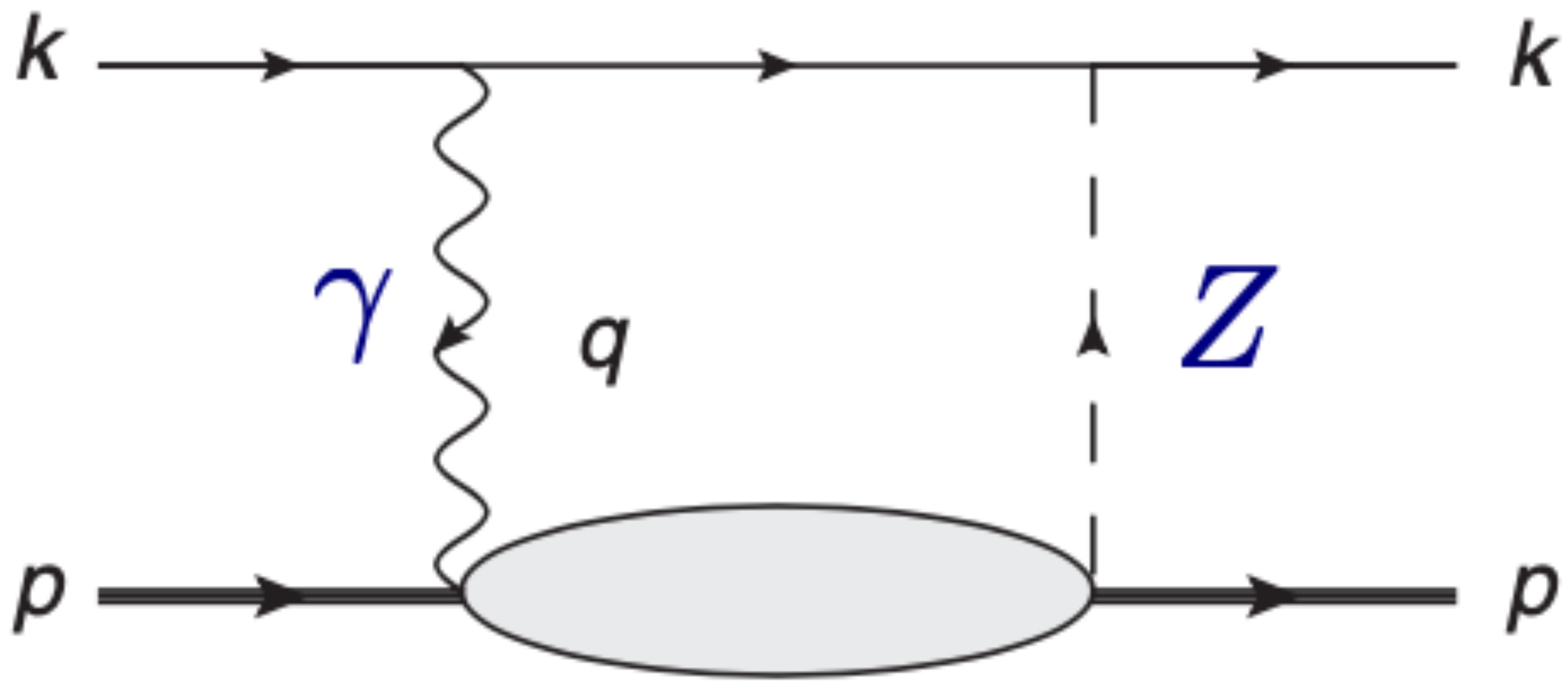
Motivation | EW Box

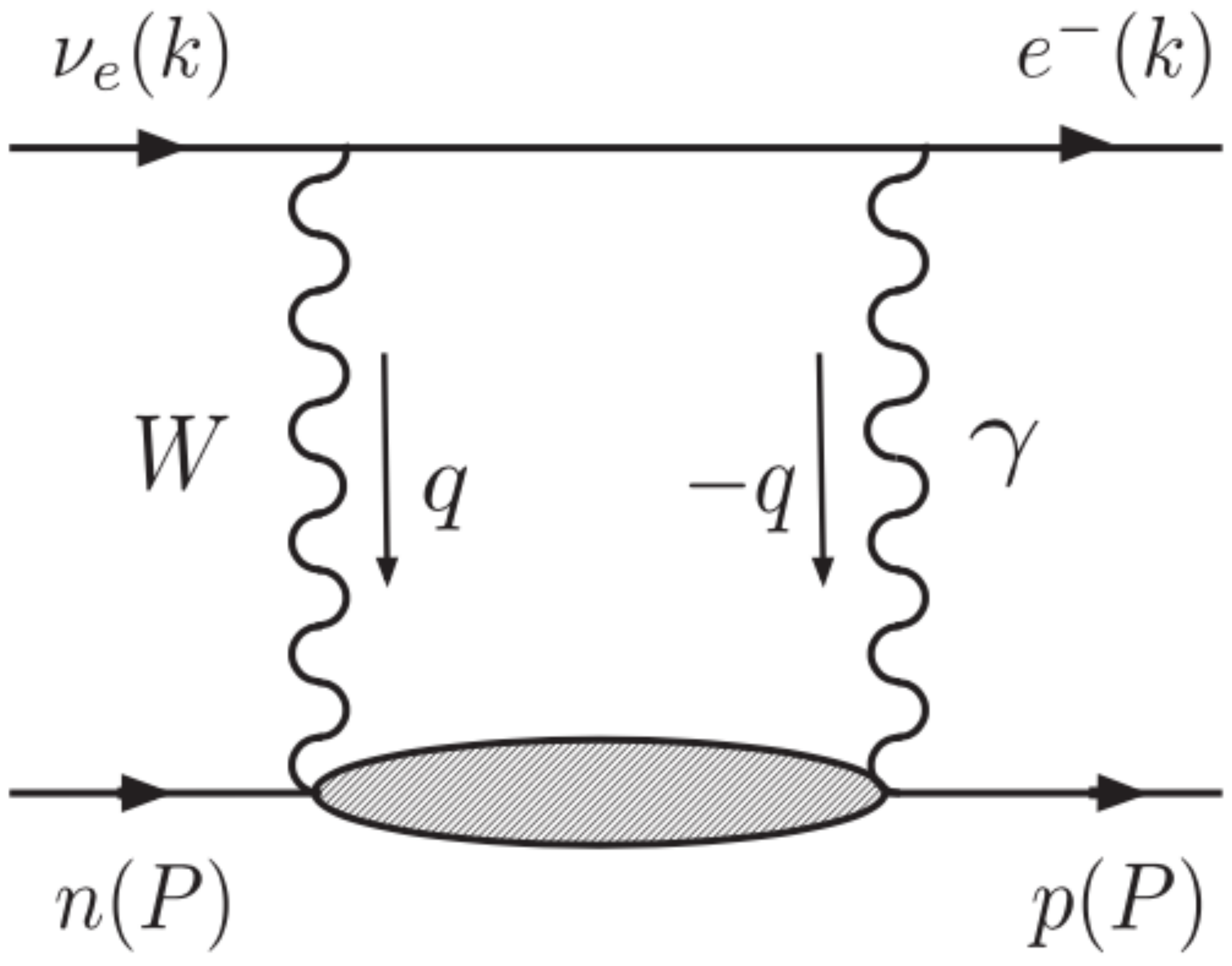
- Leading theoretical uncertainty in:
- Weak charge of the proton,

$$Q_W = (1+\Delta_\rho+\Delta_e)(1-4\sin^2\theta_W(0)+\Delta_e') + \Box^{WW}_{AA} + \Box^{ZZ}_{AA} + \Box^{\gamma Z}_{VA}$$

- CKM matrix element extracted from superallowed β decays,

$$|V_{ud}|^2 = \frac{2984.432(3)\,\mathrm{s}}{\mathscr{F}t(1+\Delta_R^V)} \qquad \Delta_R^V \longrightarrow \propto \Box^{\gamma W}_{VA}$$





Lattice QCD landscape

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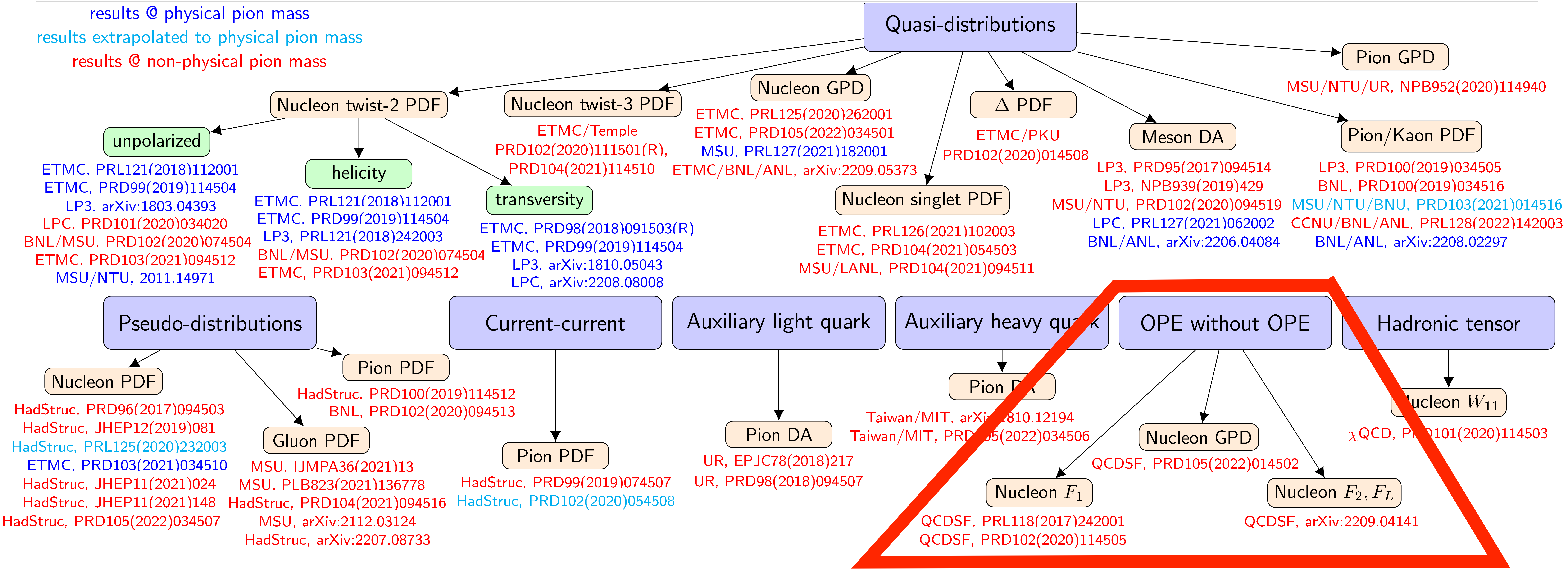


results @ physical pion mass

results extrapolated to physical pion mass

results @ non-physical pion mass

Quasi-distributions

Nucleon twist-2 PDF

unpolarized

ETMC, PRL121(2018)112001
ETMC, PRD99(2019)114504
LP3, arXiv:1803.04393
LPC, PRD101(2020)034020
BNL/MSU, PRD102(2020)074504
ETMC, PRD103(2021)094512
MSU/NTU, 2011.14971

helicity

ETMC, PRL121(2018)112001
ETMC, PRD99(2019)114504
LP3, PRL121(2018)242003
BNL/MSU, PRD102(2020)074504
ETMC, PRD103(2021)094512

transversity

ETMC, PRD98(2018)091503(R)
ETMC, PRD99(2019)114504
LP3, arXiv:1810.05043
LPC, arXiv:2208.08008

Nucleon twist-3 PDF

ETMC/Temple
PRD102(2020)111501(R),
PRD104(2021)114510

Nucleon GPD

ETMC, PRL125(2020)262001
ETMC, PRD105(2022)034501
MSU, PRL127(2021)182001
ETMC/BNL/ANL, arXiv:2209.05373

Nucleon singlet PDF

ETMC, PRL126(2021)102003
ETMC, PRD104(2021)054503
MSU/LANL, PRD104(2021)094511

Δ PDF

ETMC/PKU
PRD102(2020)014508

Meson DA

LP3, PRD95(2017)094514
LP3, NPB939(2019)429
MSU/NTU, PRD102(2020)094519
LPC, PRL127(2021)062002
BNL/ANL, arXiv:2206.04084

Pion GPD

MSU/NTU/UR, NPB952(2020)114940

Pion/Kaon PDF

LP3, PRD100(2019)034505
BNL, PRD100(2019)034516
MSU/NTU/BNU, PRD103(2021)014516
CCNU/BNL/ANL, PRL128(2022)142003
BNL/ANL, arXiv:2208.02297

Pseudo-distributions

Nucleon PDF

HadStruc, PRD96(2017)094503
HadStruc, JHEP12(2019)081
HadStruc, PRL125(2020)232003
ETMC, PRD103(2021)034510
HadStruc, JHEP11(2021)024
HadStruc, JHEP11(2021)148
HadStruc, PRD105(2022)034507

Gluon PDF

MSU, IJMPA36(2021)13
MSU, PLB823(2021)136778
HadStruc, PRD104(2021)094516
MSU, arXiv:2112.03124
HadStruc, arXiv:2207.08733

Pion PDF

HadStruc, PRD100(2019)114512
BNL, PRD102(2020)094513

Current-current

Pion PDF

HadStruc, PRD99(2019)074507
HadStruc, PRD102(2020)054508

Auxiliary light quark

Pion DA

UR, EPJC78(2018)217
UR, PRD98(2018)094507

Auxiliary heavy qu k

Pion DA

Taiwan/MIT, arXi 810.12194
Taiwan/MIT, PRD 05(2022)034506

OPE without OPE

Nucleon F_1

QCDSF, PRL118(2017)242001
QCDSF, PRD102(2020)114505

Nucleon GPD

QCDSF, PRD105(2022)014502

Nucleon F_2, F_L

QCDSF, arXiv:2209.04141

Hadronic tensor

ucleon W_{11}

χQCD, P D101(2020)114503

Hadronic Tensor EW:

Nucleon $\square^{\gamma W}_{VA}$: P-X Ma et al., arXiv:2308.16755 [hep-lat]

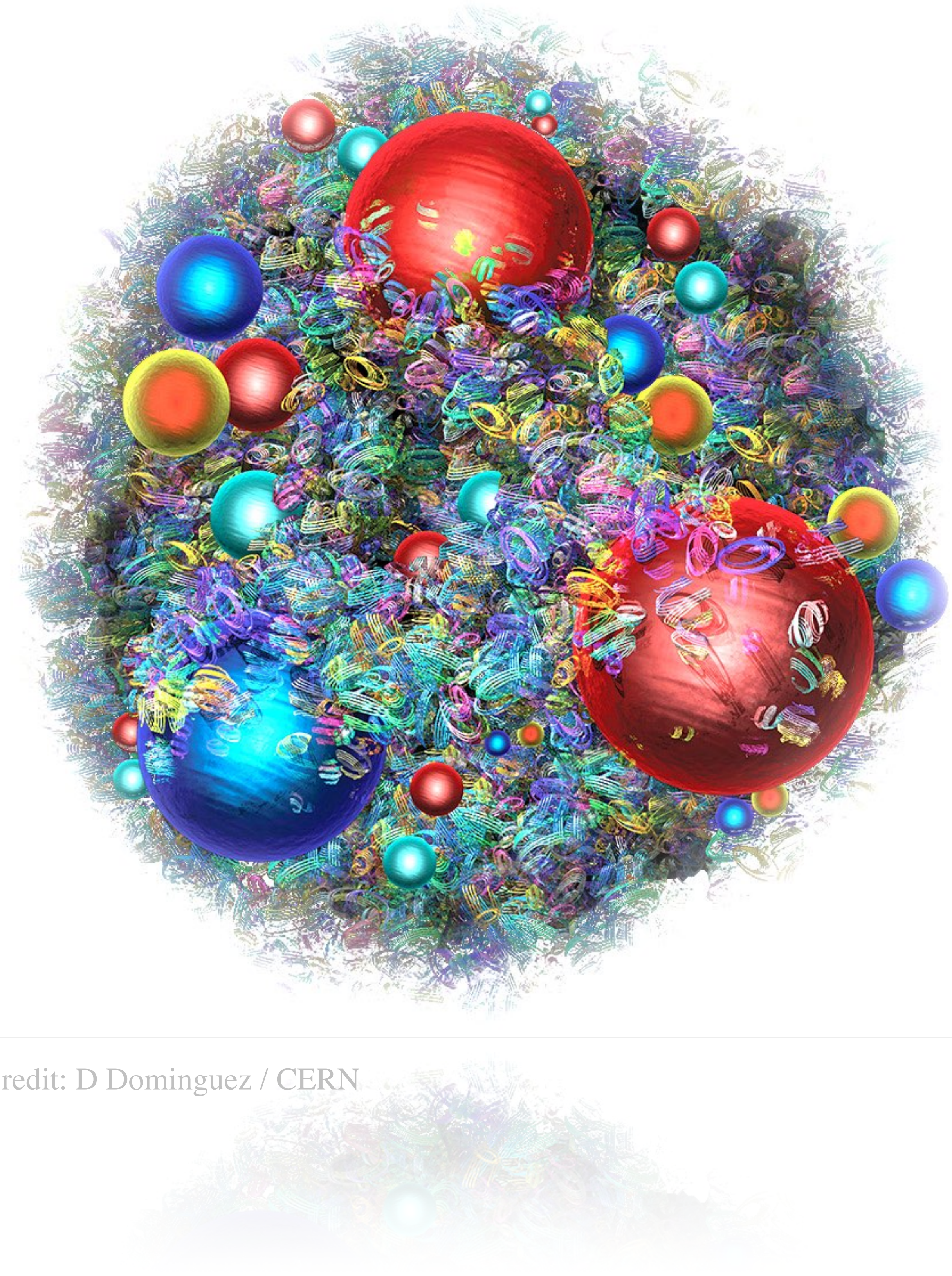
Pion $\square^{\gamma W}_{VA}$: X Feng et al., PRL124, 192002 (2020)

Kaon $\square^{\gamma W}_{VA}$: P-X Ma et al., PRD103, 114503 (2021)

Pion & Kaon $\square^{\gamma W}_{VA}$: J-S Yoo et al., arXiv:2305.03198 [hep-lat]

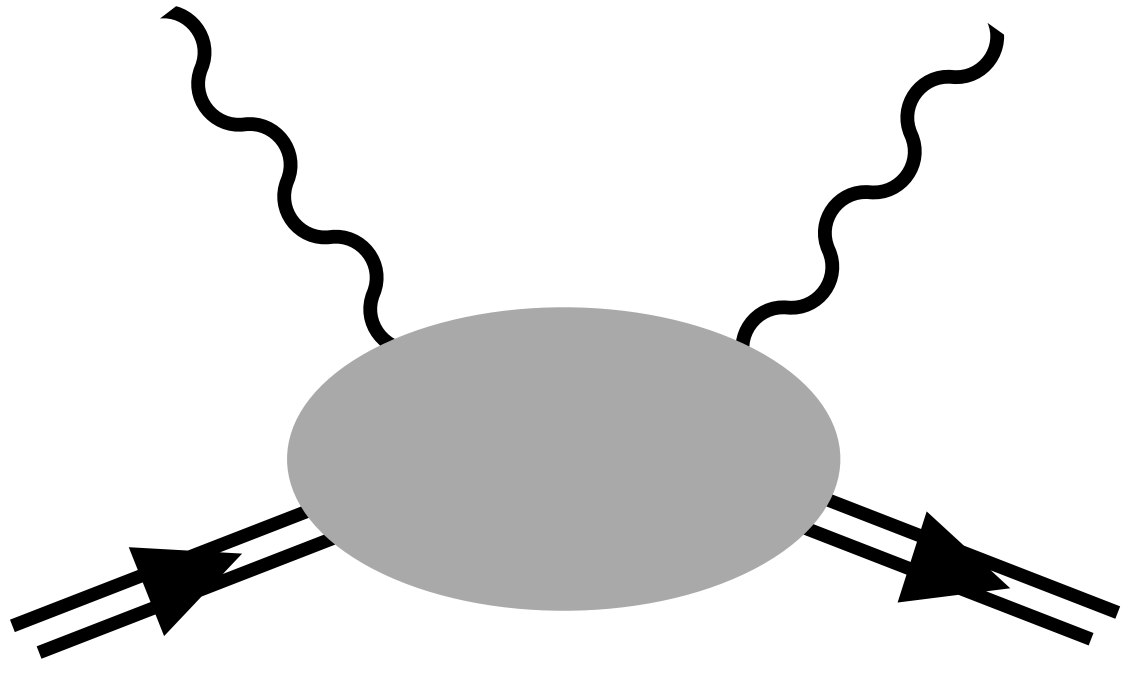
- **QCDSF-UKQCD-CSSM Collaboration**
- **Extended to nucleon F_3, and g_1, g_2**
- **Study of power corrections**

Outline

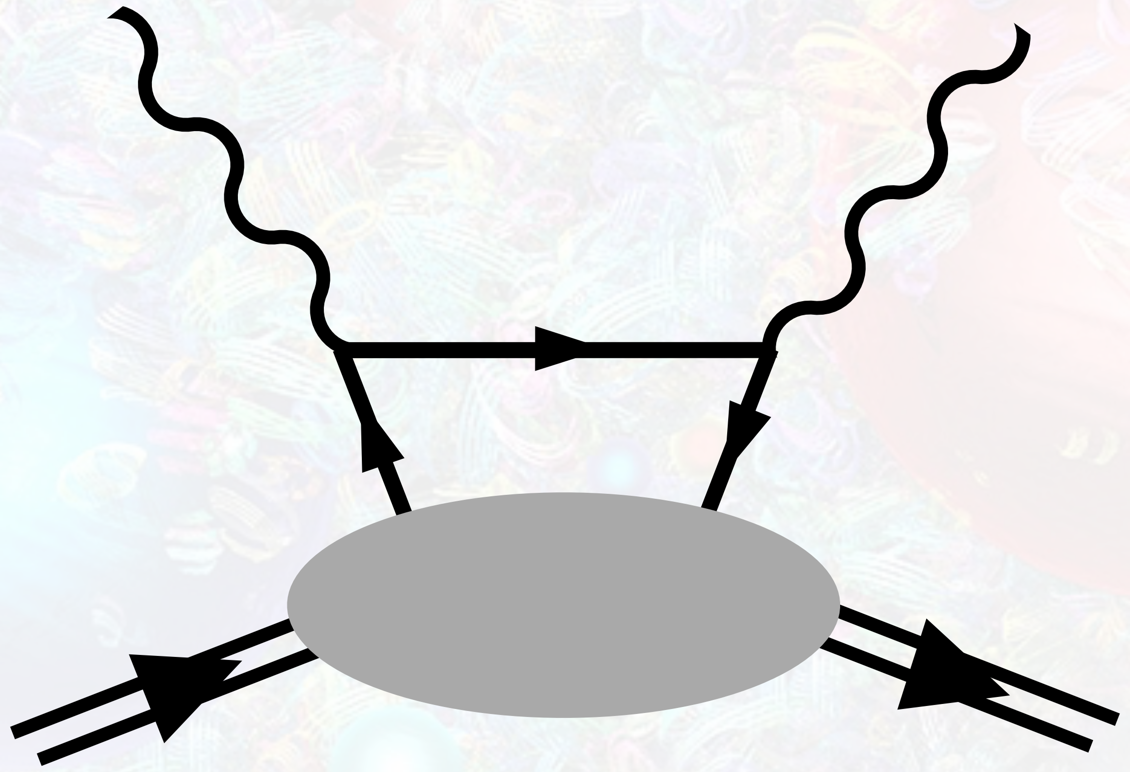

Credit: D Dominguez / CERN

- Forward Compton Amplitude
- Feynman-Hellmann Theorem on the Lattice
- Moments of the Nucleon Structure Functions
  - Unpolarised | F_1, F_2, F_L
  - Higher twist
  - Polarised | g_1, g_2
  - Parity-violating | F_3

Forward Compton Amplitude



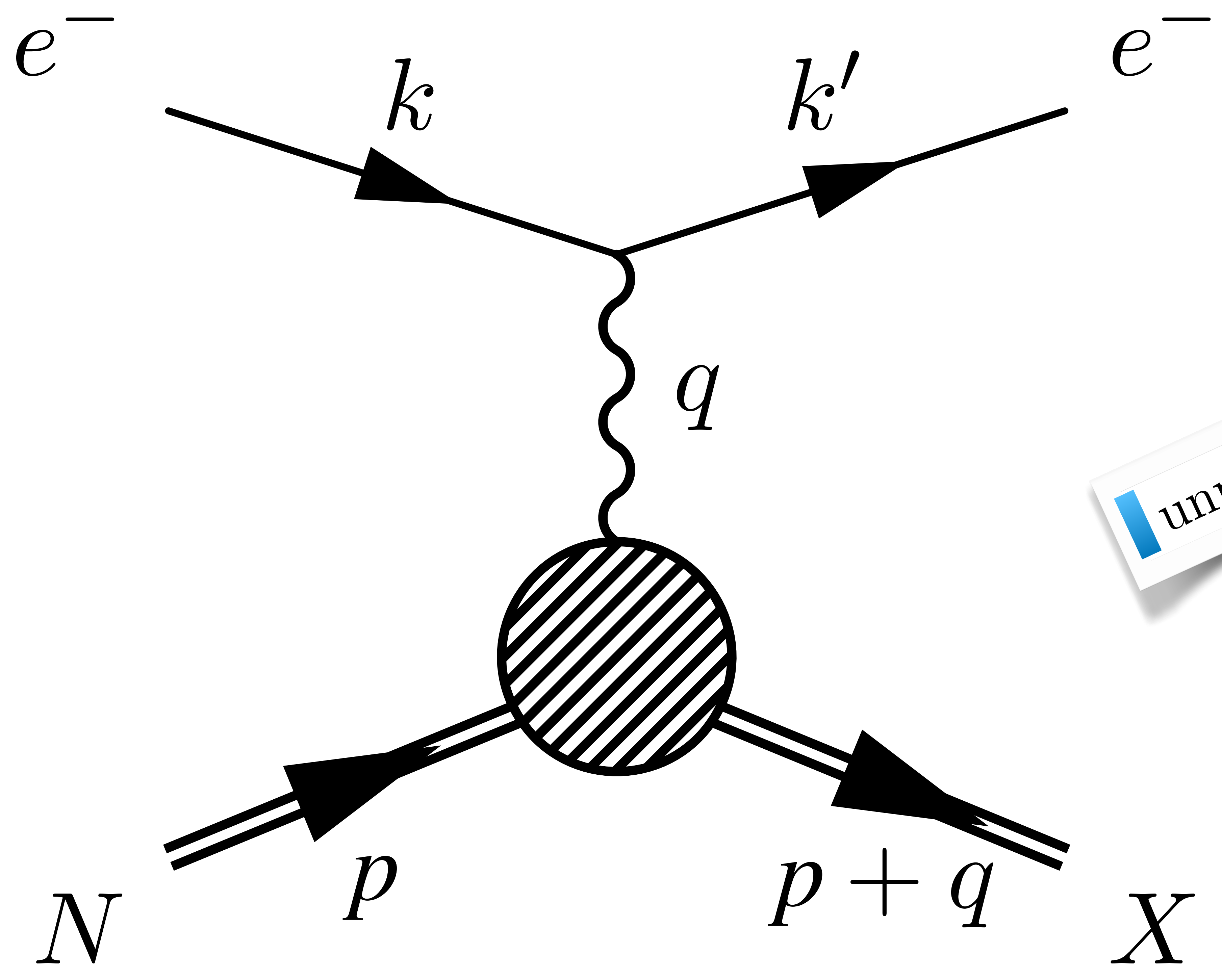
$$=$$



$$+\ \mathcal{O}\left(\frac{M_N^2}{Q^2},\frac{1}{Q^2}\right)$$

DIS and the Hadronic Tensor

Deep ($Q^2 \gg M^2$) inelastic ($W^2 \gg M^2$) scattering (DIS)



$$d\sigma \sim L_j^{\mu\nu} W_{\mu\nu}^j$$

$j = \gamma, Z,$ and γZ (neutral) or W (charged)

leptonic tensor ***hadronic tensor***

$$W_{\mu\nu} = \frac{1}{4\pi}\int d^4z e^{iq\cdot z}\rho_{ss'}\langle p, s'|[J_\mu(z), J_\nu(0)]|p, s\rangle$$

$$\rho_{ss'} = \frac{1}{2}\delta_{ss'}$$

unpolarised

$$W_{\mu\nu} = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right)\boxed{F_1(x, Q^2)} + \left(p_\mu - \frac{p\cdot q}{q^2}q_\mu\right)\left(p_\nu - \frac{p\cdot q}{q^2}q_\nu\right)\frac{\boxed{F_2(x, Q^2)}}{p\cdot q}$$

Structure Functions

Forward Compton Amplitude

$$T_{\mu\nu}(p,q) = i\int d^4z\, e^{iq\cdot z}\rho_{ss'}\langle p,s'|\,\mathscr{T}\{J_\mu(z)J_\nu(0)\}\,|p,s\rangle \quad \text{, spin avg. } \rho_{ss'} = \frac{1}{2}\delta_{ss'}$$

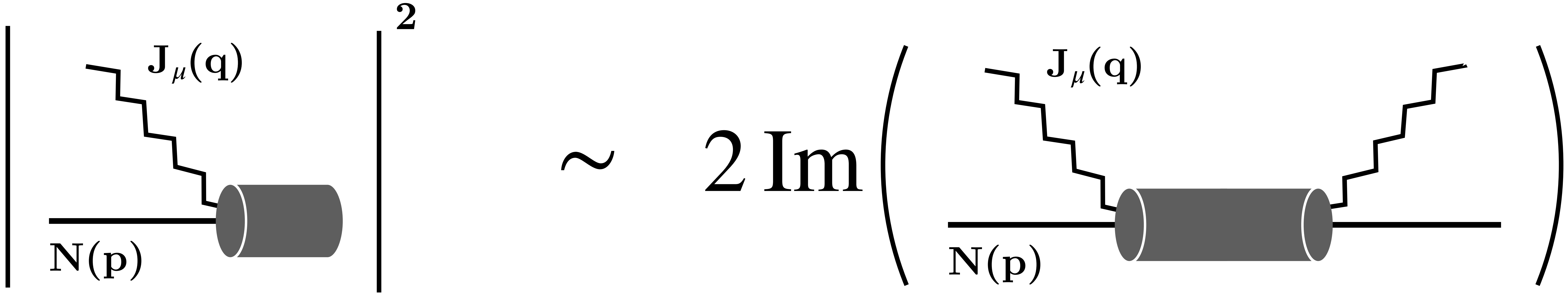
$\omega = \frac{2p\cdot q}{Q^2}$

Same Lorentz decomposition as the Hadronic Tensor

$$= \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right)\mathscr{F}_1(\omega, Q^2) + \left(p_\mu - \frac{p\cdot q}{q^2}q_\mu\right)\left(p_\nu - \frac{p\cdot q}{q^2}q_\nu\right)\frac{\mathscr{F}_2(\omega, Q^2)}{p\cdot q}$$

Compton Structure Functions (SF)

Optical theorem



$$W_{\mu\nu} \sim \int d^4x\langle p|\,[J_\mu(x), J_\nu(0)]\,|p\rangle \qquad T_{\mu\nu} \sim \int d^4x\langle p|\,T\{J_\mu(x)\,J_\nu(0)\}\,|p\rangle$$

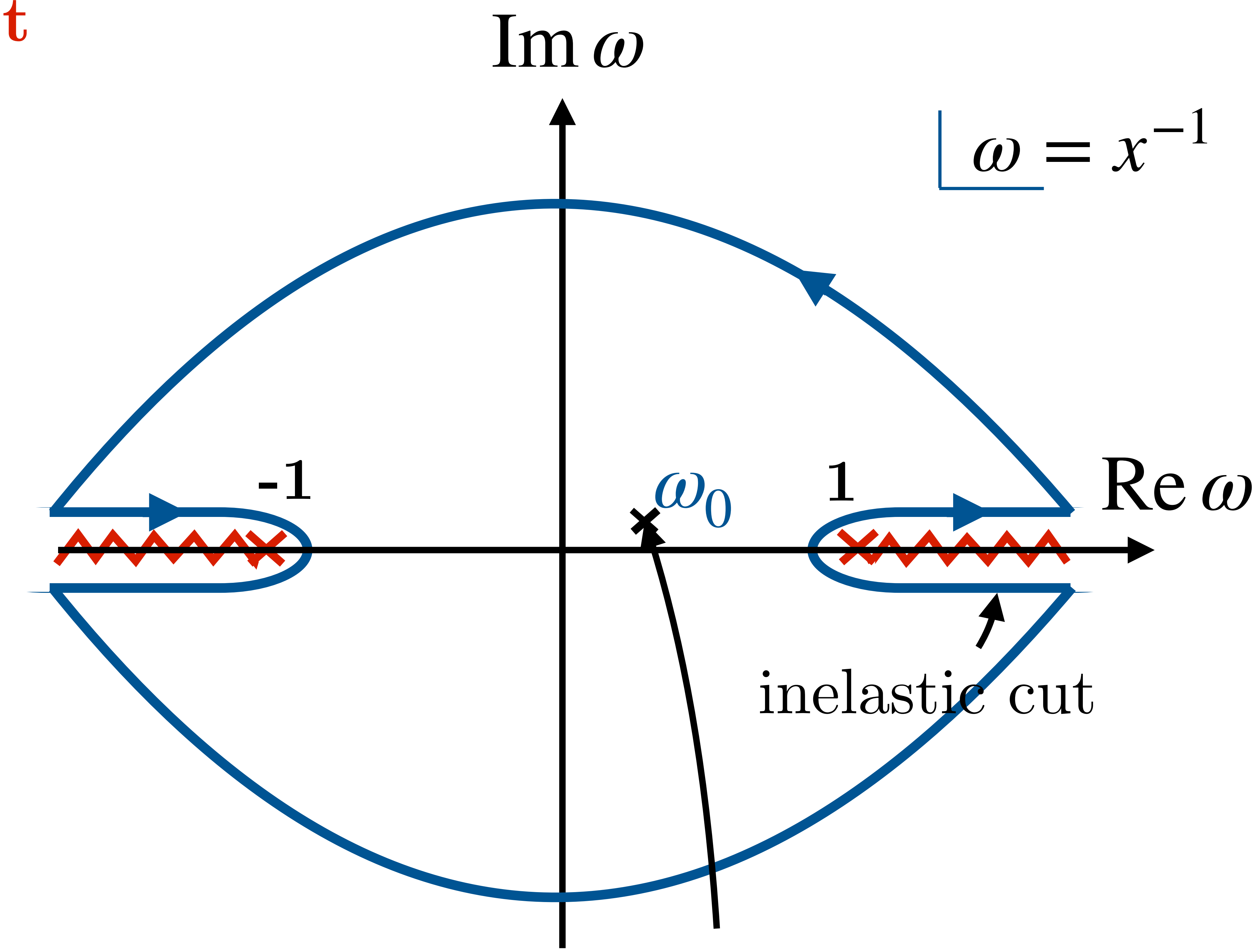
Structure Functions: $F_{1,2}(x, Q^2)$ **Compton Structure Functions:** $\mathscr{F}_{1,2}(p\cdot q, Q^2)$

Nucleon Structure Functions

- **we can write down dispersion relations and connect Compton SFs to DIS SFs:**

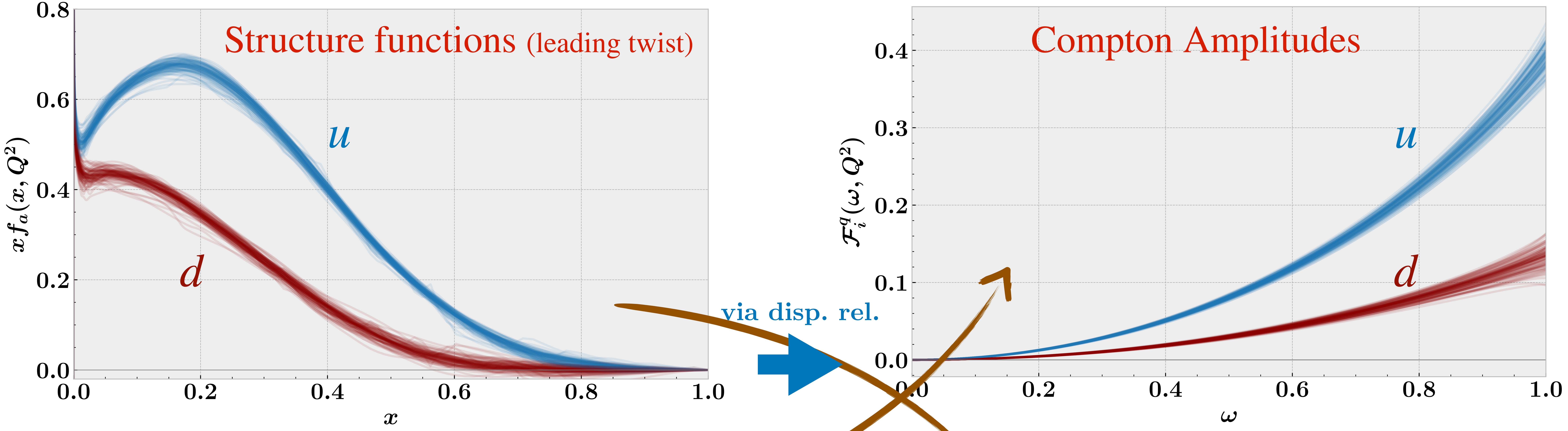
$$\overline{\mathcal{F}}_1(\omega, Q^2) = 2\omega^2 \int_0^1 dx \frac{2x\,F_1(x, Q^2)}{1 - x^2\omega^2 - i\epsilon}$$

$$\mathcal{F}_2(\omega, Q^2) = 4\omega \int_0^1 dx \frac{F_2(x, Q^2)}{1 - x^2\omega^2 - i\epsilon}$$



Compton Amplitude is an analytic function in the unphysical region $|\omega_0| < 1$

Shape of the Compton Amplitude



NNPDF3.1 NNLO

100 sets

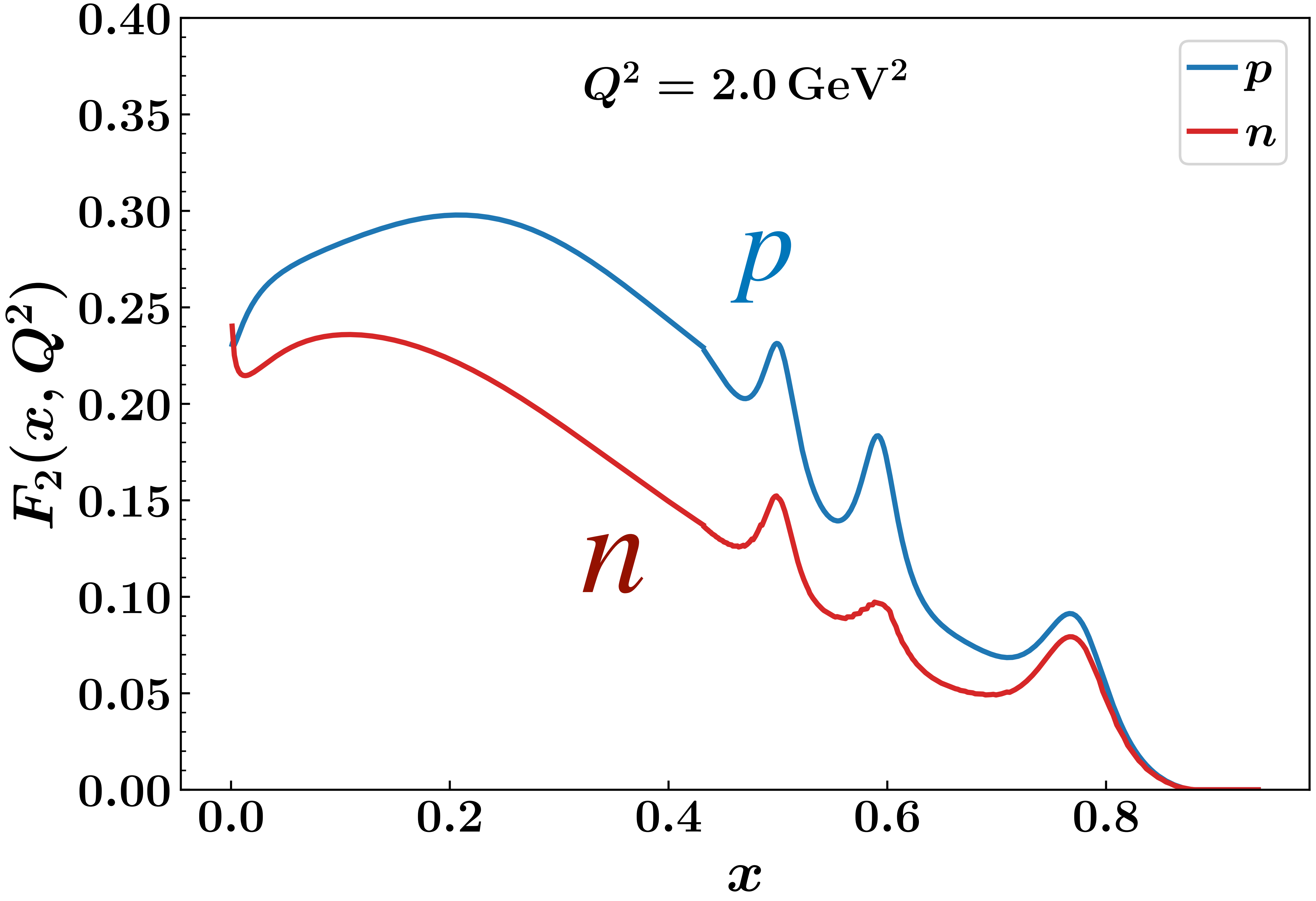
$Q^2 = 9\,GeV^2$

(DIS region)

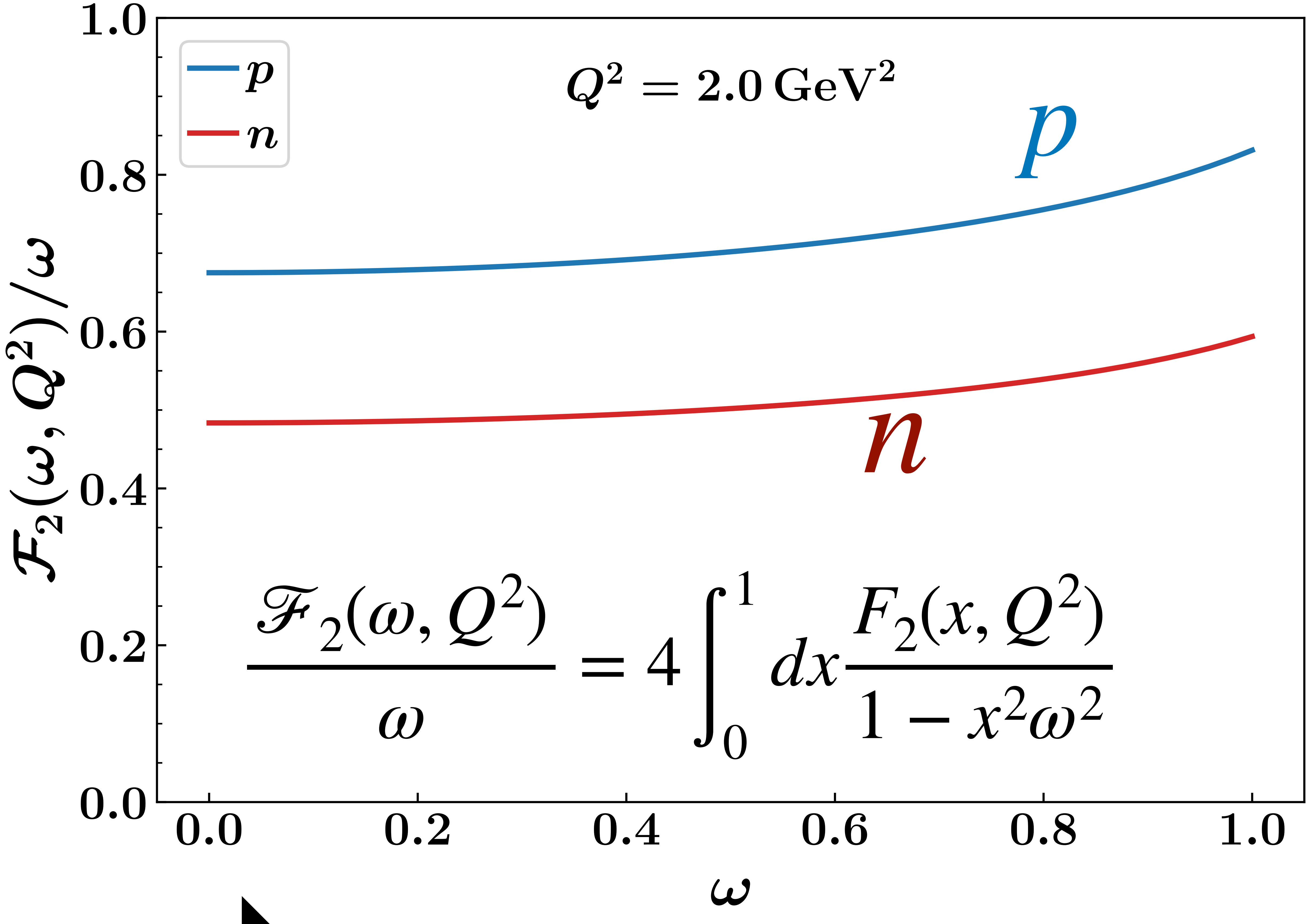
$$\overline{\mathcal{F}}_1(\omega, Q^2) = 2\omega^2 \int_0^1 dx \frac{2x\,F_1(x, Q^2)}{1 - x^2\omega^2}$$

Shape of the Compton Amplitude

Structure functions



Compton Amplitudes

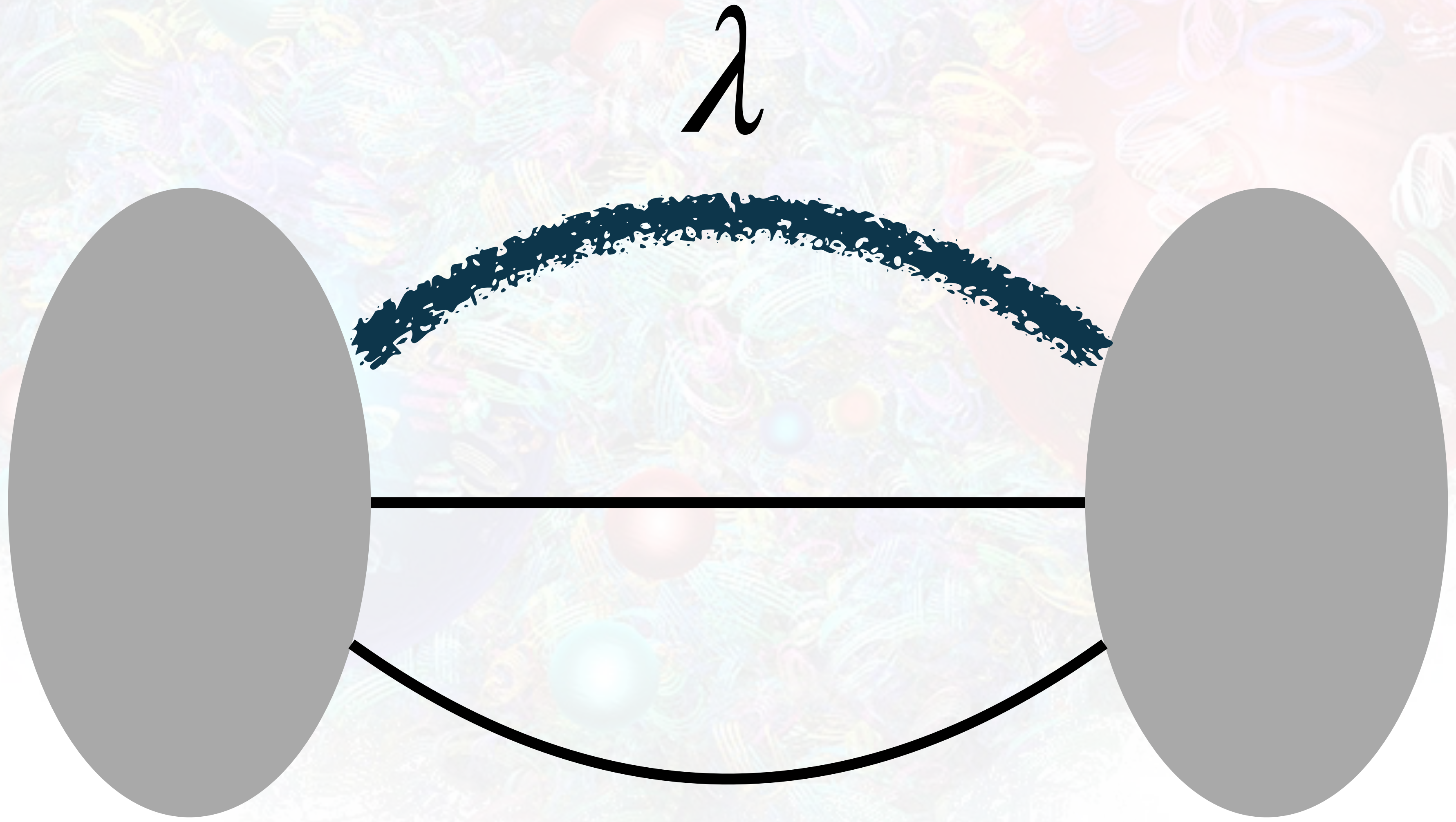


$$\frac{\mathscr{F}_2(\omega, Q^2)}{\omega} = 4\int_0^1 dx \frac{F_2(x, Q^2)}{1 - x^2\omega^2}$$

High-W: M. Arneodo et al. [NMC],
PLB364, 107-115 (1995), [hep-ph/9509406]
Low-W: M.E. Christy and P.E. Bosted,
PRC81, 055213 (2010), [0712.3731]

dispersion relation

Feynman-Hellmann Theorem on the Lattice



FH Theorem at 1st order

in Quantum Mechanics:

$$\frac{\partial E_\lambda}{\partial \lambda} = \langle \phi_\lambda | \frac{\partial H_\lambda}{\partial \lambda} | \phi_\lambda \rangle$$

H_λ: perturbed Hamiltonian of the system

E_λ: energy eigenvalue of the perturbed system

ϕ_λ: eigenfunction of the perturbed system

- **expectation value of the perturbed system is related to the shift in the energy eigenvalue**

in Lattice QCD: energy shifts in the presence of a weak external field

e.g. local bilinear operator

$$S \to S(\lambda) = S + \underset{\nearrow}{\lambda} \int d^4x\, \mathcal{O}(x) \longrightarrow \bar{q}(x)\Gamma_\mu q(x) \quad , \Gamma_\mu \in \{\mathbf{1}, \gamma_\mu, \gamma_5\gamma_\mu, \ldots\}$$

real parameter

@ 1st order

$$\frac{\partial E_\lambda}{\partial \lambda} = \frac{1}{2E_\lambda} \langle 0 | \mathcal{O} | 0 \rangle$$

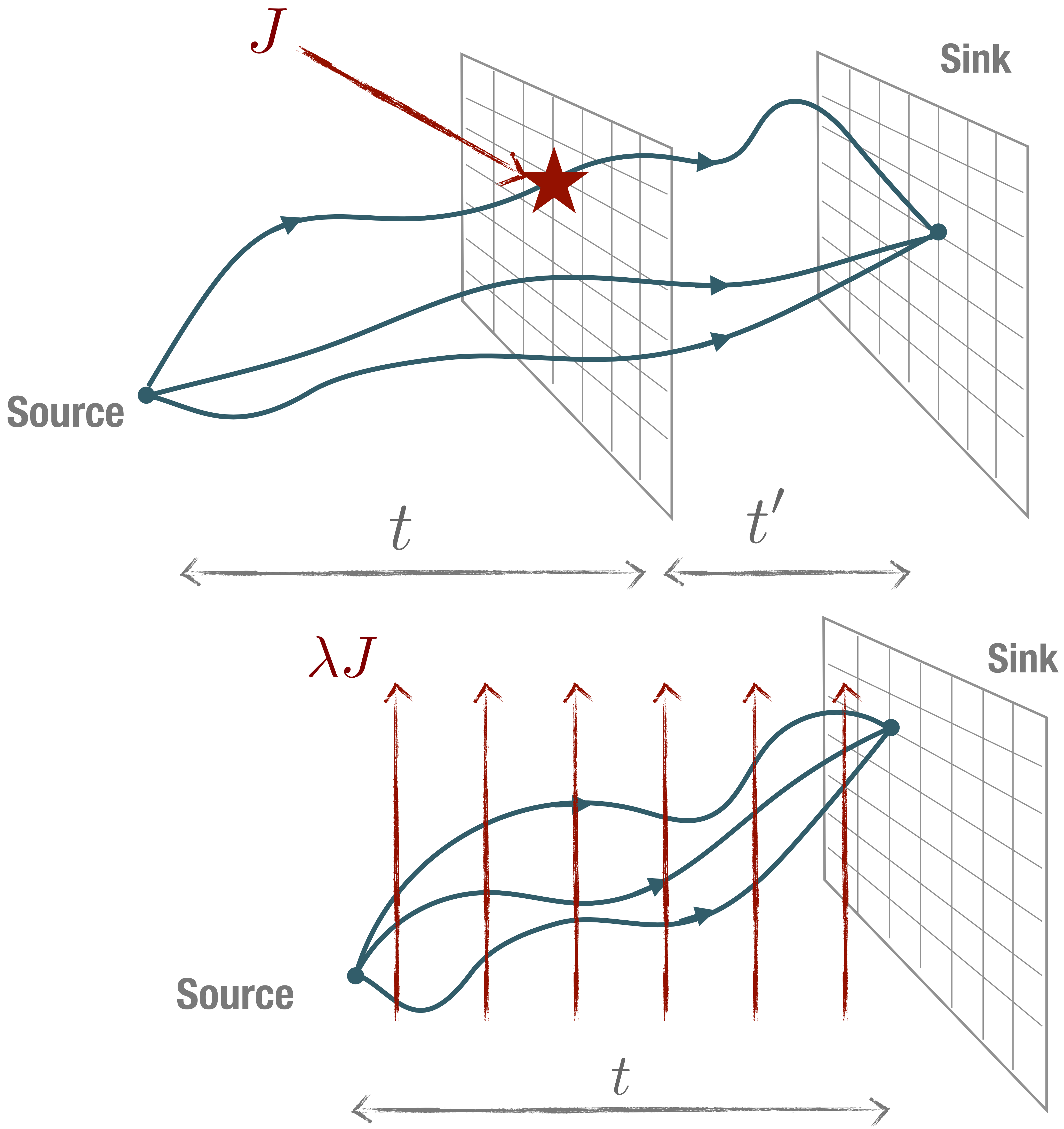
E_λ → spectroscopy, 2-pt function

$\langle 0 | \mathcal{O} | 0 \rangle$ → determine 3-pt

Applications:

- σ **- terms**
- **Form factors**

Matrix elements



- **3-pt functions**

$$t, t' \gg \frac{1}{\Delta E}$$

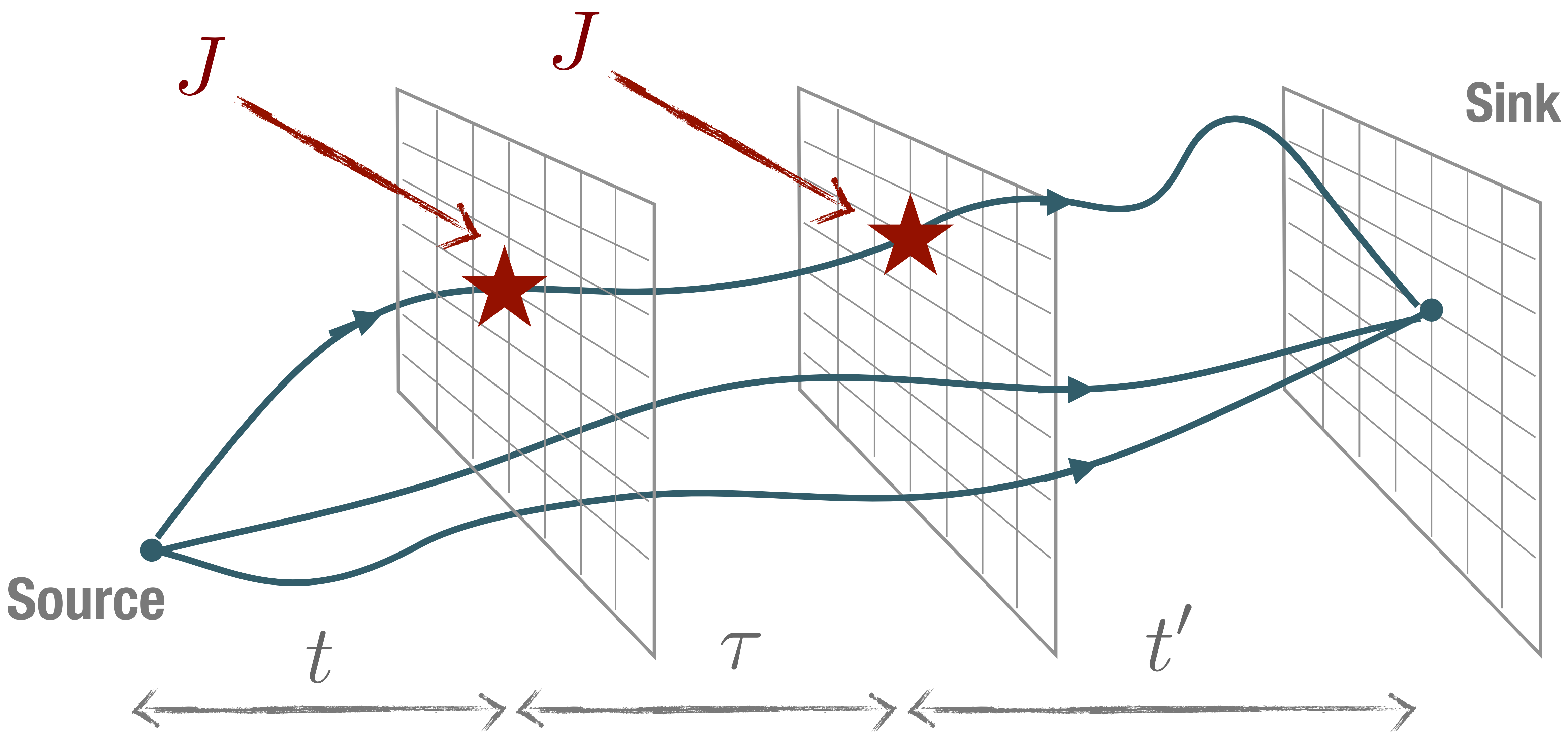
← energy gap to the lowest excitation

- **Feynman—Hellmann**

$$t \gg \frac{1}{\Delta E}$$

$$\left.\frac{\partial E}{\partial \lambda}\right|_{\lambda \to 0} \propto \langle N | J | N \rangle$$

Compton amplitude

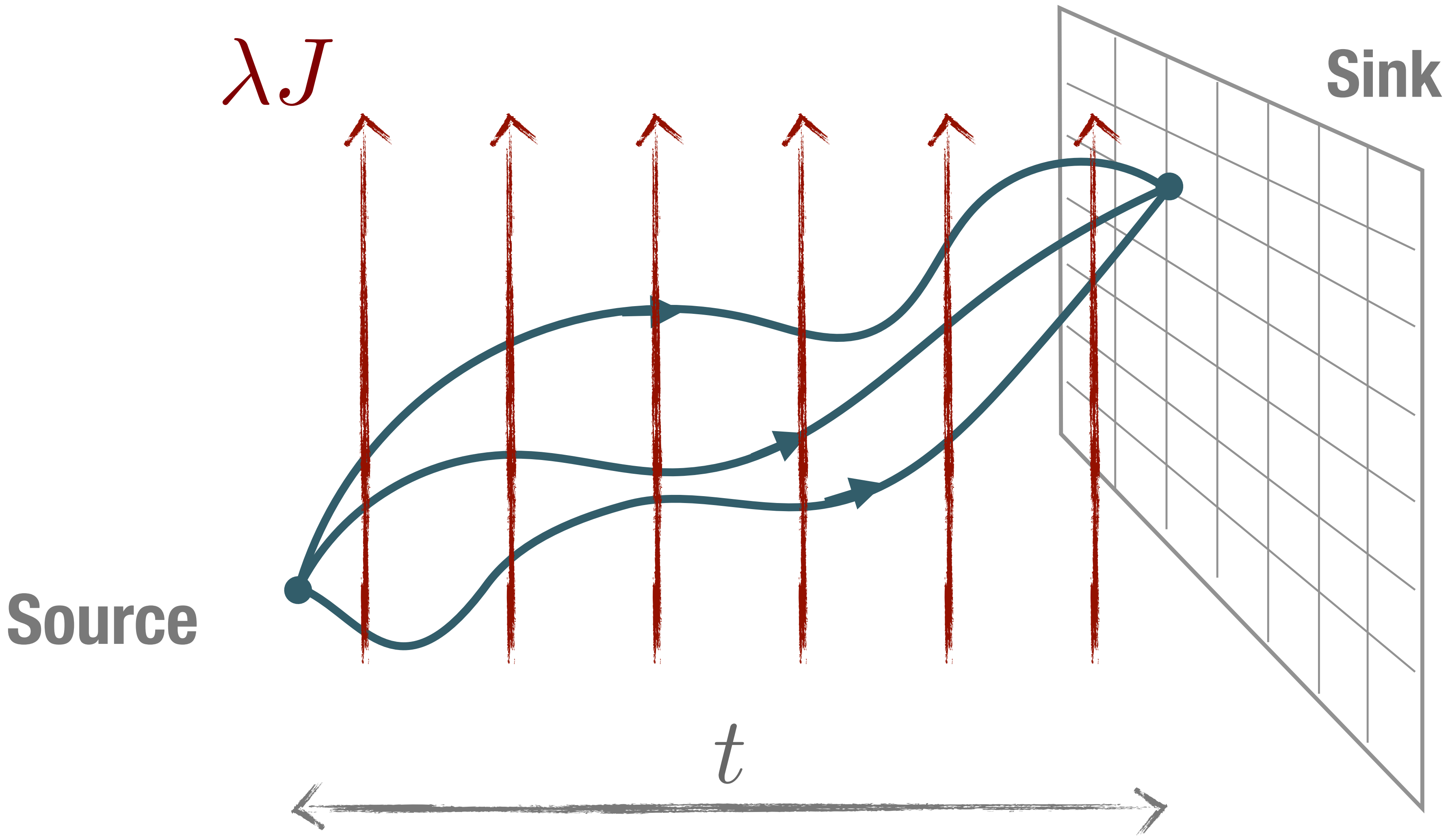


- **4-pt functions**

$$t, t' \gg \frac{1}{\Delta E}$$

$$\frac{\langle C_4(t,\tau,t')\rangle}{\langle C_2(t)\rangle\langle C_2(t')\rangle} \propto \langle N | J(\tau_E) J | N \rangle$$

$$\int_0^\infty d\tau_E \to \langle N | JJ | N \rangle$$

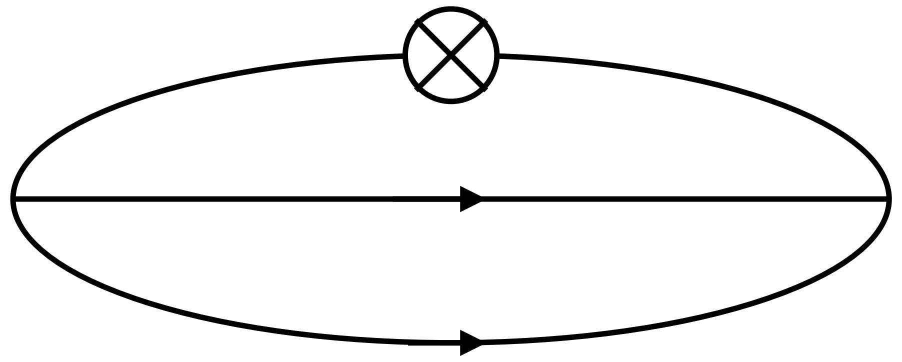


- **Feynman—Hellmann**

$$t \gg \frac{1}{\Delta E}, \qquad \left.\frac{\partial^2 E}{\partial \lambda^2}\right|_{\lambda \to 0} \propto \langle N | JJ | N \rangle$$

QCDSF Applications of FH

- Can modify fermion action in 2 places:
  - quark propagators
  - fermion determinant



Connected

g_A, $\Delta\Sigma$ [PRD90 (2014)]

NPR [PLB740 (2015)]

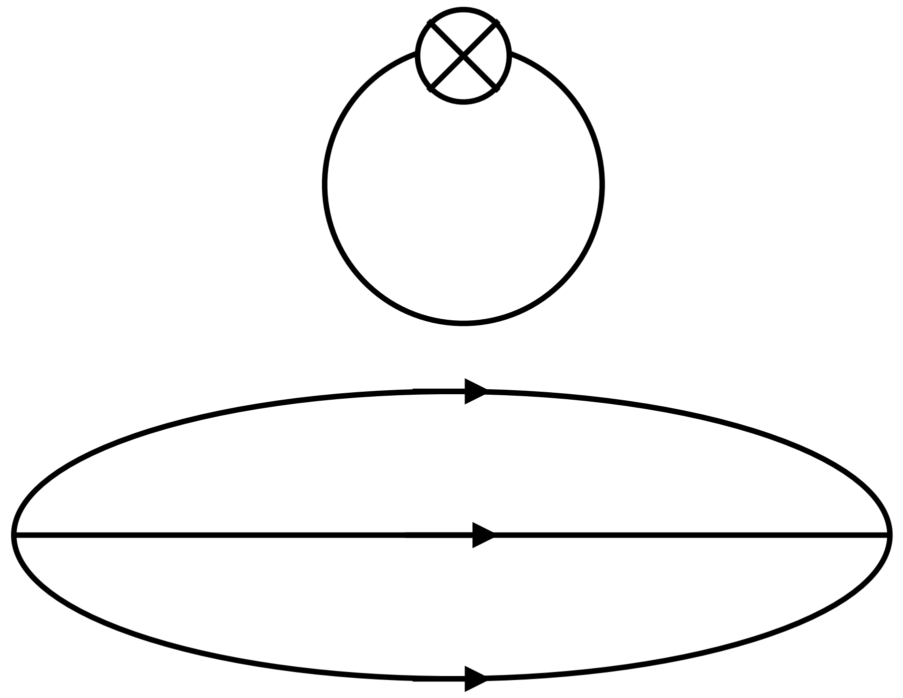
G_E,G_M [PRD96 (2017)]

$F_{1,2}(\omega, Q^2)$ [PRL118 (2017), PRD102 (2020), PRD107 (2023)]

GPDs [PRD104 (2022)]

$\Sigma \to n$ [PRD108 (2023) 3, 034507]

g_A, g_T, g_S [PRD108 (2023) 9, 094511]



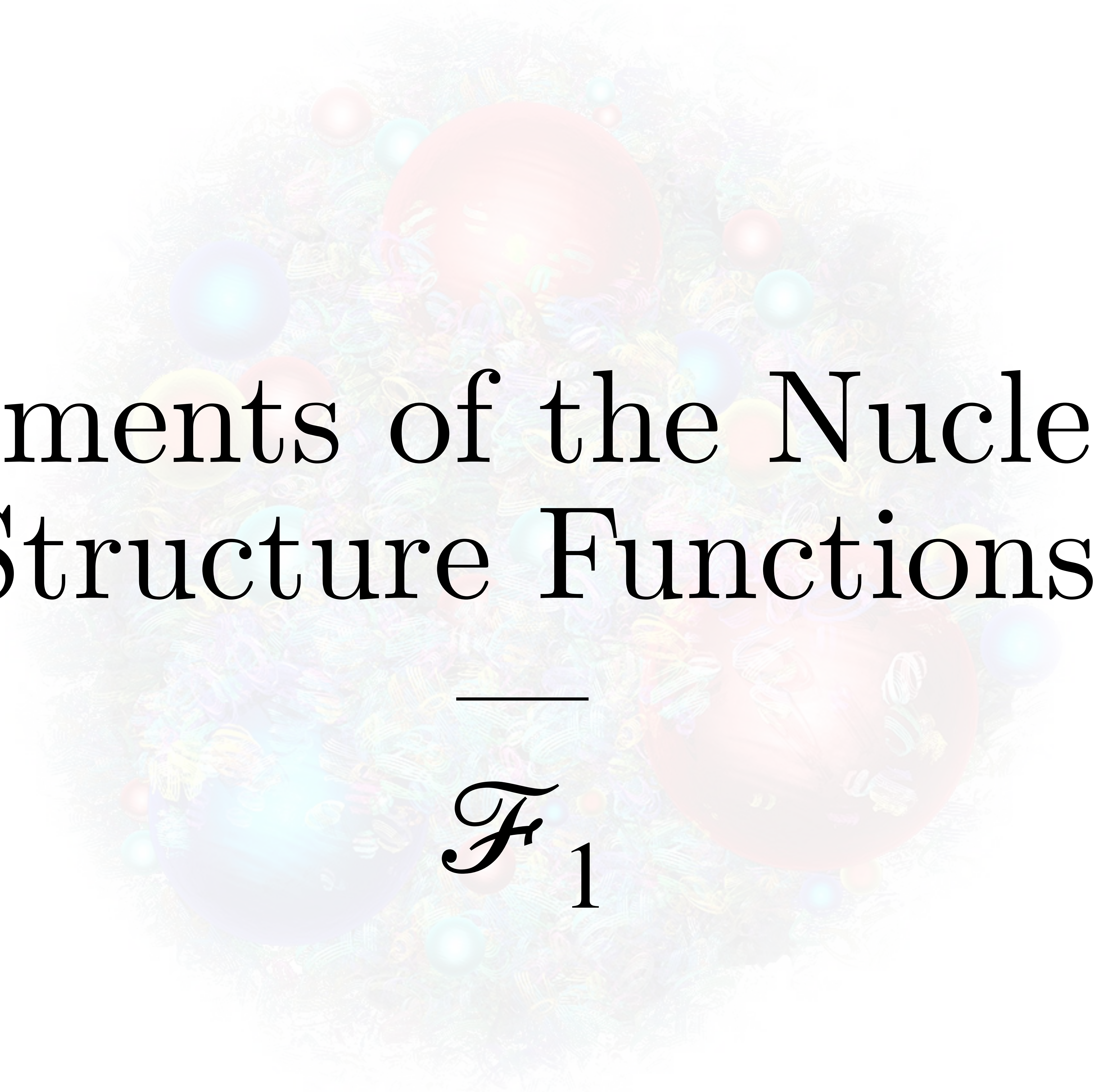
Disconnected

(Requires new gauge configurations)

$\langle x \rangle_g$ [PLB714 (2012)]

NPR [PLB740 (2015)]

Δs [PRD92 (2015)]



Moments of the Nucleon Structure Functions

$$\mathscr{F}_1$$

Forward Compton Amplitude

$$T_{\mu\nu}(p,q) = i\int d^4z\, e^{iq\cdot z}\rho_{ss'}\langle p,s'|\mathcal{T}\{J_\mu(z)J_\nu(0)\}|p,s\rangle \quad \text{, spin avg. } \rho_{ss'} = \frac{1}{2}\delta_{ss'}$$

$$\omega = \frac{2p\cdot q}{Q^2}$$

$$= \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right)\mathcal{F}_1(\omega,Q^2) + \left(p_\mu - \frac{p\cdot q}{q^2}q_\mu\right)\left(p_\nu - \frac{p\cdot q}{q^2}q_\nu\right)\frac{\mathcal{F}_2(\omega,Q^2)}{p\cdot q}$$

Simplest kinematics to directly isolate $\mathcal{F}_1$

$$J_3 J_3 \text{ and } p_3 = q_3 = 0$$

$$T_{33}(p,q) = \mathcal{F}_1(\omega,Q^2)$$

Simulation Details

QCDSF/UKQCD configurations

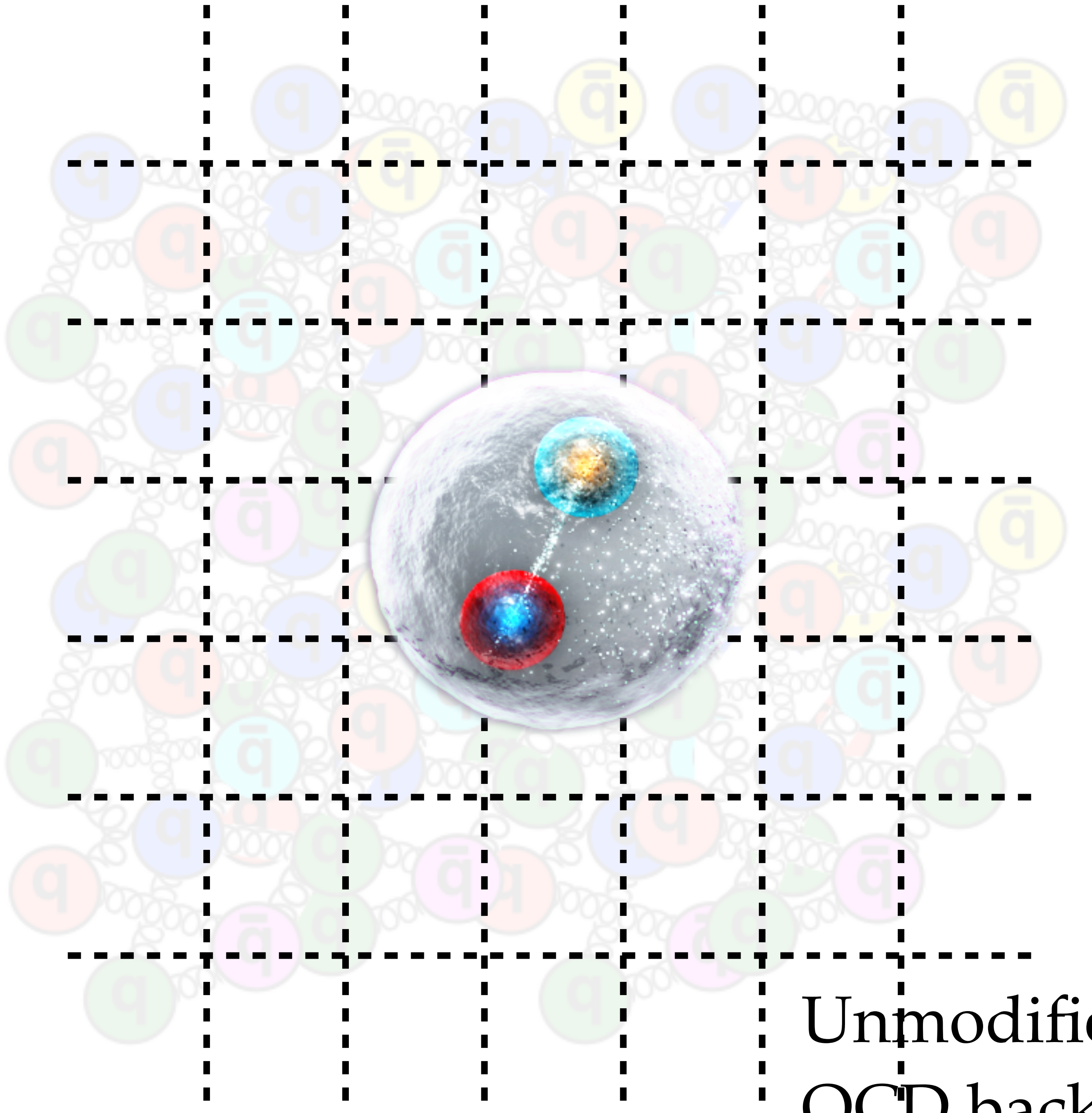
$\begin{pmatrix} 32^3 \times 64 \\ 48^3 \times 96 \end{pmatrix}$, 2+1 flavour (u/d+s)

$\beta = \begin{pmatrix} 5.50 \\ 5.65 \end{pmatrix}$, NP-improved Clover action

PRD 79, 094507 (2009), arXiv:0901.3302 [hep-lat]

$$m_\pi \sim \begin{bmatrix} 470 \\ 420 \end{bmatrix} \text{MeV, ~SU(3) sym.}$$

$$m_\pi L \sim \begin{bmatrix} 5.6 \\ 6.9 \end{bmatrix} \qquad a = \begin{bmatrix} 0.074 \\ 0.068 \end{bmatrix} \text{ fm}$$



Unmodified QCD background

- Local EM current insertion, $J_\mu(x) = Z_V \bar{q}(x)\gamma_\mu q(x)$ (valence only)
- 4 Distinct field strengths, $\lambda = [\pm 0.0125, \pm 0.025]$
- Up to $\mathcal{O}(10^4)$ measurements for each pair of Q^2 and λ

Strategy | Energy shifts

Isolate the 2nd-order energy shift

$$G^{(2)}_{\lambda}(\mathbf{p};t) \sim A_{\lambda}(\mathbf{p})e^{-E_{N_\lambda}(\mathbf{p})t}$$

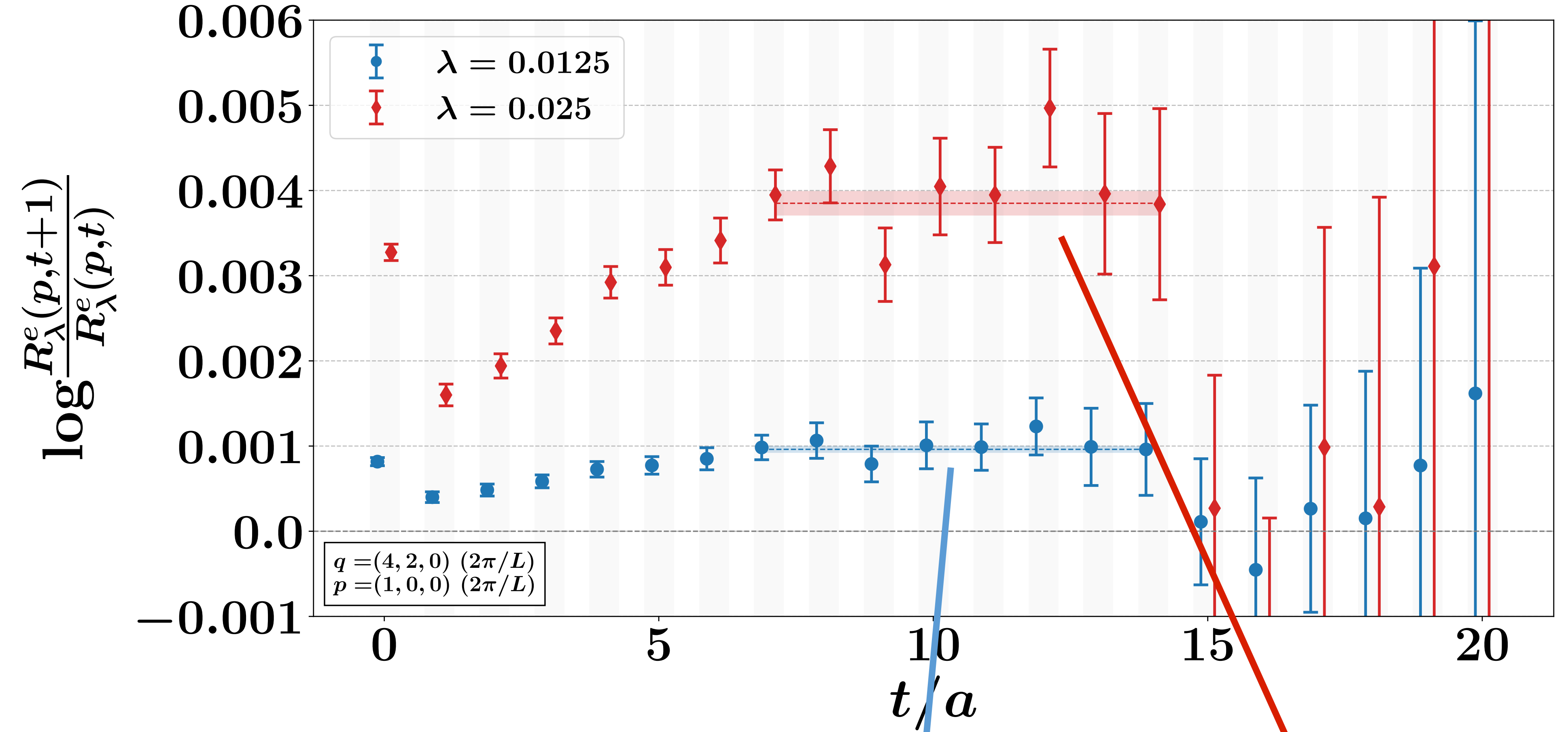
$$\begin{aligned} E_{N_\lambda}(\mathbf{p}) &= E_N(\mathbf{p}) + \lambda \left.\frac{\partial E_{N_\lambda}(\mathbf{p})}{\partial \lambda}\right|_{\lambda=0} + \frac{\lambda^2}{2!}\left.\frac{\partial^2 E_{N_\lambda}(\mathbf{p})}{\partial^2 \lambda}\right|_{\lambda=0} + \mathcal{O}(\lambda^3) \\ &= E_N(\mathbf{p}) + \Delta E^o_N(\mathbf{p}) + \Delta E^e_N(\mathbf{p}) \end{aligned}$$

Ratio of perturbed to unperturbed 2-pt functions

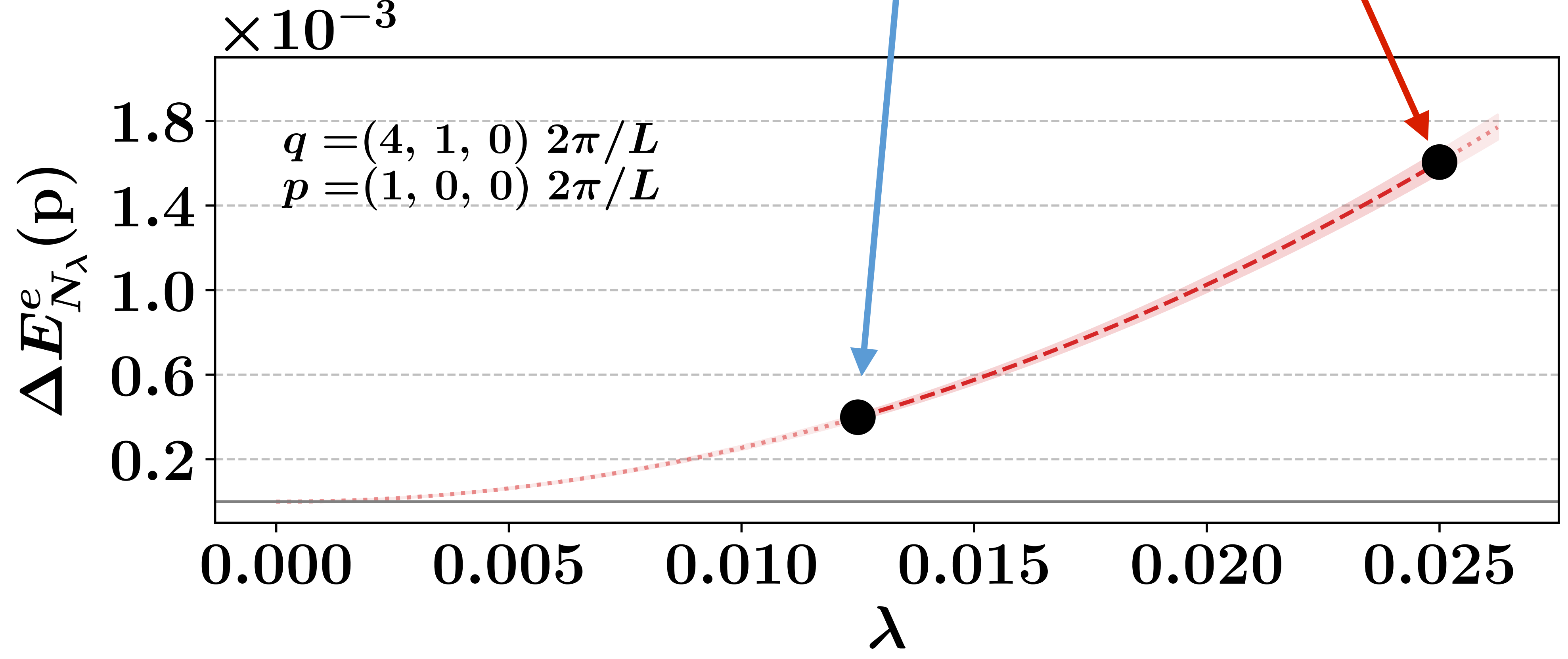
$$R^e_\lambda(\mathbf{p},t) \equiv \frac{G^{(2)}_{+\lambda}(\mathbf{p},t)G^{(2)}_{-\lambda}(\mathbf{p},t)}{\left(G^{(2)}(\mathbf{p},t)\right)^2}$$

$$\xrightarrow{t\gg 0} A_\lambda(\mathbf{p})e^{-2\Delta E^e_{N_\lambda}(\mathbf{p})t}$$

- **Extract energy shifts for each λ**

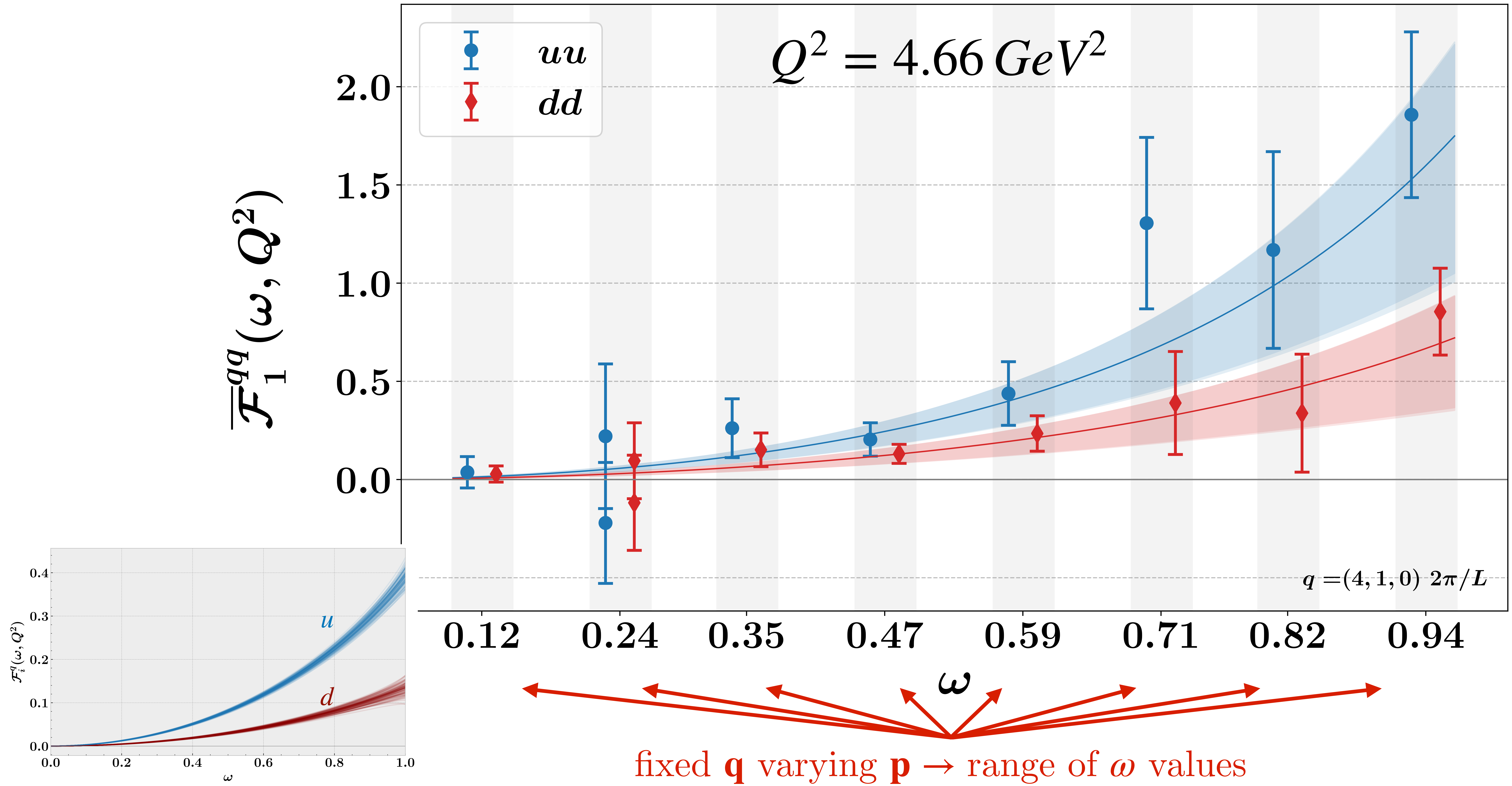


- **Get the 2nd order derivative**



$\mathcal{F}_1$

$a = 0.074\,\mathrm{fm}$
$m_\pi \sim 470\,\mathrm{MeV}$
32^3x64, 2+1 flavour



fixed **q** varying **p** → range of ω values

$\omega = 1/x = \dfrac{2p \cdot q}{Q^2}$

Nucleon Structure Functions

- using the Taylor expansion, $\frac{1}{1-(x\omega)^2} = \sum_{n=1}^{\infty} (x\omega)^{2n-2}$

$\omega = \frac{2p \cdot q}{Q^2} \equiv x^{-1}$

$$\overline{\mathscr{F}}_1(\omega, Q^2) = 2\omega^2 \int_0^1 dx \frac{2x\,F_1(x,Q^2)}{1 - x^2\omega^2} = \sum_{n=0}^{\infty} 2\omega^{2n} \underbrace{2\int_0^1 dx\, x^{2n-1} F_1(x,Q^2)}_{M_{2n}^{(1)}(Q^2)\ \equiv\ \text{Moments}}$$

- Enforce monotonic decreasing of moments

$$M_2^{(1)}(Q^2) \geq M_4^{(1)}(Q^2) \geq M_6^{(1)}(Q^2) \geq \cdots \geq M_{2n}^{(1)}(Q^2) \geq \cdots \geq 0$$

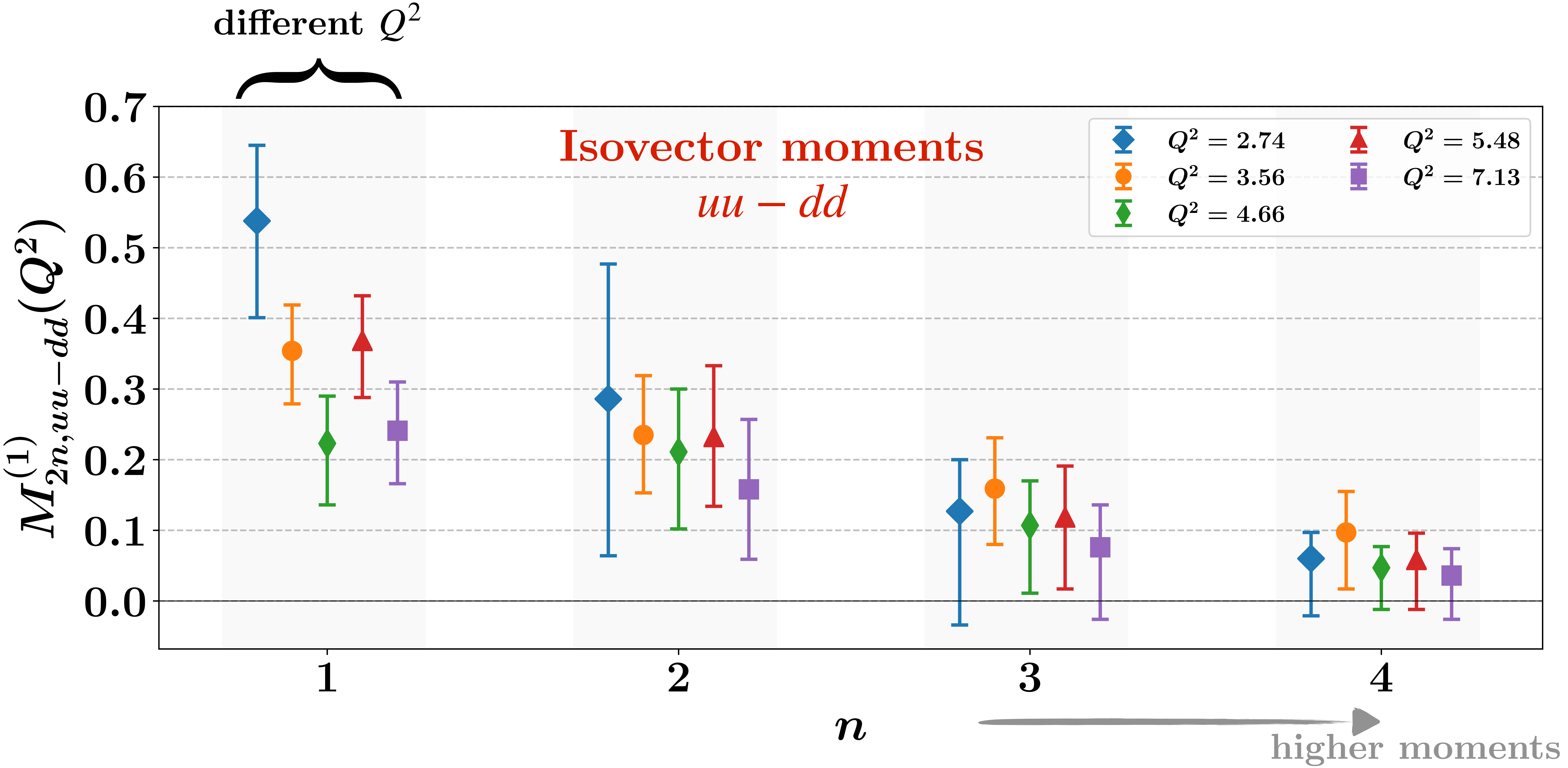
- Bayesian approach by MCMC method

Uniform priors: $$M_{2n}^{(1)}(Q^2) \sim \mathscr{U}\left(0, M_{2n-2}^{(1)}(Q^2)\right)$$

low moments insensitive to truncation order

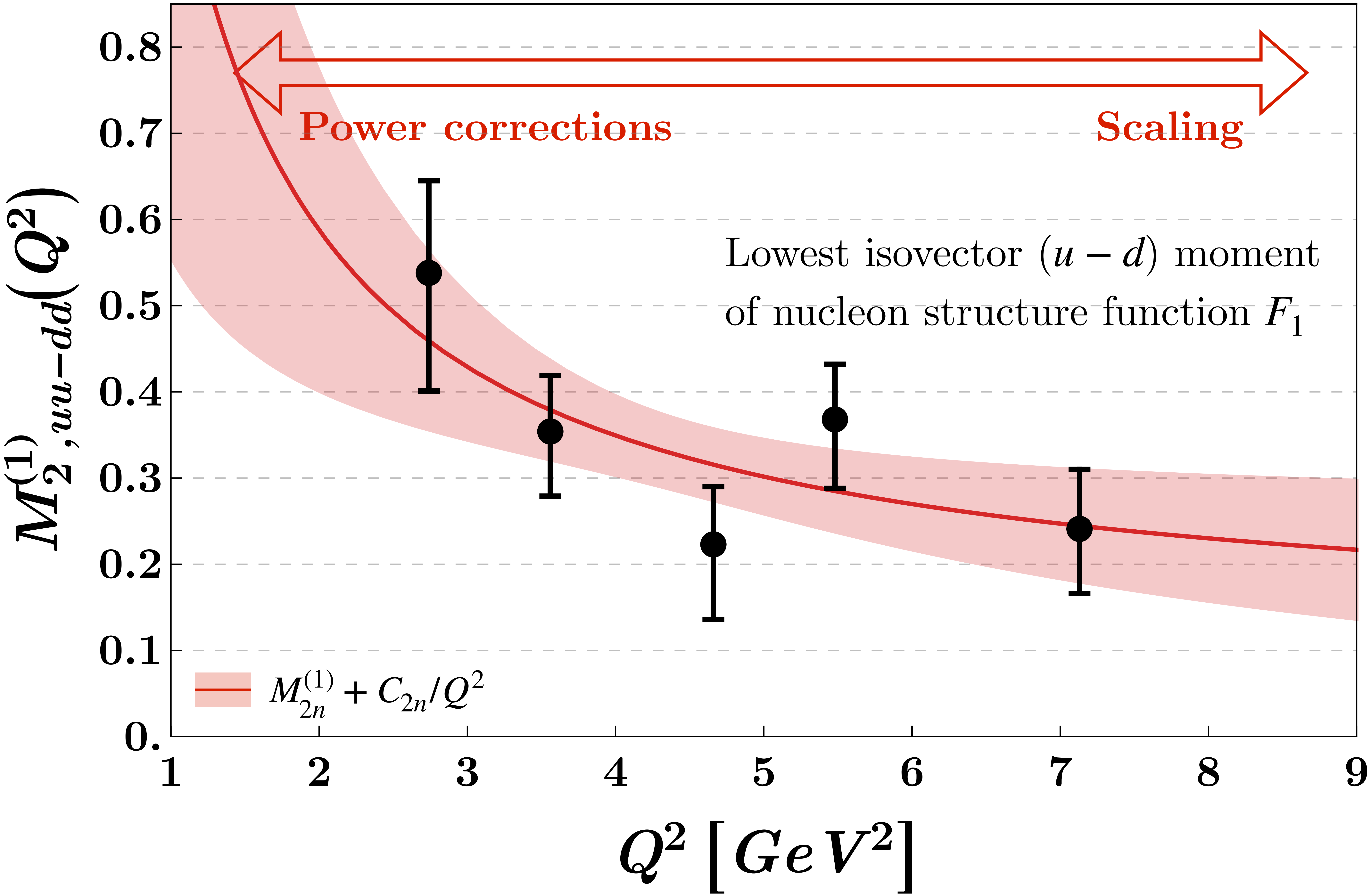
Moments

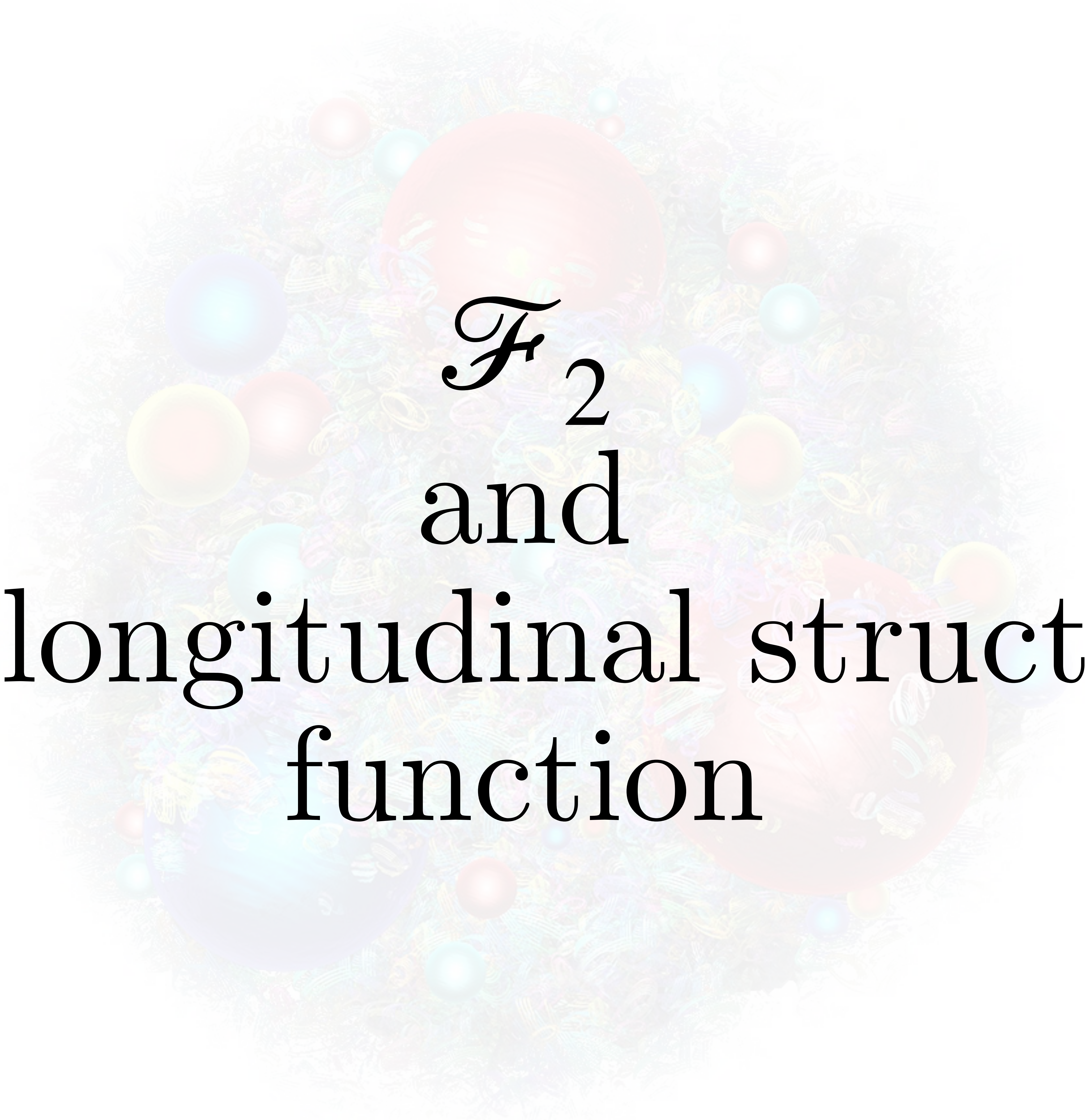
$a = 0.074\,\mathrm{fm}$
$m_\pi \sim 470\,\mathrm{MeV}$
32^3x64, 2+1 flavour



Power corrections

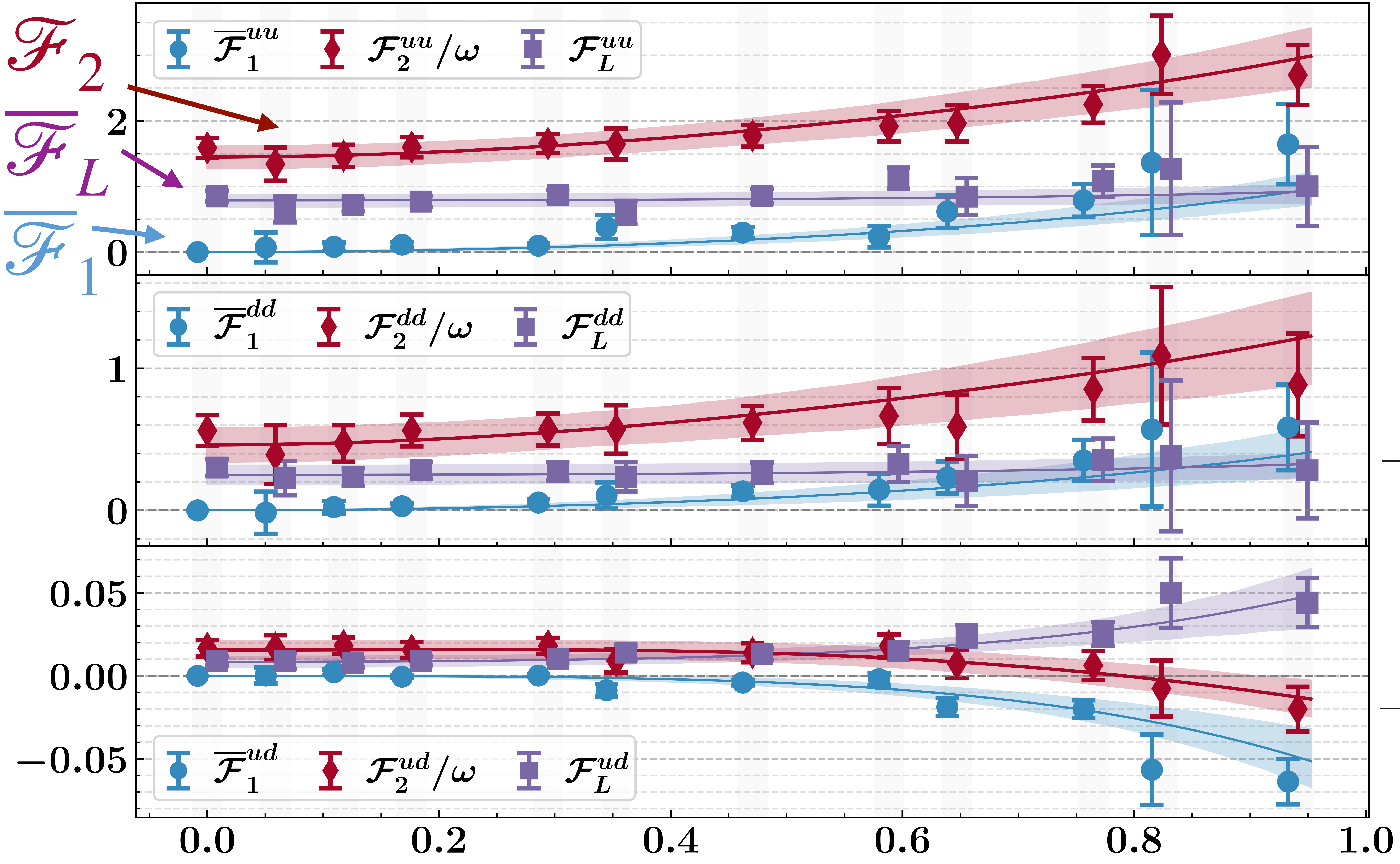
$a = 0.074\,\mathrm{fm}$
$m_\pi \sim 470\,\mathrm{MeV}$
32^3x64, 2+1 flavour





$\mathscr{F}_2$ and the longitudinal structure function

$\mathcal{F}_{2,L}$



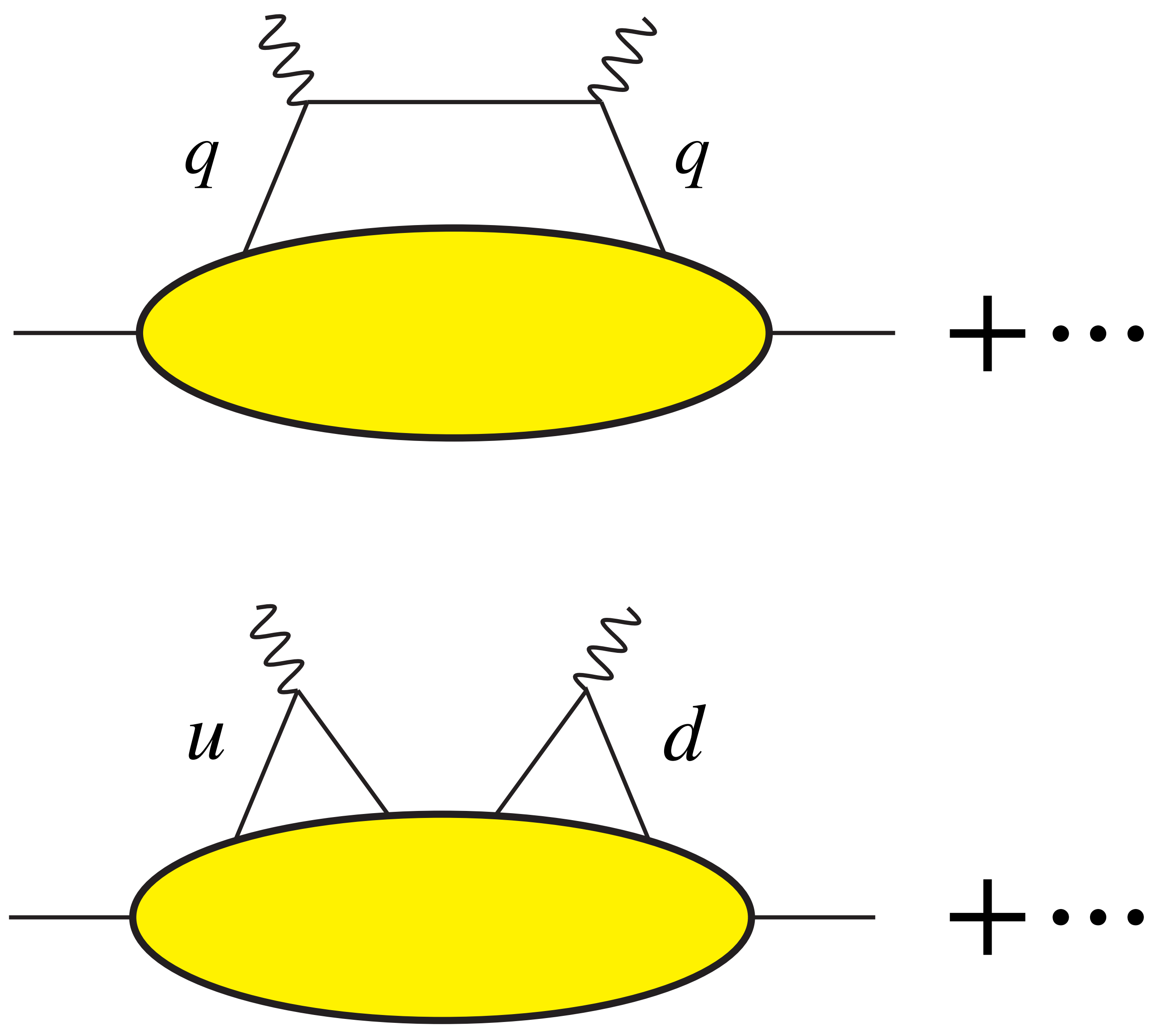
$\omega = 2p \cdot q/Q^2$

48^3x96, 2+1 flavour

$a = 0.068\,\mathrm{fm}$

$m_\pi \sim 420\,\mathrm{MeV}$

$Q^2 \sim 4.9\,GeV^2$



$\mathcal{F}_{2,L}$ | Moments

- **Dispersion relation for F_L**

$$\overline{\mathcal{F}}_L(\omega, Q^2) = \frac{8M_N^2}{Q^2}\int_0^1 dx F_2(x, Q^2) + 2\omega^2 \int_0^1 dx \frac{F_L(x, Q^2)}{1 - x^2\omega^2 - i\epsilon}$$

- **Parametrise F_2 in terms of moments of F_1 and F_L**

$$M_2^{(1)}(Q^2) \geq M_4^{(1)}(Q^2) \geq M_6^{(1)}(Q^2) \geq \cdots \geq 0$$

$$M_0^{(1)}(Q^2) \geq M_2^{(1)}(Q^2) \geq M_4^{(1)}(Q^2) \geq \cdots \geq 0$$

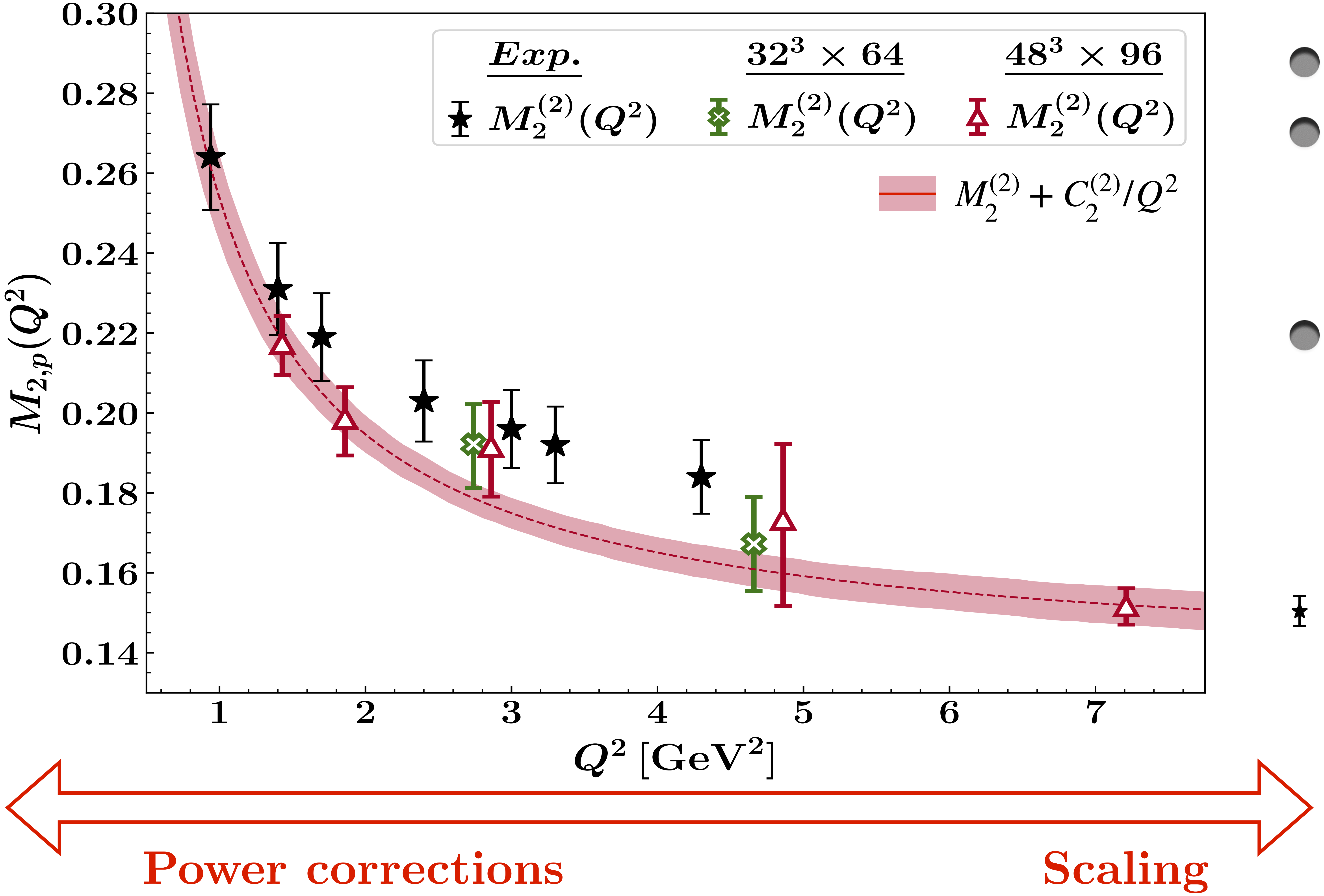
- **Fit two independent amplitudes**

$$\overline{\mathcal{F}}_1^{qq}(\omega, Q^2) = 2\sum_{n=1}^{\infty} M_{2n}^{(1)}(Q^2)\,\omega^{2n}$$

$$\frac{\mathcal{F}_2^{qq}(\omega, Q^2)}{\omega} = \frac{\tau}{1+\tau\omega^2}\sum_{n=0}^{\infty} 4\omega^{2n}\left[M_{2n}^{(1)} + M_{2n}^{(L)}\right](Q^2), \text{where} \quad \tau = \frac{Q^2}{4M_N^2}$$

Moments | proton F_2

- **Unique ability to study the Q^2 dependence of the moments!**



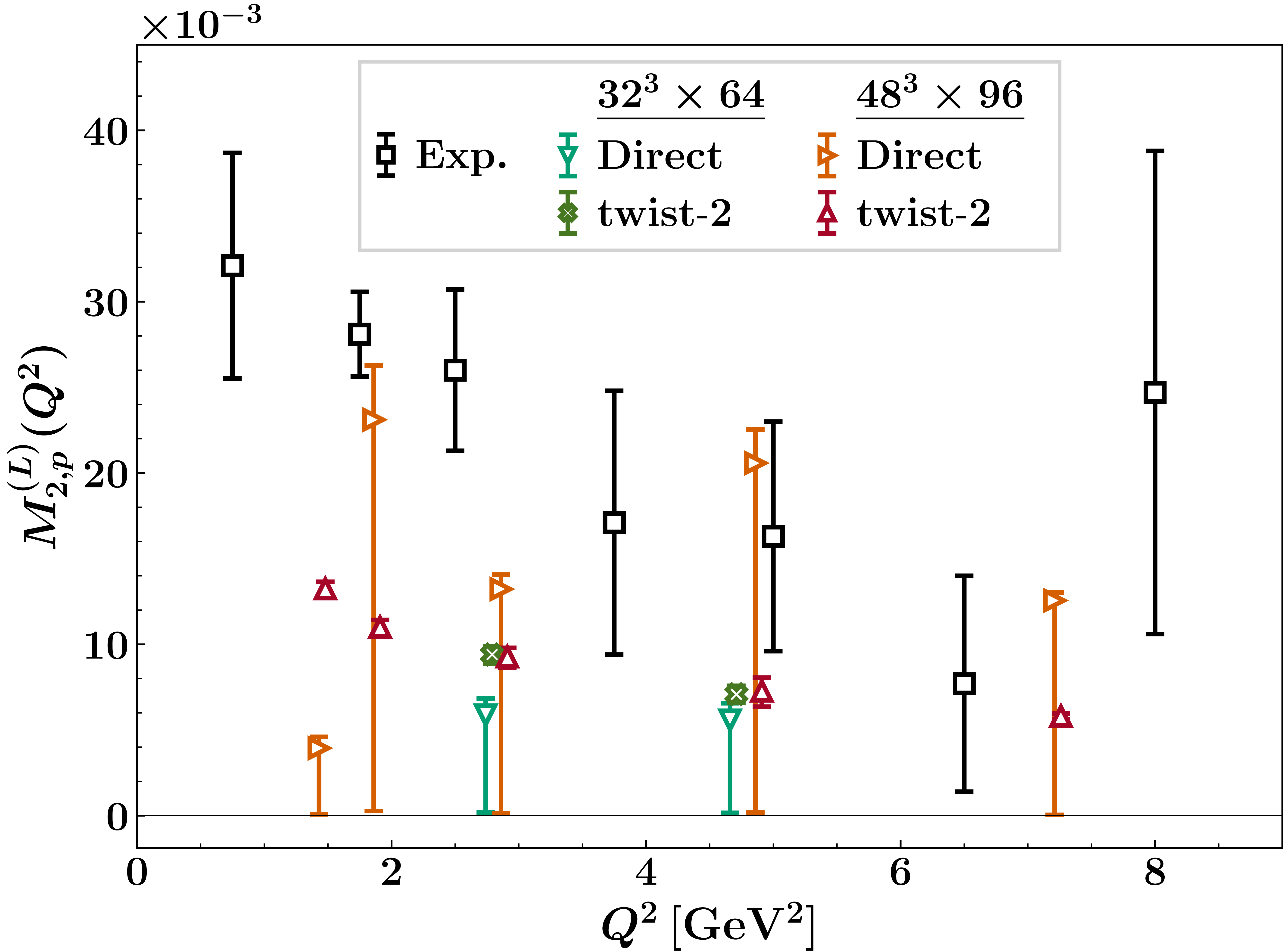
- Global PDF-fit cuts $\sim 1 - 10$ GeV^2
- Need $Q^2 > 10$ GeV^2 data to reliably extract partonic moments

- Power corrections below ~ 3 GeV^2 ?
  - Modelling via
  - $M_2^{(2)}(Q^2) = M_2^{(2)} + C_2^{(2)}/Q^2$

Exp $M_2^{(2)}$: C. S. Armstrong, R. Ent, C. E. Keppel, S. Liuti, G. Niculescu, and I. Niculescu, Phys. Rev. D 63, 094008 (2001), arXiv:hep-ph/0104055.

Moments | proton F_L

- **Unique ability to study the moments of F_L!**

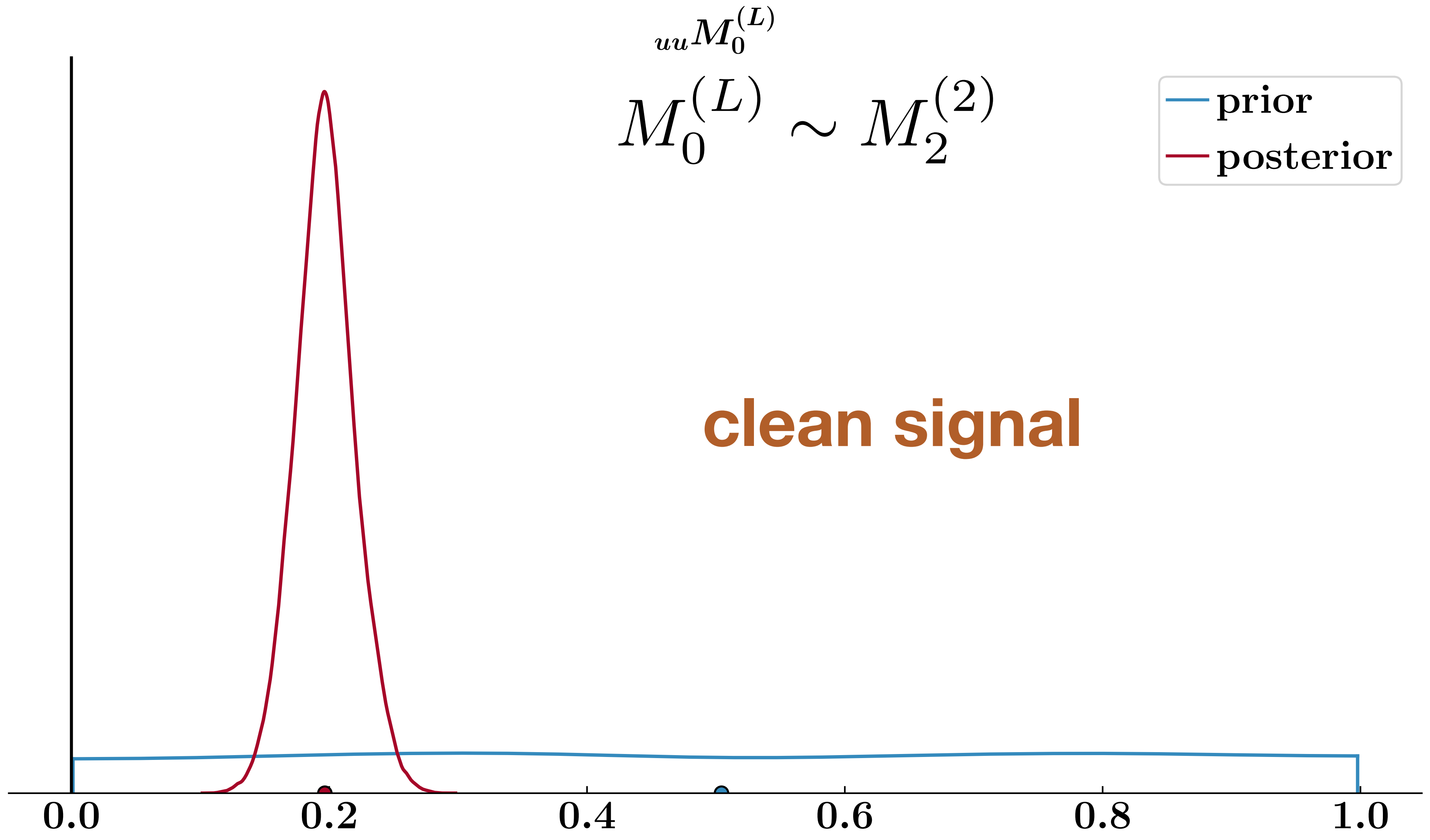


Possible for the first time in a lattice QCD simulation!

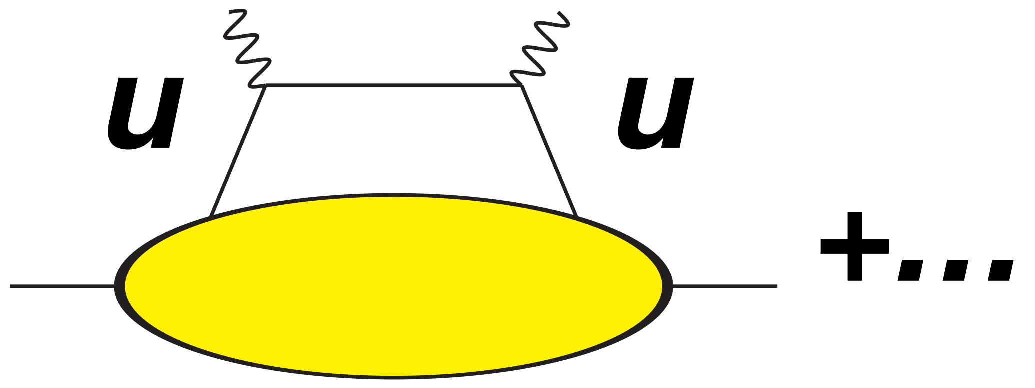
- **Direct:** Fit to data points
  - Determines upper bounds
- **Twist-2:** Use the moments of F_2:
  - $M_2^{(L),QCD}(Q^2) = \dfrac{4}{9\pi}\alpha_s(Q^2)\, M_2^{(2)}(Q^2)$
  - Better precision, good agreement with exp. behaviour

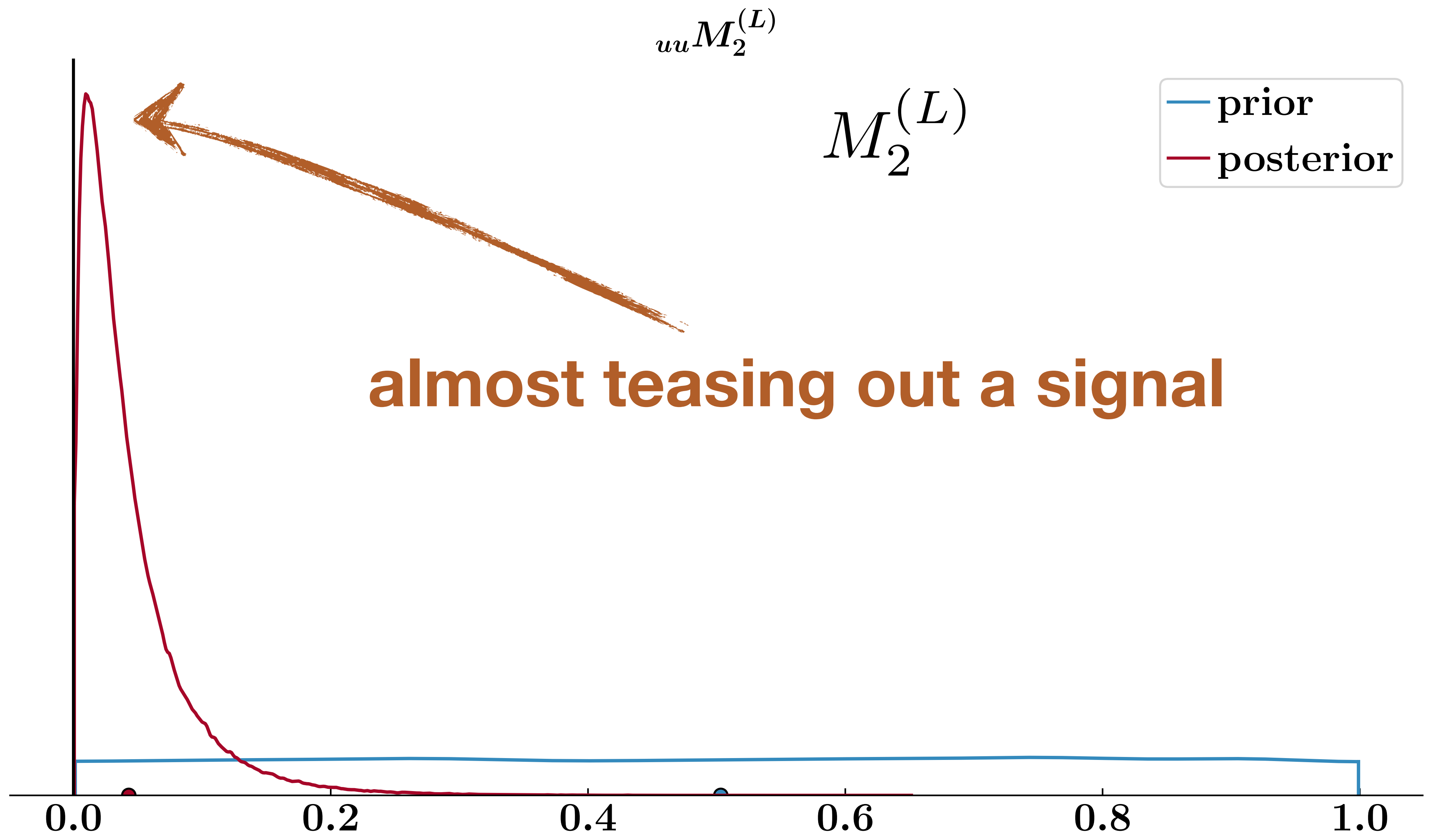
Exp Nachtmann $M_2^{(L)}$: P. Monaghan, A. Accardi, M. E. Christy, C. E. Keppel, W. Melnitchouk, and L. Zhu, Phys. Rev. Lett. 110, 152002 (2013), arXiv:1209.4542 [nucl-ex].

Moments | proton F_L

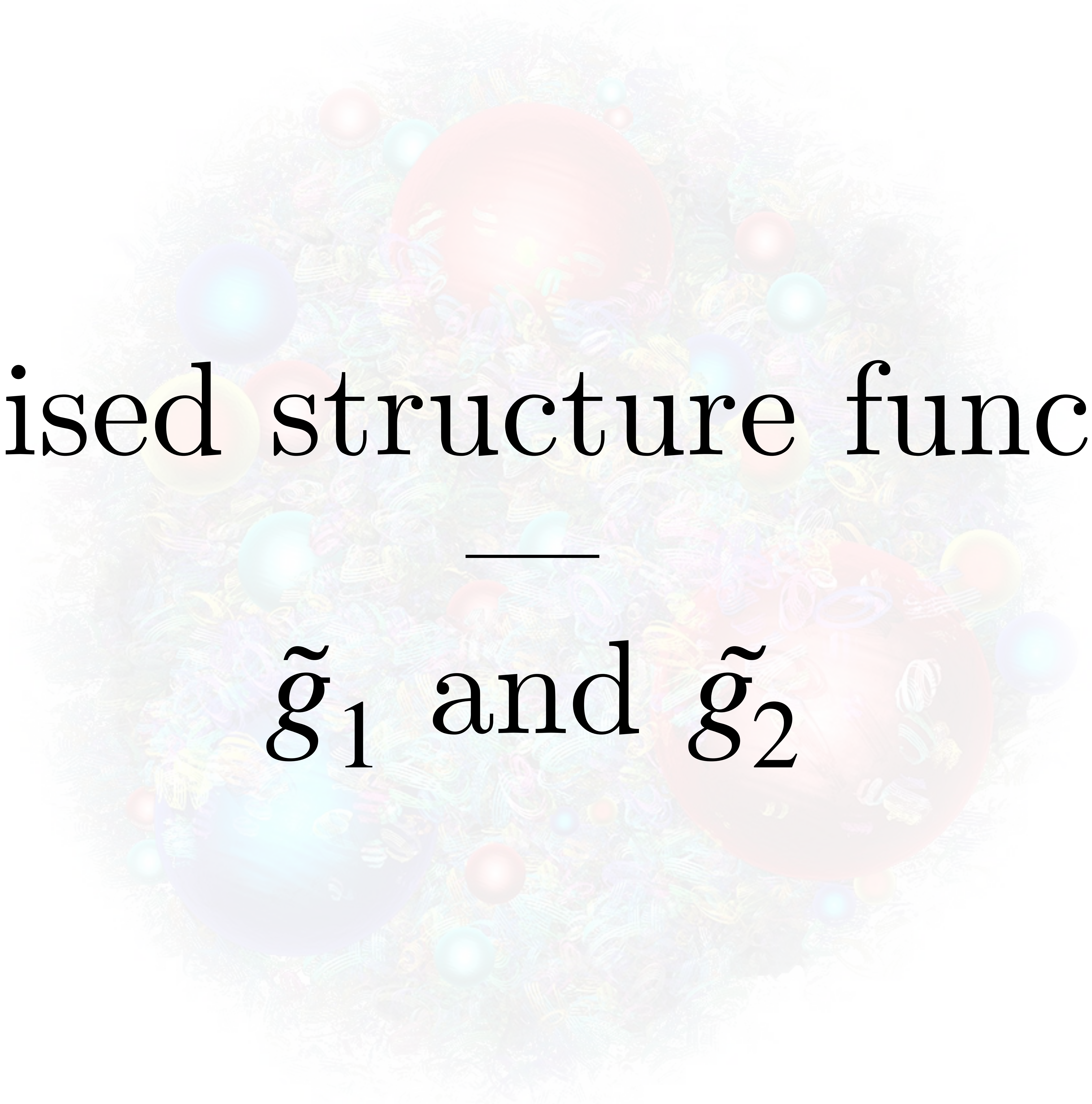


clean signal





almost teasing out a signal



Polarised structure functions

$\tilde{g}_1$ and $\tilde{g}_2$

Polarised Structure Functions

$$T_{[\mu\nu]}(p,q,s) = i\varepsilon^{\mu\nu\alpha\beta}\frac{q_\alpha}{p\cdot q}\left[s_\beta\,\tilde{g}_1(\omega,Q^2) + \left(s_\beta - \frac{s\cdot q}{p\cdot q}p_\beta\right)\tilde{g}_2(\omega,Q^2)\right]$$

- Similar to the unpolarised case, we can extract $\tilde{g}_1$ and $\tilde{g}_2$
- via an OPE analysis: the first moment of $g_1(x)$ is related to axial current matrix elements

$$\Gamma_1(Q^2) = \int_0^1 g_1^{(u-d)}(x,Q^2)\,dx = \underbrace{\left(\Delta u - \Delta d\right)}_{\equiv g_A} C_1(\alpha_s(Q^2))$$

where, $C_1(\alpha_s(Q^2)) = 1 - \dfrac{\alpha_s(Q^2)}{\pi} - \mathcal{O}(\alpha_s^2)$

- $g_2(x)$ is twist-3, holds information on quark-gluon correlations
- Wandzura-Wilczek decomposition

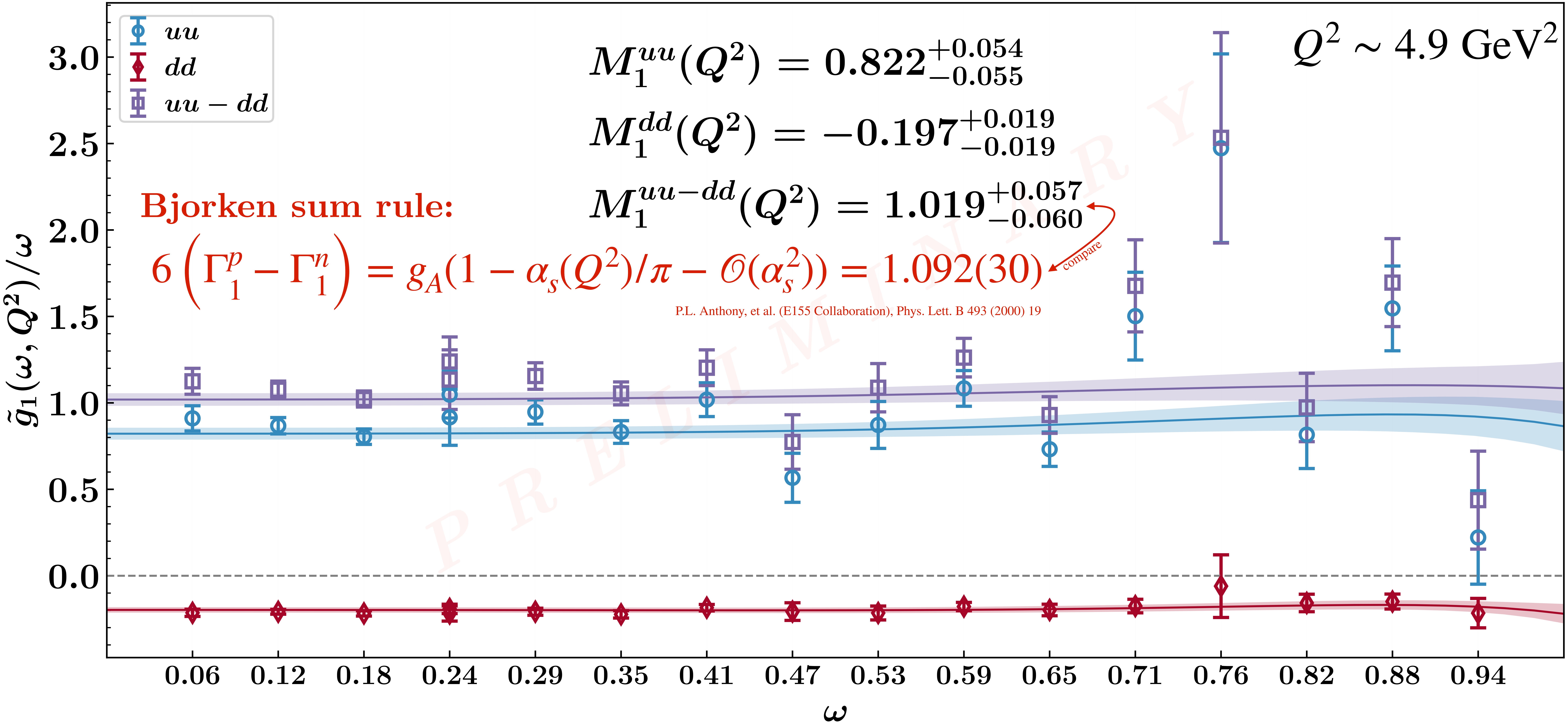
$$g_2(x,Q^2) = -\,g_1(x,Q^2) + \int_x^1 g_1(y,Q^2)\,dy + \bar{g}_2(x,Q^2)$$

- The Buckhardt — Cottingham sum rule

$$\int_0^1 g_2(x,Q^2)\,dx = 0$$

$\tilde{g}_1$ | Work in progress

48^3x96, 2+1 flavour
$a = 0.068$ fm
$m_\pi \sim 420$ MeV



$$M_1^{uu}(Q^2) = 0.822^{+0.054}_{-0.055}$$

$$M_1^{dd}(Q^2) = -0.197^{+0.019}_{-0.019}$$

$$M_1^{uu-dd}(Q^2) = 1.019^{+0.057}_{-0.060}$$

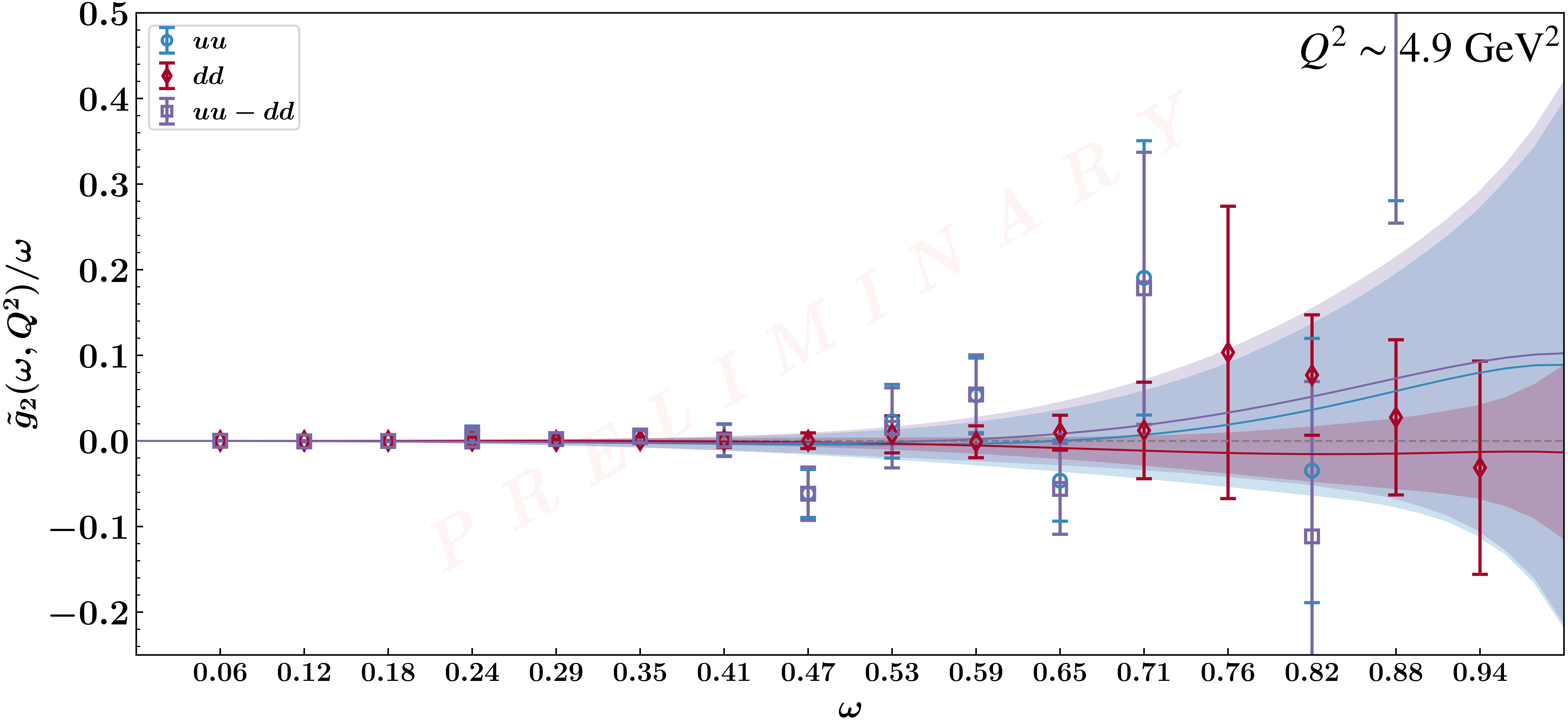
Bjorken sum rule:

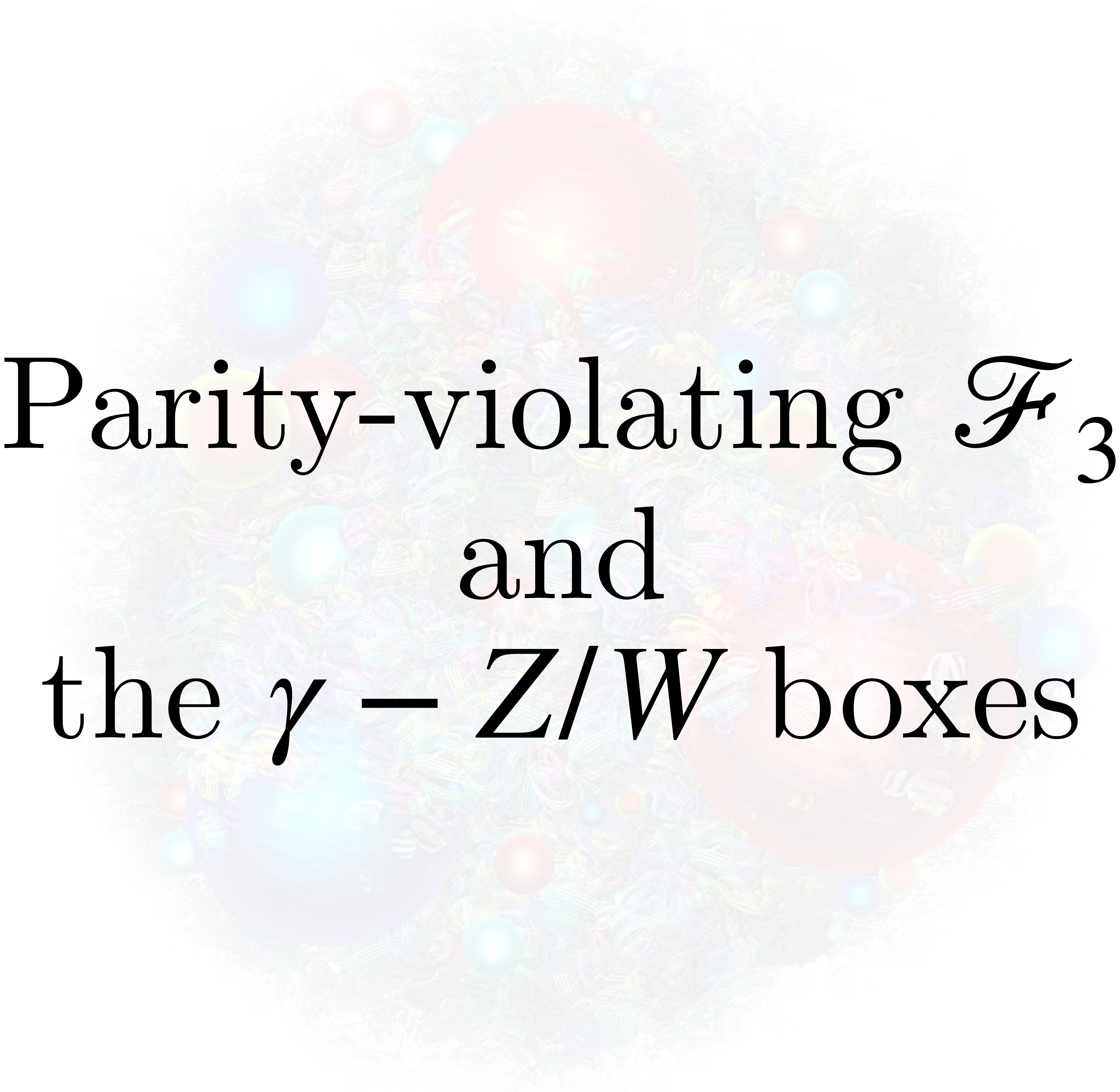
$$6\left(\Gamma_1^p - \Gamma_1^n\right) = g_A(1 - \alpha_s(Q^2)/\pi - \mathcal{O}(\alpha_s^2)) = 1.092(30)$$

P.L. Anthony, et al. (E155 Collaboration), Phys. Lett. B 493 (2000) 19

$\tilde{g}_2$ | Work in progress

48^3x96, 2+1 flavour
$a = 0.068\,\mathrm{fm}$
$m_\pi \sim 420\,\mathrm{MeV}$





Parity-violating $\mathscr{F}_3$ and the $\gamma - Z/W$ boxes

Parity Violating Forward Compton Amplitude

$$T_{\mu\nu}(p,q) = i\int d^4z\, e^{iq\cdot z}\rho_{ss'}\langle p,s'|\mathcal{T}\{J^V_\mu(z)J^A_\nu(0)\}|p,s\rangle \text{ , spin avg. } \rho_{ss'} = \frac{1}{2}\delta_{ss'}$$

$$= -g_{\mu\nu}\mathcal{F}_1(\omega,Q^2) + \frac{p_\mu p_\nu}{p\cdot q}\mathcal{F}_2(\omega,Q^2) + i\,\varepsilon^{\mu\nu\alpha\beta}\frac{p_\alpha q_\beta}{2p\cdot q}\mathcal{F}_3(\omega,Q^2)$$

$$+\frac{q_\mu q_\nu}{p\cdot q}\mathcal{F}_4(\omega,Q^2) + \frac{p_{\{\mu}q_{\nu\}}}{p\cdot q}\mathcal{F}_5(\omega,Q^2) + \frac{p_{[\mu}q_{\nu]}}{p\cdot q}\mathcal{F}_6(\omega,Q^2)$$

$\varepsilon^{0123} = 1$

allowed terms
because parity
is violated

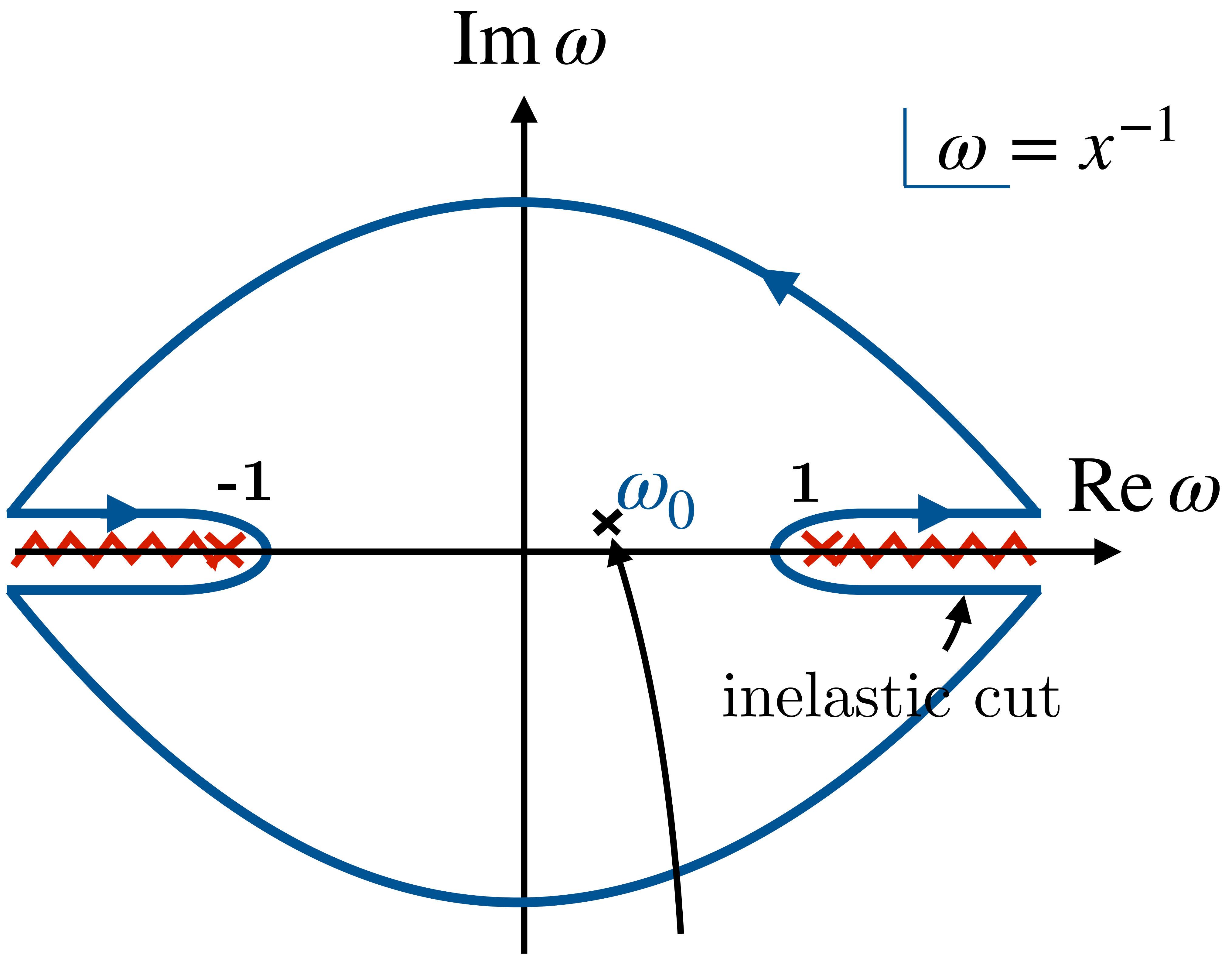
Nucleon Structure Functions | F_3

- for $\mu \neq \nu$ and $p_\mu = q_\mu = 0$, and $\beta \neq 0$, we isolate,

$$T_{\mu\nu}(p,q) = i\,\varepsilon^{\mu\nu\alpha\beta}\frac{p_\alpha q_\beta}{2p\cdot q}\mathscr{F}_3(\omega, Q^2)$$

- we can write down dispersion relations and connect Compton SFs to DIS SFs:

$$\mathscr{F}_3(\omega, Q^2) = 4\omega\int dx\frac{F_3(x,Q^2)}{1-x^2\omega^2}$$



Compton Amplitude is an analytic function in the unphysical region $|\omega_0| < 1$

Parity Violating Forward Compton Amplitude

- **The 1st moment**

$$M_1^{(3)}(Q^2) = \int_0^1 dx F_3(x, Q^2) = \left. \frac{\mathscr{F}_3(\omega, Q^2)}{4\omega} \right|_{\omega=0}$$

allows for a test of the Gross-Llewellyn-Smith sum rule $(a_s = \alpha_s(Q^2)/\pi)$

$$M_{1,uu}^{(3)}(Q^2) = \int_0^{1^-} dx F_3(x, Q^2) = 2\left(1 + \sum_{i=1}^{3} a_s^i c_i(n_f)\right) + \frac{\Delta_{HT}}{Q^2} + \mathcal{O}\left(\frac{1}{Q^4}\right)$$

known coeffs.

Higher-twist

- **Also allows for a determination of the EW box diagram**

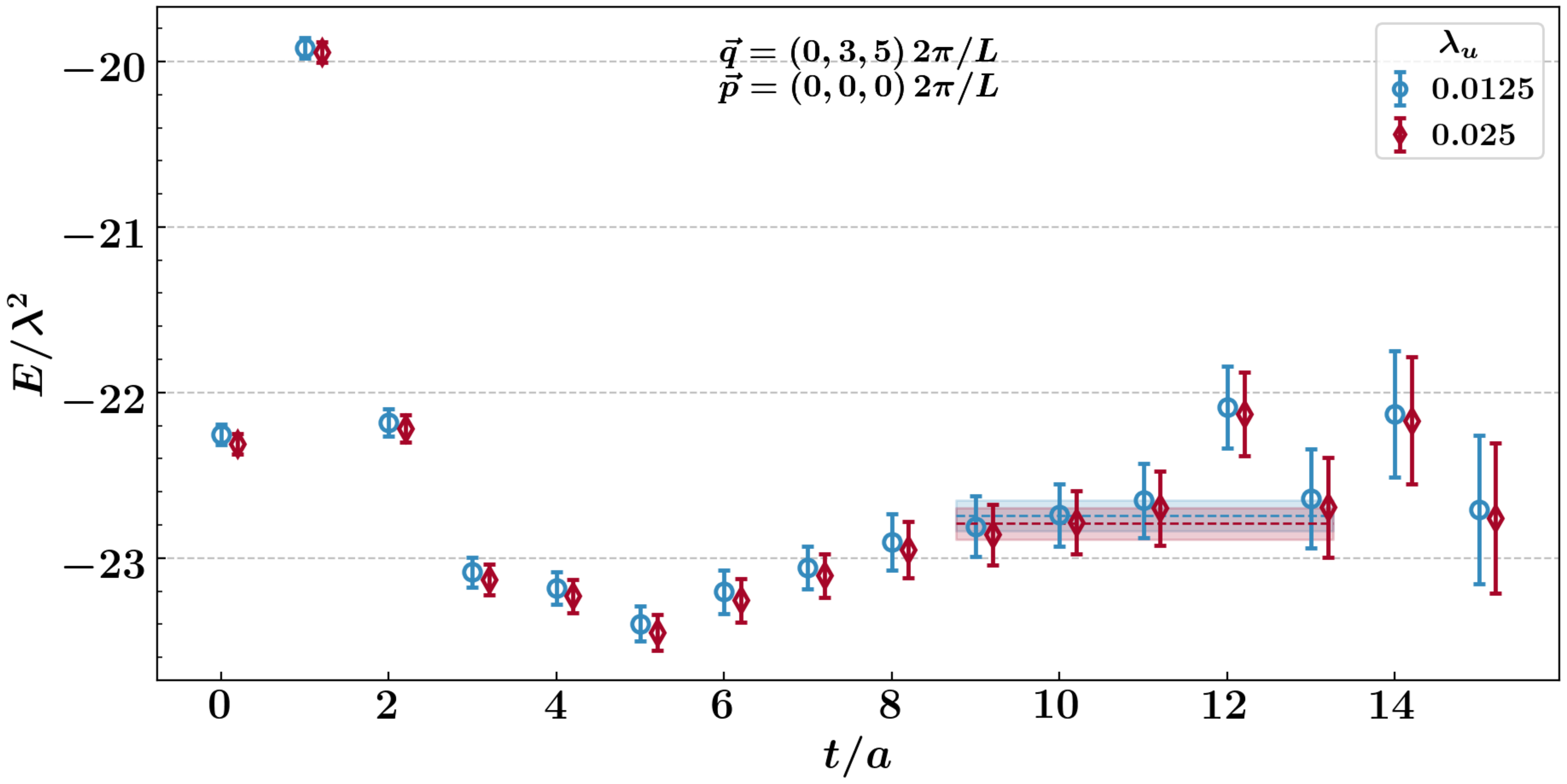
$$\Box_{VA}^{\gamma W} = \frac{3\alpha_{EM}}{2\pi} \int_0^\infty \frac{dQ^2}{Q^2} \frac{M_W^2}{M_W^2 + Q^2} M_1^{(3)}(Q^2)$$

Strategy | Energy shifts

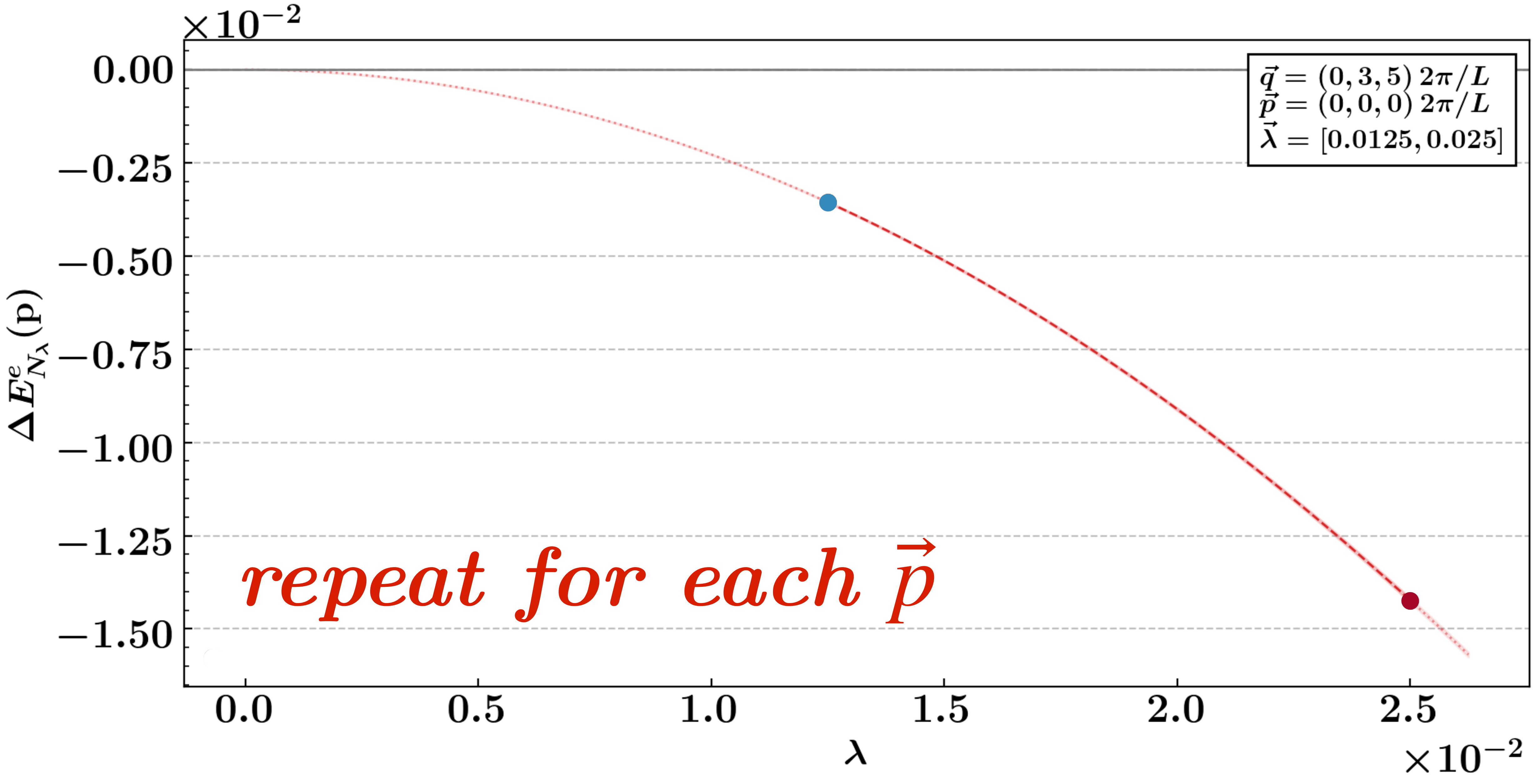
- **Ratio of perturbed to unperturbed 2-pt functions**

$$\mathcal{R}^{qq}_{\lambda}(p,t) \equiv \frac{G^{(2)}_{+\lambda_1^q,+\lambda_2^q}(p,t)\,G^{(2)}_{-\lambda_1^q,-\lambda_2^q}(p,t)}{G^{(2)}_{+\lambda_1^q,-\lambda_2^q}(p,t)\,G^{(2)}_{-\lambda_1^q,+\lambda_2^q}(p,t)} \to A_\lambda e^{-4\Delta E_{N_\lambda}(p)\,t}$$

- **Extract energy shifts for each $|\lambda|$**

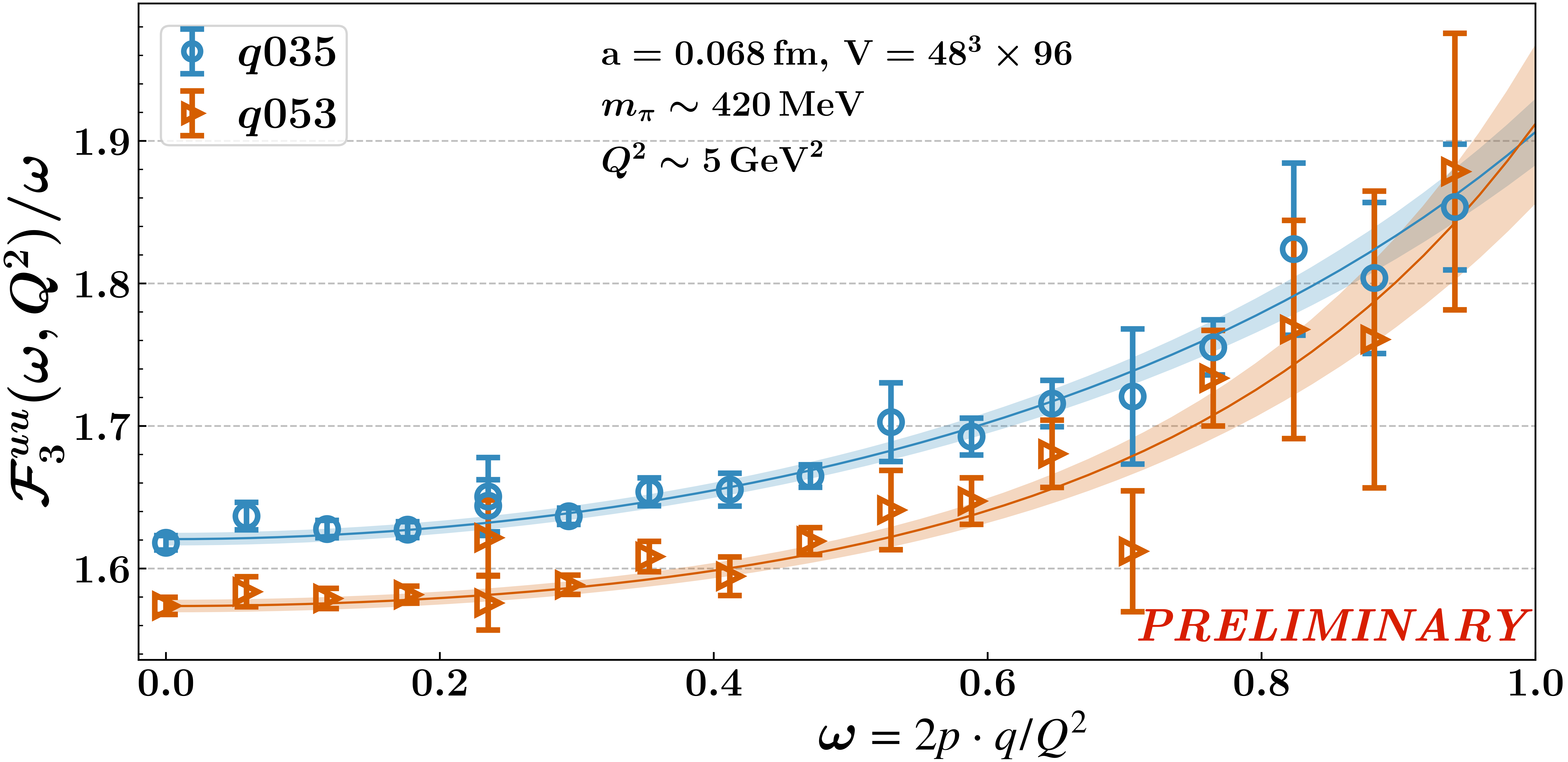


- **Get the 2nd order derivative**

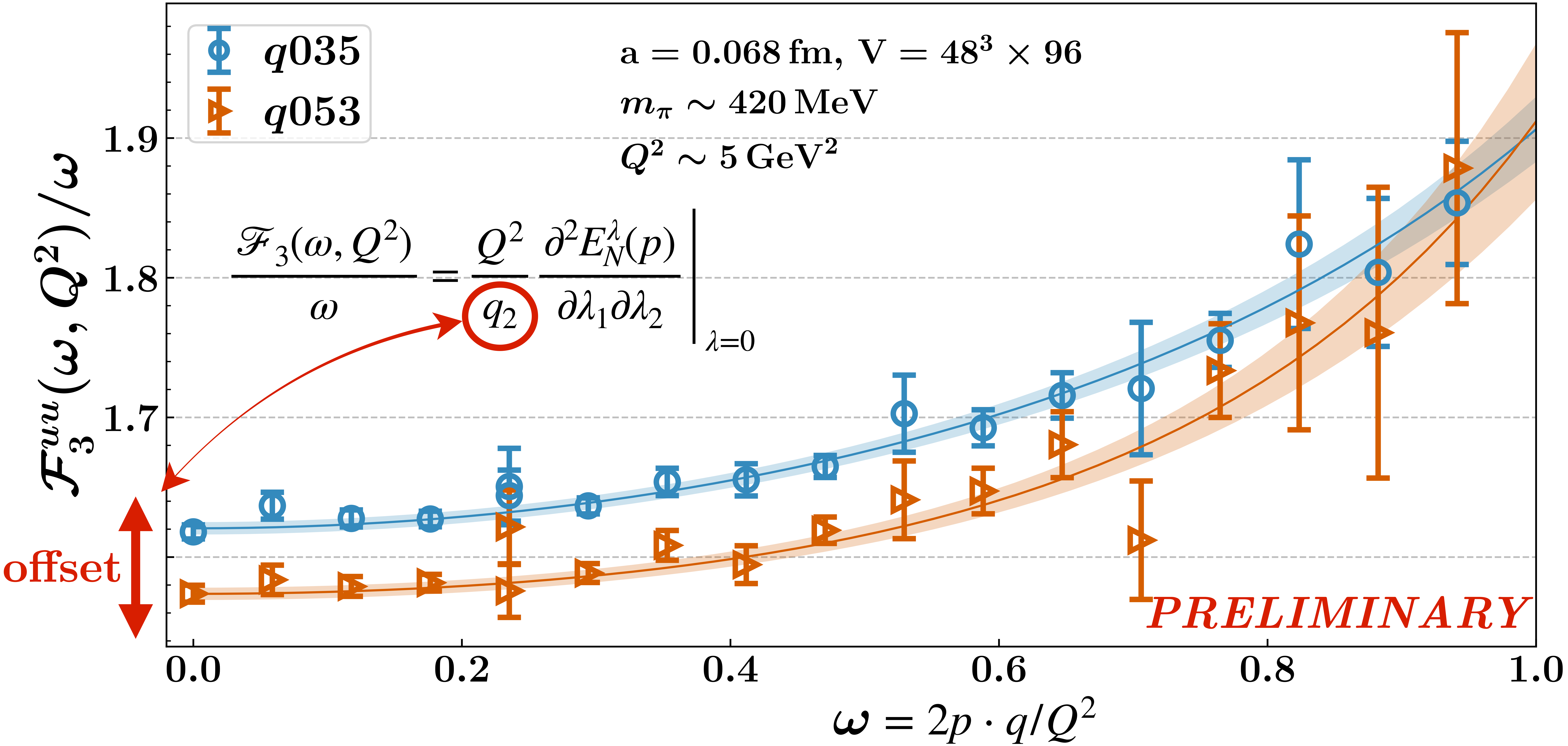


repeat for each $\vec{p}$

$\mathscr{F}_3$


q035
q053
a = 0.068 fm, V = $48^3 \times 96$
$m_\pi \sim 420$ MeV
$Q^2 \sim 5\,\mathrm{GeV}^2$
$\mathcal{F}_3^{uu}(\omega, Q^2)/\omega$
1.6
1.7
1.8
1.9
0.0
0.2
0.4
0.6
0.8
1.0
$\omega = 2p \cdot q/Q^2$
PRELIMINARY

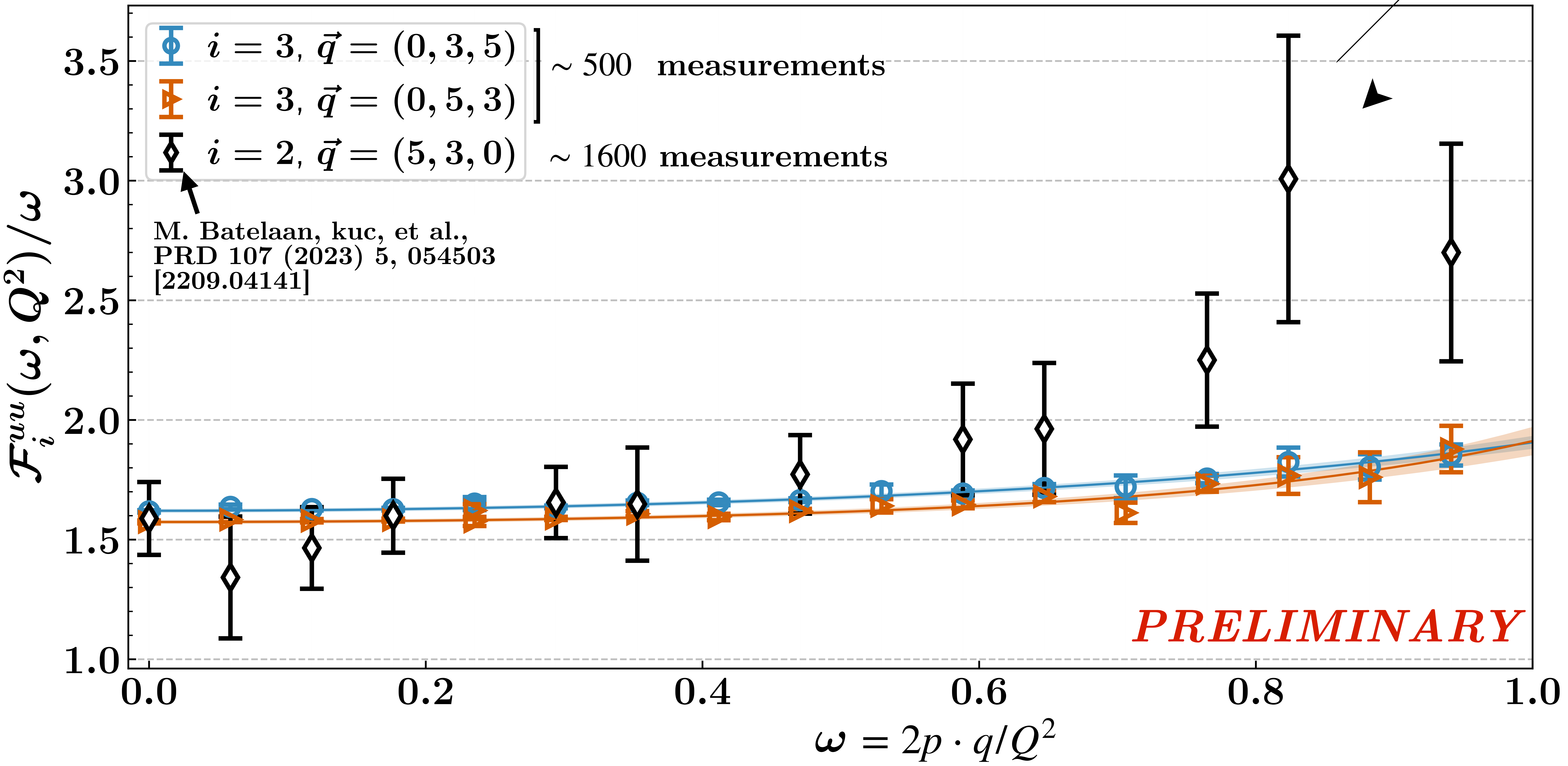
$\mathscr{F}_3$



$$\frac{\mathscr{F}_3(\omega, Q^2)}{\omega} = \frac{Q^2}{\boxed{q_2}} \frac{\partial^2 E_N^\lambda(p)}{\partial\lambda_1 \partial\lambda_2}\bigg|_{\lambda=0}$$

$\mathscr{F}_3$ vs. $\mathscr{F}_2$

Black points: $4\,\mathscr{F}_2$



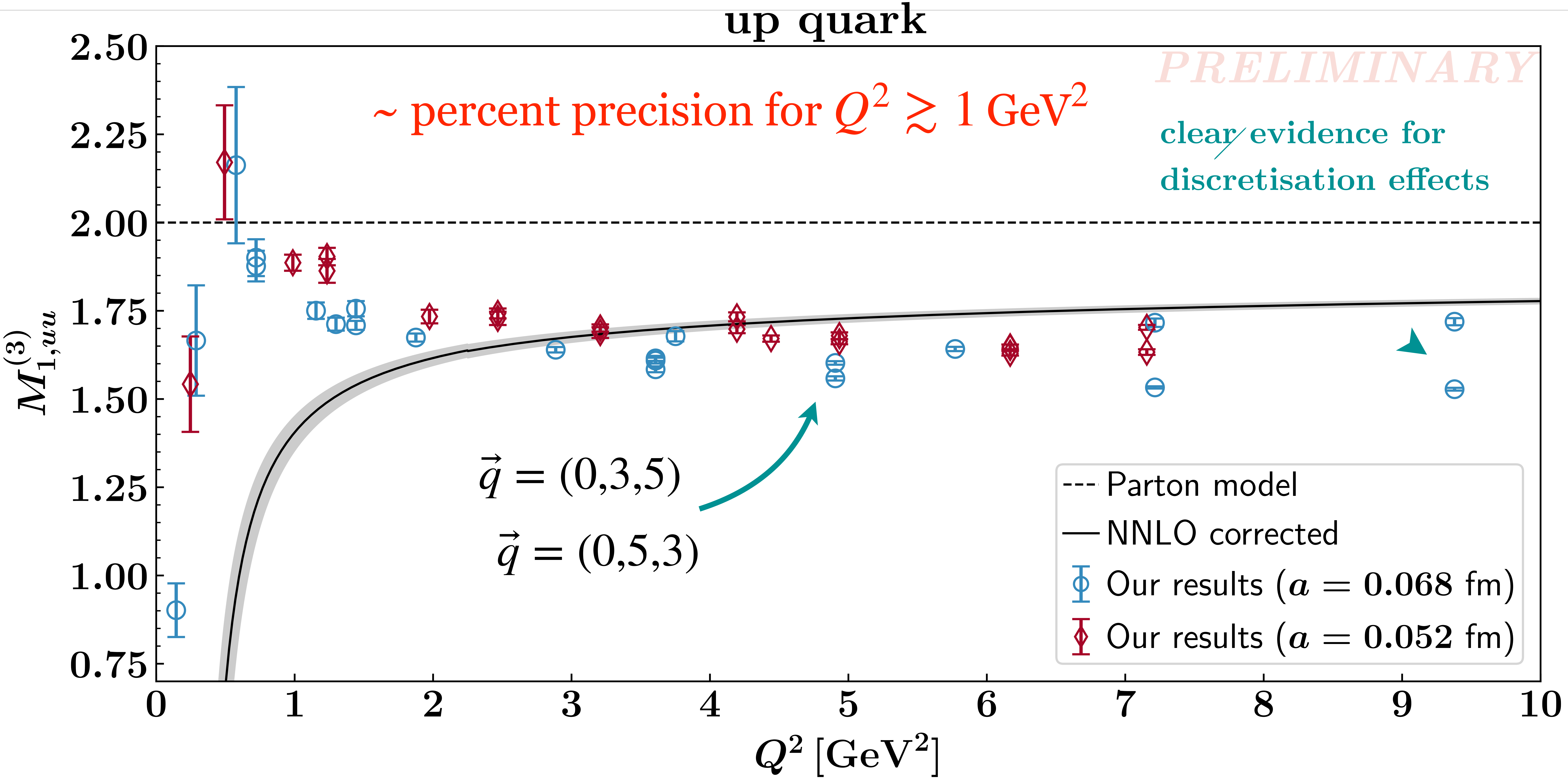
~ 500 measurements

~ 1600 measurements

M. Batelaan, kuc, et al.,
PRD 107 (2023) 5, 054503
[2209.04141]

$\mathscr{F}_3$ | First moment

$a = 0.068, 0.052\,\text{fm}$
$m_\pi \sim 410\,\text{MeV}$
48^3x96, 2+1 flavour



~ percent precision for $Q^2 \gtrsim 1\,\text{GeV}^2$

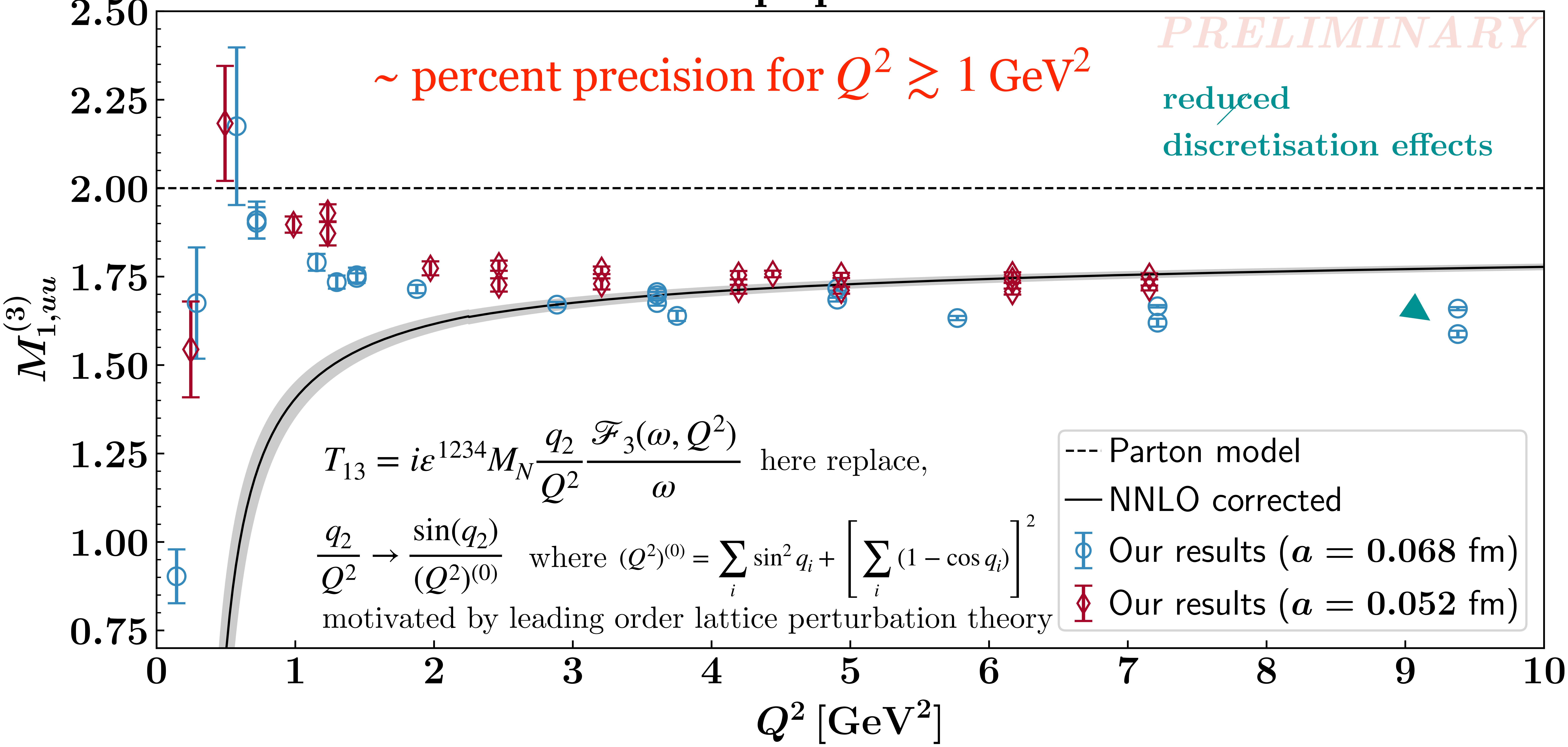
clear evidence for discretisation effects

$\vec{q} = (0,3,5)$

$\vec{q} = (0,5,3)$

$\mathscr{F}_3$ | First moment

$a = 0.068, 0.052\,\mathrm{fm}$
$m_\pi \sim 410\,\mathrm{MeV}$
48^3x96, 2+1 flavour



up quark

~ percent precision for $Q^2 \gtrsim 1\,\mathrm{GeV}^2$

~~reduced~~
discretisation effects

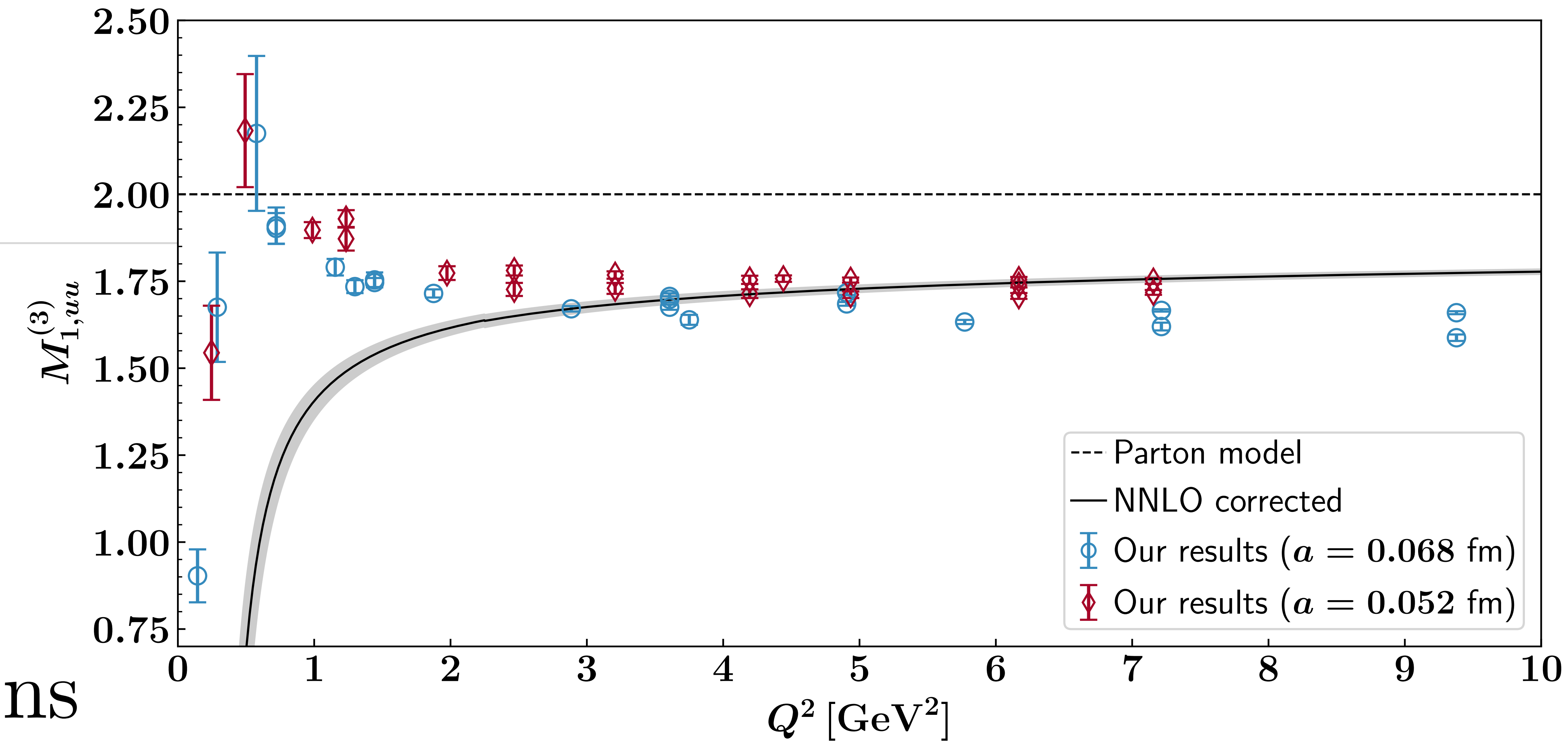
$$T_{13} = i\varepsilon^{1234} M_N \frac{q_2}{Q^2} \frac{\mathscr{F}_3(\omega, Q^2)}{\omega}$$ here replace,

$$\frac{q_2}{Q^2} \to \frac{\sin(q_2)}{(Q^2)^{(0)}}$$ where $(Q^2)^{(0)} = \sum_i \sin^2 q_i + \left[\sum_i (1 - \cos q_i)\right]^2$

motivated by leading order lattice perturbation theory

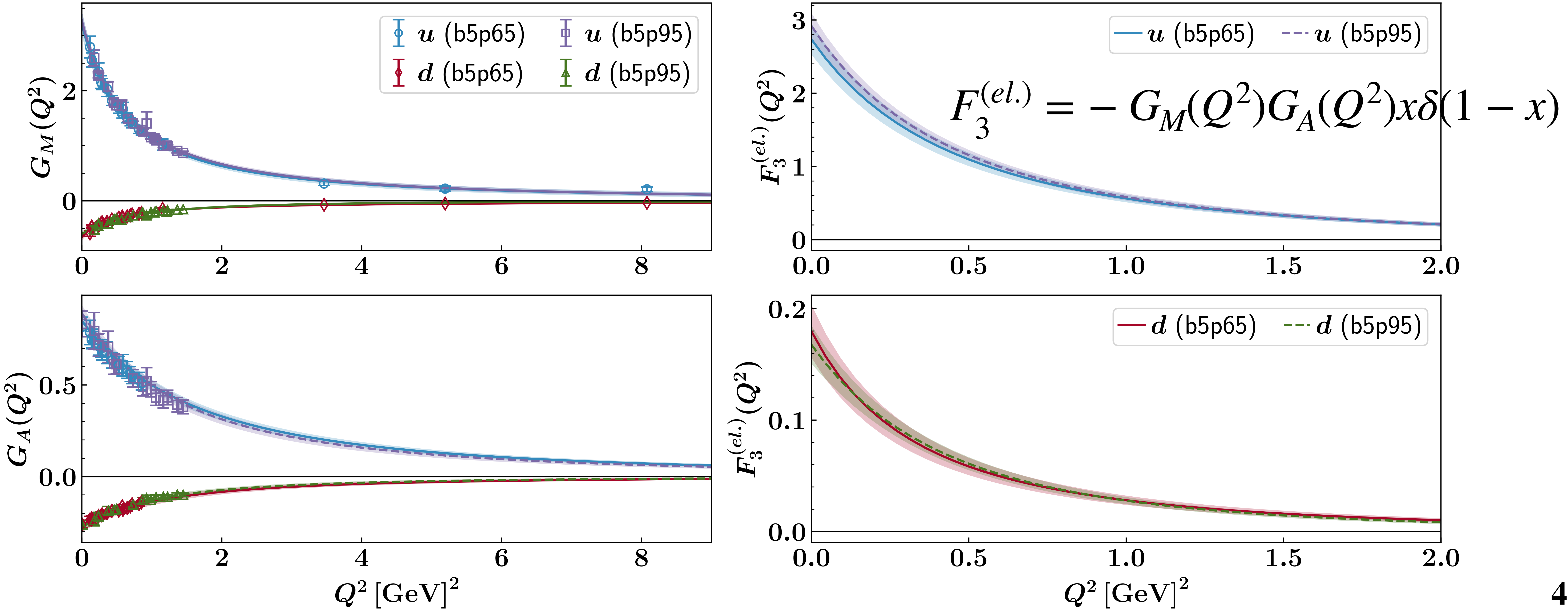
GLS sum rule

- **GLS sum rule is the inelastic part only**
  - must subtract elastic contribution
  - provides insights into higher twist contributions



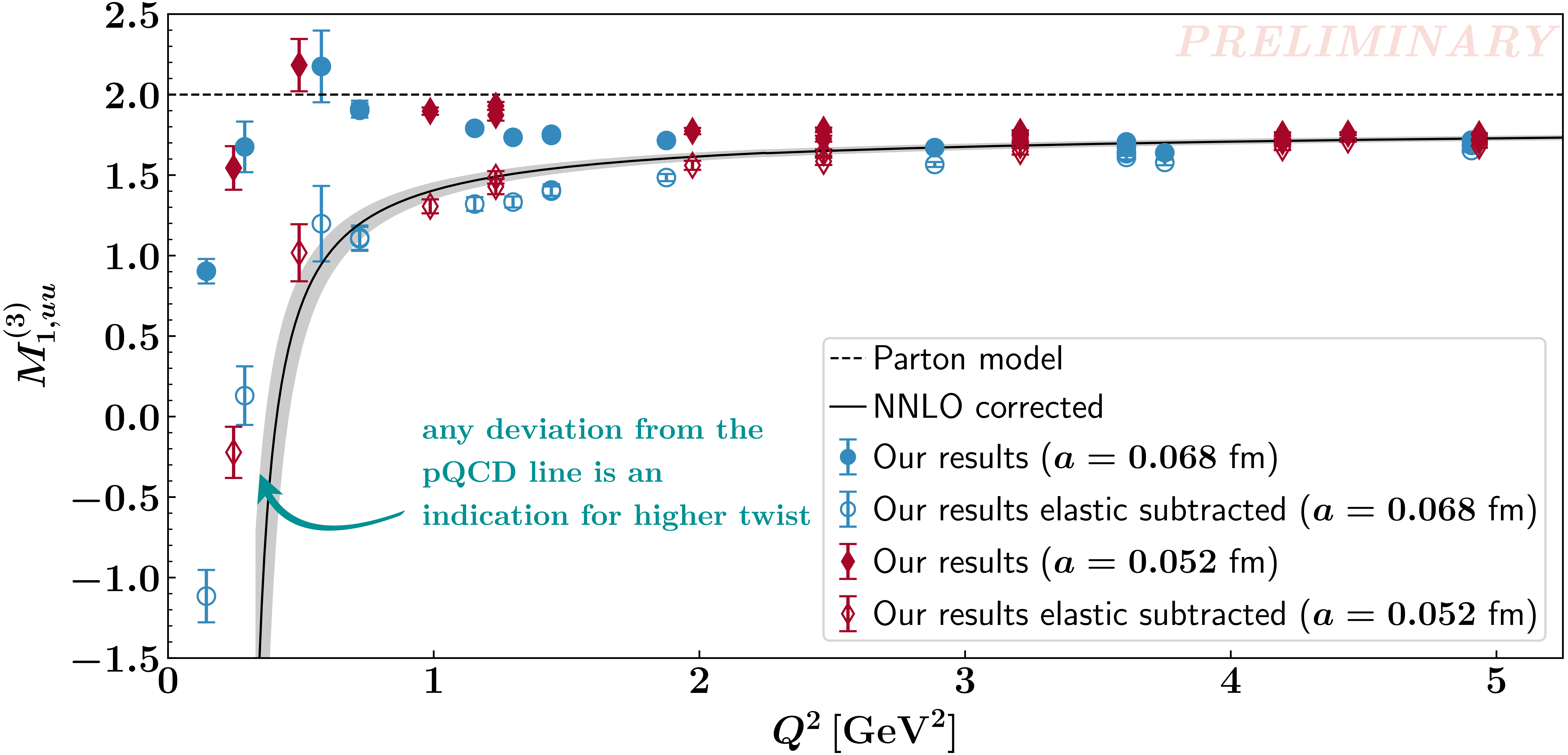
low-Q^2: 3-pt functions
high-Q^2: Feynman-Hellmann

low-Q^2: 3-pt functions
dipole parametrisation



$$F_3^{(el.)} = -G_M(Q^2)G_A(Q^2)x\delta(1-x)$$

$\mathscr{F}_3$ | First moment elastic subtracted

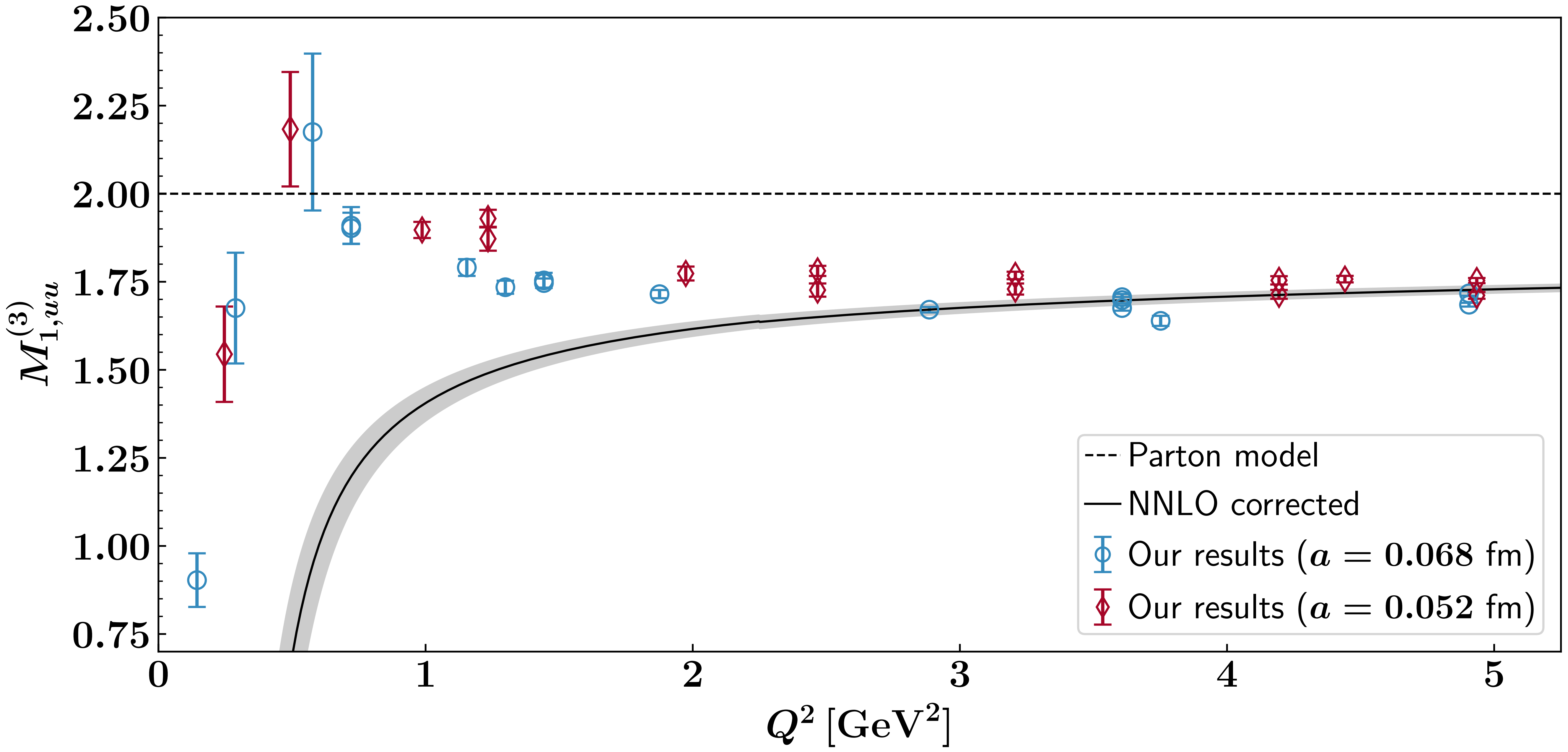


any deviation from the pQCD line is an indication for higher twist

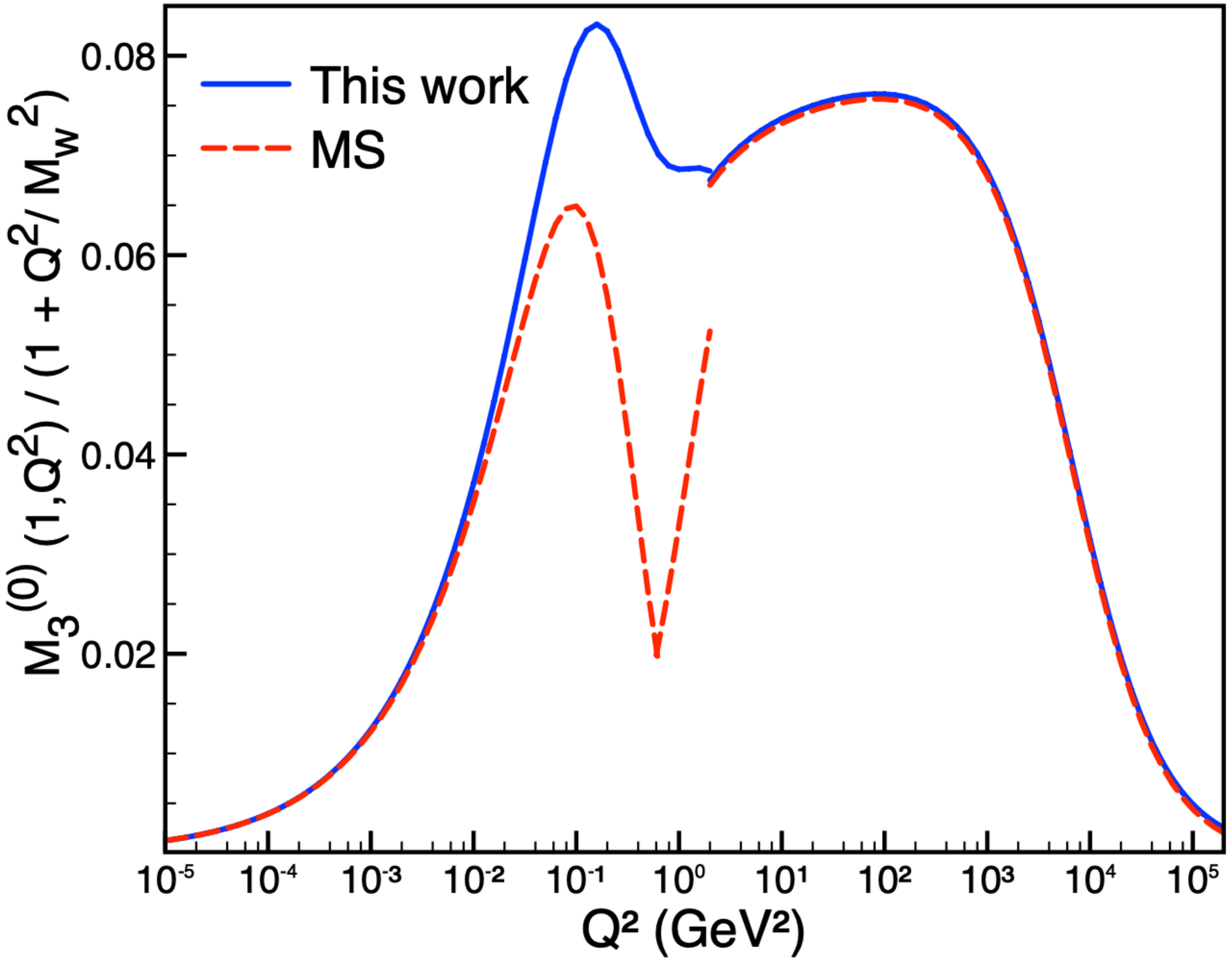
$\mathscr{F}_3$ | EW box

- **Electroweak box diagram contribution**

$$\Box_{VA}^{\gamma W} = \frac{3\alpha_{EM}}{2\pi}\int_0^\infty \frac{dQ^2}{Q^2}\frac{M_W^2}{M_W^2+Q^2}M_1^{(3)}(Q^2)$$



C.-Y. Seng, M. Gorchtein, H. H. Patel, and M. J. Ramsey-Musolf, PRL 121, 241804 (2018)



$\mathscr{F}_3$ | EW box

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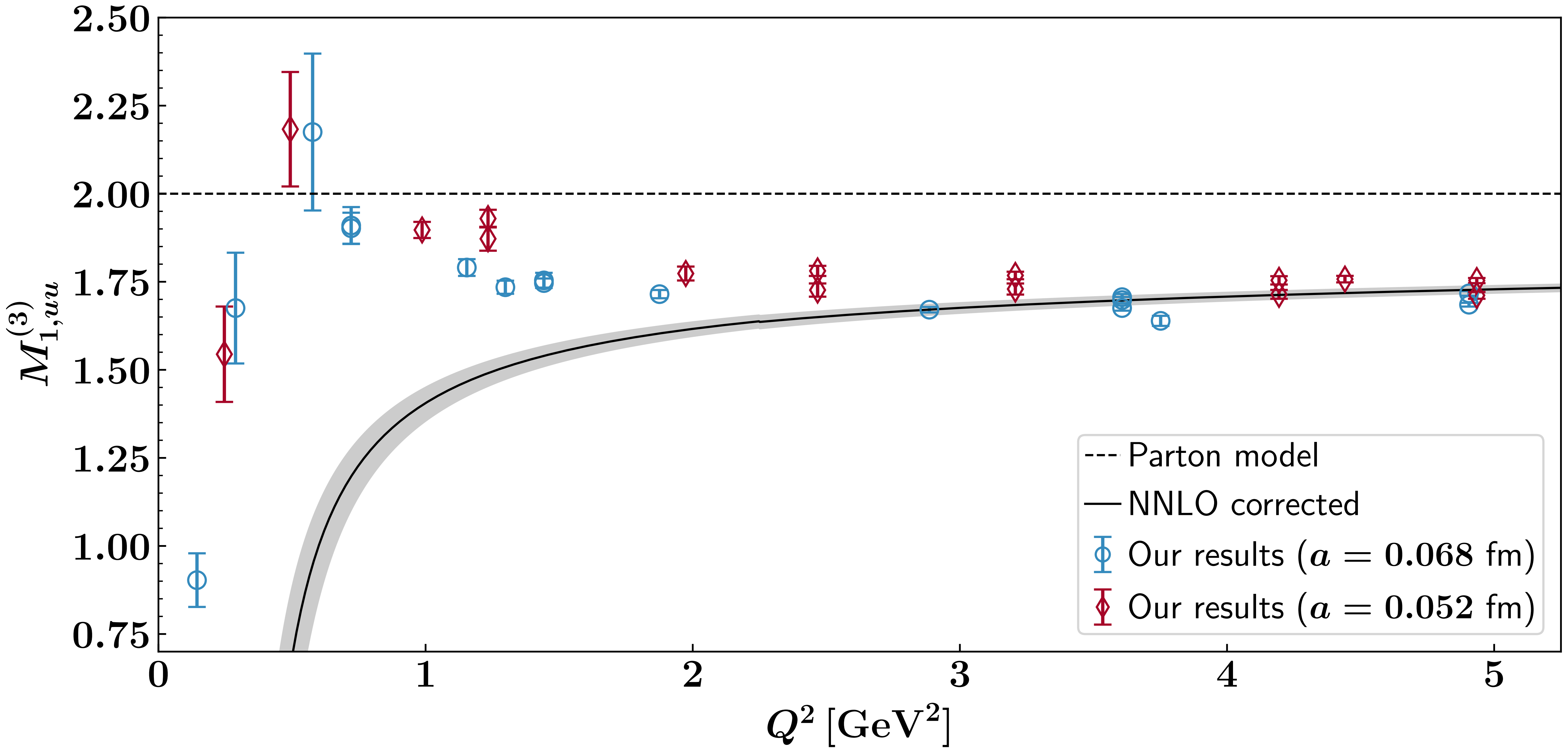

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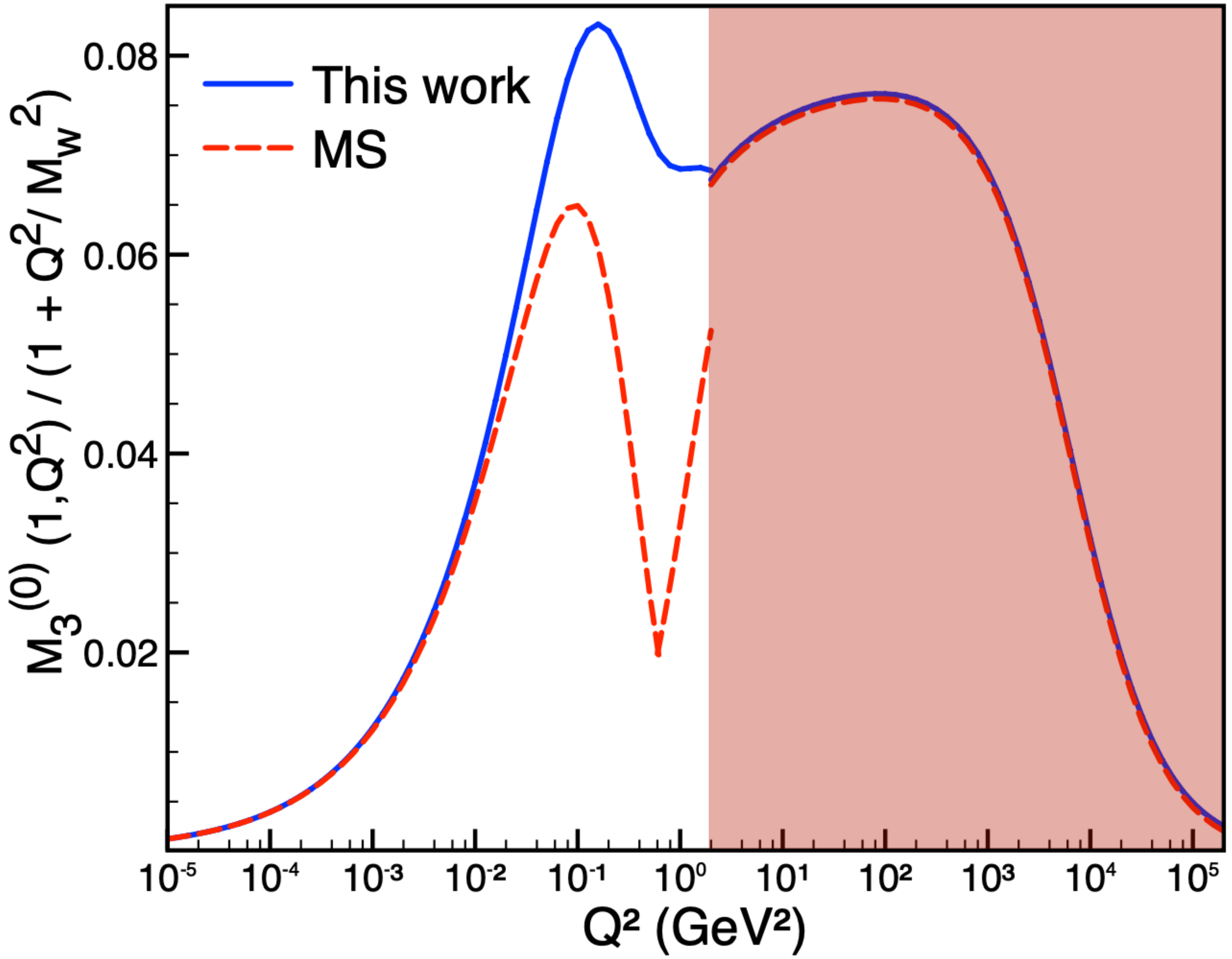
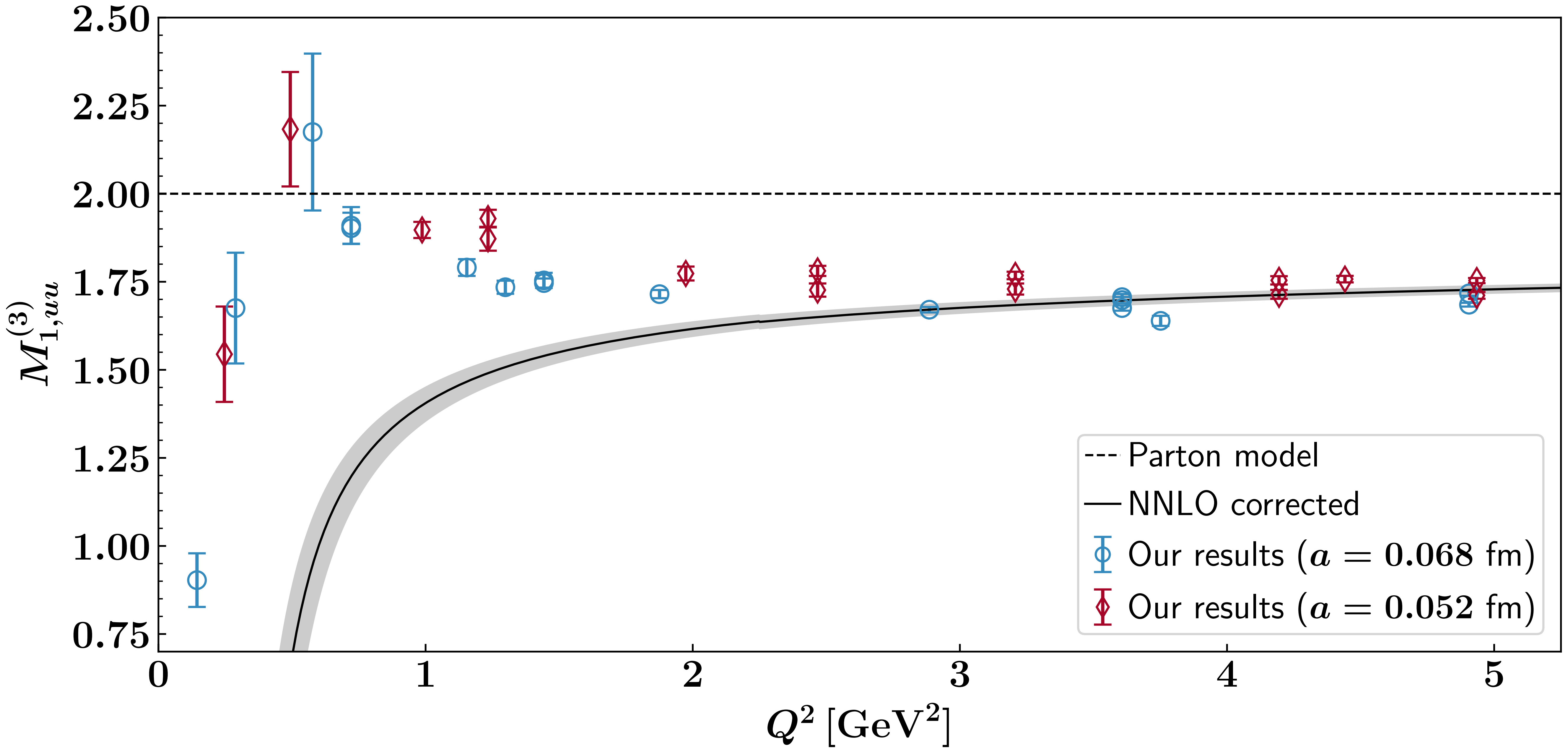


perturbation theory

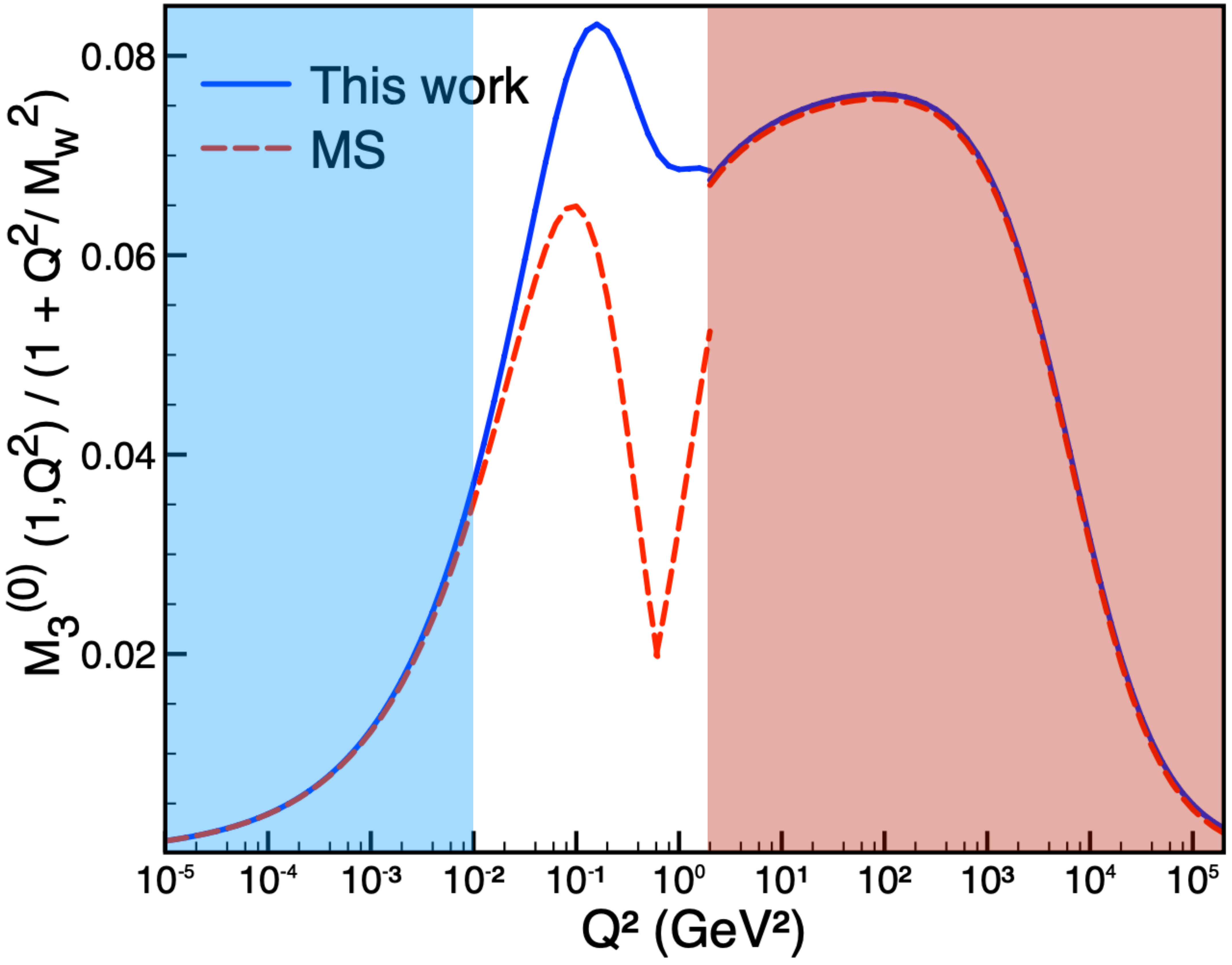
$\mathscr{F}_3$ | EW box

- **Electroweak box diagram contribution**

$$\Box^{\gamma W}_{VA} = \frac{3\alpha_{EM}}{2\pi}\int_0^\infty \frac{dQ^2}{Q^2}\frac{M_W^2}{M_W^2+Q^2}M_1^{(3)}(Q^2)$$



C.-Y. Seng, M. Gorchtein, H. H. Patel, and M. J. Ramsey-Musolf, PRL 121, 241804 (2018)



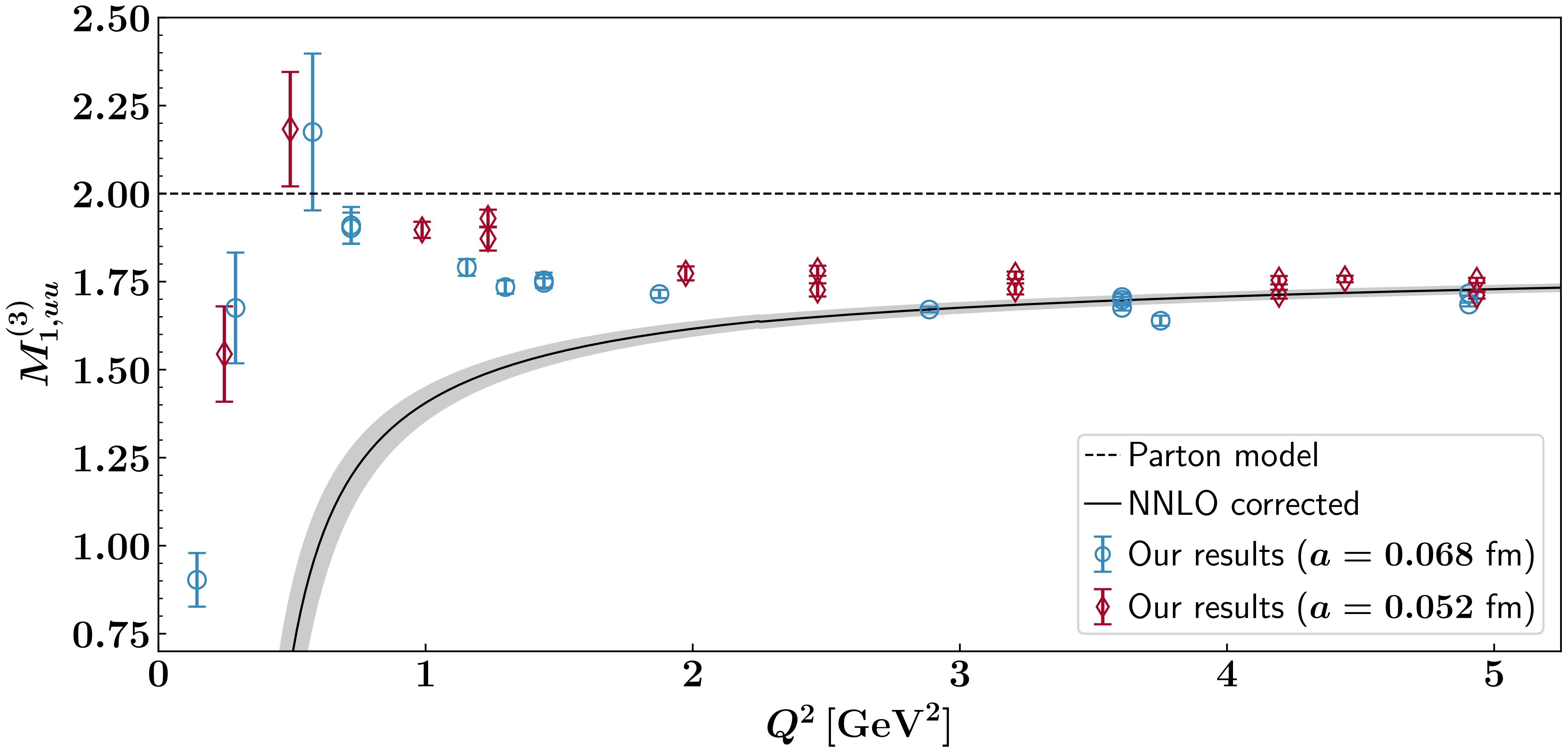
weak form factors

perturbation theory

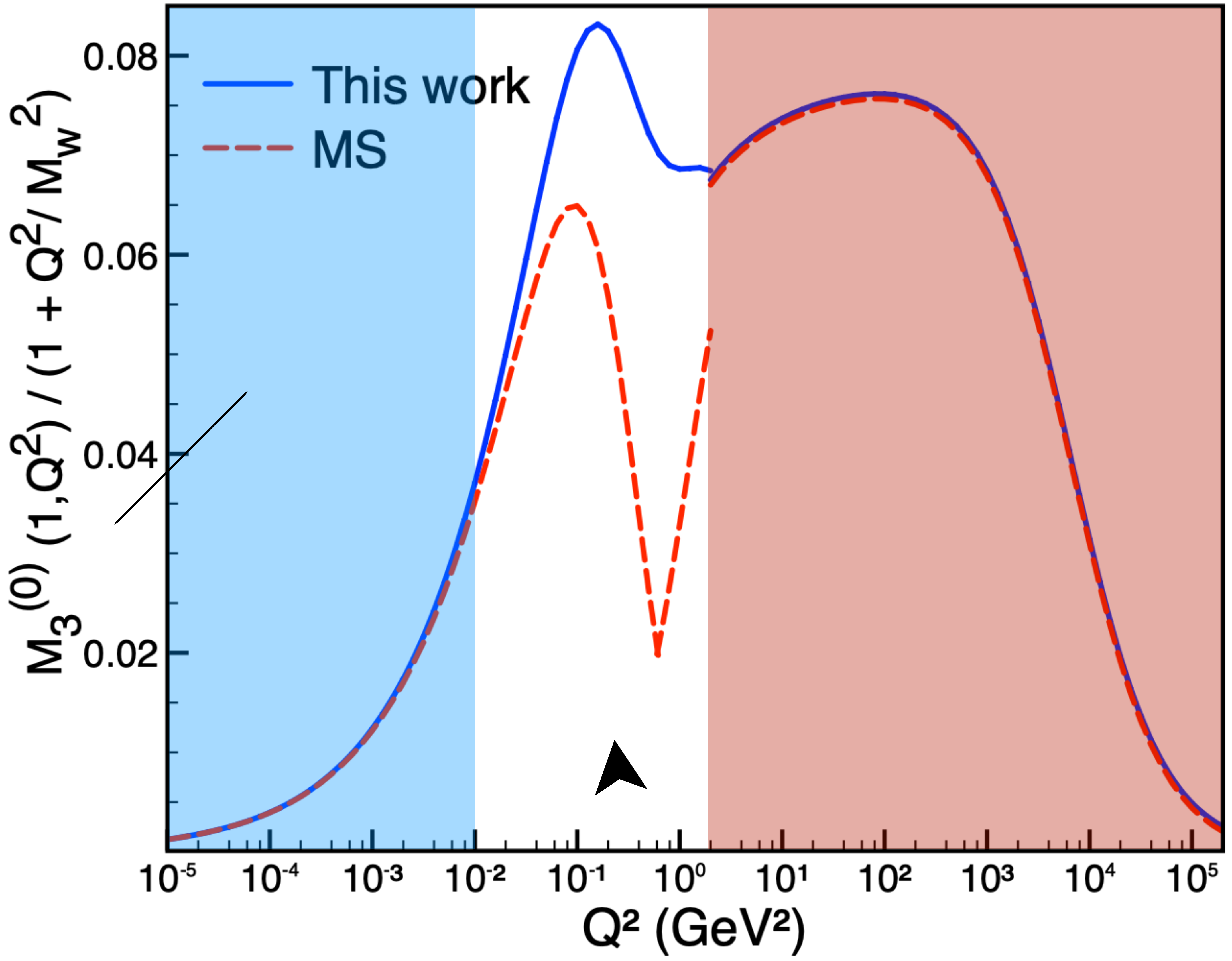
$\mathcal{F}_3$ | EW box

- **Electroweak box diagram contribution**

$$\Box^{\gamma W}_{VA} = \frac{3\alpha_{EM}}{2\pi}\int_0^\infty \frac{dQ^2}{Q^2}\frac{M_W^2}{M_W^2+Q^2}M_1^{(3)}(Q^2)$$



C.-Y. Seng, M. Gorchtein, H. H. Patel, and M. J. Ramsey-Musolf, PRL 121, 241804 (2018)



weak form factors

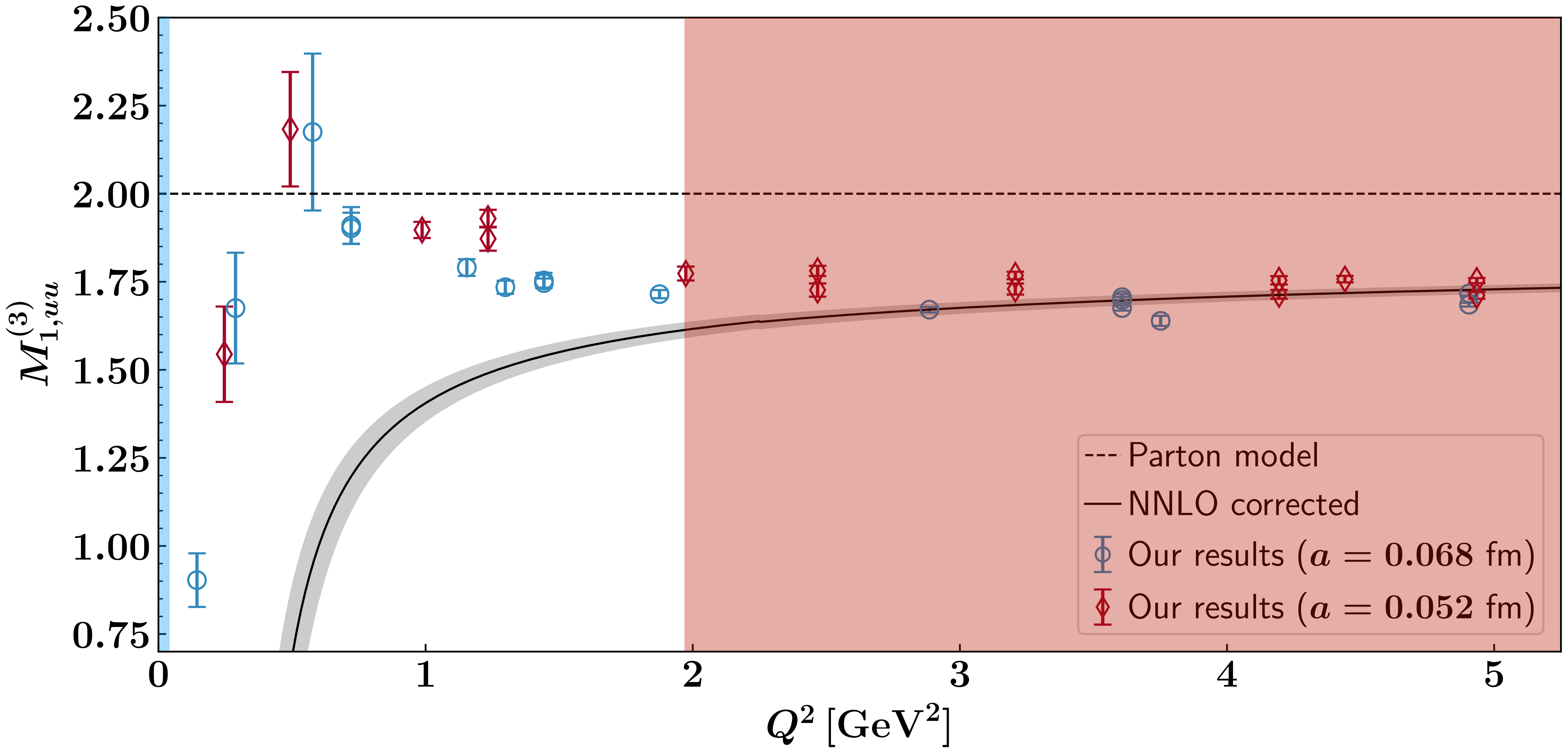
perturbation theory

input from LQCD required

$\mathscr{F}_3$ | EW box

- **Electroweak box diagram contribution**

$$\Box^{\gamma W}_{VA} = \frac{3\alpha_{EM}}{2\pi}\int_0^\infty \frac{dQ^2}{Q^2}\frac{M_W^2}{M_W^2+Q^2}M_1^{(3)}(Q^2)$$



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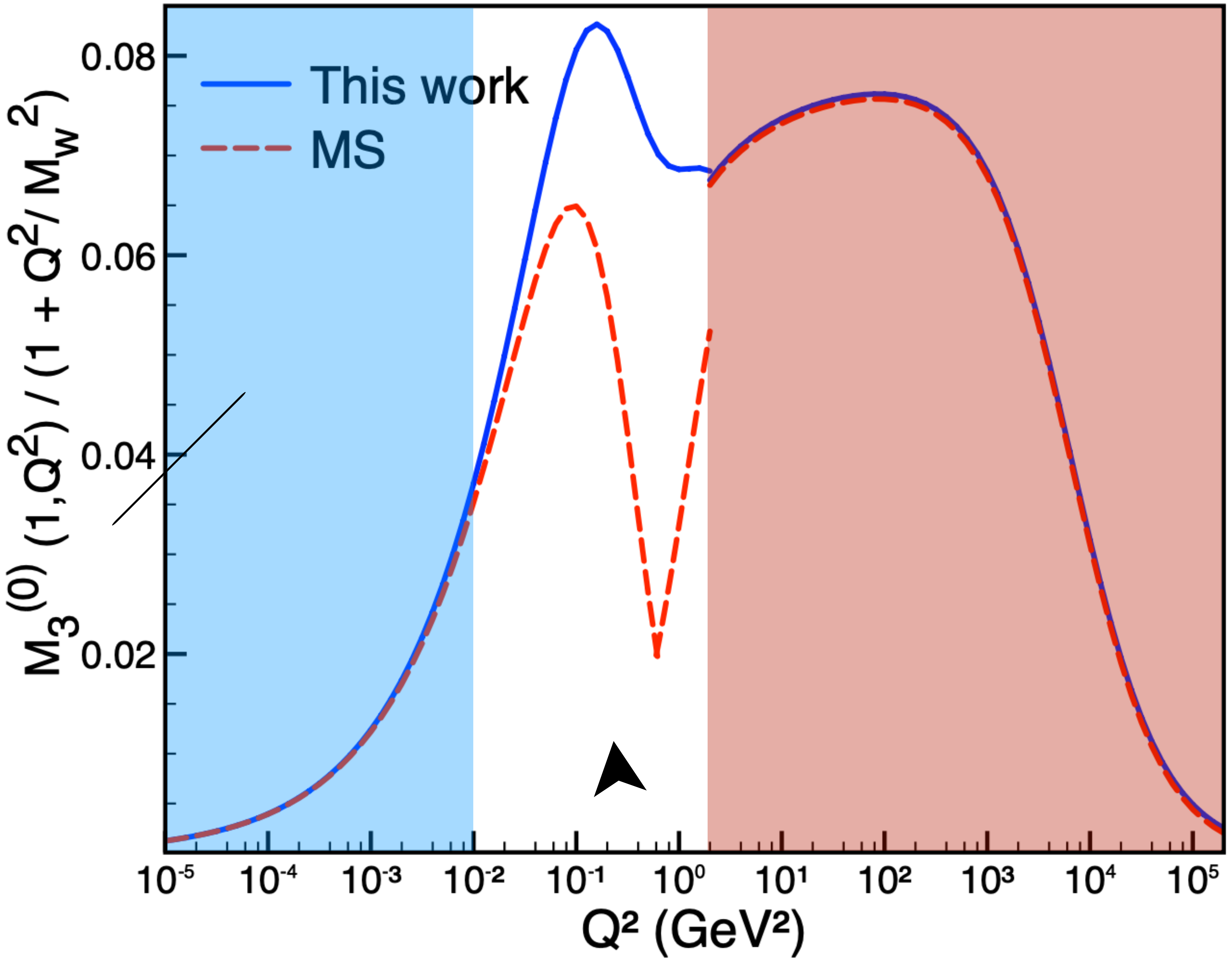


weak form factors

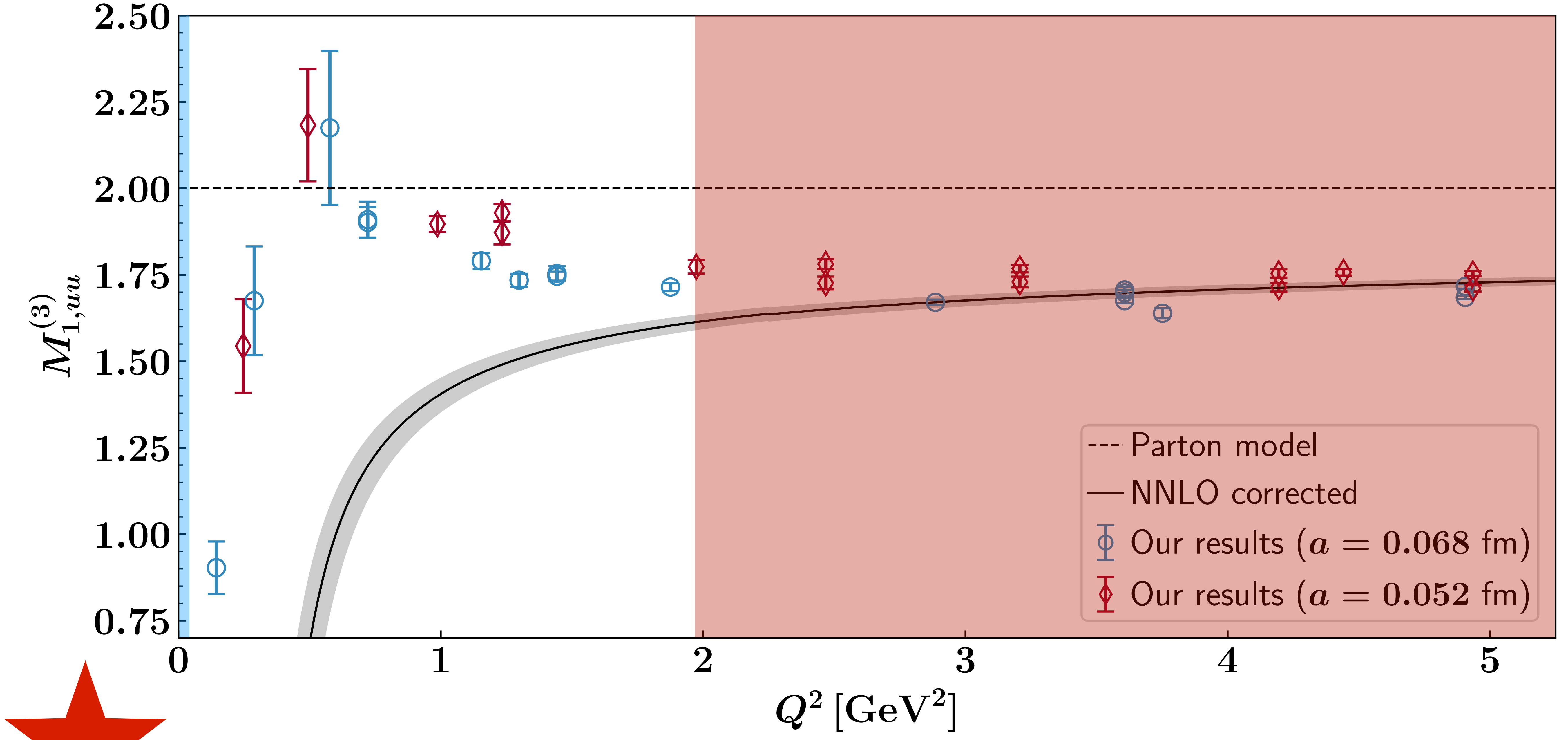
perturbation theory

input from LQCD required

$\mathscr{F}_3$ | EW box

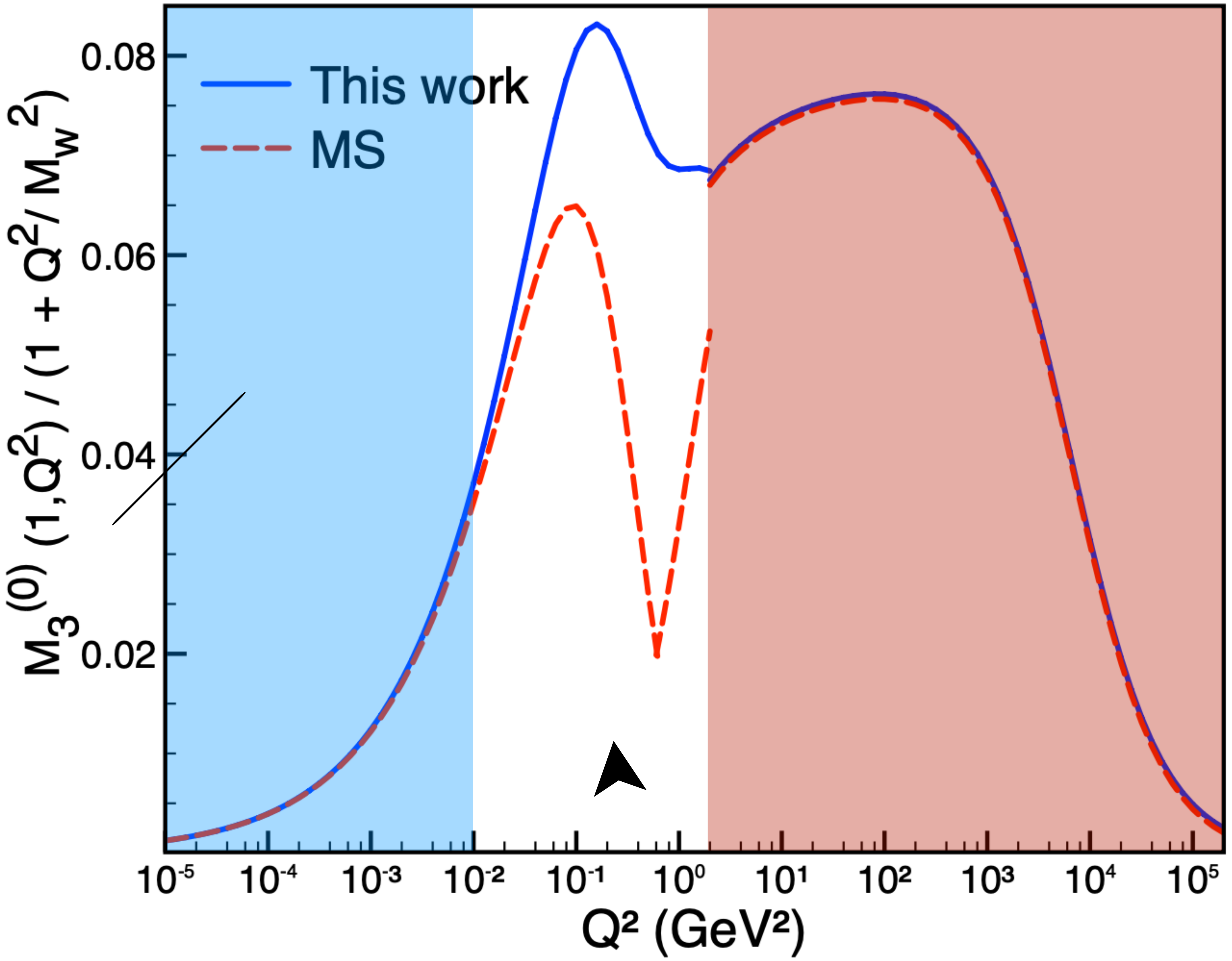
- **Electroweak box diagram contribution**

$$\Box^{\gamma W}_{VA} = \frac{3\alpha_{EM}}{2\pi}\int_0^\infty \frac{dQ^2}{Q^2}\frac{M_W^2}{M_W^2+Q^2}M_1^{(3)}(Q^2)$$



★ motivates large volumes with finer lattice spacings

C.-Y. Seng, M. Gorchtein, H. H. Patel, and M. J. Ramsey-Musolf, PRL 121, 241804 (2018)



weak form factors

perturbation theory

input from LQCD required

Future lattices

Currently thermalising/generating

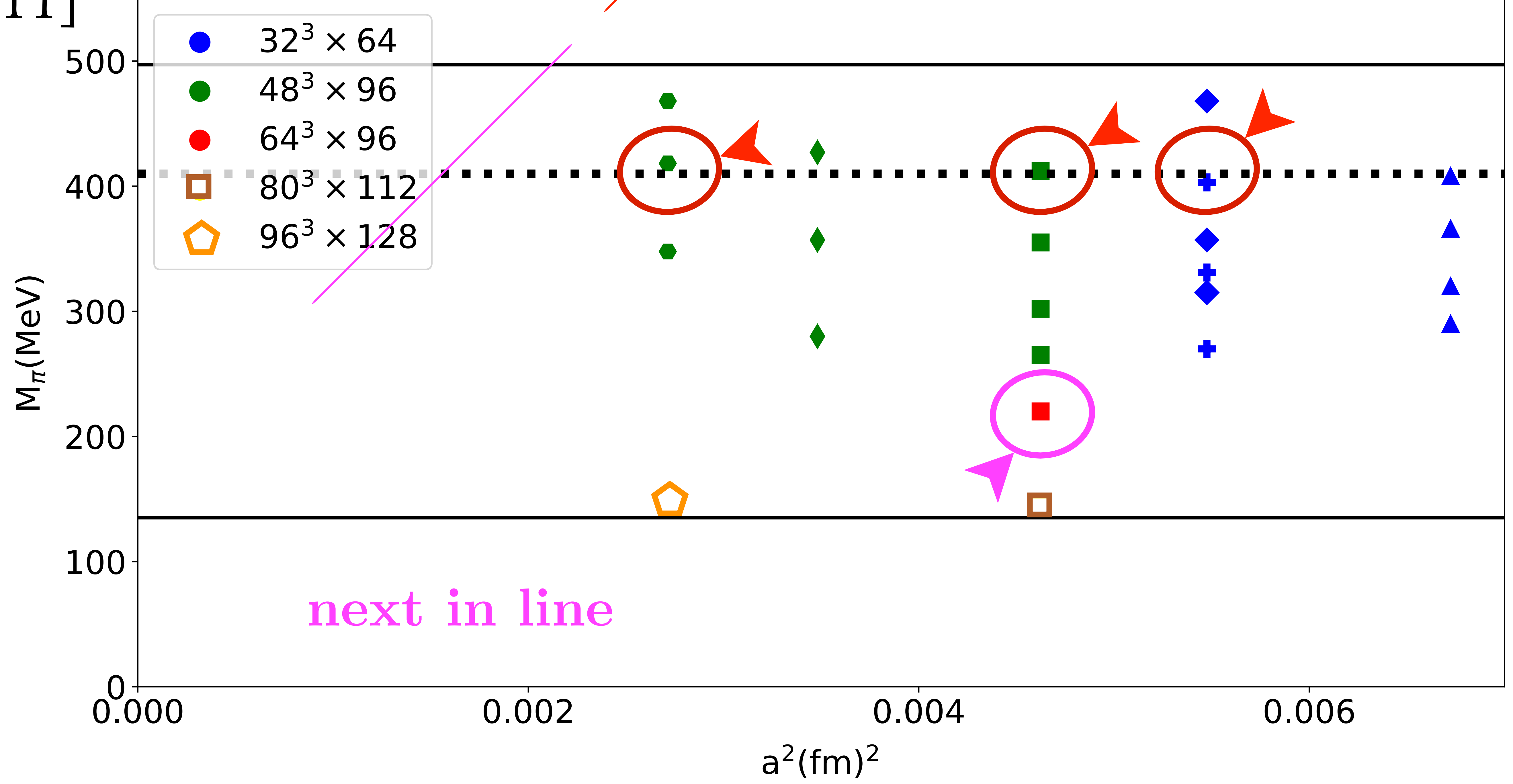
- $64^3 \times 96$, $a = (0.068,0.052)\,\text{fm}$, $m_\pi = (220,270)$ MeV *(completed - early 2024)*
- $80^3 \times 114$, $a = 0.068\,\text{fm}$, $m_\pi = 150$ MeV *(still thermalising)*
- $96^3 \times 128$, $a = 0.052\,\text{fm}$, $m_\pi = 140$ MeV *(thermalised + O(50) trajectories)*

Using BQCD [EPJ Web Conf. 175 (2018) 14011]

on

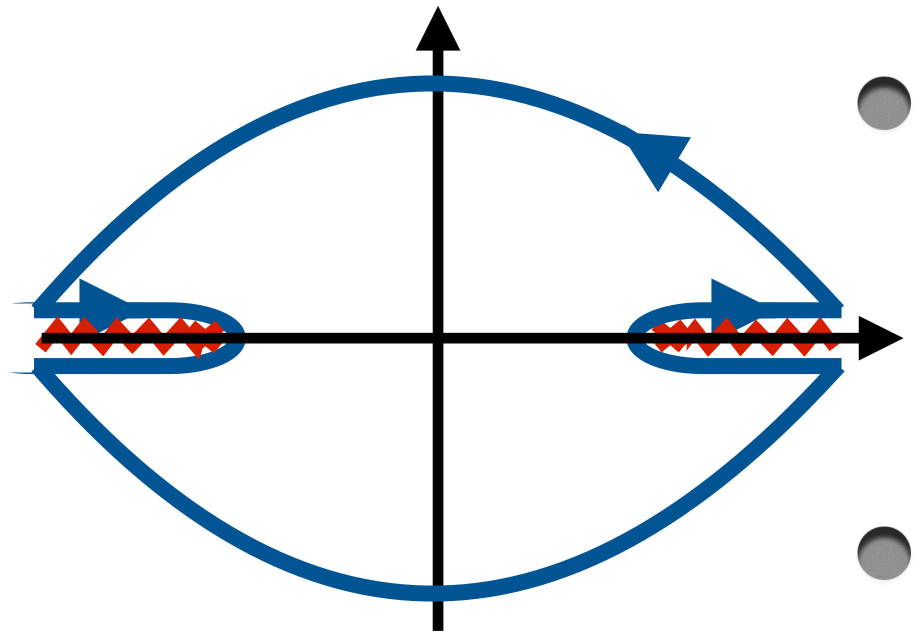
- JUWELS (Jülich, Germany)
- CSD3 (Cambridge, UK)
- Tursa (Edinburgh, UK)

Compton results so far

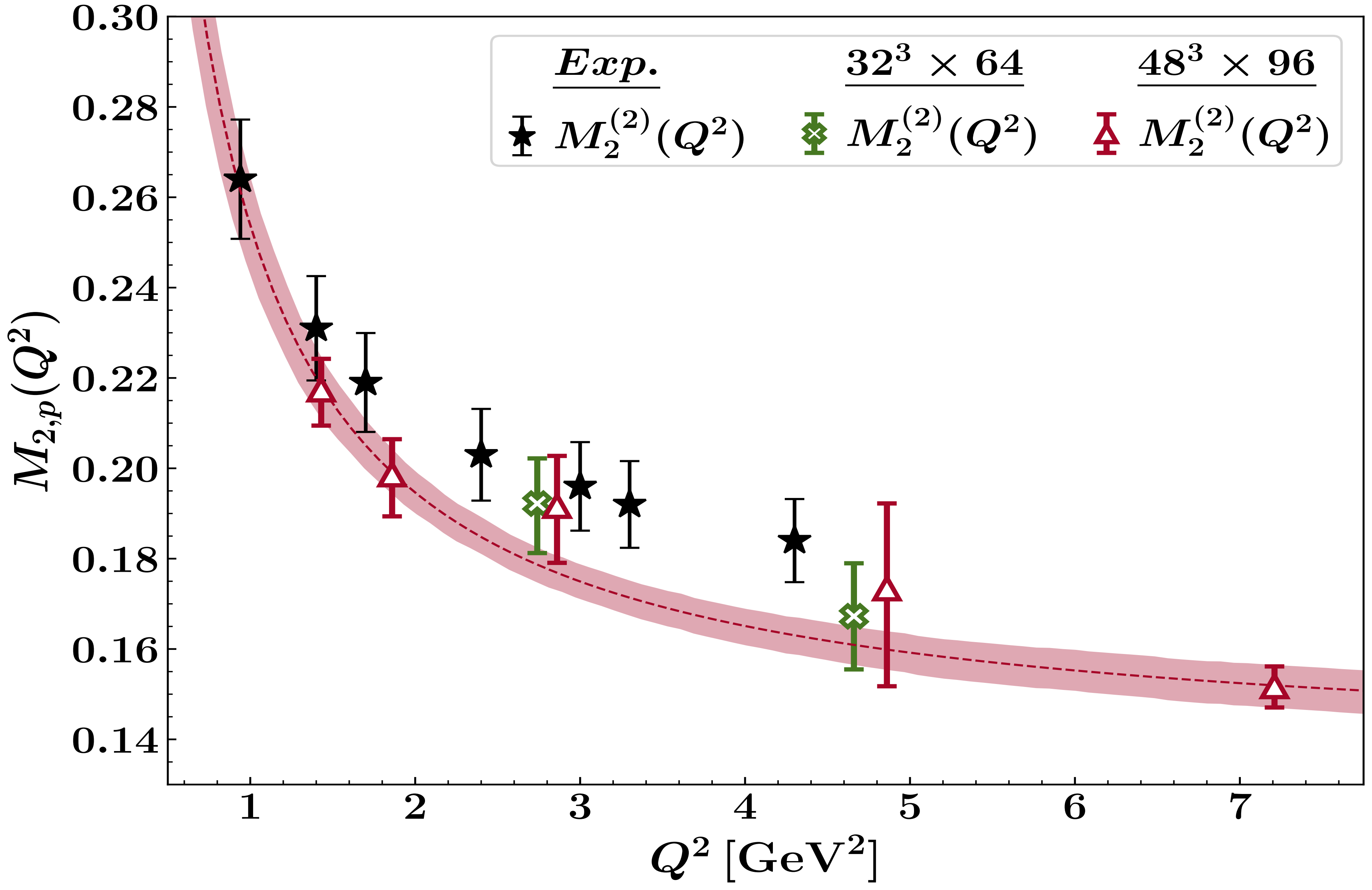


next in line

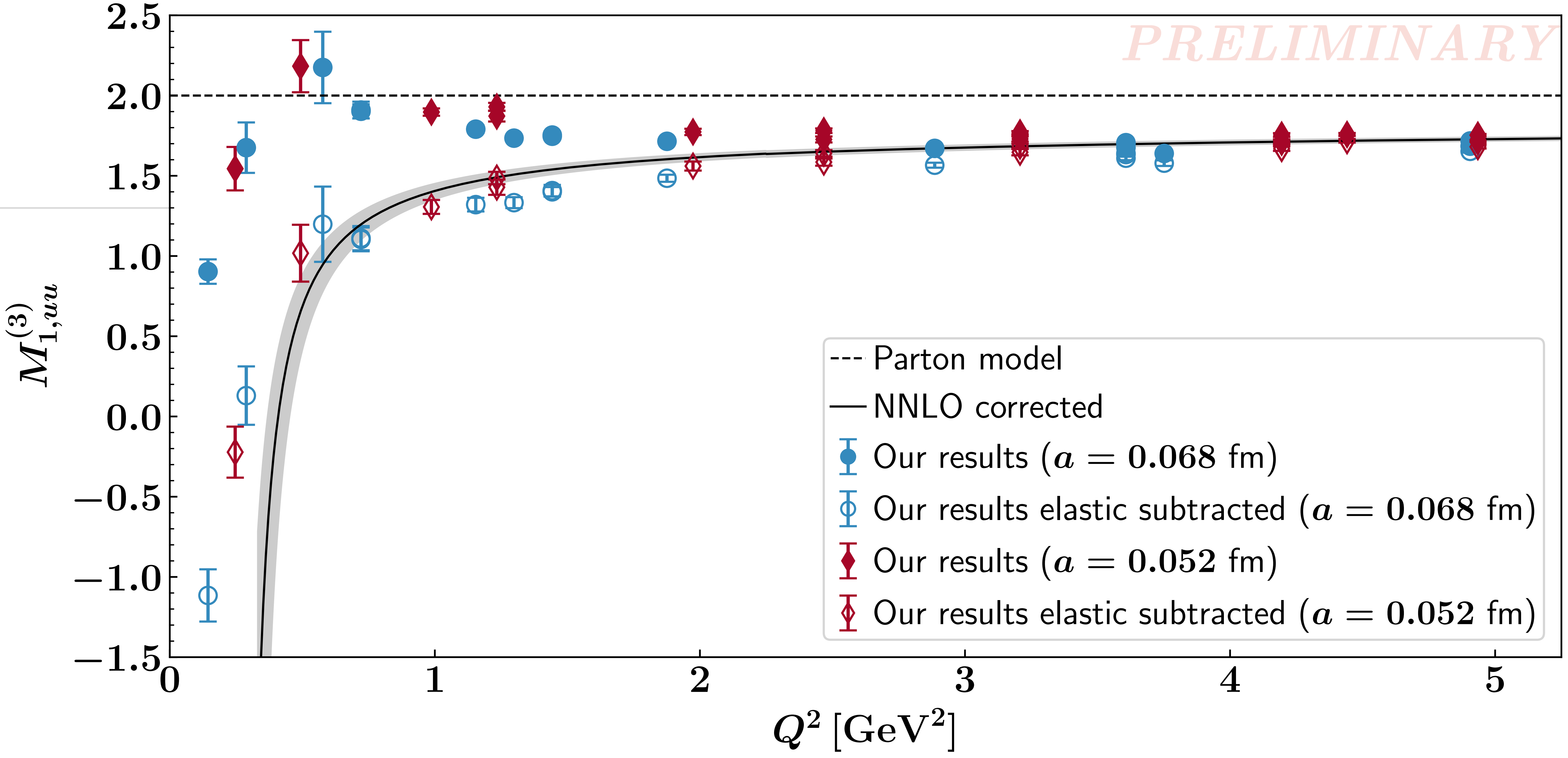
Summary



- Compton in unphysical region is spacelike
  - can be studied on the lattice
- **Versatile method:** $F_{1,2,3,L}$ and $g_{1,2}$



- First look at the **Q^2 dependence** of leading moments
- Emerging signal for **longitudinal structure function**



- **radiative corrections:** potential to offer new constraints on $\gamma Z/W$ box

Outlook:

- Control discretisation effects (larger V/β, PT)
- High-precision EW box estimates
- Polarised structure functions (coming soon)
- Ultimately, integrate Compton results into phenom. analyses

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  - resources provided by HLRN (The North-German Supercomputer Alliance),
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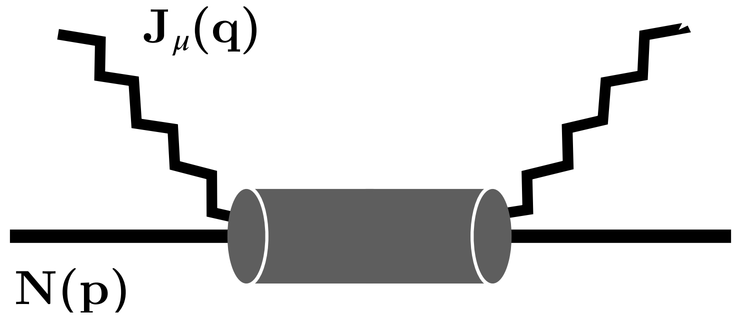
Backup

kuc et al. (CSSM/QCDSF/UKQCD) PRD102, 114505 (2020), arXiv:2007.01523 [hep-lat]

Compton amplitude via the FH relation at 2nd order

- **unpolarised Compton Amplitude**

$$T_{\mu\mu}(p,q) = \int d^4z e^{i\mathbf{q}\cdot\mathbf{z}} \langle N(p) \,|\, \mathcal{T}\{J_\mu(z) J_\mu(0)\} \,|\, N(p) \rangle$$



- **Action modification**

local EM current

$$J_\mu(z) = \sum_q e_q \bar{q}(z) \gamma_\mu q(z)$$

$$S \to S(\lambda) = S + \lambda \int d^4z \,(e^{i\mathbf{q}\cdot\mathbf{z}} + e^{-i\mathbf{q}\cdot\mathbf{z}})\, J_\mu(z)$$

- **2nd order derivatives of the 2-pt correlator, $G_\lambda^{(2)}(\mathbf{p};t)$, in the presence of the external field**

$$\left.\frac{\partial^2 G_\lambda^{(2)}(\mathbf{p};t)}{\partial\lambda^2}\right|_{\lambda=0} = \left(\frac{\partial^2 A_\lambda(\mathbf{p})}{\partial\lambda^2} - tA(\mathbf{p}) \frac{\partial^2 E_{N_\lambda}(\mathbf{p})}{\partial\lambda^2} \right) e^{-E_N(\mathbf{p})t}$$

from spectral decomposition

$$\left.\frac{\partial^2 G_\lambda^{(2)}(\mathbf{p};t)}{\partial\lambda^2}\right|_{\lambda=0} = \frac{A(\mathbf{p})}{2E_N(\mathbf{p})} t e^{-E_N(\mathbf{p})t} \int d^4z (e^{iq\cdot z} + e^{-iq\cdot z}) \langle N(\mathbf{p}) | \mathcal{T}\{\mathcal{J}(z)\mathcal{J}(0)\} | N(\mathbf{p}) \rangle$$

from path integral

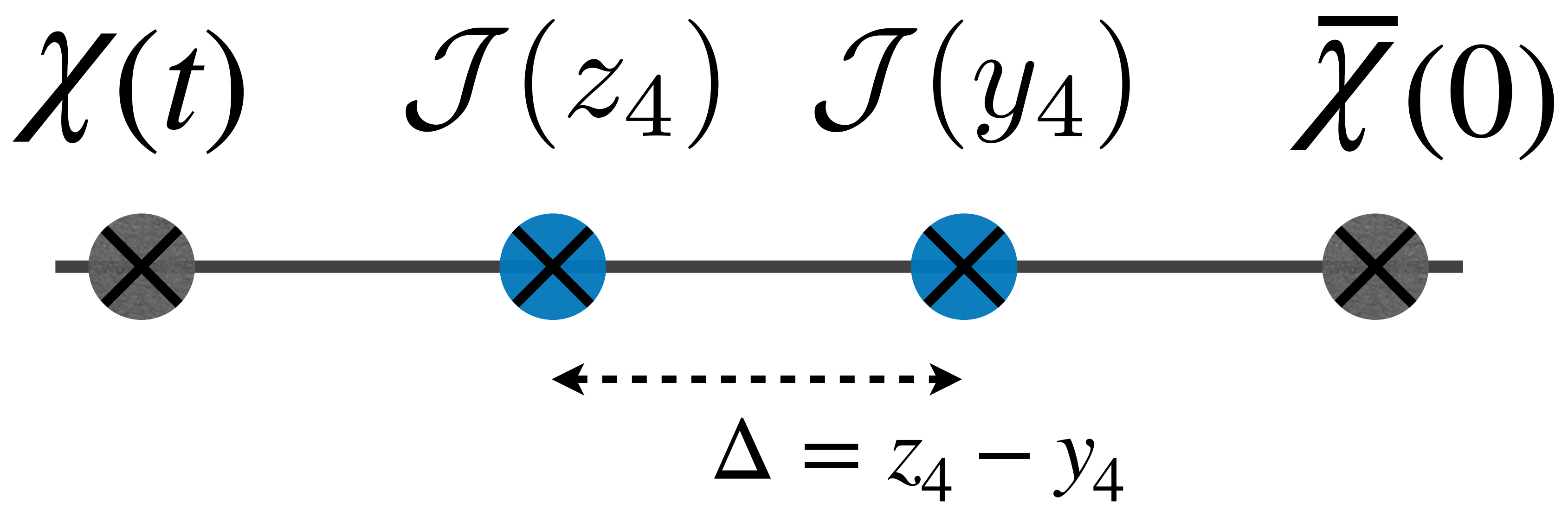
- **equate the time-enhanced terms:**

$$\left.\frac{\partial^2 E_{N_\lambda}(\mathbf{p})}{\partial\lambda^2}\right|_{\lambda=0} = -\frac{1}{2E_N(\mathbf{p})} \overbrace{\int d^4z (e^{iq\cdot z} + e^{-iq\cdot z}) \langle N(\mathbf{p}) | \mathcal{J}(z)\mathcal{J}(0) | N(\mathbf{p}) \rangle}^{T_{\mu\mu}(p,q)} + (q \to -q)$$

Compton amplitude is related to the second-order energy shift

Compton amplitude via the FH relation at 2nd order

- **relevant contribution comes from the ordering where the currents are sandwiched**



$$\sim e^{-E_N(\mathbf{p})t} \int^t d\Delta\, e^{-\left(E_X(\mathbf{p}+\mathbf{q}) - E_N(\mathbf{p})\right)\Delta} (t-\Delta)$$

- under the condition $|\omega| < 1$, $E_X(\mathbf{p}+n\mathbf{q}) \gtrsim E_N(\mathbf{p})$, so the intermediate states <u>cannot go on-shell</u>
- ground state dominance is ensured in the large time limit