Energy spectra and correlations in compressible quantum fluids: Turbulent BEC, and Quantum Fluids of Light

Ashton Bradley

Compressible Turbulence: From Cold Atoms to Neutron Star Mergers, 26 June 2025

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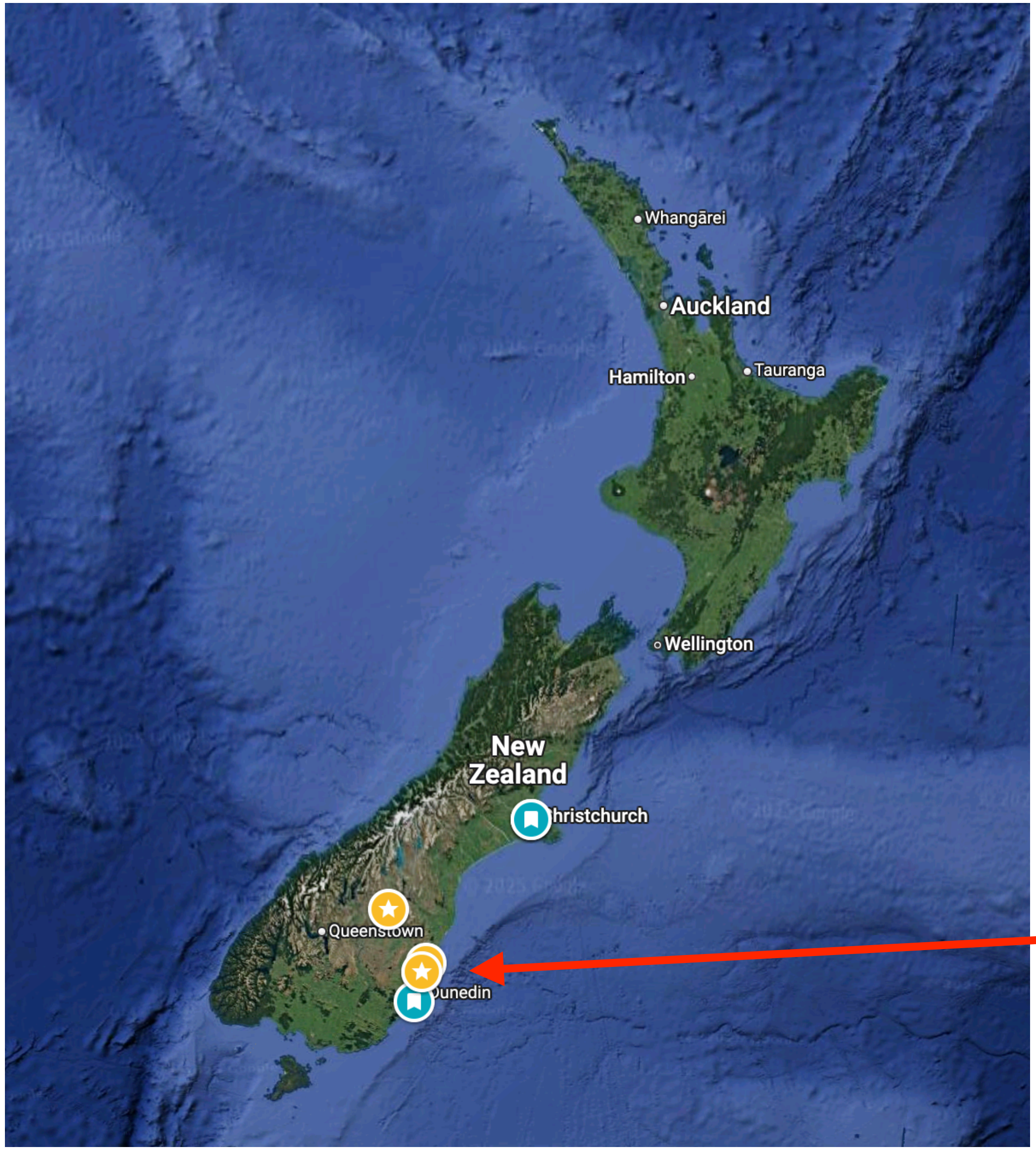
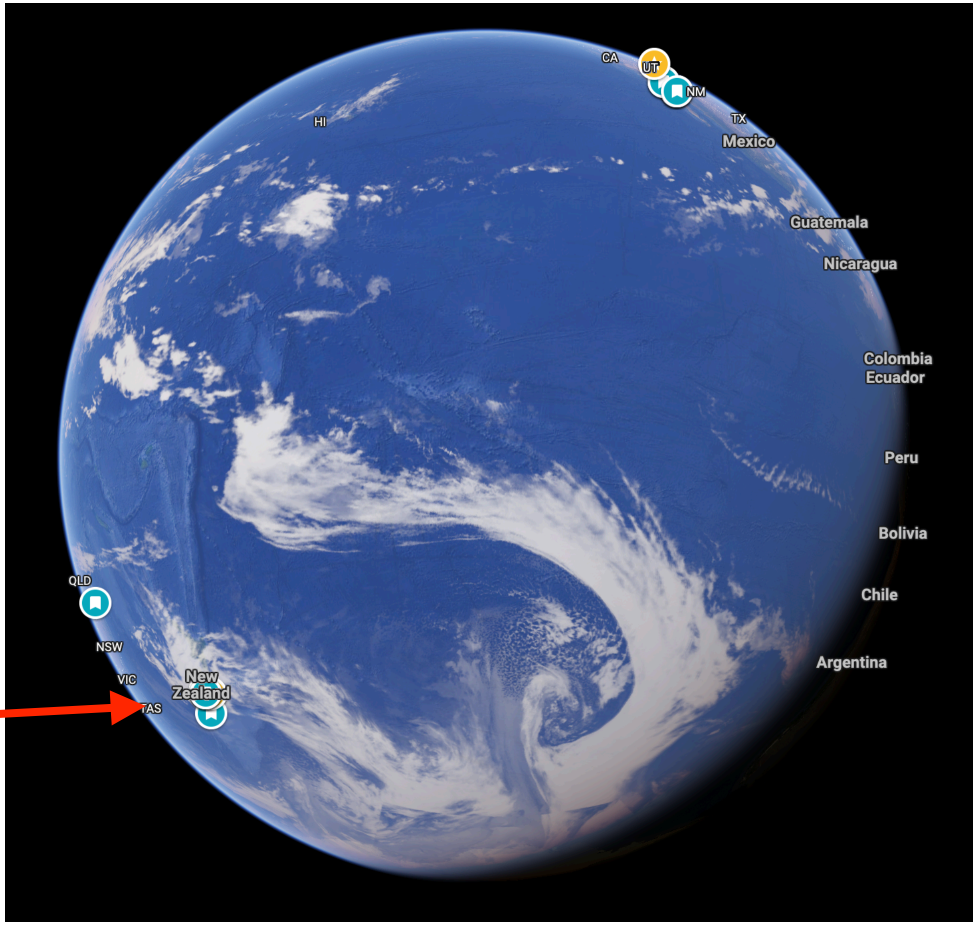
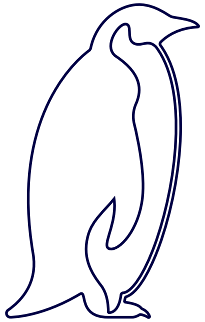
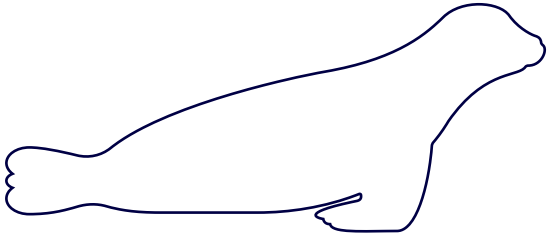




Animals?





People

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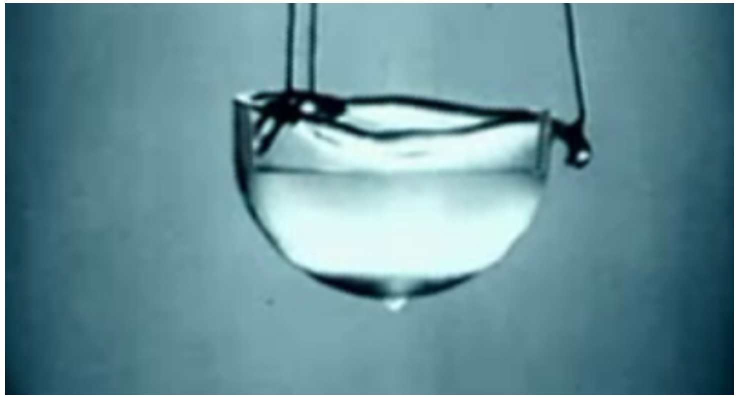
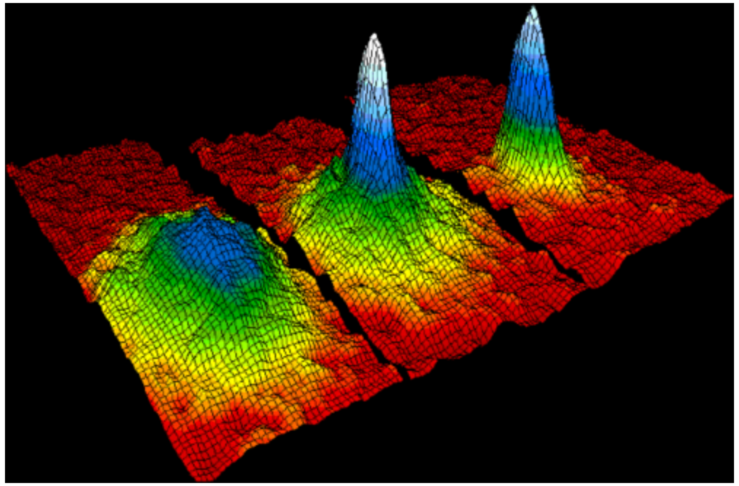
Motivation

Classical Fluid

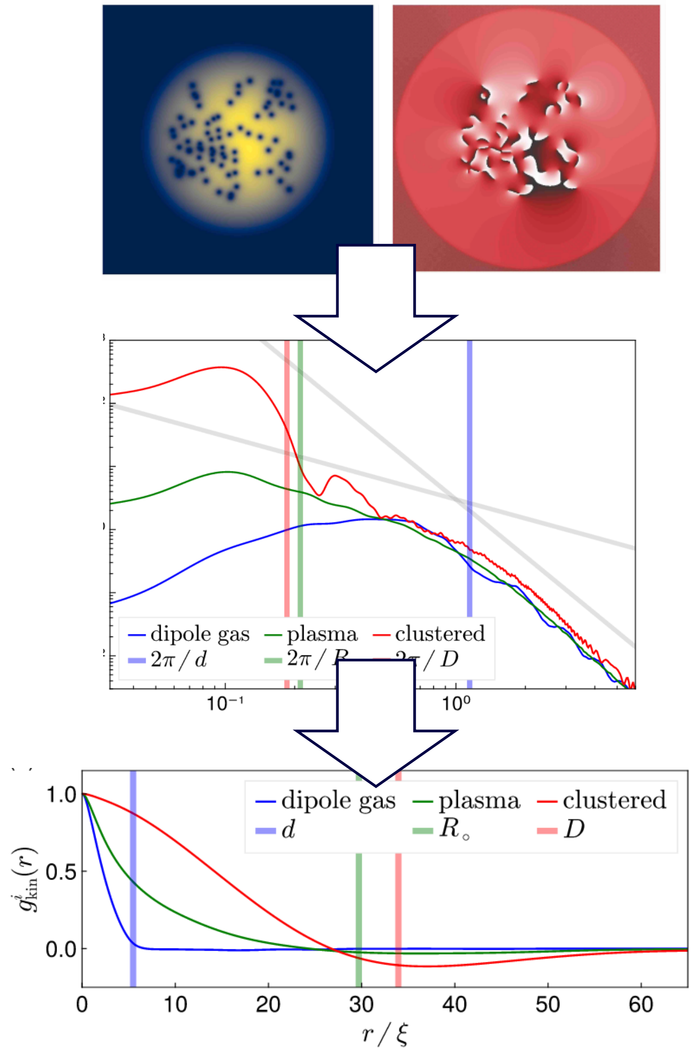
Quantum Fluid



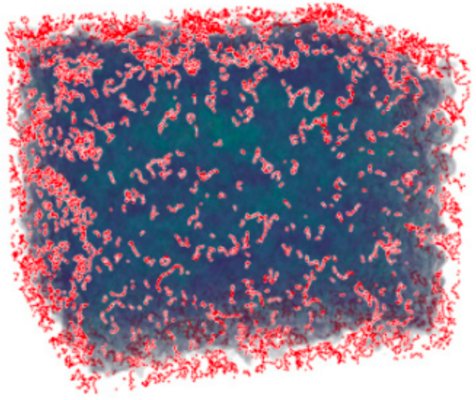


Outline

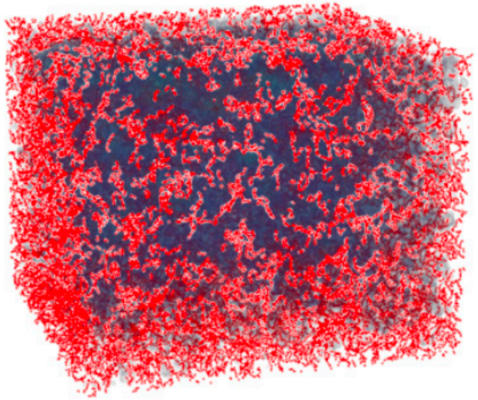
Spectral Analysis



I: Turbulent BEC

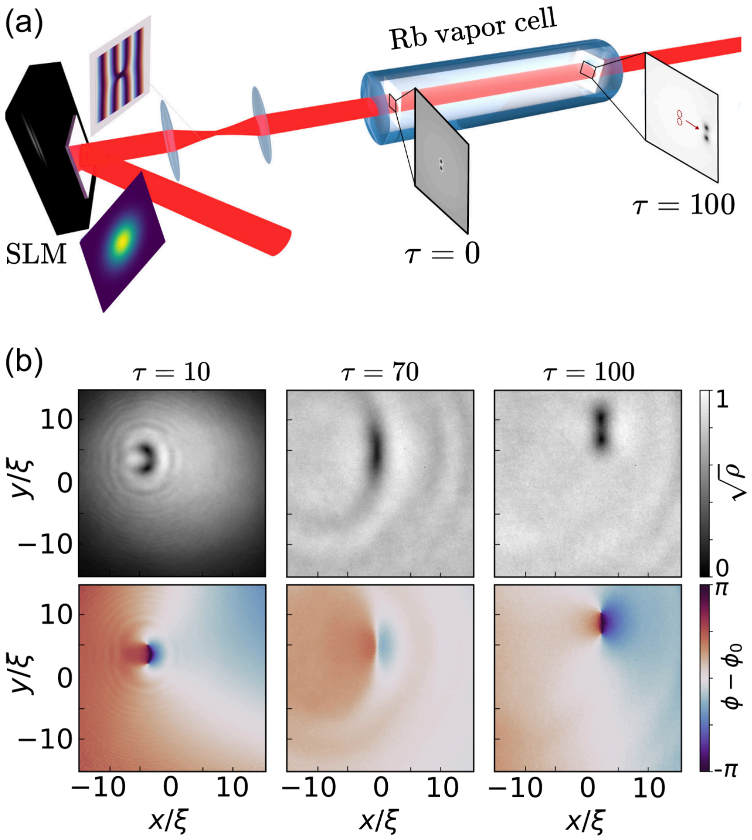


$U_F = 0.6\mu$



$U_F = 1.5\mu$

II: Quantum Fluids of Light



Papers

Turbulent BEC

Spectral Analysis

A. S. Bradley and B. P. Anderson, PRX (2012).

M. T. Reeves, T. P. Billam, B. P. Anderson, and A. S. Bradley, PRA (2014).

A. S. Bradley, R. K. Kumar, S. Pal, and X. Yu, PRA (2022).

Box Turbulence

T. Z. Fischer and A. S. Bradley, PRA (2025).

Jones-Roberts solitons

Dissipation of JRS

N. A. Krause and A. S. Bradley, PRA (2024).

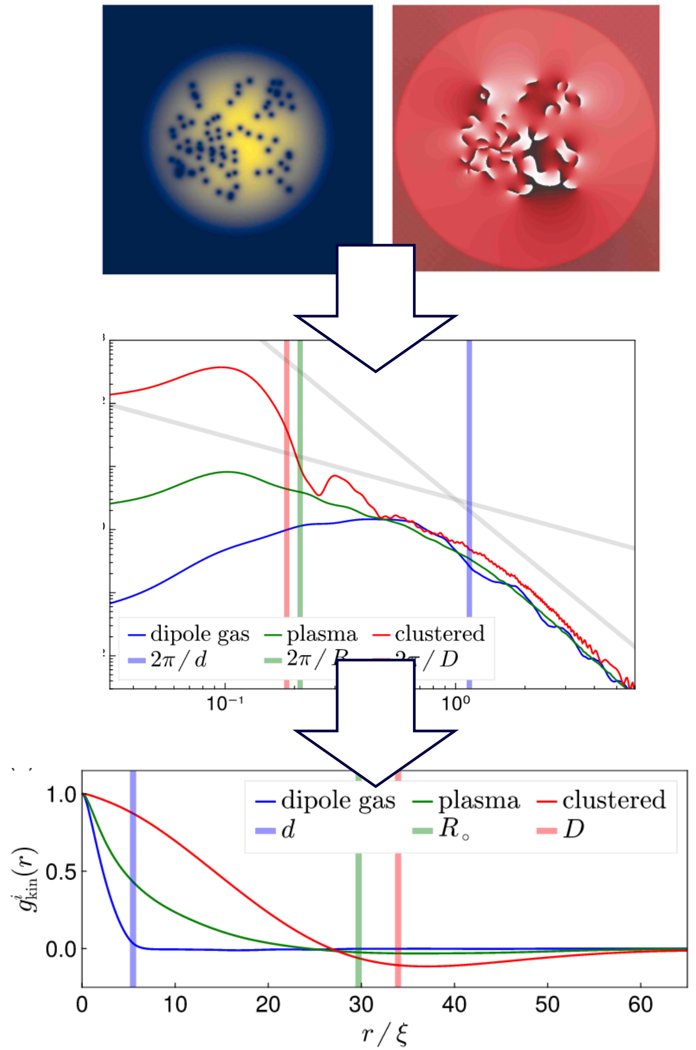
Observation of JRS

M. Baker-Rasooli, T. Aladjidi, N. A. Krause,
A. S. Bradley, and Q. Glorieux, PRL (2025).

Velocity correlations of vortices and JRS

A. S. Bradley and N. A. Krause, arXiv:2502.08930.

Spectral Analysis



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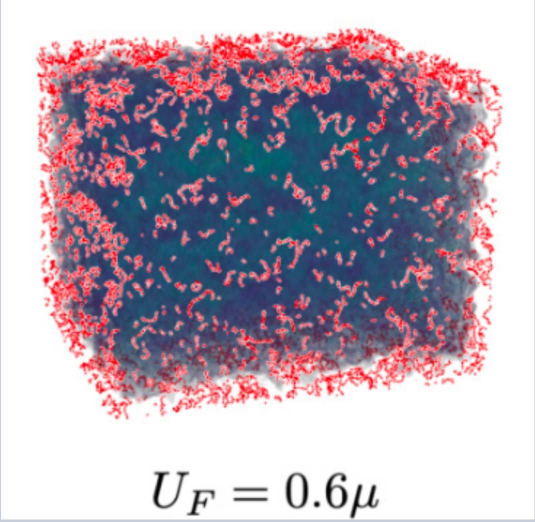

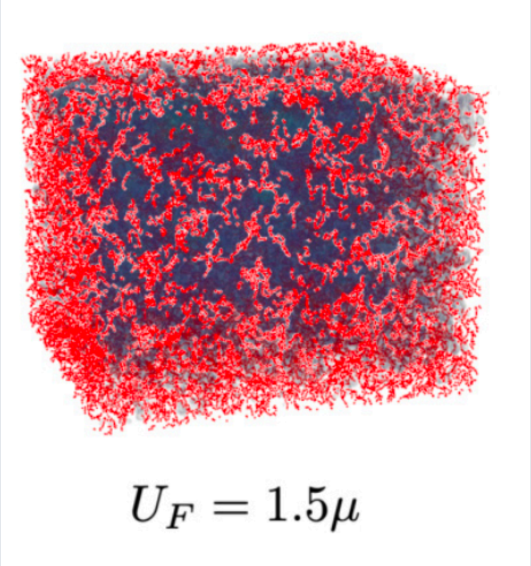

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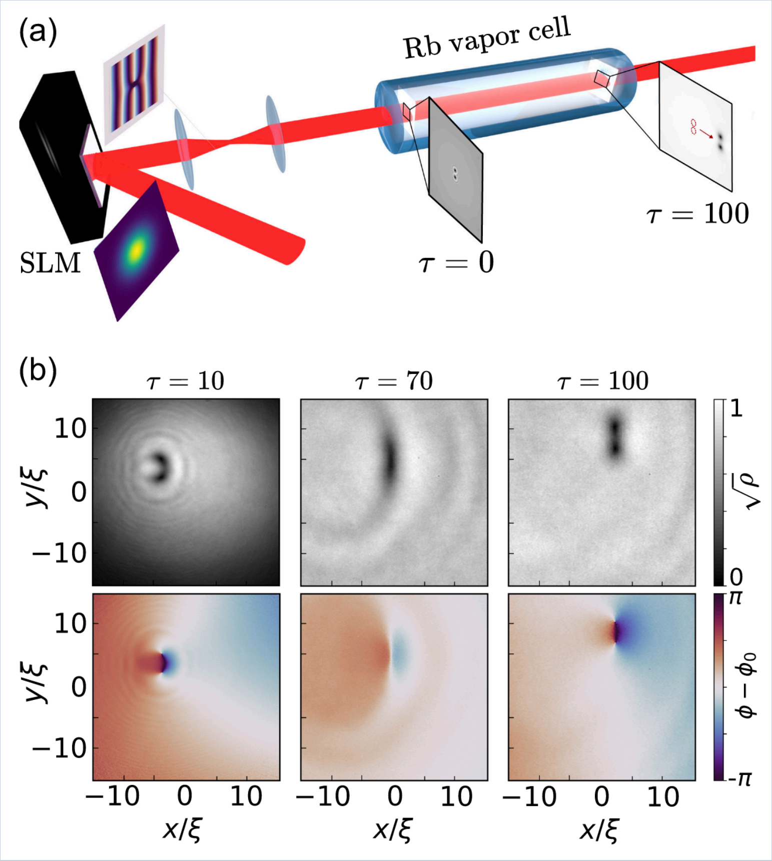
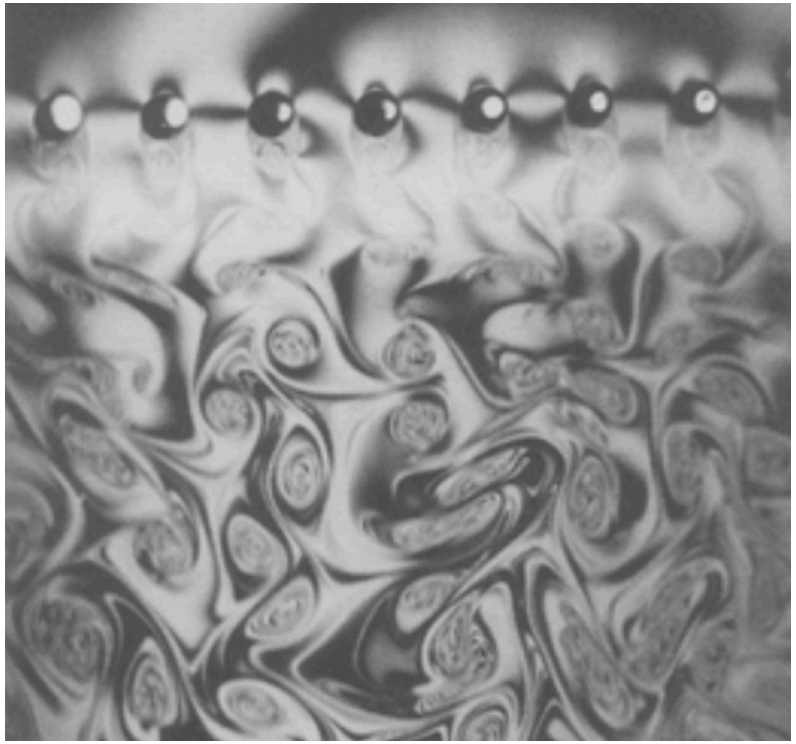


Planar Classical Fluids

Statistical Mechanics

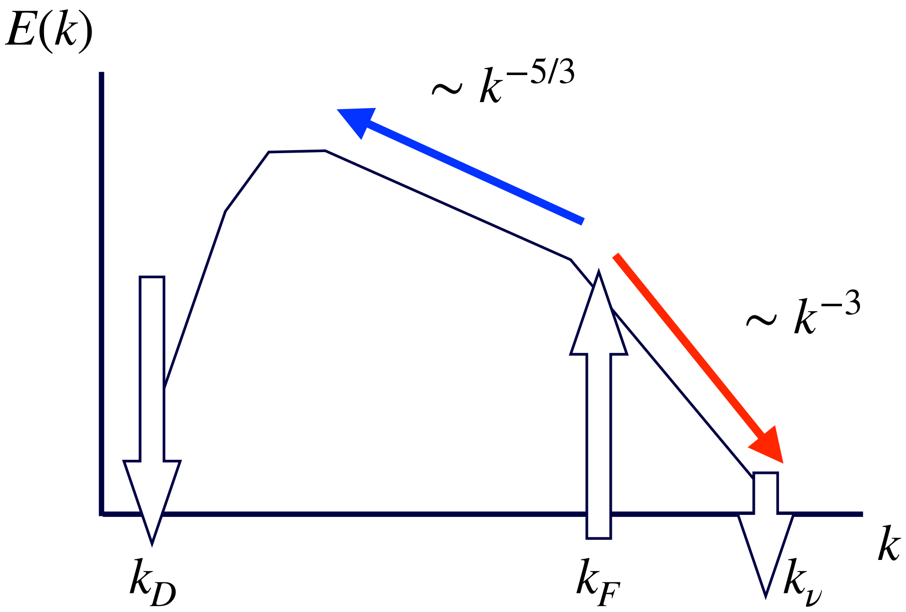
Dynamics

Decaying turbulence



Goldburg *et al*, Physica A (1997)

Dual cascade in 2D Navier-Stokes



Kraichnan, Phys Fluids (1967)

Planar Quantum Fluids

Statistical Mechanics

Dynamics

Negative Temperature

of the individual vortices has been subtracted. In an unbounded fluid, H has the form [5, 6]:

$$(2) \quad \begin{cases} H = -\dfrac{1}{2\pi} \sum_{i>j} k_i k_j \log r_{ij} \,; \\ r_{ij}^2 = (x_j - x_i)^2 + (y_i - y_j)^2 \,. \end{cases}$$

The equations of motion (1) still apply when the liquid is restrained by boundaries, in which case the Hamiltonian (2) is modified so as to allow for image forces, and may be constructed in terms of the GREEN's functions of LAPLACE's equation [6].

Now let us consider the liquid enclosed by a boundary, so that the vortices are confined to an area A. We note that our dynamical system has some unusual properties. In effect, the x and y coordinates of each vortex are canonical conjugates, so that the phase-space is identical with the configuration-space of the vortices:

$$(3) \quad d\Omega = dx_1 dy_1 \dots dx_n dy_n \,.$$

Moreover, this phase-space is finite

$$(4) \quad \int d\Omega = \left(\int dx\, dy\right)^n = A^n \,.$$

Onsager, Il Nuovo Cimento (1949)

-5/3 power law in point vortex spectrum

Using (4.2) we get for the energy spectrum from (3.16)

$$E(k) = \varkappa^2 \sigma / 4\pi k. \qquad (4.3)$$

When energy is transferred from small to large scale lengths a similarity regime may be established in which, in particular,

$$g_2(\tau, r) = g(\tau) \ (r\sigma^{1/2})^{-\alpha}, \ \tau = \varkappa \sigma t.$$

The condition that the integral in (3.16) converges gives

$$1/2 < \alpha < 2. \qquad (4.4)$$

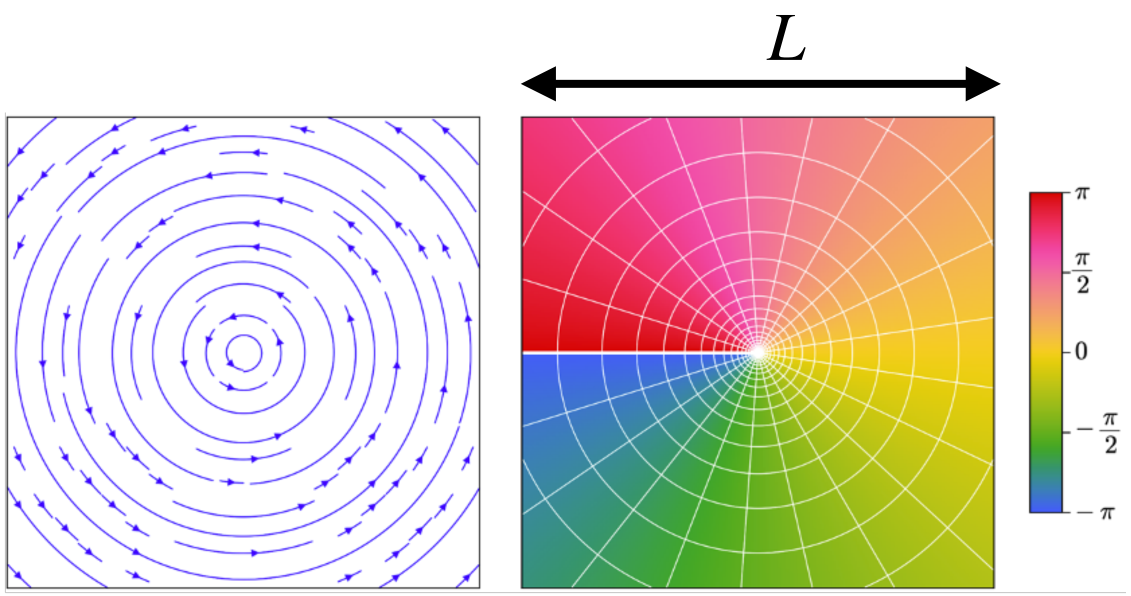
Then

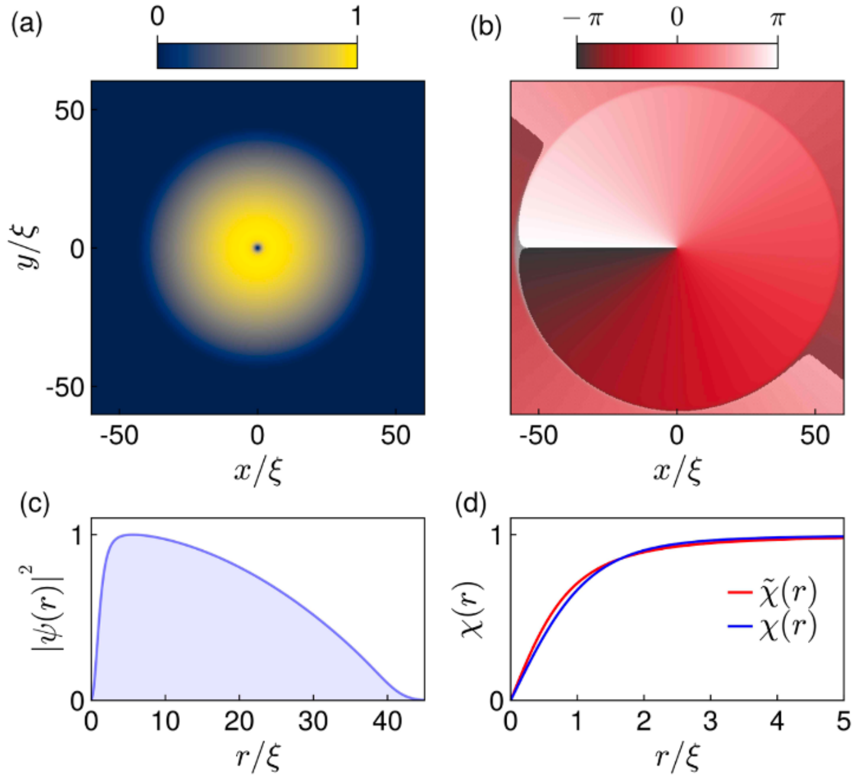
$$E(\tau, k) = \frac{\varkappa^2 \sigma}{4\pi} k^{-1} + \varkappa^2 \sigma^{2-\alpha/2} \Gamma\left(1 - \frac{\alpha}{2}\right) \left[\Gamma\left(\frac{\alpha}{2}\right)\right]^{-1} g(\tau) k^{-3+\alpha} \qquad (4.5)$$

where $\Gamma(x)$ is the gamma function. By virtue of (4.4) the second term in (4.5) is the determining one in the small wavenumber region. One sees easily that the transfer of energy from small to large scale lengths corresponds to a decrease of $g(\tau)$. As $g(\tau) \to 0$ the spectrum (4.5) goes over into (4.3). The value $\alpha = 4/3$, corresponding to the "$5/3$-law" for the energy spectrum, lies in the range (4.4).

Novikov, Zh. Eksp. Teor. Fiz (1975)

Quantum Vortices





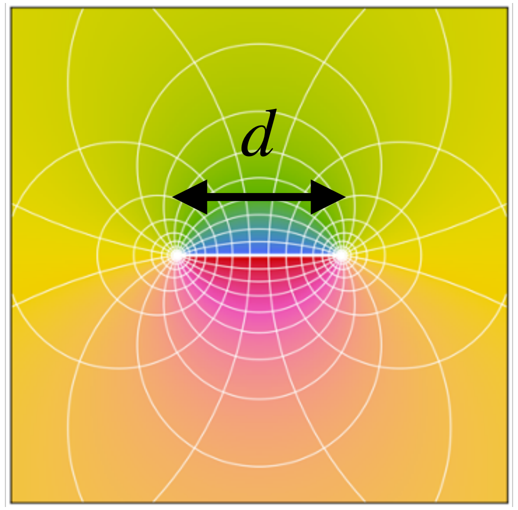
- Wavefunction $\psi = \sqrt{n}e^{i\Theta} \sim e^{i\theta}$
- Global flow $\mathbf{v} = \frac{\hbar}{m}\nabla\Theta = \frac{\hbar}{mr}\hat{\theta}$
- Energy $E \sim \log(L)$

- Core scale is healing length

$$\mu = gn = \frac{\hbar^2}{m\xi^2}$$

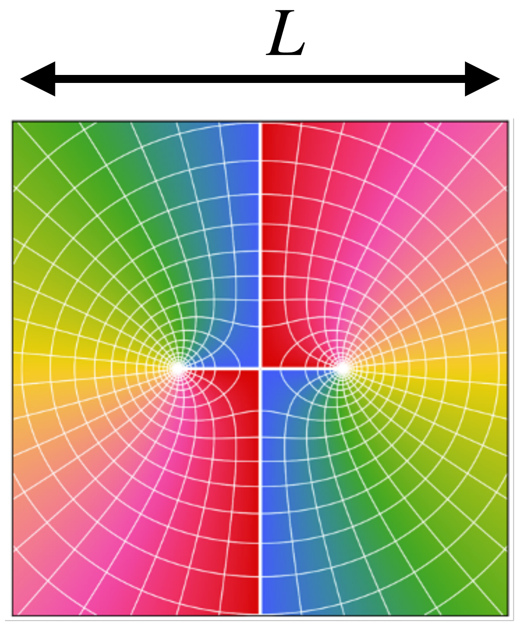
- Trapped BEC, size L, typically $\xi \ll L$

Vortex structures



Dipole

- Local flow $|\mathbf{v}| \sim 1/r^2$
- Energy $E \sim \log(d)$

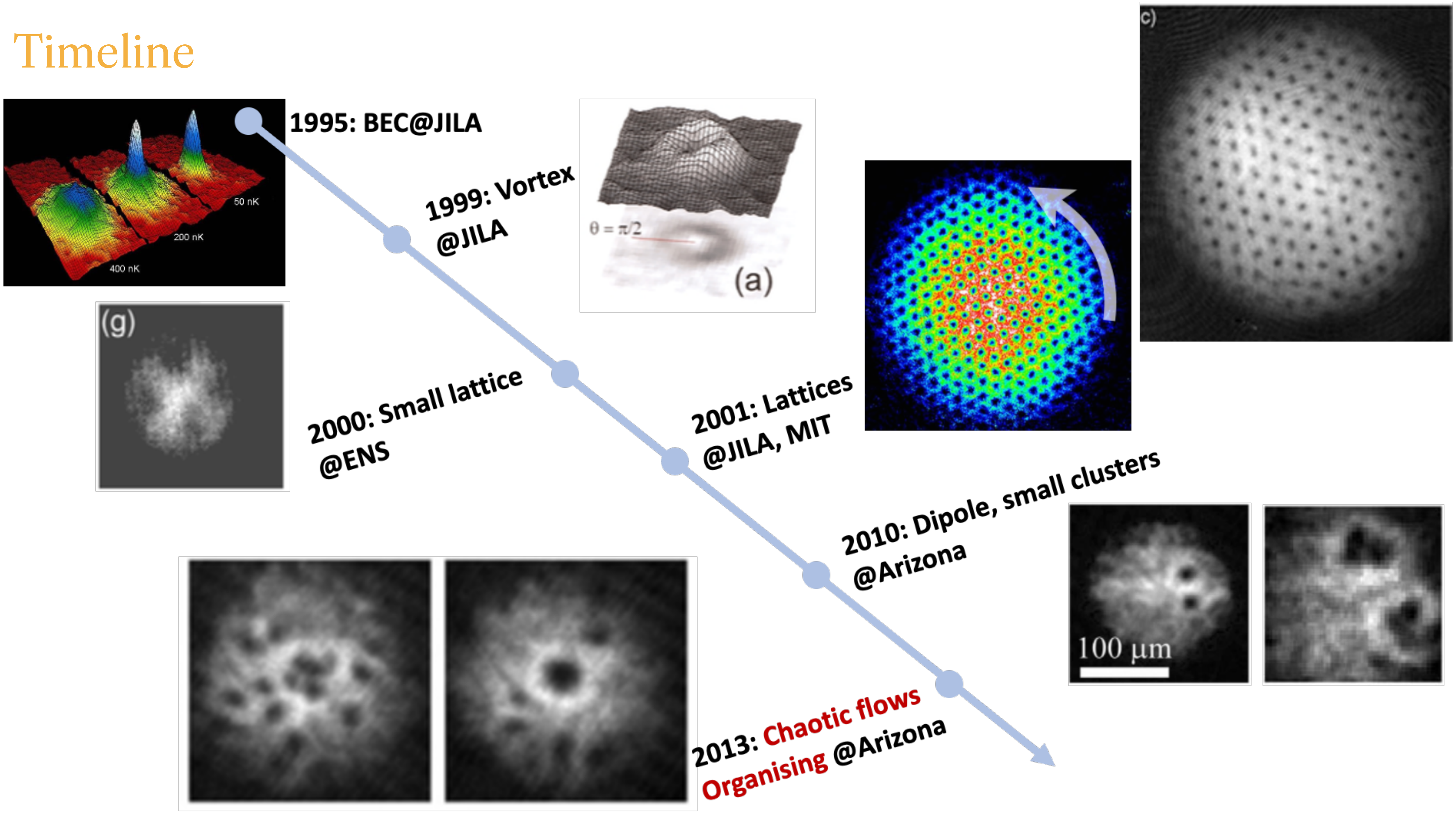


Charge 2 cluster

- Global flow $|\mathbf{v}| \sim 1/r$
- Energy $E \sim \log(L)$

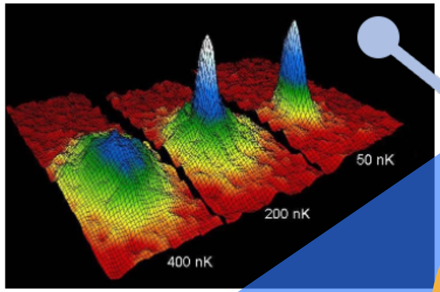
Vortices in BEC

Timeline



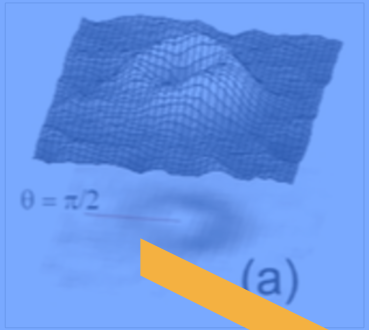
Vortices in BEC

Timeline



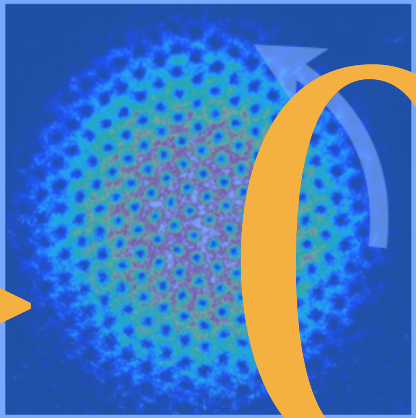
1995: BEC@JILA

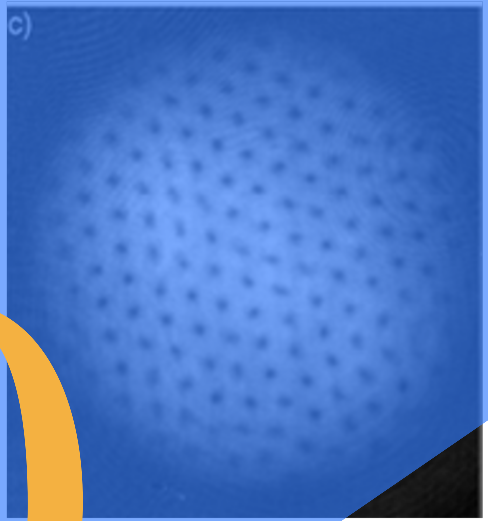
…999: Vortex @…A



…Sma… …ttice @EN…

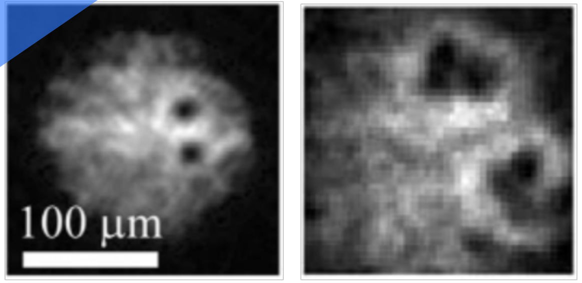
$\Omega > 0$

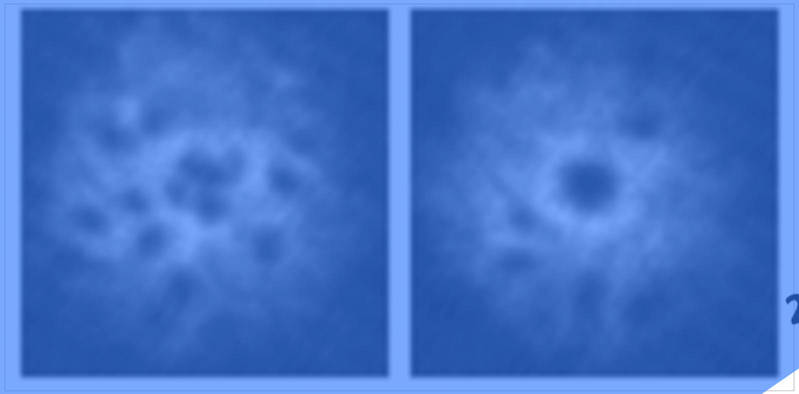




200… …ices @JILA, MIT

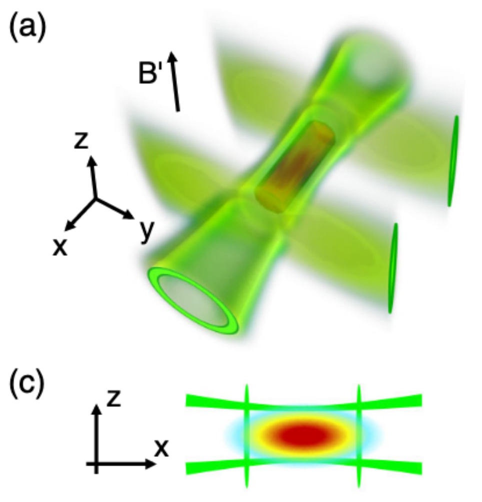
2010: Dipole, small clusters @Arizona



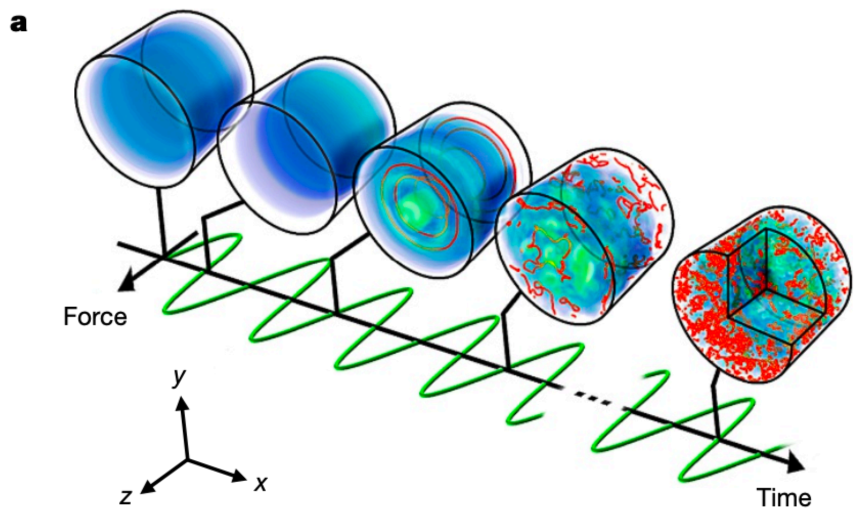


2013: Chaotic flows Organising @Arizona

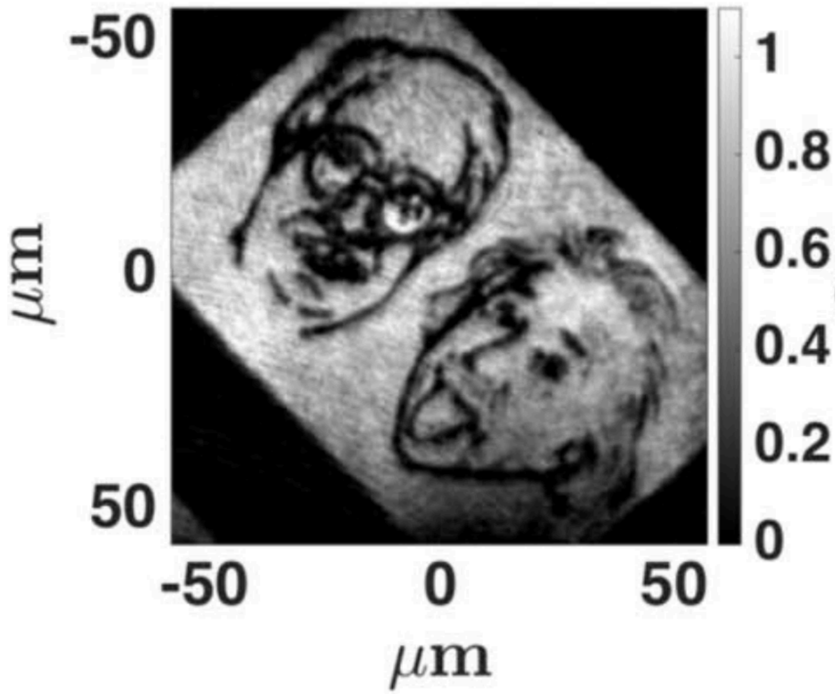
Box traps



Gaunt *et al*, PRL (2013)



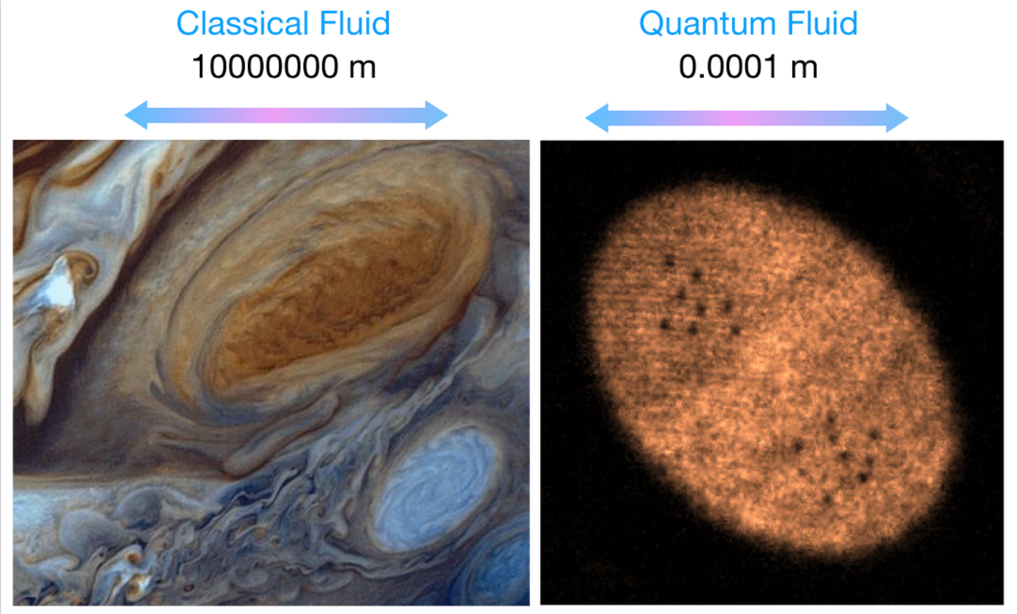
Navon *et al*, Nature (2016)

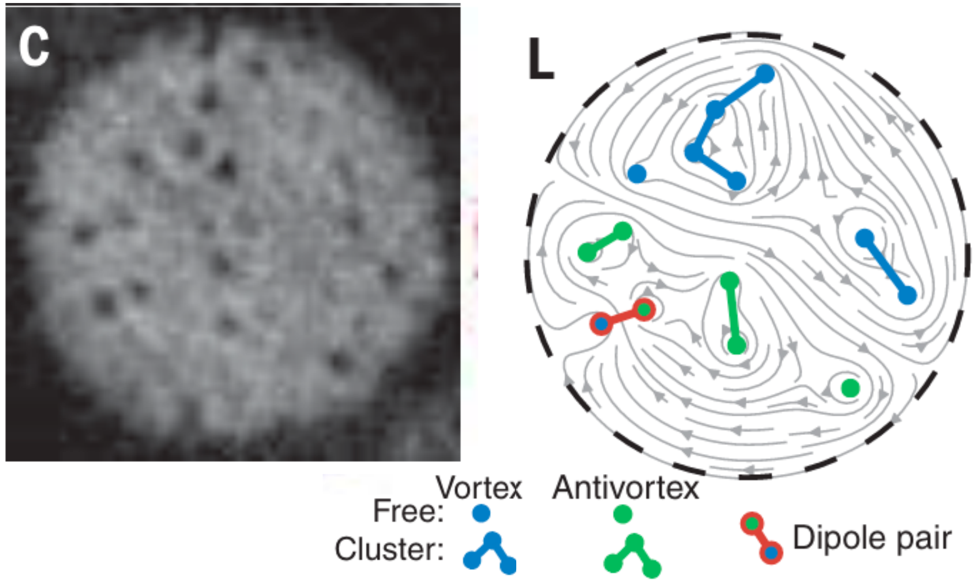


Gauthier *et al*, Optica (2016)

Onsager Vortex Clusters

Statistical Mechanics of **Point Vortices**





G. Gauthier, M. T. Reeves, X. Yu, A. S. Bradley, M. A. Baker, T. A. Bell, H. Rubinsztein-Dunlop, M. J. Davis, and T. W. Neely, Giant Vortex Clusters in a Two-Dimensional Quantum Fluid, Science 364, 1264 (2019).

S. P. Johnstone, A. J. Groszek, P. T. Starkey, C. J. Billington, T. P. Simula, and K. Helmerson, Evolution of Large-Scale Flow from Turbulence in a Two-Dimensional Superfluid, Science 364, 1267 (2019).

Compressible Planar Quantum Fluids

Quantum vortices

Healing length: ξ sets finite core size

Dynamics: point-vortex evolution for low Mach number flows

Dipoles carry linear momentum

Clusters carry angular momentum

Compressible excitations

Bogoliubov phonons including long wavelength sound modes.

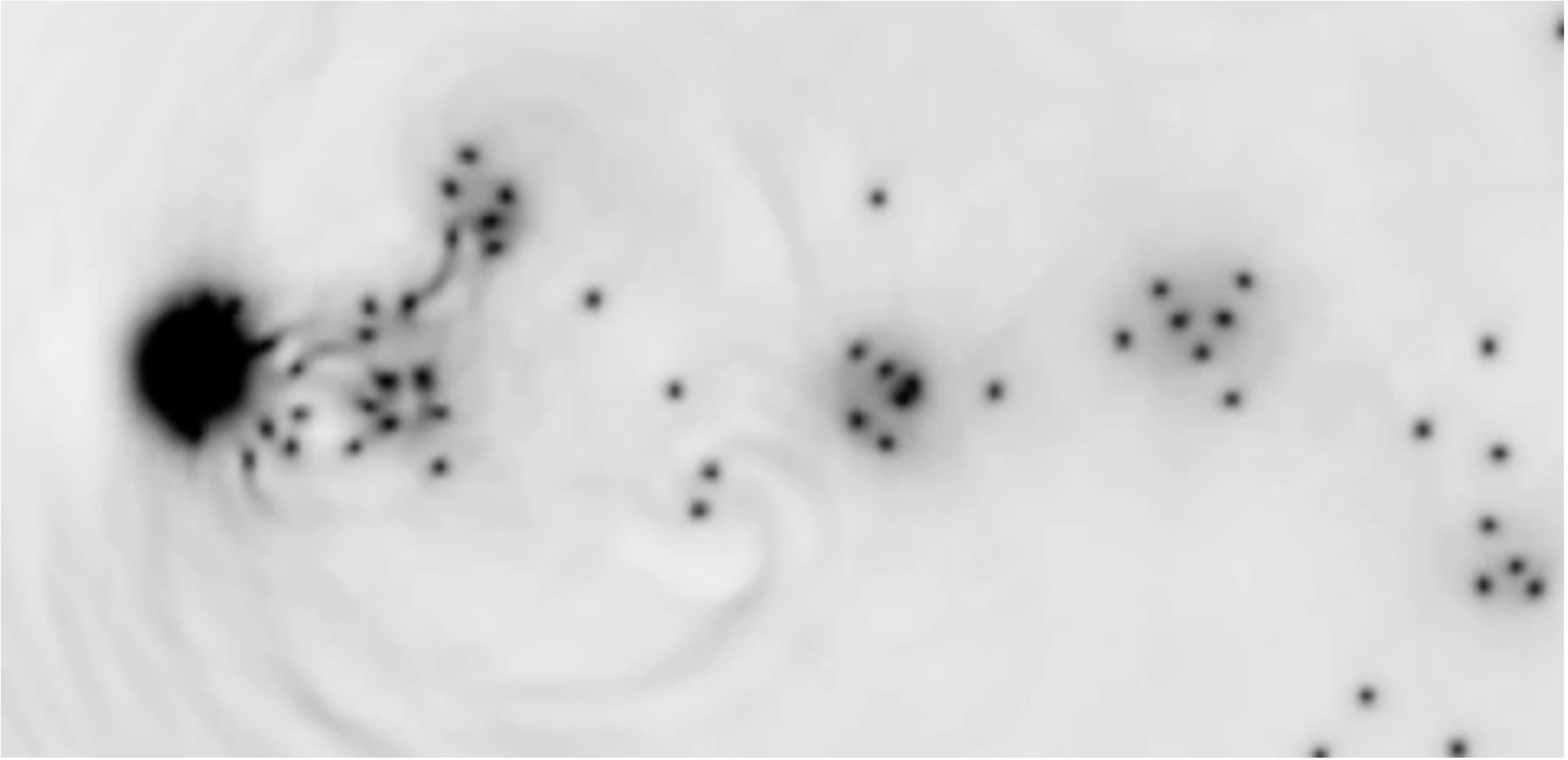
Jones-Roberts solitons purely compressible, carry linear momentum

Quantum pressure $\frac{\nabla\sqrt{n}}{\sqrt{n}}$ in vortices

2D Quantum Turbulence

Inverse Energy Cascade

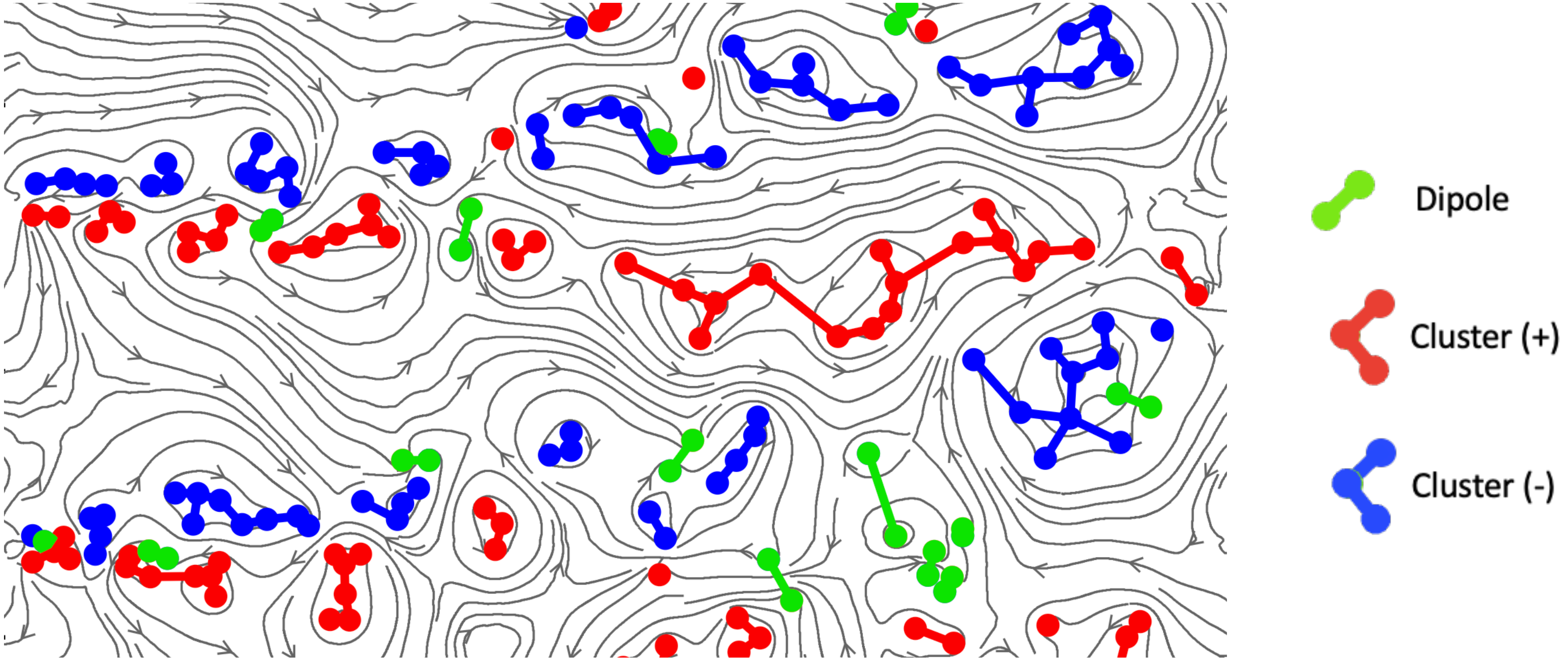
M. T. Reeves *et al*, PRL (2013).



2D Quantum Turbulence

Vortex Structures

M. T. Reeves *et al*, PRL (2013).



JRS-Vortex collision

M. T. Reeves *et al*, PRL (2013).

Kinetic Energy Spectra

C. Nore *et al*, PRL (1997)

Helmholtz decomposition

$$E_{\text{kin}}(k) = \frac{1}{2}\int k^2 \sin\theta \, d\theta \, d\phi \times \left| \frac{1}{(2\pi)^3}\int d^3r \, e^{ir_j k_j} \sqrt{\rho} \, v_j \right|^2$$

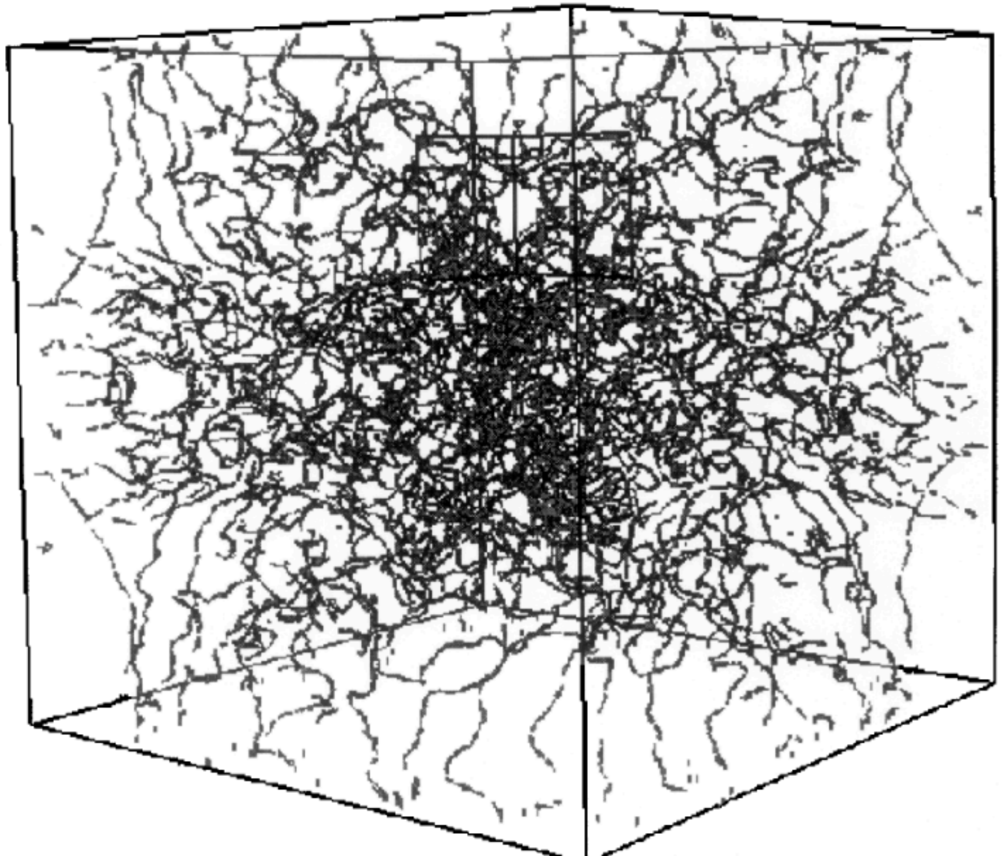
Vortex and Sound Spectra

- Separate hydrodynamic kinetic energy

$$E = \frac{m}{2}\int d\mathbf{r} \, n \, |\mathbf{v}|^2 = \frac{m}{2}\int d\mathbf{r} \, |\mathbf{u}|^2$$

- **Singular cores:** decomposition of $\mathbf{u} \equiv \sqrt{n}\mathbf{v}$
- **Helmholtz Decomposition:** $\mathbf{u} \equiv \mathbf{u}^i + \mathbf{u}^c$
    - Incompressible $\nabla \cdot \mathbf{u}^i = 0$
    - Compressible $\nabla \times \mathbf{u}^c = \mathbf{0}$

Orthogonal in Fourier space.



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  - Incompressible $\nabla \cdot \mathbf{u}^i = 0$
  - Compressible $\nabla \times \mathbf{u}^c = \mathbf{0}$

Orthogonal in Fourier space.

Parseval's theorem: the integrals are equal in either position or Fourier domain

$$E = E^c + E^i = \frac{m}{2}\int d\mathbf{r}\,|\mathbf{u}^i(\mathbf{r})|^2 + \frac{m}{2}\int d\mathbf{r}\,|\mathbf{u}^c(\mathbf{r})|^2$$

$$= \frac{m}{2}\int d\mathbf{k}\,|\tilde{\mathbf{u}}^i(\mathbf{k})|^2 + \frac{m}{2}\int d\mathbf{k}\,|\tilde{\mathbf{u}}^c(\mathbf{k})|^2$$

Not so for the spectral densities $|\tilde{\mathbf{u}}^\alpha(\mathbf{k})|^2$

Note that $\mathbf{u}^i \cdot \mathbf{u}^c \neq 0$, even though $\tilde{\mathbf{u}}^i \cdot \tilde{\mathbf{u}}^c \equiv 0$.

Relation to total spectrum $n(k)$?

What about quantum pressure?

What about quantum phase?

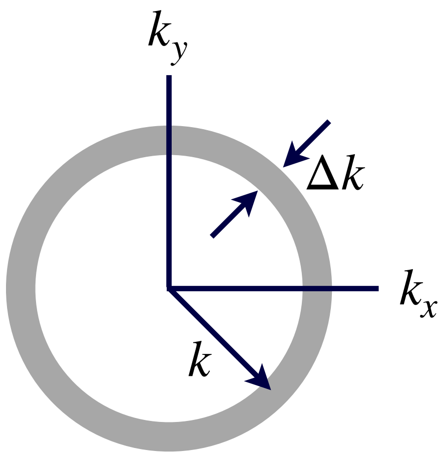
Kinetic Energy Spectra

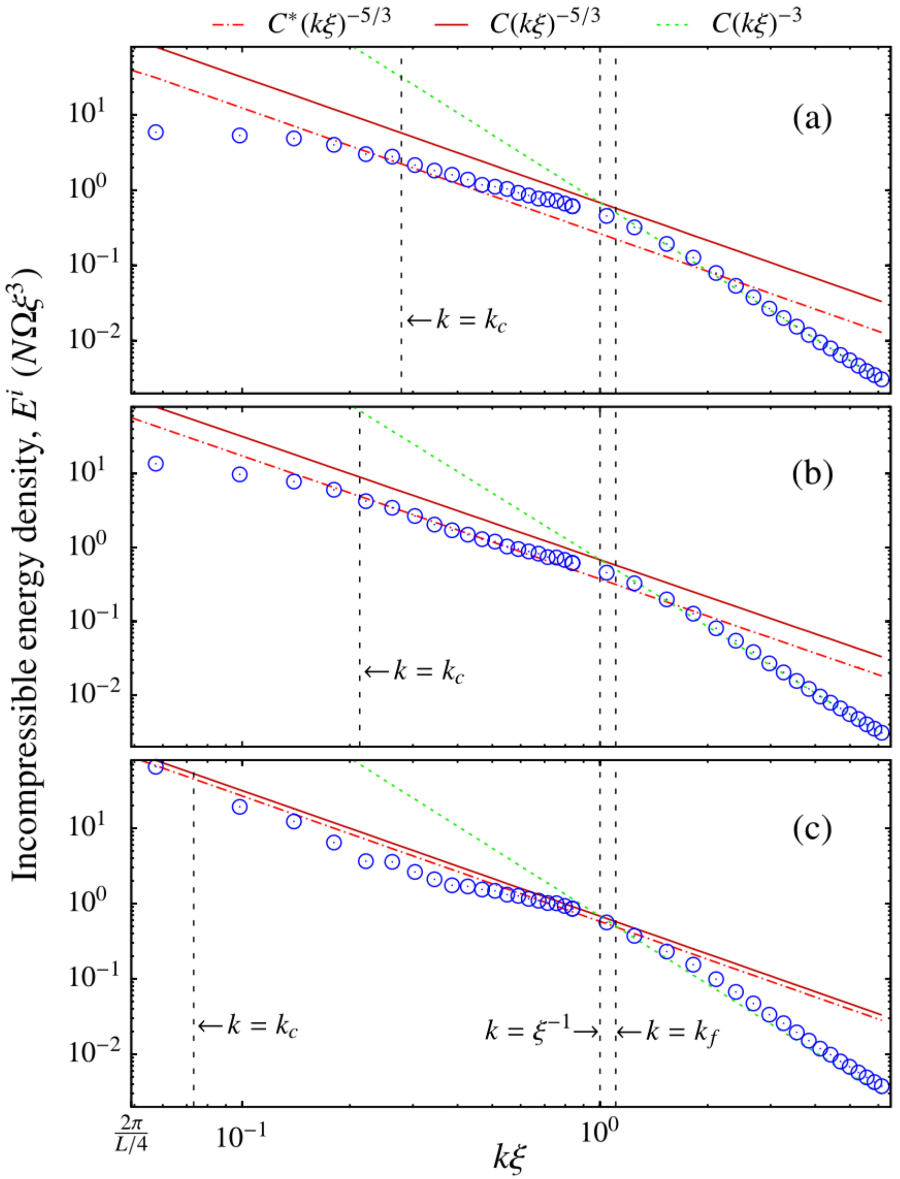
Numerical Evaluation

How to find energy spectrum of vortices?

1. Helmholtz decomposition of $\mathbf{u} = \sqrt{\rho}\,\mathbf{v}$ in GPE

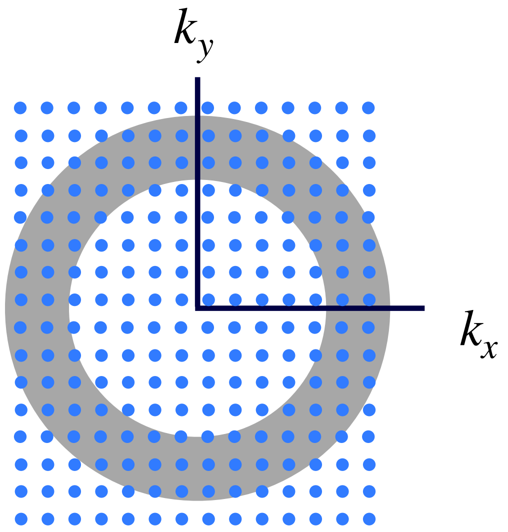
2. Sum over all points in annulus at $|k|$ to find $E^i(k)$, i.e. **"Binning"**.





Spectral analysis of GPE: binning and beyond?

Binning



Fourier method

Integrate the kernel-weighted two-point correlator

- In k space, sum annulus of (k_x, k_y) to get $E(k)$.
- **Little choice** of k grid for given cartesian data
- **Poor resolution** at low k, esp in 2D
- **Can't reuse spectrum** for e.g. correlations

- Analytically integrate $\int d\Omega_k$ to find kernel
- Integrate weighted correlation function (FFT)
- **Total flexibility** of k range and resolution

Energy spectra and correlations

Fourier method

A. S. Bradley *et al*, PRA (2022)

Two **complex vector fields** $\mathbf{u}$, $\mathbf{v}$

Inner product $\langle \mathbf{u} \,|\, \mathbf{v} \rangle \equiv \int_0^\infty dk\, \langle \mathbf{u} \| \mathbf{v} \rangle(k)$

defines **spectral density** $\langle \mathbf{u} \| \mathbf{v} \rangle(k)$.

In 3D

$$\langle \mathbf{u} \| \mathbf{v} \rangle(k) = \frac{k^2}{2\pi^2} \int d^3\mathbf{r} \frac{\sin(k\,|\,\mathbf{r}\,|)}{k\,|\,\mathbf{r}\,|} C[\mathbf{u}, \mathbf{v}](\mathbf{r})$$

Integrate the kernel-weighted two-point correlator <u>for each</u> k

But we can choose any *k*!

Two-point correlator

$$C[\mathbf{u}, \mathbf{v}](\mathbf{r}') = \int d^3\mathbf{R} \langle \mathbf{u} \,|\, \mathbf{R} - \mathbf{r}/2 \rangle \langle \mathbf{R} + \mathbf{r}/2 \,|\, \mathbf{v} \rangle$$

may be easily computed using at 3D FFT.

Angle-averaged correlator is just an sinc-weighted integral transform:

$$G_{uv}(r) \equiv \frac{1}{4\pi} \int d^3\mathbf{r}' \delta(r - |\,\mathbf{r}'\,|) C[\mathbf{u}, \mathbf{v}](\mathbf{r}')$$

$$= \int_0^\infty dk\, \frac{\sin(kr)}{kr} \langle \mathbf{u} \| \mathbf{v} \rangle(k)$$

A reasonable choice of *k* gives accurate *G(r)*

Examples: momentum, kinetic densities

$\mathbf{u} = \mathbf{v} = \psi$, and

$$N = \int_0^\infty dk\, \langle\psi\|\psi\rangle(k) \equiv \int_0^\infty 4\pi k^2 n(k)$$

also defines the **waveaction spectrum**

$$n(k) = \frac{\langle\psi\|\psi\rangle(k)}{4\pi k^2}.$$

The spectral density is

$$\langle\psi\|\psi\rangle(k) = \frac{k^2}{2\pi^2}\int d^3\mathbf{r}\frac{\sin(k|\mathbf{r}|)}{k|\mathbf{r}|}C[\psi,\psi](\mathbf{r})$$

with angle averaged two-point correlator

$$G_{\psi\psi}(r) = \int_0^\infty \frac{\sin(kr)}{kr}\langle\psi\|\psi\rangle(k)$$

$\propto g_1(r)$ the first-order field correlation function.

$\mathbf{u} = \mathbf{v} = \nabla\psi$, related to total kinetic energy by

$$E_{\text{kin}} = \frac{\hbar^2}{2m}\int_0^\infty dk\,\langle\nabla\psi\|\nabla\psi\rangle(k)$$

and the **kinetic energy spectral density**

$$e_{\text{kin}}(k) = \frac{\hbar^2}{8m\pi k^2}\langle\nabla\psi\|\nabla\psi\rangle(k)$$

Isotropy is not assumed, rather angular variation is built into the C integral.

Locally k-space additive decomposition

Atoning for crimes semiclassical

Retain

1. **all quantum phase information**
2. **quantum pressure**

Using the Helmholtz decomposition, define

$$\frac{\hbar}{m}\nabla\psi = (i\mathbf{u}^i + i\mathbf{u}^c + \mathbf{u}^q)e^{i\Theta},$$

where $\mathbf{u}^q = (\hbar/m)\nabla\sqrt{n}$. Note these are not in general orthogonal.

As usual $E_{kin} = E^i_{kin} + E^c_{kin} + E^q_{kin}$ where

$$E^\alpha_{kin} = \frac{m}{2}\int d^3\mathbf{r}\,|\mathbf{u}^\alpha|^2, \text{ for } \alpha \in \{i, c, q\}$$

Kinetic energy densities

$$e^\alpha_{kin}(k) \equiv \frac{\hbar^2}{8\pi k^2}\langle \mathbf{u}^\alpha e^{i\Theta} \| \mathbf{u}^\alpha e^{i\Theta}\rangle(k)$$

retain all quantum phase information, however the normal semiclassical velocity spectra do not:

$$\epsilon^\alpha_{kin}(k) \equiv \frac{\hbar^2}{8\pi k^2}\langle \mathbf{u}^\alpha \| \mathbf{u}^\alpha\rangle(k)$$

***k*-space additivity** an important property of the first definition is it gives a way to recover local k-space additivity [at the cost of cross terms $\langle \mathbf{u}^\alpha e^{i\Theta} \| \mathbf{u}^{\alpha'} e^{i\Theta}\rangle(k)$], while for second definition there is no cure.

Summary

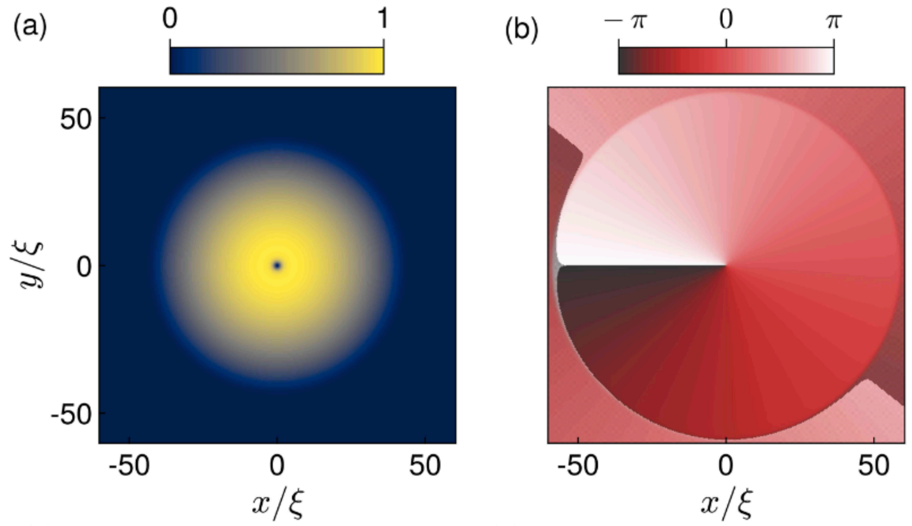
Angle-averaged Wiener-Khinchin theorem

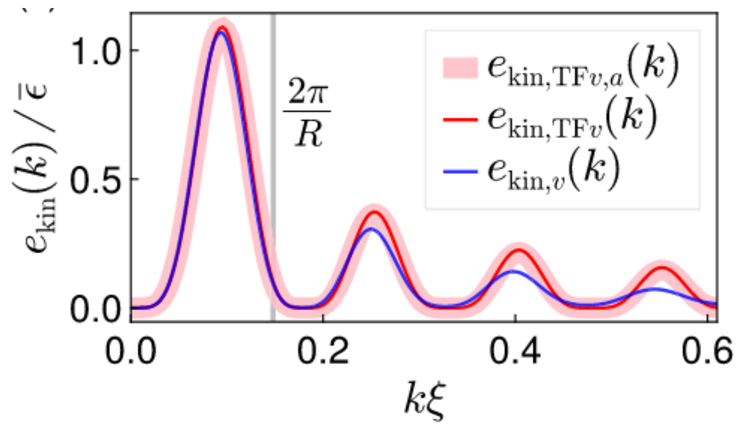
$$\int d\Omega_k$$

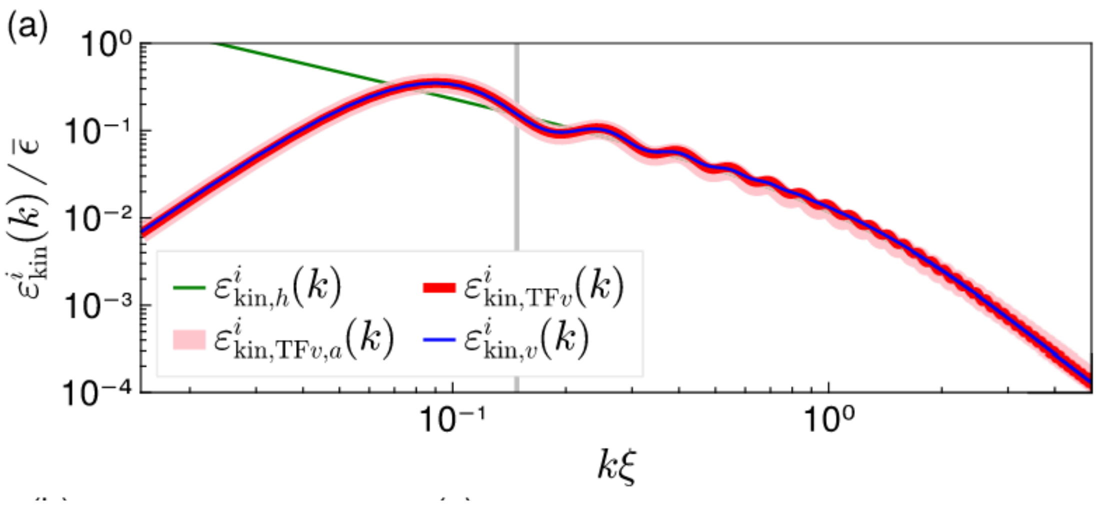
$$(\mathbf{u}, \mathbf{v}) \longrightarrow \langle \mathbf{u} \| \mathbf{v} \rangle (k) \longleftrightarrow g_{uv}(r)$$

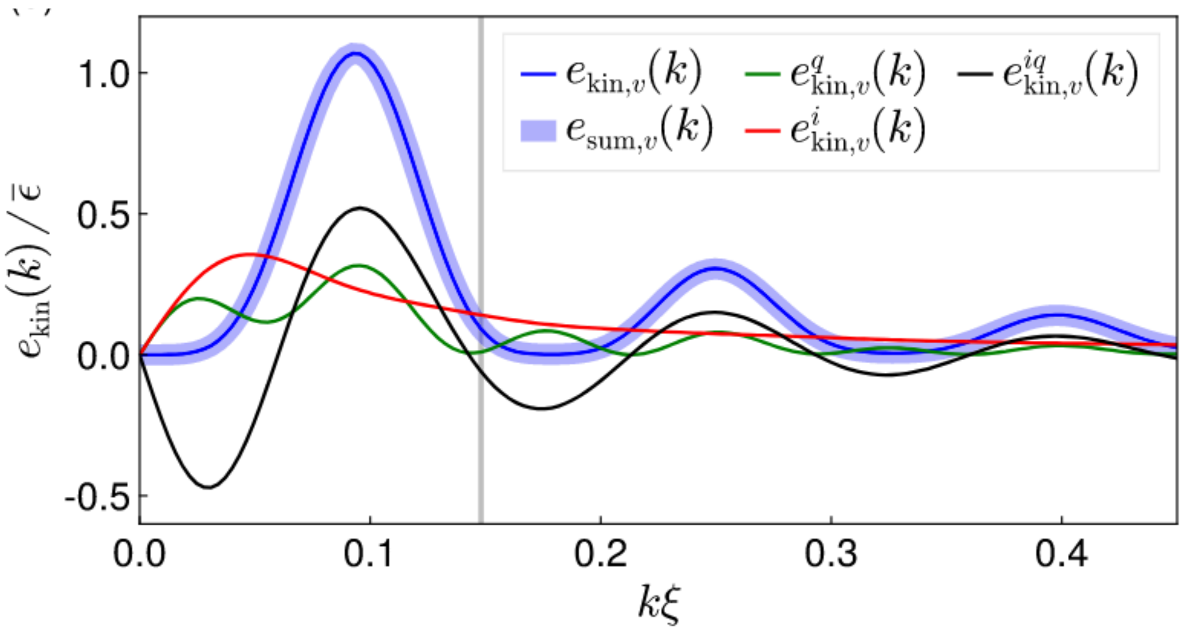
Flexible, accurate power spectra, kinetic densities for compressible quantum fluids

Central vortex

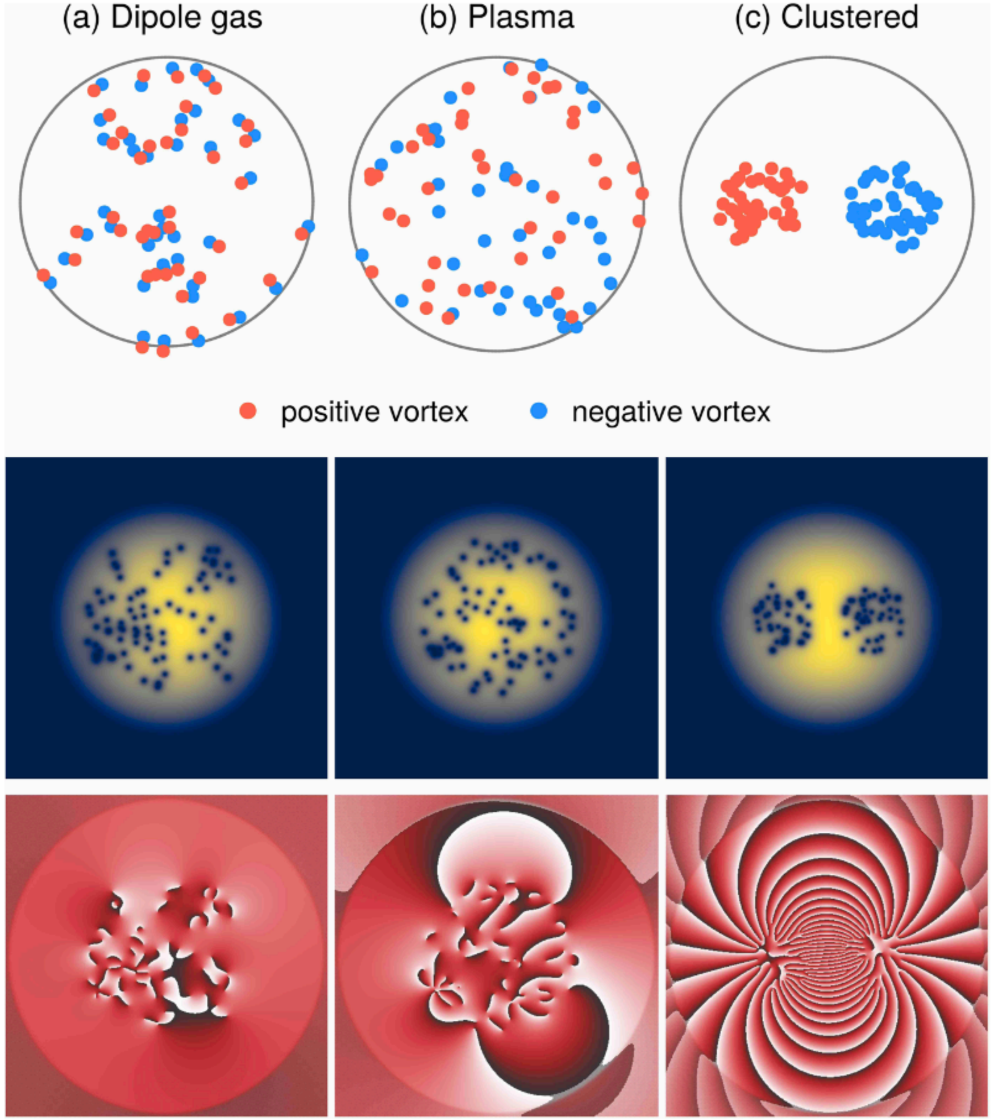


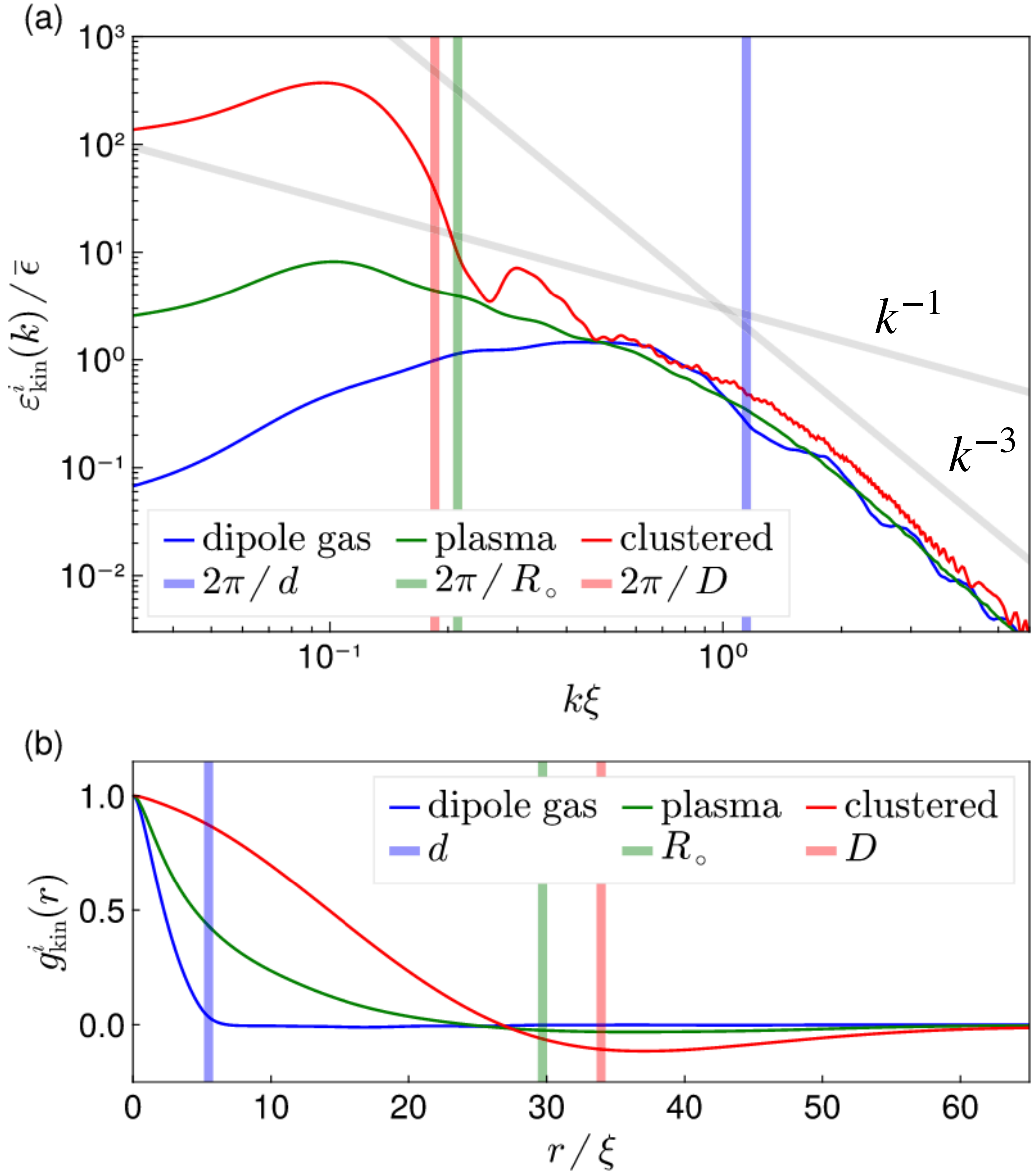




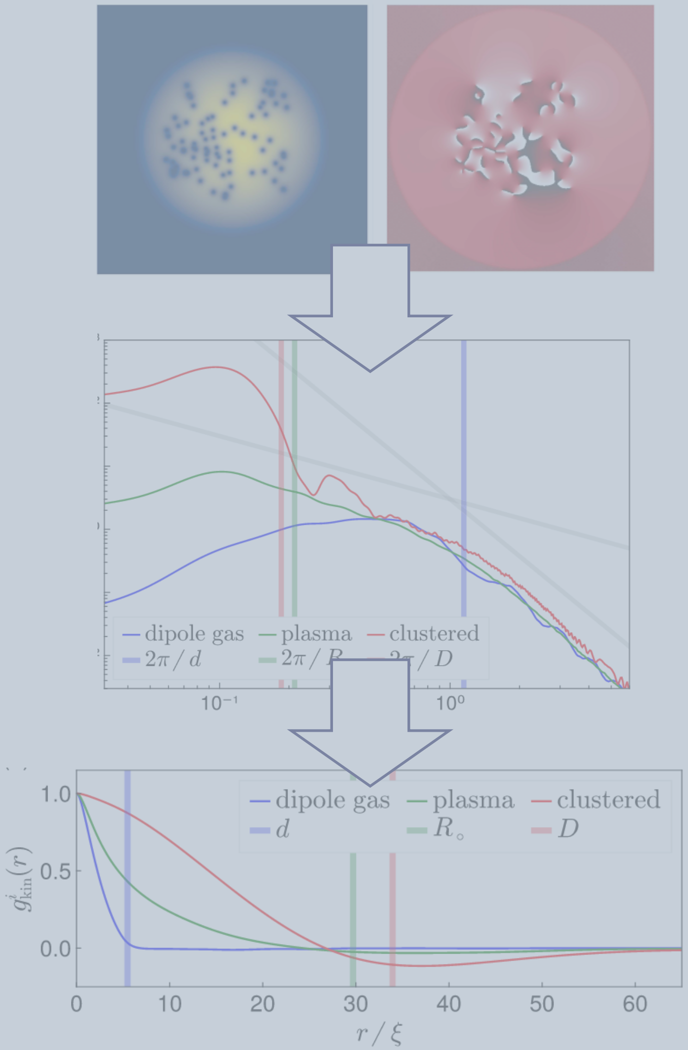


Vortex Phases

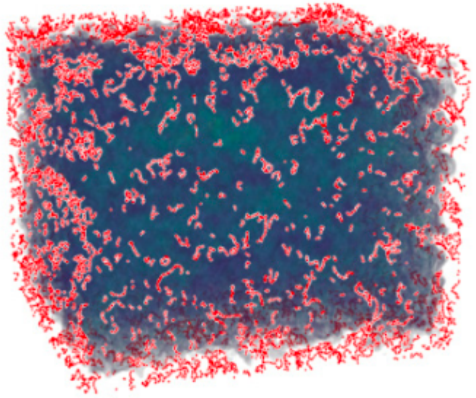




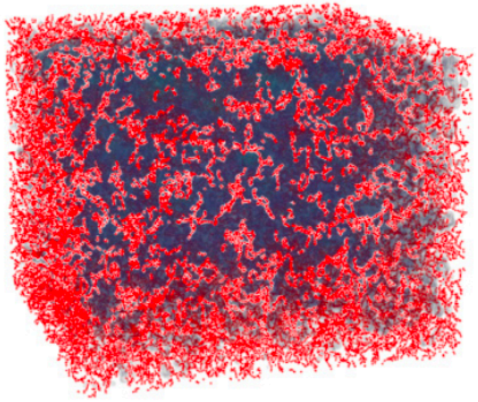
Spectral Analysis



I: Turbulent BEC

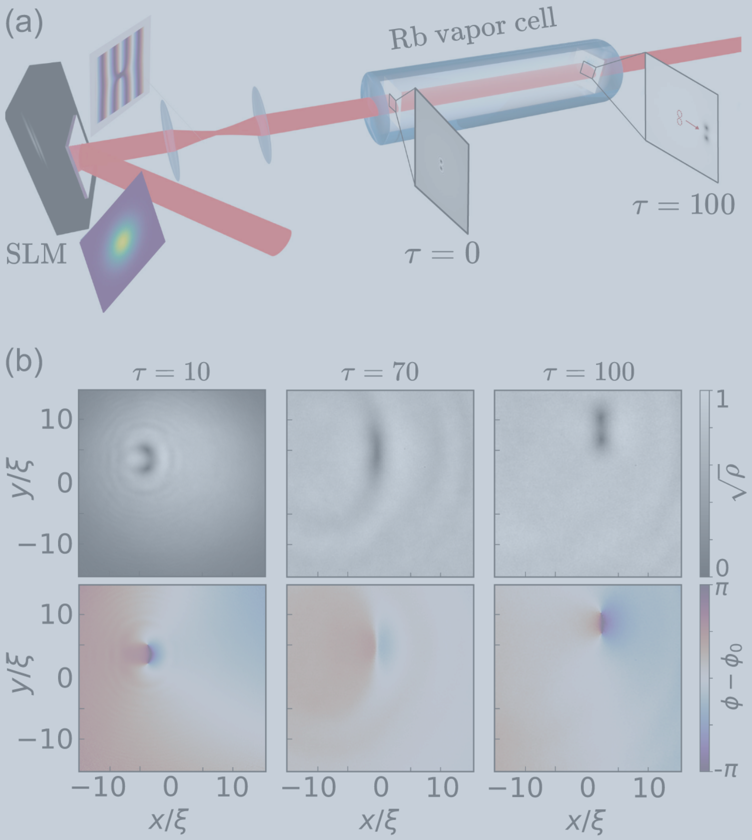


$U_F = 0.6\mu$



$U_F = 1.5\mu$

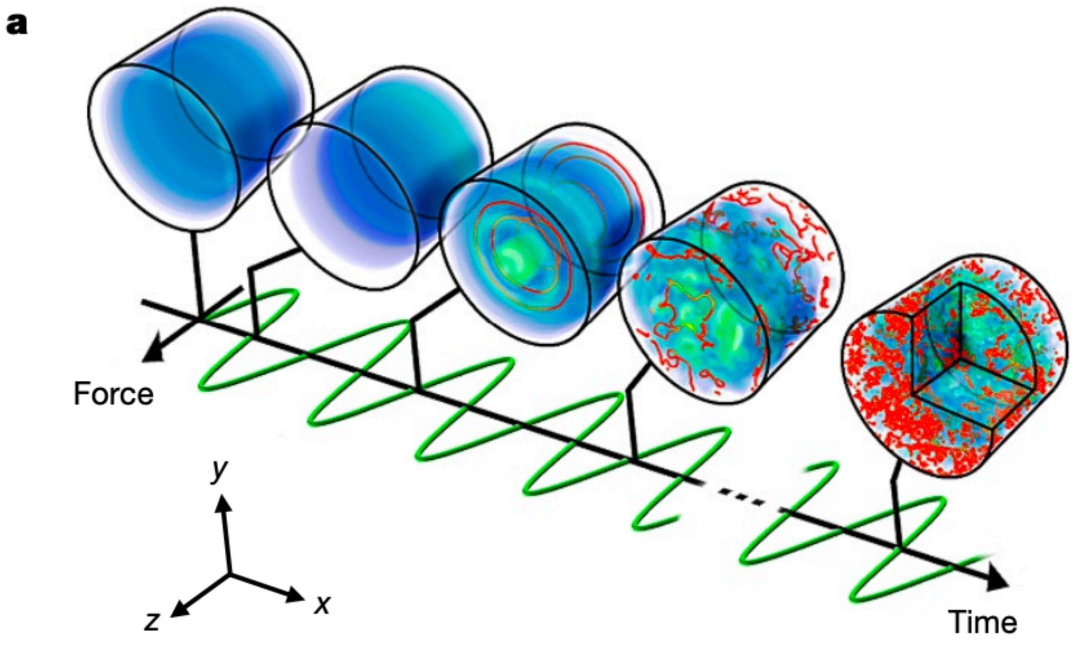
II: Quantum Fluids of Light



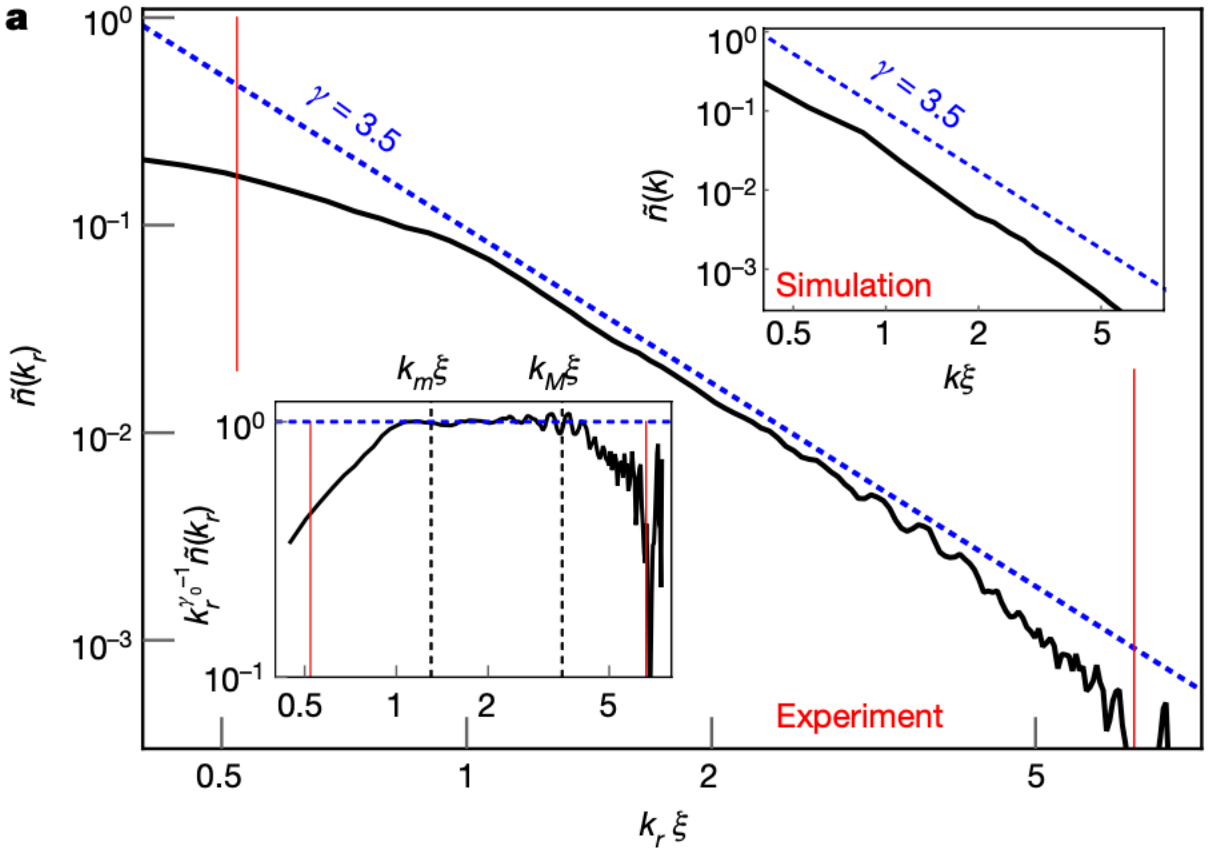
Direct Energy Cascade in 3D

Experiment!

Shake a Box of Superfluid



Wave Action Spectrum

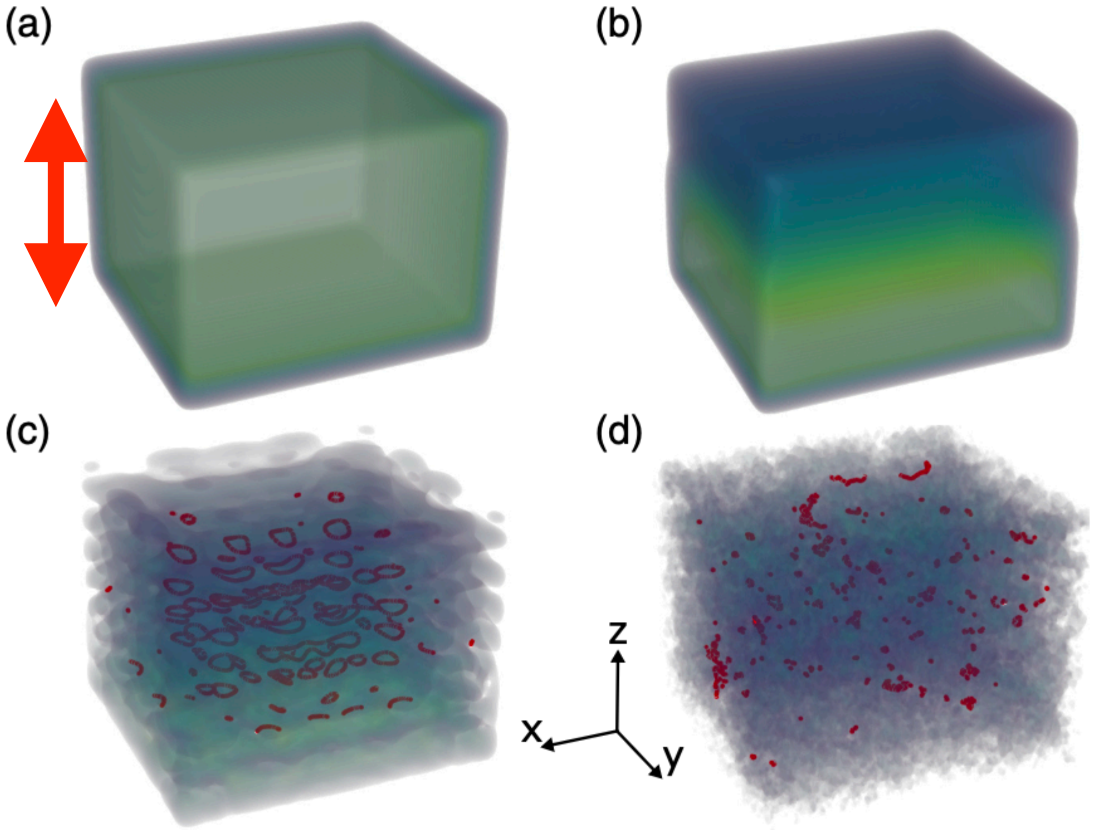


N. Navon, A. L. Gaunt, R. P. Smith, and Z. Hadzibabic, Emergence of a turbulent cascade in a quantum gas, Nature **539**, 7627 (2016).

Creating steady turbulence

GPE simulations

Anisotropic Shaking



Isotropic Turbulence

System

Box trap: $(L_x, L_y, L_z) = (40,30,20)\xi$

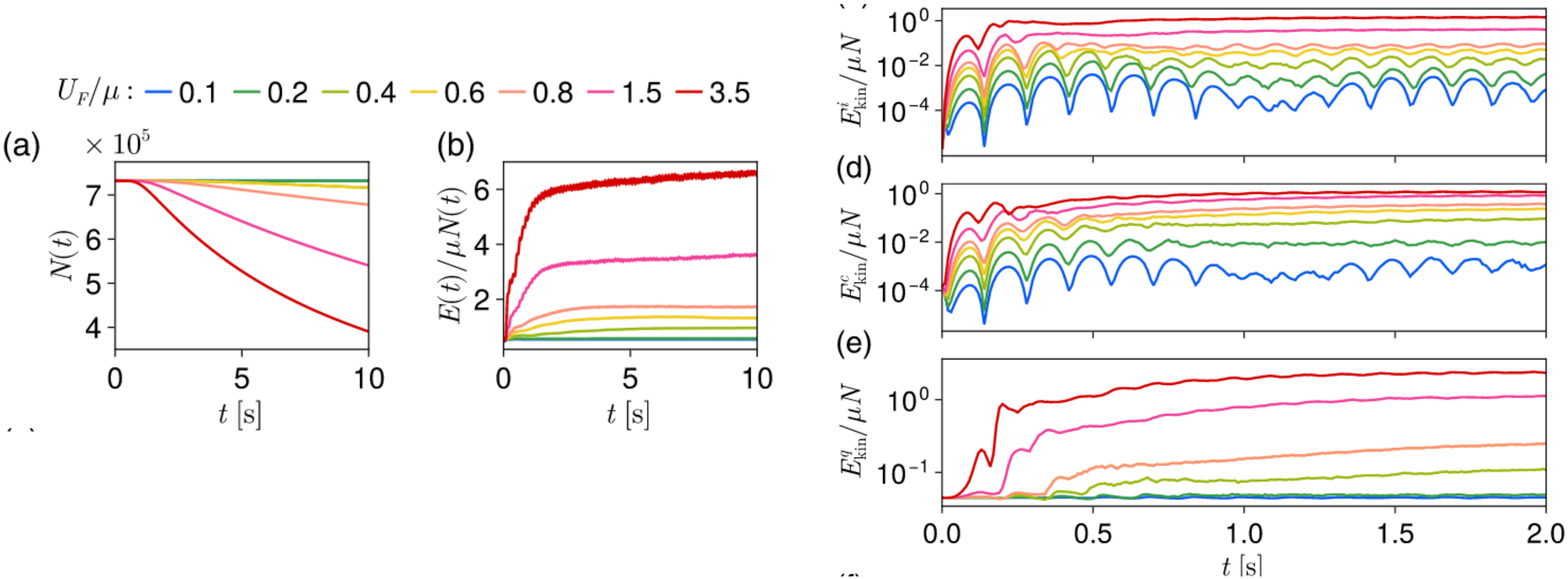
Potential height $V_b = 30\mu$

Forcing: $V_F(\mathbf{r}, t) = U_F \dfrac{z}{L_z} \sin(\omega_F t)$

Forcing range $0.1\mu \leq U_F \leq 5\mu$

Dissipation: Atoms have to scale the walls to energy V_b, with dissipation scale $k_D\xi = \sqrt{2V_b/\mu} = 7.7$

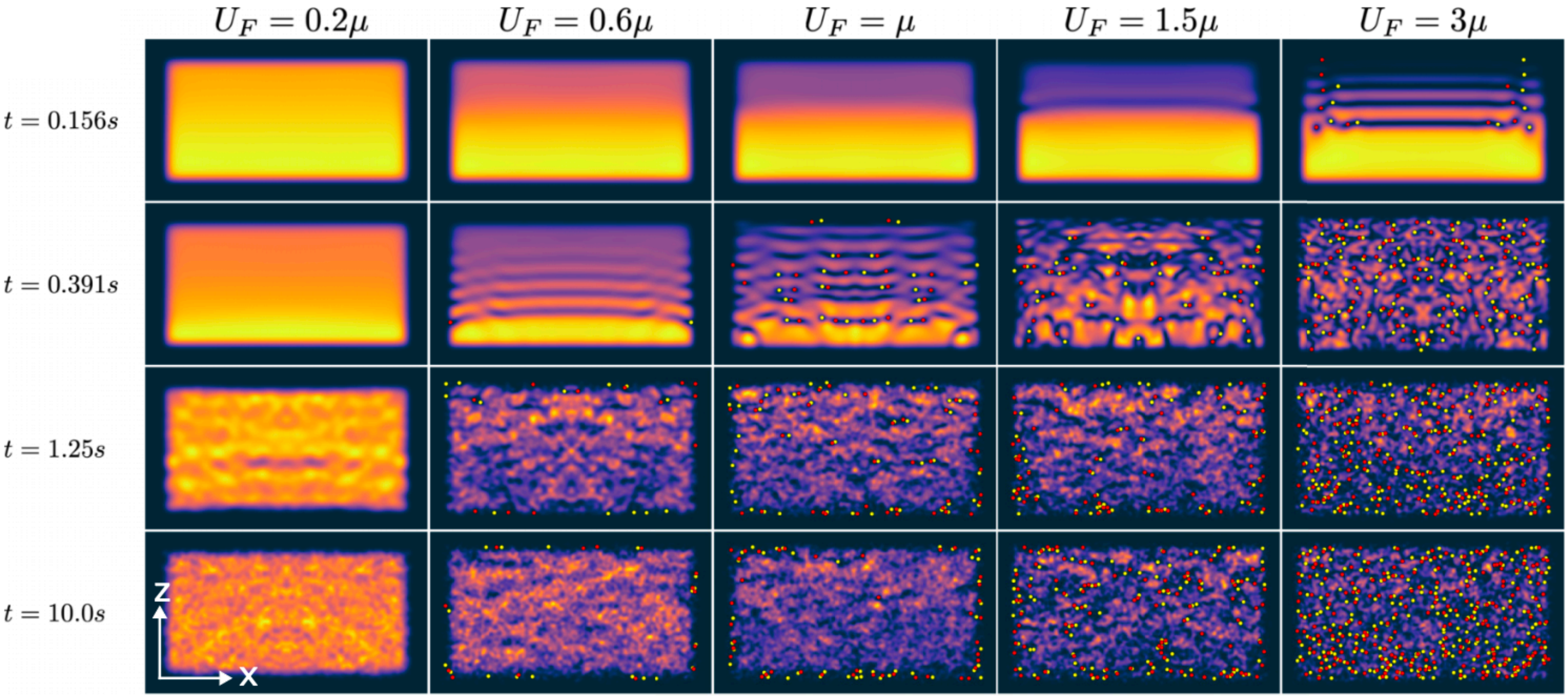
Dynamics



U_F/μ : 0.1 0.2 0.4 0.6 0.8 1.5 3.5
(a)
$\times 10^5$
$N(t)$
7
6
5
4
0
5
10
t [s]
(b)
$E(t)/\mu N(t)$
6
4
2
0
5
10
t [s]
$E^i_{\rm kin}/\mu N$
10^0
10^{-2}
10^{-4}
(d)
$E^c_{\rm kin}/\mu N$
10^0
10^{-2}
10^{-4}
(e)
$E^q_{\rm kin}/\mu N$
10^0
10^{-1}
0.0
0.5
1.0
1.5
2.0
t [s]

Evolution to steady state

Slice at y=0



Vortices in steady state

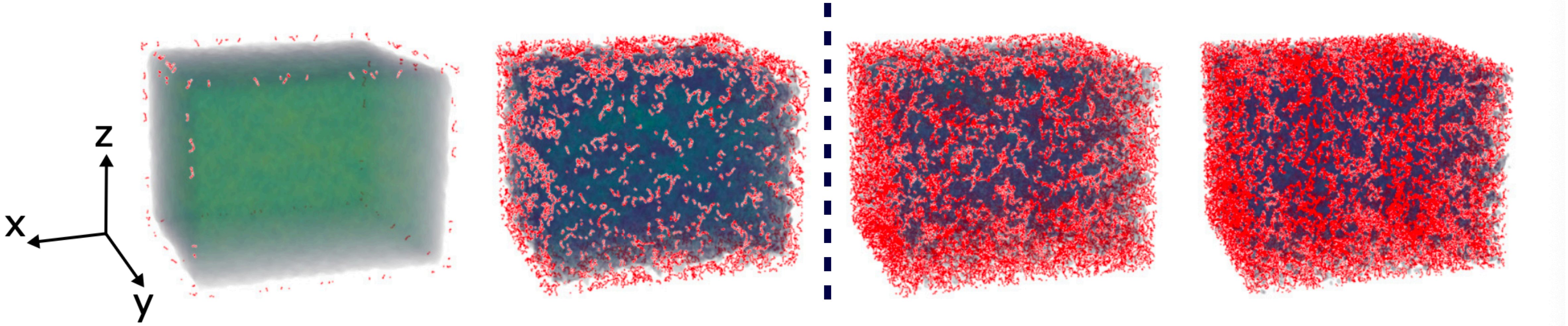
Dilute surface vortices

$U_F \lesssim \mu$

Dense bulk vortices

$U_F \gtrsim 2\mu$

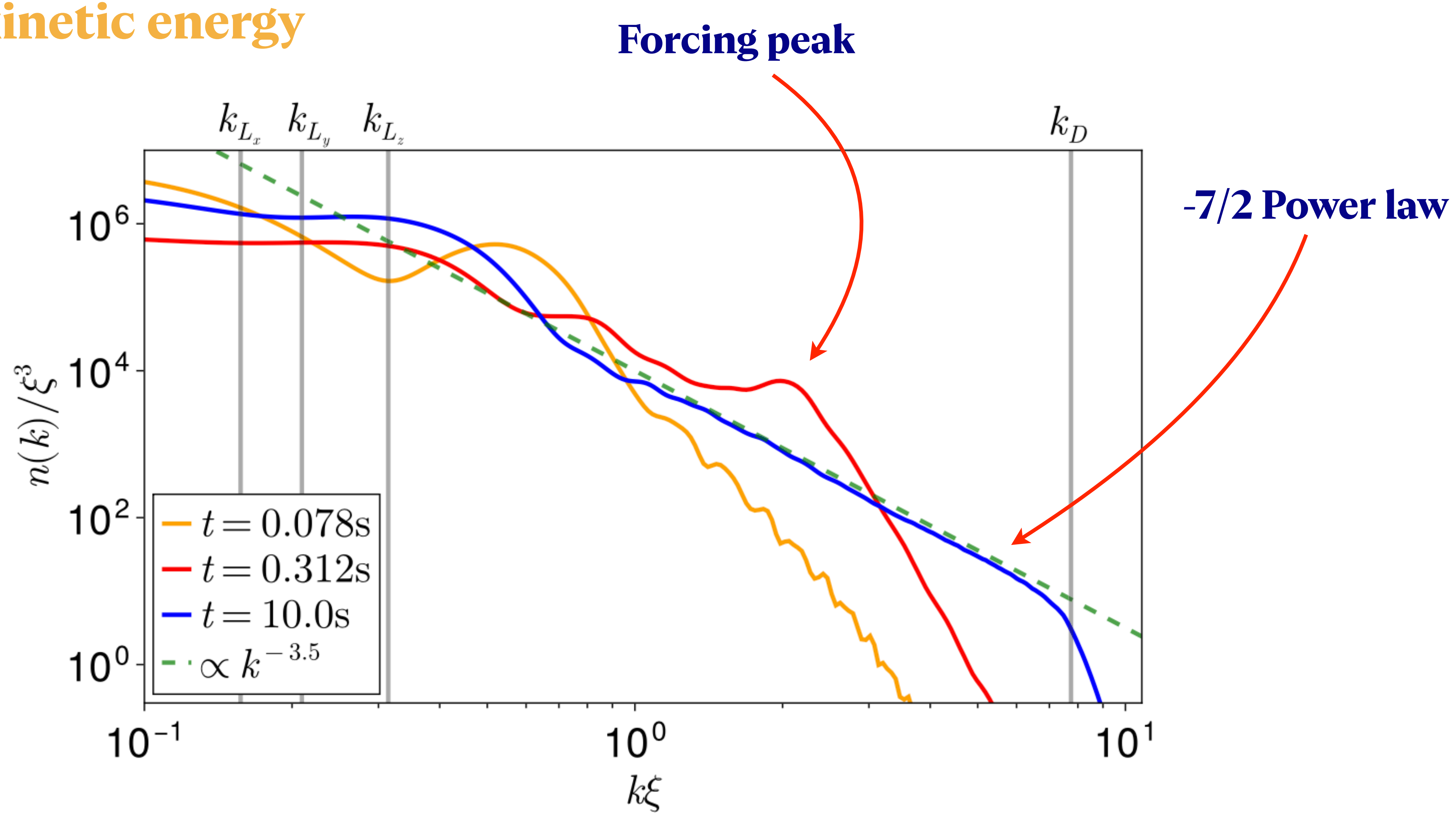
$U_F \sim \mu$



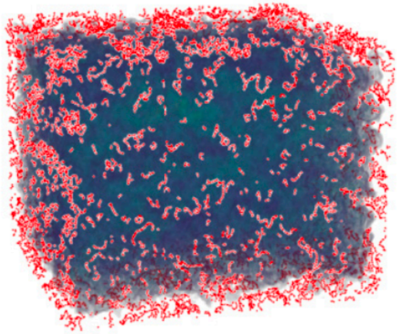
$U_F = 0.2\mu$ $U_F = 0.6\mu$ $U_F = 1.5\mu$ $U_F = 3\mu$

Waveaction spectra (no averaging)

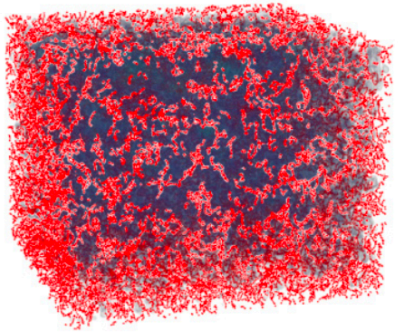
Total kinetic energy



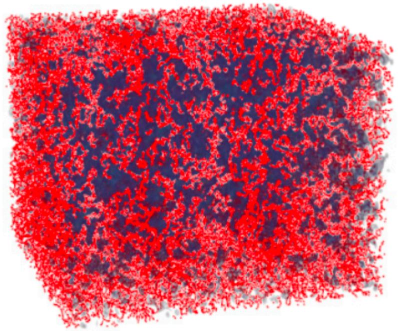
Decomposed waveaction (cycle averaged)



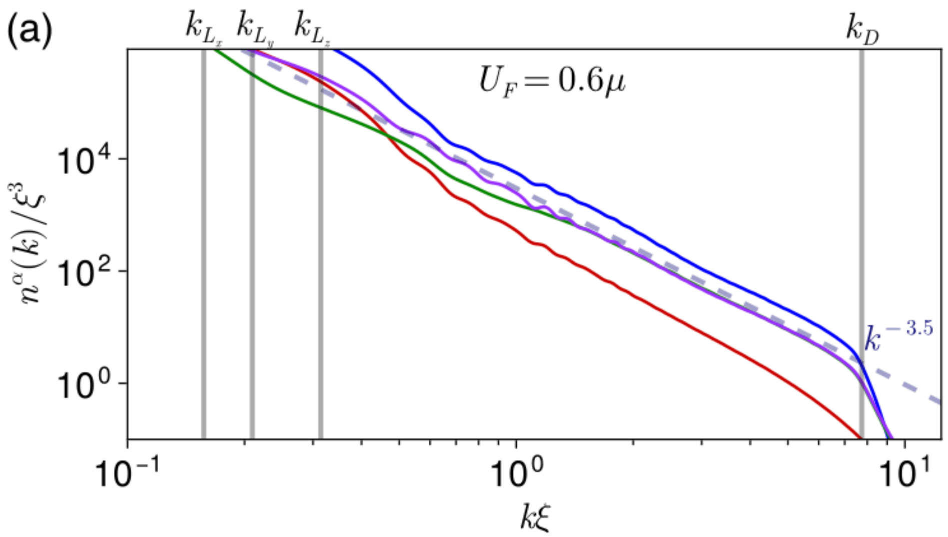
$U_F = 0.6\mu$

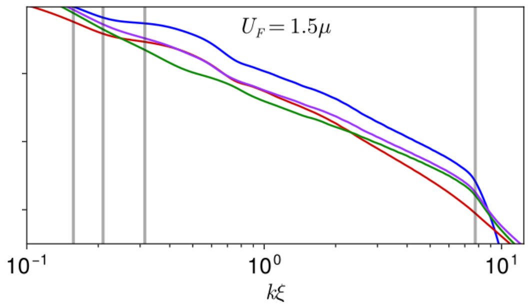


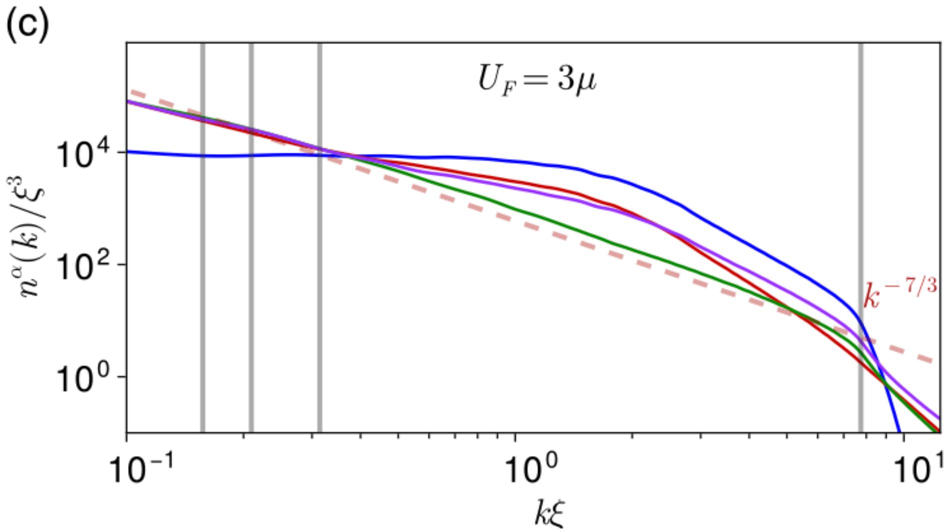
$U_F = 1.5\mu$



$U_F = 3\mu$





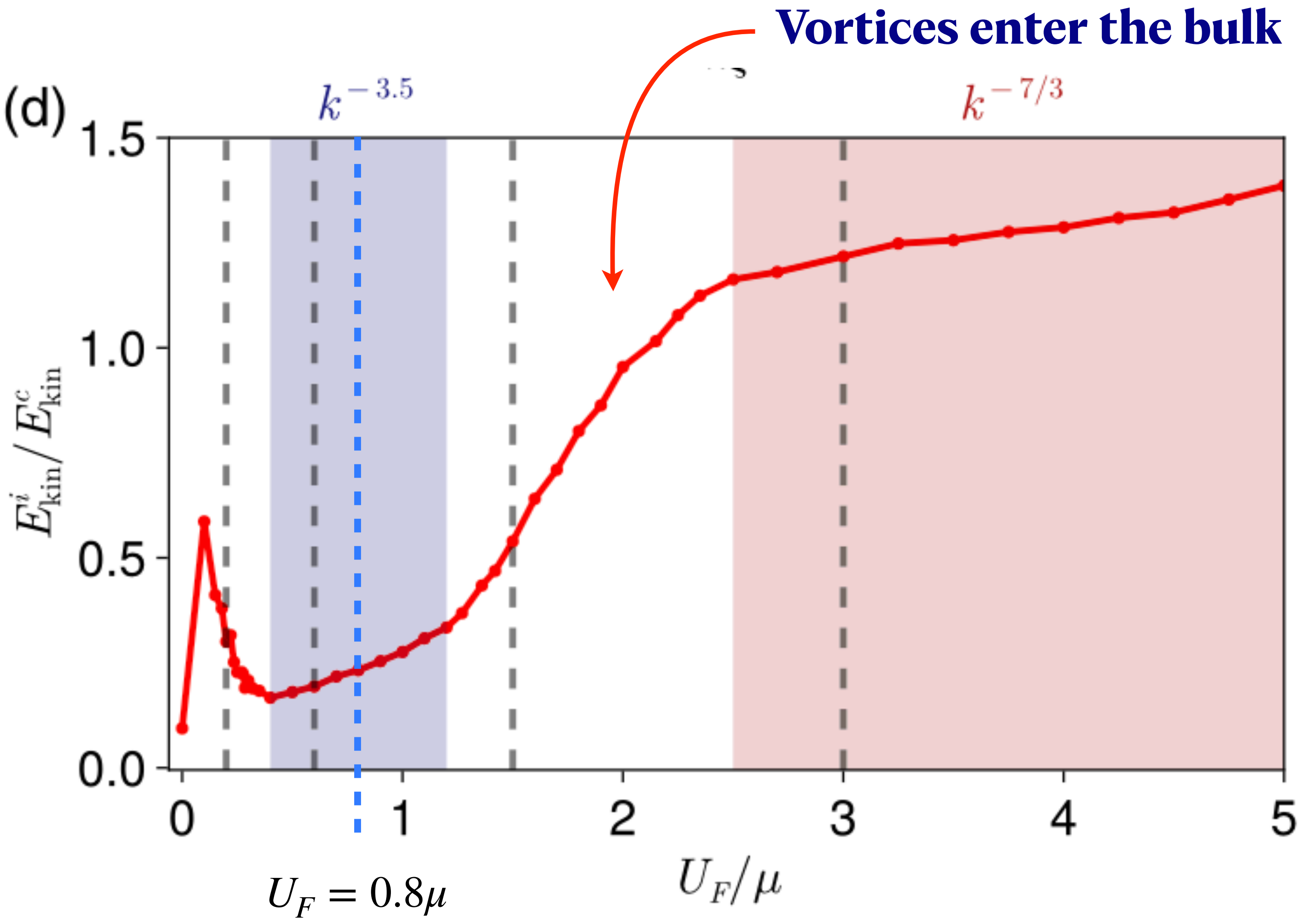


$n^c(k) \approx n^q(k) \sim k^{-7/2}$

$n(k)$ — $n^i(k)$ — $n^c(k)$ — $n^q(k)$

$n^c(k) \sim k^{-7/3}$

Regimes of energy injection

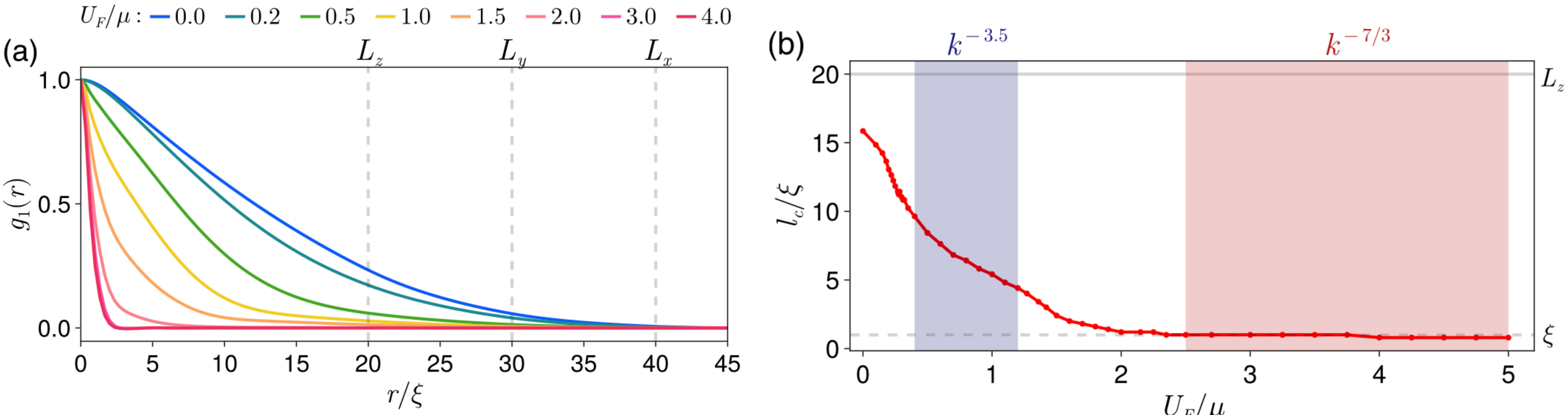


$n(k) \sim k^{-7/2}$ **reported by Navon *et al*, Nature (2016)**

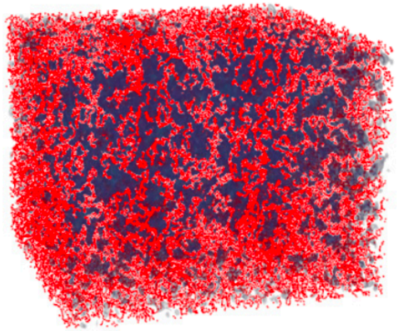
Global phase coherence?

What is happening to the BEC phase?

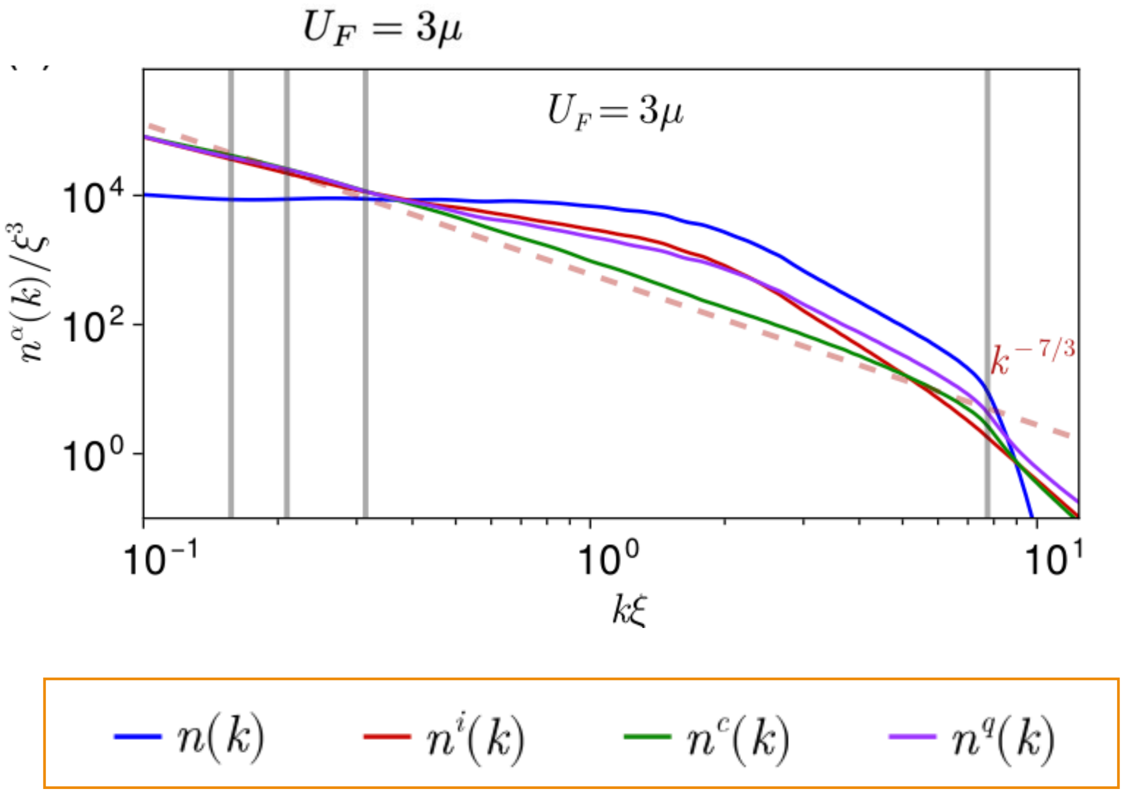
Sinc transform of $4\pi k^2 n(k) \rightarrow G_1(r) = \frac{1}{4\pi}\int d\Omega \delta(r - |\mathbf{r}|) C[\psi, \psi](\mathbf{r})$



How to understand -7/3 for strong vortex turbulence?



$$n^c(k) \sim k^{-7/3}$$
$$l_c \sim \xi$$



Inverse particle cascade prediction: for WWT without a condensate, the Kolmogorov-Zakharov solution is $n(k) \sim k^{-7/3}$ [Y. Zhu et al, PRL (2023)]

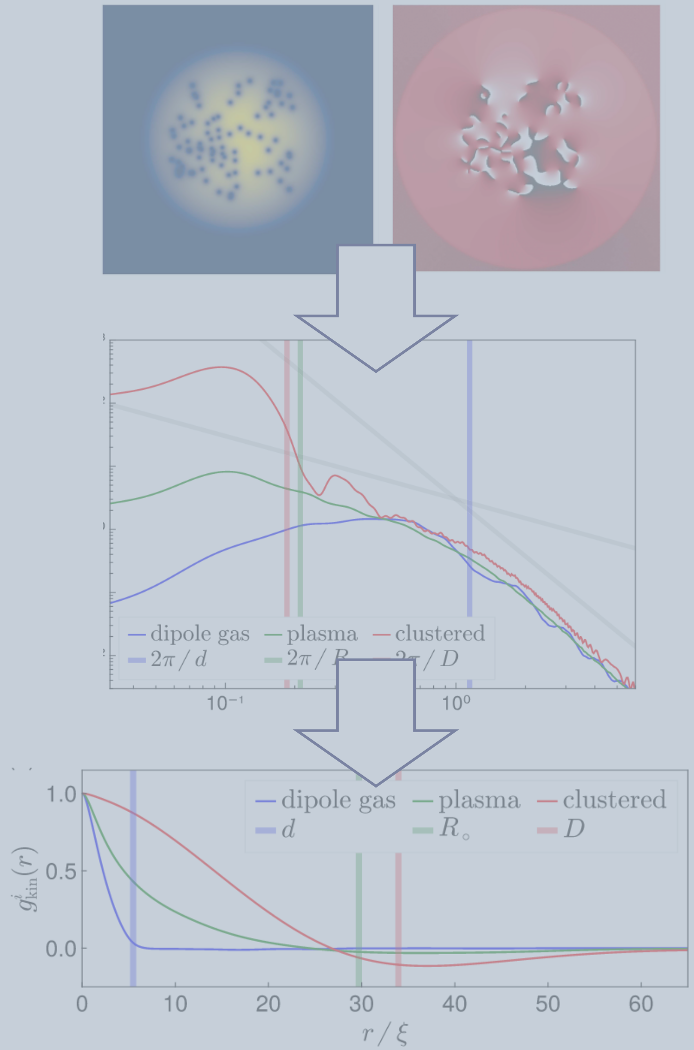
We don't see this power law in total $n(k)$!

Hidden **WWT IEC?**

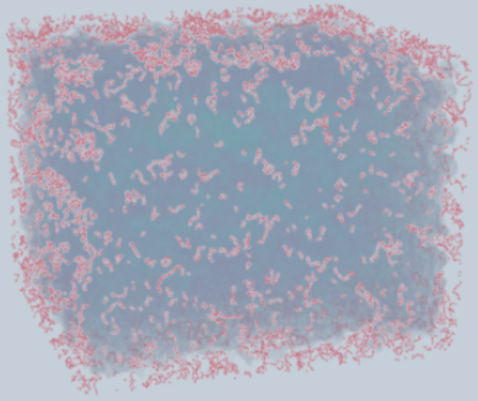
What is the compressible source at high *k*? Most likely vortex-antivortex annihilation generating short wavelength sound. Indeed, log corrections indicate a forcing scale of $k_F \sim 1/\xi$.

G. Martirosyan, K. Fujimoto, and N. Navon, arXiv:2407.08738

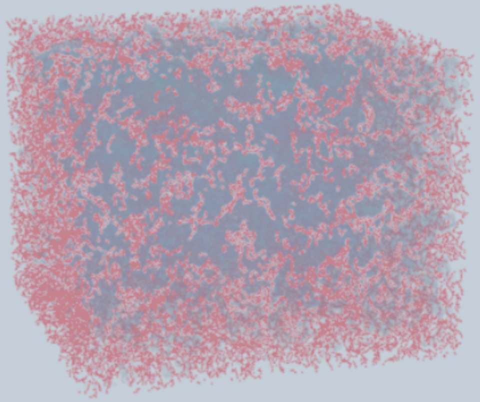
Spectral Analysis



I: Turbulent BEC

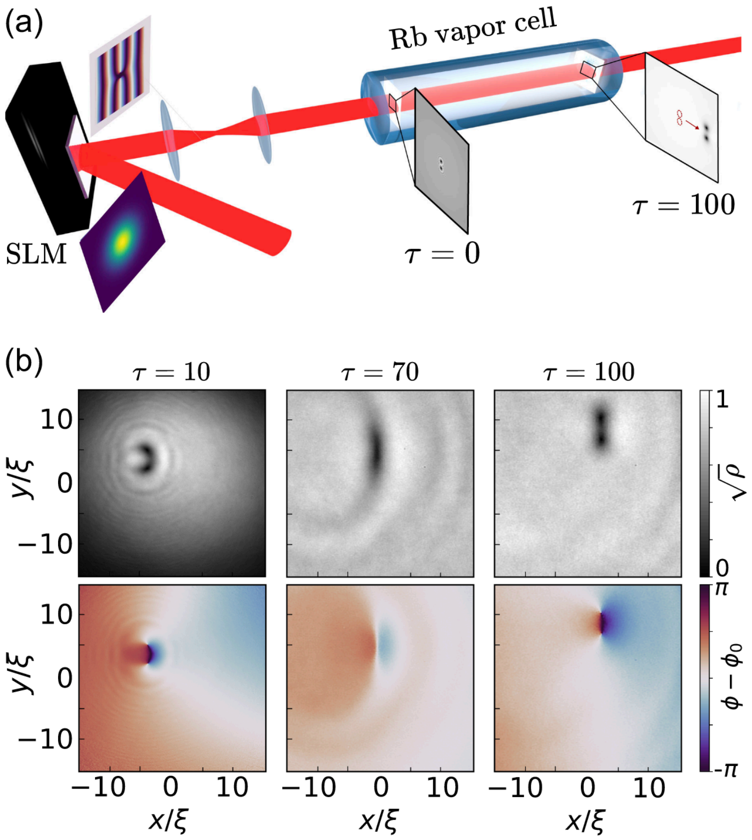


$U_F = 0.6\mu$



$U_F = 1.5\mu$

II: Quantum Fluids of Light



Motivation

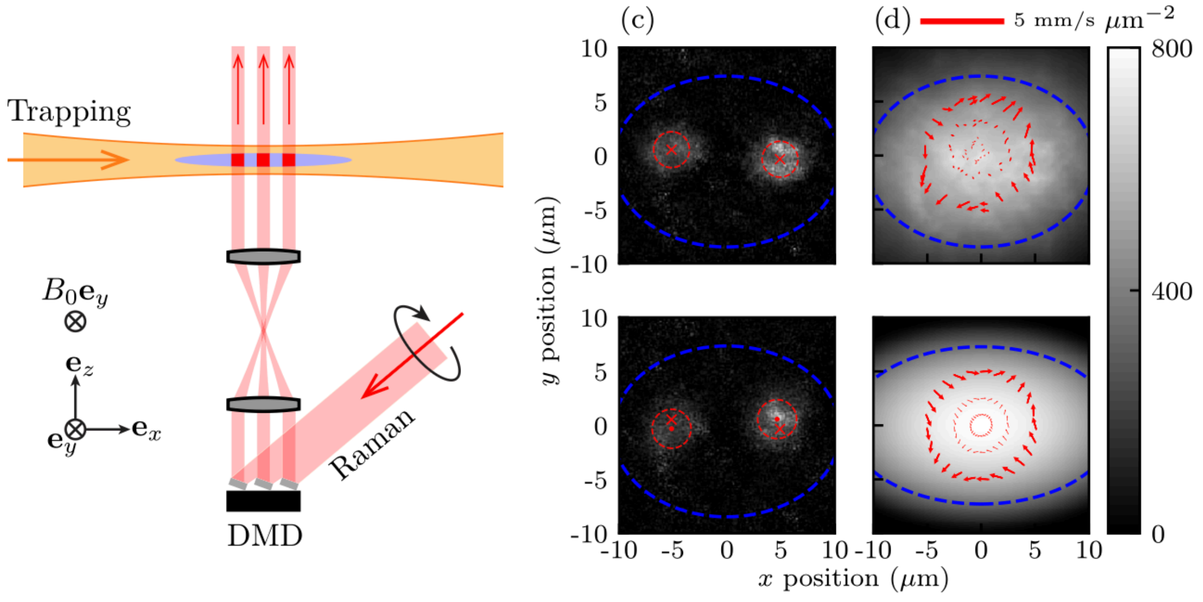
Density-phase

Challenge in BEC: high resolution density and velocity (ideally also quantum phase).

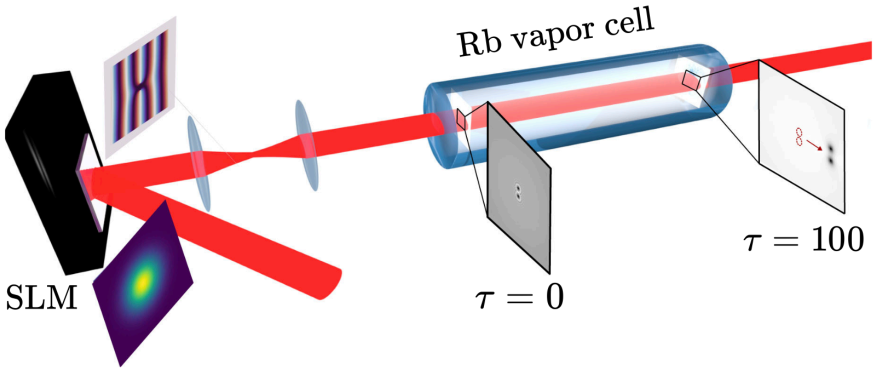
Very promising approach: **particle image velocimetry.** What resolutions in time and space are possible?

Some challenges of BEC can be addressed in other platforms using analogue quantum fluids.

They have their own challenges!

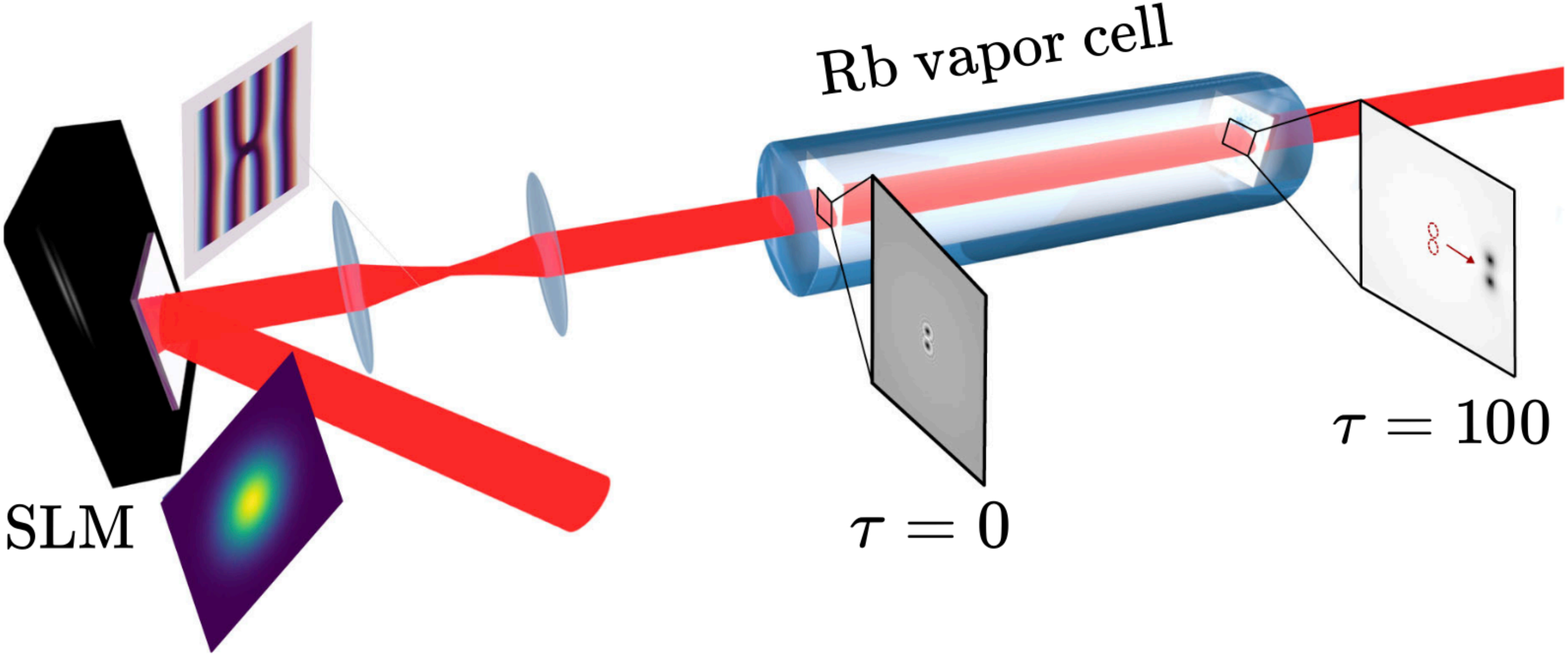


M. Zhao *et al*, PRL (2025).



M. Baker-Rasooli *et al*, PRL (2025).

Experiment



M. Baker-Rasooli *et al*, PRL (2025).

Basic Idea

Quantum Fluid of Light

Fully exploit space-time mapping of the nonlinear Schrodinger equation for electric field

$$i\frac{\partial \mathcal{E}(\mathbf{r}_\perp, z)}{\partial z} = \left[-\frac{1}{2n_0k_0}\nabla_\perp^2 - \frac{k_0\chi^{(3)}}{2n_0}|\mathcal{E}(\mathbf{r}_\perp, z)|^2\right]\mathcal{E}(\mathbf{r}_\perp, z)$$

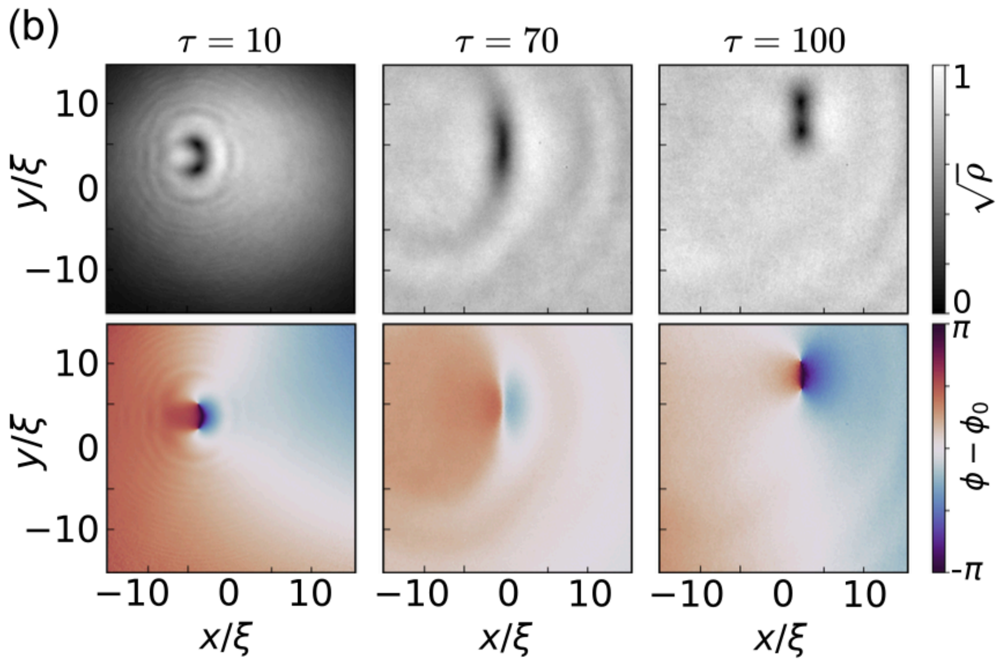
Overcome limitation of fixed propagation length (time in NSLE) by tuning the χ^3 nonlinearity.

Complete space-time mapping to the 2D GPE

$$i\frac{\partial \psi}{\partial \tau} = \left(-\frac{1}{2}\tilde{\nabla}_\perp^2 + |\psi|^2\right)\psi$$

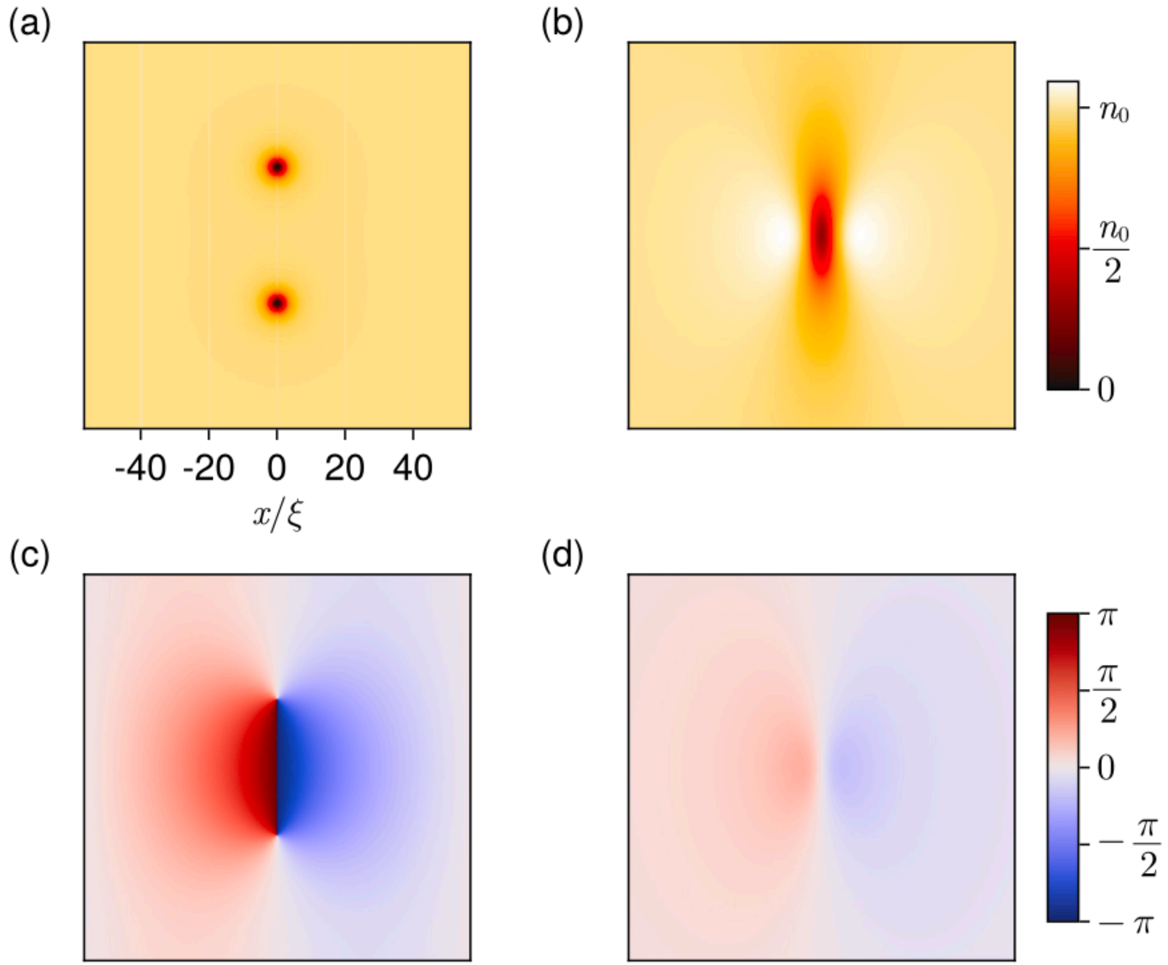
Extreme high precision for both injecting initial states, and measuring density and phase of the analogue quantum fluid

Short time evolution ($\sim 100/\mu$) so focus on fast physics!



Jones-Roberts solitons (JRS)

Eigenstates of the GPE in translating frame



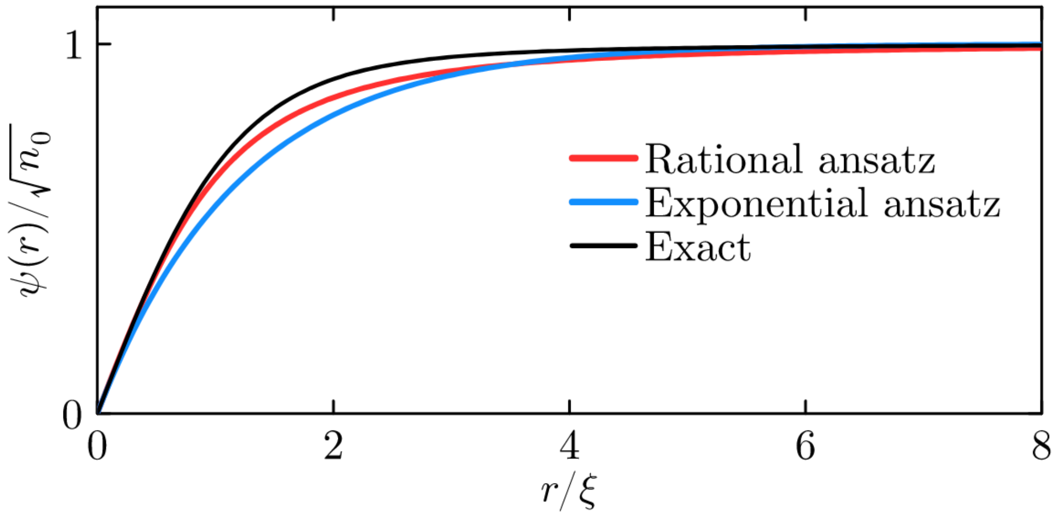
Dissipation of JRS

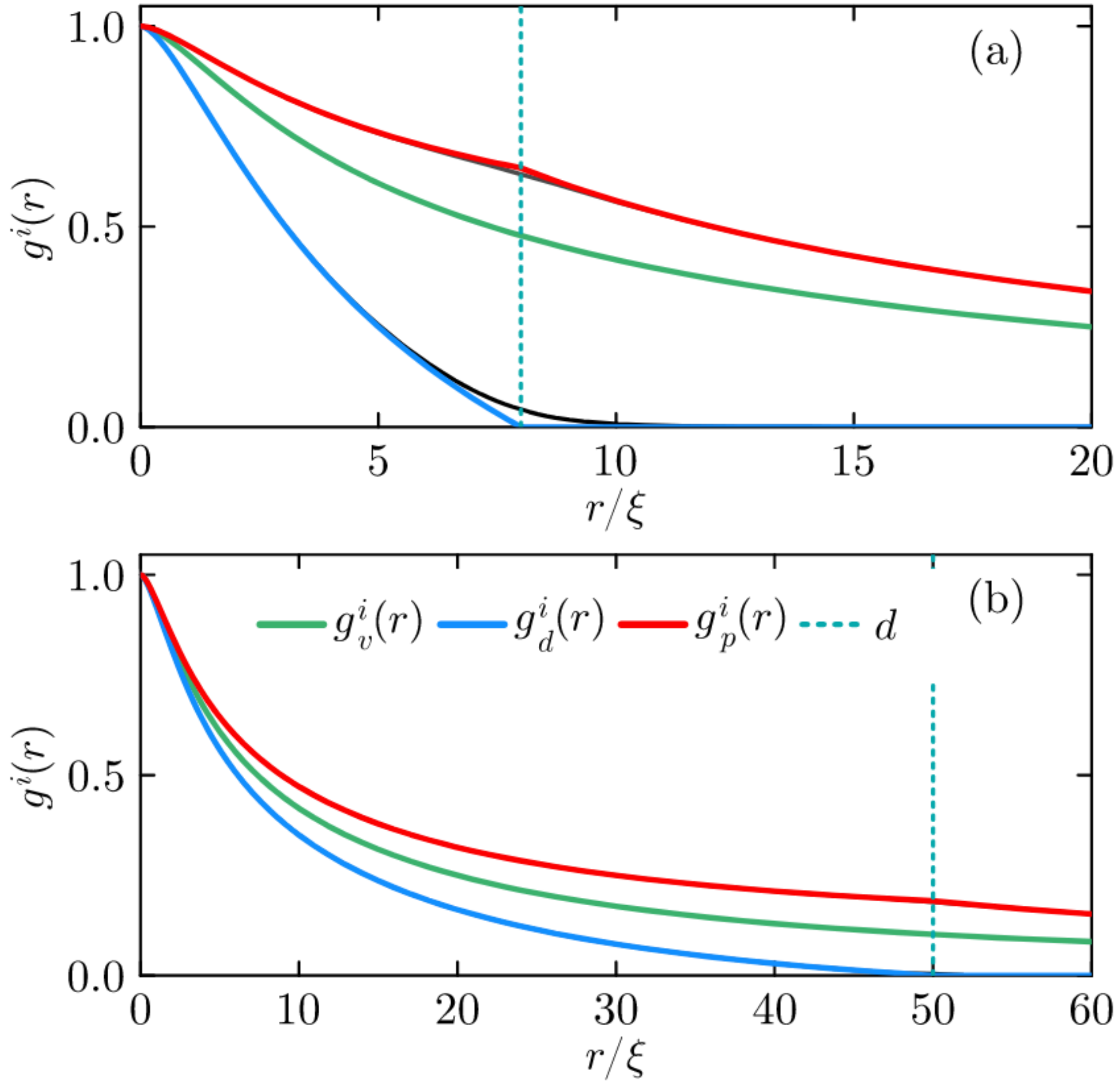
N. A. Krause and A. S. Bradley, PRA (2024).

Velocity correlations of vortices and JRS

A. S. Bradley and N. A. Krause, arXiv:2502.08930.

“Bad” vortex ansatz





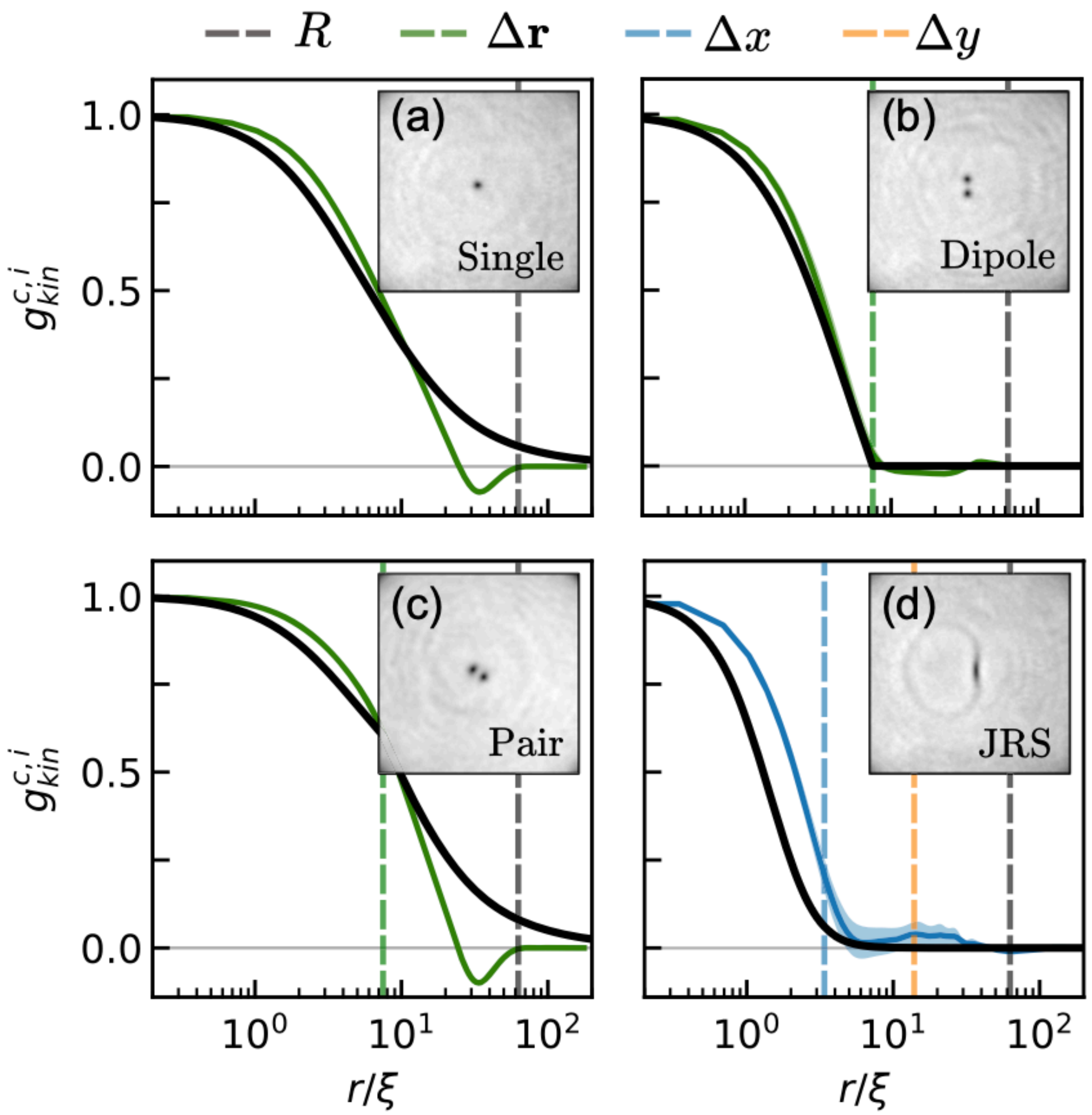
Compressible/incompressible correlations

Short time evolution

Extreme precision and resolution for **experimental spectra** using techniques described in this talk.

Develop **new analytical techniques** for spectra and velocity correlations, for vortices (incompressible) and JRS (compressible).

Inject single vortex, vortex dipole, vortex pair, JRS, and after evolution, **measure** velocity correlations, **compare** with analytical results.



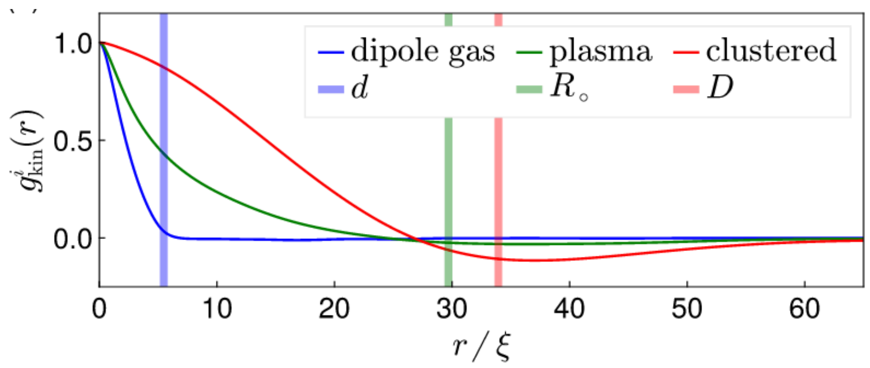
M. Baker-Rasooli *et al*, PRL (2025).

Summary, Perspectives

Angle-averaged Wiener-Khinchin

Integrate the kernel-weighted two-point correlator

Library: QuantumFluidSpectra.jl

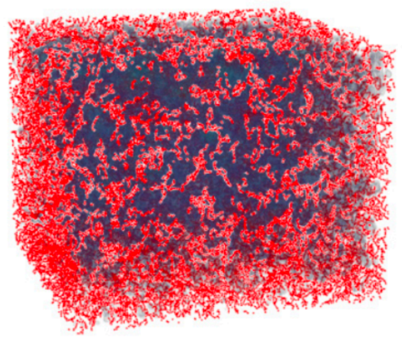


Turbulent BEC

Confirm direct cascade scenario of Navon *et al* (2016).

Bulk vortices mark onset of strong vortex turbulence

Hidden **weak wave IEC in strong vortex turbulence**, forced at the vortex annihilation scale



Quantum Fluids of Light

High resolution compressible fluid dynamics