

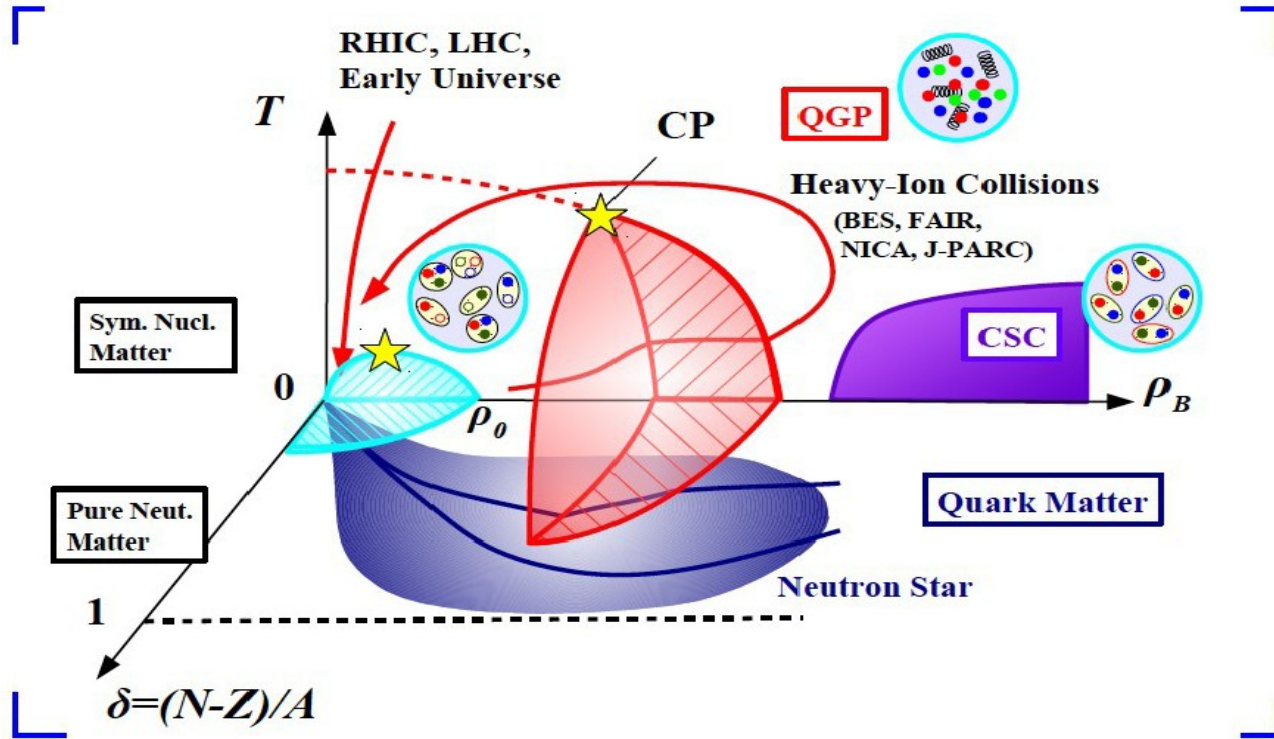
# Thermal twin stars & confining density functional approach

**David Blaschke**

University of Wroclaw, Poland & HZDR/CASUS Görlitz, Germany



# A trigger question:



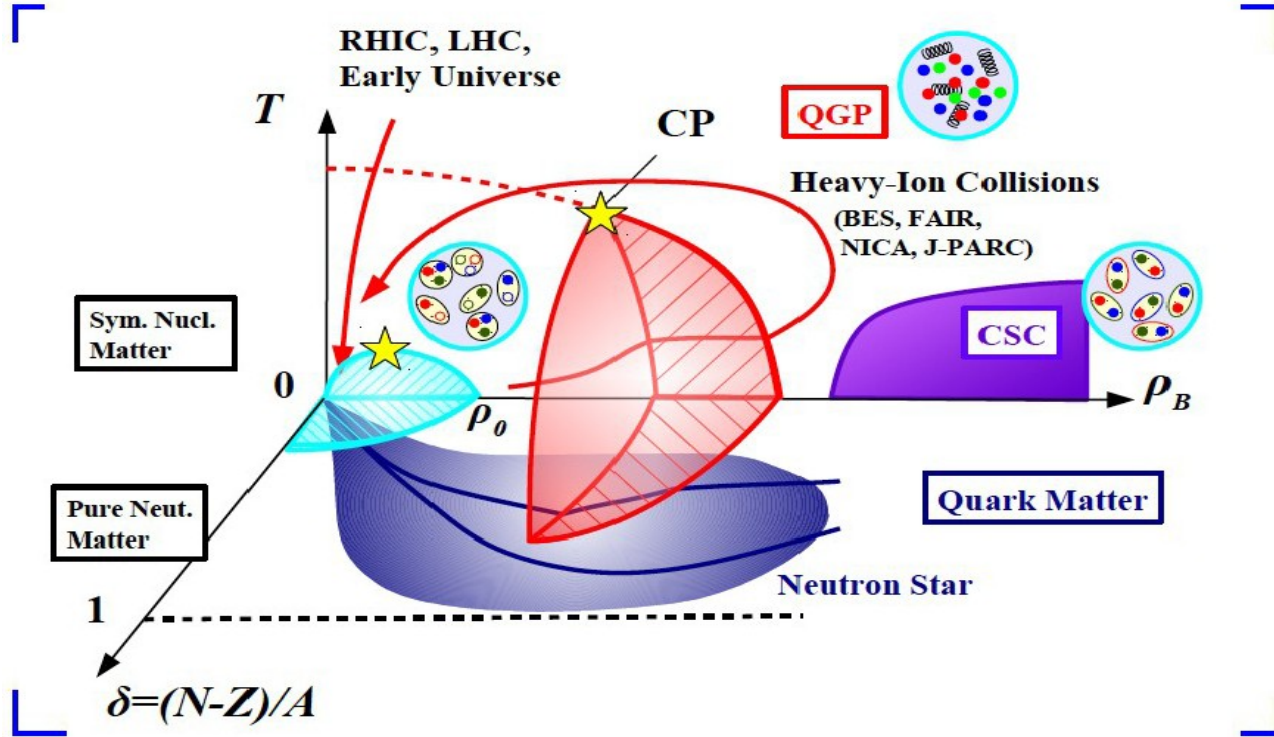
At CPOD 2013, Jorgen Randrup asks:

„Can astrophysics colleagues find out whether in neutron stars the quark deconfinement transition is first order?“



The QCD Phase Diagram by Akira Ohnishi († 16.03.2023)

# The idea



At CPOD 2013, Jorgen Randrup asks:

„Can astrophysics colleagues find out whether in neutron stars the quark deconfinement transition is first order?“



David Blaschke answers:

In principle – yes !

If they can find twin compact stars !

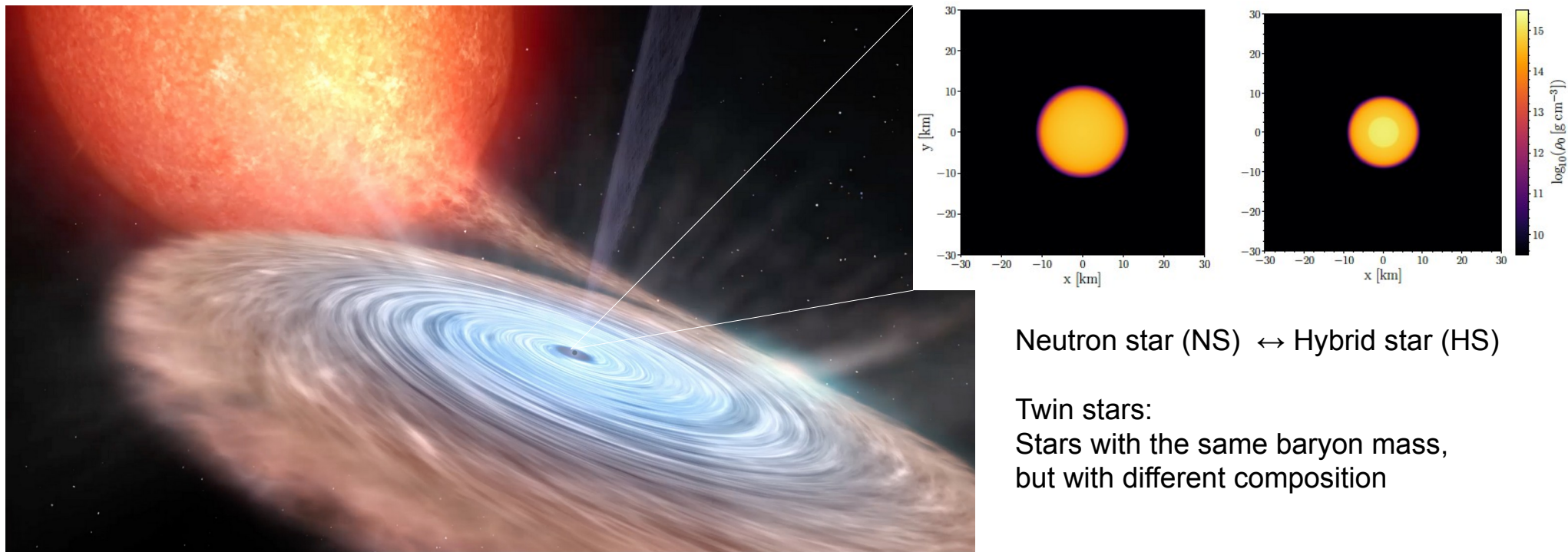
PoS CPOD2013 (2013)

The QCD Phase Diagram by Akira Ohnishi († 16.03.2023)

# Thermal twin stars & confining density functional approach

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Neutron star (NS) ↔ Hybrid star (HS)

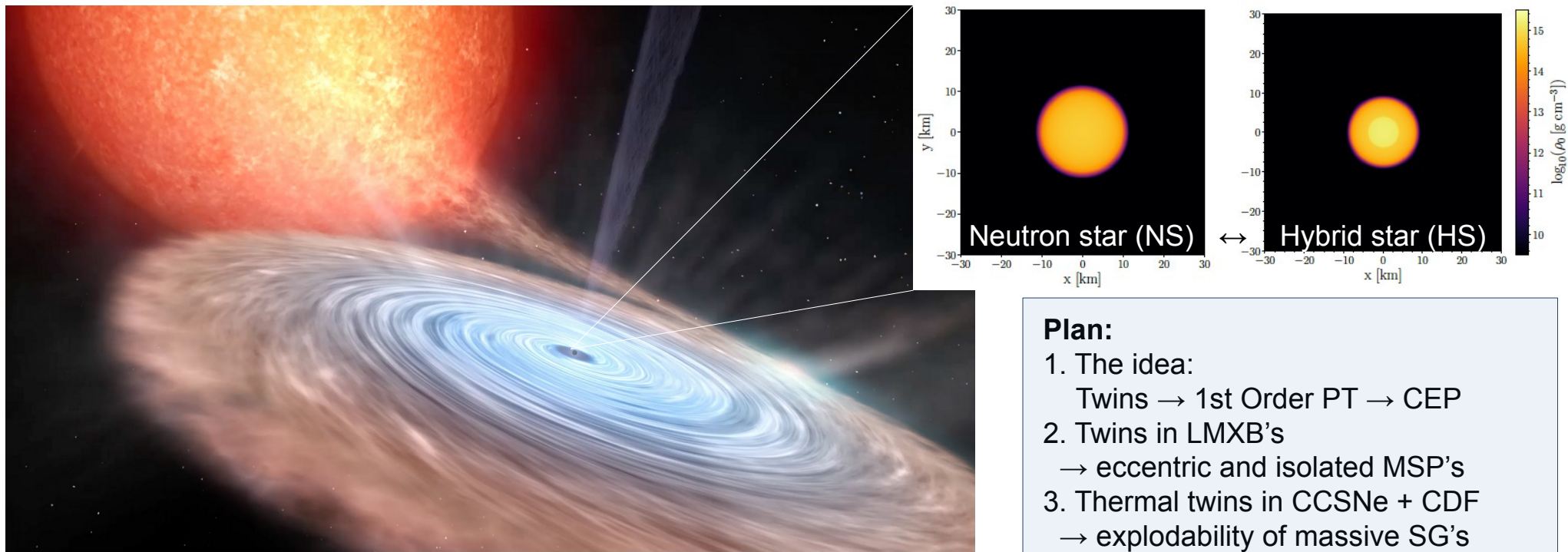
Twin stars:  
Stars with the same baryon mass,  
but with different composition

Artistic view of a neutron star accreting mass from a companion star in a low-mass X-ray binary (LMXB) system

# Thermal twin stars & confining density functional approach

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Artistic view of a neutron star accreting mass from a companion star in a low-mass X-ray binary (LMXB) system

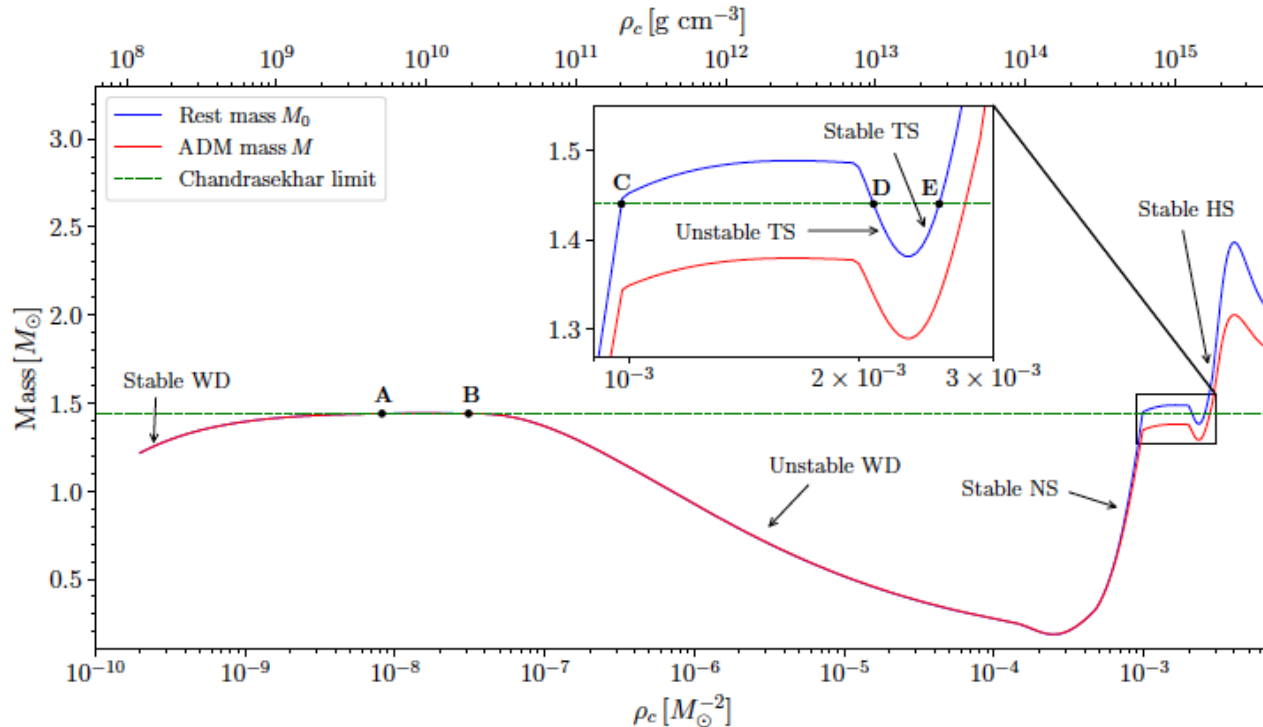
## Plan:

1. The idea:
  - Twins  $\rightarrow$  1st Order PT  $\rightarrow$  CEP
2. Twins in LMXB's
  - $\rightarrow$  eccentric and isolated MSP's
3. Thermal twins in CCSNe + CDF
  - $\rightarrow$  explodability of massive SG's
4. Discussion of CSC phase effect

# The idea

Three families of stable compact stars, separated by unstable configurations:  
1. White Dwarfs (WD), 2. Neutron Stars (NS), 3. Hybrid Stars (HS)

Twin Stars (TS): subclass of HS for which a NS with same baryon mass exists



From: M. Naser, G. Bozzola, V. Paschalidis, arXiv:2406.15544

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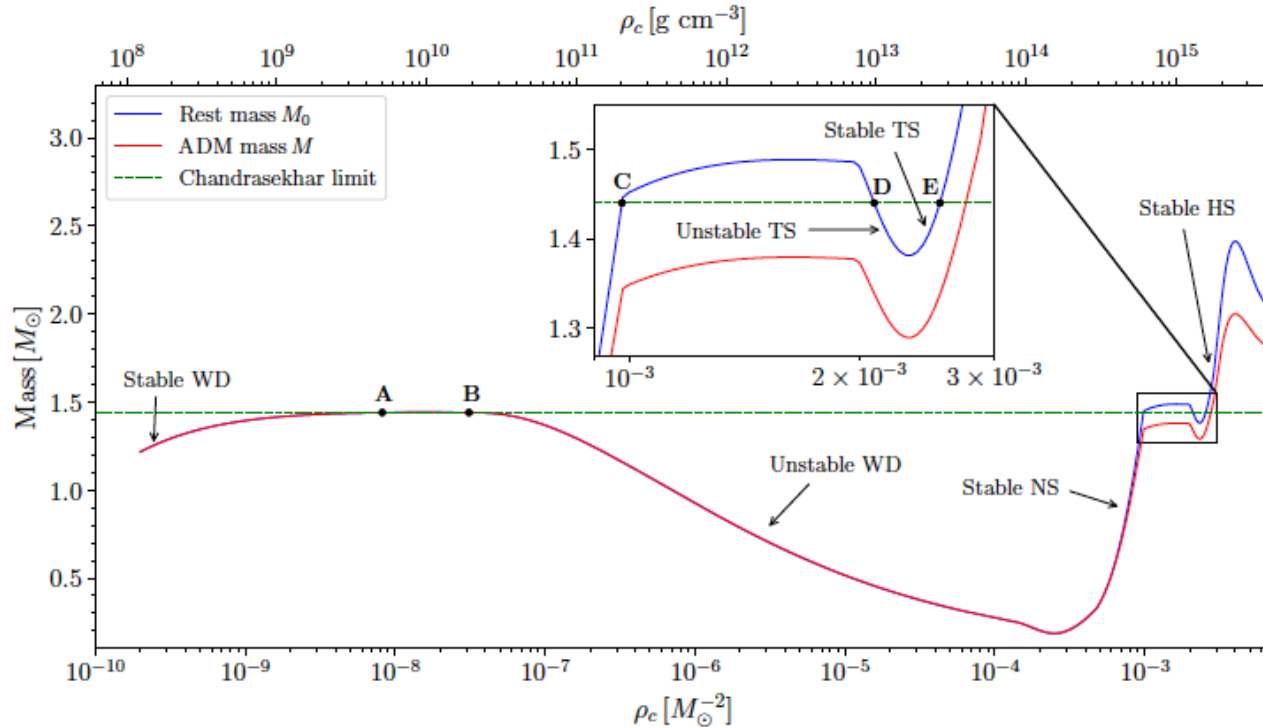
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PoS CPOD2013 (2013)

# The idea

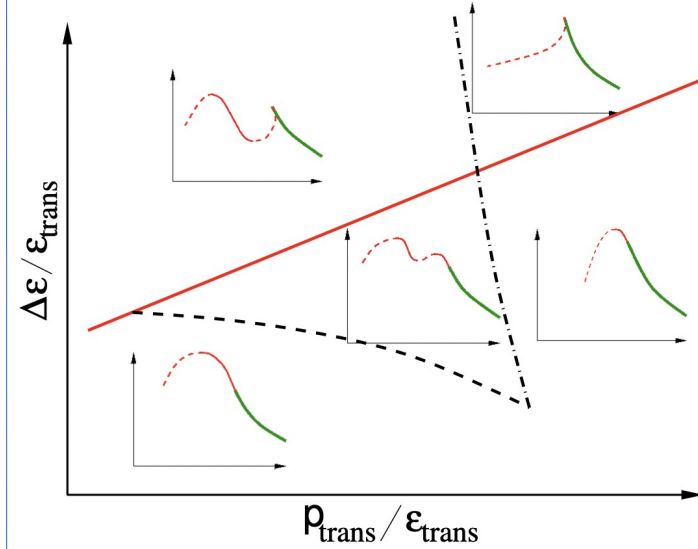
Three families of stable compact stars, separated by unstable configurations:  
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Disconnected HS branch indicates a Strong (1st-order) phase transition



Seidov criterion for gravitational Instability (red line):

$$\frac{\Delta \epsilon_{\text{crit}}}{\epsilon_{\text{trans}}} = \frac{1}{2} + \frac{3}{2} \frac{p_{\text{trans}}}{\epsilon_{\text{trans}}}$$

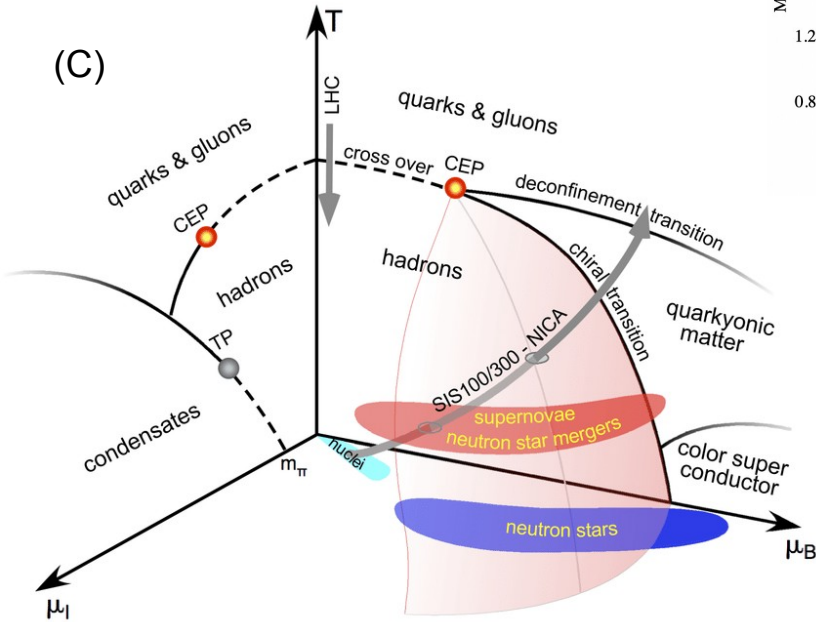
From: M. Alford, S. Han, M. Prakash, PRD (2013):  
 „Generic conditions for stable hybrid stars“

# The idea

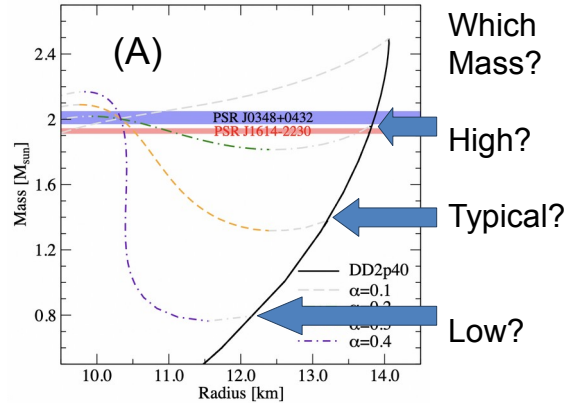
Twin Stars (A)

→ Strong Phase Transition (B)

→ Existence of CEP in Phase Diagram (C)



From: M. Kaltenborn, N. Bastian, D.B., PRD 96 (2017) 056024

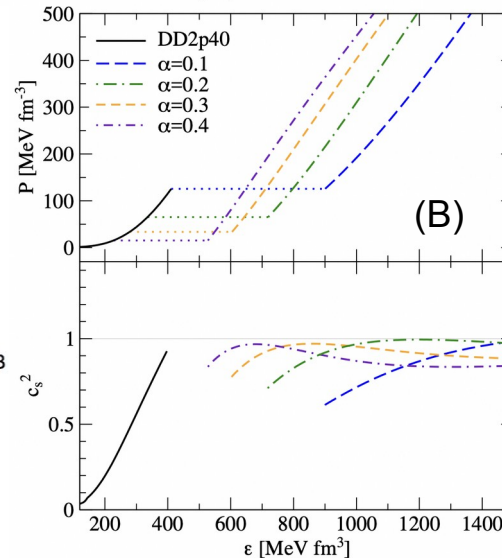


Which Mass?

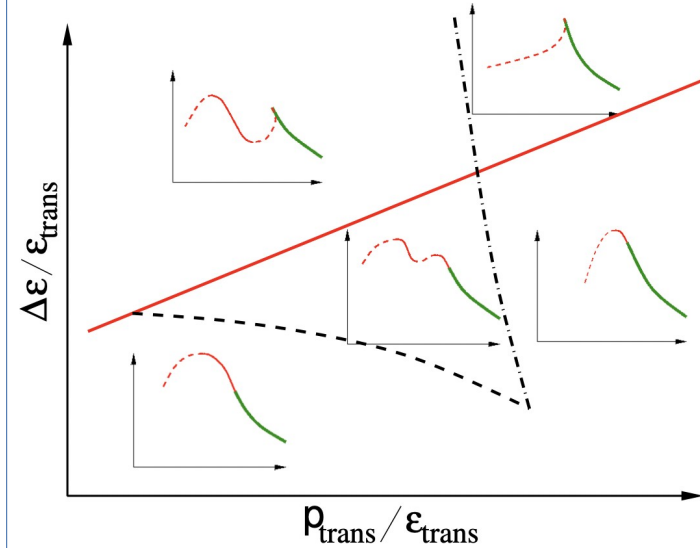
High?

Typical?

Low?



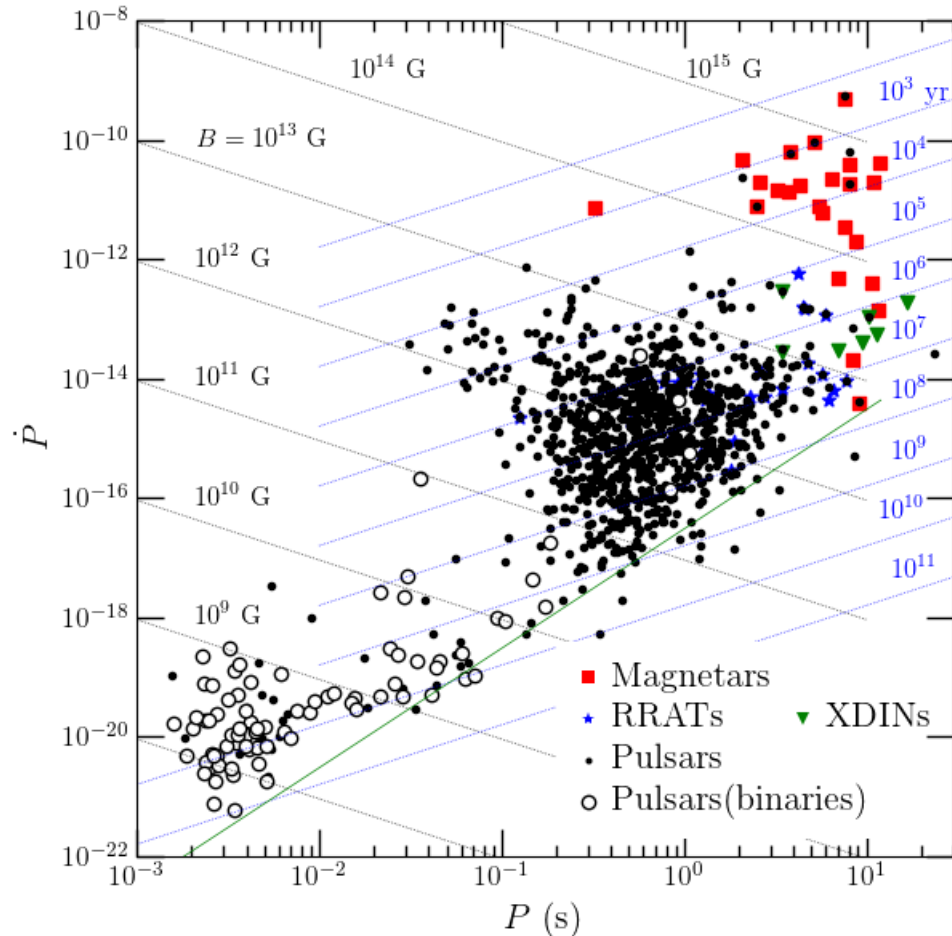
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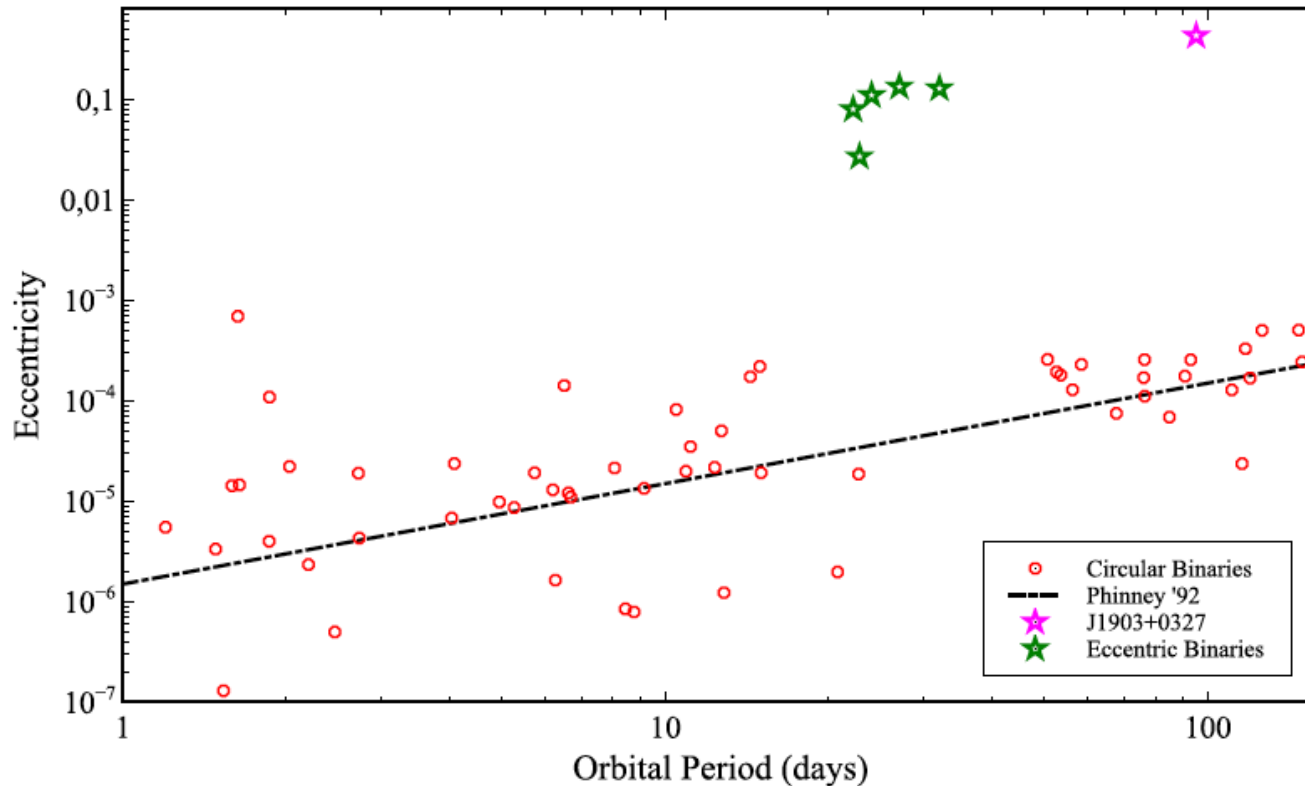
$$\frac{\Delta \epsilon_{\text{crit}}}{\epsilon_{\text{trans}}} = \frac{1}{2} + \frac{3 p_{\text{trans}}}{2 \epsilon_{\text{trans}}}$$

# Introduction: Millisecond Pulsars in the $\dot{P}$ -P Diagram



- Pulsars are born with periods  $P \sim 0.1 - 1.0$  seconds and magnetic fields  $B \sim 10^{12}$  G
- They spin down for millions of years and cross the „death-line“ (green) to the pulsar graveyard, where their dipole radiation is „switched off“
- Eventually, they accrete matter from a companion star in a binary system and spin-up to periods of milliseconds (MSPs)
- Presently  $\sim 600$  MSPs are known, most of them in binaries ( $\sim 500$ ) but some are isolated ( $\sim 20\%$ ) and 5 are a Puzzle!

# Introduction: The Puzzle of Eccentric MSPs



- Most MSPs are in binaries with circular orbits:  $e \sim 10^{-6} - 10^{-4}$   
→ „Phinney line“

- Period gap 20 – 50 d, with 5 eccentric binaries with white dwarfs:  $e \sim 10^{-1}$

- One binary with a main sequence star (J1903+0327) with larger period, 100 d, and  $e \sim 1$

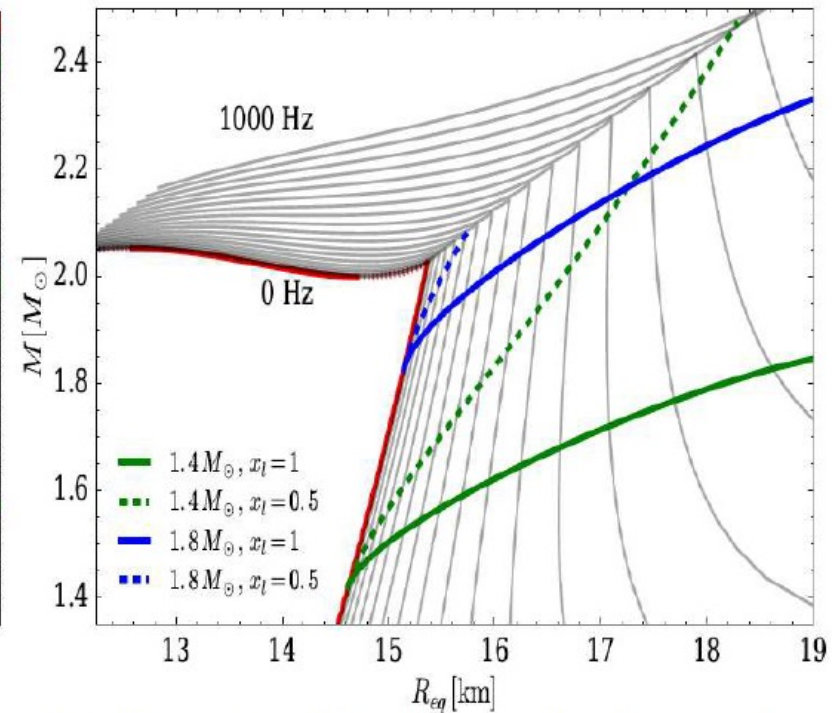
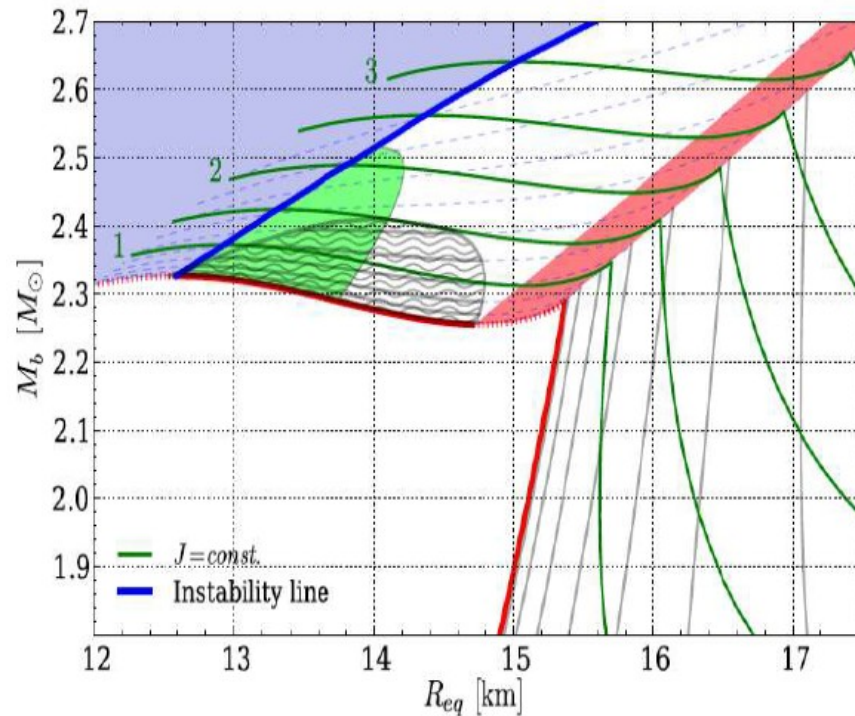
J. Antoniadis, *Astrophys. J.* 797, L24 (2014): „On the Formation of Eccentric Millisecond Pulsars with He-WD Companions“  
D. Alvarez-Castillo, J. Antoniadis, A. Ayriyan, D. Blaschke, V. Danchev et al., *Astron. Nachr.* 340 (2019) 878,  
„Accretion-induced collapse to third family compact stars as trigger for eccentric orbits of millisecond pulsars in binaries“

→ **Puzzle!**

# Let us discover the 3<sup>rd</sup> family of compact stars!

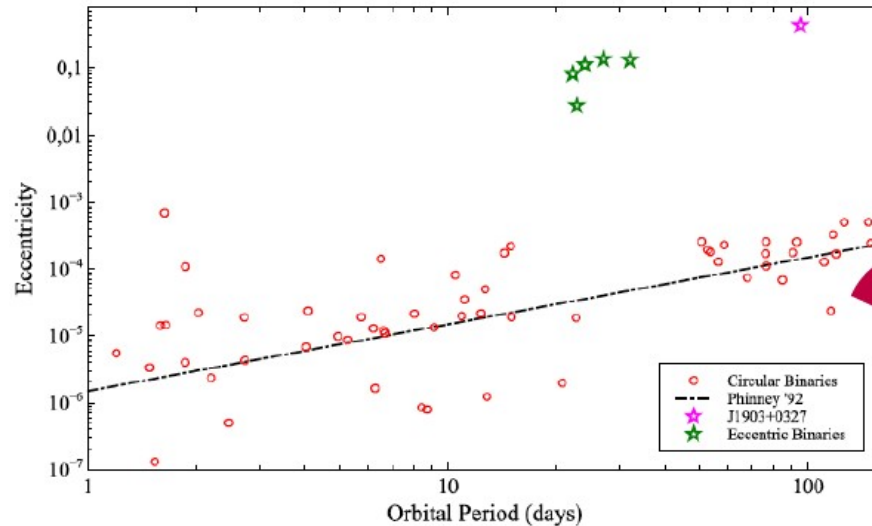
## Observation:

With a strong PT (mass twins), a sudden transition NS  $\rightarrow$  HS is possible, Triggered by accretion, under simultaneous conservation of  $M_b$  and  $J$



# Let us discover the 3<sup>rd</sup> family of compact stars!

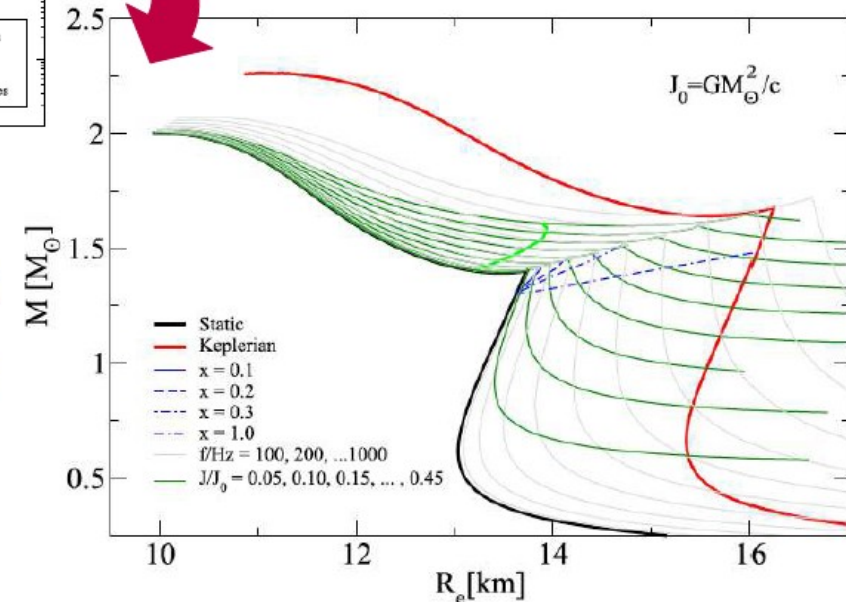
## Antoniadis-puzzle



J. Antoniadis, ApJ Lett. 797, L24 (2014)

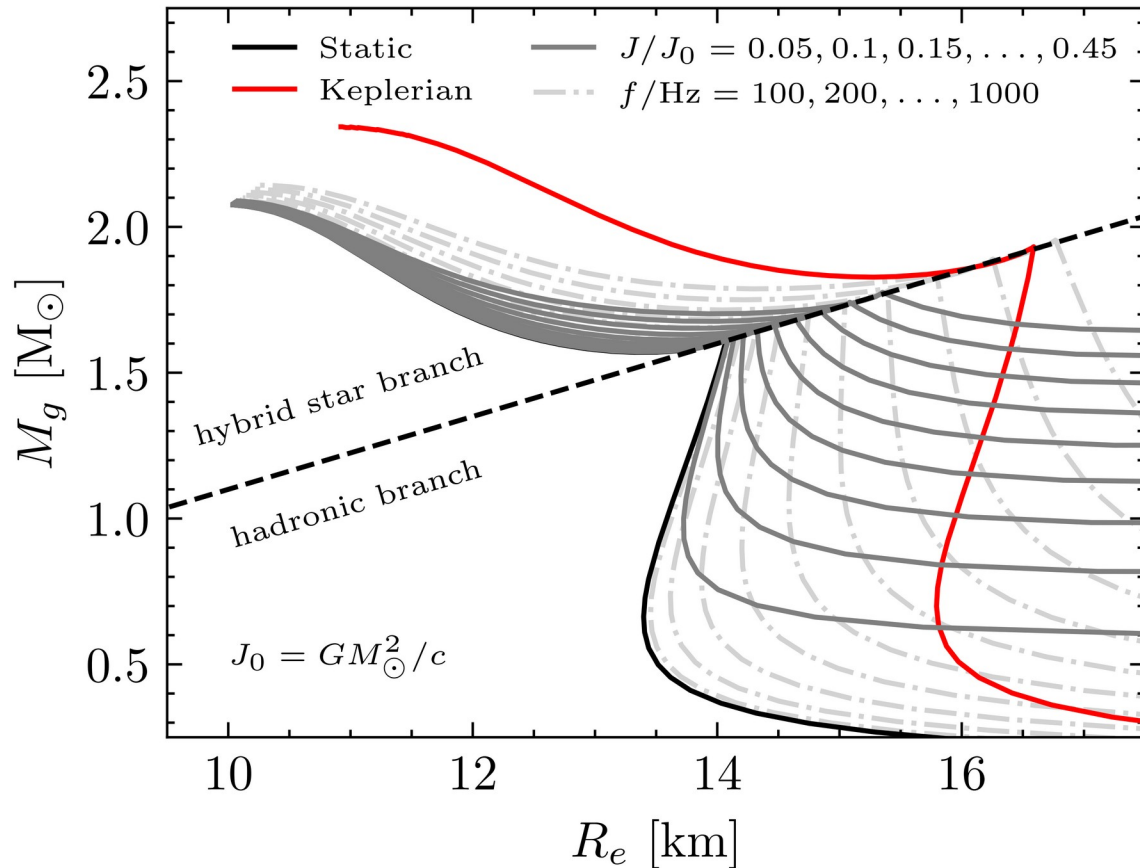
K. Stovall, P.C.C. Freire, J. Antoniadis,  
ApJ 870(2), 74 (2019)

How to relate this ?



D.E. Alvarez-Castillo, J. Antoniadis, A. Ayriyan,  
D. Blaschke, V. Danchev, H. Grigorian, N. Khosravi  
Largani, F. Weber,  
*Accretion-induced collapse to third family compact  
stars as trigger for eccentric orbits of  
Millisecond pulsars in binaries,*  
Astron. Nachr. 340 (2019) 878;  
arXiv:1912.08782 [astro-ph.HE]

# Mass Defect



- Multi-polytrope EoS [ACB-5]

$$P(n) = \kappa_i \left( \frac{n}{n_0} \right)^{\Gamma_i}$$

- Seidov criterion for instability:

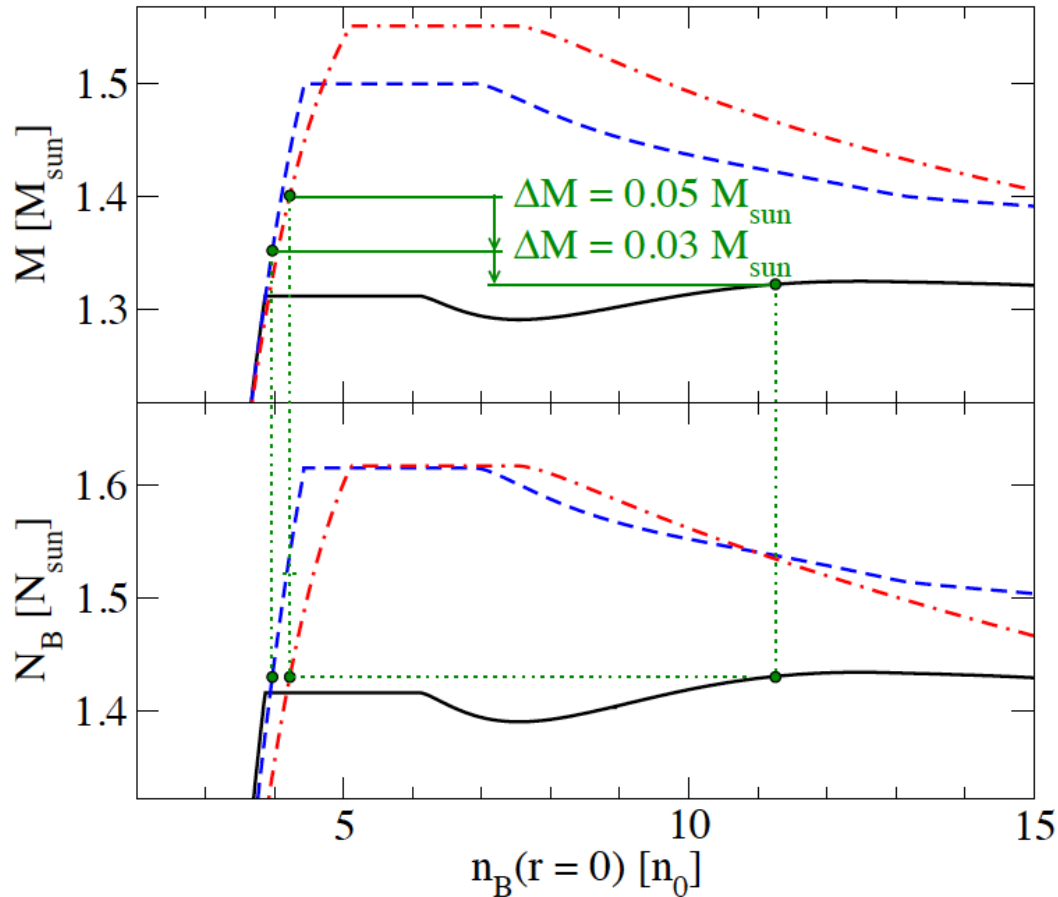
$$\Delta \varepsilon > \frac{\varepsilon_c + 3P_c}{2}$$

- Large jump in energy density (latent heat)  $\rightarrow$  instability  $\rightarrow$  3rd family of compact stars = mass twin stars

D.E. Alvarez Castillo, D. Blaschke, Phys. Rev. C 96 (2017) 045809 „High-mass twin stars with a multipolytrope equation of state“

V. Paschalidis, K. Yagi, D. Alvarez-Castillo, D. B., A. Sedrakian, Phys. Rev. D 97 (2018) 084038, „Implications from GW170817 and I-Love-Q relations for relativistic hybrid stars“

# Mass Defect

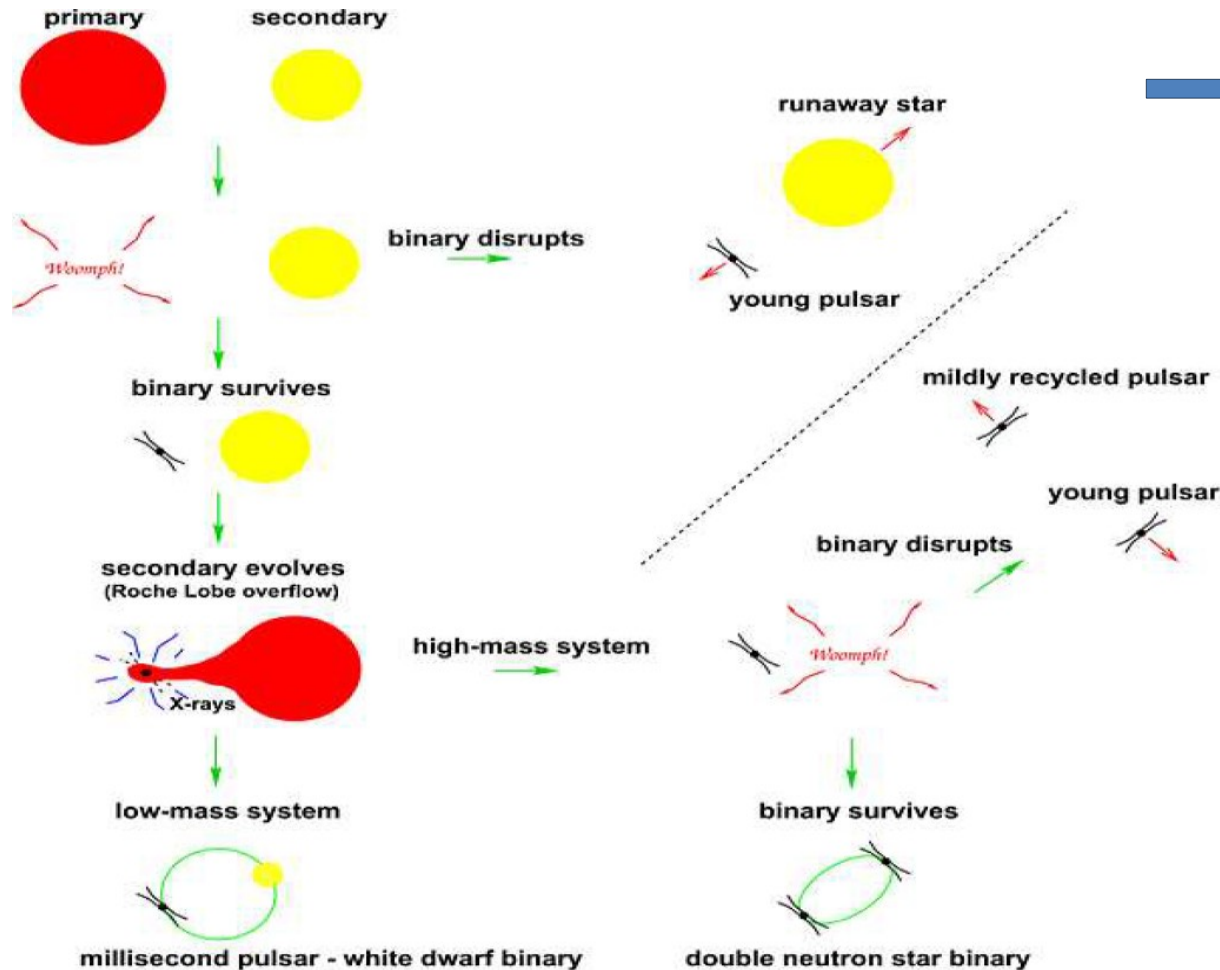


How to calculate the mass defect?

- Solve TOV equations for  $M(n_B(0))$  and  $N_B(n_B(0))$
- Consider transition at constant baryon mass  $N_B$  and constant angular momentum  $J$
- Compare the gravitational masses after the transition
- Account for neutrino trapping/untrapping by a finite chemical potential

F. Sandin, D. Blaschke, Phys. Rev. D 75 (2007) 125013  
„The quark core of protoneutron stars in the phase diagram of quark matter“

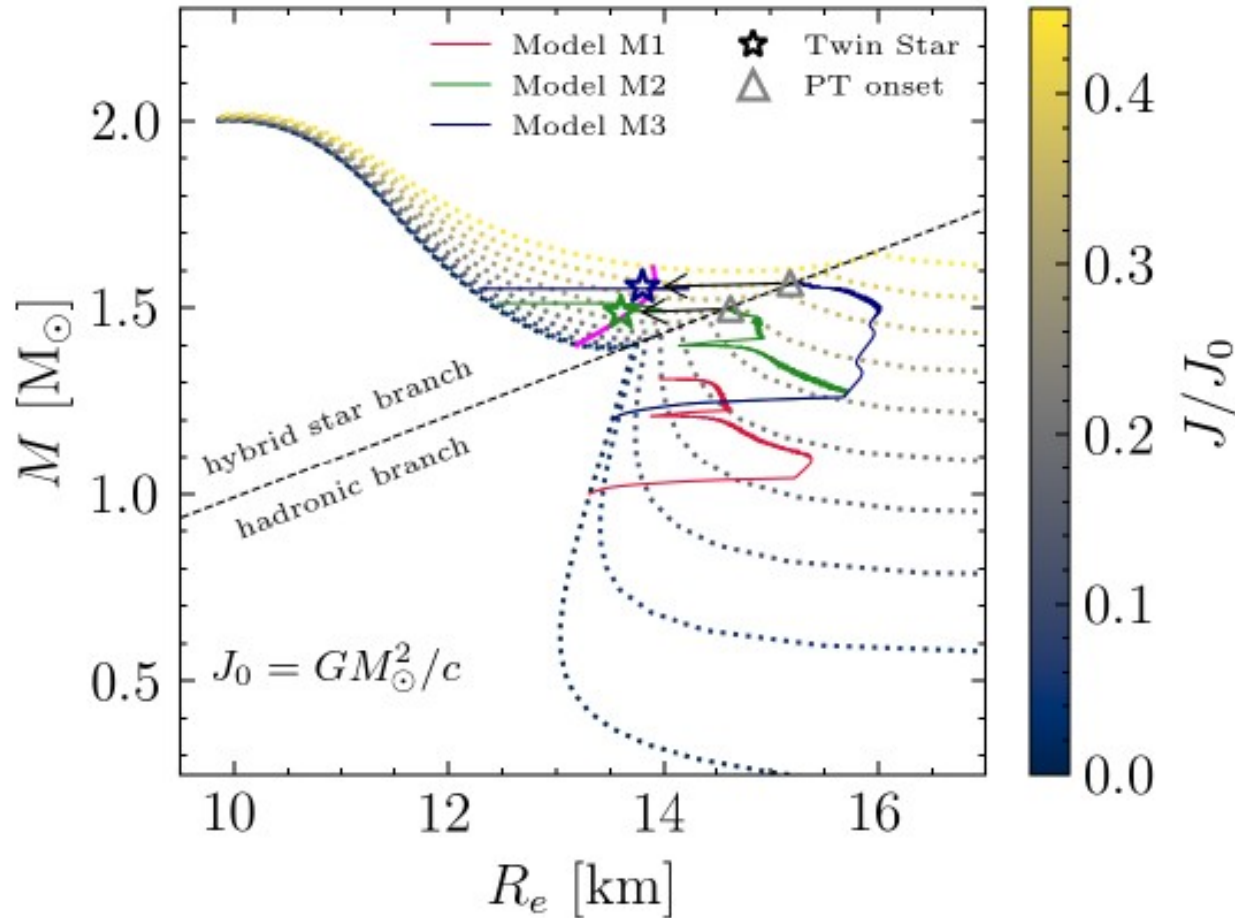
# Another kick explanation: Strong QCD phase transition in NS evolution!



➔ Disruption or anomalous eccentricity of a binary system caused by mass defect (grav. binding) in one NS due to a strong phase transition

See also:  
 “Catastrophic rearrangement of a compact star ...”  
 Mishustin et al.  
 hep-ph/0210422 (2002)

# Simulation of binary evolution



S. Chanlaridis, D. Ohse,  
J. Antoniadis, D. Blaschke,  
D.E. Alvarez-Castillo,  
V. Danchev, D. Misra and  
N. Langer:

„Formation of twin stars in  
low-mass X-ray binaries“

(in preparation for A&A)

# Simulation of binary evolution

$$\frac{a_f}{a_i} = \frac{1 - \Delta M/M}{1 - 2\Delta M/M - (w/v_{\text{rel}})^2 - 2 \cos \theta (w/v_{\text{rel}})},$$

$$v_{\text{rel}} = \sqrt{GM/a_i}$$

$w$  is the magnitude of the kick velocity,  
 $\theta$  is the kick angle

The eccentricity of the post-transition binary system is given by

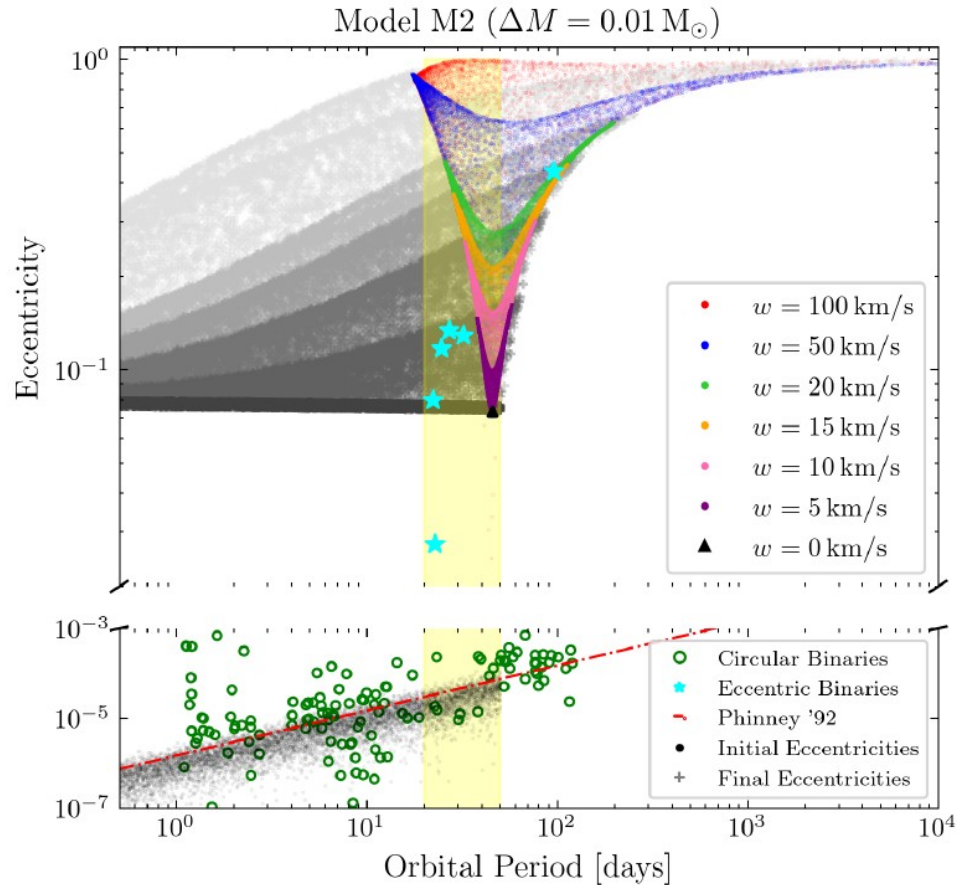
$$e = \sqrt{1 + \frac{2E_{\text{orb},f} L_{\text{orb},f}^2}{\mu_f G^2 M_{f,1}^2 M_{f,2}^2}},$$

$$L_{\text{orb},f} = a_i \mu_f \sqrt{(v_{\text{rel}} + w \cos \theta)^2 + (w \sin \theta \sin \phi)^2}$$

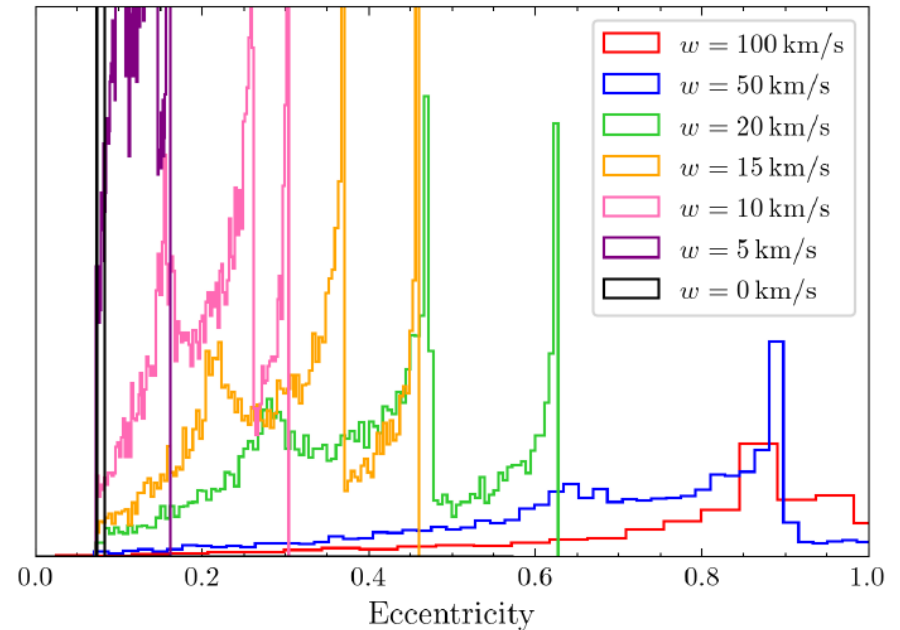
$$E_{\text{orb},f} = -GM_{f,1}M_{f,2}/2a_f$$

Model	$M_{\text{don}} [M_{\odot}]$	$M_{\text{ns}} [M_{\odot}]$	$P_{\text{orb}} [\text{days}]$
m1	1.0	1.0	8
m2	1.0	1.2	8
m3	1.0	1.2	22.627

# Simulation of binary evolution

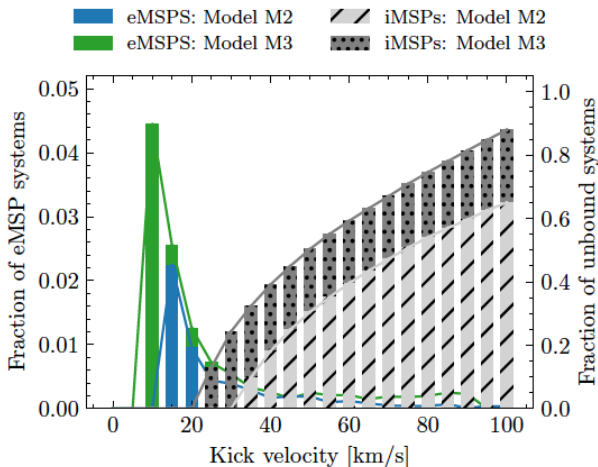
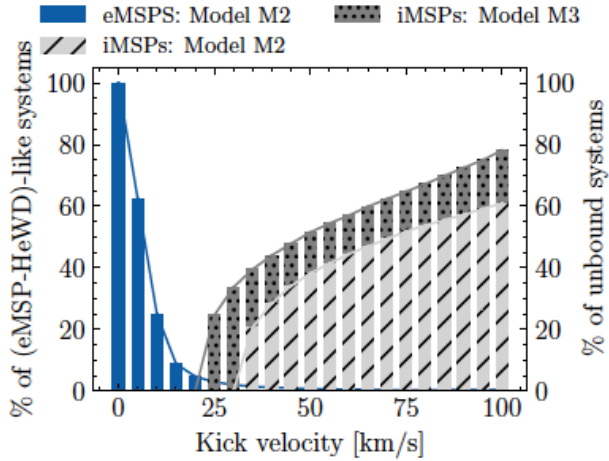


Monte-Carlo simulation of posttransition distribution orbital parameters for a mass defect of 1% solar mass due to the transition.



S. Chanlaridis, D. Ohse, J. Antoniadis, D. Blaschke, D.E. Alvarez-Castillo, V. Danchev, D. Misra and N. Langer:  
 „Formation of twin stars in low-mass X-ray binaries“ (in preparation for A&A)

# Results for T=0 twin signals in LMXB's



- First results are promising:
- Period gap 20 – 50 d, with  $e \sim 10^{-1}$  can be addressed
- One binary with a main sequence star (J1903+0327) with larger period, 100 d, and  $e \sim 1$  can also be met
- Appearance of unbound systems → isolated MSPs

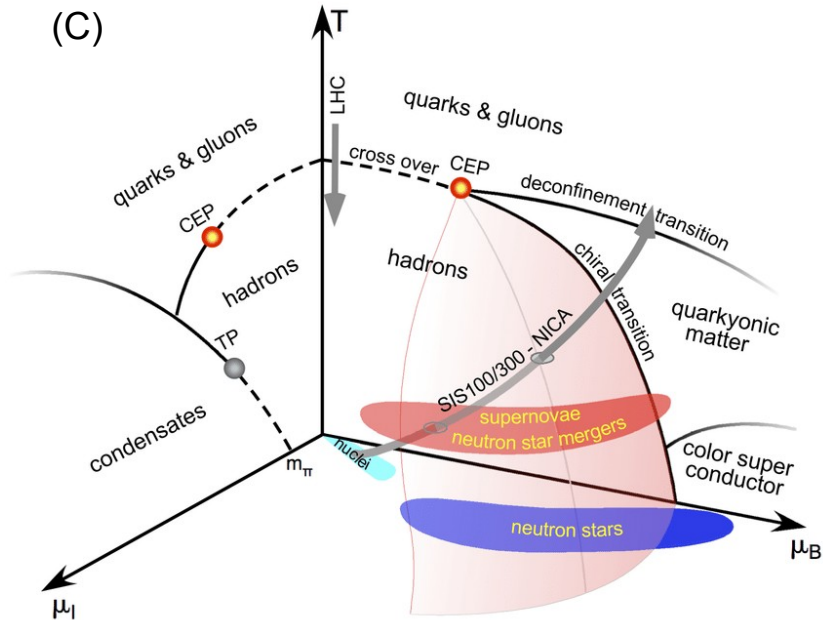
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# Thermal twin stars

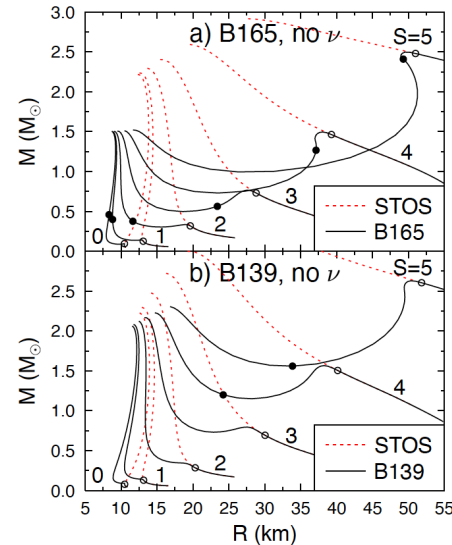
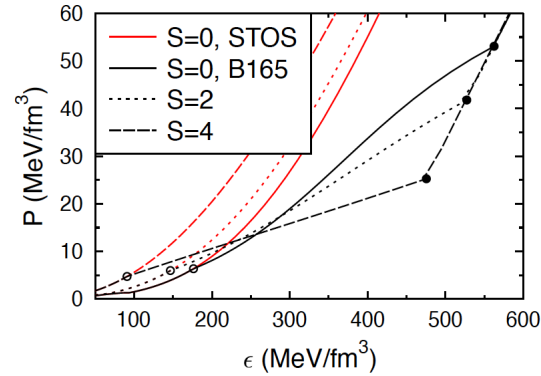
Observation\*:

When the transition at  $T=0$  is weak, with a connected hybrid star branch, At finite  $T$  ( $s/n > 0$ ) a third family (thermal twin stars) can emerge !!

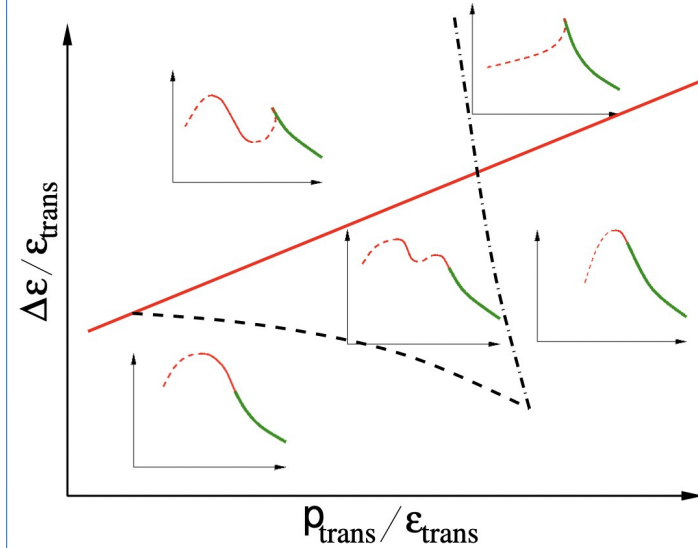
(C)



\*) M. Hempel et al., PRD 94 (2016) 103001



Disconnected HS branch indicates a Strong (1st-order) phase transition

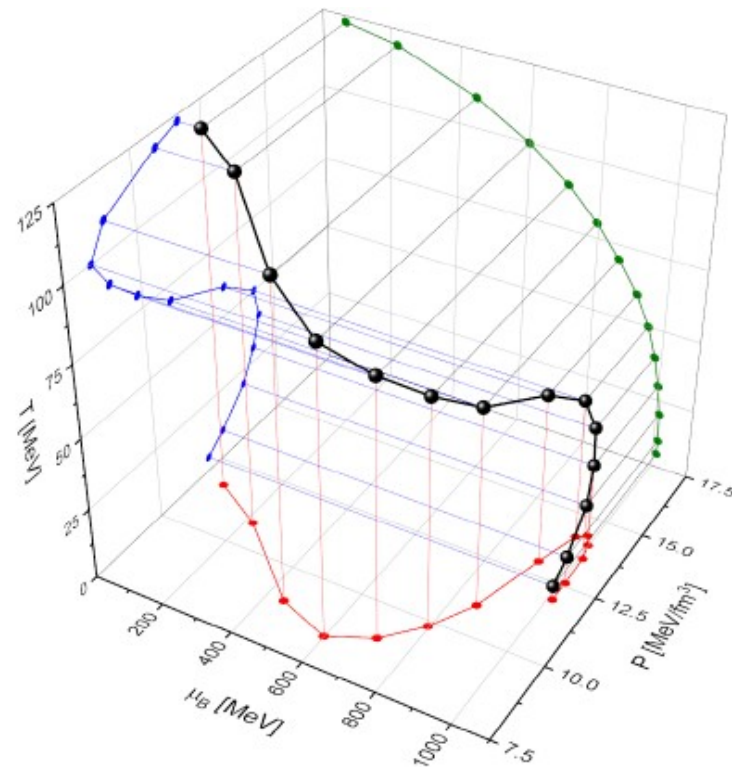
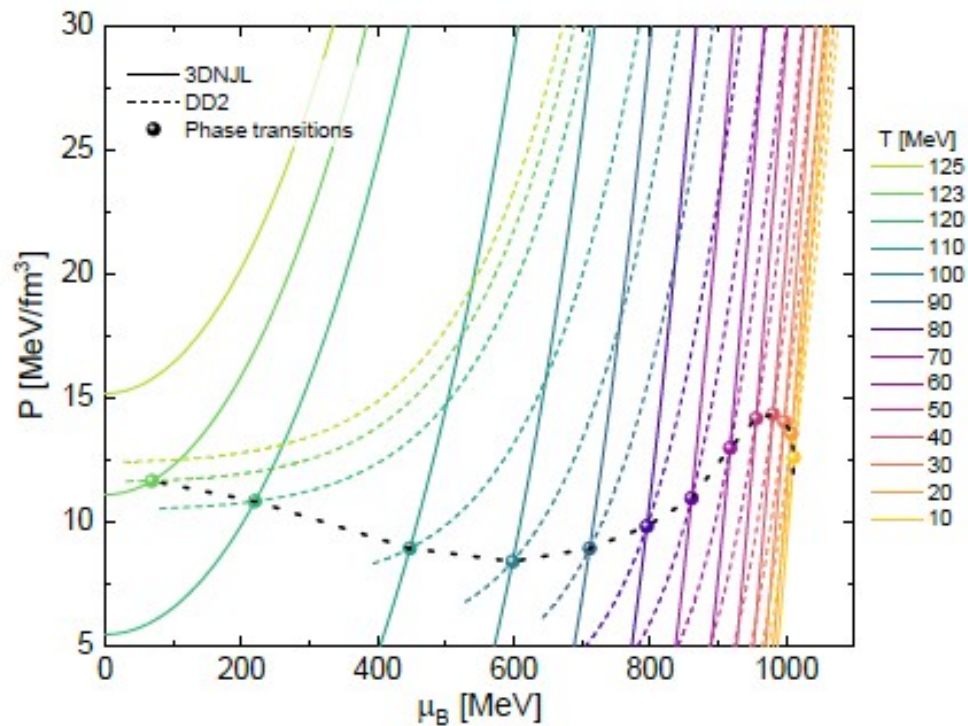


Seidov criterion for gravitational Instability (red line):

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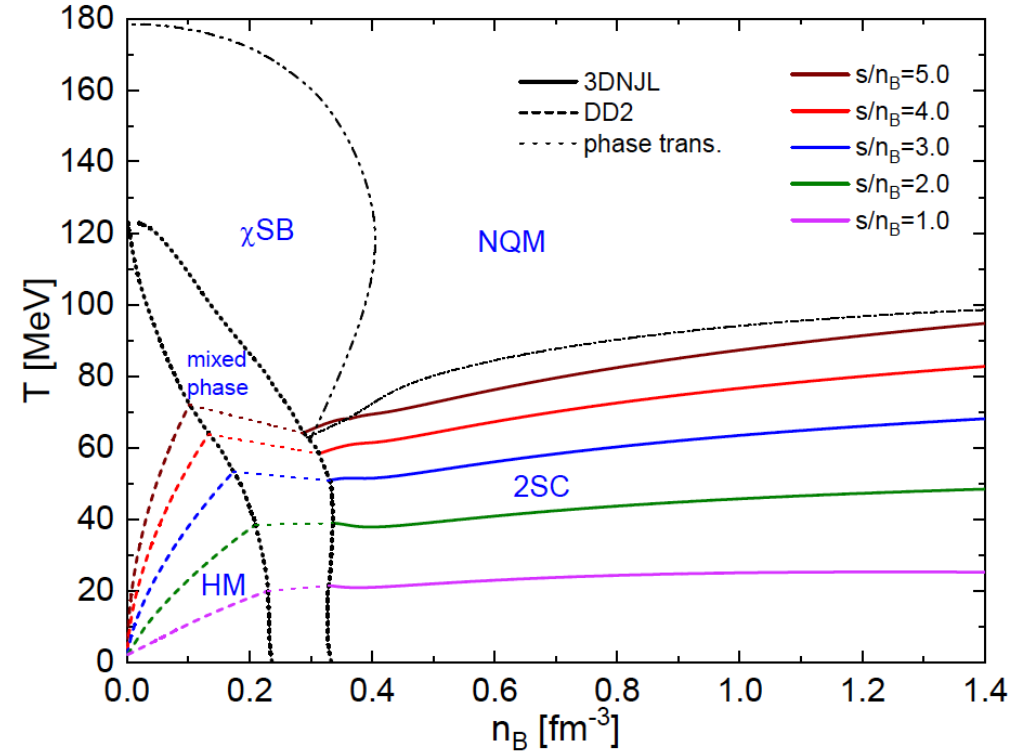
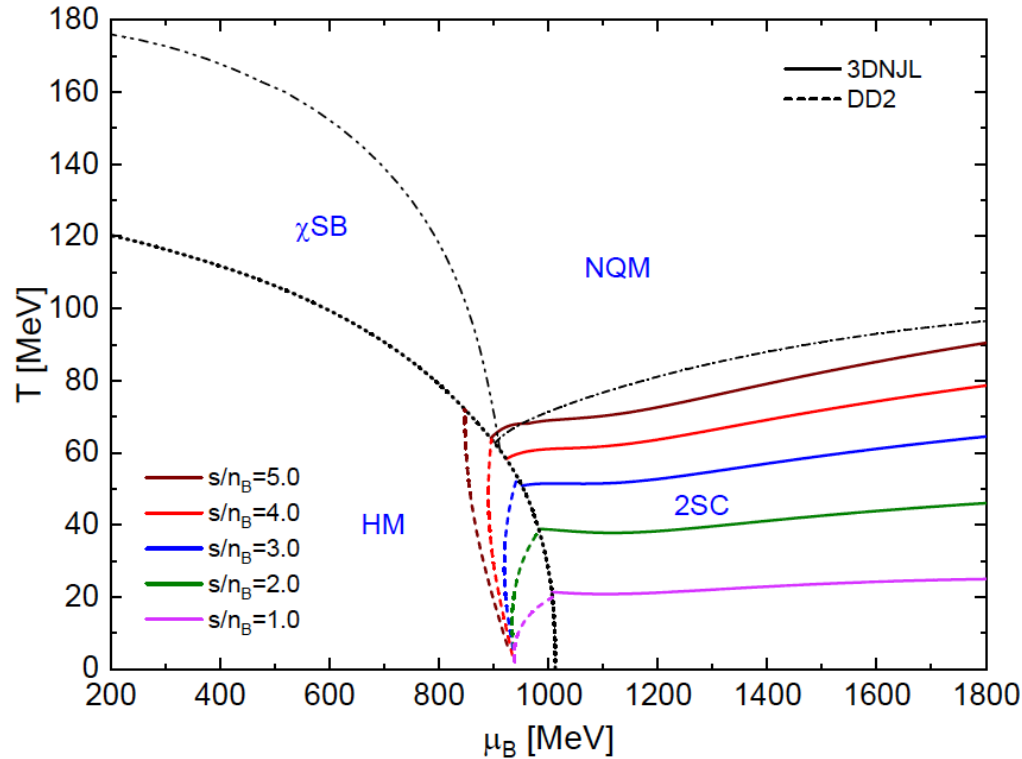
Investigation within a color superconducting, nonlocal chiral quark model\*:



\*) G. Contrera, D.B., J.P. Carlomagno, A.G. Grunfeld, PRC 105 (2022) 045808  
J.P. Carlomagno, G. Contrera, A.G. Grunfeld, D.B., PRD 109 (2024) 043050; arXiv:2406.17193

# Thermal twin stars

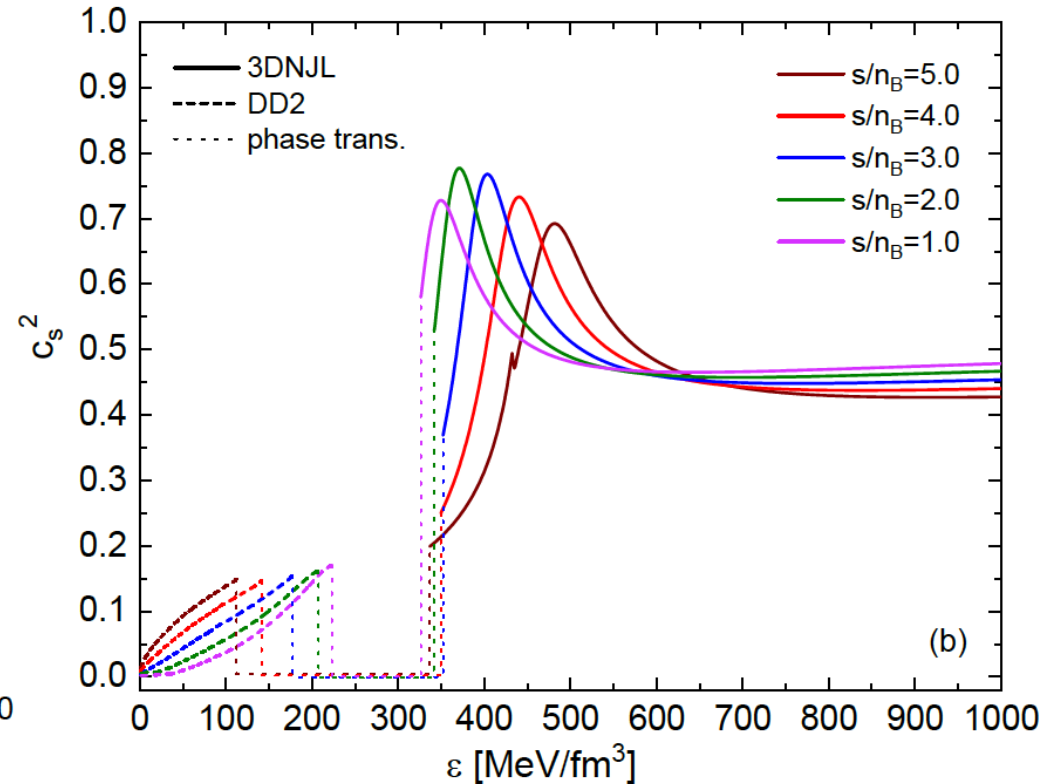
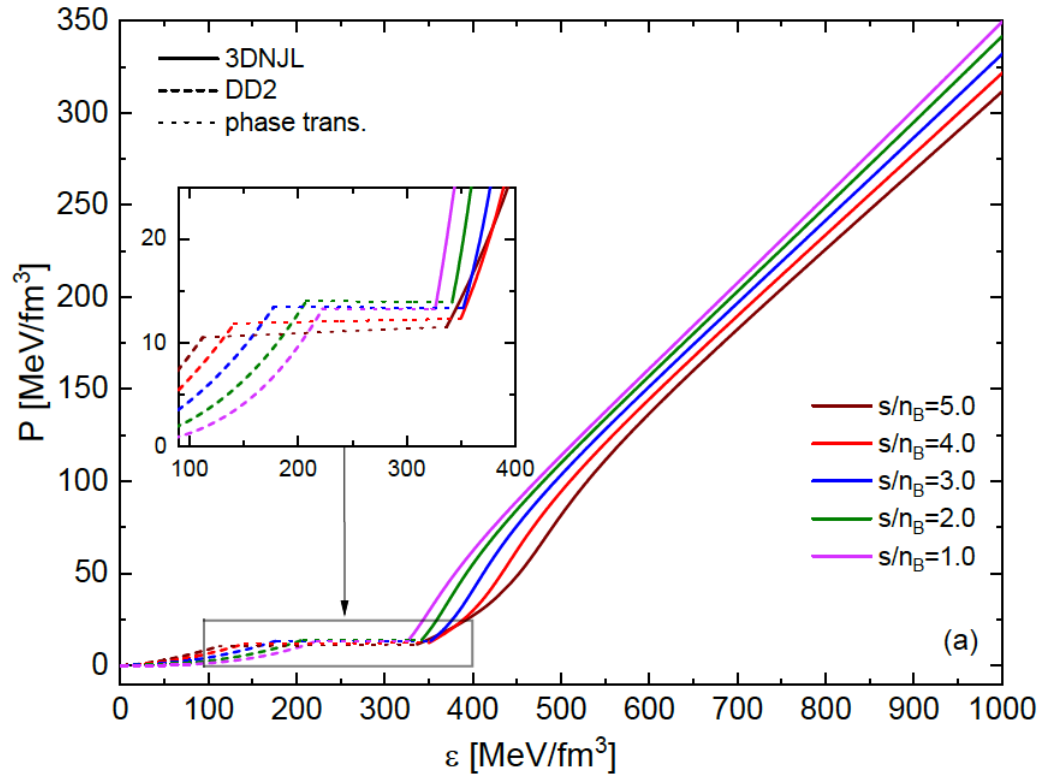
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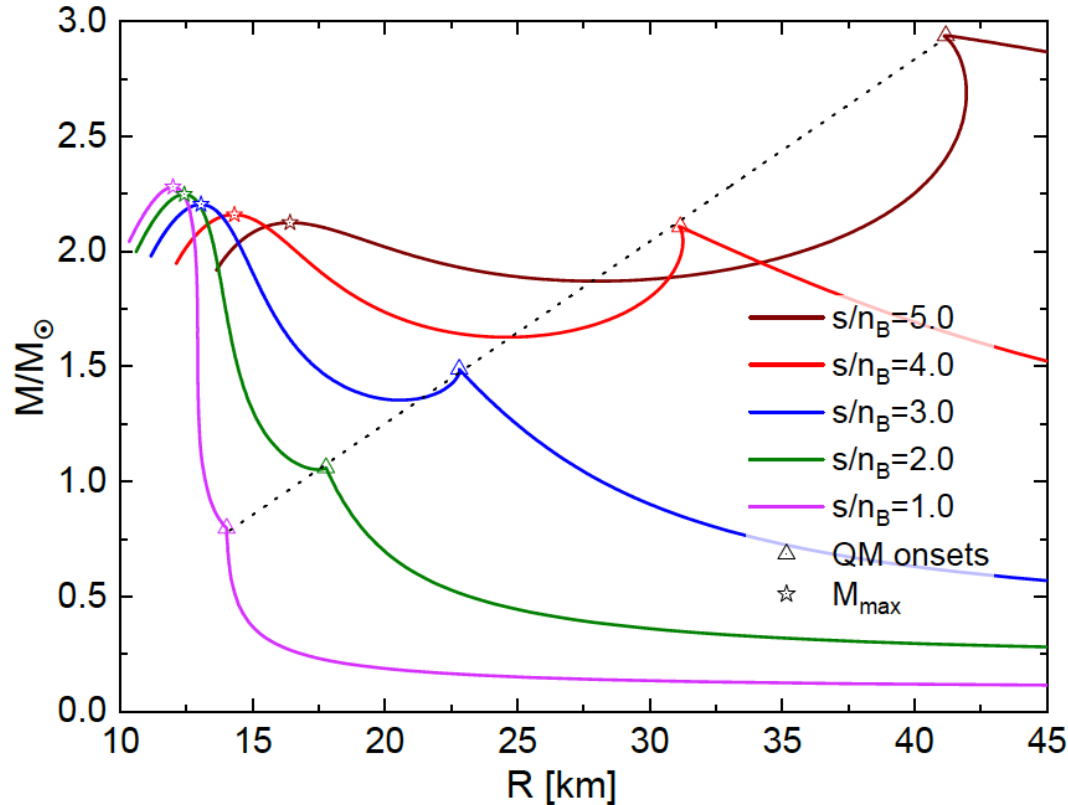
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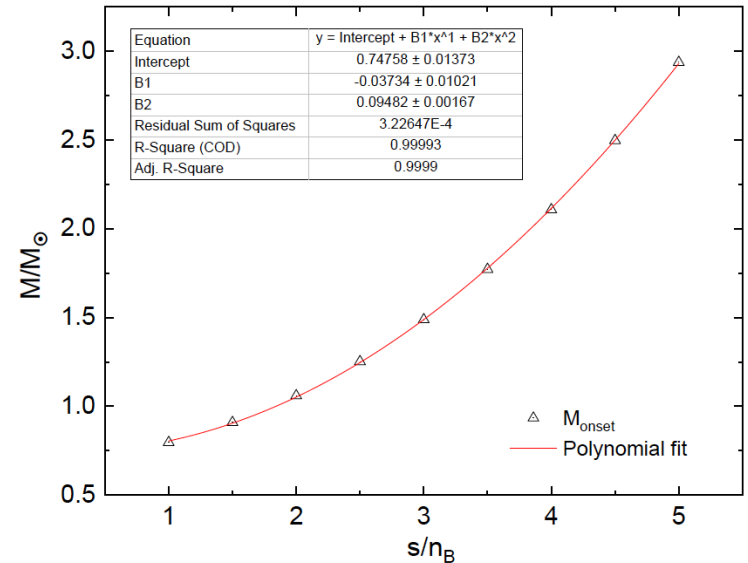
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# Thermal twin stars

Investigation within a color superconducting, nonlocal chiral quark model\*:



Systematics for the onset:  $M = C(R - R_0)$ ,  
 where  $R_0 = 4.20 \pm 0.35$  km is the radius offset  
 and  $C = dM_{\text{onset}}/dR = 0.0792 \pm 0.0009 M_{\odot}/\text{km}$   
 is the critical compactness of the star



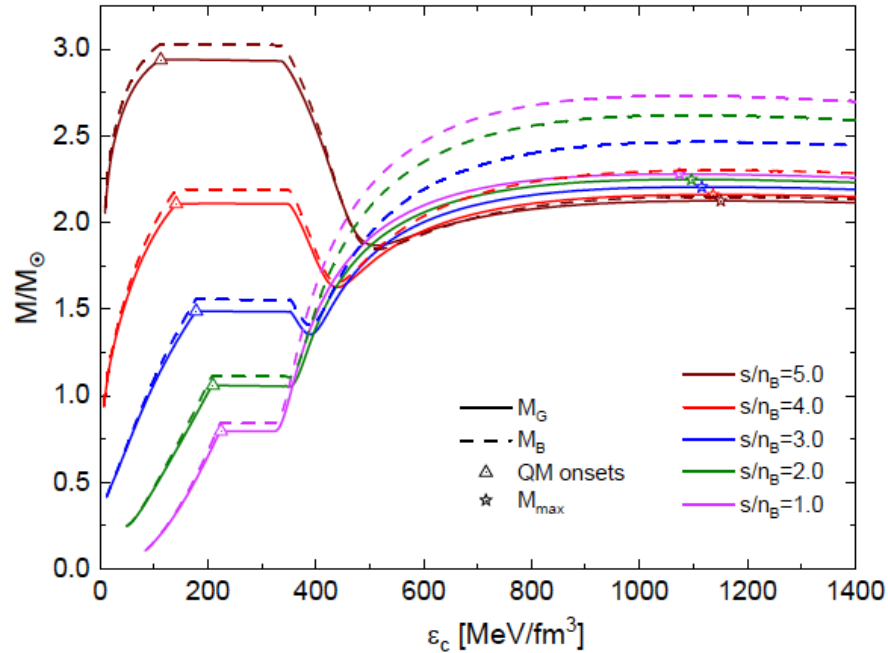
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 J.P. Carlomagno, G. Contrera, A.G. Grunfeld, D.B., PRD 109 (2024) 043050; arXiv:2406.17193

$$\frac{M_{\text{onset}}}{M_{\odot}} = 0.747 - 0.0373 \frac{s}{n_B} + 0.0948 \left( \frac{s}{n_B} \right)^2$$

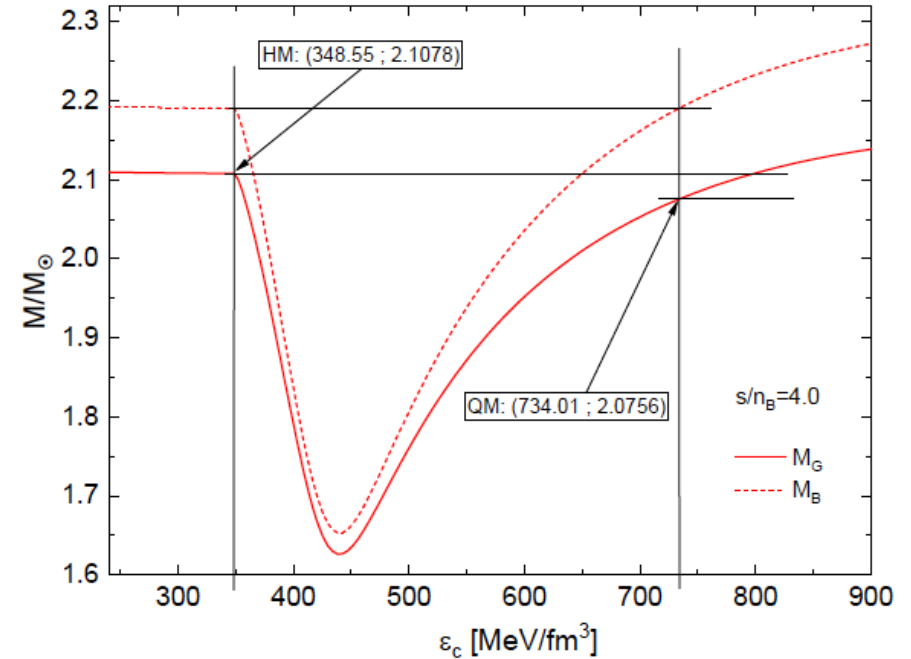
# Thermal twin stars

Investigation within a color superconducting, nonlocal chiral quark model\*:

Gravitational mass (solid) and baryon mass (dashed)



Extraction of the mass defect (energy release)

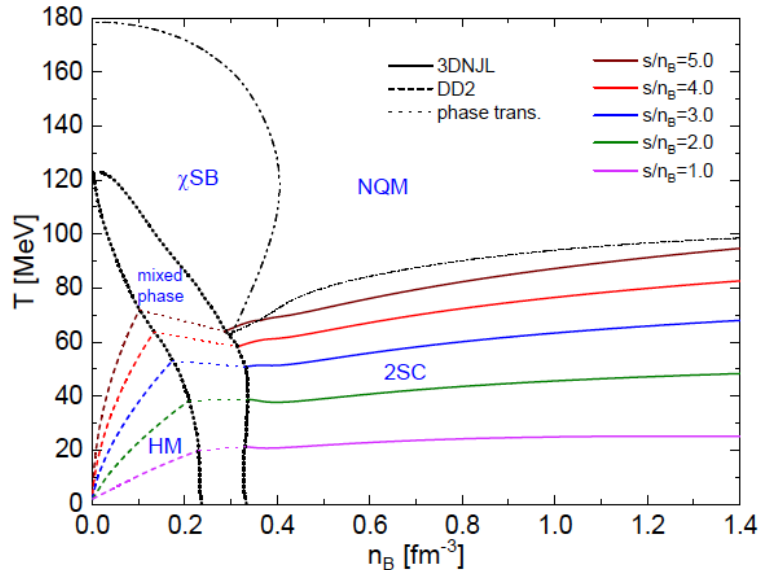


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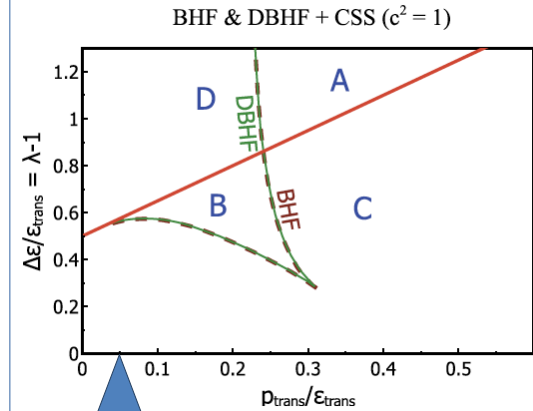
Characterization of the accretion-induced transition to the thermal twin star\*

$s/n_B$	$M_{B,tr}$	$M_{G,HM}$	$M_{G,QM}$	$\Delta M$	$T(0)_{HM}$	$T(0)_{QM}$	$\Delta T(0)$	character
2.0	1.114	1.0582	1.0582	0.0000	38.36	38.87	0.51	enthalpic
2.5	1.311	1.2490	1.2487	0.0003	46.33	45.57	-0.76	entropic
3.0	1.557	1.4878	1.4853	0.0025	53.16	50.84	-2.32	entropic
3.5	1.849	1.7726	1.7621	0.0105	58.89	55.01	-3.88	entropic
4.0	2.190	2.1078	2.0756	0.0322	63.74	58.28	-5.46	entropic

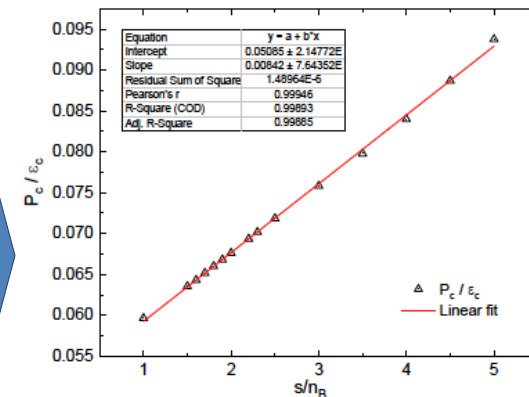


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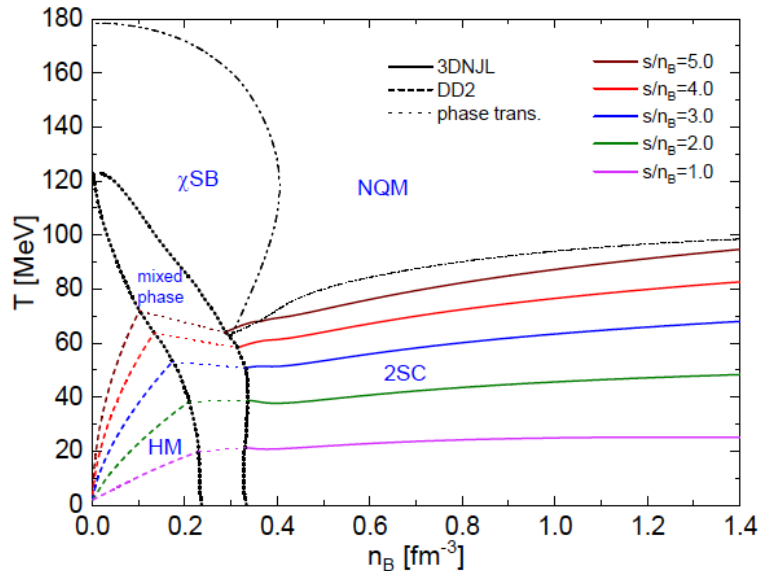
Alford & Han,  
Eur. Phys. J. A  
52, 62 (2016)



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Characterization of the accretion-induced transition to the thermal twin star\*

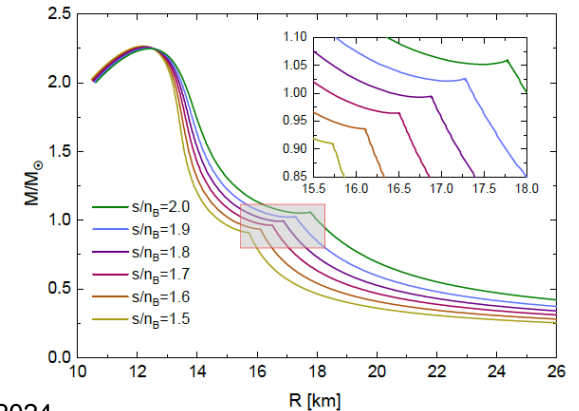
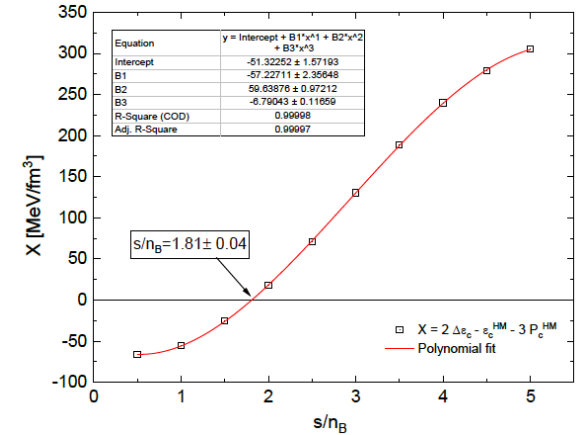
$s/n_B$	$R(M_{HYB}^{max})$	$M_{BJr}$	$M_{G,HM}$	$M_{G,QM}$	$\Delta M$	$T(0)_{HM}$	$T(0)_{QM}$	$\Delta T(0)$	character
2.0	12.41	1.114	1.0582	1.0582	0.0000	38.36	38.87	0.51	enthalpic
2.5	12.61	1.311	1.2490	1.2487	0.0003	46.33	45.57	-0.76	entropic
3.0	13.06	1.557	1.4878	1.4853	0.0025	53.16	50.84	-2.32	entropic
3.5	13.58	1.849	1.7726	1.7621	0.0105	58.89	55.01	-3.88	entropic
4.0	14.29	2.190	2.1078	2.0756	0.0322	63.74	58.28	-5.46	entropic



\*) J.P. Carlomagno, G. Contrera, A.G. Grunfeld, D.B., arXiv:2406.17193

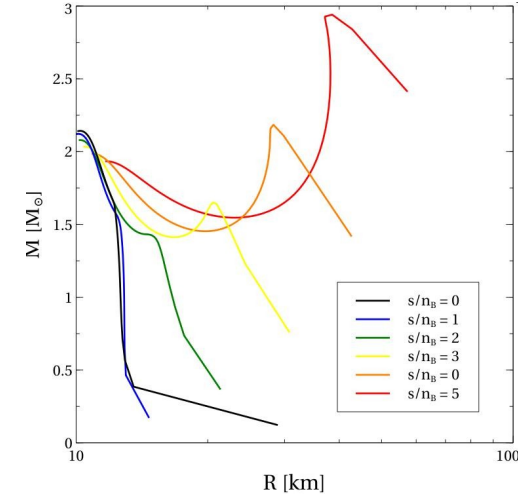
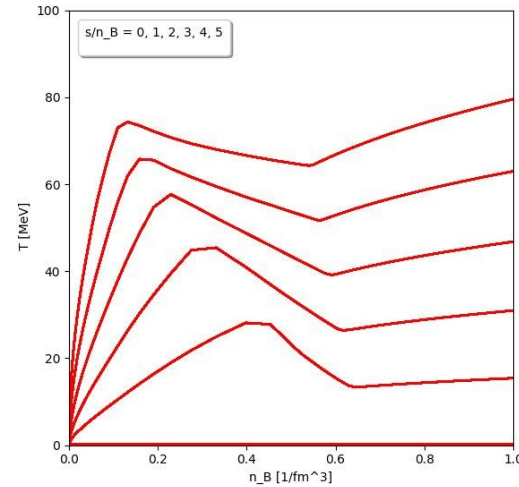
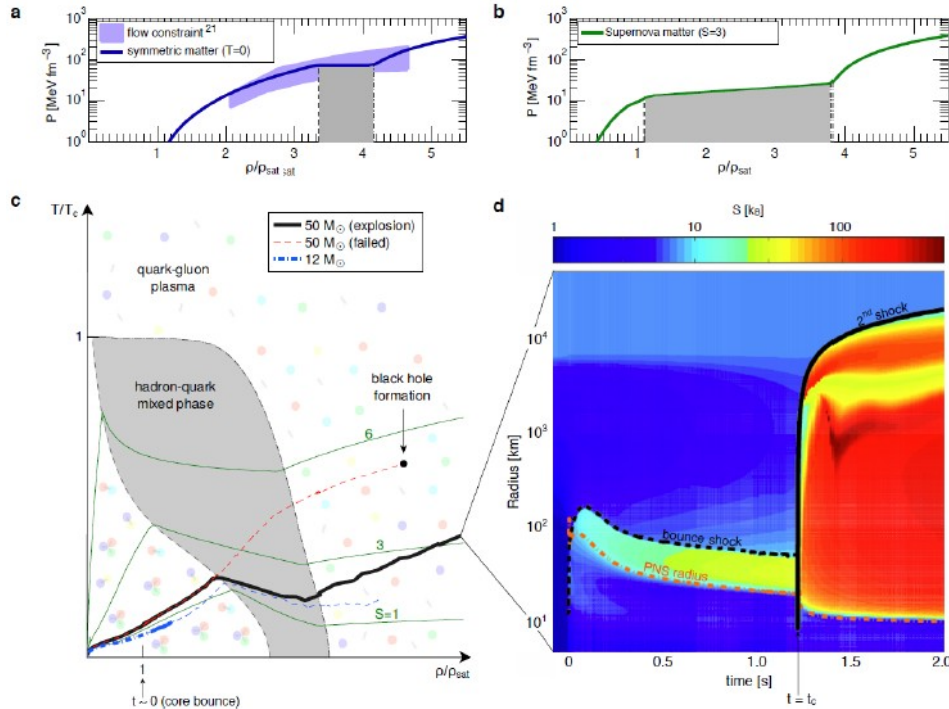
Seidov criterion for instability:

$$X = 2\Delta\epsilon - \epsilon_c^{HM} - 3P_c^{HM} > 0$$



# Thermal twin stars – Indicators of CCSN explodability ?

Successful application of hybrid EoS with entropic first-order transition (thermal twin stars) as an explosion mechanism for massive blue supergiant stars\*:



Courtesy: Oleksii Ivanytskyi

Progenitor:  
 $M = 50 M_{\odot}$

**Increase of  $s/n_B$  lowers the onset density for deconfinement:**

→ supports the fulfillment of the Seidov criterion (mass twins)

→ deconfinement is reached faster (explodability)

\*) T. Fischer et al., Nature Astron. 2, 980 (2018)

# Confining Density Functional (CDF) & CSC Quark Matter

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**Lagrangian**  $\mathcal{L} = \bar{q}(i\not{\partial} - \hat{m})q - \mathcal{U} + \mathcal{L}_V + \mathcal{L}_I + \mathcal{L}_D$

- **Scalar & pseudoscalar interaction channels**

$$\mathcal{U} = G_0 \left[ (1 + \alpha) \langle \bar{q}q \rangle_0^2 - (\bar{q}q)^2 - (\bar{q}i\vec{\tau}\gamma_5 q)^2 \right]^{\frac{1}{3}}$$

(motivated by String Flip Model,  $\chi$ -dynamics, quark "confinement")

- **Vector-isoscalar interaction channel**

$$\mathcal{L}_V = -G_V (\bar{q}\gamma_\mu q)^2$$

(motivated by gluon exchange, stiff EoS needed to reach  $2M_\odot$ )

- **Vector-isovector interaction channel**

$$\mathcal{L}_I = -G_I (\bar{q}\gamma_\mu \vec{\tau} q)^2$$

(motivated by gluon exchange, isospin sensitive interaction)

- **Diquark interaction channel**

$$\mathcal{L}_D = G_D \sum_{A=2,5,7} (\bar{q}i\gamma_5\tau_2\lambda_A q^c)(\bar{q}^c i\gamma_5\tau_2\lambda_A q)$$

(motivated by Cooper theorem, color superconductivity)

# Confining Density Functional (CDF) & CSC Quark Matter

## What is new?

O. Ivanytskyi & D.B., Phys. Rev. D 105 (2022) 114042

**Interaction**  $\mathcal{U} = D_0 [(1 + \alpha)\langle\bar{q}q\rangle_0^2 - (\bar{q}q)^2 - (\bar{q}i\vec{\tau}\gamma_5 q)^2]^\varkappa$



- **Parameters**

$D_0$  - dimensionfull coupling, controls interaction strength

$\alpha$  - dimensionless constant, controls vacuum quark mass

$\langle\bar{q}q\rangle_0$  -  $\chi$ -condensate in vacuum (introduced for the sake of convenience)

$$\varkappa = 1/3$$



motivated by String Flip model

$$\mathcal{U}_{SFM} \propto \langle q^+ q \rangle^{2/3}$$

$$\Sigma_{SFM} = \frac{\partial \mathcal{U}_{SFM}}{\partial \langle q^+ q \rangle} \propto \langle q^+ q \rangle^{-1/3} \propto \text{separation}$$

$$\varkappa = 1$$



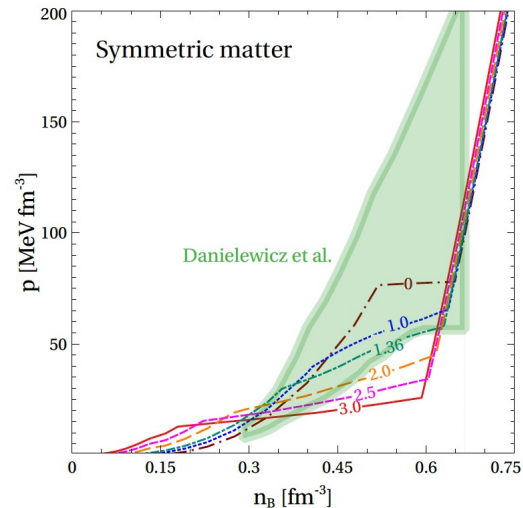
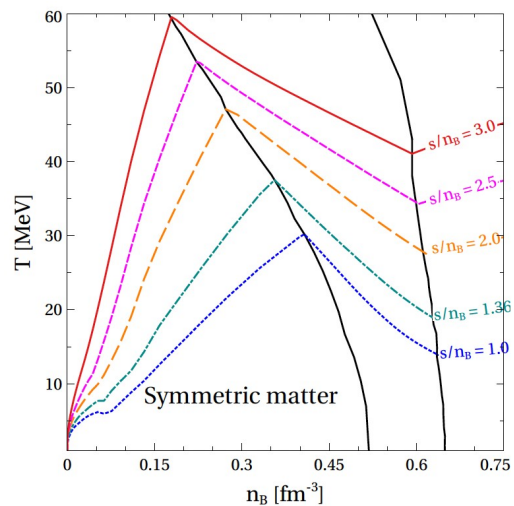
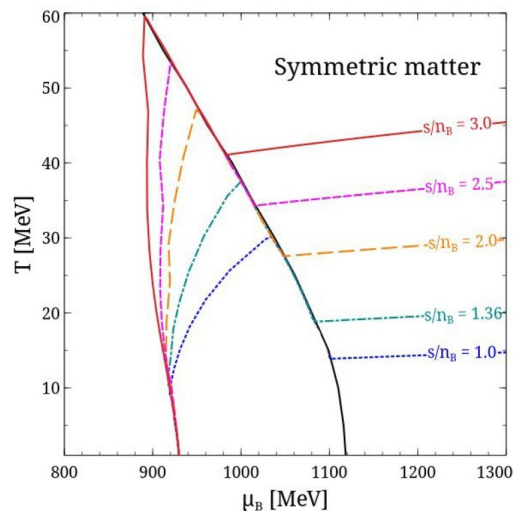
Nambu–Jona-Lasinio model

- **Dimensionality**

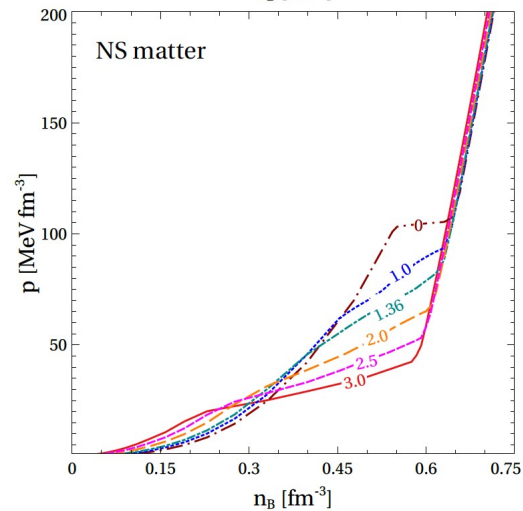
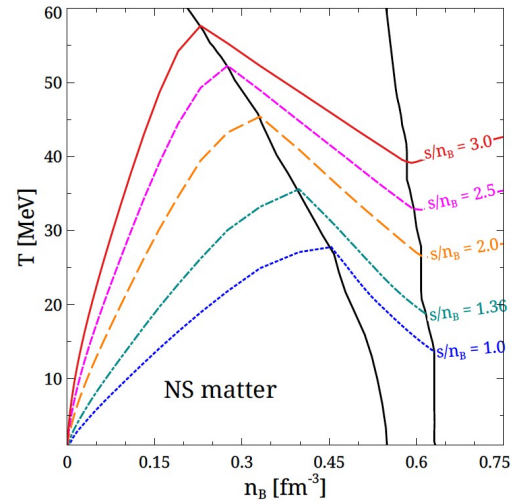
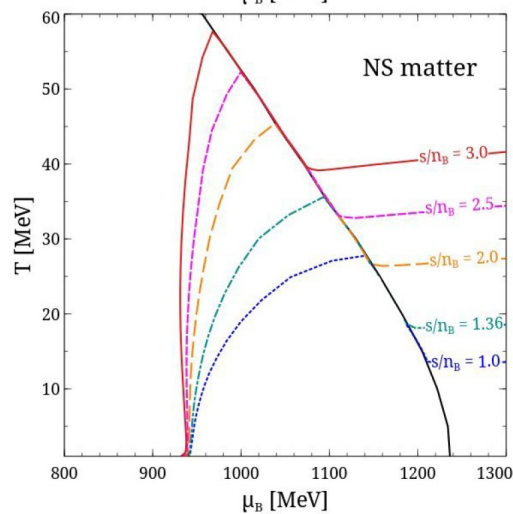
$$\begin{aligned} [U] &= \text{energy}^4 \\ [\bar{q}q] &= \text{energy}^3 \end{aligned} \Rightarrow [D_0]_{\varkappa=1/3} = \text{energy}^2 = [\text{string tension}]$$

self energy = string tension  $\times$  separation  $\Rightarrow$  **confinement**

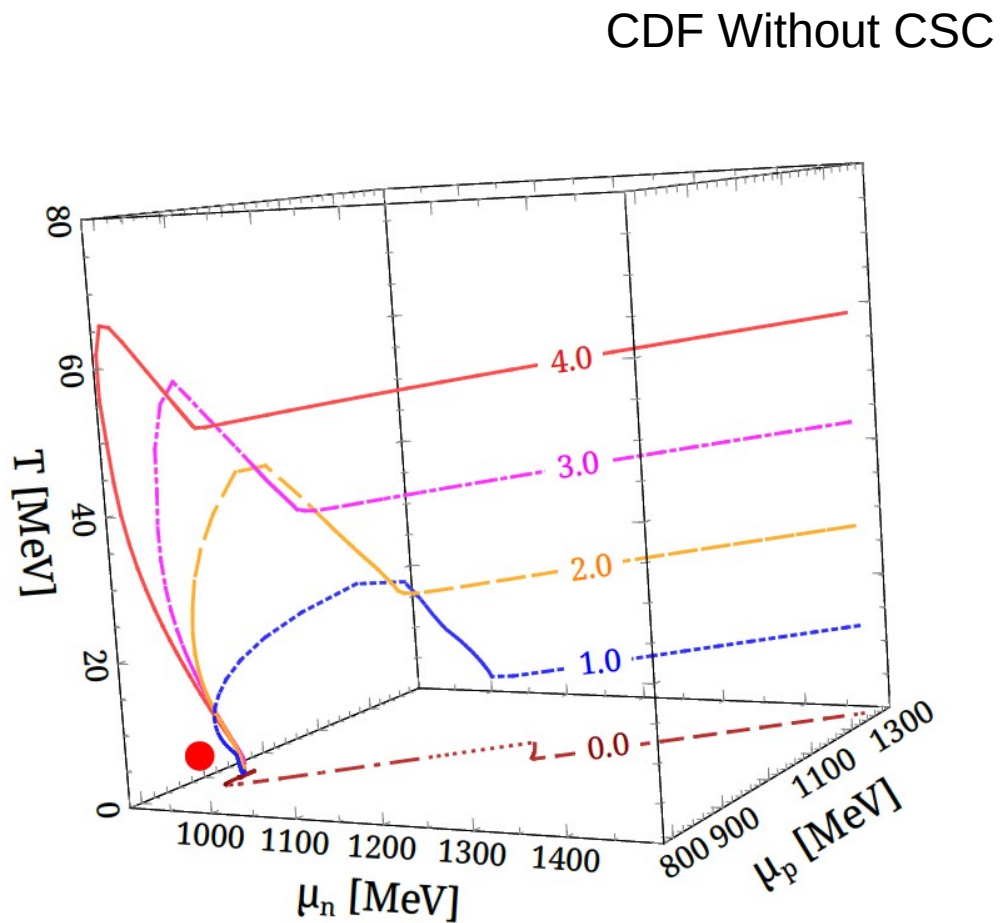
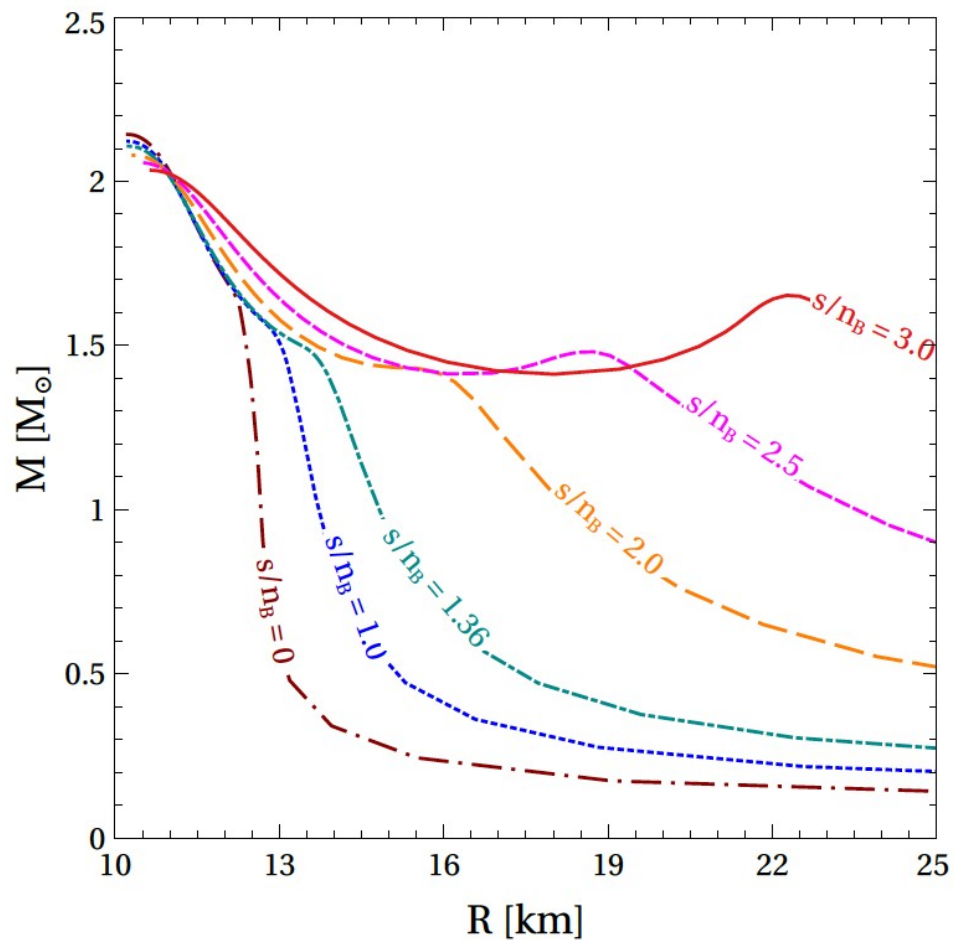
# Confining Density Functional (CDF) & CSC Quark Matter



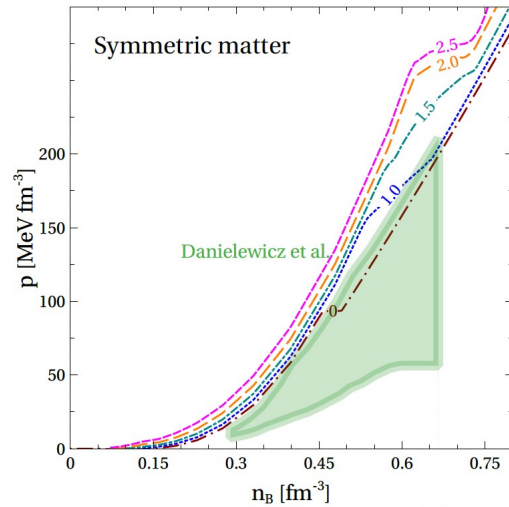
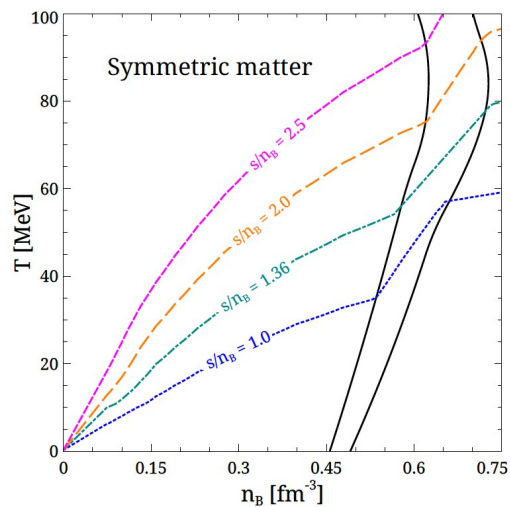
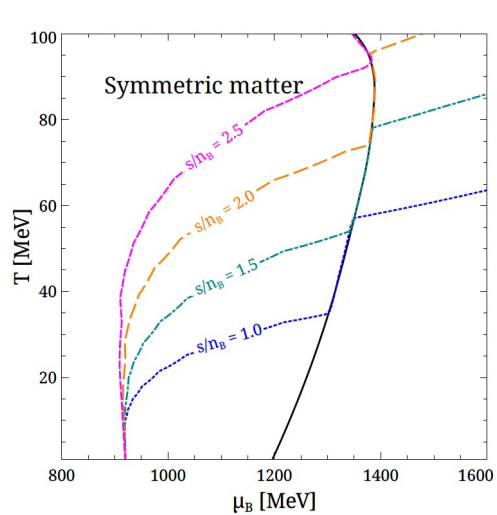
CDF Without CSC



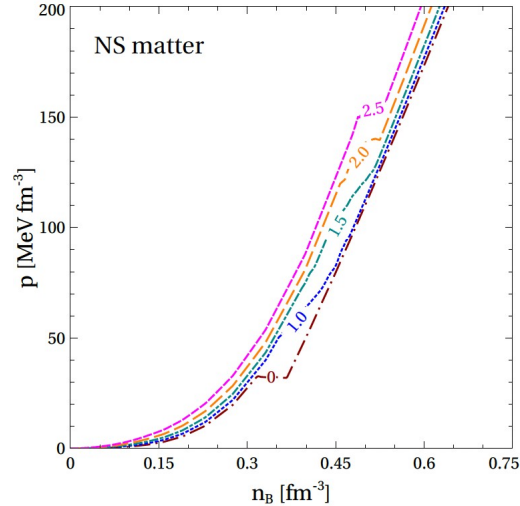
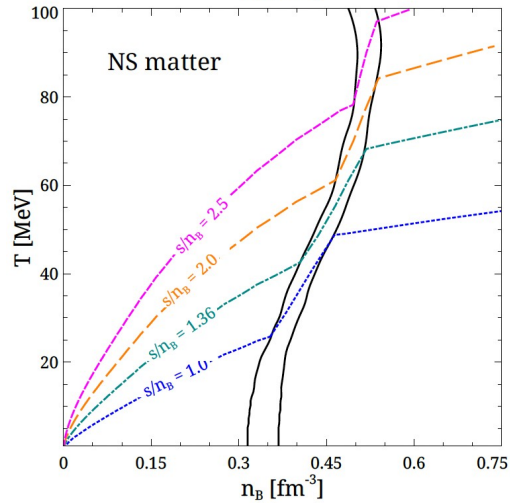
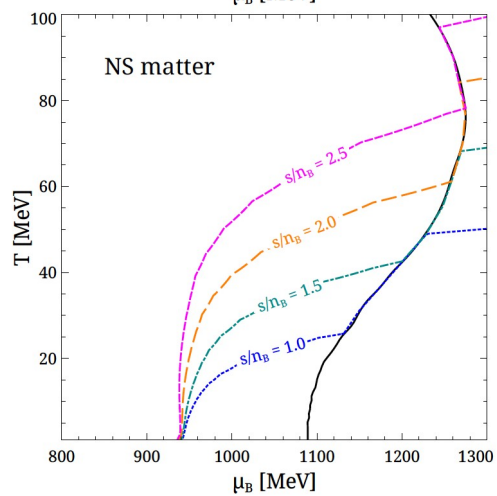
# Confining Density Functional (CDF) & CSC Quark Matter



# Confining Density Functional (CDF) & CSC Quark Matter

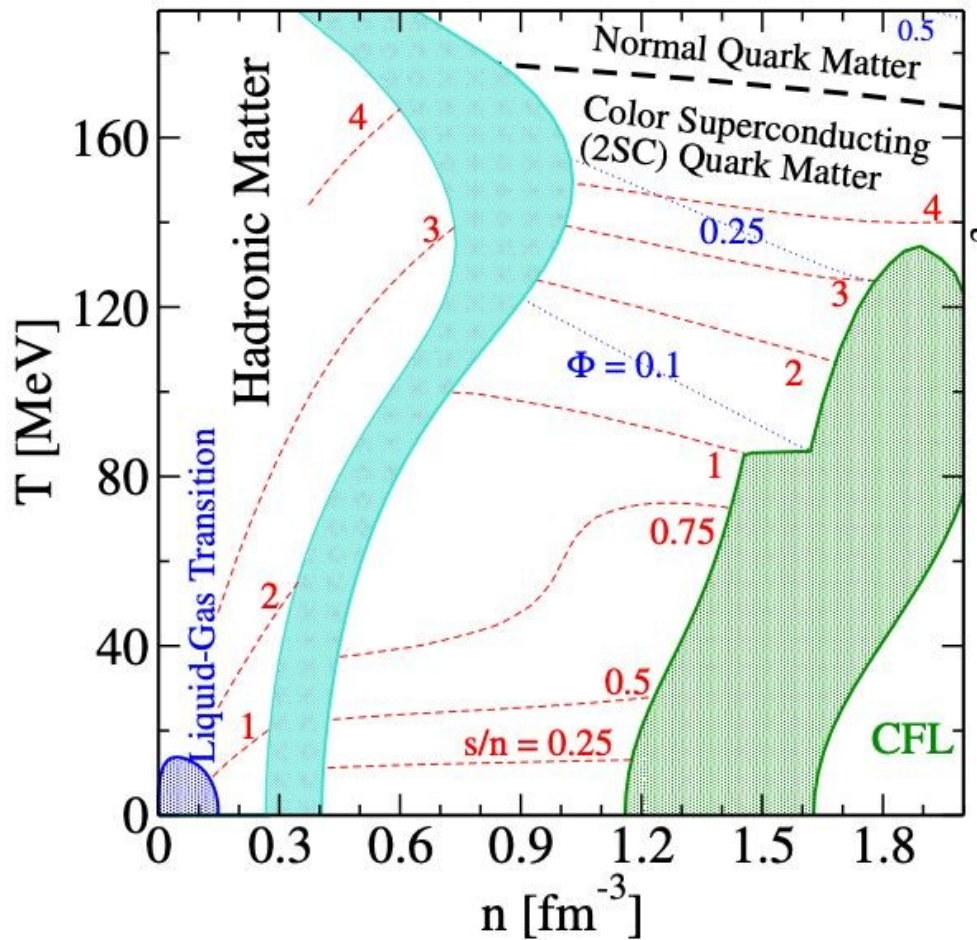
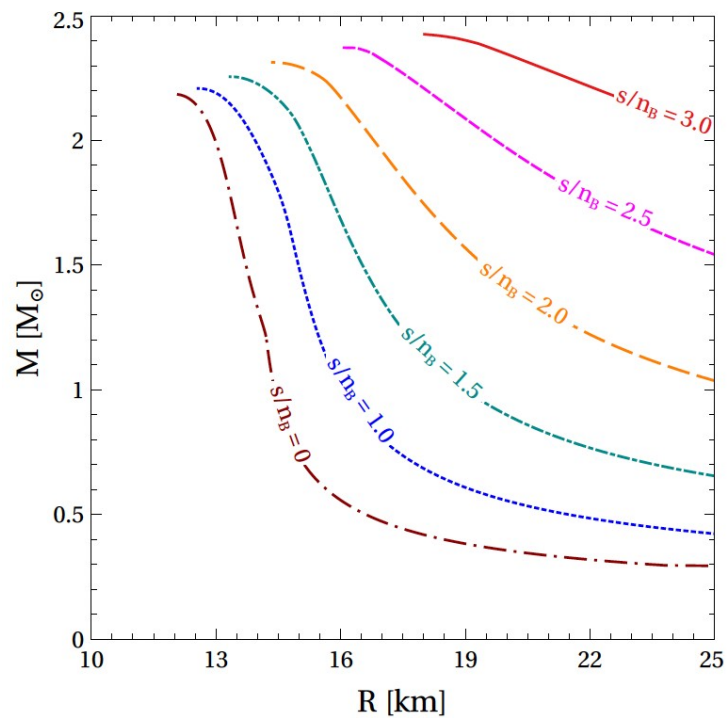


CDF With CSC

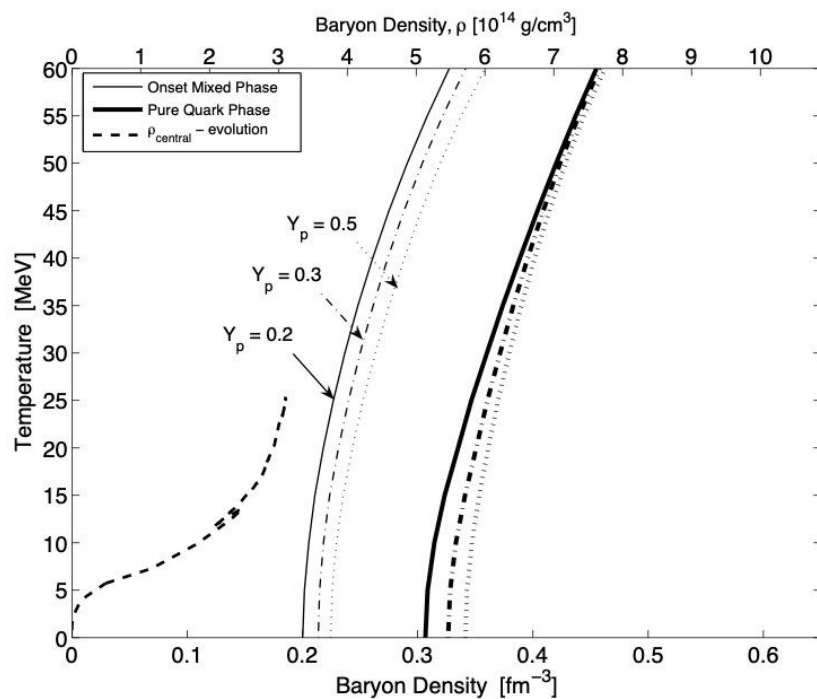
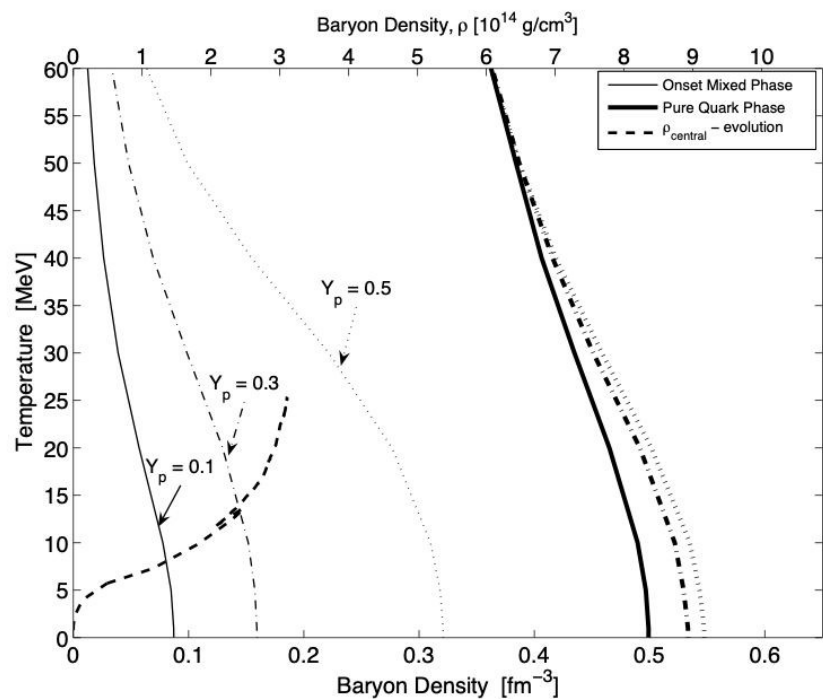


# Confining Density Functional (CDF) & CSC Quark Matter

CDF With CSC



# Confining Density Functional (CDF) & CSC Quark Matter

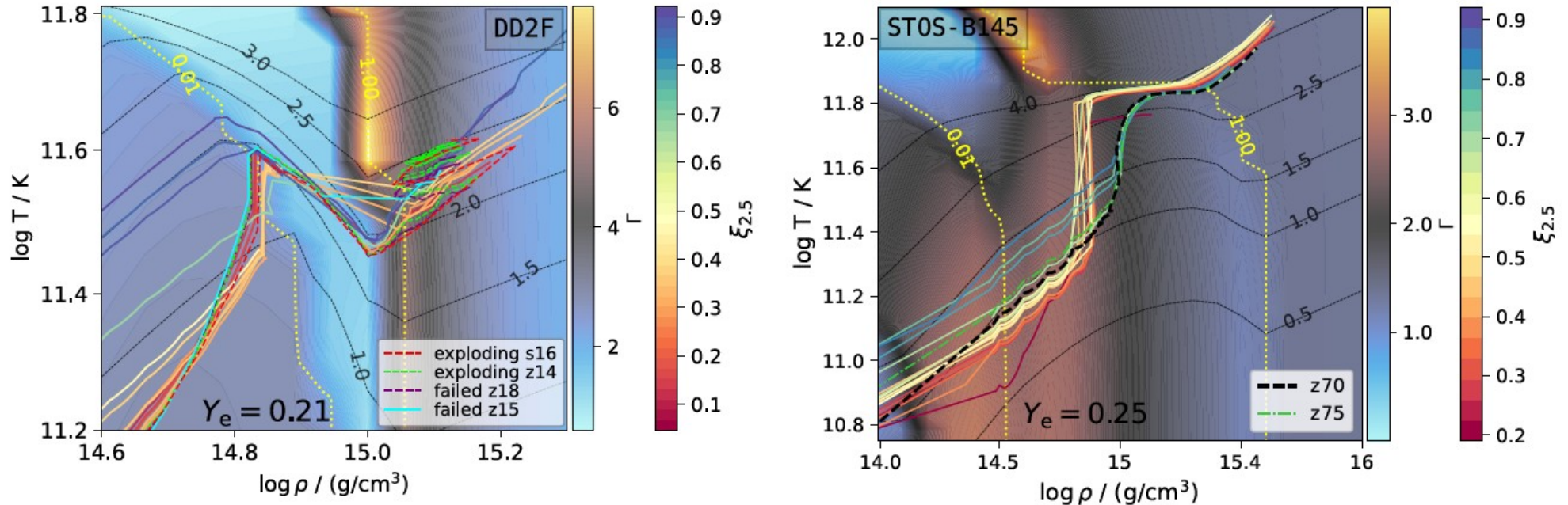


(a) Quark bag model ( $B^{1/4} = 165 \text{ MeV}$ ), for  $Y_p = 0.1$  (solid lines),  $Y_p = 0.3$  (dash-dotted lines) and  $Y_p = 0.5$  (dotted lines)

(b) PNJL model ( $\eta_D = 1.02$ ,  $\eta_V = 0.25$ ), for  $Y_p = 0.2$  (solid lines),  $Y_p = 0.3$  (dash-dotted lines) and  $Y_p = 0.5$  (dotted lines)

# Thermal twin stars – Indicators of CCSN explodability ?

Successful explosion of massive progenitor stars\* for hybrid EoS with entropic first-order transition (thermal twin stars)



Important for explodability: Postbounce mass accretion rate (metallicity) vs. Time to reach the onset of deconfinement\*\*)

\*) Pia Jakobus et al., MNRAS 516, 2554 (2022); arxiv:2204.10397 [astro-ph.HE]

\*\*\*) Noshad Khosravi Largani et al., Astrophys. J. 964, 143 (2024); arxiv:2304.12316 [astro-ph.HE]

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David E. Alvarez-Castillo (IFJ PAN Cracow, Poland & Incubator CSSF Wroclaw, Poland)

On eccentric MSPs in low-mass X-ray binaries:

Savvas Chanlaridis, John Antoniadis (FORTH Heraklion, Greece), David Ohse (MPIfR Bonn, Germany)

On the color-superconducting quark matter EOS, QCD phase diagram, CDF and thermal twin stars:

Juan Pablo Carlomagno, Gustavo Contrera (CONICET, Univ. La Plata, Argentina),

Gabriela Grunfeld (CONICET & CNEA Buenos Aires, Argentina),

Oleksii Ivanytskyi (University of Wroclaw, Poland & Incubator CSSF Wroclaw, Poland)

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2021/43/P/ST2/03319: Bayesian analysis of the dense matter equation of state



Critical Point & Onset of  
Deconfinement (CPOD)

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