Threshold states in nuclei
with Phenomenological R-Matrix

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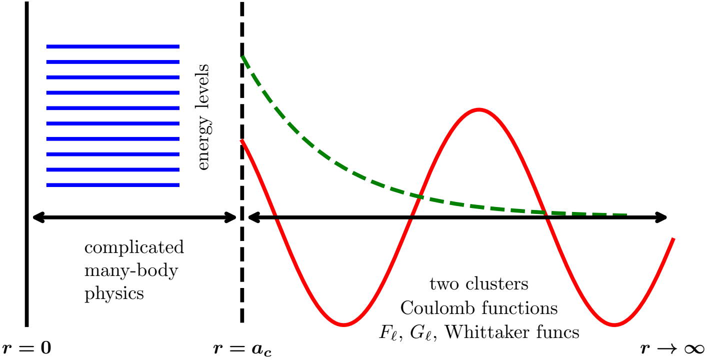
Outline

- R-matrix effects near thresholds
- Recent developments and test of phenomenological R-matrix

Basic Idea of the R-Matrix Approach



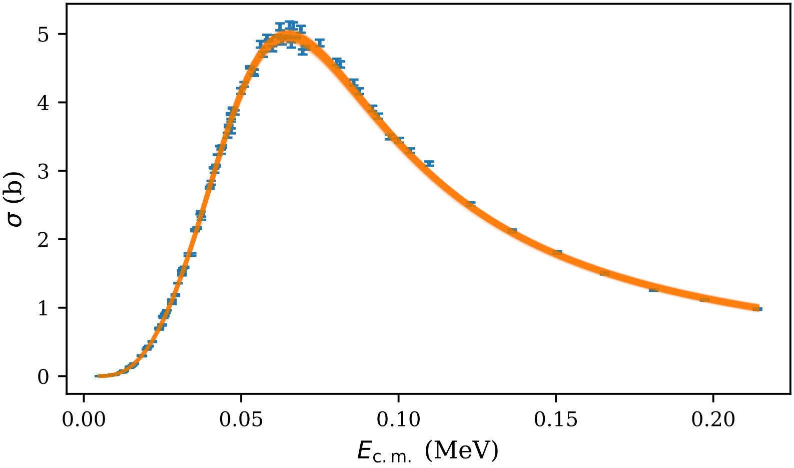
- Inside the channel radii, a basis of states $|\lambda\rangle$ is used.
- The basis must be truncated, depending on the range of excitation energy under consideration, available constraints,...
- Bound and scattering states enter on the same footing – an ideal tool for threshold physics!

R-matrix effects near thresholds

Let me start with a statement:

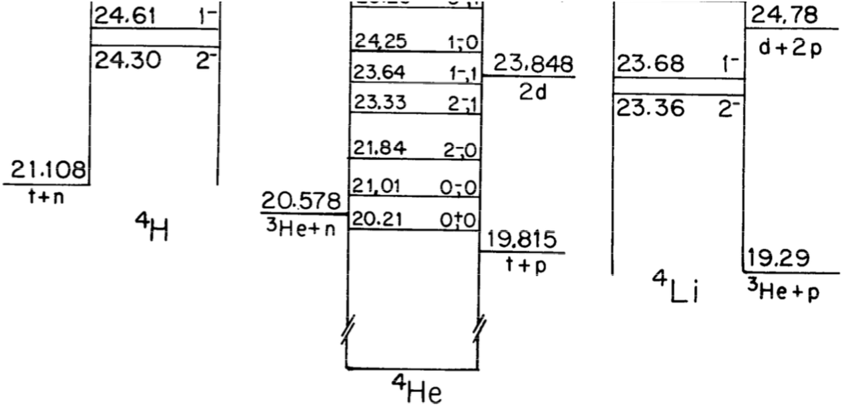
There is an enhanced probability for a single-particle or cluster state to lie near the threshold for that channel configuration.

Example: $^3\mathrm{H} + {}^2\mathrm{H} \to {}^4\mathrm{He} + n$



Odell, Brune, & Phillips (2022) https://doi.org/10.1103/PhysRevC.105.014625

Example: neutron detection with $^3He(n,p)^3H$



http://www.tunl.duke.edu/nucldata/

Similar scenarios are found for $^6Li(n,t)^4He$ and $^{10}B(n,\alpha)^7Li$. All three have thermals cross sections of 1000s of barns and are of great practical importance.

Astrophysical examples

- The famous α-capture reactions:
  - $3\alpha \rightarrow {}^{12}$C via the Hoyle state.
  - ${}^{12}C(\alpha,\gamma){}^{16}O$ enhancement.
- But wait, there's more: ${}^{14}N(p,\gamma){}^{15}O$, ${}^{13}C(\alpha,n){}^{16}O$, ${}^{20}Ne(p,\gamma){}^{21}Na$,...
- However, it not a sure thing: ${}^{16}O(\alpha,\gamma){}^{20}Ne$ is not enhanced by near-threshold resonances.

What is going on here?

- There are many ways of looking at this effect. I am going to emphasize the phenomenological R-matrix approach.
- The phenomenological R-matrix theory of Wigner, Lane, Thomas *et al.* is a "go-to" method for describing near-threshold resonances, interference effects, etc...
- Long-ranged Coulomb interacts and coupling to the the continuum are explicitly taken into account.
- It does not, however, explicitly model many-body dynamical effects.

PROC. PHYS. SOC., 1964, VOL. 84

A model for nuclear threshold levels

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MS. received 30*th June* 1964

Abstract. The observation of some levels in light nuclei closer to thresholds than one might expect from average level densities suggests the investigation of models which might produce such an effect. One model based on the *R*-matrix theory of nuclear reactions is shown to predict that near thresholds of channels with small barriers there should be an increased density of levels with large reduced widths. The experimental evidence for such an effect is meagre.

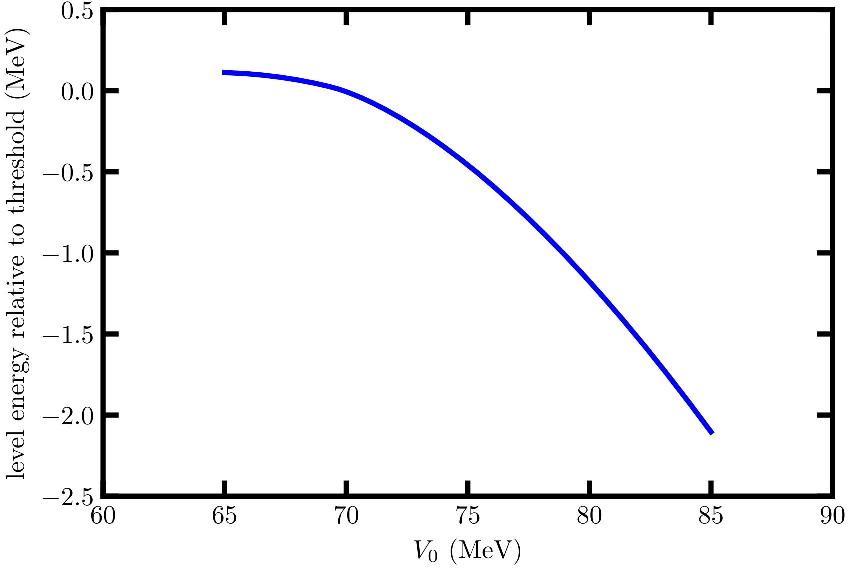
What is the idea here?

- Energy levels have a boundary condition requirement outside the nuclear surface: exponential decay (bound states) or some version of an outgoing wave (unbound states).
- Near a threshold, this condition is strongly non-linear in energy and leads an "attraction" of levels to the threshold. The effect may also be termed "excitation energy compression."
- The effect is strongest for low orbital angular momentum and when the wavefunction has a large component in the two-body channel which has a nearby threshold.

This can also be studied using a potential model

- As an example, I have studied $\ell = 0$ $^3\text{H} + {}^2\text{H}$ using a Woods-Saxon potential. Specifically, one can see how the level energy, relative to the threshold, depends on potential parameters.
- Here, I vary the depth, but you would see the same thing with the radius or diffuseness.
- One sees a non-linear dependence near the threshold that corresponds to compression.
- See also: *Behaviour of Single-Particle States and Their Eigenvalues near Zero Energy*, A.M. Lane and P.E. Hodgson, Annals of Physics **106**, 44-78 (1977).

Level energy versus potential depth



Further Remarks

- It is easy to lose sight of this compression if one is just doing phenomenology.
- The Thomas-Ehrman shift has the same physical origin.
- This effect is also closely related to the change of basis between the Wigner-Lane-Thomas boundary condition (constant logarithmic derivative) and the energy level boundary condition.
- There may also be additional dynamical enhancements.
- When estimating nuclear reaction rates via Monte Carlo methods, caution should be employed, as this effect non-statistical.

Testing the phenomenological R-matrix

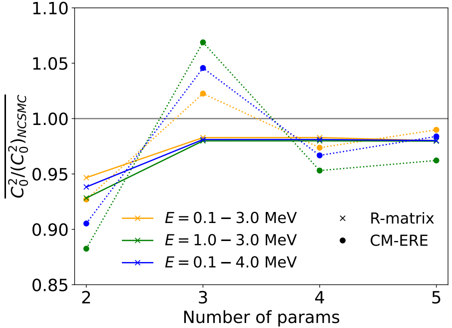
The phenomenological R-matrix requires the use of channel radii that enclose most, but not all, of the nuclear interactions. Background poles are also necessary because of truncation. How does one test these aspects of the theory?

- Ab initio calculations for light nuclear systems are now available, e.g. for $\alpha + d$ scattering and the ^{6}Li bound state. These are in some sense "perfect data."
  - What are the limiting factors on fit accuracy?
  - Can bound state properties be extracted from scattering data?

- The ^{6}Li bound state Asymtotic Normalization Constant (ANC) determined by fitting experimental scattering data does not agree with current ab initio theoretical predictions [Hebborn *et al.*, **PRL** 129, 042503 (2022) `https://doi.org/10.1103/PhysRevLett.129.042503`].

Testing the phenomenological R-matrix, continued

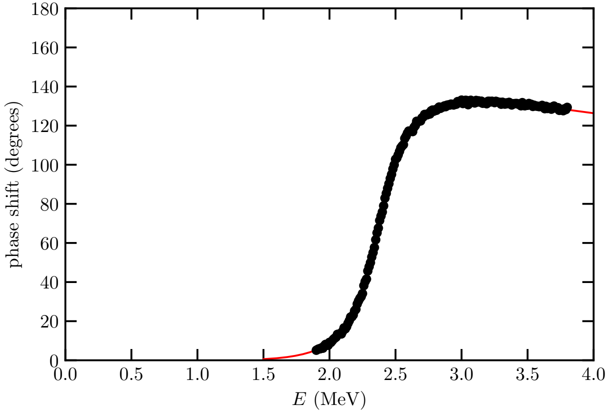
We recently did this: see Hebborn, Brune, and Phillips, J. Phys. G **53** 025101 (2026) https://doi.org/10.1088/1361-6471/ae38fc . We fitted $\ell = 0$ $\alpha + d$ phase shifts from 11 ab initio No-Core Shell Model with Continuum (NCSMC) caclulations to determine the ANC. Both phenomenological R-matrix and effective range theory methods were used. Both approaches were found to yield the "correct" ANCs – those calculated directly by NCSMC.



Alternative basis

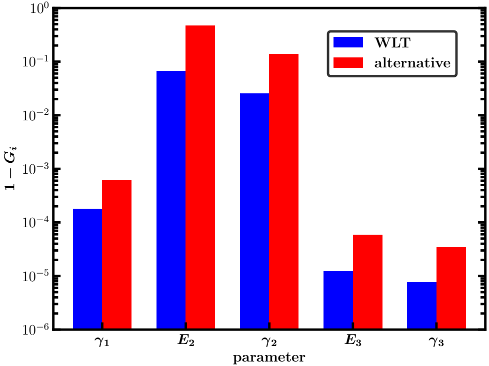
- Over 20 years ago, I showed that an alternative basis using $b_c = S_c(E)$ boundary conditions was practical to use: Phy. C **66**, 044611 (2002) https://doi.org/10.1103/PhysRevC.66.044611 .
- It is mathematically equivalent to the WLT formalism.
- Level shifts are eliminated, so that exact excitation energies can be easily implemented.
- Interpretation of fit parameters in terms of excitation energies, partial widths, and/or ANCs is straightforward.
- Parameter correlations are reduced. This feature is advantageous for any fitting and/or random-walk parameter search algorithm.

Demonstration of reduced correlations



- "Experiment" with fitting 191 pseudodata $^{12}\mathrm{C} + \alpha\ \ell = 1$ phase shifts.
- Use a three-level R-matrix fit, with the energy of the subthreshold state fixed.

Comparison of Correlations: WLT to Alternative



- Results for the Global Correlation Coefficient

$$G_i = \sqrt{1 - 1/(V_{ii}W_{ii})},$$

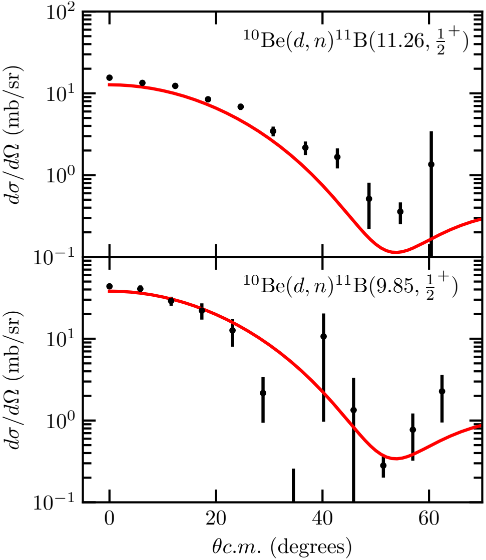
where V is the error matrix and $W = V^{-1}$ and $0 \leq G_i \leq 1$.

Bayesian versus χ^2 fitting

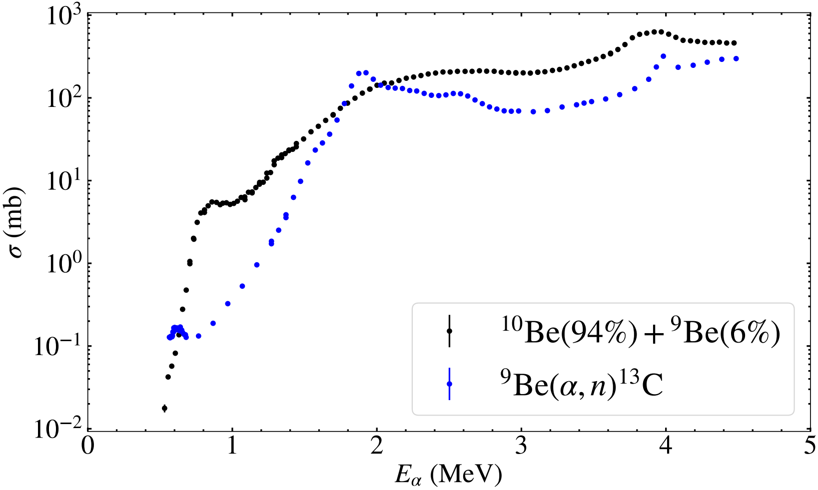
- Bayesian:
  - fine-grained modeling of uncertainties
  - simple error estimation for any calculated quantity
  - computationally demanding
- χ^2 minimization:
  - essentially limited to uncorrelated Gaussian uncertainties
  - emphasis on goodness of fit

^{10}Be$(d,n)^{11}$B cross sections at $E_d = 4$ MeV

- States at $E_x = 9.85$ and 11.26 MeV in ^{11}B are strongly populated.
- Both appears to be $\ell_p = 0$, $J^\pi = 1/2^+$ states.
- In the case of the 9.85-MeV state, the TUNL (2012) compilation lists it as a $3/2^+$ state.



Preliminary (α, n) cross sections on ^{9}Be and ^{10}Be



Threshold states in light nuclei remains an interesting topic
and are important in astrophysics
and other applications

Thank you for your attention.

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U.S. DEPARTMENT OF ENERGY
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The “World Data” for the 9.85-MeV state are puzzling

- Cusson (1966), ^{7}Li(α, α), determined
  $E_x = 9.88(2)$ MeV, $\Gamma_{\text{tot}} = 130(30)$ keV and $J^\pi = (3/2^+)$.

- Van der Steenhoven *et al.* (1988), ^{12}C$(e, e'p)^{11}$B* determined
  $E_x = 9.820(25)$ MeV and $J^\pi = (1/2^+)$.

- The TUNL (2012) compilation provides
  $E_x = 9.873(4)$ MeV, $\Gamma_{\text{tot}} = 109(14)$ keV, and $J^\pi = 3/2^+$.

- Refsgaard *et al.* (2019), ^{11}Be$(\beta\alpha)$ determined $E_x = 9.850$ MeV, $\Gamma_\alpha = 240$ keV, and $\Gamma_{\alpha'} = 21$ keV.

- Our work indicates $J^\pi = 1/2^+$, with E_x and $\Gamma_{\text{tot}} = \Gamma_\alpha + \Gamma_{\alpha'}$ consistent with Refsgaard *et al.*.

Comments on the 9.85-MeV state

- Both Cusson (1966) Refsgaard *et al.* (2019) remarked that it was not possible to describe Γ_α for this state with with a reduced α width below the Wigner limit, assuming conventional channel radii.
- We have just performed further measurements of ^{7}Li(α, α) around this resonance ($E_\alpha \approx 1.9$ MeV) to confirm the Cusson data. This is part of the Ph.D. project of Bikash Chauhan. Measurements have also been conducted at higher energies, in collaboration with the University of Notre Dame.