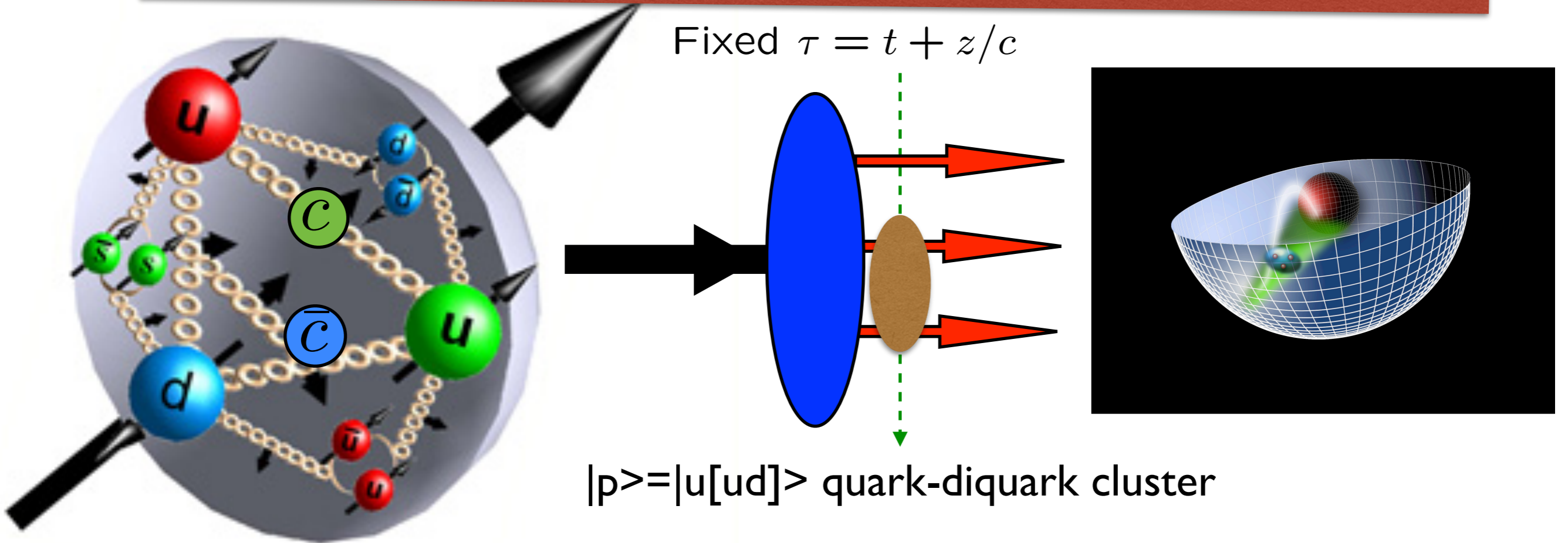


# Supersymmetric Properties of Hadron Physics, Color Transparency, Intrinsic Heavy Quarks, and other Novel Features of QCD



with Guy de Tèramond, Hans Günter Dosch, Marina Nielsen, Ivan Schmidt, F. Navarra, Jennifer Rittenhouse West, G. Miller, Keh-Fei Liu, Tianbo Liu, Liping Zou, S. Groote, S. Koshkarev, Xing-Gang Wu, Sheng-Quan Wang, Cedric Lorcè, R. S. Sufian, A. Deur, R. Vogt, G. Lykasov, S. Gardner, S. Liuti



Stan Brodsky

SLAC

NATIONAL  
ACCELERATOR  
LABORATORY

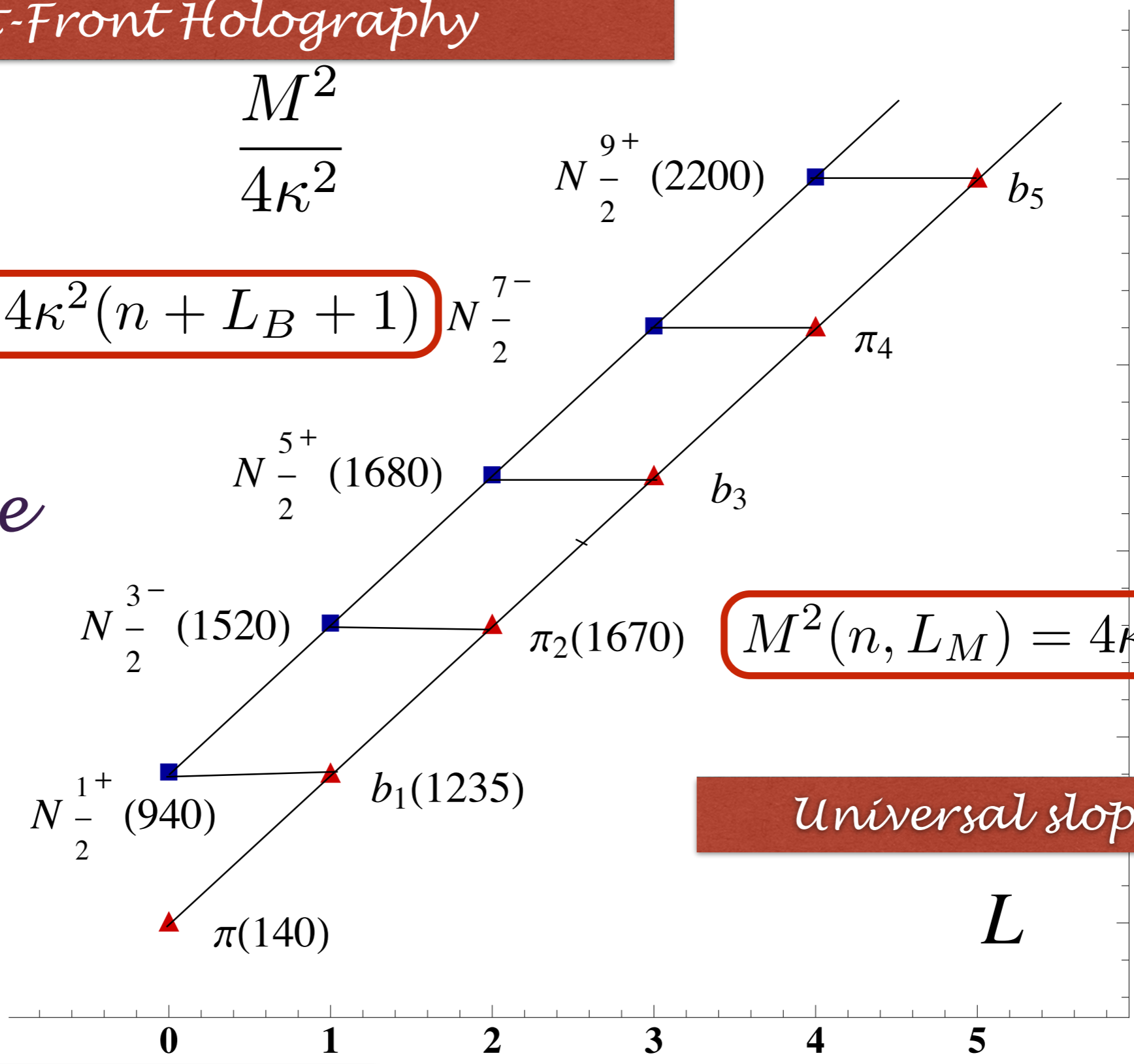


INT October 10, 2022

$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$

*Same slope*

$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$



*Universal slopes in n, L*

$$\frac{M_{meson}^2}{M_{nucleon}^2} = \frac{n + L_M}{n + L_B + 1}$$

**Meson-Baryon  
Mass Degeneracy  
for  $L_M=L_B+1$**

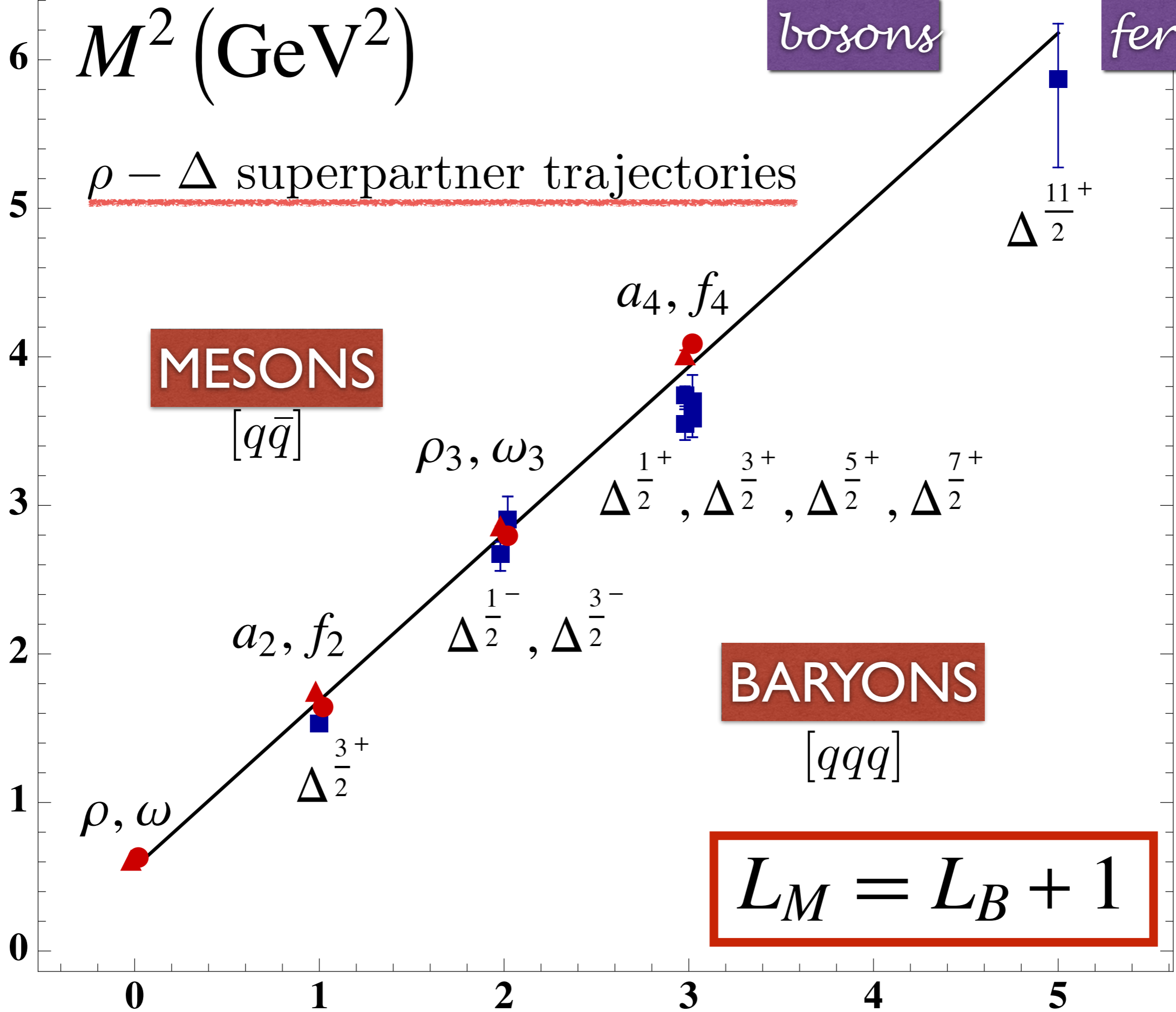


$M^2$  (GeV<sup>2</sup>)

bosons

fermions

$\rho - \Delta$  superpartner trajectories



BARYONS

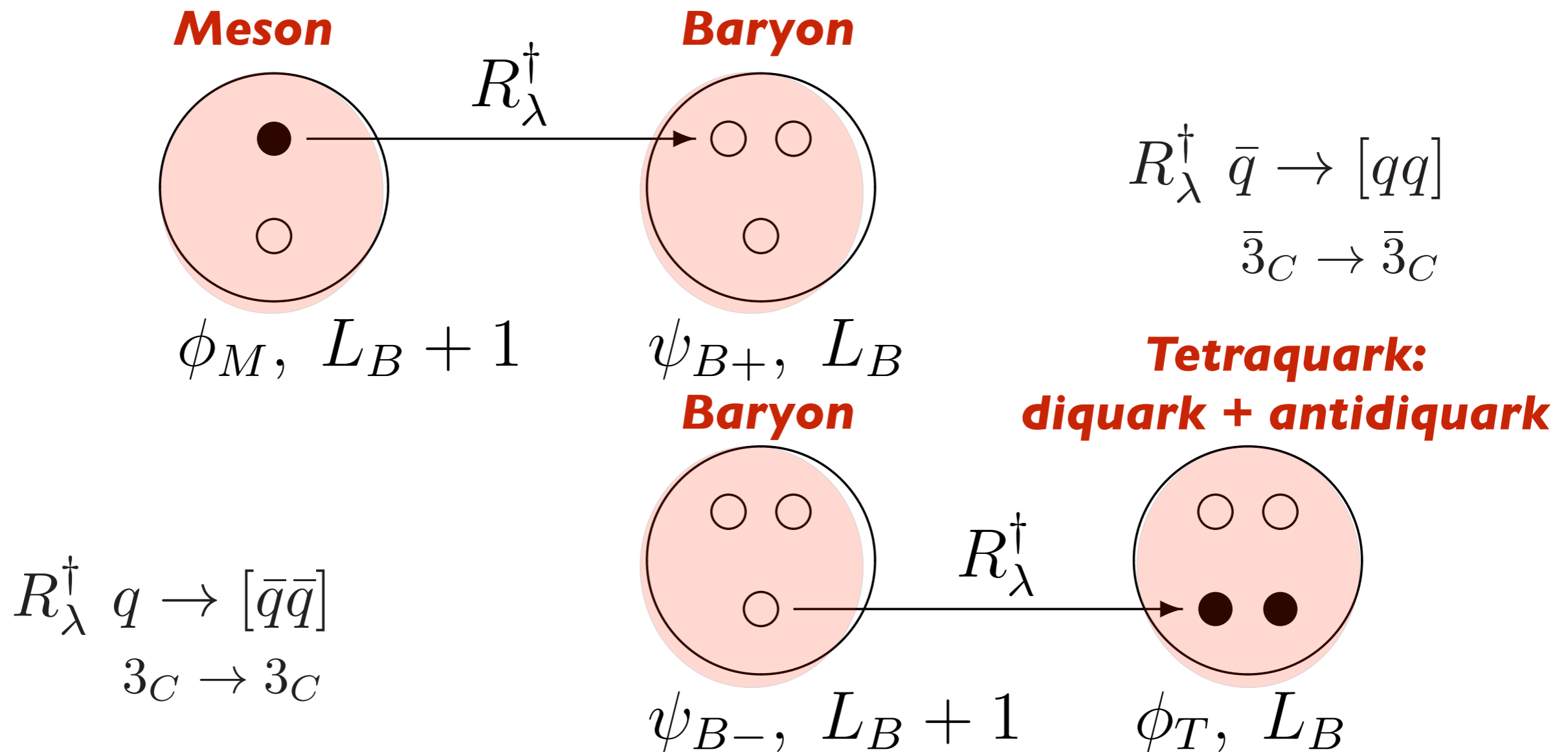
MESONS

$L_M = L_B + 1$

# Superconformal Algebra

## 2X2 Hadronic Multiplets

Bosons, Fermions with Equal Mass!



Proton:  $|u[ud]\rangle$  Quark + Scalar Diquark  
 Equal Weight:  $L=0, L=1$



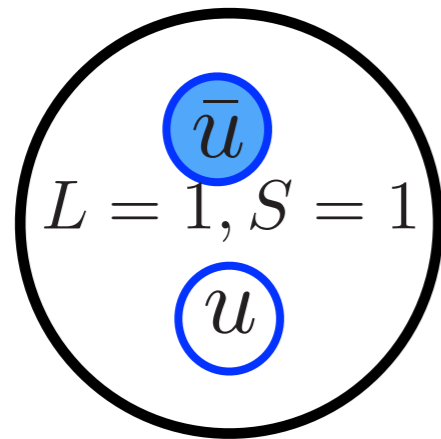
# Superconformal Algebra 4-Plet

$$R_\lambda^\dagger \bar{q} \rightarrow (qq) \quad S = 1$$

$$\bar{3}_C \rightarrow \bar{3}_C$$

Vector ( ) + Scalar [ ] Diquarks

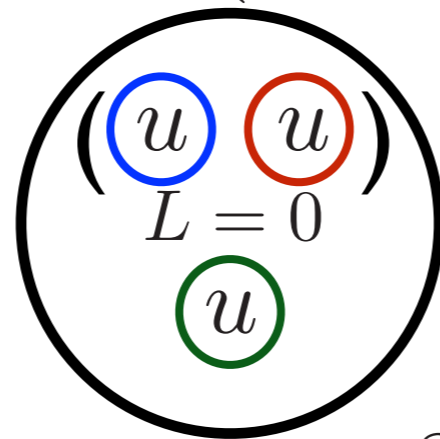
$f_2(1270)$



$$J^{PC} = 2^{++}$$

**Meson**

$\Delta^+(1232)$



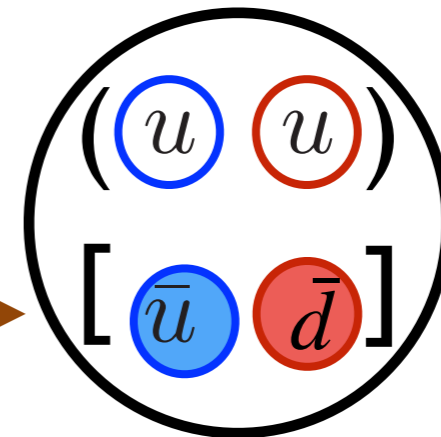
$$J^P = \frac{3}{2}^+$$

**Baryon**

**Tetraquark**

$$J^{PC} = 1^{++}$$

$a_1(1260)$



$$S = 0$$

$$L = 0$$

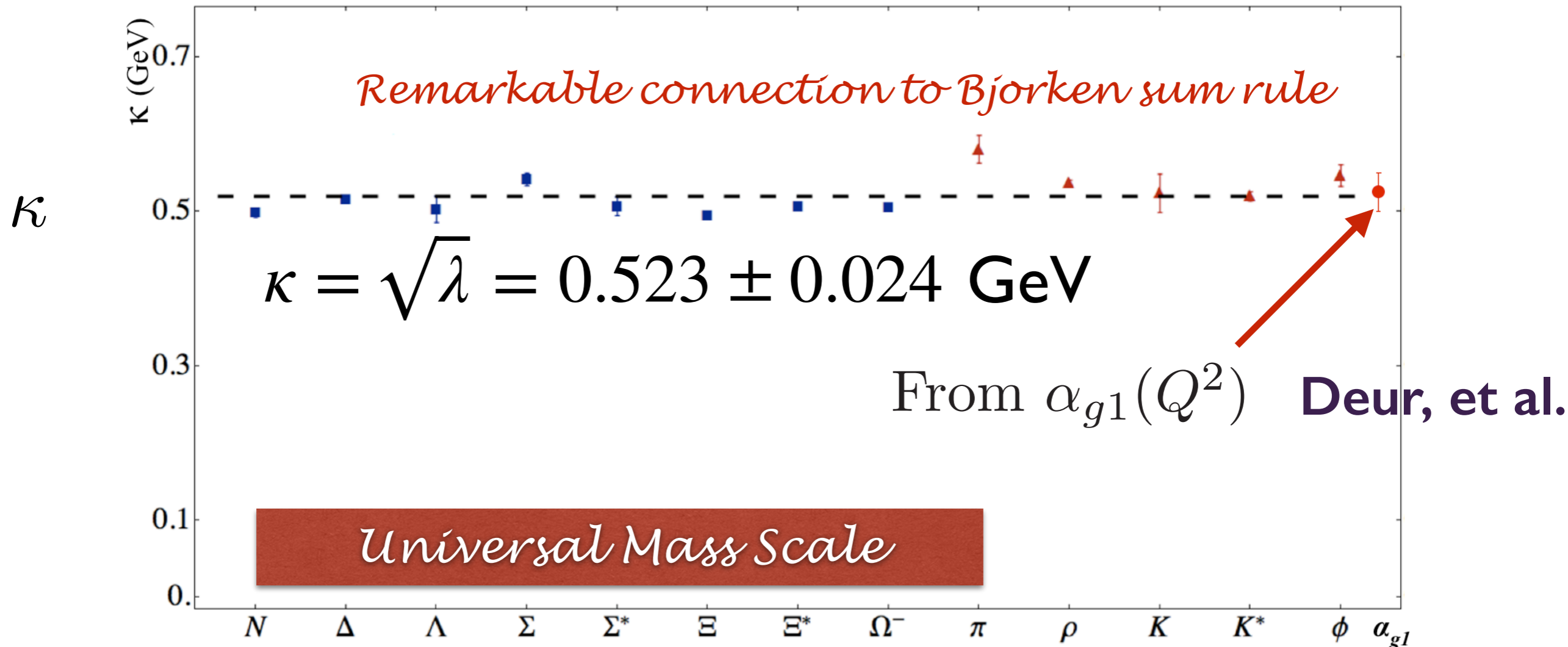
$$R_\lambda^\dagger q \rightarrow [\bar{q}\bar{q}]$$

$$3_C \rightarrow 3_C$$

$$\lambda = \kappa^2$$

*de Tèramond, Dosch, Lorce', sjb*

$$m_u = m_d = 46 \text{ MeV}, m_s = 357 \text{ MeV}$$



**Fit to the slope of Regge trajectories,  
including radial excitations**

**Same Regge Slope for Meson, Baryons:  
Supersymmetric feature of hadron physics**



# Challenge: Compute Hadron Structure, Spectroscopy, and Dynamics from QCD!

- **Color Confinement**
- **Origin of the QCD Mass Scale**
- **Meson and Baryon Spectroscopy**
- **Exotic States: Tetraquarks, Pentaquarks, Gluonium,**
- **Universal Regge Slopes:  $n$ ,  $L$ , Mesons and Baryons**
- **Almost Massless Pion: GMOR Chiral Symmetry Breaking**  
$$M_\pi^2 f_\pi^2 = -\frac{1}{2}(m_u + m_d) \langle \bar{u}u + \bar{d}d \rangle + \mathcal{O}((m_u + m_d)^2)$$
- **QCD Coupling at all Scales  $\alpha_s(Q^2)$**
- **Eliminate Scale Uncertainties and Scheme Dependence**

$$\mathcal{L}_{QCD} \rightarrow \psi_n^H(x_i, \vec{k}_{\perp i}, \lambda_i) \quad \text{Valence and Higher Fock States}$$

# Supersymmetry in QCD

- A hidden symmetry of Color  $SU(3)_c$  in hadron physics:
- Relates meson and baryon spectroscopy
- QCD: No squarks or gluinos!
- Emerges from Light-Front Holography and Super-Conformal Algebra
- Color Confinement

*de Téramond, Dosch, Lorcé, sjb*

**Input: one fundamental mass scale**

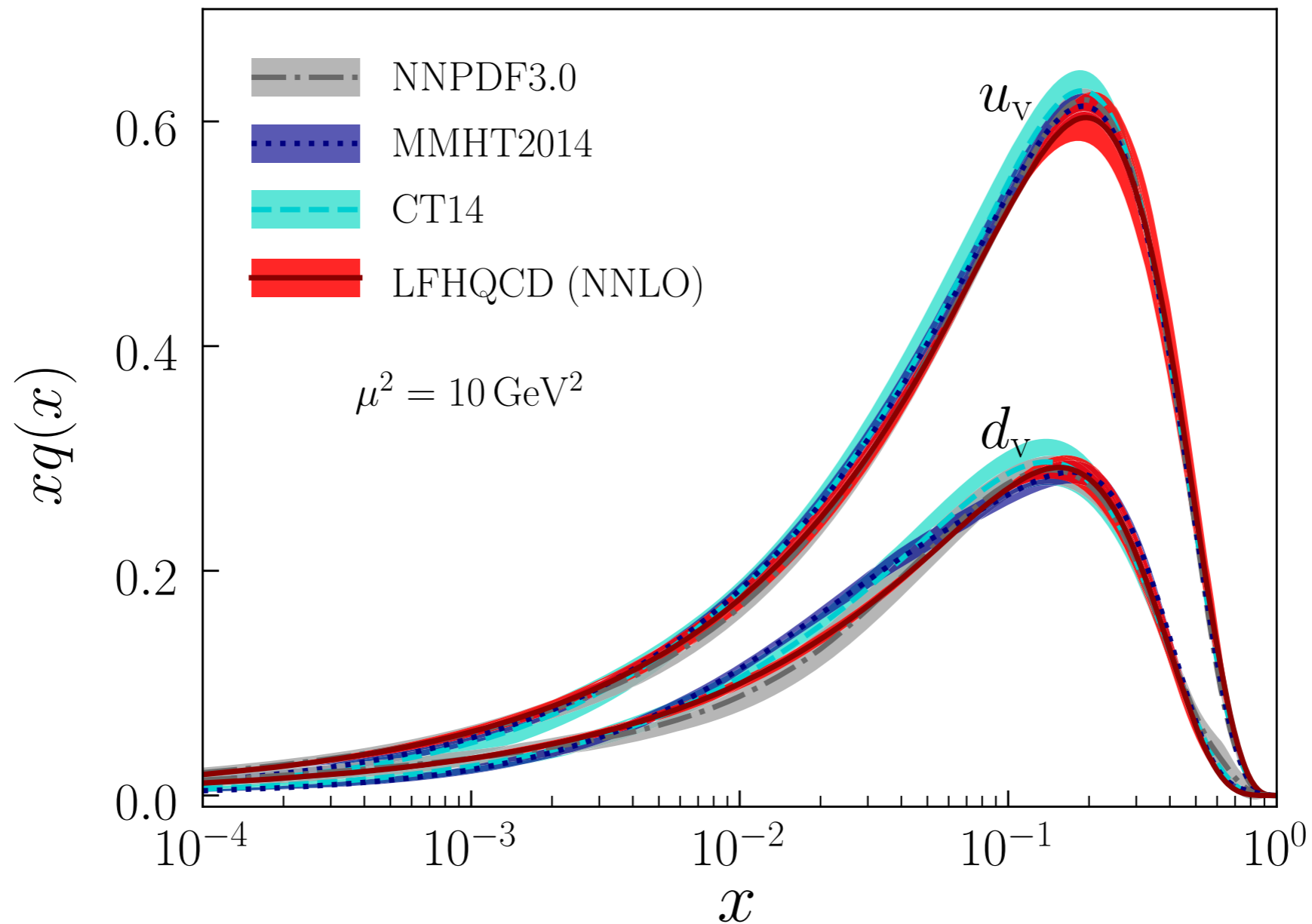
$$\kappa = \sqrt{\lambda} = 0.523 \pm 0.024 \text{ GeV}$$



# *Light-Front Holography: First Approximation to QCD*

- **Color Confinement, Analytic form of confinement potential**
- **Retains underlying conformal properties of QCD despite mass scale (DeAlfaro-Fubini-Furlan Principle)**
- **Massless quark-antiquark pion bound state in chiral limit, GMOR**
- **QCD coupling at all scales**
- **Connection of perturbative and nonperturbative mass scales**
- **Poincarè Invariant**
- **Hadron Spectroscopy-Regge Trajectories with universal slopes in  $n$ ,  $L$**
- **Supersymmetric 4-Plet: Meson-Baryon-Tetraquark Symmetry**
- **Light-Front Wavefunctions**
- **Form Factors, Structure Functions, Hadronic Observables**
- **OPE: Constituent Counting Rules**
- **Hadronization at the Amplitude Level: Many Phenomenological Tests**
- **Systematically improvable: Basis LF Quantization (BLFQ)**

*Supersymmetric Features of Hadron Physics  
from Superconformal Algebra  
and Light-Front Holography*



Comparison for  $xq(x)$  in the proton from LFHQCD (red bands) and global fits: MMHT2014 (blue bands) [5], CT14 [6] (cyan bands), and NNPDF3.0 (gray bands) [77]. LFHQCD results are evolved from the initial scale  $\mu_0 = 1.06 \pm 0.15$  GeV.

*Universality of Generalized Parton Distributions in Light-Front Holographic QCD*

*Guy F. de Téramond, Tianbo Liu, Raza Sabbir Sufian, Hans Günter Dosch, Stanley J. Brodsky, and Alexandre Deur*

*PHYSICAL REVIEW LETTERS 120, 182001 (2018)*



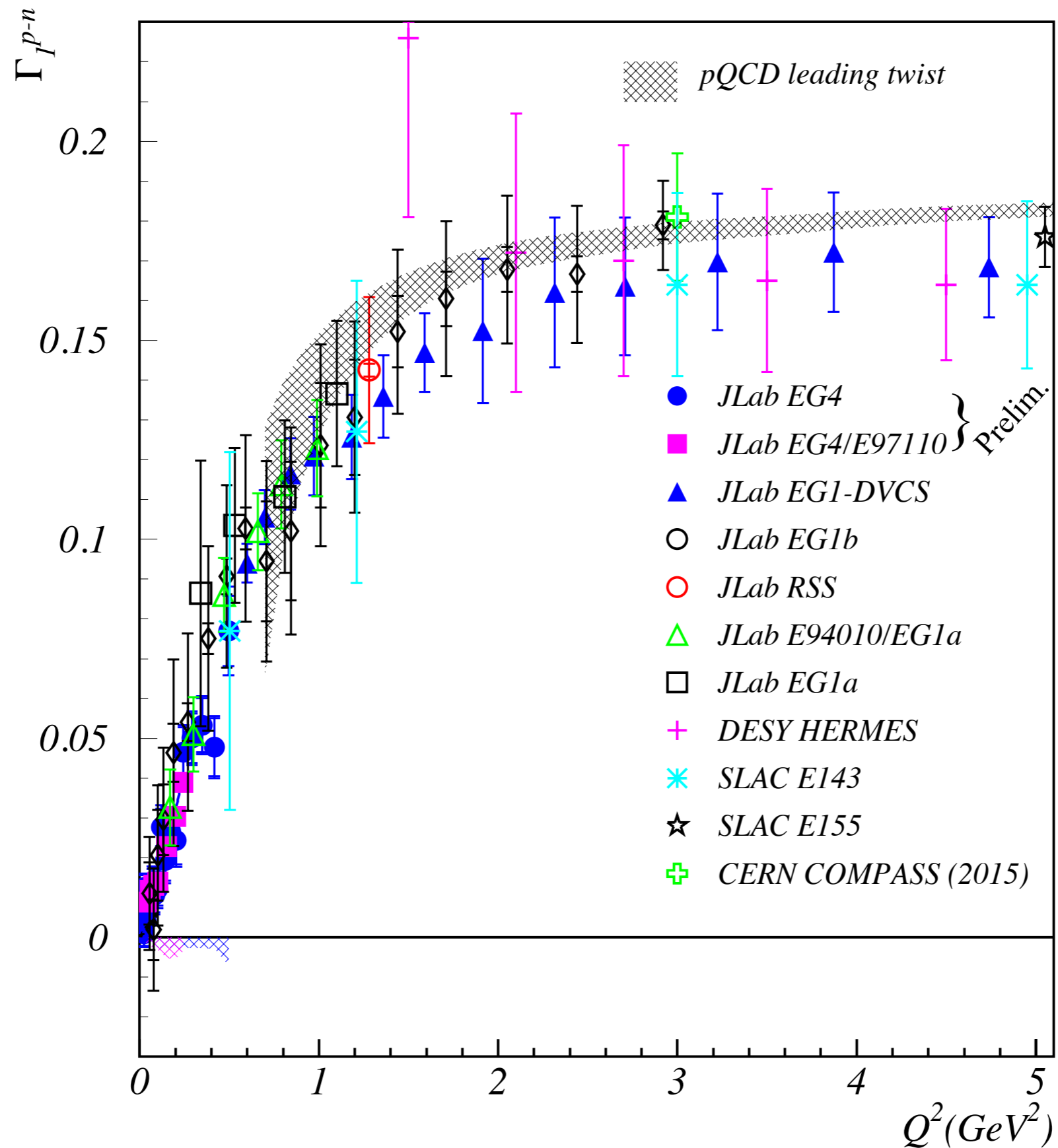
Bjorken sum rule defines effective charge

$$\alpha_{g1}(Q^2)$$

$$\int_0^1 dx [g_1^{ep}(x, Q^2) - g_1^{en}(x, Q^2)] \equiv \frac{g_a}{6} \left[ 1 - \frac{\alpha_{g1}(Q^2)}{\pi} \right]$$

- **Can be used as standard QCD coupling**
- **Well measured**
- **Asymptotic freedom at large  $Q^2$**
- **Computable at large  $Q^2$  in any pQCD scheme**
- **Universal  $\beta_0, \beta_1$**

# Bjorken sum $\Gamma_1^{p-n}$ measurements



$$m_\rho = \sqrt{2}\kappa$$

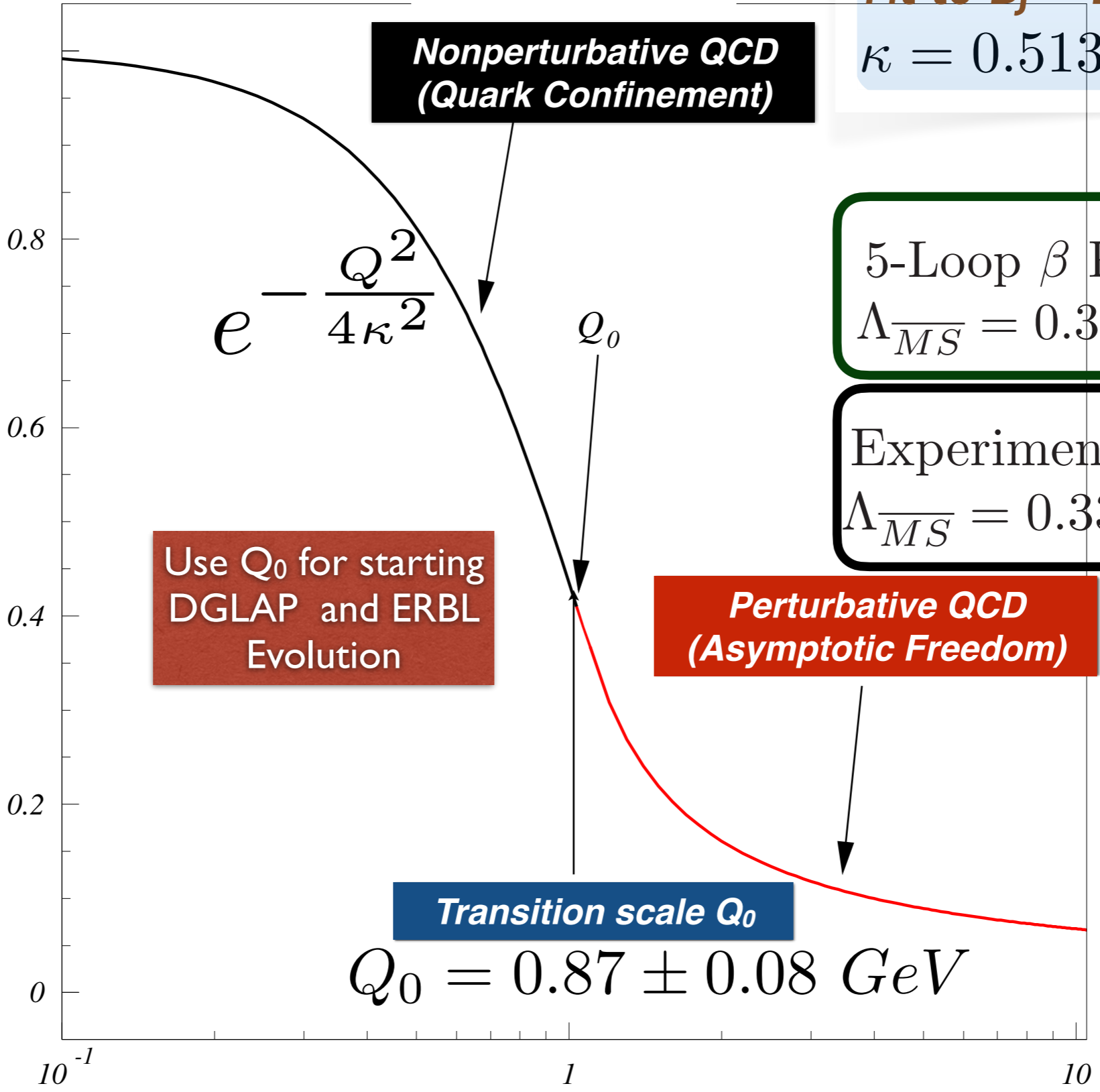
$$m_p = 2\kappa$$

Deur, de Tèramond, sjb

**All-Scale QCD Coupling**

Fit to Bj + DHG Sum Rules:  
 $\kappa = 0.513 \pm 0.007 \text{ GeV}$

$$\frac{\alpha_{g_1}^s(Q^2)}{\pi}$$



5-Loop  $\beta$  Prediction:  
 $\Lambda_{\overline{MS}} = 0.339 \pm 0.019 \text{ GeV}$

Experiment:  
 $\Lambda_{\overline{MS}} = 0.332 \pm 0.017 \text{ GeV}$

**Perturbative QCD  
(Asymptotic Freedom)**

**Transition scale  $Q_0$**

$$Q_0 = 0.87 \pm 0.08 \text{ GeV}$$

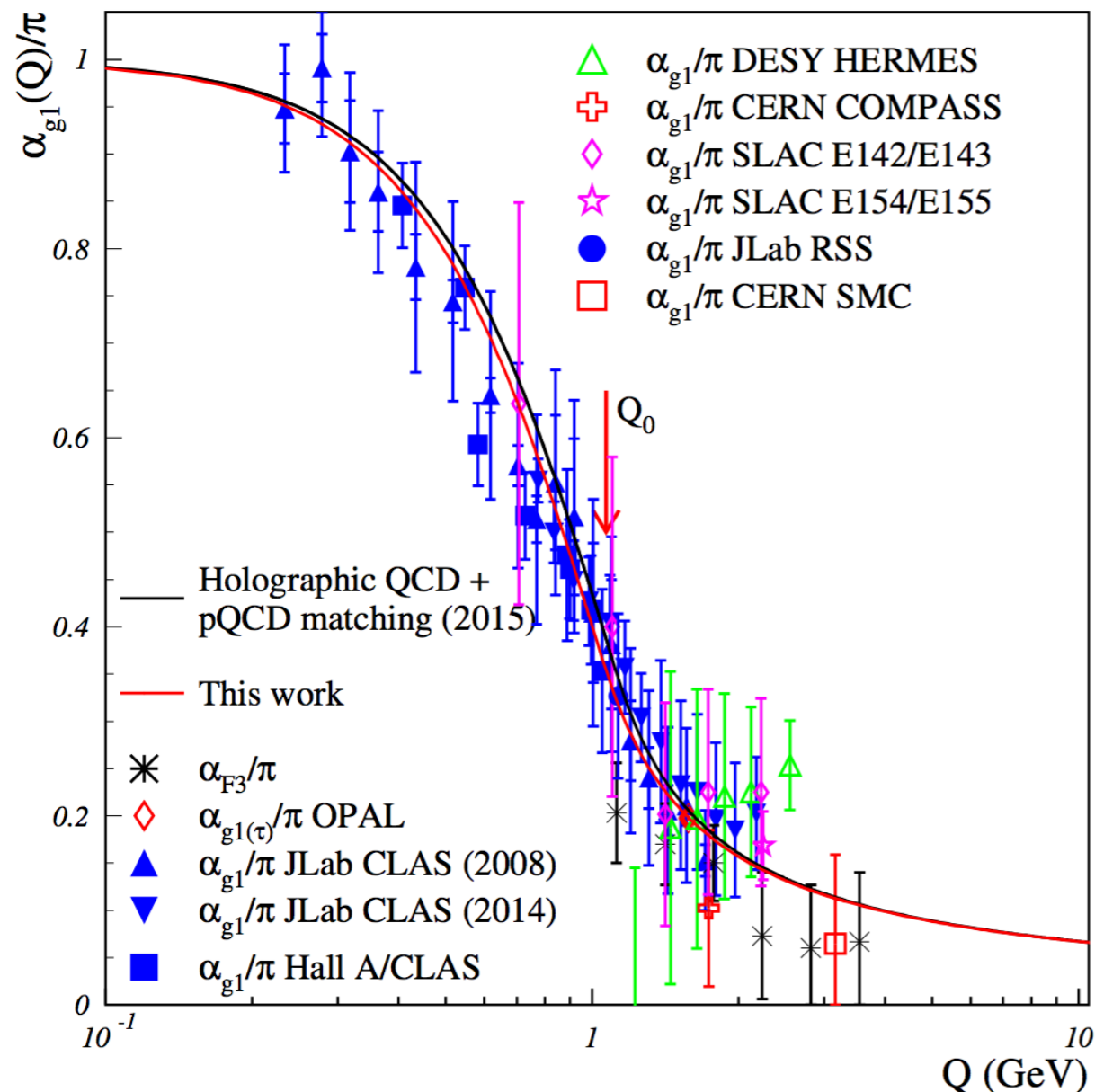
Use  $Q_0$  for starting  
DGLAP and ERBL  
Evolution

$$\lambda \equiv \kappa^2$$

*Reverse Dimensional Transmutation!*

Q (GeV)  $\overline{MS}$  scheme

# Running Coupling from AdS/QCD



Bjorken sum rule:

$$\frac{\alpha_{g_1}(Q^2)}{\pi} = 1 - \frac{6}{g_A} \int_0^1 dx g_1^{p-n}(x, Q^2)$$

Effective coupling in LFHQCD  
(valid at low- $Q^2$ )

$$\alpha_{g_1}^{AdS}(Q^2) = \pi \exp(-Q^2/4\kappa^2)$$

Imposing continuity for  $\alpha$   
and its first derivative

A. Deur, S.J. Brodsky, G.F. de Téramond,  
Phys. Lett. B 750, 528 (2015); J. Phys. G 44, 105005 (2017).

**Analytic, defined at all scales, IR Fixed Point**

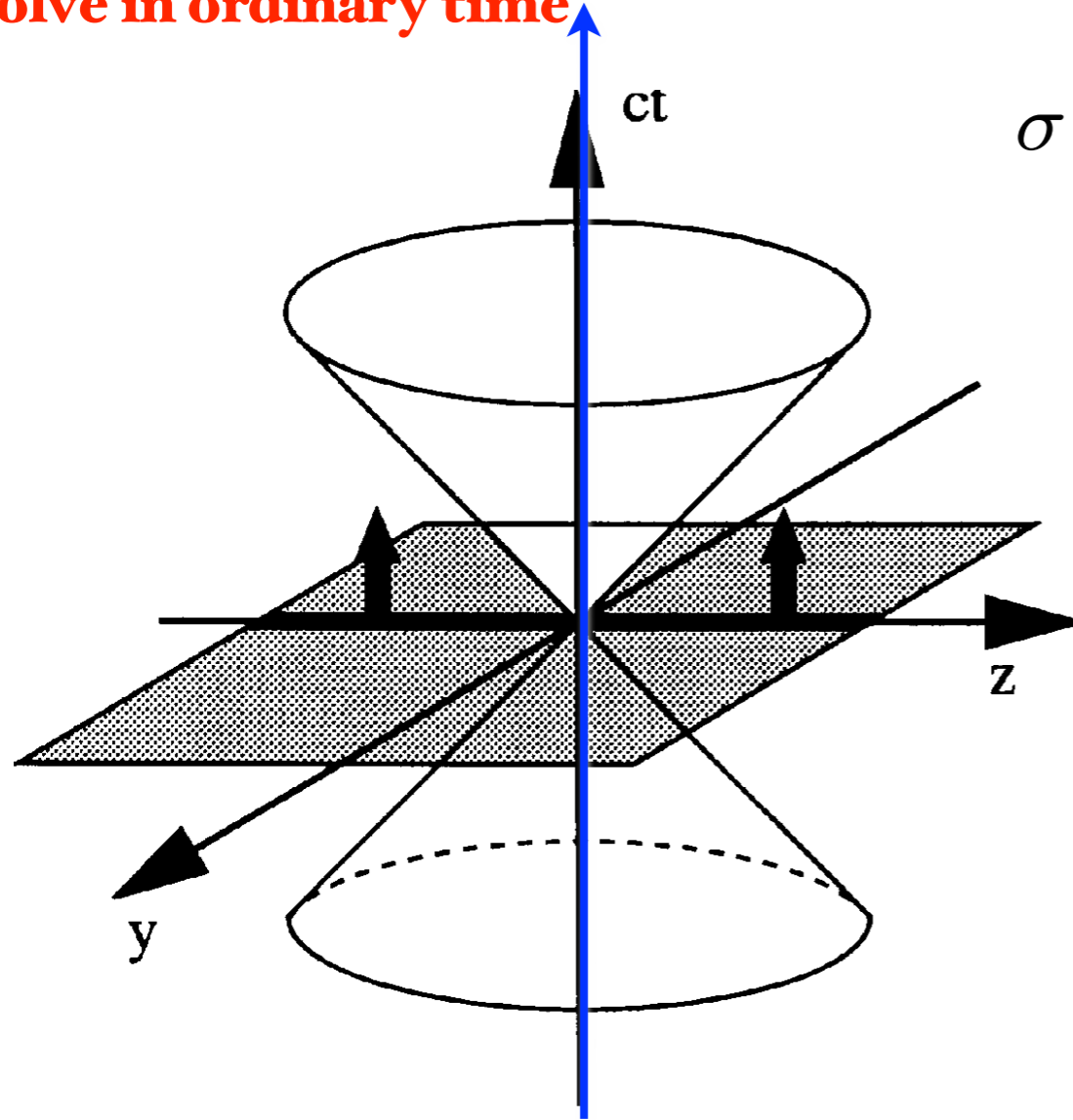




**P.A.M Dirac, Rev. Mod. Phys. 21, 392 (1949)**

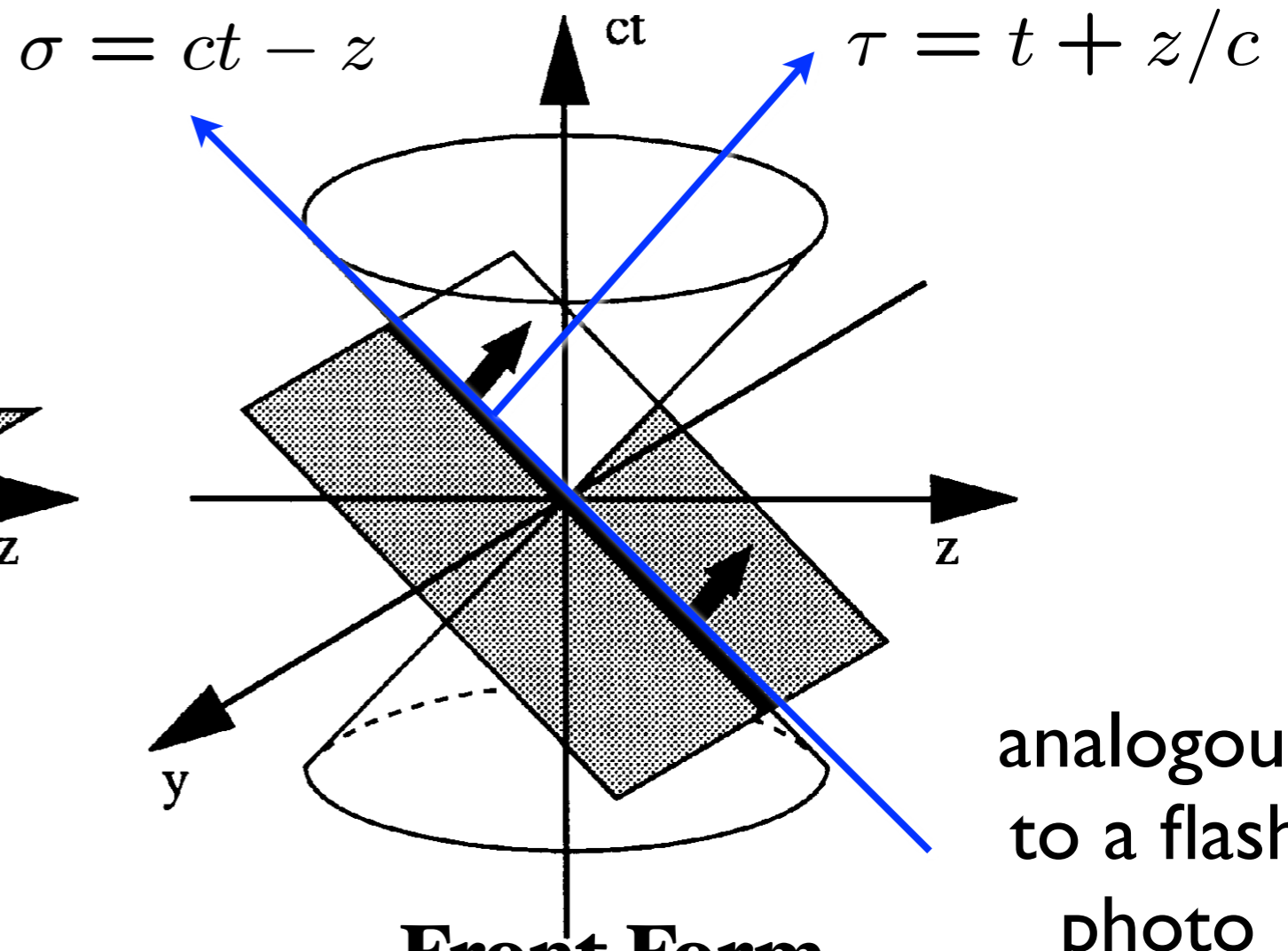
*Dirac's Amazing Idea:  
The "Front Form"*

**Evolve in ordinary time**



**Instant Form**

**Evolve in light-front time!**



**Front Form**

analogous  
to a flash  
photo

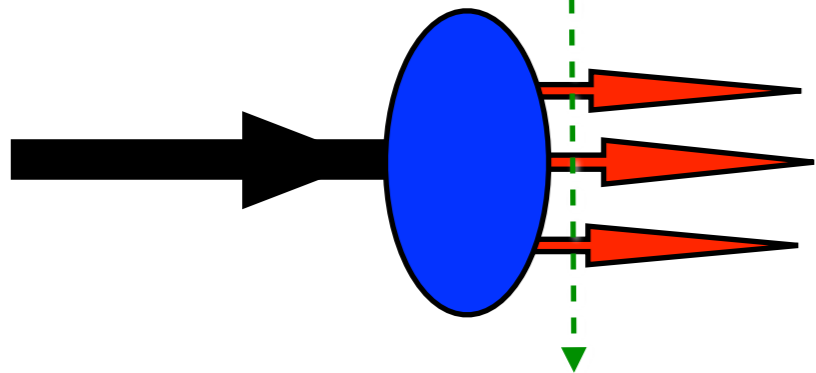
*Casual, Boost Invariant!*

# Bound States in Relativistic Quantum Field Theory:

## *Light-Front Wavefunctions*

Dirac's Front Form: Fixed  $\tau = t + z/c$

Fixed  $\tau = t + z/c$



$$\psi(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

***Invariant under boosts. Independent of  $P^\mu$***

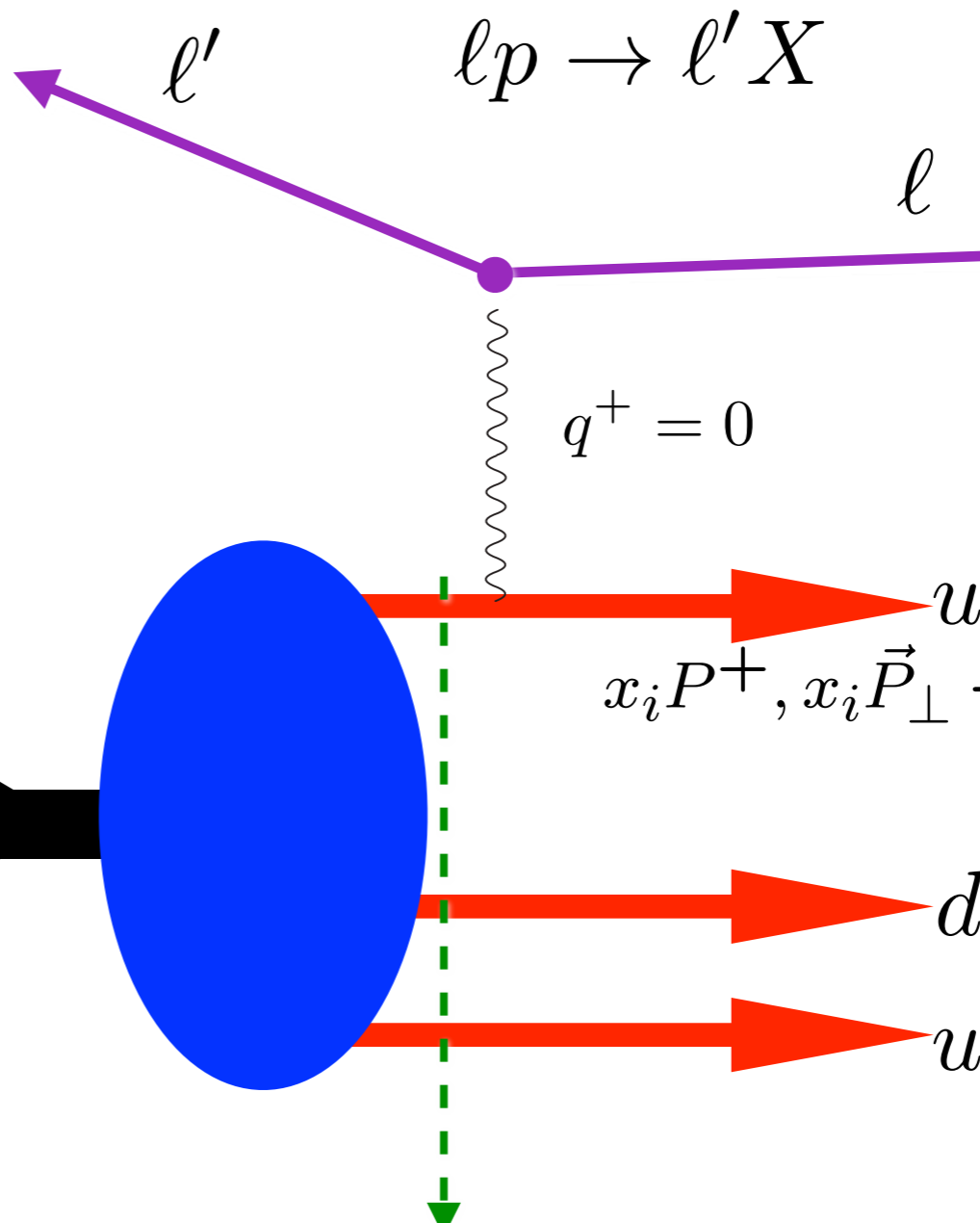
$$H_{LF}^{QCD} |\psi\rangle = M^2 |\psi\rangle$$

**Direct connection to QCD Lagrangian**

***LF Wavefunction: off-shell in invariant mass***

*Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space*

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$



**Dirac: Front Form**

*Measurements of hadron LF wavefunction are at fixed LF time*

Fixed  $\tau = t + z/c$

*Like a flash photograph*

$$x_{bj} = x = \frac{k^+}{P^+}$$

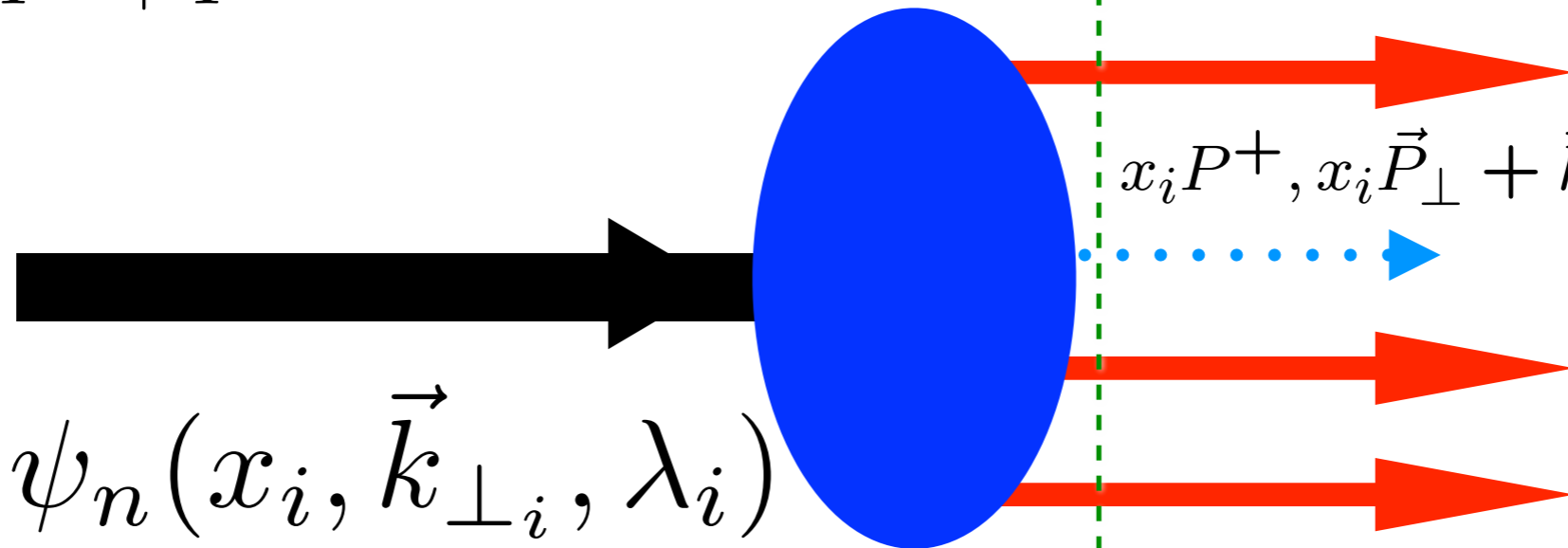
*Invariant under boosts! Independent of  $P^\mu$*

# Light-Front Wavefunctions: **rigorous** representation of composite systems in quantum field theory

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

Fixed  $\tau = t + z/c$

$P^+, \vec{P}_\perp$



$$\sum_i^n x_i = 1$$

$$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$$

$$\psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$$H_{LF}^{QCD} |\Psi_h\rangle = \mathcal{M}_h^2 |\Psi_h\rangle \quad \text{Eigenstate of LF Hamiltonian}$$

$$|p, J_z\rangle = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; x_i, \vec{k}_{\perp i}, \lambda_i\rangle$$

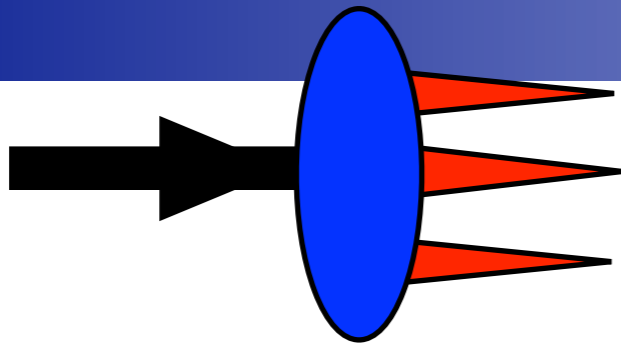
LFWF: Projection on free Fock state:  $\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) = \langle p | n \rangle$

*Invariant under boosts! Independent of  $P^\mu$*

**Structure Function is square of LFWFs, summed over all Fock states. Causal, Frame-independent. Creation Operators on Simple Vacuum, Current Matrix Elements are Overlaps of LFWFs**



Light-Front Wavefunctions  
underly hadronic observables

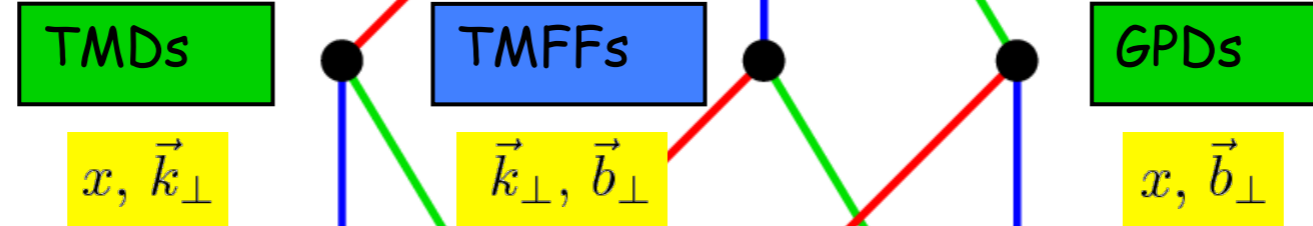


$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

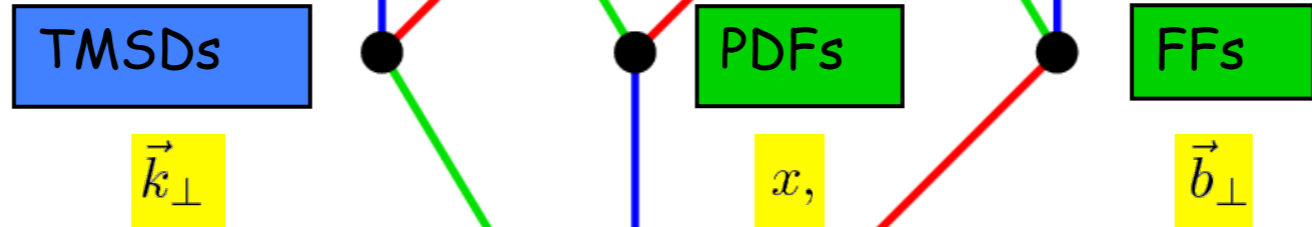
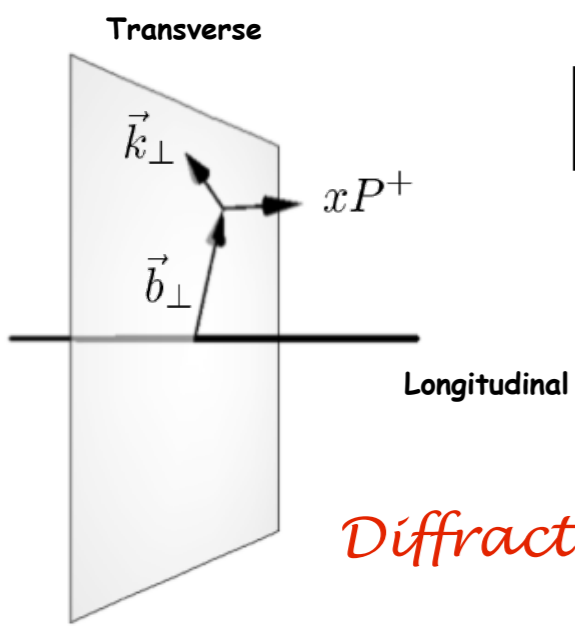
Transverse density in  
momentum space

Momentum space  $\vec{k}_{\perp} \leftrightarrow \vec{z}_{\perp}$  Position space  
 $\vec{\Delta}_{\perp} \leftrightarrow \vec{b}_{\perp}$   
 Transverse density in position  
space

Weak transition  
form factors



*DGLAP, ERBL Evolution  
Factorization Theorems*



→  $\int d^2 b_{\perp}$   
 →  $\int dx$   
 →  $\int d^2 k_{\perp}$

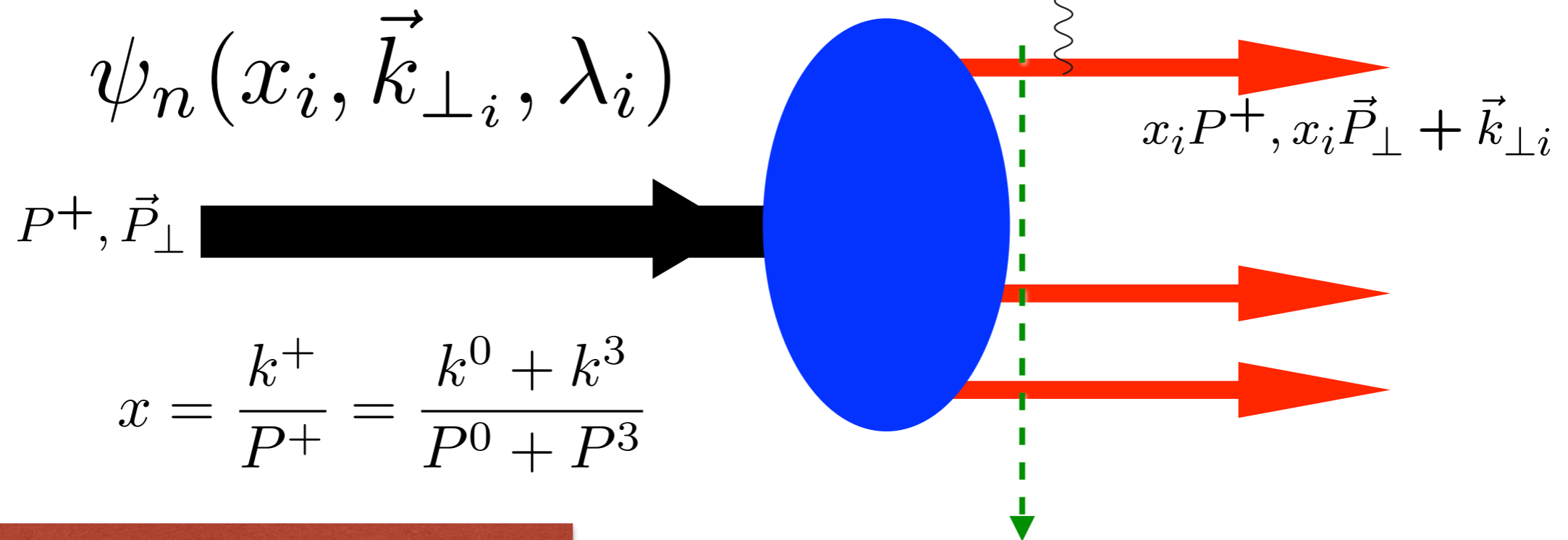
*Diffractive DIS from FSI*

Charges

*Sivers, T-odd from lensing*

$$q^\mu = (q^+, \vec{q}_\perp, q^-) = (0, \vec{q}_\perp, \frac{q_\perp^2}{P^+})$$

$$q_\perp^2 = Q^2 = -q^2$$



## Dirac: Front Form

[1912.08911](#) [hep-ph]

G. A. Miller, sjb:

$$x_{bj} = x = \frac{k^+}{P^+}$$

Ioffe Time:  $\tilde{z}$  Third spatial LF coordinate.

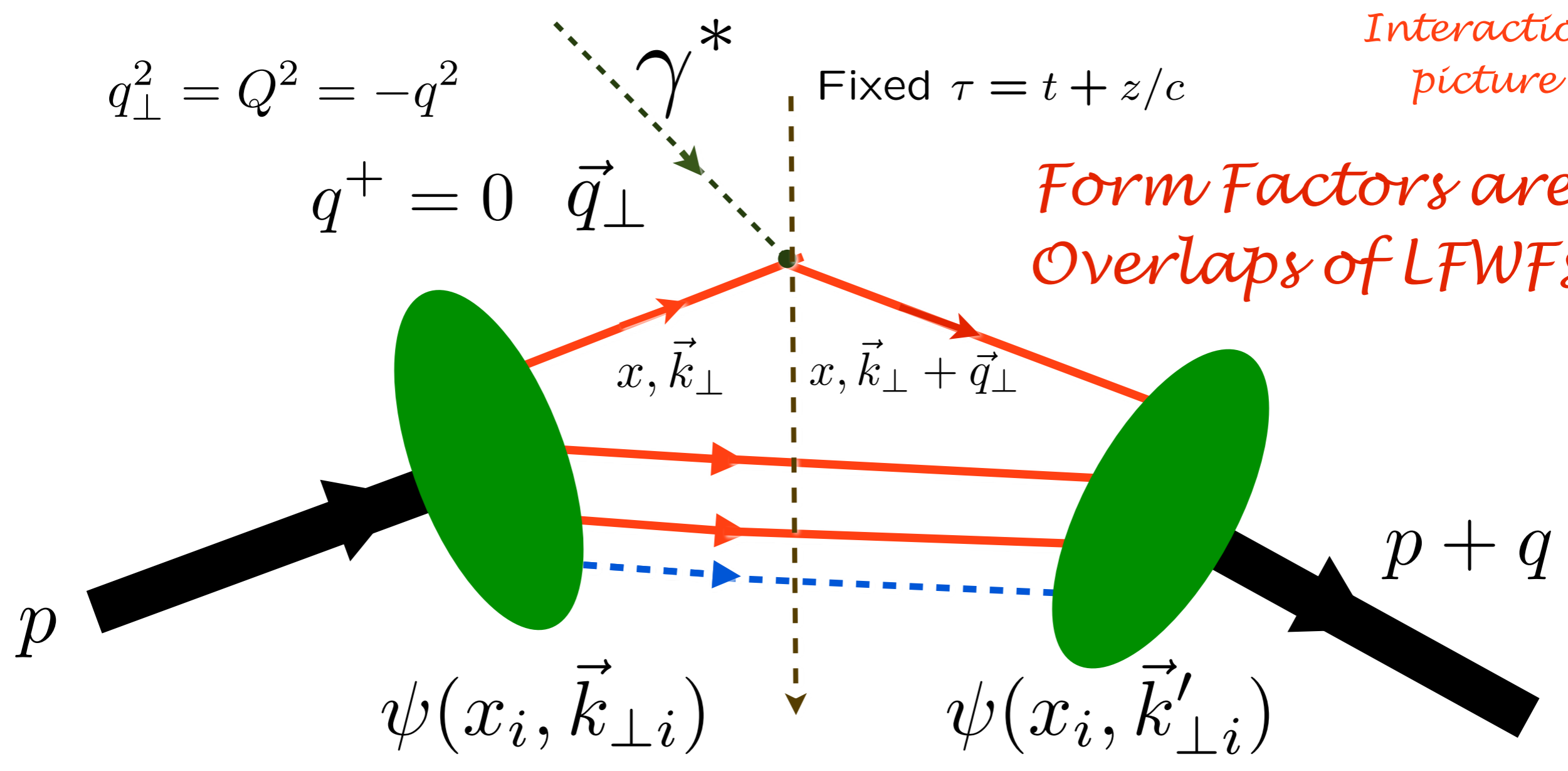
Fourier Transform of  $x$  in LFWFs

$$\langle p + q | j^+(0) | p \rangle = 2p^+ F(q^2)$$

**Front Form**

*Interaction picture*

*Form Factors are Overlaps of LFWFs*



$$q_{\perp}^2 = Q^2 = -q^2$$

$$q^+ = 0 \quad \vec{q}_{\perp}$$

Fixed  $\tau = t + z/c$

$$x, \vec{k}_{\perp}$$

$$x, \vec{k}_{\perp} + \vec{q}_{\perp}$$

$$p + q$$

$$\psi(x_i, \vec{k}_{\perp i})$$

$$\psi(x_i, \vec{k}'_{\perp i})$$

*struck*  $\vec{k}'_{\perp i} = \vec{k}_{\perp i} + (1 - x_i)\vec{q}_{\perp}$

*spectators*  $\vec{k}'_{\perp i} = \vec{k}_{\perp i} - x_i\vec{q}_{\perp}$

**Drell & Yan, West  
Exact LF formula!**

Drell, sjb

Transverse size  $\propto \frac{1}{Q}$

# Exact LF Formula for Pauli Form Factor

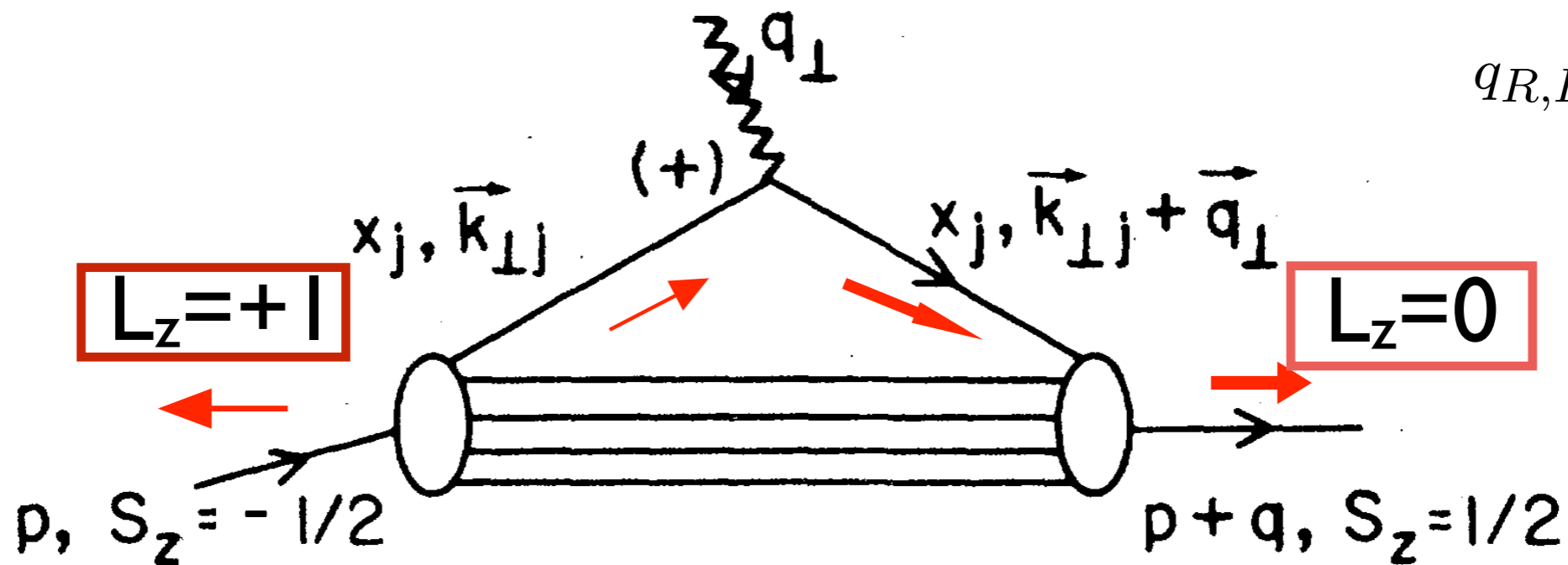
$$\frac{F_2(q^2)}{2M} = \sum_a \int [dx][d^2\mathbf{k}_\perp] \sum_j e_j \frac{1}{2} \times \quad \text{Drell, sjb}$$

$$\left[ -\frac{1}{q^L} \psi_a^{\uparrow*}(x_i, \mathbf{k}'_{\perp i}, \lambda_i) \psi_a^\downarrow(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R} \psi_a^{\downarrow*}(x_i, \mathbf{k}'_{\perp i}, \lambda_i) \psi_a^\uparrow(x_i, \mathbf{k}_{\perp i}, \lambda_i) \right]$$

$$\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i \mathbf{q}_\perp$$

$$\mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j) \mathbf{q}_\perp$$

$$q_{R,L} = q^x \pm iq^y$$



Must have  $\Delta l_z = \pm 1$  to have nonzero  $F_2(q^2)$

Nonzero Proton Anomalous Moment  $\rightarrow$   
Nonzero orbital quark angular momentum

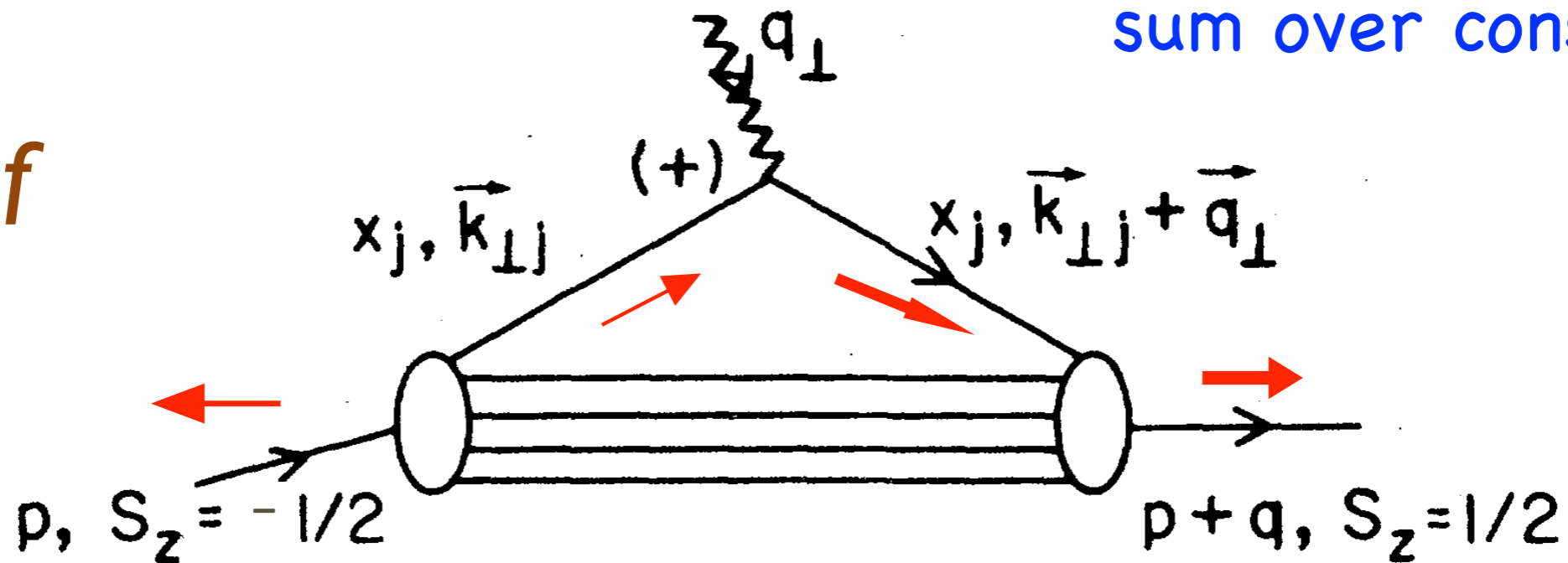


**Terayev, Okun:**  $B(0)$  Must vanish because of Equivalence Theorem

graviton

sum over constituents

LF Proof

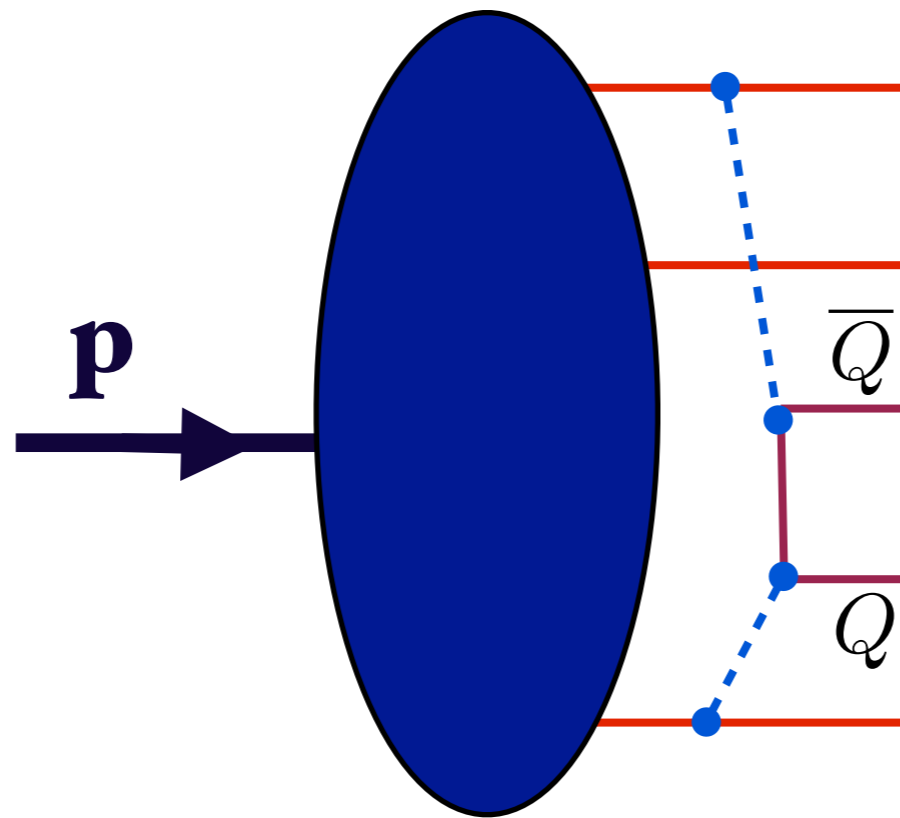


$$B(0) = 0$$

Each Fock State

Vanishing Anomalous gravitomagnetic moment  $B(0)$

*Proton 5-quark Fock State:  
Intrinsic Heavy Quarks*



*QCD predicts  
Intrinsic  
Heavy Quarks  
at high  $x$ !*

Perturbative contribution

$$x_Q \propto (m_Q^2 + k_{\perp}^2)^{1/2}$$

$$\text{Probability (QED)} \propto \frac{1}{M_{\ell}^4}$$

**Minimal off-shellness**

$$\text{Probability (QCD)} \propto \frac{1}{M_Q^2}$$

**Hoyer, Peterson, Sakai, Collins, Ellis, Gunion, Mueller, sjb  
Polyakov, et al.**



## Intrinsic Chevrolets at the SSC

Stanley J. Brodsky (SLAC), John C. Collins (IIT, Chicago and Argonne), Stephen D. Ellis (Washington U., Seattle), John F. Gunion (UC, Davis), Alfred H. Mueller (Columbia U.) (Aug, 1984)

Published in: , Snowmass Summer Study 1984:0227 • Contribution to: 1984 DPF Summer Study on the Design and Utilization of the Superconducting Super Collider (SSC) (Snowmass 84), 227

Quantum Mechanics Uncertainty Principle on the Light Front:  
Arbitrarily off-shell in invariant mass squared

$$\mathcal{M}^2 = \sum_i \frac{m_i^2 + \vec{k}_{\perp i}^2}{x_i} \text{ at fixed LF time } \tau = t + z/c$$

## Intrinsic Heavy Quark States

Stanley J. Brodsky (SLAC), C. Peterson (SLAC), N. Sakai (Fermilab) (Jan, 1981)

Published in: *Phys.Rev.D* 23 (1981) 2745

## The Intrinsic Charm of the Proton

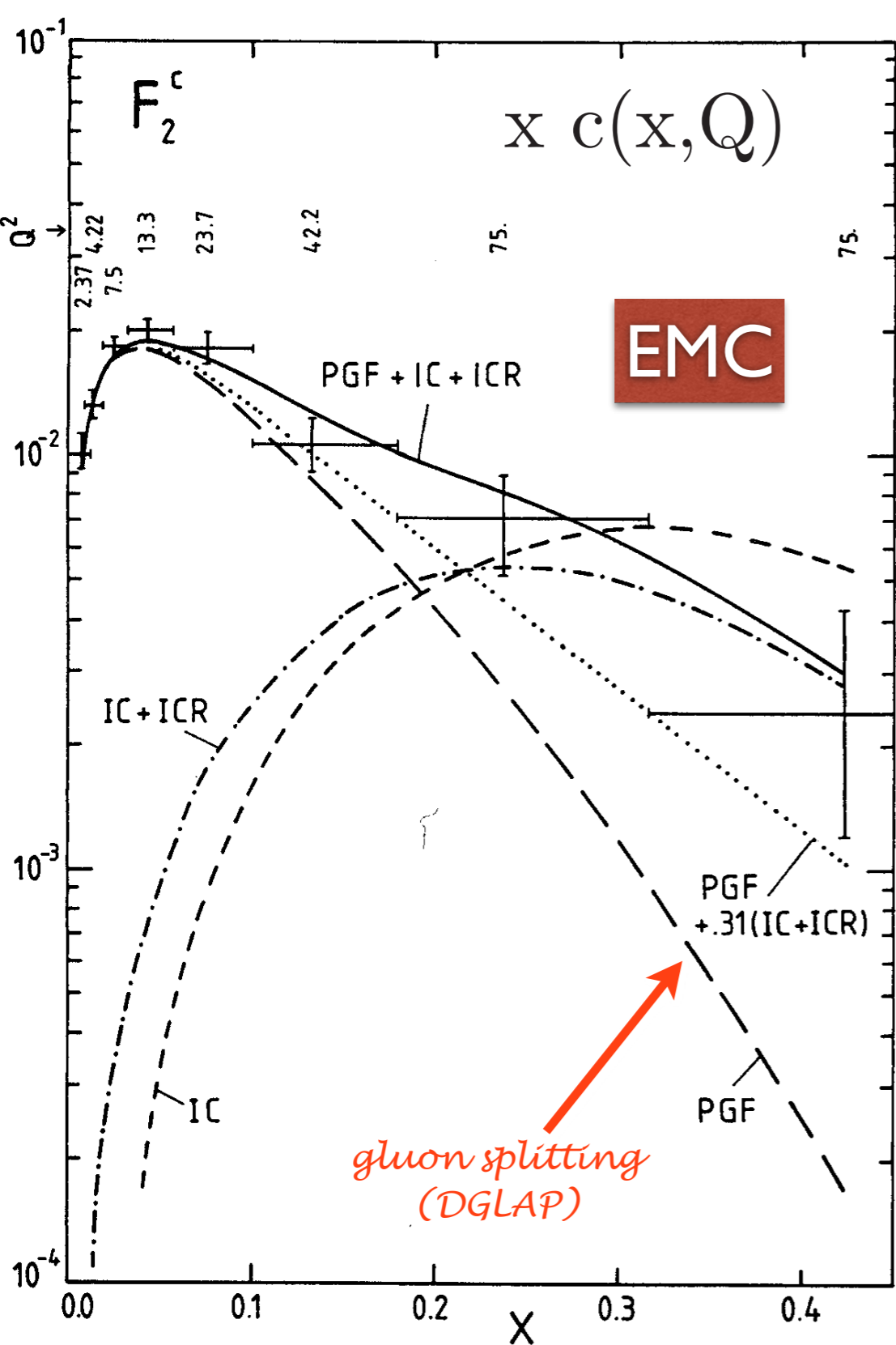
S.J. Brodsky (SLAC), P. Hoyer (Nordita), C. Peterson (Nordita), N. Sakai (Nordita) (Apr, 1980)

Published in: *Phys.Lett.B* 93 (1980) 451-455

42 years ago!

# Measurement of Charm Structure Function!

J. J. Aubert et al. [European Muon Collaboration], "Production Of Charmed Particles In 250-GeV Mu+ - Iron Interactions," Nucl. Phys. B 213, 31 (1983).

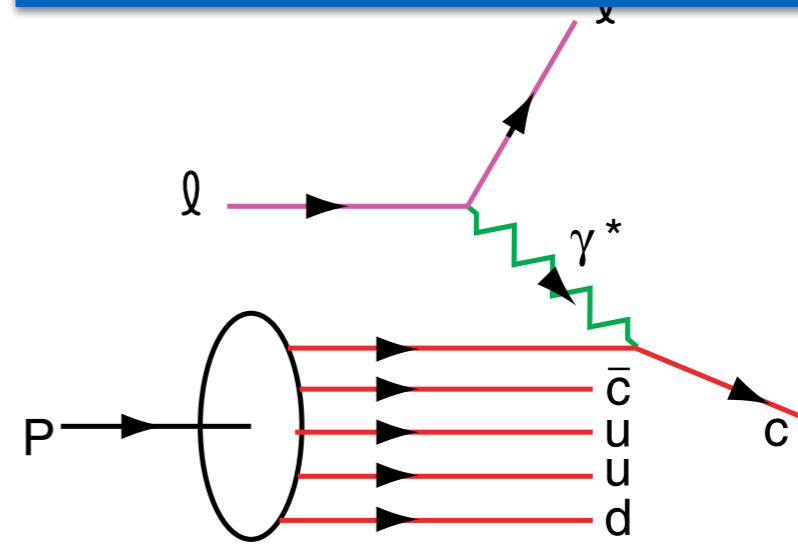


## First Evidence for Intrinsic Charm

$$\langle x_{c\bar{c}} \rangle_p \simeq 1\%$$

New Analysis:  
 R.D. Ball, et al. [NNPDF Collaboration],  
 "A Determination of the Charm Content  
 of the Proton,"  
 arXiv:1605.06515 [hep-ph].

factor of 30!



**DGLAP / Photon-Gluon Fusion: factor of 30 too small**

Two Components (separate evolution):

$$c(x, Q^2) = c(x, Q^2)_{\text{extrinsic}} + c(x, Q^2)_{\text{intrinsic}}$$



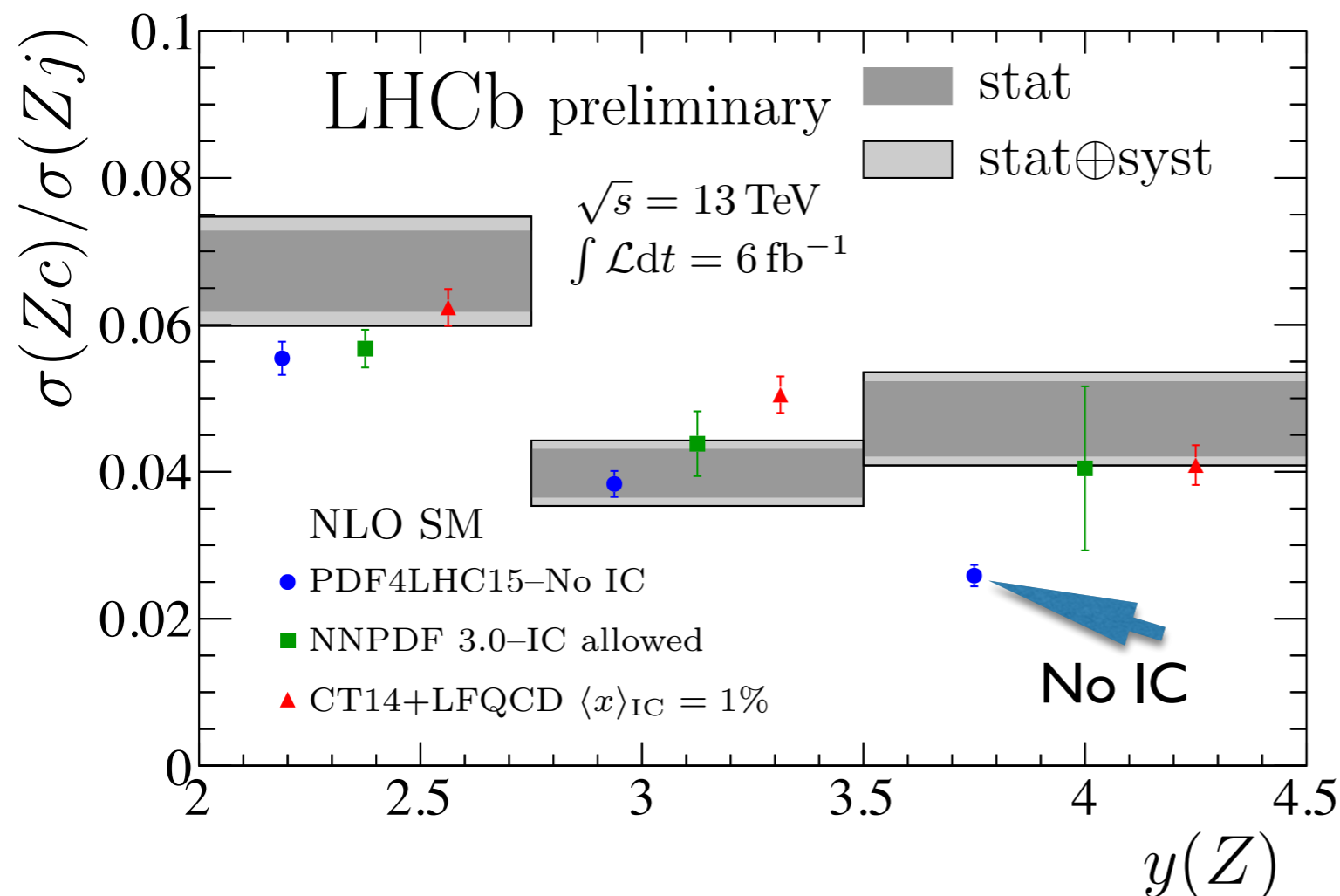
$$pp \rightarrow Z + c + X$$

$$g + c \rightarrow Z + c$$

## Z + c: results



LHCb-PAPER-2021-029



► Clear enhancement in highest- $y$  bin

► Consistent with expected effect from  $|uudc\bar{c}\rangle$  component predicted by LFQCD

► Inconsistent with No-IC theory at  $\sim 3$  standard deviations

► Global PDF analysis required to determine true significance

QCD physics measurements at the LHCb experiment

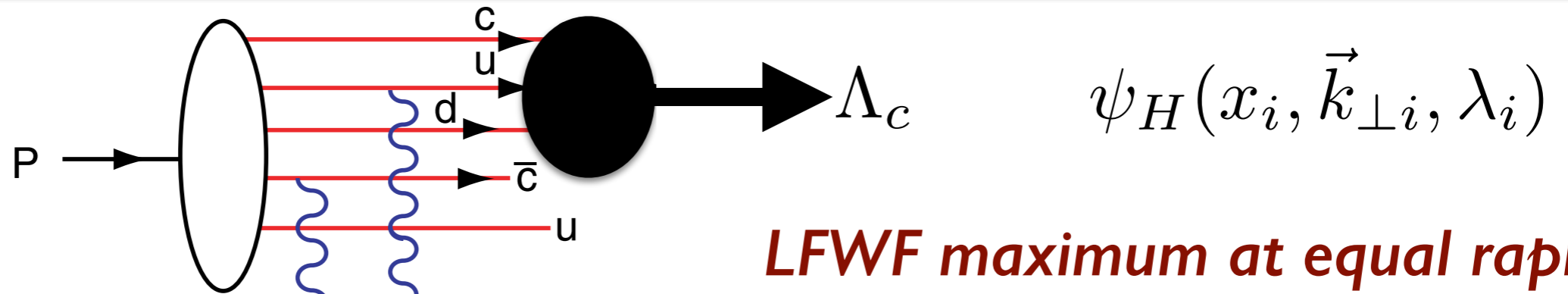
BOOST 2021

Daniel Craik  
 on behalf of the LHCb collaboration



# Coalescence of comovers produces high $x_F$ heavy hadrons

High  $x_F$  hadrons combine most of the comovers, fewest spectators



*LFWF maximum at equal rapidity*

*maximum at minimal invariant mass*

**X**  $\rightarrow$  *Asymmetries of leading hadrons*

**Spectator counting rules**

$$\frac{dN}{dx_F} \propto (1 - x_F)^{2n_{spect} - 1}$$

Coalescence of Comoving Charm and Valence Quarks  
Produce  $J/\psi$ ,  $\Lambda_c$  and other Charm Hadrons at High  $x_F$

Vogt, sjb

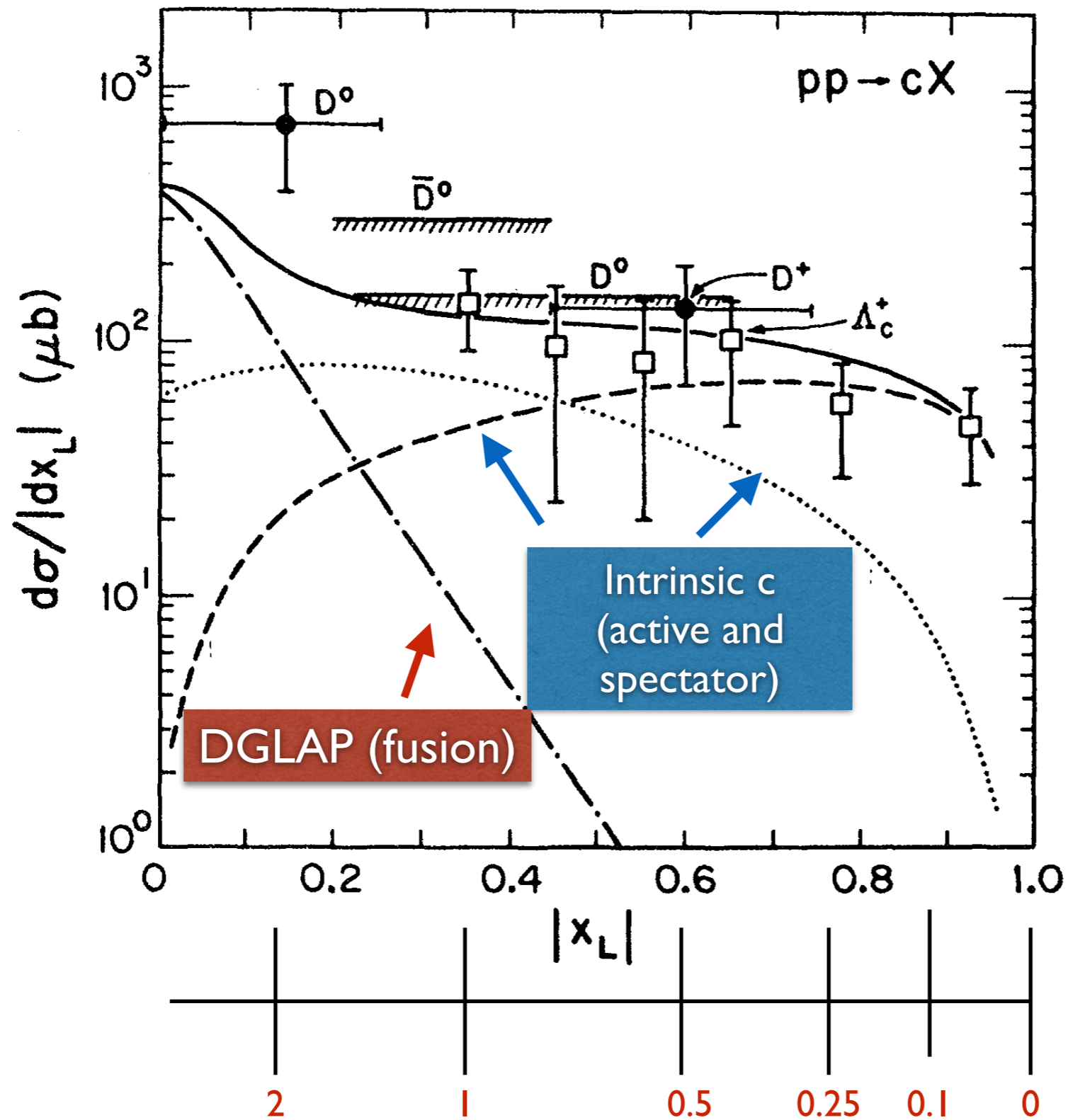
Stan Brodsky



Supersymmetric Features of Hadron Physics  
from Superconformal Algebra  
and Light-Front Holography



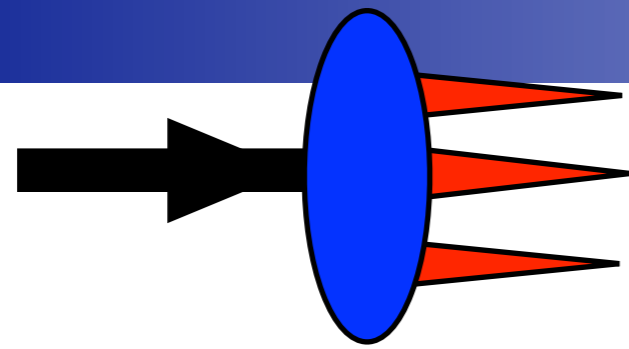
19 April 2021



$$\Delta y = \log x$$

*Lorce,  
Pasquini*

Light-Front Wavefunctions  
underly hadronic observables



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

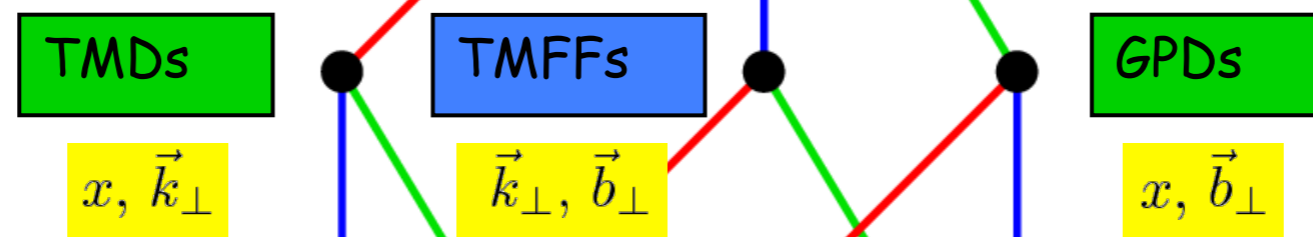
Transverse density in  
momentum space

Momentum space  $\vec{k}_{\perp} \leftrightarrow \vec{z}_{\perp}$  Position space  
 $\vec{\Delta}_{\perp} \leftrightarrow \vec{b}_{\perp}$

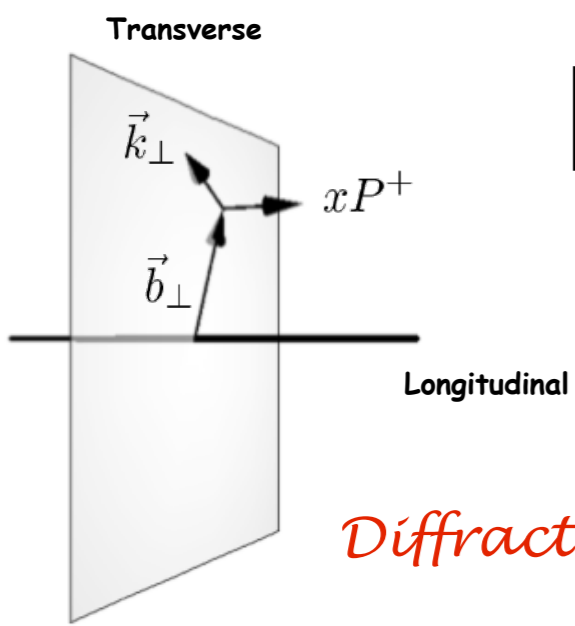
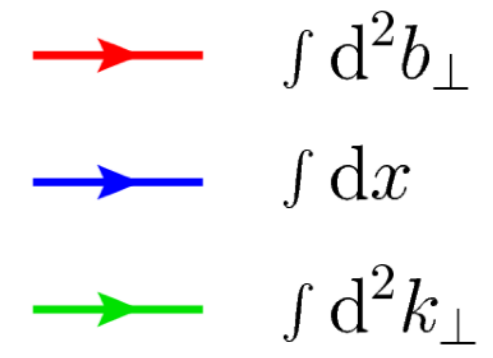
Transverse density in position  
space

Structure Functions  
computed from squares of LFWFs

Weak transition  
form factors



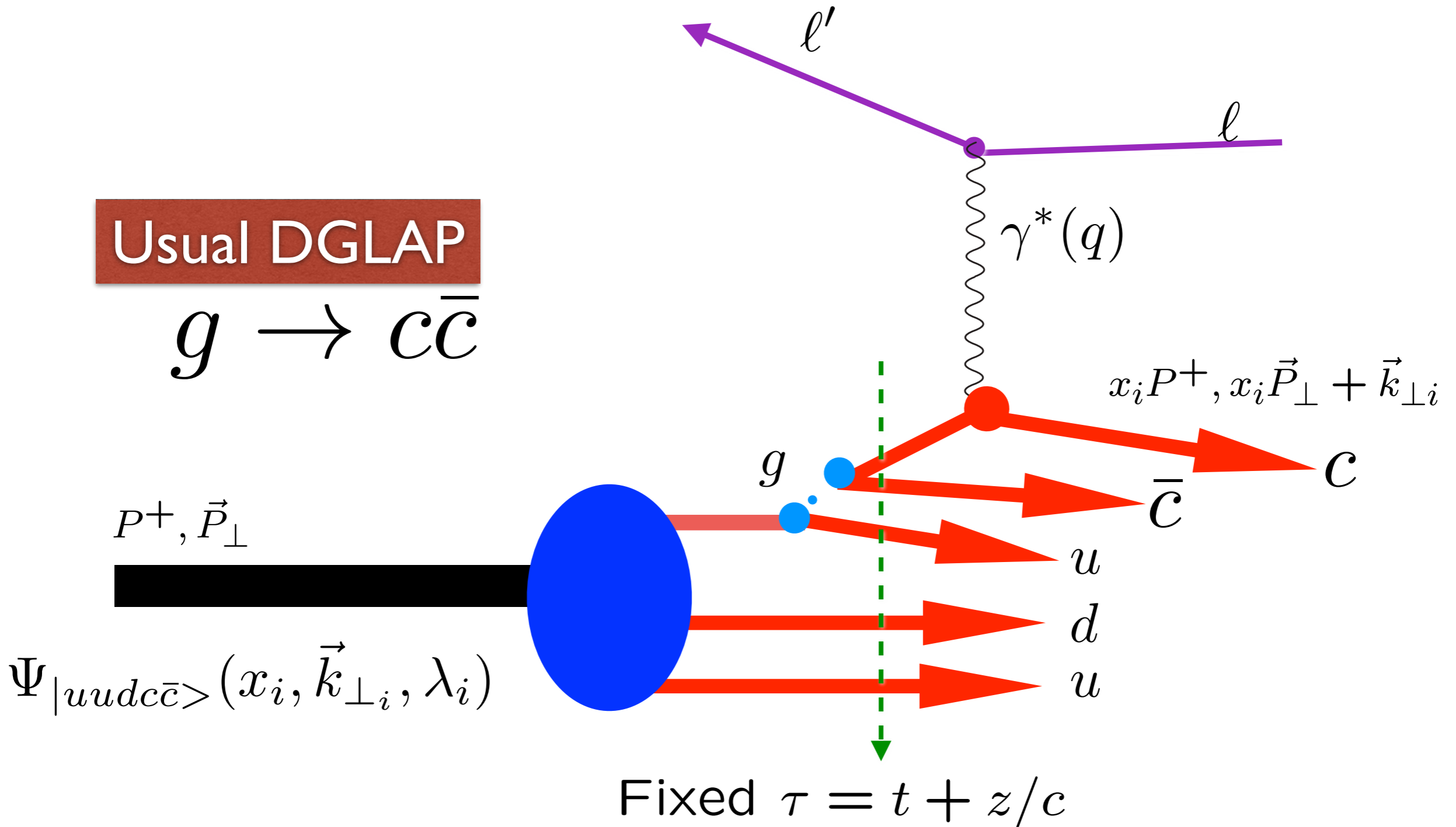
*DGLAP, ERBL Evolution  
Factorization Theorems*



*Diffractive DIS from FSI*

*Sivers, T-odd from lensing*

Charges



Probability  $P_{uudc\bar{c}} \sim \log \frac{Q^2 + M_c^2}{\Lambda_{QCD}^2}$

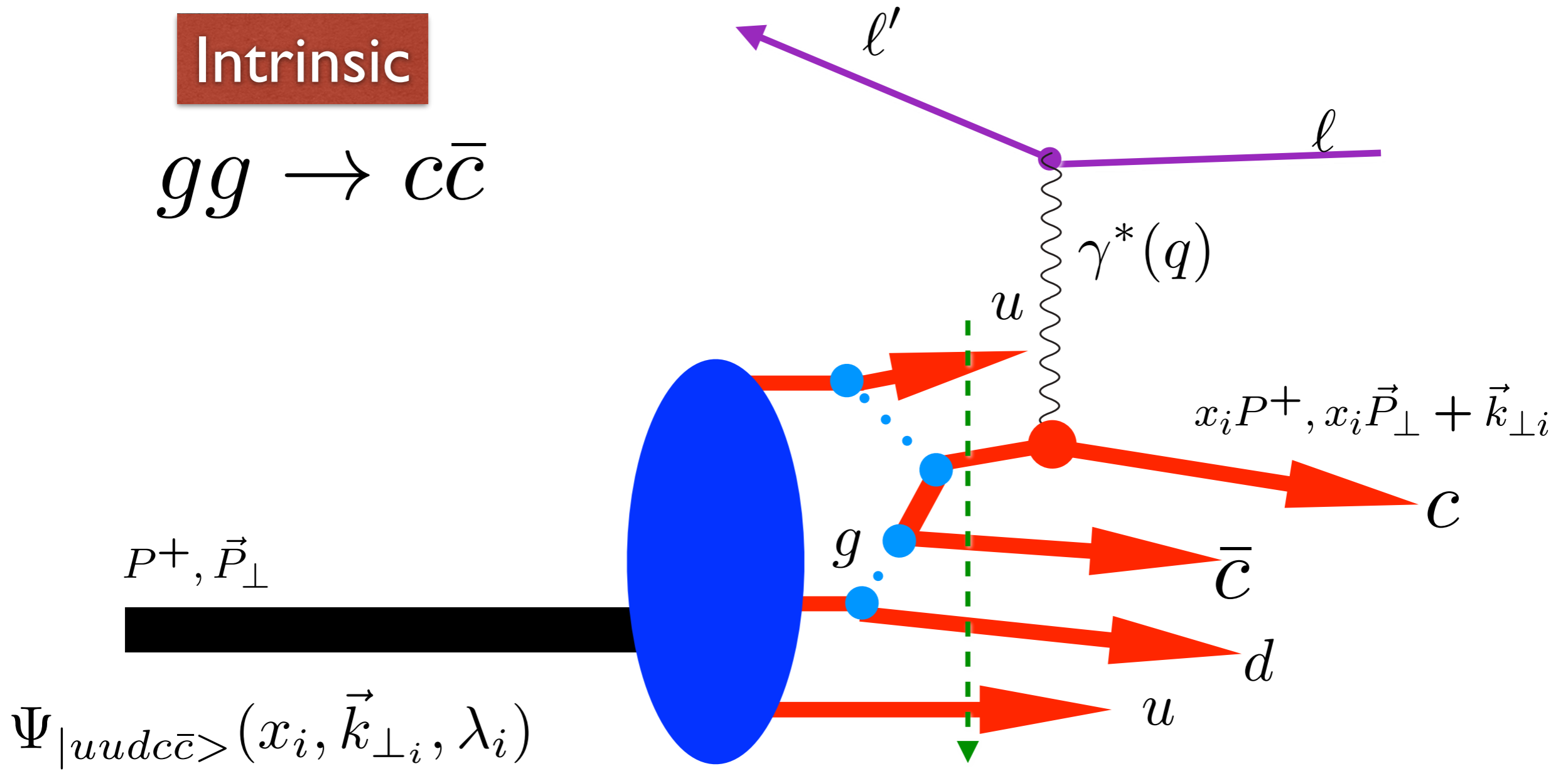
low  $x$ :  $c(x) \sim (1-x)g(x) \sim (1-x)^4, (1-x)^6$

*Low  $x$  extrinsic charm!*



Intrinsic

$$gg \rightarrow c\bar{c}$$



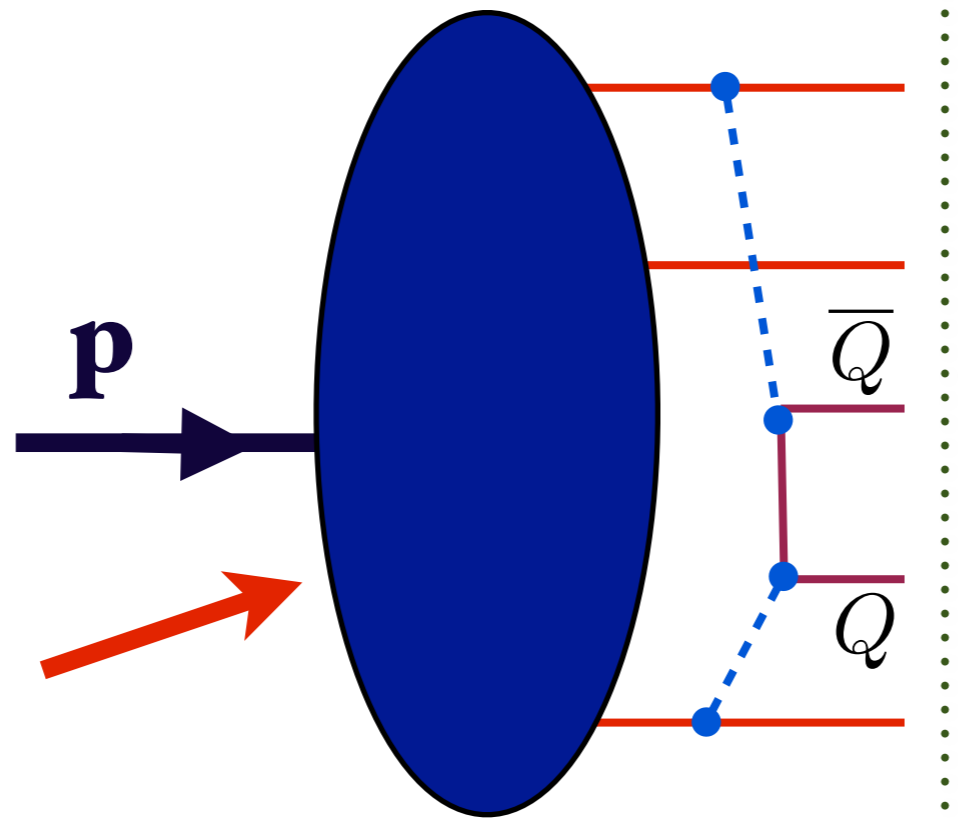
$$\Psi_{|uudc\bar{c}\rangle}(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$$\text{Probability } P_{uudc\bar{c}} \propto \frac{1}{M_c^2}$$

$$\hat{x}_c \sim \frac{m_{\perp c}}{\sum_{i=1}^5 m_{\perp i}} \quad m_{\perp i}^2 = m_i^2 + \vec{k}_{\perp i}^2$$

*High  $x$  intrinsic charm!*

*Proton 5-quark Fock State:  
Intrinsic Heavy Quarks*



*QCD predicts  
Intrinsic  
Heavy Quarks  
at high  $x$ !*

**Minimal off-shellness!**

$$x_Q \propto (m_Q^2 + k_{\perp}^2)^{1/2}$$

$$\text{Probability (QED)} \propto \frac{1}{M_{\ell}^4}$$

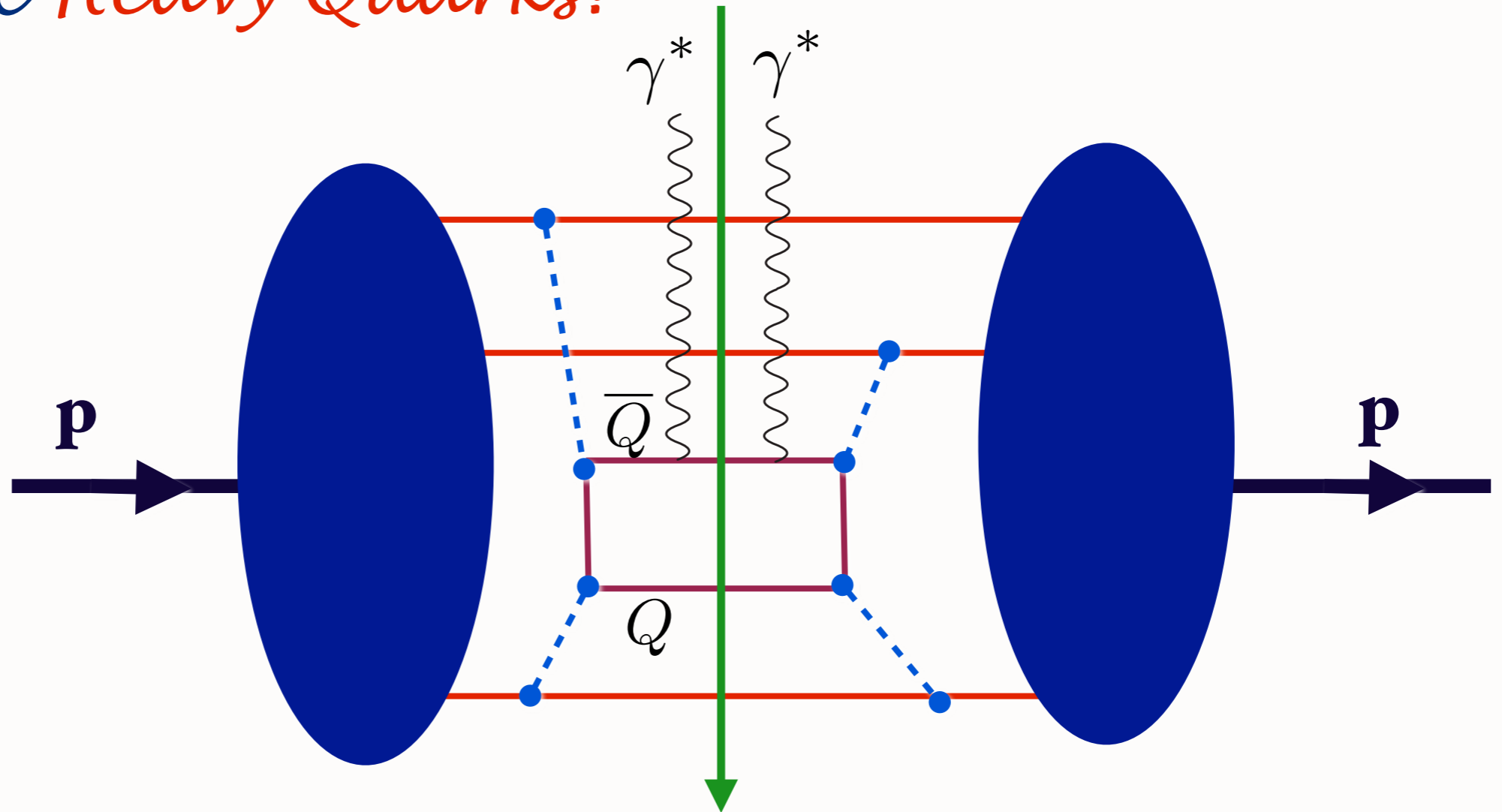
$$\text{Probability (QCD)} \propto \frac{1}{M_Q^2}$$

**Hoyer, Peterson, Sakai, Collins, Ellis, Gunion, Mueller, sjb  
Polyakov, et al.**

# Cut of Proton Self Energy:

QCD predicts

Intrinsic Heavy Quarks!



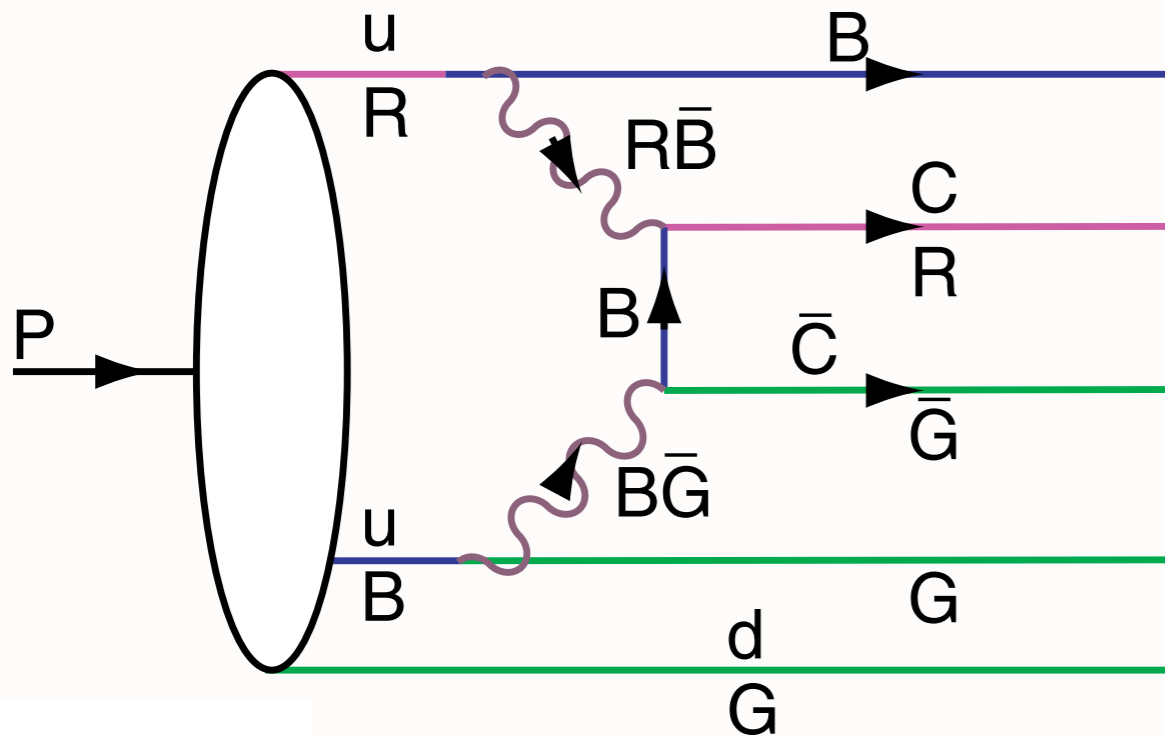
$$\text{Probability (QED)} \propto \frac{1}{M_\ell^4}$$

$$\text{Probability (QCD)} \propto \frac{1}{M_Q^2}$$

$$x_Q \propto (m_Q^2 + k_\perp^2)^{1/2}$$

**Hoyer, Peterson, Sakai, Collins, Ellis, Gunion, Mueller, sjb  
Polyakov, et al.**

BHPS: Hoyer, Peterson, Sakai, sjb



$|uudc\bar{c}\rangle$  Fluctuation in Proton

QCD: Probability  $\frac{\sim \Lambda_{QCD}^2}{M_Q^2}$

$|e^+e^-\ell^+\ell^-\rangle$  Fluctuation in Positronium

QED: Probability  $\frac{\sim (m_e\alpha)^4}{M_\ell^4}$

OPE derivation - M.Polyakov et al.

$$\langle p | \frac{G_{\mu\nu}^3}{m_Q^2} | p \rangle \text{ vs. } \langle p | \frac{F_{\mu\nu}^4}{m_\ell^4} | p \rangle$$

$c\bar{c}$  in Color Octet

Distribution peaks at equal rapidity (velocity)  
Therefore heavy particles carry the largest momentum fractions

$$\hat{x}_i = \frac{m_{\perp i}}{\sum_j^n m_{\perp j}}$$

*High x charm!*

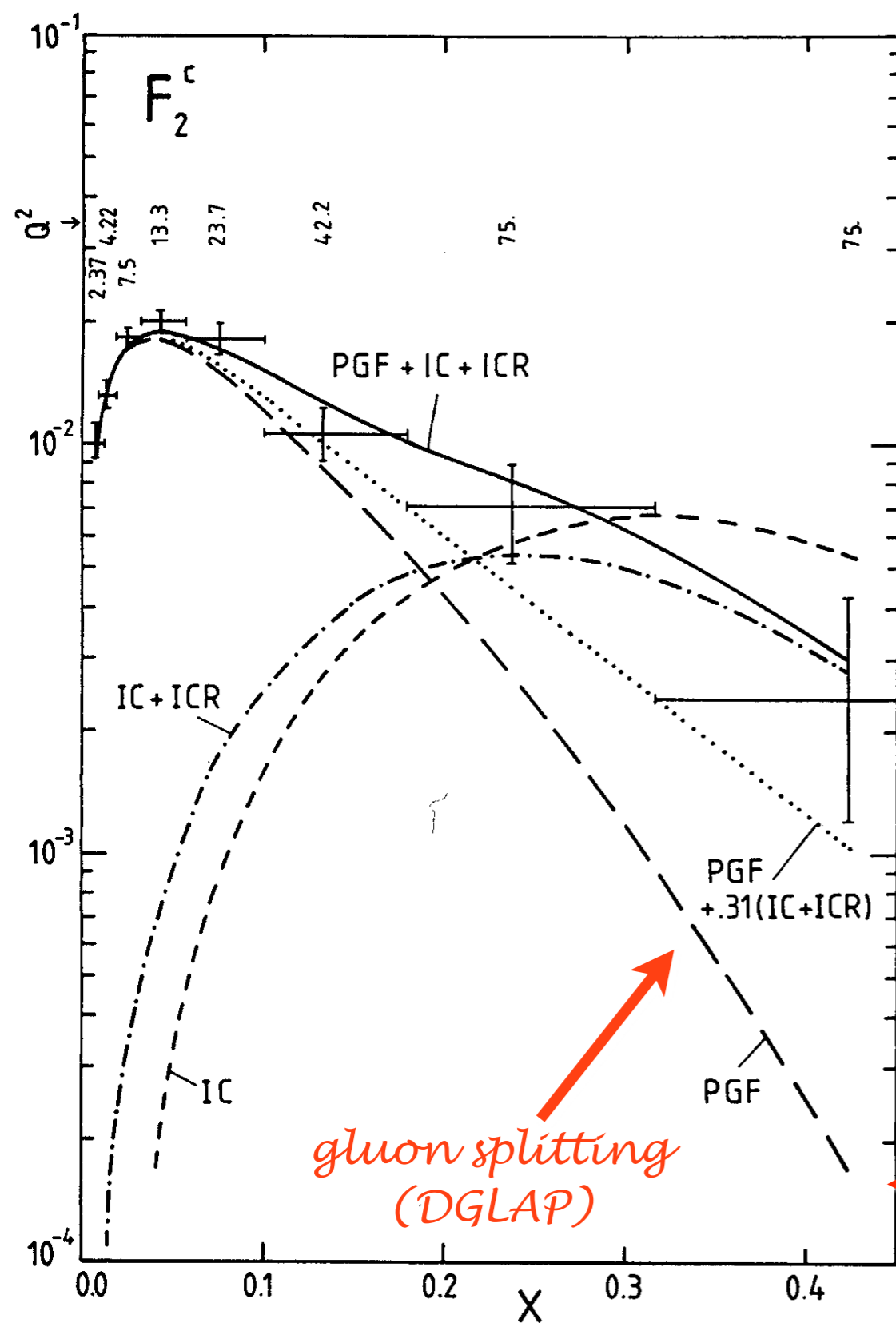
*Charm at Threshold*

**Action Principle: Minimum KE, maximal potential**

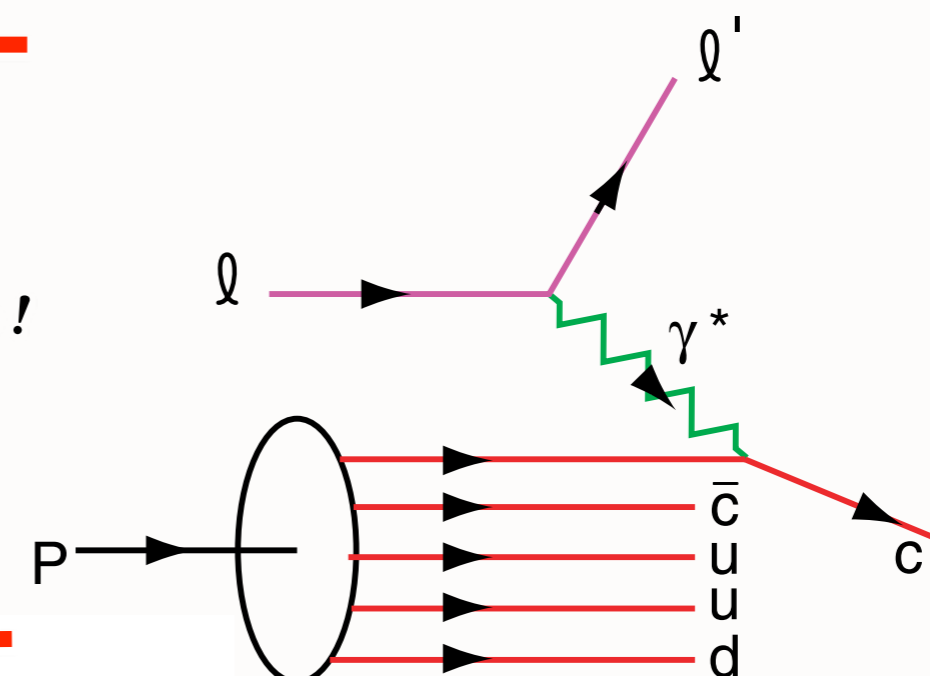
# Measurement of Charm Structure Function

J. J. Aubert et al. [European Muon Collaboration], "Production Of Charmed Particles In 250-GeV Mu+ - Iron Interactions," Nucl. Phys. B 213, 31 (1983).

## First Evidence for Intrinsic Charm



factor of 30!



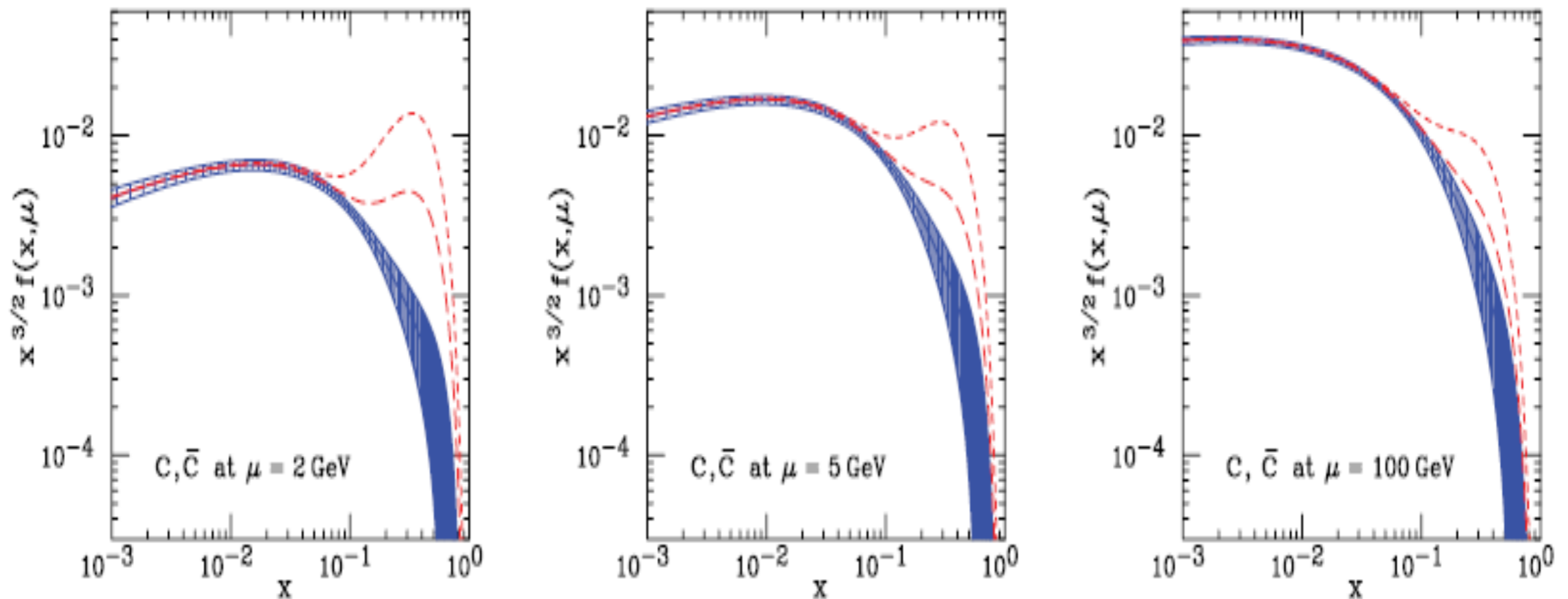
**DGLAP / Photon-Gluon Fusion: factor of 30 too small**

*Two Components (separate evolution):*

$$c(x, Q^2) = c(x, Q^2)_{\text{extrinsic}} + c(x, Q^2)_{\text{intrinsic}}$$



## CHARM QUARK DISTRIBUTIONS IN PROTON

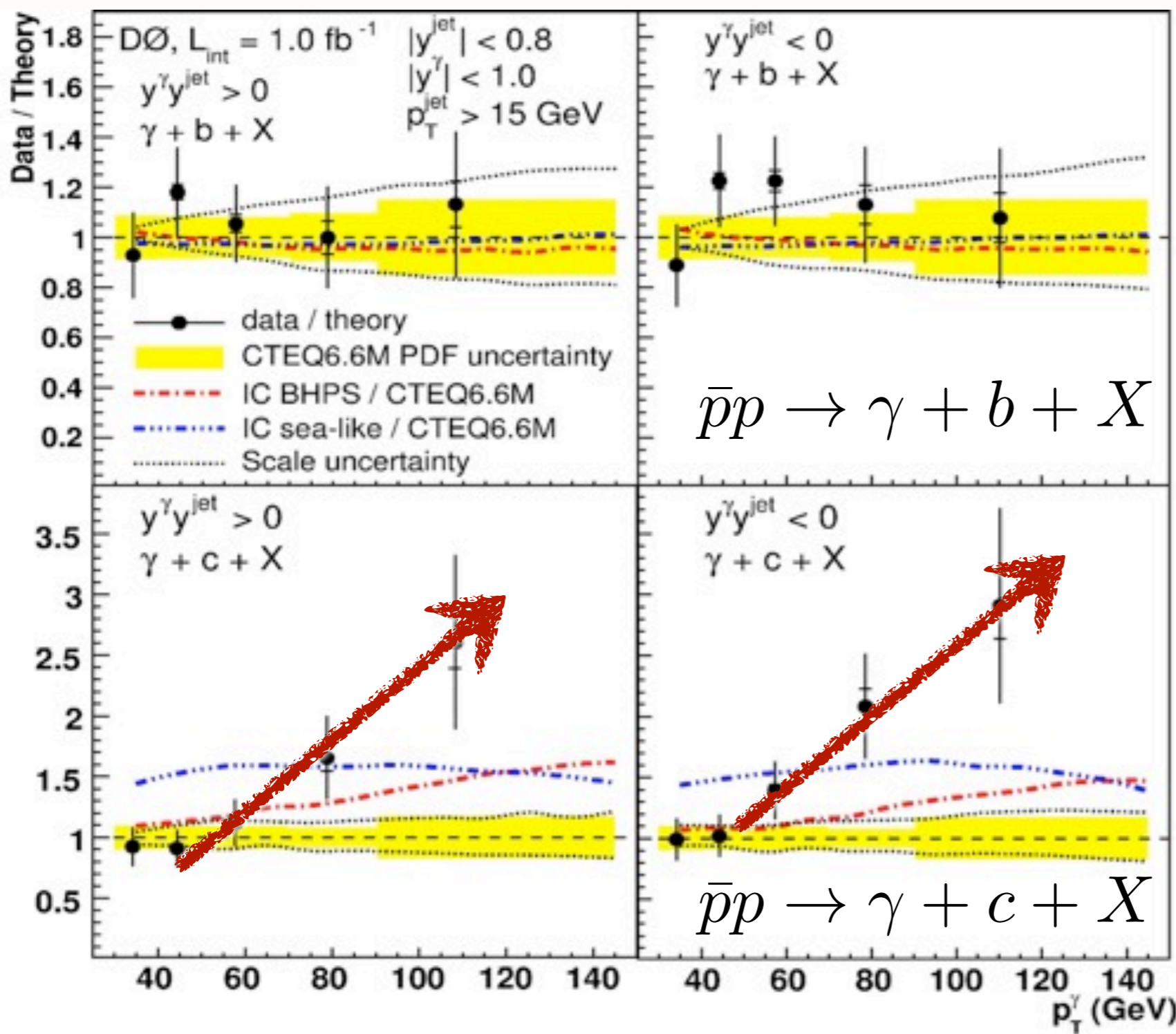


Charm quark distributions within the BHPS model.

**D0**

Measurement of  $\gamma + b + X$  and  $\gamma + c + X$  Production Cross Sections  
in  $p\bar{p}$  Collisions at  $\sqrt{s} = 1.96$  TeV

$$p\bar{p} \rightarrow \gamma + Q + X$$



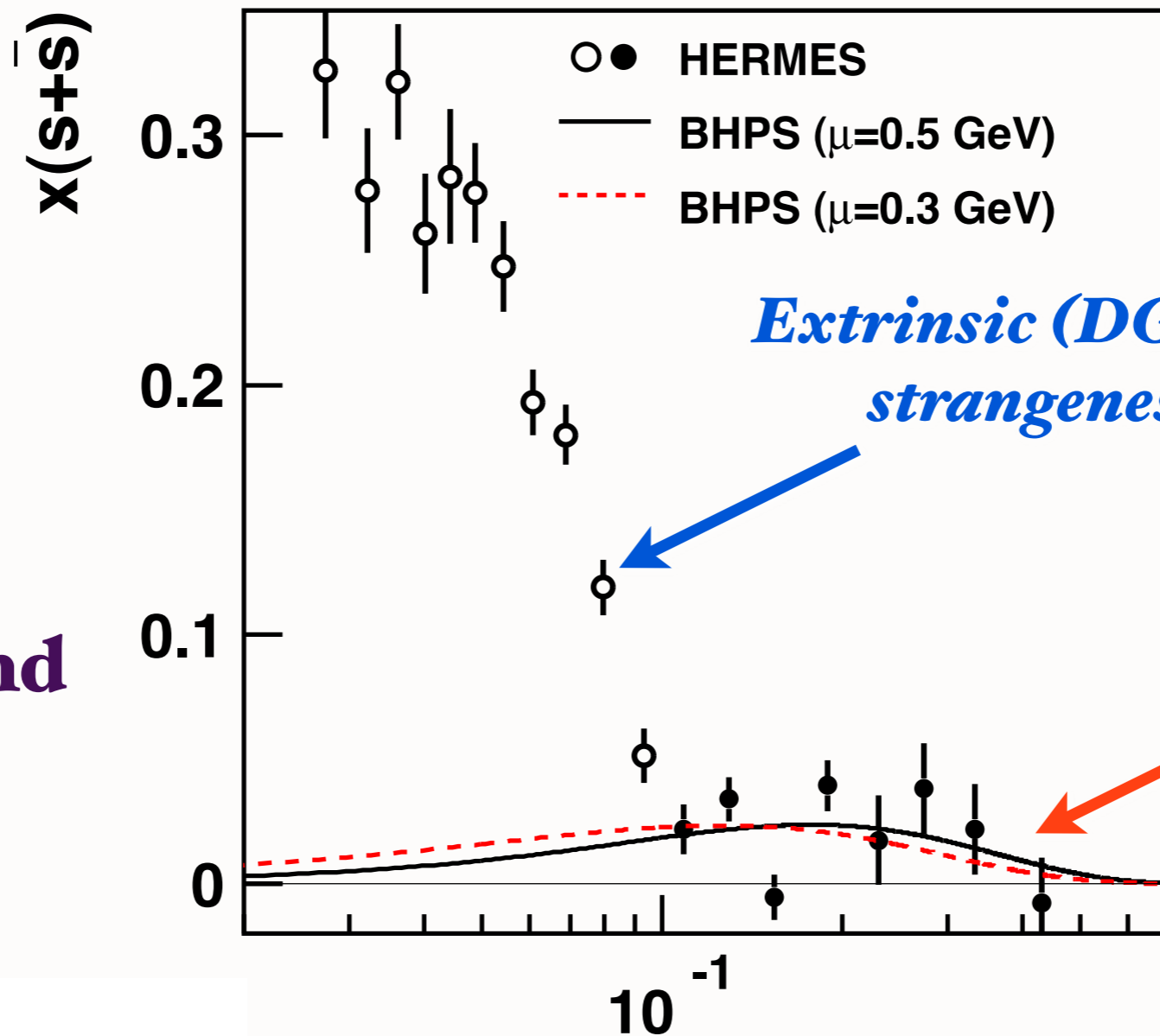
$$\frac{\Delta\sigma(\bar{p}p \rightarrow \gamma c X)}{\Delta\sigma(\bar{p}p \rightarrow \gamma b X)}$$

**Ratio is insensitive to gluon PDF, scales**

Consistent with  $\frac{m_c^2}{m_b^2}$   
relative suppression of intrinsic bottom

$$c(x, Q^2) = c(x, Q^2)_{\text{extrinsic}} + c(x, Q^2)_{\text{intrinsic}}$$

# HERMES: Two components to $s(x, Q^2)$ !

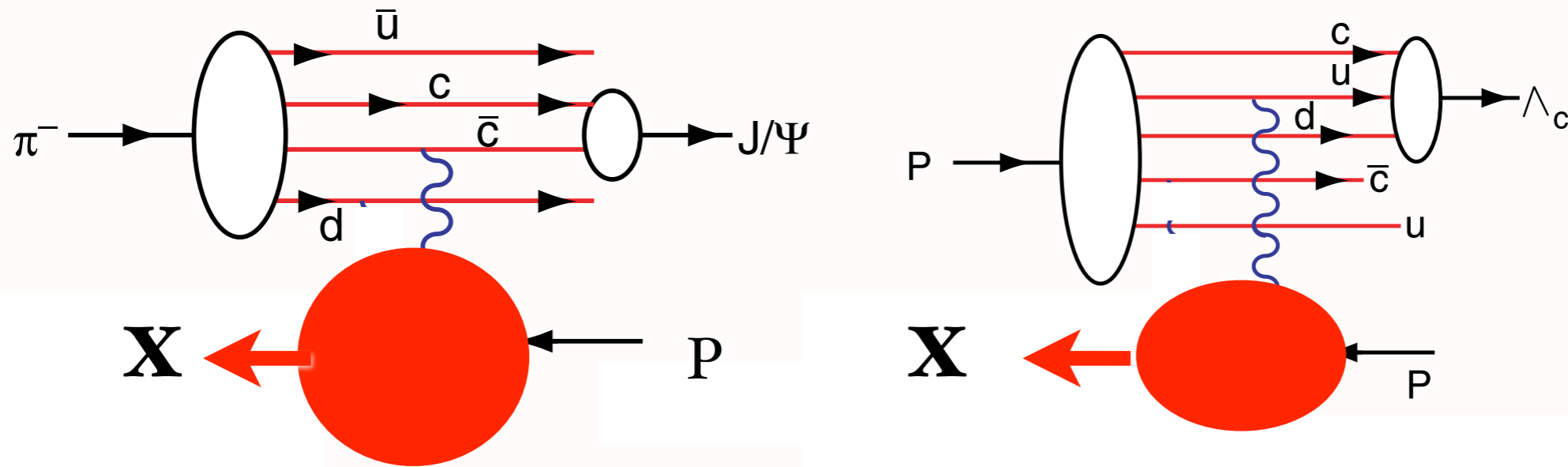


W. C. Chang and  
J.-C. Peng  
arXiv:1105.2381

Comparison of the HERMES  $x(s(x) + \bar{s}(x))$  data with the calculations based on the BHPS model. The solid and dashed curves are obtained by evolving the BHPS result to  $Q^2 = 2.5 \text{ GeV}^2$  using  $\mu = 0.5 \text{ GeV}$  and  $\mu = 0.3 \text{ GeV}$ , respectively. The normalizations of the calculations are adjusted to fit the data at  $x > 0.1$  with statistical errors only, denoted by solid circles.

$$s(x, Q^2) = s(x, Q^2)_{\text{extrinsic}} + s(x, Q^2)_{\text{intrinsic}}$$

# Leading Hadron Production from Intrinsic Charm



Coalescence of Comoving Charm and Valence Quarks  
Produce  $J/\psi$ ,  $\Lambda_c$  and other Charm Hadrons at High  $x_F$

- EMC data:  $c(x, Q^2) > 30 \times \text{DGLAP}$   
 $Q^2 = 75 \text{ GeV}^2, x = 0.42$

- High  $x_F$   $pp \rightarrow J/\psi X$

**CERN NA3**

- High  $x_F$   $pp \rightarrow J/\psi J/\psi X$

- High  $x_F$   $pp \rightarrow \Lambda_c X$

**ISR**

- High  $x_F$   $pp \rightarrow \Lambda_b X$

**Intrinsic Bottom!**  
**Zichichi, Cifarelli, et al.**

- High  $x_F$   $pp \rightarrow \Xi(ccd) X$  (SELEX)

**FermiLab**

**IC Structure Function: Critical Measurement for EIC**

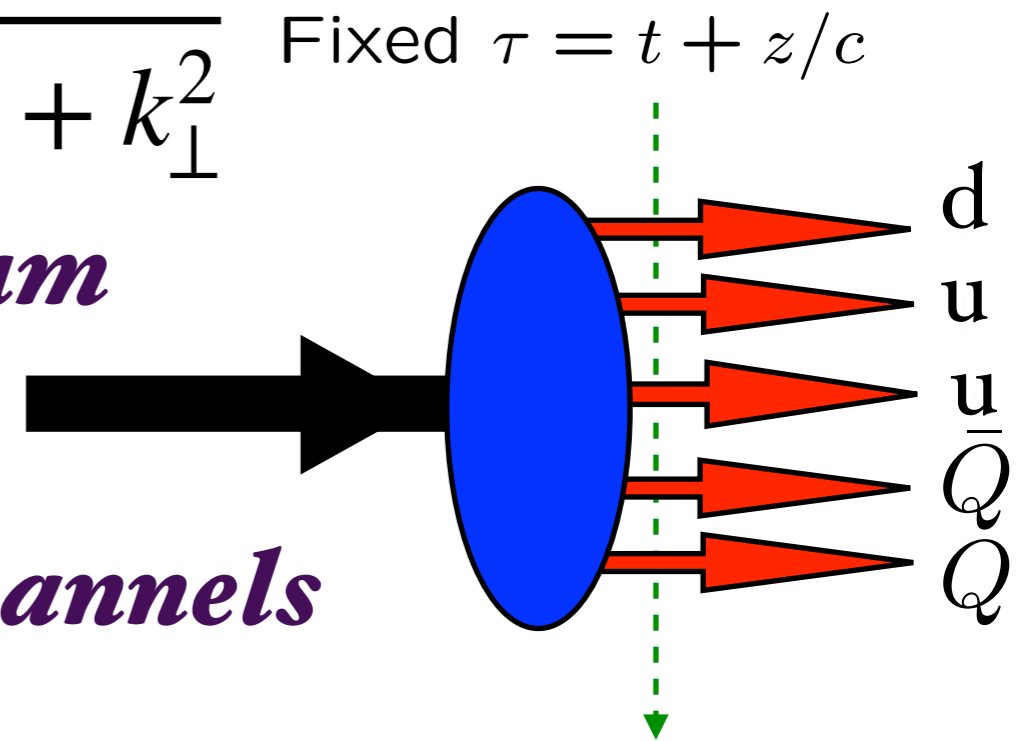
**Many interesting spin, charge asymmetry, spectator effects**



# Properties of Non-Perturbative Five-Quark Fock-State

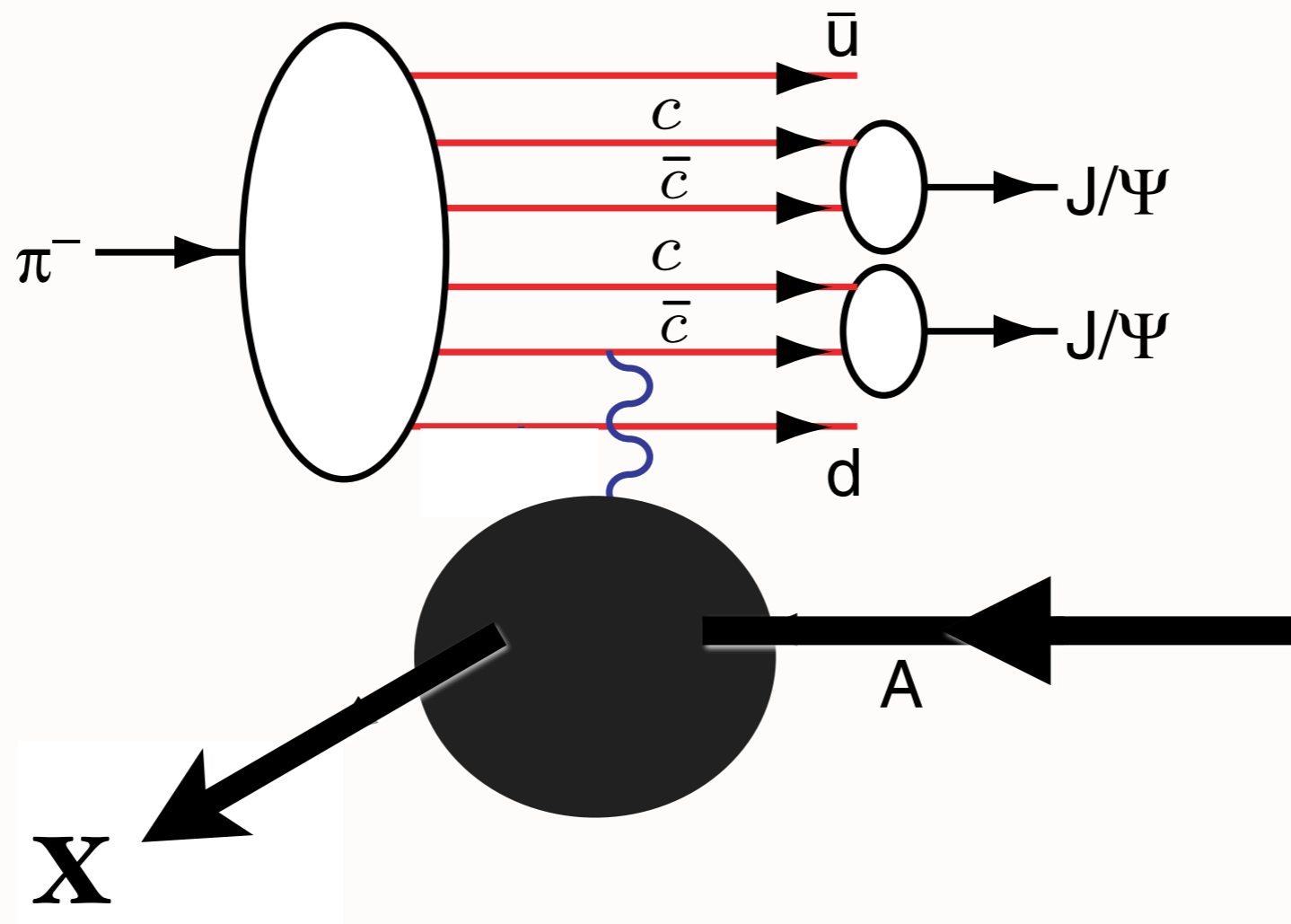
- **Dominant configuration: minimum off-shell, same rapidity**
- **Heavy quarks have most of the LF momentum**

$$\langle x_Q \rangle \propto \sqrt{m_Q^2 + k_{\perp}^2}$$
- **Correlated with proton quantum numbers**
- **Duality with meson-baryon channels**
- **Strangeness, charm asymmetry at  $x > 0.1$**

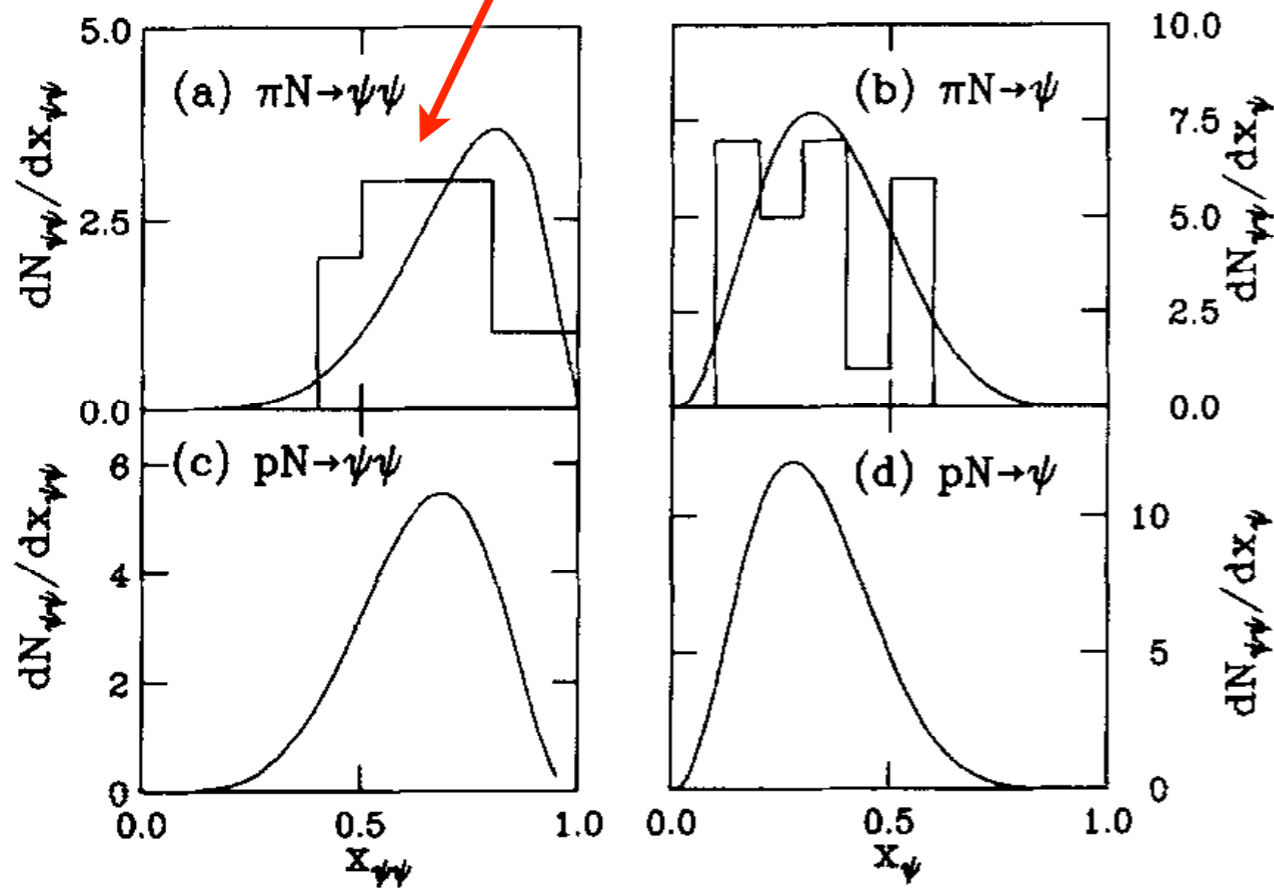


$$s_p(x) \neq \bar{s}_p(x) \quad c_p(x) \neq \bar{c}_p(x)$$

# Production of Two Charmonia at High $x_F$



All events have  $x_{\psi\psi}^F > 0.4$  !



# Double $J/\psi$ Production

$$\pi A \rightarrow J/\psi J/\psi X$$

R. Vogt, sjb

The probability distribution for a general  $n$ -particle intrinsic  $c\bar{c}$  Fock state as a function of  $x$  and  $k_T$  is written as

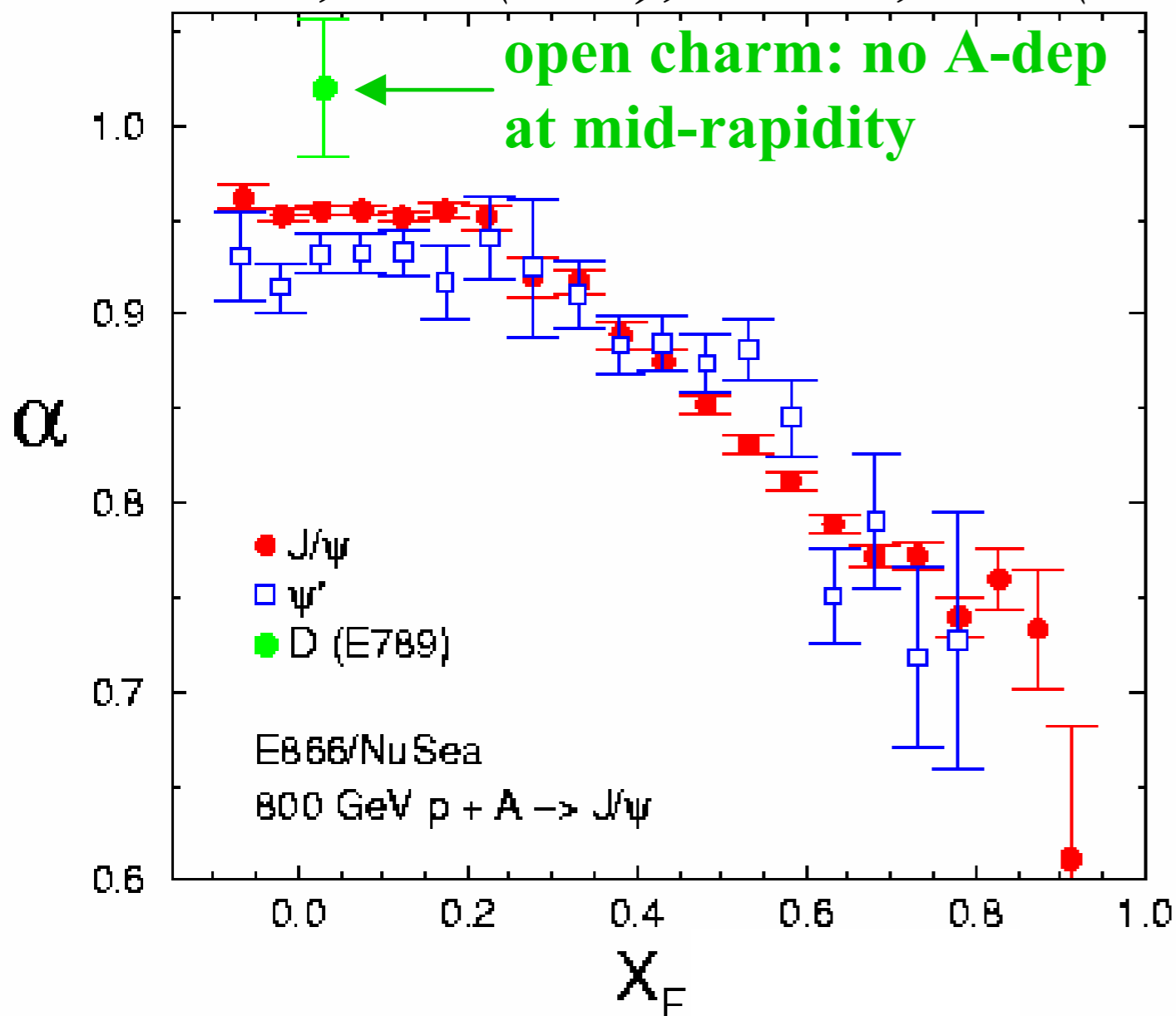
$$\frac{dP_{ic}}{\prod_{i=1}^n dx_i d^2 k_{T,i}} = N_n \alpha_s^4 (M_{c\bar{c}}) \frac{\delta(\sum_{i=1}^n k_{T,i}) \delta(1 - \sum_{i=1}^n x_i)}{(m_h^2 - \sum_{i=1}^n (m_{T,i}^2/x_i))^2},$$

The  $\psi\psi$  pair distributions are shown in (a) and (c) for the pion and proton projectiles. Similarly, the distributions of  $J/\psi$ 's from the pairs are shown in (b) and (d). Our calculations are compared with the  $\pi^- N$  data at 150 and 280 GeV/c [1]. The  $x_{\psi\psi}$  distributions are normalized to the number of pairs from both pion beams (a) and the number of pairs from the 400 GeV proton measurement (c). The number of single  $J/\psi$ 's is twice the number of pairs.

NA3 Data

Excludes PYTHIA  
'color drag' model

800 GeV p-A (FNAL)  $\sigma_A = \sigma_p * A^\alpha$   
 PRL 84, 3256 (2000); PRL 72, 2542 (1994)



*Remarkably Strong Nuclear Dependence for Fast Charmonium*

$$\frac{\frac{d\sigma}{dx_F}(pA \rightarrow J/\psi X)}{\frac{d\sigma}{dx_F}(pp \rightarrow J/\psi X)} = A^{\alpha(x_F)}$$

*Violation of PQCD Factorization.*

Violation of factorization in charm hadroproduction.

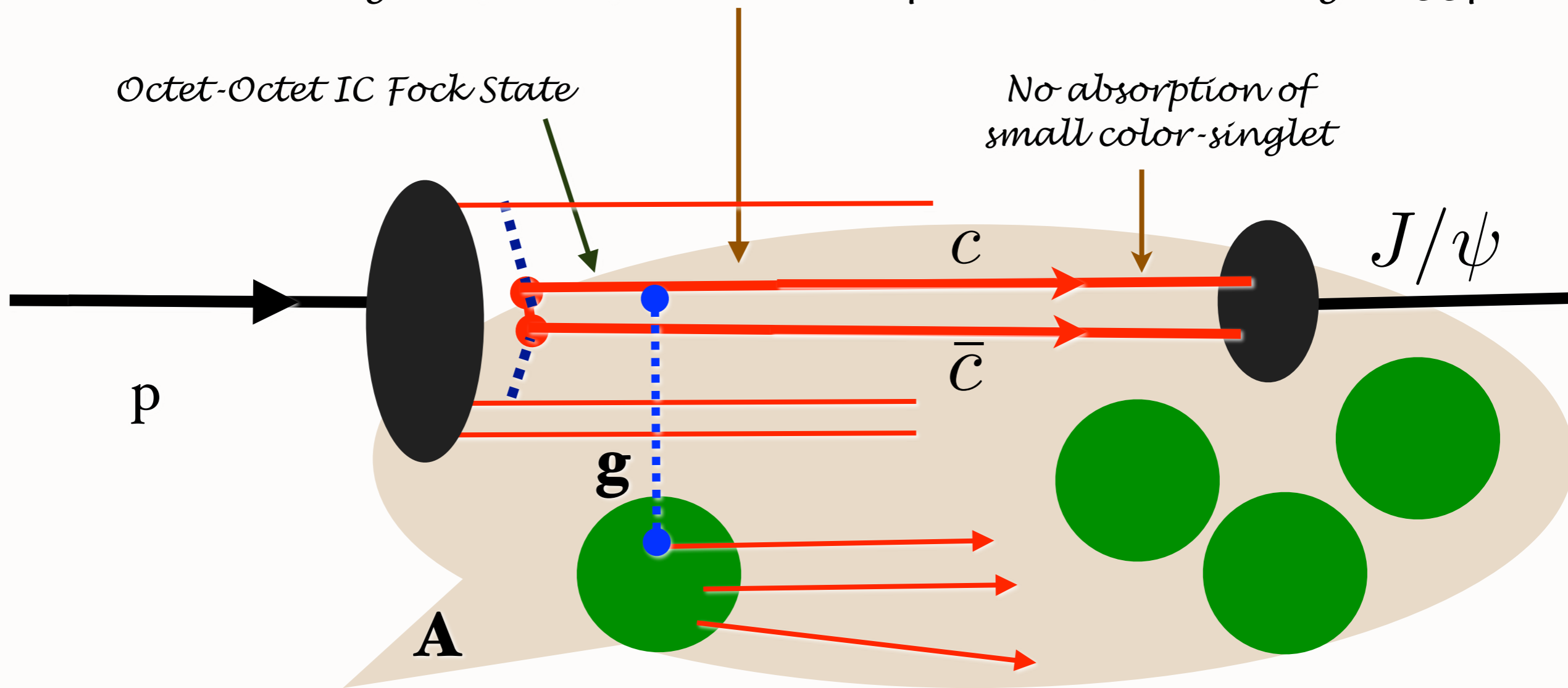
[P. Hoyer](#), [M. Vanttinen](#) (Helsinki U.), [U. Sukhatme](#) (Illinois U., Chicago) . HU-TFT-90-14, May 1990. 7pp.

**IC Explains large excess of quarkonia at large  $x_F$ , A-dependence**

E866/NuSea data for the nuclear A dependence of  $J/\psi$  and  $\psi'$  hadroproduction.

*Color-Opaque IC Fock state  
interacts on nuclear front surface*

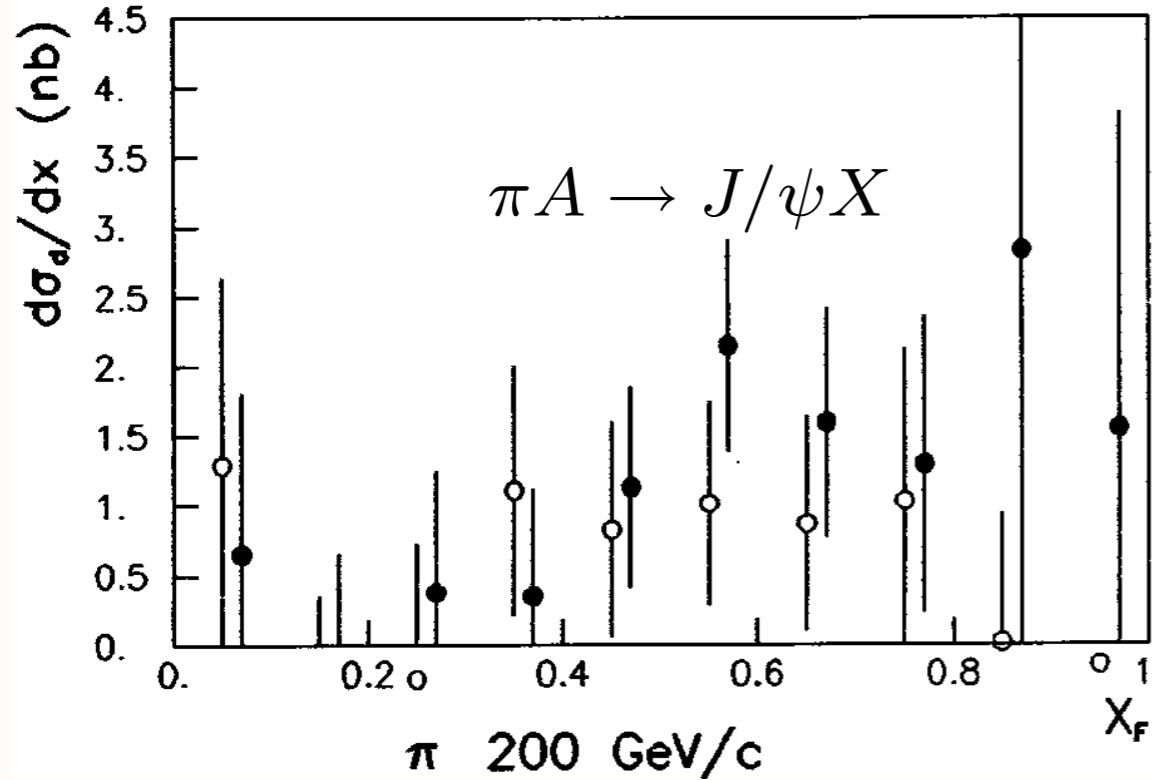
*Scattering on front-face nucleon produces color-singlet  $c\bar{c}$  pair*



$$\frac{d\sigma}{dx_F}(pA \rightarrow J/\psi X) = A^{2/3} \times \frac{d\sigma}{dx_F}(pN \rightarrow J/\psi X)$$

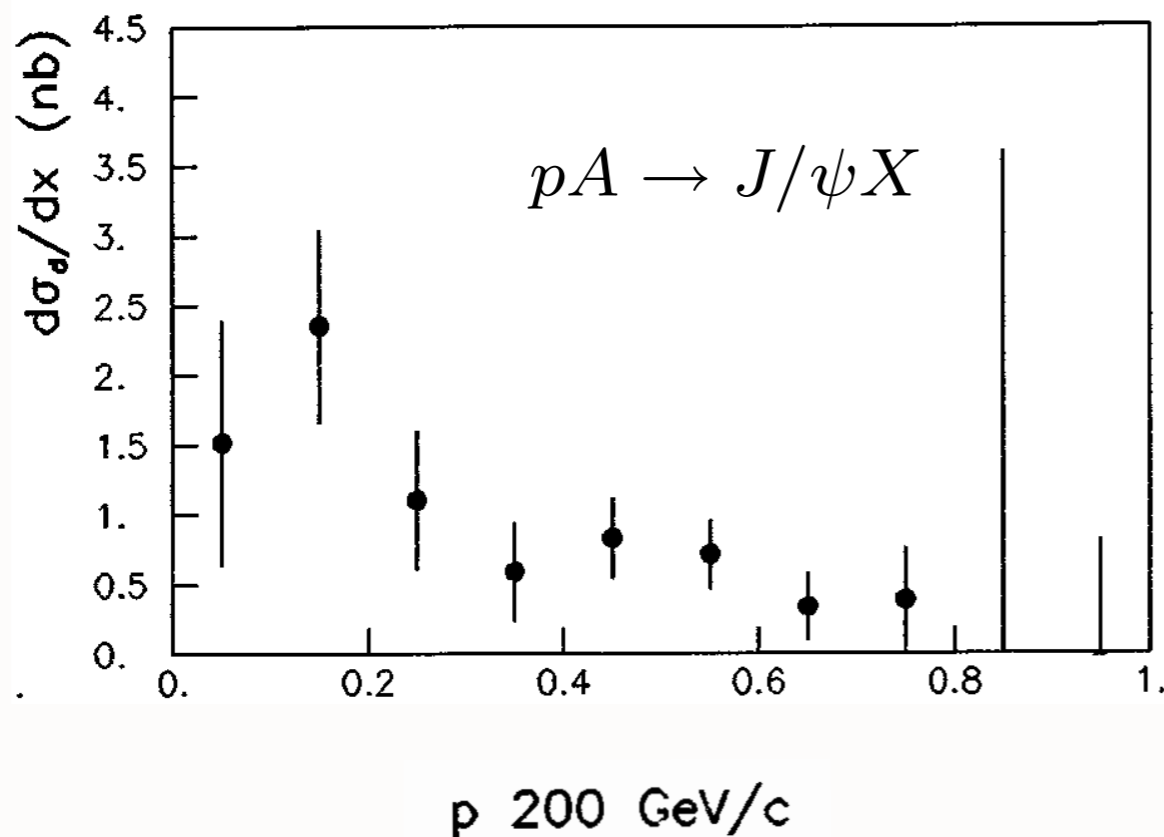


# J. Badier et al, NA3



$$\frac{d\sigma}{dx_F}(pA \rightarrow J/\psi X) = A^1 \frac{d\sigma_1}{dx_F} + A^{2/3} \frac{d\sigma_2}{dx_F}$$

$A^{2/3}$  component

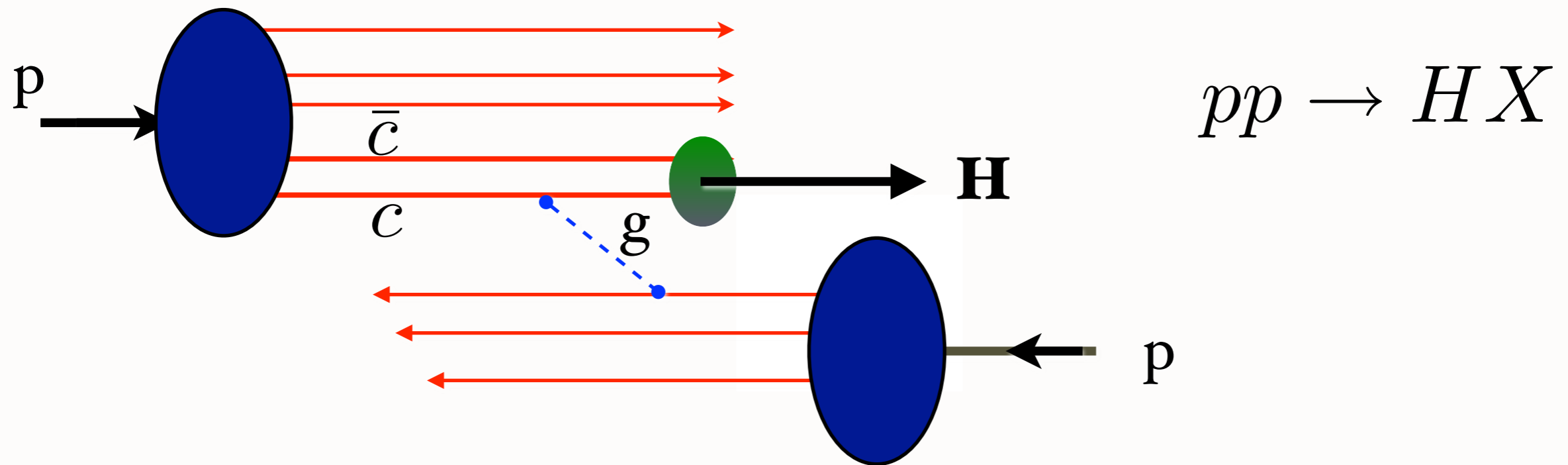


*High  $x_F$ :*

*Consistent with  
color-octet intrinsic  
charm*

**Excess beyond conventional gluon-splitting PQCD subprocesses**

# *Intrinsic Charm Mechanism for Inclusive High- $x_F$ Higgs Production*



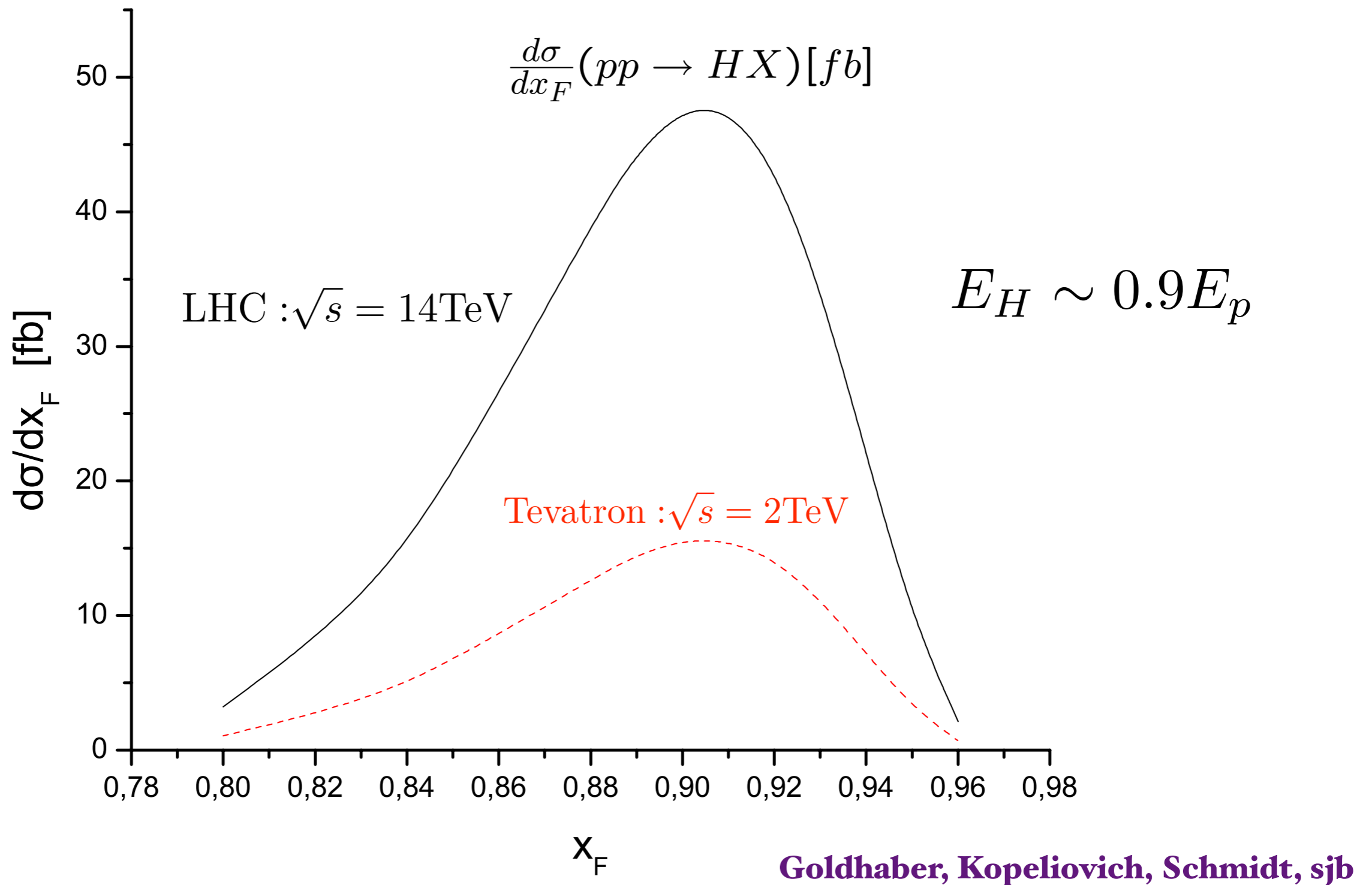
**Also: intrinsic bottom, top**

**Goldhaber, Kopeliovich,  
Schmidt, sjb**

**Higgs can have 80% of Proton Momentum!**

*New search strategy for Higgs*

# Intrinsic Heavy Quark Contribution to Inclusive Higgs Production



Measure  $H \rightarrow ZZ^* \rightarrow \mu^+ \mu^- \mu^+ \mu^-$ .

# Constraints on charm-anticharm asymmetry in the nucleon from lattice QCD

Raza Sabbir Sufian<sup>a</sup>, Tianbo Liu<sup>a</sup>, Andrei Alexandru<sup>b,c</sup>, Stanley J. Brodsky<sup>d</sup>, Guy F. de Téramond<sup>e</sup>,  
Hans Günter Dosch<sup>f</sup>, Terrence Draper<sup>g</sup>, Keh-Fei Liu<sup>g</sup>, Yi-Bo Yang<sup>h</sup>

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<sup>b</sup>*Department of Physics, The George Washington University, Washington, DC 20052, USA*

<sup>c</sup>*Department of Physics, University of Maryland, College Park, MD 20742, USA*

<sup>d</sup>*SLAC National Accelerator Laboratory, Stanford University, Stanford, CA 94309, USA*

<sup>e</sup>*Laboratorio de Física Teórica y Computacional, Universidad de Costa Rica, 11501 San José, Costa Rica*

<sup>f</sup>*Institut für Theoretische Physik der Universität, D-69120 Heidelberg, Germany*

<sup>g</sup>*Department of Physics and Astronomy, University of Kentucky, Lexington, Kentucky 40506, USA*

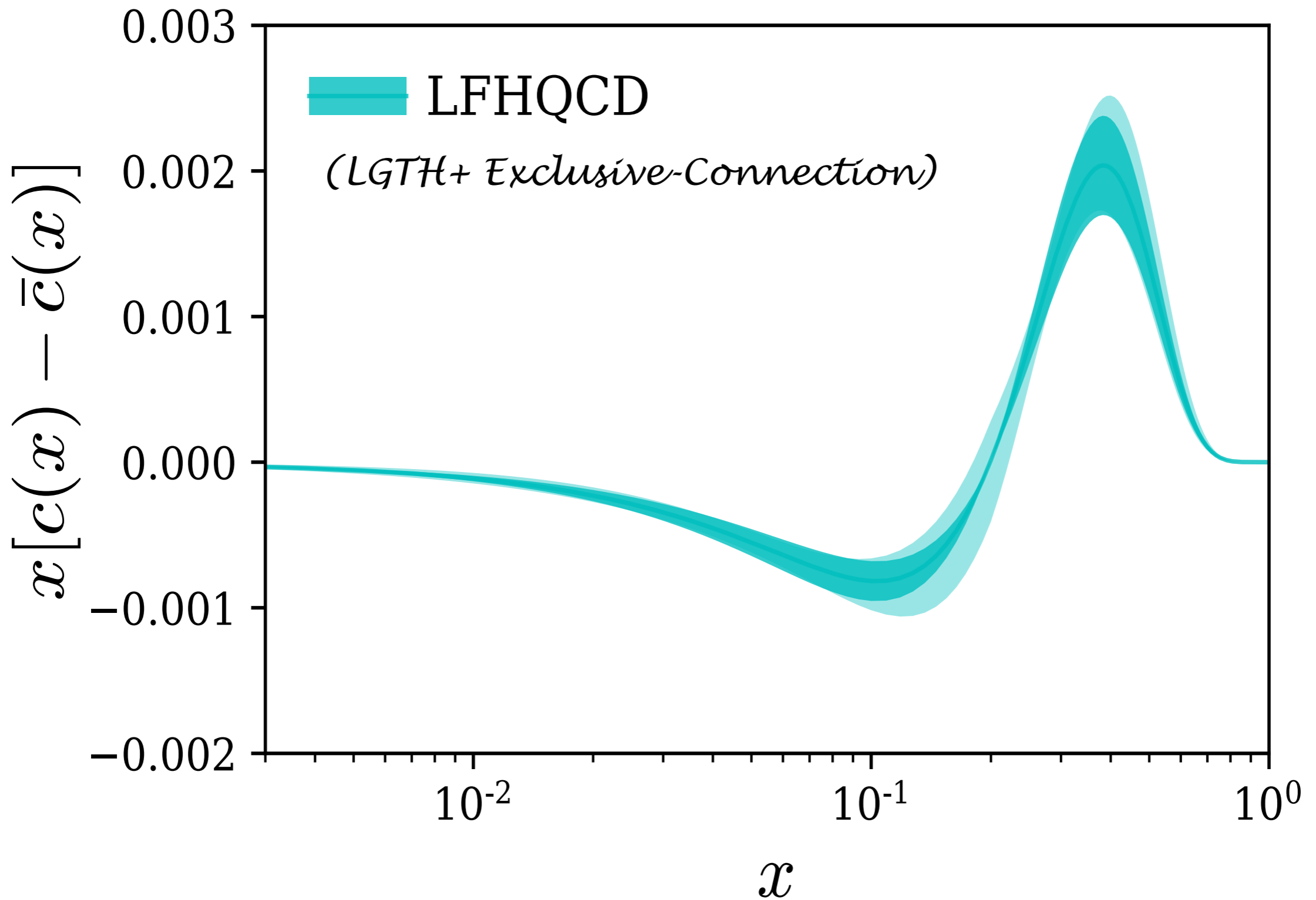
<sup>h</sup>*CAS Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, China*

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## Abstract

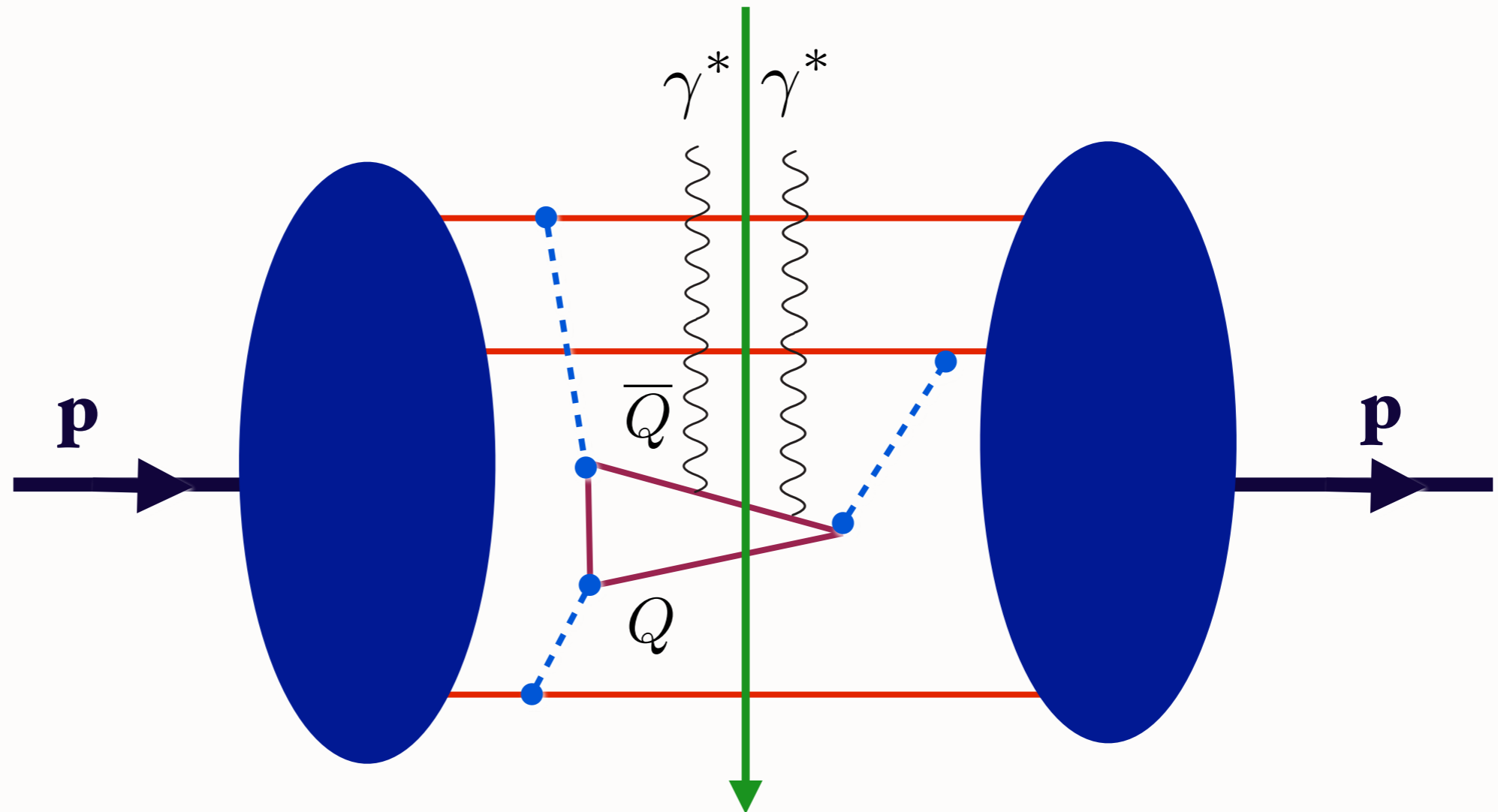
We present the first lattice QCD calculation of the charm quark contribution to the nucleon electromagnetic form factors  $G_{E,M}^c(Q^2)$  in the momentum transfer range  $0 \leq Q^2 \leq 1.4 \text{ GeV}^2$ . The quark mass dependence, finite lattice spacing and volume corrections are taken into account simultaneously based on the calculation on three gauge ensembles including one at the physical pion mass. The nonzero value of the charm magnetic moment  $\mu_M^c = -0.00127(38)_{\text{stat}}(5)_{\text{sys}}$ , as well as the Pauli form factor, reflects a nontrivial role of the charm sea in the nucleon spin structure. The nonzero  $G_E^c(Q^2)$  indicates the existence of a nonvanishing asymmetric charm-anticharm sea in the nucleon. Performing a non-perturbative analysis based on holographic QCD and the generalized Veneziano model, we study the constraints on the  $[c(x) - \bar{c}(x)]$  distribution from the lattice QCD results presented here. Our results provide complementary information and motivation for more detailed studies of physical observables that are sensitive to intrinsic charm and for future global analyses of parton distributions including asymmetric charm-anticharm distribution.

*Keywords:* Intrinsic charm, Form factor, Parton distributions, Lattice QCD, Light-front holographic QCD, JLAB-THY-20-3155, SLAC-PUB-17515



The distribution function  $x[c(x) - \bar{c}(x)]$  obtained from the LFHQCD formalism using the lattice QCD input of charm electromagnetic form factors  $G_{E,M}^c(Q^2)$ . The outer cyan band indicates an estimate of systematic uncertainty in the  $x[c(x) - \bar{c}(x)]$  distribution obtained from a variation of the hadron scale  $\kappa_c$  by 5%.

## Interference of Intrinsic and Extrinsic Heavy Quark Amplitudes

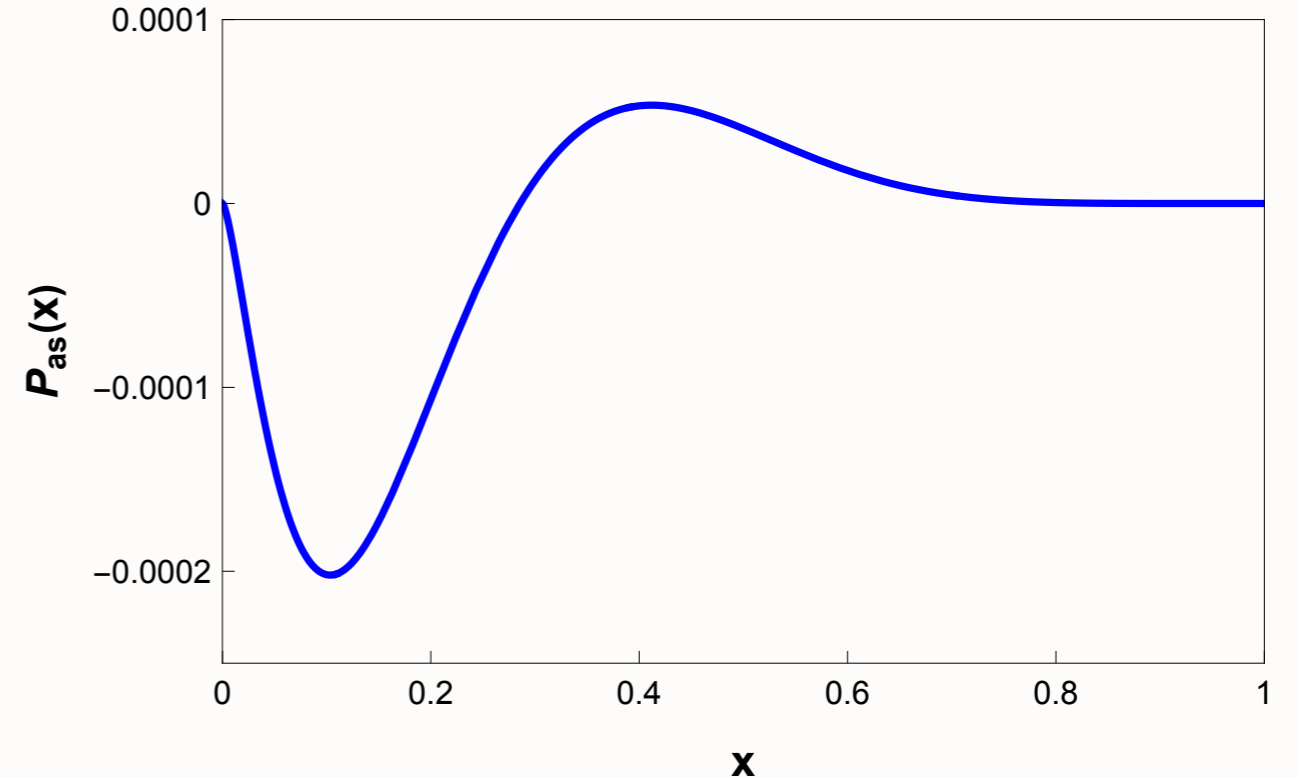
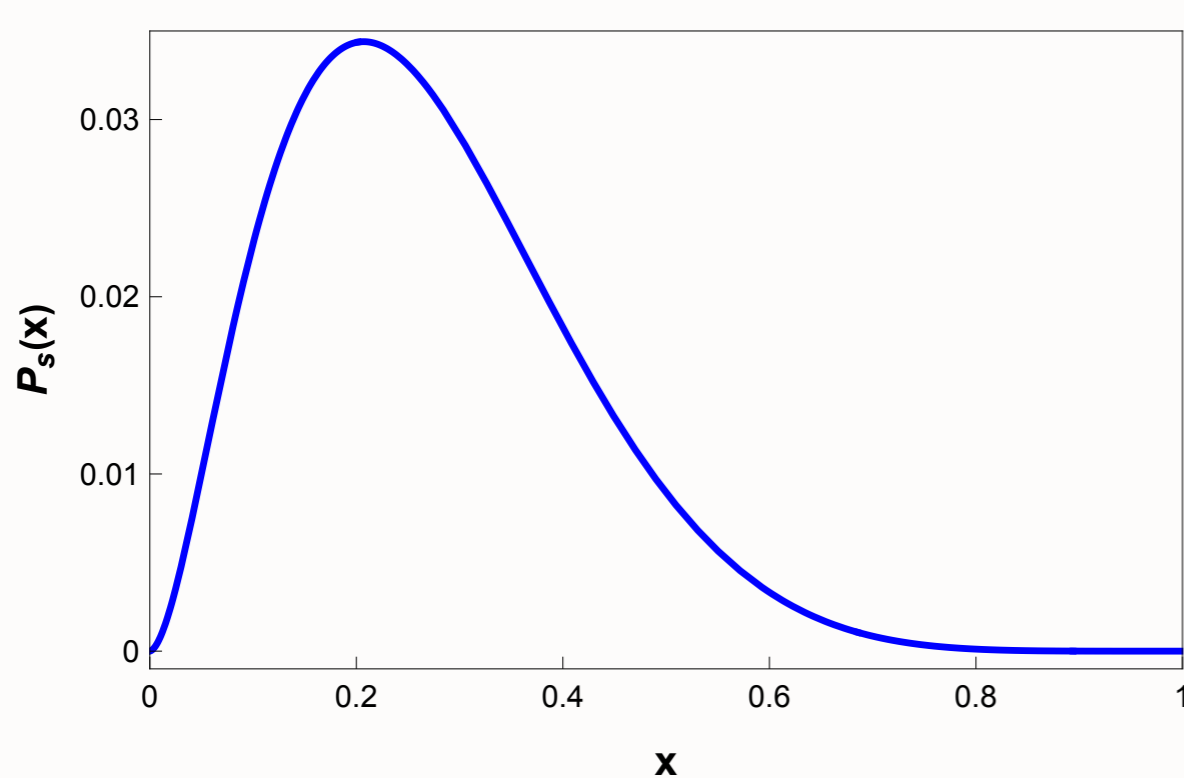


Interference predicts  $Q(x) \neq \bar{Q}(x)$   
 $\frac{d\sigma}{dydp_T^2}(pp \rightarrow D^+ cdX) \neq \frac{d\sigma}{dydp_T^2}(pp \rightarrow D^- \bar{c}dX)$

QED Analog: J. Gillespie, sjb (1968)



$$e^- + p \rightarrow e^- + Q + \bar{Q} + X.$$



## Interference of DGLAP and Intrinsic Heavy Quark Amplitudes

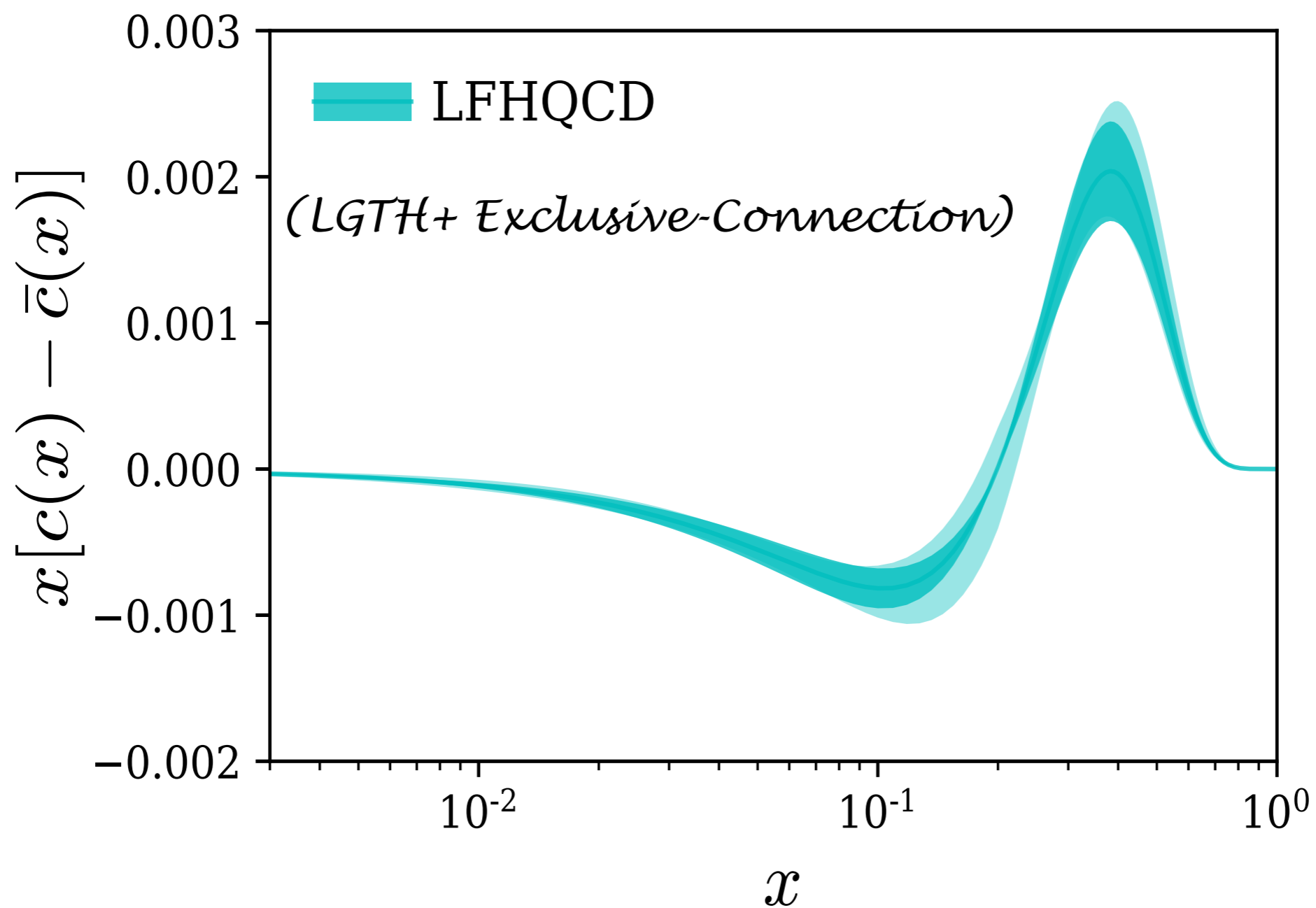
Also: Novel Asymmetry odd in transverse momentum

but CP invariant

$$\psi_{Q/\bar{Q};+\frac{1}{2}}^\uparrow(x, \mathbf{k}_\perp) = -\left[\psi_{Q/\bar{Q};-\frac{1}{2}}^\downarrow(x, \mathbf{k}_\perp)\right]^\dagger = \frac{\alpha_s C_F}{2\pi} \frac{k^1 - ik^2}{\kappa} x(1-x) \varphi^{(2)}(x, \mathbf{k}_\perp) \quad (L_z = -1),$$

$$\psi_{Q/\bar{Q};+\frac{1}{2}}^\uparrow(x, \mathbf{k}_\perp) = +\left[\psi_{-1+\frac{1}{2}}^\downarrow(x, \mathbf{k}_\perp)\right]^\dagger = \frac{\alpha_s C_F}{2\pi} x(1-x) \varphi^{(1)}(x, \mathbf{k}_\perp^2) \quad (L_z = 0),$$

$$\psi_{Q/\bar{Q};-\frac{1}{2}}^\uparrow(x, \mathbf{k}_\perp) = -\left[\psi_{Q/\bar{Q};+\frac{1}{2}}^\downarrow(x, \mathbf{k}_\perp)\right]^\dagger = -\frac{\alpha_s C_F}{2\pi} \frac{k^1 + ik^2}{\kappa} x(1-x)^2 \varphi^{(2)}(x, \mathbf{k}_\perp^2) \quad (L_z = 1),$$



Predict charm hadron asymmetries

$$\frac{d\sigma}{dx_F dp_T^2}(pp \rightarrow D^+(c\bar{d})X) >$$

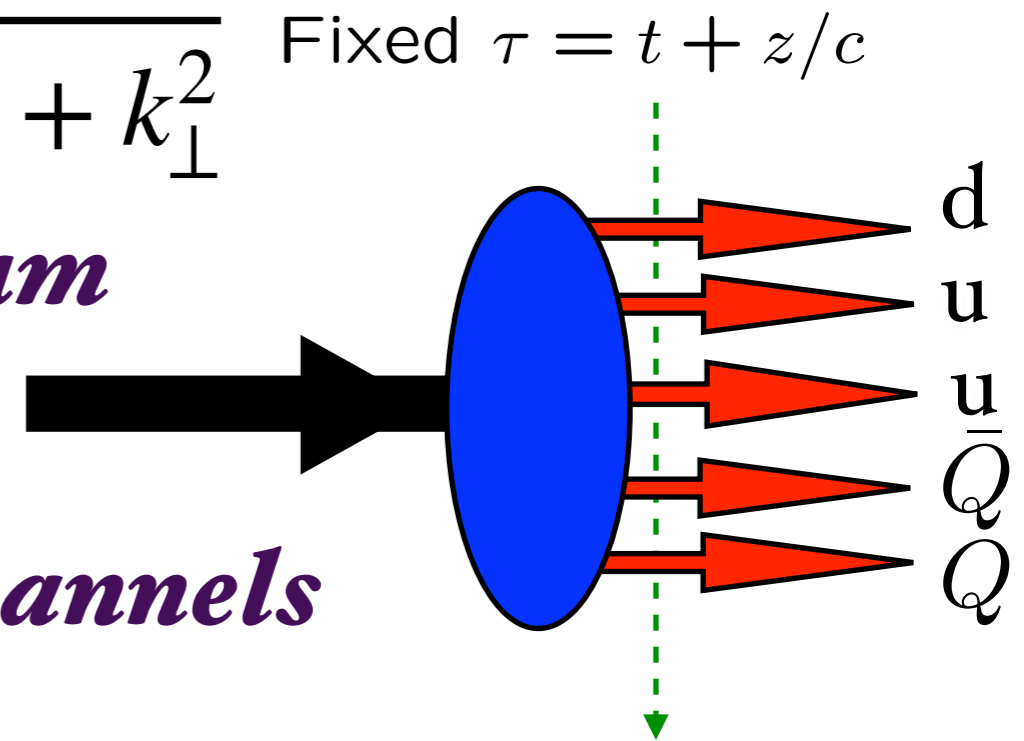
$$\frac{d\sigma}{dx_F dp_T^2}(pp \rightarrow D^-(\bar{c}d)X)$$

at high  $p_T$  and high  $x_F$

# Properties of Non-Perturbative Five-Quark Fock-State

- **Dominant configuration: minimum off-shell, same rapidity**
- **Heavy quarks have most of the LF momentum**

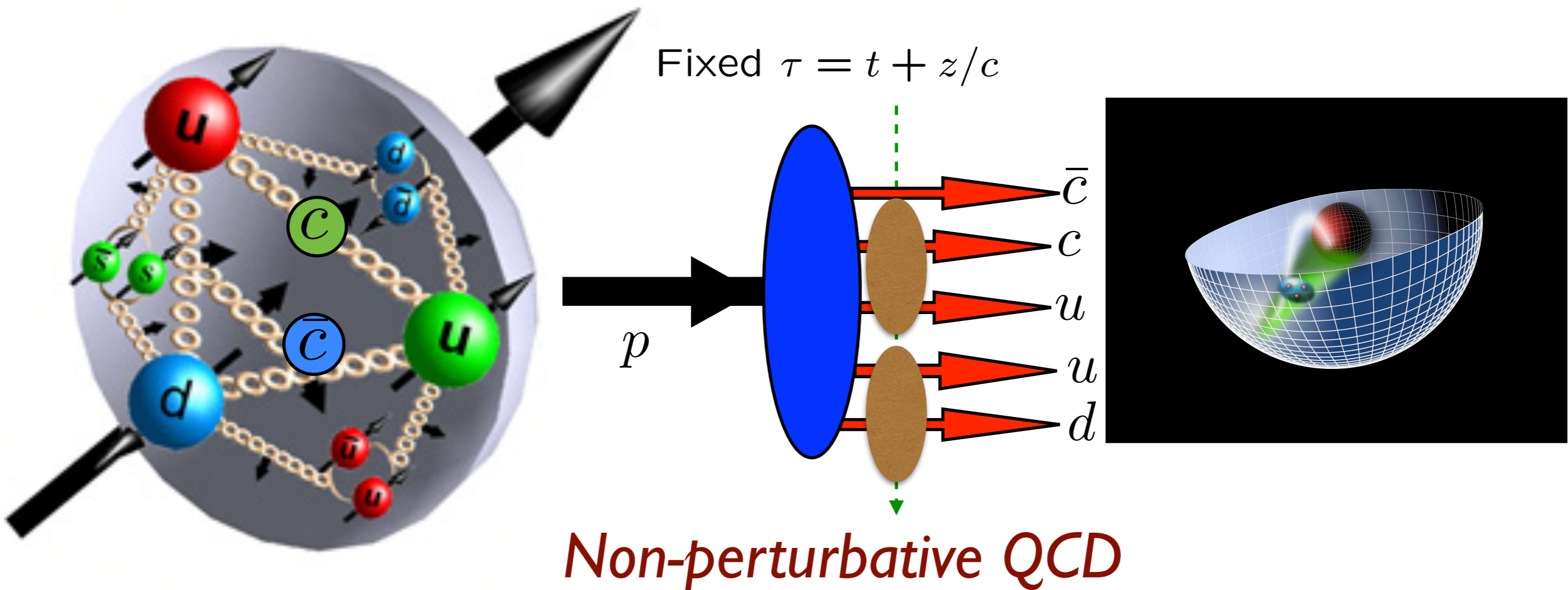
$$\langle x_Q \rangle \propto \sqrt{m_Q^2 + k_{\perp}^2}$$
- **Correlated with proton quantum numbers**
- **Duality with meson-baryon channels**
- **Strangeness, charm asymmetry at  $x > 0.1$**



$$s_p(x) \neq \bar{s}_p(x) \quad c_p(x) \neq \bar{c}_p(x)$$

# Intrinsic Heavy Quark Phenomena

## A Novel Property of QCD



$$|p\rangle = C_{valence} |u[ud]\rangle + C_{intrinsic} |\bar{c}[cu][ud]\rangle$$

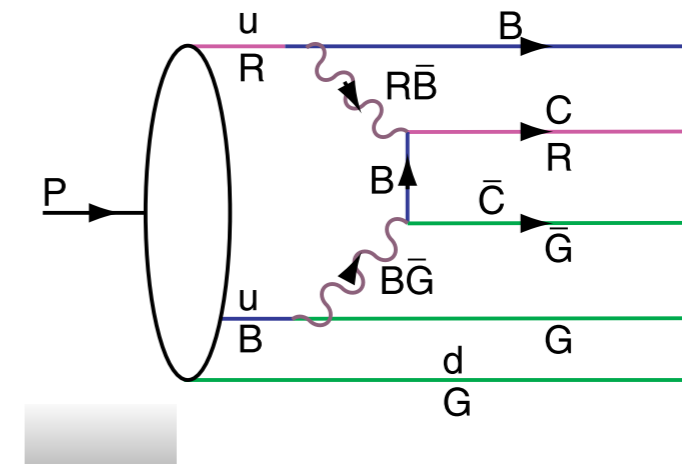
$$[du]_{\bar{3}_C} \text{ and } [cu]_{\bar{3}_C} \quad J = 0 \text{ diquark dominance}$$

$$c(x) \neq \bar{c}(x)$$

$\bar{c}(x)$  carries proton spin in the  $|[ud][uc]\bar{c}\rangle$  intrinsic charm Fock state.

# Intrinsic Heavy-Quark Fock States

- Rigorous prediction of QCD, OPE
- Color-Octet Color-Octet Fock State!

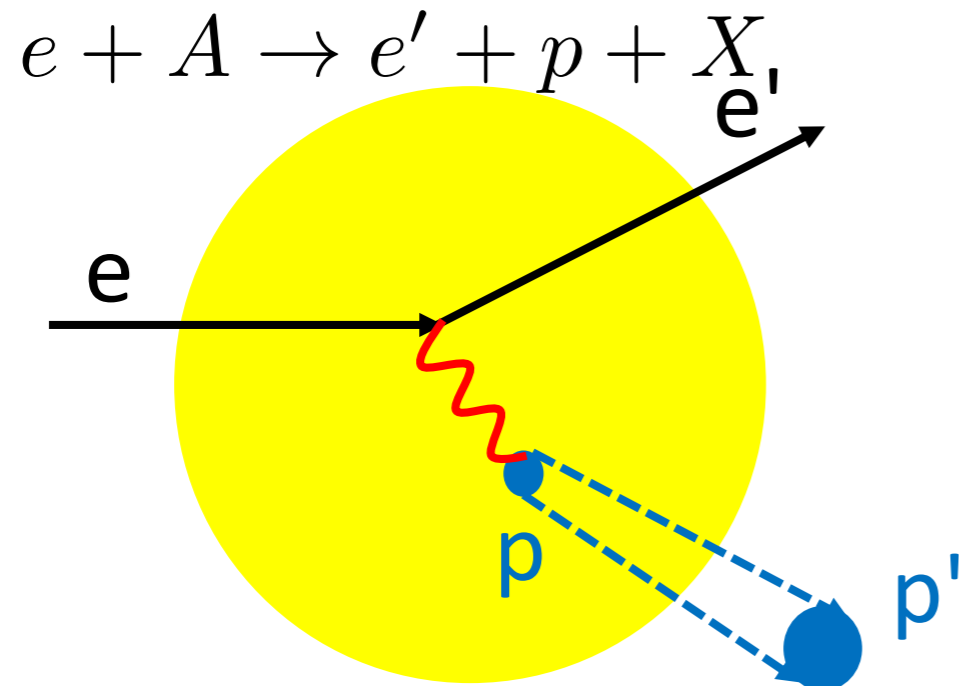


- Probability  $P_{Q\bar{Q}} \propto \frac{1}{M_Q^2}$   $P_{QQ\bar{Q}\bar{Q}} \sim \alpha_s^2 P_{Q\bar{Q}}$   $P_{c\bar{c}/p} \simeq 1\%$

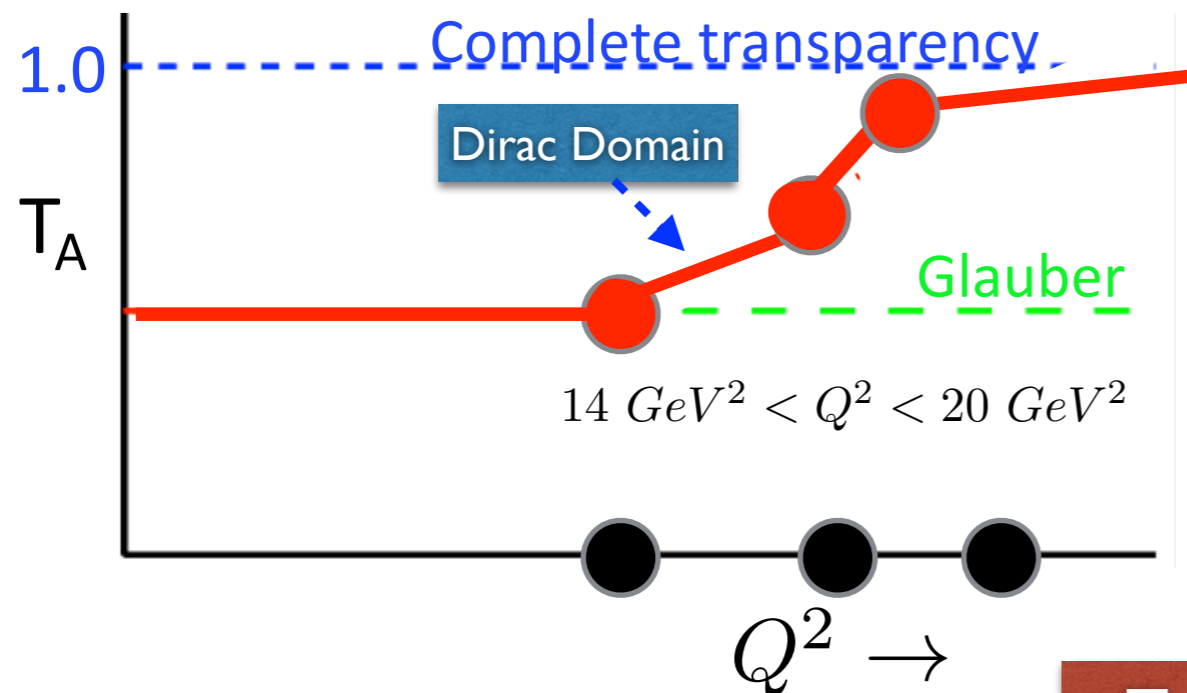
- Large Effect at high x
- Greatly increases kinematics of colliders such as Higgs production at high  $x_F$  (Kopeliovich, Schmidt, Soffer, Goldhaber, sjb)
- Severely underestimated in conventional parameterizations of heavy quark distributions (Pumplin, Tung)
- Many empirical tests (Gardner, Karliner, ..)

Review: G. Lykasov, et al

# Color transparency: fundamental prediction of QCD



- Not predicted by strongly interacting hadronic picture  $\rightarrow$  arises in picture of quark-gluon interactions
- QCD: color field of singlet objects vanishes as size is reduced
- Signature is a rise in nuclear transparency,  $T_A$ , as a function of the momentum transfer,  $Q^2$



$$T_A = \frac{\sigma_A \text{ (nuclear cross section)}}{A \sigma_N \text{ (free nucleon cross section)}}$$

G. de Teramond, sjb



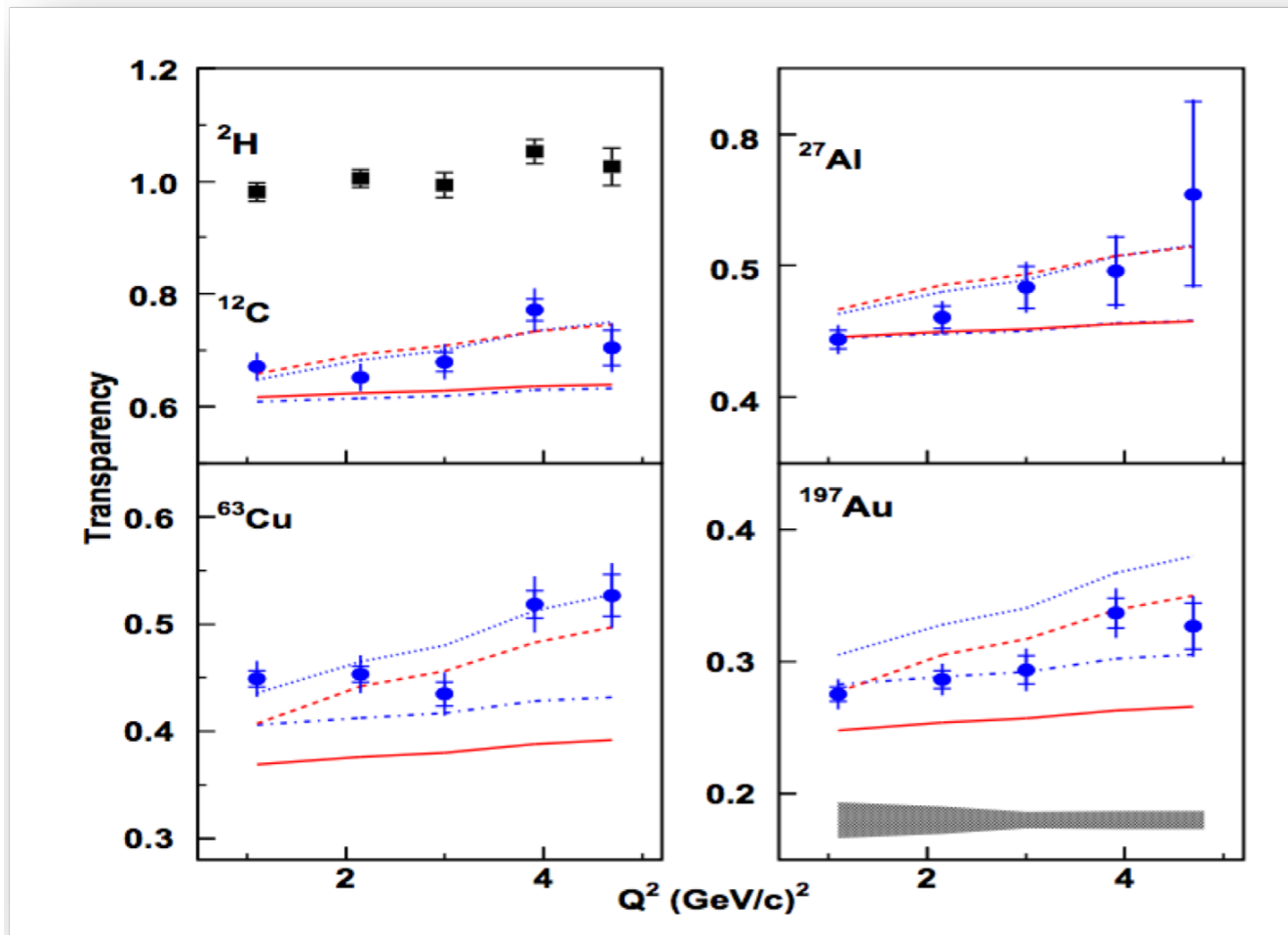
# Color Transparency verified for $\pi^+$ and $\rho$ electroproduction

Hall C E01-107 pion electro-production

$A(e, e' \pi^+)$

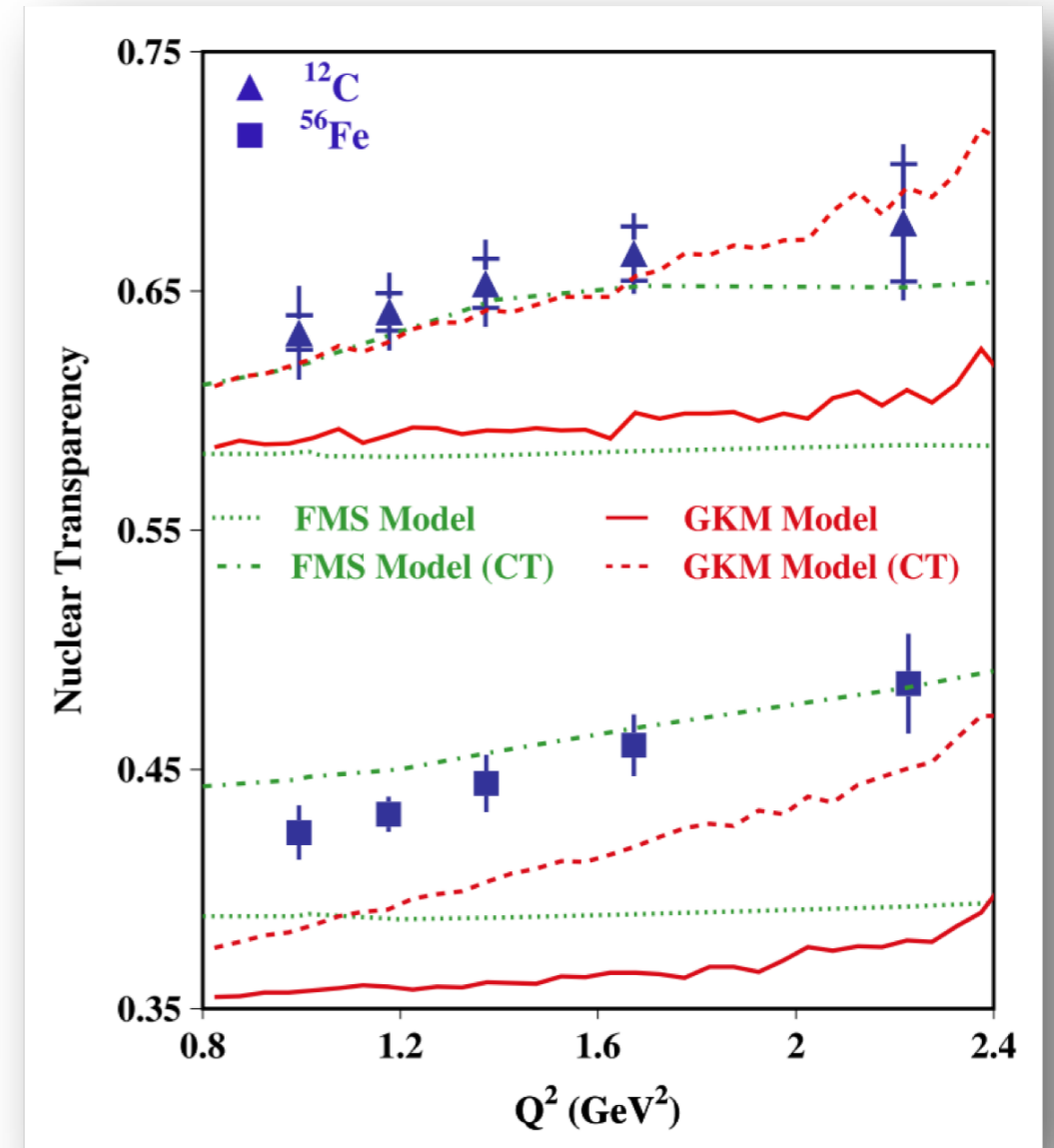
CLAS E02-110 rho electro-production

$A(e, e' \rho^0)$



B. Clasie *et al.* PRL 99:242502 (2007)

X. Qian *et al.* PRC81:055209 (2010)



L. El Fassi *et al.* PLB 712,326 (2012)

$$T_A = \frac{\frac{d\sigma}{dQ^2} (pA \rightarrow \pi^+ X)}{\frac{d\sigma}{dQ^2} (pp \rightarrow \pi^+ X)}$$

$$T_A = \frac{\frac{d\sigma}{dQ^2} (pA \rightarrow \rho^0 X)}{\frac{d\sigma}{dQ^2} (pp \rightarrow \rho^0 X)}$$

## Drell-Yan-West Formula in Impact Space

$$\begin{aligned}
 F(q^2) &= \sum_n \prod_{i=1}^n \int dx_i \int \frac{d^2 \mathbf{k}_{\perp i}}{2(2\pi)^3} 16\pi^3 \delta\left(1 - \sum_{j=1}^n x_j\right) \delta^{(2)}\left(\sum_{j=1}^n \mathbf{k}_{\perp j}\right) \\
 &\quad \sum_j e_j \psi_n^*(x_i, \mathbf{k}'_{\perp i}, \lambda_i) \psi_n(x_i, \mathbf{k}_{\perp i}, \lambda_i), \\
 &= \sum_n \prod_{i=1}^{n-1} \int dx_j \int d^2 \mathbf{b}_{\perp j} \exp\left(i \mathbf{q}_{\perp} \cdot \sum_{i=1}^{n-1} x_j \mathbf{b}_{\perp j}\right) |\psi_n(x_j, \mathbf{b}_{\perp j})|^2
 \end{aligned}$$

$$\sum_{i=1}^n x_i = 1 \quad \text{and} \quad \sum_{i=1}^n \mathbf{b}_{\perp i} = 0.$$

$$F(q^2) = \int_0^1 dx \int d^2 \mathbf{a}_{\perp} e^{i \mathbf{a}_{\perp} \cdot \mathbf{q}_{\perp}} q(x, \mathbf{a}_{\perp}),$$

where  $\mathbf{a}_{\perp} = \sum_{j=1}^{n-1} x_j \mathbf{b}_{\perp j}$  is the  $x$ -weighted transverse position coordinate of the  $n - 1$  spectators.

$$F(q^2) = \sum_n \prod_{j=1}^{n-1} \int dx_j \int d^2 \mathbf{b}_{\perp j} \exp\left(i \mathbf{q}_{\perp} \cdot \sum_{j=1}^{n-1} x_j \mathbf{b}_{\perp j}\right) |\psi_n(x_j, \mathbf{b}_{\perp j})|^2$$

$$\sum_i x_i = 1$$

$$\vec{a}_{\perp} \equiv \sum_{j=1}^{n-1} x_j \vec{b}_{\perp j}$$

$$\vec{a}_{\perp}^2(Q^2) = -4 \frac{\frac{d}{dQ^2} F(Q^2)}{F(Q^2)}$$

Proton radius squared at  $Q^2 = 0$

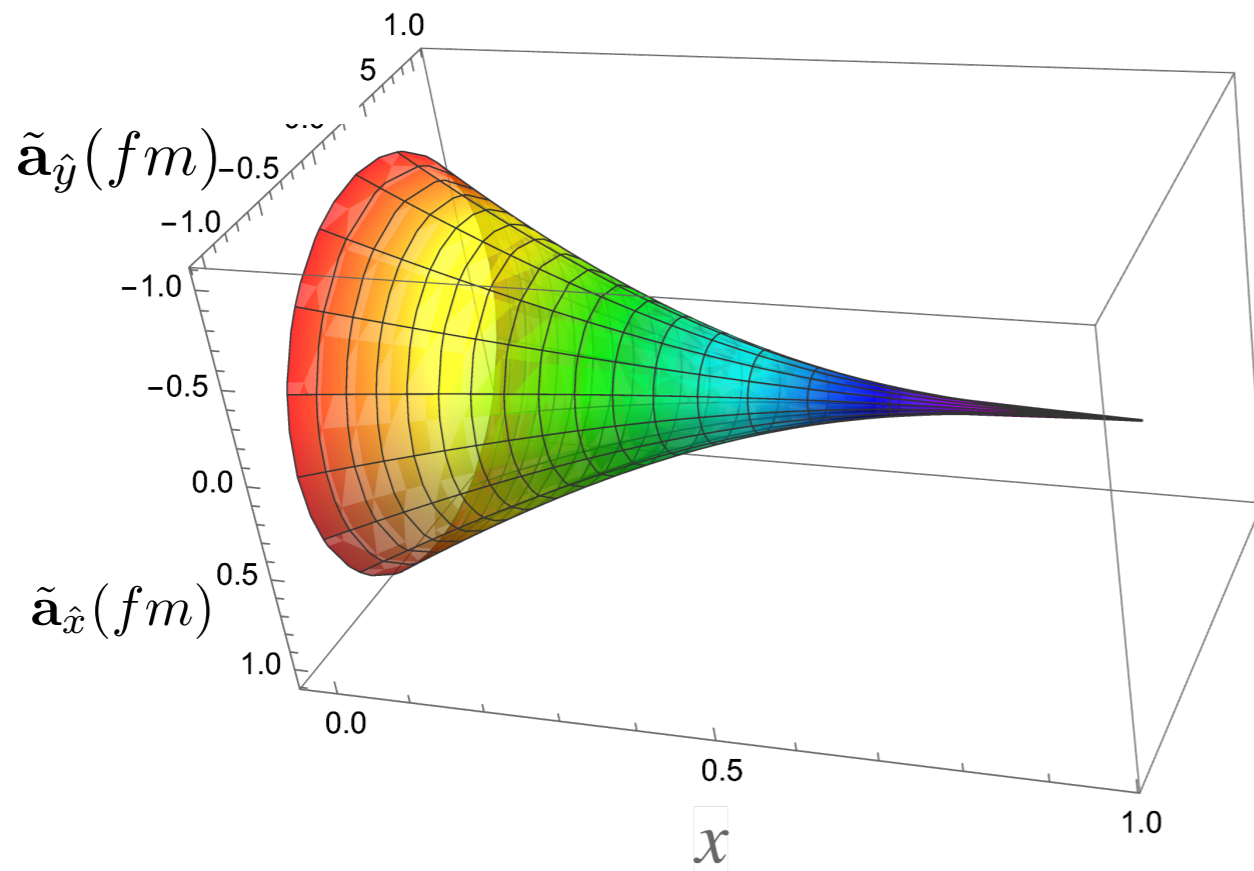
Color Transparency is controlled by the transverse-spatial size  $\vec{a}_{\perp}^2$  and its dependence on the momentum transfer  $Q^2 = -t$  :  
The scale  $Q_{\tau}^2$  required for Color Transparency grows with twist  $\tau$

Light-Front Holography:

For large  $Q^2$  :

$$\langle \mathbf{a}_{\perp}^2(t) \rangle_{\tau} = \frac{1}{\lambda} \sum_{j=1}^{\tau-1} \frac{1}{j - \alpha(t)}$$

$$\langle \mathbf{a}_{\perp}^2(Q^2) \rangle_{\tau} \rightarrow \frac{4(\tau - 1)}{Q^2}.$$



$$\langle \tilde{\mathbf{a}}_{\perp}^2(x) \rangle = \frac{\int d^2 \mathbf{a}_{\perp} \mathbf{a}_{\perp}^2 q(x, \mathbf{a}_{\perp})}{\int d^2 \mathbf{a}_{\perp} q(x, \mathbf{a}_{\perp})}$$

At large light-front momentum fraction  $x$ , and equivalently at large values of  $Q^2$ , the transverse size of a hadron behaves as a point-like color-singlet object. This behavior is the origin of color transparency in nuclei.

Although the dependence of the transverse impact area as a function of  $x$  is universal, the behavior in  $Q^2$  depends on properties of the hadron, such as its twist.

$$\langle \mathbf{a}_{\perp}^2(Q^2) \rangle_{\tau} \rightarrow \frac{4(\tau - 1)}{Q^2}.$$

*Mean transverse size  
as a function of  $Q$  and Twist*

# Universality of Generalized Parton Distributions in Light-Front Holographic QCD

Guy F. de Téramond, Tianbo Liu, Raza Sabbir Sufian, Hans Günter Dosch, Alexandre Deur, SJB

(HLFHS Collaboration)

$$F_\tau(t) = \frac{1}{N_\tau} B\left(\tau - 1, \frac{1}{2} - \frac{t}{4\lambda}\right), \quad N_\tau = B(\tau - 1, 1 - \alpha(0))$$

$$B(u, v) = \int_0^1 dy y^{u-1} (1-y)^{v-1} = [\Gamma(u)\Gamma(v)/\Gamma(u+v)]$$

$$F_\tau(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{M_0^2}\right)\left(1 + \frac{Q^2}{M_1^2}\right) \cdots \left(1 + \frac{Q^2}{M_{\tau-2}^2}\right)}$$

$$F_\tau(Q^2) \sim \left(\frac{1}{Q^2}\right)^{\tau-1}$$

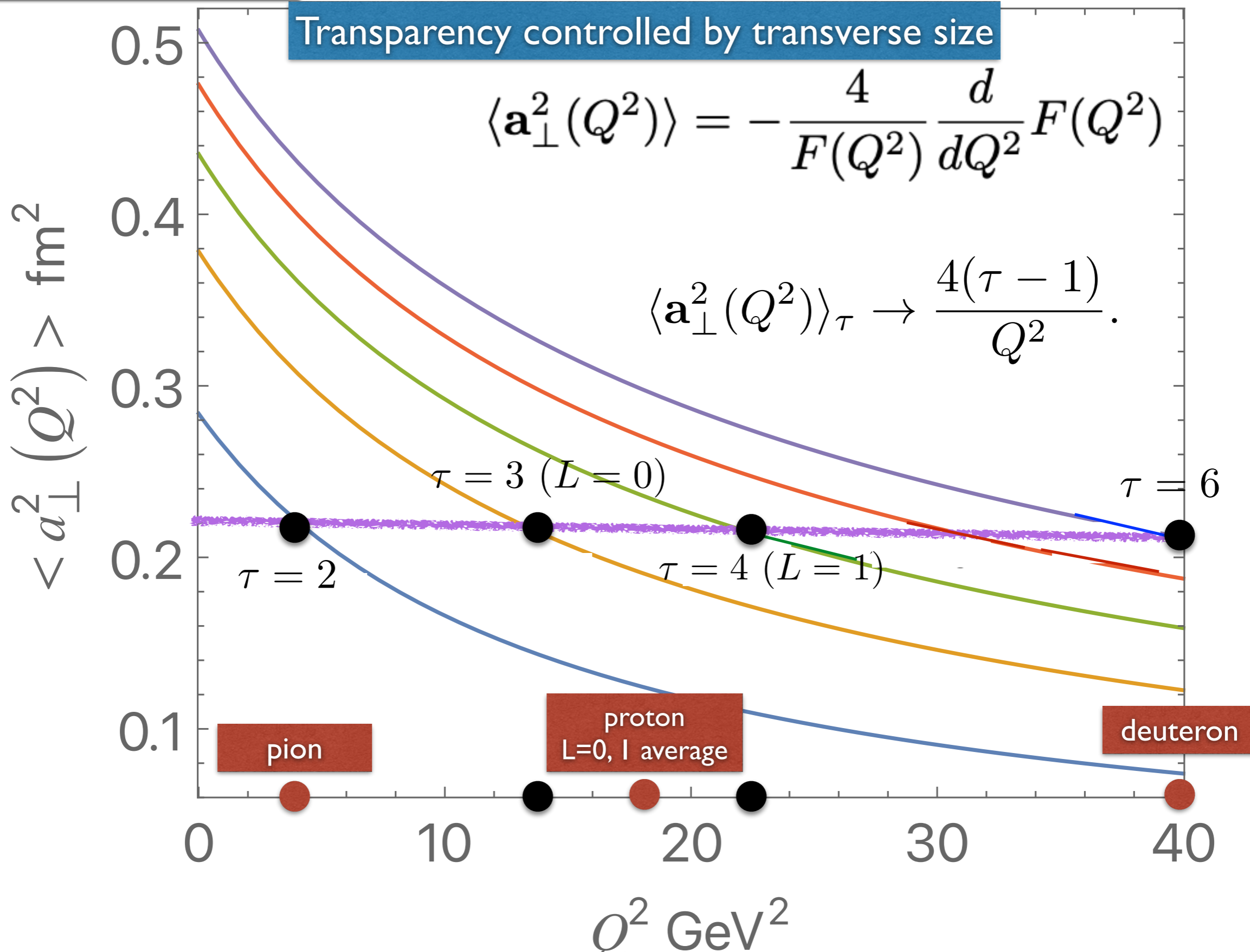
$$M_n^2 = 4\lambda\left(n + \frac{1}{2}\right), n = 0, 1, 2, \dots, \tau - 2, \quad M_0 = m_\rho$$

$$\sqrt{\lambda} = \kappa = \frac{m_\rho}{\sqrt{2}} = 0.548 \text{ GeV} \quad \frac{1}{2} - \frac{t}{4\lambda} = 1 - \alpha_R(t)$$

$\alpha_R(t) = \rho$  Regge Trajectory

Transparency scale  $Q$   
increases with twist

Light-Front Holography



Proton has equal probability for  $\tau = 3$  and  $\tau = 4$



# *Color Transparency and Light-Front Holography*

- Essential prediction of QCD
- LF Holography: Spectroscopy, dynamics, structure
- Transverse size predicted by LF Holography as a function of Q
- Q scale for CT increases with twist, number of constituents
- Two-Stage Proton Transparency: Equal probability L=0,1
- No contradiction with present experiments

$Q_0^2(p) \simeq 18 \text{ GeV}^2$  vs.  $Q_0^2(\pi) \simeq 4 \text{ GeV}^2$  for onset of color transparency in  $^{12}\text{C}$

# Two-Stage Color Transparency

$$14 \text{ GeV}^2 < Q^2 < 20 \text{ GeV}^2$$

If  $Q^2$  is in the intermediate range, then the twist-3 state will propagate through the nuclear medium with minimal absorption, and the protons which survive nuclear absorption will only have  $L = 0$  (twist-3).

The twist-4  $L = 1$  state which has a larger transverse size will be absorbed.

Thus 50% of the events in this range of  $Q^2$  will have full color transparency and 50% of the events will have zero color transparency ( $T = 0$ ).

The  $ep \rightarrow e'p'$  cross section will have the same angular and  $Q^2$  dependence as scattering of the electron on an unphysical proton which has no Pauli form factor.

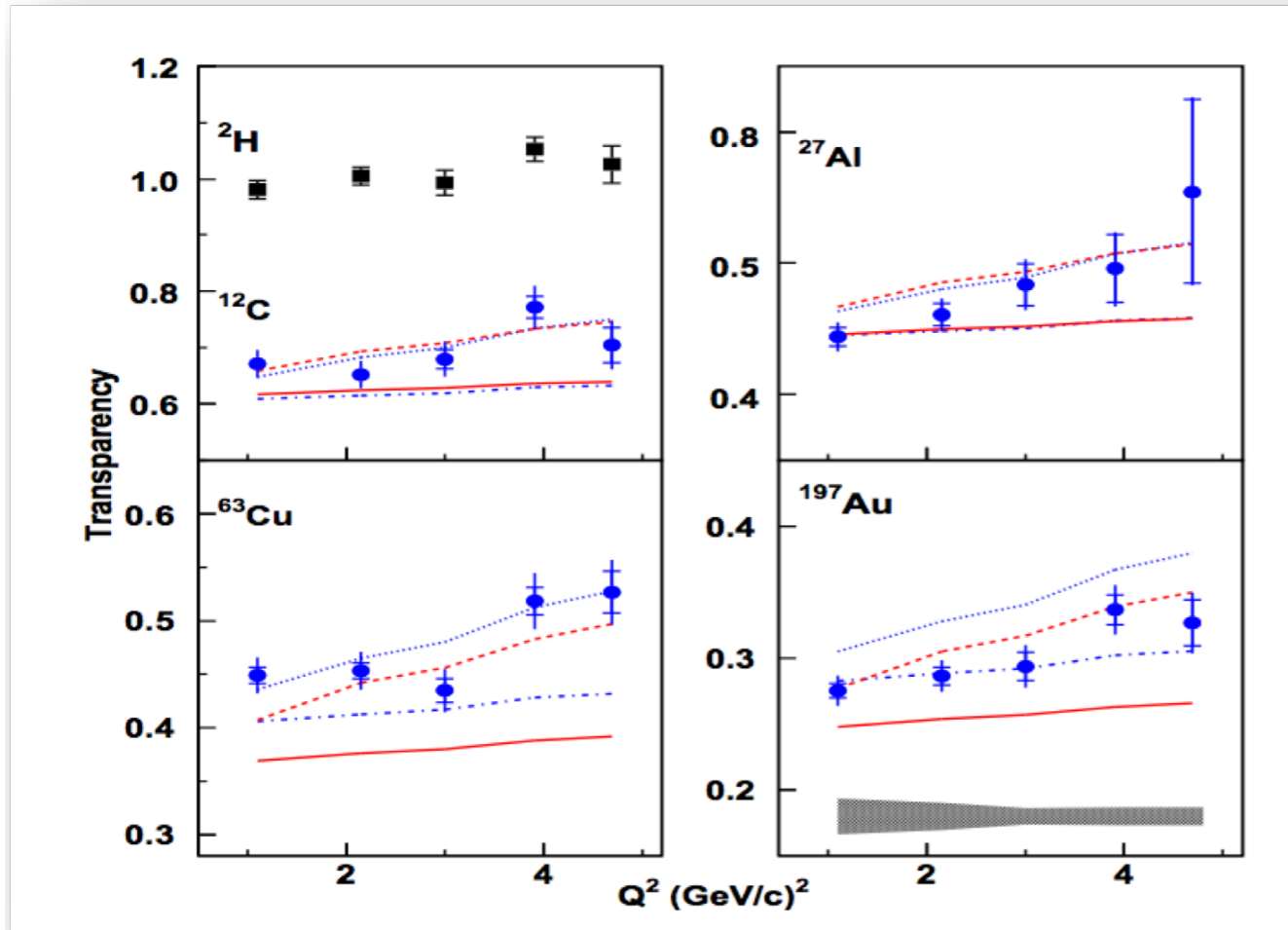
$$Q^2 > 20 \text{ GeV}^2$$

However, if the momentum transfer is increased to  $Q^2 > 20 \text{ GeV}^2$ , all events will have full color transparency, and the  $ep \rightarrow e'p'$  cross section will have the same angular and  $Q^2$  dependence as scattering of the electron on a physical proton eigenstate, with both Dirac and Pauli form factor components.

Hall C E01-107 pion electro-production

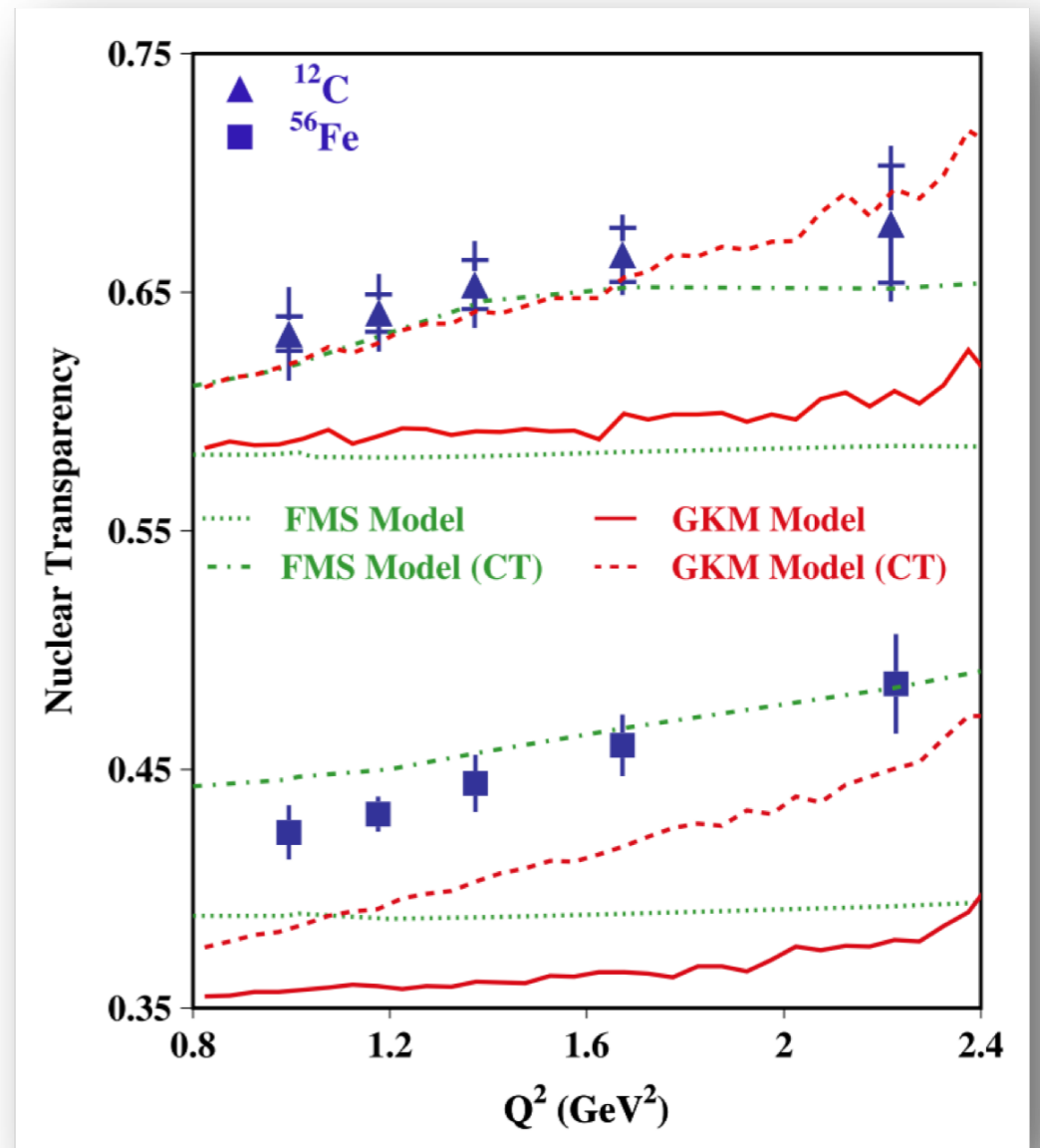
$$A(e, e' \pi^+)$$

$$A(e, e' \rho^0)$$



B.Glasie *et al.* PRL 99:242502 (2007)

X. Qian *et al.* PRC81:055209 (2010)

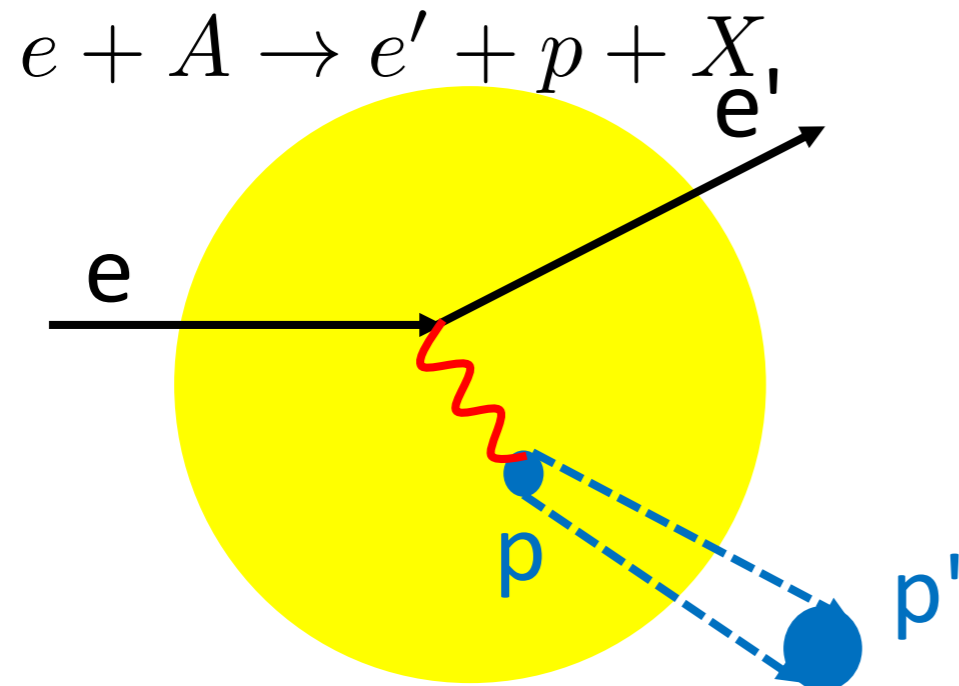


L. El Fassi *et al.* PLB 712,326 (2012)

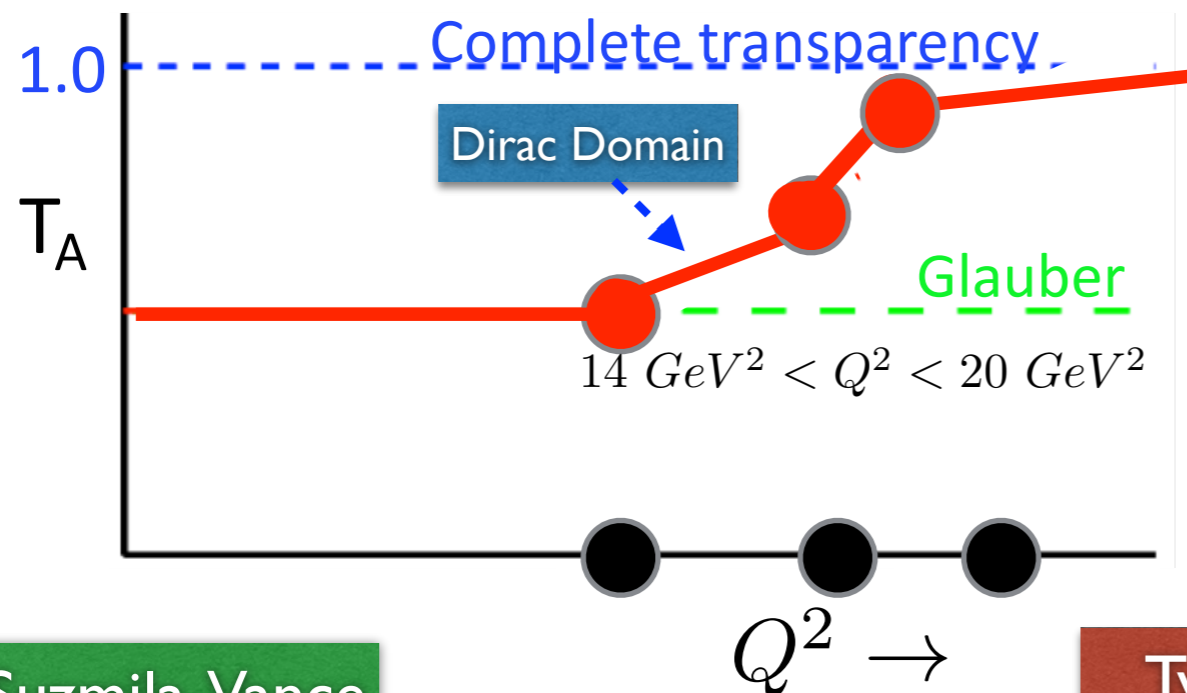
$$\langle a_{\perp}^2(Q^2 = 4 \text{ GeV}^2) \rangle_{\tau=2} \simeq \langle a_{\perp}^2(Q^2 = 14 \text{ GeV}^2) \rangle_{\tau=3} \simeq \langle a_{\perp}^2(Q^2 = 22 \text{ GeV}^2) \rangle_{\tau=4} \simeq 0.24 \text{ fm}^2$$

5% increase for  $T_{\pi}$  in  $^{12}\text{C}$  at  $Q^2 = 4 \text{ GeV}^2$  implies 5% increase for  $T_p$  at  $Q^2 = 18 \text{ GeV}^2$

# Color transparency fundamental prediction of QCD



- Not predicted by strongly interacting hadronic picture  $\rightarrow$  arises in picture of quark-gluon interactions
- QCD: color field of singlet objects vanishes as size is reduced
- Signature is a rise in nuclear transparency,  $T_A$ , as a function of the momentum transfer,  $Q^2$



$$T_A = \frac{\sigma_A}{A \sigma_N} \quad \begin{array}{l} \text{(nuclear cross section)} \\ \text{(free nucleon} \\ \text{cross section)} \end{array}$$

# *Color Transparency and Light-Front Holography*

- Essential prediction of QCD
- LF Holography: Spectroscopy, dynamics, structure
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$Q_0^2(p) \simeq 18 \text{ GeV}^2$  vs.  $Q_0^2(\pi) \simeq 4 \text{ GeV}^2$  for onset of color transparency in  $^{12}\text{C}$

**Feynman domain also incorporated**

# Supersymmetry in QCD

- A hidden symmetry of Color  $SU(3)_c$  in hadron physics
- QCD: No squarks or gluinos!
- Emerges from Light-Front Holography and Super-Conformal Algebra
- Color Confinement
- Massless Pion in Chiral Limit

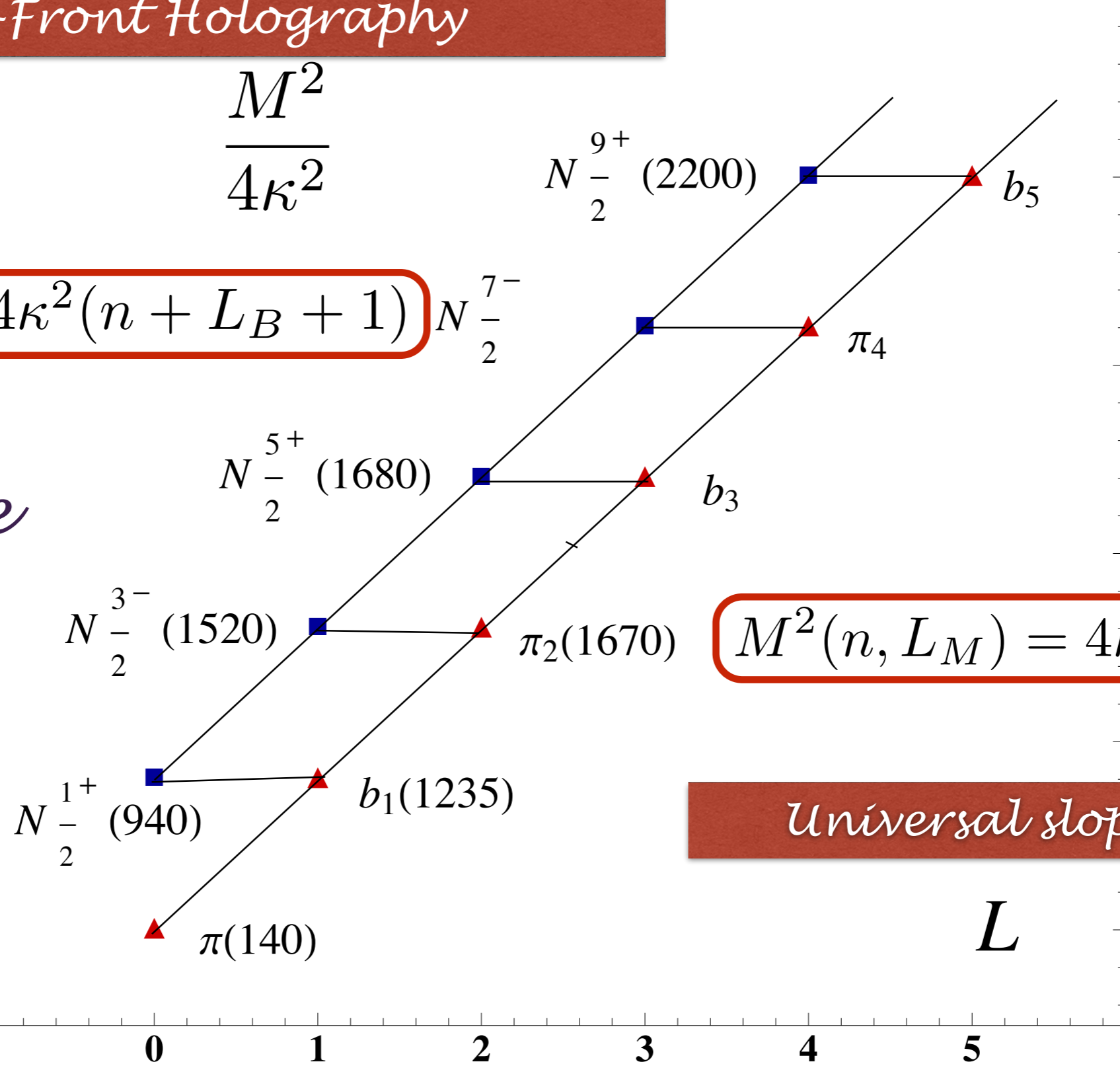
*de Téramond, Dosch, Lorcé, sjb*



$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$

$$\frac{M^2}{4\kappa^2}$$

*Same slope*



$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$

*Universal slopes in  $n, L$*

$$\frac{M_{meson}^2}{M_{nucleon}^2} = \frac{n + L_M}{n + L_B + 1}$$

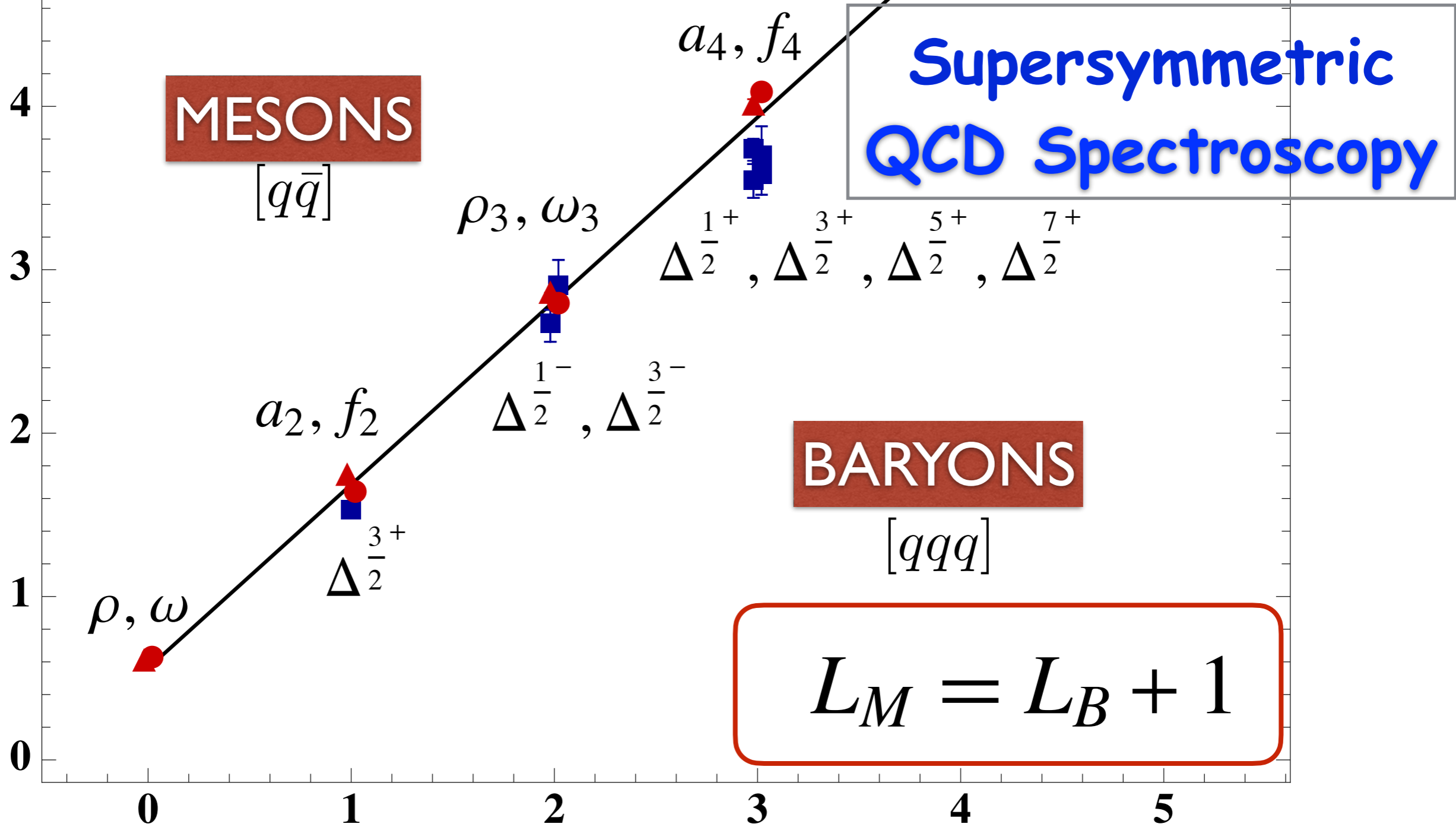
**Meson-Baryon  
Mass Degeneracy  
for  $L_M=L_B+1$**

$M^2$  (GeV<sup>2</sup>)

bosons

fermions

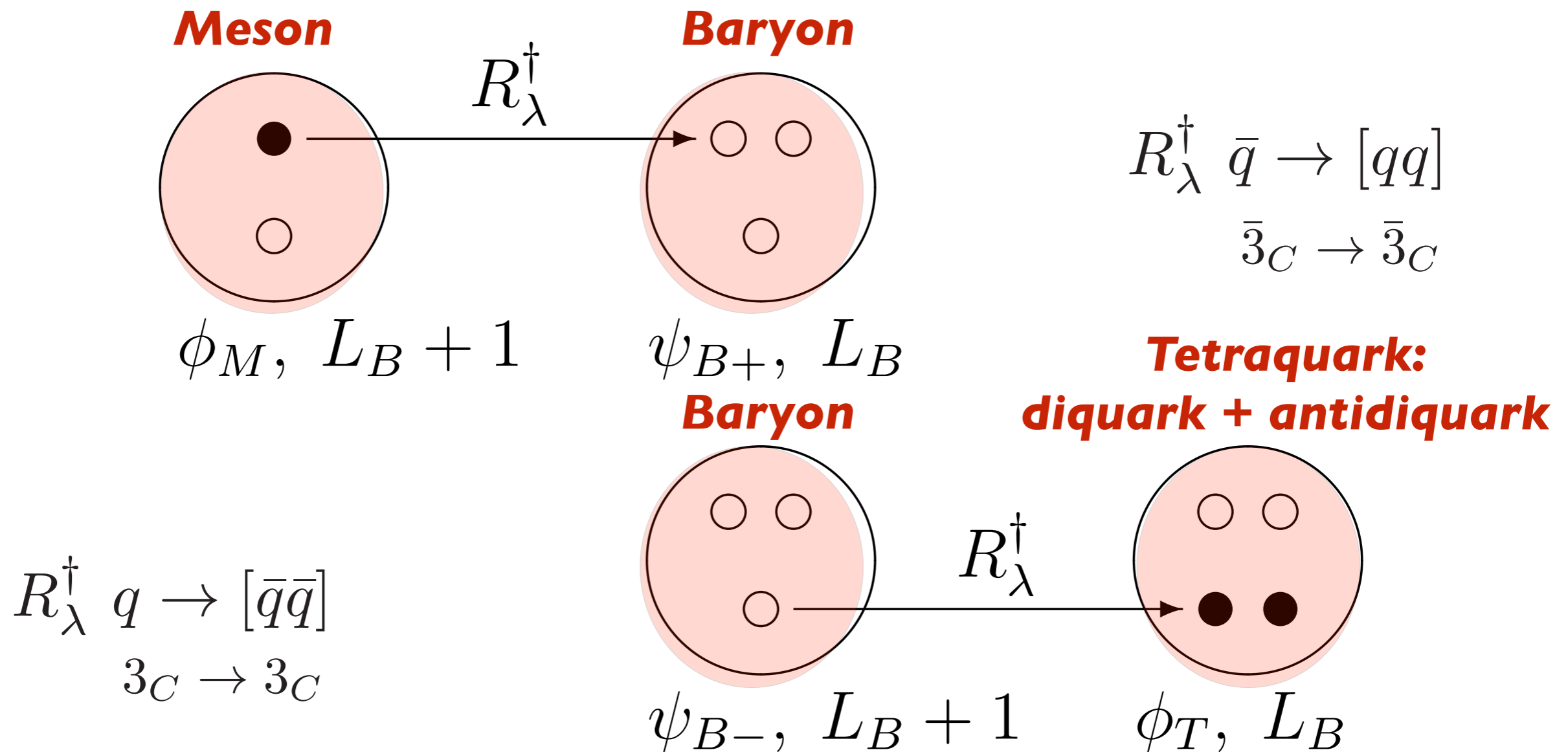
$\rho - \Delta$  superpartner trajectories



# Superconformal Algebra

## Four-Plet Representations

Bosons, Fermions with Equal Mass!



Proton:  $|u[ud]\rangle$  Quark + Scalar Diquark  
 Equal Weight:  $L=0, L=1$

# Challenge: Compute Hadron Structure, Spectroscopy, and Dynamics from QCD!

- **Color Confinement**
- **Origin of the QCD Mass Scale**
- **Meson and Baryon Spectroscopy**
- **Exotic States: Tetraquarks, Pentaquarks, Gluonium,**
- **Universal Regge Slopes:  $n$ ,  $L$ , Mesons and Baryons**
- **Almost Massless Pion: GMOR Chiral Symmetry Breaking**  
$$M_\pi^2 f_\pi^2 = -\frac{1}{2}(m_u + m_d) \langle \bar{u}u + \bar{d}d \rangle + \mathcal{O}((m_u + m_d)^2)$$
- **QCD Coupling at all Scales  $\alpha_s(Q^2)$**
- **Eliminate Scale Uncertainties and Scheme Dependence**

$$\mathcal{L}_{QCD} \rightarrow \psi_n^H(x_i, \vec{k}_{\perp i}, \lambda_i) \quad \text{Valence and Higher Fock States}$$

Exact frame-independent formulation of nonperturbative QCD!

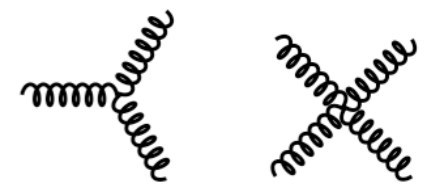
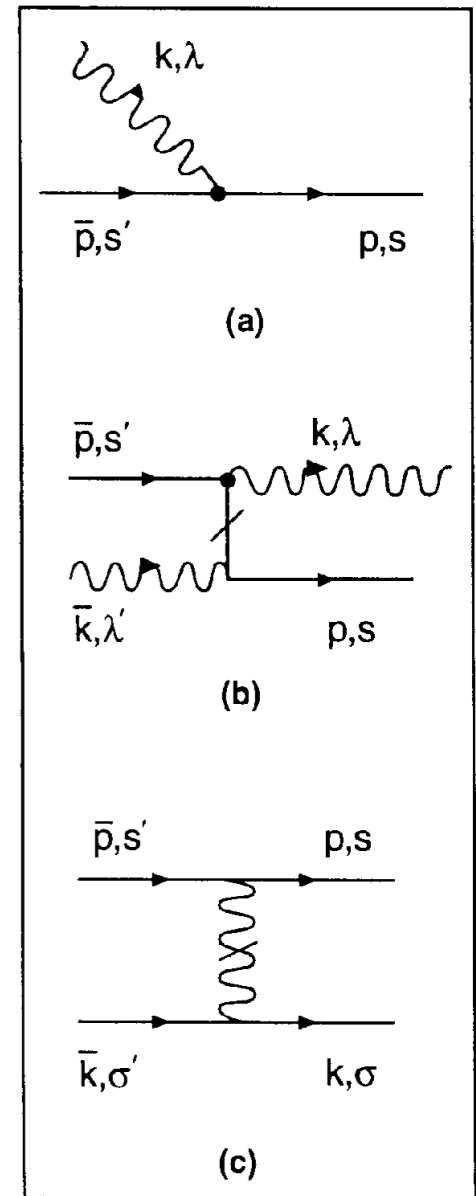
$$L^{QCD} \rightarrow H_{LF}^{QCD}$$

$$H_{LF}^{QCD} = \sum_i \left[ \frac{m^2 + k_{\perp}^2}{x} \right]_i + H_{LF}^{int}$$

$H_{LF}^{int}$ : Matrix in Fock Space

$$H_{LF}^{QCD} |\Psi_h\rangle = \mathcal{M}_h^2 |\Psi_h\rangle$$

$$|p, J_z\rangle = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; x_i, \vec{k}_{\perp i}, \lambda_i\rangle$$



$H_{LF}^{int}$

Eigenvalues and Eigensolutions give Hadronic Spectrum and Light-Front wavefunctions

**LFWFs: Off-shell in P- and invariant mass**

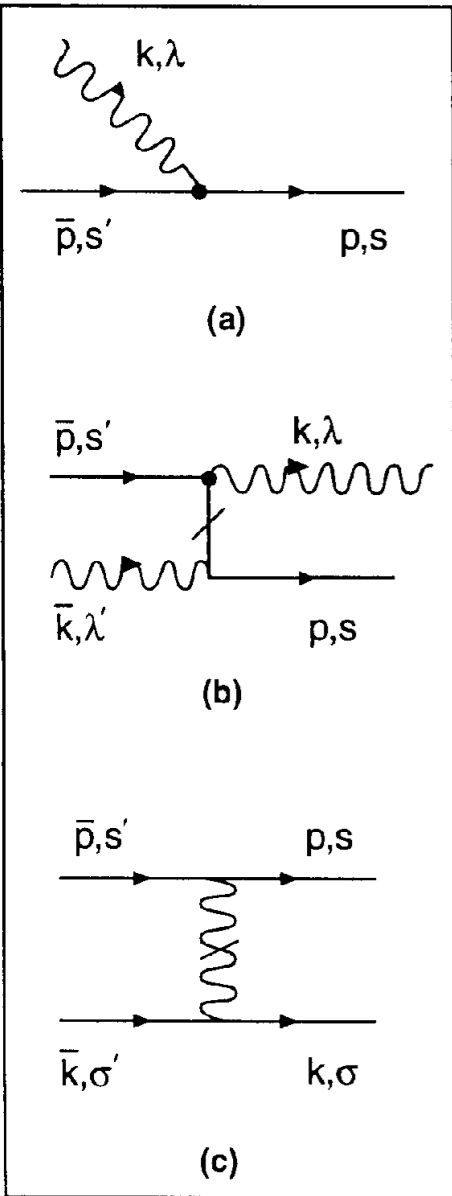


Light-Front QCD  
Heisenberg Equation

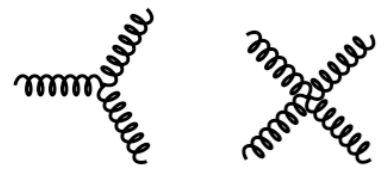
$$H_{LC}^{QCD} |\Psi_h\rangle = \mathcal{M}_h^2 |\Psi_h\rangle$$

DLCQ: Solve QCD(1+1) for any quark mass and flavors

Hornbostel, Pauli, sjb



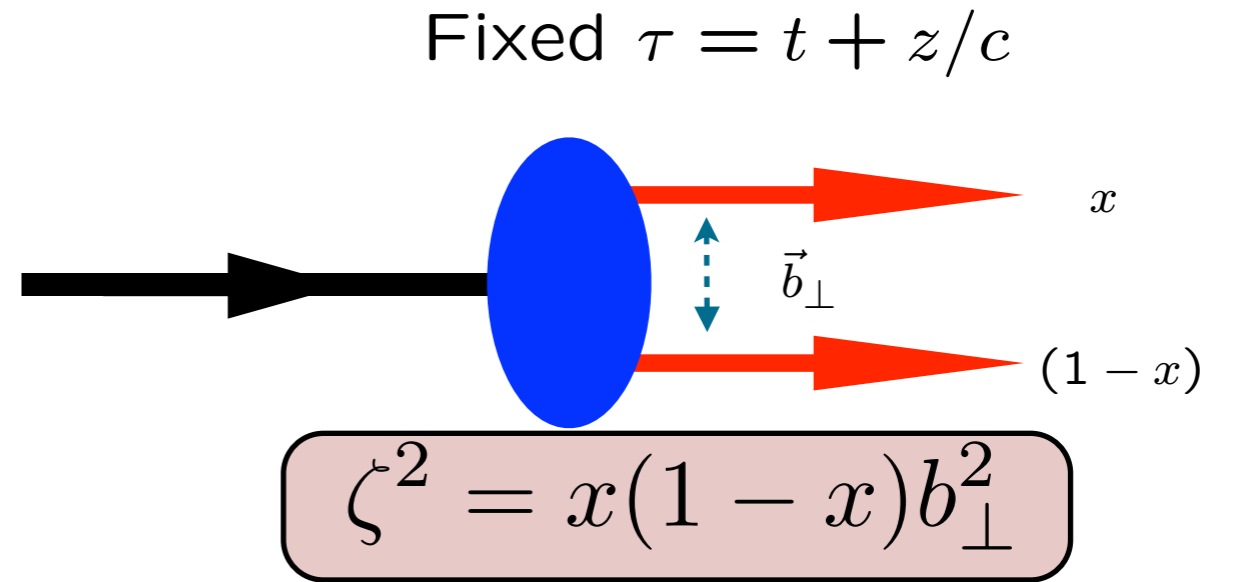
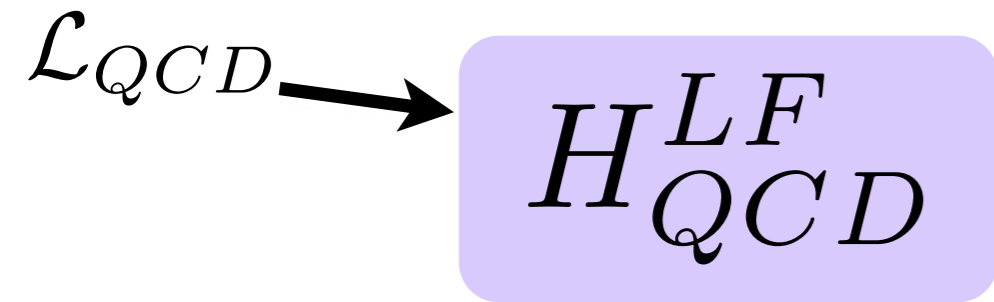
n	Sector	1 q $\bar{q}$	2 gg	3 q $\bar{q}$ g	4 q $\bar{q}$ q $\bar{q}$	5 gg g	6 q $\bar{q}$ gg	7 q $\bar{q}$ q $\bar{q}$ g	8 q $\bar{q}$ q $\bar{q}$ q $\bar{q}$	9 gg gg	10 q $\bar{q}$ gg g	11 q $\bar{q}$ q $\bar{q}$ gg	12 q $\bar{q}$ q $\bar{q}$ q $\bar{q}$ g	13 q $\bar{q}$ q $\bar{q}$ q $\bar{q}$ q $\bar{q}$
1	q $\bar{q}$					.		.	.	.	.	.	.	.
2	gg				.			.	.		.	.	.	.
3	q $\bar{q}$ g								.	.		.	.	.
4	q $\bar{q}$ q $\bar{q}$		.			.				.	.		.	.
5	gg g	.			.			.	.			.	.	.
6	q $\bar{q}$ gg								.				.	.
7	q $\bar{q}$ q $\bar{q}$ g	.	.			.				.				.
8	q $\bar{q}$ q $\bar{q}$ q $\bar{q}$	.	.	.		.	.			.	.			
9	gg gg	.		.	.			.	.			.	.	.
10	q $\bar{q}$ gg g	.	.		.				.				.	.
11	q $\bar{q}$ q $\bar{q}$ gg	.	.	.		.				.				.
12	q $\bar{q}$ q $\bar{q}$ q $\bar{q}$ g	.	.	.	.	.	.			.	.			
13	q $\bar{q}$ q $\bar{q}$ q $\bar{q}$ q $\bar{q}$	.	.	.	.	.	.			.	.	.		



Minkowski space; frame-independent; no fermion doubling; no ghosts  
trivial vacuum



# Light-Front QCD



$$(H_{LF}^0 + H_{LF}^I) |\Psi\rangle = M^2 |\Psi\rangle$$

*Coupled Fock states*

*Eliminate higher Fock states and retarded interactions*

$$\left[ \frac{\vec{k}_{\perp}^2 + m^2}{x(1-x)} + V_{\text{eff}}^{LF} \right] \psi_{LF}(x, \vec{k}_{\perp}) = M^2 \psi_{LF}(x, \vec{k}_{\perp})$$

*Effective two-particle equation*

$$\left[ -\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta) \right] \psi(\zeta) = \mathcal{M}^2 \psi(\zeta)$$

*Azimuthal Basis  $\zeta, \phi$*

**Single variable Equation**

$$m_q = 0$$

**AdS/QCD:**

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)$$

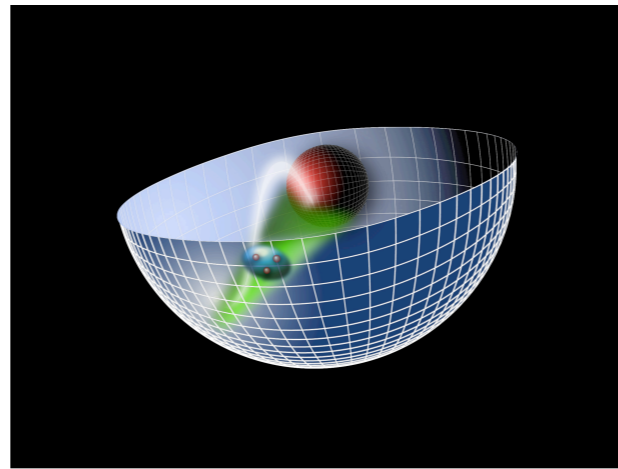
*Confining AdS/QCD potential!*

*Semiclassical first approximation to QCD*

*Sums an infinite # diagrams*

*AdS/QCD  
Soft-Wall Model*

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$



$$\zeta^2 = x(1-x)b_{\perp}^2$$

*Light-Front Holography*

$$\left[ -\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta) \right] \psi(\zeta) = M^2 \psi(\zeta)$$



***Light-Front Schrödinger Equation***

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)$$

*Single variable  $\zeta$*

***Unique  
Confinement Potential!***  
*Conformal Symmetry  
of the action*

***Confinement scale:***

$$\kappa \simeq 0.5 \text{ GeV}$$

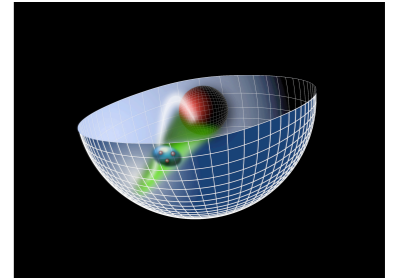
- **de Alfaro, Fubini, Furlan:**
- **Fubini, Rabinovici:**

***Scale can appear in Hamiltonian and EQM  
without affecting conformal invariance of action!***

*GeV units external to QCD: Only Ratios of Masses Determined*

# Dilaton-Modified AdS

$$ds^2 = e^{\varphi(z)} \frac{R^2}{z^2} (\eta_{\mu\nu} x^\mu x^\nu - dz^2)$$



- **Soft-wall dilaton profile breaks conformal invariance**  $e^{\varphi(z)} = e^{+\kappa^2 z^2}$
- **Color Confinement in z**
- **Introduces confinement scale  $\kappa$**
- **Uses AdS<sub>5</sub> as template for conformal theory**

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

**Positive-sign dilaton**

• de Teramond, sjb

*AdS Soft-Wall Schrödinger Equation for bound state of two scalar constituents:*

$$\left[ -\frac{d^2}{dz^2} - \frac{1 - 4L^2}{4z^2} + U(z) \right] \Phi(z) = \mathcal{M}^2 \Phi(z)$$

$$U(z) = \kappa^4 z^2 + 2\kappa^2 (L + S - 1)$$

*Derived from variation of Action for Dilaton-Modified AdS<sub>5</sub>*

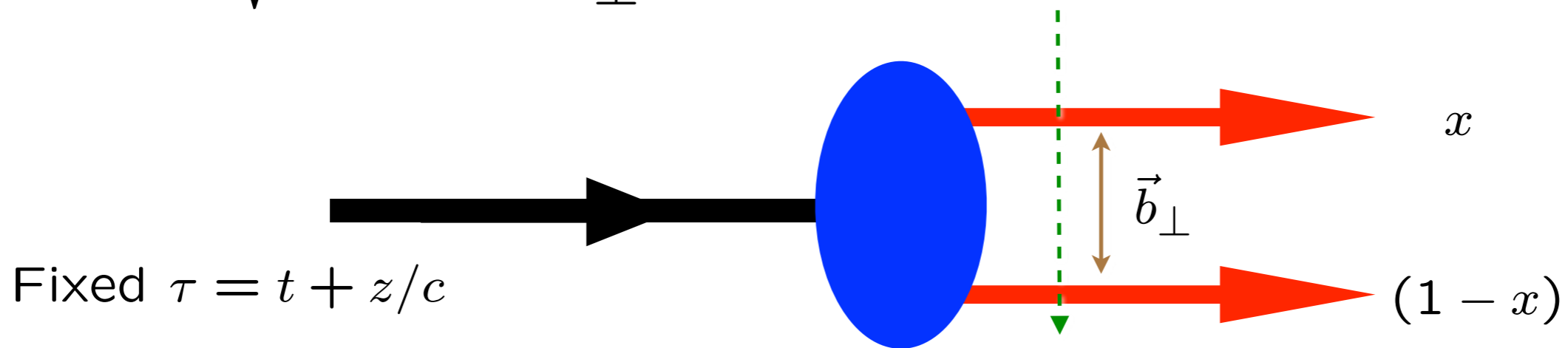
***Identical to Single-Variable Light-Front Bound State Equation in  $\zeta$ !***

$$z \quad \longleftrightarrow \quad \zeta = \sqrt{x(1-x)} \vec{b}_\perp^2$$

***Light-Front Holography***

$LF(3+1) \longleftrightarrow AdS_5$ 

# Light-Front Holographic Dictionary

 $\psi(x, \vec{b}_\perp) \longleftrightarrow \phi(z)$ 
 $\zeta = \sqrt{x(1-x)b_\perp^2} \longleftrightarrow z$ 


$$\psi(x, \zeta) = \sqrt{x(1-x)} \zeta^{-1/2} \phi(\zeta)$$

$$(\mu R)^2 = L^2 - (J - 2)^2$$

**Light-Front Holography:** Unique mapping derived from equality of LF and AdS formula for EM and gravitational current matrix elements and identical equations of motion

## Holographic Mapping of AdS Modes to QCD LFWFs

*Drell-Yan-West: Form Factors are Convolution of LFWFs*

- Integrate Soper formula over angles:

$$F(q^2) = 2\pi \int_0^1 dx \frac{(1-x)}{x} \int \zeta d\zeta J_0 \left( \zeta q \sqrt{\frac{1-x}{x}} \right) \tilde{\rho}(x, \zeta),$$

with  $\tilde{\rho}(x, \zeta)$  QCD effective transverse charge density.

- Transversality variable

$$\zeta = \sqrt{x(1-x)} \vec{b}_\perp^2$$

- Compare AdS and QCD expressions of FFs for arbitrary  $Q$  using identity:

$$\int_0^1 dx J_0 \left( \zeta Q \sqrt{\frac{1-x}{x}} \right) = \zeta Q K_1(\zeta Q),$$

the solution for  $J(Q, \zeta) = \zeta Q K_1(\zeta Q)$  !

**de Teramond, sjb**

*Identical to Polchinski-Strassler Convolution of AdS Amplitudes*



$$\left( -\partial_{\zeta}^2 + \kappa^4 \zeta^2 + 2\kappa^2(L_B + 1) + \frac{4L_B^2 - 1}{4\zeta^2} \right) \psi_J^+ = M^2 \psi_J^+ \quad \left. \vphantom{\psi_J^+} \right\}$$

$$\left( -\partial_{\zeta}^2 + \kappa^4 \zeta^2 + 2\kappa^2 L_B + \frac{4(L_B + 1)^2 - 1}{4\zeta^2} \right) \psi_J^- = M^2 \psi_J^- \quad \left. \vphantom{\psi_J^-} \right\}$$

$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1) \quad \mathbf{S=1/2, P=+}$$

## Meson Equation

$$\lambda = \kappa^2$$

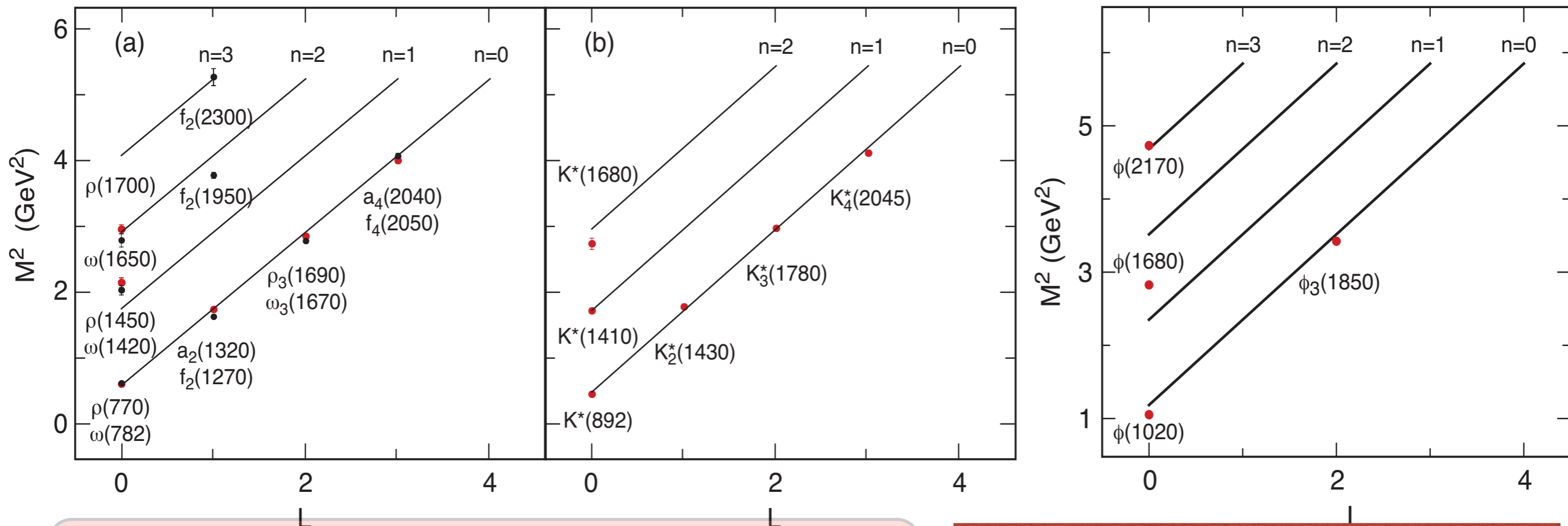
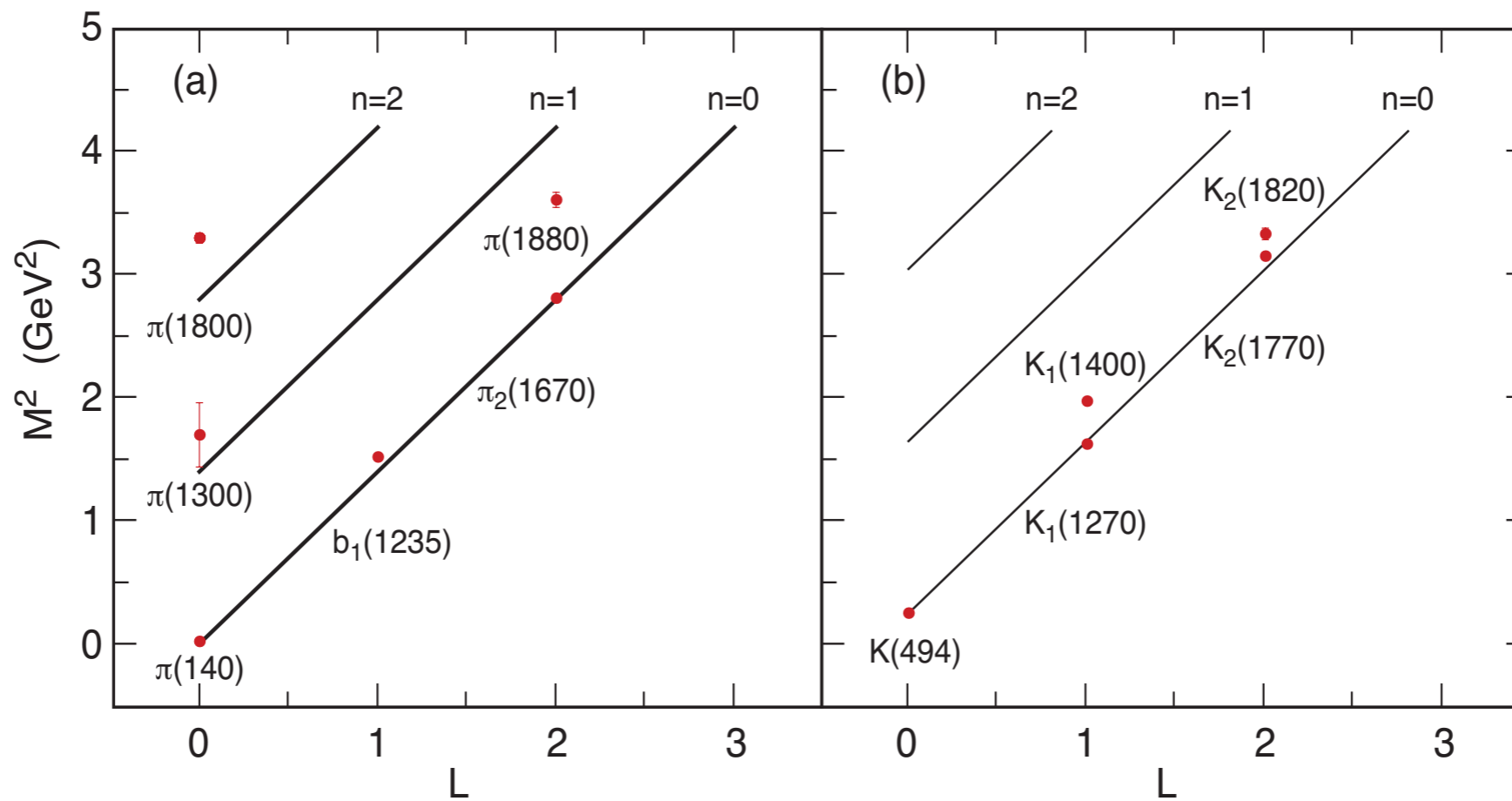
$$\left( -\partial_{\zeta}^2 + \kappa^4 \zeta^2 + 2\kappa^2(J - 1) + \frac{4L_M^2 - 1}{4\zeta^2} \right) \phi_J = M^2 \phi_J$$

$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$

$\mathbf{S=0, P=+}$   
*Same  $\kappa$ !*

**$S=0, I=I$  Meson is superpartner of  $S=1/2, I=I$  Baryon**

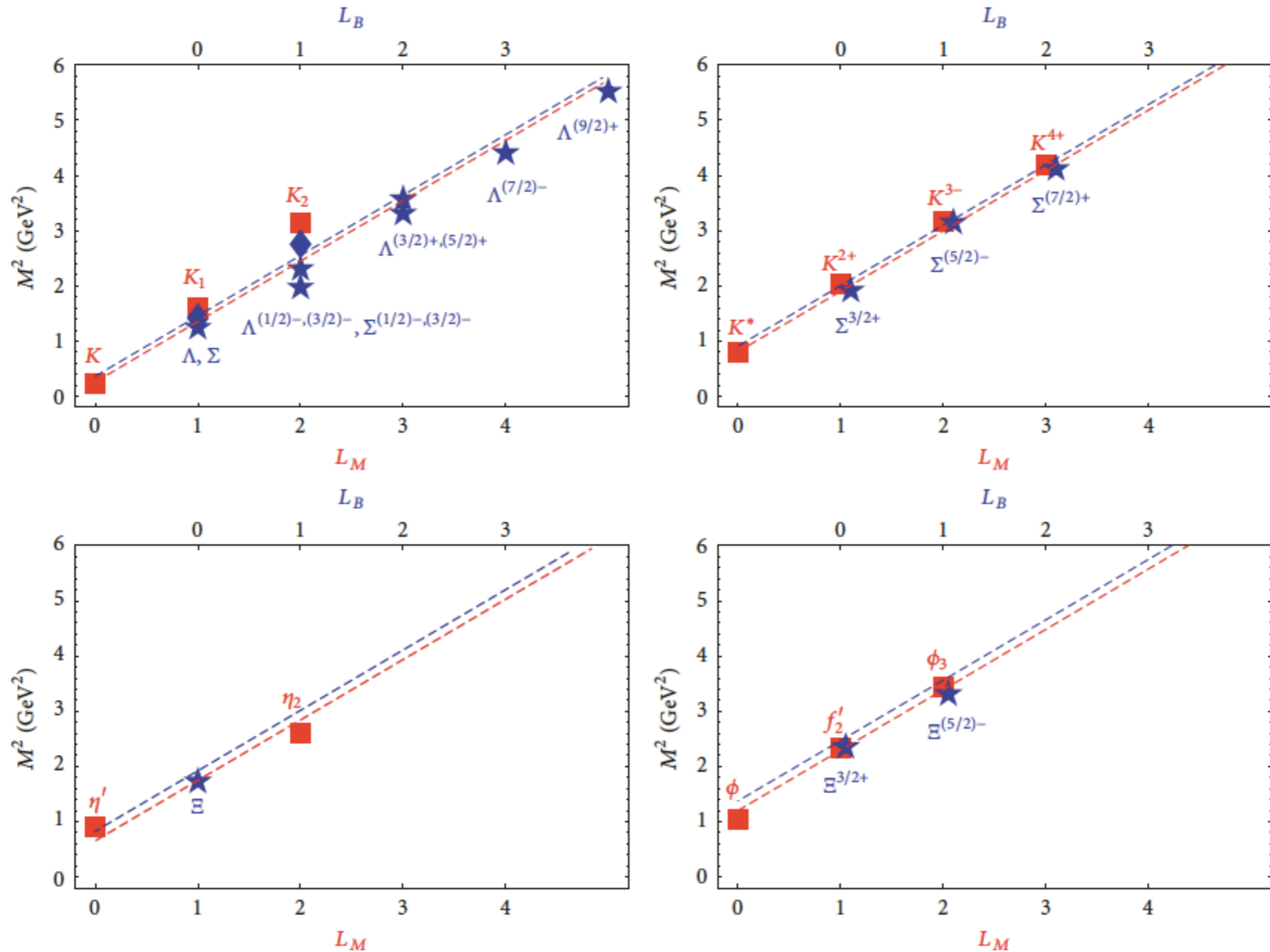
**Meson-Baryon Degeneracy for  $L_M=L_B+1$**



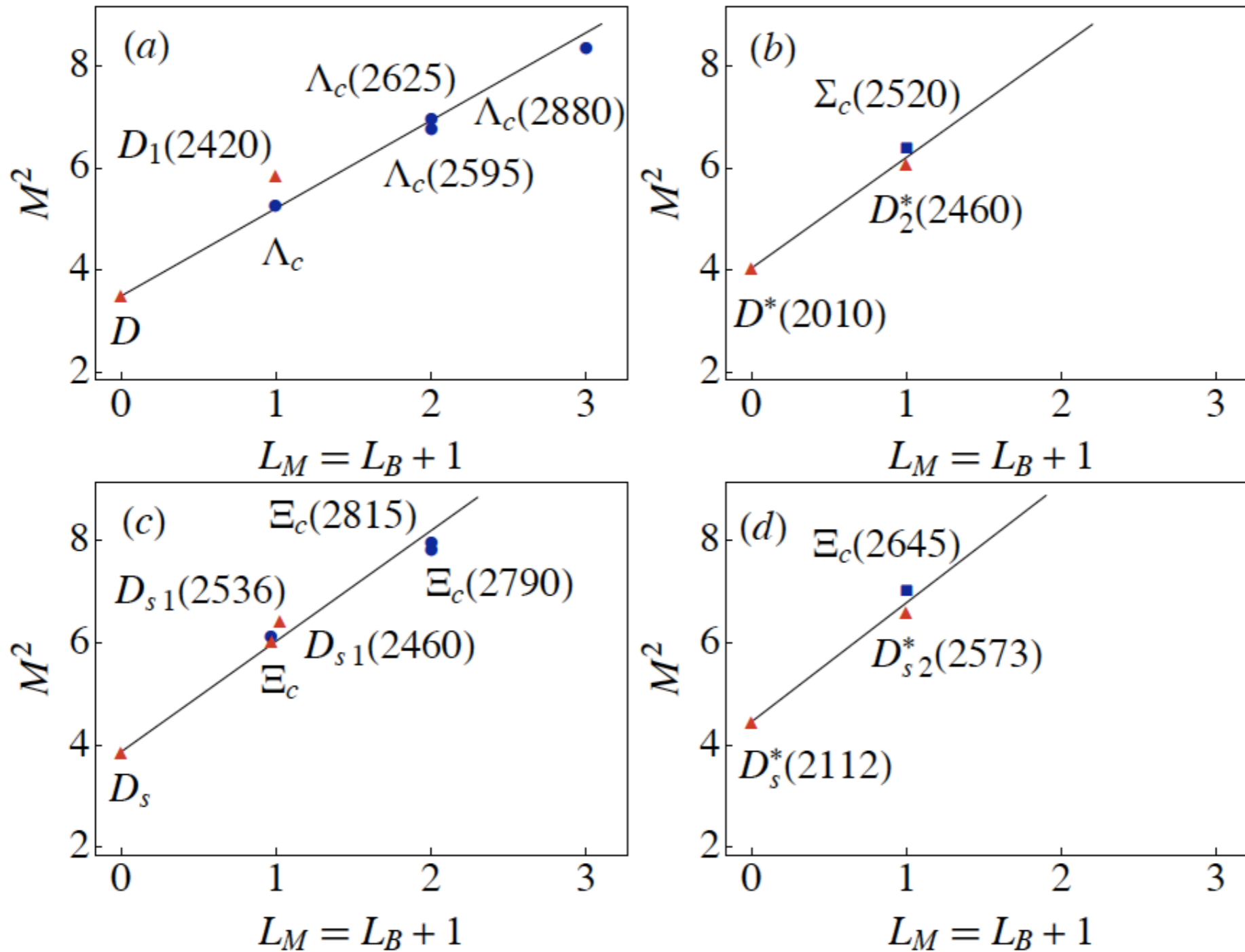
$$M^2(n, L, S) = 4\kappa^2(n + L + S/2)$$

*Equal Slope in  $n$  and  $L$*

# Supersymmetry across the light and heavy-light spectrum



# Supersymmetry across the light and heavy-light spectrum



**Heavy charm quark mass does not break supersymmetry**

# Connection to the Linear Instant-Form Potential

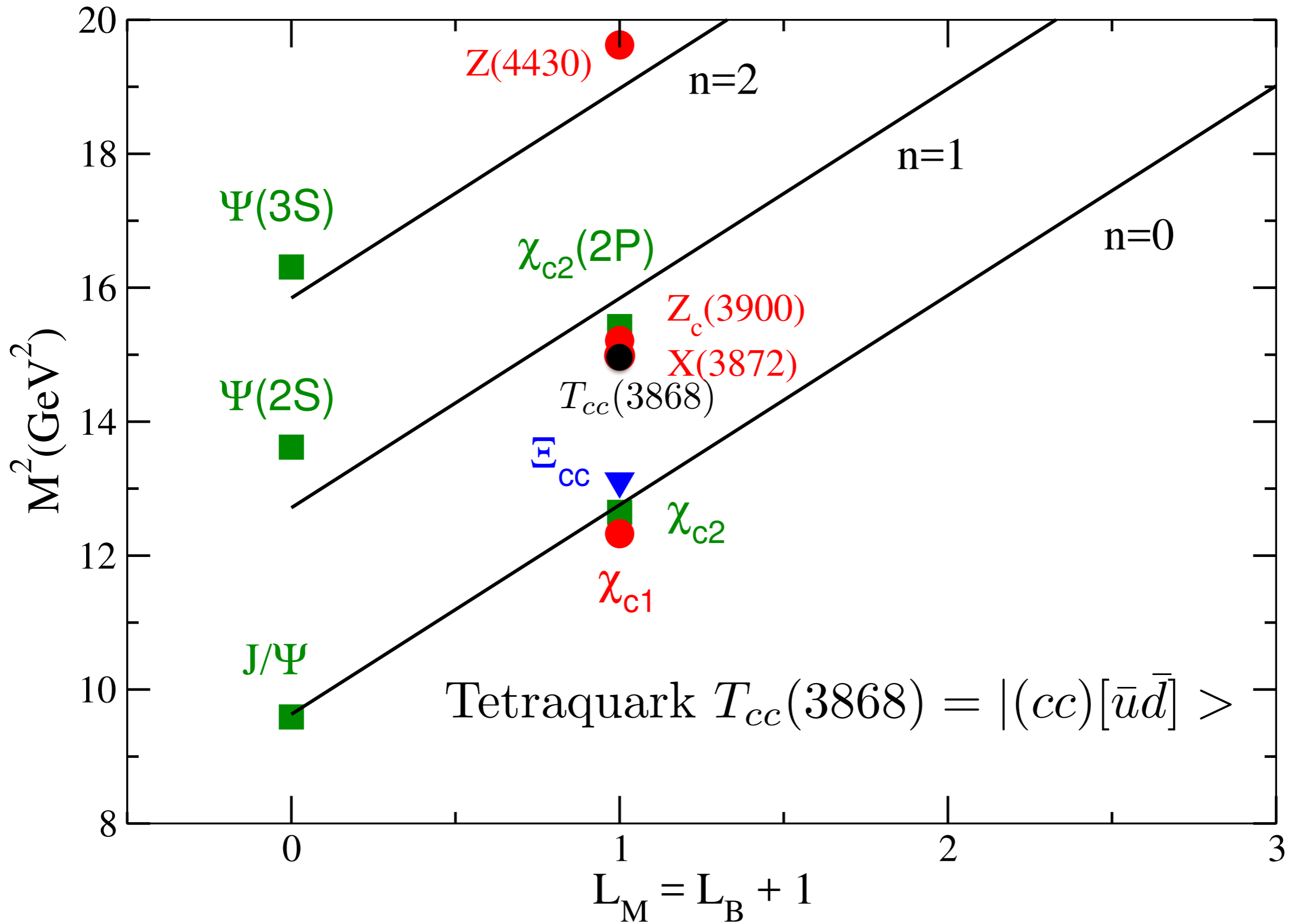
Linear instant nonrelativistic form  $V(r) = Cr$  for heavy quarks



Harmonic Oscillator  $U(\zeta) = \kappa^4 \zeta^2$  LF Potential for relativistic light quarks

A.P. Trawinski, S.D. Glazek, H. D. Dosch, G. de Teramond, sjb





Mesons : *GreenSquare*, Baryons (*BlueTriangle*), Tetraquarks (*RedCircle*)



# Universal Hadronic Decomposition

$$\frac{\mathcal{M}_H^2}{\kappa^2} = (1 + 2n + L) + (1 + 2n + L) + (2L + 4S + 2B - 2)$$

- **Universal quark light-front kinetic energy**

$$\Delta\mathcal{M}_{LFKE}^2 = \kappa^2(1 + 2n + L)$$

- **Universal quark light-front potential energy**

$$\Delta\mathcal{M}_{LFPE}^2 = \kappa^2(1 + 2n + L)$$

- **Universal Constant Contribution from AdS and Superconformal Quantum Mechanics**

$$\Delta\mathcal{M}_{spin}^2 = 2\kappa^2(L + 2S + B - 1)$$

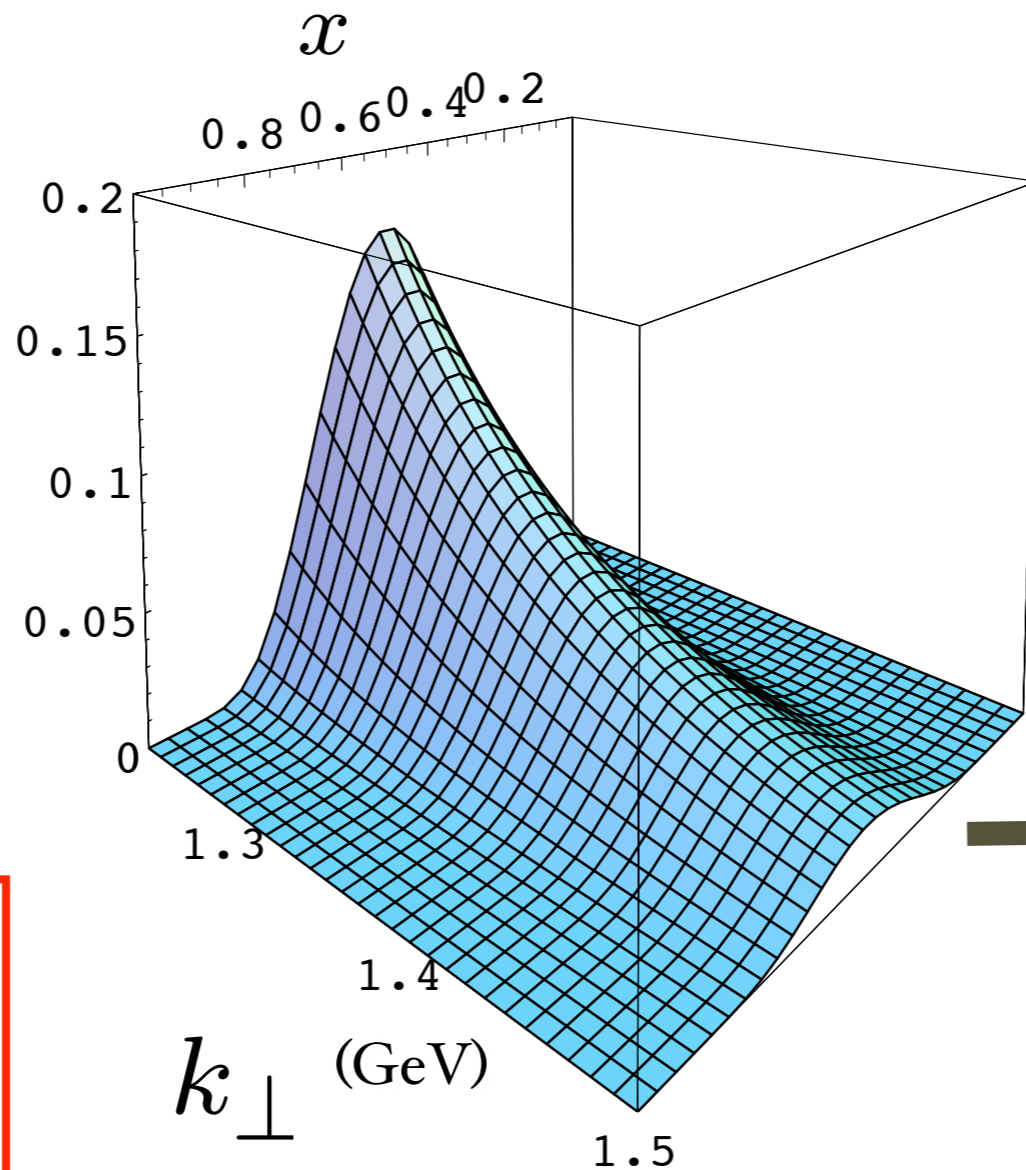
hyperfine spin-spin

**Equal:  
Virial  
Theorem**

# Prediction from AdS/QCD: Meson LFWF

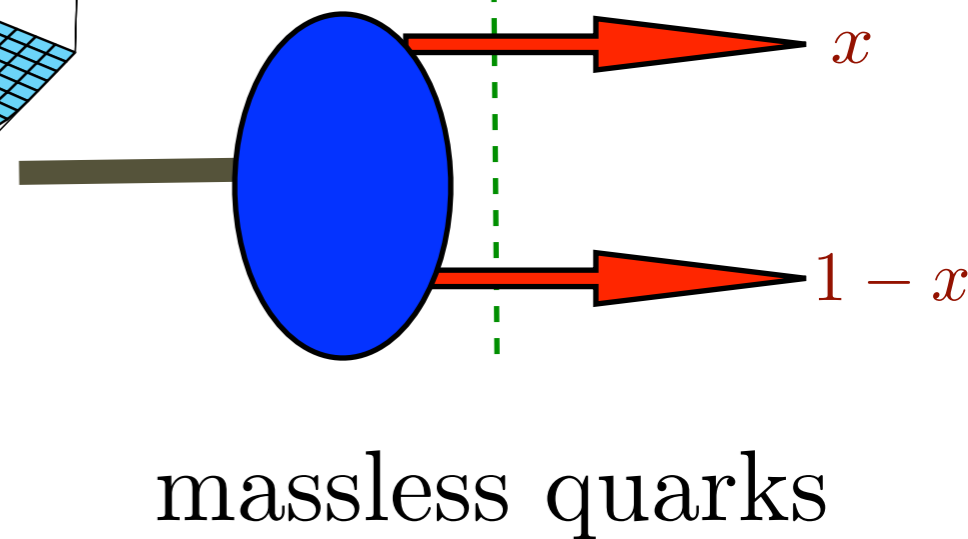
$$e^{\varphi(z)} = e^{+\kappa^2 z}$$

$$\psi_M(x, k_{\perp}^2)$$



de Teramond,  
Cao, sjb

“Soft Wall”  
model



**Note coupling**

$$k_{\perp}^2, x$$

$$\psi_M(x, k_{\perp}) = \frac{4\pi}{\kappa \sqrt{x(1-x)}} e^{-\frac{k_{\perp}^2}{2\kappa^2 x(1-x)}}$$

$$\phi_{\pi}(x) = \frac{4}{\sqrt{3}\pi} f_{\pi} \sqrt{x(1-x)}$$

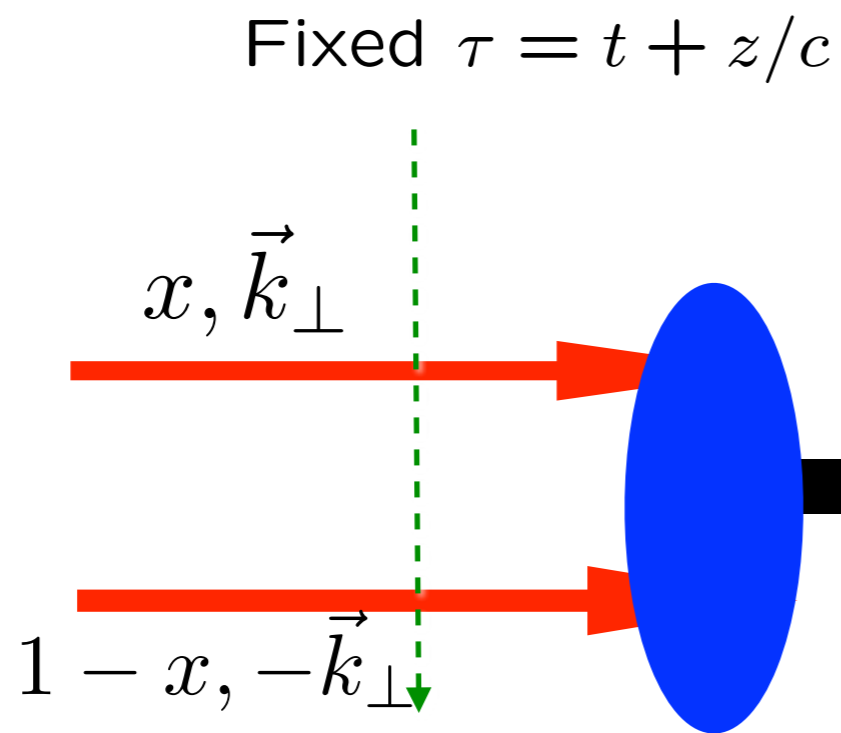
$$f_{\pi} = \sqrt{P_{q\bar{q}}} \frac{\sqrt{3}}{8} \kappa = 92.4 \text{ MeV}$$

**Same as DSE!** C. D. Roberts et al.

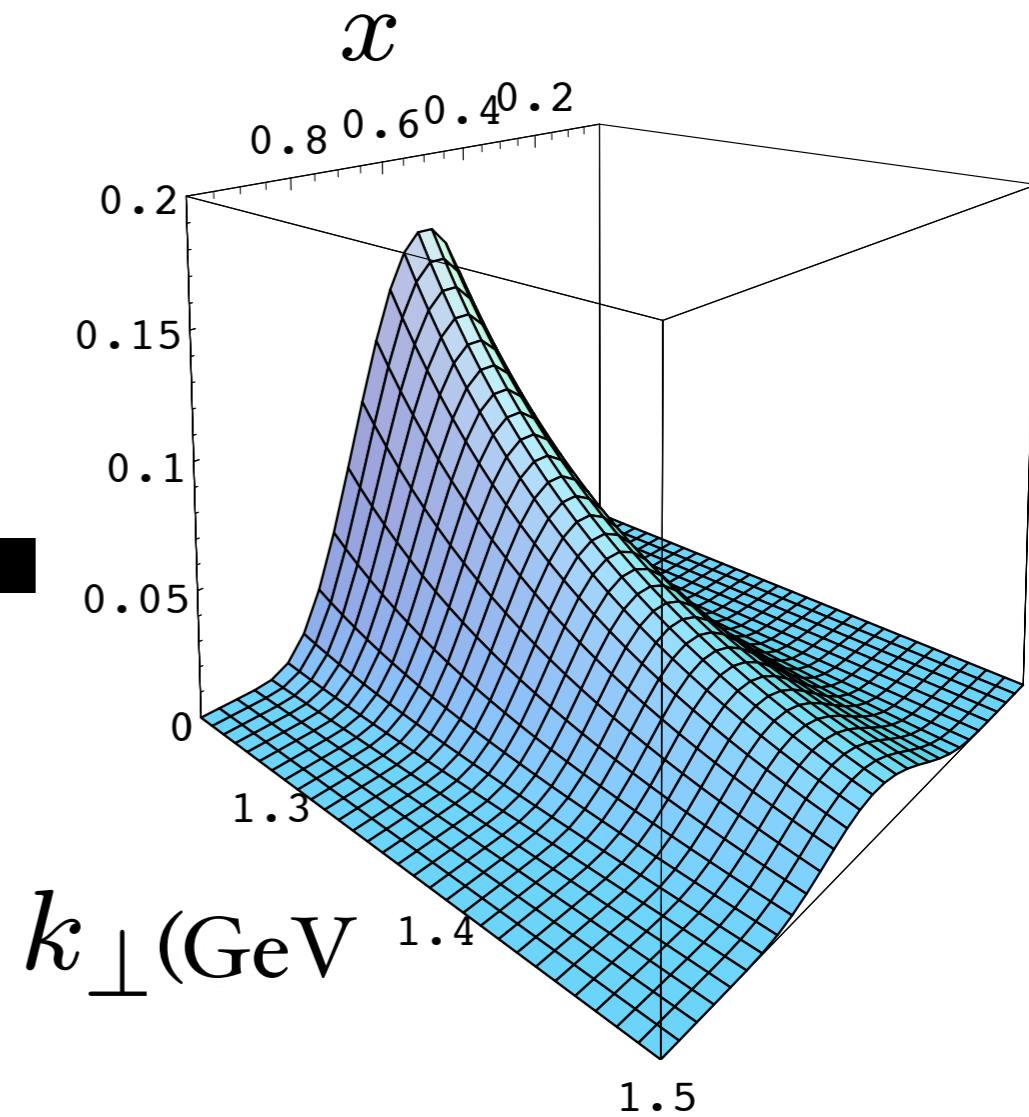
*Provides Connection of Confinement to Hadron Structure*

- *Light Front Wavefunctions:*  $\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$

off-shell in  $P^-$  and invariant mass  $\mathcal{M}_{q\bar{q}}^2$



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$



***“Hadronization at the Amplitude Level”***

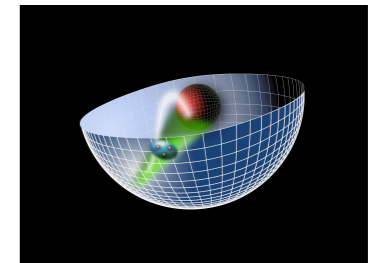
**Boost-invariant LFWF connects confined quarks and gluons to hadrons**

**Proceeds in LF time  $\tau$  within casual horizon  
Instant time violates causality**

# LFHQCD: Underlying Principles

- **Poincarè Invariance: Independent of the observer's Lorentz frame: Quantization at Fixed Light-Front Time  $\tau$**
- **Causality: Information within causal horizon: Light-Front**
- **Light-Front Holography:  $AdS_5 = LF (3+1)$**

$$z \leftrightarrow \zeta \text{ where } \zeta^2 = b_{\perp}^2 x(1-x)$$



- **Introduce Mass Scale  $\kappa$  while retaining the Conformal Invariance of the Action (dAFF)**
- **Unique Dilaton in  $AdS_5$ :  $e^{+\kappa^2 z^2}$**
- **Unique color-confining LF Potential  $U(\zeta^2) = \kappa^4 \zeta^2$**
- **Superconformal Algebra: Mass Degenerate 4-Plet:**

Meson  $q\bar{q} \leftrightarrow$  Baryon  $q[qq] \leftrightarrow$  Tetraquark  $[qq][\bar{q}\bar{q}]$



# Remarkable Features of Light-Front Schrödinger Equation

## Dynamics + Spectroscopy!

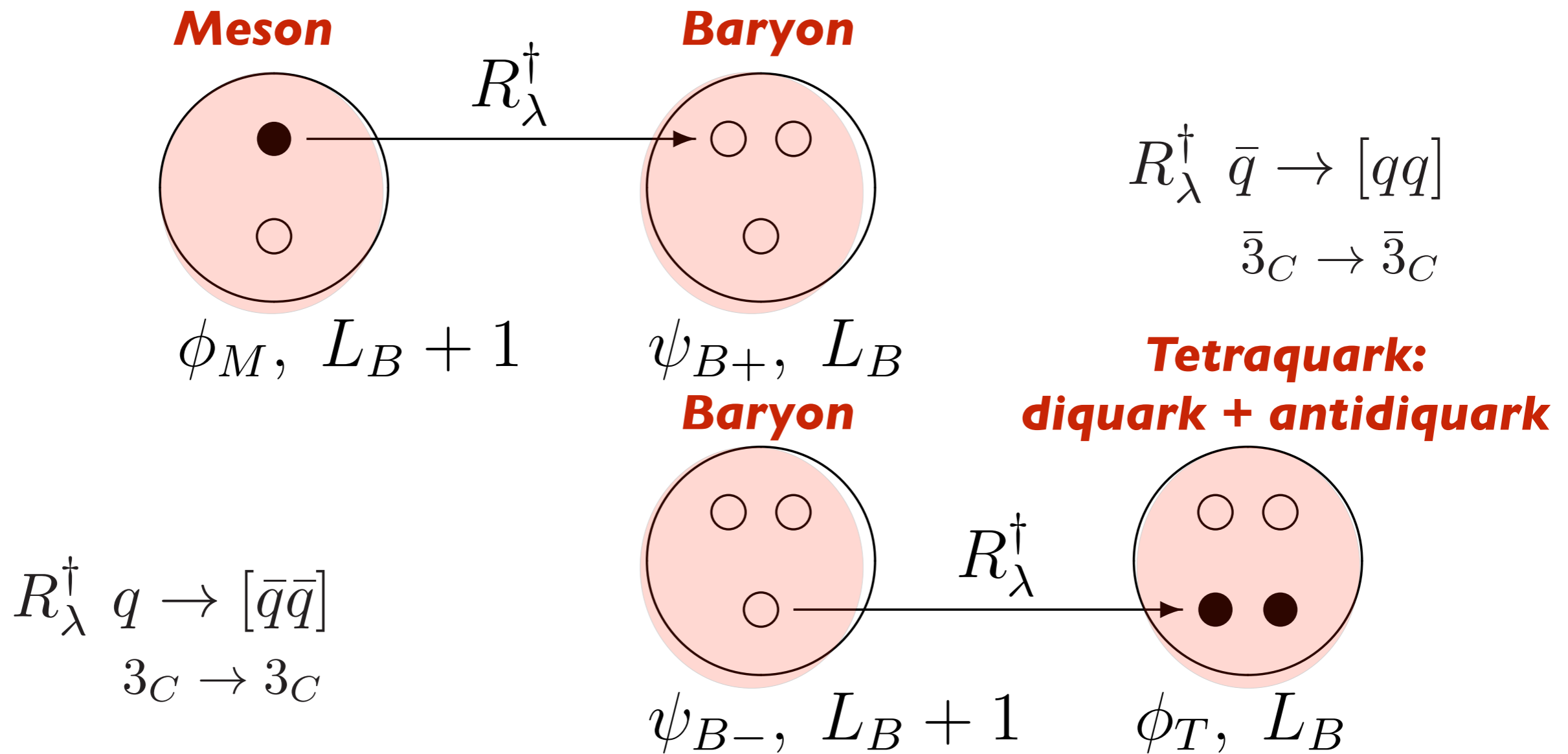
- **Relativistic, frame-independent**
- **QCD scale appears - unique LF potential**
- **Reproduces spectroscopy and dynamics of light-quark hadrons with one parameter**
- **Zero-mass pion for zero mass quarks!**
- **Regge slope same for  $n$  and  $L$  -- not usual HO**
- **Splitting in  $L$  persists to high mass -- contradicts conventional wisdom based on breakdown of chiral symmetry**
- **Phenomenology: LFWFs, Form factors, electroproduction**
- **Extension to heavy quarks**

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)$$

# Superconformal Algebra

## 2X2 Hadronic Multiplets

Bosons, Fermions with Equal Mass!



Proton:  $|u[ud]\rangle$  Quark + Scalar Diquark  
 Equal Weight:  $L=0, L=1$

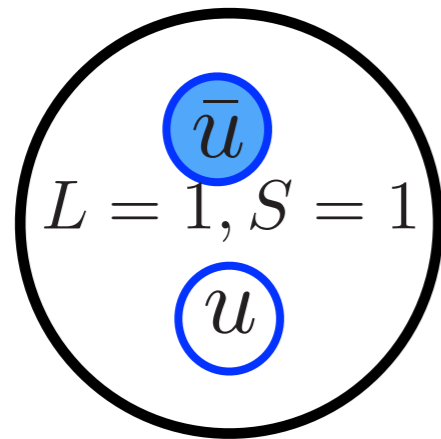


# Superconformal Algebra 4-Plet

$$R_\lambda^\dagger \begin{array}{l} \bar{q} \rightarrow (qq) \\ \bar{3}_C \rightarrow \bar{3}_C \end{array} S = 1$$

Vector ( ) + Scalar [ ] Diquarks

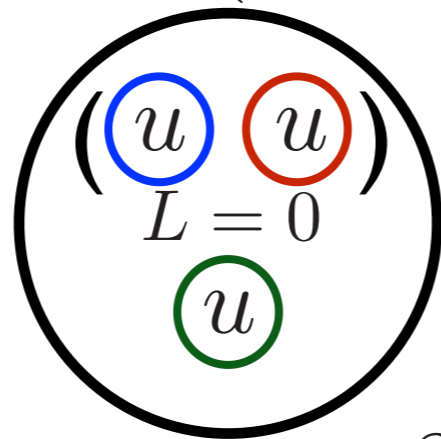
$f_2(1270)$



$$J^{PC} = 2^{++}$$

**Meson**

$\Delta^+(1232)$



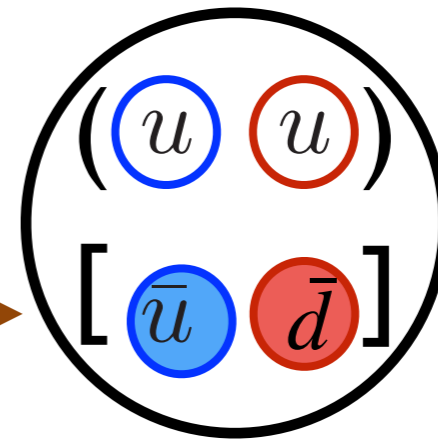
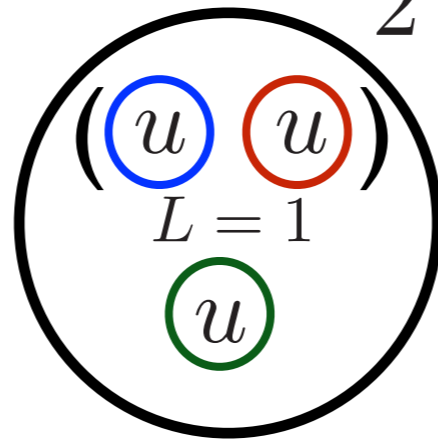
$$J^P = \frac{3}{2}^+$$

**Baryon**

**Tetraquark**

$$J^{PC} = 1^{++}$$

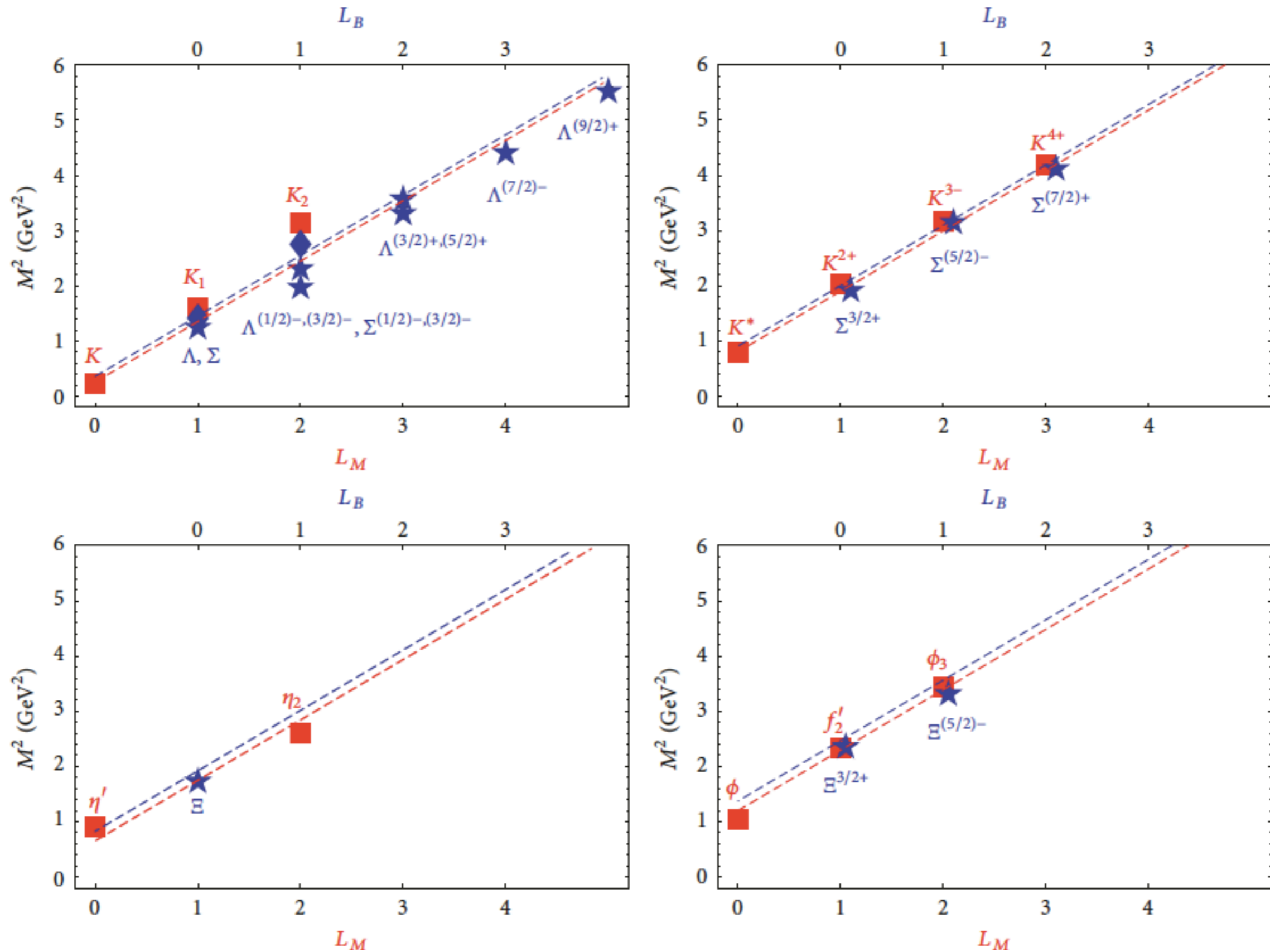
$a_1(1260)$



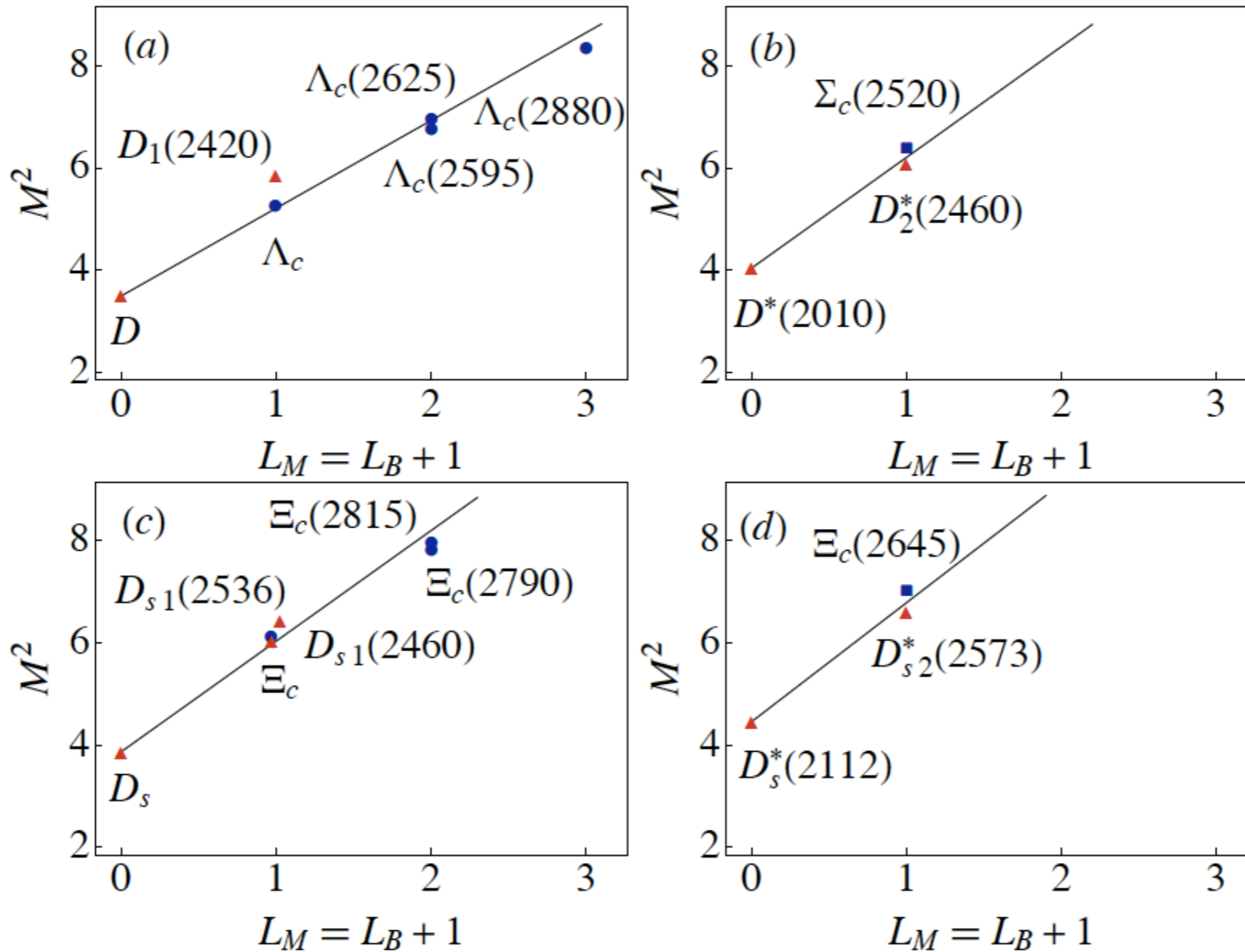
$$\begin{array}{l} S = 0 \\ L = 0 \end{array}$$

$$R_\lambda^\dagger \begin{array}{l} q \rightarrow [\bar{q}\bar{q}] \\ 3_C \rightarrow 3_C \end{array}$$

# Supersymmetry across the light and heavy-light spectrum



# Supersymmetry across the light and heavy-light spectrum



**Heavy charm quark mass does not break supersymmetry**

Meson			Baryon			Tetraquark		
$q\text{-cont}$	$J^{P(C)}$	Name	$q\text{-cont}$	$J^P$	Name	$q\text{-cont}$	$J^{P(C)}$	Name
$\bar{q}q$	$0^{-+}$	$\pi(140)$	—	—	—	—	—	—
$\bar{q}q$	$1^{+-}$	$b_1(1235)$	$[ud]q$	$(1/2)^+$	$N(940)$	$[ud][\bar{u}\bar{d}]$	$0^{++}$	$f_0(980)$
$\bar{q}q$	$2^{-+}$	$\pi_2(1670)$	$[ud]q$	$(1/2)^-$	$N_{\frac{1}{2}}^-(1535)$	$[ud][\bar{u}\bar{d}]$	$1^{-+}$	$\pi_1(1400)$
				$(3/2)^-$	$N_{\frac{3}{2}}^-(1520)$			$\pi_1(1600)$
$\bar{q}q$	$1^{--}$	$\rho(770), \omega(780)$	—	—	—	—	—	—
$\bar{q}q$	$2^{++}$	$a_2(1320), f_2(1270)$	$[qq]q$	$(3/2)^+$	$\Delta(1232)$	$[qq][\bar{u}\bar{d}]$	$1^{++}$	$a_1(1260)$
$\bar{q}q$	$3^{--}$	$\rho_3(1690), \omega_3(1670)$	$[qq]q$	$(1/2)^-$	$\Delta_{\frac{1}{2}}^-(1620)$	$[qq][\bar{u}\bar{d}]$	$2^{--}$	$\rho_2(\sim 1700)?$
				$(3/2)^-$	$\Delta_{\frac{3}{2}}^-(1700)$			
$\bar{q}q$	$4^{++}$	$a_4(2040), f_4(2050)$	$[qq]q$	$(7/2)^+$	$\Delta_{\frac{7}{2}}^+(1950)$	$[qq][\bar{u}\bar{d}]$	$3^{++}$	$a_3(\sim 2070)?$
$\bar{q}s$	$0^{-+}$	$K(495)$	—	—	—	—	—	—
$\bar{q}s$	$1^{+-}$	$\bar{K}_1(1270)$	$[ud]s$	$(1/2)^+$	$\Lambda(1115)$	$[ud][\bar{s}\bar{q}]$	$0^{++}$	$K_0^*(1430)$
$\bar{q}s$	$2^{-+}$	$K_2(1770)$	$[ud]s$	$(1/2)^-$	$\Lambda(1405)$	$[ud][\bar{s}\bar{q}]$	$1^{-+}$	$K_1^*(\sim 1700)?$
				$(3/2)^-$	$\Lambda(1520)$			
$\bar{s}q$	$0^{-+}$	$K(495)$	—	—	—	—	—	—
$\bar{s}q$	$1^{+-}$	$K_1(1270)$	$[sq]q$	$(1/2)^+$	$\Sigma(1190)$	$[sq][\bar{s}\bar{q}]$	$0^{++}$	$a_0(980)$ $f_0(980)$
$\bar{s}q$	$1^{-+}$	$K^*(890)$	—	—	—	—	—	—
$\bar{s}q$	$2^{++}$	$K_2^*(1430)$	$[sq]q$	$(3/2)^+$	$\Sigma(1385)$	$[sq][\bar{q}\bar{q}]$	$1^{++}$	$K_1(1400)$
$\bar{s}q$	$3^{-+}$	$K_3^*(1780)$	$[sq]q$	$(3/2)^-$	$\Sigma(1670)$	$[sq][\bar{q}\bar{q}]$	$2^{-+}$	$K_2(\sim 1700)?$
$\bar{s}q$	$4^{++}$	$K_4^*(2045)$	$[sq]q$	$(7/2)^+$	$\Sigma(2030)$	$[sq][\bar{q}\bar{q}]$	$3^{++}$	$K_3(\sim 2070)?$
$\bar{s}s$	$0^{-+}$	$\eta(550)$	—	—	—	—	—	—
$\bar{s}s$	$1^{+-}$	$h_1(1170)$	$[sq]s$	$(1/2)^+$	$\Xi(1320)$	$[sq][\bar{s}\bar{q}]$	$0^{++}$	$f_0(1370)$ $a_0(1450)$
$\bar{s}s$	$2^{-+}$	$\eta_2(1645)$	$[sq]s$	$(?)^?$	$\Xi(1690)$	$[sq][\bar{s}\bar{q}]$	$1^{-+}$	$\Phi'(1750)?$
$\bar{s}s$	$1^{--}$	$\Phi(1020)$	—	—	—	—	—	—
$\bar{s}s$	$2^{++}$	$f_2'(1525)$	$[sq]s$	$(3/2)^+$	$\Xi^*(1530)$	$[sq][\bar{s}\bar{q}]$	$1^{++}$	$f_1(1420)$
$\bar{s}s$	$3^{--}$	$\Phi_3(1850)$	$[sq]s$	$(3/2)^-$	$\Xi(1820)$	$[sq][\bar{s}\bar{q}]$	$2^{--}$	$\Phi_2(\sim 1800)?$
$\bar{s}s$	$2^{++}$	$f_2(1950)$	$[ss]s$	$(3/2)^+$	$\Omega(1672)$	$[ss][\bar{s}\bar{q}]$	$1^{++}$	$K_1(\sim 1700)?$

Meson

Baryon

Tetraquark

**New Organization of the Hadron Spectrum**

M. Nielsen,  
sjb



# Superpartners for states with one c quark

Meson			Baryon			Tetraquark		
$q$ -cont	$J^{P(C)}$	Name	$q$ -cont	$J^P$	Name	$q$ -cont	$J^{P(C)}$	Name
$\bar{q}c$	$0^-$	$D(1870)$	—	—	—	—	—	—
$\bar{q}c$	$1^+$	$D_1(2420)$	$[ud]c$	$(1/2)^+$	$\Lambda_c(2290)$	$[ud][\bar{c}\bar{q}]$	$0^+$	$\bar{D}_0^*(2400)$
$\bar{q}c$	$2^-$	$D_J(2600)$	$[ud]c$	$(3/2)^-$	$\Lambda_c(2625)$	$[ud][\bar{c}\bar{q}]$	$1^-$	—
$\bar{c}q$	$0^-$	$\bar{D}(1870)$	—	—	—	—	—	—
$\bar{c}q$	$1^+$	$\bar{D}_1(2420)$	$[cq]q$	$(1/2)^+$	$\Sigma_c(2455)$	$[cq][\bar{u}\bar{d}]$	$0^+$	$D_0^*(2400)$
$\bar{q}c$	$1^-$	$D^*(2010)$	—	—	—	—	—	—
$\bar{q}c$	$2^+$	$D_2^*(2460)$	$(qq)c$	$(3/2)^+$	$\Sigma_c^*(2520)$	$(qq)[\bar{c}\bar{q}]$	$1^+$	$D(2550)$
$\bar{q}c$	$3^-$	$D_3^*(2750)$	$(qq)c$	$(3/2)^-$	$\Sigma_c(2800)$	$(qq)[\bar{c}\bar{q}]$	—	—
$\bar{s}c$	$0^-$	$D_s(1968)$	—	—	—	—	—	—
$\bar{s}c$	$1^+$	$D_{s1}(2460)$	$[qs]c$	$(1/2)^+$	$\Xi_c(2470)$	$[qs][\bar{c}\bar{q}]$	$0^+$	$\bar{D}_{s0}^*(2317)$
$\bar{s}c$	$2^-$	$D_{s2}(\sim 2860)?$	$[qs]c$	$(3/2)^-$	$\Xi_c(2815)$	$[sq][\bar{c}\bar{q}]$	$1^-$	—
$\bar{s}c$	$1^-$	$D_s^*(2110)$	—	—	—	—	—	—
$\bar{s}c$	$2^+$	$D_{s2}^*(2573)$	$(sq)c$	$(3/2)^+$	$\Xi_c^*(2645)$	$(sq)[\bar{c}\bar{q}]$	$1^+$	$D_{s1}(2536)$
$\bar{c}s$	$1^+$	$\bar{D}_{s1}(\sim 2700)?$	$[cs]s$	$(1/2)^+$	$\Omega_c(2695)$	$[cs][\bar{s}\bar{q}]$	$0^+$	??
$\bar{s}c$	$2^+$	$D_{s2}^*(\sim 2750)?$	$(ss)c$	$(3/2)^+$	$\Omega_c(2770)$	$(ss)[\bar{c}\bar{s}]$	$1^+$	??

M. Nielsen, sjb

predictions

beautiful agreement!

# Heavy-light and heavy-heavy hadronic sectors

- Extension to the heavy-light hadronic sector

[H. G. Dosch, GdT, S. J. Brodsky, PRD **92**, 074010 (2015), PRD **95**, 034016 (2017)]

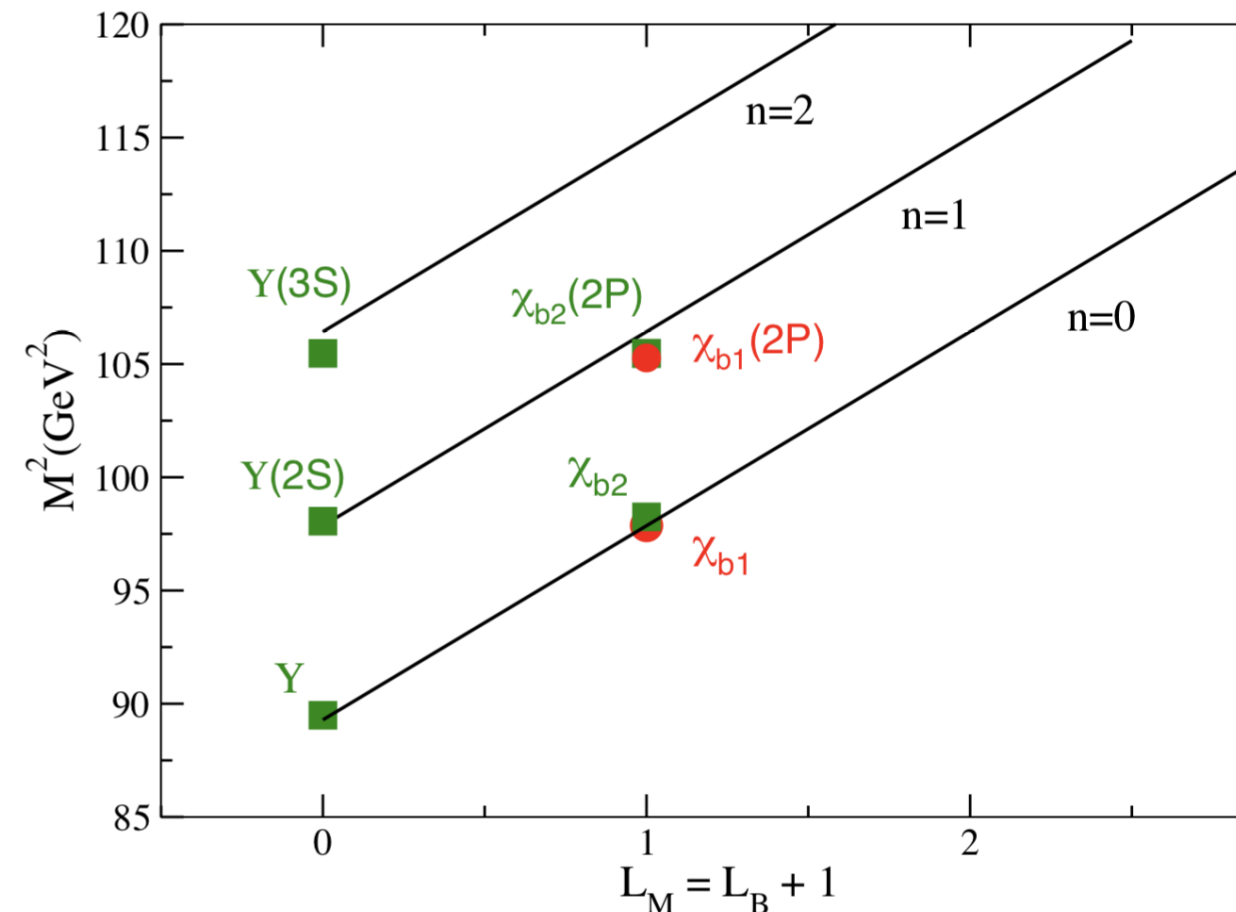
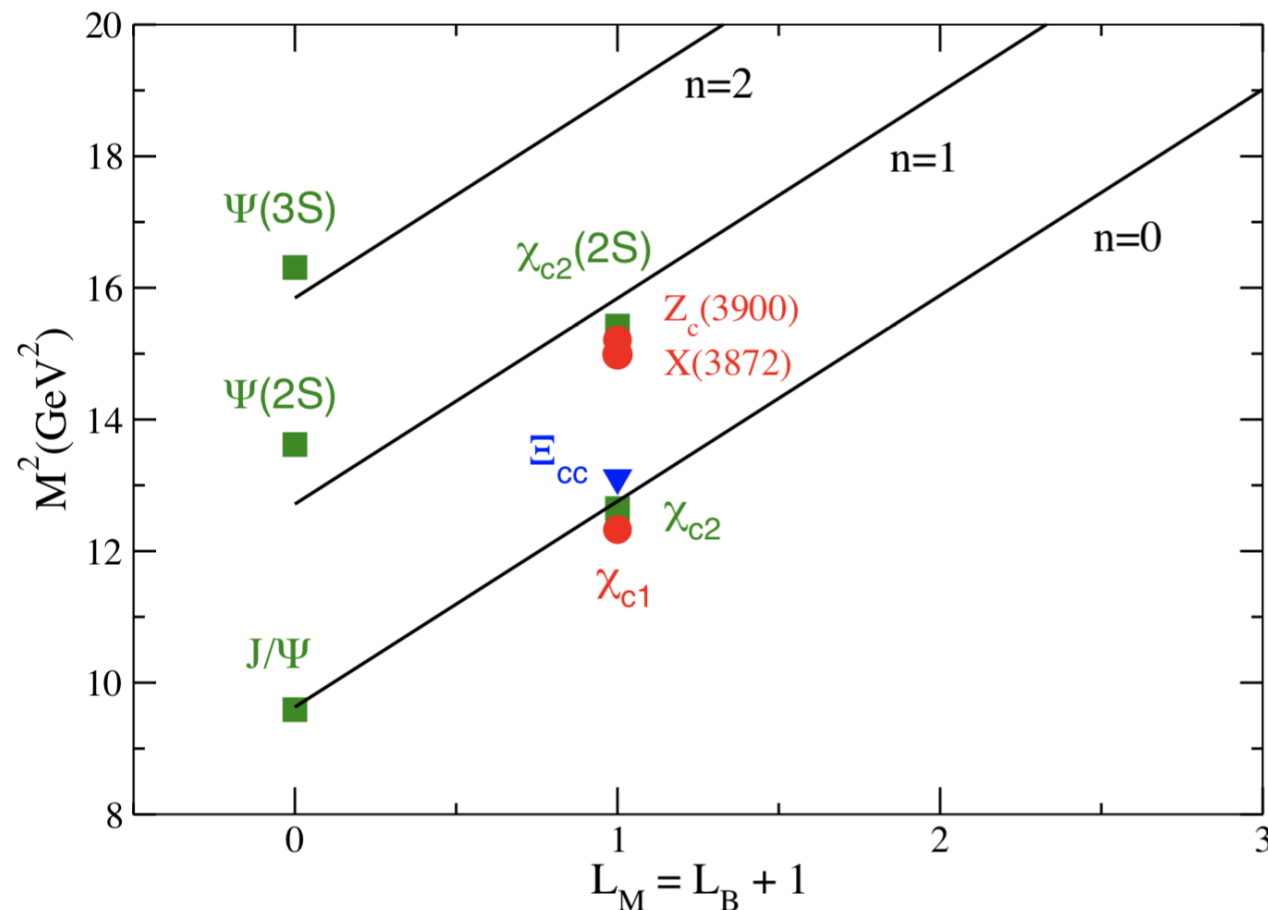
- Extension to the double-heavy hadronic sector

[M. Nielsen and S. J. Brodsky, PRD, 114001 (2018)]

[M. Nielsen, S. J. Brodsky, GdT, H. G. Dosch, F. S. Navarra, L. Zou, PRD **98**, 034002 (2018)]

- Extension to the isoscalar hadronic sector

[L. Zou, H. G. Dosch, GdT, S. J. Brodsky, arXiv:1901.11205 [hep-ph]]





# Massless pion!

## Meson Spectrum in Soft Wall Model

$$m_\pi = 0 \text{ if } m_q = 0$$

*Pion: Negative term for  $J=0$  cancels positive terms from LFKÉ and potential*



- Effective potential:  $U(\zeta^2) = \kappa^4 \zeta^2 + 2\kappa^2(J - 1)$

- LF WE

$$\left( -\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + \kappa^4 \zeta^2 + 2\kappa^2(J - 1) \right) \phi_J(\zeta) = M^2 \phi_J(\zeta)$$

- Normalized eigenfunctions  $\langle \phi | \phi \rangle = \int d\zeta \phi^2(z)^2 = 1$

$$\phi_{n,L}(\zeta) = \kappa^{1+L} \sqrt{\frac{2n!}{(n+L)!}} \zeta^{1/2+L} e^{-\kappa^2 \zeta^2 / 2} L_n^L(\kappa^2 \zeta^2)$$

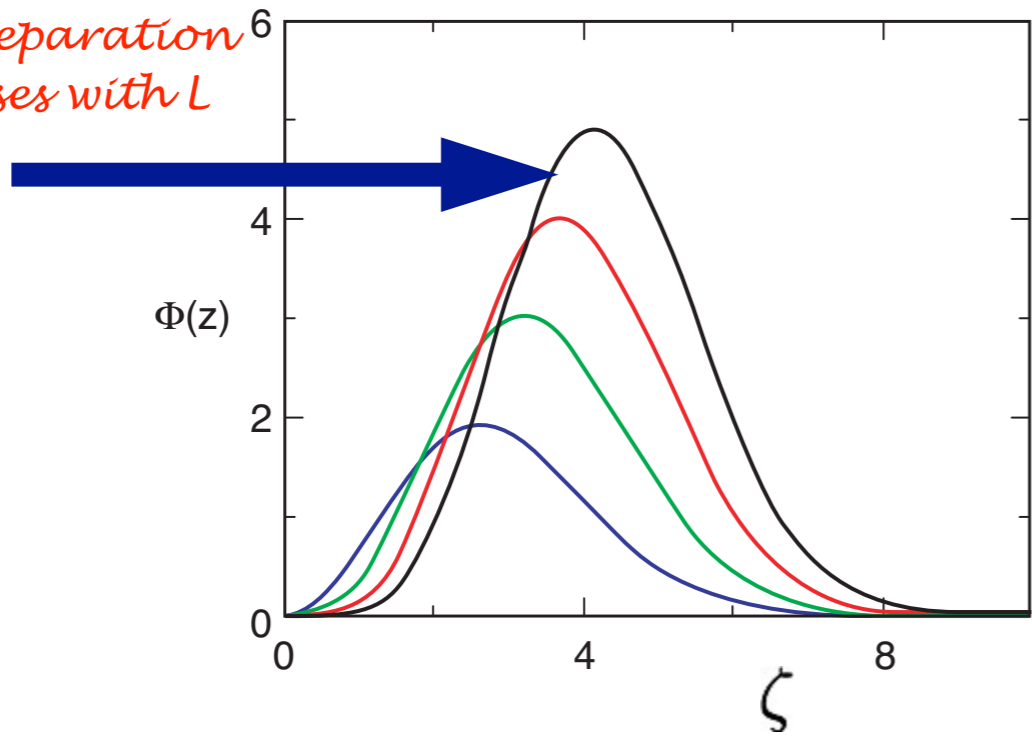
- Eigenvalues

$$M_{n,J,L}^2 = 4\kappa^2 \left( n + \frac{J+L}{2} \right)$$

$$\vec{\zeta}^2 = \vec{b}_\perp^2 x(1-x)$$

**G. de Teramond, H. G. Dosch, sjb**

Quark separation increases with  $L$



2-2007  
8721A21

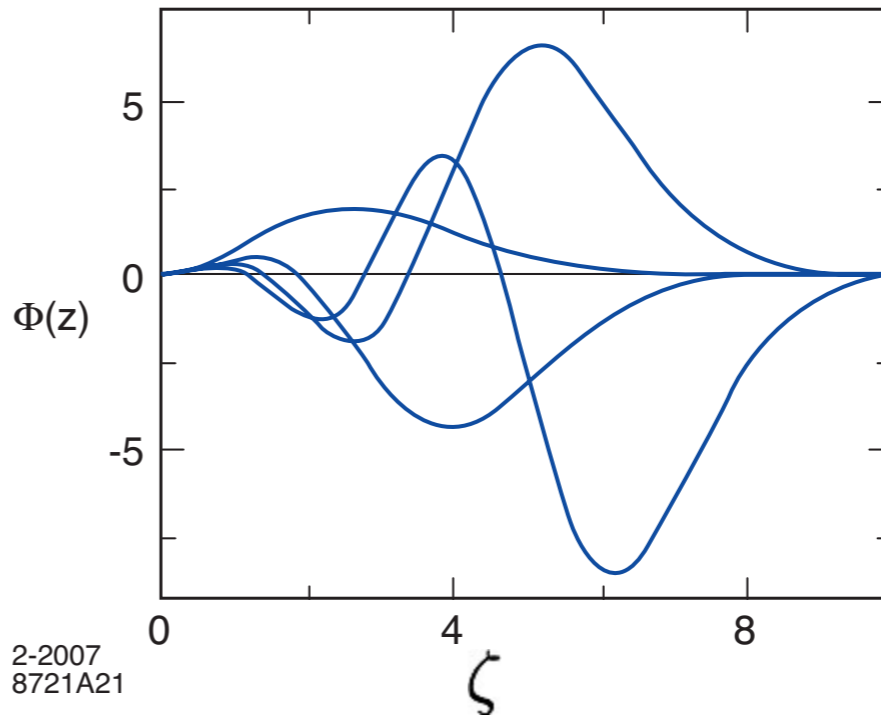
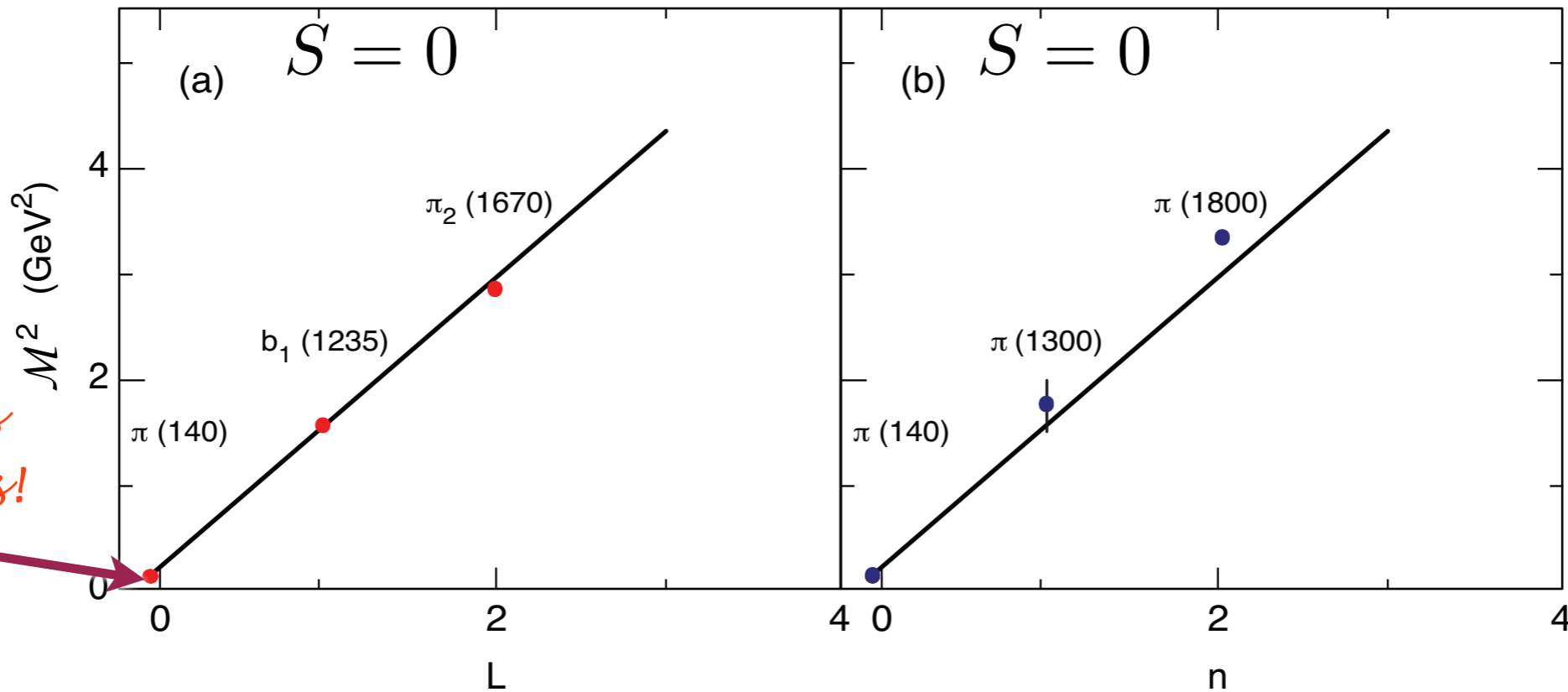


Fig: Orbital and radial AdS modes in the soft wall model for  $\kappa = 0.6$  GeV .

*Same slope in  $n$  and  $L$ !*

*Soft Wall Model*



*Pion has zero mass!*

$$m_q = 0$$

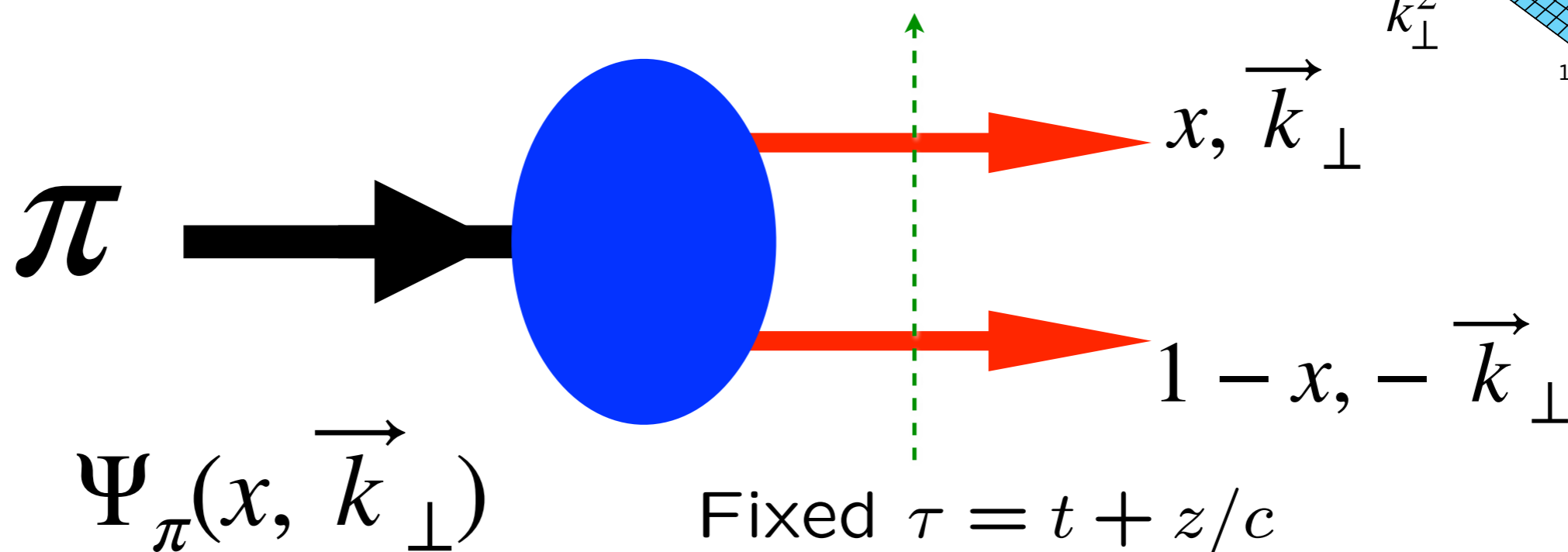
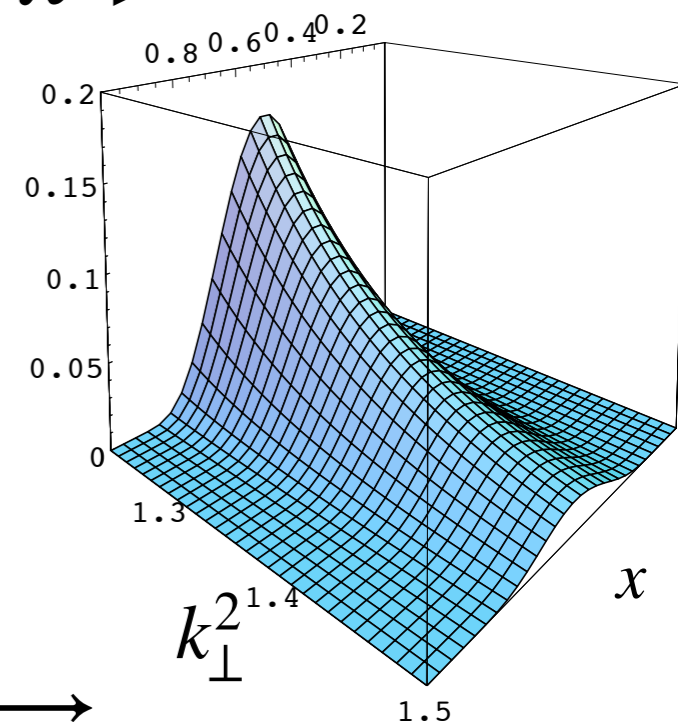
Light meson orbital (a) and radial (b) spectrum for  $\kappa = 0.6$  GeV.

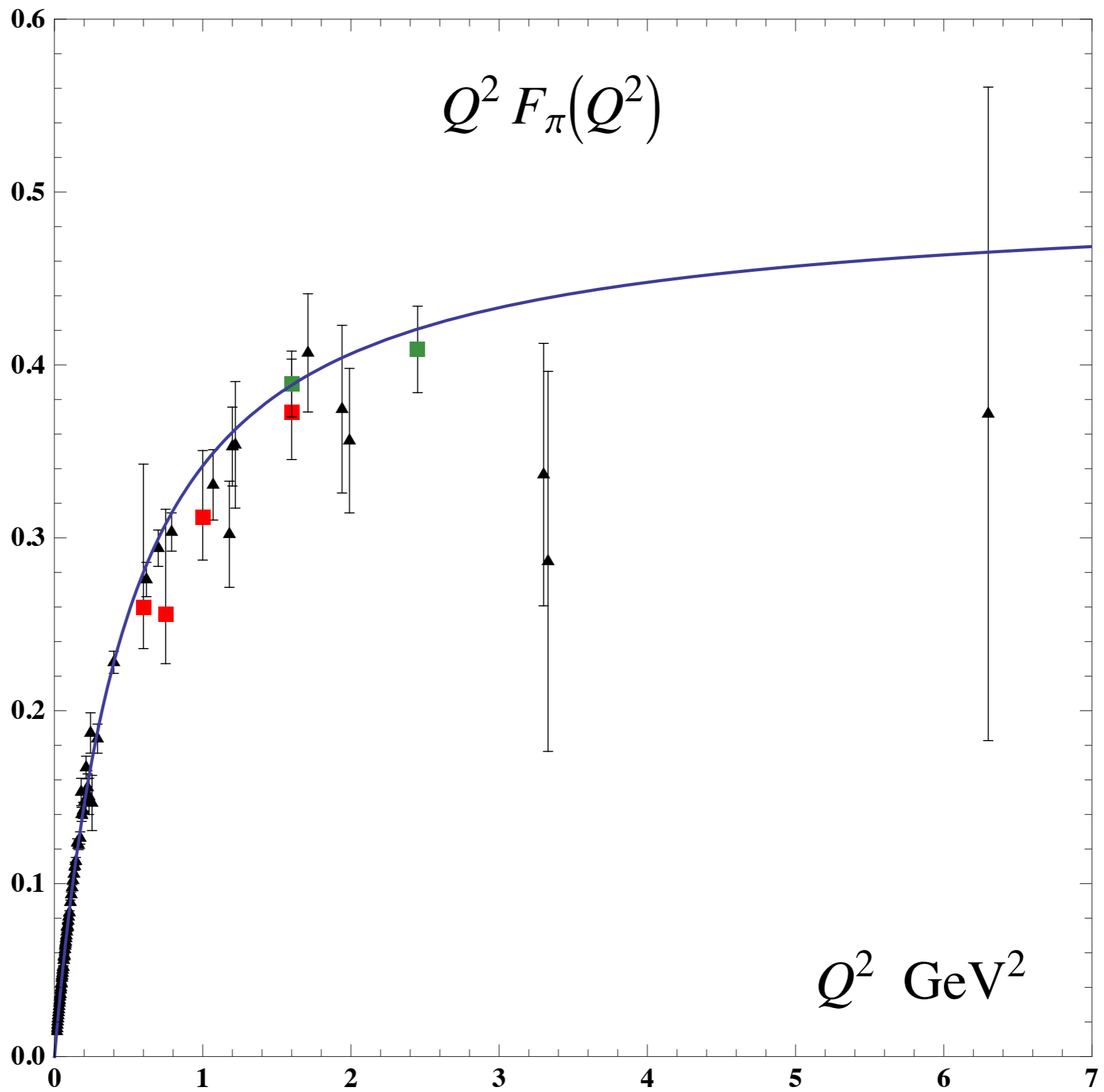
# The Pion's Valence Light-Front Wavefunction

- Relativistic Quantum-Mechanical Wavefunction of the pion eigenstate  $H_{LF}^{QCD} |\pi\rangle = m_\pi^2 |\pi\rangle$

$$\Psi_\pi(x, \vec{k}_\perp) = \langle q(x, \vec{k}_\perp) \bar{q}(1-x, -\vec{k}_\perp) | \pi \rangle$$

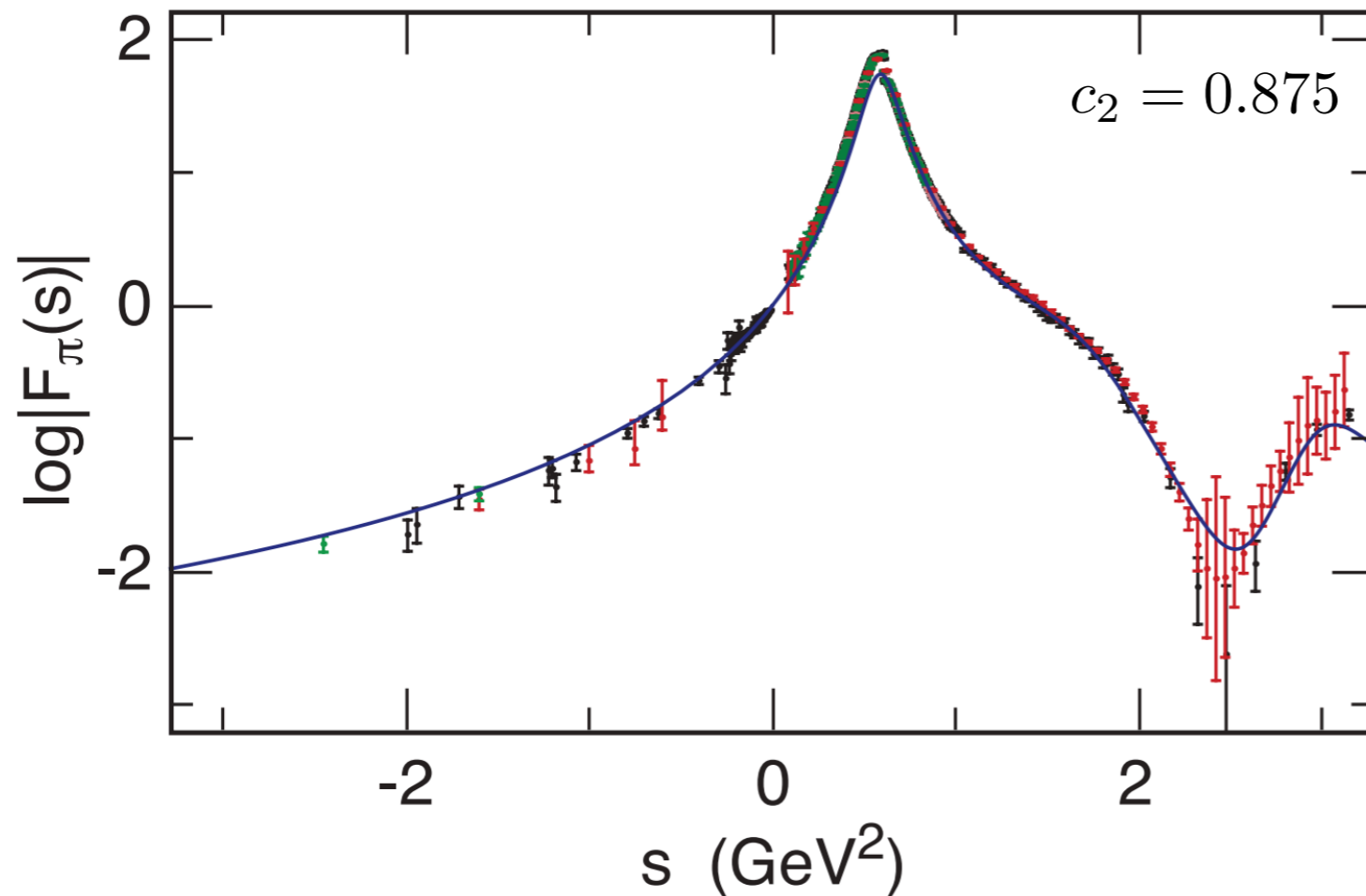
- Independent of the observer's or pion's motion
- No Lorentz contraction; causal
- **Confined** quark-antiquark bound state





# Pion EM Form Factor

Pion form factor compared with data



$$F_\pi(t) = \sum_{\tau} P_\tau F_\tau(t) \quad \sum_{\tau} P_\tau = 1$$

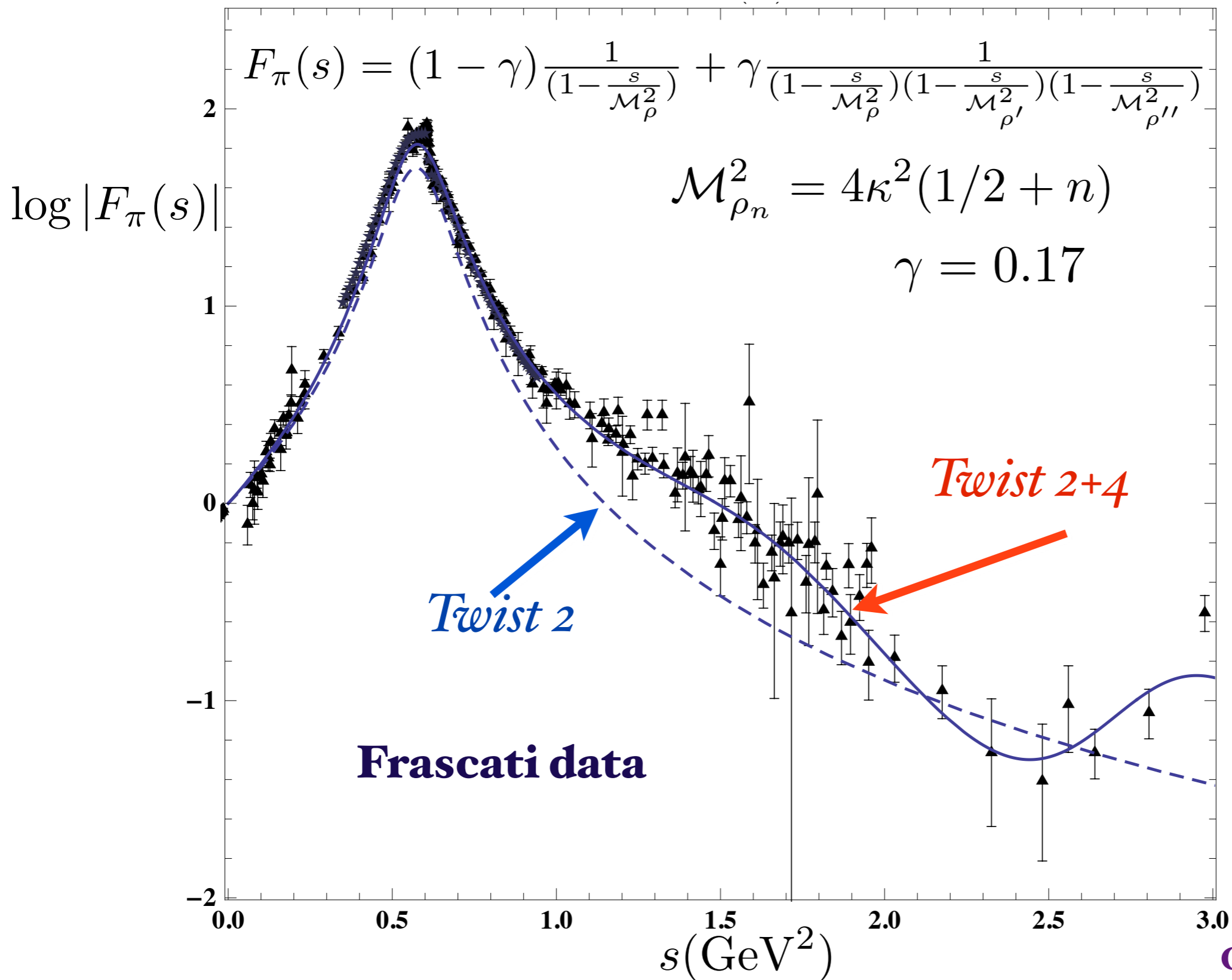
Truncated at twist- $\tau = 4$

$$F_\pi(t) = c_2 F_{\tau=2}(t) + (1 - c_2) F_{\tau=4}(t)$$

**G.F. de Téramond and S.J. Brodsky, Proc. Sci. LC2010 (2010) 029.**

**S.J. Brodsky, G.F. de Téramond, H.G. Dosch, J. Erlich, Phys. Rep. 584, 1 (2015). [Sec. 6.1.5]**

# Timelike Pion Form Factor from AdS/QCD and Light-Front Holography



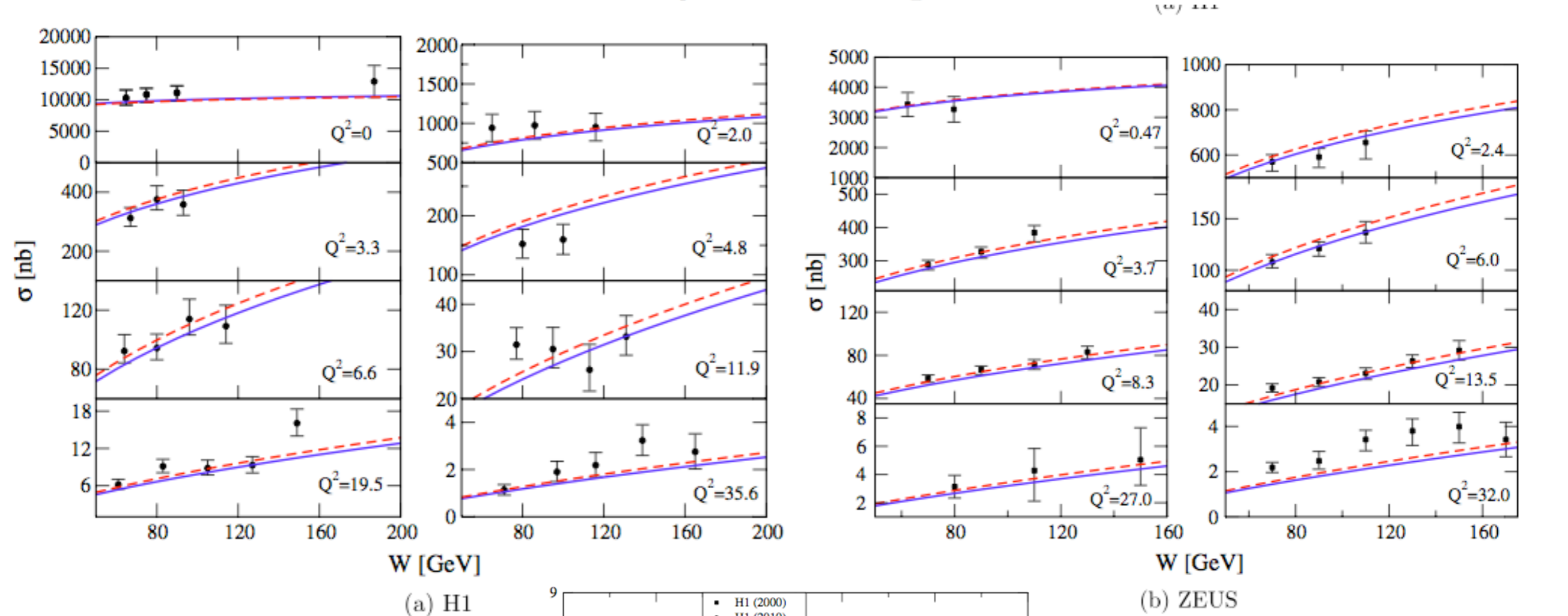
**Prescription for Timelike poles :**

$$\frac{1}{s - M^2 + i\sqrt{s}\Gamma}$$

**14% four-quark probability**

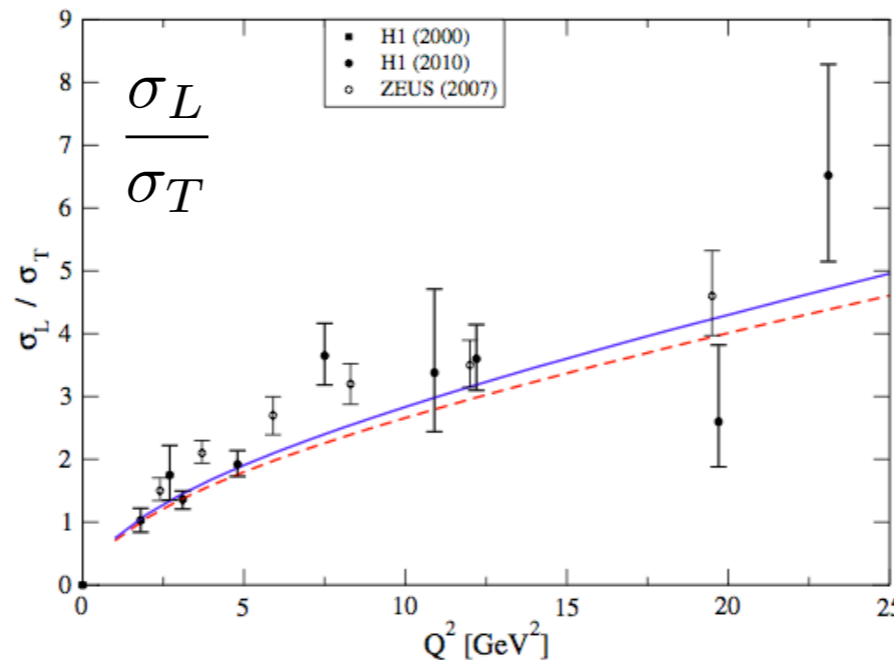


### AdS/QCD Holographic Wave Function for the $\rho$ Meson and Diffractive $\rho$ Meson Electroproduction

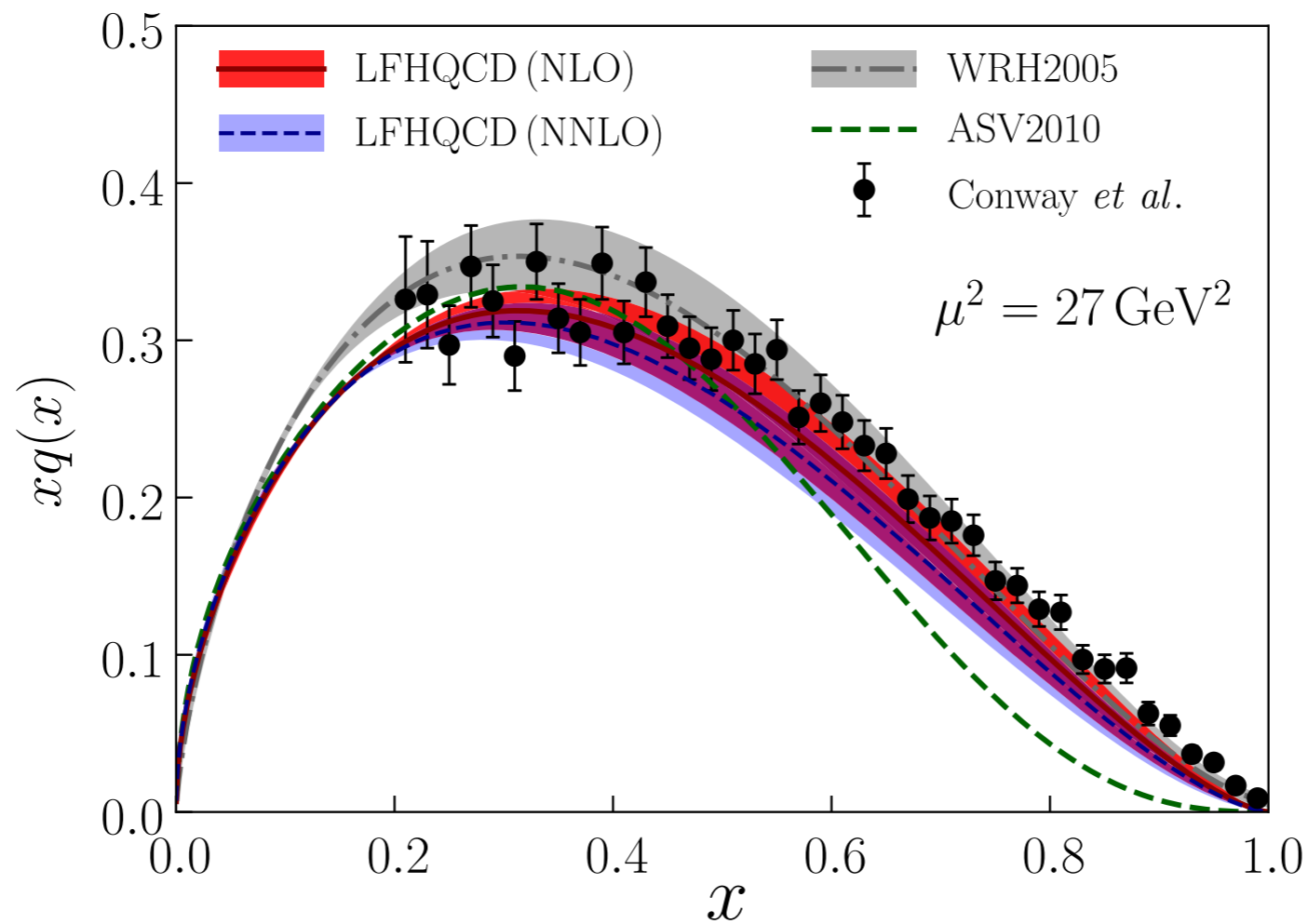


**J. R. Forshaw,  
R. Sandapen**

$$\gamma^* p \rightarrow \rho^0 p'$$



$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

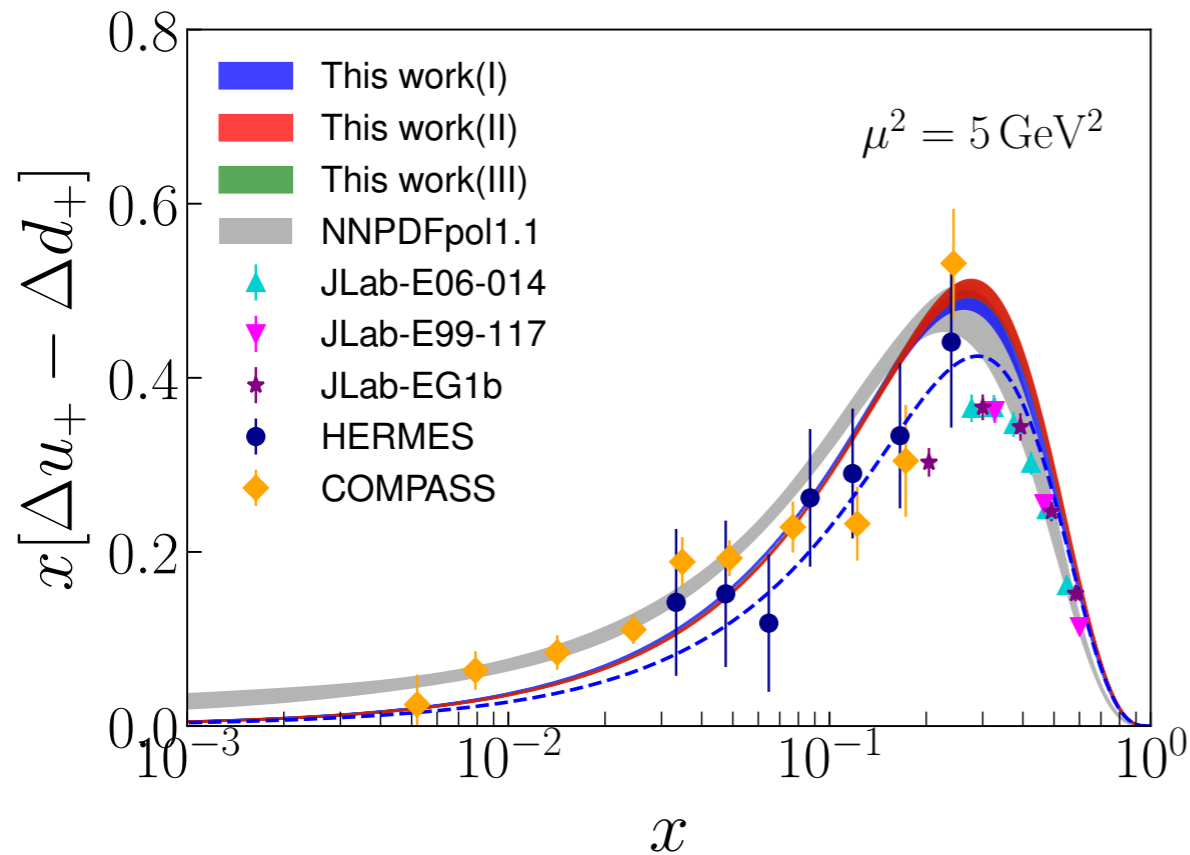


Comparison for  $xq(x)$  in the pion from LFHQCD (red band) with the NLO fits [82,83] (gray band and green curve) and the LO extraction [84]. NNLO results are also included (light blue band). LFHQCD results are evolved from the initial scale  $\mu_0 = 1.1 \pm 0.2 \text{ GeV}$  at NLO and the initial scale  $\mu_0 = 1.06 \pm 0.15 \text{ GeV}$  at NNLO.

### *Universality of Generalized Parton Distributions in Light-Front Holographic QCD*

*Guy F. de Téramond, Tianbo Liu, Raza Sabbir Sufian, Hans Günter Dosch, Stanley J. Brodsky, and Alexandre Deur* *PHYSICAL REVIEW LETTERS* 120, 182001 (2018)

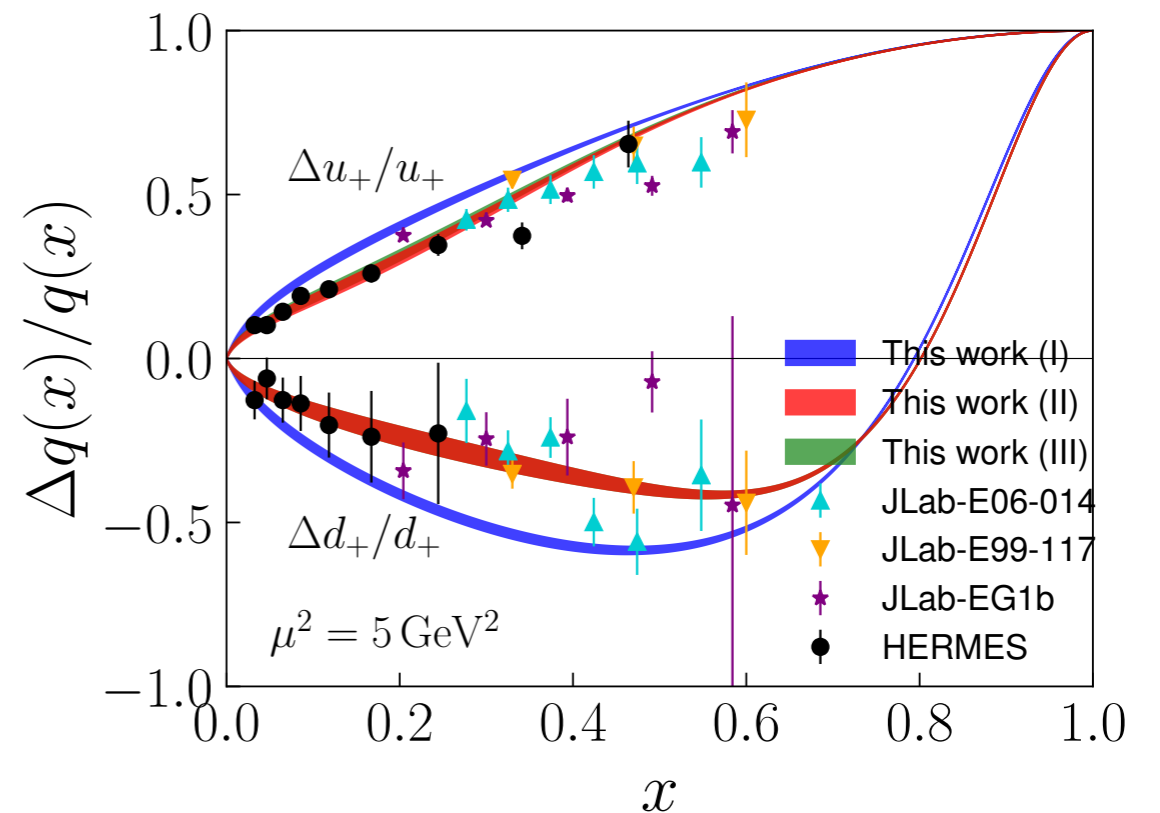
# Tianbo Liu, Raza Sabbir Sufian, Guy F. de T'era mond, Hans Gunter Dös ch, Alexandre Deur, s j b



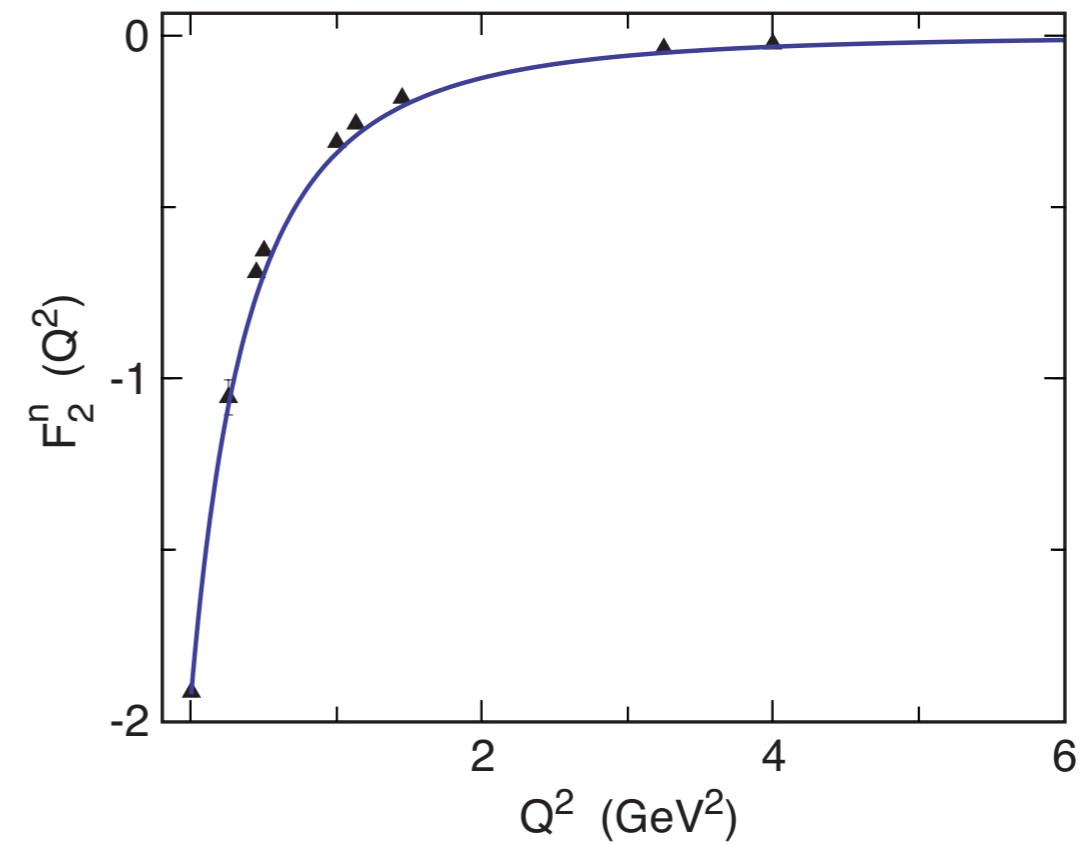
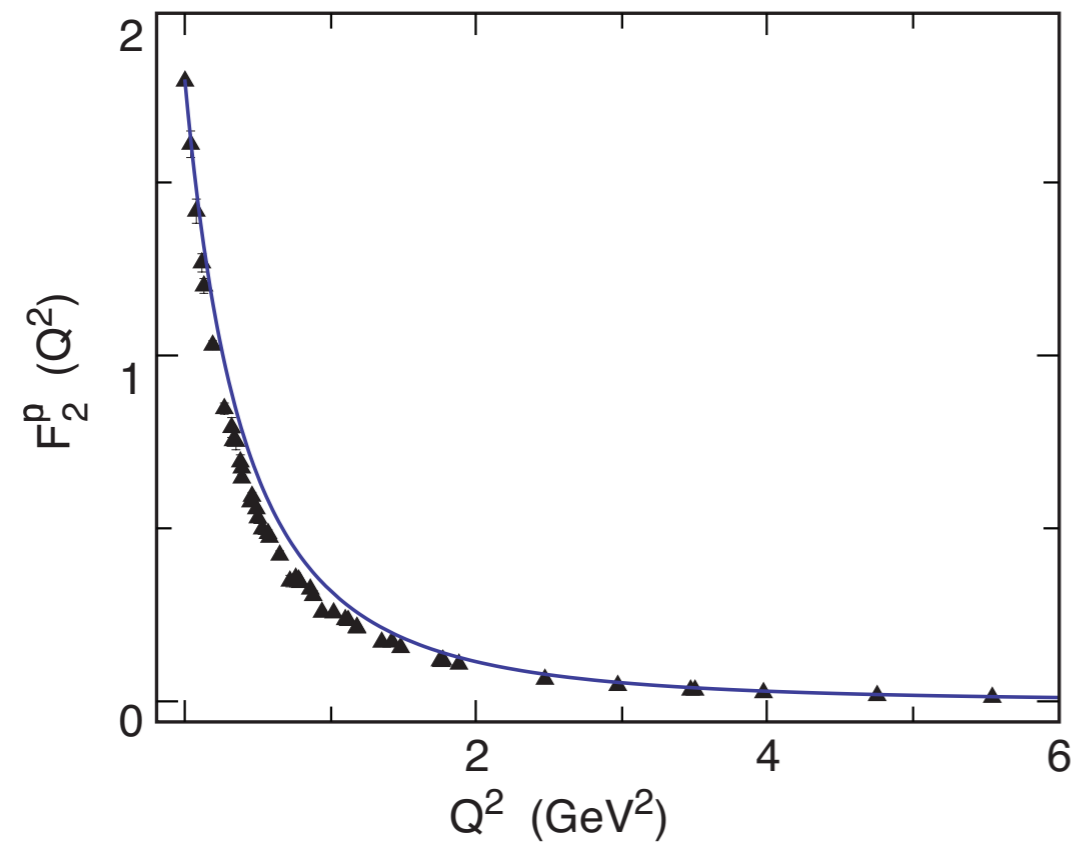
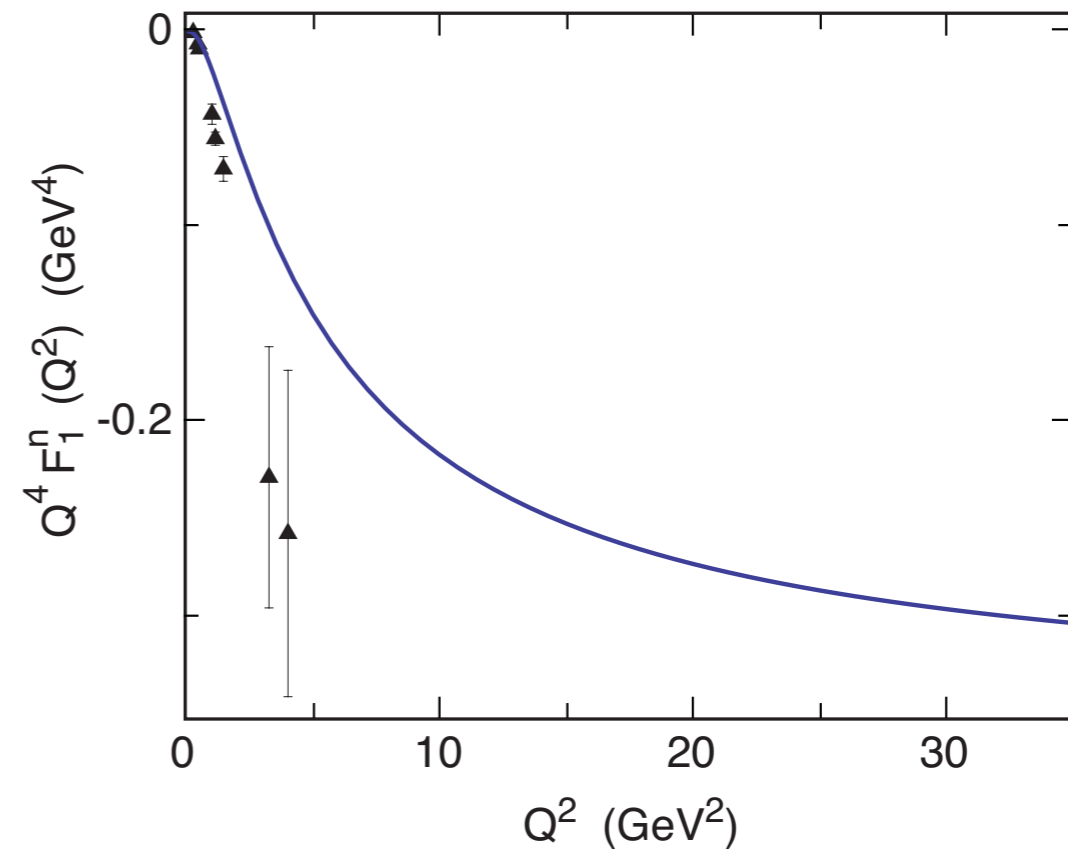
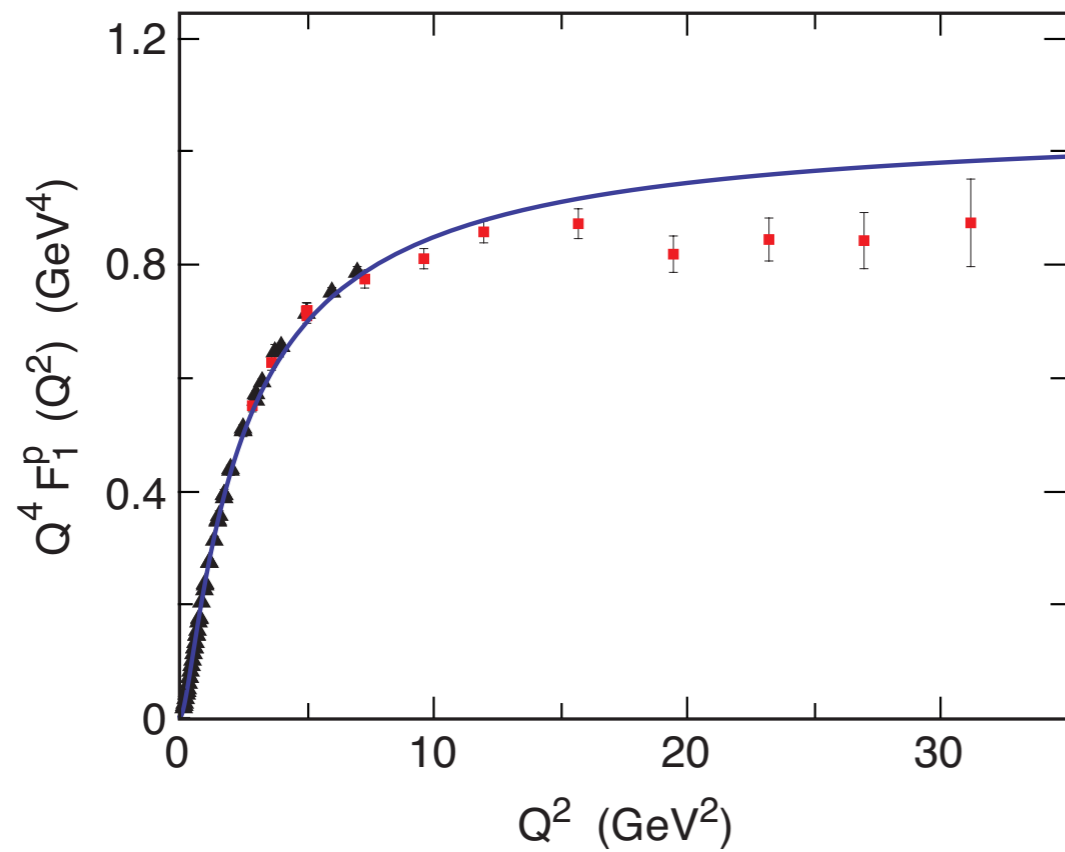
Polarized distributions for the isovector combination  $x[\Delta u_+(x) - \Delta d_+(x)]$

$$d_+(x) = d(x) + \bar{d}(x) \quad u_+(x) = u(x) + \bar{u}(x)$$

$$\Delta q(x) = q_{\uparrow}(x) - q_{\downarrow}(x)$$

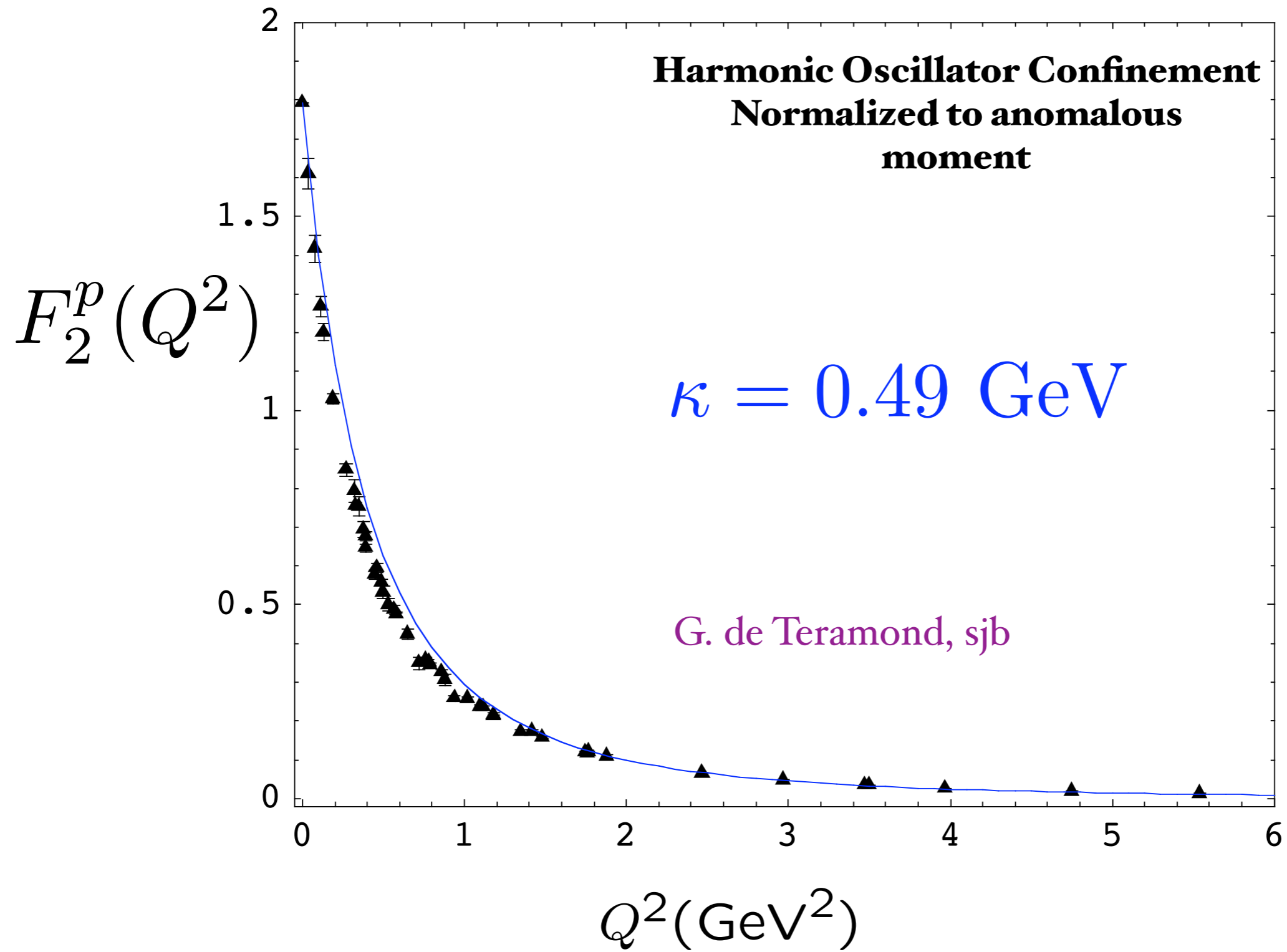


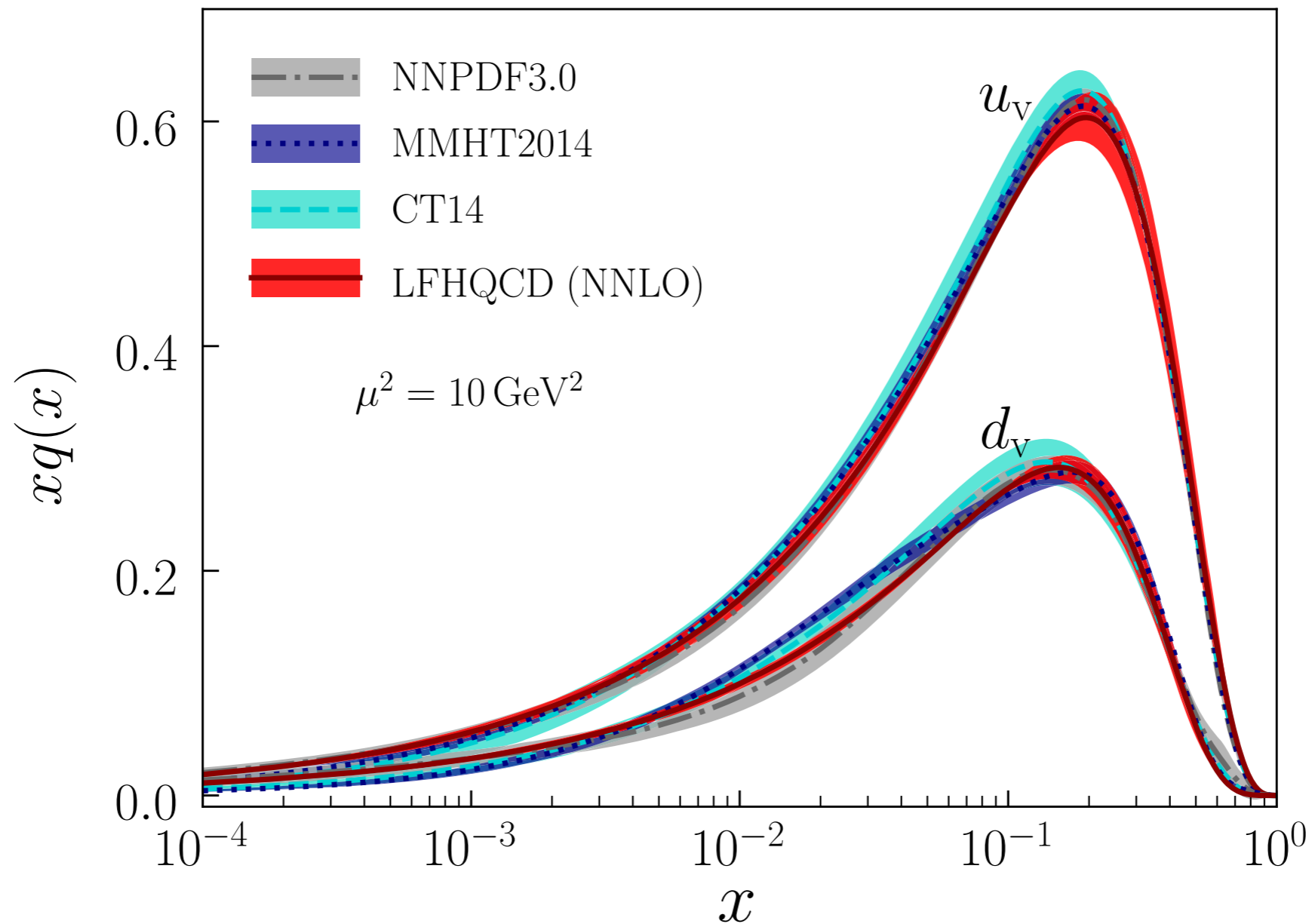
Using  $SU(6)$  flavor symmetry and normalization to static quantities



# Spacelike Pauli Form Factor

From overlap of  $L = 1$  and  $L = 0$  LFWFs





Comparison for  $xq(x)$  in the proton from LFHQCD (red bands) and global fits: MMHT2014 (blue bands) [5], CT14 [6] (cyan bands), and NNPDF3.0 (gray bands) [77]. LFHQCD results are evolved from the initial scale  $\mu_0 = 1.06 \pm 0.15$  GeV.

*Universality of Generalized Parton Distributions in Light-Front Holographic QCD*

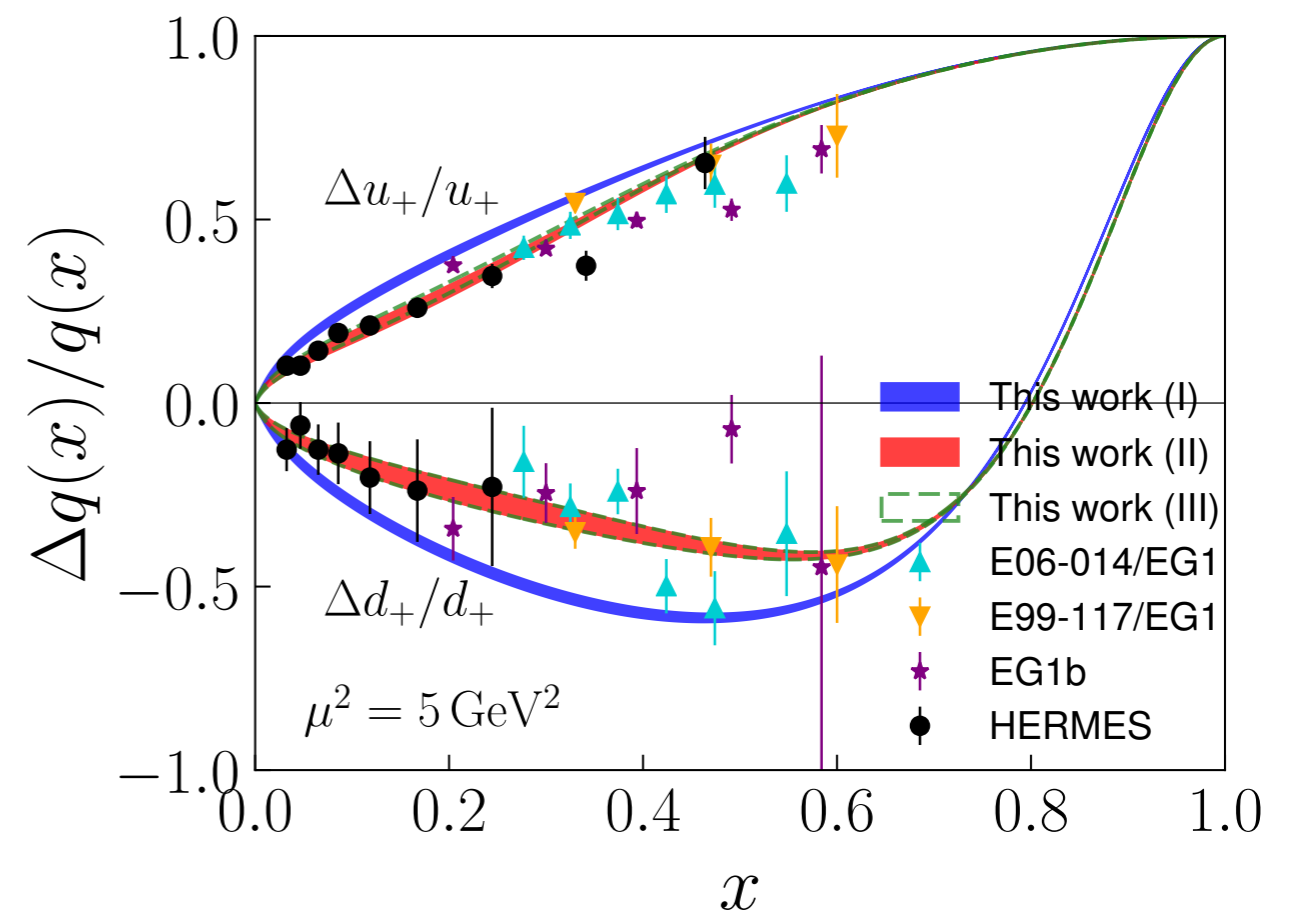
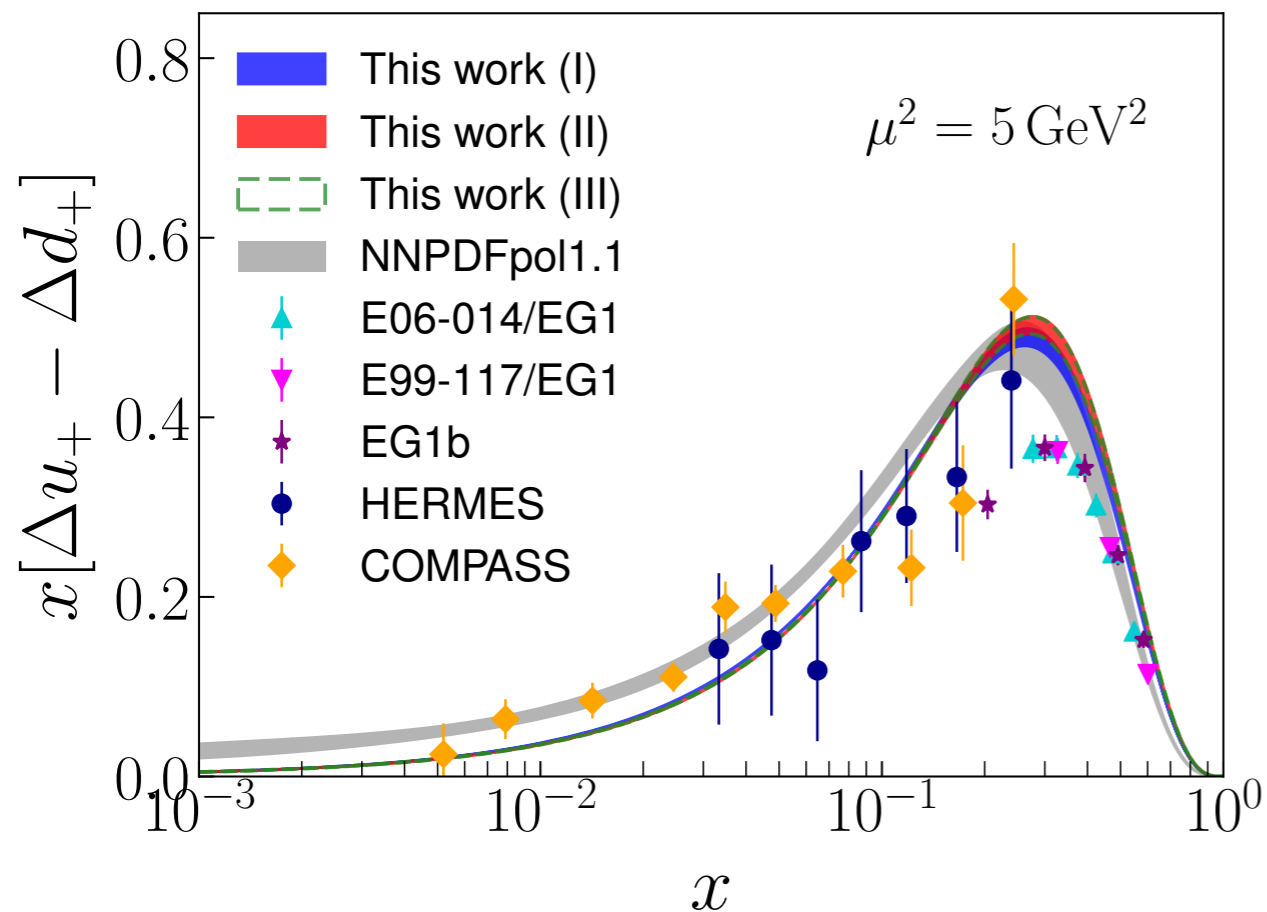
*Guy F. de Téramond, Tianbo Liu, Raza Sabbir Sufian, Hans Günter Dosch, Stanley J. Brodsky, and Alexandre Deur*

*PHYSICAL REVIEW LETTERS 120, 182001 (2018)*



## Polarized GPDs and PDFs (HLFHS Collaboration, 2019)

- Separation of chiralities in the AdS action allows computation of the matrix elements of the axial current including the correct normalization, once the coefficients  $c_T$  are fixed for the vector current
- Helicity retention between quark and parent hadron (pQCD prediction):  $\lim_{x \rightarrow 1} \frac{\Delta q(x)}{q(x)} = 1$
- No spin correlation with parent hadron:  $\lim_{x \rightarrow 0} \frac{\Delta q(x)}{q(x)} = 0$



Other Consequences of  $[ud]_{\bar{3}_C, I=0, J=0}$  diquark cluster

## QCD Hidden-Color Hexadiquark in the Core of Nuclei

J. Rittenhouse West, G. de Teramond, A. S. Goldhaber, I. Schmidt, sjb

$$|\Psi_{HDQ}\rangle = |[ud][ud][ud][ud][ud][ud]\rangle$$

mixes with

$${}^4He|npnp\rangle$$

Increases alpha binding energy, EMC effects

## Diquarks Can Dominate Five-Quark Fock State of Proton

$$|p\rangle = \alpha|[ud]u\rangle + \beta|[ud][ud]\bar{d}\rangle \quad \text{J. Rittenhouse West, sjb}$$

Natural explanation why  $\bar{d}(x) \gg \bar{u}(x)$  in proton

Excitations and Decay of HdQ in Alpha-Nuclei  
may explain ATOMKI X17 signal

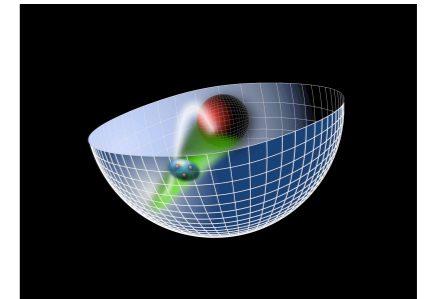
# Underlying Principles

- **Polncarè Invariance: Independent of the observer's Lorentz frame: Quantization at Fixed Light-Front Time  $\tau$**

- **Causality: Information within causal horizon: Light-Front**

- **Light-Front Holography:  $AdS_5 = LF (3+1)$**

$$z \leftrightarrow \zeta \text{ where } \zeta^2 = b_{\perp}^2 x(1-x)$$



- **Introduce mass scale  $\kappa$  while retaining conformal invariance of the Action (dAFF)**

*“Emergent Mass”*

- **Unique Dilaton in  $AdS_5$ :  $e^{+\kappa^2 z^2}$**

- **Unique color-confining LF Potential  $U(\zeta^2) = \kappa^4 \zeta^2$**

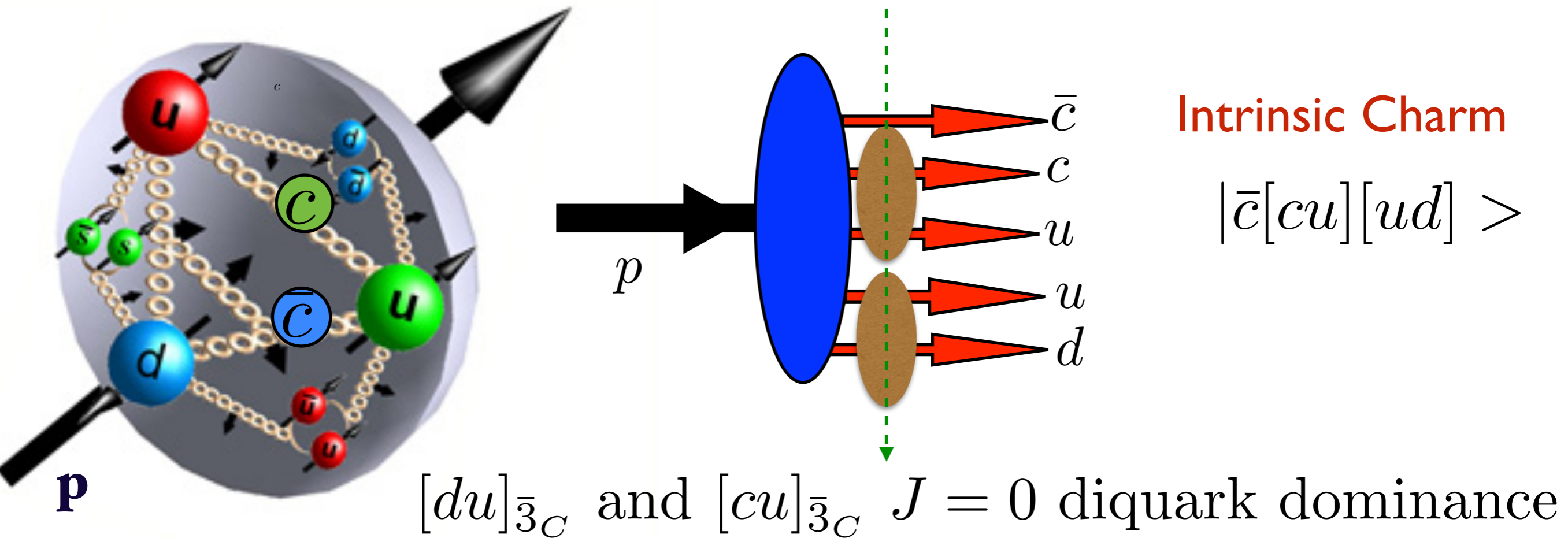
- **Superconformal Algebra: Mass Degenerate 4-Plet:**

Meson  $q\bar{q} \leftrightarrow$  Baryon  $q[qq] \leftrightarrow$  Tetraquark  $[qq][\bar{q}\bar{q}]$

# Color confinement potential from AdS/QCD

$$U(\zeta^2) = \kappa^4 \zeta^2, \zeta^2 = b_{\perp}^2 x(1-x)$$

Fixed  $\tau = t + z/c$



$$\psi_n(\vec{k}_{\perp i}, x_i) \propto \frac{1}{\kappa^{n-1}} e^{-\mathcal{M}_n^2/2\kappa^2} \prod_{j=1}^n \frac{1}{\sqrt{x_j}}$$

$$\mathcal{M}_n^2 = \sum_{i=1}^n \left( \frac{k_{\perp}^2 + m^2}{x} \right)_i$$

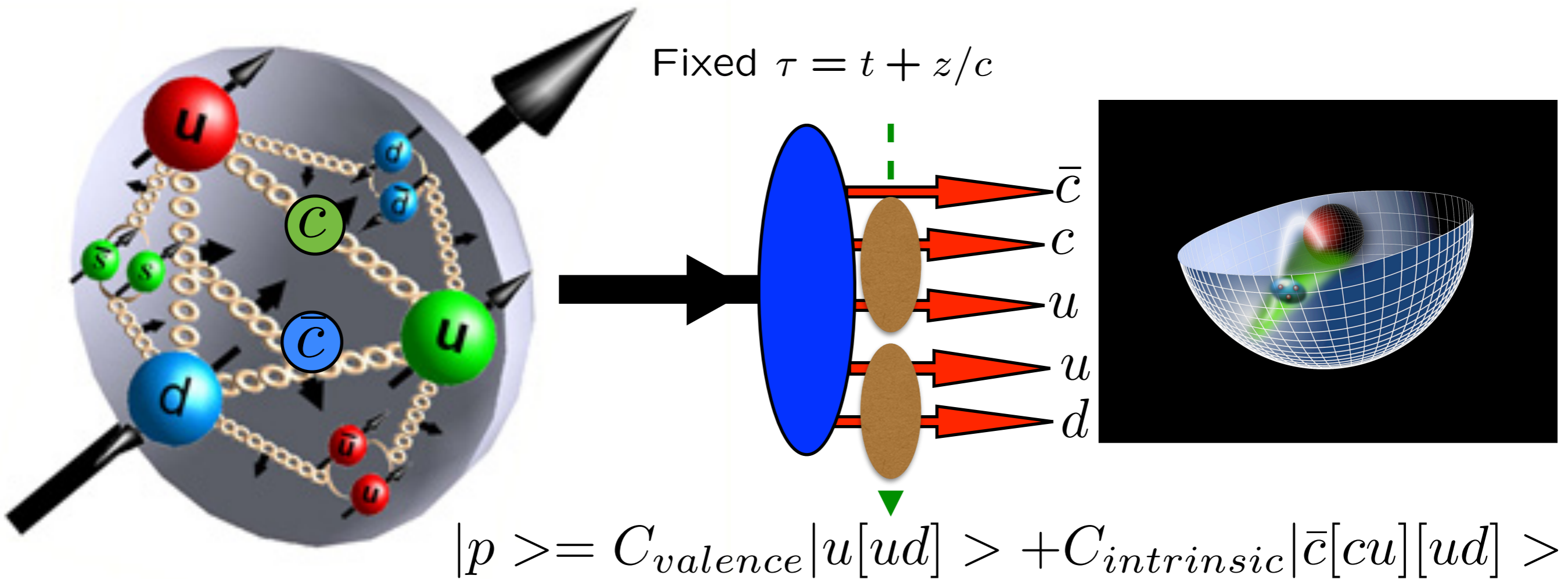
# *Light-Front Holography: First Approximation to QCD*

- **Color Confinement, Analytic form of confinement potential**
- **Retains underlying conformal properties of QCD despite mass scale (DeAlfaro-Fubini-Furlan Principle)**
- **Massless quark-antiquark pion bound state in chiral limit, GMOR**
- **QCD coupling at all scales**
- **Connection of perturbative and nonperturbative mass scales**
- **Poincarè Invariant**
- **Hadron Spectroscopy-Regge Trajectories with universal slopes in  $n$ ,  $L$**
- **Supersymmetric 4-Plet: Meson-Baryon-Tetraquark Symmetry**
- **Light-Front Wavefunctions**
- **Form Factors, Structure Functions, Hadronic Observables**
- **OPE: Constituent Counting Rules**
- **Hadronization at the Amplitude Level: Many Phenomenological Tests**
- **Systematically improvable: Basis LF Quantization (BLFQ)**

*Supersymmetric Features of Hadron Physics  
from Superconformal Algebra  
and Light-Front Holography*



# Intrinsic Heavy Quark Phenomena A Novel Property of QCD



with P. Hoyer, N. Sakai, C. Peterson, A. Mueller, J. Collins, S. Ellis, J. Gunion, G. Lykasov

Implications of LHCb measurements and future prospects



Stan Brodsky

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INT October 10, 2022