Global analyses of polarized PDFs beyond NLO

Ignacio Borsa

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In collaboration with Daniel de Florian, Rodolfo Sassot, Marco Stratmann and Werner Vogelsang

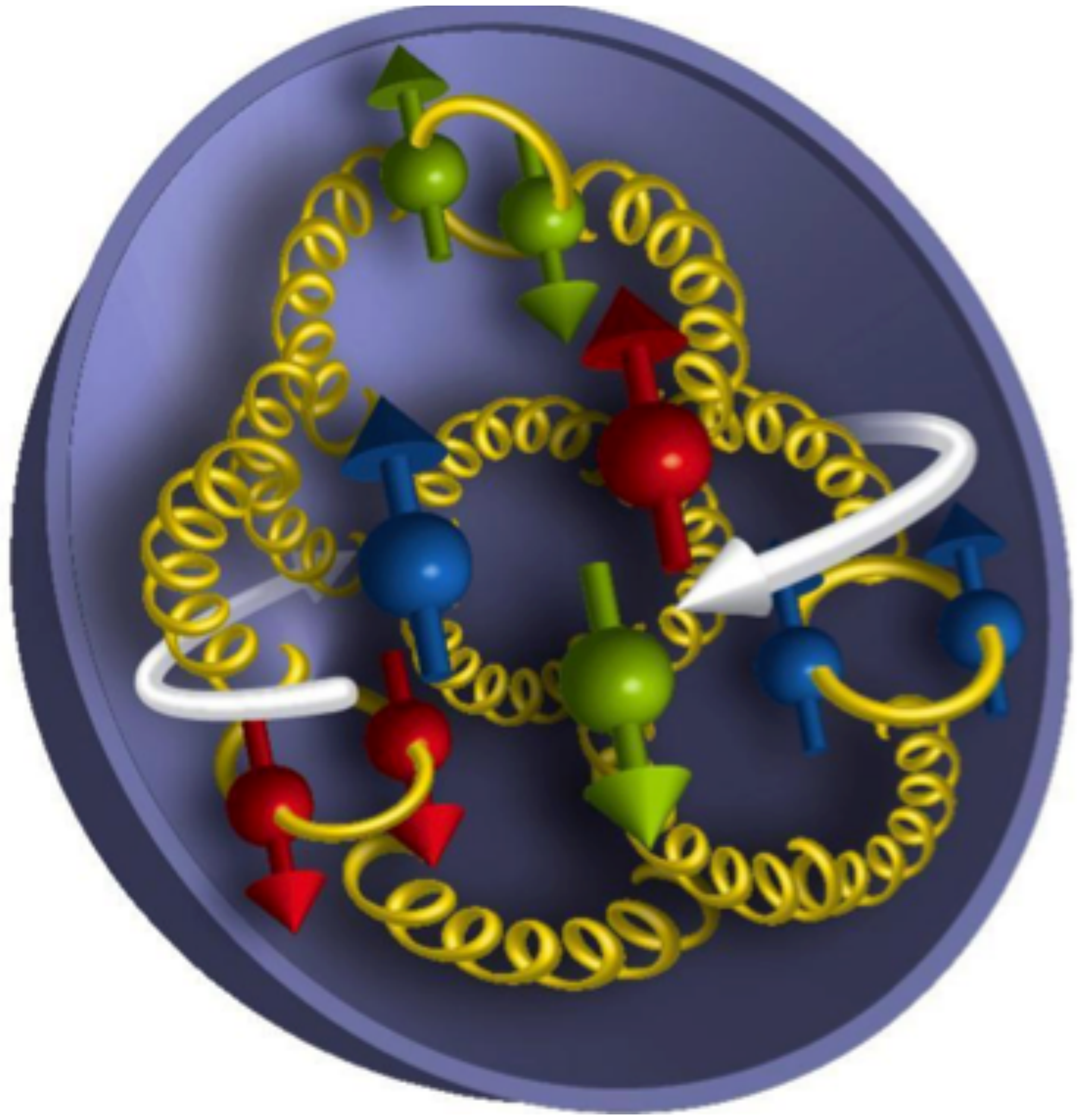
Precision QCD with the Electron Ion Collider
Seattle- May 13th

EBERHARD KARLS UNIVERSITÄT TÜBINGEN

Introduction

Introduction - The proton's spin structure

How is the proton's spin distributed among its constituents?



$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta g + L_q + L_g$$

Jaffe, Manohar (1990)

- Surprisingly low amount of spin carried by intrinsic quarks, $\Delta\Sigma \sim 0.25 \ll 1$ [European Muon Collaboration (1989)] → "Proton spin crisis".

- Significant progress both from experiment and theory [for a review: Aidala, Bass, Hasch, Mallot (2013)]. First evidence of positive polarization of gluons from polarized proton-proton collisions at RHIC [de Florian, Sassot, Stratmann, Vogelsang (2014); Nocera, Ball Forte, Ridolfi, Rojo (2014)].

- Still, rather incomplete picture of the spin structure in terms of the contribution from gluons or flavor decomposition.

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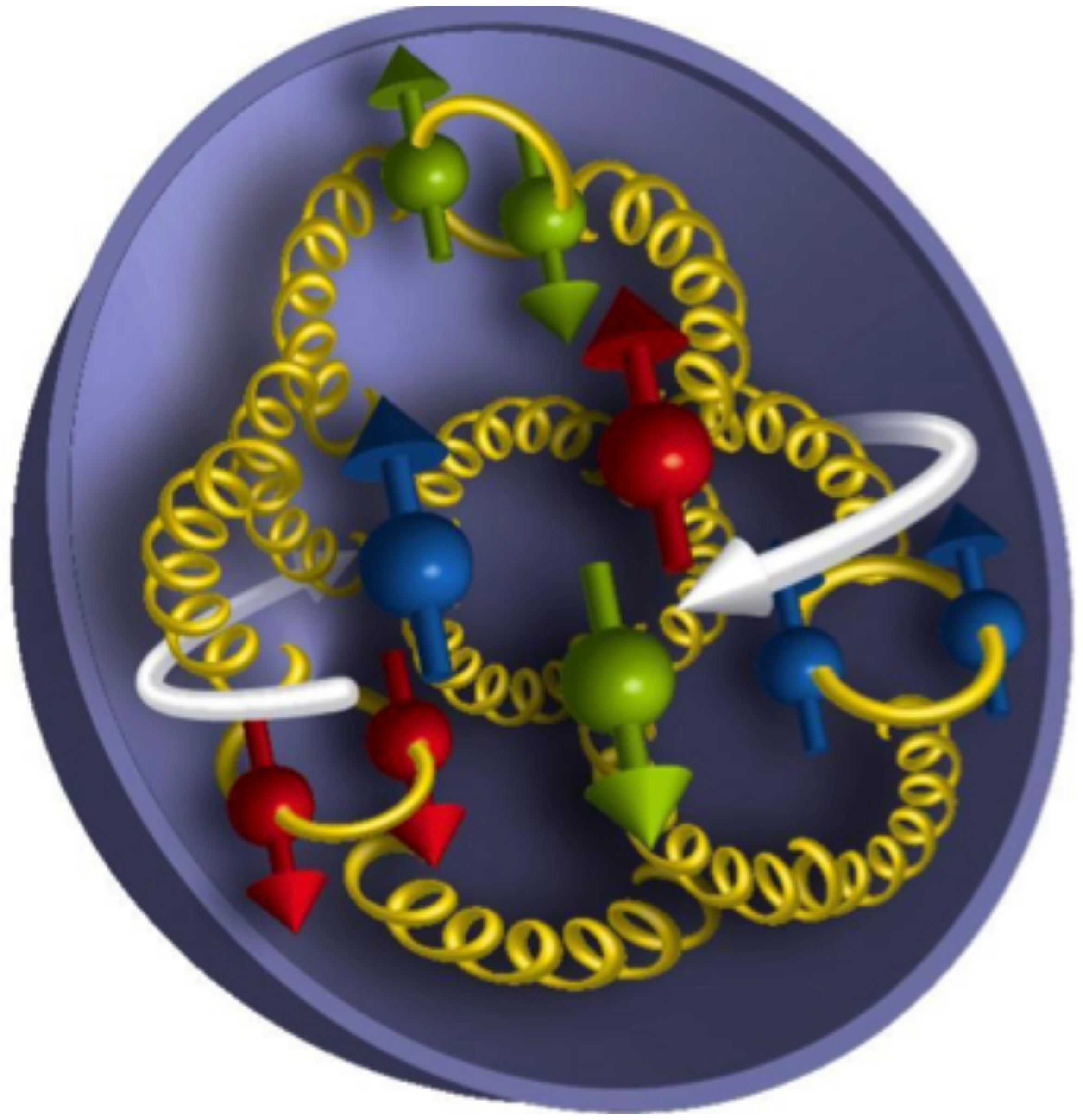

Quark's spin   Gluon's spin   OAM

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Introduction - The proton's spin structure

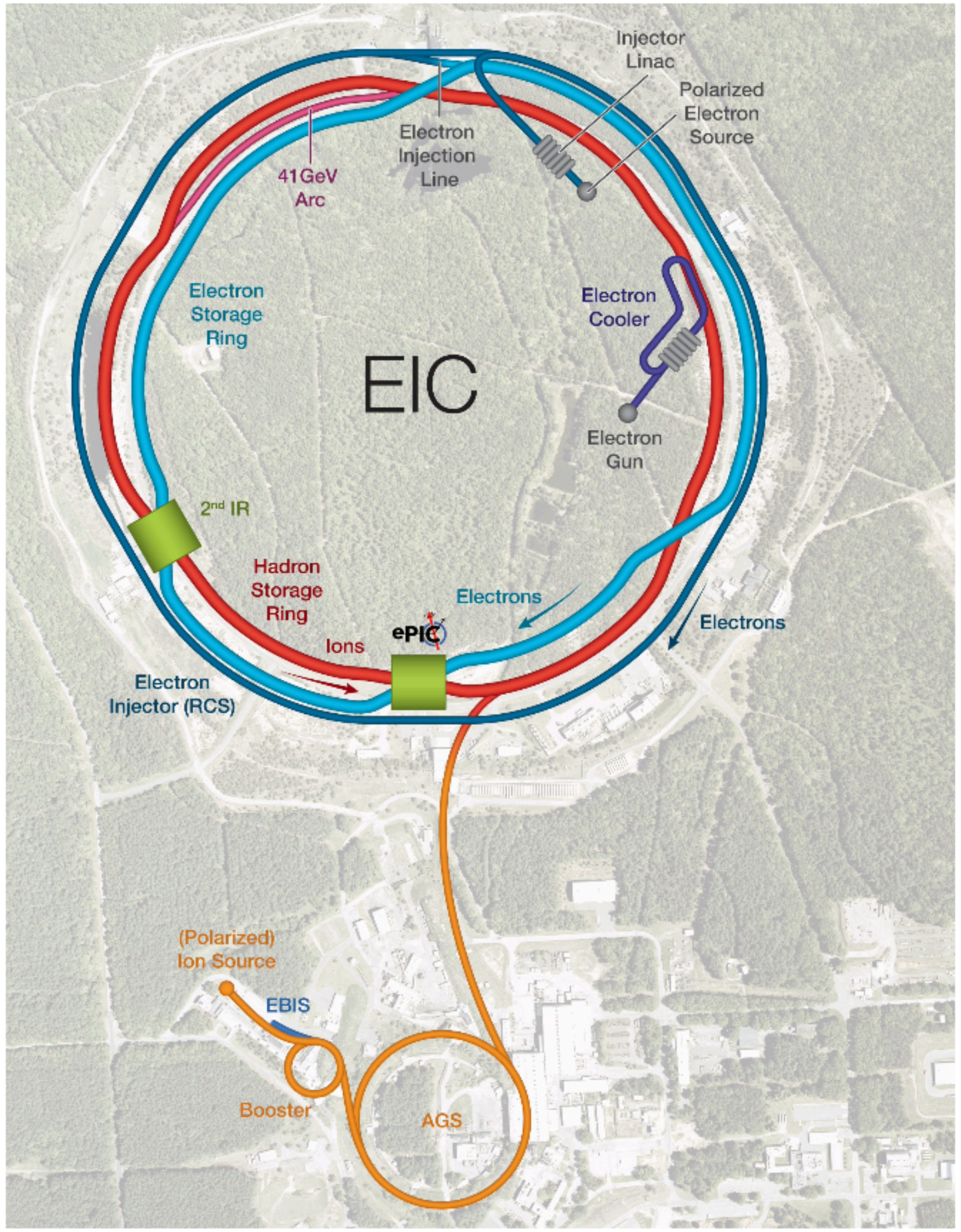
Why NNLO? - Precision physics at the Electron-Ion Collider

BNL-based EIC on its path towards construction

- High Luminosity: $\mathscr{L} = 10^{33} - 10^{34}\,\mathrm{cm}^{-2}\,\mathrm{sec}^{-1}$
- Center-of-mass energy range: $20 - 140\,\mathrm{GeV}$
- Highly polarized electron & light hadron beams

Unique access to the proton's spin structure in terms of helicity parton distributions!

Physical interpretation of EIC data will require an increased precision of theory predictions



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Calculations for polarized eP observables beyond NLO:

- NNLO structure functions g_1 (photon exchange)
  van Neerven, Zijlstra (1994)
- NNLO NC & CC structure functions g_1, g_4, g_5
  IB, de Florian, Pedron (2022)
- Approx. NNLO and N3LO Semi-Inclusive DIS
  Abele, de Florian, Vogelsang (2022)
- NNLO Single-Jet production
  NC and CC- IB, de Florian, Pedron (2023)
- NNLO Semi-Inclusive DIS
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This talk

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Parton distribution functions:

- NNLO polarized PDFs
  Taghavi-Shahri, Khanpour, Atashbar Tehrani, Alizadeh Yazdi (2016)
  Bertone, Chiefa, Nocera (2024)
  IB, de Florian, Sassot, Stratmann, Vogelsang (2024)
  J. Cruz-Martínez et al. (2025)

Introduction

Why NNLO?

BNL-b

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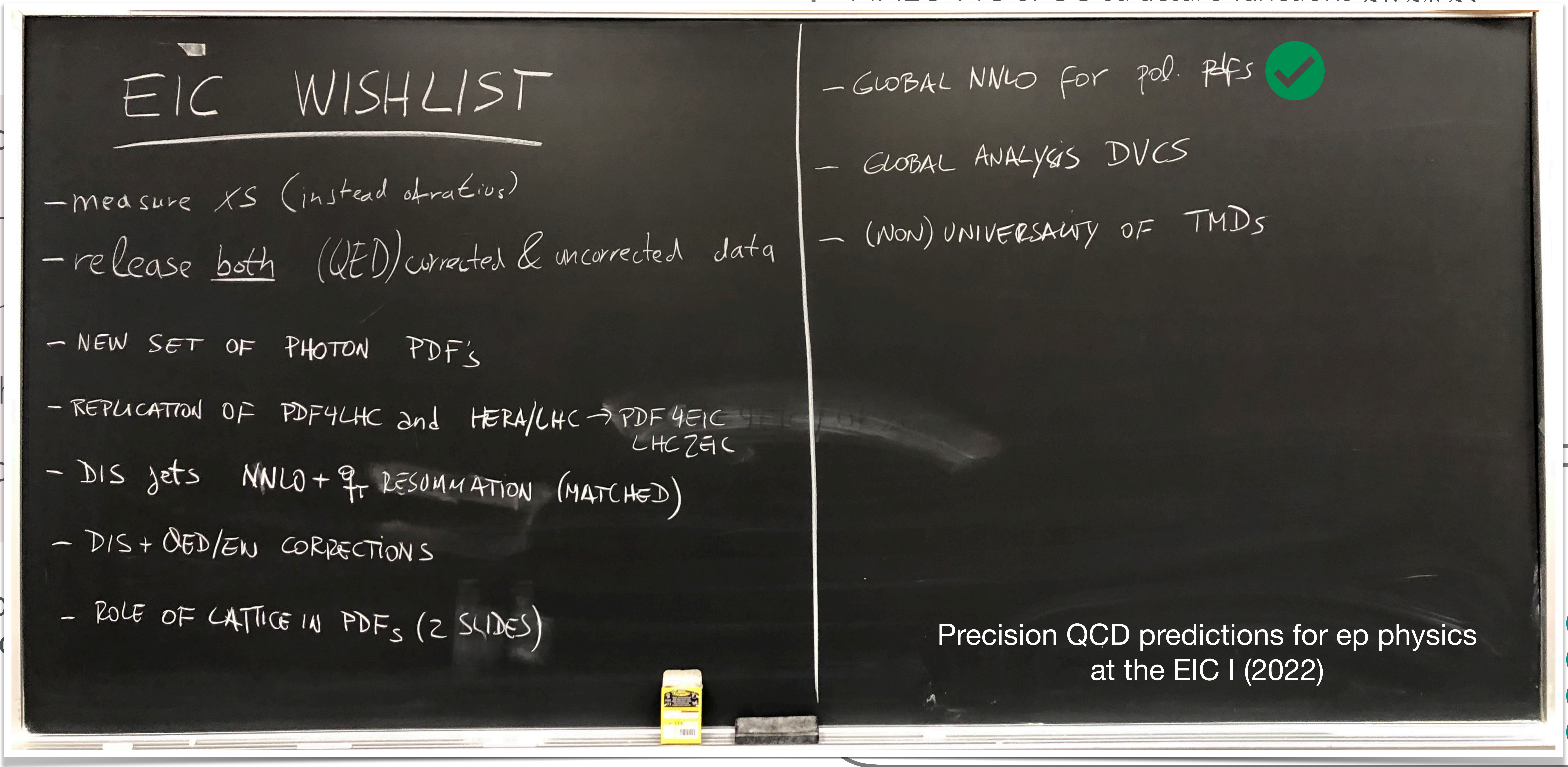

Precision QCD predictions for ep physics at the EIC I (2022)

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(2022)

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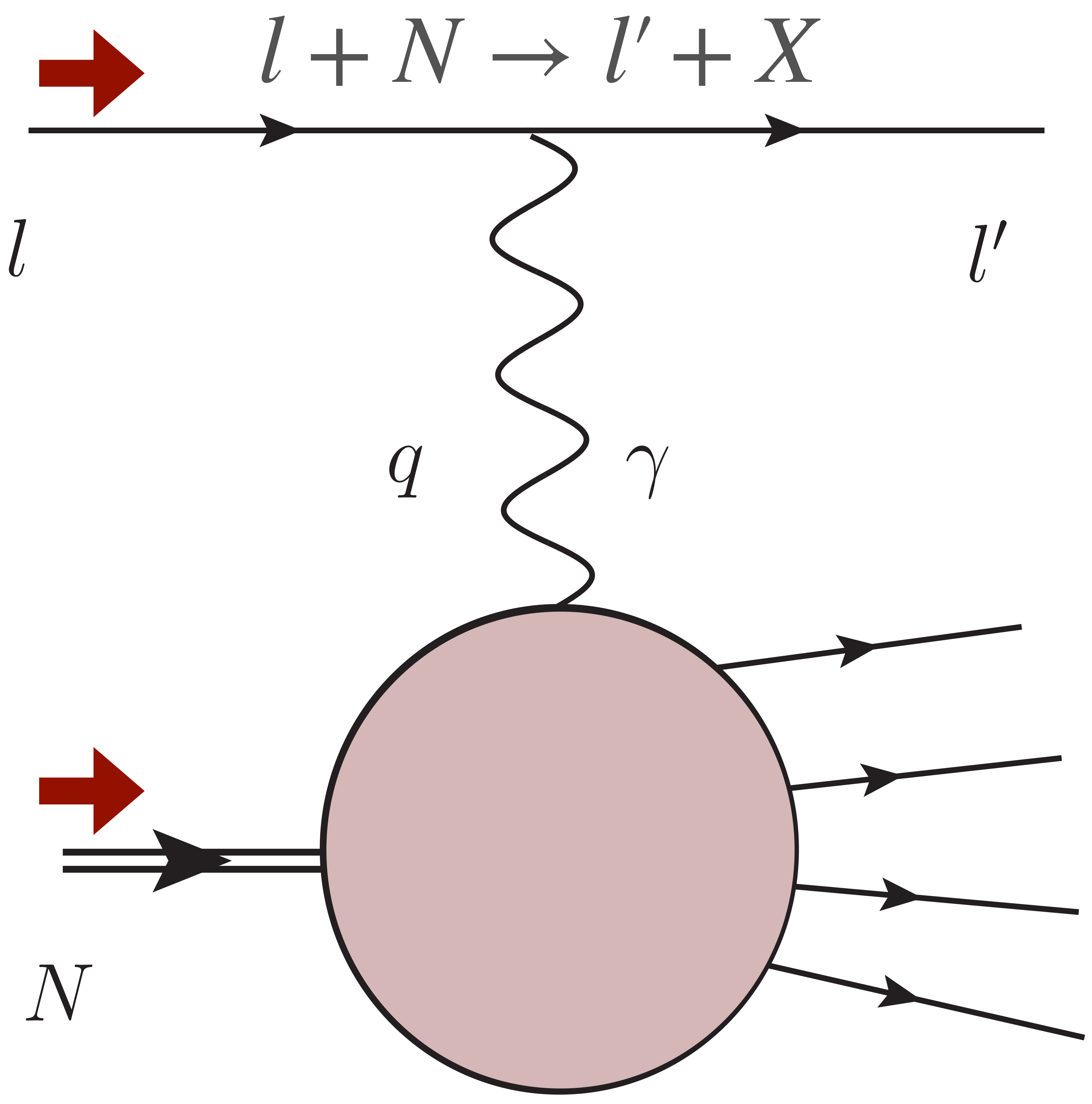
Global analysis of pPDFS

Global analyses of helicity PDFs

Accessing pPDFs in polarized high-energy scattering processes



$$\Delta\sigma \equiv \frac{1}{2}\left[\sigma^{\uparrow\uparrow} - \sigma^{\uparrow\downarrow}\right]$$

$$\Delta\sigma = \sum_a \int dz\, \Delta f_a(z, \mu_F^2)\, \Delta\hat{\sigma}_i(\alpha_S(\mu_R), \mu_F^2, \mu_R^2)$$

Polarized PDFs

$$\Delta f_a \equiv f_a^{\uparrow} - f_a^{\downarrow}$$

Polarized Partonic cross-section

$$\Delta\hat{\sigma} \equiv \frac{1}{2}[\hat{\sigma}^{\uparrow\uparrow} - \hat{\sigma}^{\uparrow\downarrow}]$$

$$\Delta f_a(\mu_F^2) = \int_0^1 \Delta f_a(x, \mu_F^2)\, dx$$

Contribution of parton a to the proton's spin

PDFs' scale dependence can be calculated perturbatively in QCD

$$\frac{\partial}{\partial \ln \mu^2} \Delta f_i(z, \mu^2) = \sum_j \int_z^1 \frac{dy}{y}\, \Delta P_{ji}(y, \alpha_S(\mu^2))\, \Delta f_j(\frac{z}{y}, \mu^2)$$

They can be determined at some input scale from a set of experimental measurements → QCD global analyses

Global analyses of helicity PDFs

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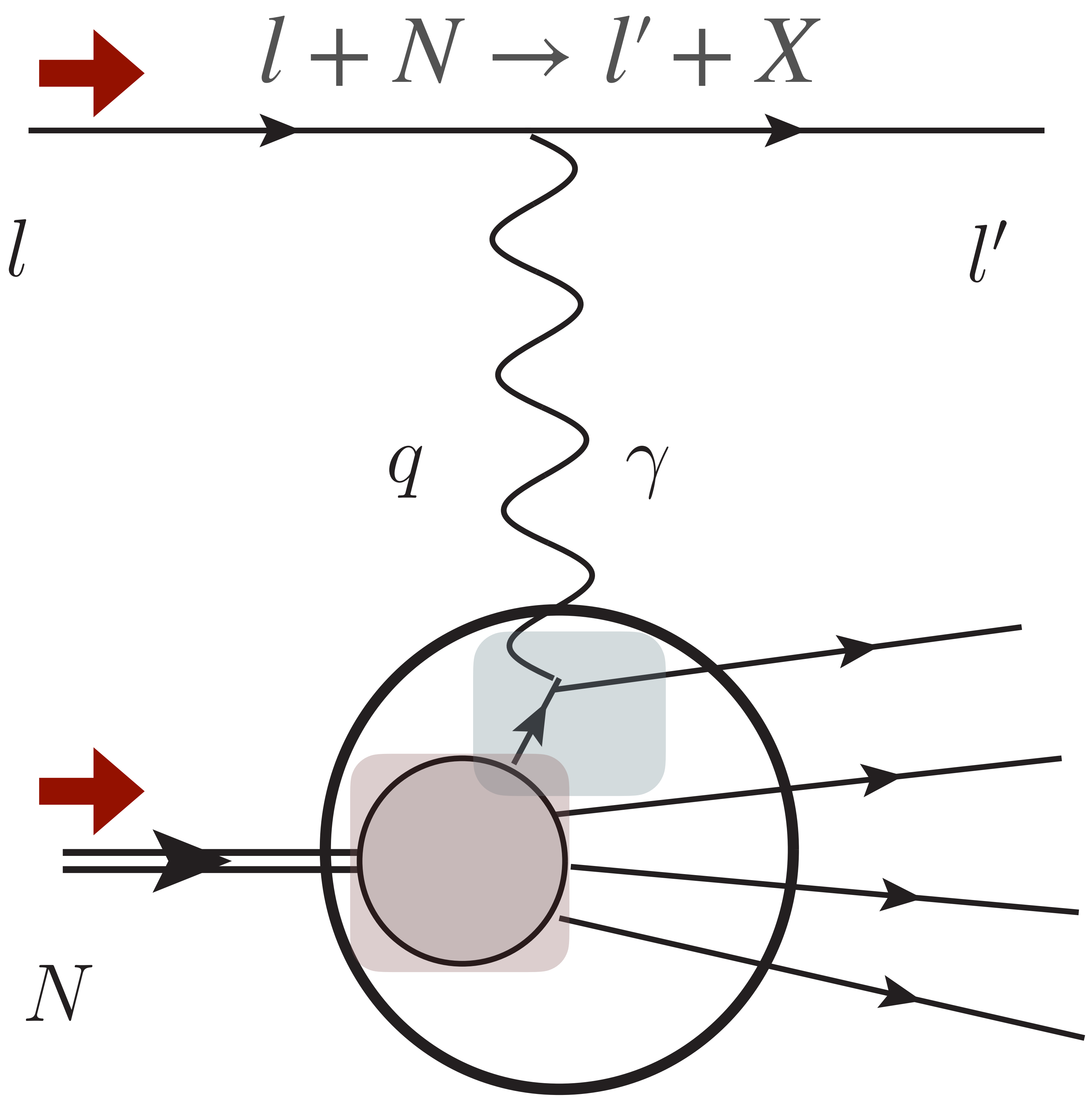

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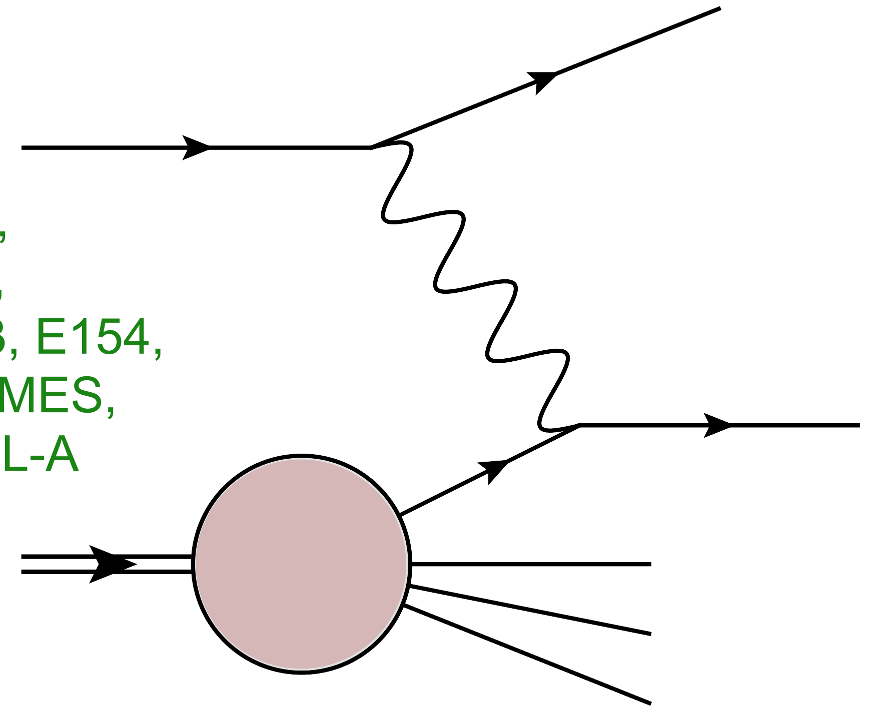
Global analyses of helicity PDFs

Accessing pPDFs in polarized high-energy scattering processes

DIS

EMC, SMC, COMPASS, E142, E143, E154, E155, HERMES, CLAS, HALL-A



- Mainly constrains $\Delta\Sigma \sim (\Delta q + \Delta\bar{q})$
- Only indirect constraints on Δg, limited flavour separation

First NLO QCD analyses based on DIS data:
[Gehrmann, Stirling; Glück, Reya, Stratmann, Vogelsang; Blümlein, Böttcher; Leader, Sidorov, Stamenov; Hirai, Kumano, Saito; Bourrely, Soffer, Bucella]

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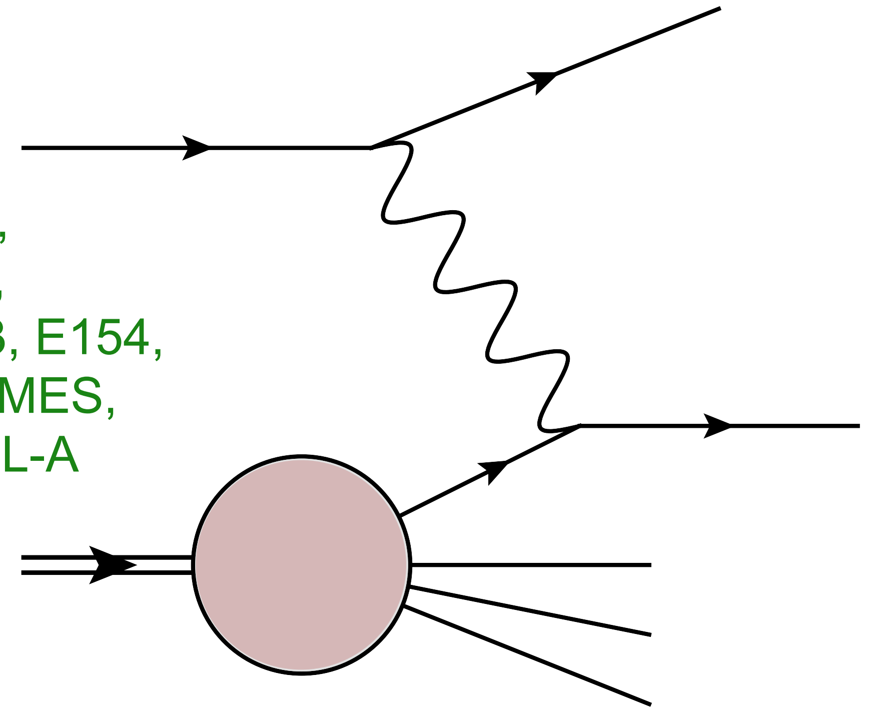

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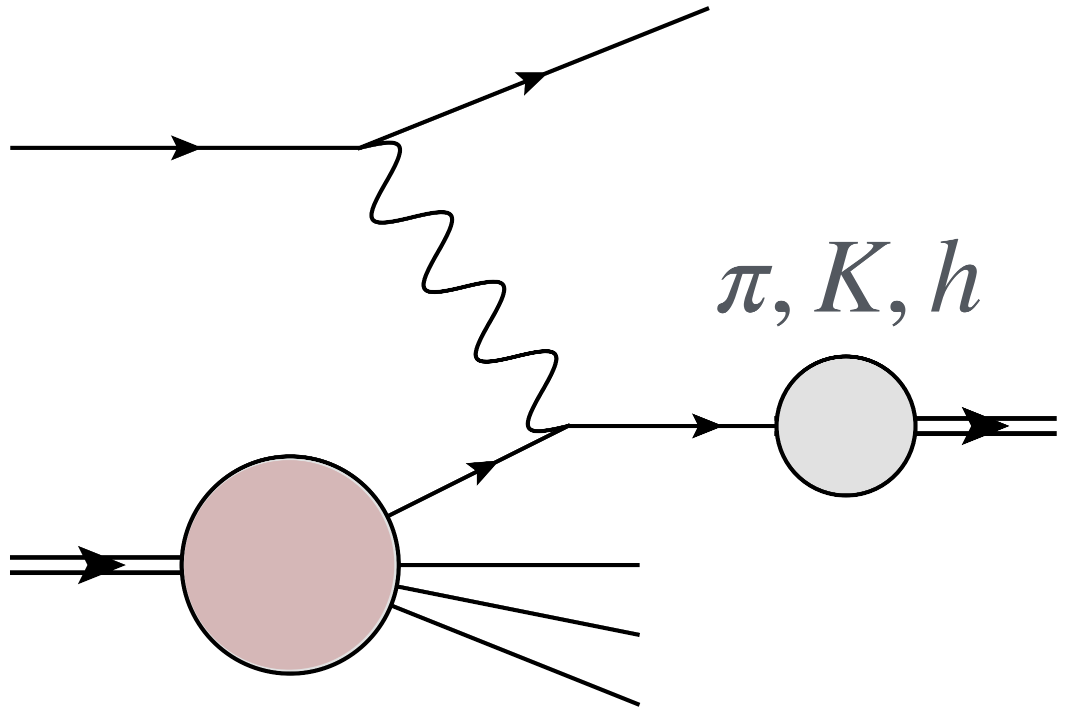

- Improved flavor discrimination
- Only indirect constraints on Δg

NLO QCD analyses based on combined DIS+SIDIS: data:

[de Florian, Sampayo, Sassot; de Florian, Navarro, Sassot]

Global analyses of helicity PDFs

Accessing pPDFs in polarized high-energy scattering processes

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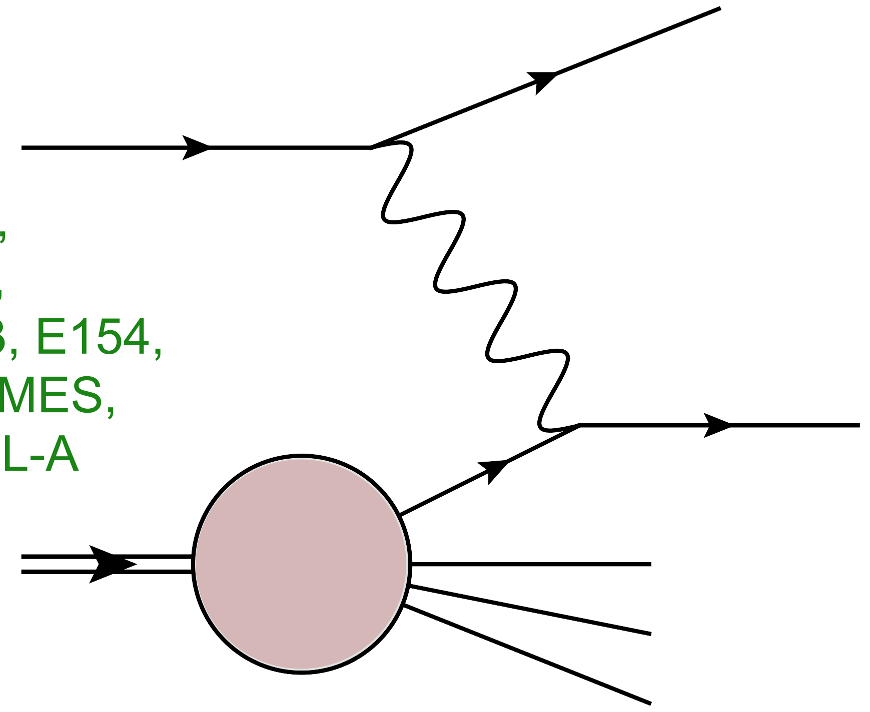
EMC, SMC, COMPASS, E142, E143, E154, E155, HERMES, CLAS, HALL-A



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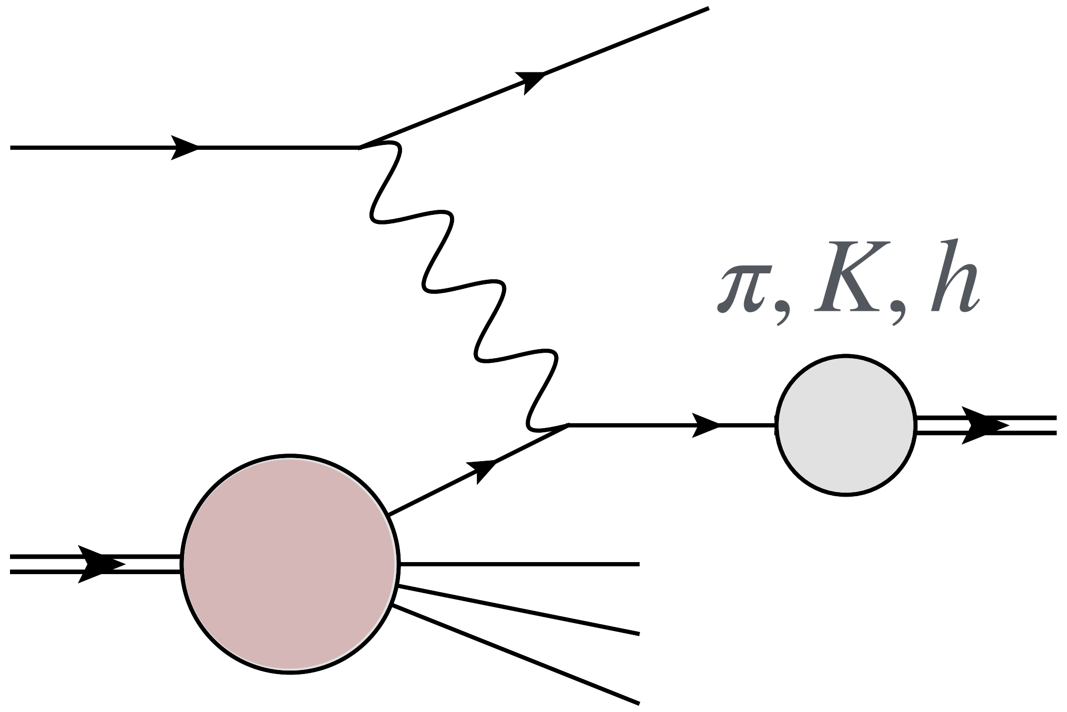
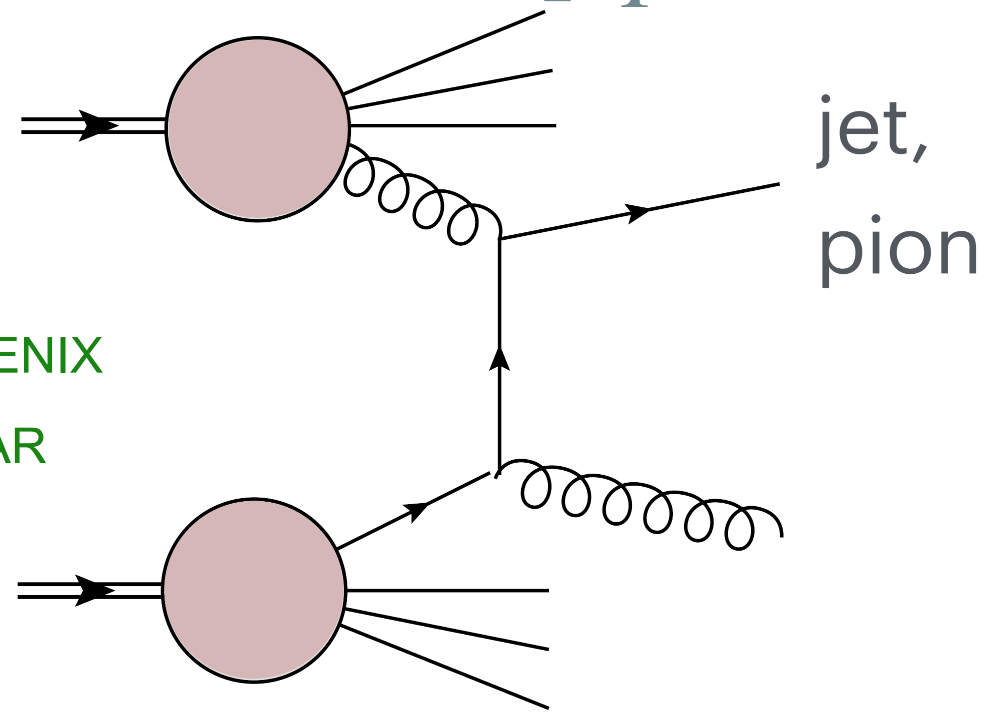


- Improved flavor discrimination
- Only indirect constraints on Δg

pp high-p_T

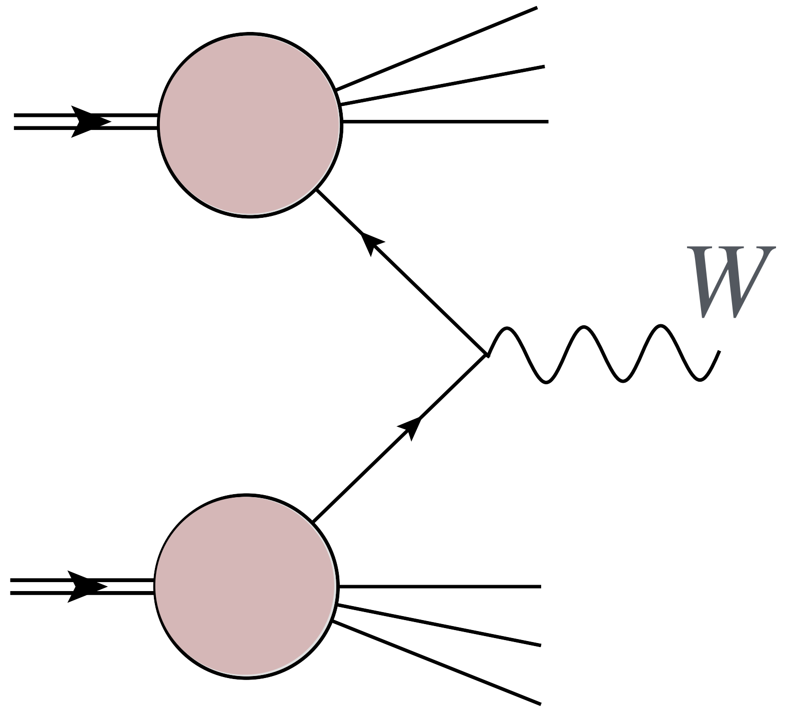
PHENIX

STAR



- Constraints on Δg
- Improved flavor discrimination

pp W boson

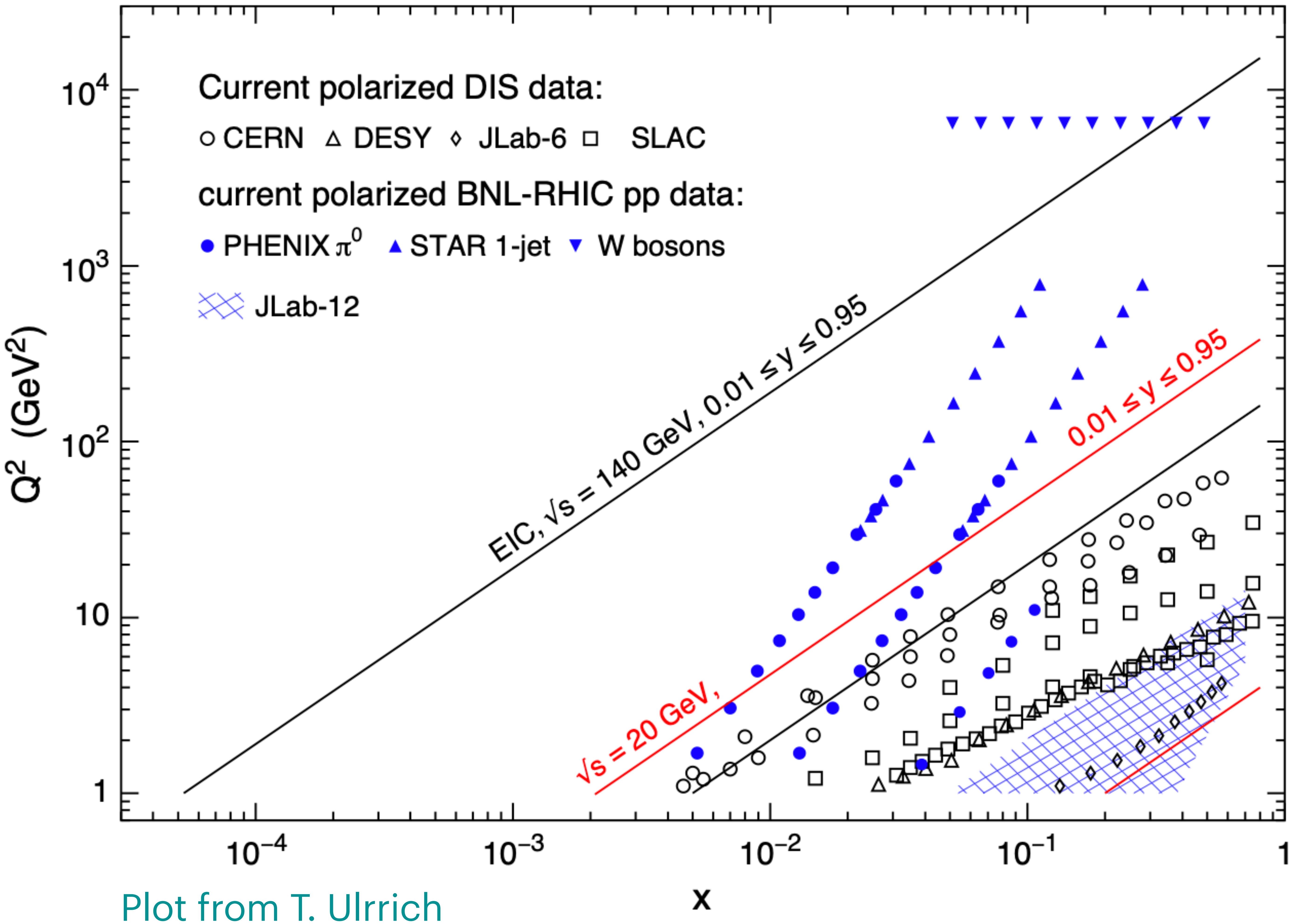


- Improved flavor discrimination

Fully global NLO analyses: [de Florian, Sassot, Stratmann, Vogelsang (2008-); Nocera, Ball Forte, Ridolfi, Rojo (2013-); Cocuzza, Ethier, Melnitchouk, Sato (2013-)] $\Rightarrow$ Mature analysis frameworks with robust estimation of uncertainties

Global analyses of helicity PDFs

Accessing pPDFs in polarized high-energy scattering processes



Plot from T. Ulrrich

Global analyses of helicity PDFs

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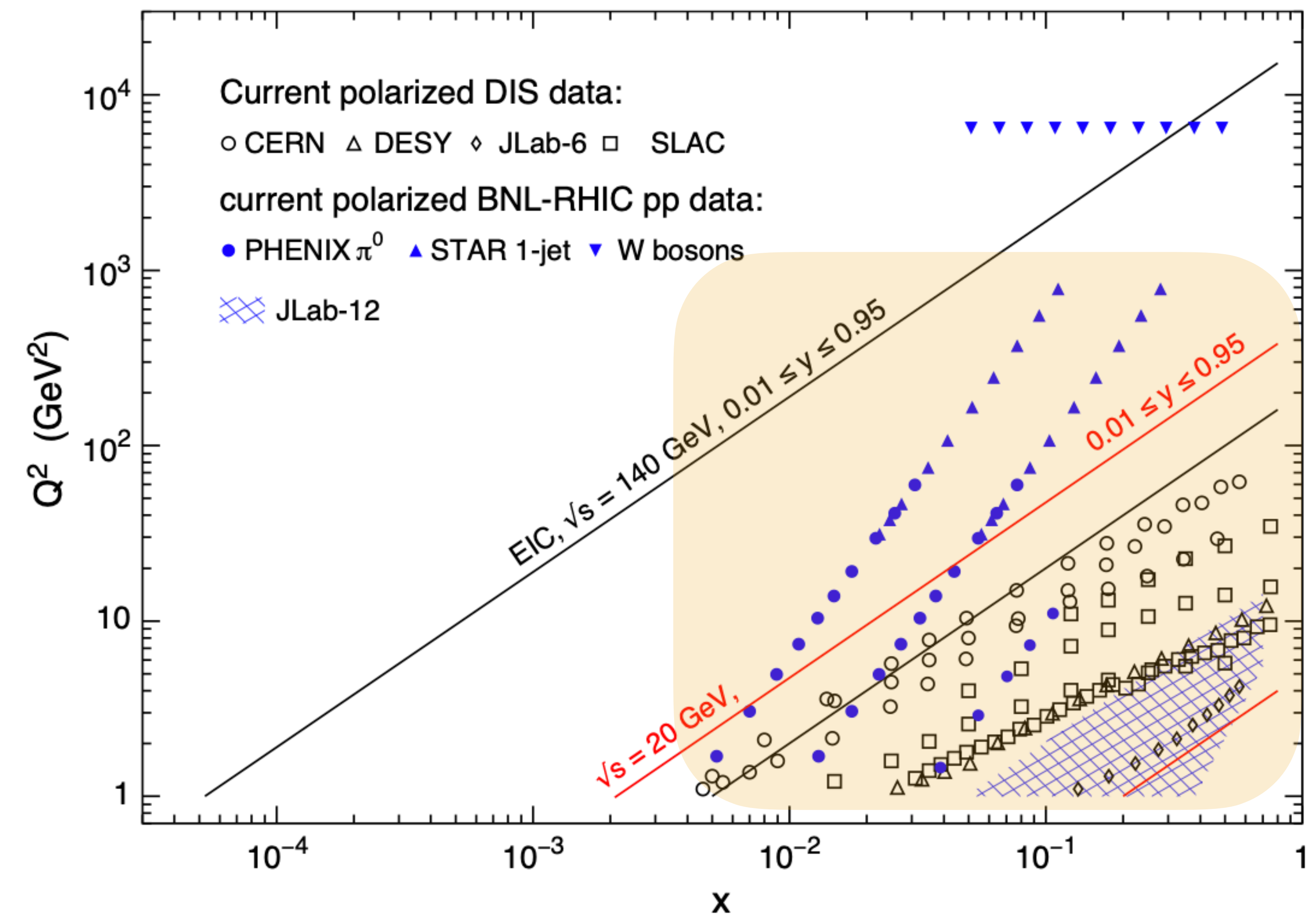
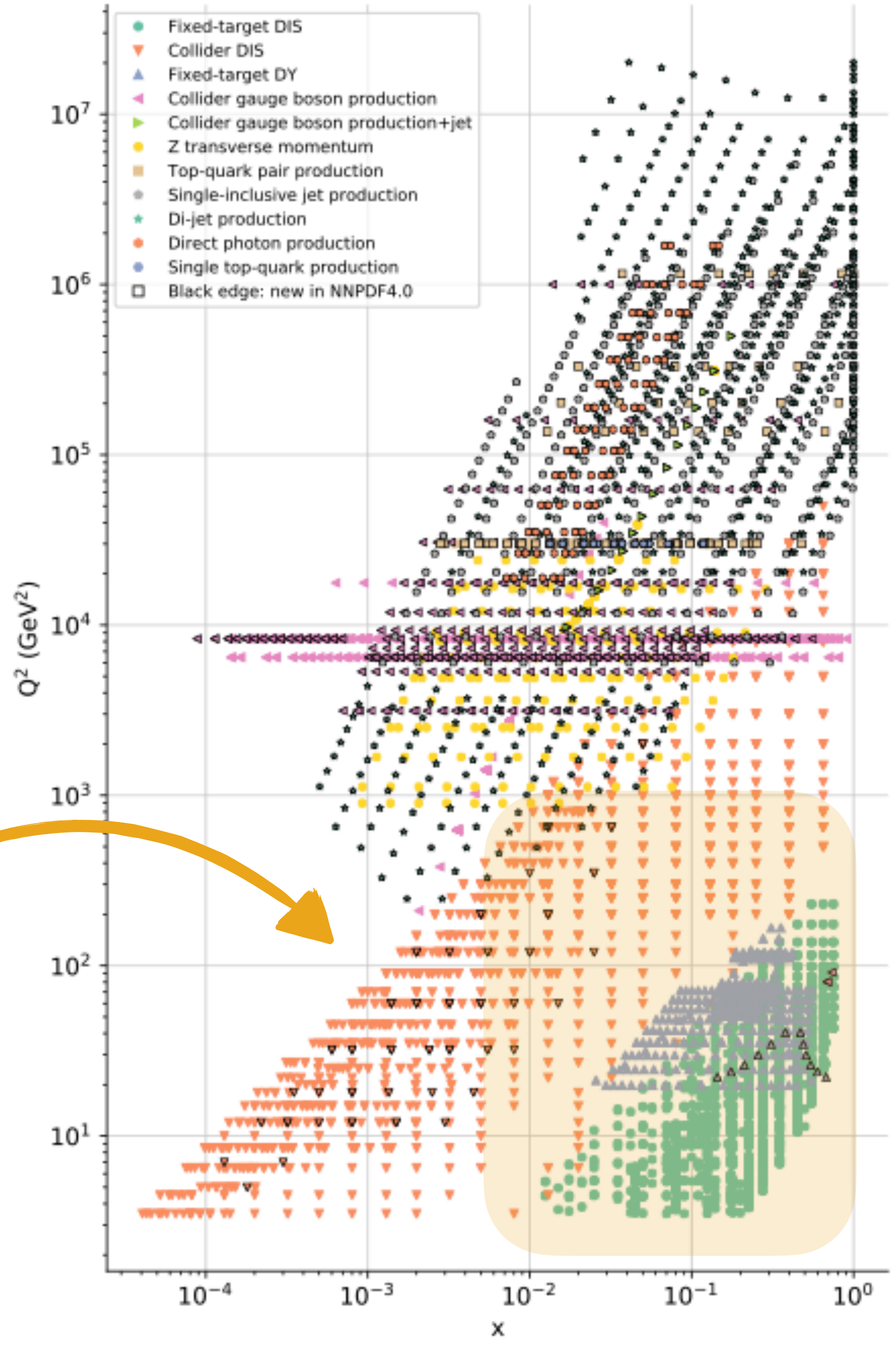
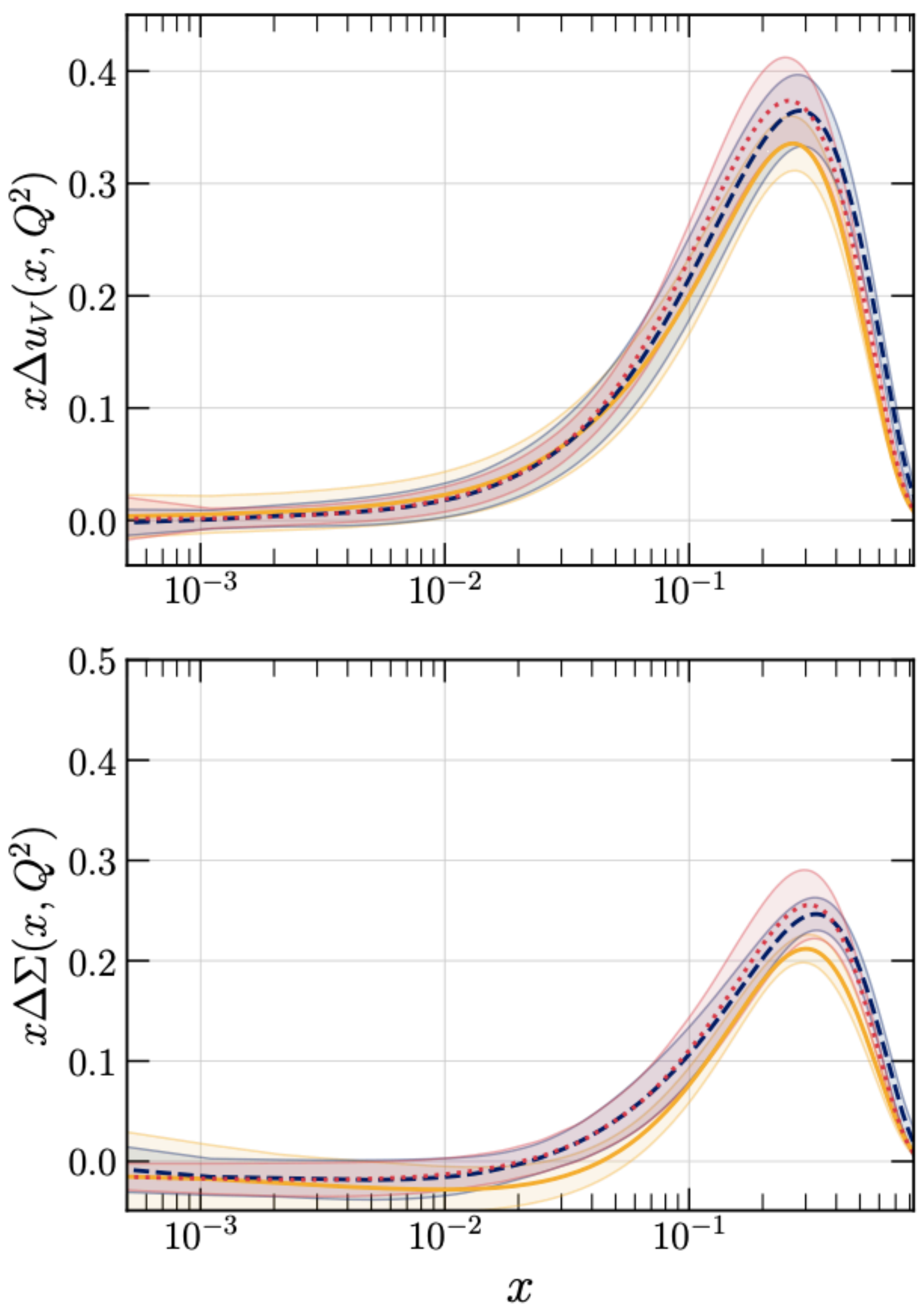




NNPDF4.0 EPJC 82 (2022)

Global analyses of helicity PDFs

How well do we know polarized PDFs?



[Hekhorn, Magni, Nocera, Rabemananjara, Rojo, Schaus, Stegeman (2024)]

- Well constrained $\Delta\Sigma$, and valence distributions

Global analyses of helicity PDFs

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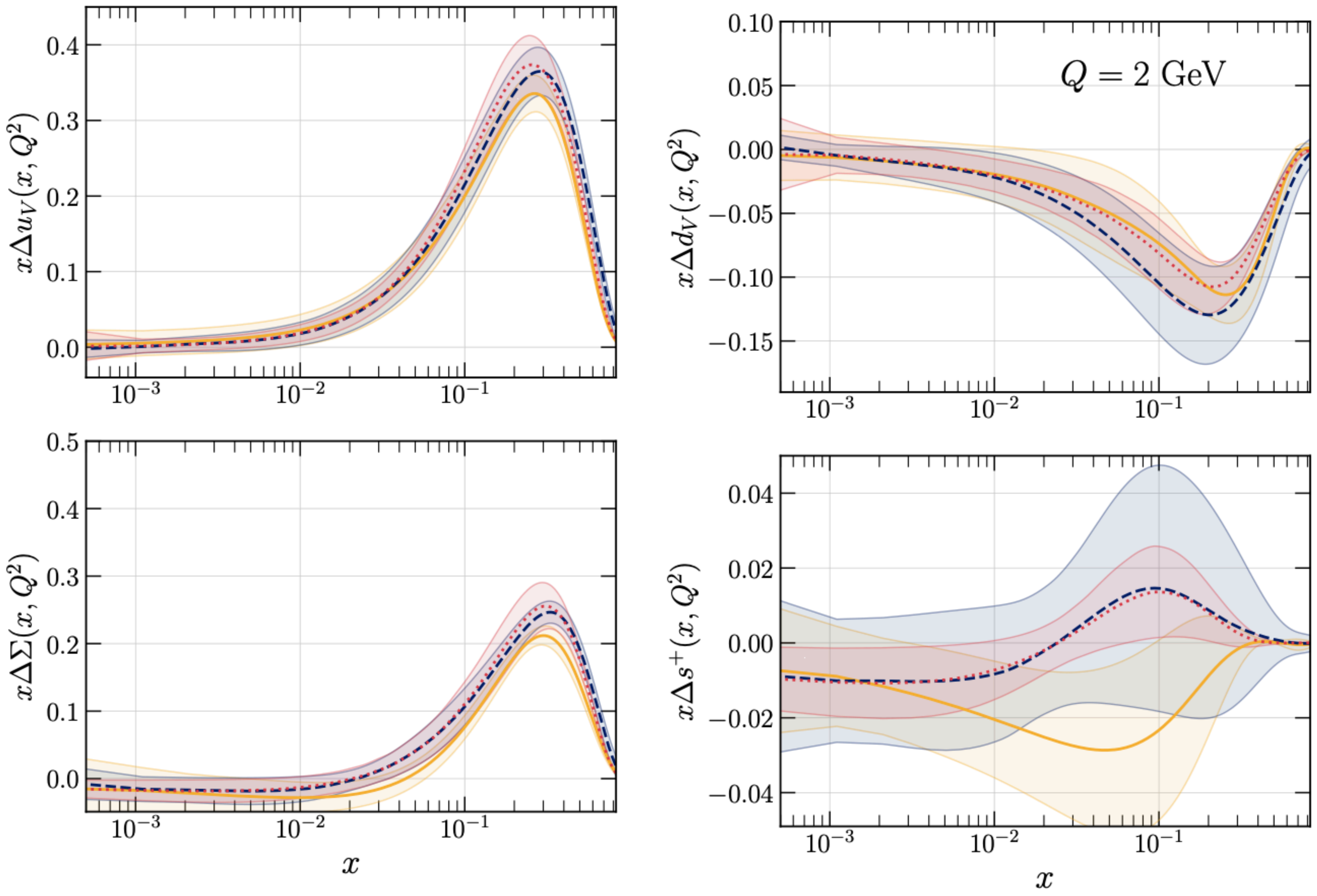

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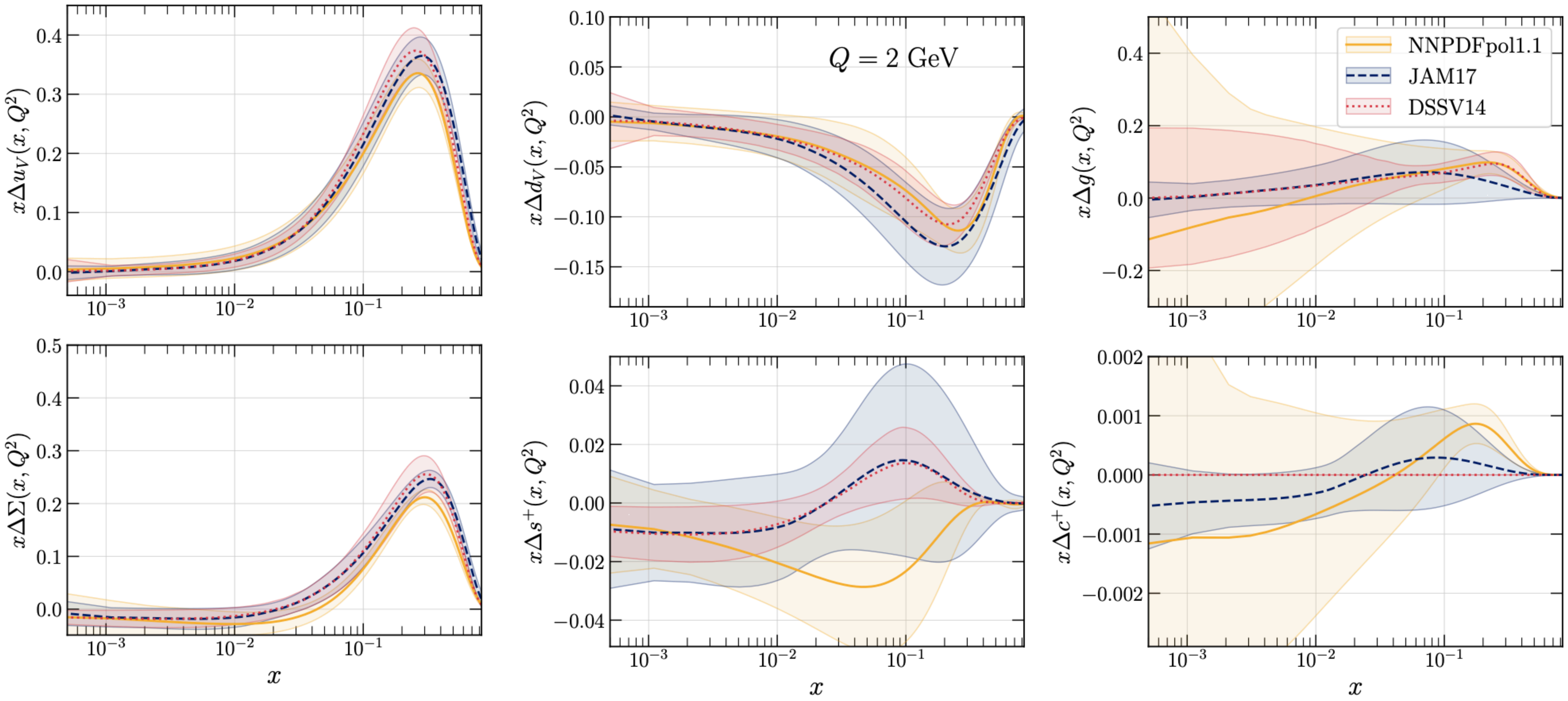
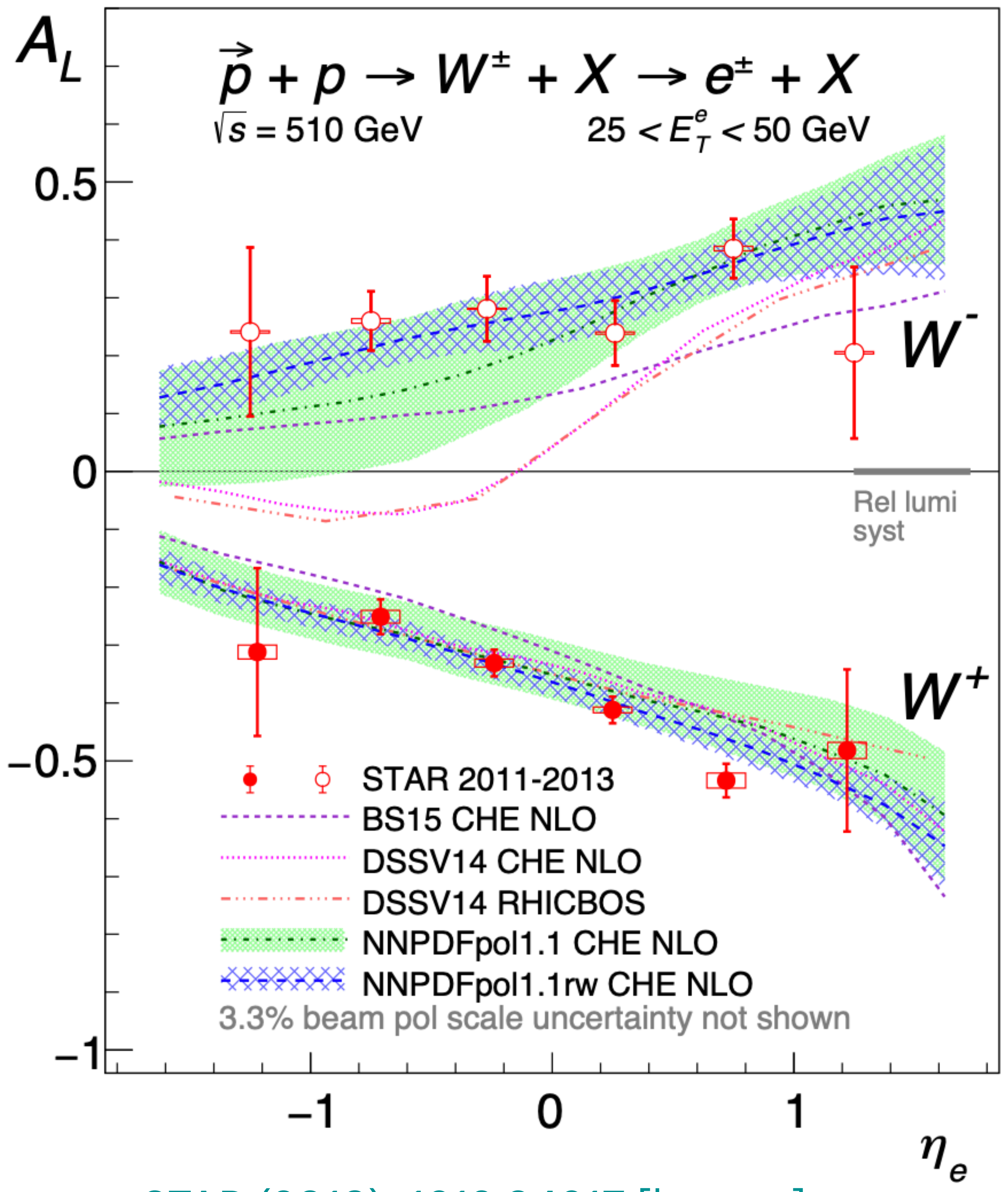


[Hekhorn, Magni, Nocera, Rabemananjara, Rojo, Schaus, Stegeman (2024)]

- Well constrained $\Delta\Sigma$, and valence distributions
- Still incomplete picture in terms of flavor separation
- Gluons determined only for $x \gtrsim 0.5$

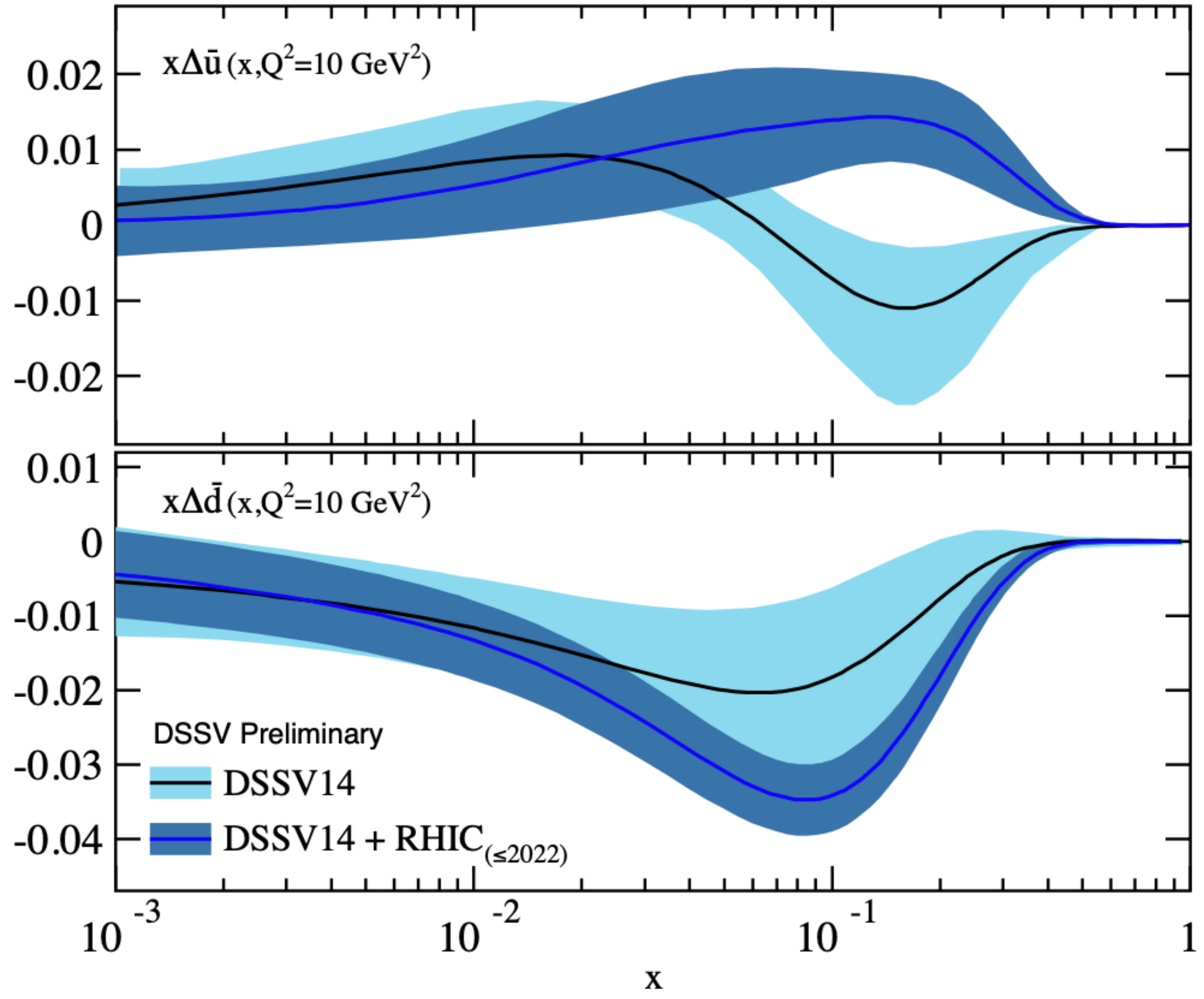
Global analyses of helicity PDFs

Recent measurements from RHIC



STAR (2019)- 1812.04817 [hep-ex]

The RHIC Cold QCD Program (2023)



Similar results in JAM analysis and NNPDpol1.1:
[Cocuzza, Melnitchouk, Metz, Sato (2023)]
[Nocera, Ball Forte, Ridolfi, Rojo (2014)]

Global analyses of helicity PDFs

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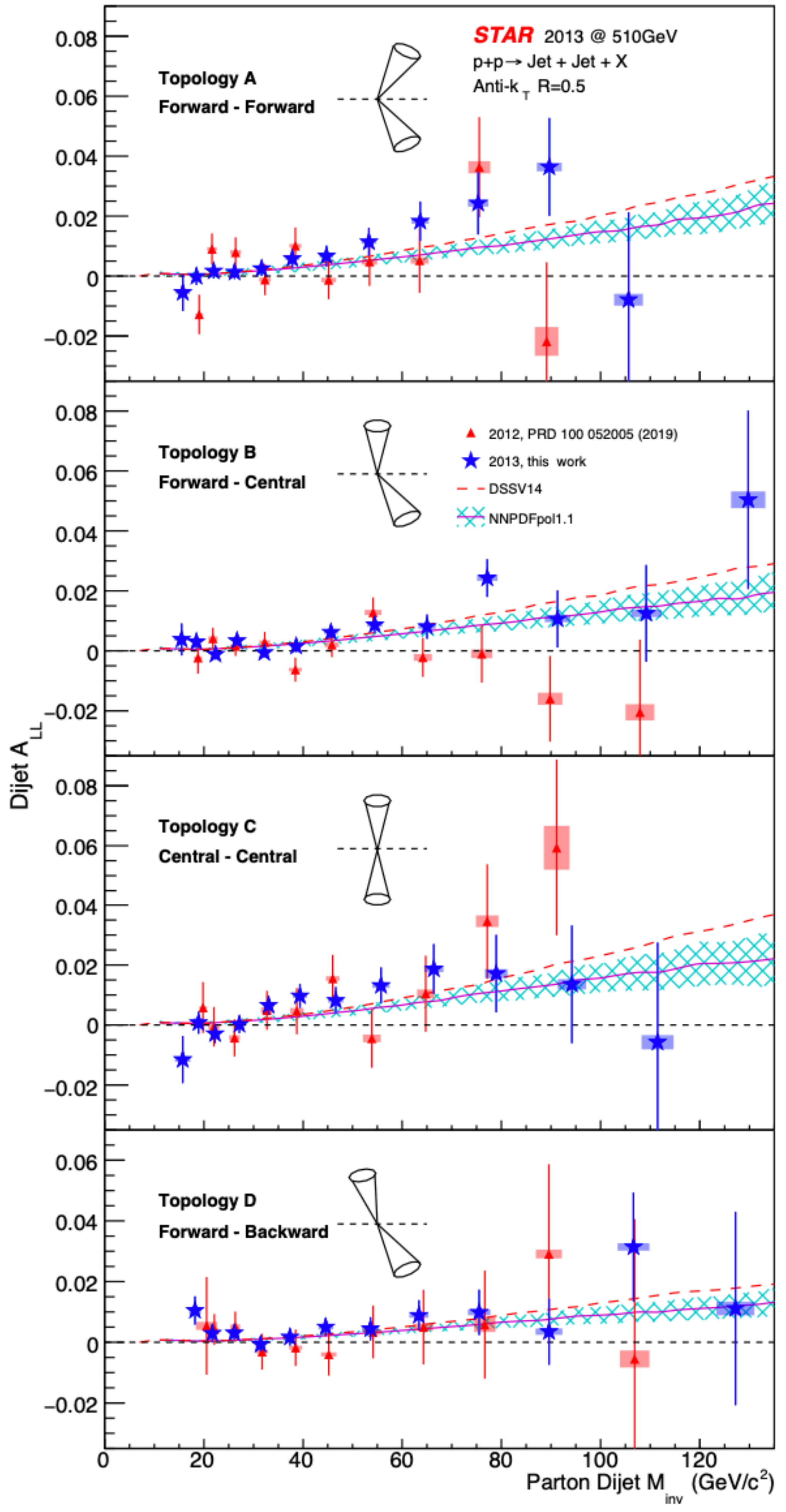
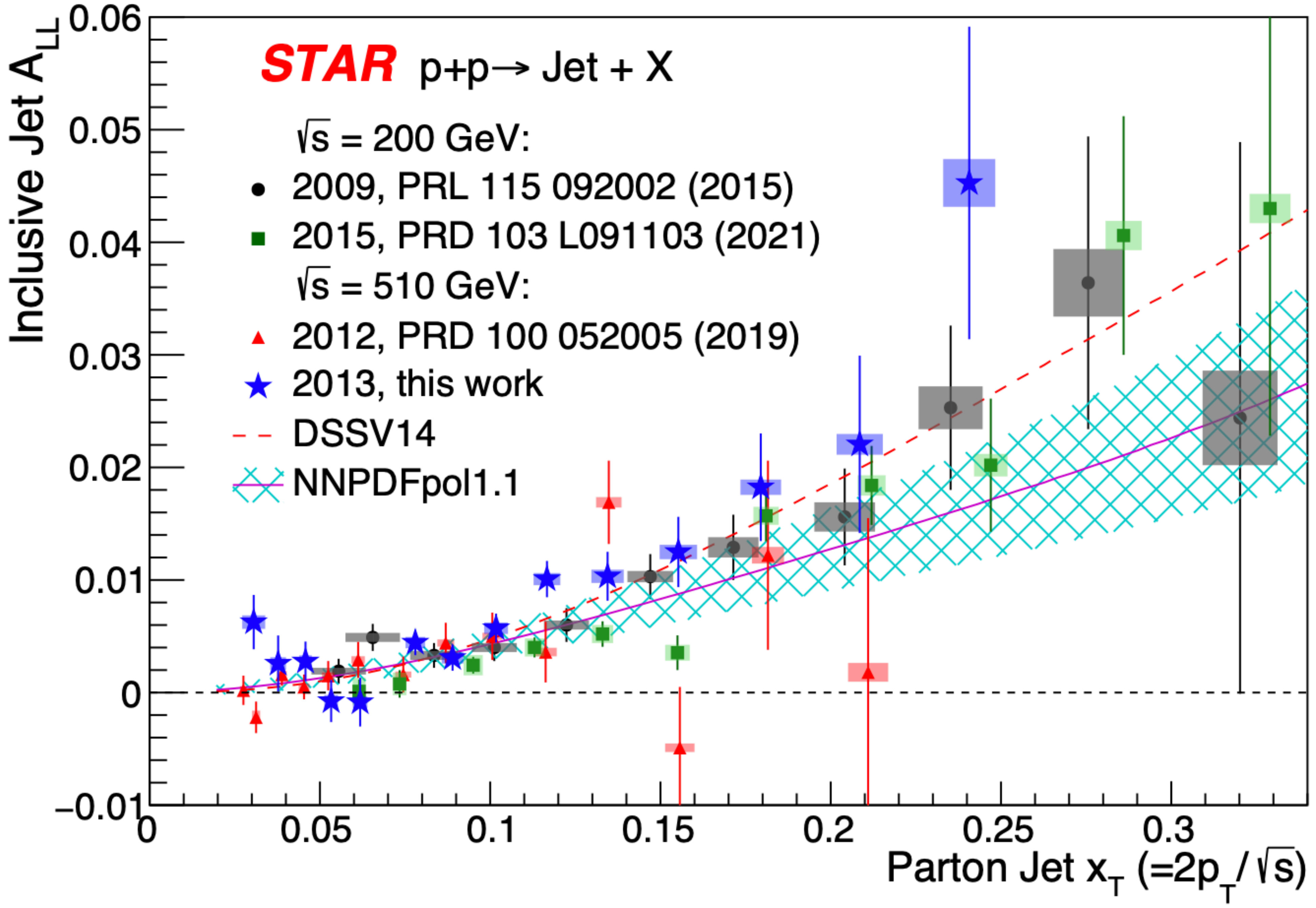
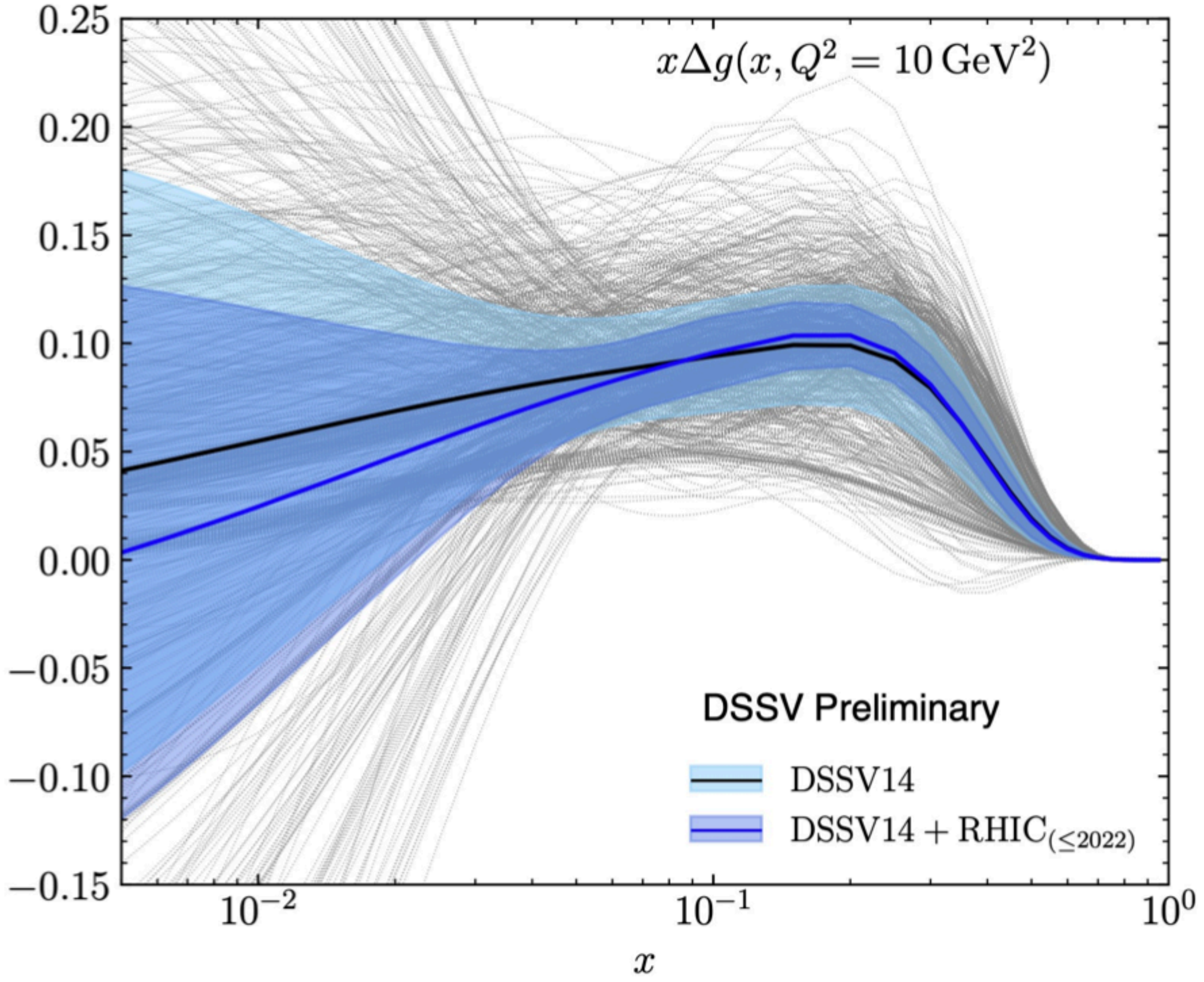




STAR (2021)- 2110.11020 [hep-ex]



The RHIC Cold QCD Program (2023)

DSSV14 : $\Delta g > 0$ driven by RHIC data → Improvement and consolidation

From NLO to NNLO

From NLO to NNLO

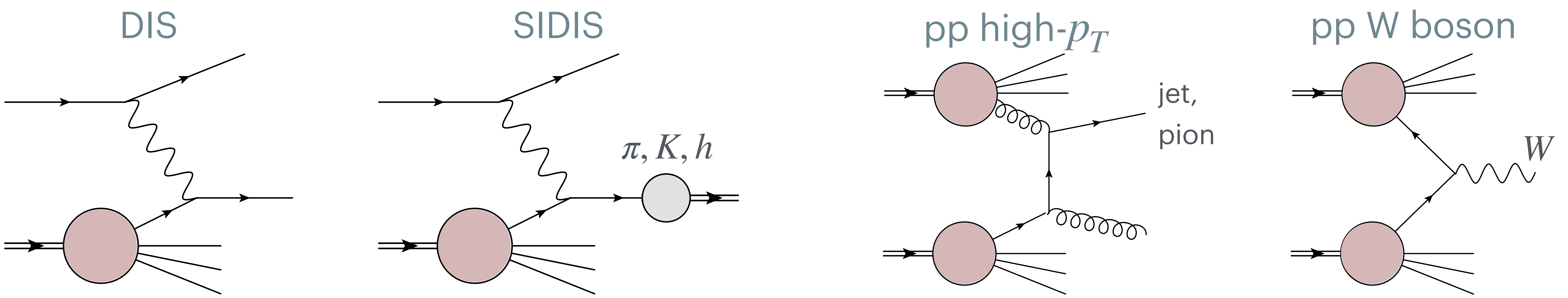
Ingredients for NNLO

- PDF evolution kernels

$$\Delta P_{ij} = \frac{\alpha_s}{2\pi}\Delta P_{ij}^{\mathrm{LO}} + \left(\frac{\alpha_s}{2\pi}\right)^2 \Delta P_{ij}^{\mathrm{NLO}} + \left(\frac{\alpha_s}{2\pi}\right)^3 \Delta P_{ij}^{\mathrm{NNLO}} + \dots$$

- Partonic hard scattering:

$$\Delta\hat{\sigma}_{ab} = \Delta\hat{\sigma}_{ab}^{\mathrm{LO}} + \frac{\alpha_s}{\pi}\Delta\hat{\sigma}_{ab}^{\mathrm{NLO}} + \left(\frac{\alpha_s}{\pi}\right)^2 \Delta\hat{\sigma}_{ab}^{\mathrm{NNLO}} + \dots$$



From NLO to NNLO

NNLO PDF evolution

$$\frac{\partial}{\partial \ln \mu^2} \Delta f_i(x,\mu^2) = \Delta P_{ji} \otimes \Delta f_j(x,\mu^2) \qquad \Delta P_{ij} = \frac{\alpha_s}{2\pi}\Delta P_{ij}^{\text{LO}} + \left(\frac{\alpha_s}{2\pi}\right)^2 \Delta P_{ij}^{\text{NLO}} + \left(\frac{\alpha_s}{2\pi}\right)^3 \Delta P_{ij}^{\text{NNLO}} + \ldots$$

- NNLO polarized evolution kernels known [Moch, Vermaseren, Vogt (2008, 2014, 2015)]; [Blümlein, Marquard, Schneider, Schönwald (2022)].
- OMEs for matching conditions at mass thresholds known [Bierenbaum, Blümlein, De Freitas, Goedicke, Klein (2022)].
- NNLO evolution of pPDFs implemented in different libraries [Vogt (2004)].
  - Extended PEGASUS [Vogt (2004)].
  - APFEL [Bertone, Carrazza, Rojo (2013)] [Bertone (2017)]
  - EKO [Candido, Hekhorn, Magni (2022)].

From NLO to NNLO

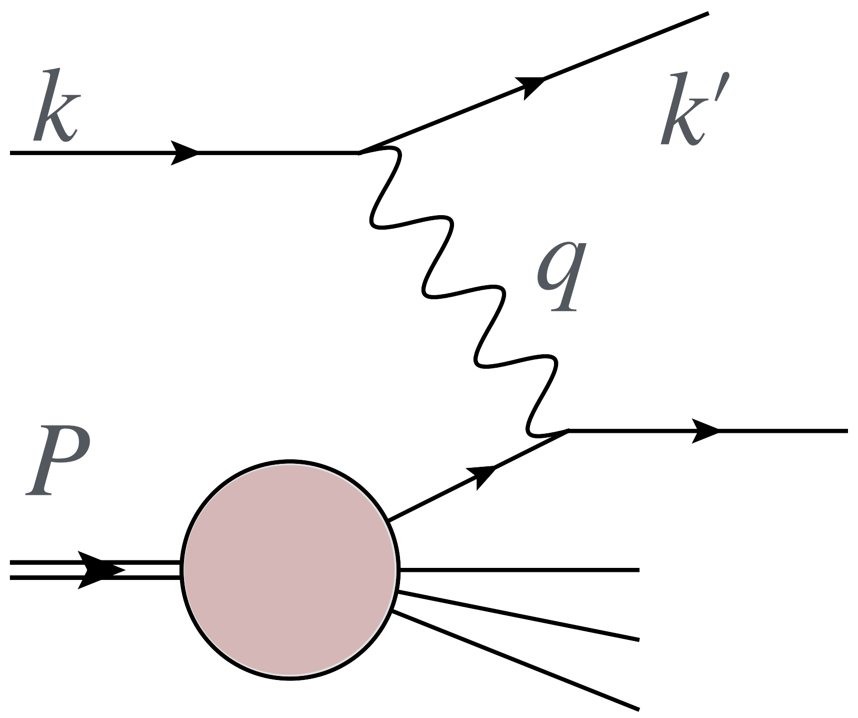
NNLO Coefficient functions for DIS

$$\frac{d^2\Delta\sigma}{dx\,dy} = \frac{8\pi\alpha^2}{Q^2}\left[(2-y)\,g_1(x,Q^2)\right] \qquad g_1(x,Q^2) = \sum_{f=q,g} \Delta C^f(x,Q^2) \otimes \Delta f(x,Q^2)$$

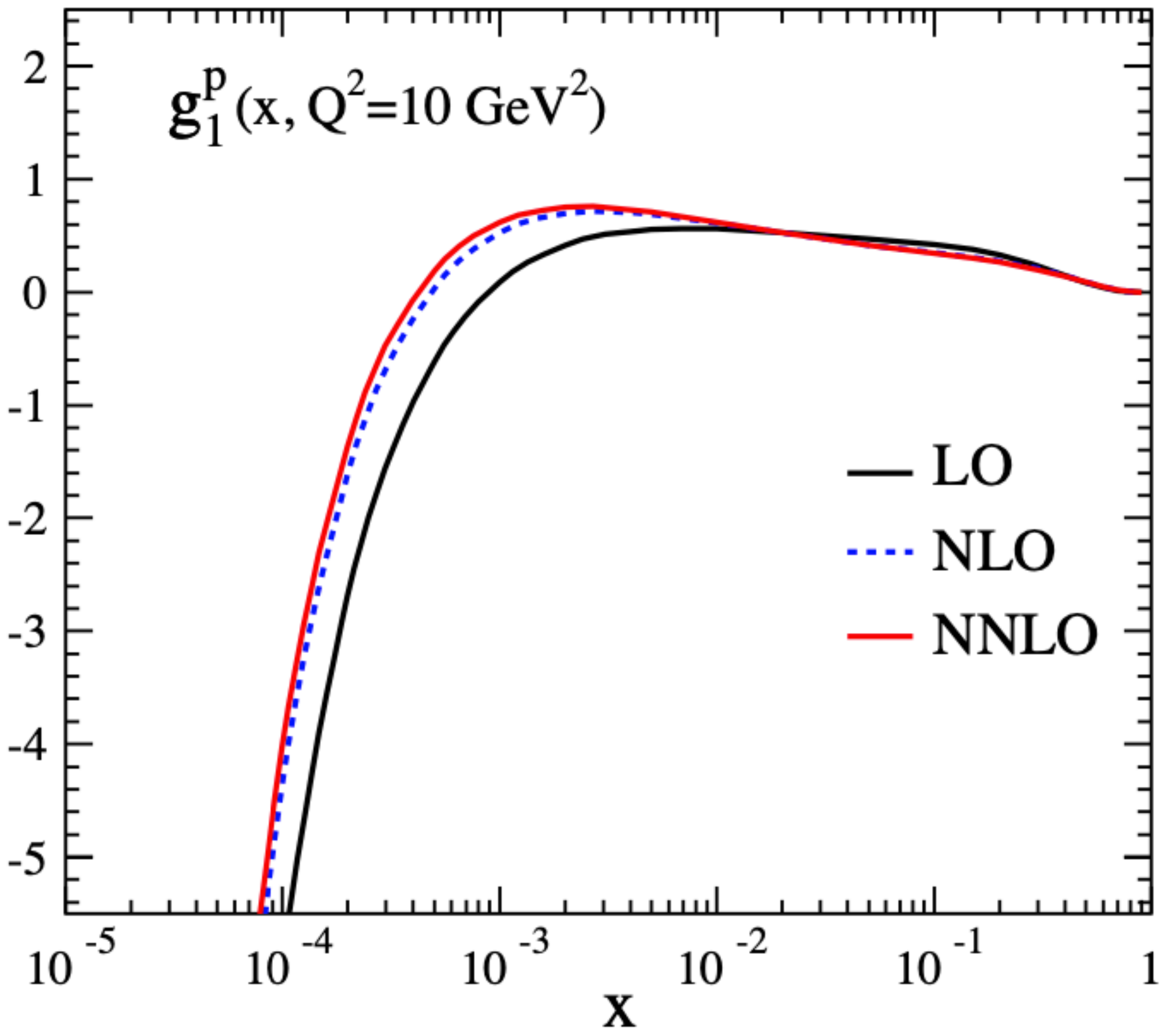
$$\Delta C^f = \Delta C^{f,(0)} + \frac{\alpha_s}{2\pi}\Delta C^{f,(1)} + \left(\frac{\alpha_s}{2\pi}\right)^2 \Delta C^{f,(2)} + \ldots$$

- NNLO corrections to $g_1(x,Q^2)$ known [van Neerven, Zijlstra (1994)] (N3LO corrections recently obtained for Larin scheme [Blümlein, Marquard, Schneider, Schönwald (2023)]).

DIS



$$Q^2 = -q^2 \qquad x = \frac{Q^2}{2P\cdot q}$$



From NLO to NNLO

NNLO Coefficient functions for SIDIS

$$\frac{d^3\Delta\sigma^h}{dx\,dy\,dz} = \frac{4\pi\alpha^2}{Q^2}\left[(2-y)\,g_1^h(x,z,Q^2)\right] \qquad g_1^h(x,z,Q^2) = \sum_{f,f'} \Delta f(x,Q^2) \otimes \Delta C_{ff'}(x,z,Q^2) \otimes D_{f'}^h$$

$$\Delta C_{ff'} = \Delta C_{ff'}^{(0)} + \frac{\alpha_s}{2\pi}\Delta C_{ff'}^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^2 \Delta C_{ff'}^{(2)} + \dots$$

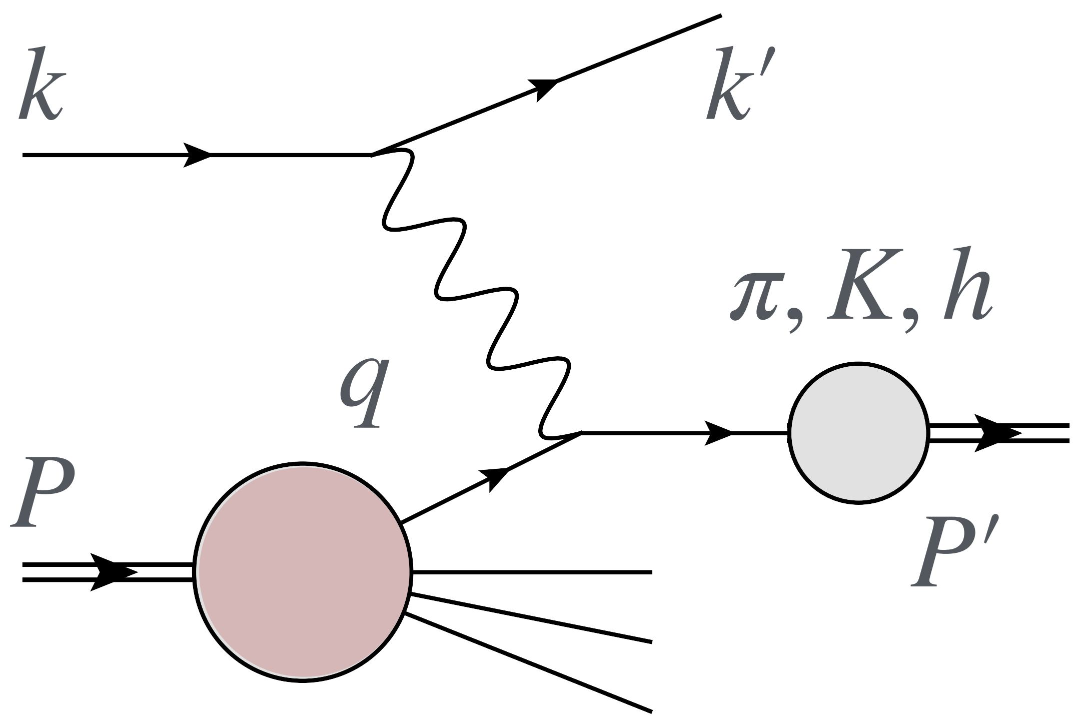
- Soft gluon approximate NNLO [Anderle, Ringer, Vogelsang (2012)]; [Abelde, de Florian, Vogelsang (2021)].

- NNLO coefficients recently obtained [Bonino, Gehrmann, Löchner, Schönwald, Stagnitto (2024)]; [Goyal, Lee, Much, Pathak, Rana, Ravindran (2024)]) ⇒ Soon to be included in global analyses of FFs and pPDFs.

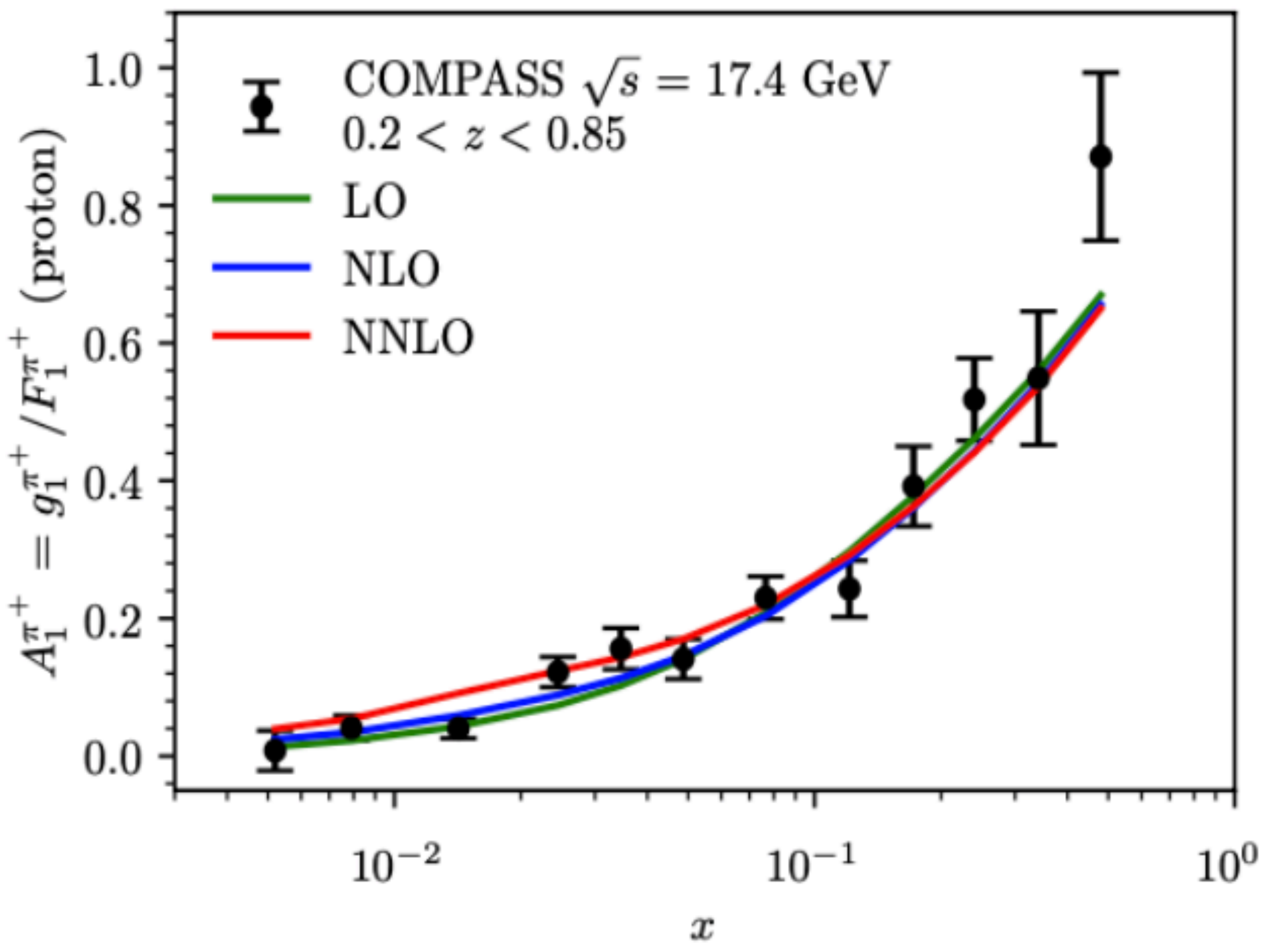
- NNLO FFs available, but ... until recently based only on SIA [Anderle, Ringer, Stratmann (2015)]; [Bertone, Carrazza, Hartland, Nocera, Rojo (2017)], or SIA+approx. SIDIS [IB, de Florian, Sassot, Stratmann, Vogelsang (2021)], [Abdul-Khalek, Bertone, Khoudii, Nocera (2021)].

⇒ FFs based on SIA+ SIDIS [Gao, Shen, Xing, Zhao, Zhou (2025)]

SIDIS



$$Q^2 = -q^2 \qquad x = \frac{Q^2}{2P\cdot q} \qquad z = \frac{P\cdot P_h}{P\cdot q}$$

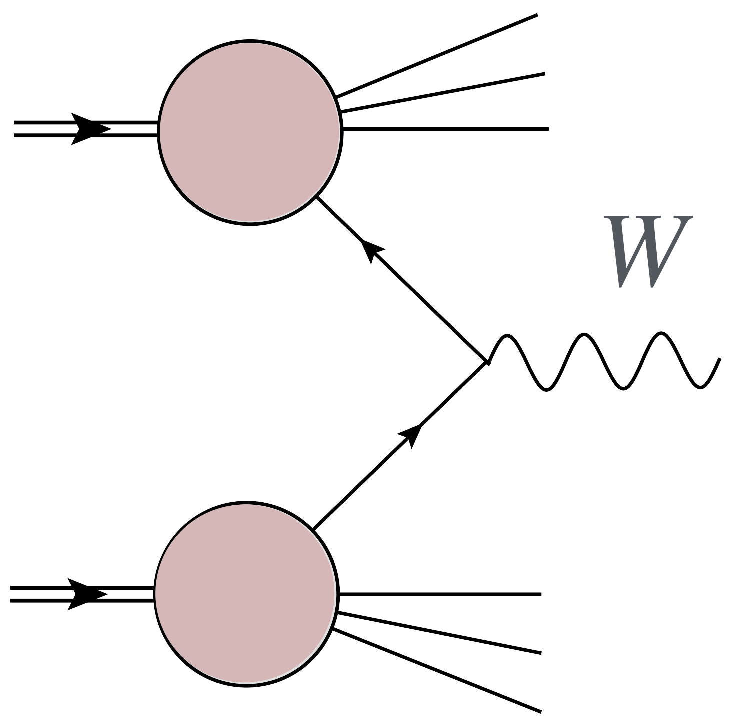


See S.Moch's talk

From NLO to NNLO

NNLO corrections for pp observables

pp W boson

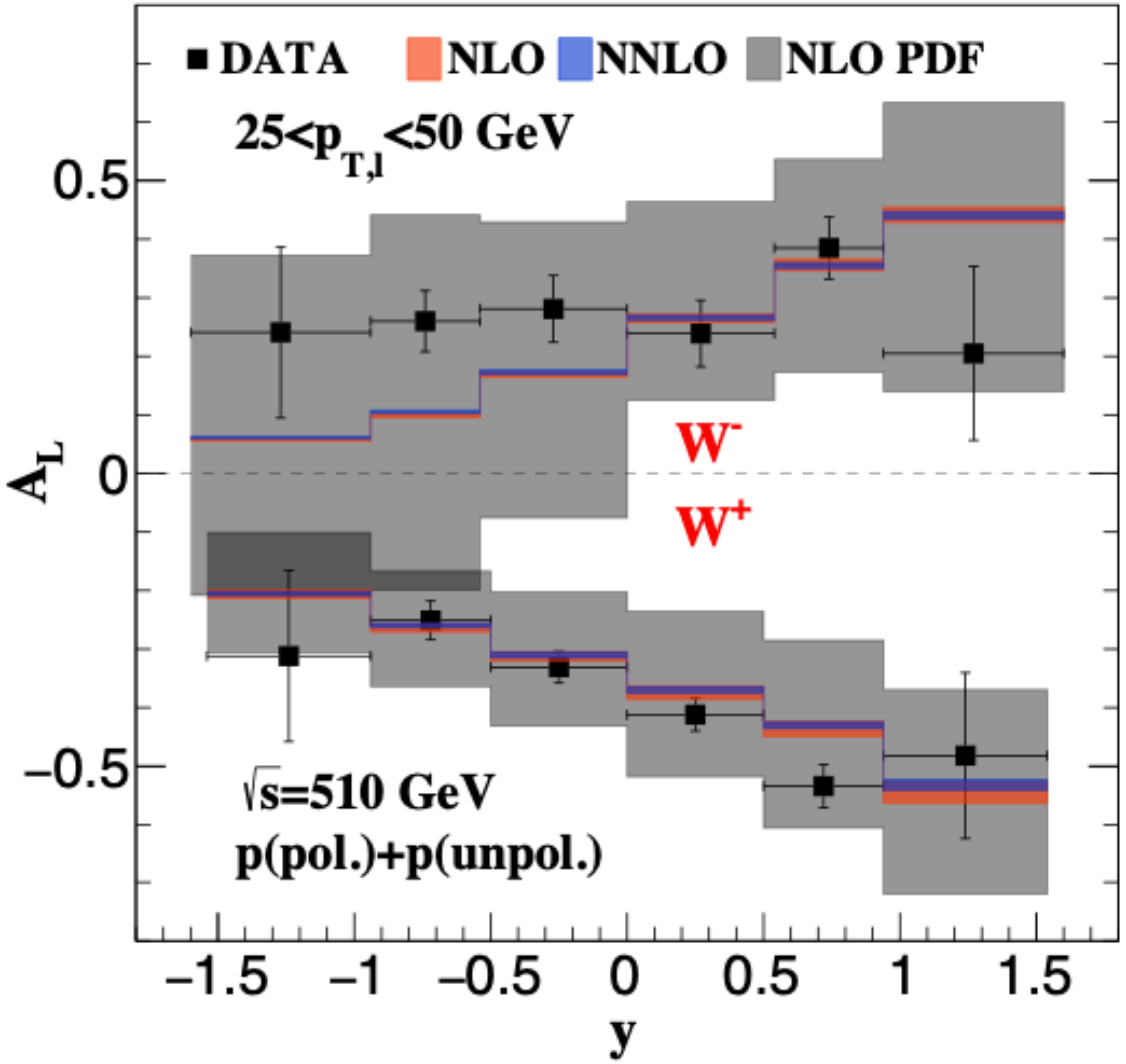


Single spin asymmetries in polarized pp $\rightarrow W^{\pm}$

$$A_L^{W^+} \propto \Delta\bar{d}\,u - \Delta u\,\bar{d}$$

$$A_L^{W^-} \propto \Delta\bar{u}\,d - \Delta d\,\bar{u}$$

- NNLO corrections for $W^{\pm}$ known [Boughezal, Li, Petriello (2021)].

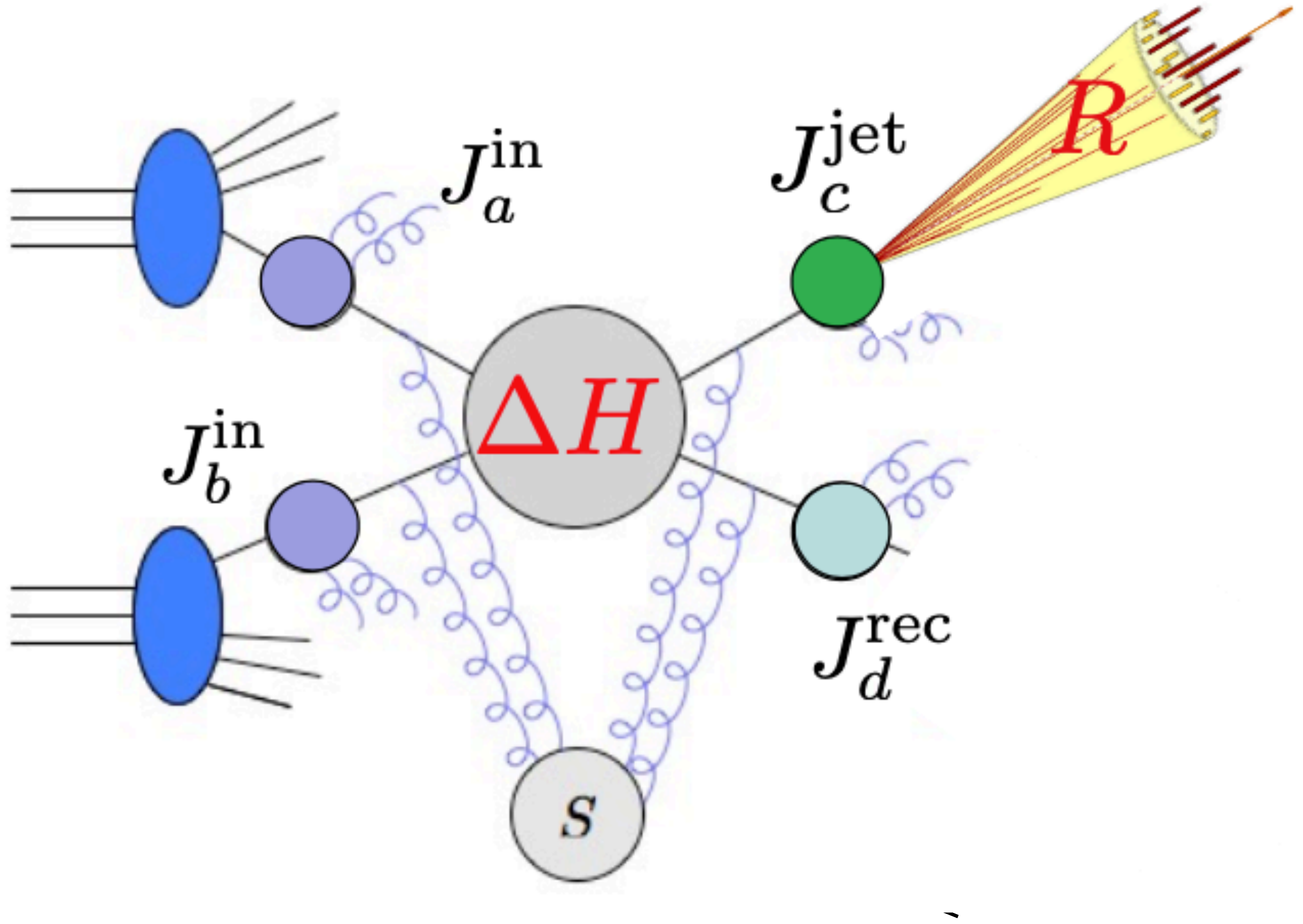


From NLO to NNLO

NNLO corrections for pp observables

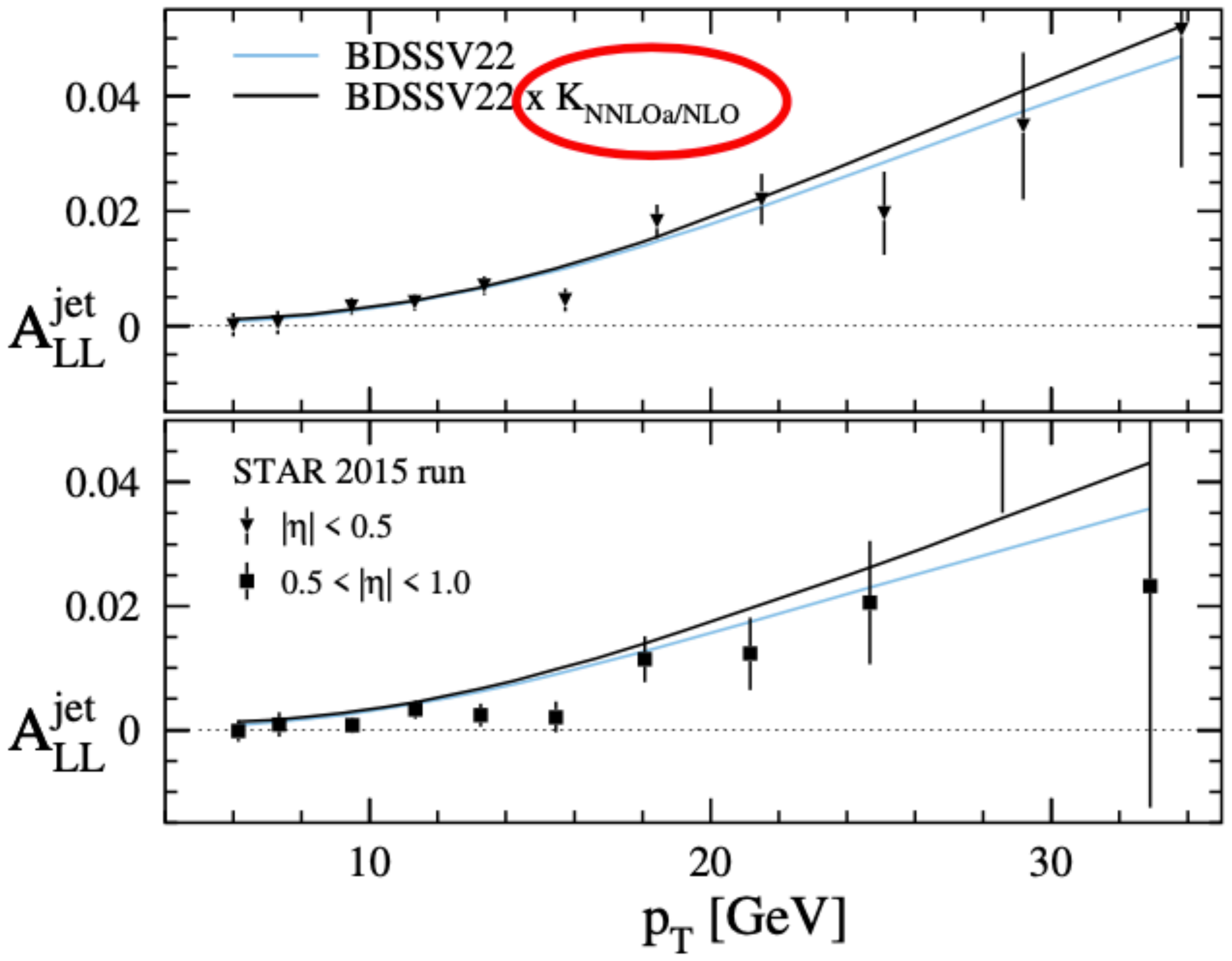
$$\Delta\sigma^{ab\to cd} \sim J_a^{in} \times J_b^{in} \times J_c^{\text{jet}} \times J_d^{\text{rec}} \times \text{Tr}[\Delta H\, S]_{ab\to cd}$$

Threshold logarithms $\ln\left(1 - \frac{s_{\text{rad}}}{s}\right)$



- NNLO corrections not known for jet nor pion production.

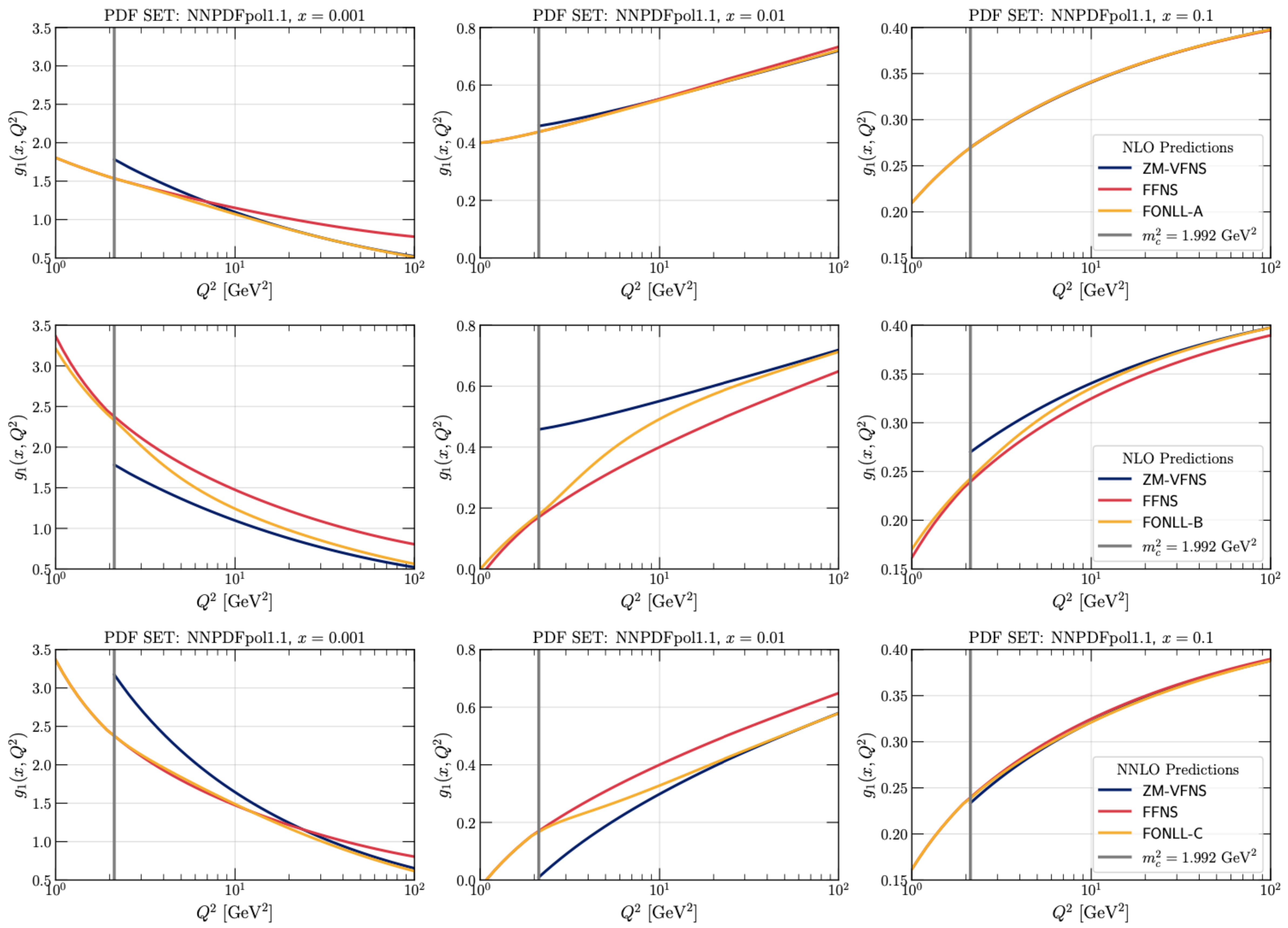
- Still possible to derive approximate NNLO corrections based on the resummation of threshold logs [Kidonakis, Oderda, Sterman, (1998); de Florian, Vogelsang (2005); Hinderer, Ringer, Sterman, Vogelsang (2019)].



From NLO to NNLO

Other theoretical/methodological developments

Extension of the FONLL scheme for polarized structure functions

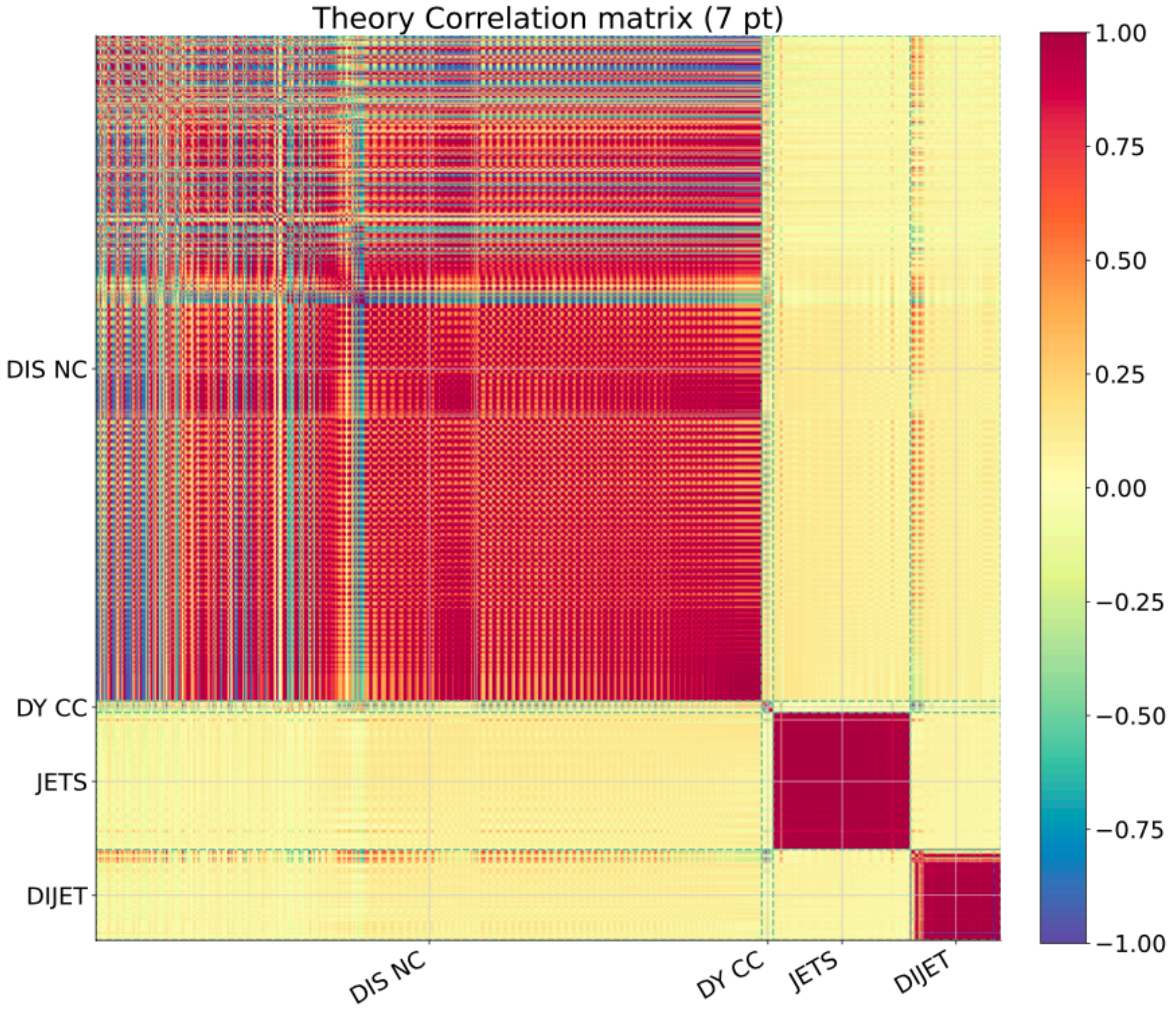


[Hekhorn, Magni, Nocera, Rabemananjara, Rojo, Schaus, Stegeman (2024)]

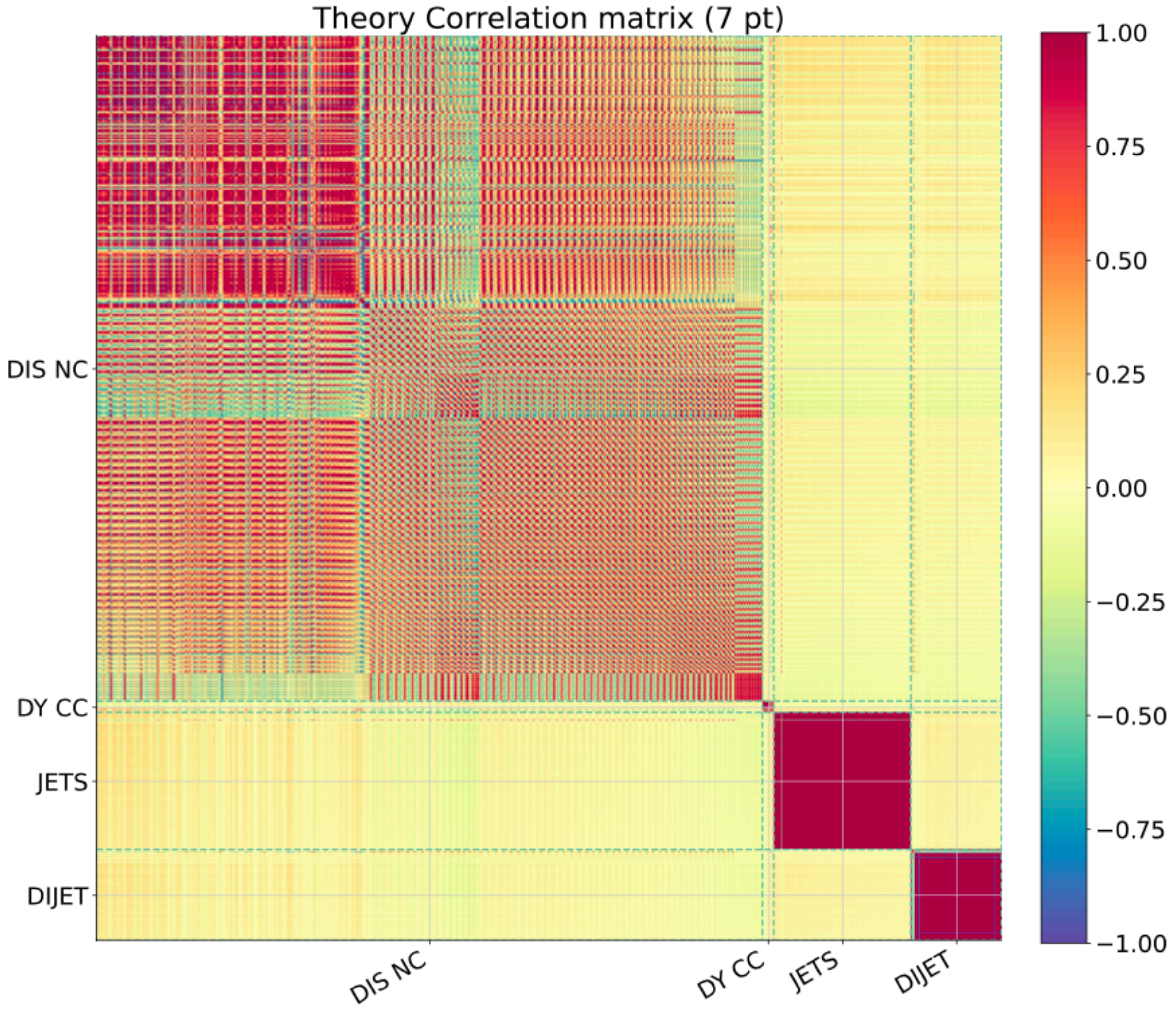
From NLO to NNLO

Other theoretical/methodological developments

Theoretical uncertainties from missing higher orders in QCD



(a) NLO



(b) NNLO

[J. Cruz-Martínez et al. (2025)]

From NLO to NNLO

The new landscape at NNLO

NNLO helicity PDFs based on DIS data

- DIS-only analysis [Taghavi-Shahri, Khanpour, Atashbar Tehrani, Alizadeh Yazdi (2016)]

From NLO to NNLO

The new landscape at NNLO

NNLO helicity PDFs based on DIS data

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NNLO global analyses of helicity PDFs

- DIS and (approximate) SIDIS [MAP: Bertone, Chiefa, Nocera (2024)]
- DIS and approximate NNLO for SIDIS and pp [BDSSV: IB, de Florian, Sassot, Stratmann, Vogelsang (2024)]
- DIS and pp $\rightarrow W^{\pm}$, jets [NNPDFpol2.0: J. Cruz-Martínez et al. (2025)]

From NLO to NNLO

The new landscape at NNLO

		MAPPDFpol1.0	BDSSV24	NNPDFpol2.0
Data sets	DIS	☑ DIS	☑ DIS	☑ DIS
	SIDIS	☑ SIDIS	☑ SIDIS	☐ SIDIS
	pp→jets	☐ pp→jets	☑ pp→jets (No di-jets)	☑ pp→jets
	pp→π	☐ pp→π	☑ pp→π	☐ pp→π
	pp→$W^{\pm}$	☐ pp→$W^{\pm}$	☑ pp→$W^{\pm}$	☑ pp→$W^{\pm}$
Method.	Uncertainties	Monte Carlo	Monte Carlo	Monte Carlo
	Parametrisation	Neuronal Network	Standard	Neuronal Network
Theory	Pert. Order	NNLO*	NNLO*	NNLO*
	HF scheme	ZM-VFNS	ZM-VFNS	GM-VFNS
	MHOU	No	No	Yes

From NLO to NNLO

The new landscape at NNLO

		MAPPDFpol1.0	BDSSV24	NNPDFpol2.0
Data sets	DIS	☑ DIS	☑ DIS	☑ DIS
	SIDIS	☑ SIDIS	☑ SIDIS	☐ SIDIS
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	pp→π	☐ pp→π	☑ pp→π	☐ pp→π
	pp→$W^{\pm}$	☐ pp→$W^{\pm}$	☑ pp→$W^{\pm}$	☑ pp→$W^{\pm}$
Method.	Uncertainties	Monte Carlo	Monte Carlo	Monte Carlo
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From NLO to NNLO

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	SIDIS	☑ SIDIS	☑ SIDIS	☐ SIDIS
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	pp→π	☐ pp→π	☑ pp→π	☐ pp→π
	pp→$W^{\pm}$	☐ pp→$W^{\pm}$	☑ pp→$W^{\pm}$	☑ pp→$W^{\pm}$
Method.	Uncertainties	Monte Carlo	Monte Carlo	Monte Carlo
	Parametrisation	Neuronal Network	Standard	Neuronal Network
Theory	Pert. Order	NNLO*	NNLO*	NNLO*
	HF scheme	ZM-VFNS	ZM-VFNS	GM-VFNS
	MHOU	No	No	Yes

Approximate NNLO SIDIS

From NLO to NNLO

The new landscape at NNLO

		MAPPDFpol1.0	BDSSV24	NNPDFpol2.0
Data sets	DIS	☑ DIS	☑ DIS	☑ DIS
	SIDIS	☑ SIDIS	☑ SIDIS	☐ SIDIS
	pp→jets	☐ pp→jets	☑ pp→jets	☑ pp→jets
	pp→π	☐ pp→π	☑ pp→π	☐ pp→π
	pp→$W^{\pm}$	☐ pp→$W^{\pm}$	☑ pp→$W^{\pm}$	☑ pp→$W^{\pm}$
Method.	Uncertainties	Monte Carlo	Monte Carlo	Monte Carlo
	Parametrisation	Neuronal Network	Standard	Neuronal Network
Theory	Pert. Order	NNLO*	NNLO*	NNLO*
	HF scheme	ZM-VFNS	ZM-VFNS	GM-VFNS
	MHOU	No	No	Yes

Approximate NNLO SIDIS, pp→ jets and pp → π

NLO fragmentation functions

From NLO to NNLO

The new landscape at NNLO

		MAPPDFpol1.0	BDSSV24	NNPDFpol2.0
Data sets	DIS	☑ DIS	☑ DIS	☑ DIS
	SIDIS	☑ SIDIS	☑ SIDIS	☐ SIDIS
	pp→jets	☐ pp→jets	☑ pp→jets	☑ pp→jets
	pp→π	☐ pp→π	☑ pp→π	☐ pp→π
	pp→$W^{\pm}$	☐ pp→$W^{\pm}$	☑ pp→$W^{\pm}$	☑ pp→$W^{\pm}$
Method.	Uncertainties	Monte Carlo	Monte Carlo	Monte Carlo
	Parametrisation	Neuronal Network	Standard	Neuronal Network
Theory	Pert. Order	NNLO*	NNLO*	NNLO*
	HF scheme	ZM-VFNS	ZM-VFNS	GM-VFNS
	MHOU	No	No	Yes

NLO results for pp→ jets

Global analysis of helicity PDFs

Technical specifications & data selection

Data:	data-points
DIS: EMC, SMC, E142, E143, E154, E155, HERMES, COMPASS, HALL-A, CLAS (p, n, d, He targets)	378
SIDIS: SMC, HERMES, COMPASS (p, n, targets; identified $\pi^{\pm}, K^{\pm}, h^{\pm}$)	277
PP-JETS: STAR run 5, 6, 9, 12, 13, 15 ($\sqrt{s} = 200, 510$ GeV)	91
PP-$\pi^0/\pi^{\pm}$: PHENIX, STAR	78
PP-$W^{\pm}$: PHENIX, STAR	22
TOTAL:	850

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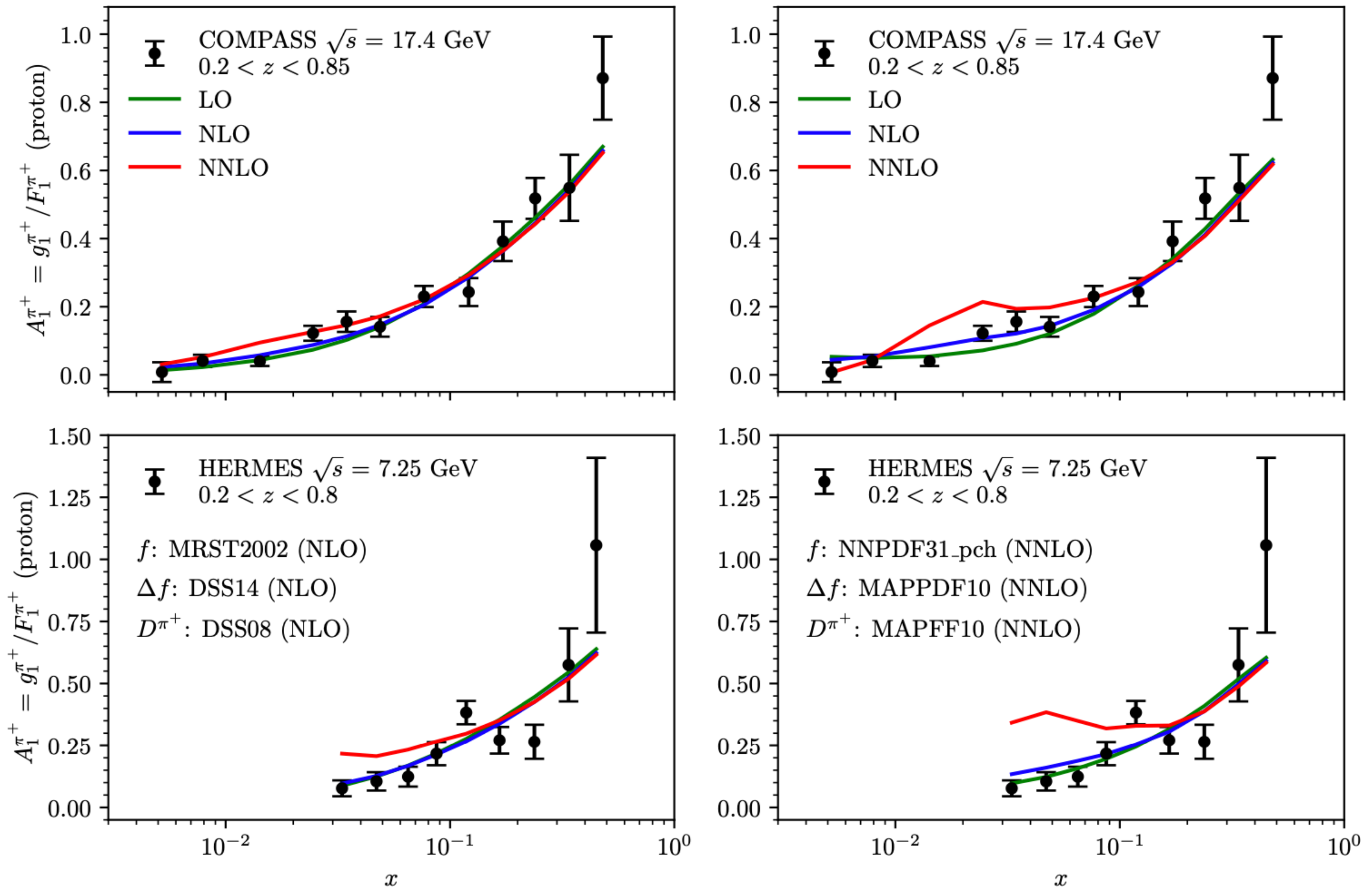
TOTAL:	850

But

Bonino, Gehrmann, Löchner, Schönwald, Stagnitto (2024)



- Bump in the asymmetry not captured by the threshold approximation

Global analysis of helicity PDFs

Technical specifications & data selection

Data:	data-points	χ^2-NLO	χ^2-NNLO
DIS: EMC, SMC, E142, E143, E154, E155, HERMES, COMPASS, HALL-A, CLAS (p, n, d, He targets)	378	304.7	308.74
SIDIS: SMC, HERMES, COMPASS (p, n, targets; identified $\pi^{\pm}, K^{\pm}, h^{\pm}$)	277	276.1	322.5
PP-JETS: STAR run 5, 6, 9, 12, 13, 15 ($\sqrt{s} = 200, 510$ GeV)	91		
PP-$\pi^0/\pi^{\pm}$: PHENIX, STAR	78		
PP-$W^{\pm}$: PHENIX, STAR	22		
TOTAL:	850		

Similar observations:

- MAP analysis [Bertone et al. (2024)]
- NNLO FFs fits [IB et al. (2022); Abdul-Khalek et al. (2022)]

⇒ Conservative cut of $x_{\text{SIDIS}} > 0.12$ imposed on SIDIS data

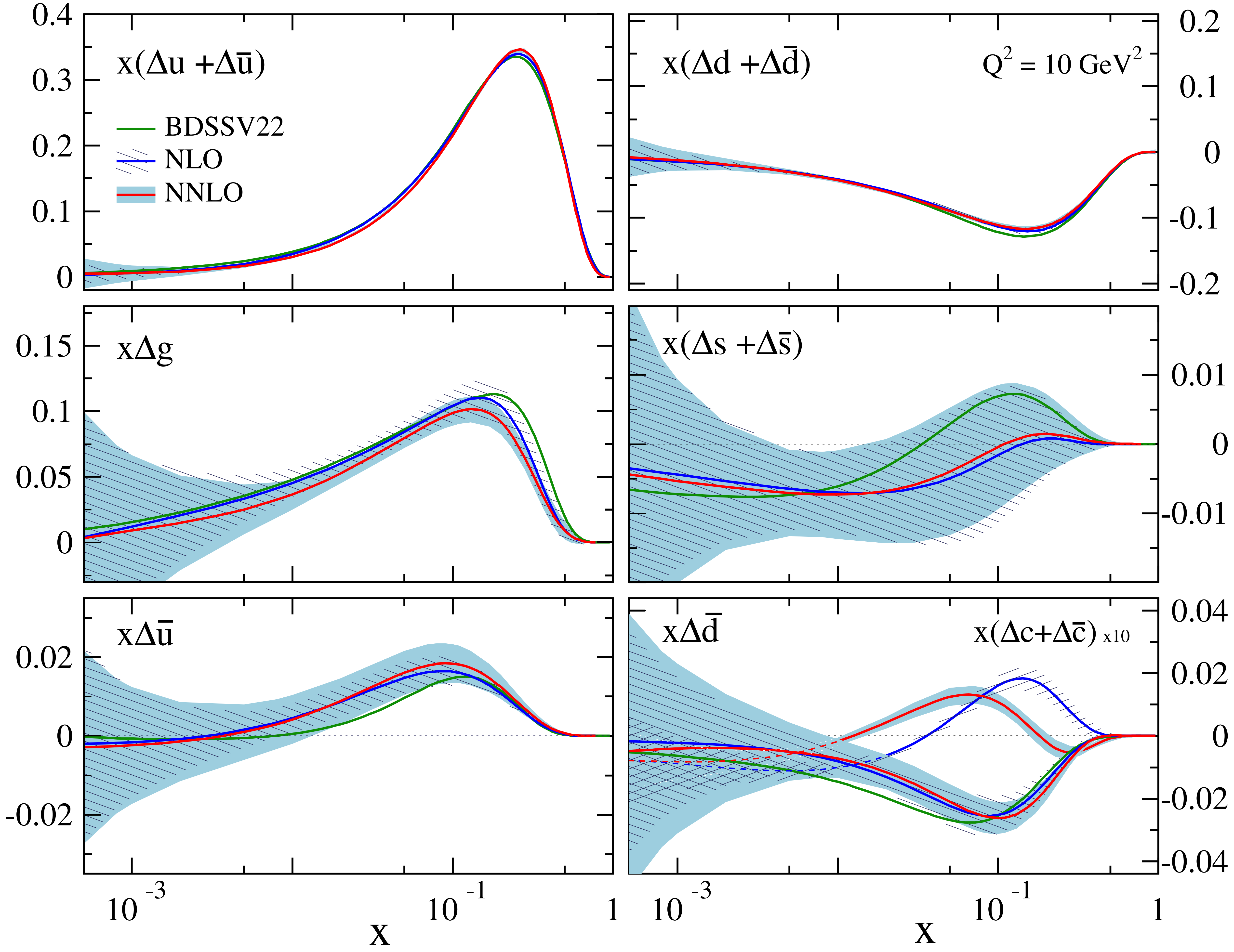
Results

χ^2-numerology

Data:	data-points	No cut on x_{SIDIS}: χ^2-NLO	No cut on x_{SIDIS}: χ^2-NNLO	$x_{SIDIS} > 0.12$: χ^2-NLO	$x_{SIDIS} > 0.12$: χ^2-NNLO
DIS: EMC, SMC, E142, E143, E154, E155, HERMES, COMPASS, HALL-A, CLAS (p, n, d, He targets)	378	304.7	308.7	302.8	294.5
SIDIS: SMC, HERMES, COMPASS (p, n, targets; identified $\pi^\pm, K^\pm, h^\pm$)	114 (277)	276.1	322.5	127.6	122.9
PP-JETS: STAR run 5, 6, 9, 12, 13, 15 ($\sqrt{s} = 200, 510$ GeV)	91			111.1	104.7
PP-$\pi^0/\pi^\pm$: PHENIX, STAR	78			63.5	66.0
PP-$W^\pm$: PHENIX, STAR	22			22.3	20.3
TOTAL:	673			627.2	607.5

Results

NNLO polarized distributions



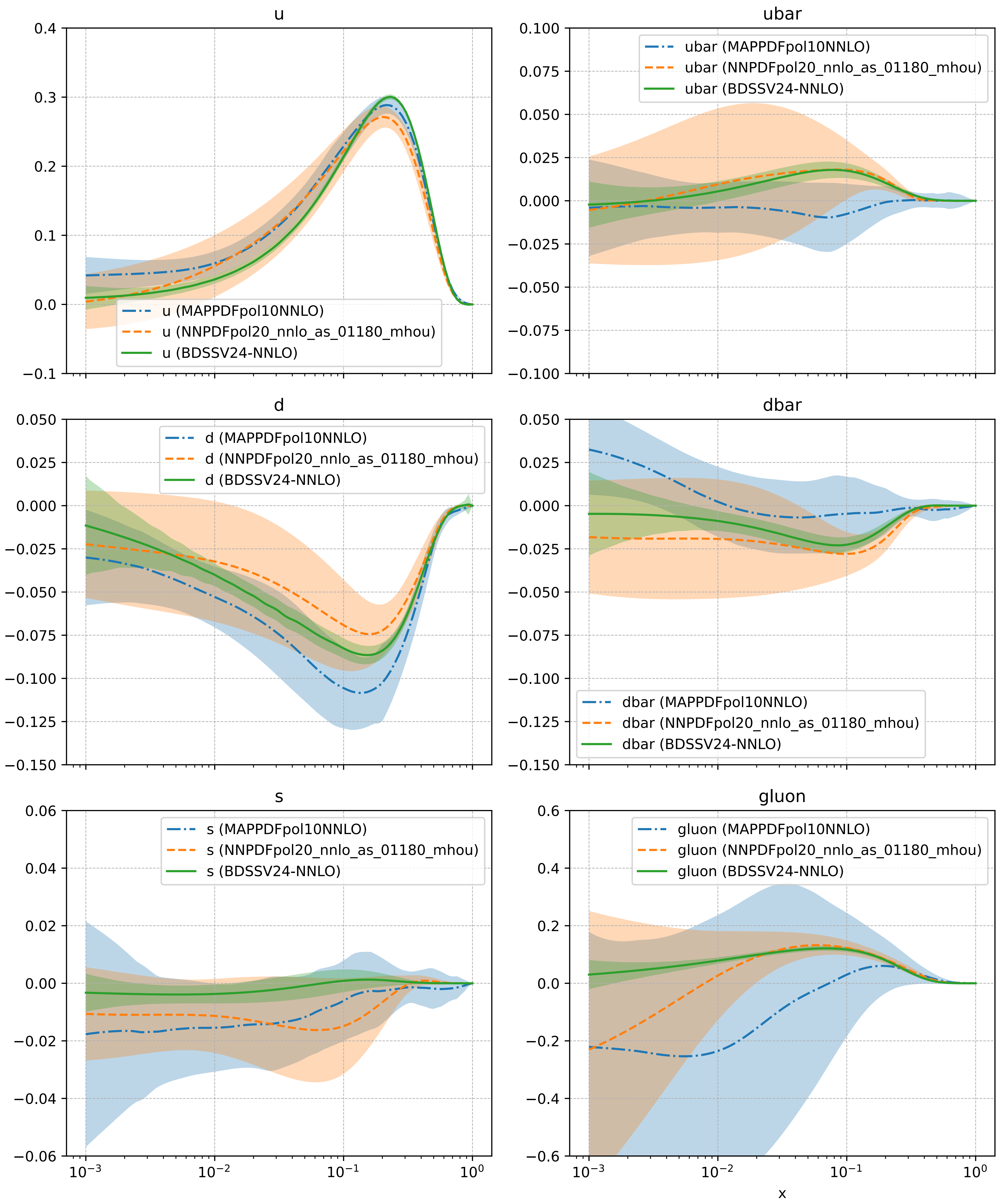
BDSSV22: NLO including dijets, $W^{\pm}$ data and no cuts on SIDIS

- $(\Delta u + \Delta \bar{u})$ and $(\Delta d + \Delta \bar{d})$ well constrained. No significant differences between NLO & NNLO.
- Δg positive, and constrained for RHIC kinematics. NLO/NNLO differences well within uncertainties.
- $(\Delta s + \Delta \bar{s})$ consistent with zero → Reduced number of data-points from SIDIS & lack of F, D constraints.
- $\Delta \bar{u}$ and $\Delta \bar{d}$ constrained by $W^{\pm}$ data.
- $(\Delta c + \Delta \bar{c})$ small, and strongly dependent on perturbative order (no intrinsic-charm).

Results

Comparison between NNLO sets

- Generally, fair agreement between the three sets within uncertainties
- Striking agreement between NNPDFpol2.0 and BDSSV24 central values
- Smaller uncertainties in the case of BDSSV (larger dataset, fixed-form parametrization, no theoretical uncertainties)



Results

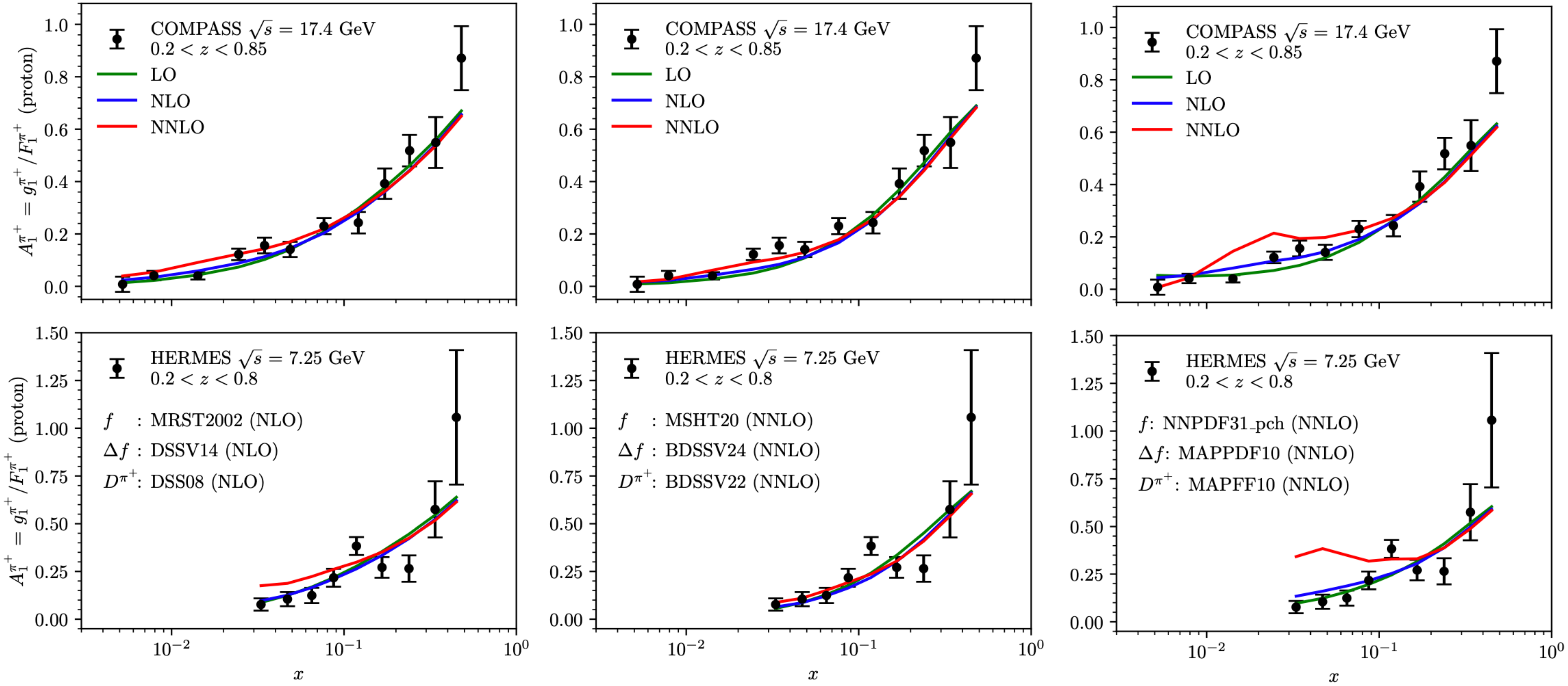
Selected data sets

- In general, good description of data; similar results for NLO and NNLO
- For SIDIS, slight suppression of the NNLO asymmetry for low-x

Results

Revisiting SIDIS data

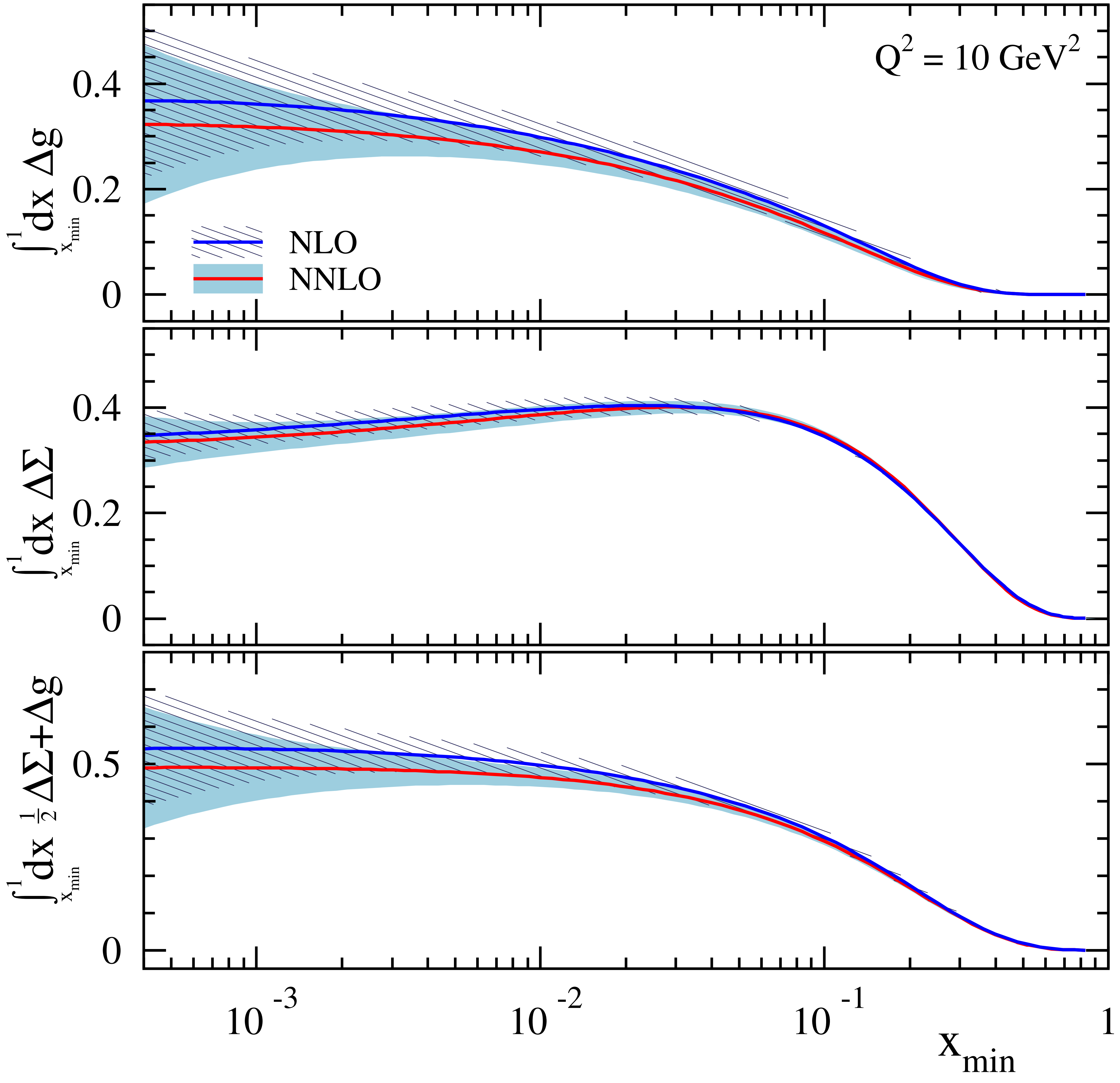
Bonino, Gehrmann, Löchner, Schönwald, Stagnitto (2024)



- Good description of the asymmetries even at low values of x
- NNLO corrections seem to solve some of the tension between HERMES and COMPASS data

Results

Revisiting the spin sum rule



$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta g + L_q + L_g$$

- Room for OAM?
- Improved picture with EIC low-x data

Outlook

Fragmentation Functions at NNLO

- Until recently, NNLO fragmentation functions based on SIA and approx. SIDIS

[IB, de Florian, Sassot, Stratmann, Vogelsang (2022)]

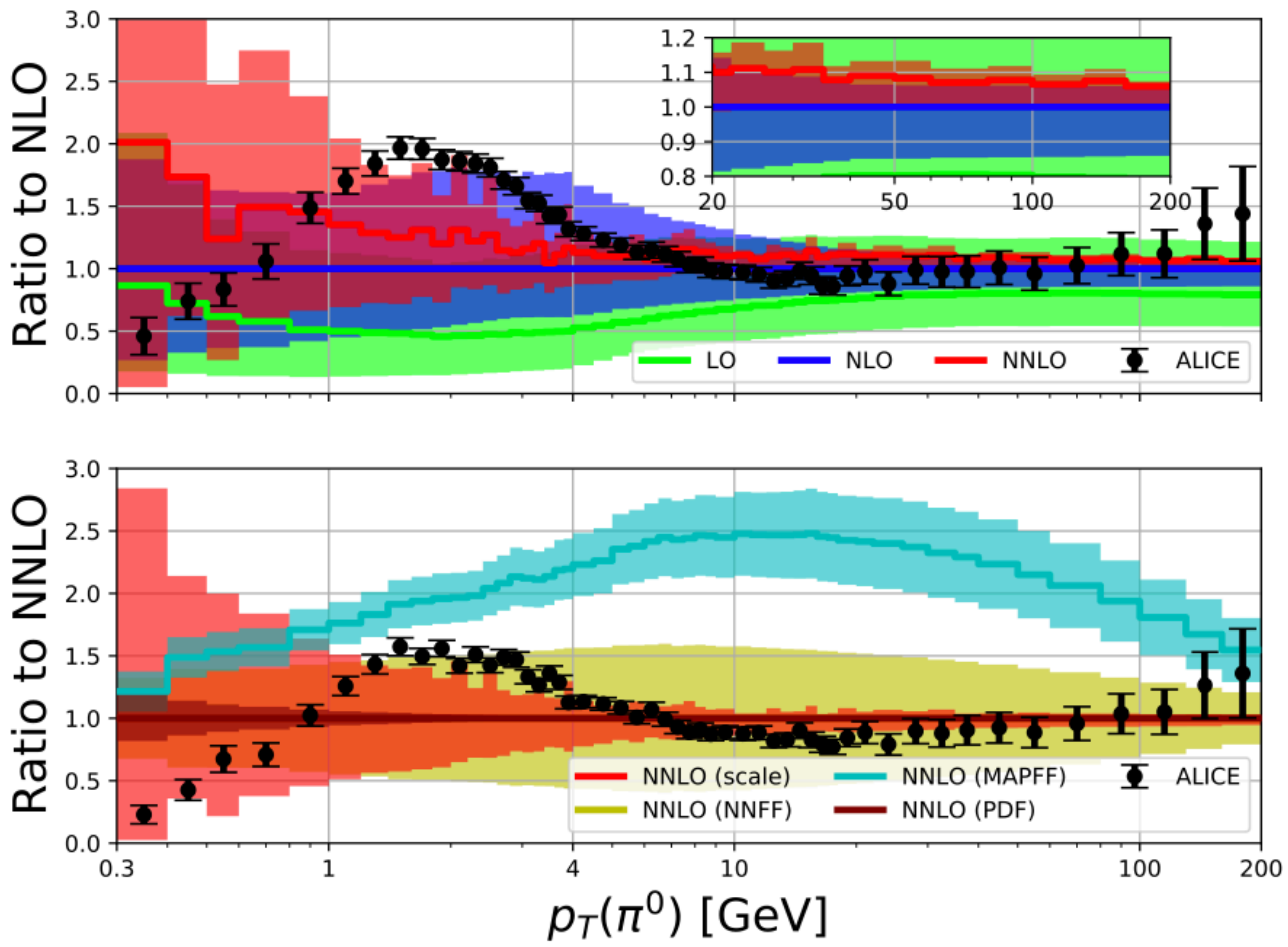
[Abdul Khalek, Bertone, Khoudli, Nocera (2022)]

- New NNLO global analyses of FFs based on SIA and SIDIS

[Gao, Shen, Xing, Zhao, Zhou (2025)]

- NNLO calculation of light hadron production at proton-proton colliders

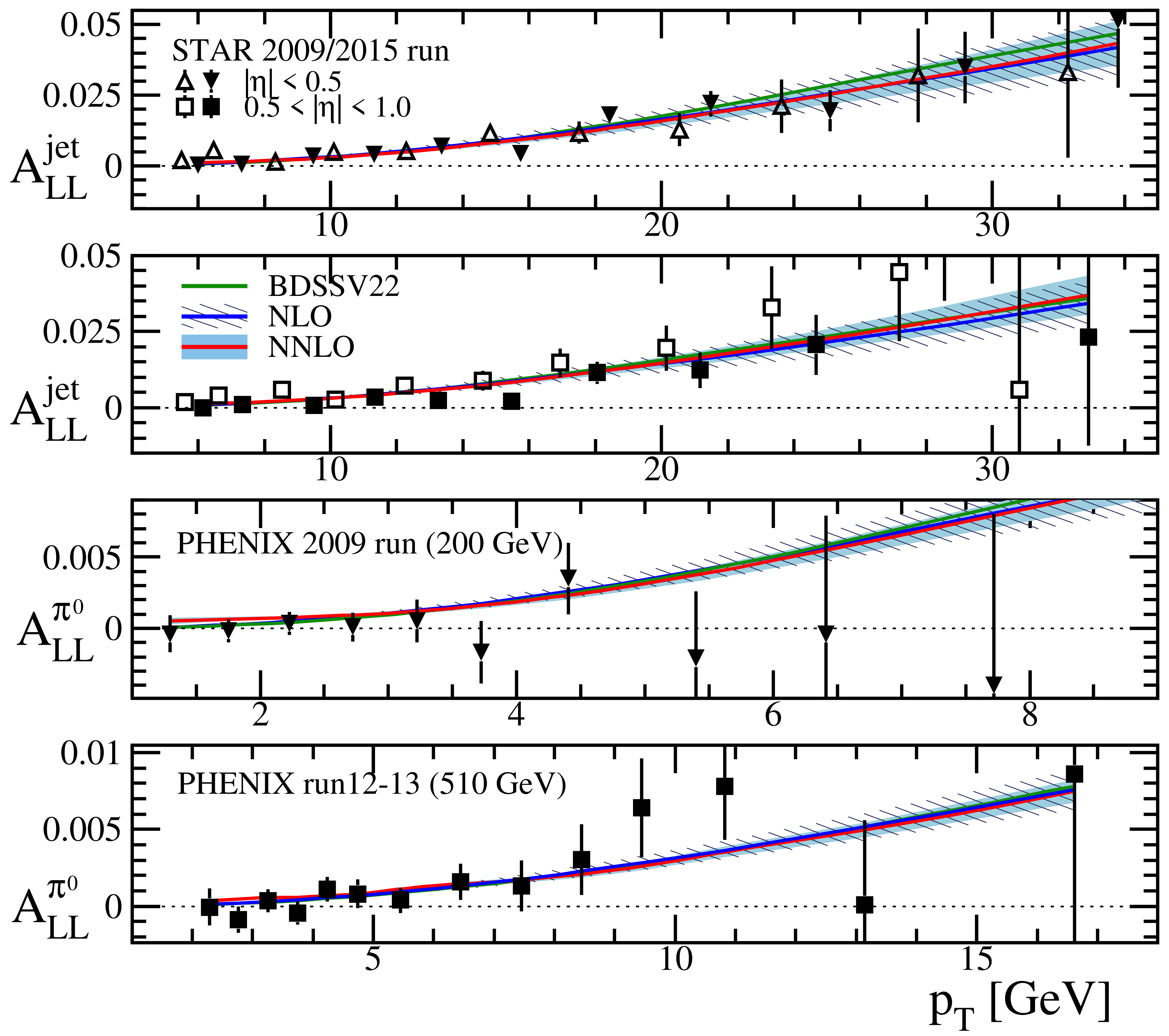
[Czakon, Generet, Mitov, Poncelet (2025)]



New results allow for fully global analyses of FFs at NNLO

Outlook

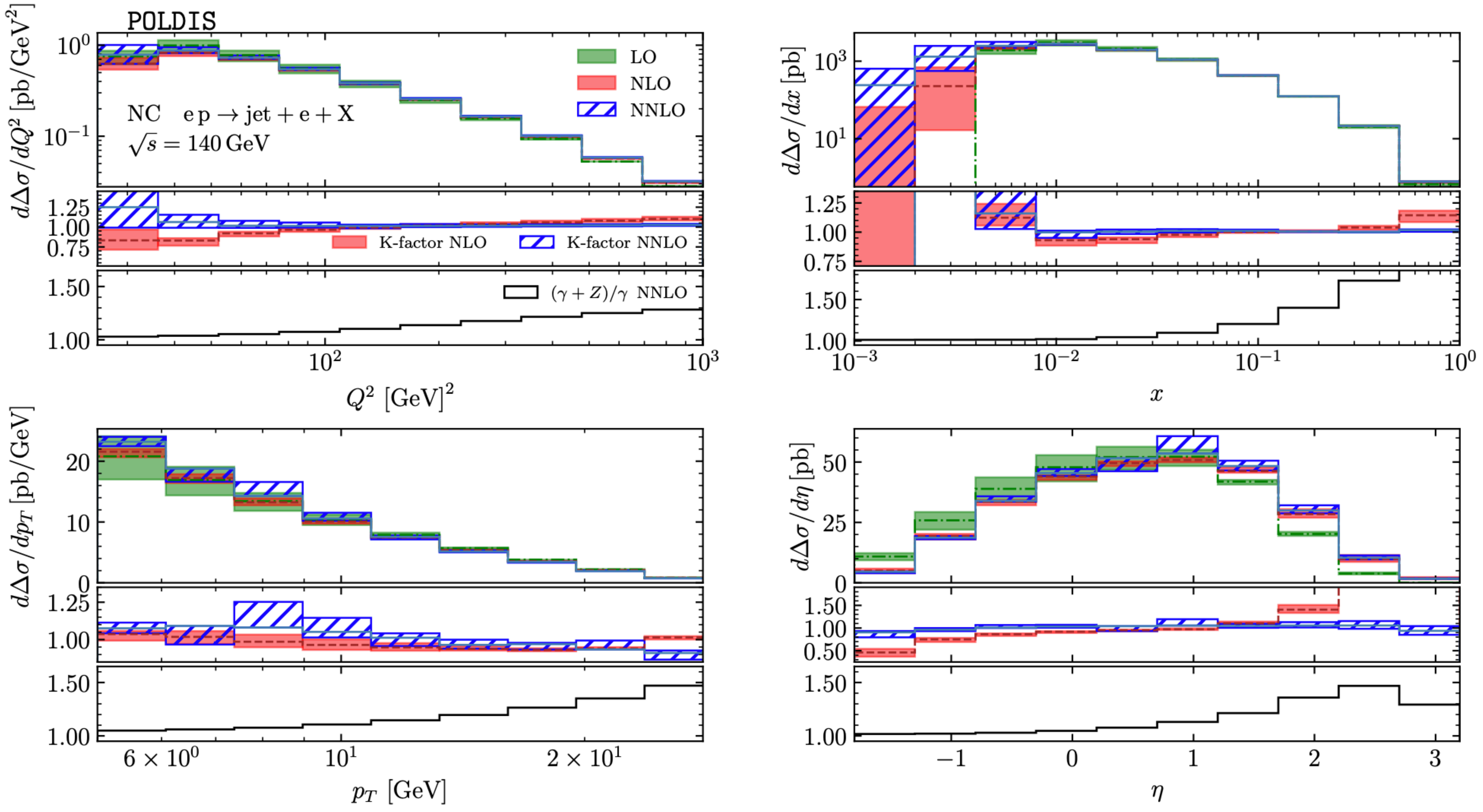
π & jet production in proton-proton collisions



- NNLO corrections for pp $\rightarrow \pi$, pp $\rightarrow$ jets still missing.
- Probably small impact with current experimental uncertainties

Outlook

Jet production in polarized DIS



- Jet observables expected to play a major role for EIC physics, complementing measurements with identified hadrons
- Single jet production in NC and CC polarized DIS known up to NNLO [IB, de Florian, Pedron (2022)]
- Di-jet production known up to NLO [IB, de Florian, Pedron (2021)]

Summary

Summary

- Still rather incomplete picture of the proton's spin in terms of the contribution from quarks, anti-quarks and gluons.
- The spin program at the future EIC expected to give unique access to the proton's spin structure.

- Remarkable effort from the theory side to set NNLO as the standard for the EIC.
- First NNLO global analyses of polarized PDFs over the last year:
    - Good perturbative stability going from NLO to NNLO.
    - Remarkable agreement between results from different collaborations.
    - SIDIS, Fragmentation Functions, pp data still work in progress.

Thank you

Global analysis of helicity PDFs

Workflow

Parametrization for $\Delta f(x, Q_0)$ at input scale Q_0

Evolve to the relevant scale using DGLAP Equations

Evaluate cross section and calculate χ^2

χ^2 minimum?

No → Change parameters

Yes

Optimum set of parameters

+ Prescription for uncertainties

- Fixed-form parametrization
- Neuronal network

- PEGASUS [Vogt (2004)]
- APFEL [Bertone, Carrazza, Rojo (2013)]
- EKO [Candido, Hekhorn, Magni (2022)]

Fast evaluation of cross sections
- Calculate in Mellin space and invert
- PineAPPL [Carrazza, Nocera, Schwan, Zaro (2020)]

Estimation of uncertainties
- Monte Carlo Sampling
- Lagrange multiplier
- Hessian

Global analysis of helicity PDFs

Mellin Technique

- Fast evaluation in Mellin N-space ⇒ Convolutions become simple products

$$\tilde{a}(N) = \int_0^1 dx\, x^{N-1} a(x) \qquad \left[a \otimes b\right](N) = \tilde{a}(N) \cdot \tilde{b}(N)$$

- Simple Gaussian integral for each convolution in the cross-section
- If analytical coefficients are known in Mellin space (e.g: DIS) ⇒ Direct application
- "Trick" if coefficients not known in N-space (e.g: $pp \to \pi X$) ⇒ write PDFs in terms of their Mellin inverse $\Delta f_i(x) = \frac{1}{2\pi i} \int_{\mathscr{C}_N} dN\, x^{-N} \Delta \tilde{f}_i(N)$

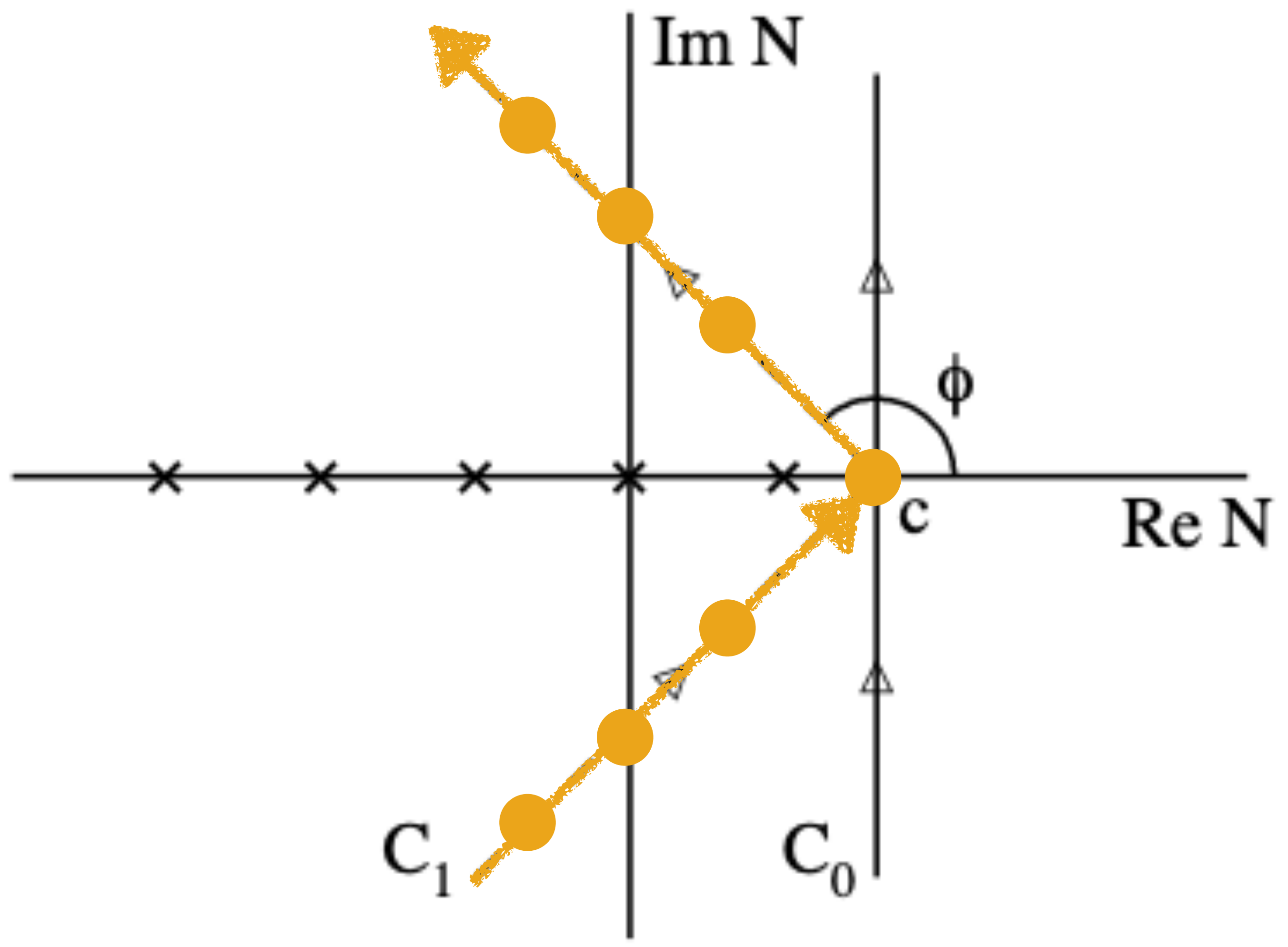
Mellin inversion

$$\frac{d\Delta\sigma}{dO} = \sum_{ab} \int_{\mathscr{C}_N} \int_{\mathscr{C}_M} \Delta\tilde{f}_a(N)\, \Delta\tilde{f}_b(M)\; \Delta\tilde{\sigma}^h_{ab}(N, M, O)$$

Fit

Precomputed grid

$$a(x) = \frac{1}{2\pi i} \int_{\mathscr{C}_N} dN\, x^{-N} \tilde{a}(N)$$



Global analysis of helicity PDFs

Technical specifications & data selection

- Parameterizations (at $Q_0^2 = 1\,\mathrm{GeV}^2$):
  - $(\Delta q + \Delta\bar{q})(x, Q_0^2) = N_q\, x^{\alpha_q}(1 - x^{\beta_q})(1 + \gamma_q x^{\delta_q} + \eta_q x)$ for (u, d)
  - $\Delta\bar{q}(x, Q_0^2) = N_{\bar{q}}\, x^{\alpha_{\bar{q}}}(1 - x^{\beta_{\bar{q}}})(1 + \gamma_{\bar{q}} x^{\delta_{\bar{q}}})$ for (u, d, s)
  - $\Delta g(x, Q_0^2) = N_g\, x^{\alpha_g}(1 - x^{\beta_g})(1 + \gamma_g x^{\delta_g})$
- Evolution:
  - Zero-Mass Variable Flavor Number Scheme (ZMVFNS). HQ Matching coefficients from [Bierenbaum et al.]
  - Extended QCD-PEGASUS library [Vogt]
- Assumptions
  - No SU(2)/SU(3) constraints
  - Positivity enforced (with respect to MSHT20 [Bailey, Cridge, Harland-Lang, Martin, Thorne (2020)])
- NLO FFs [IB, de Florian, Sassot, Stratmann (2021, 2024)]
- Uncertainties
  - Monte Carlo Sampling

Global analysis of helicity PDFs

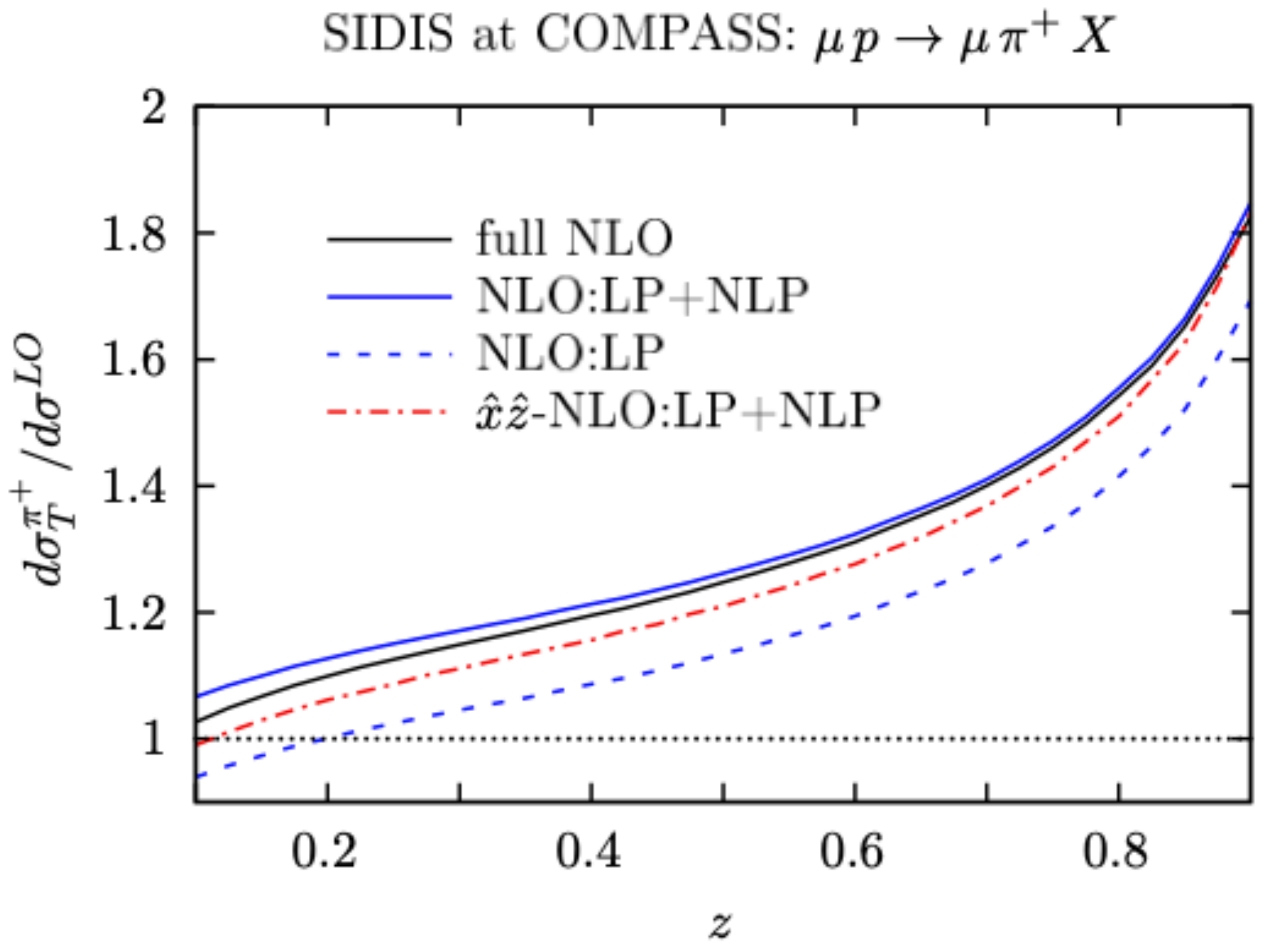
Approximate NNLO Coefficient functions for SIDIS

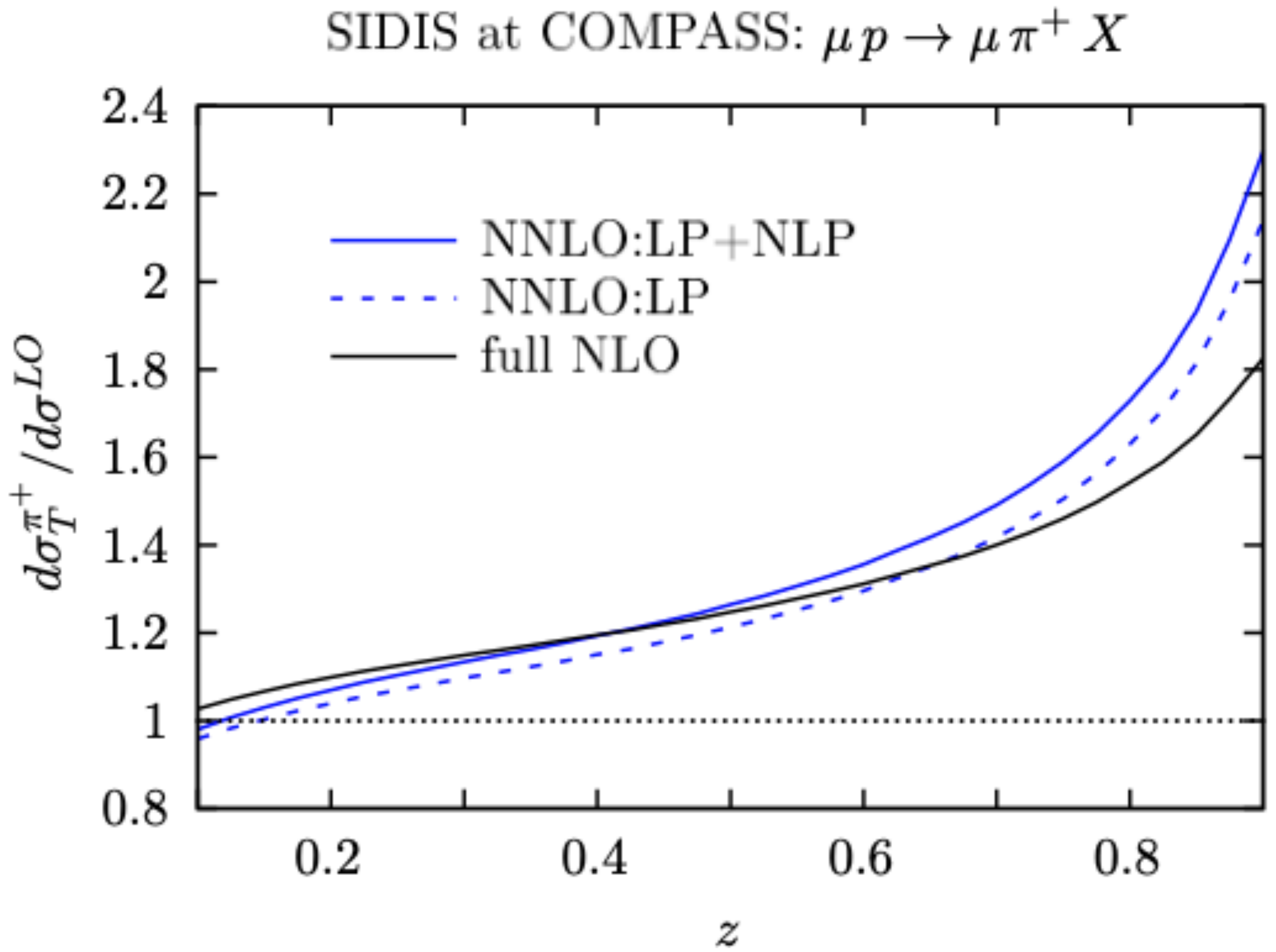
k^{th} order of perturbation theory:

$$\Delta\hat{\sigma}_{qq}^{\mathrm{N}^k\mathrm{LO}}(\hat{x},\hat{z}) \sim \alpha_s^k \Bigg[\delta(1-\hat{x})\left(\frac{\ln^{2k-1}(1-\hat{z})}{1-\hat{z}}\right)_+ + \delta(1-\hat{z})\left(\frac{\ln^{2k-1}(1-\hat{x})}{1-\hat{x}}\right)_+ + \frac{1}{(1-\hat{x})_+}\left(\frac{\ln^{2k-2}(1-\hat{z})}{1-\hat{z}}\right)_+ + \frac{1}{(1-\hat{z})_+}\left(\frac{\ln^{2k-2}(1-\hat{x})}{1-\hat{x}}\right)_+ + \ldots\Bigg]$$

- Near the threshold for hadronic production $x, z \to 1$, logs can be resumed to all orders: **threshold resummation** [Anderle, Ringer, Vogelsang (2012)]; [Abelde, de Florian, Vogelsang (2021)].

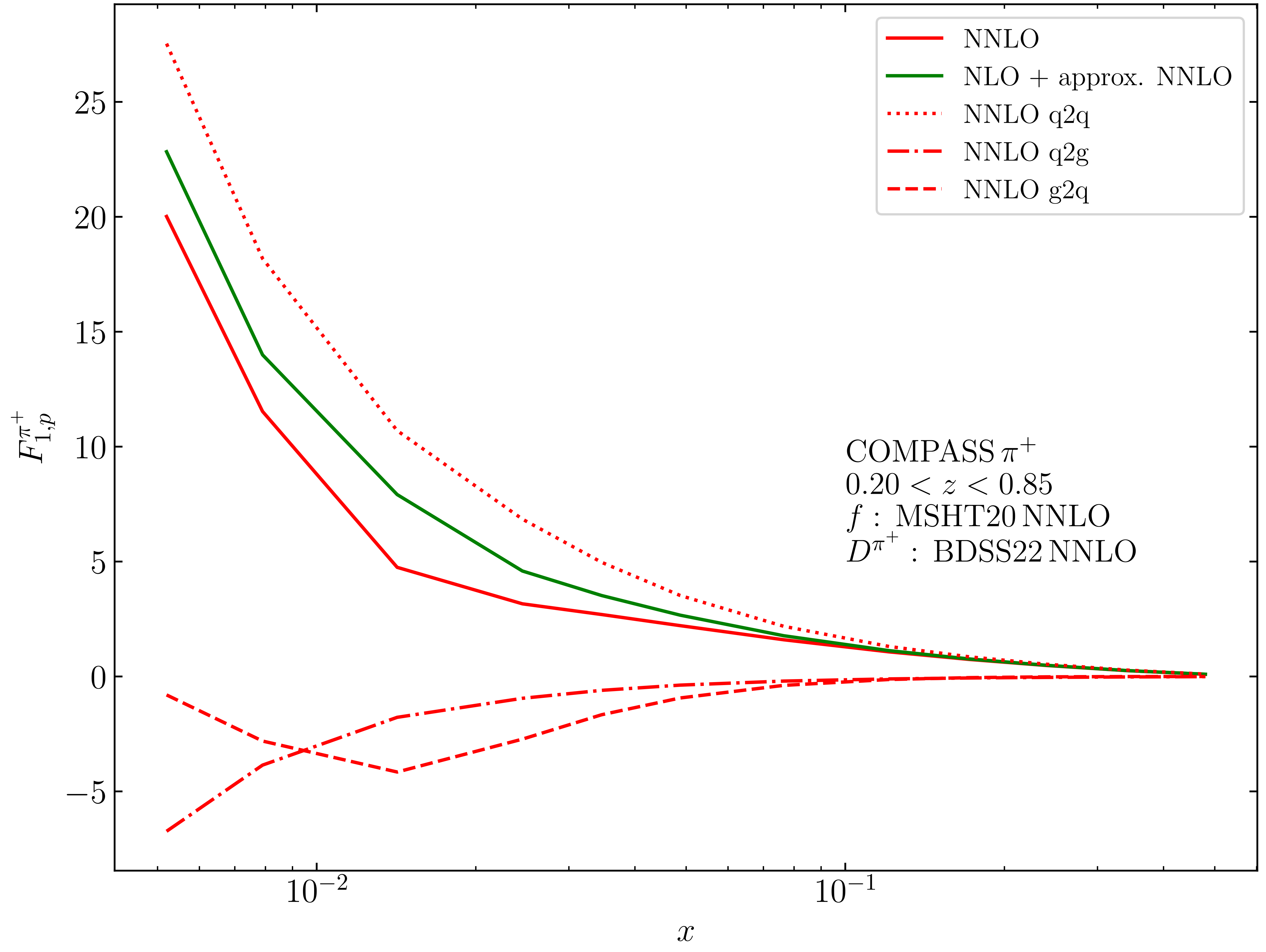
- Approximate NNLO coefficients derived for the $q \to q$ channel.

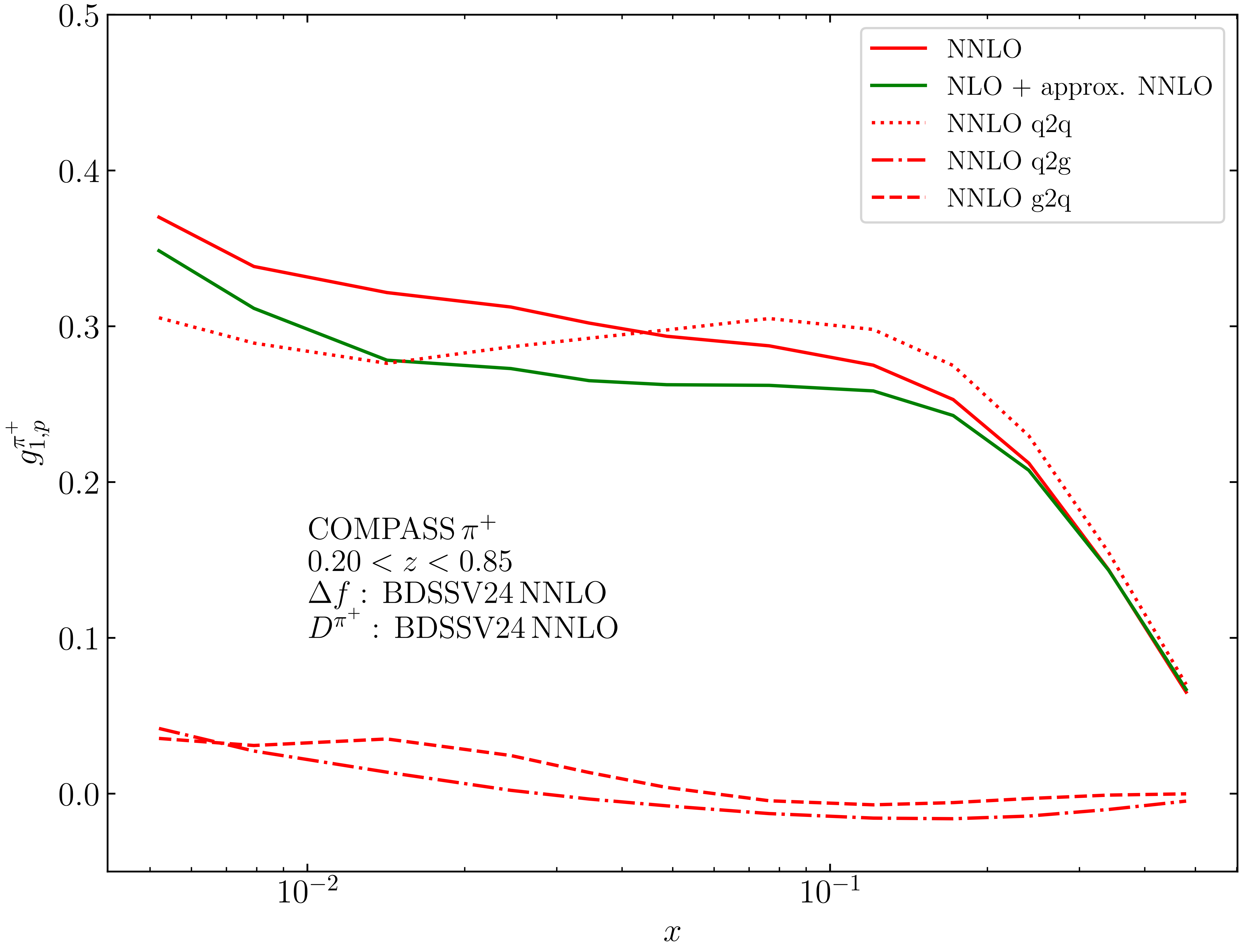




Results

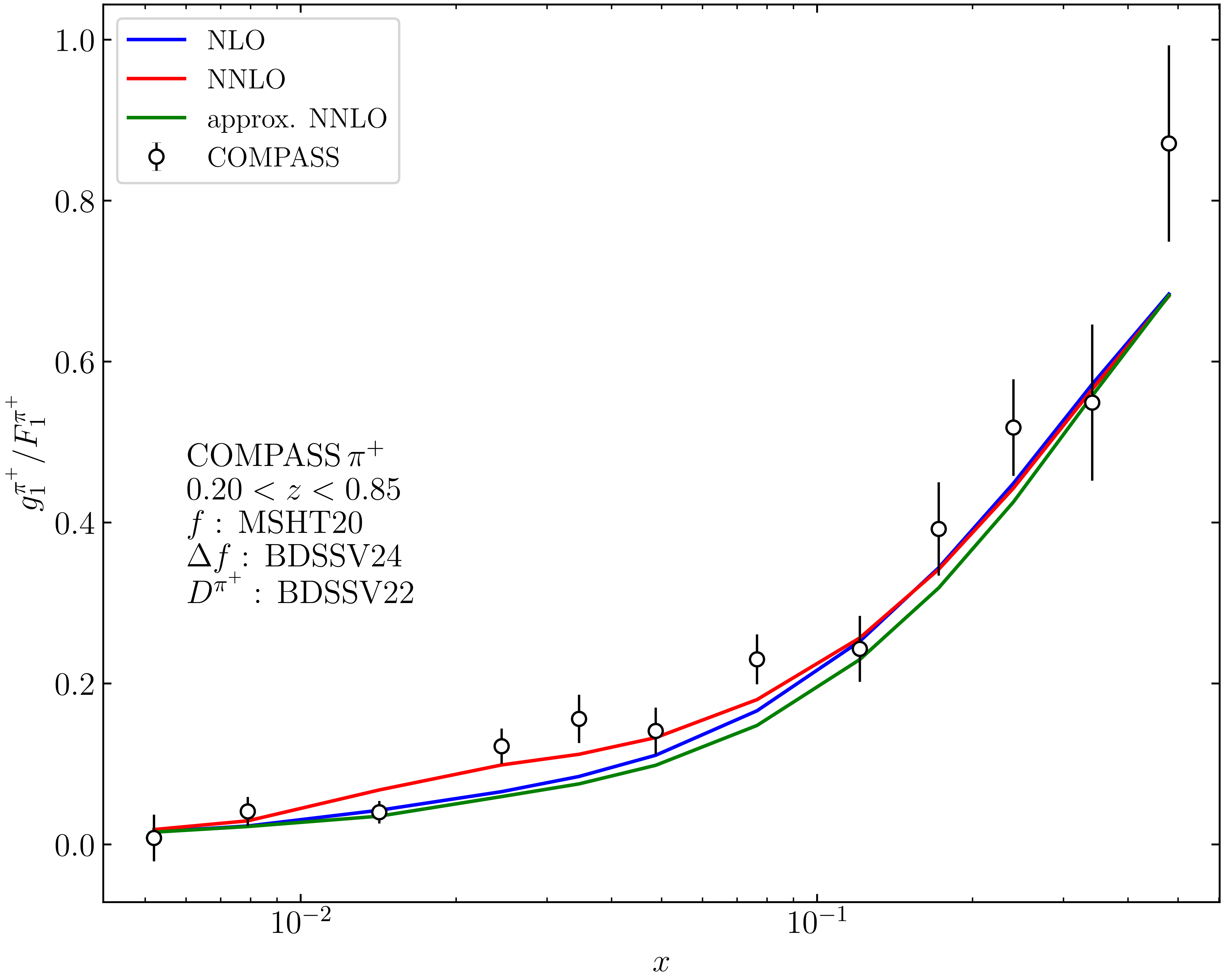
Post validation





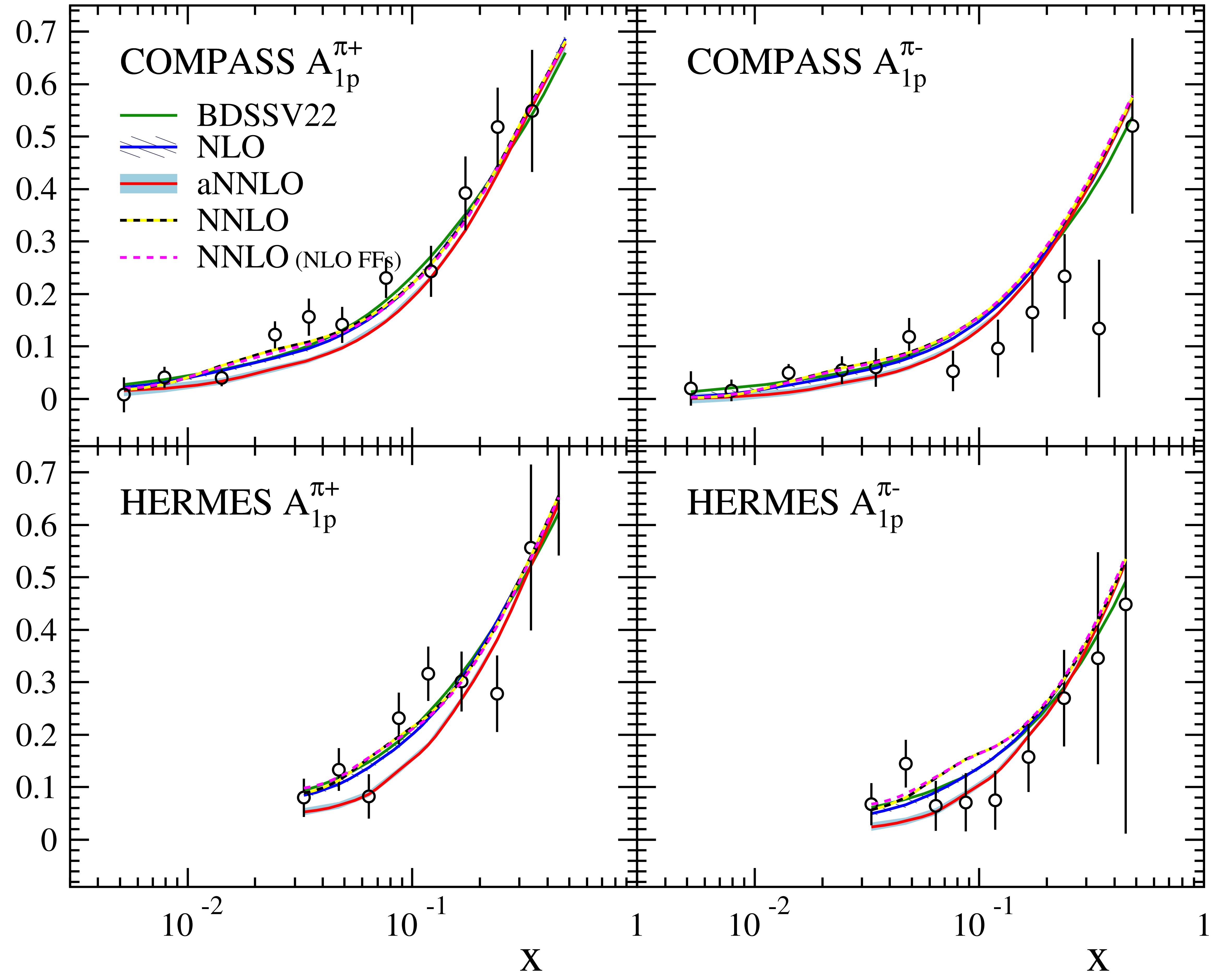
Results

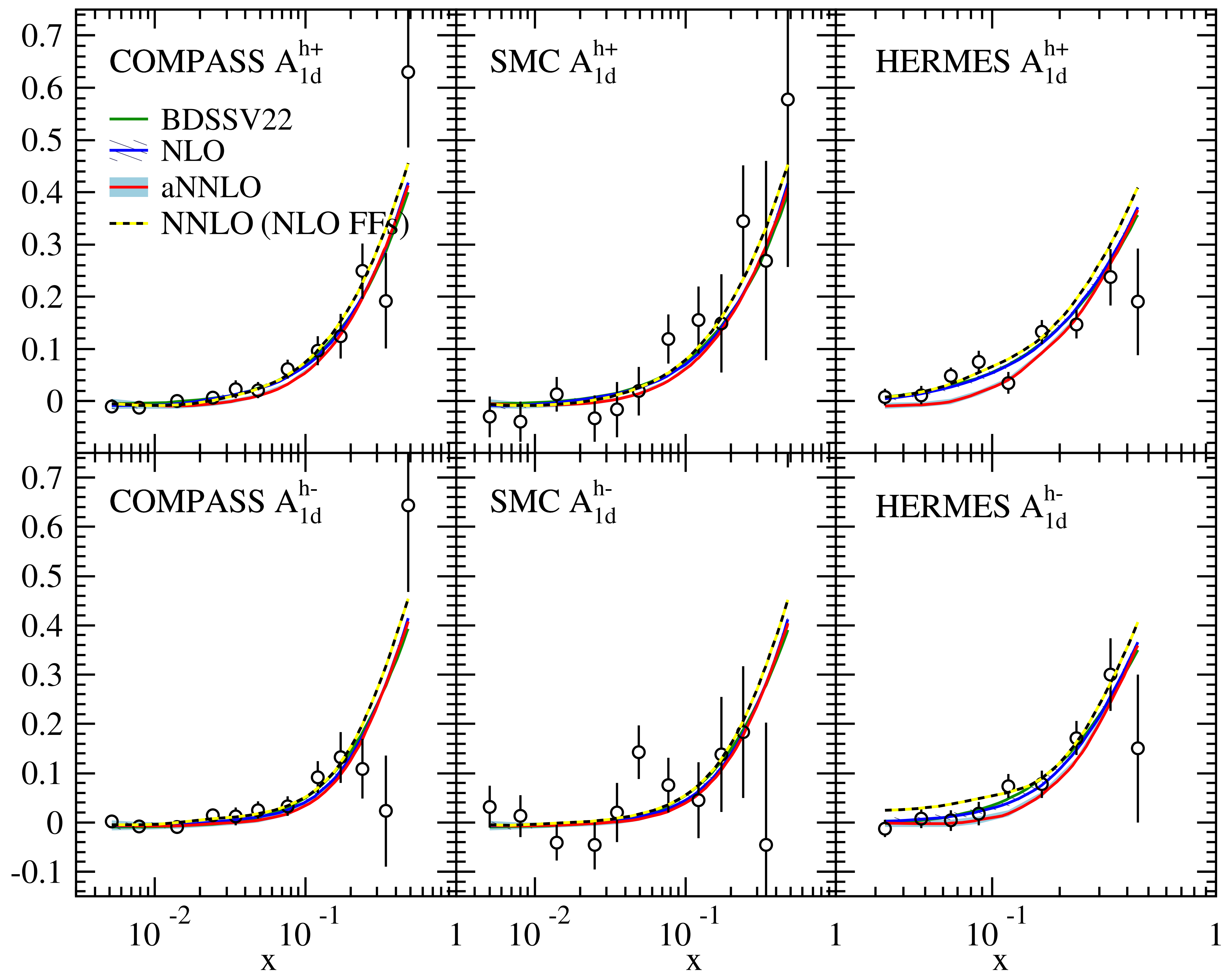
Post validation



Results

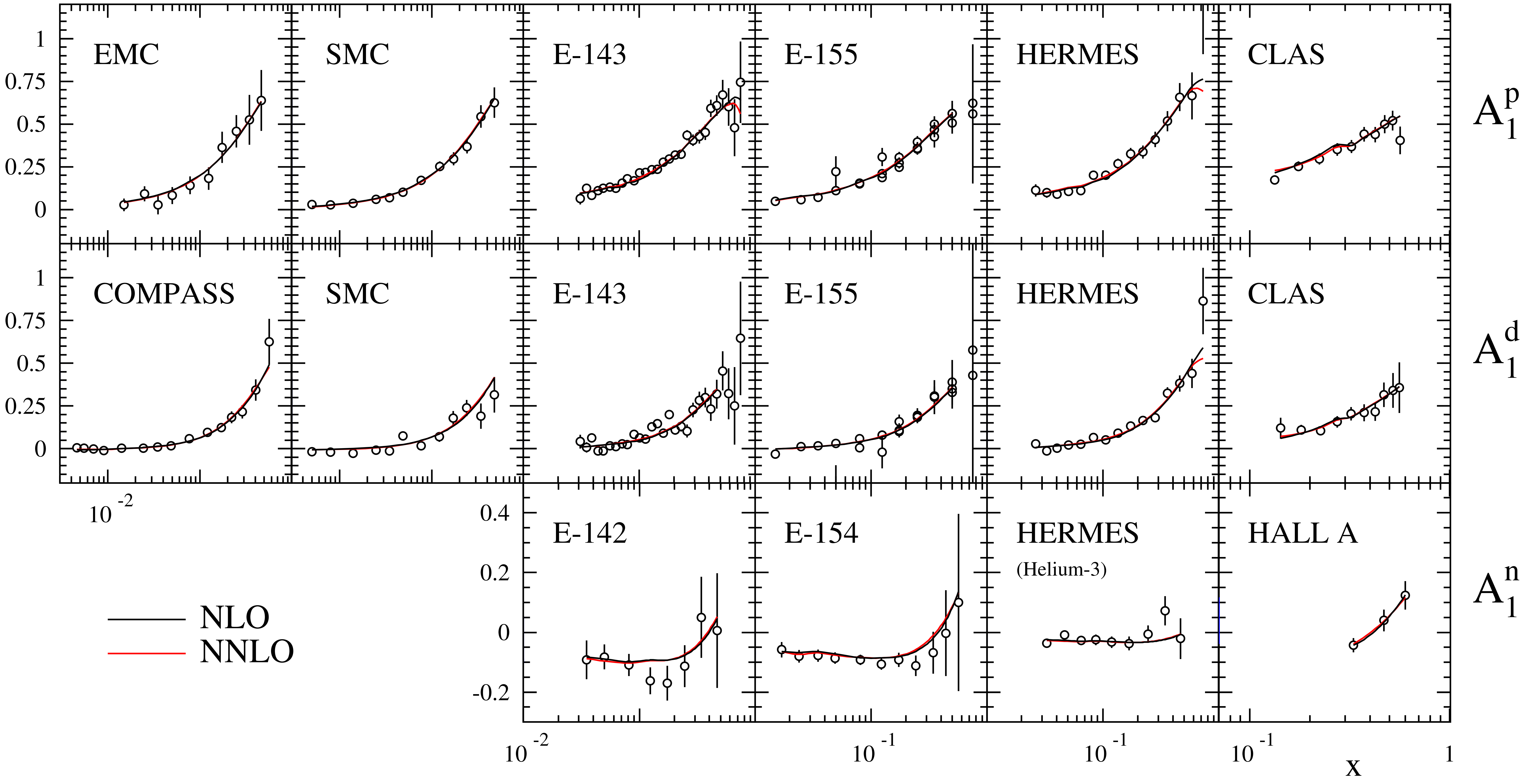
Post validation





Results

Other data sets : DIS



Results

Other data sets : Full SIDIS