Spin polarization in heavy ion collisions

Chirality and Criticality Workshop

Matteo Buzzegoli

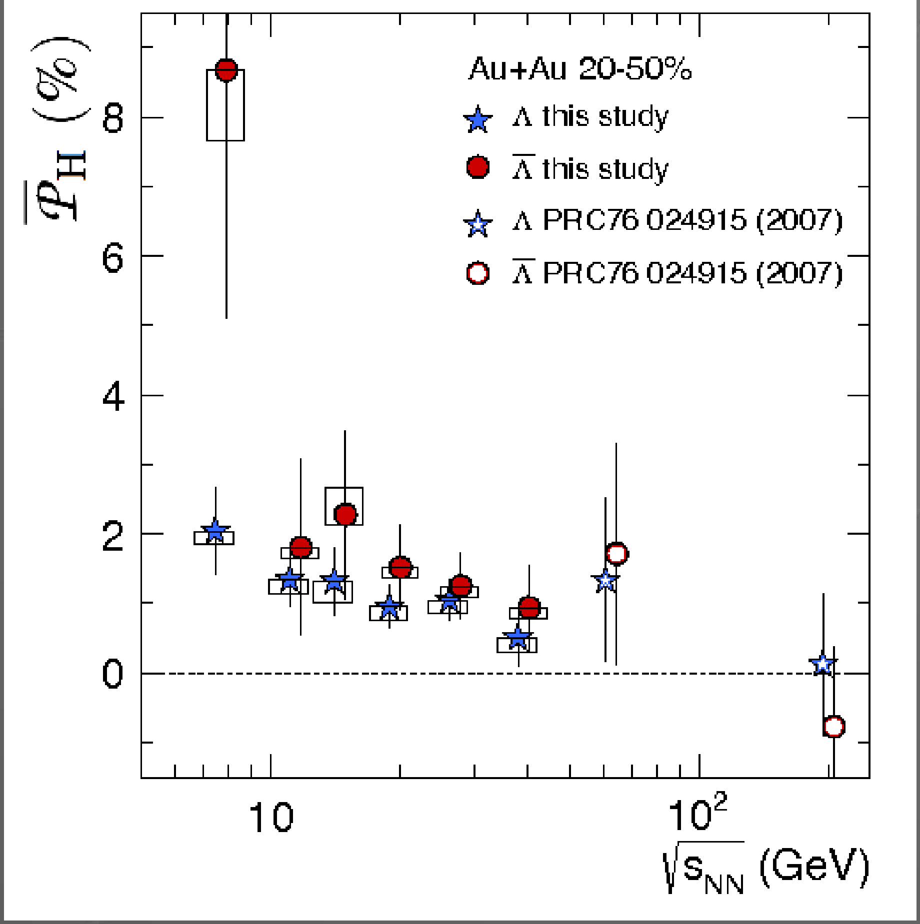
24 August 2023

IOWA STATE UNIVERSITY

INSTITUTE for NUCLEAR THEORY

Discovery of global Λ polarization

STAR Collaboration, Global Lambda hyperon polarization in nuclear collisions, Nature 548 6265, 2017



Particle and antiparticle have the same polarization sign.
This shows that the phenomenon cannot be driven
by a mean field (such as EM) whose coupling is C-*odd*.
Definitely favors the thermodynamic (equipartition) interpretation

Analogy between Rotation Ω and Magnetic field B

[MB, Nucl. Phys. A 1036, 122674 (2023)]

Magnetization

Barnett Effect (1915)

$$\frac{qB^{\mu}_{\rm Eff}}{E} = \Omega^{\mu}$$

Spin polarization

$$S^{\mu}_{\Omega} = \frac{1}{4m}(1-n_F)\beta E\left(\Omega^{\mu} - u^{\mu}\frac{\Omega\cdot p}{E}\right) \qquad S^{\mu}_{B} = \frac{1}{4m}(1-n_F)\beta\left(qB^{\mu} - u^{\mu}\frac{qB\cdot p}{E}\right)$$

[F. Becattini, I. Karpenko, M. Lisa, I. Upsal, S. Voloshin, PRC 95 (2017)]

Electromagnetic radiation

Classical total radiation intensity from circular motion [Schott (1912)]

$$W = \frac{q^2\Omega^2}{c}\sum_{\nu=1}^{\infty}\int_0^{\pi}\sin\theta d\theta\left[\cot^2\theta J_{\nu}(\nu\beta\sin\theta) + \beta^2 {J'_{\nu}}^2(\nu\beta\sin\theta)\right]$$

In magnetic field:

$$\Omega \to \omega_B = \frac{qB}{E}$$

Prompt photon puzzle: EM radiation in B and Ω

[MB, J.D. Kroth, K. Tuchin and N. Vijayakumar, PRD107 (2023)]

Gravitational anomaly?

The Axial Vortical Effect (AVE) conductivity

Massless AVE $\langle j_A^z \rangle \propto \mathcal{N} T^2 \Omega$

$$\nabla_\mu j_A^\mu = \mathcal{N} \epsilon^{\mu\nu\alpha\beta} R_{\mu\nu\lambda\rho} R_{\alpha\beta}{}^{\lambda\rho}$$

K. Landsteiner, E. Megias, F. Pena-Benitez, PRL 107 (2011)
K. Jensen, R. Loganayagam, A. Yarom, JHEP 02 (2013)
M. Stone, J. Kim, PRD 98 (2) (2018)
G.Y. Prokhorov, O.V. Teryaev, V.I. Zakharov, PRL 129 (2022)

Barnett effect: Classical model for spin polarization

[MB, Nucl. Phys. A 1036, 122674 (2023)]

Angular momentum-rotation coupling

Classical inertial effect

$$H \to H - \boldsymbol{J} \cdot \boldsymbol{\Omega}$$

$$\langle \boldsymbol{S} \cdot \hat{\boldsymbol{\Omega}} \rangle = \langle \boldsymbol{\mu} \cdot \hat{\boldsymbol{\Omega}} \rangle \simeq \frac{1}{3} \frac{\mu}{\gamma} \frac{\Omega}{T}$$

γ gyromagnetic ratio
μ magnetic moment

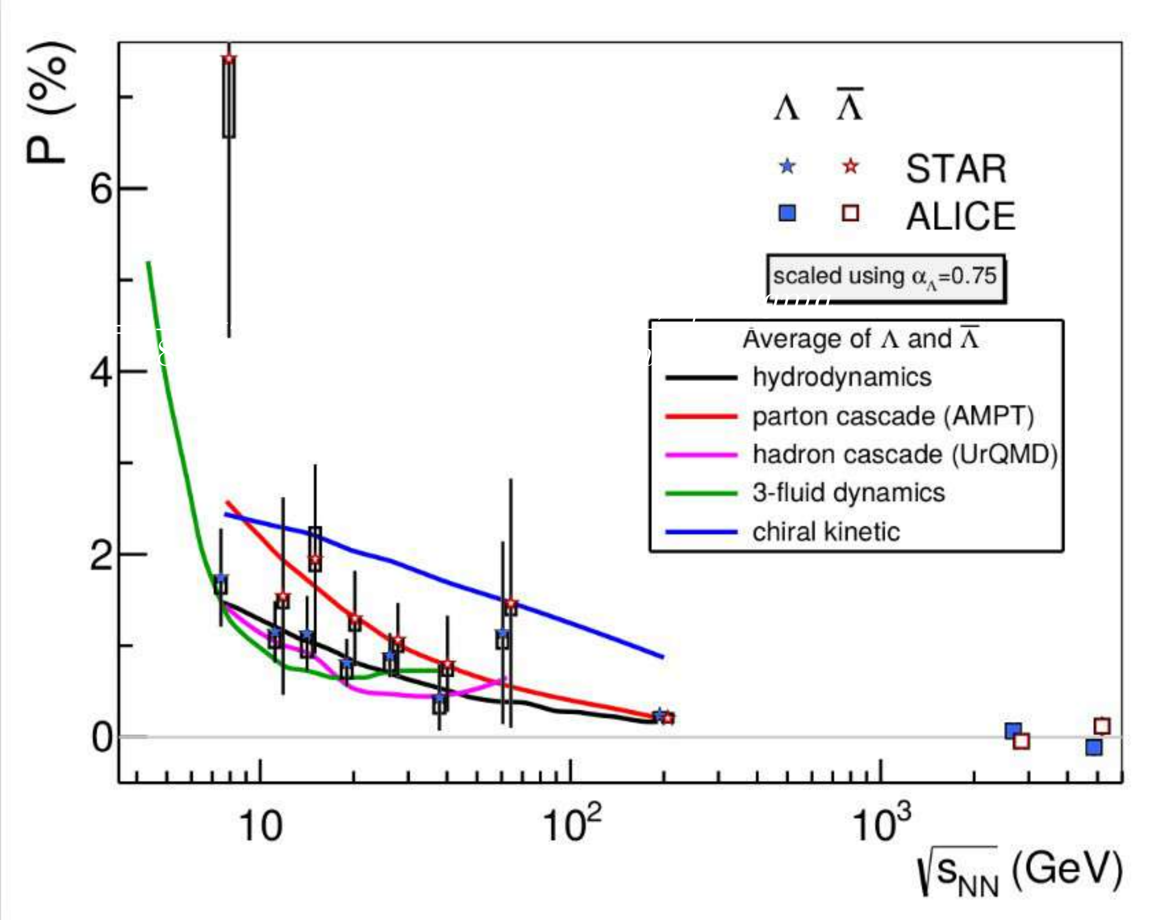
Furthermore:
Unlike the CVE/CME the (massive) AVE is allowed at the actual equilibrium (no chiral imbalance)
No connection between the massive AVE and the gravitational anomaly

$$\langle j_A^z \rangle \simeq \frac{\Omega}{T} \left(1 + 2\frac{T}{m}\right) \frac{(mT)^{3/2}}{\sqrt{2}\,\pi^{3/2}} e^{|\beta|(\mu - m)} \qquad T \ll m$$

[MB, Lect. Notes Phys. 987 (2021)]

Agreement between hydrodynamic predictions and the data

F. Becattini, M. Lisa, Polarization and vorticity in the QGP, Ann. Rev. Part, Nucl. Sc. 70, 395 (2020)



F. Becattini, V. Chandra, L. Del Zanna, E. Grossi, Ann. Phys. 338:32 (2013)

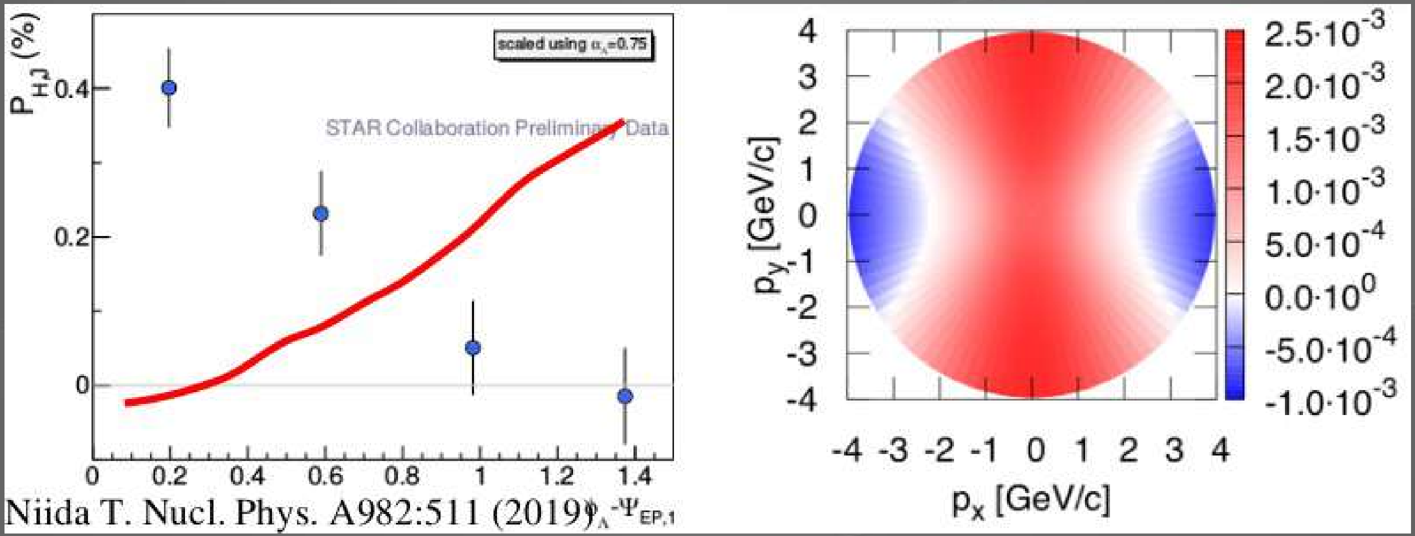
$$S^\mu(p) = \frac{1}{8m}\epsilon^{\mu\rho\sigma\tau}p_\tau \frac{\int_\Sigma d\Sigma \cdot p\, n_F(1-n_F)\partial_\rho \beta_\sigma}{\int_\Sigma d\Sigma \cdot p\, n_F}$$

$$n_F = \left(e^{\beta\cdot p - \zeta} + 1\right)^{-1}$$

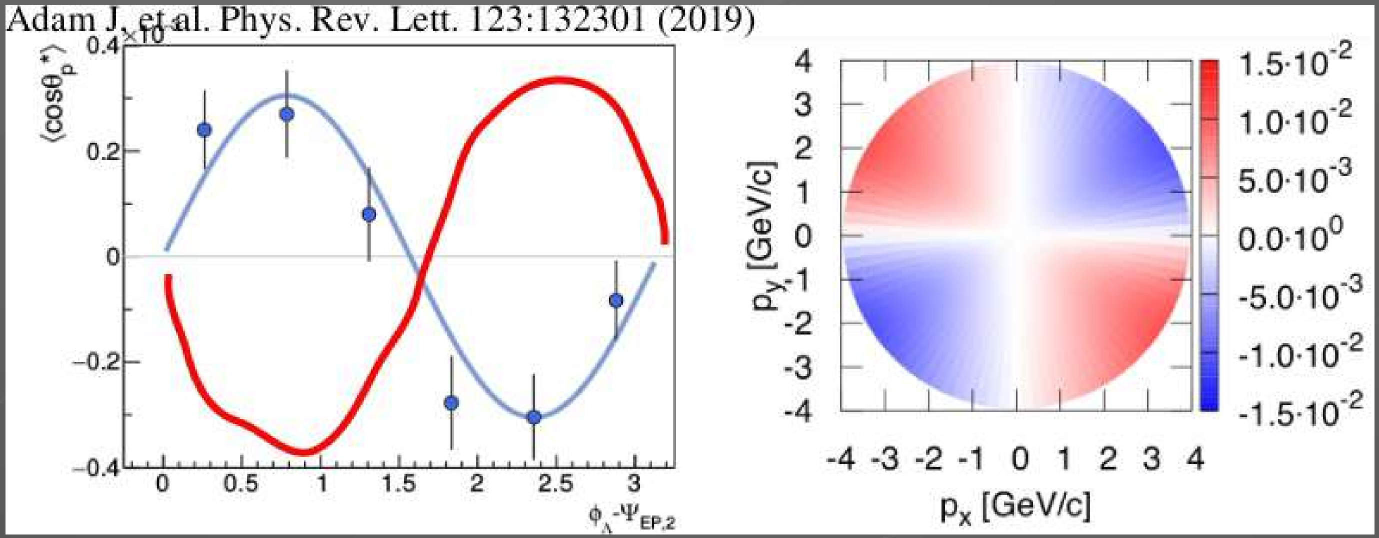
$$\beta = \frac{1}{T}u$$

Different models of the collision, same formula for polarization

Puzzles: momentum dependence of polarization



Theory prediction



Not the effect of decays:

X. L. Xia, H. Li, X.G. Huang and H. Z. Huang, Phys. Rev. C 100 (2019), 014913

F. Becattini, G. Cao and E. Speranza, Eur. Phys. J. C 79 (2019) 741

Theory

The spin polarization vector for spin ½ particles:

$$S^\mu(p) = \frac{1}{2}\frac{\int \mathrm{d}\Sigma \cdot p \; \mathrm{tr}_4\left[\gamma^\mu \gamma^5 W_+(x,p)\right]}{\int \mathrm{d}\Sigma \cdot p \; \mathrm{tr}_4 W_+(x,p)}$$

Wigner function is an expectation value of an integrated two point function of the Dirac field

$$W(x,k) = \mathrm{tr}\left(\widehat{\rho}\,\widehat{W}(x,k)\right)$$

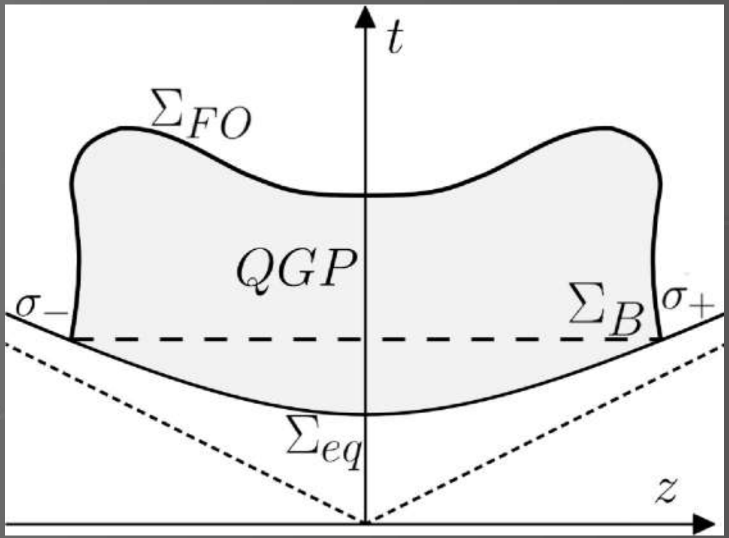
One needs to know the statistical operator to calculate mean values

$$\langle \widehat{X} \rangle = \mathrm{tr}\left(\widehat{\rho}\,\widehat{X}\right)$$

F. Becattini, Lect. Notes Phys. 987 (2021) 15-52.

The actual statistical operator (Zubarev theory)

F. Becattini, M. B., E. Grossi, *Reworking the Zubarev's approach to nonequilibrium quantum statistical mechanics*, Particles 2 (2019) 2, 197-207; 1902.01089



In the covariant Zubarev theory, this is the LTE at some initial "time":

$$\widehat{\rho} = \frac{1}{Z} \exp\left[- \int_{\Sigma(\tau_0)} \mathrm{d}\Sigma_\mu \left(\widehat{T}_B^{\mu\nu} \beta_\nu - \zeta \, \widehat{j}^\mu \right) \right]$$

With the Gauss theorem:

NOTE: T_B stands for the symmetrized Belinfante stress-energy tensor

$$\widehat{\rho} = \frac{1}{Z} \exp\left[- \int_{\Sigma(\tau)} \mathrm{d}\Sigma_\mu \left(\widehat{T}_B^{\mu\nu} \beta_\nu - \zeta \, \widehat{j}^\mu \right) + \int_\Theta \mathrm{d}\Theta \left(\widehat{T}_B^{\mu\nu} \nabla_\mu \beta_\nu - \widehat{j}^\mu \nabla_\mu \zeta \right) \right]$$

Local equilibrium, non-dissipative terms

Dissipative terms

Kubo formulas

$$\widehat{\rho} = \frac{1}{Z} \exp \left[- \int_{\Sigma(\tau)} \mathrm{d}\Sigma_\mu \left(\widehat{T}_B^{\mu\nu} \beta_\nu - \zeta \,\widehat{j}^\mu \right) + \int_\Theta \mathrm{d}\Theta \left(\widehat{T}_B^{\mu\nu} \nabla_\mu \beta_\nu - \widehat{j}^\mu \nabla_\mu \zeta \right) \right]$$

$\widehat{A} =$ Local equilibrium

$\widehat{B} =$ Dissipation

Kubo Identity:

$$\exp\left[\widehat{A} + \widehat{B}\right] = \exp\left[\widehat{A}\right] + \int_0^1 \mathrm{d}z \, \exp\left[z\left(\widehat{A} + \widehat{B}\right)\right] \widehat{B} \exp\left[-z\widehat{A}\right] \exp\left[\widehat{A}\right]$$

Linear response:

$$\widehat{\rho} \simeq \widehat{\rho}_{\rm LE} + \int_0^1 \mathrm{d}z \, \exp\left[z\widehat{A}\right] \widehat{B} \exp\left[-z\widehat{A}\right] \widehat{\rho}_{\rm LE} - \langle \widehat{B} \rangle_{\rm LE} \, \widehat{\rho}_{\rm LE}$$

This is the method to generate the so-called Kubo formulas

A. Hosoya, M. Sakagami and M. Takao, Annals Phys. 154 (1984), 229.

Local thermodynamic equilibrium approximation

$$\widehat{\rho} \simeq \widehat{\rho}_{\rm LE} = \frac{1}{Z} \exp\left[- \int_{\Sigma(\tau)} {\rm d}\Sigma_\mu \left(\widehat{T}_B^{\mu\nu} \beta_\nu - \zeta \,\widehat{j}^\mu \right) \right]$$
$$= \frac{1}{Z} \exp\left[- \int_{\Sigma_{FO}} {\rm d}\Sigma_\mu \left(\widehat{T}_B^{\mu\nu} \beta_\nu - \zeta \,\widehat{j}^\mu \right) \right]$$

Corresponds to the ideal fluid. Neglecting dissipative term in the exponent of the density operator

$$W(x,k) \simeq W(x,k)_{\rm LE} = {\rm tr}\left(\widehat{\rho}_{\rm LE} \widehat{W}(x,k) \right)$$

$$W(x,k)_{LE} = \frac{1}{Z} {\rm tr}\left(\exp\left[- \int_{\Sigma_{FO}} {\rm d}\Sigma_\mu(y) \left(\widehat{T}_B^{\mu\nu}(y) \beta_\nu(y) - \zeta(y) \,\widehat{j}^\mu(y) \right) \right] \widehat{W}(x,k) \right)$$

Even the local equilibrium value of the Wigner function is hard to calculate for general four-temperature and chemical potential/T fields.

Hydrodynamic limit: Taylor expansion

Expand the β and ζ fields from the point x where the Wigner operator is to be evaluated

$$\beta_\nu(y) \simeq \beta_\nu(x) + \partial_\lambda \beta_\nu(x)(y-x)^\lambda + \cdots$$

$$\widehat{\rho}_{LE} \simeq \frac{1}{Z} \exp\left[-\beta_\nu(x)\widehat{P}^\nu + \zeta(x)\widehat{Q} + \frac{1}{2}\varpi_{\mu\nu}(x)\widehat{J}^{\mu\nu}_x - \frac{1}{2}\xi_{\mu\nu}(x)\widehat{Q}^{\mu\nu}_x + \cdots \right]$$

$$\widehat{J}^{\mu\nu}_x = \int \mathrm{d}\Sigma_\lambda \left[(y-x)^\mu \widehat{T}^{\lambda\nu}(y) - (y-x)^\nu \widehat{T}^{\lambda\mu}(y) \right] \qquad \widehat{Q}^{\mu\nu}_x = \int \mathrm{d}\Sigma_\lambda \left[(y-x)^\mu \widehat{T}^{\lambda\nu}(y) + (y-x)^\nu \widehat{T}^{\lambda\mu}(y) \right]$$

$$\varpi_{\mu\nu} = -\frac{1}{2}\left(\partial_\mu \beta_\nu - \partial_\nu \beta_\mu\right)$$

Thermal vorticity

Adimensional in natural units

Equilibrium

$$\xi_{\mu\nu} = \frac{1}{2}\left(\partial_\mu \beta_\nu + \partial_\nu \beta_\mu\right)$$

Thermal shear

Adimensional in natural units

Non-equilibrium

At global equilibrium the thermal shear vanishes because of the Killing equation

Spin polarization at local thermal equilibrium

Linear response theory ⇨ $S^\mu(p) = S^\mu_\varpi + S^\mu_\xi + \cdots$

Vorticity:

$$S^\mu_\varpi(p) = -\frac{1}{8m}\epsilon^{\mu\rho\sigma\tau}p_\tau \frac{\int_\Sigma \mathrm{d}\Sigma\cdot p\, n_F(1-n_F)\varpi_{\rho\sigma}}{\int_\Sigma \mathrm{d}\Sigma\cdot p\, n_F}$$

Shear:

$$S^\mu_\xi(p) = -\frac{1}{4m}\epsilon^{\mu\nu\sigma\tau}\frac{p_\tau p^\rho}{\varepsilon}\frac{\int_\Sigma \mathrm{d}\Sigma\cdot p\, n_F(1-n_F)\hat{t}_\nu\xi_{\sigma\rho}}{\int_\Sigma \mathrm{d}\Sigma\cdot p\, n_F}$$

F. Becattini, MB, A. Palermo, Phys. Lett. B 820 (2021) 136519

Same (not precisely the same) formula obtained by Liu and Yin with a different method:
S. Liu, Y. Yin, JHEP 07 (2021) 188

Spin-thermal shear in kinetic theory

Derivations by using expansions of the Wigner functions in h:

C. Yi, S. Pu, D. L. Yang, Phys.Rev.C 104 (2021) 6, 064901
Y. C. Liu, X. G. Huang, Sci. China Phys.Mech.Astron. 65 (2022) 7, 272011

Need of a local equilibrium ansatz

$$\left[\gamma\cdot\left(p+\mathrm{i}\frac{\hbar}{2}\partial\right)-m\right]W_{\alpha\beta}=\hbar\mathcal{C}_{\alpha\beta}$$

From relativistic kinetic theory:

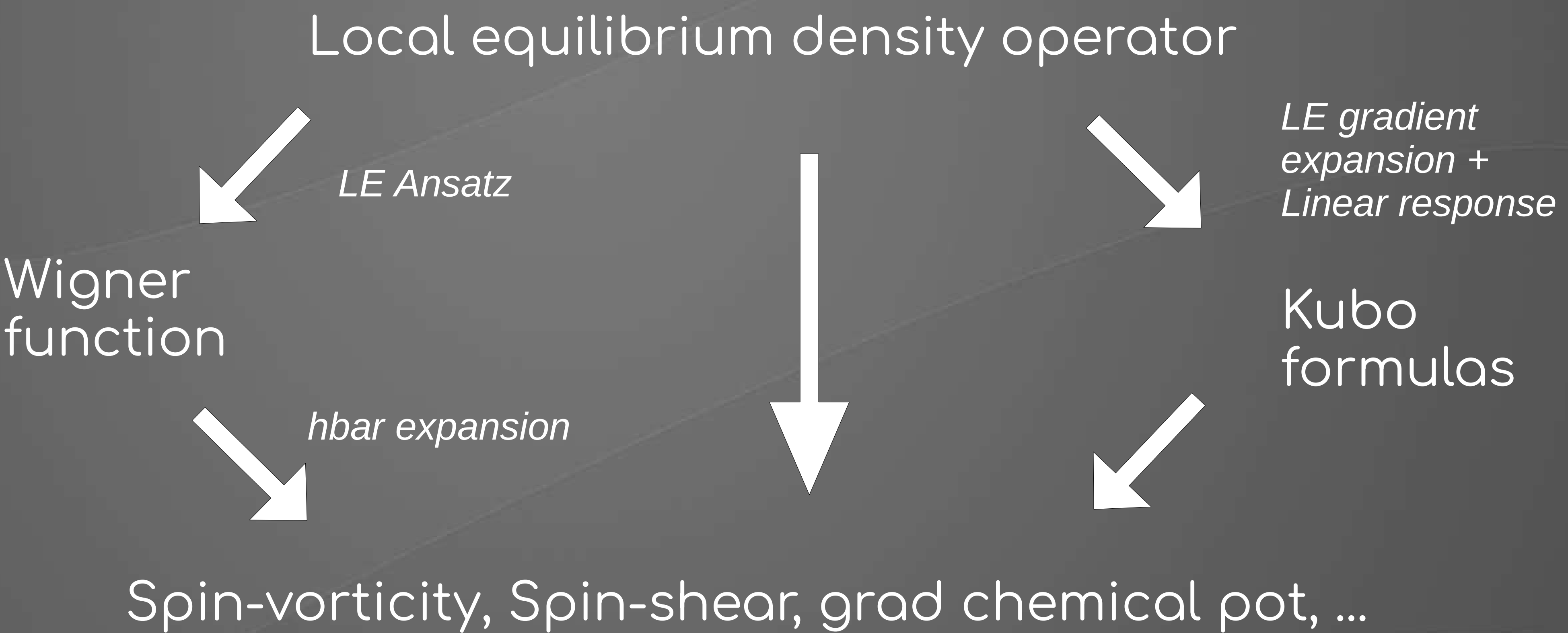
N. Weickgenannt, D. Wagner, E. Speranza and D. H. Rischke, Phys. Rev. D 106 (2022), L091901

$$\Pi^{\mu}_{\mathrm{NS}} \sim \int_{\Sigma_{\mathrm{f.o.}}} d\Sigma\cdot k\, f_{0\mathbf{k}}\Big\{\epsilon^{\mu\nu\rho\sigma}k_\nu\Omega_{\rho\sigma}+\left(\delta^\mu_\nu-\frac{u^\mu k_{\langle\nu\rangle}}{E_{\mathbf{k}}}\right) \times\left[\kappa_{\mathfrak{p}}\epsilon^{\nu\rho\alpha\beta}\left(\Omega_{\alpha\beta}-\varpi_{\alpha\beta}\right)u_\rho+\frac{\kappa_{\mathfrak{q}}}{T}\sigma_\alpha{}^{\langle\rho}\epsilon^{\sigma\rangle\nu\alpha\beta}u_\beta k_{\langle\rho}k_{\sigma\rangle}\right]\Big\}$$

Dissipative ?

Note: derived in the Hilgevoord-Wouthuysen (HW) pseudo gauge

Theory summary



Note: there is no principle distinction between “statistical method” or “Quantum field theory method” or “Kubo-formula method” or “Wigner function method”

Application to relativistic heavy ion collisions

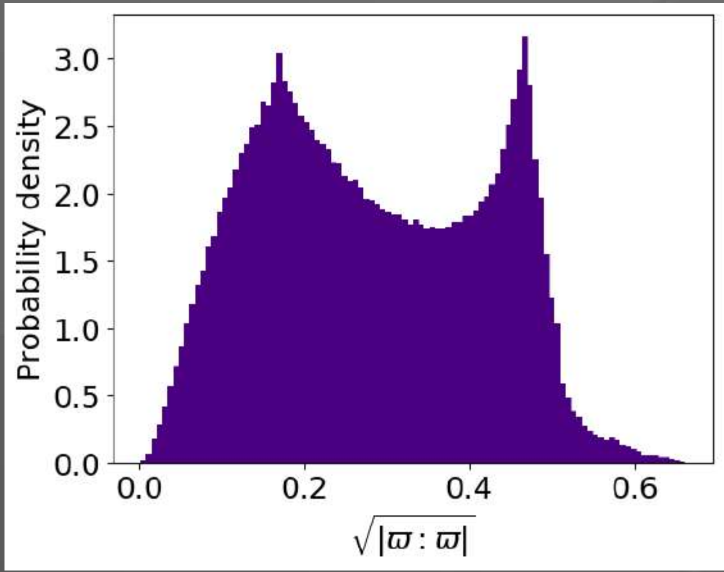
F. Becattini, M.B., A. Palermo, G. Inghirami and I. Karpenko, Phys. Rev. Lett. 127 (2021) 27, 272302

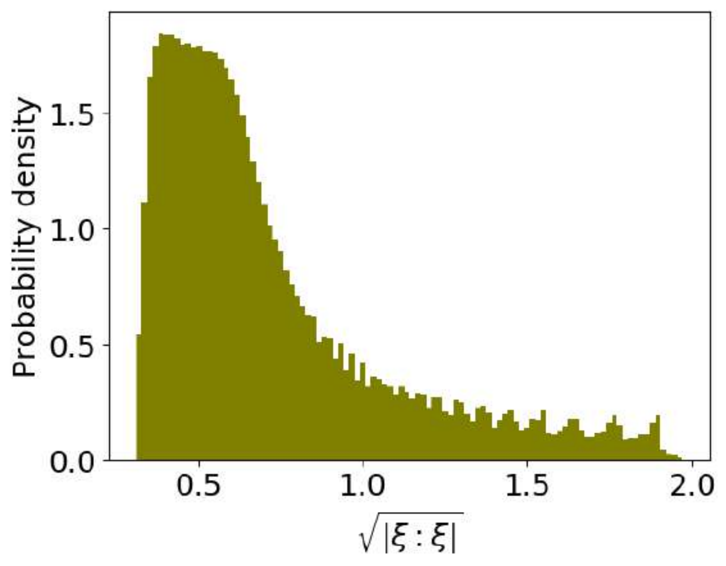
$$S^\mu = S^\mu_\varpi + S^\mu_\xi$$

$$S^\mu_\varpi(p) = -\frac{1}{8m}\epsilon^{\mu\rho\sigma\tau}p_\tau \frac{\int_\Sigma \mathrm{d}\Sigma\cdot p\, n_F(1-n_F)\varpi_{\rho\sigma}}{\int_\Sigma \mathrm{d}\Sigma\cdot p\, n_F}$$

$$S^\mu_\xi(p) = -\frac{1}{4m}\epsilon^{\mu\nu\sigma\tau}\frac{p_\tau p^\rho}{\varepsilon}\frac{\int_\Sigma \mathrm{d}\Sigma\cdot p\, n_F(1-n_F)\hat{t}_\nu\xi_{\sigma\rho}}{\int_\Sigma \mathrm{d}\Sigma\cdot p\, n_F}$$

Comparison between thermal vorticity and shear





Comparison between different calculations

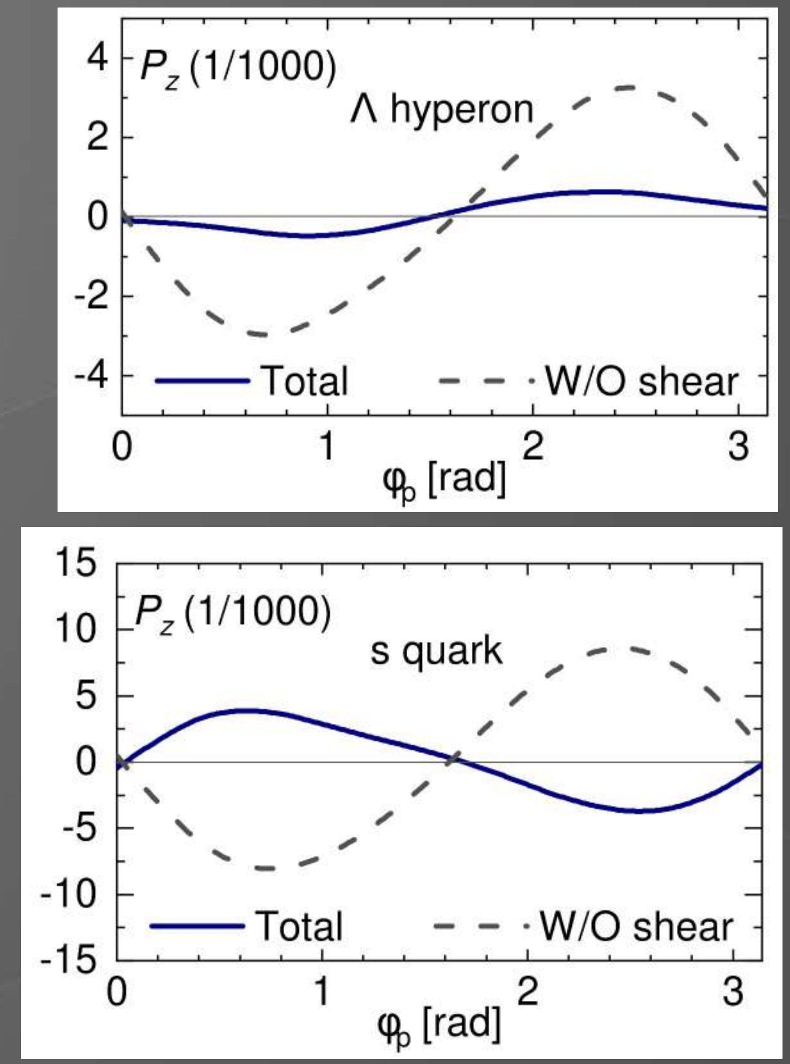
$$S^\mu(p) = \frac{1}{4m}\frac{\int d\Sigma\cdot p\, n_F(1-n_F)\mathcal{A}^\mu}{\int d\Sigma \cdot p\, n_F}$$

$$\mathcal{A}^\mu_{\rm LY} = -\varepsilon^{\mu\rho\sigma\tau}\left[\frac{1}{2}\omega_{\rho\sigma}p_\tau + \frac{1}{E}u_\rho\xi_{\sigma\lambda}p^\lambda_\perp p_\tau + \frac{b_i}{\beta E}u_\rho p^\perp_\sigma \partial^\perp_\tau(\beta\mu_B)\right].$$

$$p^\lambda_\perp = p^\lambda - (u\cdot p)u^\lambda$$

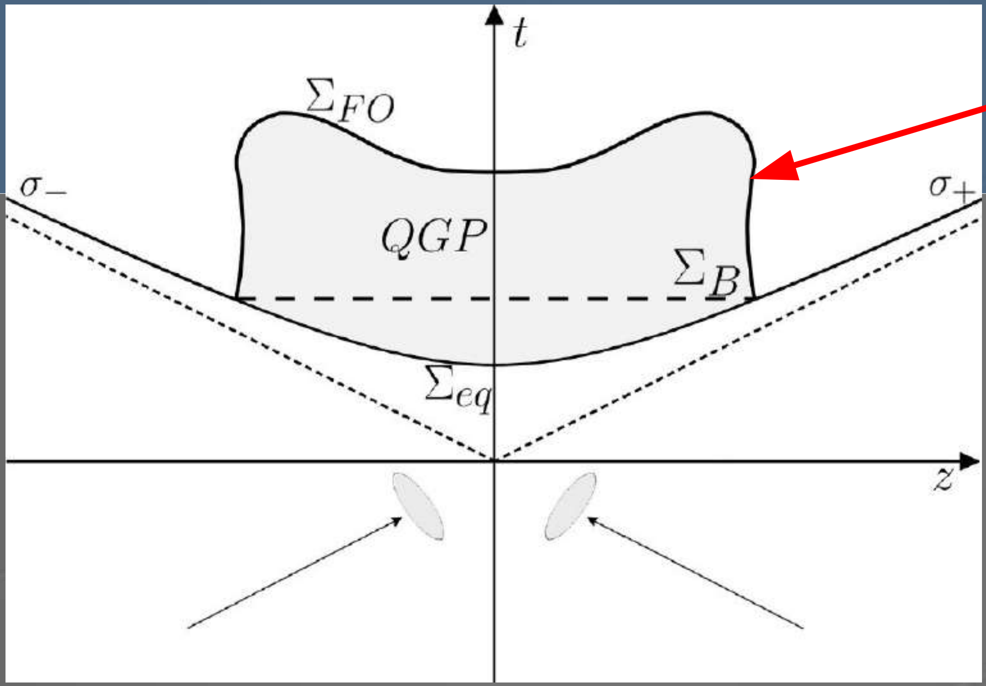
S. Liu, Y. Yin, JHEP 07 (2021) 188

Qualitative agreement found in the "strange memory" scenario



B. Fu, S. Liu, L. Pang, H. Song and Y. Yin, PRL 127 (2021) 14, 142301

Isothermal hadronization



At high energy, Σ_{FO} expected to be T= constant!

$$\beta^\mu = (1/T)u^\mu$$

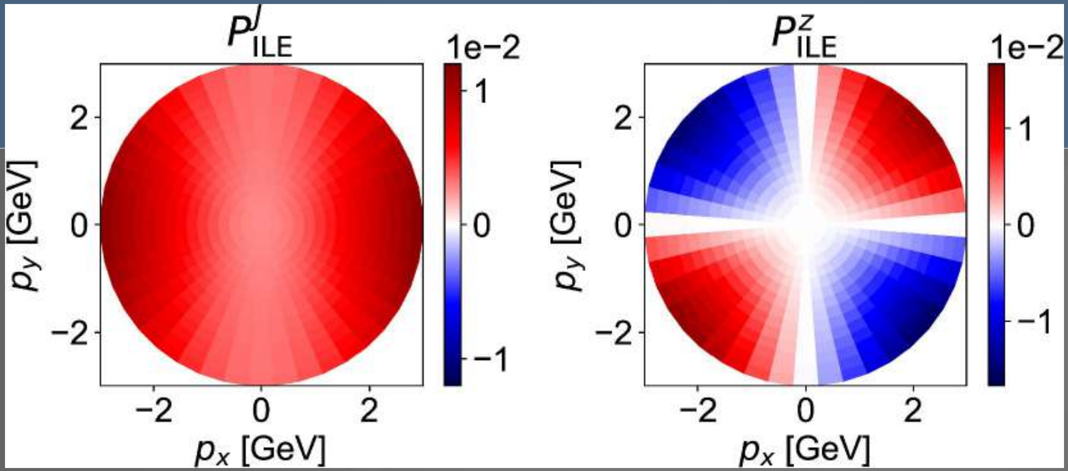
$$\widehat{\rho}_{LE} = \frac{1}{Z}\exp\left[-\int_{\Sigma_{FO}} \mathrm{d}\Sigma_\mu \widehat{T}^{\mu\nu}\beta_\nu\right] = \frac{1}{Z}\exp\left[-\frac{1}{T}\int_{\Sigma_{FO}} \mathrm{d}\Sigma_\mu \widehat{T}^{\mu\nu}u_\nu\right]$$

$$S^\mu(p) = \frac{1}{4m}\frac{\int \mathrm{d}\Sigma\cdot p\, n_F(1-n_F)\mathcal{A}^\mu}{\int \mathrm{d}\Sigma\cdot p\, n_F}$$

$$\mathcal{A}^\mu_{\rm BBP} = -\varepsilon^{\mu\rho\sigma\tau}\left(\frac{1}{2}\omega_{\rho\sigma}p_\tau + \frac{1}{E}\,\hat{t}_\rho\xi_{\sigma\lambda}p^\lambda p_\tau\right)$$

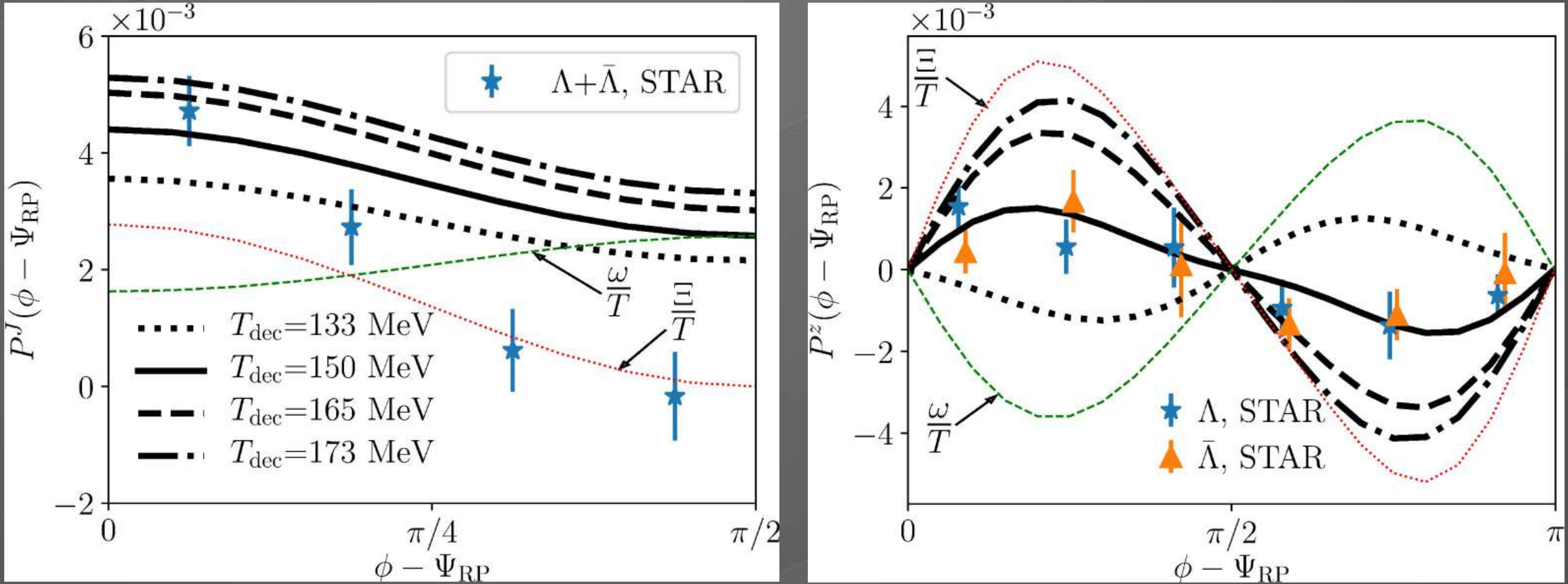
F. Becattini, M.B., A. Palermo, G. Inghirami and I. Karpenko, Phys. Rev. Lett. 127 (2021) 27, 272302

Isothermal local equilibrium: result



Apply the new formula (for primary hadrons)

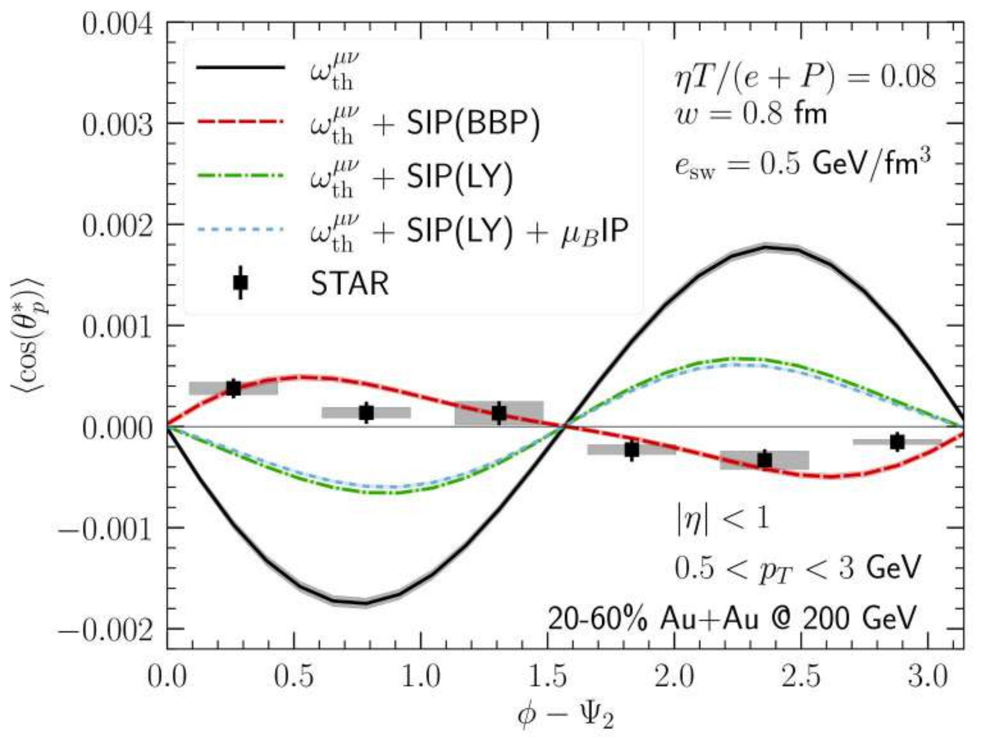
$$S^\mu_{\rm ILE}(p) = -\epsilon^{\mu\rho\sigma\tau} p_\tau \frac{\int_\Sigma {\rm d}\Sigma \cdot p\, n_F(1-n_F)\left[\omega_{\rho\sigma} + 2\,\hat{t}_\rho \frac{p^\lambda}{\varepsilon}\Xi_{\lambda\sigma}\right]}{8 m T_{\rm FO} \int_\Sigma {\rm d}\Sigma \cdot p\, n_F}$$



F. Becattini, M.B., A. Palermo, G. Inghirami and I. Karpenko, Phys. Rev. Lett. 127 (2021) 27, 272302

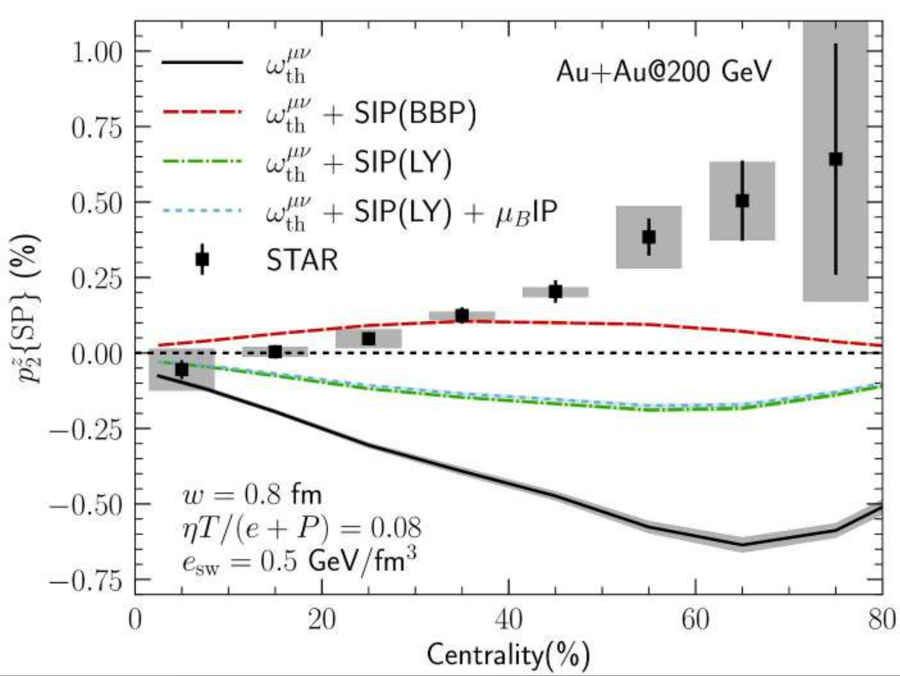
Comparison between different calculations

S. Alzhrani, S. Ryu, C. Shen, PRC 106 (2022) 1, 014905



Agreement with the BBP formula WITH thermal gradients!

Attributed to different initial conditions...



Dependence of the dominant Fourier component (sin 2phi) on centrality

Isothermal local equilibrium (ILE)

At ***high energy***, *Σ_{FO} expected to be T_{FO}= constant!* ⇨ Improved approximation

$$S^{\mu}_{\rm ILE}(p) = -\epsilon^{\mu\rho\sigma\tau} p_\tau \frac{\int_\Sigma {\rm d}\Sigma \cdot p \, n_F (1-n_F) \left[\omega_{\rho\sigma} + 2\, \hat{t}_\rho \frac{p^\lambda}{\varepsilon} \Xi_{\lambda\sigma} \right]}{8 m T_{\rm FO} \int_\Sigma {\rm d}\Sigma \cdot p \, n_F}$$

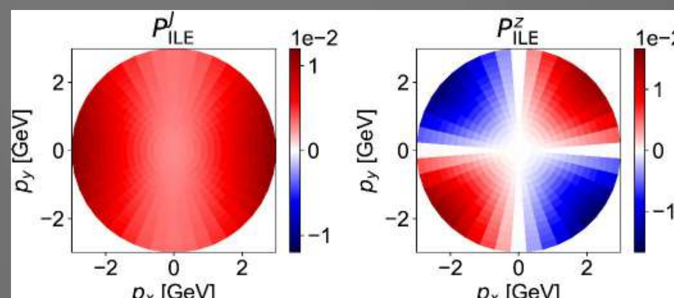
Kinematic vorticity

$$\omega_{\rho\sigma} = \frac{1}{2}\left(\partial_\sigma u_\rho - \partial_\rho u_\sigma\right)$$

Kinematic shear

$$\Xi_{\rho\sigma} = \frac{1}{2}\left(\partial_\sigma u_\rho + \partial_\rho u_\sigma\right)$$

Quantitative agreement:



F. Becattini, MB, A. Palermo, G. Inghirami and I. Karpenko, PRL 127 (2021) 27, 272302

Recent analysis of shear induced polarization:
C. Yi, S. Pu, D. L. Yang, PRC 104, 064901 (2021);
Y. Sun, Z. Zhang, C. M. Ko and W. Zhao, PRC 105 (2022) 3, 034911;
S. Ryu, V. Jupic, C. Shen, PRC 104 (2021);
S. Alzhrani, S. Ryu, C. Shen, PRC 106 (2022);
X.-Y. Wu, C. Yi, G.-Y. Qin, S. Pu, PRC 105 (2022);

- Global polarization is not significantly affected by thermal shear
- Sensitivity to initial conditions
- Isothermal equilibrium should be implemented at high energies

Pseudo-gauge in spin polarization

MB, PRC 105, 044907 (2022)

Role of pseudo-gauge transformations

F. Becattini, W. Florkowski, and E. Speranza, Phys. Lett. B 789, 419 (2019)
E. Speranza and N. Weickgenannt, Eur. Phys. J. A 57, 155 (2021)
K. Fukushima and S. Pu, Phys. Lett. B 817, 136346 (2021)
A. Das, W. Florkowski, R. Ryblewski and R. Singh, Phys. Rev. D 103, L091502 (2021)
S. Li, M. A. Stephanov and H. U. Yee, Phys. Rev. Lett. 127, 082302 (2021)
N. Weickgenannt, D. Wagner and E. Speranza, Phys. Rev. D 105 (2022) 11

Angular momentum decomposition

Noether current: Canonical form

$$\partial_\mu \widehat{T}_{\mathrm{C}}^{\mu\nu} = 0 \qquad \widehat{J}_{\mathrm{C}}^{\lambda,\mu\nu} = x^\mu \widehat{T}_{\mathrm{C}}^{\lambda\nu} - x^\nu \widehat{T}_{\mathrm{C}}^{\lambda\mu} + \widehat{\mathcal{S}}_{\mathrm{C}}^{\lambda,\mu\nu}$$

Energy momentum tensor (EMT) and spin tensor are not unique in special relativity

Pseudo-gauge transformations

F. W. Hehl, Rept. Math. Phys. 9 (1976) 55

$$\widehat{T}'^{\mu\nu} = \widehat{T}^{\mu\nu} + \frac{1}{2}\nabla_\lambda \left(\widehat{\Phi}^{\lambda,\mu\nu} - \widehat{\Phi}^{\mu,\lambda\nu} - \widehat{\Phi}^{\nu,\lambda\mu} \right)$$

$$\widehat{\mathcal{S}}'^{\lambda,\mu\nu} = \widehat{\mathcal{S}}^{\lambda,\mu\nu} - \widehat{\Phi}^{\lambda,\mu\nu} + \partial_\rho \widehat{Z}^{\mu\nu,\lambda\rho}$$

$$\widehat{\Phi}^{\lambda,\mu\nu} = -\widehat{\Phi}^{\lambda,\nu\mu}, \quad \widehat{Z}^{\mu\nu,\lambda\rho} = -\widehat{Z}^{\nu\mu,\lambda\rho} = -\widehat{Z}^{\mu\nu,\rho\lambda}$$

They leave the conservation equations and spacial integrals (=generators, or total energy, momentum and angular momentum operators) invariant

Belinfante EMT and spin tensor

From canonical choosing $\widehat{\Phi} = \widehat{\mathcal{S}}_{\mathrm{C}},\ \widehat{Z} = 0$ ⇨ $\widehat{T}_{\mathrm{B}}^{\mu\nu} - \widehat{T}_{\mathrm{B}}^{\nu\mu} = 0 \qquad \widehat{\mathcal{S}}_{\mathrm{B}}^{\lambda,\mu\nu} = 0$

What spin-tensor should we use?

Out-of-equilibrium dynamics of spin, relaxation time of spin

Hydrodynamic and kinetic theory with a spin tensor

W. Florkowski, A. Kumar, R. Ryblewski, Prog. Part. Nucl. Phys. 108 (2019) 103709;
M. Hongo, X. G. Huang, M. Kaminski, M. Stephanov, H.U. Yee, JHEP 11 (2021), 150 and JHEP 08 (2022) 263;
K. Hattori et al, Phys.Lett.B 795 (2019) 100;
S. Bhadury et al, Eur.Phys.J.ST 230 (2021) 3, 655;
D. She, A. Huang, D. Hou and J. Liao, Sci. Bull. 67 (2022);
S. Bhadury et al., Phys.Rev.D 103 (2021) 1, 014030;

What spin-tensor should we use?

- Canonical spin tensor

F. W. Hehl and Y. N. Obukhov,Fundam. Theor. Phys. 199 (2020), 217-252

$$\widehat{\mathcal{S}}_{\mathrm{C}}^{\lambda,\mu\nu} = \frac{\mathrm{i}}{8}\bar{\Psi}\left\{\gamma^{\lambda},[\gamma^{\mu},\gamma^{\nu}]\right\}\Psi$$

- de Groot-van Leeuwen-van Weert (GLW)

S. R. De Groot, Relativistic Kinetic Theory. Principles and Applications, 1980
W. Florkowski, A. Kumar, R. Ryblewski, Prog. Part. Nucl. Phys. 108 (2019) 103709

$$\widehat{\mathcal{S}}_{\mathrm{GLW}}^{\lambda,\mu\nu} = \frac{\mathrm{i}}{4m}\left(\bar{\Psi}\sigma^{\mu\nu}\overleftrightarrow{\partial}^{\lambda}\Psi - \partial_{\rho}\epsilon^{\mu\nu\lambda\rho}\bar{\Psi}\gamma^{5}\Psi\right)$$

- Hilgevoord-Wouthuysen (HW)

J. Hilgevoord, S.A. Wouthuysen, Nuclear Physics 40, 1 (1963)
E. Speranza and N. Weickgenannt, Eur. Phys. J. A 57, 155 (2021)

$$\widehat{\mathcal{S}}_{\mathrm{HW}}^{\lambda,\mu\nu} = \frac{\mathrm{i}}{4m}\bar{\Psi}\sigma^{\mu\nu}\overleftrightarrow{\partial}^{\lambda}\Psi$$

S., Dey, W. Florkowski, A. Jaiswal and R. Ryblewsky, 2303.0527
Only the canonical form satisfies the SO(3) algebra!

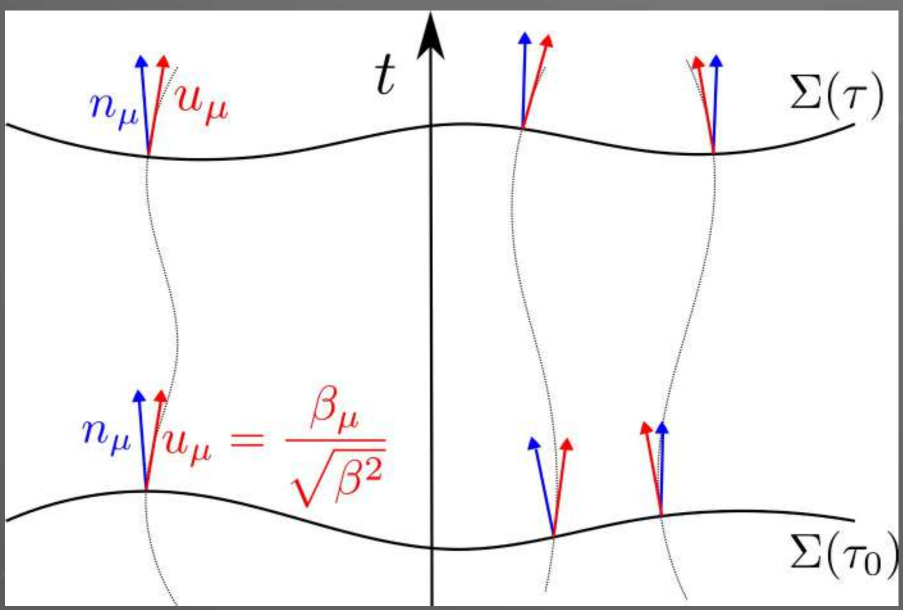
Local equilibrium density operator

The operator is obtained by maximizing the entropy $S = -\mathrm{tr}\left(\widehat{\rho}\log\widehat{\rho}\right)$
with the constraints of fixed energy momentum density

$$n_\mu \mathrm{tr}\left(\widehat{\rho}\,\widehat{T}^{\mu\nu}\right) = n_\mu T^{\mu\nu}$$

$$n_\mu \mathrm{tr}\left(\widehat{\rho}\,\widehat{\mathcal{J}}^{\mu,\lambda\nu}\right) = n_\mu \mathcal{J}^{\mu,\lambda\nu} \Longrightarrow n_\mu \mathrm{tr}\left(\widehat{\rho}\,\widehat{\mathcal{S}}^{\lambda,\mu\nu}\right) = n_\lambda \mathcal{S}^{\lambda,\mu\nu}$$

$$n_\mu \mathrm{tr}\left(\widehat{\rho}\,\widehat{j}^{\mu}\right) = n_\mu j^\mu$$



General covariant Local thermodynamic Equilibrium density operator

$$\widehat{\rho} = \frac{1}{\mathcal{Z}}\exp\left[-\int_{\Sigma(\tau_0)} \mathrm{d}\Sigma_\mu \left(\widehat{T}^{\mu\nu}\beta_\nu - \frac{1}{2}\Omega_{\lambda\nu}\widehat{\mathcal{S}}^{\mu,\lambda\nu} - \widehat{j}^\mu\zeta\right)\right]$$

$$\beta = \frac{1}{T}u \quad \zeta = \frac{\mu}{T} \quad \Omega = \text{ Spin Potential}$$

Zubarev, 1979, Ch, Van Weert 1982, see also Becattini, L. Bucciantini, E. Grossi, L. Tinti, Eur. Phys. J. C 75 (2015)
T. Hayata, Y. Hidaka, T. Noumi, M. Hongo, Phys. Rev. D 92 (2015)

Local equilibrium density operator

F. Becattini, W. Florkowski, and E. Speranza, Phys. Lett. B 789, 419 (2019)
E. Speranza and N. Weickgenannt, Eur. Phys. J. A 57, 155 (2021)

For Belinfante decomposition: $\widehat{T}_{\rm B}^{\mu\nu}$, $\widehat{\mathcal{S}}_{\rm B}^{\lambda,\mu\nu} = 0$

$$\widehat{\rho}_{\rm LTE}^{\rm B} = \frac{1}{\mathcal{Z}} \exp\left[- \int {\rm d}\Sigma_\mu \left(\widehat{T}_{\rm B}^{\mu\nu}\beta_\nu - \widehat{j}^\mu \zeta \right) \right]$$

For a general decomposition:

$$\widehat{\rho}_{\rm LTE}^{\Phi} = \frac{1}{\mathcal{Z}} \exp\left[- \int {\rm d}\Sigma_\mu \left(\widehat{T}_{\Phi}^{\mu\nu}\beta_\nu - \frac{1}{2}\Omega_{\lambda\nu}\widehat{\mathcal{S}}_{\Phi}^{\mu,\lambda\nu} - \widehat{j}^\mu \zeta \right) \right]$$

$$\widehat{T}_{\Phi}^{\mu\nu} = \widehat{T}_{\rm B}^{\mu\nu} + \frac{1}{2}\nabla_\lambda \left(\widehat{\Phi}^{\lambda,\mu\nu} - \widehat{\Phi}^{\mu,\lambda\nu} - \widehat{\Phi}^{\nu,\lambda\mu} \right), \; \widehat{\mathcal{S}}_{\Phi}^{\lambda,\mu\nu} = -\widehat{\Phi}^{\lambda,\mu\nu} + \partial_\rho \widehat{Z}^{\mu\nu,\lambda\rho}$$

We can highlight the difference:

$$\widehat{\rho}_{\rm LTE}^{\Phi} = \frac{1}{\mathcal{Z}} \exp\left\{ - \int {\rm d}\Sigma_\mu \left[\widehat{T}_{\rm B}^{\mu\nu}\beta_\nu - \frac{1}{2}\left(\varpi_{\lambda\nu} - \Omega_{\lambda\nu}\right)\widehat{\Phi}^{\mu,\lambda\nu} + \xi_{\lambda\nu}\widehat{\Phi}^{\lambda,\mu\nu} - \frac{1}{2}\Omega_{\lambda\nu}\nabla_\rho \widehat{Z}^{\lambda\nu,\mu\rho} - \widehat{j}^\mu\zeta \right] \right\}$$

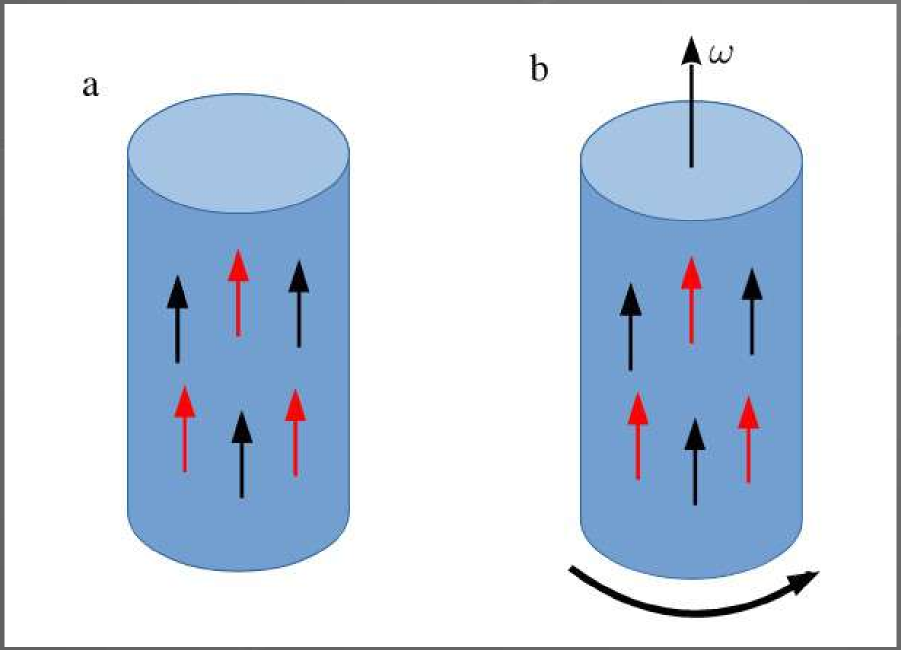
Thermal vorticity $\varpi_{\mu\nu} = -\frac{1}{2}\left(\partial_\mu\beta_\nu - \partial_\nu\beta_\mu\right)$ Thermal shear $\xi_{\mu\nu} = \frac{1}{2}\left(\partial_\mu\beta_\nu + \partial_\nu\beta_\mu\right)$

At global equilibrium the density operator does not depend on pseudo-gauge!

Different descriptions

F. Becattini, W. Florkowski, and E. Speranza, Phys. Lett. B 789, 419 (2019)

Consider a fluid temporarily at rest with a constant temperature T, hence β=(1/T)(1,0,0,0) , wherein both particles and anti-particles are polarized in the same direction.



a) Zero thermal vorticity.
Impossible with Belinfante decomposition
Possible with spin tensor

b) With thermal vorticity.
Possible with Belinfante decomposition

$\widehat{S}^{\lambda,\mu\nu} \neq 0$

"Slow" evolution of spin
Weak spin-rotation coupling

$\widehat{S}^{\lambda,\mu\nu} = 0$

"Fast" evolution of spin
Strong spin-rotation coupling

Spin polarization in other pseudo-gauges

Spin polarization predictions are pseudo-gauge dependent MB, PRC 105, 044907 (2022)

$$\widehat{\rho}^{\Phi}_{\rm LTE} = \frac{1}{\mathcal{Z}} \exp\left\{-\int \mathrm{d}\Sigma_\mu \left[\widehat{T}^{\mu\nu}_{\rm B}\beta_\nu - \frac{1}{2}\left(\varpi_{\lambda\nu} - \Omega_{\lambda\nu}\right)\widehat{\Phi}^{\mu,\lambda\nu} + \xi_{\lambda\nu}\widehat{\Phi}^{\lambda,\mu\nu} - \frac{1}{2}\Omega_{\lambda\nu}\nabla_\rho \widehat{Z}^{\lambda\nu,\mu\rho} - \widehat{j}^\mu\zeta\right]\right\}$$

- Belinfante

$$S^\mu_{\rm B}(k) \simeq S^\mu_\varpi(k) + S^\mu_\xi(k)$$

$$S^\mu_\varpi(k) = -\frac{1}{8m}\epsilon^{\mu\rho\sigma\tau}k_\tau \frac{\int_\Sigma \mathrm{d}\Sigma\cdot k\, n_{\rm F}\,(1-n_{\rm F})\,\varpi_{\rho\sigma}}{\int_\Sigma \mathrm{d}\Sigma\cdot k\, n_{\rm F}}$$

$$S^\mu_\xi(k) = -\frac{1}{4m}\epsilon^{\mu\lambda\sigma\tau}\frac{k_\tau k^\rho}{\varepsilon_k}\frac{\int_\Sigma \mathrm{d}\Sigma\cdot k\, n_{\rm F}\,(1-n_{\rm F})\,\hat{t}_\lambda\xi_{\rho\sigma}}{\int_\Sigma \mathrm{d}\Sigma\cdot k\, n_{\rm F}}$$

- Canonical spin tensor

$$S^\mu_{\rm C}(k) \simeq S^\mu_\varpi(k) + S^\mu_\xi(k) + \Delta^{\rm C}_\Theta S^\mu(k)$$

$$\Delta^{\rm C}_\Theta S^\mu(k) = \frac{\epsilon^{\lambda\rho\sigma\tau}\hat{t}_\lambda(k^\mu k_\tau - \eta^\mu_\tau m^2)}{m\varepsilon_k}\frac{\int_\Sigma \mathrm{d}\Sigma(x)\cdot k\, n_{\rm F}\,(1-n_{\rm F})\,(\varpi_{\rho\sigma}-\Omega_{\rho\sigma})}{\int_\Sigma \mathrm{d}\Sigma\cdot k\, n_{\rm F}}$$

- GLW and HW

Also confirmed in Y. C. Liu, X. G. Huang, Sci.China Phys.Mech.Astron. 65 (2022)

$$S^\mu_{\rm GLW,HW}(k) \simeq S^\mu_\varpi(k) + \Delta^{\rm C}_\Theta S^\mu(k) + \Delta^{\rm GLW,HW}_\Theta S^\mu(k)$$

$$\Delta^{\rm GLW,HW}_\Theta S^\mu(k) = -\frac{1}{8m}\epsilon^{\mu\lambda\rho\tau}\hat{t}_\lambda\frac{k_\tau k^\sigma}{\varepsilon_k}\frac{\int_\Sigma \mathrm{d}\Sigma\cdot k\, n_{\rm F}\,(1-n_{\rm F})\,(\varpi_{\rho\sigma}-\Omega_{\rho\sigma})}{\int_\Sigma \mathrm{d}\Sigma\cdot k\, n_{\rm F}}$$

Similar to N. Weickgenannt, D. Wagner, E. Speranza and D. H. Rischke, Phys. Rev. D 106 (2022), L091901

No (non-dissipative) shear induced polarization in GLW and HW!
The shear contribution can be tuned by choosing the PG.

Arbitrary pseudo-gauge

Spin polarization predictions are pseudo-gauge dependent MB, PRC 105, 044907 (2022)

$$\widehat{\rho}^{\Phi}_{\rm LTE} = \frac{1}{\mathcal{Z}} \exp\left\{ - \int {\rm d}\Sigma_\mu \left[\widehat{T}^{\mu\nu}_{\rm B} \beta_\nu - \frac{1}{2} \left(\varpi_{\lambda\nu} - \Omega_{\lambda\nu} \right) \widehat{\Phi}^{\mu,\lambda\nu} + \xi_{\lambda\nu} \widehat{\Phi}^{\lambda,\mu\nu} - \frac{1}{2} \Omega_{\lambda\nu} \nabla_\rho \widehat{Z}^{\lambda\nu,\mu\rho} - \widehat{j}^\mu \zeta \right] \right\}$$

Thermal shear Contribution

$$S^\mu_{B/C,\,\xi}(k) = -\frac{1}{4m} \epsilon^{\mu\lambda\sigma\tau} \frac{k_\tau k^\rho}{\varepsilon_k} \frac{\int_\Sigma \Sigma \cdot k \, n_{\rm F} \left(1 - n_{\rm F}\right) \hat{t}_\lambda \xi_{\rho\sigma}}{\int_\Sigma \Sigma \cdot k \, n_{\rm F}}$$

$$S^\mu_{{\rm GLW/HW},\,\xi}(k) = 0$$

$$\Delta^{\partial\Sigma} S^\mu_\xi(k) = \mp 2K \frac{1}{4m} \epsilon^{\mu\lambda\sigma\tau} \frac{k_\tau k^\rho}{\varepsilon_k} \frac{\int_\Sigma {\rm d}\Sigma \cdot k \, n_{\rm F} \left(1 - n_{\rm F}\right) \hat{t}_\lambda \xi_{\rho\sigma}}{\int_\Sigma {\rm d}\Sigma \cdot k \, n_{\rm F}}$$

- Choose a pseudo-gauge with $\widehat{\Phi}^{\lambda,\mu\nu}_{\partial\Sigma} = \frac{\rm i}{m} K \bar{\Psi} \overleftrightarrow{\partial}^\lambda \sigma^{\mu\nu} \Psi$

  where K is an arbitrary real parameter.
  You can **tune** the shear contribution by choosing K!

Pseudo-gauge dependence
Relativistic kinetic theory with spin

Trying to solve the dynamical Wigner equation for interacting fermions without the introduction of the density operator

$$\left[\gamma\cdot\left(p+\mathrm{i}\frac{\hbar}{2}\partial\right)-m\right]W_{\alpha\beta}=\hbar\,\mathcal{C}_{\alpha\beta}$$

Equilibrium ansatz

Equilibrium form of the Wigner function is an *ansatz* and for consistency it depends on the chosen pseudo-gauge.

- Wigner-function formalism, recent developments

  N. Weickgenannt, X.-L. Sheng, E. Speranza, Q. Wang, and D. H. Rischke, PRD100, 056018 (2019)
  J.-H. Gao and Z.-T. Liang, PRD100, 056021(2019)
  K. Hattori, Y. Hidaka, and D.-L. Yang, PRD100, 096011 (2019)
  Y.-C. Liu, K. Mameda, and X.-G. Huang, Chin.Phys.C 44 (2020) 9, 094101
  N. Weickgenannt, E. Speranza, X.-l. Sheng, Q. Wang, and D.H. Rischke, PRL127 (2021) 5, 052301
  X.L. Sheng, N. Weickgenannt, E. Speranza, D. H. Rischke , Q. Wang PRD104 (2021) 1, 016022

Summary and outlook

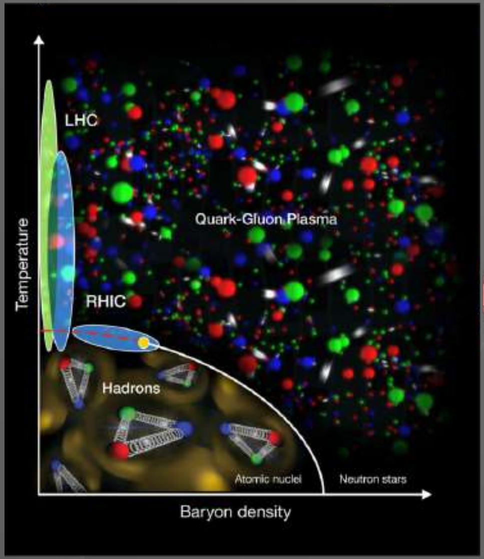
- Quantum statistical mechanics is an essential tool to properly deal with spin physics in relativistic fluids (and not just spin)
- Spin-thermal shear coupling: new unexpected, non-dissipative phenomenon, not dependent on unknown dynamical transport coefficient. Also a dissipative contribution?
- Local polarization puzzle solved by isothermal hadronization
- Important phenomenological consequences: local equilibrium ("ideal fluid" picture) seems to hold in the spin sector too!

- What is the role of pseudo-gauge transformations?

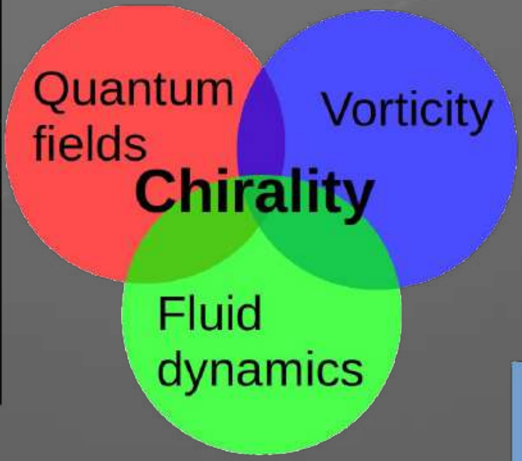
- Polarization has a great potential to pin down the initial conditions and the QGP evolution which is yet unexploited to a large extent.

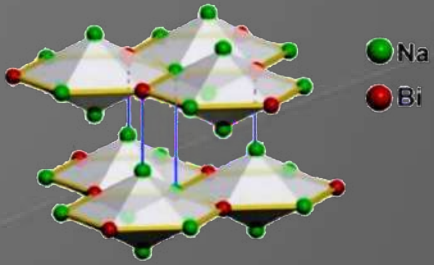
Thank you!

Nuclear physic

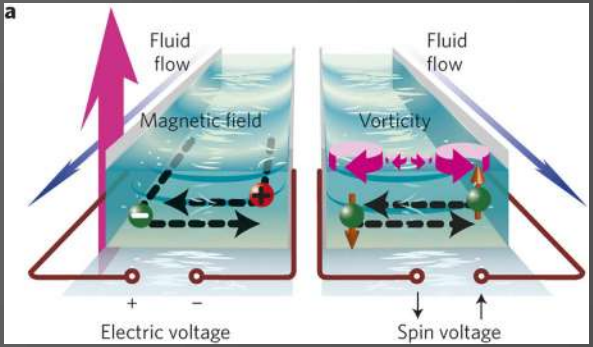


Particle physic





Condensed matter physic



Takahashi et al, Nature Physics 12, 52–56 (2016)

Applications

Cosmology

Astrophysic

Backup

What is this new term?

Does it have a non-relativistic limit? Let us decompose it

$$\xi_{\sigma\rho} = \frac{1}{2}\partial_\sigma\left(\frac{1}{T}\right)u_\rho + \frac{1}{2}\partial_\rho\left(\frac{1}{T}\right)u_\sigma + \frac{1}{2T}\left(A_\rho u_\sigma + A_\sigma u_\rho\right) + \frac{1}{T}\sigma_{\rho\sigma} + \frac{1}{3T}\theta\Delta_{\rho\sigma}$$

A is the acceleration field

$$\sigma_{\mu\nu} = \frac{1}{2}(\nabla_\mu u_\nu + \nabla_\nu u_\mu) - \frac{1}{3}\Delta_{\mu\nu}\theta$$

$$\theta = \nabla\cdot u \qquad \Delta_{\mu\nu} = g_{\mu\nu} - u_\mu u_\nu$$

All terms are relativistic (they vanish in the infinite c limit)
EXCEPT grad T terms, which give rise to

$$\mathbf{S}_\xi = \frac{1}{8}\mathbf{v}\times\frac{\int \mathrm{d}^3\mathrm{x}\, n_F(1-n_F)\nabla\left(\frac{1}{T}\right)}{\int \mathrm{d}^3\mathrm{x}\, n_F}$$

There is an equal contribution in the NR limit from thermal vorticity

Dissipative vs Non-dissipative

$$S^\mu_\xi(p) = -\frac{1}{4m}\epsilon^{\mu\nu\sigma\tau}\frac{p_\tau p^\rho}{\varepsilon}\frac{\int_\Sigma \mathrm{d}\Sigma\cdot p\, n_F(1-n_F)\hat{t}_\nu\xi_{\sigma\rho}}{\int_\Sigma \mathrm{d}\Sigma\cdot p\, n_F}$$

is a non-dissipative term

Gradient corrections, even those proportional to the shear tensor, may be non-dissipative if they stem from the gradient expansion of the LE operator.

Two important signatures:
1) they do not involve any unknown, dynamical, transport coefficient (fulfilled)
2) the two sides have the same time-reversal transformation (fulfilled)

Dissipative: correlators of operators at different time (e.g. Kubo formula):

From operator $\widehat{B}$

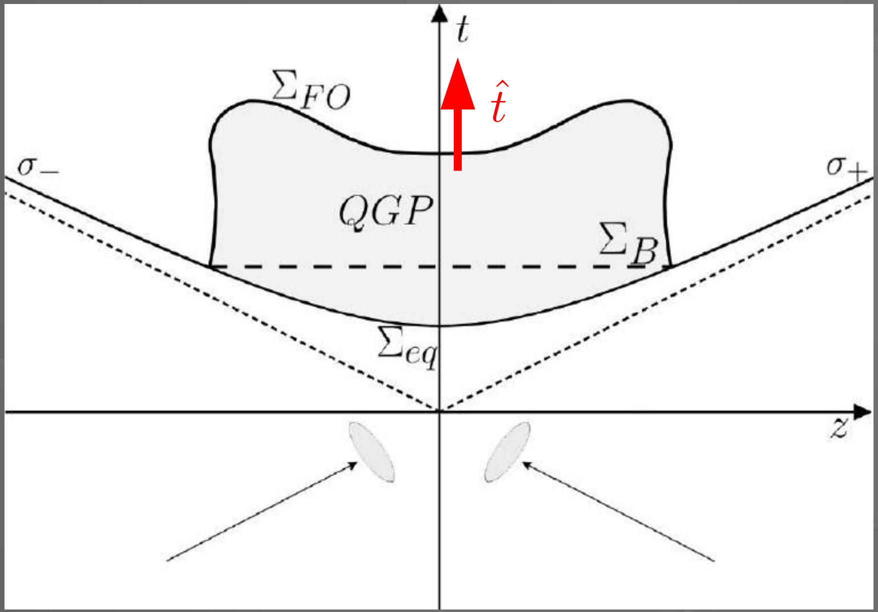
$$\left(\widehat{X},\widehat{Y}\right) \equiv n^\alpha \frac{\partial}{\partial K^\alpha}\Big|_{n\cdot k=0}\lim_{k_T\to 0}\mathrm{Im}\, iT\int_{-\infty}^{t}\mathrm{d}^4x'\,\langle[\widehat{X}(x),\widehat{Y}(x')]\rangle_{\beta(x)}\mathrm{e}^{-iK\cdot(x'-x)}$$

Non-dissipative: correlators of operator on the SAME hypersurface (or same time)

From operator $\widehat{A}$

$$\int_0^1 \mathrm{d}z\int_\Sigma \mathrm{d}\Sigma_\mu(y)\left(\langle\widehat{O}(x)\widehat{T}^{\mu\nu}(y+iz\beta(x))\rangle_{\beta(x)} - \langle\widehat{O}(x)\rangle_{\beta(x)}\langle\widehat{T}^{\mu\nu}(y+iz\beta(x))\rangle_{\beta(x)}\right)\delta\beta_\nu$$

Why do we have a dependence on Σ?



The thermal shear term depends on the correlator

$$\langle \widehat{Q}_x^{\mu\nu} \widehat{W}(x,k) \rangle$$

$$\widehat{J}_x^{\mu\nu} = \int \mathrm{d}\Sigma_\lambda \left[(y-x)^\mu \widehat{T}^{\lambda\nu}(y) - (y-x)^\nu \widehat{T}^{\lambda\mu}(y) \right]$$

$$\widehat{Q}_x^{\mu\nu} = \int \mathrm{d}\Sigma_\lambda \left[(y-x)^\mu \widehat{T}^{\lambda\nu}(y) + (y-x)^\nu \widehat{T}^{\lambda\mu}(y) \right]$$

The divergence of the integrand of $J^{\mu\nu}$ vanishes, therefore it does not depend on the integration hypersurface (it is a constant of motion) and

$$\widehat{\Lambda} \widehat{J}_x^{\mu\nu} \widehat{\Lambda}^{-1} = \Lambda_\alpha^{-1\mu} \Lambda_\beta^{-1\nu} \widehat{J}_x^{\alpha\beta}$$

The divergence of the integrand of $Q^{\mu\nu}$ does not vanish, therefore it does depend on the integration hypersurface and

$$\widehat{\Lambda} \widehat{Q}_x^{\mu\nu} \widehat{\Lambda}^{-1} \neq \Lambda_\alpha^{-1\mu} \Lambda_\beta^{-1\nu} \widehat{Q}_x^{\alpha\beta}$$

Is it the best Approximation?

The formulas we have derived are based on a Taylor expansion of the density operator

$$W(x,k)_{LE} = \frac{1}{Z}\mathrm{tr}\left(\exp\left[-\int_{\Sigma_{FO}} \mathrm{d}\Sigma_\mu(y)\left(\widehat{T}_B^{\mu\nu}(y)\beta_\nu(y) - \zeta(y)\,\widehat{j}^\mu(y)\right)\right]\widehat{W}(x,k)\right)$$

$$\beta_\nu(y) \simeq \beta_\nu(x) + \partial_\lambda\beta_\nu(x)(y-x)^\lambda + \cdots$$

$$\widehat{\rho}_{LE} \simeq \frac{1}{Z}\exp\left[-\beta_\nu(x)\widehat{P}^\nu - \frac{1}{2}(\partial_\mu\beta_\nu(x) - \partial_\nu\beta_\mu(x))\widehat{J}_x^{\mu\nu} - \frac{1}{2}(\partial_\mu\beta_\nu(x) + \partial_\nu\beta_\mu(x))\widehat{Q}_x^{\mu\nu} + \cdots\right]$$

This is generally correct, but it is an approximation after all.

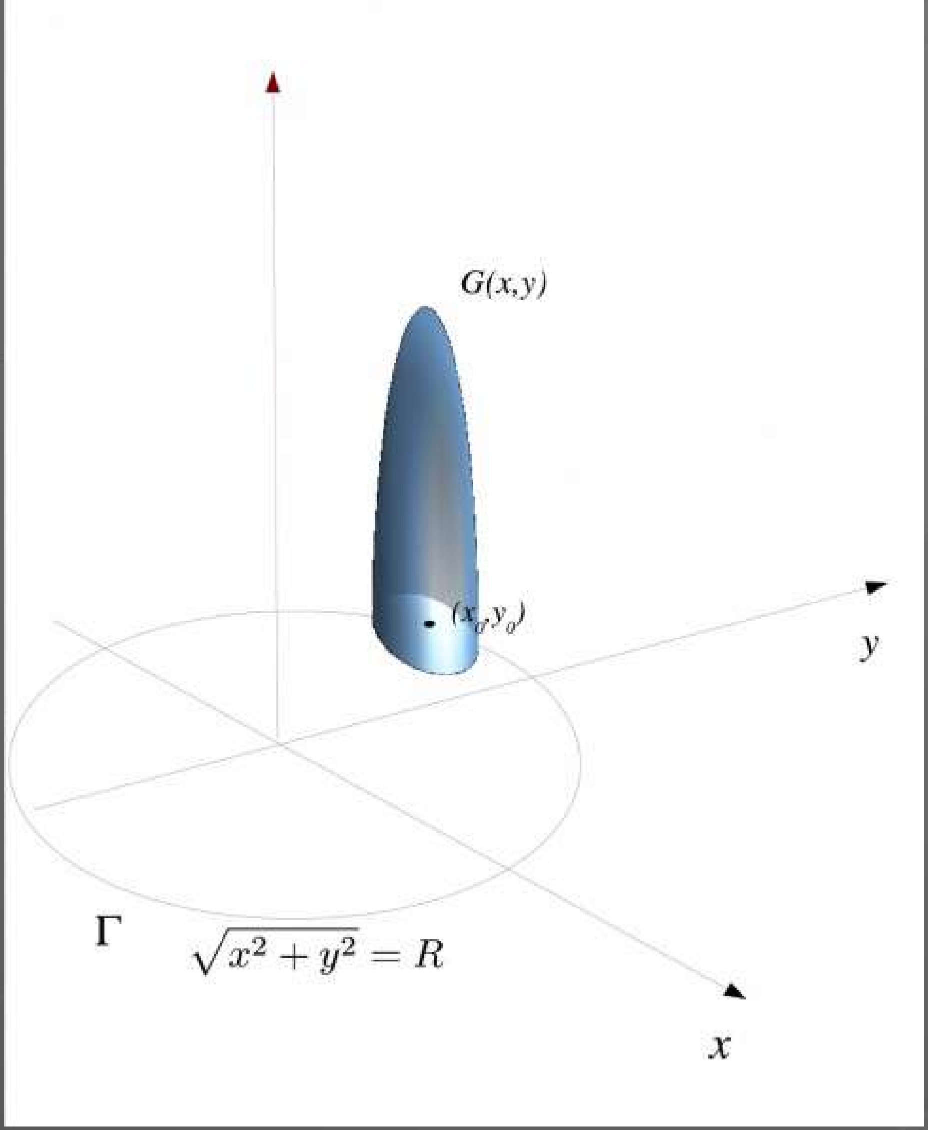
Can we find a better approximation for our specific case?

Experimental data for local polarization:

STAR, Au+Au at 200 GeV [PRL 123, 132301 (2019), Nucl. Phys. A 982, 511 (2019)]
Alice, Pb+Pb at 5 TeV S. [PRL 128, 172005 (2022)]

Understand the point: a simple example

Task: approximate the integral:



$$W = \int_\Gamma e^{\sqrt{x^2+y^2}} G(x,y)\mathrm{d}s$$

Where G(x,y) is a peaked function around the point (x_0 ,y_0) on the circle.
Since G is peaked, one can Taylor expand the exponent about (x_0 ,y_0)

$$W \simeq e^{\sqrt{x_0^2+y_0^2}} \int_\Gamma e^{x_0(x-x_0)/R+y_0(y-y_0)/R} G(x,y)\mathrm{d}s$$

$$= e^R \int_\Gamma e^{x_0(x-x_0)/R+y_0(y-y_0)/R} G(x,y)\mathrm{d}s$$

$$= e^R \int_\Gamma e^{\nabla r|_{(x_0,y_0)}\cdot(\boldsymbol{x}-\boldsymbol{x}_0)} G(x,y)\mathrm{d}s$$

But it is just pointless if we integrate over the circle!

$$W = e^R \int_\Gamma G(x,y)\mathrm{d}s$$

In the previous example, the Taylor expansion at first order introduces an undesired term:

$$W = \mathrm{e}^{R} \int_{\Gamma} G(x,y) \mathrm{d}s \qquad W \simeq \mathrm{e}^{R} \int_{\Gamma} \mathrm{e}^{\boldsymbol{\nabla} r|_{(x_0,y_0)} \cdot (\boldsymbol{x}-\boldsymbol{x}_0)} G(x,y) \mathrm{d}s$$

Exact

With gradient of r expansion

which is proportional to the gradient of the constant quantity on the circle, perpendicular to the integration line. This term does not vanish in the integration!

Similarly, for an isothermal hadronization, the inclusion of temperature gradients results in an additional, undesirable contribution proportional to the gradient of T, perpendicular to Σ_{FO}:

$$\frac{1}{2}\left[(\partial_\mu T)u_\nu(x) - (\partial_\nu T)u_\mu(x)\right] \widehat{J}^{\mu\nu}_x + \frac{1}{2}\left[(\partial_\mu T)u_\nu(x) + (\partial_\nu T)u_\mu(x)\right] \widehat{Q}^{\mu\nu}_x$$