Causality and instability of the relativistic hydrodynamics from holography

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What holography can add to turbulence?

$\Longrightarrow$ How do we compute entropy production in decaying turbulence (see G.Falkovich, part2)?

■ **Answered:** in "Holographic turbulence" by Adams-Chesler-Liu (ACL, arXiv:1307.7267)

- Imagine you have $T^{\mu\nu}$ of a theory of relativistic hydrodynamics (truncation+transport coefficients) $D+1$ spacetime dimensions:

$$T^{\mu\nu}(t,\boldsymbol{x}) = \underbrace{T^{\mu\nu}_{(0)}}_{\mathcal{O}(\partial^0 u)} + \underbrace{T^{\mu\nu}_{(1)}}_{\mathcal{O}(\partial^1 u)} + \underbrace{T^{\mu\nu}_{(2)}}_{\mathcal{O}(\partial^2 u,(\partial^1 u)^2)} + \cdots$$

- You simulate the turbulent hydro from the theory:

$$\nabla_\mu T^{\mu\nu} = 0$$

- The holographic magic is **gravity-fluid** correspondence:
  - given $T^{\mu\nu}(t, \boldsymbol{x})$ we can reconstruct geometry of a boosted black hole in D + 1 dimensions, $X = \{ t, \boldsymbol{x} = x^i, \ r\}$,

$$ds^2_{D+1} = \Sigma^2(X)\hat{g}_{ij}(X)\ dx^i dx^j + 2dt\Big[dr - A(X)\ dt - F_i(X)\ dx^i\Big]$$

where as $r \to \infty$, *e.g.*,

$$\Sigma^2(X)\hat{g}_{ij}(X) = \frac{r^2}{L^2}\left[\delta_{ij} + \frac{1}{r^D}\langle T_{ij}\rangle + \cdots\right],\ -2A = \frac{r^2}{L^2}\left[-1 + \frac{1}{r^D}\langle T_{00}\rangle + \cdots\right]$$

  - The reconstruction is more accurate the more accurate is the truncation in gradients
  - $\Longrightarrow$ large Reynolds numbers + inverse cascade $(\nabla u << T)$

- Given a geometry, we can find an **apparent (dynamical) horizon** (AH) of the black hole:

$$(\partial_t + A(X)\ \partial_r)\ \Sigma\Big|_{r_h} = -\frac{1}{2}\Sigma' F^2 - \frac{1}{D-1}\Sigma\ \nabla\cdot F\Big|_{r_h}$$

- Associate with the apparent horizon, there is a **non-equilibrium entropy density** s and an **entropy current** s^μ

$$s \equiv \underbrace{\frac{2c}{\pi}}_{\#\ DOF}\ \Sigma^{D-1}\Big|_{r_h}\,, \qquad s^\mu = s\ u^\mu\,, \qquad T^\mu{}_\nu u^\nu = -\mathcal{E} u^\mu$$

which has non-negative divergence

$$\nabla_\mu s^\mu \geq 0$$

$\Longrightarrow$ You can study what the entropy does in a turbulent regime (ACL paper)

$\Longrightarrow$ <u>To summarize</u>:

- Holography defines all-gradient relativistic hydrodynamics
- Computes all-order transport coefficients (there is some leverage in engineering the hydro)
- Gives an out-of-equilibrium definition of the entropy

$\Longrightarrow$ In this talk I want to discuss some general properties of all-gradient theories of relativistic hydrodynamics from holography

Outline:

- Instability and causality violation in first-order dissipative relativistic fluids
- Why holography?
  - Gauge theory plasma EOS
  - Transport coefficients from holography
  - Beyond hydrodynamics - QNM of black branes
- Holographic models with controlled causality violation
- All-order hydrodynamic instabilities in acausal plasma
  - Minimal boost velocity
- Conclusion and future directions

$\Longrightarrow$ <u>Hiscock and Lindblom (1985)</u>:

Consider *general* theory of 1st-order **relativistic hydrodynamics**

- EF of conserved stress-energy tensor $T^{\mu\nu}$ and a current J^μ

$$\nabla_\mu T^{\mu\nu} = 0\,, \qquad \nabla_\mu J^\mu = 0$$

- constitutive relations $(\Delta^{\mu\nu} \equiv \eta^{\mu\nu} + u^\mu u^\nu)$

$$T^{\mu\nu} = \underbrace{\mathcal{E} u^\mu u^\nu + P\Delta^{\mu\nu}}_{\mathcal{O}(\partial^0 u)} + \underbrace{\left[\tau\Delta^{\mu\nu} + q^\mu u^\nu + q^\nu u^\mu + \tau^{\mu\nu}\right]}_{\mathcal{O}(\partial^1 u)} + \underbrace{[\,\cdots\,]}_{\mathcal{O}(\partial^2 u, (\partial u)^2)} + \cdots$$

$$J^\mu = \underbrace{\rho u^\mu}_{\mathcal{O}(\partial^0 u)} + \underbrace{\nu^\mu}_{\mathcal{O}(\partial^1 u)} + \cdots$$

  - energy and charge diffusion currents $\{q^\mu, \nu^\mu\}$
  - bulk and shear viscosity stresses $\{\tau, \tau^{\mu\nu}\}$

- The 1st order-in-gradients terms are fixed from the positivity of the entropy production current

$$\mathcal{S}^\mu = su^\mu + \frac{1}{T}q^\mu - \frac{\mu}{T}\ \nu^\mu\,, \qquad \nabla_\mu \mathcal{S}^\mu \geq 0$$

$\Longrightarrow$

$$\tau = -\zeta \nabla_\mu u^\mu\,, \qquad q^\mu = -\kappa T \Delta^{\mu\nu}\left[\frac{1}{T}\nabla_\nu T + u^\lambda \nabla_\lambda u_\nu\right]$$

$$\tau^{\mu\nu} = -2\eta\ \langle \nabla^\mu u^\nu \rangle\,, \qquad \nu^\mu = -\sigma T \Delta^{\mu\nu}\nabla_\nu \left[\frac{\mu}{T}\right]$$

$\Longrightarrow$

$$\nabla_\mu \mathcal{S}^\mu = \frac{\tau^2}{\zeta} + \frac{q^\mu q_\mu}{kT} + \frac{\nu^\mu \nu_\mu}{\sigma T} + \frac{\tau^{\mu\nu}\tau_{\mu\nu}}{2\eta} \geq 0$$

– : shear and bulk viscosities $\{\eta, \zeta\}$

– : thermal conductivity κ and charge diffusion σ

must be non-negative

- First-order hydrodynamics is frame-dependent: no local definition of $\{T(x), \mu(x), u^\mu(x)\}$
- "Eckart frame": $\nu_\mu = 0$ (no charge flow in local rest frame, effectively $\sigma = 0$)
- "Landau-Lifshitz" frame (LL): $q_\mu = 0$ (no energy flow in the local rest frame, effectively $\kappa = 0$)
- one can stay in the *general* frame where $\sigma \cdot \kappa \neq 0$

$\Longrightarrow$ Consider linearized perturbations $\delta Q \propto \exp(-i\omega t + i\boldsymbol{k}\boldsymbol{x})$

- The dispersion relations are found from

$$\boldsymbol{M} \cdot \vec{Q} = 0 \qquad \Longrightarrow \qquad \det \boldsymbol{M} = (\det \boldsymbol{M}_{sound}) \cdot (\det \boldsymbol{M}_{shear})^2 = 0$$

where determinants are polynomials of $(\omega, |\boldsymbol{k}|^2)$, *e.g.*,

$$\det \boldsymbol{M}_{shear} = \mathcal{P}_{shear}(\omega, |\boldsymbol{k}|^2)$$

- In the shear channel:

$$-i\omega_{\pm} = \frac{1}{2\kappa T}\left[P + \mathcal{E} \pm \sqrt{(P+\mathcal{E})^2 + 4\eta\kappa T \boldsymbol{k}\cdot\boldsymbol{k}}\right]$$

- $\mathrm{Im}\,[\omega_+] > 0$ — the mode is always unstable in a frame with $\kappa \neq 0$; it is gapped (as $|\boldsymbol{k}| \to 0$)
- In the Landau-Lifshitz frame, ω_+ mode disappears from the spectrum ($\mathrm{Im}\,[\omega_+] \to +\infty$ as $\kappa \to 0_+$); the remaining mode is stable

$$\omega_- = -i\,\frac{\eta}{\mathcal{E}+P}\,|\boldsymbol{k}|^2 + \mathcal{O}(|\boldsymbol{k}|^4)$$

- Similar situation in the sound channel: $\kappa \neq 0 \iff$ instability; no-instabilities in LL frame

$\Longrightarrow$ So, LL frame is stable?

$\Longrightarrow$ NO!, but to see the instability one needs to look in the boosted frame

- consider a boosted frame of an equilibrium fluid flow in the direction $\boldsymbol{k}$

$$\tilde{t} = \gamma\, t - v\gamma\, x\,, \qquad \tilde{x} = -v\gamma\, t + \gamma\, x\,, \qquad \gamma = \frac{1}{\sqrt{1-v^2}}\,, \qquad v < 1$$

$\Longrightarrow$ Consider linearized perturbations $\delta Q \propto \exp(-i\tilde{\omega}\tilde{t} + i\tilde{k}\tilde{x})$

- The appropriate boosted polynomials can be determined from the unboosted one, with the corresponding Lorentz transformation on $(\omega, \boldsymbol{k})$:

$$\tilde{\mathcal{P}}(\tilde{\omega}, (\tilde{k})^2) = \mathcal{P}\Big(\omega \equiv \gamma\tilde{\omega} - v\gamma\, \tilde{k} \qquad , \qquad (k \equiv \gamma\tilde{k} - v\gamma\tilde{\omega})^2\Big) = 0$$

- The dispersion relation produces two complex roots for ω with the property:

$$\mathrm{Im}\left[\omega_1\right]\cdot\mathrm{Im}\left[\omega_2\right] = -\left[\pm\mathrm{Re}\left[\omega_1\right]-\frac{\tilde{k}}{v}\right]^2 < 0\,, \qquad \mathrm{Im}\left[\omega_1\right]+\mathrm{Im}\left[\omega_2\right] = \frac{P+\mathcal{E}}{\gamma v^2\eta} > 0$$

$\Longrightarrow$

▪ when $v \neq 0$ (no matter how small), there is always an unstable mode at finite $\tilde{k}$

▪ in the limit $v \to 0$ the unstable mode is removed from the spectrum as $\mathrm{Im}\left[w\right] \to +\infty$

- There are instabilities in the sound channel even at $\tilde{k} = 0$, as long as $v \neq 0$.

Moral of the story: sometimes we must boost the fluid to see the instability

$\Longrightarrow$ <u>Causality violation of the 1st-order relativist hydrodynamics</u>

- Consider 2nd-order relativistic conformal hydrodynamics (MIS - like, with the relaxation time τ_Π, LL frame)

$$T^{\mu\nu} = \underbrace{\mathcal{E} u^\mu u^\nu + P\Delta^{\mu\nu}}_{\mathcal{O}(\partial^0 u)} + \underbrace{\tau^{\mu\nu}}_{\mathcal{O}(\partial^1 u)} \underbrace{+ \tau_\Pi \left[\langle u^\lambda \nabla_\lambda \tau^{\mu\nu} \rangle + \frac{1}{3} \nabla_\lambda u^\lambda \ \tau^{\mu\nu} \right] + \cdots}_{\mathcal{O}(\partial^2 u, (\partial u)^2)}$$

▪ The dispersion relation of the shear channel:

$$0 = -\mathfrak{w}^2 \ \tau_\Pi T - \frac{i\mathfrak{w}}{2\pi} + \mathfrak{q}^2 \ \frac{\eta}{s}$$

where $\mathfrak{w} = \omega/(2\pi T)$ and $\mathfrak{q} = k/(2\pi T)$

▪ Speed with which a wave-front propagates out from a discontinuity in any initial data is governed by

$$\lim_{|\mathfrak{q}| \to \infty} \ \frac{\mathrm{Re}(\mathfrak{w})}{\mathfrak{q}} \bigg|_{[\mathrm{shear}]} = \sqrt{\frac{\eta}{s \ \tau_\Pi T}} \equiv v^{front}_{[\mathrm{shear}]} \leq 1$$

$$\tau_\Pi T \ \geq \frac{\eta}{s}$$

$\Longrightarrow$ reverting back to first order hydrodynamics

$$\tau_\Pi T \to 0$$

results in causality violation

<u>**Thus**</u>: 1st-order hydrodynamics (in either Eckart or LL frames) are both acausal and unstable

$\Longrightarrow$ <u>Objections:</u>

- The results are frame dependent (because we consider derivative truncation of the all-order in gradients relativistic hydrodynamics)
  - there exist frames (BDNK) where 1st order hydro is causal and stable
- We never really established that

$$\text{acausal theory} \qquad \Longleftrightarrow \qquad \text{relativistic hydro unstable}$$

- instabilities are the gapped (non-hydro modes), and thus outside the expected regime of validity of hydro
- likewise, the causality criteria

$$\lim_{|\mathfrak{q}|\to\infty} \frac{\mathrm{Re}(\mathfrak{w})}{\mathfrak{q}} < 1$$

is sensitive to large $\mathfrak{q}$, hence high-order gradients of the truncation

$\Longrightarrow$ To resolve all the above objections is the reason

Why holography?

$\Longrightarrow$ Specifically:

- We can study all-orders in gradients hydrodynamics
- We can study theories with tunable parameter that controls there causality

$\Longrightarrow$ AdS/CFT correspondence — a primer

- **conformal models**: $\mathcal{N}=4$ supersymmetric Yang-Mills theory

■ $SU(N)$ gauge theory A_μ + bosons ϕ_i + fermions ψ_a = maximally supersymmetric and scale invariant $\Longrightarrow \mathcal{L}_{SYM}[A_\mu, \psi_a, \phi_i]$

■

$$Z_{SYM}[\mathcal{M}_4] \equiv \underbrace{\int [dAd\psi d\phi] \epsilon^{i \int_{\mathcal{M}_4} d^4x \mathcal{L}_{SYM}}}_{\text{gauge theory}} = \underbrace{e^{i\ S_5[\partial\mathcal{M}_5=\mathcal{M}_4]}}_{\text{dual ``gravity'' in 5-dim}}$$

■ classical gravity approximation:

$$\begin{cases} {}'\text{t Hooft limit}: & N \to \infty\,, g_{YM}^2 \to 0 \text{ with } \lambda \equiv g_{YM}^2 N = \text{const} \\ \text{strong coupling}: & \lambda \to \infty \end{cases}$$

$\Longrightarrow$

$$S_5 = \frac{1}{16\pi G_5} \int_{\mathcal{M}_5} \mathrm{d}^5x \sqrt{-g}\,(R+12)\,, \qquad G_5 = \frac{\pi}{2N^2}$$

$$\text{SYM thermal states} \qquad \Longleftrightarrow \qquad \text{black holes of } S_5$$

- AdS-Schwarzschild black hole:

$$ds_5^2 = \frac{r_0^2}{u}\left(-(1-u^2)dt^2 + d\boldsymbol{x}^2\right) + \frac{du^2}{4u^2(1-u^2)}$$

  - $u \to 1$ BH horizon
  - $u \to 0 \;\Longleftrightarrow\; \mathcal{M}_5 \to \partial\mathcal{M}_5 = \mathcal{M}_4 = \mathbb{R}^{3,1}:, \qquad ds^2_{\mathcal{M}_4} = -dt^2 + d\boldsymbol{x}^2$

- BH temperature T and the entropy density s:

$$T = \frac{r_0}{\pi}\,, \qquad s \equiv \frac{\text{horizon area density}}{4G_5} = \frac{r_0^3}{4G_5}$$

$\Longrightarrow$ Thermal properties of BH are interpreted as thermal properties of strongly coupled $\mathcal{N} = 4$ SYM plasma (trade $r_0 \leftrightarrow T$):

- the energy density

$$\mathcal{E} = \frac{3}{8}\pi^2 N^2 T^4 = \frac{3}{4}\epsilon_{SB}$$

- the pressure

$$P = \frac{1}{8}\pi^2 N^2 T^4$$

- the entropy density

$$s = \frac{1}{2}\pi^2 N^2 T^3$$

$$\text{all} - \text{order hydrodynamic mode spectra}$$

$$\Updownarrow$$

$$\text{QNMs of AdS} - \text{Schwarzschildblack holes}$$

Specifically:

- $g_{ab} \to g_{ab}^{\text{AdS}-\text{Schwarzschild}} \left(1 + \boxed{h_{ab}(u)}\, e^{-i\omega t + ikz}\right)$
  - (scalar channel): $\{h_{xy}\}$, $\{h_{xx} - h_{yy}\}$
  - (shear channel): $\{h_{tx}, h_{xz}, h_{xr}\}$, $\{h_{ty}, h_{yz}, h_{yr}\}$
  - (sound channel): $\{h_{tt}, h_{tz}, h_{zz}, h_{xx} + h_{yy}, h_{tr}, h_{zr}, h_{rr}\}$
- in each channel, the decoupled fluctuations can be combined in a single gauge-invariant variable $Z(u)$, leading to a QNM equation:

$$0 = \frac{d^2 Z}{du^2} + C_1(u, \mathfrak{w}^2, \mathfrak{q}^2)\, \frac{dZ}{du} + C_2(u, \mathfrak{w}^2, \mathfrak{q}^2)$$

  - Dirichlet condition at AdS boundary: $\lim_{u \to 0} Z = 0$
  - Incoming-wave boundary condition at the horizon: $Z \propto (1-u)^{-i\mathfrak{w}/2}$

- Solving QNM eqs produce the spectra $\mathfrak{w} = \mathfrak{w}(\mathfrak{q})$, *e.g.*, in the hydro limit, $\{\mathfrak{w}, \mathfrak{q}\} \to 0$,
  - shear channel:
$$\mathfrak{w} = -\frac{i\mathfrak{q}^2}{2} - \frac{i(1-\ln 2)\mathfrak{q}^4}{4} + \mathcal{O}(\mathfrak{q}^6)$$
    when interpreted in LL frame hydro leads to
$$\frac{\eta}{s} = \frac{1}{4\pi}$$
  - sound channel:
$$\mathfrak{w} = \pm\frac{\mathfrak{q}}{\sqrt{3}} - \frac{i\mathfrak{q}^2}{3} \pm \frac{(3-2\ln 2)\mathfrak{q}^3}{6\sqrt{3}} + \mathcal{O}(\mathfrak{q}^4)$$
    when interpreted in LL frame hydro leads to
$$c_s^2 = \frac{1}{3}\,, \qquad \frac{\zeta}{s} = 0\,, \qquad T\tau_\Pi = \frac{2-\ln 2}{2\pi}$$

$\Longrightarrow$ **Important:** holography provides <u>full</u> spectral relation $\mathfrak{w} = \mathfrak{w}(\mathfrak{q})$, not just a few terms of hydro approximation!

- We can construct boosted black hole solution:

$$ds^2 = -2u_\mu dx^\mu dr - r^2 f(br)\; u_\mu u_\nu dx^\mu dx^\nu + r^2\; P_{\mu\nu} dx^\mu dx^\nu$$

$$f(r) = 1 - \frac{1}{r^4}\,, \qquad P_{\mu\nu} = u_\mu u_\nu + \eta_{\mu\nu}\,, \qquad u_\mu u^\mu = -1$$

where $T = \frac{1}{\pi b}$, and u_μ is a boost 4-velocity:

$$u_\mu = \gamma\left(-1, \vec{\beta}\right)\,, \qquad \gamma \equiv (1 - \vec{\beta}\cdot\vec{\beta})^{-1/2}$$

- As in boosted hydro,

$$0 = Z'' + \tilde{C}_1(u, \tilde{\mathfrak{w}}^2, \tilde{\mathfrak{q}}^2)\; Z' + \tilde{C}_2(u, \tilde{\mathfrak{w}}^2, \tilde{\mathfrak{q}}^2)$$

$$\Updownarrow$$

$$0 = Z'' + C_1(u, \mathfrak{w}^2, \mathfrak{q}^2)\; Z' + C_2(u, \mathfrak{w}^2, \mathfrak{q}^2)$$

where $(\tilde{\mathfrak{w}}, \tilde{\mathfrak{q}})$ and $(\mathfrak{w}, \mathfrak{q})$ are related by the Lorentz transformation, producing the boost (as in hydro)

$\Longrightarrow$ <u>How do we produce the theories with controlled causality violation?</u>

- Consider a CFT in curved-space time (on $\mathcal{M}_4$):

$$\langle T_\mu{}^\mu \rangle = \frac{c}{16\pi^2}\Big(\underbrace{\mathrm{Riem}^2 - 2\mathrm{Ric}^2 + \frac{1}{3}\mathrm{R}^2}_{I_4-\text{Weyl curvature}}\Big) - \frac{a}{16\pi^2}\Big(\underbrace{\mathrm{Riem}^2 - 4\mathrm{Ric}^2 + \mathrm{R}^2}_{E_4-\text{Euler density}}\Big)$$

  where $\{c, a\}$ are the central charges, and the curvatures inv are those of $\mathcal{M}_4$

- $\mathcal{N} = 4$ SYM: $c = a = \frac{N^2}{4}$

- from Hofman and Maldacena (2008): the causal conformal theories must have

$$-\frac{1}{2} \le \frac{c-a}{c} \le \frac{1}{2}$$

$\Longrightarrow$ Holography of $c \neq a$ theories:

- the Gauss-Bonnet model

$$S_5 = \frac{1}{16\pi G_5}\int_{\mathcal{M}_5} d^5x\sqrt{-g}\left[R + 12 + \frac{\lambda_{\rm GB}}{2}\left(R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\right)\right]$$

with

$$\frac{c-a}{c} = \frac{2}{\sqrt{1-4\lambda_{\rm GB}}} - 2$$

$$-\frac{1}{2} \leq \frac{c-a}{c} \leq \frac{1}{2} \qquad \Longleftrightarrow \qquad -\frac{7}{36} \leq \lambda_{\rm GB} \leq \frac{9}{100}$$

$\Longrightarrow$ We can now explore causality and hydrodynamic stability of Gauss-Bonnet models for general $\lambda_{\rm GB}$

$\Longrightarrow$ the scalar channel (Brigante-Liu-Myers-Shenker-Yaida (2008) and Buchel-Myers (2009)):

- the QNM equation:

$$0 = Z'' + C_1(u, \mathfrak{w}^2, \mathfrak{q}^2, \lambda_{\rm GB})\ Z' + C_2(u, \mathfrak{w}^2, \mathfrak{q}^2, \lambda_{\rm GB})$$

- QNM equation $\rightarrow$ 1d bound-state QM problem:

$$\frac{dy}{du} = Y(u, \lambda_{\rm GB})\,, \qquad Z \equiv \frac{1}{\mathcal{B}(u, \lambda_{\rm GB})}\ \psi$$

$\Longrightarrow$

$$-\frac{1}{\mathfrak{q}^2}\ \partial_y^2\psi(y) + \left(U_0^{scalar} + \frac{1}{\mathfrak{q}^2}\ U_1^{scalar}\right)\ \psi(y)\ =\ \frac{\mathfrak{w}^2}{\mathfrak{q}^2}\ \psi(y)$$

where $U_i^{scalar} = U_i^{scalar}(u, \lambda_{\rm GB})$

$\Longrightarrow$ Causality:

$$\lim_{\mathfrak{q}\to\infty} \frac{\mathrm{Re}\,[\mathfrak{w}]}{\mathfrak{q}} \equiv \alpha \leq 1$$

- Effective $\hbar \equiv \frac{1}{\mathfrak{q}}$; $\hbar \to 0$,

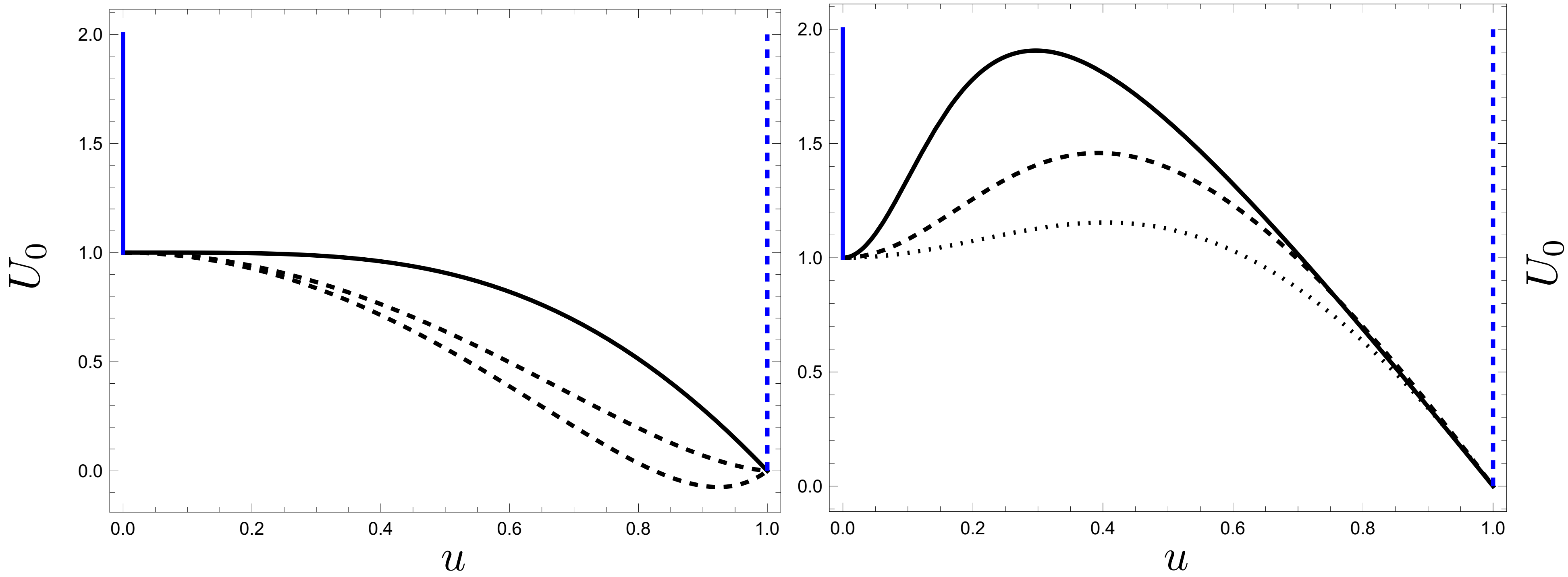
$$-\hbar^2\ \partial_y^2 \psi(y) + U_0\ \psi(y) = E\ \psi(y)\,, \qquad E \equiv \alpha^2$$

where $\hbar^2 U_1$ can be ignored, except as $u \to 0$, where its role is to set an impenetrable wall

- For **causality violation** we search for bound state with

$$\mathrm{Re}\,[E] > 1$$

- $U_0(u, \lambda_{\mathrm{GB}})$ crucially depends on λ_{GB}:



$0 < \lambda_{\mathrm{GB}} \leq \frac{9}{100}$

$\frac{1}{4} > \lambda_{\mathrm{GB}} > \frac{9}{100}$

NO bound state with $E > 1$

NO causality violation

YES bound state with $E > 1$

YES causality violation

$\Longrightarrow$ Instability in the boosted frame (boost velocity v):

$$\mathrm{Im}\,[\tilde{\mathfrak{w}}] > 0$$

- We search for modes with (real) $\Gamma > 0$

$$\left(\tilde{\mathfrak{w}} \equiv i\Gamma + 0\,, \tilde{\mathfrak{q}}^2 = 0\right) \qquad \Longleftrightarrow \qquad \left(\mathfrak{w} \equiv \frac{i\Gamma}{\sqrt{1-v^2}}\,, \mathfrak{q}^2 = -\frac{\Gamma^2 v^2}{(1-v^2)}\right)$$

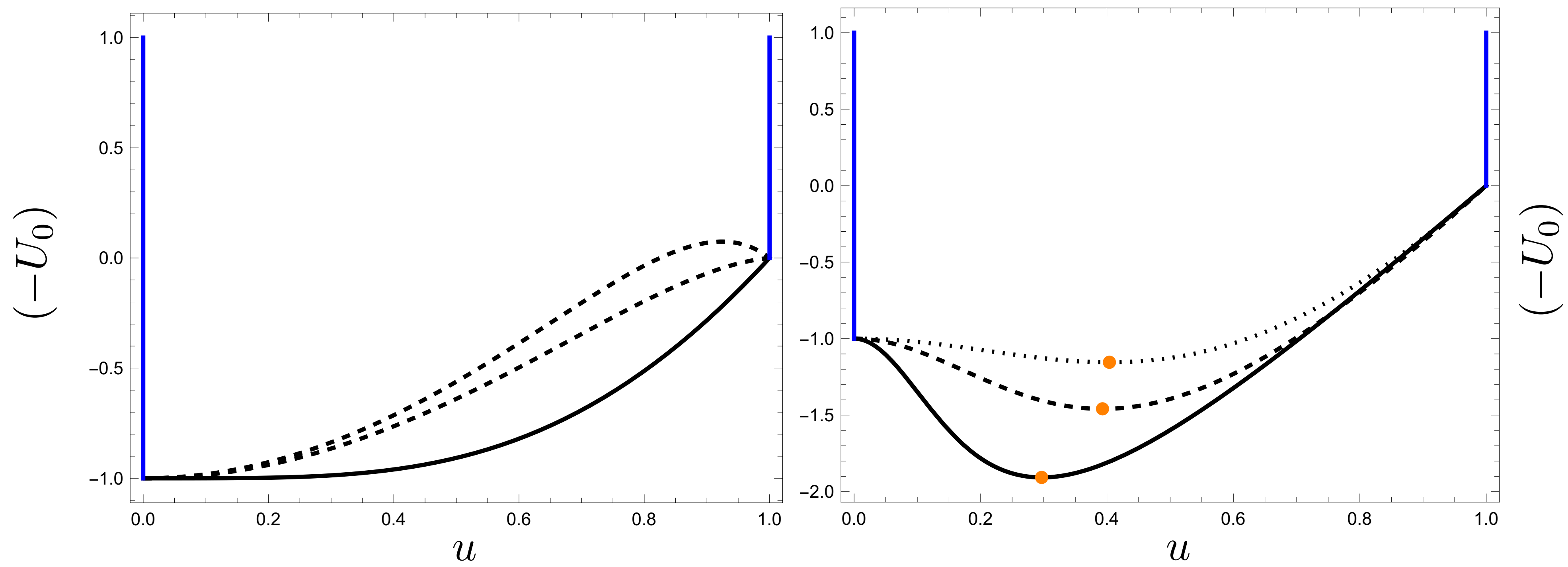
- Effective $\hbar^2 \equiv -1/\mathfrak{q}^2$; consider $\hbar \to 0$ ($\Gamma \to +\infty$ or $v \to 0$)

$$-\hbar^2\, \partial_y^2 \psi(y) + \Big(-U_0\Big)\, \psi(y) = E\, \psi(y)\,, \qquad E \equiv -\frac{1}{v^2}$$

- For **instability** we search for bound state with

$$E < -1$$

- note that in this problem the 1D QM potential is **minus** the potential of the causality problem!



$$0 < \lambda_{\rm GB} \leq \frac{9}{100} \qquad\qquad \frac{1}{4} > \lambda_{\rm GB} > \frac{9}{100}$$

NO bound state with $E < -1$ **YES** bound state with $E < -1$

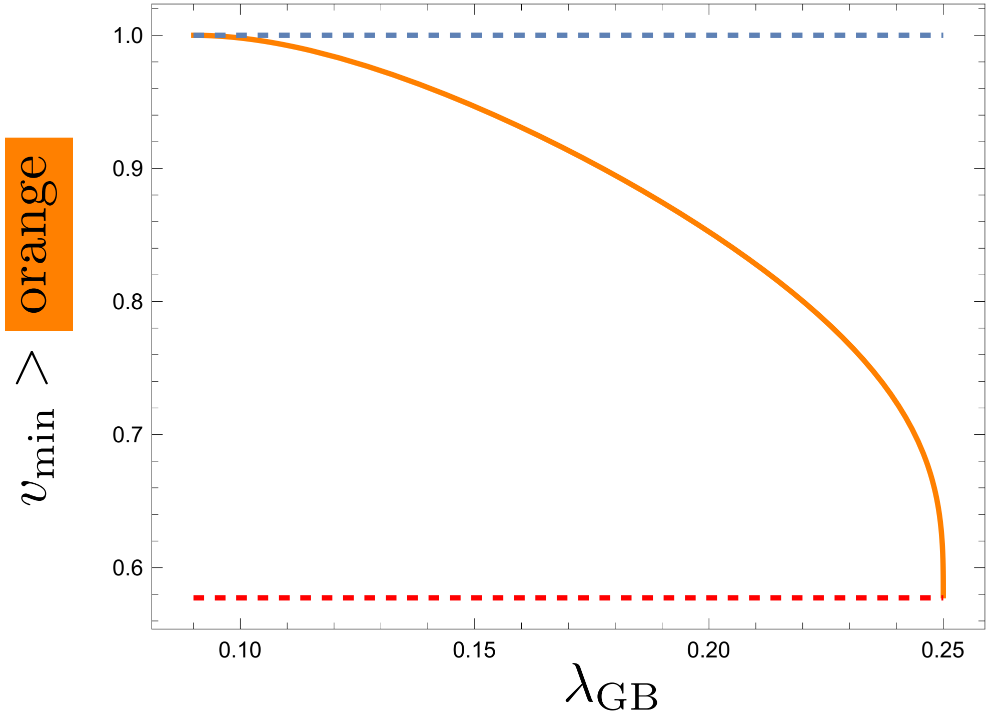
NO instability **YES** instability

$\Longrightarrow$ If the bound state ($\equiv$ the instability) exists, it must be

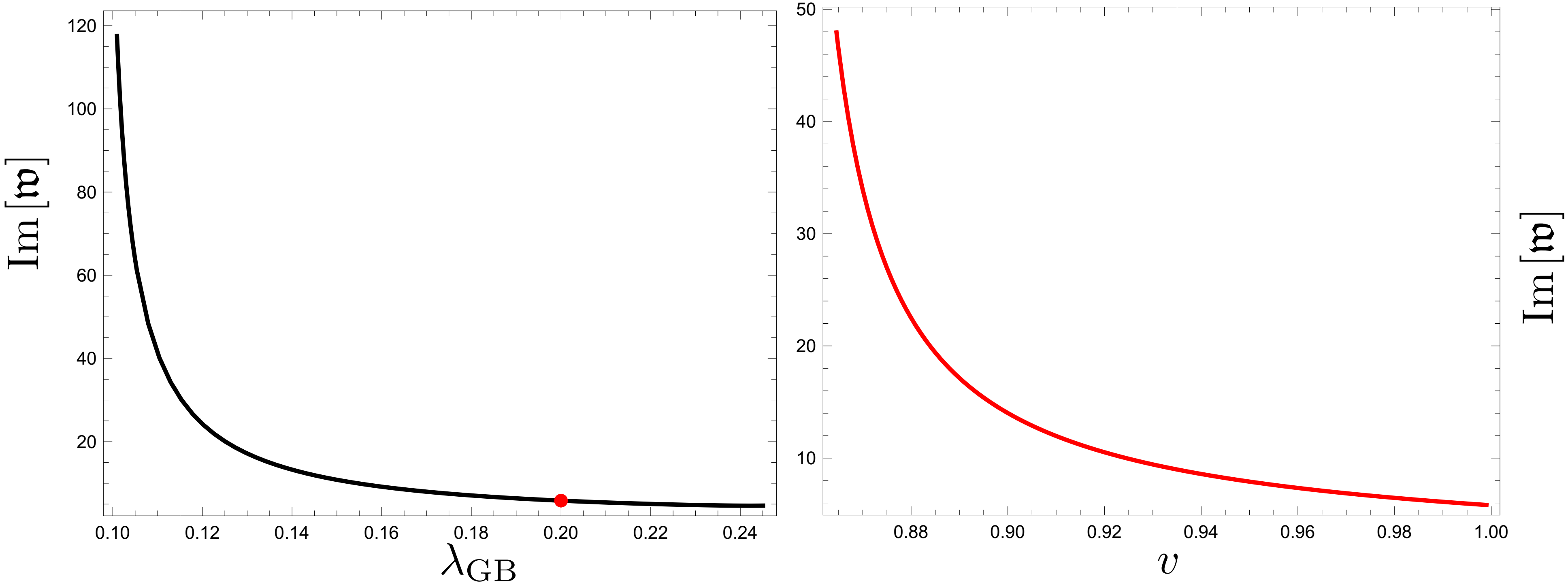
$$E\Big|_{bound\ state} = -\frac{1}{v^2} > \min_u[-U_0]$$

$\Longrightarrow$

$$v > v_{\min}(\lambda_{\mathrm{GB}}) > \frac{1}{\sqrt{-\min_u[-U_0]}}$$



$\Longrightarrow$ Explicit construction of boost unstable QNMs:



The left panel: unstable QNMs of the metric fluctuations in the helicity-2 (the scalar) sector of the boosted GB black branes with the boost velocity $v = 0.999$. The right panel: the boost velocity dependence of the red dot of the left panel.

Conclusions and future directions:

- We established a precise correspondence between all-derivative causality-violating relativistic theories of hydrodynamics and instabilities of the corresponding fluids
- All channels (scalar, shear and sound) must be considered to fully identify acausal/unstable regime
- Instability of acausal theories can be seen only if the equilibrium plasma fluid is sufficiently boosted

$\Longrightarrow$

- ▪ A particular class of discussed holographic models exhibit causality violation in scattering of plasma shock-waves for **any finite** $\lambda_{\rm GB}$ (Camanho-Edelstein-Maldacena-Zhiboedov (2014))
- ▪ The only way to fix the causality of such theories is to add additional fields of spin $J > 2$ (go to String Theory!)
- ▪ **Question:** Can such causality violation can also be seen as an instability?