Electroweak structure functions as input to precision radiative corrections at low energies

Peter Blunden† (University of Manitoba)

Parity-Violation and other Electroweak Physics at JLab 12 GeV and Beyond, INT, June 2022

†collaborators: Kyle Shiells (UM), Wally Melnitchouk (JLab),
Nathan Hall, Anthony Thomas and Ross Young (Adelaide)

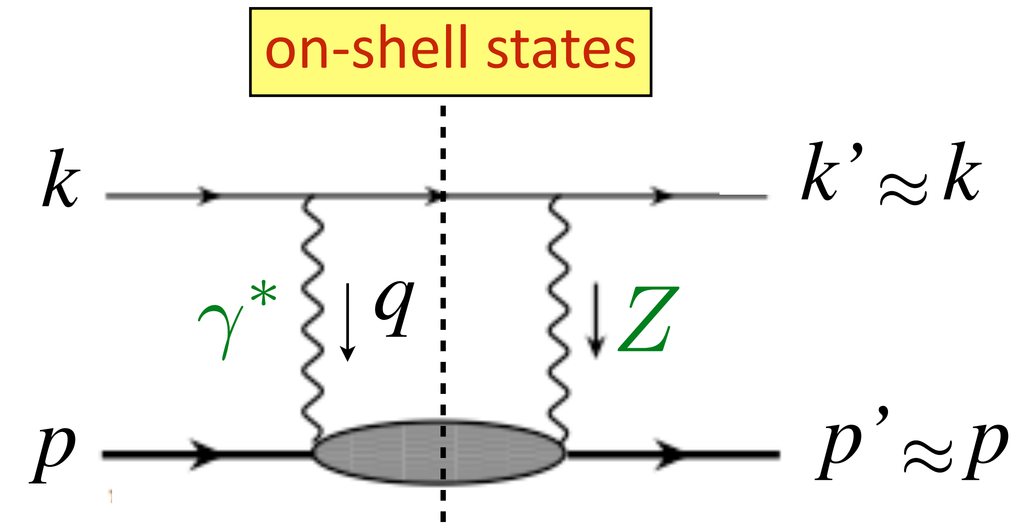
Outline

- Update AJM model predictions for inelastic PVES in SOLID kinematics
- Discuss what improvements could be made
- Discuss improvements to models of $F_3(W^2, Q^2)$ structure functions and their role in precision electroweak physics

"Modern" approach to box diagrams

- GH formulated box corrections in terms of forward angle dispersion relations using γZ structure functions
- Data-driven theoretical analysis

Gorchtein, Horowitz, PRL ***102*** *(2009) 091806*



Forward scattering limit: $|\, f\,\rangle \approx |\, i\,\rangle$

$$\mathcal{I}m\langle i|\mathcal{M}^{\gamma Z}|i\rangle \sim \int d^3\, k_1\, \frac{L_{\mu\nu}W^{\mu\nu}}{q^2(q^2 - M_Z^2)}$$

vector *h*   axial *h*

hadronic tensor: $$MW^{\mu\nu}_{\gamma Z} = -g^{\mu\nu} F_1^{\gamma Z} + \frac{p^\mu p^\nu}{p\cdot q} F_2^{\gamma Z} - i\varepsilon^{\mu\nu\lambda\rho}\frac{p_\lambda q_\rho}{2p\cdot q} F_3^{\gamma Z}$$

Vector h correction

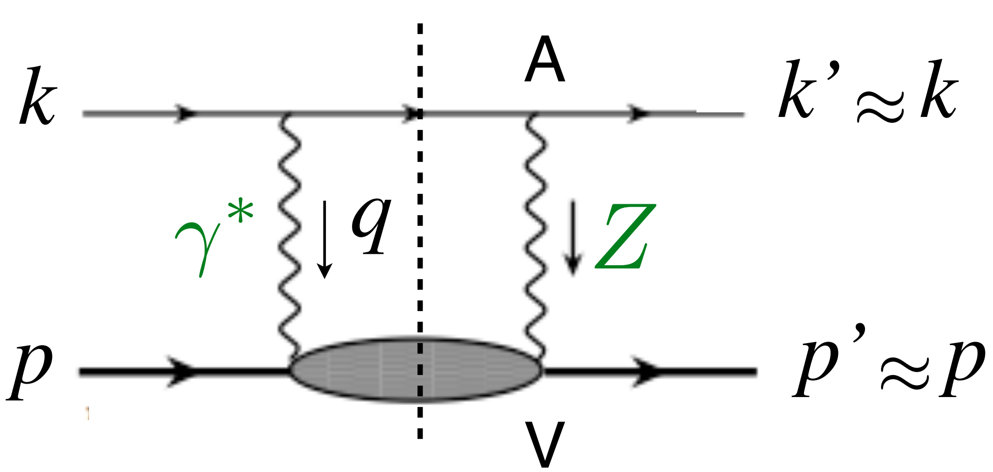
- vector h correction $\Box^V_{\gamma Z}$ vanishes at $E = 0$ (atomic PV limit), but PVES is at finite energy (*e.g.* Qweak $E \sim 1$ GeV)

$\longrightarrow$ Use forward limit dispersion relation

$$\Re e\, \Box^V_{\gamma Z}(E) = \frac{2E}{\pi} \int_0^\infty dE' \frac{1}{E'^2 - E^2}\, \Im m\, \Box^V_{\gamma Z}(E')$$

$\longrightarrow$ imaginary part given by

$$\Im m\, \Box^V_{\gamma Z}(E) = \frac{\alpha}{(s - M^2)^2} \int_{W_\pi^2}^{s} dW^2 \int_0^{Q^2_{\max}} \frac{dQ^2}{1 + Q^2/M_Z^2} \times \left(F_1^{\gamma Z} + F_2^{\gamma Z} \frac{s\,(Q^2_{\max} - Q^2)}{Q^2(W^2 - M^2 + Q^2)} \right)$$



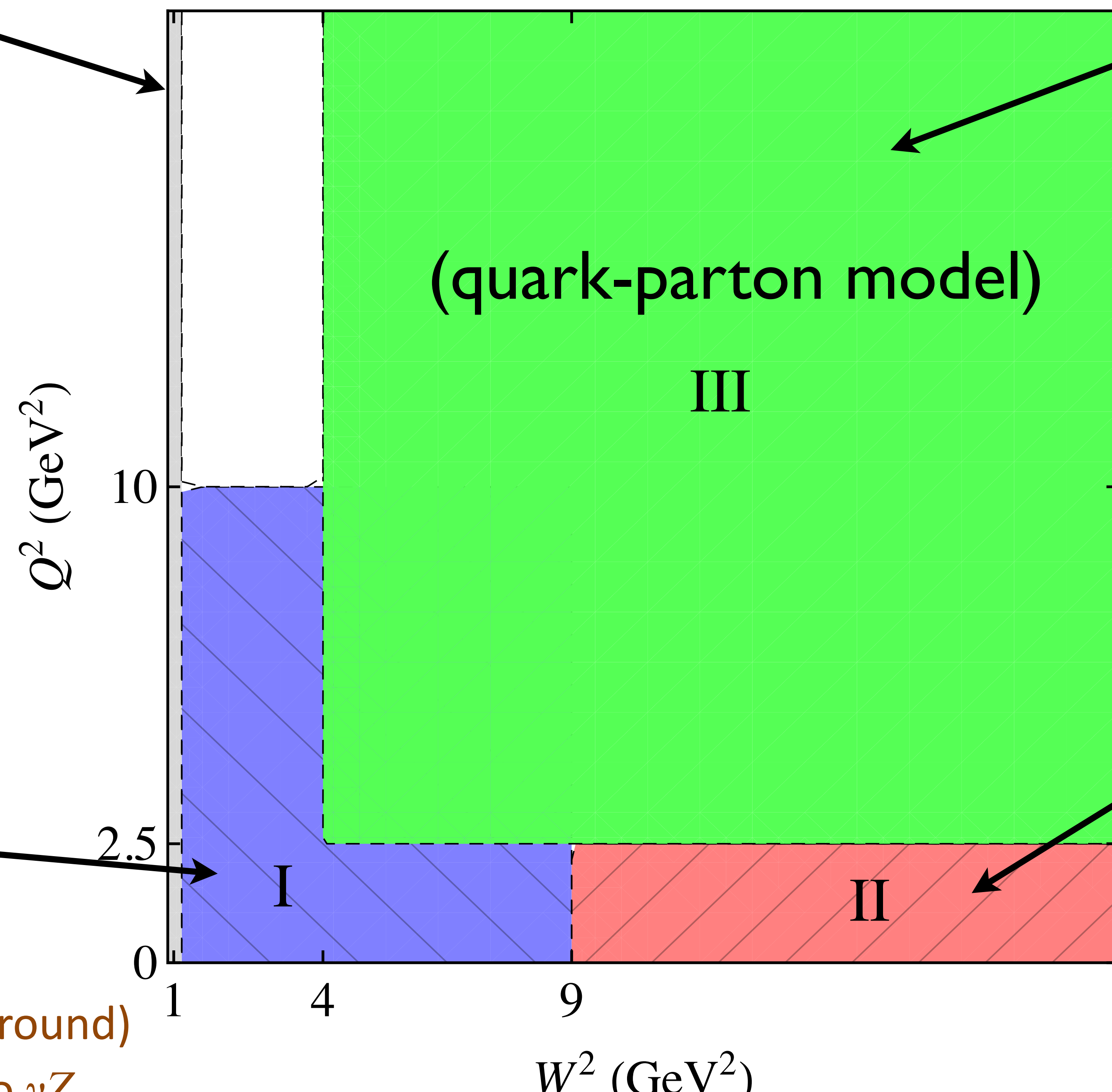
Structure function map: 3 distinct regions

Need structure functions for $A_e \times V_h$ $(F_{1,2}^{\gamma Z})$ and $V_e \times A_h$ $(F_3^{\gamma Z})$

"elastic"

"DIS"

Leading twist PDFs



"Regge"

VMD + Regge model

"resonance"

Christy-Bosted $\gamma\gamma$ fit
(resonances + background)
+ isospin rotation to γZ

Main issues

- Relatively little known about $F_i^{\gamma Z}$ interference structure functions below HERA measurements, with $Q^2 \geq 1500\ \mathrm{GeV}^2$

- Fit $F_i^{\gamma\gamma}$ over all kinematics in Q^2 and W, then "rotate" to $F_i^{\gamma Z}$ using available theoretical/phenomenological constraints

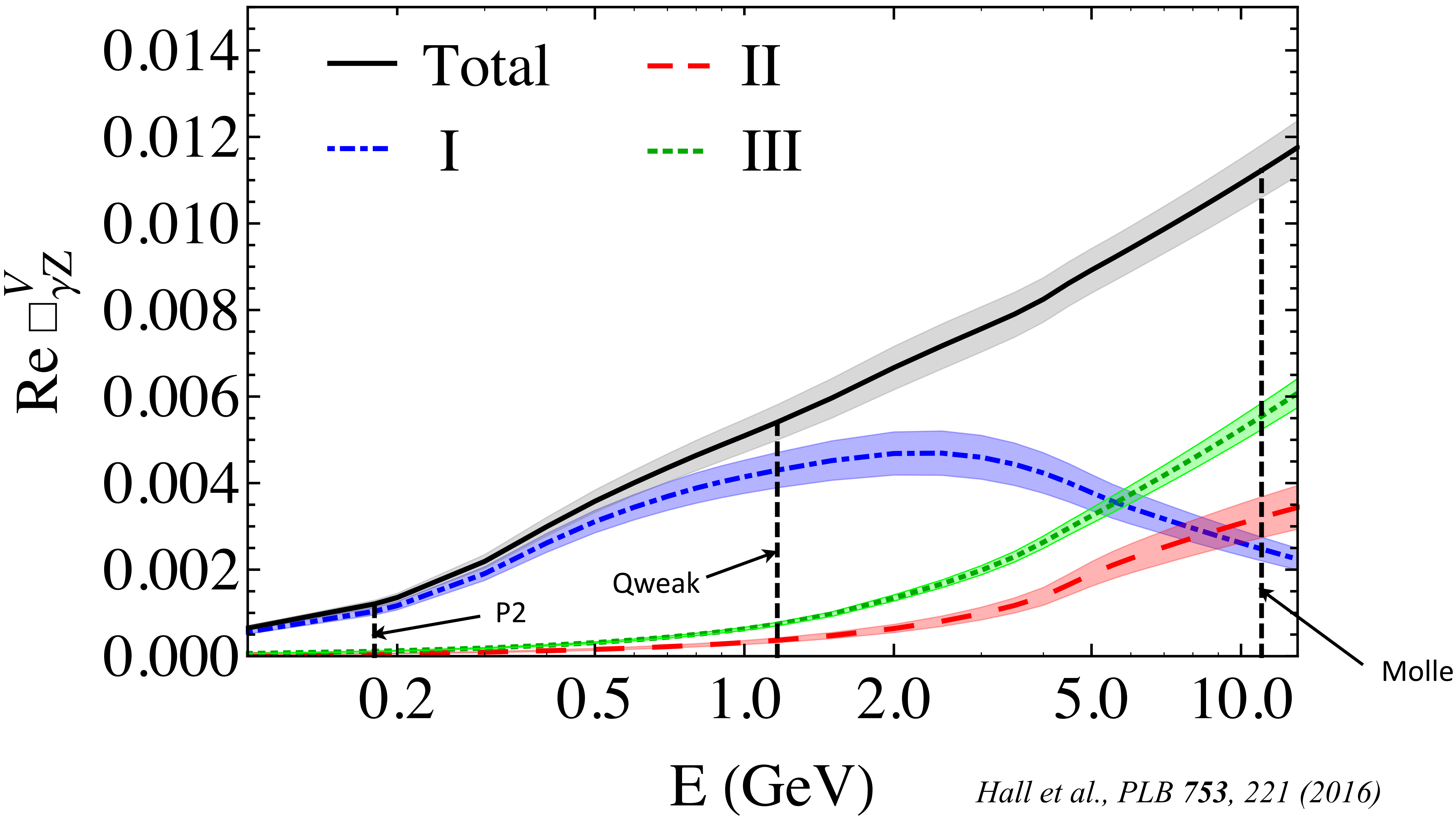
$\longrightarrow$ *e.g.* isospin symmetry

$$\langle N^* | J_Z^\mu | p \rangle = (1 - 4\sin^2\theta_W) \langle N^* | J_\gamma^\mu | p \rangle - \langle N^* | J_\gamma^\mu | n \rangle$$

- AJM model (approach): require structure functions to match across different kinematic regions (constrain free parameters)

Contribution from different regions to $\square^{V}_{\gamma Z}$

(relative to weak charge of 0.0713)

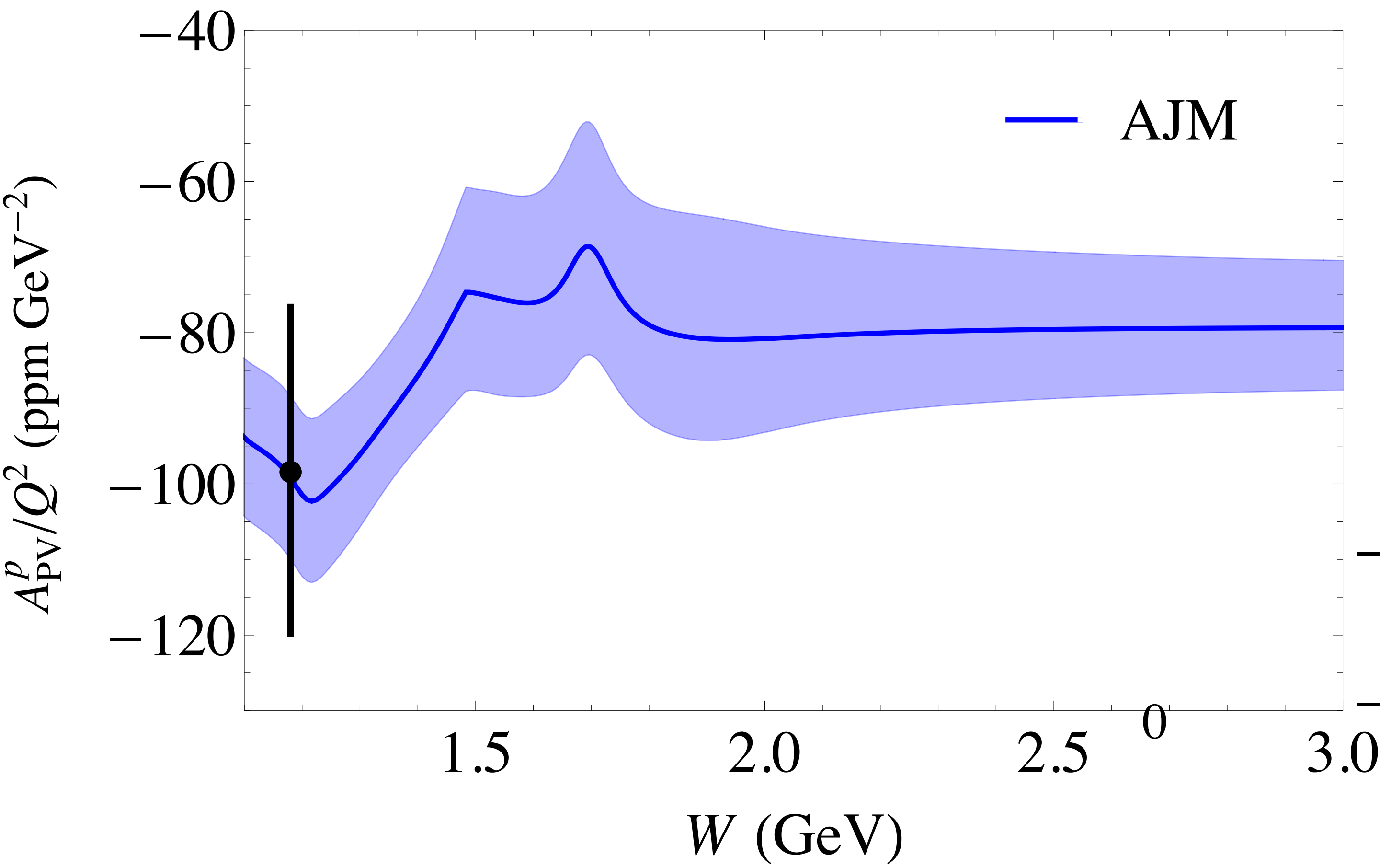


*Hall et al., PLB **753**, 221 (2016)*

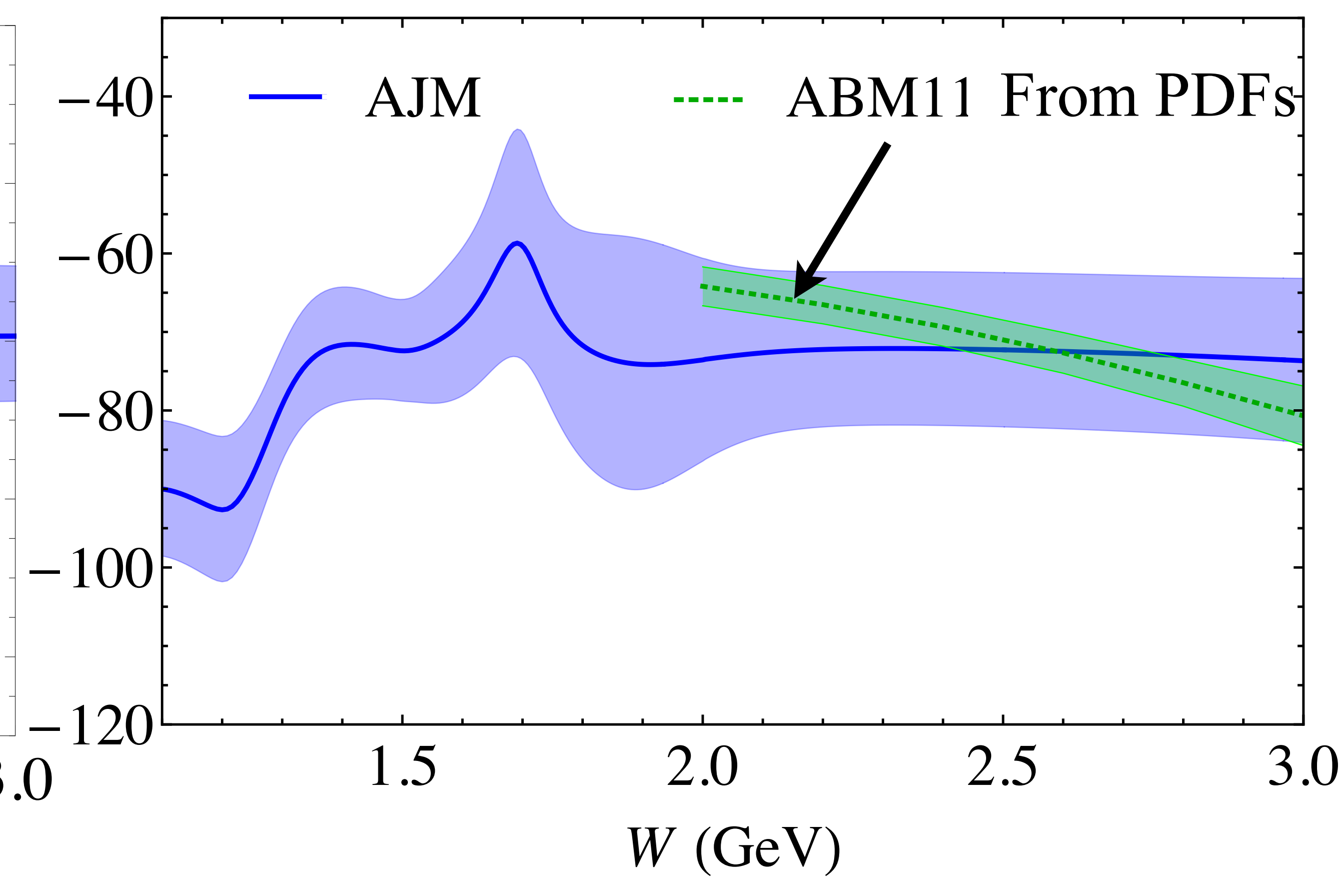
γZ model direct test

Parity-violating Deep Inelastic Scattering (PVDIS) asymmetry allows a <u>direct</u> measurement of the γZ structure functions

$$A_{\rm PV} = g_A^e \left(\frac{G_F Q^2}{2\sqrt{2}\pi\alpha}\right) \frac{xy^2 F_1^{\gamma Z} + (1-y)F_2^{\gamma Z} + \frac{g_V^e}{g_A^e}(y - y^2/2)xF_3^{\gamma Z}}{xy^2 F_1^{\gamma\gamma} + (1-y)F_2^{\gamma\gamma}}$$



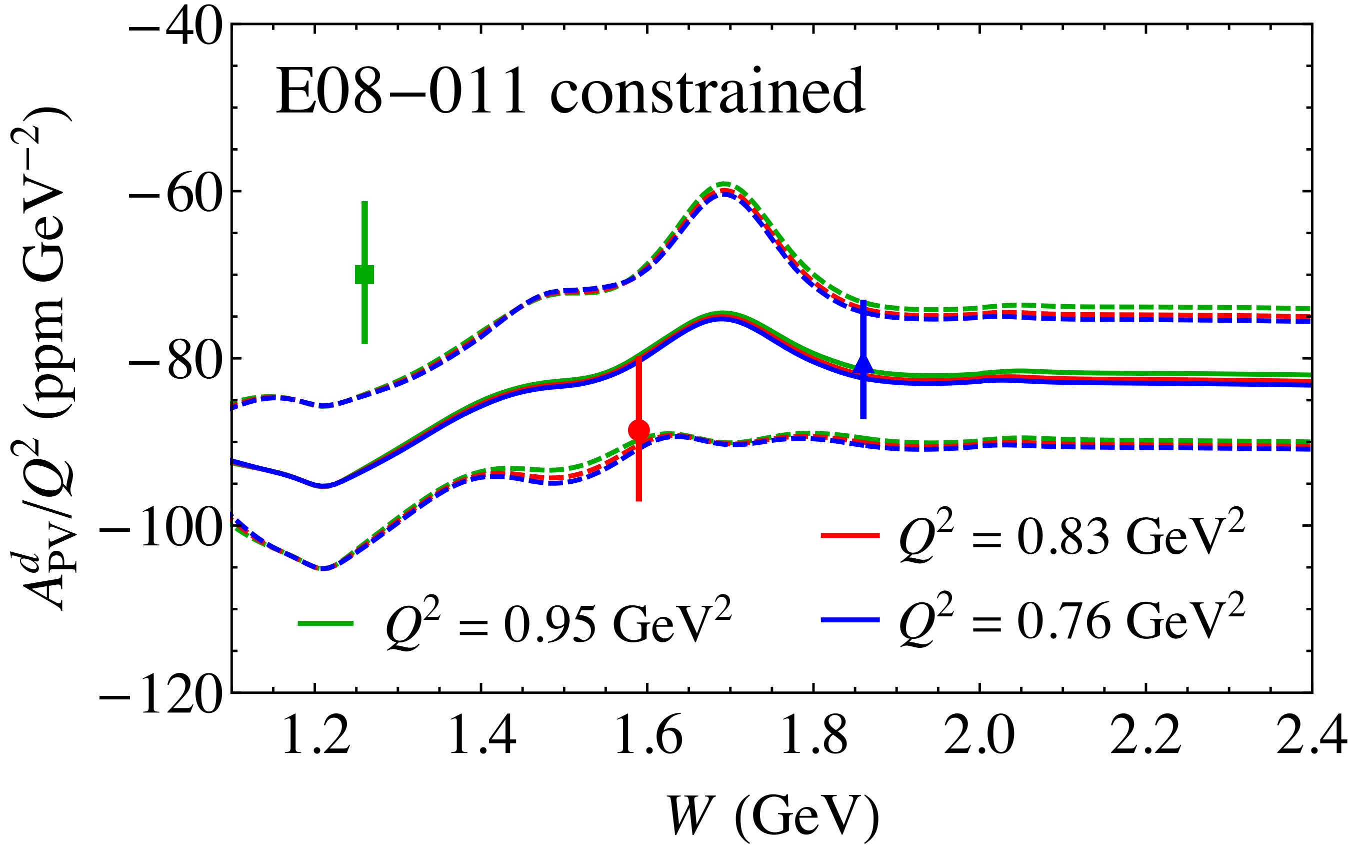
Q^2=0.34 GeV², E=0.69 GeV



Q^2 = 2.5 GeV², E = 6 GeV

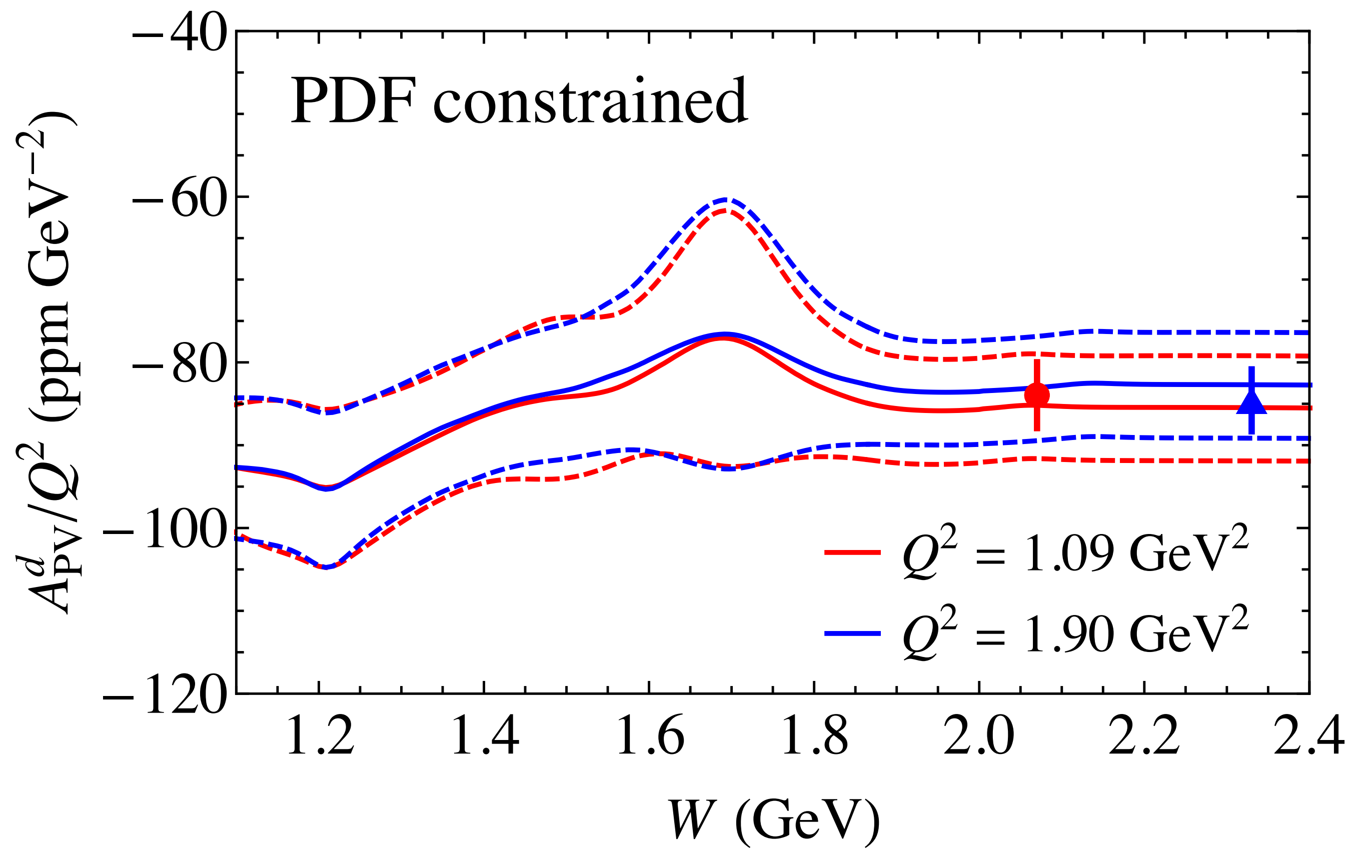
Androic et al. (G0 collaboration), arXiv:1212.1637

Constraints from PV inelastic asymmetries



*Wang et al. PRL **111**, 082501 (2013)*

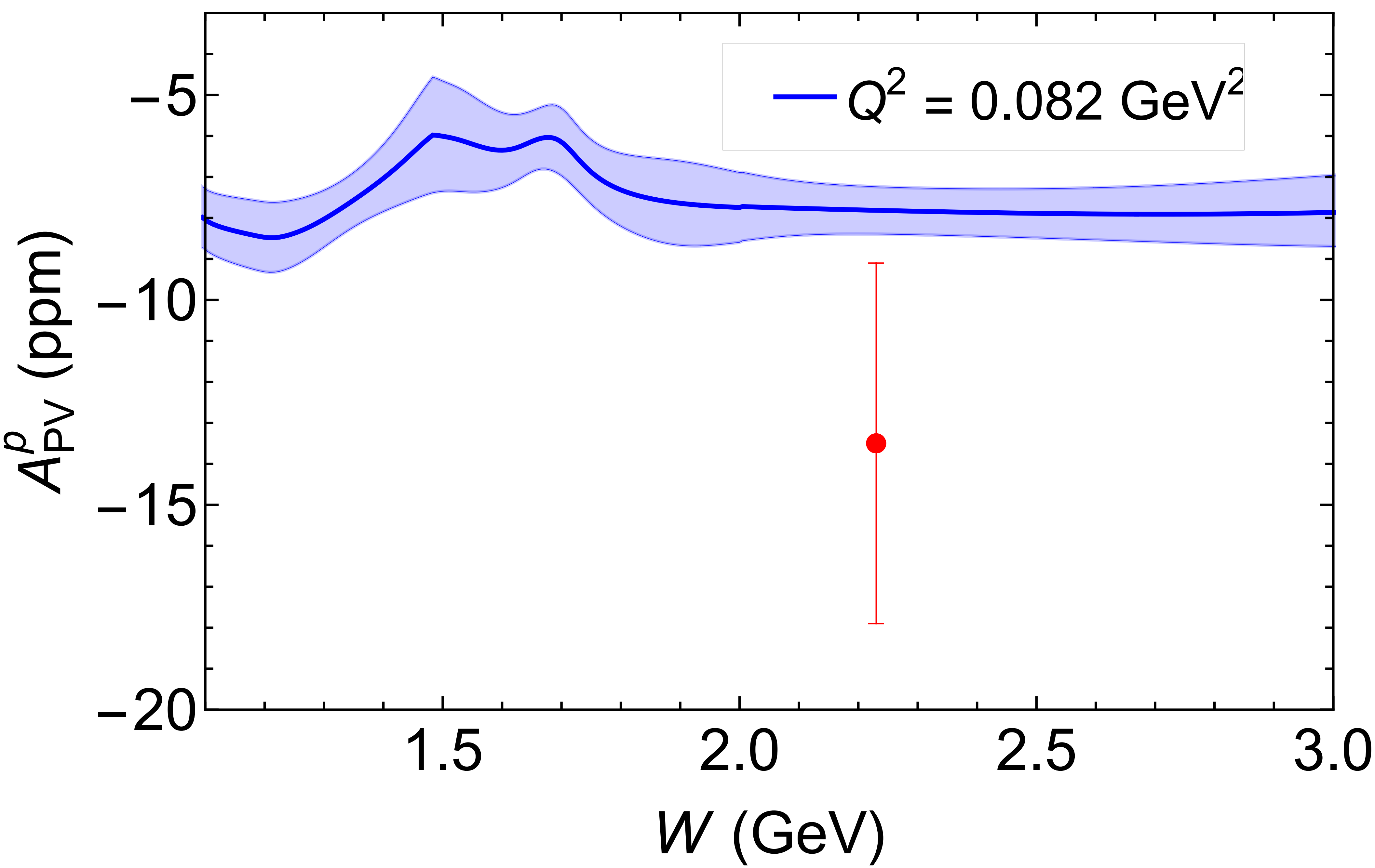
AJM model asymmetries and uncertainties for PV deuteron asymmetry constrained by fit to E08-011 data *Hall et al. (2013)*



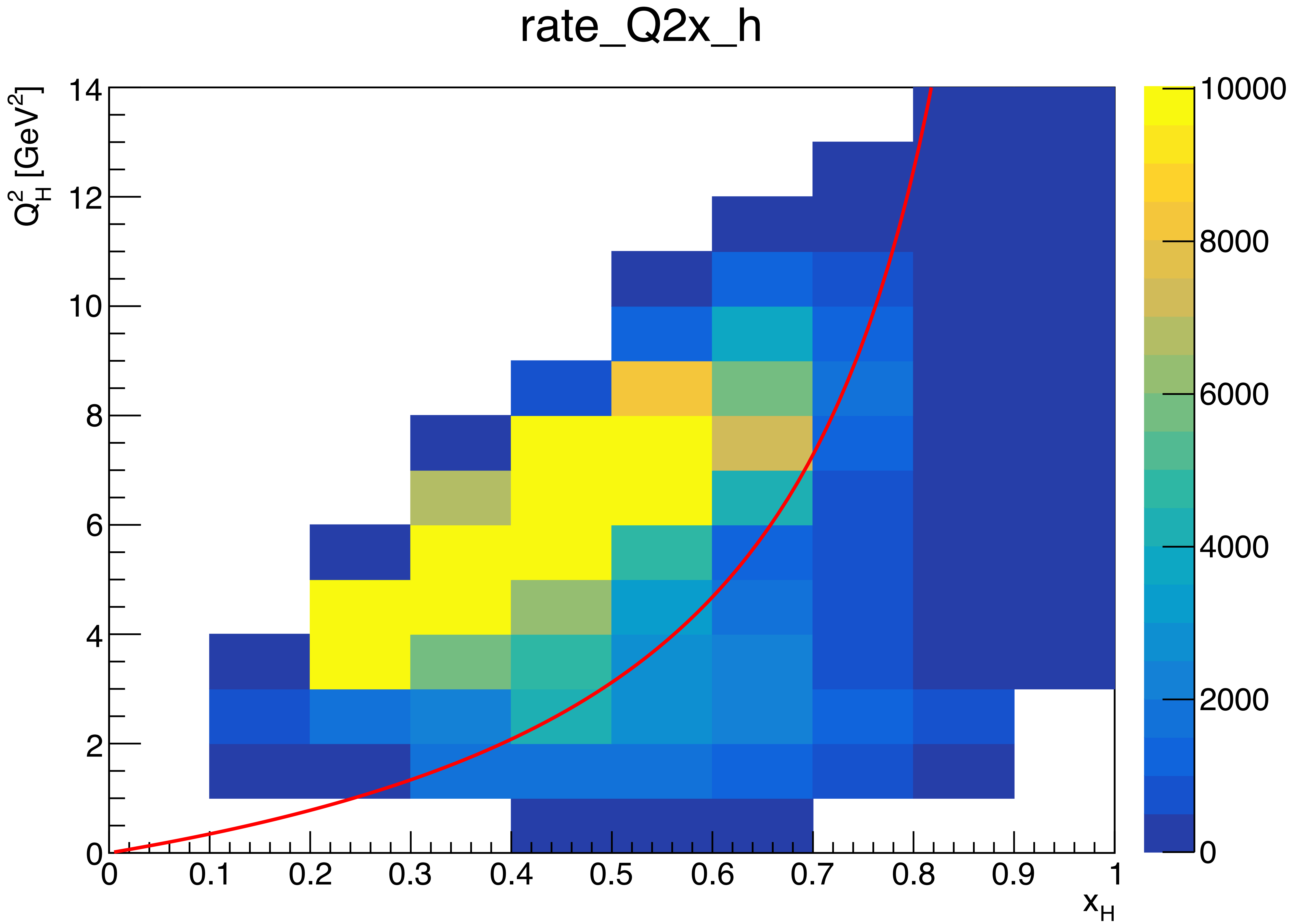
E08-011: *Wang et al. Nature **506**, 67 (2014)*

Inelastic asymmetry data from Qweak

*Androić et al., PRC **101**, 055503 (2020)*



SOLID $A_{PV}(d)$ kinematics



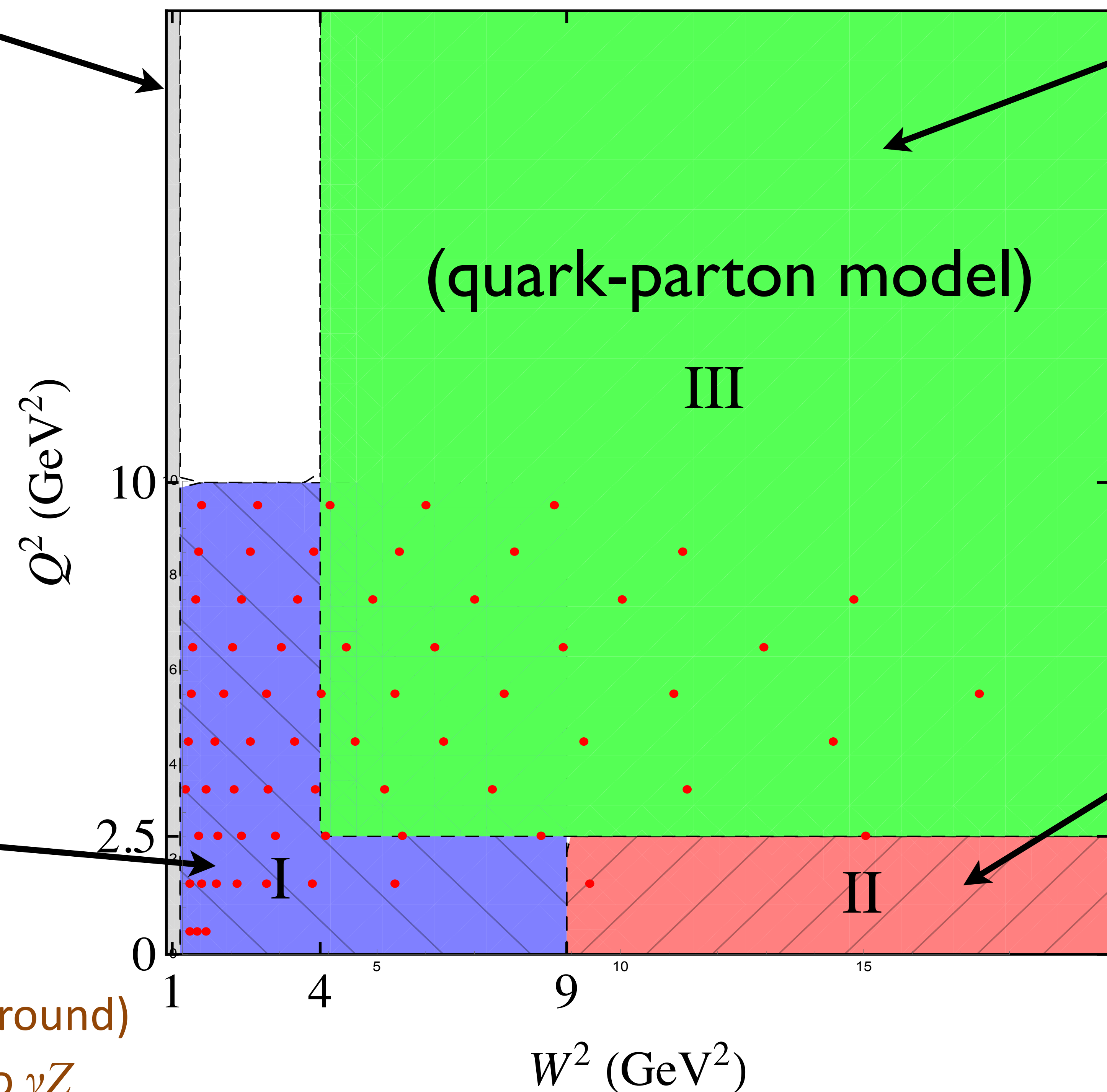
Structure function map with SOLID kinematics

"elastic"

"DIS"

Leading twist PDFs

- SOLID PVDIS kinematics



"Regge"

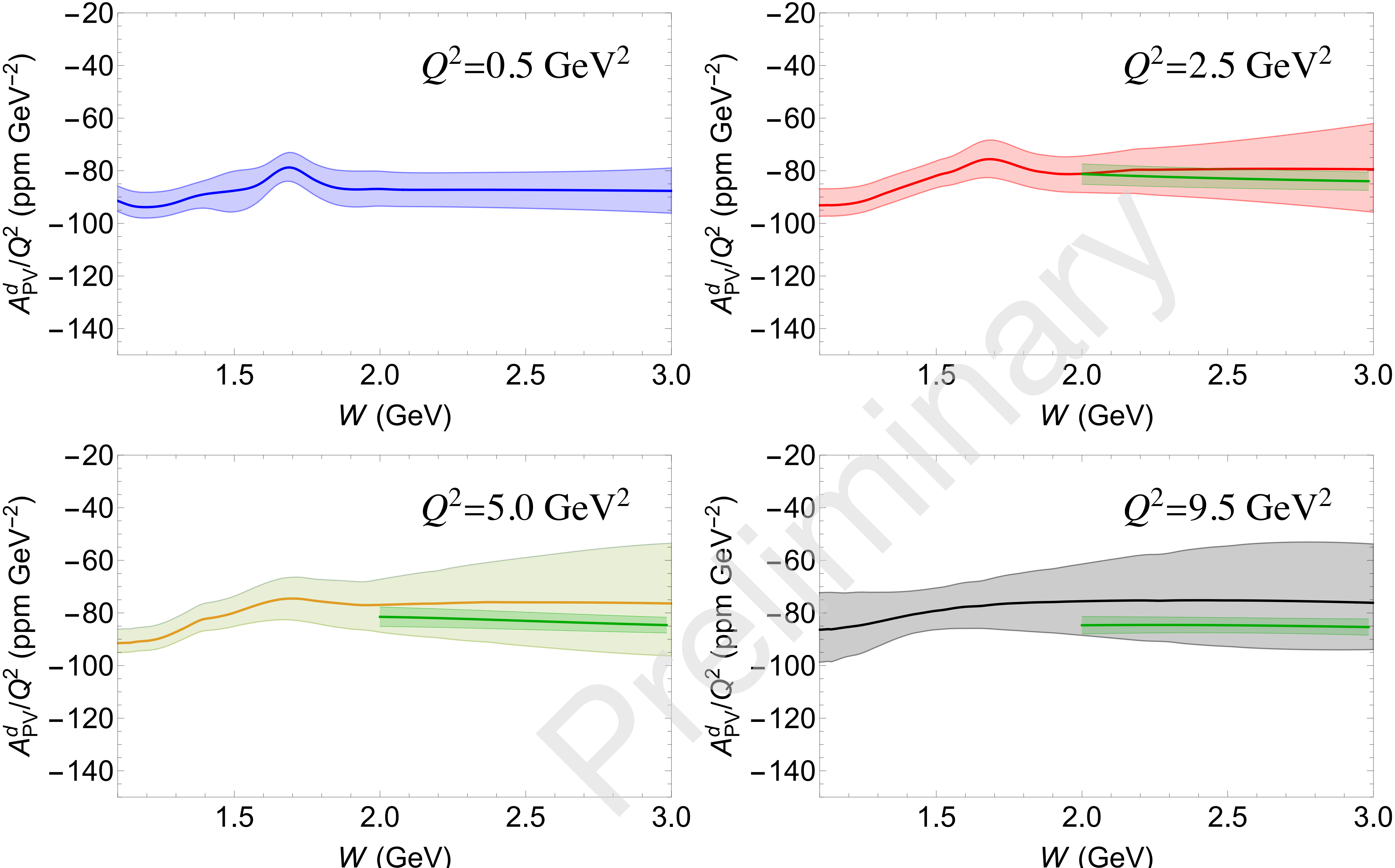
VMD + Regge model

"resonance"

Christy-Bosted $\gamma\gamma$ fit
(resonances + background)
+ isospin rotation to γZ

AJM model A_{PV}(d) for selected SOLID kinematics

(E=11 GeV)



“To do” list

- Use other PDF datasets (*e.g.* JAM)
- All analyses (Gorchtein *et al.*, Rislow-Carlson, AJM collaboration) use Christy-Bosted parametrization of resonance region (pre 2010).
  - This is a parametrization of *N** states to fit inclusive cross section data
  - It is **not** based on transition form factors and branching ratios of produced resonant states, for which exclusive meson production is suited.
  - Suggests that other more recent parametrizations should be explored (*e.g.* CLAS)
- Incorporate recent improvements to $F_3(W^2, Q^2)$ structure function into the predictions

Axial h correction

$$\Box_A^{\gamma Z}(E=0) \sim g_V^e \int_0^\infty dQ^2 \frac{\alpha(Q^2)}{Q^2(1+Q^2/M_Z^2)} \int_0^{x_{\max}} dx\ F_{3p}^{\gamma Z}(x,Q^2)\,\frac{1+2r}{(1+r)^2},$$

$$x = \frac{Q^2}{W^2 - M^2 + Q^2}, \quad r = \sqrt{1+4M^2x^2/Q^2}.$$

Blunden *et al.* PRL **107**, 181861 (2011)

→ in DIS region $(Q^2 > Q_0^2)$ integrand can be expressed in terms of moments of structure functions by letting $x_{\max} \to 1$

$$M_3^{\gamma Z(n)} = \int_0^1 dx\, x^{n-1} F_3^{\gamma Z}(x,Q^2)$$

Allows for systematic small corrections to a model-independent leading term

<u>$n=1$</u> $\quad M_3^{\gamma Z(1)}(Q^2) = \frac{5}{3}\left(1 - \frac{\alpha_s(Q^2)}{\pi}\right)$

→ γZ analog of Gross-Llewellyn Smith sum rule for F_3^W

→ leads to old result of Marciano & Sirlin

F_3 structure function landscape

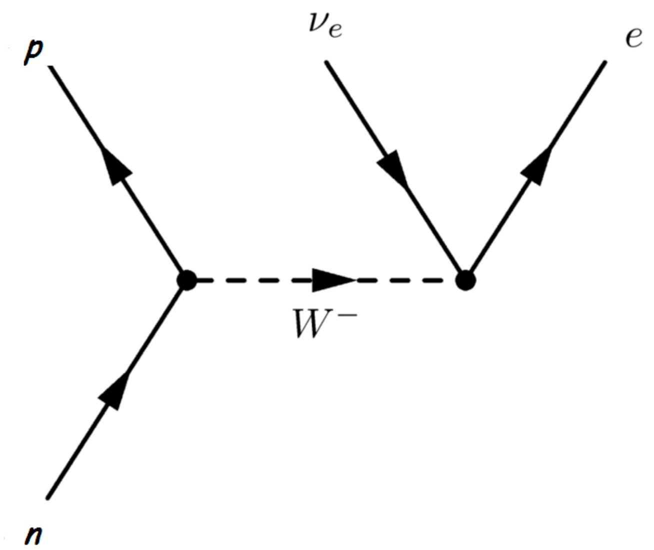
F_3 structure functions	PDF (DIS region)	Observable/Experiment
$F_{3p}^{\gamma Z}$	$\frac{2}{3}u_v + \frac{1}{3}d_v + \frac{2}{3}c_v + \frac{1}{3}s_v$	$A_{\mathrm{PV}}(\mathrm{p})$ (Qweak): $\Box_A^{\gamma Z}(E)$
$F_{3n}^{\gamma Z}$	$\frac{1}{3}u_v + \frac{2}{3}d_v + \frac{2}{3}c_v + \frac{1}{3}s_v$	$A_{\mathrm{PV}}(\mathrm{Cs})$ (atomic PV)
$F_{3p}^{\gamma Z} + F_{3n}^{\gamma Z}$	$u_v + d_v + \frac{4}{3}c_v + \frac{2}{3}s_v$	$A_{\mathrm{PV}}(\mathrm{d})$ (SOLID)
$F_3^W = \frac{1}{2}\left(F_{3p}^{W^+} + F_{3p}^{W^-}\right)$	$u_v + d_v + c_v + s_v$	$\nu p + \bar{\nu} p$: GLS sum rule
$F_{3(0)}^{\gamma W} = F_{3p}^{\gamma Z} - F_{3n}^{\gamma Z}$	$\frac{1}{3}u_v - \frac{1}{3}d_v$	superallowed β-decay: $\Box_A^{\gamma W}(E=0)$
$F_{3(1)}^{\gamma W} = F_{1p}^{\gamma} - F_{1n}^{\gamma}$	$(u+\bar{u}) - (d+\bar{d})$	Gottfried sum rule: $\Box_A^{\gamma W}(E \neq 0)$

$\boldsymbol{\Box_A^{\gamma Z}}$ **($\times 10^{-3}$)**	P2	Qweak
E (GeV)	0.155	1.165
Marciano and Sirlin (1984)	5.2(5)	5.2(5)
Blunden *et al.* (2011)	4.4(4)	3.7(4)
Erler *et al.* (2019)	4.46(21)	3.97(22)

No background term in resonance region

Rewrite in terms of isoscalar + isovector currents and use experimental constraints from F_3^W

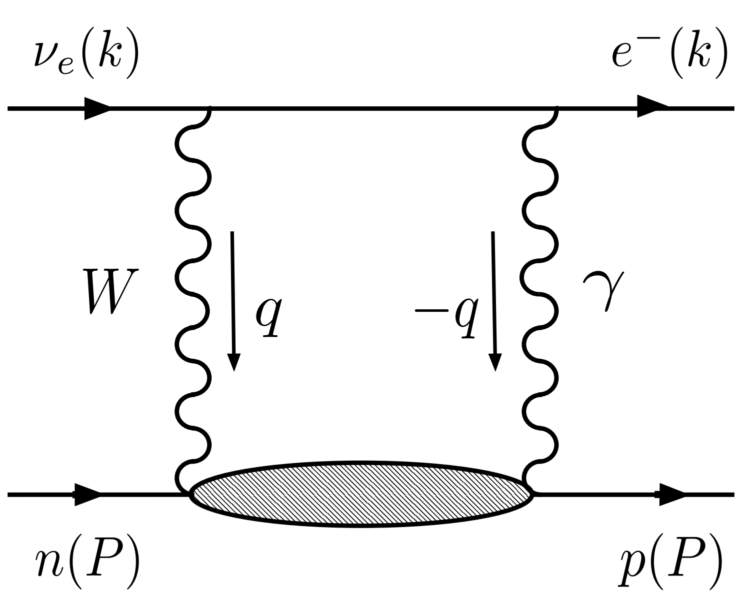
Superallowed 0^+ to 0^+ β decays



$$|V_{ud}|^2 = \frac{0.97148(20)}{1+\Delta_R^V},$$

Inner radiative correction term (nucleus-independent)

$$\Delta_R^V = \frac{\alpha}{2\pi}\left[3\ln\frac{M_Z}{M} - \ln\frac{M_W}{M_Z}\right] + 2\,\Box_A^{\gamma W},$$



γW box is largest source of hadronic uncertainty

Seng *et al.* (2018, 2019) found the correct dispersion representation of $\Box_A^{\gamma W}$

Analogous to the contribution to the weak axial charge. At zero energy

$$\Box_A^{\gamma W} = \frac{\alpha}{2\pi}\int_0^\infty dQ^2 \, \frac{1}{Q^2(1+Q^2/M_W^2)}\int_0^{x_{\max}} dx \; F_3^{(0)}(x,Q^2)\,\frac{1+2r}{(1+r)^2}.$$

Isospin symmetry implies $F_3^{(0)} = F_{3p}^{\gamma Z} - F_{3n}^{\gamma Z}$

In DIS region: $F_3^{(0)} = \frac{1}{3}u_v - \frac{1}{3}d_v$ $\quad$ (not $F_3^{(0)} = \frac{1}{6}(u - \bar{d})$ as originally written)

$F_3^W \approx u_v + d_v$

Used sum rules plus data on F_3^W from $\nu p + \bar{\nu} p$

$\longrightarrow$ Found a shift in $\Box_A^{\gamma W}$ from $3.52(11) \times 10^{-3}$ to $3.79(10) \times 10^{-3}$.

Shiells *et al.* (2021): calculation based on constructing $F_3^{(0)}(W^2, Q^2)$
and imposing continuity across kinematic boundaries (AJM approach)

Elastic contribution

$$F_{3\,(\mathrm{el})}^{(0)}(Q^2) = -\left[G_M^p(Q^2) + G_M^n(Q^2)\right] G_A(Q^2)\, Q^2 \delta(W^2 - M^2)$$

- Isoscalar magnetic form factor from recent fits (including TPE)
- Axial form factors from νN scattering (major source of uncertainty)

$$G_A(Q^2) = -\frac{g_A}{(1 + Q^2/m_A^2)^2} \qquad m_A = 1.05(5)\ \mathrm{GeV}$$

Resonance contributions (isoscalar, so very small)

- Electromagnetic $e{+}p$ and $e{+}n$ helicity amplitudes from MAID2009 (can also use CLAS for $e{+}p$ with similar results)
- Axial transition form factors from Leitner (νN scattering)

DIS contribution

- Convolution at NLO

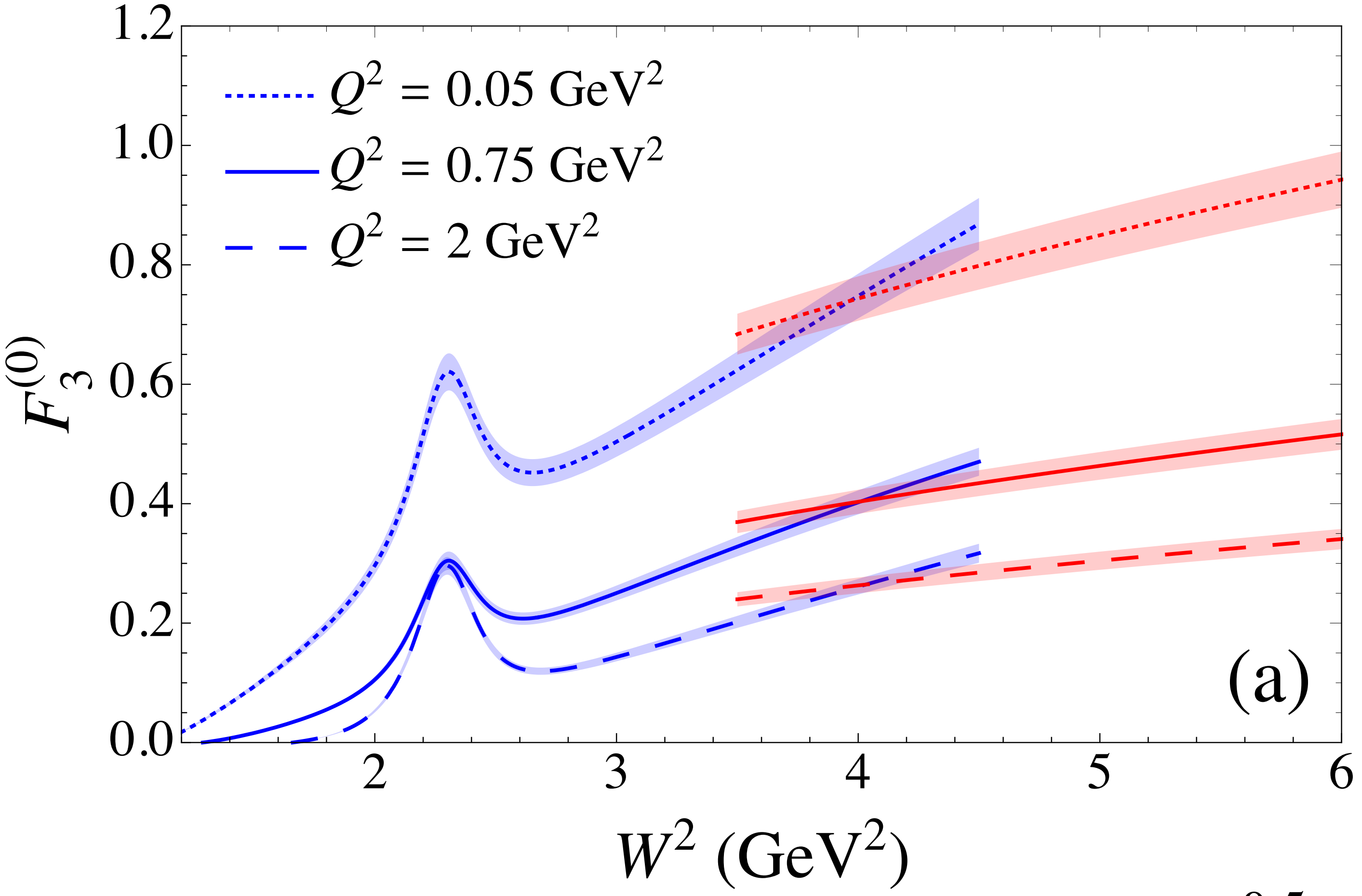
$$F_{3,\mathrm{DIS}}^{(0)}(x, Q^2) = \int_x^1 \frac{dz}{z} C_3^{(1)}(z) \times \frac{1}{3}\left(u_v(x/z, Q^2) - d_v(x/z, Q^2)\right)$$

$\boldsymbol{\Box_A^{\gamma W}}$ $\mathbf{(\times 10^{-3})}$

Q_0^2 (GeV^2)	**2.0**	1.0
elastic	1.05(4)	1.05(4)
resonance	0.04(1)	0.04(1)
DIS + ($Q^2 > Q_0^2$) bgd	2.29(3)	2.43(5)
Regge + ($Q^2 < Q_0^2$) bgd		
Total	**3.90(9)**	3.91(9)

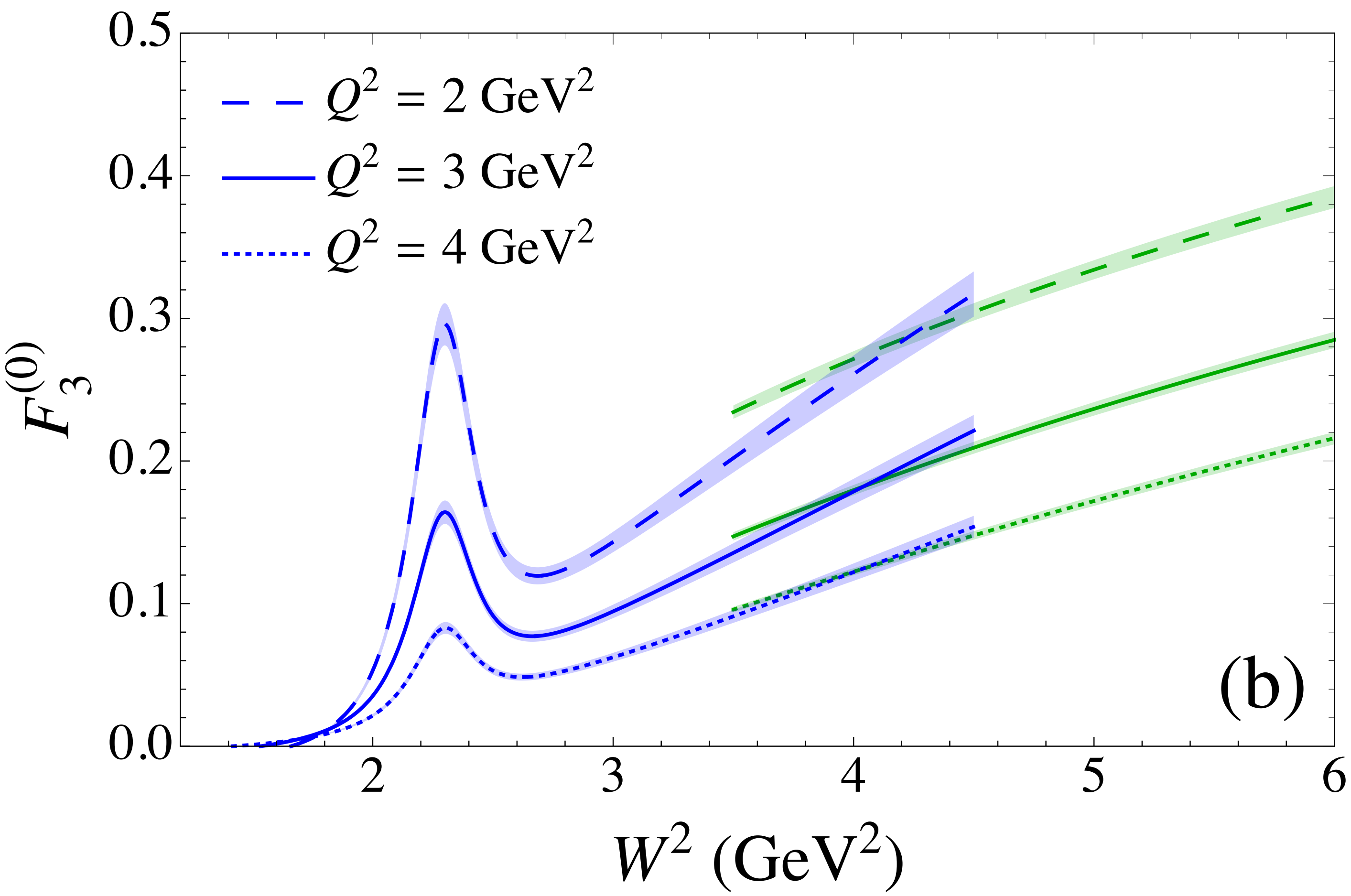
Matching CB resonant background to Regge and DIS

Shiells, PB & Melnitchouk, PRD **104**, 033101 (2021)



Match to Regge

Match to DIS



Regge contribution

At low Q^2 and high W^2, the strong interaction becomes nonperturbative.

Model of Capella et al. from ν-N scattering fits:

$$F^{(0)}_{3\,(\mathrm{Reg})}(x, Q^2) = A^{p-n}\, x^{-\alpha_R}(1-x)^c \left(\frac{Q^2}{Q^2+\Lambda_R^2}\right)^{\alpha_R}$$

The true Q^2 dependence of this structure function is not well-determined by theory.

Match this function to the well-known value in the DIS region around $Q^2 = 2\ \mathrm{GeV}^2$ AND constrain it from available data on F_3^W from $\nu p + \bar{\nu} p$

$$A^{p-n} = A^{p+n}\,\frac{1}{9}$$

Sum-rule constrained

OR

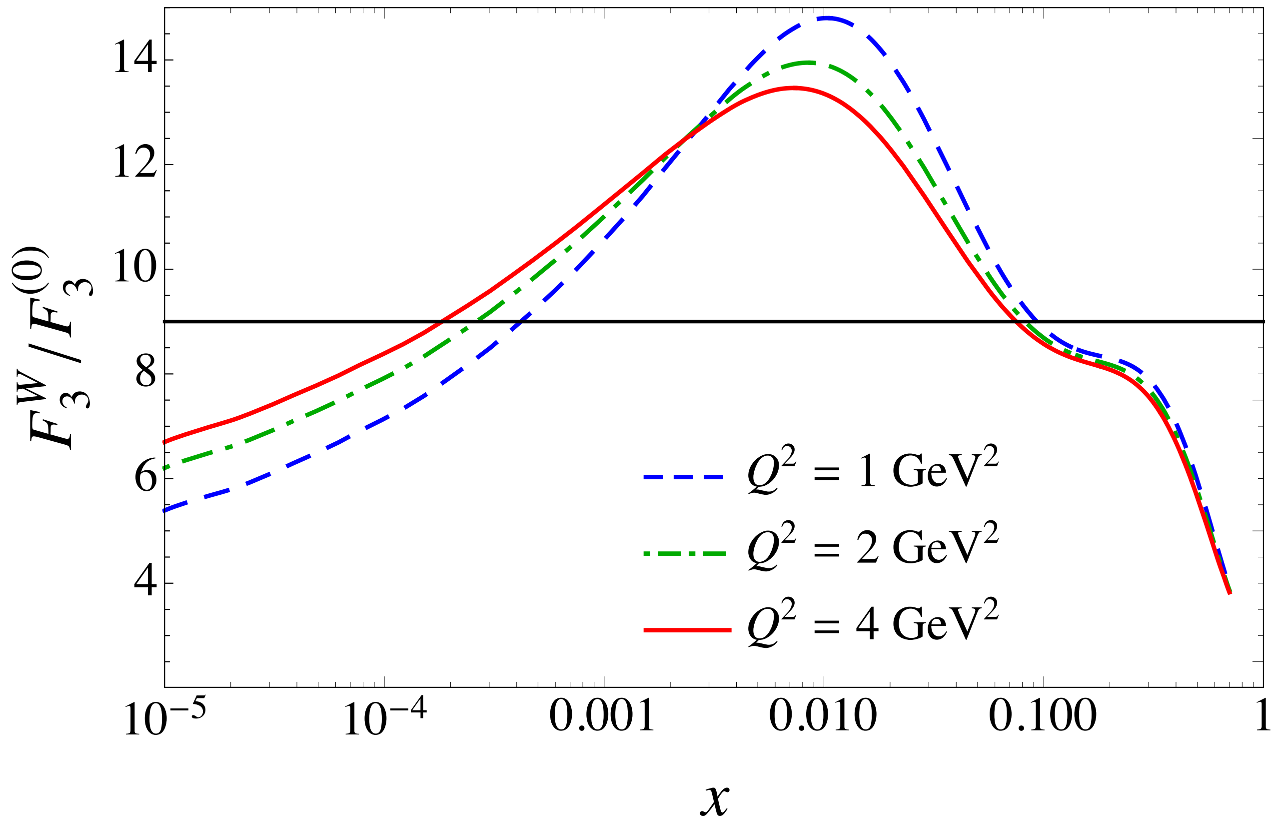
$$A^{p-n} = A^{p+n}\,\left.\frac{F^{(0)}_{3\,(\mathrm{Reg})}}{F^{W}_{3\,(\mathrm{Reg})}}\right|_{Q^2=Q_0^2}$$

PDF constrained

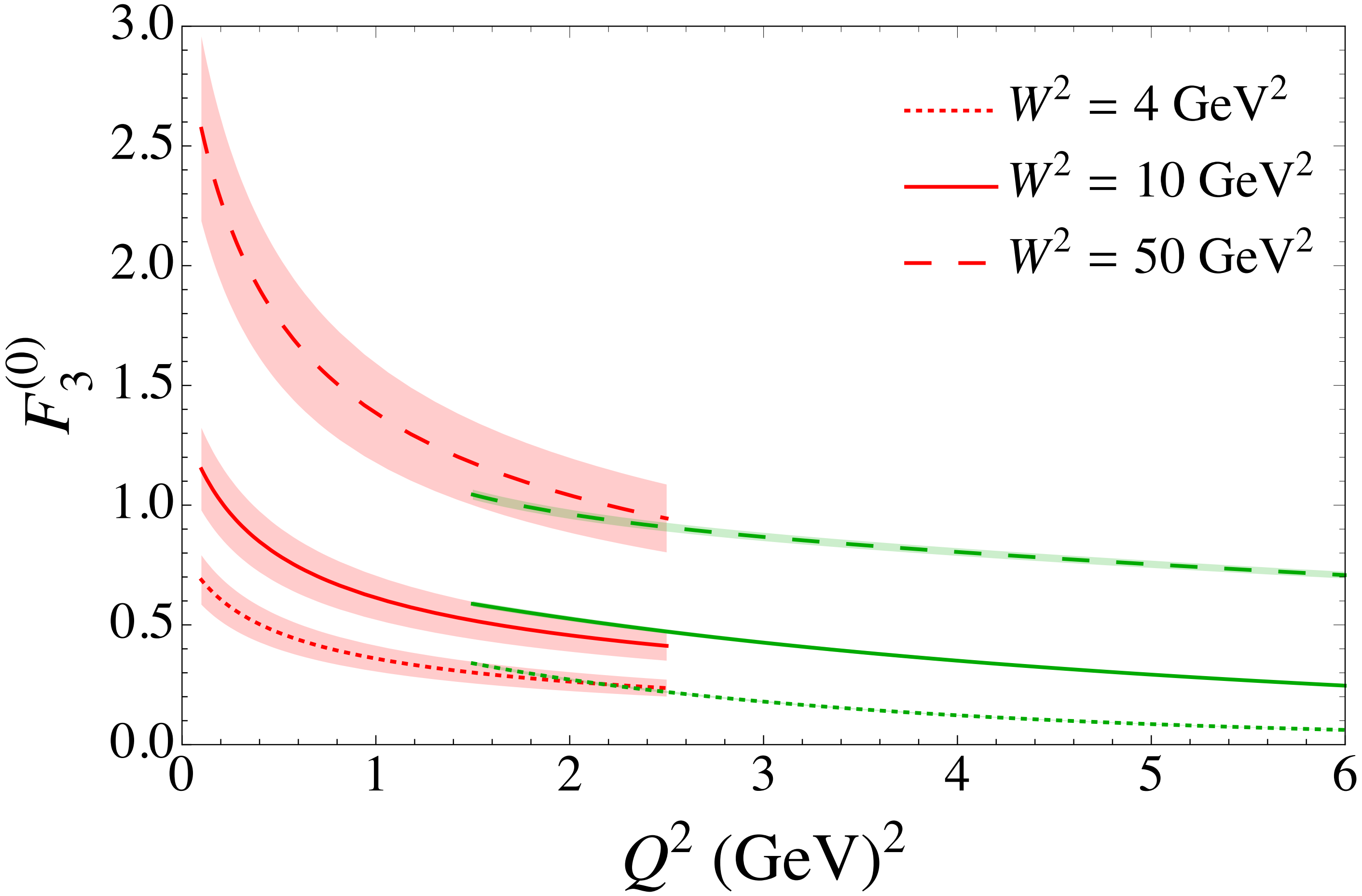
Regge contribution

Is the ratio 9 as a function of x?

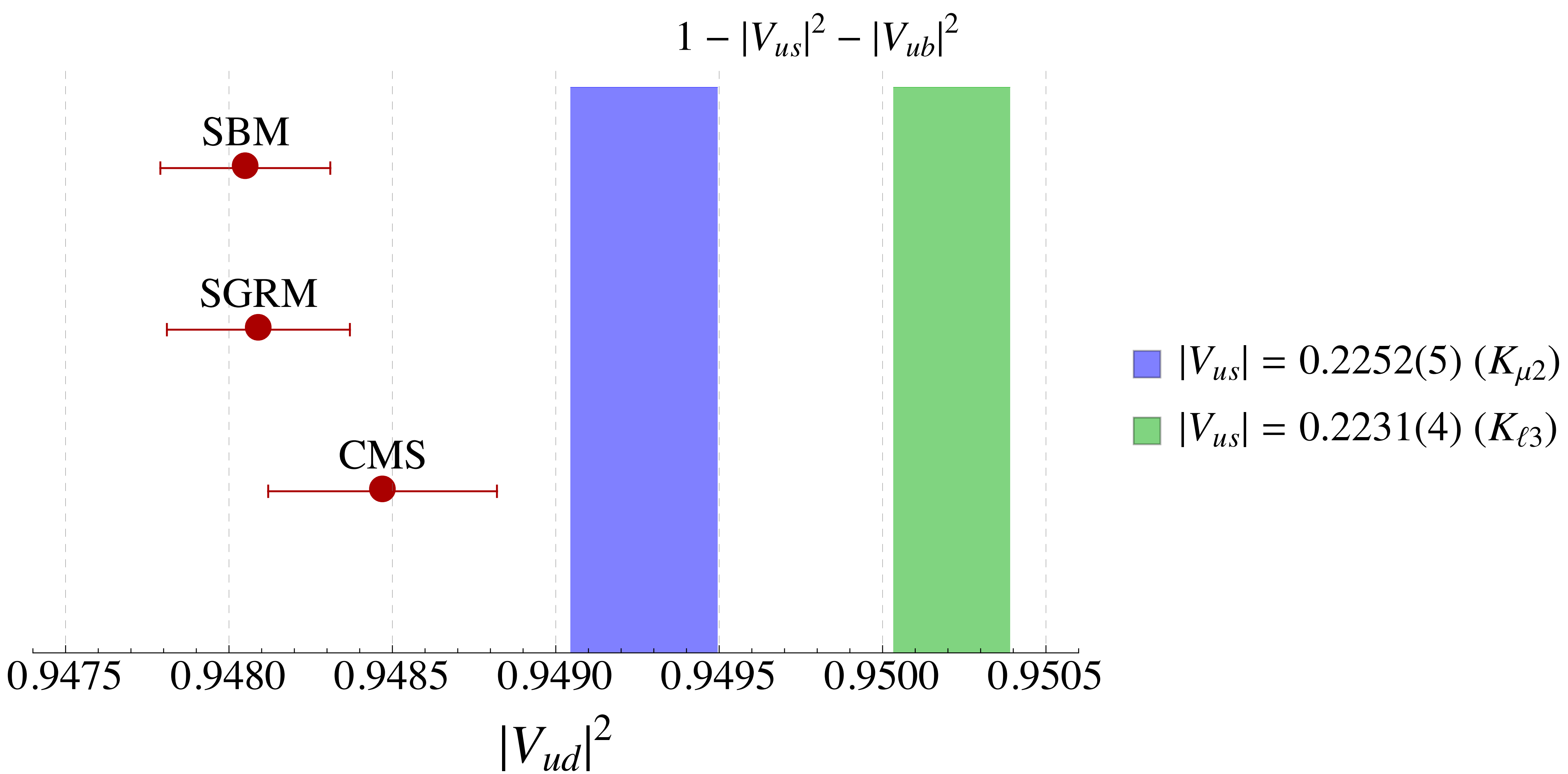
No, but the observable correction is proportional to $\int_0^1 dx F_3(x, Q^2)$
so taking a ratio of 9 is still meaningful.



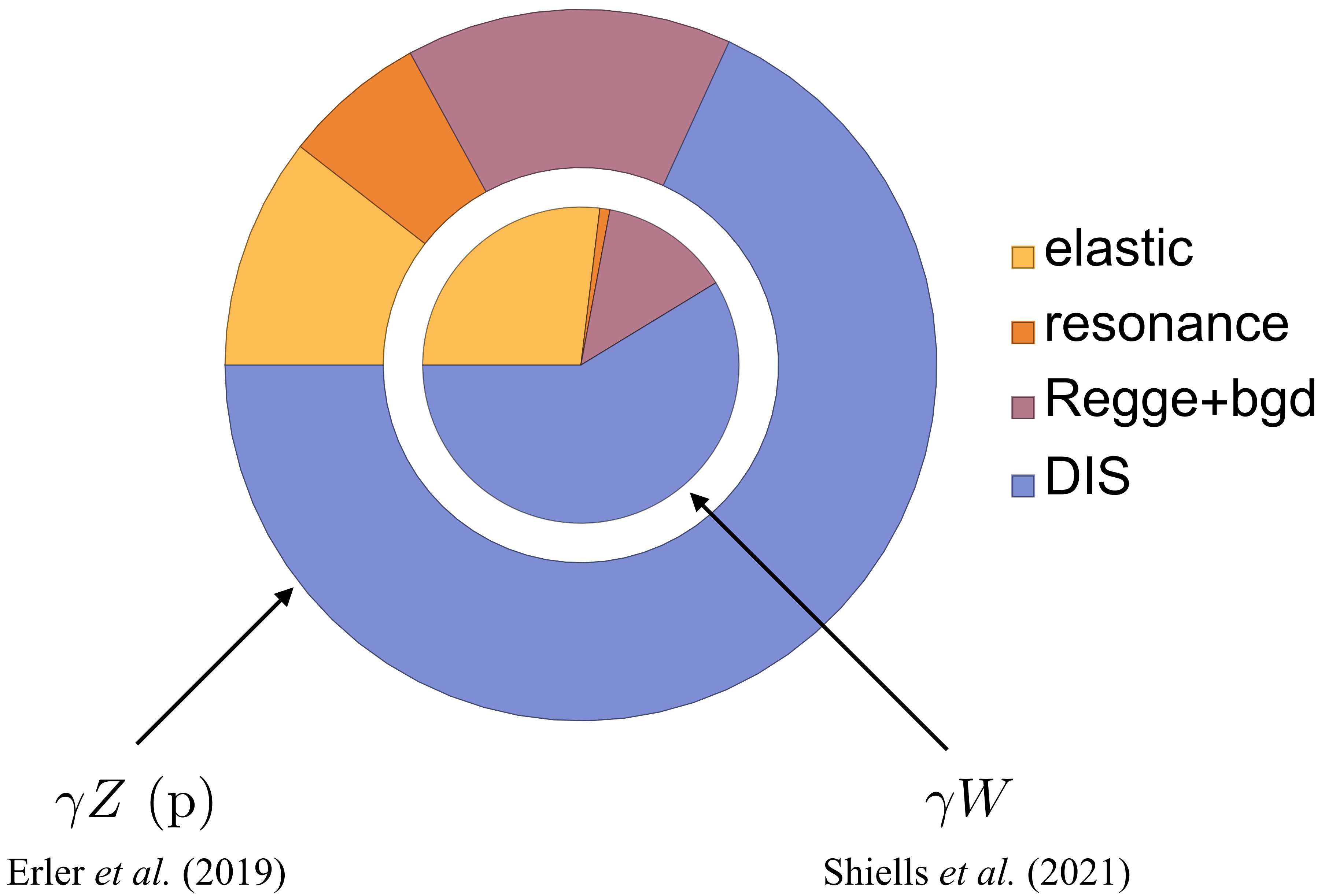
Matching Regge to DIS at low Q^2



$\Box_A^{\gamma W}$ ($\times \mathbf{10^{-3}}$)	SBM		SGRM [7]	CMS [5]
Q_0^2 [GeV^2]	**2.0**	1.0	2.0	1.1
Elastic	1.05(4)	1.05(4)	1.06(6)	† 0.99(10)
Resonance	0.04(1)	0.04(1)	...	...
DIS + ($Q^2 > Q_0^2$) bgd	2.29(3)	2.43(5)	*2.17(0)	*2.29(2)
Regge + ($Q^2 < Q_0^2$) bgd	0.52(7)	0.39(5)	0.56(8)	0.25(2)
Total	**3.90**(**9**)	3.91(9)	3.79(10)	3.52(11)
Δ_R^V	**0.02472**(**18**)	0.02474(18)	0.02467(22)	0.02426(32)
$\lvert V_{ud}\rvert^2$	**0.94805**(**26**)	0.94803(26)	0.94809(28)	0.94847(35)



Comparison of $\Box_A^{\gamma Z}(0)$ and $\Box_A^{\gamma W}(0)$ contributions from different regions



Summary

- Dispersion approach significant improvement over old methods
- PDF region provides constraints on model-dependence of isospin rotation
- Direct comparison with PV inelastic data in resonance and DIS regions
- *e-d* PVDIS asymmetry strongly constrains the uncertainty in $\Box_V^{\gamma Z}$
- checking Δ region at Mainz or JLab would be useful
- Regge and background contributions show largest uncertainties in $\Box_A^{\gamma Z}$