

# Transverse Momentum Distributions Phenomenology With NN



Chiara Bissolotti

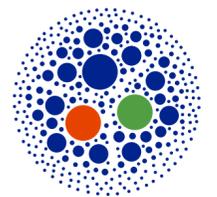
Argonne National Laboratory



with

*Map Collaboration*

Valerio Bertone, Matteo Cerutti  
Simone Rodini

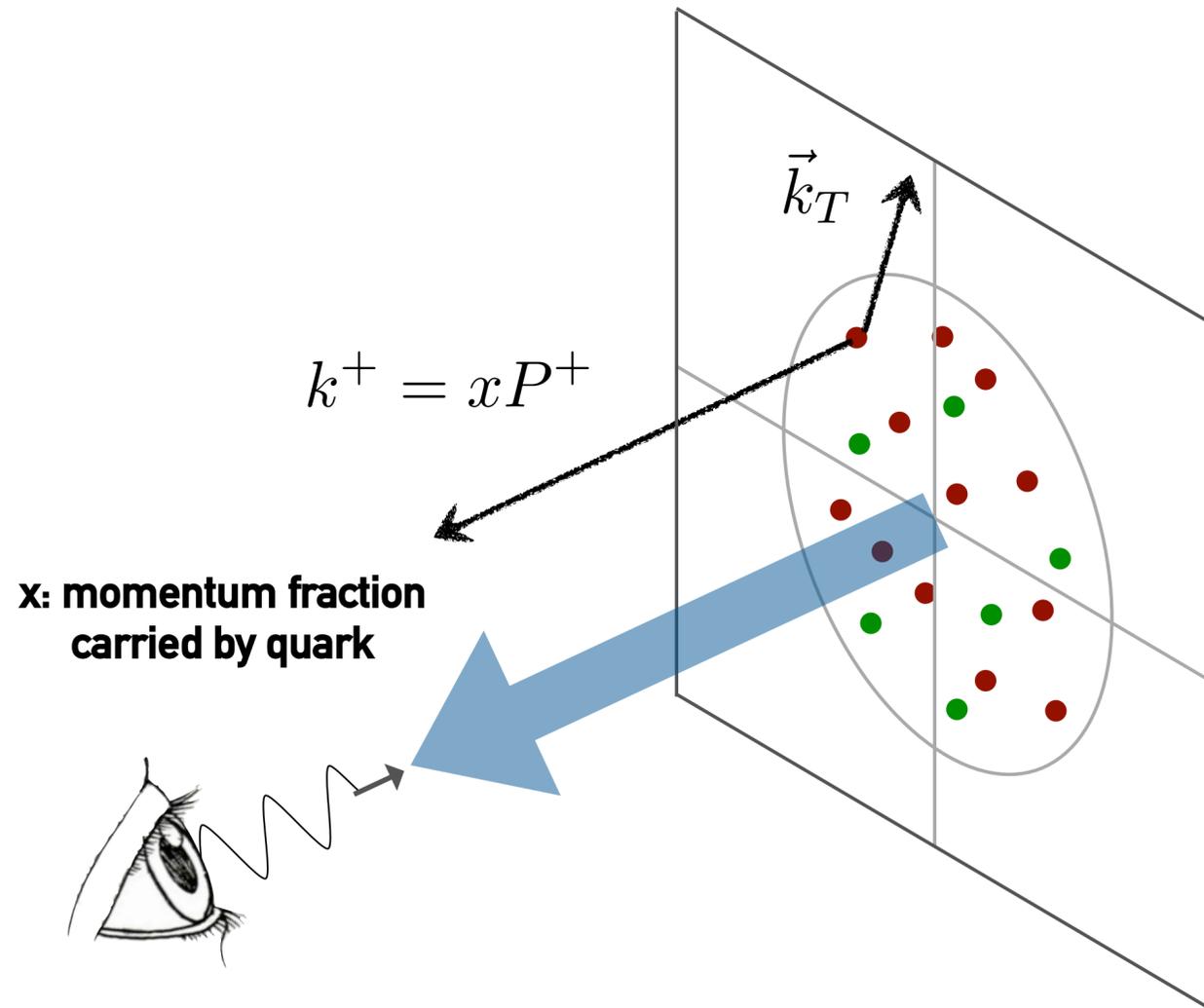


**HAS QCD**

HADRONIC STRUCTURE AND  
QUANTUM CHROMODYNAMICS

# TMDs: 3D maps

in momentum space



Transverse Momentum Distributions

TMDs

$$f^q(x, \mathbf{k}_T)$$

$$\int d^2\mathbf{k}_T$$

collinear

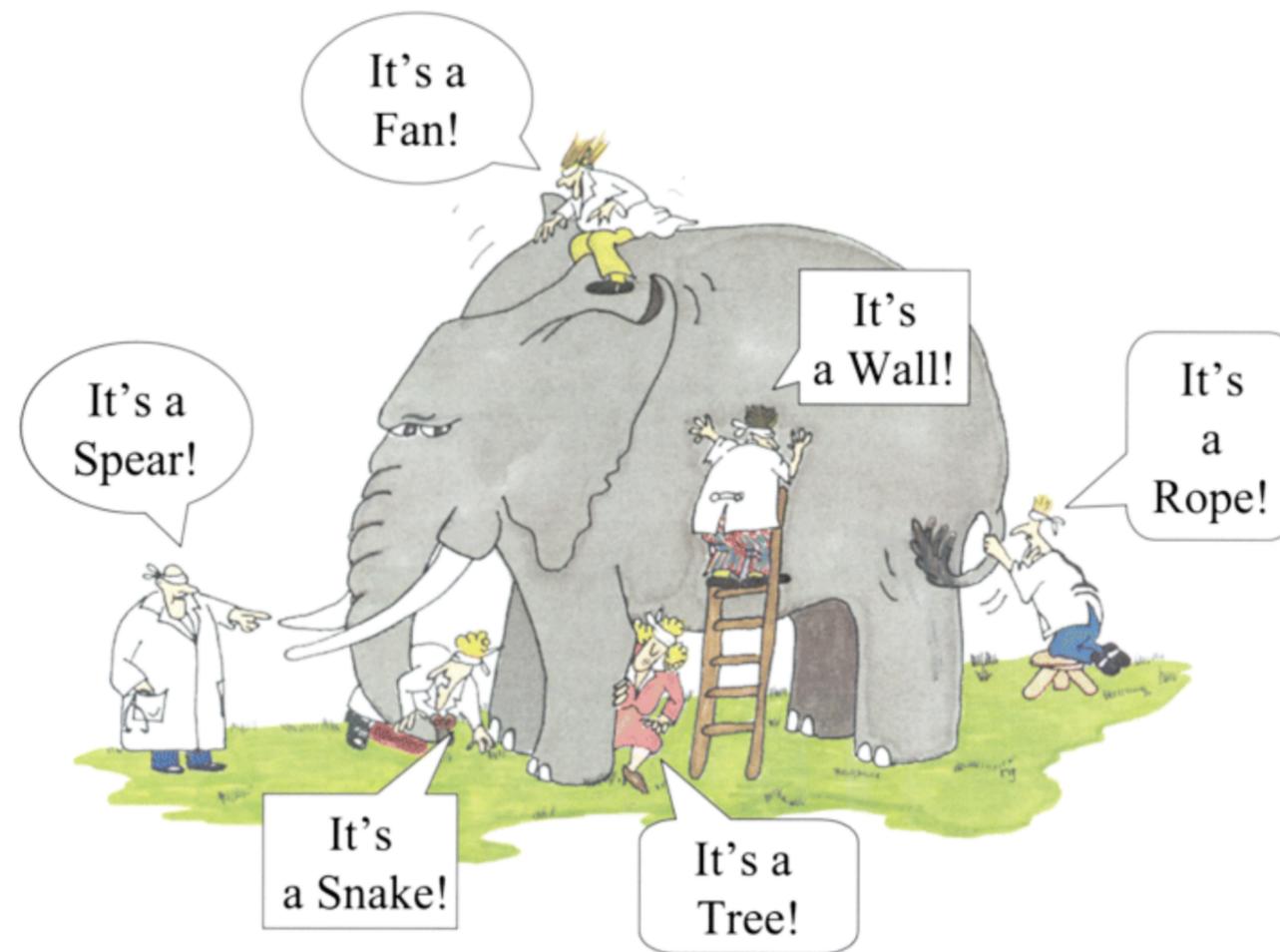
Parton Distribution Functions

1D maps

PDFs

# Nucleon tomography

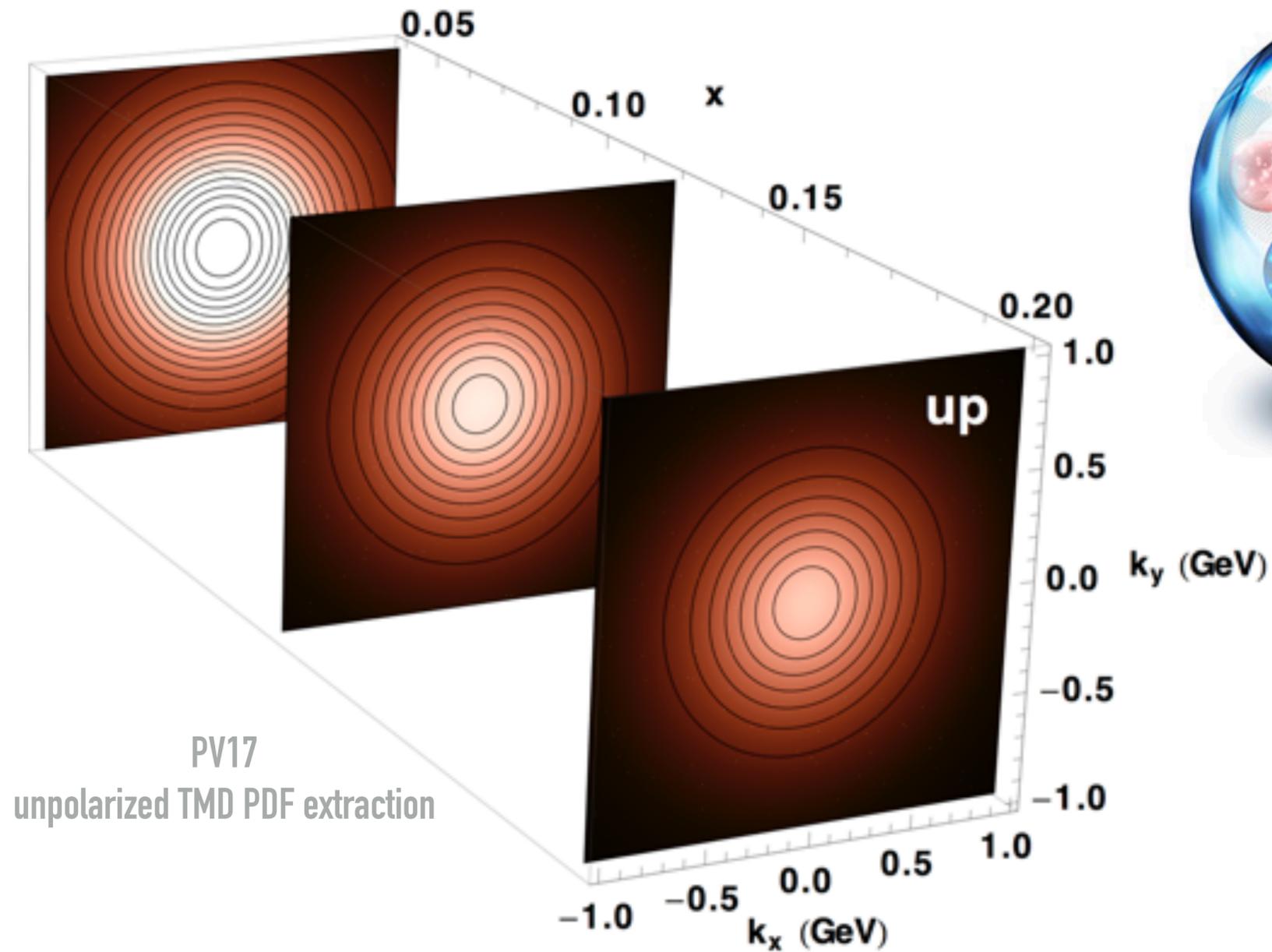
## Mapping the proton



...what you see depends on how you look!

# TMDs: 3D maps

in momentum space



Transverse Momentum  
Distributions

TMDs

$$f^q(x, \mathbf{k}_T)$$

$$\int d^2\mathbf{k}_T$$

collinear

Parton Distribution  
Functions

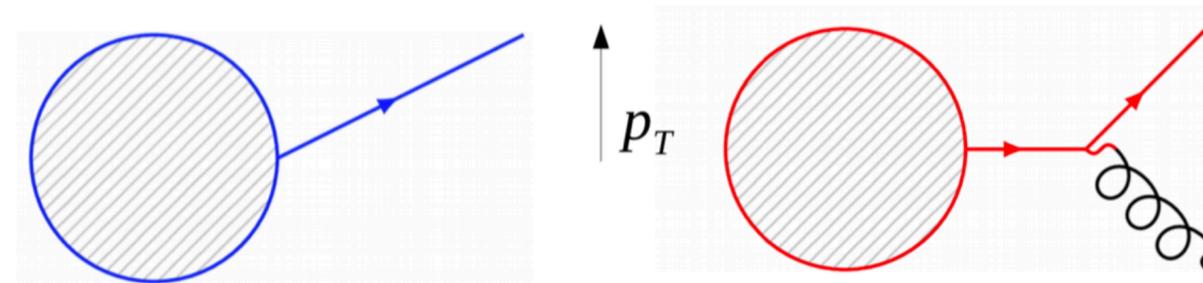
PDFs

# Complications!

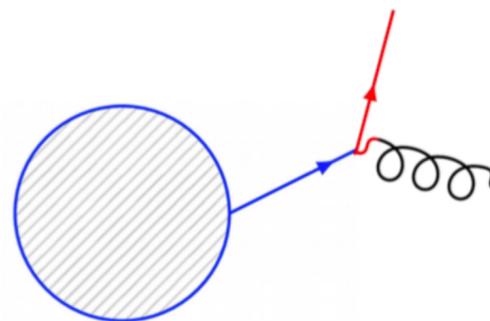
Transverse momentum can be ...

intrinsic

perturbative



what we measure is:



# TMD PDFs

*unpolarized Transverse Momentum Dependent  
Parton Distribution Functions*

$$f_1^q(x, b; \mu, \zeta) = \sum_j (C_j \otimes f^j)(x, b_*; \mu_b) e^{R(b_*; \mu_b, \mu)} f_{\text{NP}}(x, b)$$

# TMD PDFs

## *unpolarized Transverse Momentum Dependent Parton Distribution Functions*

collinear PDFs

$$f_1^q(x, b; \mu, \zeta) = \sum_j (C_j \otimes f^j)(x, b_*; \mu_b) e^{R(b_*; \mu_b, \mu)} f_{\text{NP}}(x, b)$$

# TMD PDFs

## *unpolarized Transverse Momentum Dependent Parton Distribution Functions*

matching to the collinear region

collinear PDFs

$$f_1^q(x, b; \mu, \zeta) = \sum_j (C_j \otimes f^j)(x, b_*; \mu_b) e^{R(b_*; \mu_b, \mu)} f_{\text{NP}}(x, b)$$

perturbative expansion  
in  $\alpha_s(\mu)$

perturbative evolution

# TMD PDFs

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perturbative expansion  
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perturbative evolution

$$L = \ln \frac{Q^2}{\mu_b^2}$$

resummation of large  
logarithms

# TMD PDFs

## *unpolarized Transverse Momentum Dependent Parton Distribution Functions*

matching to the collinear region

collinear PDFs

$$f_1^q(x, b; \mu, \zeta) = \sum_j (C_j \otimes f^j)(x, b_*; \mu_b) e^{R(b_*; \mu_b, \mu)} f_{\text{NP}}(x, b)$$

perturbative expansion  
in  $\alpha_s(\mu)$

perturbative evolution

non perturbative  
transverse content

$$L = \ln \frac{Q^2}{\mu_b^2}$$

resummation of large  
logarithms

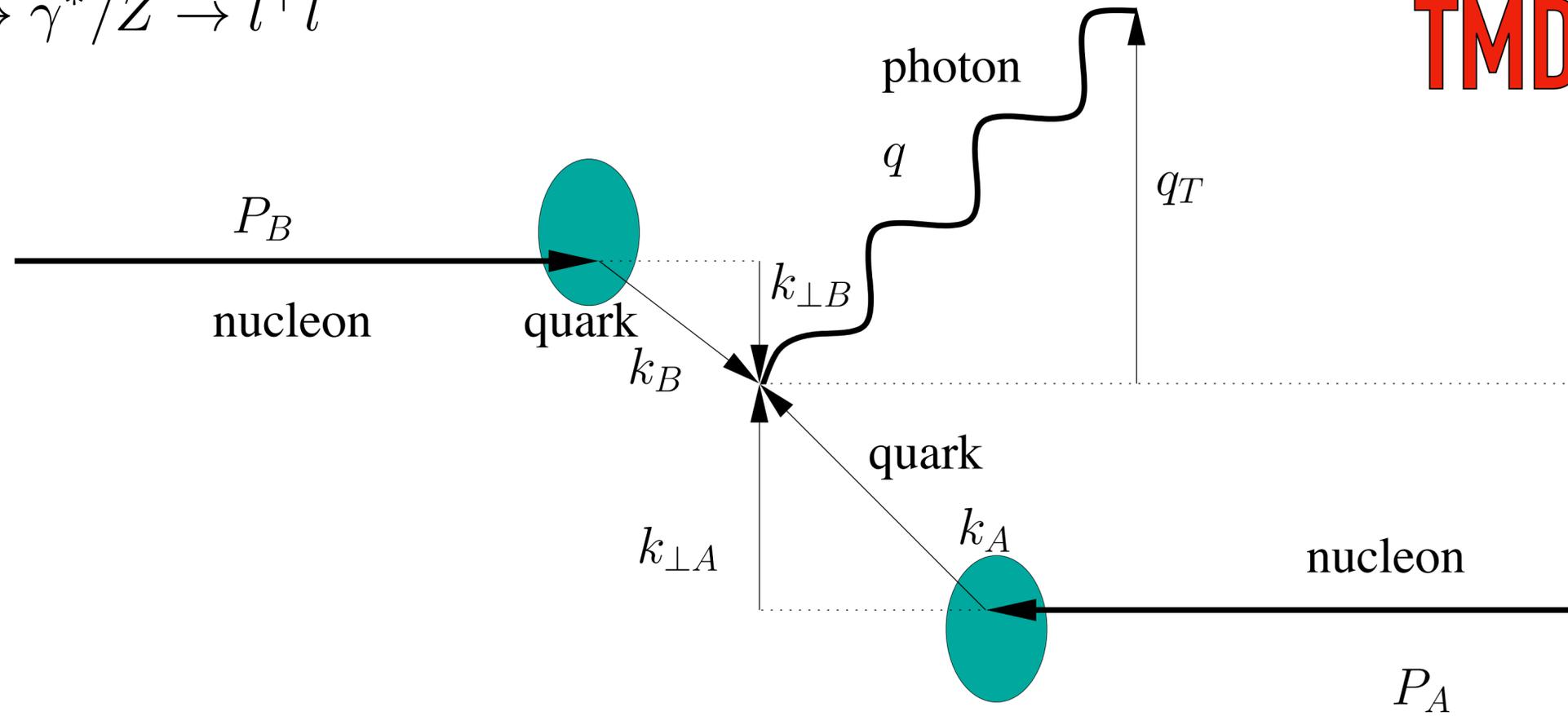
parametrized  
and fitted to data

# Drell-Yan

$$N(P_A) + N(P_B) \rightarrow \gamma^*/Z \rightarrow l^+l^-$$

for  $q_T \ll Q$

**TMD factorization**



$$F_{UU}^1(x_A, x_B, \mathbf{q}_T^2, Q^2)$$

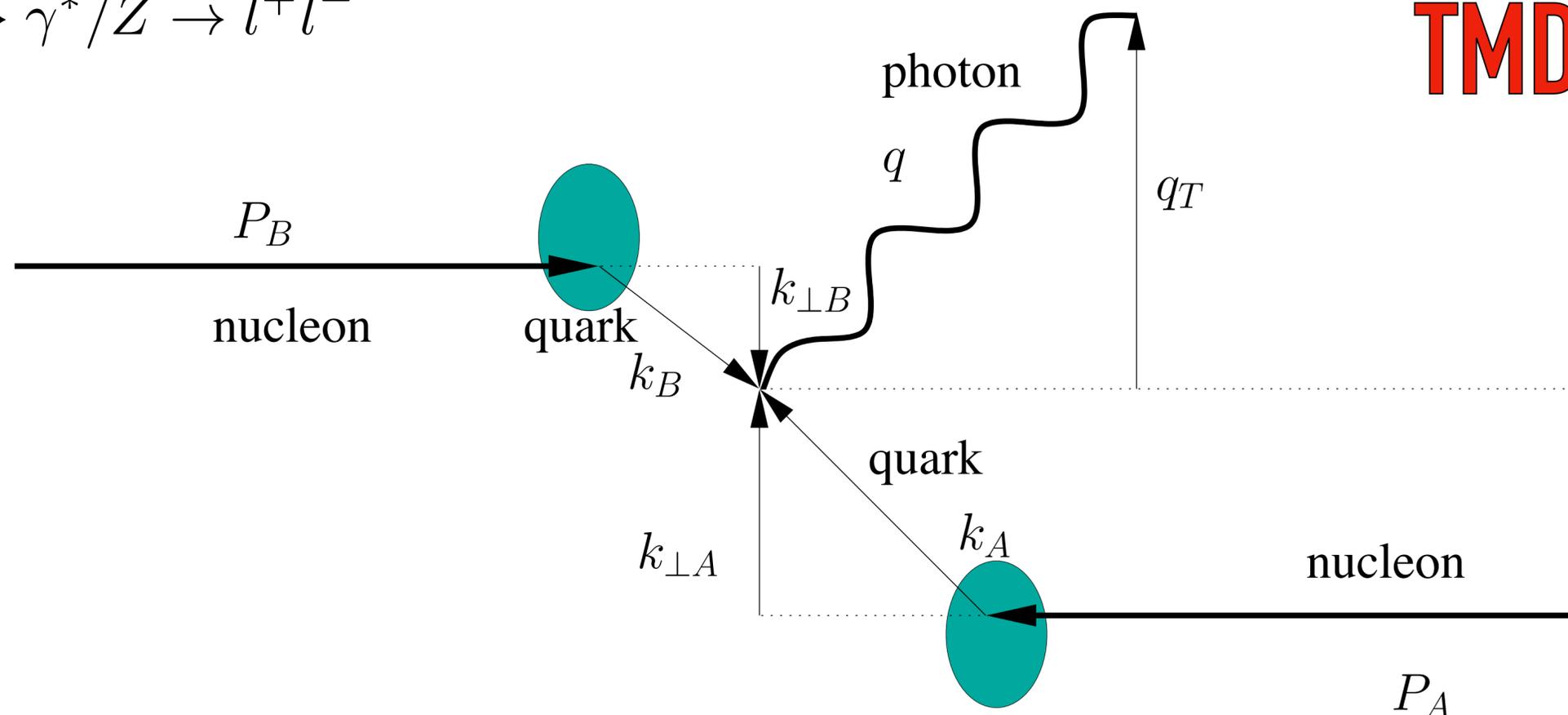
$$= \sum_a \mathcal{H}_{UU}^{1a}(Q^2, \mu^2) \int d^2\mathbf{k}_{\perp A} d^2\mathbf{k}_{\perp B} f_1^a(x_A, \mathbf{k}_{\perp A}^2; \mu^2) f_1^{\bar{a}}(x_B, \mathbf{k}_{\perp B}^2; \mu^2) \delta^{(2)}(\mathbf{k}_{\perp A} - \mathbf{q}_T + \mathbf{k}_{\perp B}) + Y_{UU}^1(Q^2, \mathbf{q}_T^2) + \mathcal{O}(M^2/Q^2)$$

# Drell-Yan

$$N(P_A) + N(P_B) \rightarrow \gamma^*/Z \rightarrow l^+l^-$$

for  $q_T \ll Q$

## TMD factorization



$$\left( \frac{d\sigma}{dq_T} \right)_{\text{res.}} \propto H(Q, \mu) \int \frac{d^2\mathbf{b}}{4\pi} e^{i\mathbf{b}\cdot\mathbf{q}_T} x_1 f_1^q(x_1, \mathbf{b}; \mu, \zeta_1) x_2 f_1^{\bar{q}}(x_2, \mathbf{b}; \mu, \zeta_2)$$

# large $b_T$

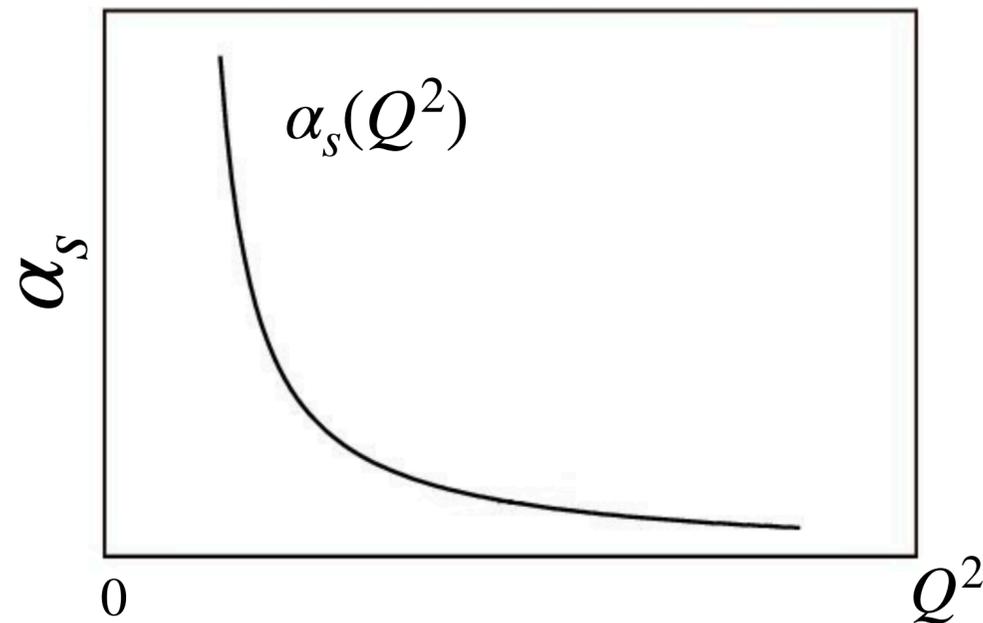
integration up to infinity

$$\frac{d\sigma}{dq_T} \propto \int \frac{d^2\mathbf{b}}{4\pi} e^{i\mathbf{b}\cdot\mathbf{q}_T} f_1^q(x_1, \mathbf{b}) f_1^{\bar{q}}(x_2, \mathbf{b})$$

**cross section**

when  $b_T$  becomes large

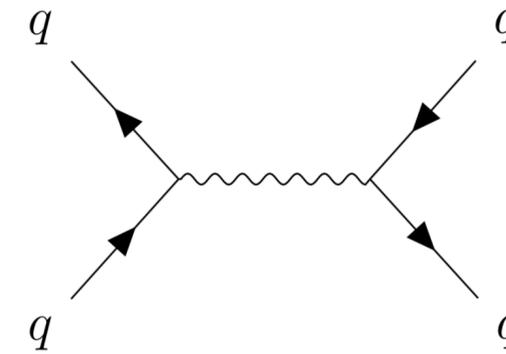
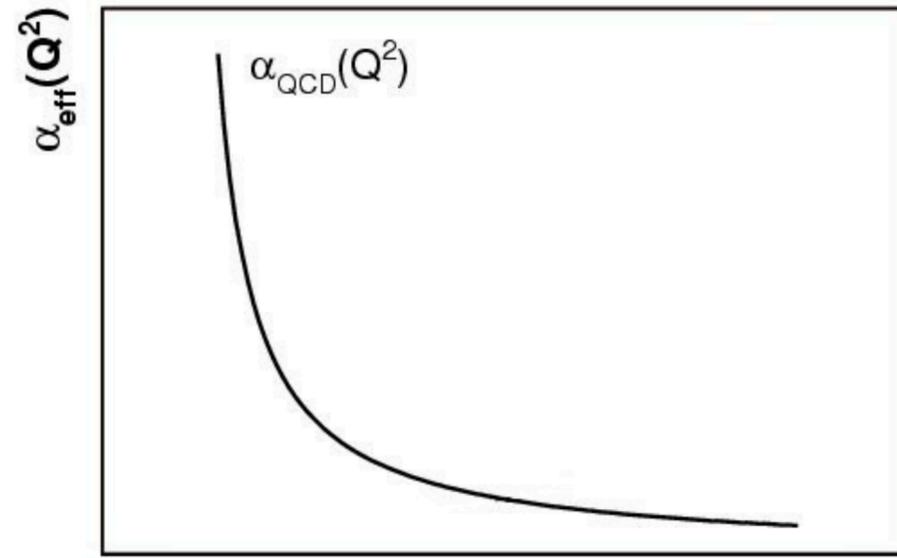
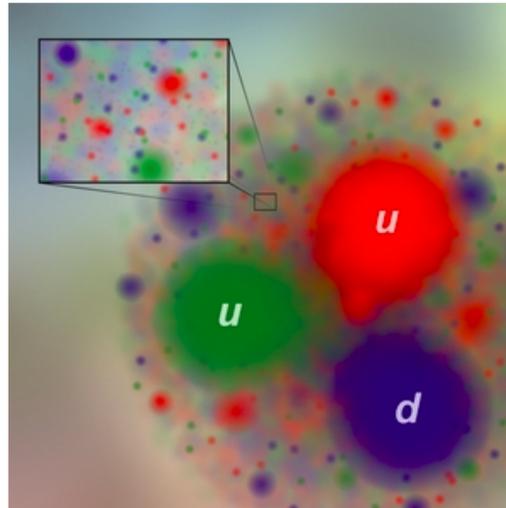
$$\alpha_s(\mu_b) = \alpha \left( \frac{2e^{-\gamma_E}}{b} \right) \gg 1$$



invalidates **perturbative** calculations

necessary to introduce a prescription

# QCD running constant $\alpha_s$



*probing small distance scales*



**non perturbative region**



quark confinement

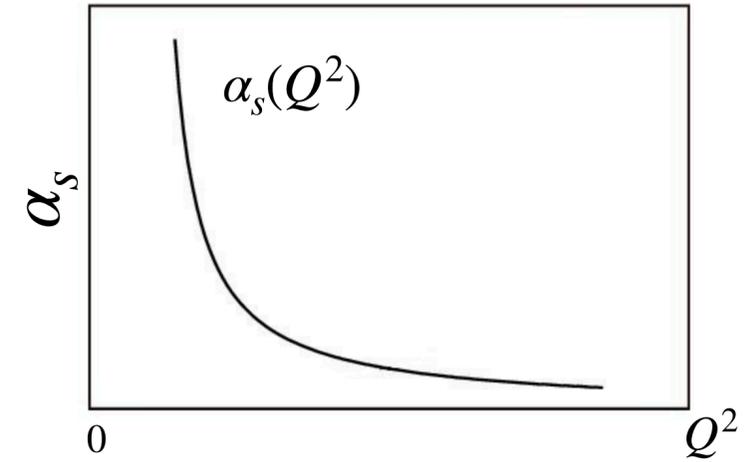
**perturbative region**



asymptotic freedom

# $b^*$ prescription

$$\frac{d\sigma}{dq_T} \propto \int \frac{d^2\mathbf{b}}{4\pi} e^{i\mathbf{b}\cdot\mathbf{q}_T} f_1^q(x_1, \mathbf{b}) f_1^{\bar{q}}(x_2, \mathbf{b})$$



when  $b_T$  becomes large

$$\alpha_s(\mu_b) = \alpha \left( \frac{2e^{-\gamma_E}}{b} \right) \gg 1$$

invalidates **perturbative** calculations

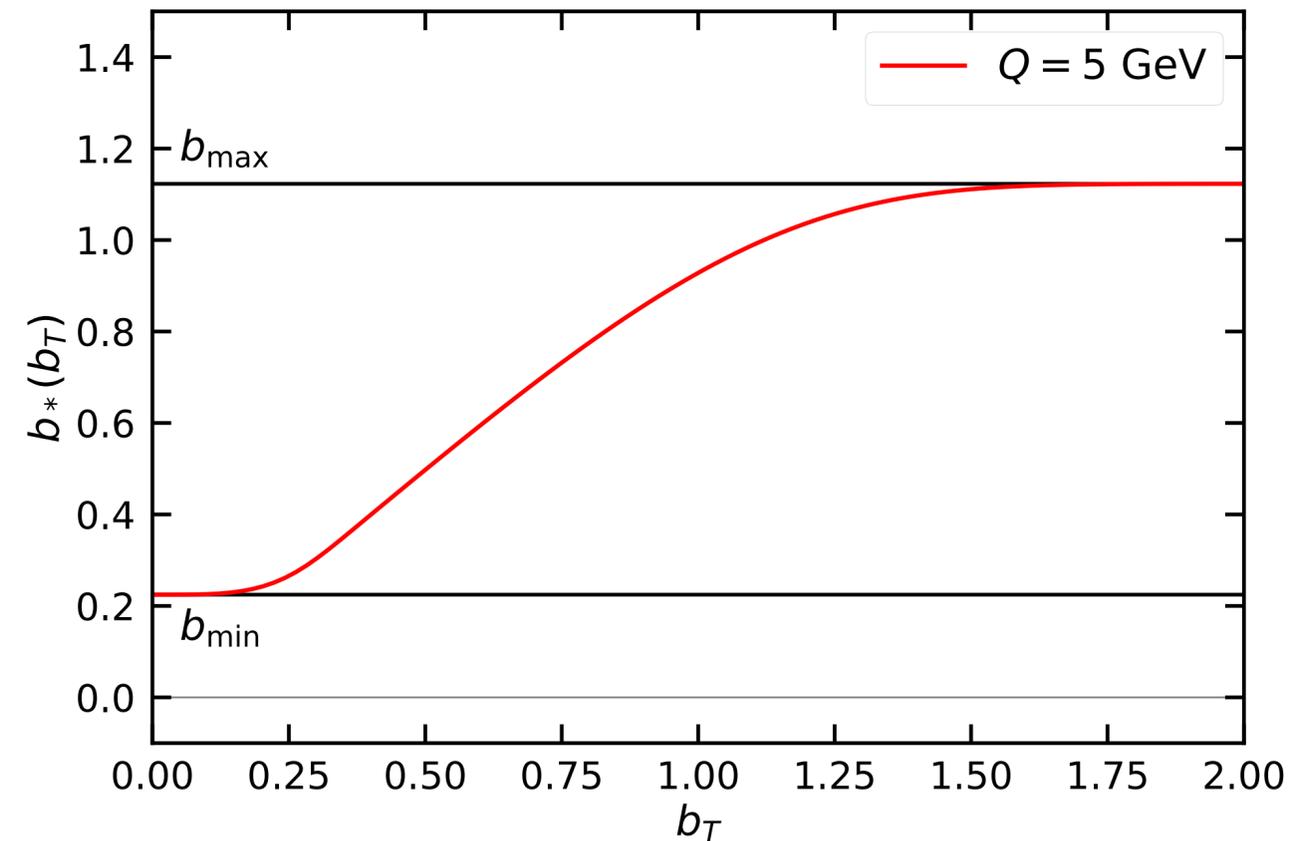
$\Rightarrow b_{\max}$

$$b_{\max} = 2e^{-\gamma_E}$$

**b-min choice**

$$b_{\min} = 2e^{-\gamma_E} / Q$$

$$b_*(b) = b_{\max} \left( \frac{1 - \exp\left(-\frac{b^4}{b_{\max}^4}\right)}{1 - \exp\left(-\frac{b^4}{b_{\min}^4}\right)} \right)^{\frac{1}{4}}$$



# $b^*$ prescription

## and definition of $f_{\text{NP}}$

 perturbative

$$f(x, b; \mu, \zeta) = \left[ \frac{f(x, b; \mu, \zeta)}{f(x, b_*(b); \mu, \zeta)} \right] f(x, b_*(b); \mu, \zeta)$$

non perturbative



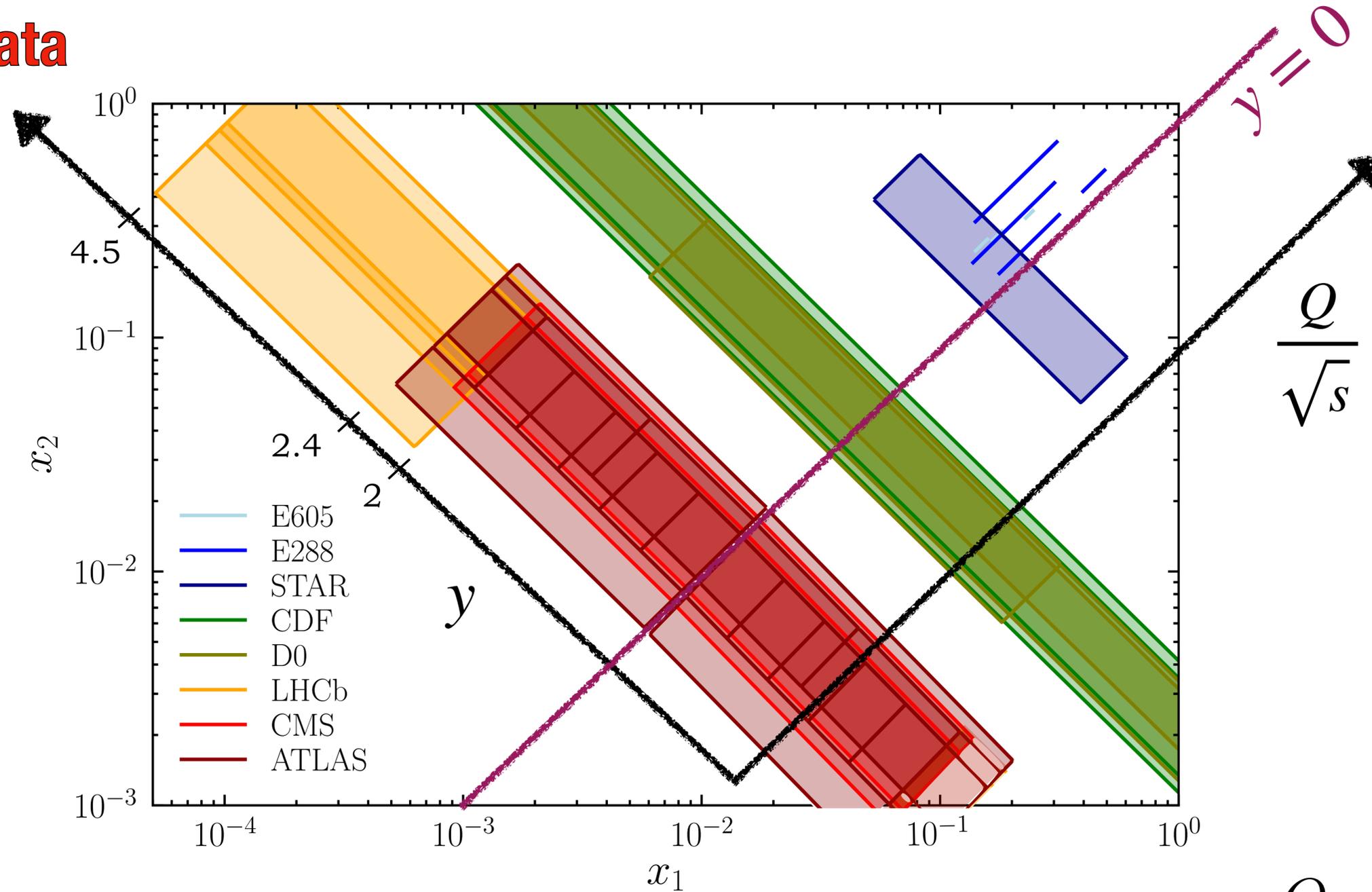
$$f_{\text{NP}}(x, b, \zeta)$$

fit to data

Non perturbative function depends on  
the choice of  $b^*$ -prescription

# x, Q, y coverage

only Drell-Yan data



$$x_{1,2} = \frac{Q}{\sqrt{s}} e^{\pm y}$$

# Integrations

In order to compare theory with data

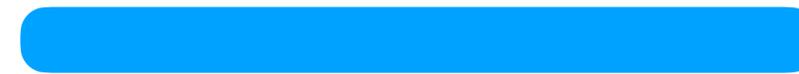
\* experiments measure the cross section  $\sigma$

integration over the range  
in momentum transfer

integration over the range  
in rapidity

integration over bins in transverse  
momentum  $q_T$

$$\sigma = \int_{Q_{\min}}^{Q_{\max}} dQ \int_{y_{\min}}^{y_{\max}} dy \int_{q_{T,\min}}^{q_{T,\max}} dq_T \left[ \frac{d\sigma}{dQ dy dq_T} \right]$$



**numerical integration**

Ogata quadrature



**analytic integration**

see PV19 paper

# Parameterization

"analytic", "with a functional form"

of the non-perturbative part of TMDs

## TMD PDF

$$f_{1\text{NP}}(x, b_T^2) \propto \text{F.T. of } \left( e^{-\frac{k_\perp^2}{g_1^1 A}} + \lambda_B k_\perp^2 e^{-\frac{k_\perp^2}{g_1^1 B}} + \lambda_C e^{-\frac{k_\perp^2}{g_1^1 C}} \right)$$

Gaussians

weighted Gaussian

$$g_1(x) = N_1 \frac{(1-x)^\alpha x^\sigma}{(1-\hat{x})^\alpha \hat{x}^\sigma}$$

## NP evolution

$$g_K(b_T^2) = -g_2^2 \frac{b_T^2}{4}$$

MAP22

12 parameters

# Parameterization

## of the non-perturbative part of TMDs

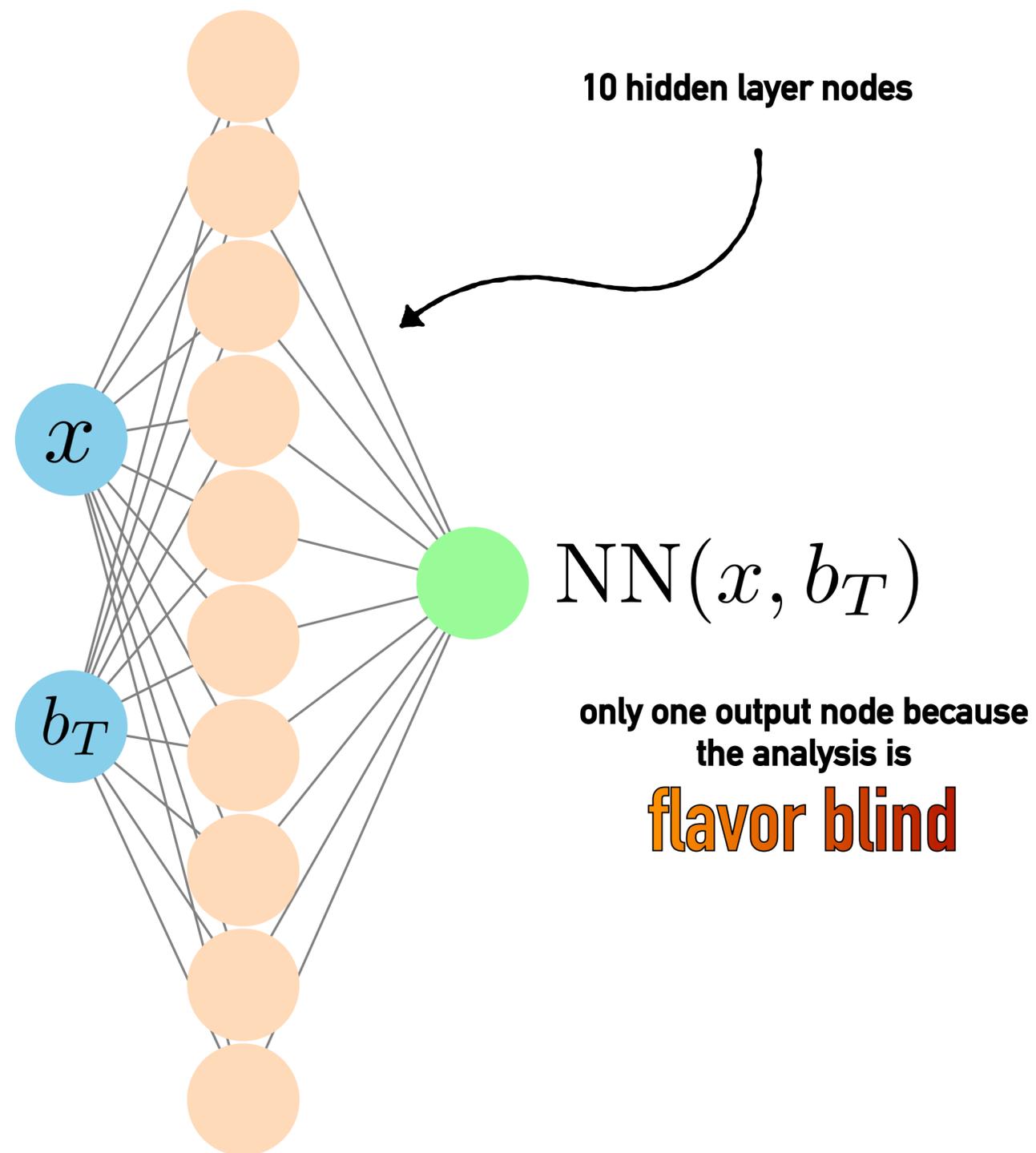
proof of concept

$$f_{NP}(x, b_T) = e^{NNAD(x, b_T)} e^{S_{NP}}$$

formula implemented in NangaParbat

$$f_{NP}(x, b_T, \zeta) = \exp \left[ - (NN(x, b_T) - NN(x, 0)) - g_2^2 \log \left( \frac{\zeta}{Q_0^2} \right) b_T^2 \right]$$

## Neural Network



# Motivation

What do we want to do? What do we want to prove?

*goal:* fit for the first time TMDs with NN

\* are NN really a more general parameterization with respect to the “analytic classics”?

e.g. ‘analytic functional forms’, like sums of Gaussian and weighted Gaussians

\* we want to/have to **test methodology**

↳ with **closure tests**

<https://docs.nnpdf.science/tutorials/closuretest.html>



The screenshot shows the NNPDF website interface. On the left is a blue header with the NNPDF logo and a search bar labeled 'Search docs'. On the right, there is a navigation menu with a home icon, 'Tutorials', and 'How to run a closure test'. Below the navigation, the title 'How to run a closure test' is displayed with a link icon. The main content area begins with the text: 'Closure tests are a way to validate methodology by fitting on pseudodata generated from pre-existing PDFs.'

# Closure tests

to validate the methodology

## \* Level 0

- \_\_\_\_\_ central pseudo-data is given by **central predictions** of the known model
- \_\_\_\_\_ **no Monte Carlo noise** is added on top of the central data  
— each replica is fitting the same set of data

## \* Level 1

- \_\_\_\_\_ central pseudo-data is shifted by some noise  $\eta$   
drawn from the experimental covariance matrix
- \_\_\_\_\_ no MC noise is added — each replica fits a subset of the same shifted data

## \* Level 2

- \_\_\_\_\_ central pseudo-data is shifted by level 1 noise  $\eta$
- \_\_\_\_\_ MC noise is added on top of the level 1 shift

# Closure tests

*only Drell-Yan data*

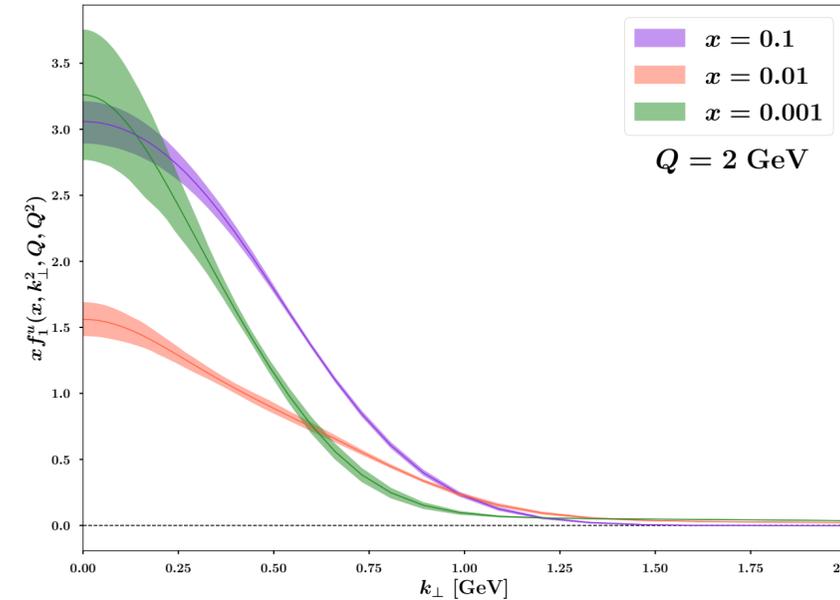
to validate the methodology

closure test of level 0

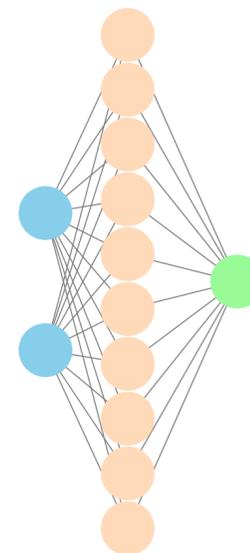
Generate **pseudodata** based on a model that we know

- \* central value: **MAP22**
- \* uncertainties: real DY data

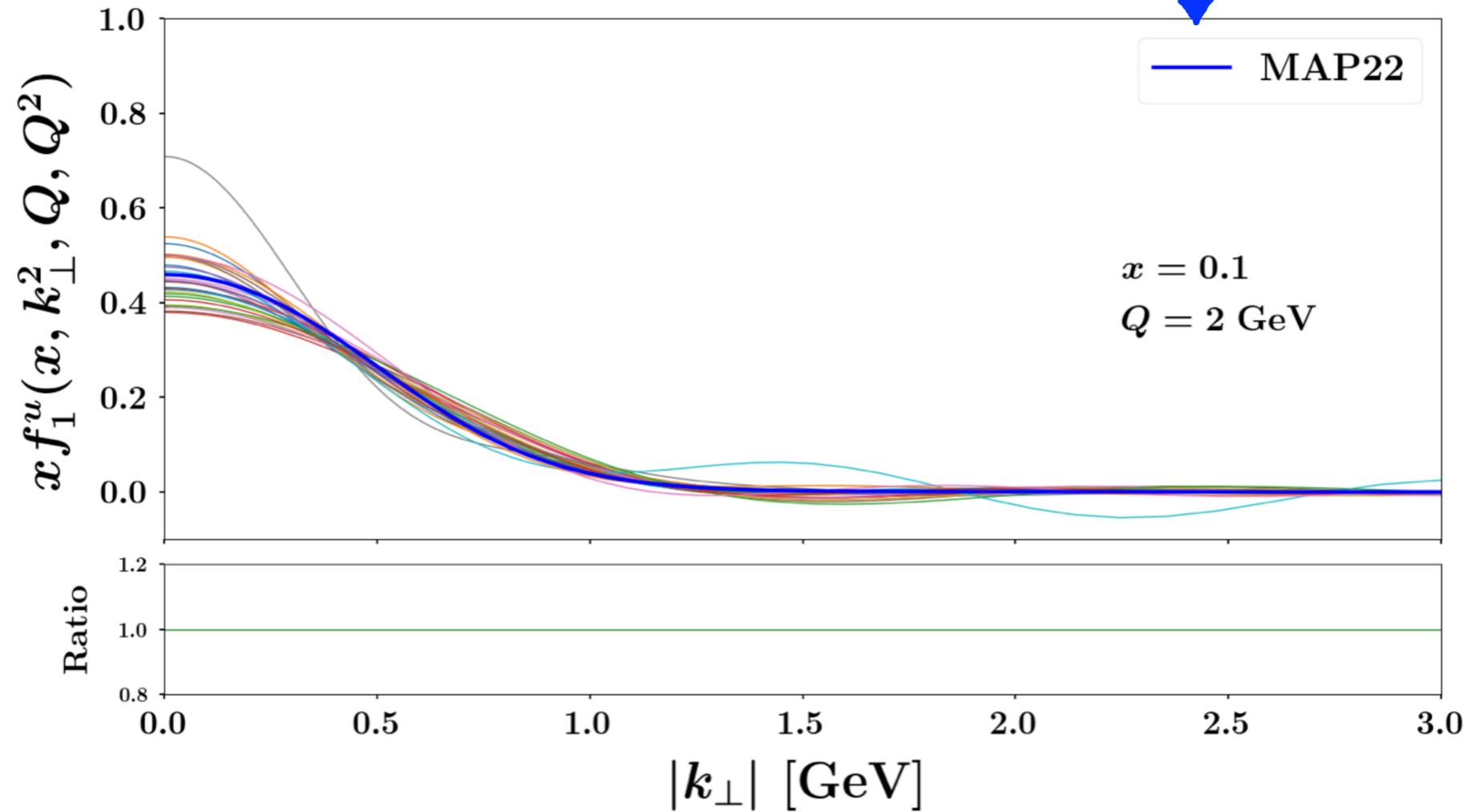
fit with NN



can we recover the MAP22 result?



# Closure test - level 0

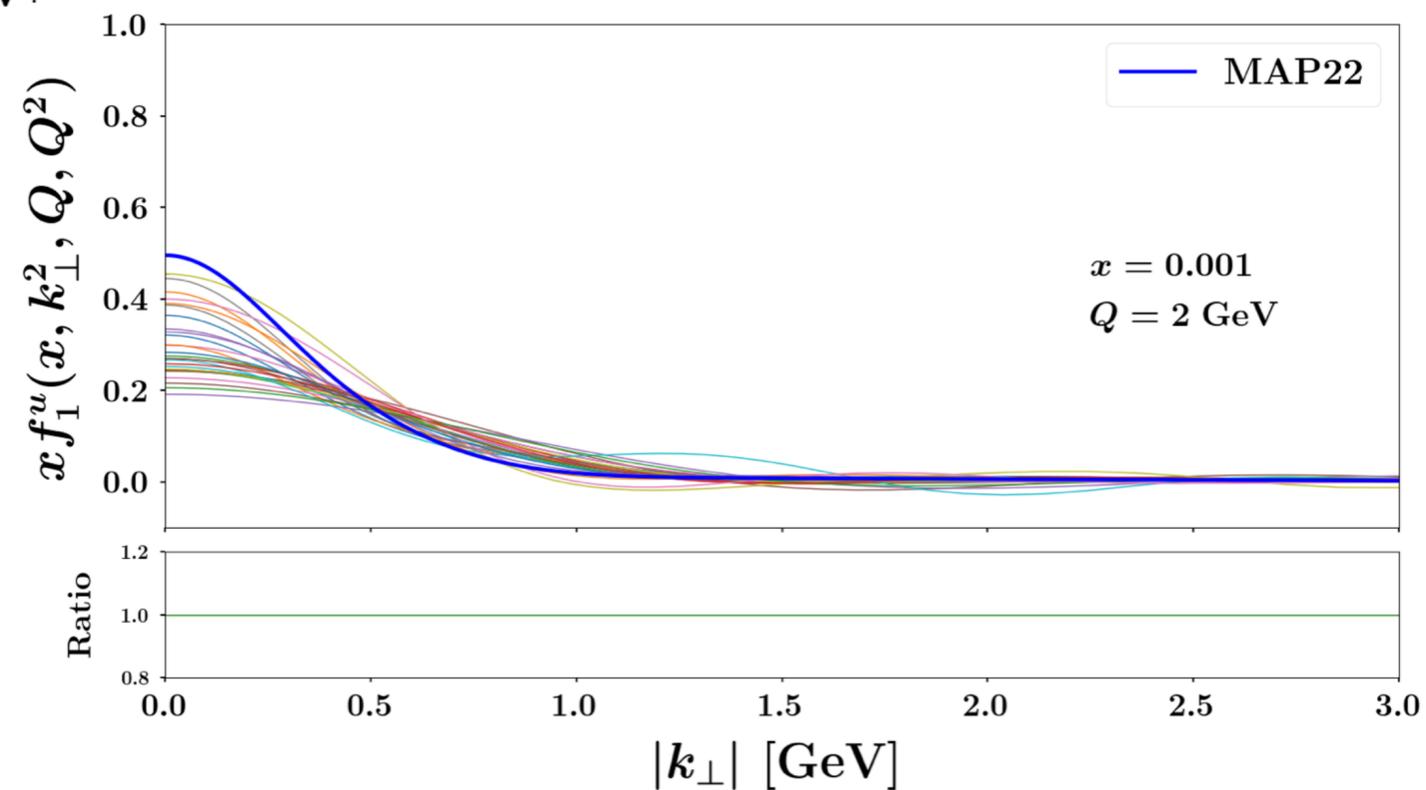
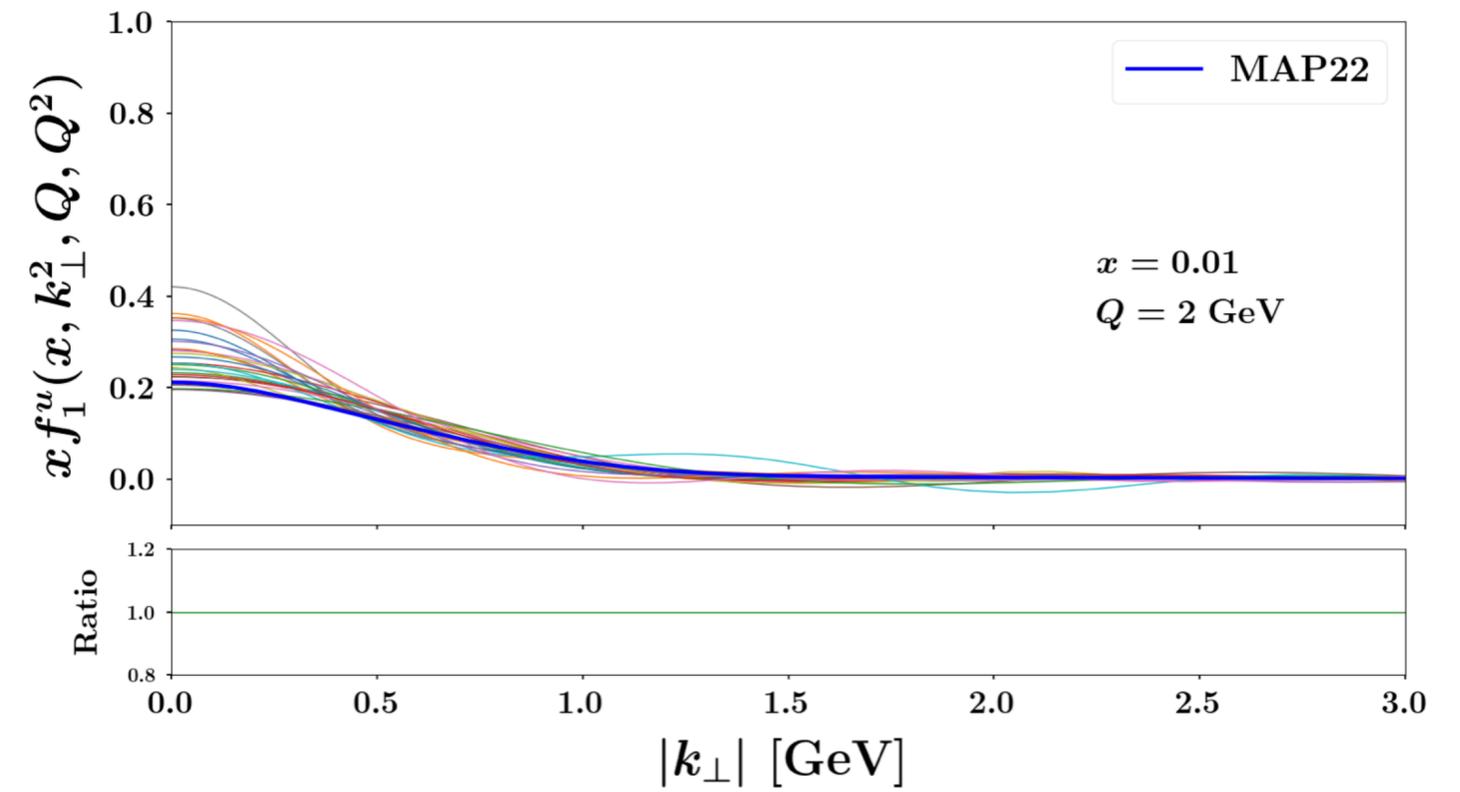
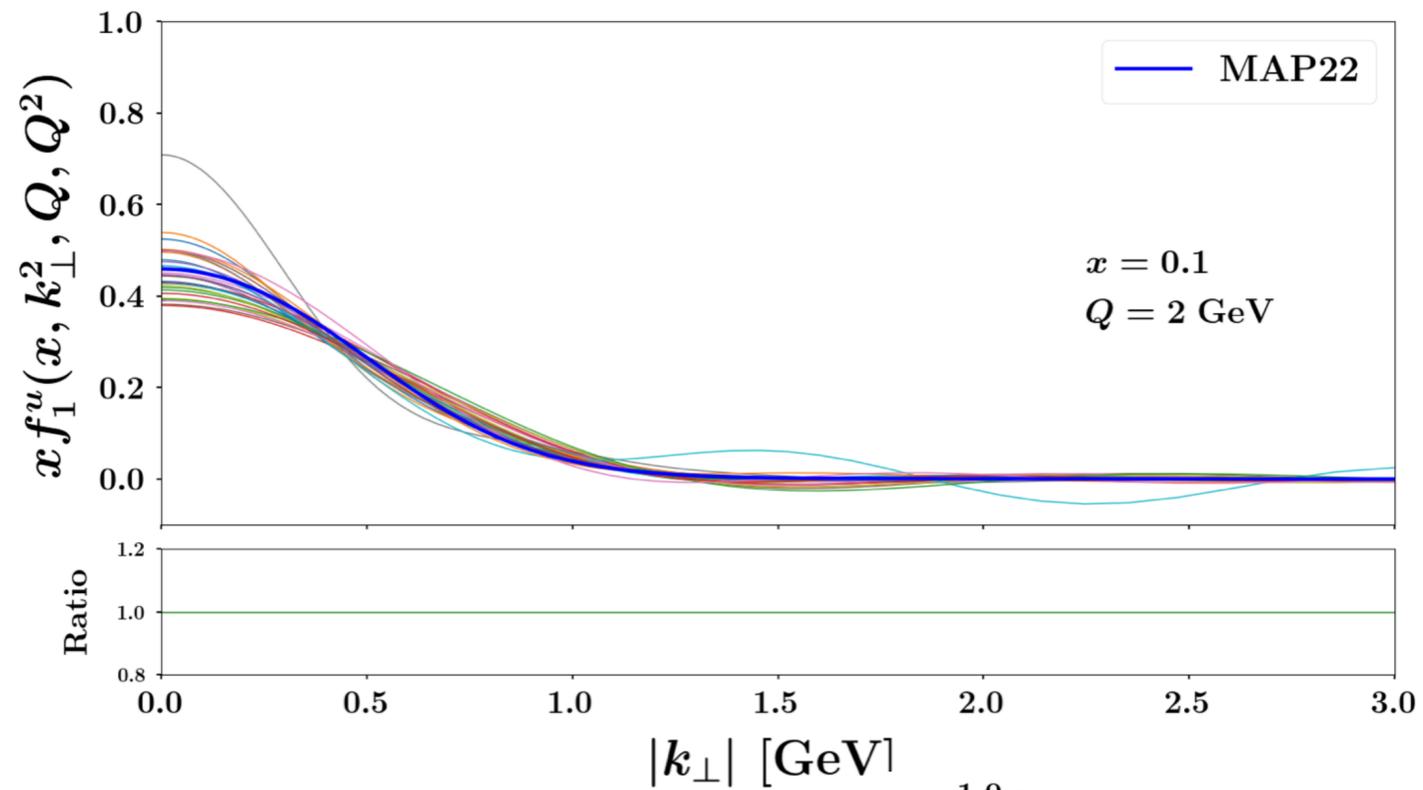


comparison with central replica  
of MAP22

replicas here come from varying  
the random seed for  
the initial values of the parameters

$$\chi^2 \sim 10^{-5}$$

# Closure test - level 0



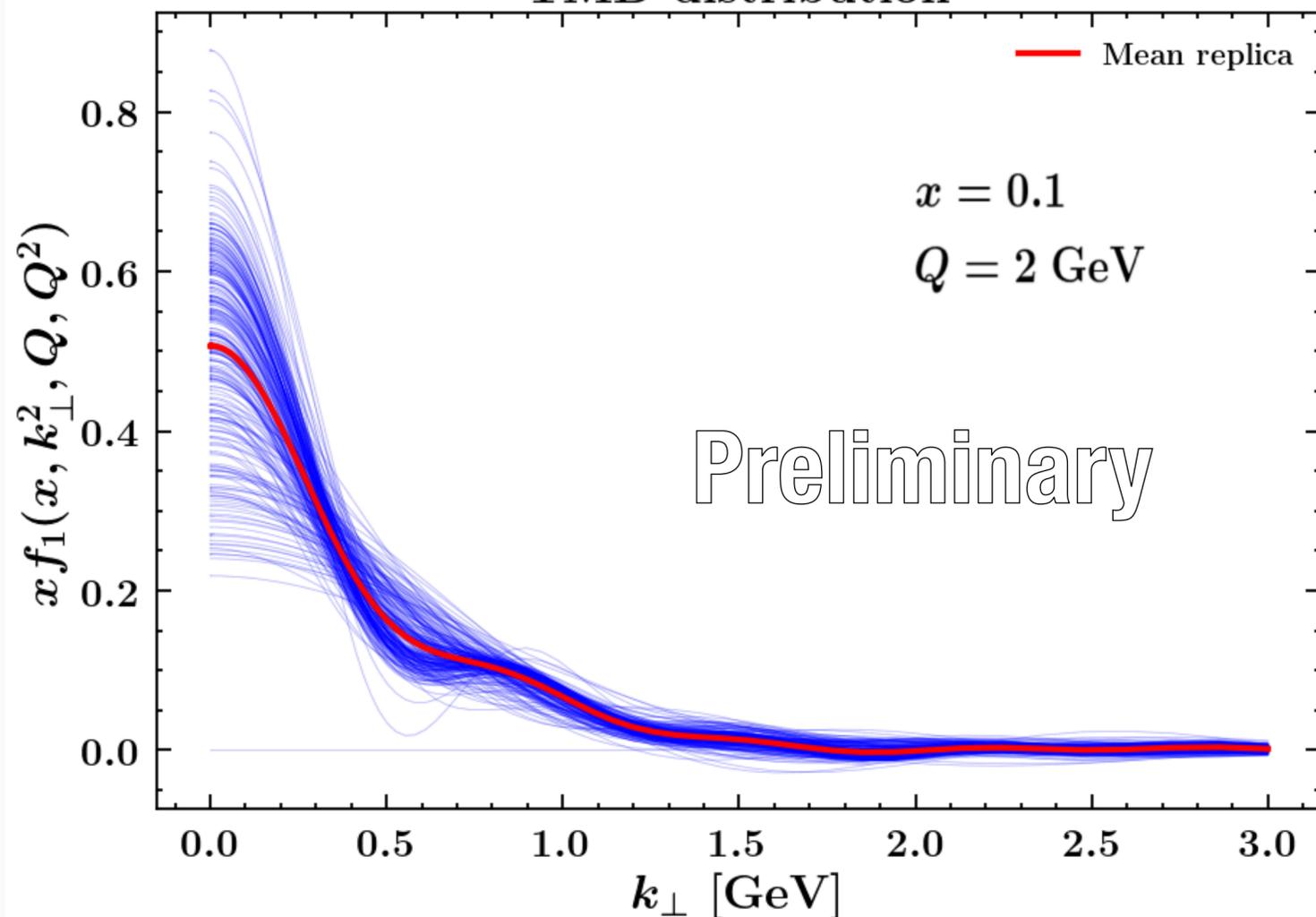
# Results of NN TMD fit

**Caveat: very preliminary results!**

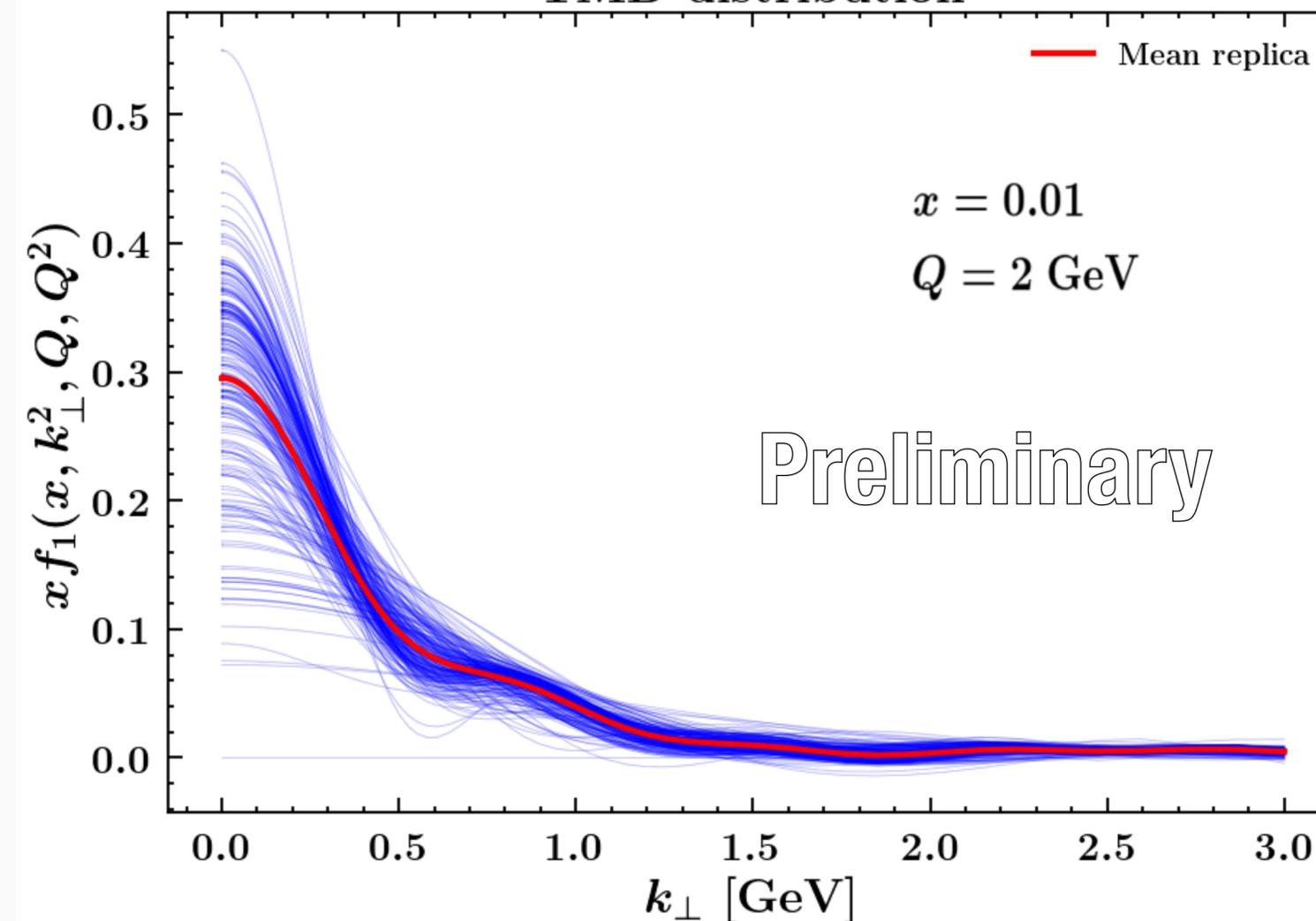
$$\chi^2 = 1.02$$

**5 nodes (hidden layer)**

TMD distribution



TMD distribution



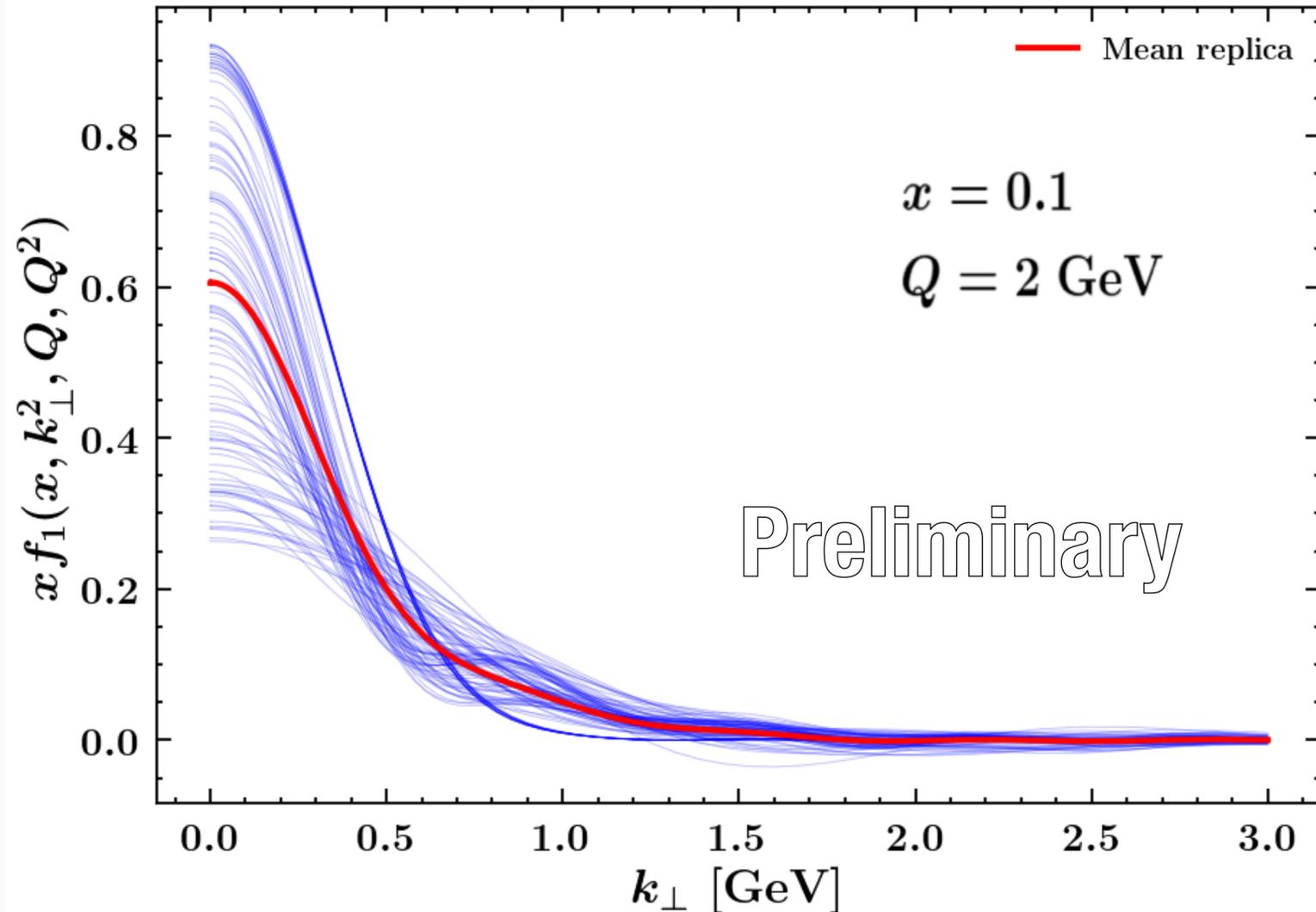
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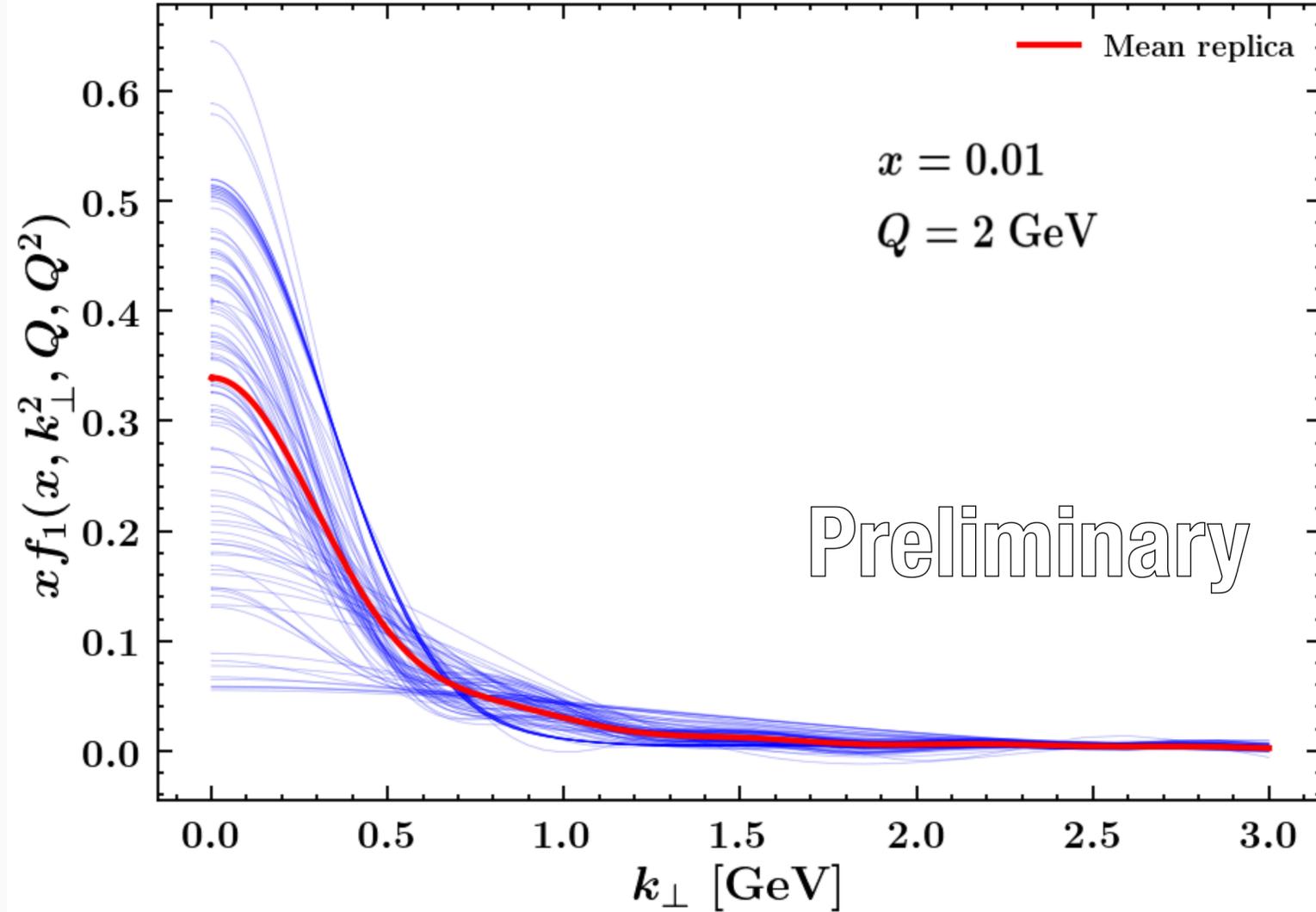
$$\chi^2 = 0,97$$

**10 nodes (hidden layer)**

TMD distribution

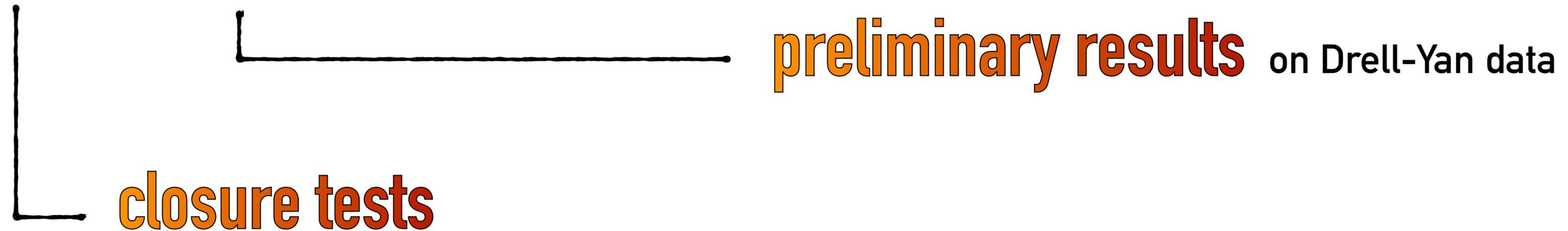


TMD distribution



# Conclusions

**Neural Networks** opened up a new realm of possibilities for fits



*to do*

- \* Study  $b^*$  prescriptions - do they have an impact on the TMDs?
- \* Perform all stages of closure tests - validate the use of NN
- \* ... keep trying to use NN in TMD fits