Phenomenological R-Matrix
Introduction

Carl R. Brune

Ohio University

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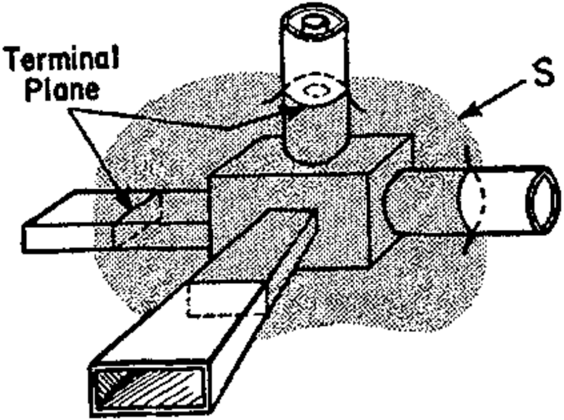
What is the Phenomenological R-matrix ?

- A way to describe two-body nuclear reactions using adjustable parameters associated with nuclear energy levels.

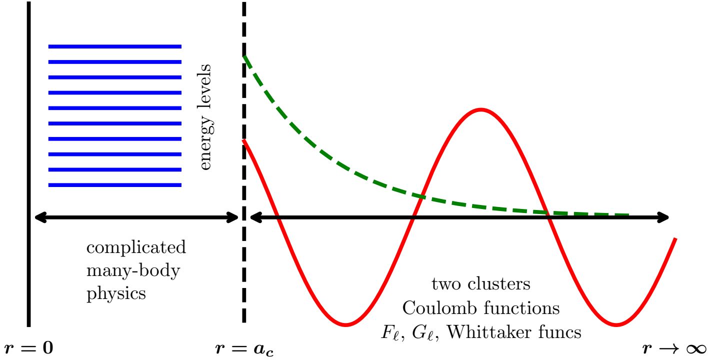
- As much quantum mechanics as possible is put in:.
  - Angular momentum and parity conservation.
  - Long-range Coulomb interaction.
  - Probability conservation (unitarity).
  - Time-reversal invariance.

An Intuitive Picture for Nuclear Reactions



"A transmission line (waveguide) junction"
E.P. Wigner, *Nuclear Reactions and Level Widths*,
American Journal of Physics **17**, 99-109 (1949).

Basic Idea of the R-Matrix Approach



- Inside the channel radii, a basis of states $|\lambda\rangle$ is used.
- The basis must be truncated, depending on the range of excitation energy under consideration, available constraints,...
- Bound and scattering states enter on the same footing – an ideal tool for threshold physics!

Phenomenological R Matrix approach

- Adjust E_λ and $\gamma_{\lambda c}$ to describe data! The $\gamma_{\lambda c}$ are projections of the wavefunction on to a particular channel configuration at the channel radius.
- These parameters are (mostly) related to to the level energies partial widths of those levels.
- Interestingly, observable quantities, i.e., the S matrix, only depend upon a few properties of discrete eigenfunctions: the energy eigenvalues and the amplitudes at the channel radii.
- The number of levels and channels must be severely truncated.
- Fortunately, the truncations in levels and channels do no destroy the unitarity and time-reversal invariance properties of the S matrix. This is a key reason why the phenomenological R-matrix approach is so useful.

When and Why to use Phenomenological R-matrix

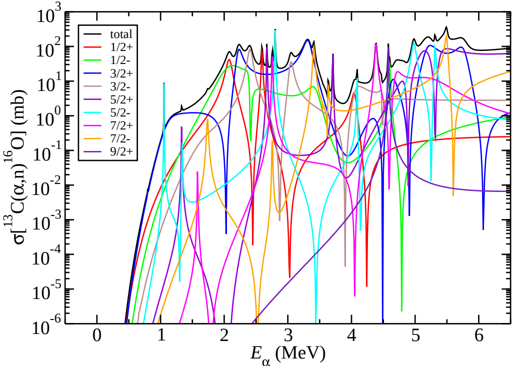
- Ab initio or other theoretical approaches may
  - lack the desired precision
  - not be possible
- Parametrization of data for astrophysics, etc...
- Extrapolation / interpolation of data into regions without data
- When dealing with resonances
  - particularly when dealing with more than one (but not *too* many)
  - particularly when resolved with widths are non-negligible
    - $\rightarrow A < 25$
    - $\rightarrow$ neutron-induced reactions at low energies
- Low energies (few channels)
- When incorporating information from multiple sources:
  - cross section data
  - spectroscopic information (excitation energy, spin,...)
  - transfer reactions (ANCs, spectroscopic factors)
  - theoretical calculations

Summary of Approximations

- A channel radius that is not "too large" must be adopted.
- Channels are assumed to be orthogonal beyond the channel radius.
- The number of levels must be truncated.
- The number of channels must be truncated.

The $^{13}C(\alpha, n)^{16}O$ Reaction

an example of the complexity that can be handled



- Important s-process neutron source and, via the time-reverse reaction, a neutron sink water-moderated power reactors.
- Partial wave decomposition of the $^{13}C(\alpha, n)$ cross section, as implemented in the 2018 ENDF/B-VIII.0 evaluation by Hale and Paris (LANL) using their `EDA` code.

R-matrix versus phenomenological alternatives

- Effective Range Theory and the modified K-matrix are alternatives.
- Effective Range Theory is not a natural tool for resonances.
- Most of the advantages of R-matrix are that it is related to a basis:
  – parameters can be connected to wave functions
  – truncation is easier to evaluate
  – isospin and mirror symmetry can be implemented
  – perturbation theory can be applied:
  γ decays, β decays, transfer reactions,...