



university of  
groningen

faculty of science  
and engineering

van swinderen institute for  
particle physics and gravity

# Gluon TMD & GTMD studies at the EIC

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Daniël Boer  
Van Swinderen Institute for  
Particle Physics and Gravity  
University of Groningen  
The Netherlands



BNL-INT Workshop: Bridging Theory & Experiment at the EIC, Seattle, June 2, 2025

# Overview

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- Gluon TMDs from semi-inclusive processes at EIC
- Quarkonium production in ep collisions
- Shape functions & how they affect the TMD extraction
- Gluon GTMDs from exclusive coherent diffractive processes at EIC

There is lots of synergy with processes at the LHC, but not the focus of this talk

# Joint ECFA-NuPECC-APPEC Activity Workshop "Synergies between the EIC and the LHC"

22–24 September 2025

Institute of Nuclear Physics Polish Academy of Sciences  
Kraków, Poland



Webpage

<https://indico.cern.ch/event/1509042>

The goal of this workshop is to stimulate and strengthen collaboration among the European particle physicists to mutually benefit from the synergies between theory developments and experiments at the planned U.S.-based Electron-Ion Collider (EIC) and the Large Hadron Collider (LHC) at CERN

## TOPICS

Proton structure including spin, Forward physics, diffraction and saturation, Nuclear effects,  
Probes: photon, quarkonia and jets, Detector developments and future facilities,  
MC generators, analysis tools, and AI for EIC/LHC

### JENA Activity coordinators:

Daniël Boer (Groningen U.)  
Pasquale di Nezza (INFN-LNF & CERN)  
Maria Vittoria Garzelli (Hamburg U.)

### Local Organising Committee:

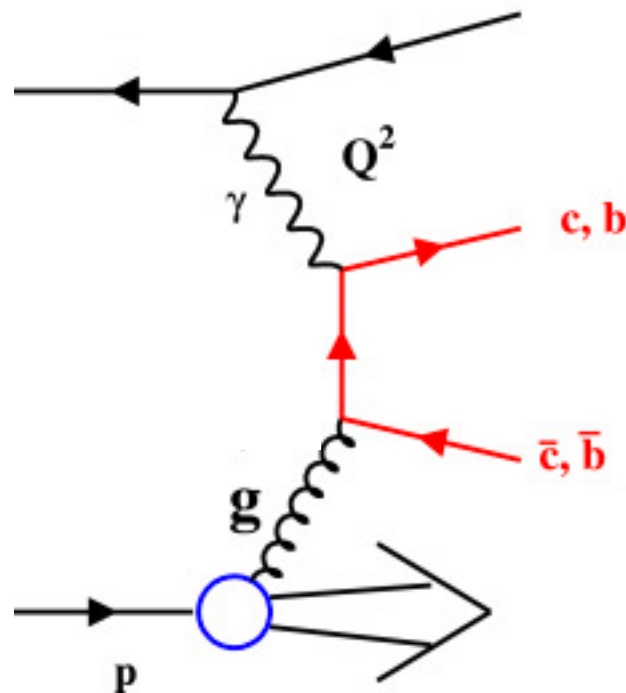
Krzysztof Kutak (IFJ PAN)      Leszek Motyka (UJ)  
Jacek Otwinowski (IFJ PAN)      Antoni Szczurek (IFJ PAN)  
Sahil Upadhyaya (IFJ PAN)      Jakub Wagner (NCBJ)

Contact info:

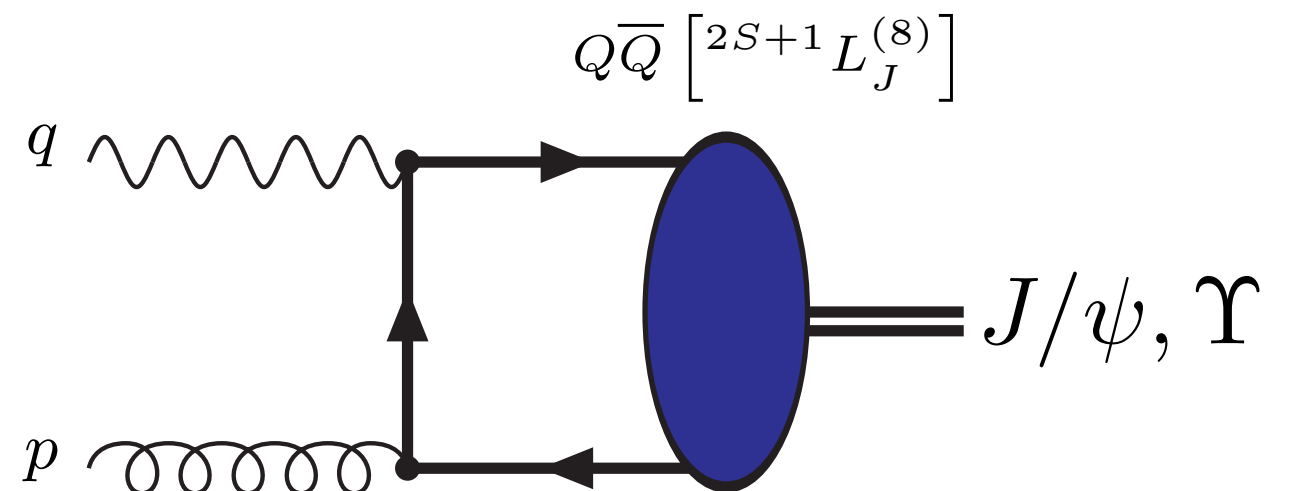
[eiclhc@ifj.edu.pl](mailto:eiclhc@ifj.edu.pl)

# Gluon TMDs

# Probing gluon TMDs



$$ep \rightarrow e' Q \bar{Q} X$$



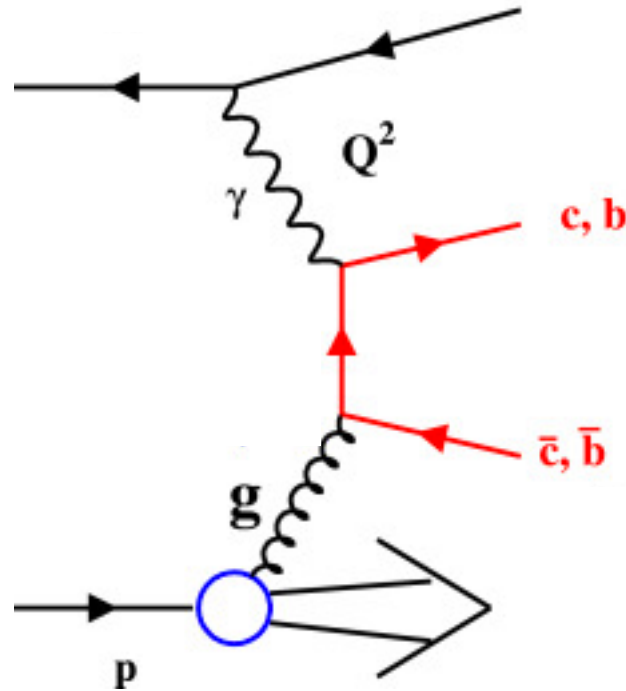
$$ep \rightarrow e' Q \bar{Q} X$$

Open heavy quark pair production and quarkonium production are among the simplest processes that are sensitive to the transverse momentum of gluons

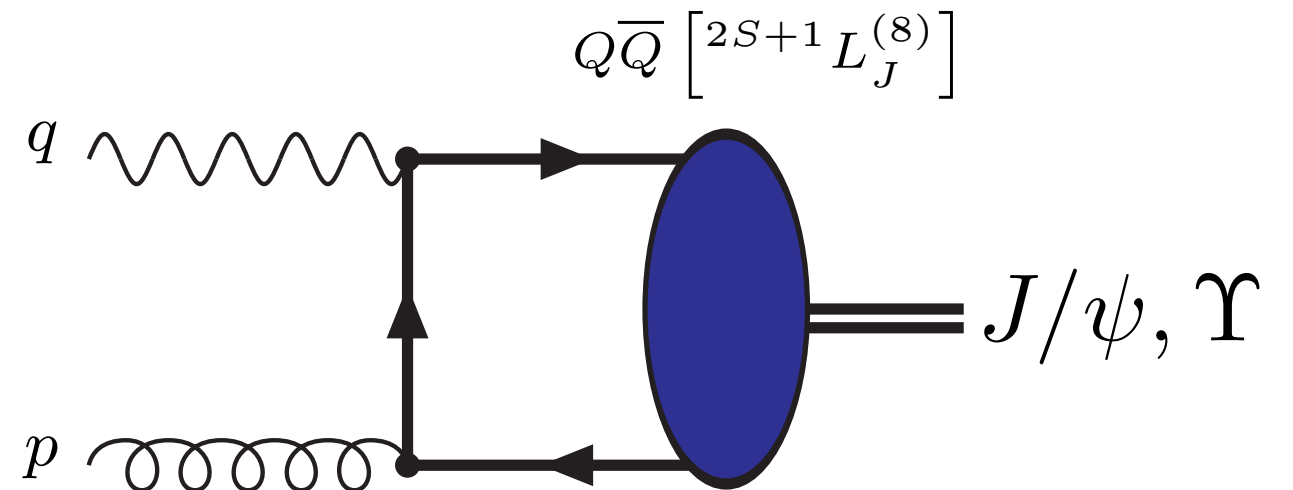
Jet pair production a good option too, especially at small x, where gluons dominate

Heavy nuclei can also help boost the gluon density, but not for the polarized case

# Probing gluon TMDs using heavy quarks in ep



$$ep \rightarrow e' Q \bar{Q} X$$



$$ep \rightarrow e' Q X$$

These “simplest” processes do have more complicated factorization expressions

TMD factorization of heavy quark pair and dijet production involves a new soft factor of 6 Wilson lines, complicating the TMD extraction

Zhu, Sun, Yuan, 2013; del Castillo, Echevarria, Makris, Scimemi, 2020

TMD factorization of quarkonium production will involve new shape functions

Echevarria, 2019; Fleming, Makris & Mehen, 2019; DB, D'Alesio, Murgia, Pisano, Taelis, 2020;  
DB, Bor, Maxia, Pisano, Yuan, 2023

# Gluons TMDs

Gluon TMD correlator:  $\Gamma_g^{\mu\nu}(x, p_T) \propto \langle P | F^{+\nu}(0) \mathcal{U} F^{+\mu}(\xi^-, \xi_T) \mathcal{U}' | P \rangle$

↑  
transverse momentum dependent (TMD)

For unpolarized protons:

$$\Gamma_U^{\mu\nu}(x, \mathbf{p}_T) = \frac{x}{2} \left\{ -g_T^{\mu\nu} f_1^g(x, \mathbf{p}_T^2) + \left( \frac{p_T^\mu p_T^\nu}{M_p^2} + g_T^{\mu\nu} \frac{\mathbf{p}_T^2}{2M_p^2} \right) h_1^{\perp g}(x, \mathbf{p}_T^2) \right\}$$

↑  
unpolarized gluon TMD

↑  
linearly polarized  
gluon TMD

Mulders, Rodrigues '01

For transversely polarized protons:

gluon Sivers TMD

$$\Gamma_T^{\mu\nu}(x, \mathbf{p}_T) = \frac{x}{2} \left\{ g_T^{\mu\nu} \frac{\epsilon_T^{\rho\sigma} p_{T\rho} S_{T\sigma}}{M_p} f_{1T}^{\perp g}(x, \mathbf{p}_T^2) + \dots \right\}$$

# Parallels between quarks and gluons

$$\Phi_U(x, \mathbf{k}) = \frac{1}{2} \left[ \not{n} f_1(x, \mathbf{k}^2) + \frac{\sigma_{\mu\nu} k_T^\mu \bar{n}^\nu}{M} h_1^\perp(x, \mathbf{k}^2) \right],$$

$$\Phi_L(x, \mathbf{k}) = \frac{1}{2} \left[ \gamma^5 \not{n} S_L g_1(x, \mathbf{k}^2) + \frac{i\sigma_{\mu\nu} \gamma^5 \bar{n}^\mu k_T^\nu S_L}{M} h_{1L}^\perp(x, \mathbf{k}^2) \right],$$

$$\Phi_T(x, \mathbf{k}) = \frac{1}{2} \left[ \frac{\not{n} \epsilon_T^{S_T k_T}}{M} f_{1T}^\perp(x, \mathbf{k}^2) + \frac{\gamma^5 \not{n} \mathbf{k} \cdot \mathbf{S}_T}{M} g_{1T}(x, \mathbf{k}^2) \right. \\ \left. + i\sigma_{\mu\nu} \gamma^5 \bar{n}^\mu S_T^\nu h_1(x, \mathbf{k}^2) - \frac{i\sigma_{\mu\nu} \gamma^5 \bar{n}^\mu k_T^{\nu\rho} S_{T\rho}}{M^2} h_{1T}^\perp(x, \mathbf{k}^2) \right]$$

For quarks the BM & Sivers TMDs are T-odd and the h-type functions are chiral-odd

$$\Gamma_U^{ij}(x, \mathbf{k}) = x \left[ \delta_T^{ij} f_1(x, \mathbf{k}^2) + \frac{k_T^{ij}}{M^2} h_1^\perp(x, \mathbf{k}^2) \right],$$

$$\Gamma_L^{ij}(x, \mathbf{k}) = x \left[ i\epsilon_T^{ij} S_L g_1(x, \mathbf{k}^2) + \frac{\epsilon_T^{\{i} k_T^{j\}\alpha} S_L}{2M^2} h_{1L}^\perp(x, \mathbf{k}^2) \right],$$

$$\Gamma_T^{ij}(x, \mathbf{k}) = x \left[ \frac{\delta_T^{ij} \epsilon_T^{S_T k_T}}{M} f_{1T}^\perp(x, \mathbf{k}^2) + \frac{i\epsilon_T^{ij} \mathbf{k} \cdot \mathbf{S}_T}{M} g_{1T}(x, \mathbf{k}^2) \right. \\ \left. - \frac{\epsilon_T^{k_T \{i} S_T^{j\}} + \epsilon_T^{S_T \{i} k_T^{j\}}}{4M} h_1(x, \mathbf{k}^2) - \frac{\epsilon_T^{\{i} k_T^{j\}\alpha} S_{T\alpha}}{2M^3} h_{1T}^\perp(x, \mathbf{k}^2) \right]$$

For gluons  $h_1^\perp$  is T-even and  $h_1$  is  $k_T$ -odd, T-odd and unrelated to transversity

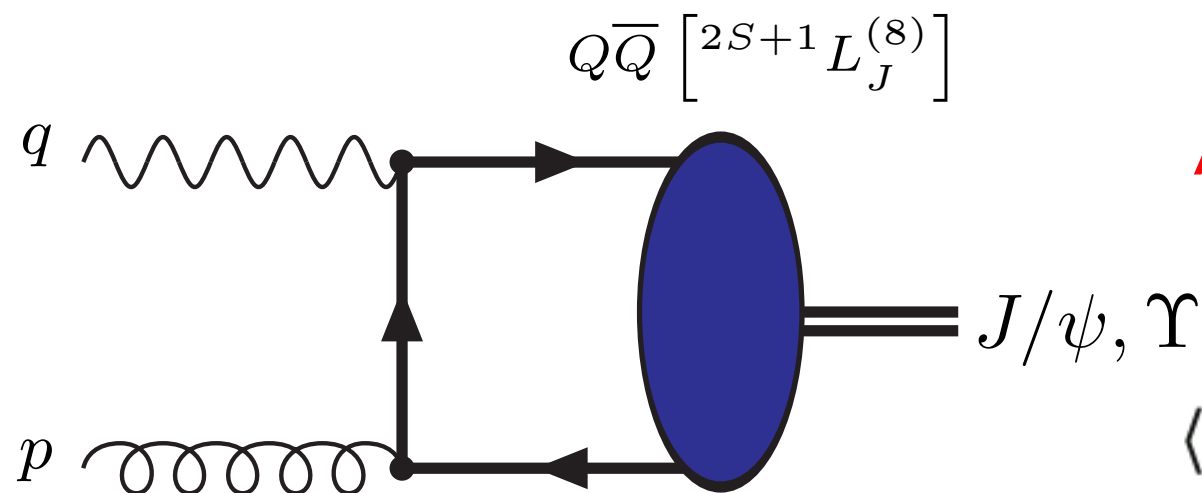
But they lead to analogous angular modulations in the production of light quarks and of heavy quark pairs

ep collisions

# Quarkonium production in ep

$ep \rightarrow e' Q X$  with  $Q$  either a  $J/\psi$  or a  $\Upsilon$  meson

Mukherjee, Rajesh, 2017; Sun, Zhang, 2017; Bacchetta, DB, Pisano, Tael, 2018;  
Kishore, Mukherjee, 2018; Kishore, Mukherjee, Siddiqah, 2021; ...



A  $\cos(2\phi_T)$  asymmetry probes  $h_1^\perp g$

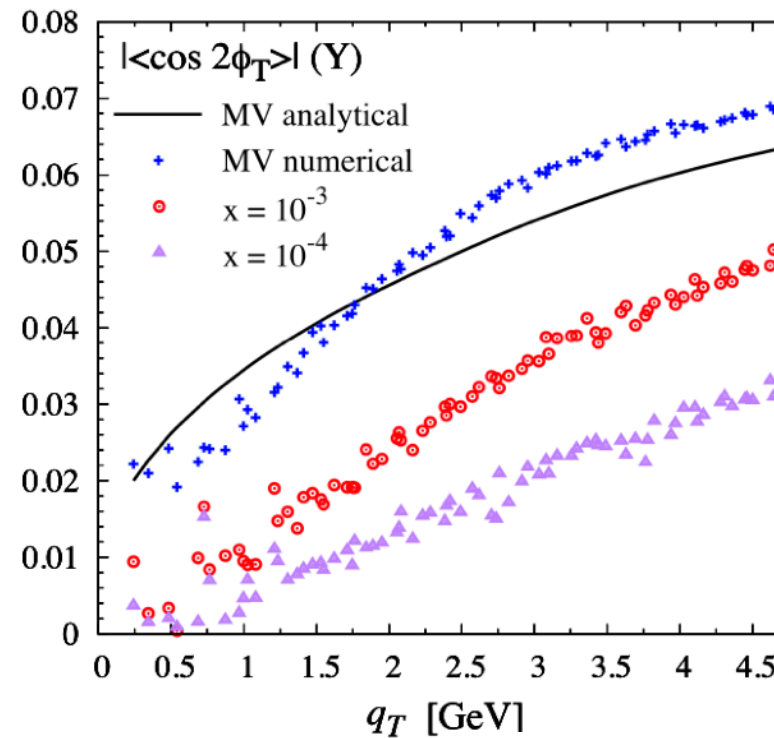
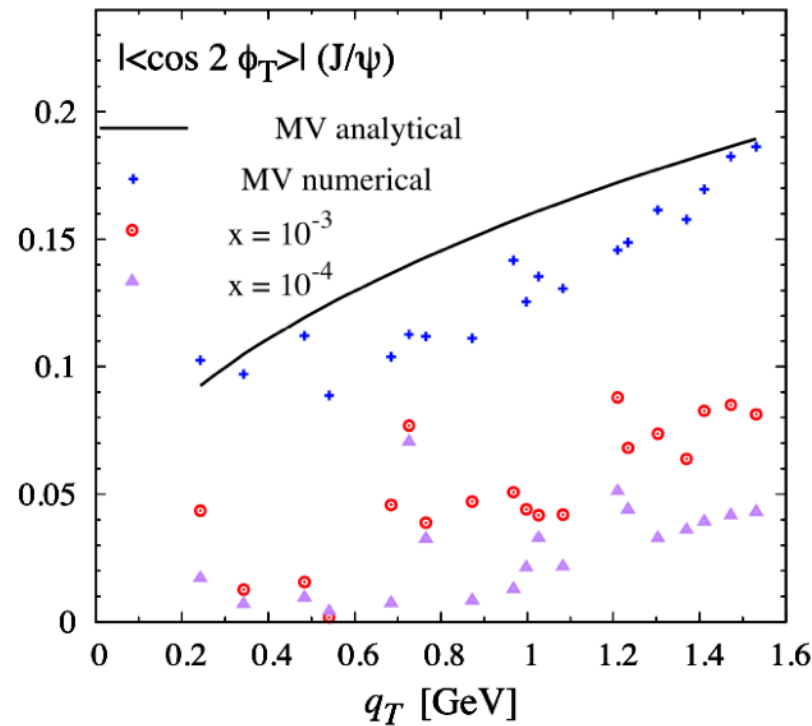
$$\langle \cos 2\phi_T \rangle = \frac{(1-y) \mathcal{B}_T^{\gamma^* g \rightarrow Q}}{[1 + (1-y)^2] \mathcal{A}_{U+L}^{\gamma^* g \rightarrow Q} - y^2 \mathcal{A}_L^{\gamma^* g \rightarrow Q}} \times \frac{\mathbf{q}_T^2}{2M_p^2} \frac{h_1^\perp g(x, \mathbf{q}_T^2)}{f_1^g(x, \mathbf{q}_T^2)}.$$

In LO NRQCD the prefactor of the asymmetry depends on kinematic variables and on two poorly known Color Octet (CO) Long Distance Matrix Elements (LDMEs)

CS is  $\alpha_s$  suppressed, but  $v^3$  enhanced; higher minimum  $Q^2$  and  $z$  cuts suppress it

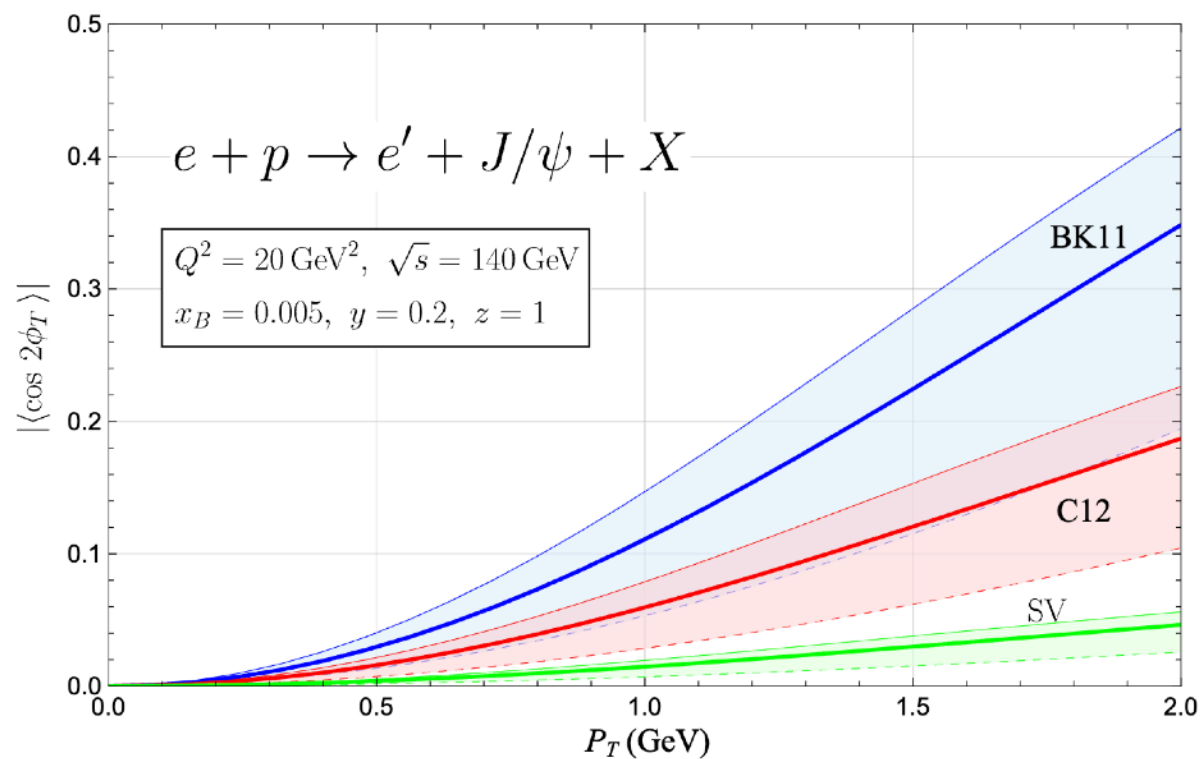
Bacchetta, DB, Pisano, Tael, 2018

# $\cos 2\phi_T$ asymmetry - predictions

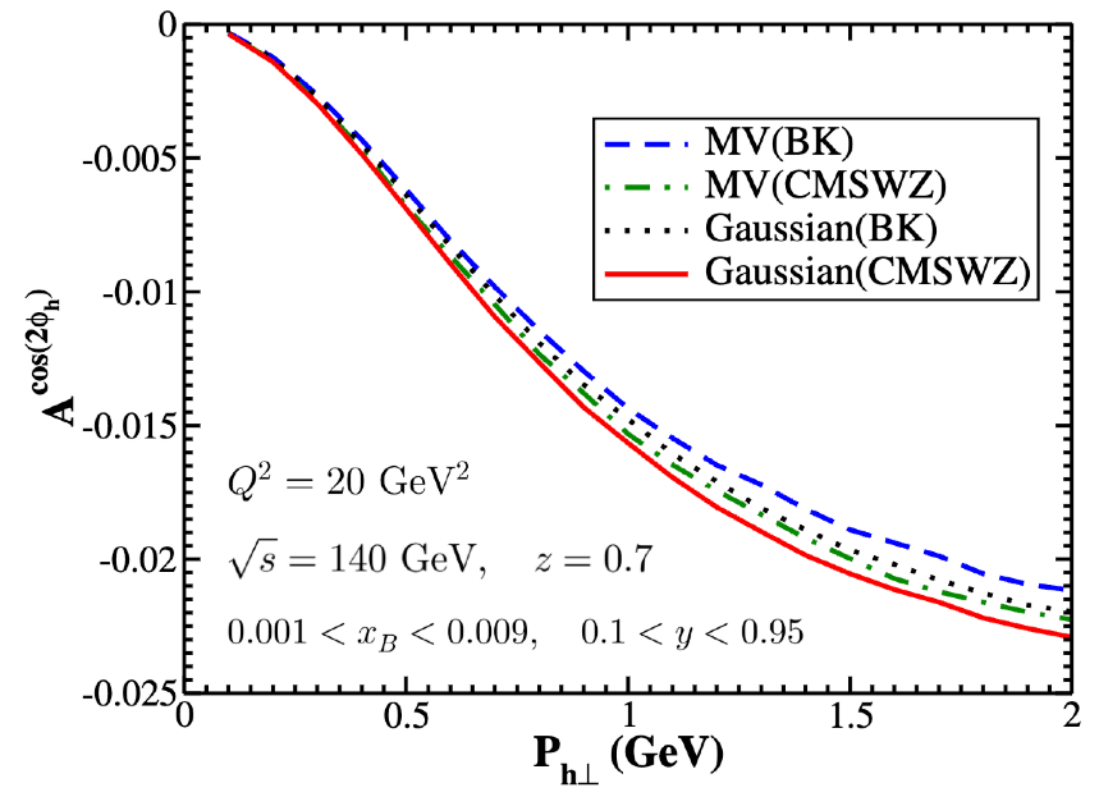


Asymmetries for  $Q=M_Q, y=0.1$   
in the small-x MV model,  
including nonlinear evolution

Bacchetta, DB, Pisano, Tael, 2018



Bor, DB, 2022



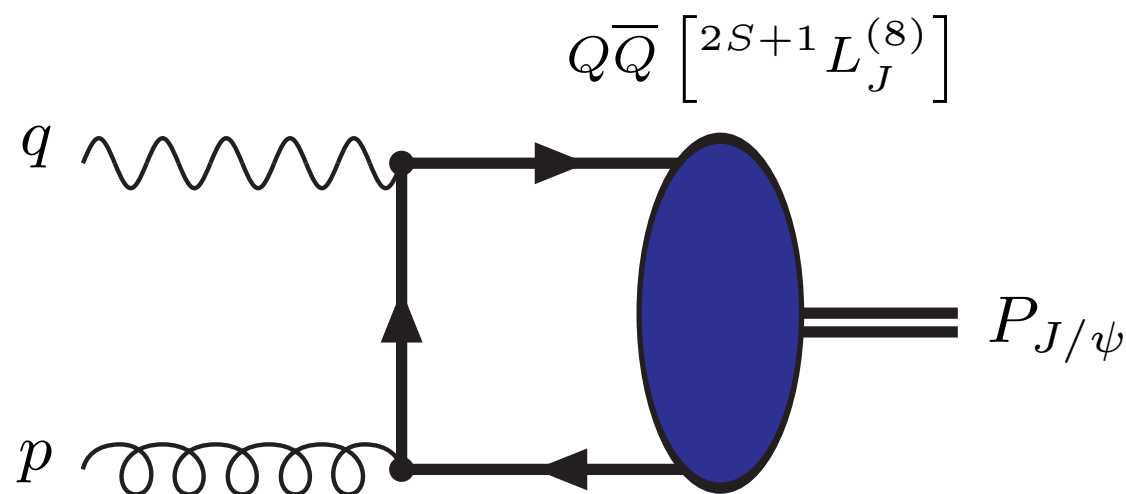
Kishore, Mukherjee, Pawar, Siddiqah, 2022

Despite the large uncertainties sizable  $\cos 2\phi_T$  asymmetries are possible at EIC

# Quarkonium production in $ep^\uparrow$

$ep^\uparrow \rightarrow e' Q X$  with  $Q$  either a  $J/\psi$  or a  $\Upsilon$  meson

Godbole, Misra, Mukherjee, Rawoot, 2012/3; Godbole, Kaushik, Misra, Rawoot, 2015;  
Mukherjee, Rajesh, 2017; Rajesh, Kishore, Mukherjee, 2018; ...



Using LO NRQCD the Sivers asymmetry is:

$$A^{\sin(\phi_S - \phi_T)} = \frac{|\mathbf{q}_T|}{M_p} \frac{f_{1T}^{\perp g}(x, \mathbf{q}_T^2)}{f_1^g(x, \mathbf{q}_T^2)}$$

$$\frac{A^{\cos 2\phi_T}}{A^{\sin(\phi_S + \phi_T)}} = \frac{\mathbf{q}_T^2}{M_p^2} \frac{h_1^{\perp g}(x, \mathbf{q}_T^2)}{h_1^g(x, \mathbf{q}_T^2)}$$

$$\frac{A^{\cos 2\phi_T}}{A^{\sin(\phi_S - 3\phi_T)}} = -\frac{1}{2} \frac{h_1^{\perp g}(x, \mathbf{q}_T^2)}{h_{1T}^{\perp g}(x, \mathbf{q}_T^2)}$$

$$\frac{A^{\sin(\phi_S - 3\phi_T)}}{A^{\sin(\phi_S + \phi_T)}} = -\frac{\mathbf{q}_T^2}{2M_p^2} \frac{h_{1T}^{\perp g}(x, \mathbf{q}_T^2)}{h_1^g(x, \mathbf{q}_T^2)}$$

CO NRQCD LDMEs cancel out in ratios of asymmetries at LO

Bacchetta, DB, Pisano, Tael, 2018

Higher order corrections and shape functions will complicate this simple picture

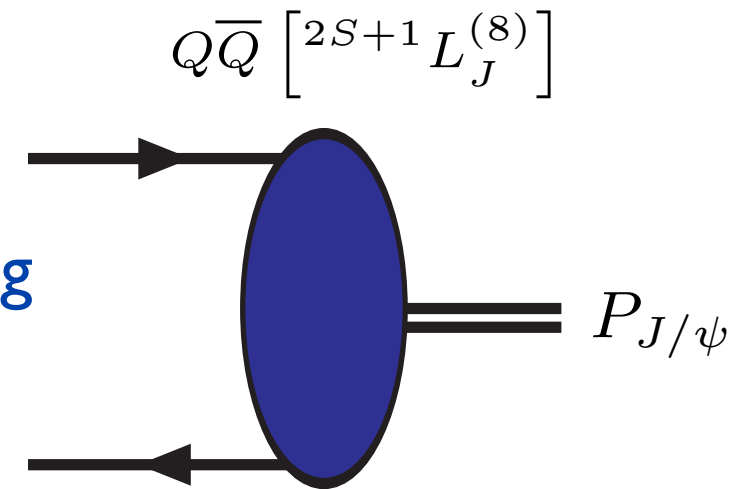
# Shape functions

# Effect of smearing

In reality the process of  $Q\bar{Q} \rightarrow J/\psi$  involves some  $k_T$ -smearing

TMD factorization requires inclusion of shape function  $\Delta$

Echevarria, 2019; Fleming, Makris & Mehen, 2019



Using LO NRQCD the unpolarized structure function becomes:

$$\mathcal{F}_{UU,T} = \frac{2\pi^2 \alpha_s e_c^2}{M_\psi (M_\psi^2 + Q^2)} \left[ \langle 0 | \mathcal{O}(^1S_0^{[8]}) | 0 \rangle + 4 \frac{(7M_\psi^4 + 2M_\psi^2 Q^2 + 3Q^4)}{M_\psi^2 (M_\psi^2 + Q^2)^2} \langle 0 | \mathcal{O}(^3P_0^{[8]}) | 0 \rangle \right] \\ \times f_1^g(x, p_T^2) \Big|_{\mathbf{p}_T = \mathbf{q}_T}$$

introducing a shape function

$$\int d^2 \mathbf{p}_T \int d^2 \mathbf{k}_T \delta^2(\mathbf{q}_T - \mathbf{p}_T - \mathbf{k}_T) f_1^g(x, \mathbf{p}_T^2; \mu^2) \Delta^{[n]}(\mathbf{k}_T^2, \mu^2)$$

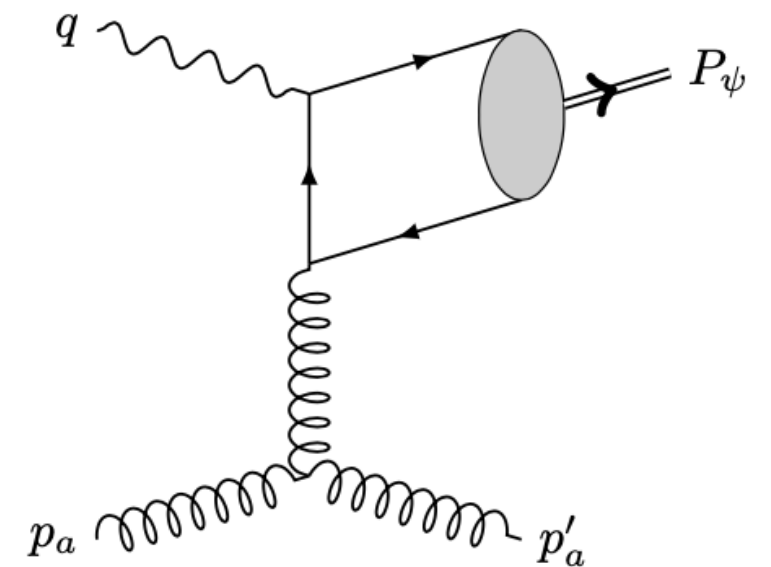
LO NRQCD:  $\Delta^{[n]}(\mathbf{k}_T^2; \mu^2) = \langle 0 | \mathcal{O}(n) | 0 \rangle \delta^2(\mathbf{k}_T)$

More generally, the shape function will be a smeared out delta function with a tail

# Perturbative tail of the shape function

LO NRQCD:  $\Delta^{[n]}(\mathbf{k}_T^2; \mu^2) = \langle 0 | \mathcal{O}(n) | 0 \rangle \delta^2(\mathbf{k}_T)$

Taking into account non-analytic behavior of the hard scattering amplitude ( $z \rightarrow 1$  and  $q_T \rightarrow 0$  limits do not commute)



Perturbative tail:

$$\Delta^{[n]}(z, \mathbf{k}_T^2; \tilde{Q}^2) = -\frac{\alpha_s}{2\pi^2 \mathbf{k}_T^2} C_A \left( 1 + \log \frac{M_\psi^2}{M_\psi^2 + Q^2} \right) \langle \mathcal{O}[n] \rangle \delta(1 - z).$$

$$\mu_H^2 \equiv \tilde{Q}^2 = M_\psi^2 + Q^2$$

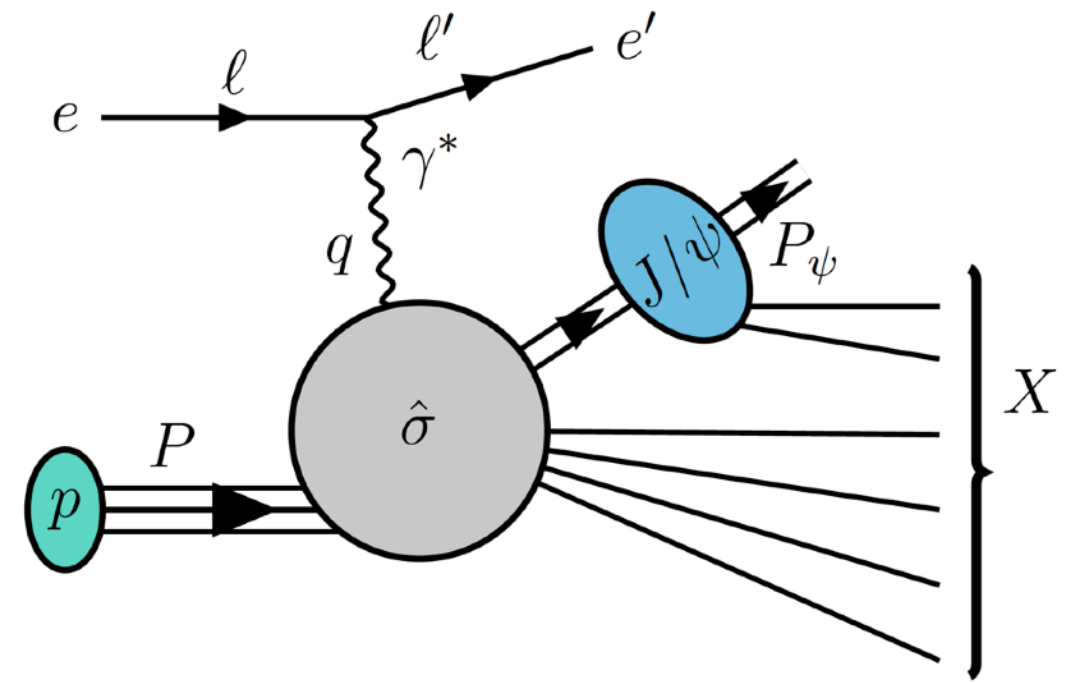
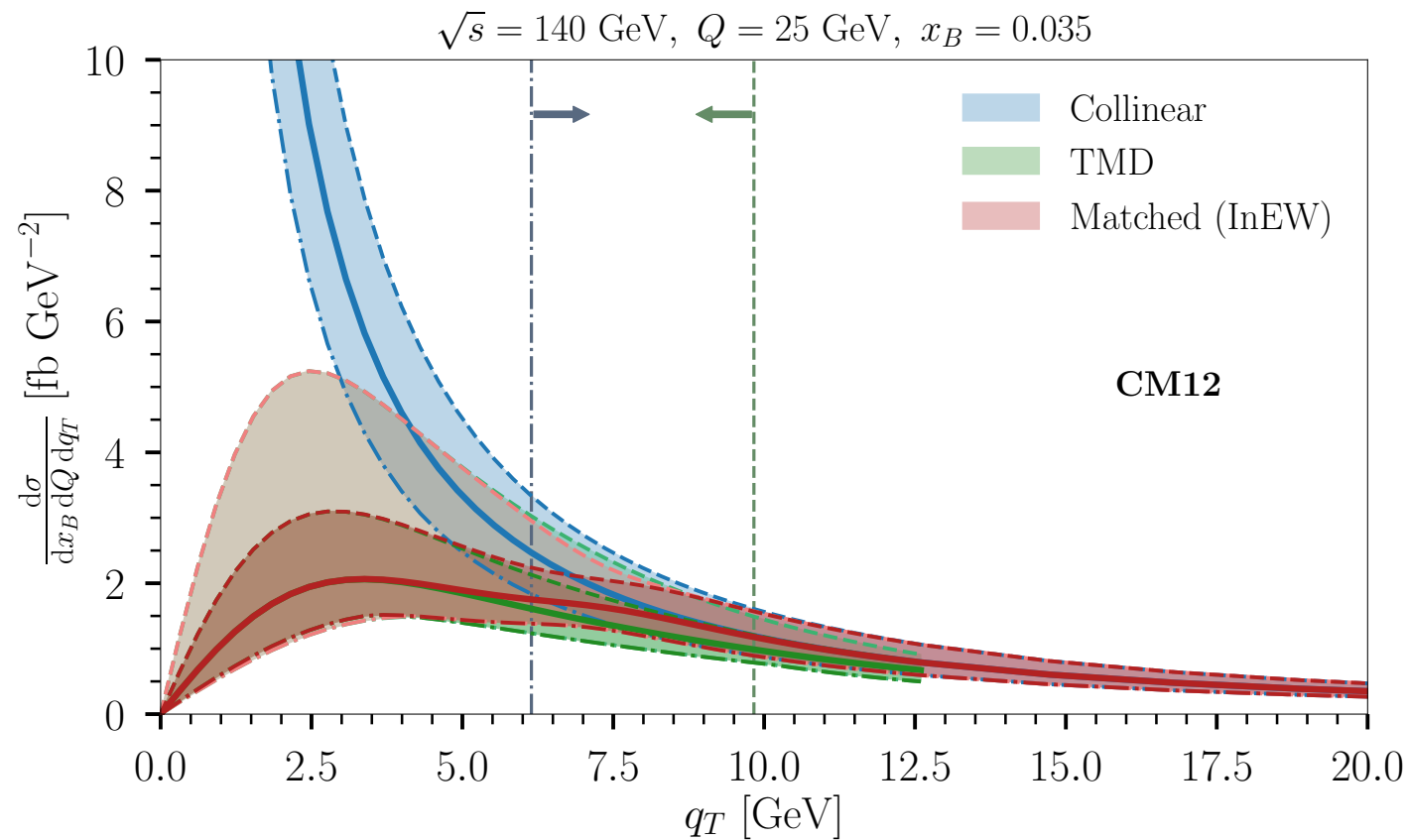
DB, Bor, Maxia, Pisano, Yuan, 2023

Even before taking the hard scale to be  $Q^2$  dependent, the tail of the shape function is  $Q^2$  dependent

Process dependence!

In agreement with open heavy quark pair expression by Zhu, Sun, Yuan, 2013

# Matching



Matching the TMD and collinear (FO) expressions with uncertainties using Inverse Error Weighting (InEW)

$$\Delta^{[n]}(z, \mathbf{k}_T^2; \tilde{Q}^2) = -\frac{\alpha_s}{2\pi^2 \mathbf{k}_T^2} C_A \left( 1 + \log \frac{M_\psi^2}{M_\psi^2 + Q^2} \right) \langle \mathcal{O}[n] \rangle \delta(1 - z).$$

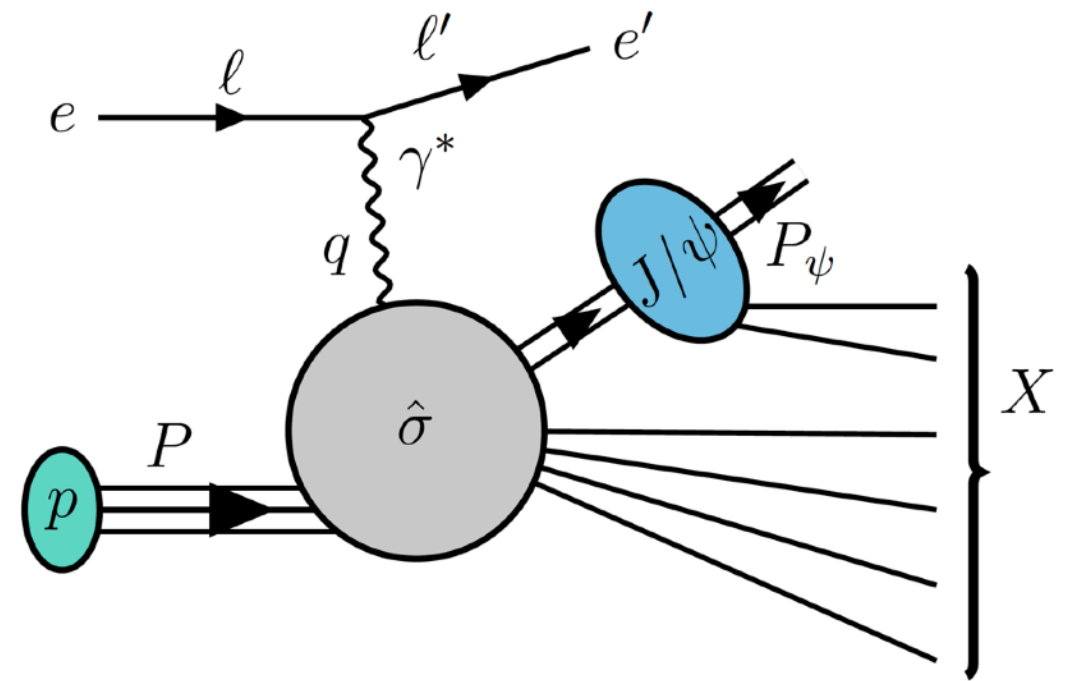
$$S_{\text{pert}} = \int_{\mu_b^2}^{\mu^2} \frac{d\eta^2}{\eta^2} \left[ A_g(\alpha_s(\eta)) \log \frac{\mu^2}{\eta^2} + B_g(\alpha_s(\eta)) \right] + \int_{\mu_b^2}^{\mu^2} \frac{d\eta^2}{\eta^2} B_{\text{CO}}(\alpha_s(\eta))$$

$B_{\text{CO}} = 0$  means no tail and no resummation

$B_{\text{CO}} = 1$  means no process dependent term

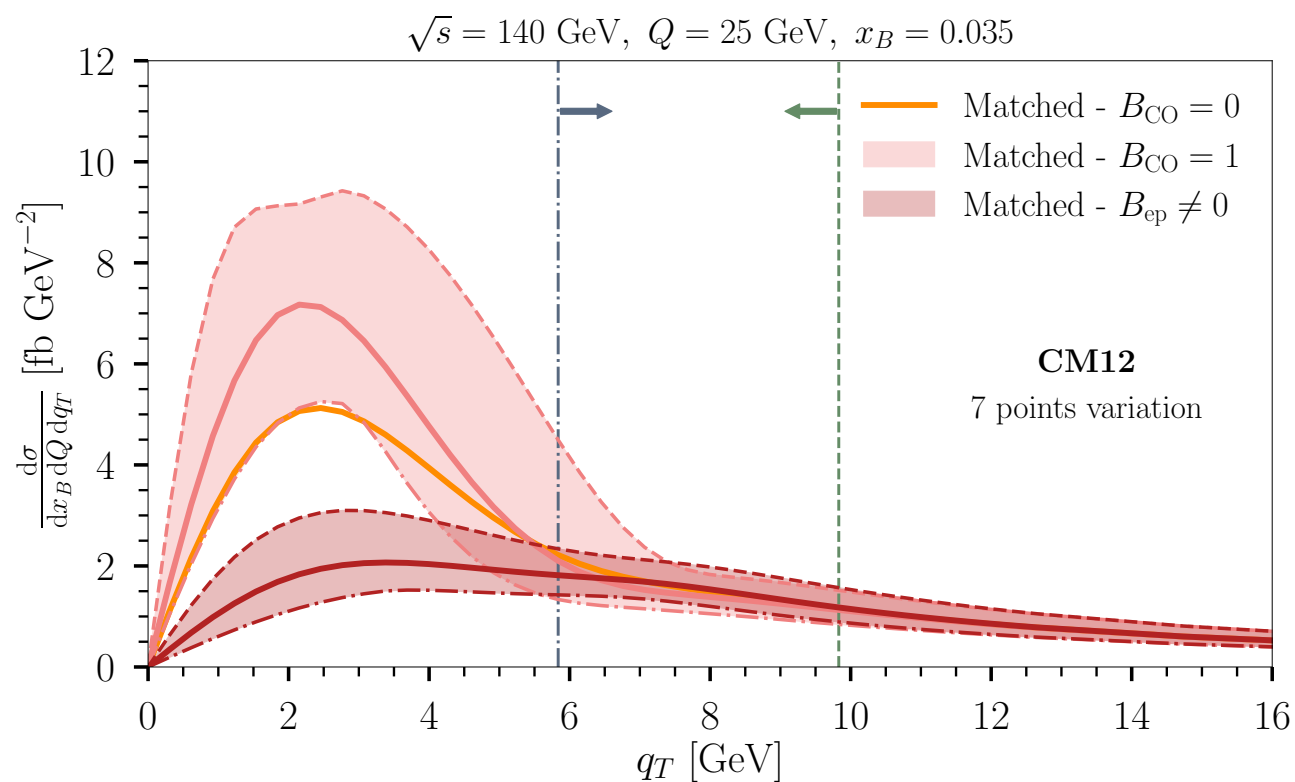
# Matching

Uncertainty bands include LDME, scale & nonperturbative Sudakov uncertainties

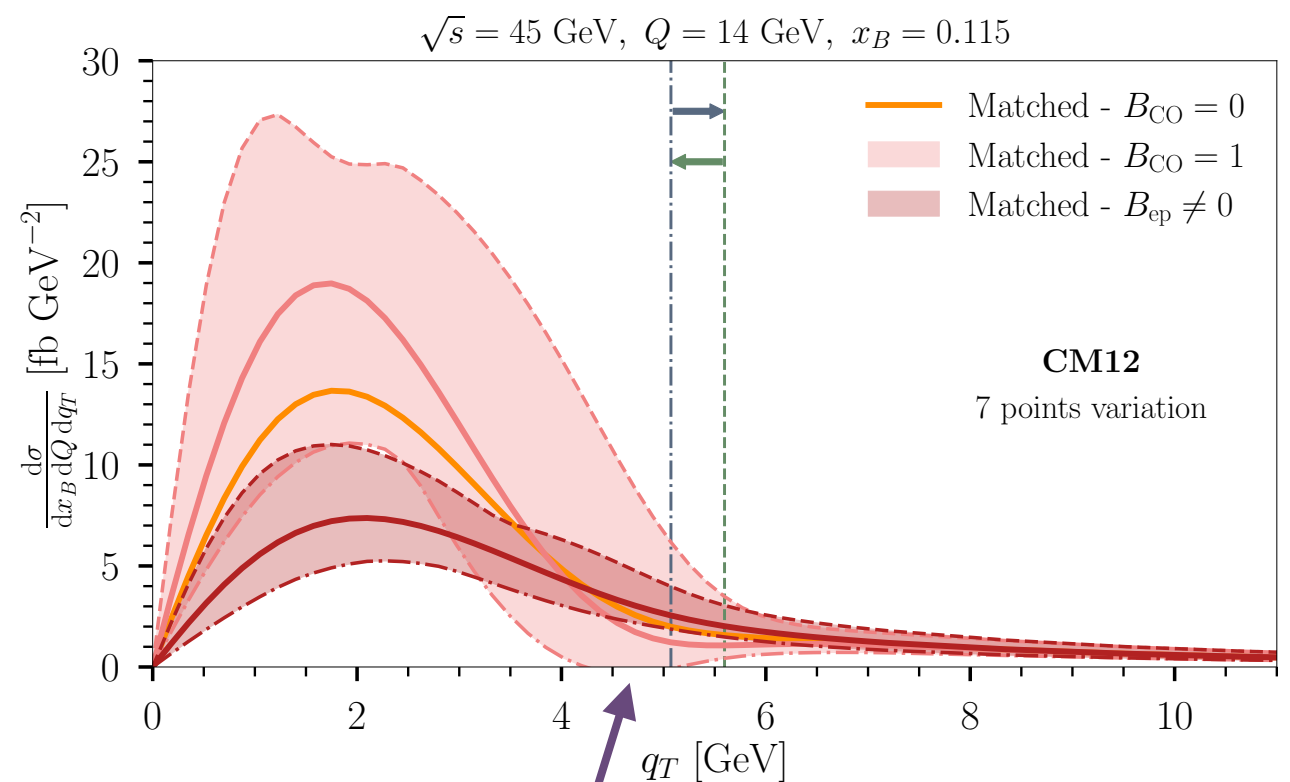


$$S_{\text{pert}} = \int_{\mu_b^2}^{\mu^2} \frac{d\eta^2}{\eta^2} \left[ A_g(\alpha_s(\eta)) \log \frac{\mu^2}{\eta^2} + B_g(\alpha_s(\eta)) \right] + \int_{\mu_b^2}^{\mu^2} \frac{d\eta^2}{\eta^2} B_{\text{CO}}(\alpha_s(\eta))$$

Maxia, DB, Bor, 2025

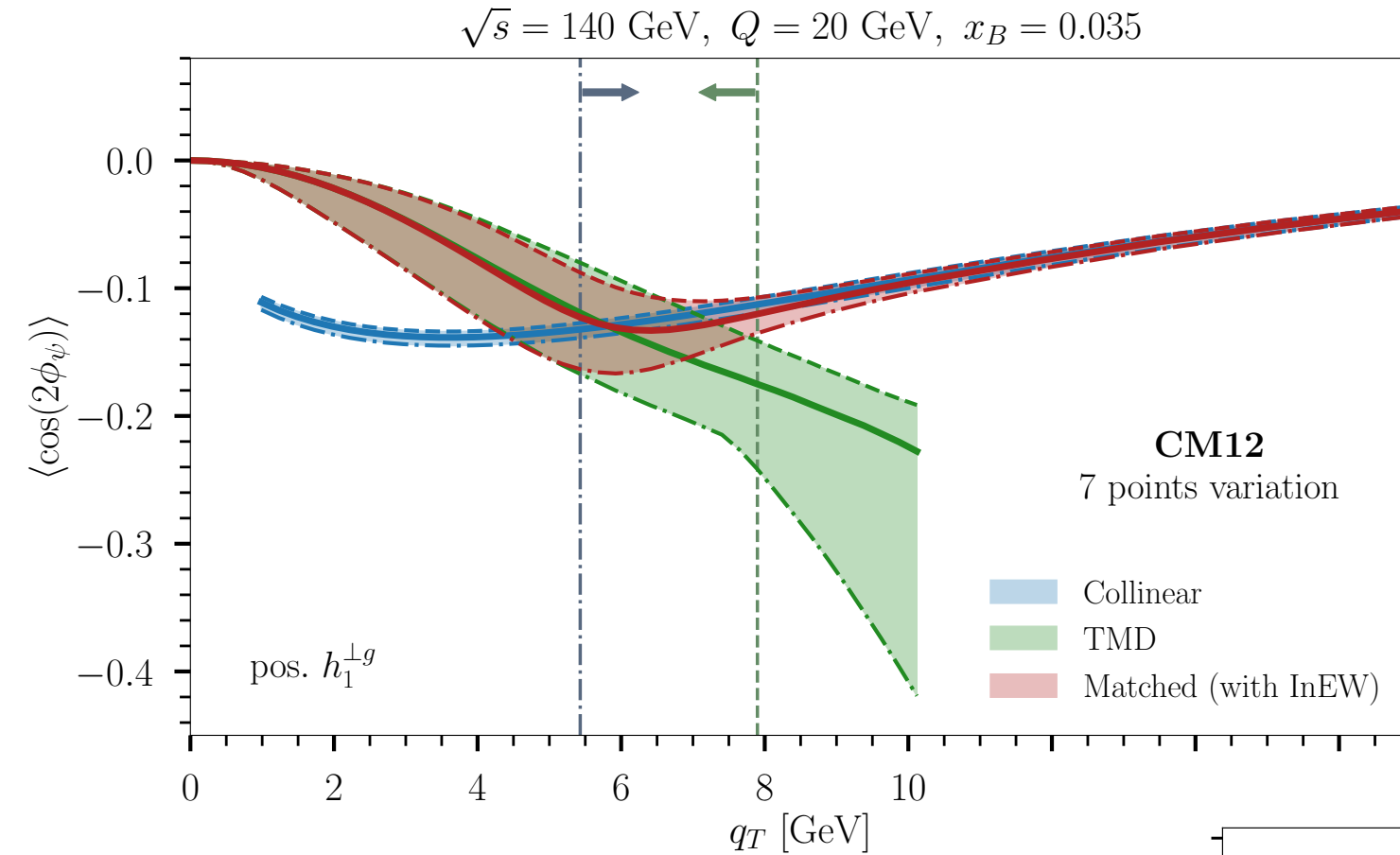
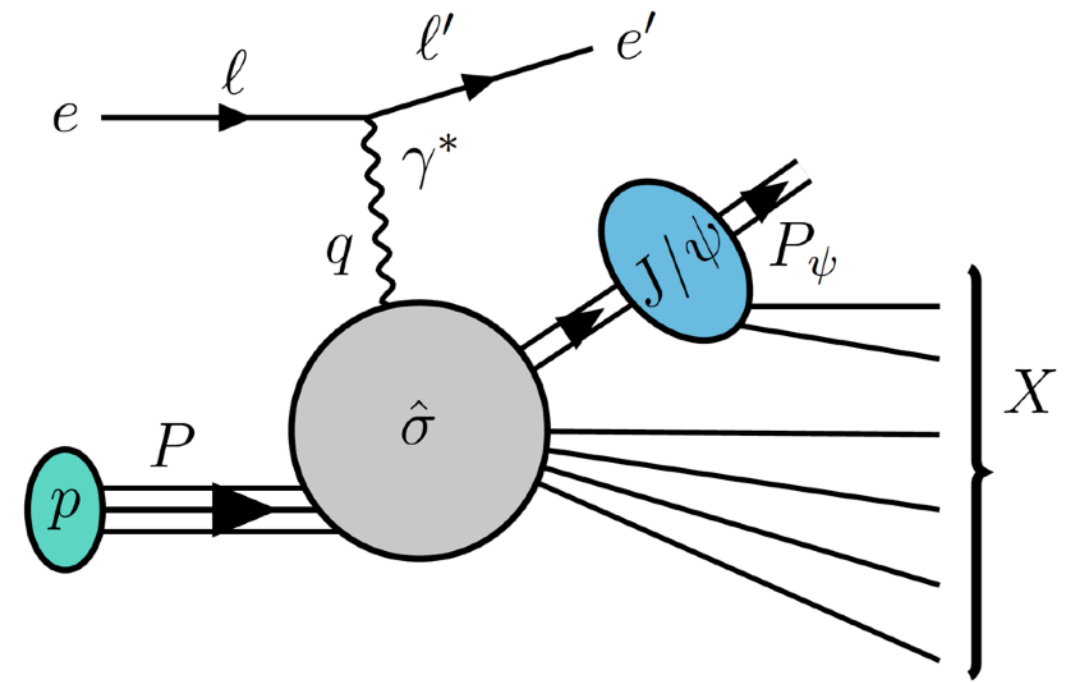


Process dependence matters



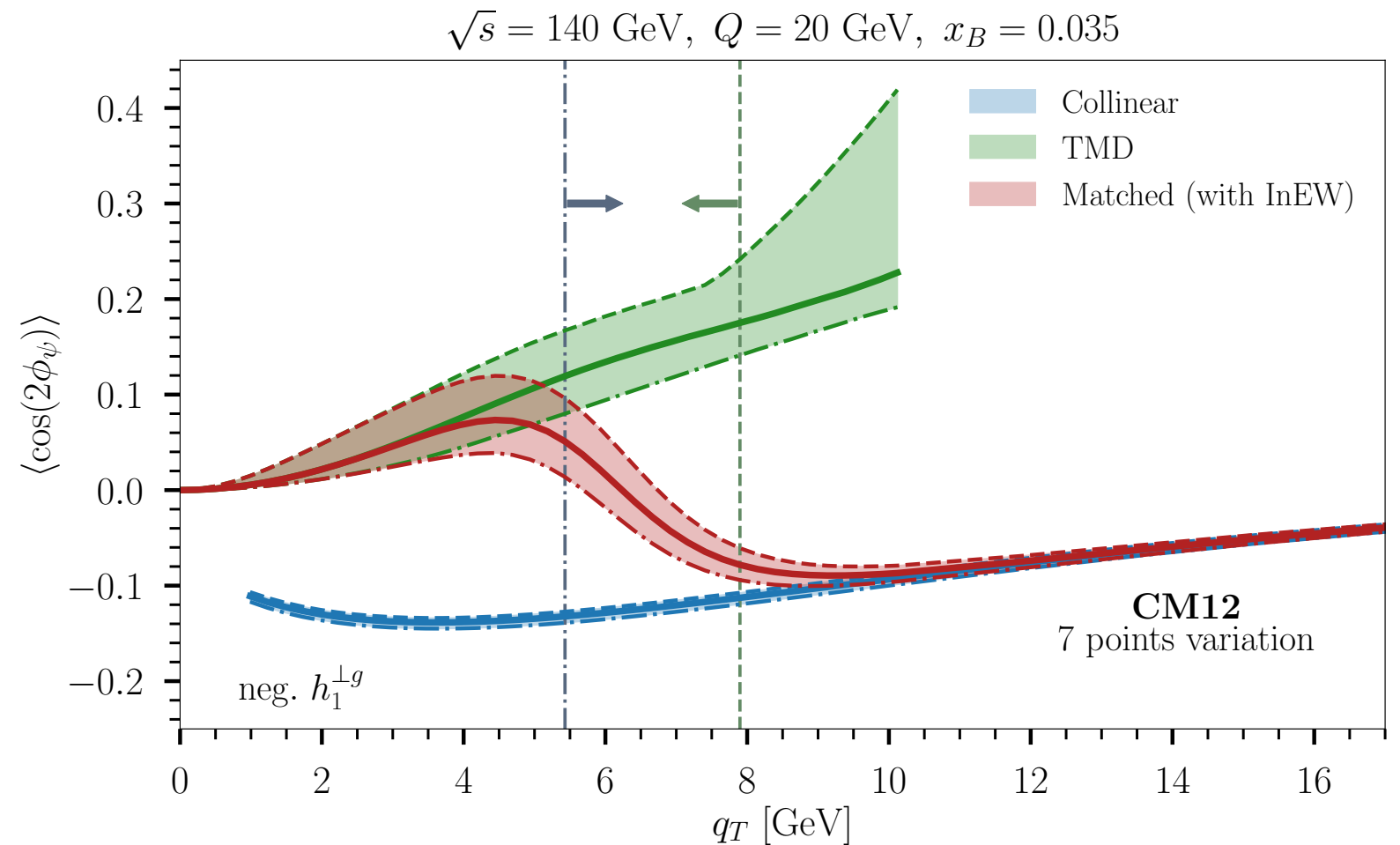
Negative cross sections can arise and asymmetries overshooting 1

# Asymmetry



Sign of  $h_1^{\perp g}$  not known

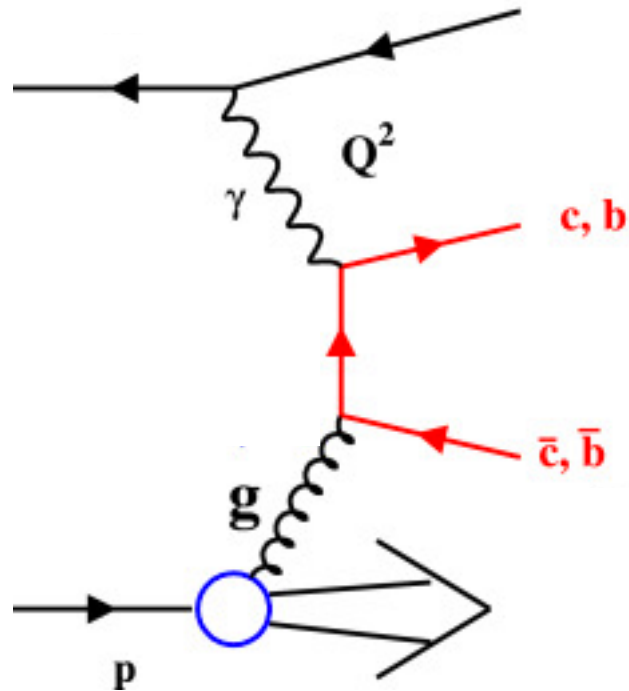
Modulations can be in the 10% range at EIC, a node is possible



Open heavy quark pairs

# Open heavy quark electro-production

Unpolarized open heavy quark production at EIC probes  $h_1^{\perp g}(x, p_T^2)$



The heavy quarks will not be exactly back-to-back in the transverse plane:

$$K_{\perp} = (K_{Q\perp} - K_{\bar{Q}\perp})/2$$

$$q_T = K_{Q\perp} + K_{\bar{Q}\perp}$$

$$|q_T| \ll |K_{\perp}|$$

$\phi_T, \phi_{\perp}$  are the angles of  $q_T, K_{\perp}$

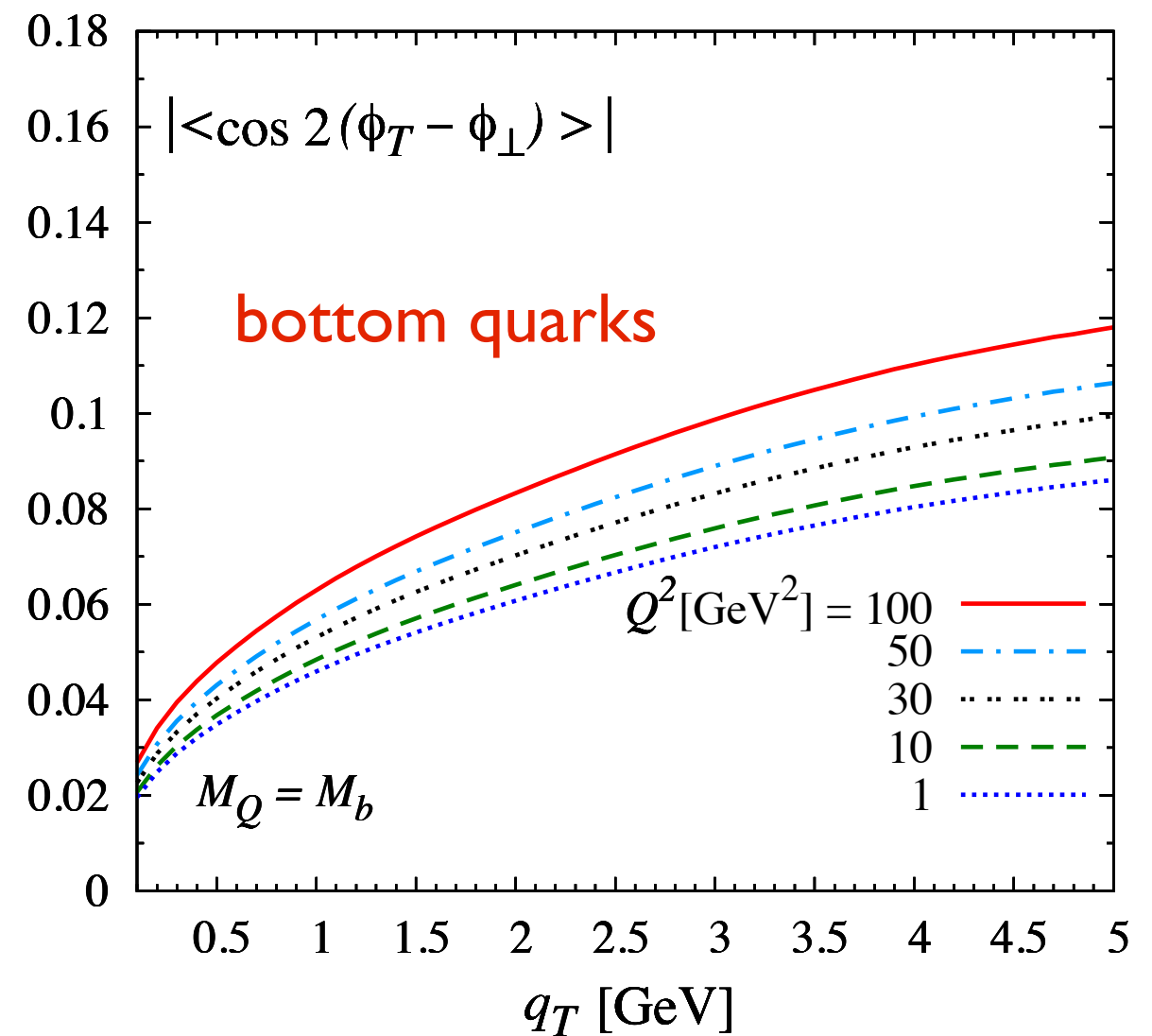
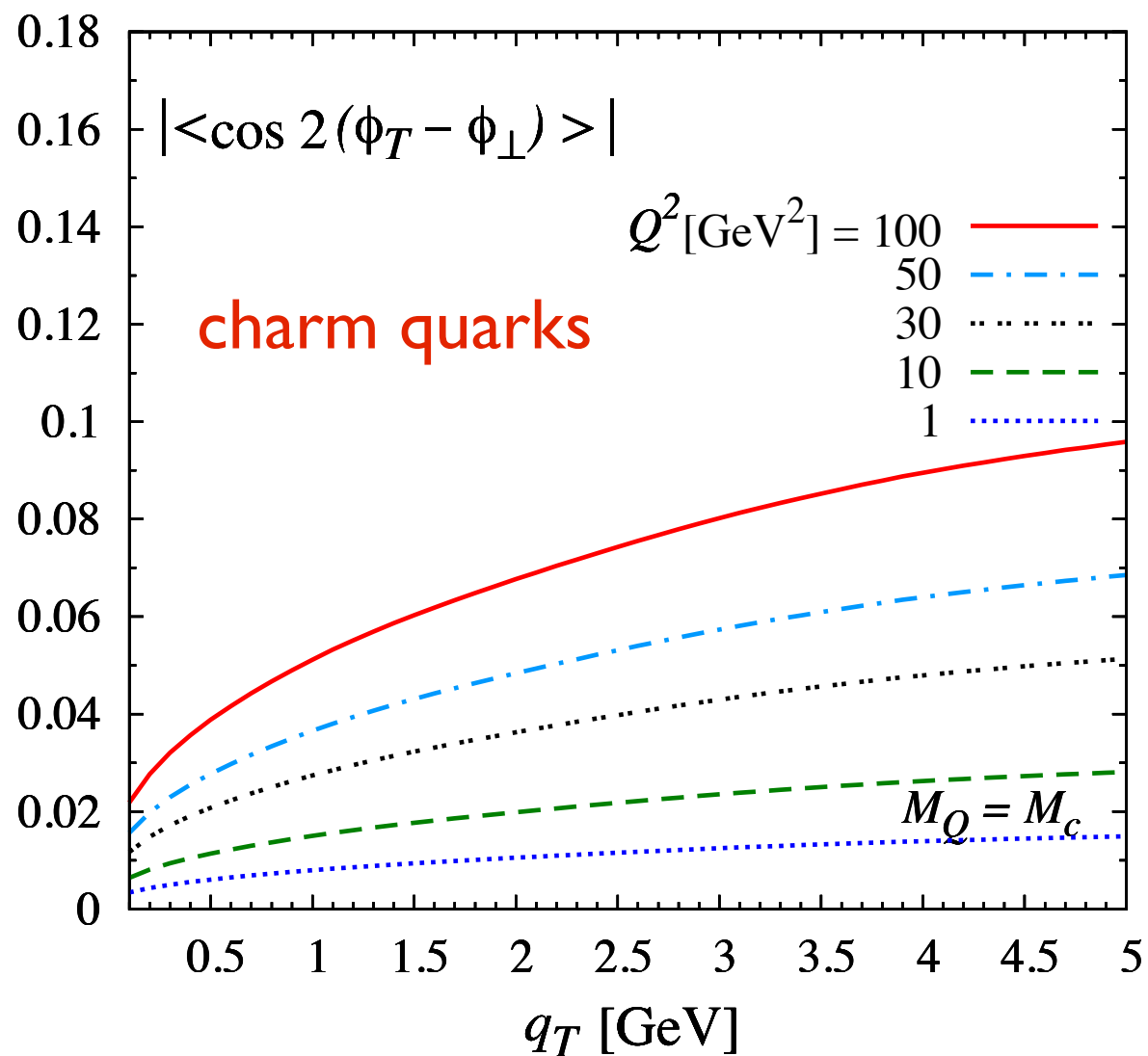
Linear gluon polarization shows up as a  $\cos 2\phi_T$  or  $\cos 2(\phi_T - \phi_{\perp})$  distribution

Boer, Brodsky, Mulders & Pisano, 2010

$h_1^{\perp g}$  is also accessible in inclusive dijet production at EIC

Metz, Zhou 2011; Pisano, Boer, Brodsky, Buffing, Mulders, 2013; Dumitru, Lappi, Skokov, 2015; ...

# Asymmetries in heavy quark pair production



small  $x$   
 MV model  
 $|\mathbf{K}_\perp| = 10 \text{ GeV}$   
 $z = 0.5$   
 $y = 0.3$

Up to 10% asymmetries at EIC

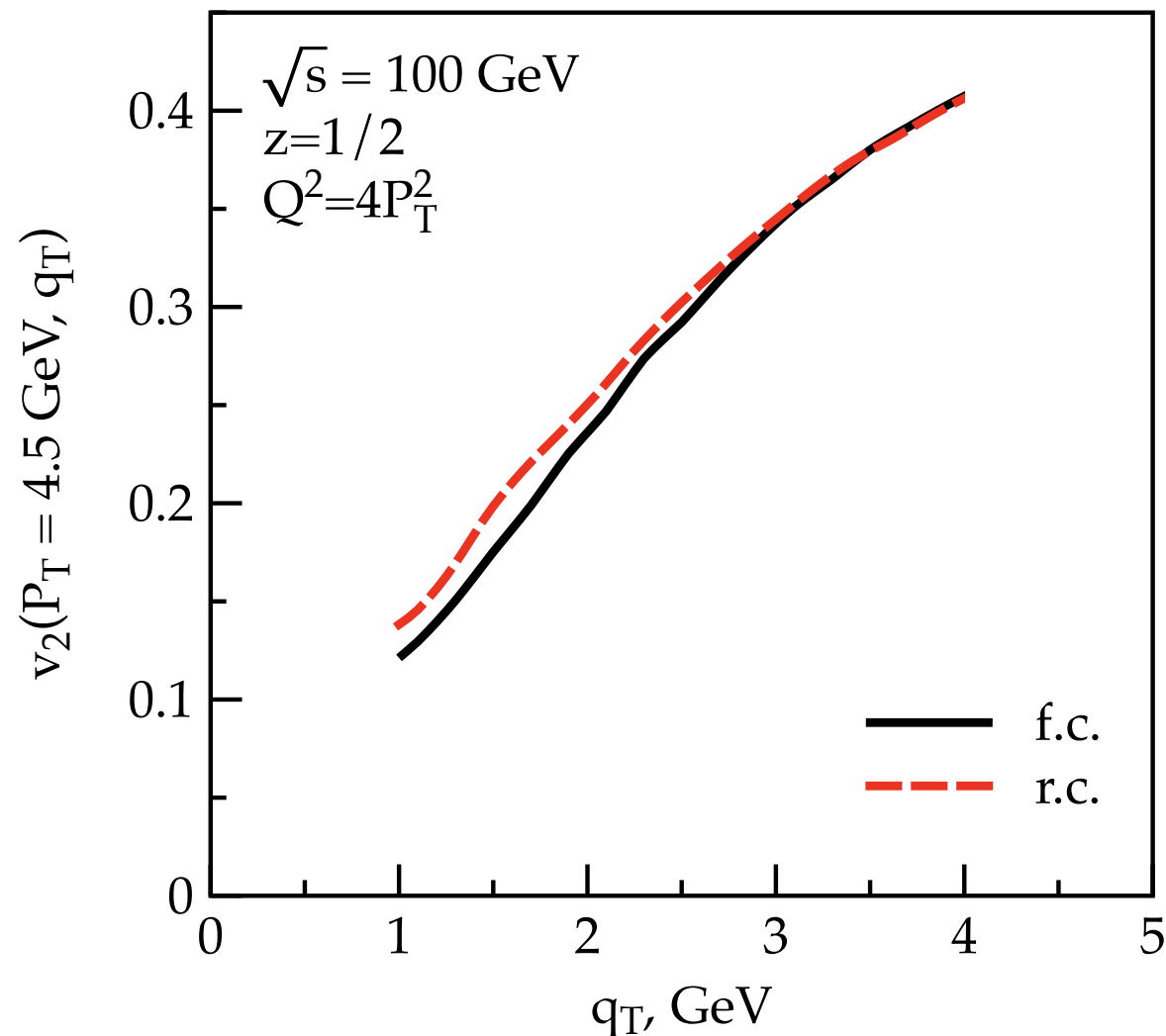
DB, Pisano, Mulders, Zhou, 2016

However, this does not include TMD or  $x$ -evolution yet

# Inclusive dijet production at EIC

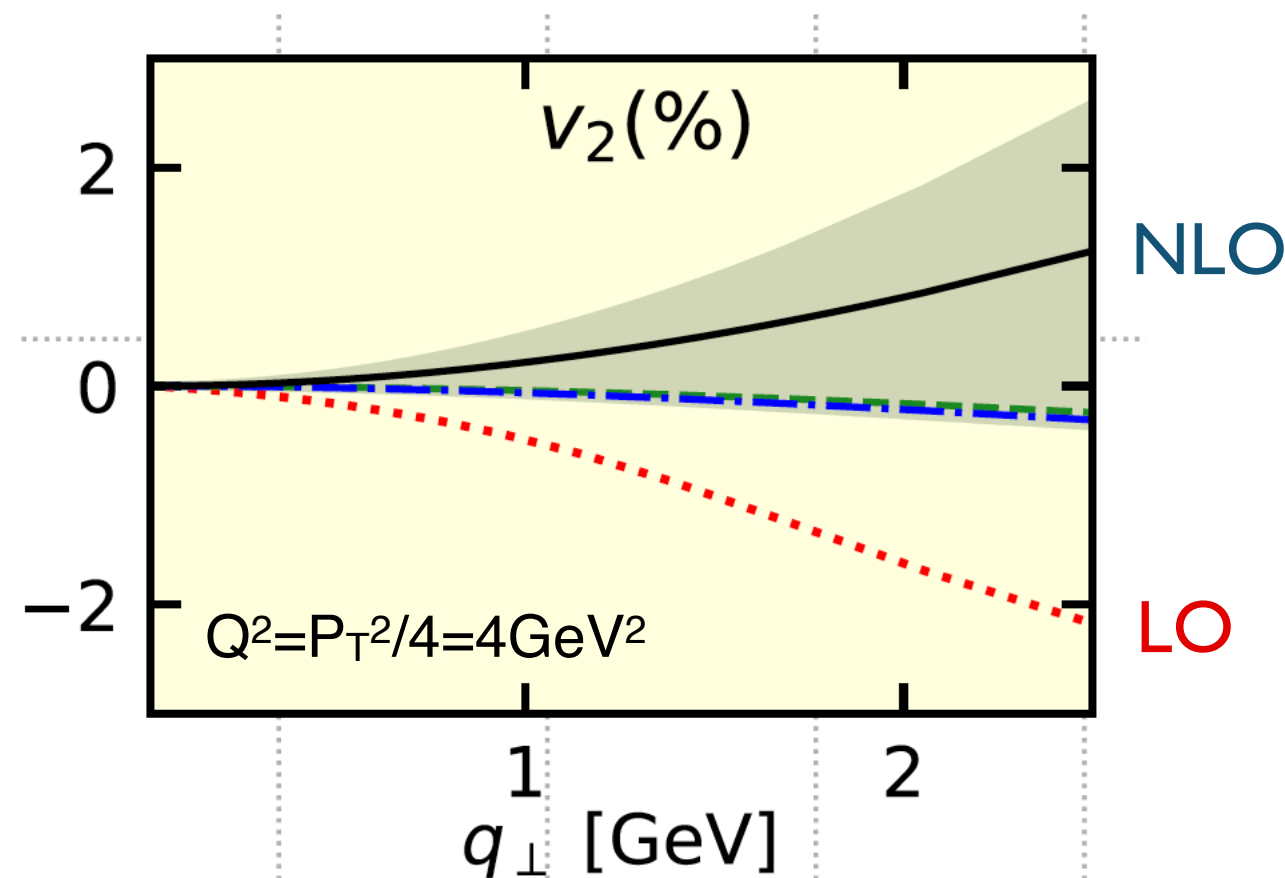
$$\phi = \phi_T - \phi_\perp$$

Linear gluon polarization shows itself through a  $\cos 2\phi$  distribution (“ $v_2$ ”)



Large effects are found

Dumitru, Lappi, Skokov, 2015



Effect of including NLO corrections

Caucal, Salazar, Schenke, Stebel,  
Venugopalan, 2024

Sign of  $v_2$  is matter of definition and depends on sign of  $h_{1\perp}$ , but it can apparently flip due to HO corrections; note that the probed (WW) distribution does not satisfy the same BK equation as  $f_1$  (unlike the DP one)

# Gluon GTMDs

# 3D momentum and spatial distributions

TMDs - 3D momentum structure ( $x$  &  $k_T$ )

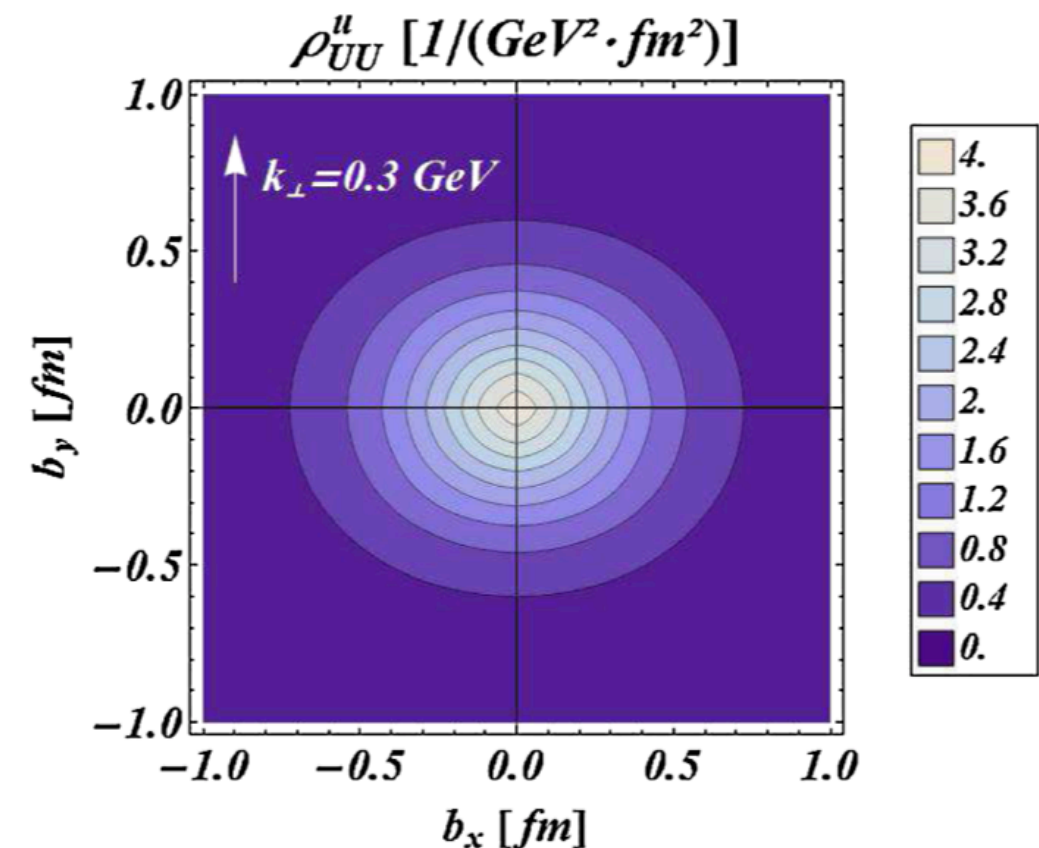
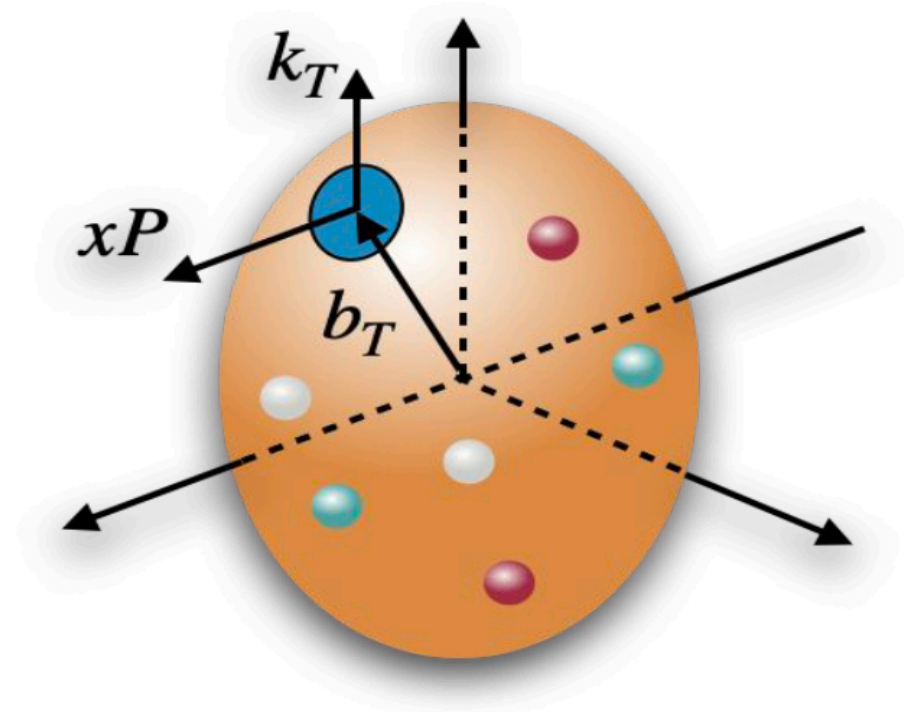
GPDs - 3D spatial structure ( $\xi$  &  $t$  or  $z$  &  $b_T$ )

GTMDs - combined 5D (or 6D) structure

This talk: only zero skewness ( $\xi=0$ ) and mostly small  $x$

Teaches us about orbital angular momentum

Lorce, Pasquini, 2011; Hatta, 2011; ...



# GTMDs - 5D parton distributions

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Off-forward distributions, like GPDs, give access to the transverse spatial distributions; here the proton stays intact but gets a momentum kick

GTMDs can be seen as:

- off-forward TMDs
- transverse momentum dependent GPDs
- Fourier transforms of Wigner distributions

$$G(x, \mathbf{k}_T, \mathbf{\Delta}_T) \xleftrightarrow{FT} W(x, \mathbf{k}_T, \mathbf{b}_T)$$

Meißner, Metz, Schlegel, 2009

Ji, 2003; Belitsky, Ji & Yuan, 2004

GTMDs combine all properties of TMDs and GPDs, such as process dependence & nontrivial impact parameter dependence

# Gluon GTMDs for unpolarized protons

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For unpolarized protons there are 2 (real valued) gluon TMDs:

$$\Gamma_U^{ij}(x, \mathbf{k}_T) = \frac{x}{2} \left[ -g_T^{ij} f_1(x, \mathbf{k}_T^2) + \frac{k_T^{ij}}{M^2} h_1^\perp(x, \mathbf{k}_T^2) \right]$$

$$a_T^{ij} \equiv a_T^i a_T^j - \frac{1}{2} a_T^2 \delta_T^{ij}$$

Mulders, Rodrigues, 2001

For GTMDs one has one more vector so more anisotropic terms can arise

For unpolarized protons there are 4 (complex valued) gluon GTMDs

$$G^{[U,U']}^{ij}(x, \mathbf{k}_T, \mathbf{\Delta}_T) = x \left( \delta_T^{ij} \mathcal{F}_1 + \frac{k_T^{ij}}{M^2} \mathcal{F}_2 + \frac{\Delta_T^{ij}}{M^2} \mathcal{F}_3 + \frac{k_T^{[i} \Delta_T^{j]}}{M^2} \mathcal{F}_4 \right)$$

DB, van Daal, Mulders, Petreska, 2018

Lorcé, Pasquini, 2013; More, Mukherjee, Nair, 2018

Also for GTMDs gauge links  $[U,U']$  matter  $\rightarrow$  WW and DP versions at small  $x$

# Dipole gluon GTMD

In the  $x \rightarrow 0$  the dipole gluon GTMD becomes a correlator of a single Wilson loop:

$$G^{[+,-]ij}(\mathbf{k}, \mathbf{\Delta}) = \frac{2N_c}{\alpha_s} \left[ \frac{1}{2} \left( \mathbf{k}^2 - \frac{\mathbf{\Delta}^2}{4} \right) \delta_T^{ij} + k_T^{ij} - \frac{\Delta_T^{ij}}{4} - \frac{k_T^{[i} \Delta_T^{j]}}{2} \right] G^{[\square]}(\mathbf{k}, \mathbf{\Delta})$$

$$G^{[\square]}(\mathbf{k}, \mathbf{\Delta}) \equiv \int \frac{d^2\mathbf{x} d^2\mathbf{y}}{(2\pi)^4} e^{-i\mathbf{k} \cdot (\mathbf{x} - \mathbf{y}) + i\mathbf{\Delta} \cdot \frac{\mathbf{x} + \mathbf{y}}{2}} \frac{\langle p' | S^{[\square]}(\mathbf{x}, \mathbf{y}) | p \rangle |_{\text{LF}}}{\langle P | P \rangle},$$

All gluon polarization states (linear & circular) become related:

$$\lim_{x, \xi \rightarrow 0} x\mathcal{F}_1 = \lim_{x, \xi \rightarrow 0} x\mathcal{F}_2^{(1)} = -4 \lim_{x, \xi \rightarrow 0} x\mathcal{F}_3^{(1)} = -2 \lim_{x, \xi \rightarrow 0} x\mathcal{F}_4^{(1)} = \mathcal{E}^{(1)}$$

$$\mathcal{F}_i^{(n)} \equiv [(\mathbf{k}^2 - \mathbf{\Delta}^2/4)/(2M^2)]^n \mathcal{F}_i \quad \text{DB, van Daal, Mulders, Petreska, 2018}$$

Not expected to hold for the WW GTMD, except at large  $k_\perp$

Real part of  $G^{[\square]}(\mathbf{k}, \mathbf{\Delta})$  only depends on  $k^2$ ,  $\Delta^2$  and  $(\mathbf{k} \cdot \mathbf{\Delta})^2$

It means that also the  $\delta_T^{ij}$  term can be anisotropic now

# Elliptic Wigner distributions

$$xW(x, \mathbf{b}, \mathbf{k}) = x\mathcal{W}_0(x, \mathbf{b}^2, \mathbf{k}^2) + 2 \cos(\phi_b - \phi_k) x\mathcal{W}_1(x, \mathbf{b}^2, \mathbf{k}^2) \\ + 2 \cos 2(\phi_b - \phi_k) x\mathcal{W}_2(x, \mathbf{b}^2, \mathbf{k}^2) + \dots$$

The  $\cos 2(\phi_b - \phi_k)$  part is called “the” elliptic Wigner distribution

Hatta, Xiao, Yuan, 2016; J. Zhou, 2016; Mäntysaari, Mueller, Schenke, 2019; Salazar, Schenke, 2019

There can be such an elliptic piece in each Wigner distribution

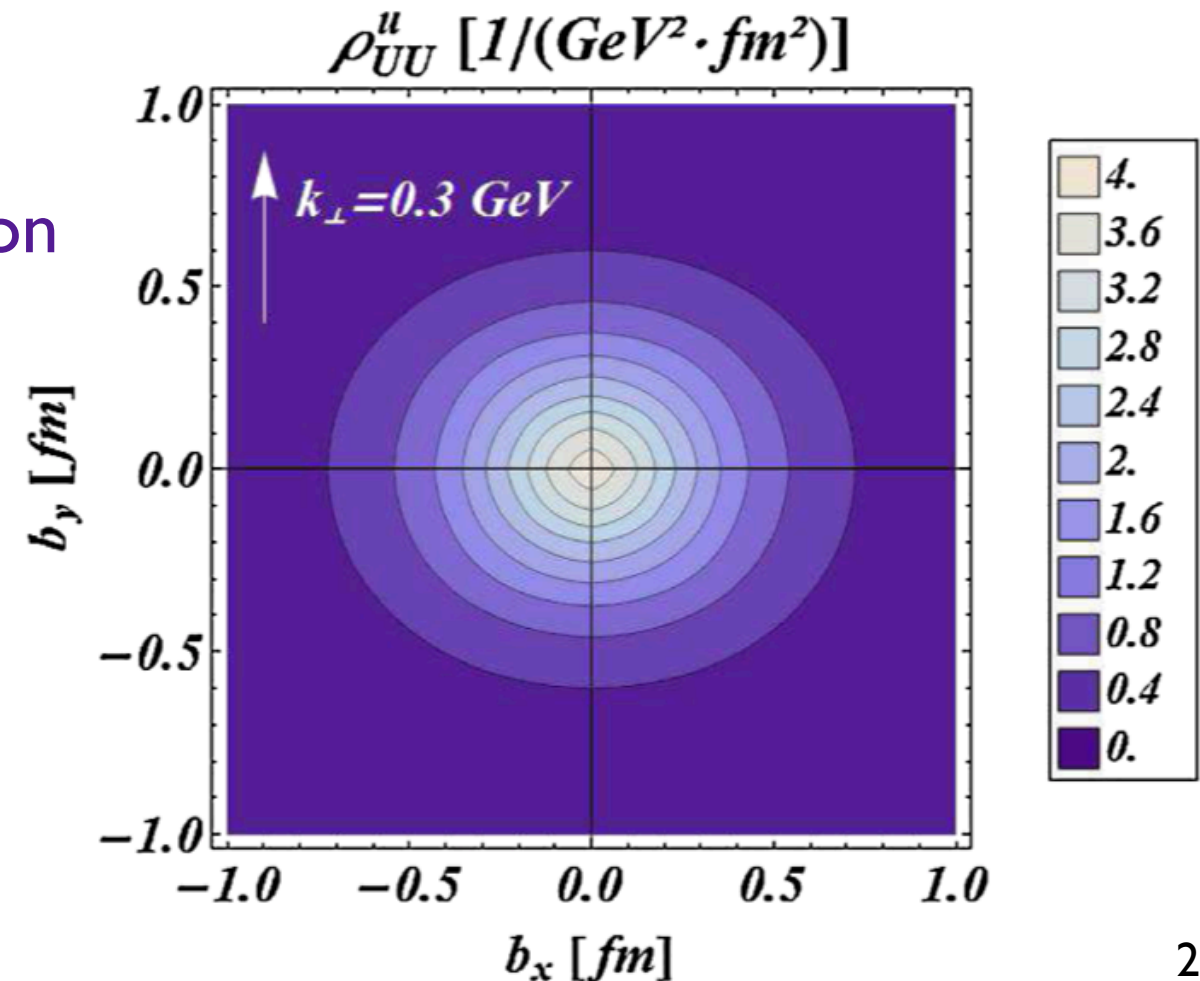
Hence 4 different ones for unpolarized protons, reducing to 1 in the small-x limit

A nonzero elliptic quark Wigner distribution  
in the lightcone constituent quark model:

Lorcé, Pasquini, 2011

Due to quark orbital angular momentum

Lorcé, Pasquini, 2011; Hatta, 2011



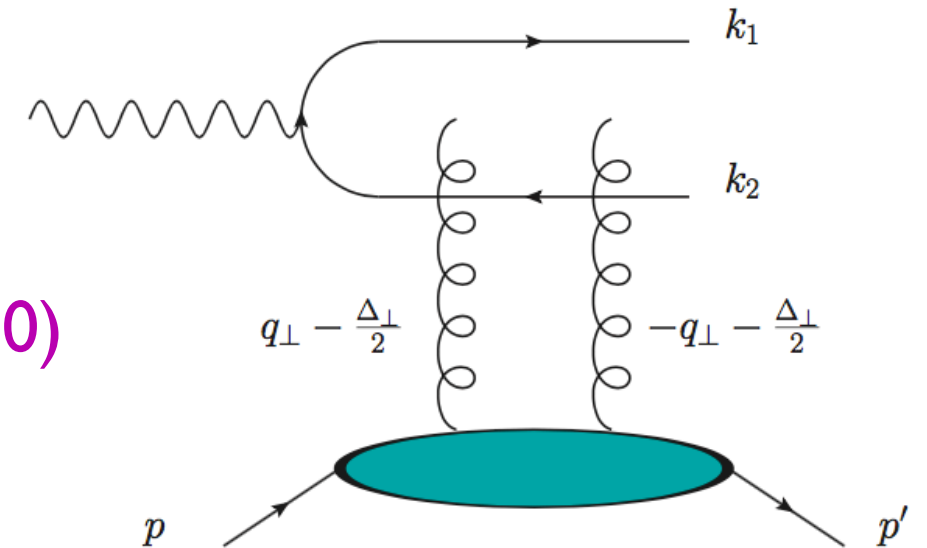
# Diffractive dijet production

Probe gluon GTMDs via hard diffractive dijet production in eA ( $\Delta_{\perp} \neq 0, \xi = 0$ )

Altinoluk, Armesto, Beuf, Rezaeian, 2016; Hatta, Xiao, Yuan, 2016

Earlier suggested to probe gluon GPDs ( $\Delta_{\perp} = 0, \xi \neq 0$ )

Braun, Ivanov, 2005



The DP GTMD correlator appears, as opposed to the inclusive case

$$\frac{d\sigma}{dy_1 dy_2 d^2 \mathbf{k}_{1\perp} d^2 \mathbf{k}_{2\perp}} \propto \int d^2 \mathbf{q}_{\perp} d^2 \mathbf{q}'_{\perp} \mathcal{F}^{[\square]}(\mathbf{q}_{\perp}, \Delta_{\perp}) \mathcal{F}^{[\square]}(\mathbf{q}'_{\perp}, \Delta_{\perp}) \mathcal{A}(\mathbf{K}_{\perp}, \mathbf{q}_{\perp}, \mathbf{q}'_{\perp}, \epsilon_f^2)$$

Altinoluk, Armesto, Beuf, Rezaeian, 2016; Hatta, Xiao, Yuan, 2016

$$\epsilon_f^2 = z(1-z)Q^2$$

$$G^{[\square]} \rightarrow \mathcal{F}^{[\square]} \quad S^{[\square]}(\mathbf{x}_{\perp}, \mathbf{y}_{\perp}) \rightarrow 1 - S^{[\square]}(\mathbf{x}_{\perp}, \mathbf{y}_{\perp})$$

The transverse momentum dependence of the GTMD is probed indirectly

# Diffraction dijet production

The transverse momentum dependence of the GTMD is probed indirectly

$$\frac{d\sigma_T^{\gamma^* p \rightarrow jjp}}{dK_\perp d\Delta_\perp^2} = \frac{(2\pi)^4 \alpha_{em}}{16N_c} \sum_f e_f^2 \int dz [z^2 + (1-z)^2] \frac{\mathcal{A}_T^2(K_\perp, \Delta_\perp, z, Q, y)}{K_\perp}$$

$$\mathcal{A}_T(K_\perp, \Delta_\perp, z, Q, y) = \int \frac{d^2 q_\perp}{(2\pi)^3} \left[ \frac{K_\perp \cdot (K_\perp - q_\perp)}{z(1-z)Q^2 + (K_\perp - q_\perp)^2} \right] \mathcal{F}_0^{[\square]}(x, q_\perp, \Delta_\perp) \Big|_{x=s/(yQ^2)}$$

By varying  $Q^2$  and  $K_\perp$  one probes differently weighted integrals over the GTMD

Idem for the longitudinal photon polarization (which requires  $Q^2 \neq 0$ ):

$$\frac{d\sigma_L^{\gamma^* p \rightarrow jjp}}{dK_\perp d\Delta_\perp^2} = \frac{(2\pi)^4 \alpha_{em}}{4N_c} \sum_f e_f^2 \int dz z^2 (1-z)^2 \frac{\mathcal{A}_L^2(K_\perp, \Delta_\perp, z, Q, y)}{K_\perp}$$

$$\mathcal{A}_L(K_\perp, \Delta_\perp, z_i, Q) = \int \frac{d^2 q_\perp}{(2\pi)^3} \left[ \frac{Q K_\perp}{z(1-z)Q^2 + (K_\perp - q_\perp)^2} \right] \mathcal{F}_0^{[\square]}(x, q_\perp, \Delta_\perp) \Big|_{x=s/(yQ^2)}$$

# Diffractional J/ψ production

Again the transverse momentum dependence of the GTMD is probed indirectly

$$\mathcal{A}_{T,L} = \frac{\pi i}{2N_c} \int_0^1 dz \int d^2 r_\perp (\Psi_V^* \Psi_\gamma)_{T,L}(r_\perp, z) \int d^2 q_\perp J_0(|q_\perp + \delta_\perp| r_\perp) \mathcal{F}_0^{[\square]}(x, q_\perp, \Delta_\perp)$$

$$\delta_\perp = \left(\frac{1}{2} - z\right) \Delta_\perp$$

Kowalski, Teaney, 2003; Kowalski, Motyka, Watt, 2006; many others

By varying  $t$  one probes (slightly) differently weighted integrals

Using different quarkonia also changes the weights ( $r_\perp \sim 1/M_V$ )

Often one considers this process to probe GPDs, which one (formally) recovers upon applying a collinear expansion:  $r_\perp \sim 1/M_V$  means  $q_\perp r_\perp \ll 1$

$$J_0(|q_\perp + \delta_\perp| r_\perp) \approx 1 - \frac{(q_\perp + \delta_\perp)^2 r_\perp^2}{4}$$

$$\begin{aligned} \mathcal{A}_{T,L} &\approx \frac{\pi i}{8N_c} \int_0^1 dz \int d^2 r_\perp r_\perp^2 (\Psi_V^* \Psi_\gamma)_{T,L}(r_\perp, z) \int d^2 q_\perp q_\perp^2 \mathcal{F}_0^{[\square]}(x, q_\perp, \Delta_\perp) \\ &= \frac{\pi^3 i \alpha_s}{N_c} \int_0^1 dz \int d^2 r_\perp r_\perp^2 (\Psi_V^* \Psi_\gamma)_{T,L}(r_\perp, z) x H_g(x, \Delta_\perp) \end{aligned}$$

See also Bertone, 2022

This yields an expression in terms of a GPD (requires regularization)

# Phenomenology

# MV-like model

We consider the MV-like model:

$$\mathcal{F}^{[\square]}(\mathbf{k}_\perp, \mathbf{\Delta}_\perp) = 4N_c \int \frac{d^2\mathbf{r}_\perp d^2\mathbf{b}_\perp}{(2\pi)^2} e^{-i\mathbf{k}_\perp \cdot \mathbf{r}_\perp} e^{i\mathbf{\Delta}_\perp \cdot \mathbf{b}_\perp} e^{-\epsilon_r r_\perp^2} \left[ 1 - \exp \left( -\frac{1}{4} r_\perp^2 \chi Q_s^2(b_\perp) \ln \left[ \frac{1}{r_\perp^2 \Lambda^2} + e \right] \right) \right]$$

Similar to Hagiwara, Hatta, Pasechnik, Tasevsky & Teryaev, 2017; Salazar, Schenke, 2019

$\chi$  sets the normalization of  $Q_s$  and is  $x$  dependent (of GBW form)

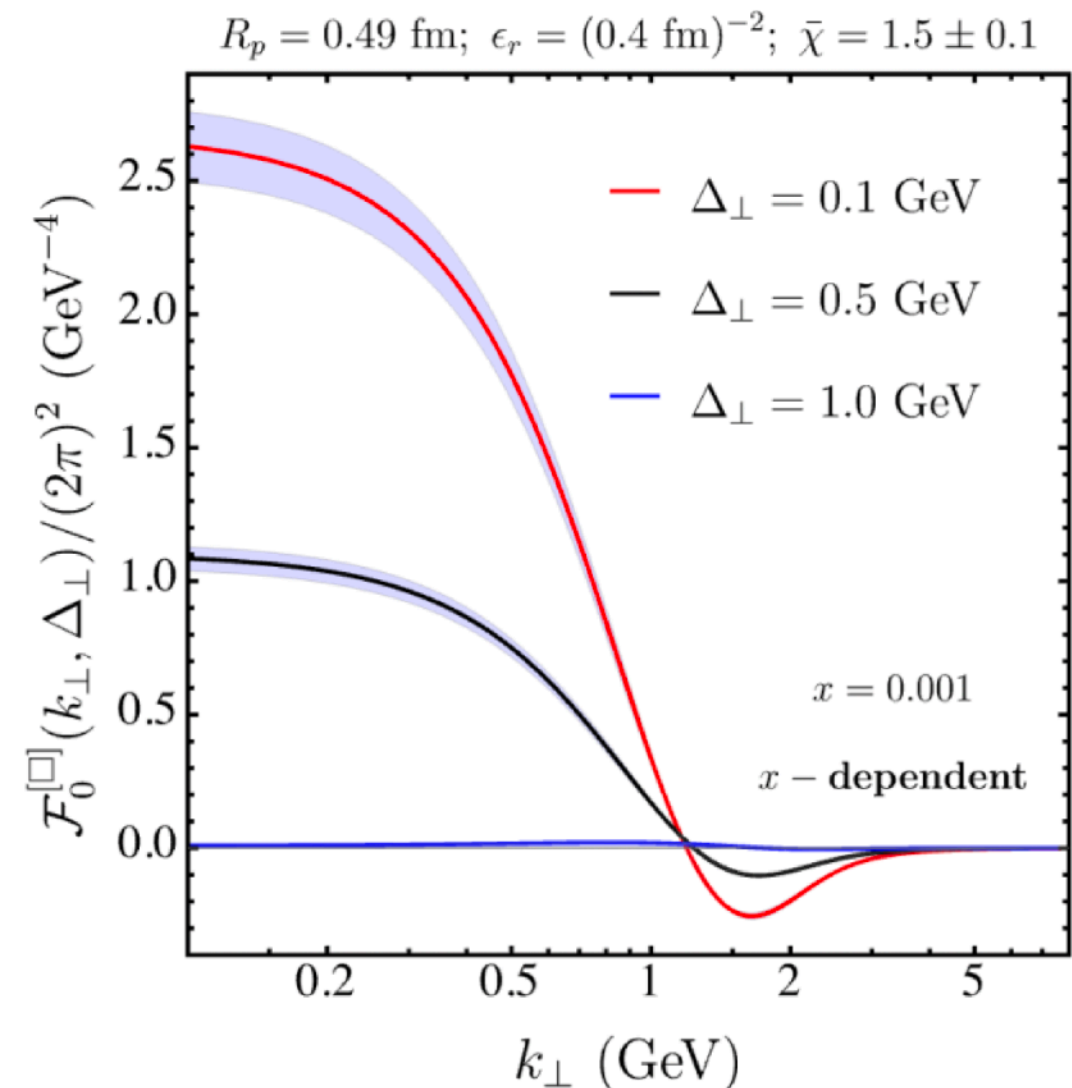
$$\chi(x) = \bar{\chi} \left( \frac{x_0}{x} \right)^\lambda \quad x_0 = 3 \times 10^{-4} \quad \lambda = 0.29$$

$Q_s$  is proportional to the proton (Gaussian) or nuclear (Woods-Saxon) profile

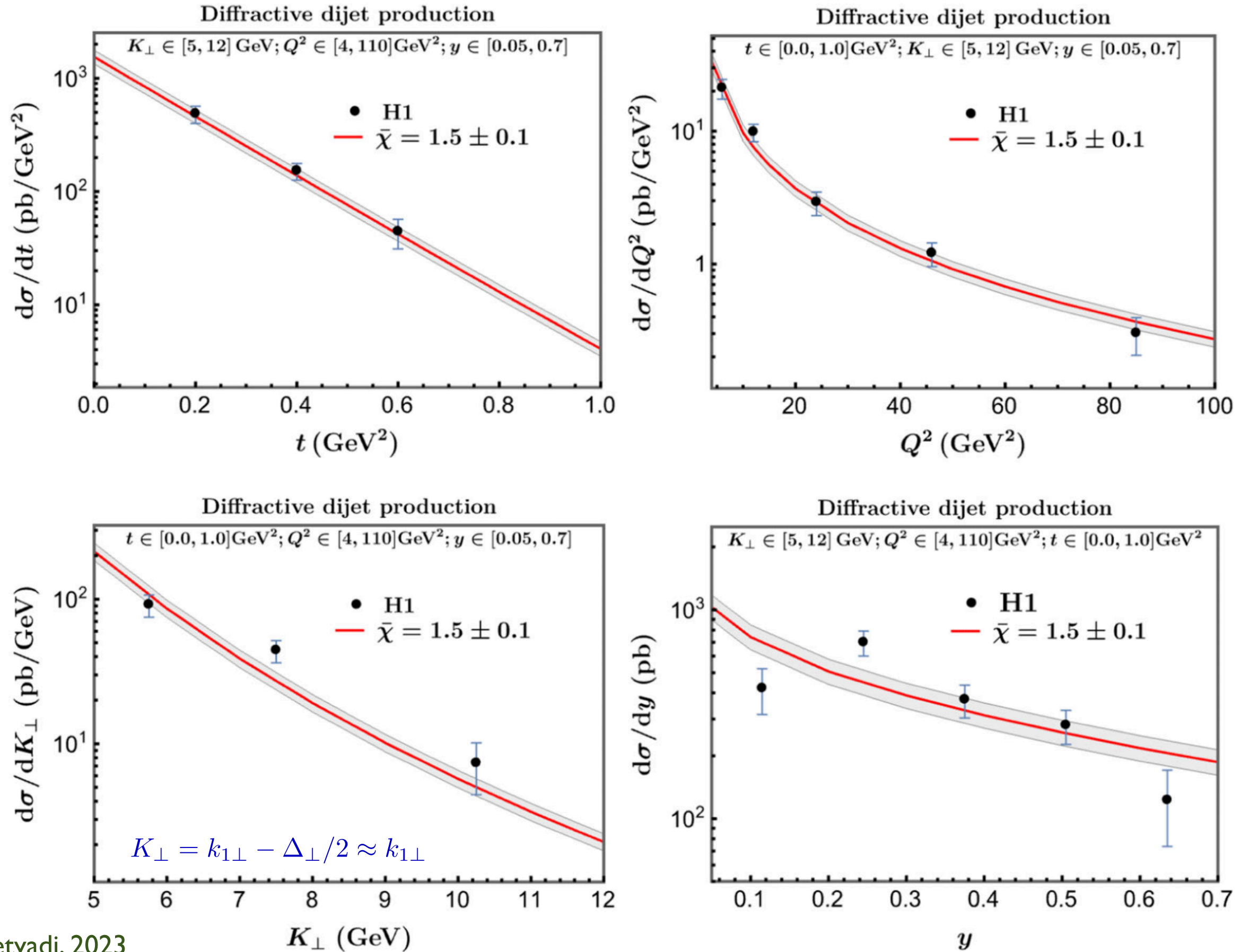
For details see DB, Setyadi, 2023

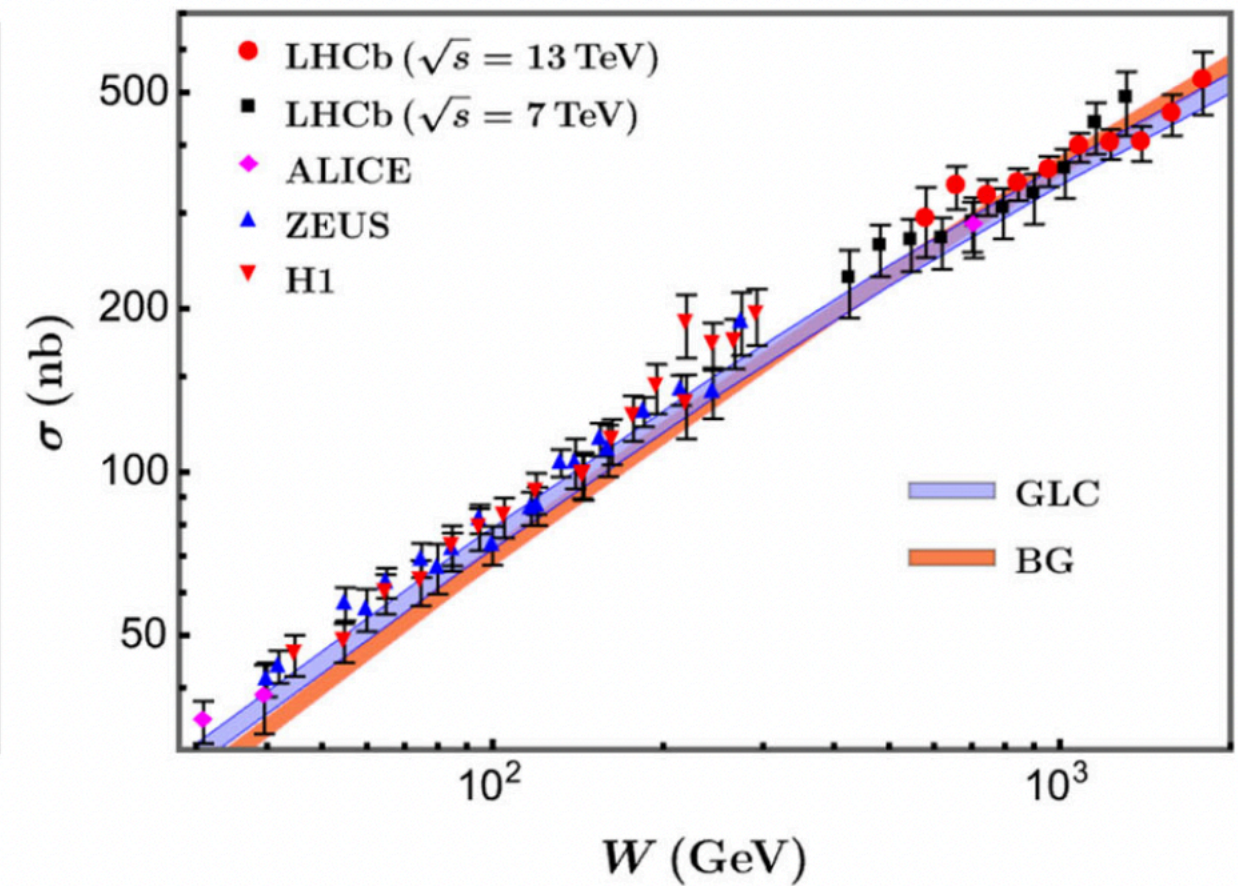
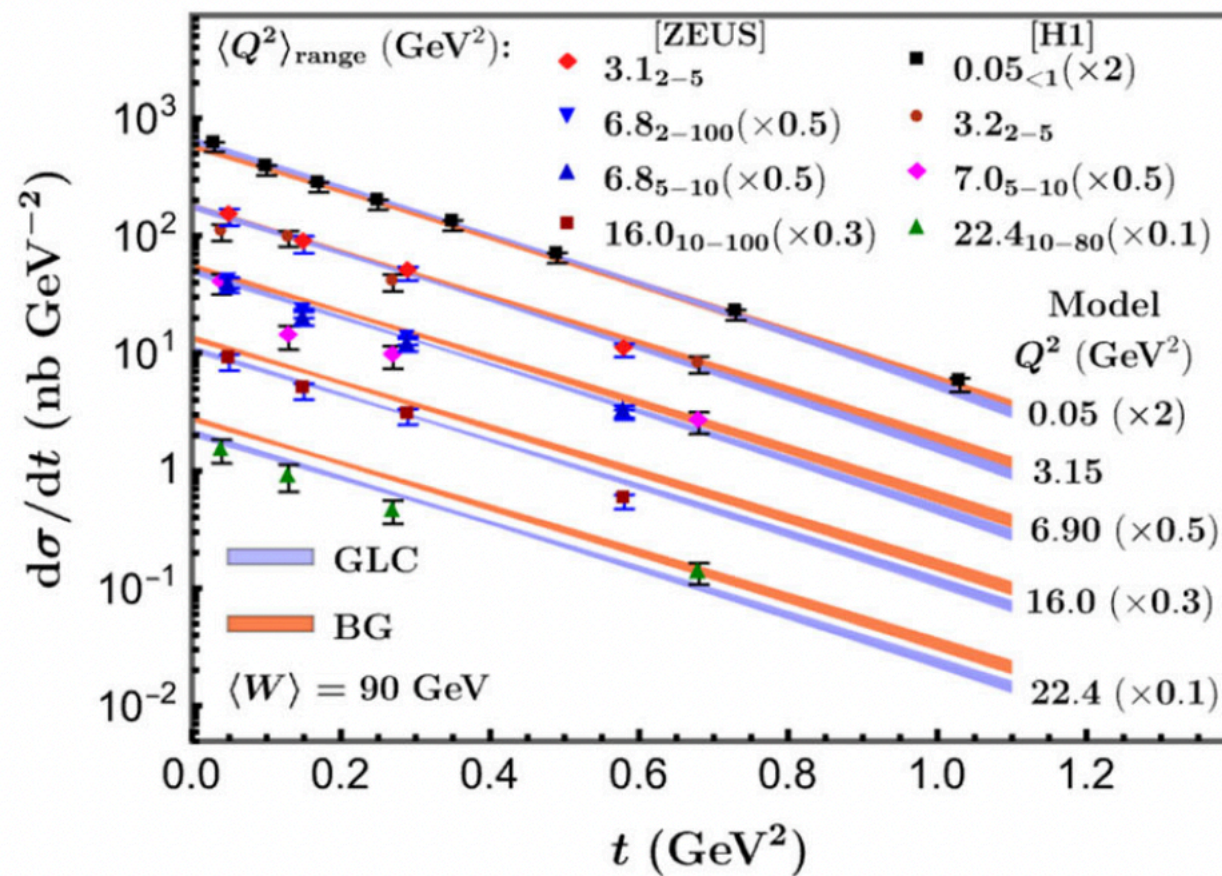
Dominant contribution from:

$$\Delta_\perp \ll K_\perp \text{ or } M_V$$



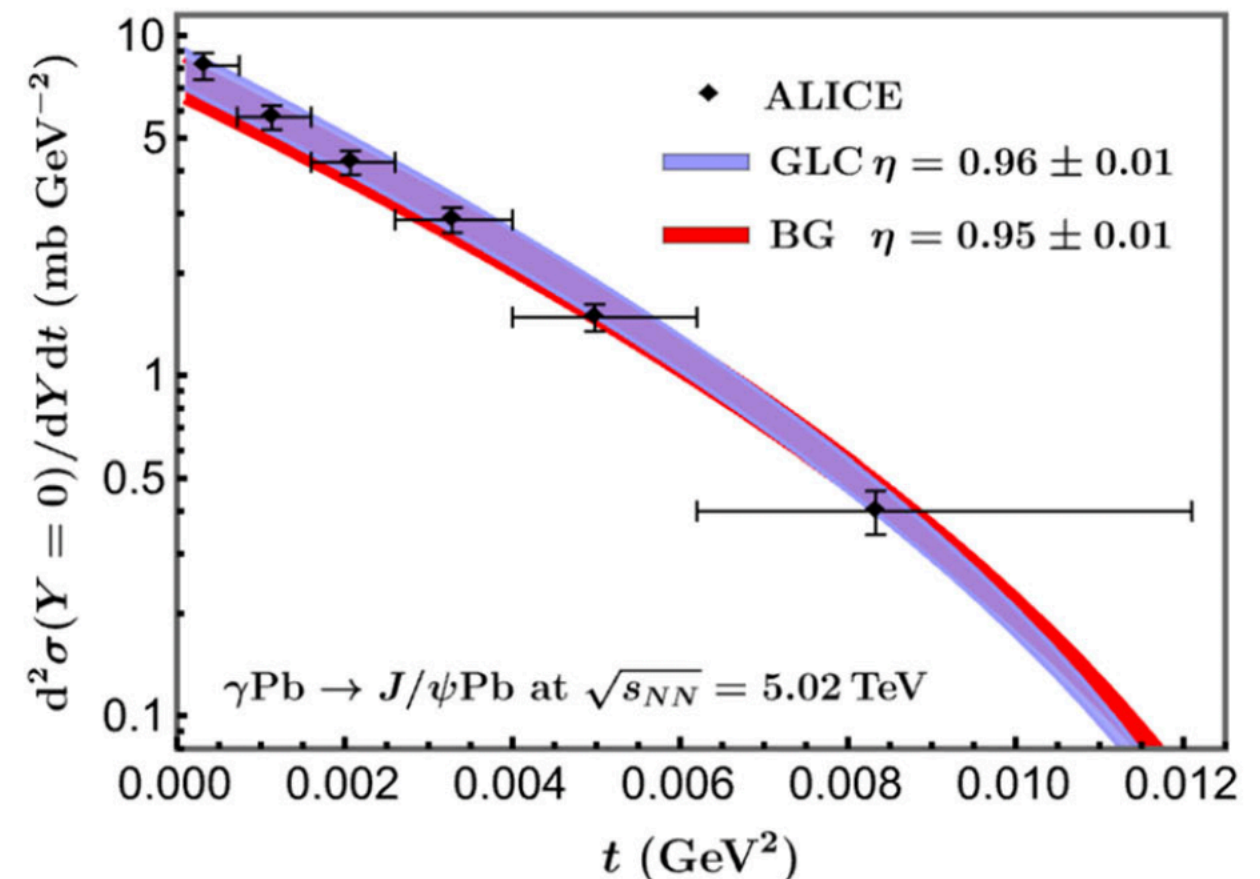
Best fit of H1 dijet data with  $R_p = 0.49$  fm,  $\lambda = 0.29$ , and  $\epsilon_r = (0.4 \text{ fm})^{-2}$





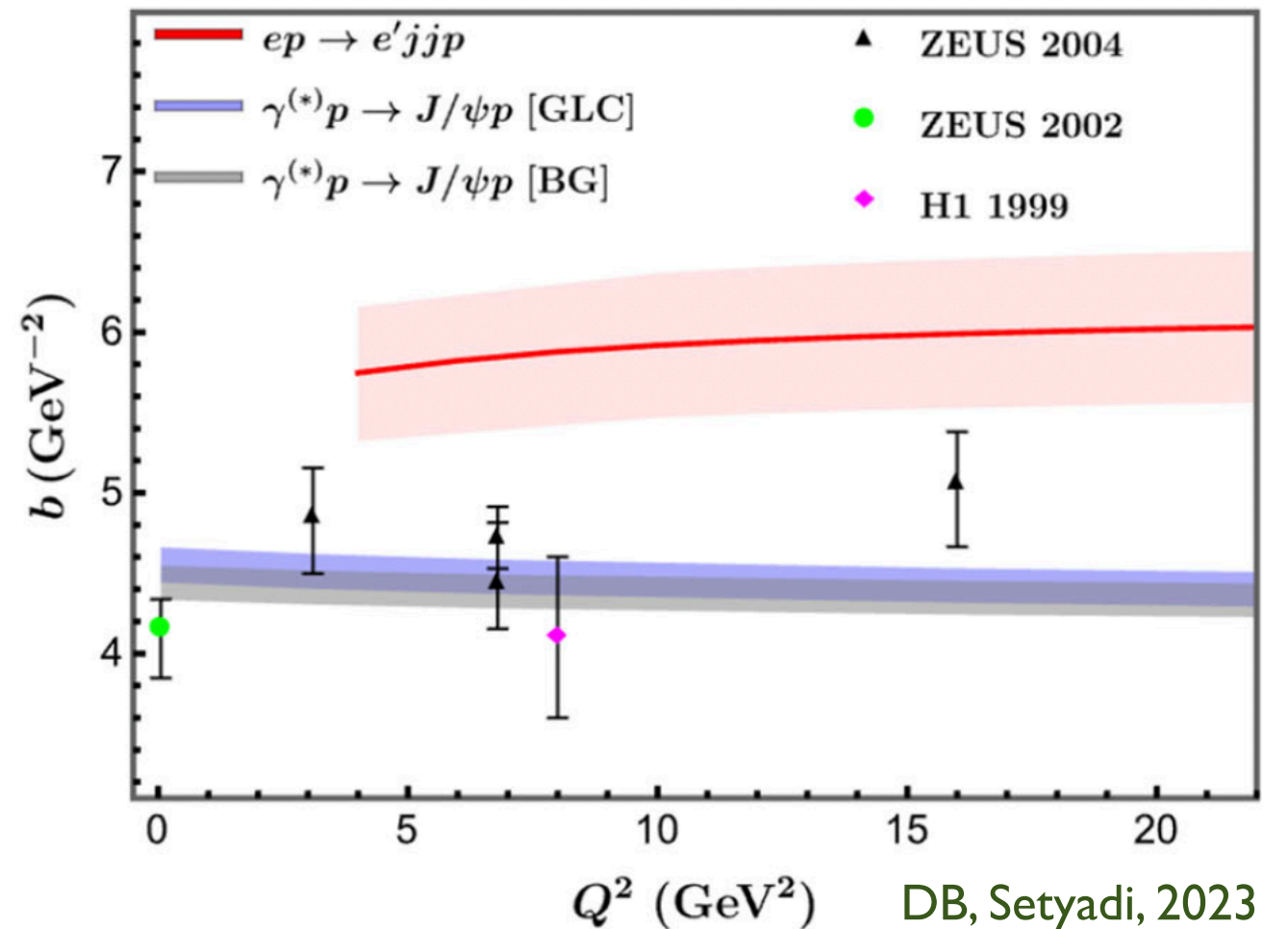
Diffractive  $J/\psi$  production data of HERA (H1 & ZEUS) prefer smaller  $R_p$  (0.40-0.41 fm), smaller  $\lambda$  (0.22), and is more sensitive to  $\epsilon_r$

Description of ALICE UPC data qualitatively fine with an  $A$  dependence somewhat smaller than  $A^{1/3}$ , but this is dependent on the profile functions



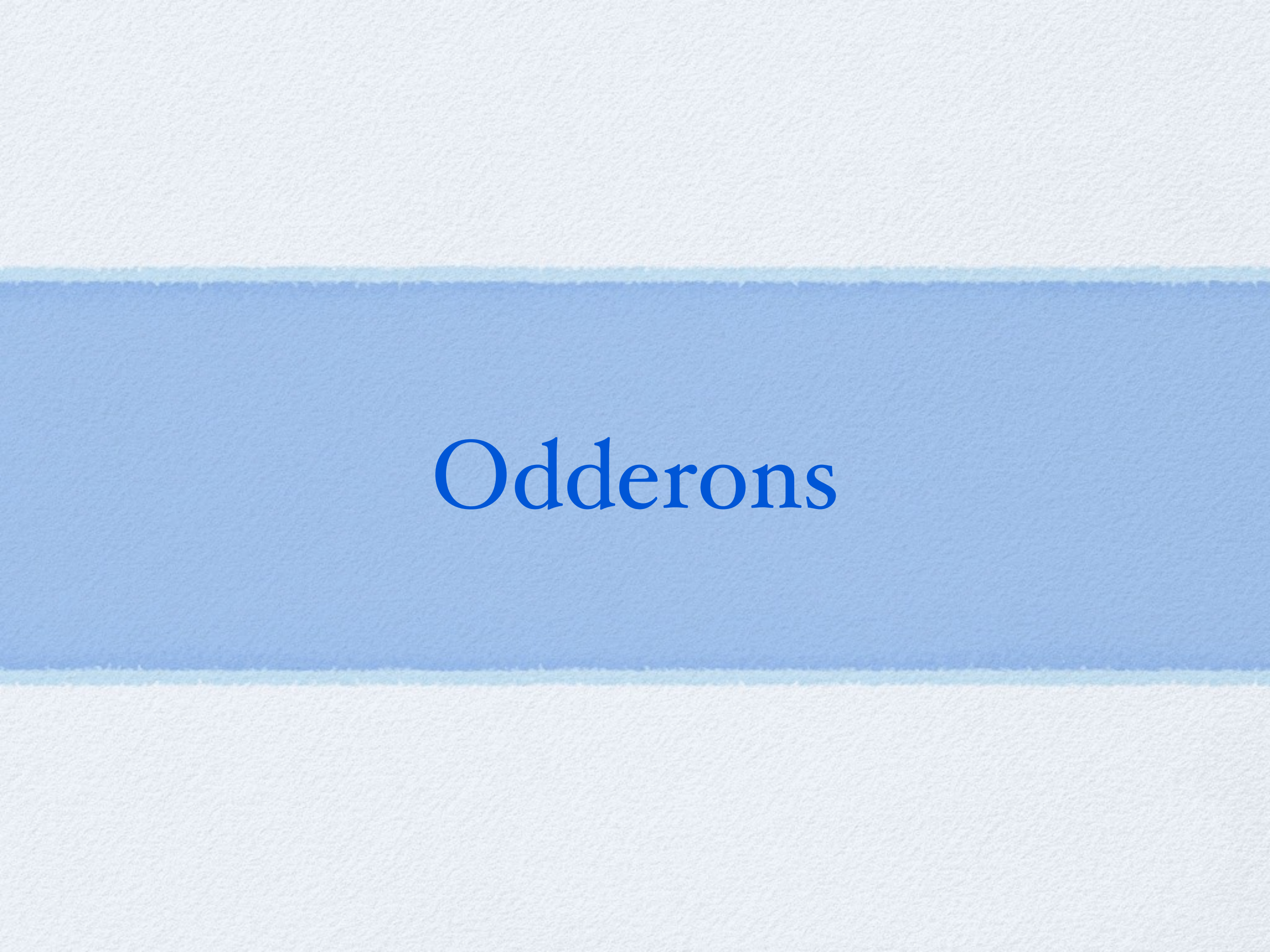
# Tension between diffractive dijet and $J/\psi$ production

There is tension between the dijet and  $J/\psi$  data regarding the steepness of the t-slope (dictated by  $R_p$ )



Applicability to H1 data (gluon dominance) is questioned by Linek, Luszczak, W. Schäfer, Szczurek, 2024  
H1 dijet data may actually be in the ERBL region ( $\xi > x$ )

UPC data from RHIC and LHC and especially EIC data can shed further light on these issues, in order to check whether a common GTMD description is possible



Odderons

# Odderon GTMDs

$S^{[\square]}$  can also have an imaginary part:

$$S^{[\square]}(\boldsymbol{x}, \boldsymbol{y}) = \mathcal{P}(\boldsymbol{x}, \boldsymbol{y}) + i\mathcal{O}(\boldsymbol{x}, \boldsymbol{y})$$

$$\mathcal{P}(\boldsymbol{x}, \boldsymbol{y}) \equiv \frac{1}{2N_c} \text{Tr} \left( U^{[\square]} + U^{[\square]\dagger} \right) \quad \mathcal{O}(\boldsymbol{x}, \boldsymbol{y}) \equiv \frac{1}{2iN_c} \text{Tr} \left( U^{[\square]} - U^{[\square]\dagger} \right)$$

This “odderon” operator is **C-odd** and **T-odd**

$$\begin{aligned} G_{(d)}^{(\text{T-odd})\,ij}(\boldsymbol{k}, \boldsymbol{\Delta}) &\equiv \frac{1}{2} \left( G^{[+,-]\,ij}(\boldsymbol{k}, \boldsymbol{\Delta}) - G^{[-,+]\,ij}(\boldsymbol{k}, \boldsymbol{\Delta}) \right) \\ &= \frac{N_c}{\alpha_s} \left[ \frac{1}{2} \left( \boldsymbol{k}^2 - \frac{\boldsymbol{\Delta}^2}{4} \right) \delta_T^{ij} + k_T^{ij} - \frac{\Delta_T^{ij}}{4} - \frac{k_T^{[i} \Delta_T^{j]}}{2} \right] \\ &\quad \times \left( G^{[\square]}(\boldsymbol{k}, \boldsymbol{\Delta}) - G^{[\square]^\dagger}(\boldsymbol{k}, \boldsymbol{\Delta}) \right) \end{aligned}$$

$$G^{[\square]}(\boldsymbol{k}, \boldsymbol{\Delta}) - G^{[\square]^\dagger}(\boldsymbol{k}, \boldsymbol{\Delta}) \propto \int \frac{d^2\boldsymbol{x} \, d^2\boldsymbol{y}}{(2\pi)^4} e^{-i\boldsymbol{k} \cdot (\boldsymbol{x} - \boldsymbol{y}) + i\boldsymbol{\Delta} \cdot \frac{\boldsymbol{x} + \boldsymbol{y}}{2}} \langle \mathcal{O}(\boldsymbol{x}, \boldsymbol{y}) \rangle$$

# Odderon GTMDs

Hermiticity and PT constraints imply:

$$G^{[\square]*}(\mathbf{k}, \Delta) = G^{[\square]}(\mathbf{k}, -\Delta) \quad G^{[\square]*}(\mathbf{k}, \Delta) = G^{[\square^\dagger]}(-\mathbf{k}, -\Delta)$$

$G^{[\square]}(\mathbf{k}, \Delta) - G^{[\square^\dagger]}(\mathbf{k}, \Delta)$  only depends on odd powers of  $\mathbf{k} \cdot \Delta$

Odderon (for  $\xi = 0$ ) involves only odd harmonics  $\cos[(2n+1)(\phi_k - \phi_\Delta)]$

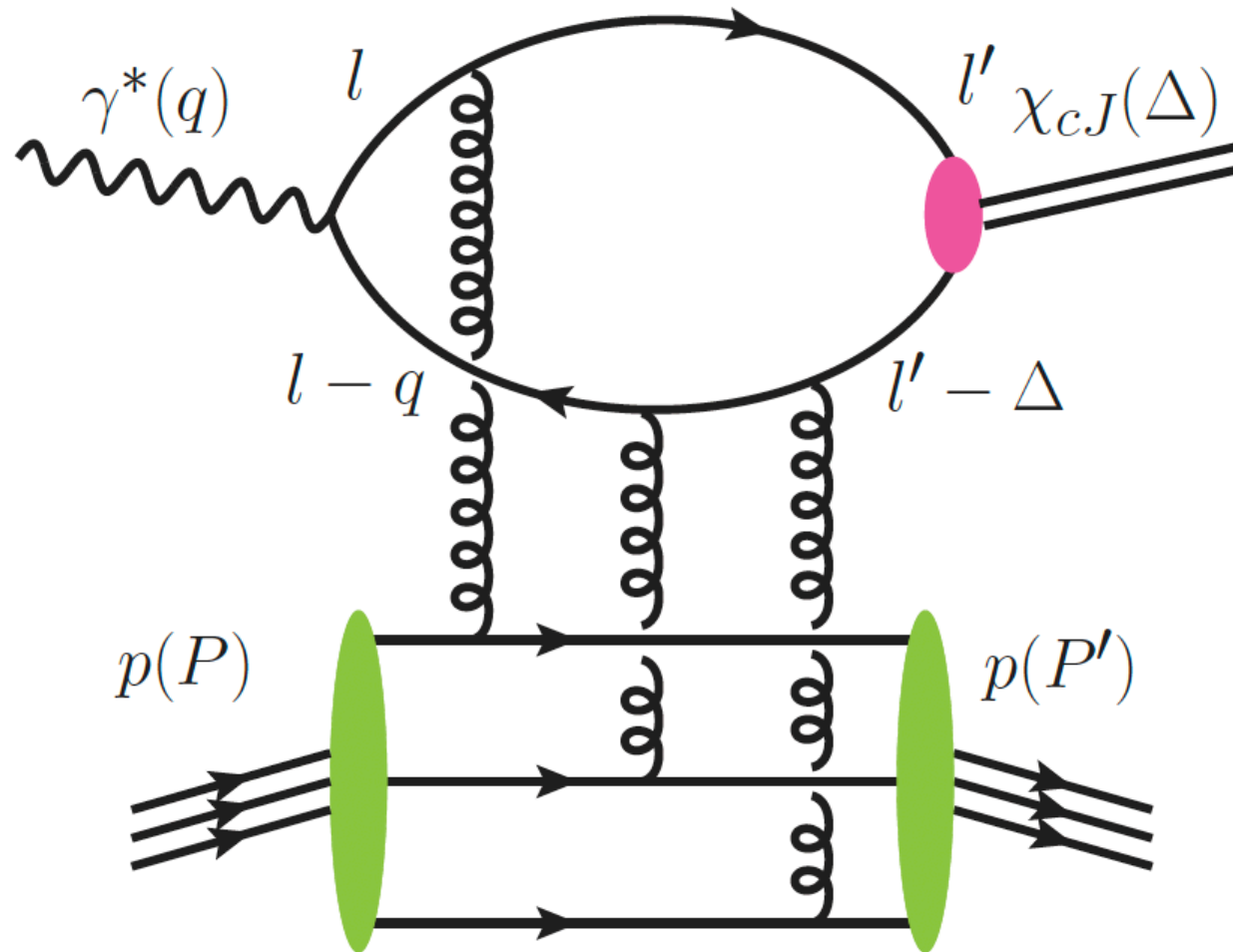
$$\begin{aligned} xW(x, \mathbf{b}, \mathbf{k}) = & x\mathcal{W}_0(x, \mathbf{b}^2, \mathbf{k}^2) + 2 \cos(\phi_b - \phi_k) x\mathcal{W}_1(x, \mathbf{b}^2, \mathbf{k}^2) \\ & + 2 \cos 2(\phi_b - \phi_k) x\mathcal{W}_2(x, \mathbf{b}^2, \mathbf{k}^2) + \dots \end{aligned}$$

For  $\xi \neq 0$  odd powers of  $\mathbf{k} \cdot \Delta$  can appear in the real parts as well

Future EIC data should specify the region (ERBL and/or DGLAP) probed and check for odd harmonics

# Exclusive $\chi_c$ production at EIC

This process also probes the odderon:

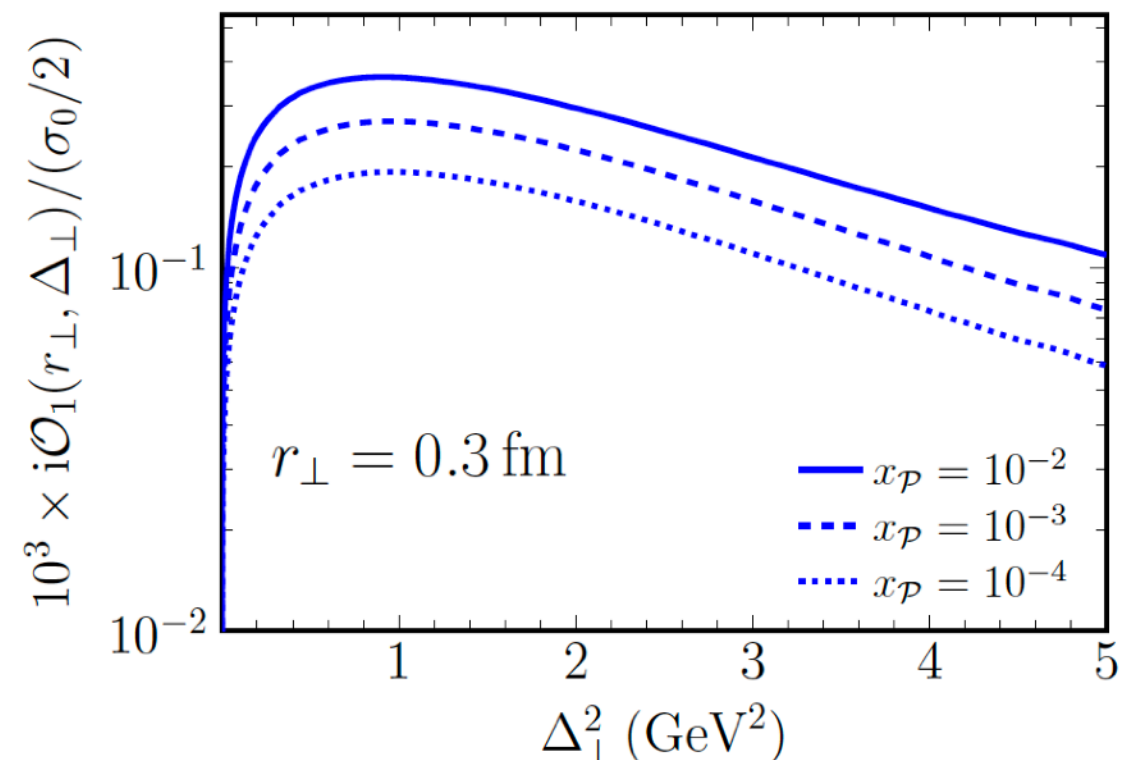


First azimuthal harmonic found to be rather small

C-even final state requires  
C-odd t-channel exchange

Constructive interference  
with photon t-channel exchange  
for  $|t| \sim 1 \text{ GeV}^2$

Benić, Dumitru, Kaushik, Motyka, Stebel, 2024



# Conclusions

# Conclusions

- Semi-inclusive open heavy quark pair, dijets & quarkonium production at EIC are well suited to study gluon TMDs, although the factorization poses challenges
- Predictions are still quite uncertain because of many factors: unknown nonperturbative Sudakov factors, scale uncertainty, poorly known LDMEs or more generally, unknown shape functions, process dependence
- The linear polarization of gluons inside unpolarized hadrons is expected to lead to sizable  $\cos 2\phi$  asymmetries in all three processes discussed, although evolution towards small  $x$  will likely have a suppressing effect on them
- Gluon GTMDs can be studied using quarkonium and dijet production in exclusive coherent diffractive processes at EIC (synergy with UPCs)
- For many cases the energy scale and/or  $x$  evolution needs to be studied still