Lattice calculations of GPDs

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BNL

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In Collaboration with:

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Jack Dodson (Temple U.)
Xiang Gao (ANL)
Andreas Metz (Temple U.)
Swagato Mukherjee (BNL)
Aurora Scapellato (Temple U.)
Fernanda Steffens (Bonn U.)
Yong Zhao (ANL)

INT Workshop:

Parton Distributions & Nucleon Structure

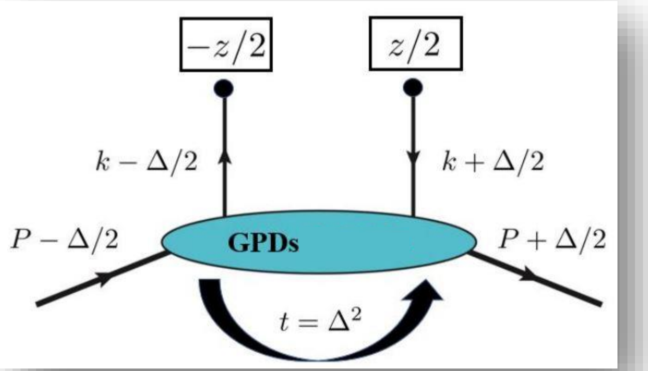
INSTITUTE for NUCLEAR THEORY

Seattle, Washington

Based on arXiv: 2209.05373

Background

What? Why? How?



Generalized Parton Distributions (GPDs): (See Diehl, arXiv: 0307382)

$$F^{[\Gamma]}(x,\Delta;\lambda,\lambda') = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ik\cdot z} \langle p';\lambda'|\bar{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p;\lambda\rangle\Big|_{z^+=0,\vec{z}_\perp=\vec{0}_\perp}$$

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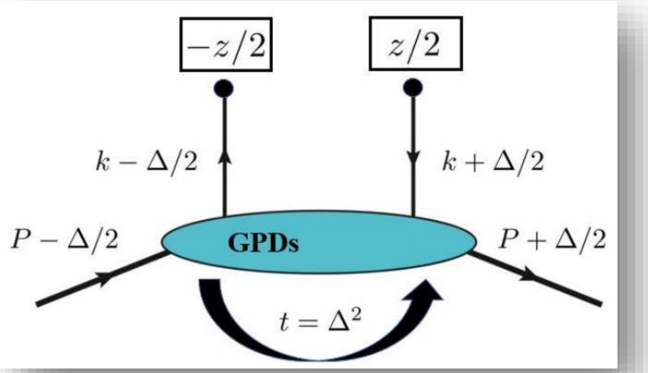

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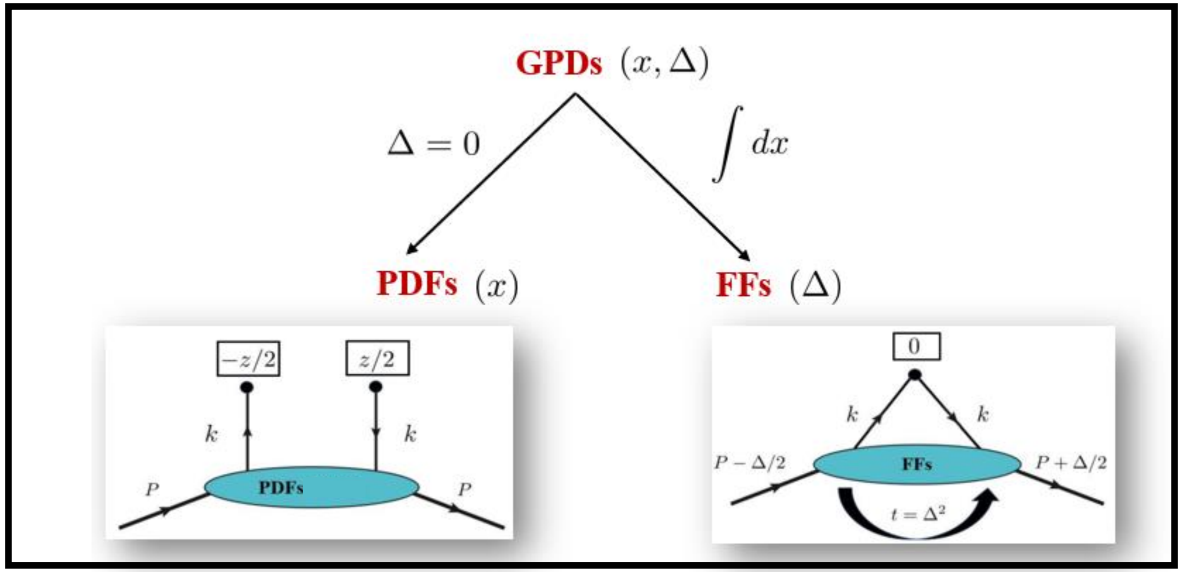
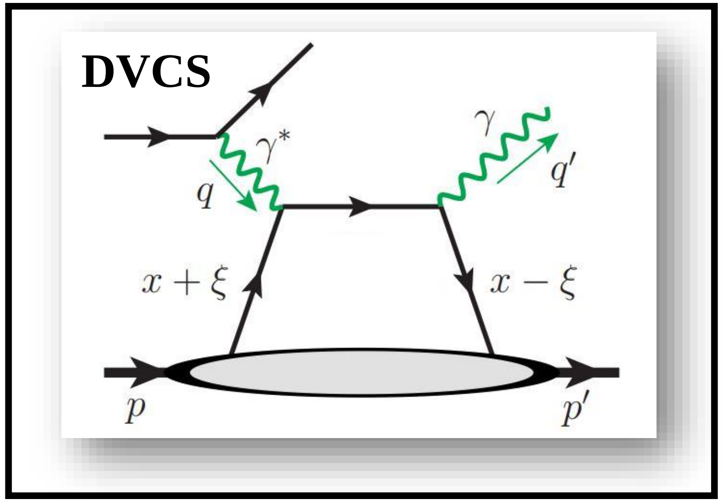
Relation with PDFs & FFs:

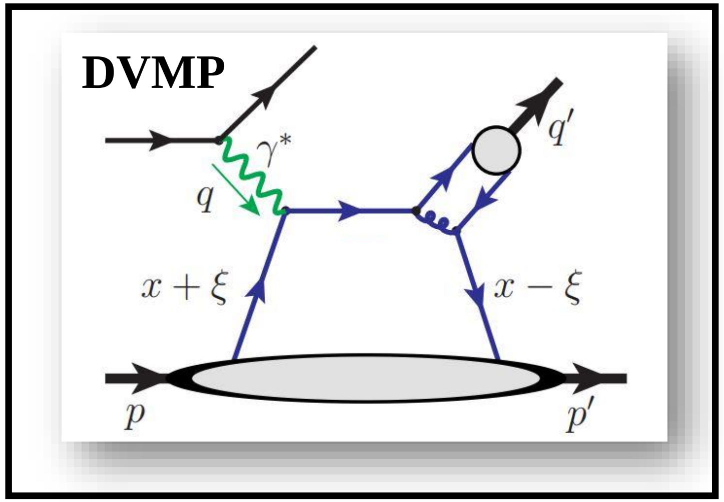


Background

What? **Why?** How?

Physical processes giving access to GPDs:





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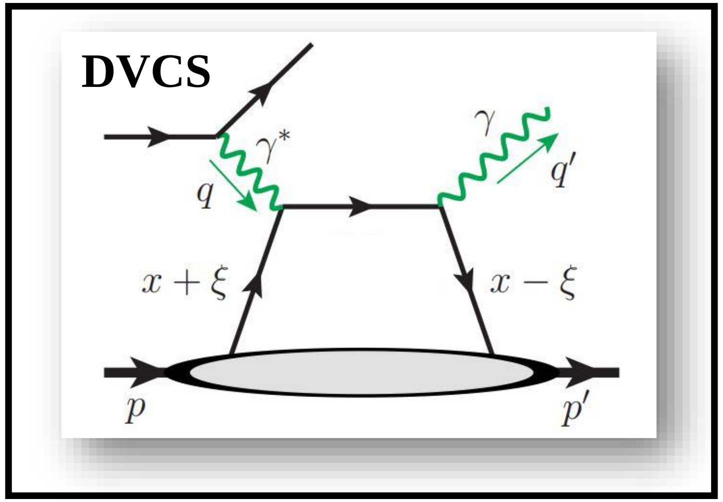
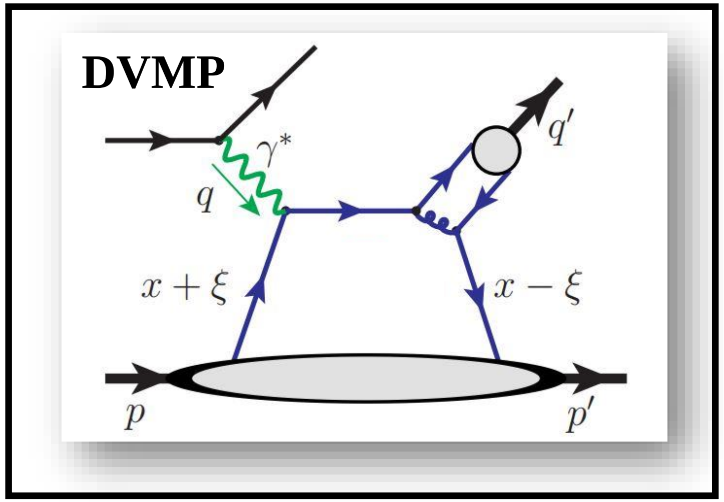




Amplitude:

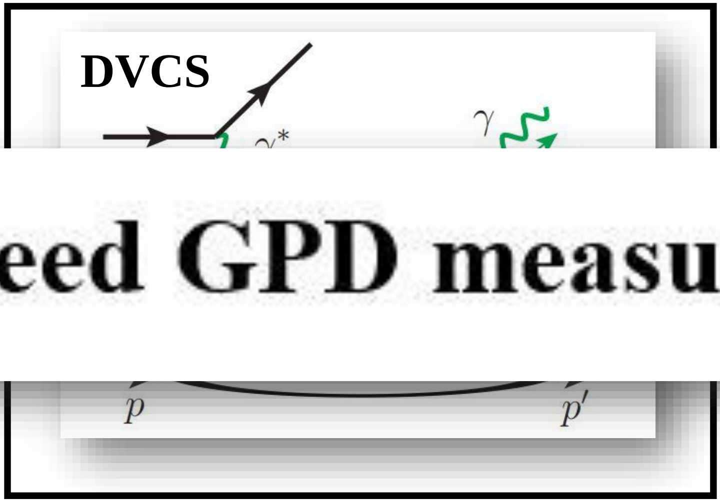
$$\mathcal{M} \propto \int_{-1}^{1} dx \frac{F(x,\xi,t)}{x \pm \xi + i\epsilon}$$

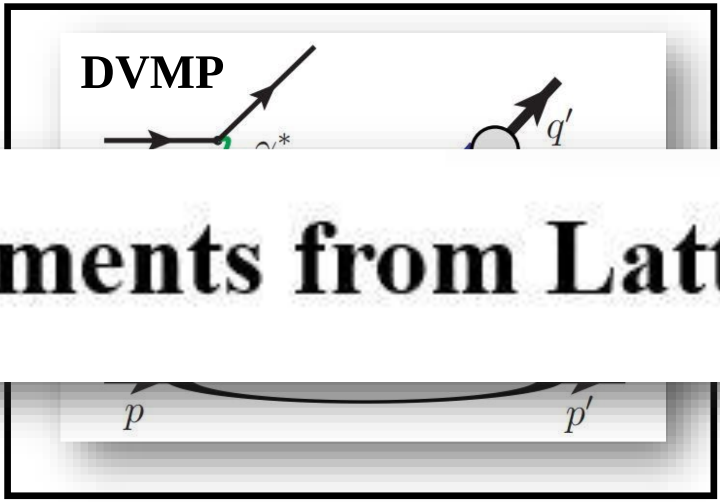
x-dependence lost!

Background

What? **Why?** How?

Physical processes giving access to GPDs:





We need GPD measurements from Lattice QCD

Amplitude:

$$\mathcal{M} \propto \int_{-1}^{1} dx \frac{F(x,\xi,t)}{x \pm \xi + i\epsilon}$$

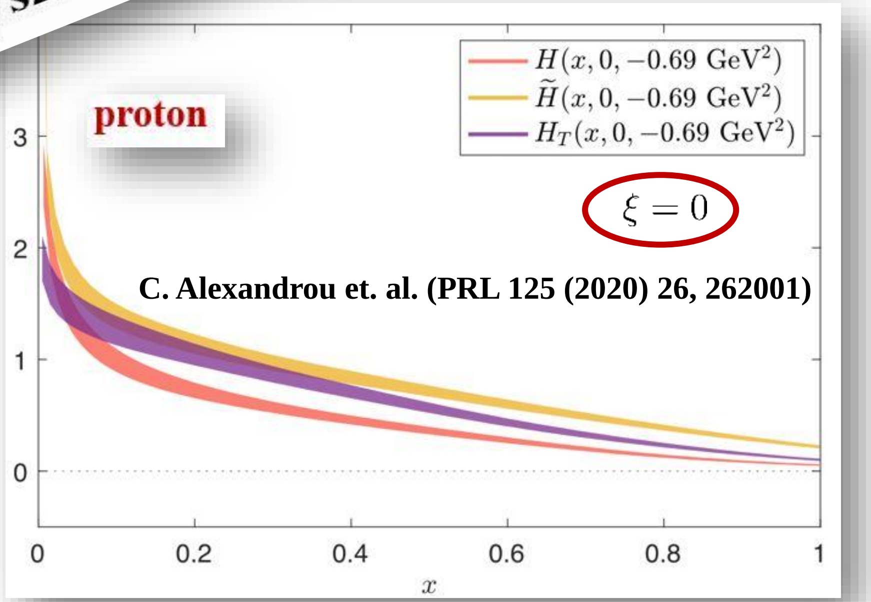
x-dependence lost!

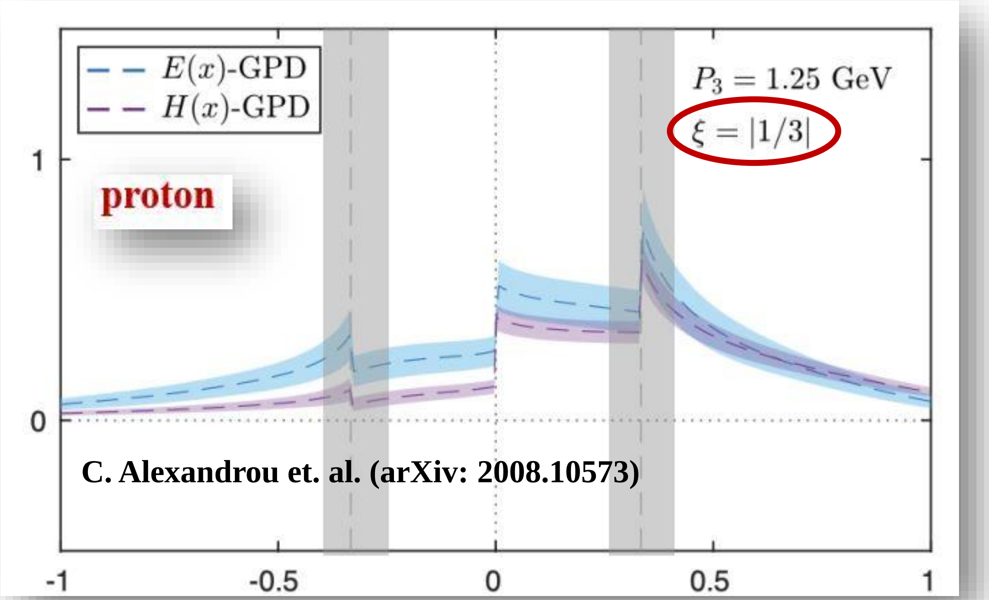
Background

What? **Why?** How?

Pioneering Lattice QCD calculations of GPDs

Krzysztof's talk



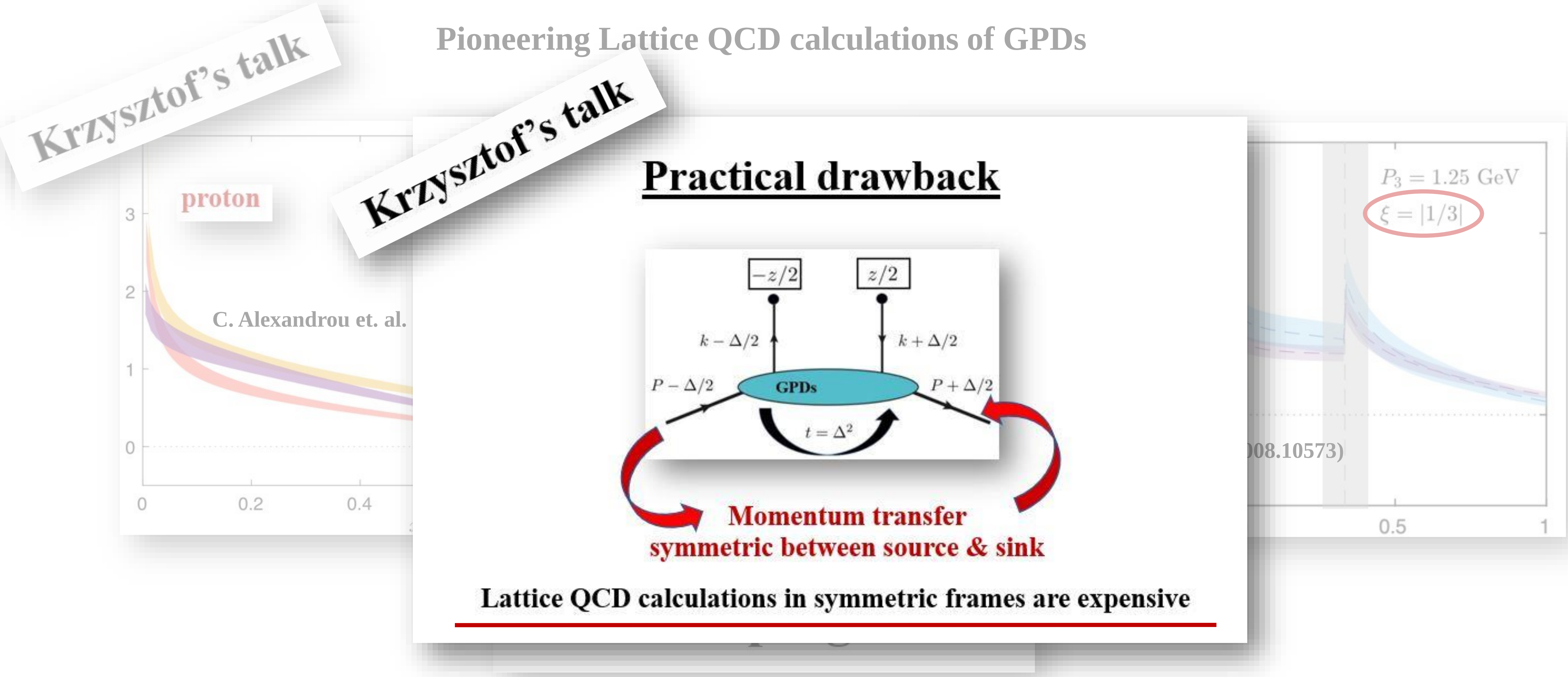


Excellent progress!!!

Background

What? **Why?** How?

Pioneering Lattice QCD calculations of GPDs

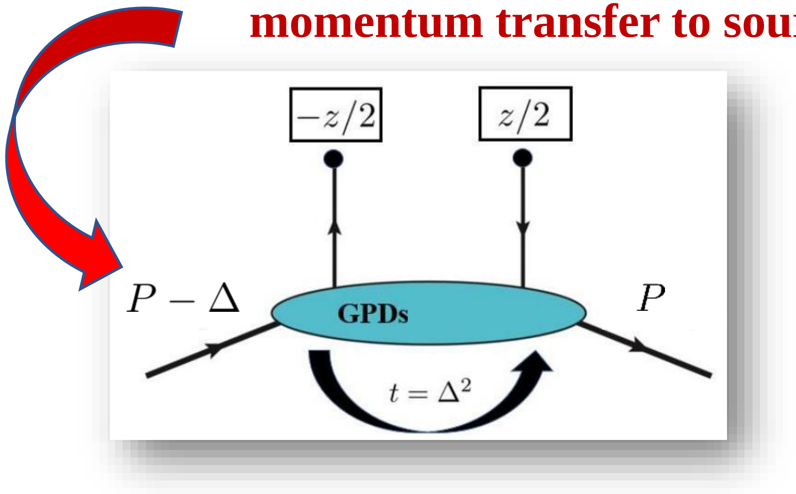


Background

What? Why? How?

Resolution:

All
momentum transfer to source



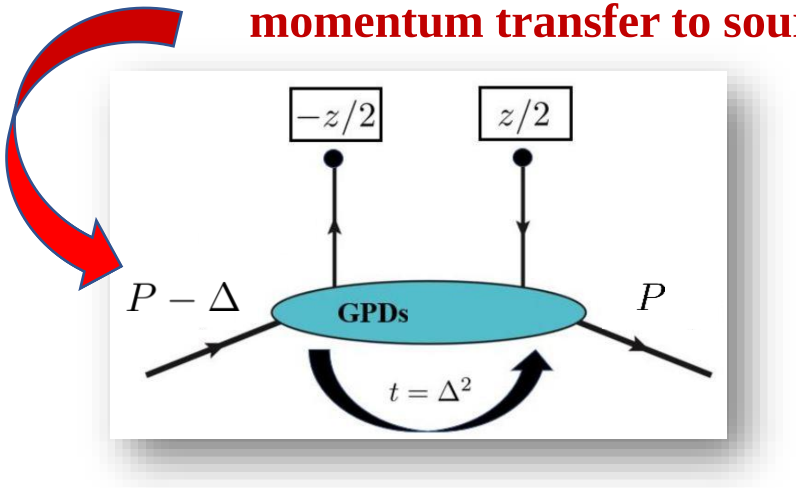
- **Perform Lattice QCD calculations of GPDs in asymmetric frames**

Background

What? Why? **How?**

Our contribution in a nutshell:

All momentum transfer to source



Key findings: QCD calculations of GPDs in asymmetric frames

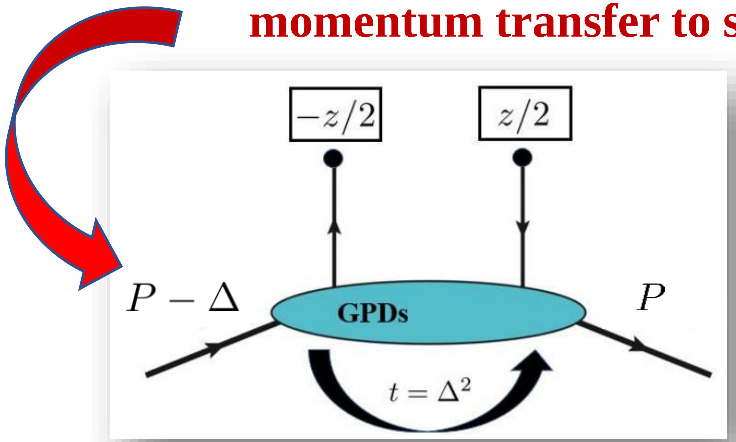
- **Lorentz covariant formalism for calculating quasi-GPDs in any frame**

Background

What? Why? How?

Our contribution in a nutshell:

All momentum transfer to source

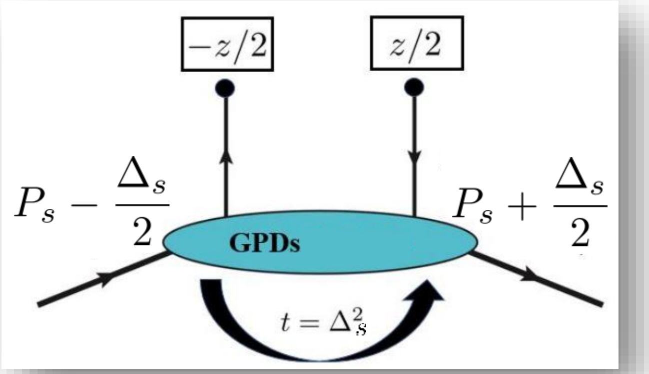


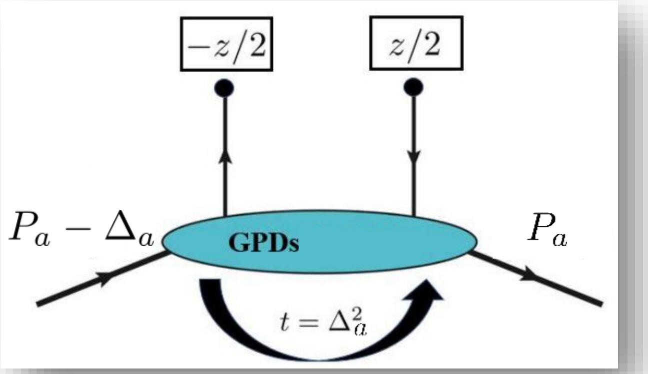
Key findings: QCD calculations of GPDs in asymmetric frames

- **Lorentz covariant formalism for calculating quasi-GPDs in any frame**
- **Elimination of power corrections potentially allowing faster convergence to light-cone GPDs at LO**

Main results

Symmetric & asymmetric frames





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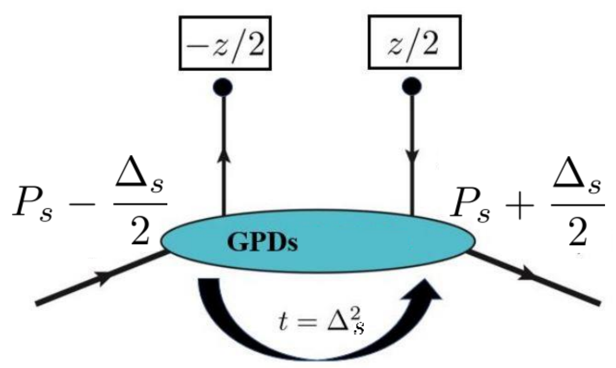
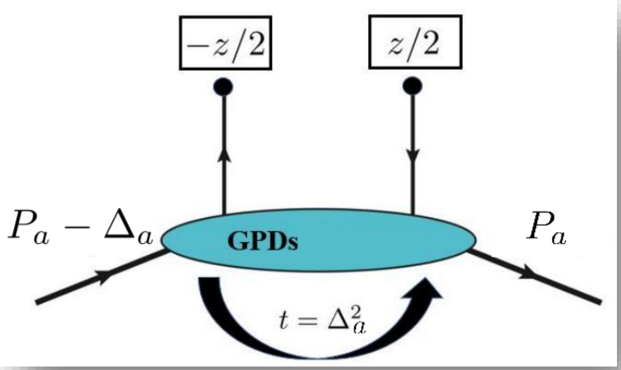
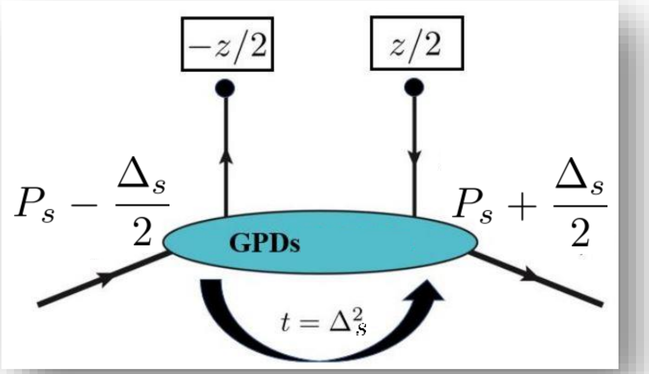




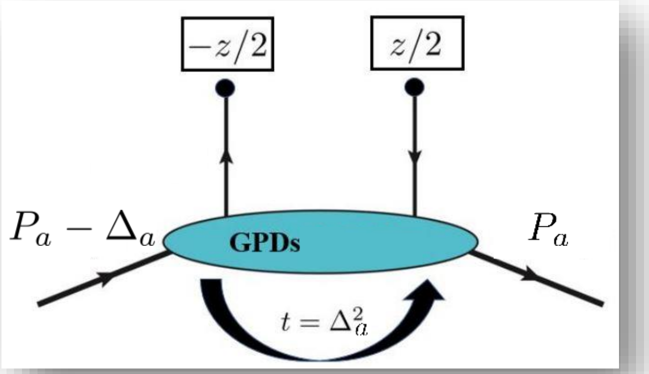
Approach 1: Can we calculate a quasi-GPD in symmetric frame through an asymmetric frame?

Main results

Symmetric & asymmetric frames



Related via Lorentz transformation?



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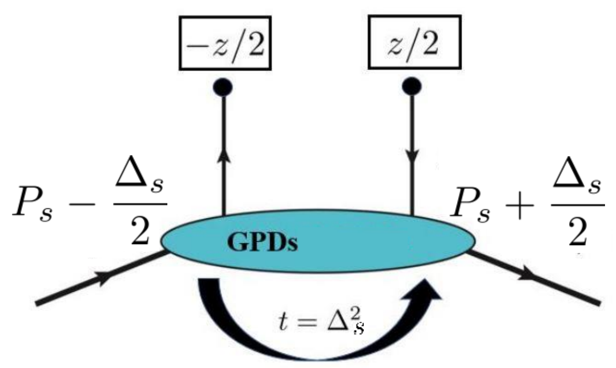

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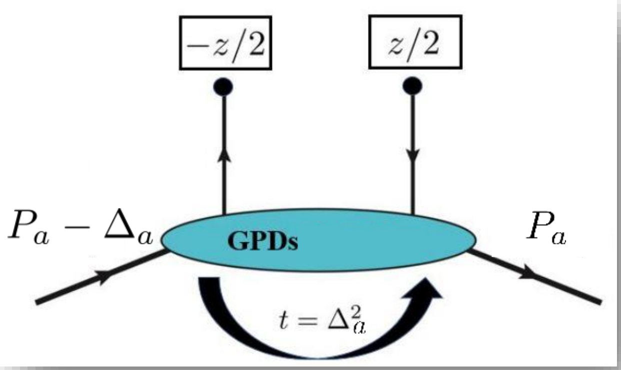
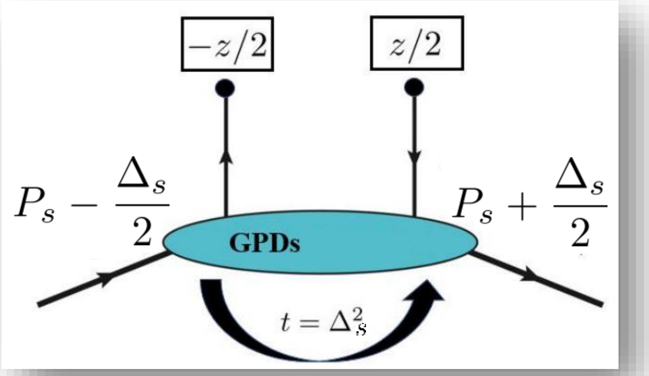
What kind?



Yes, since symmetric & asymmetric frames are connected via Lorentz transformation

Main results

Symmetric & asymmetric frames



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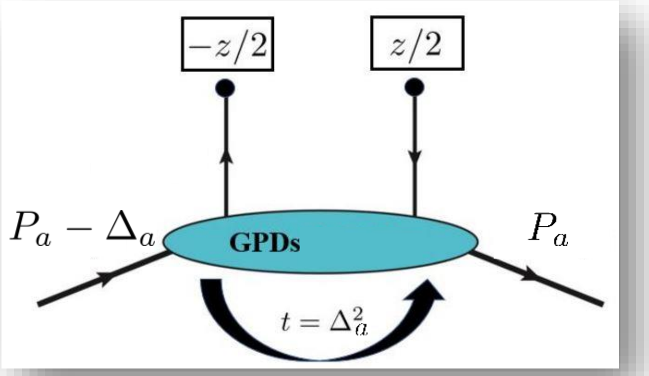


Case 1: Lorentz transformation in the z-direction

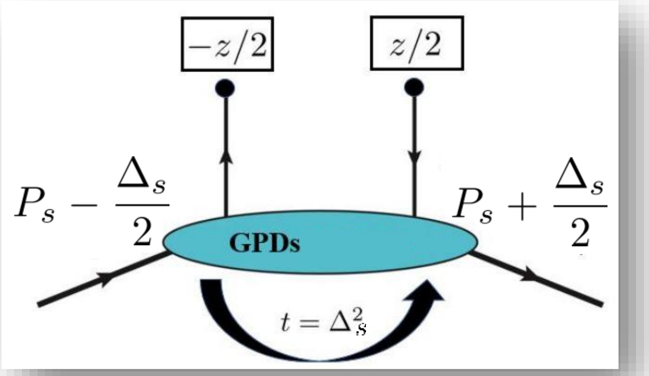
$$\begin{pmatrix} z_s^0 \\ z_s^x \\ z_s^z \end{pmatrix} = \begin{pmatrix} \gamma & 0 & -\gamma\beta \\ 0 & 1 & 0 \\ -\gamma\beta & 0 & \gamma \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ z_a^z \end{pmatrix}$$

$\bar{\psi}$ ψ

$-z^z/2$ $z^z/2$

Main results

Symmetric & asymmetric frames



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↔

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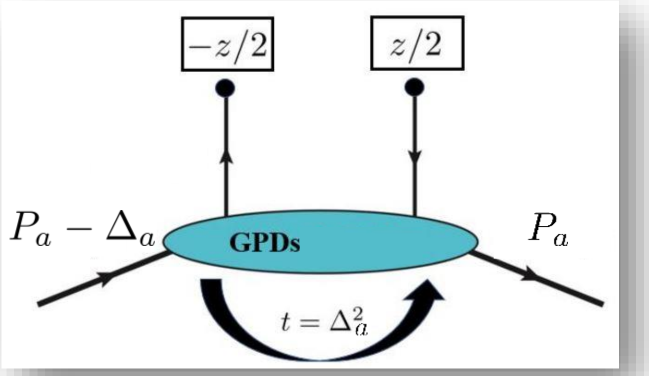

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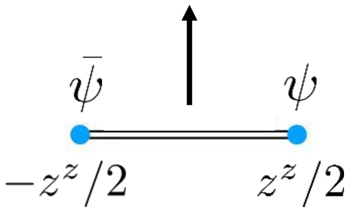


Results:

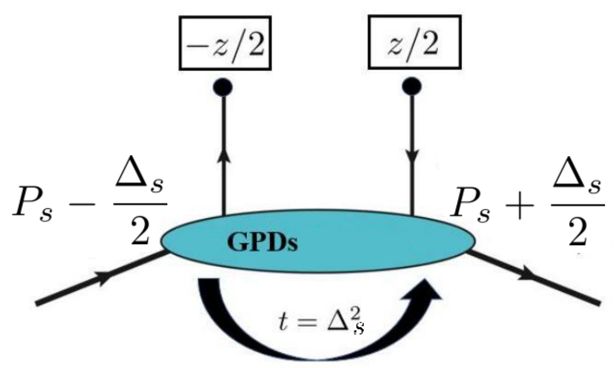
$$z_s^0 = -\gamma\beta z_a^z$$

$$z_s^z = \gamma z_a^z$$

Operator distance develops a non-zero temporal component

Main results

Symmetric & asymmetric frames



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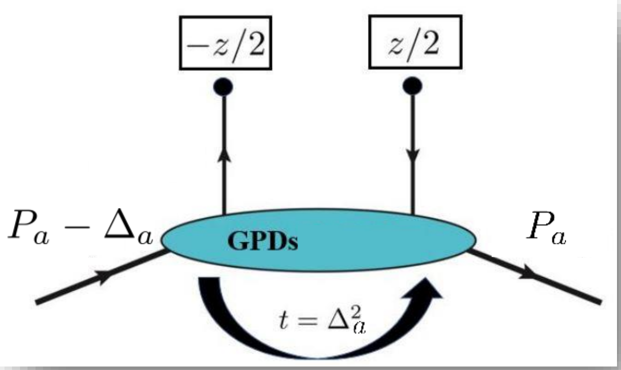
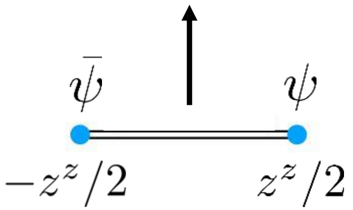


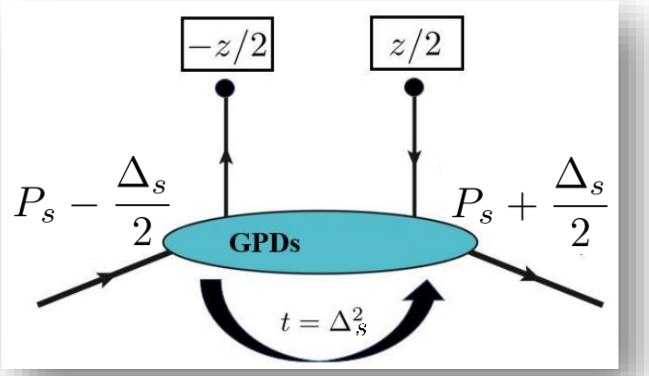
Case 2: Transverse boost in the x-direction

$$\begin{pmatrix} z_s^0 \\ z_s^x \\ z_s^z \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta & 0 \\ -\gamma\beta & \gamma & 0 \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ z_a^z \end{pmatrix}$$



Main results

Symmetric & asymmetric frames



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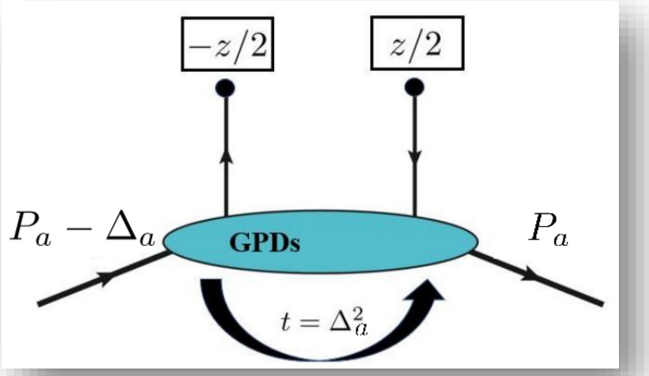

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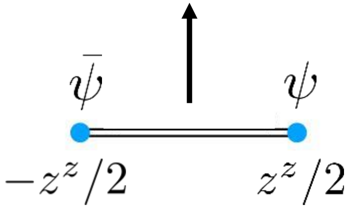


Results:

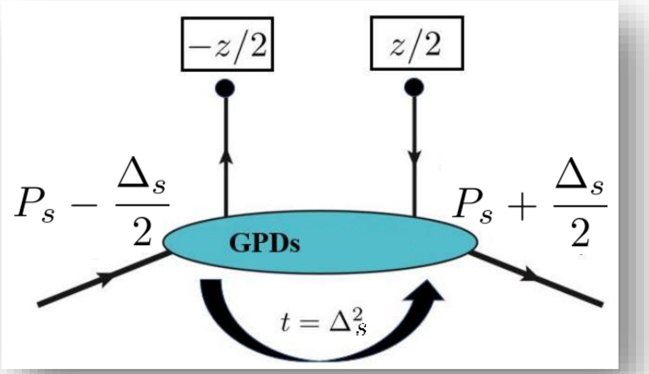
$$z_s^0 = 0$$

$$z_s^z = z_a^z$$

Operator distance remains spatial (& same)

Main results

Symmetric & asymmetric frames



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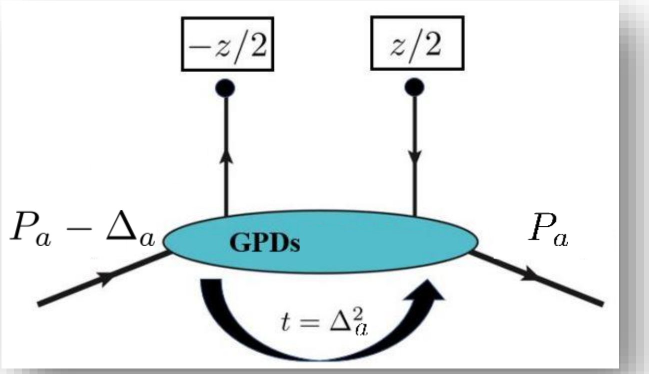


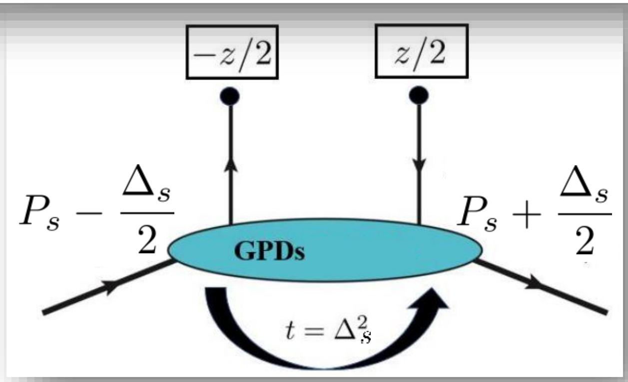
Case 2: Transver

Approach 1: Can we calculate a quasi-GPD in symmetric frame through an asymmetric frame?

Transverse boost: This Lorentz transformation allows for an exact calculation of quasi-GPDs in symmetric frame through matrix elements of asymmetric frame

Main results

Approach 2: Why does it matter in which frame quasi-GPDs are calculated?



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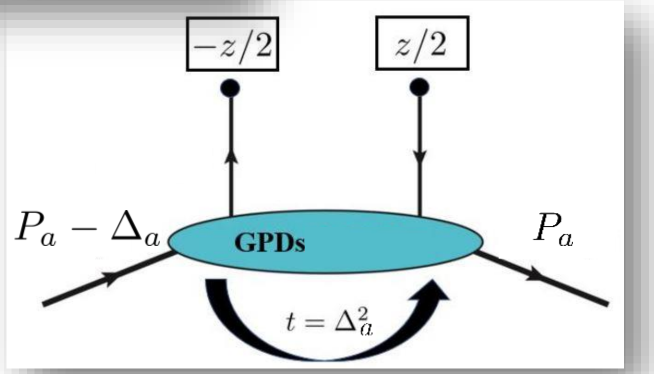

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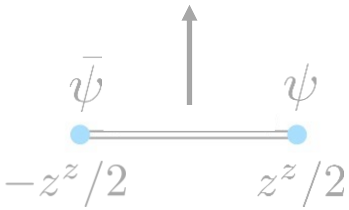

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Key points:

P_s

GPDs on the light-cone:

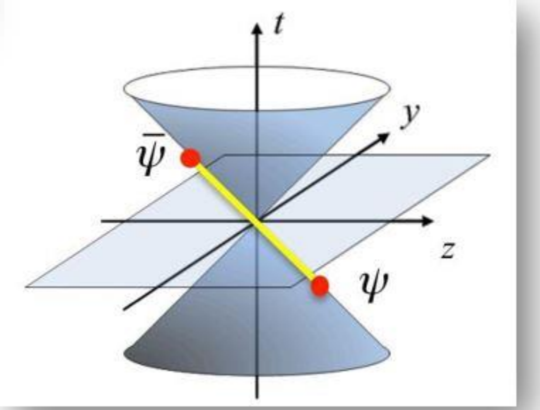

Figure courtesy: Yong Zhao

$$H(x,\xi,t) \to \int \frac{dz^-}{4\pi} e^{ixP\cdot z} \langle p'|\bar{q}\,\gamma^+ q|p\rangle \qquad z = (0, z^-, 0_\perp)$$

$$H(x,\xi,t) \to \int \frac{d(P\cdot z)}{4\pi} e^{ixP\cdot z} \frac{1}{P\cdot z} \langle p'|\bar{q}\,\not{z}\, q|p\rangle \qquad \text{Arbitrary light-like } z$$

GPDs on the light-cone can be defined in a Lorentz-invariant way

Case 2: Trans

$\begin{pmatrix} z_s^0 \\ z_s^x \\ z_s^z \end{pmatrix}$ ... $0 \quad 0 \quad 1$ ... z_a^z

$z_s^z = z_a^z$

Operator distance remains spatial (& same)

$\bar{\psi}$ ψ

$-z^z/2$ $z^z/2$

Main results

Approach 2: Why does it matter in which frame quasi-GPDs are calculated?

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P_s

Case 2: Trans

tor distance remains
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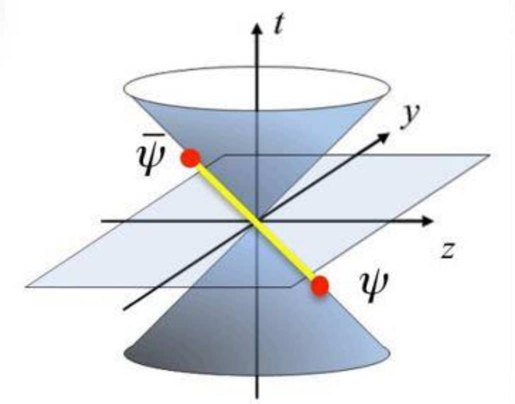

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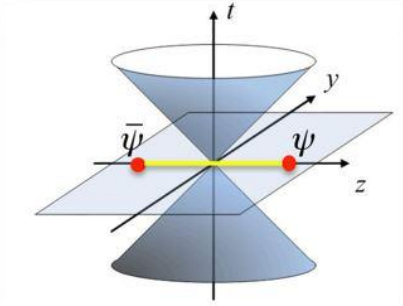


Figure courtesy: Yong Zhao

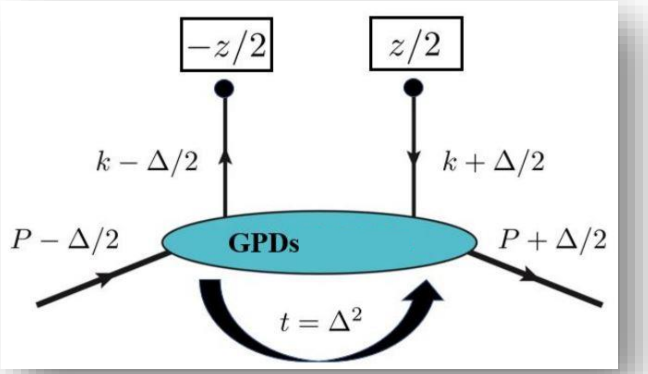
Are quasi-GPDs Lorentz-invariant?

Main results

Definitions of quasi-GPDs

Main results

Definitions of quasi-GPDs



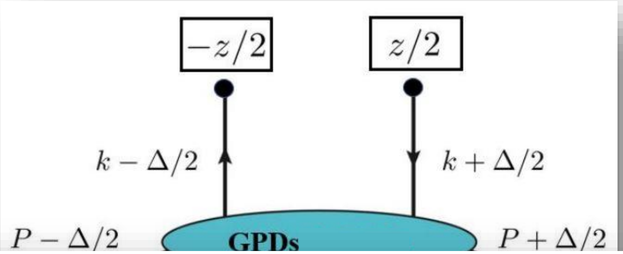
Definition of quasi-GPDs in symmetric frames: (Historical)

$$F^0_{\lambda,\lambda'}\Big|_s = \langle p'_s, \lambda' | \bar{q}(-z/2)\gamma^0 q(z/2) | p_s, \lambda\rangle \Big|_{z=0,\vec{z}_\perp=\vec{0}_\perp}$$

$$= \bar{u}_s(p'_s,\lambda')\left[\gamma^0 H_{\mathrm{Q}(0)}(z,P_s,\Delta_s)\Big|_s + \frac{i\sigma^{0\mu}\Delta_{\mu,s}}{2M} E_{\mathrm{Q}(0)}(z,P_s,\Delta_s)\Big|_s\right] u_s(p_s,\lambda)$$

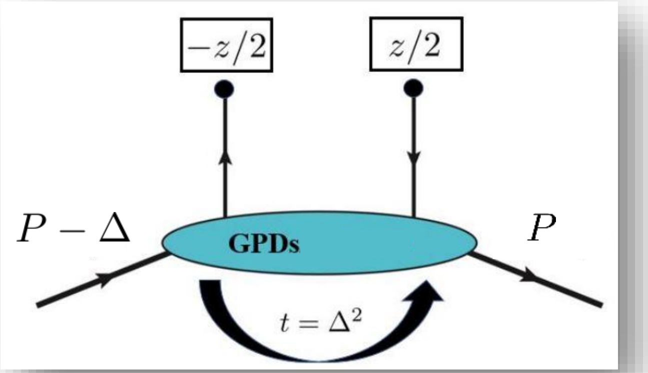
Main results

Definitions of quasi-GPDs



If this definition is Lorentz covariant:

Use γ^0



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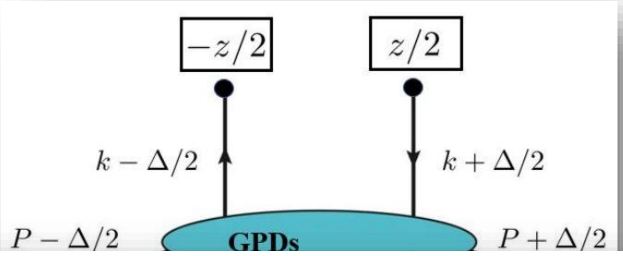
Definition of quasi-GPDs in asymmetric frames:

$$F^0_{\lambda,\lambda'}\Big|_a = \langle p'_a,\lambda'|\bar{q}(-z/2)\gamma^0 q(z/2)|p_a,\lambda\rangle\Big|_{z=0,\vec{z}_\perp=\vec{0}_\perp}$$

$$= \bar{u}_a(p'_a,\lambda')\left[\gamma^0 H_{Q(0)}(z,P_a,\Delta_a)\Big|_a + \frac{i\sigma^{0\mu}\Delta_{\mu,a}}{2M}E_{Q(0)}(z,P_a,\Delta_a)\Big|_a\right]u_a(p_a,\lambda)$$

Main results

Definitions of quasi-GPDs



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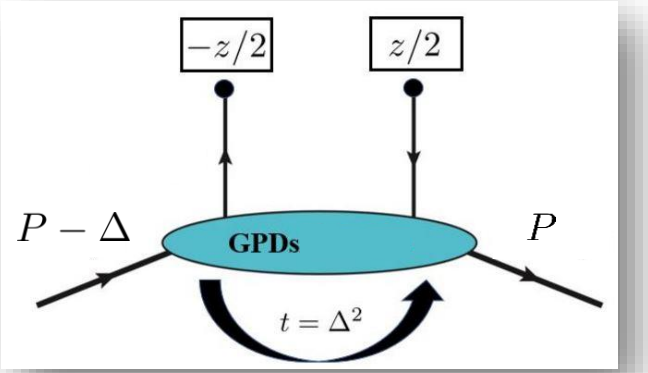
Use γ^0

Use kinematics of asymmetric frame

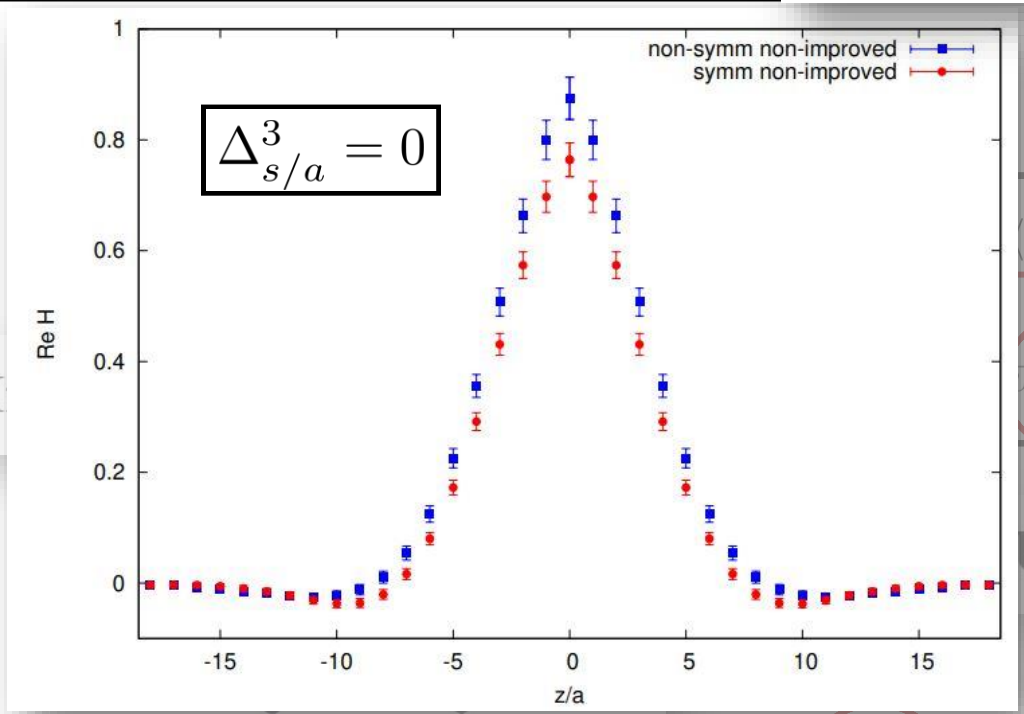
Definition of quasi-GPDs in asymmetric frames:

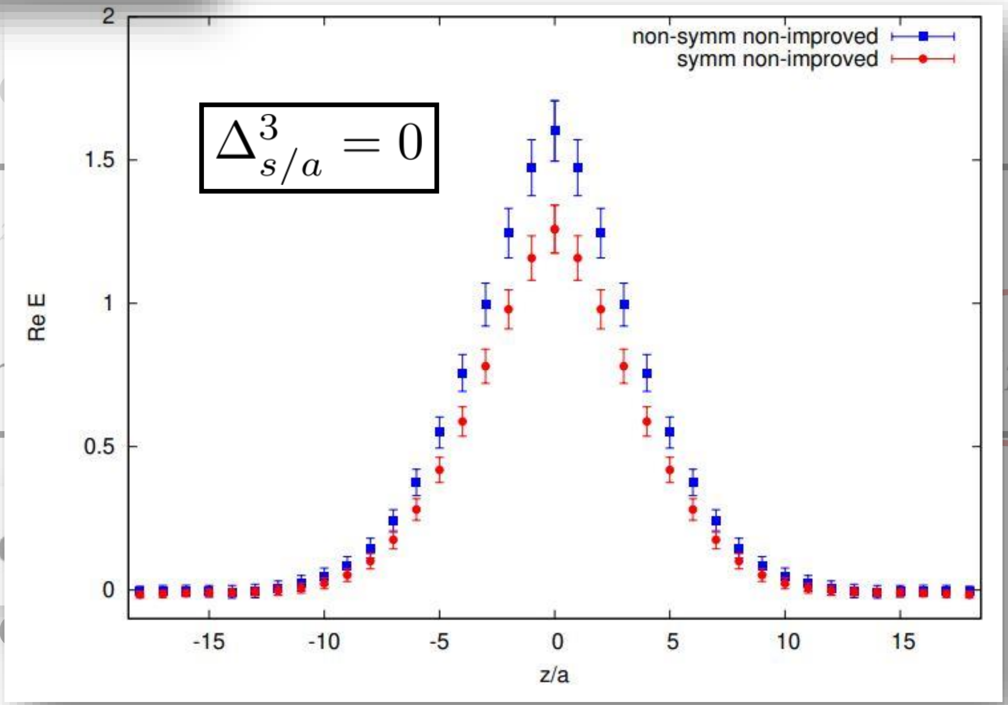


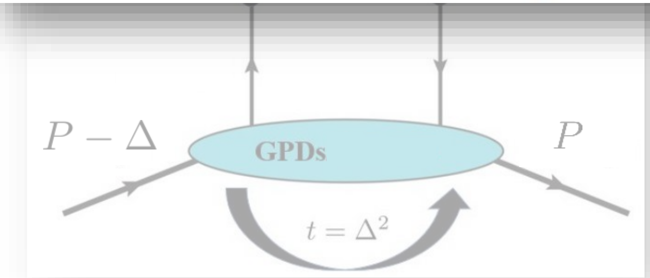
$$F^0_{\lambda,\lambda'}\Big|_a = \langle p'_a,\lambda'|\bar{q}(-z/2)\gamma^0 q(z/2)|p_a,\lambda\rangle\Big|_{z=0,\vec{z}_\perp=\vec{0}_\perp}$$
$$= \bar{u}_a(p'_a,\lambda')\left[\gamma^0 H_{Q(0)}(z,P_a,\Delta_a)\Big|_a + \frac{i\sigma^{0\mu}\Delta_{\mu,a}}{2M}E_{Q(0)}(z,P_a,\Delta_a)\Big|_a\right]u_a(p_a,\lambda)$$

Main results

Lattice QCD results







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Main results

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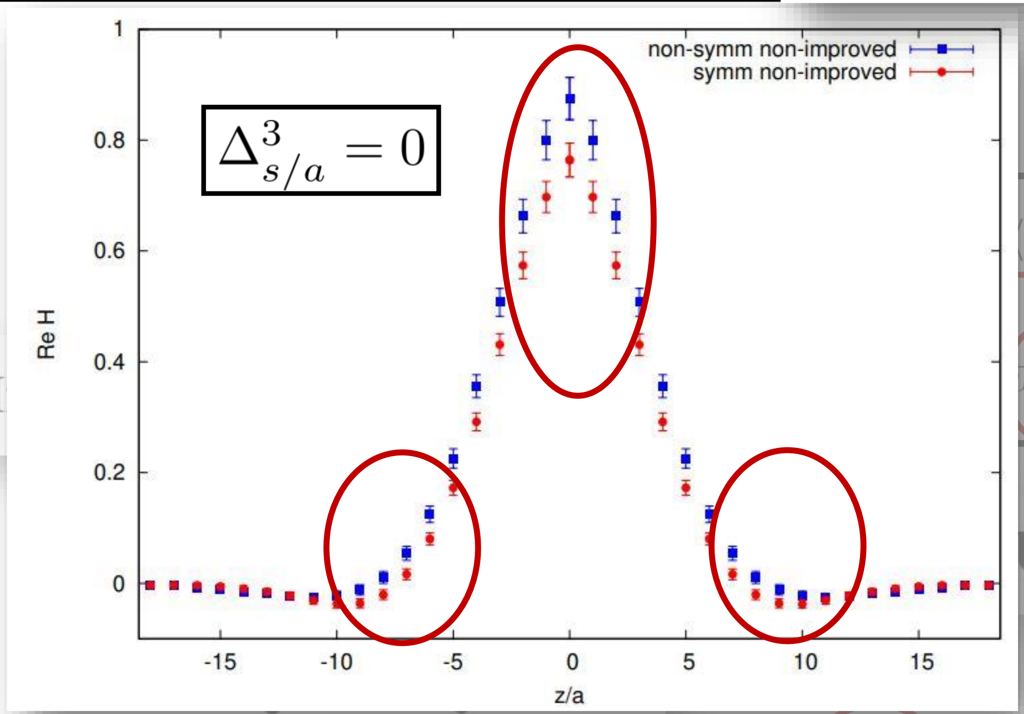
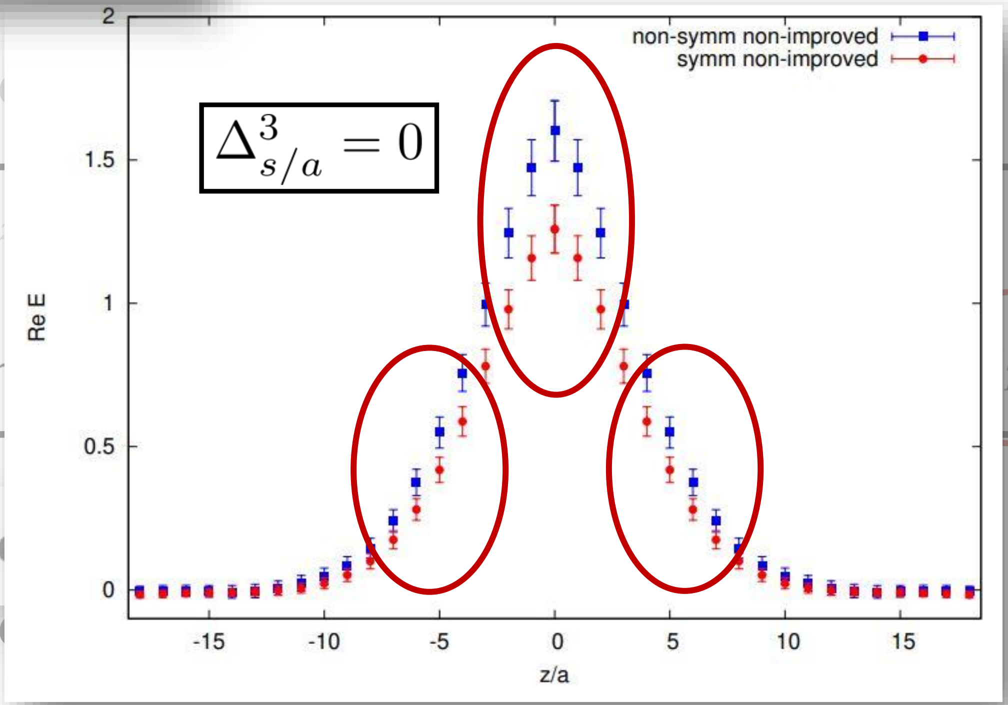


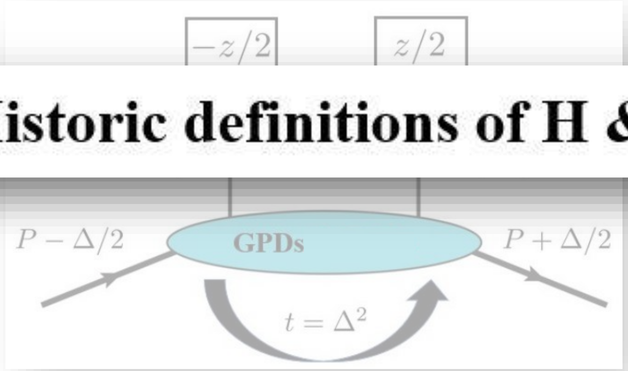


Qualitatively similar results for imaginary parts

Frame-dependence of quasi-GPDs

Main results

Definitions of quasi-GPDs

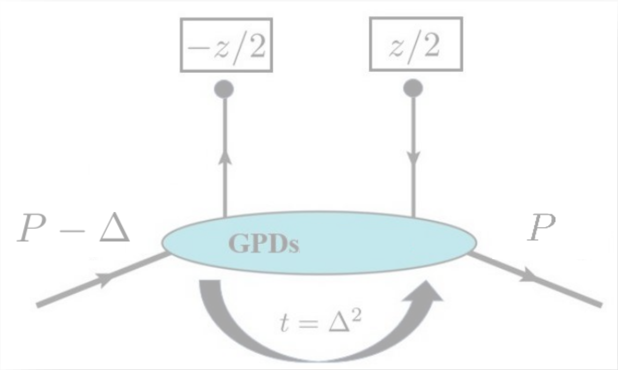


Definition of quasi-GPDs in symmetric frames: (Historical)

Historic definitions of H & E quasi-GPDs are not manifestly Lorentz invariant

$$\Big|_{z=0,\vec{z}_\perp=\vec{0}_\perp}$$

$$= \bar{u}_s(p'_s,\lambda')\left[\gamma^0 H_{\mathrm{Q}(0)}(z,P_s,\Delta_s)\Big|_s + \frac{i\sigma^{0\mu}\Delta_{\mu,s}}{2M} E_{\mathrm{Q}(0)}(z,P_s,\Delta_s)\Big|_s\right] u_s(p_s,\lambda)$$



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Main results

Definitions of quasi-GPDs

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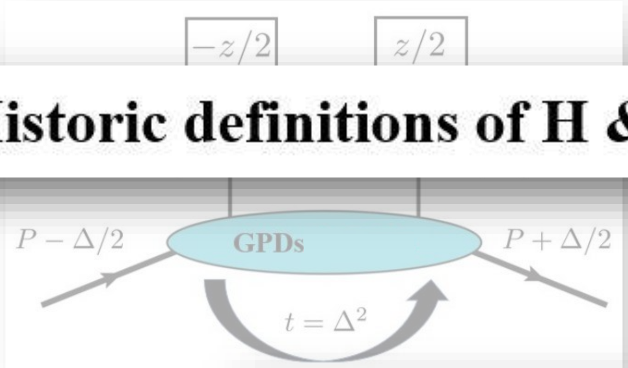

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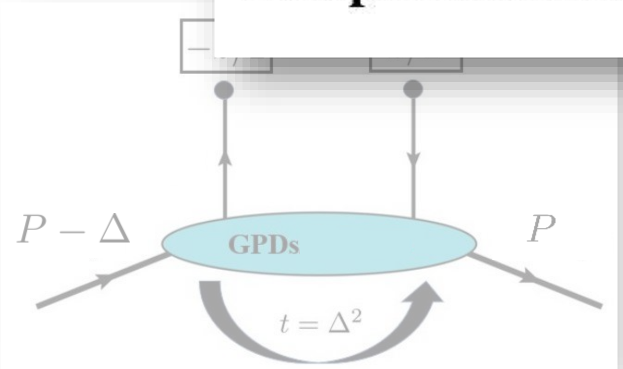
This means that the basis vectors $(\gamma^0, i\sigma^{0\Delta_{s/a}})$ do not form a complete basis for a spatially-separated bi-local operator at finite momentum



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Main results

Definitions of quasi-GPDs

Definition of quasi-GPDs in symmetric frames: (Historical)

Historic definitions of H & E quasi-GPDs are not manifestly Lorentz invariant

We do not dismiss these definitions
since they do work in the large-momentum limit (I will show this formally later)

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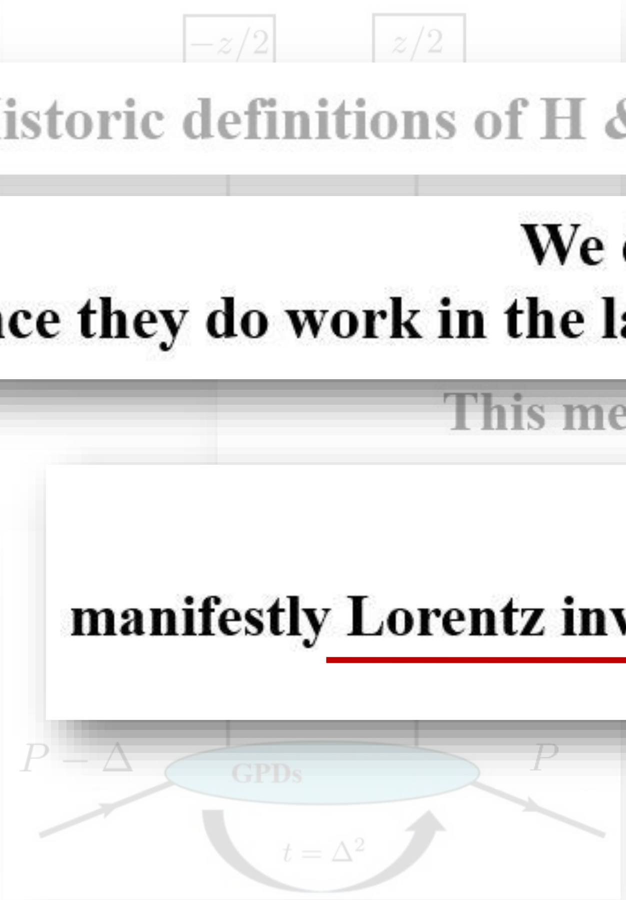
Main results

Definitions of quasi-GPDs



Definition of quasi-GPDs in symmetric frames: (Historical)

Historic definitions of H & E quasi-GPDs are not manifestly Lorentz invariant

We do not dismiss these definitions
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$u_s(p_s, \lambda)$

This means that the basis vectors $(\gamma^0, i\sigma^{0\Delta_{s/a}})$ do not form a

Can we come up with a

manifestly Lorentz invariant definition of quasi-GPDs for finite values of momentum?

$z=0, \vec{z}_\perp = \vec{0}_\perp$

$$= \bar{u}_a(p'_a, \lambda') \left[\gamma^0 H_{Q(0)}(z, P_a, \Delta_a)\Big|_a + \frac{i\sigma^{0\mu}\Delta_{\mu,a}}{2M} E_{Q(0)}(z, P_a, \Delta_a)\Big|_a \right] u_a(p_a, \lambda)$$

Main results

Lorentz covariant formalism

Novel parameterization of position-space matrix element: (Inspired from Meissner, Metz, Schlegel, 2009)

$$F^{\mu}_{\lambda,\lambda'} = \bar{u}(p',\lambda')\left[\frac{P^{\mu}}{M}\boldsymbol{A_1} + \frac{z^{\mu}}{M}\boldsymbol{A_2} + \frac{\Delta^{\mu}}{M}\boldsymbol{A_3} + \frac{i\sigma^{\mu z}}{M}\boldsymbol{A_4} + \frac{i\sigma^{\mu\Delta}}{M}\boldsymbol{A_5} + \frac{P^{\mu}i\sigma^{z\Delta}}{M^3}\boldsymbol{A_6} + \frac{z^{\mu}i\sigma^{z\Delta}}{M^3}\boldsymbol{A_7} + \frac{\Delta^{\mu}i\sigma^{z\Delta}}{M^3}\boldsymbol{A_8}\right]u(p,\lambda)$$

Vector operator $F^{\mu}_{\lambda,\lambda'} = \langle p',\lambda'|\bar{q}(-z/2)\gamma^{\mu}q(z/2)|p,\lambda\rangle\Big|_{z=0,\vec{z}_{\perp}=\vec{0}_{\perp}}$

Main results

Lorentz covariant formalism

Novel parameterization of position-space matrix element: (Vector operator)

$$F^{\mu}_{\lambda,\lambda'} = \bar{u}(p',\lambda')\left[\frac{P^\mu}{M}\boldsymbol{A_1} + \frac{z^\mu}{M}\boldsymbol{A_2} + \frac{\Delta^\mu}{M}\boldsymbol{A_3} + \frac{i\sigma^{\mu z}}{M}\boldsymbol{A_4} + \frac{i\sigma^{\mu\Delta}}{M}\boldsymbol{A_5} + \frac{P^\mu i\sigma^{z\Delta}}{M^3}\boldsymbol{A_6} + \frac{z^\mu i\sigma^{z\Delta}}{M^3}\boldsymbol{A_7} + \frac{\Delta^\mu i\sigma^{z\Delta}}{M^3}\boldsymbol{A_8}\right]u(p,\lambda)$$

Features:

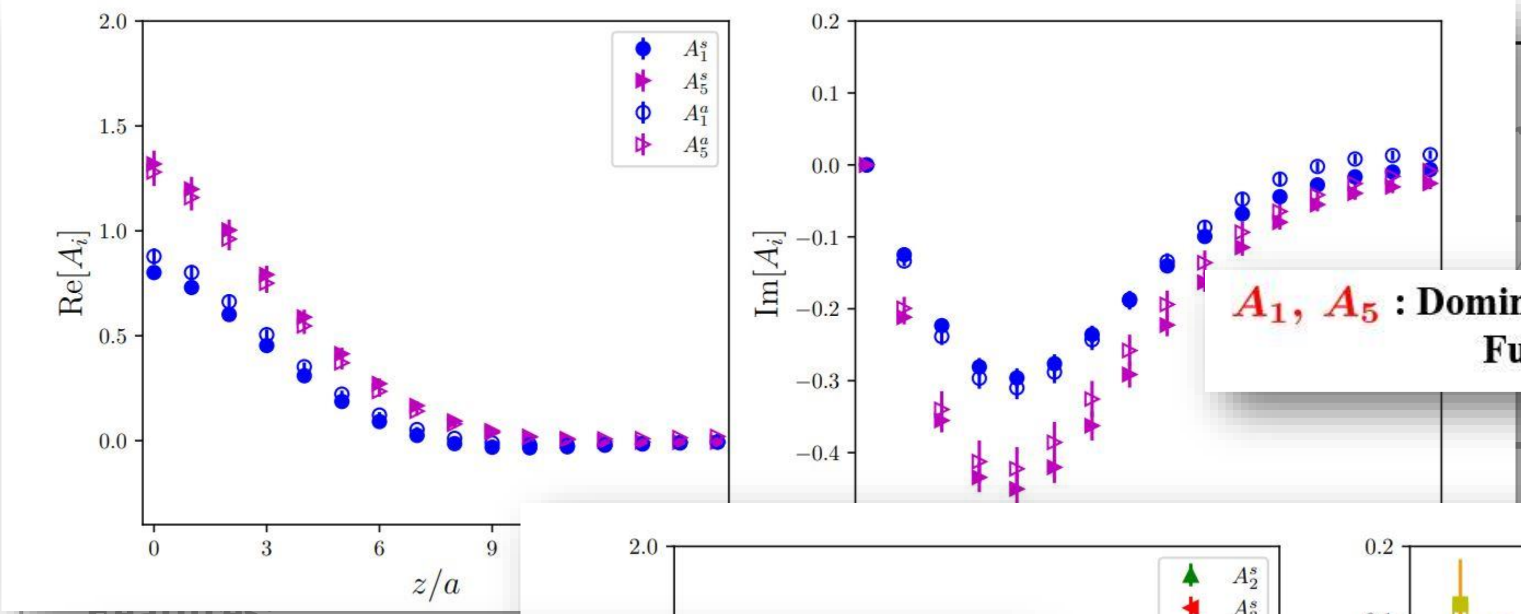
- **General structure of matrix element based on constraints from Parity**
- **8 linearly-independent Dirac structures**
- **8 Lorentz-invariant amplitudes (or Form Factors)** $A_i \equiv A_i(z\cdot P, z\cdot\Delta, t=\Delta^2, z^2)$

Validating the frame-independence of A's from Lattice QCD

Lorentz covariant formalism

Krzysztof's talk

rator)



A_1, A_5 : Dominant contributions;
Full agreement in two frames for both Re & Im parts

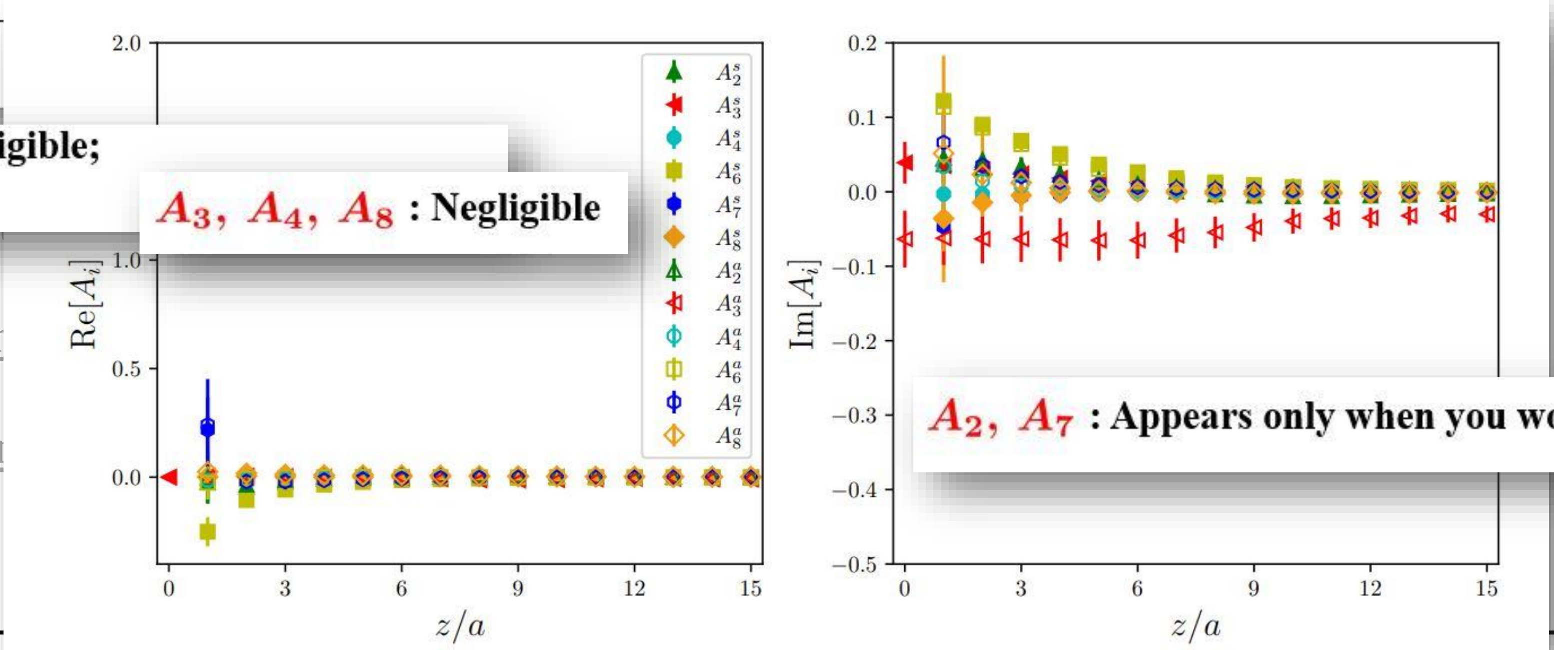
Features:

A_6 : Small but non-negligible;

General str

- 8 linearly-in
- 8 Lorentz-in

A_3, A_4, A_8 : Negligible



A_2, A_7 : Appears only when you work with γ^3 ; Negligible

Main results

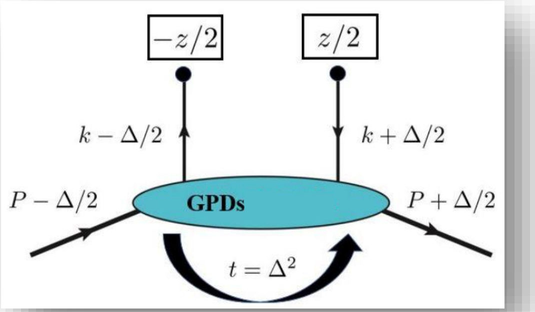
Exploring historical definitions of quasi-GPDs

Mapping Form Factors to the historical definitions of quasi-GPDs:

Main results

Exploring historical definitions of quasi-GPDs

Mapping Form Factors to the historical definitions of quasi-GPDs:



Symmetric frame:

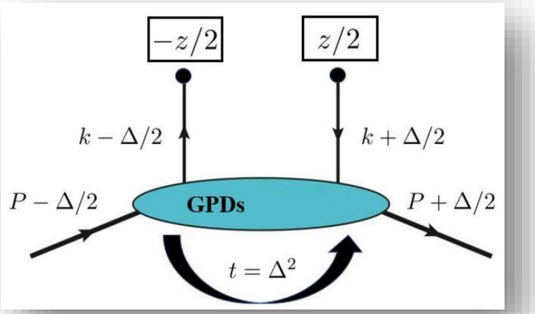
$$H_{\mathrm{Q}(0)}(z,P_s,\Delta_s)\big|_s = A_1 + \frac{\Delta_s^0}{P_s^0}A_3 - \frac{\Delta_s^0 z^3}{2P_s^0 P_s^3}A_4 + \left(\frac{(\Delta_s^0)^2 z^3}{2M^2 P_s^3} - \frac{\Delta_s^0\Delta_s^3 z^3 P_s^0}{2M^2 (P_s^3)^2} - \frac{z^3\Delta_\perp^2}{2M^2 P_s^3}\right)A_6 + \left(\frac{(\Delta_s^0)^3 z^3}{2M^2 P_s^0 P_s^3} - \frac{(\Delta_s^0)^2\Delta_s^3 z^3}{2M^2 (P_s^3)^2} - \frac{\Delta_s^0 z^3 \Delta_\perp^2}{2M^2 P_s^0 P_s^3}\right)A_8$$

Main results

Exploring historical definitions of quasi-GPDs

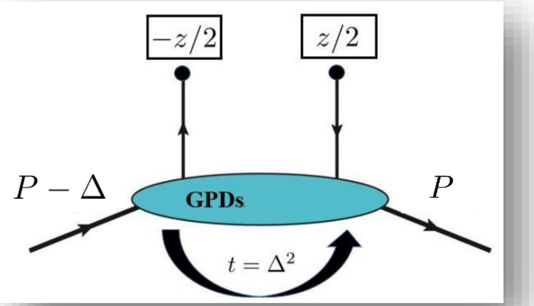
Mapping Form Factors to the historical definitions of quasi-GPDs:

Symmetric frame:



$$H_{\mathrm{Q}(0)}(z,P_s,\Delta_s)\big|_s = \boldsymbol{A_1} + \frac{\Delta_s^0}{P_s^0}\boldsymbol{A_3} - \frac{\Delta_s^0 z^3}{2P_s^0 P_s^3}\boldsymbol{A_4} + \left(\frac{(\Delta_s^0)^2 z^3}{2M^2 P_s^3} - \frac{\Delta_s^0 \Delta_s^3 z^3 P_s^0}{2M^2 (P_s^3)^2} - \frac{z^3 \Delta_\perp^2}{2M^2 P_s^3}\right)\boldsymbol{A_6} + \left(\frac{(\Delta_s^0)^3 z^3}{2M^2 P_s^0 P_s^3} - \frac{(\Delta_s^0)^2 \Delta_s^3 z^3}{2M^2 (P_s^3)^2} - \frac{\Delta_s^0 z^3 \Delta_\perp^2}{2M^2 P_s^0 P_s^3}\right)\boldsymbol{A_8}$$

Asymmetric frame:

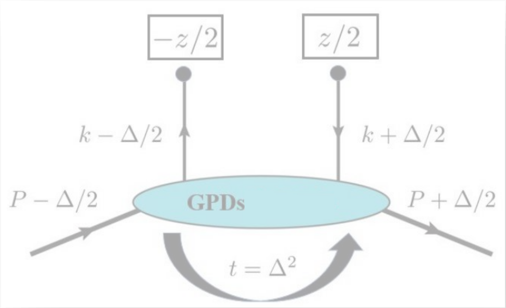


$$H_{\mathrm{Q}(0)}\big|_a(z,P_a,\Delta_a) = \boldsymbol{A_1} + \frac{\Delta_a^0}{P_{avg,a}^0}\boldsymbol{A_3} - \left(\frac{\Delta_a^0 z^3}{2P_{avg,a}^0 P_{avg,a}^3} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{\Delta_a^0 \Delta_a^3 z^3}{4P_{avg,a}^0 (P_{avg,a}^3)^2}\right)\boldsymbol{A_4} + \left(\frac{(\Delta_a^0)^2 z^3}{2M^2 P_{avg,a}^3} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{(\Delta_a^0)^2 \Delta_a^3 z^3}{4M^2 (P_{avg,a}^3)^2} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{P_{avg,a}^0 \Delta_a^0 \Delta_a^3 z^3}{2M^2 (P_{avg,a}^3)^2} - \frac{z^3 \Delta_\perp^2}{2M^2 P_{avg,a}^3}\right)\boldsymbol{A_6} + \left(\frac{(\Delta_a^0)^3 z^3}{2M^2 P_{avg,a}^0 P_{avg,a}^3} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{(\Delta_a^0)^3 \Delta_a^3 z^3}{4M^2 P_{avg,a}^0 (P_{avg,a}^3)^2} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{(\Delta_a^0)^2 \Delta_a^3 z^3}{2M^2 (P_{avg,a}^3)^2} - \frac{z^3 \Delta_\perp^2 \Delta_a^0}{2M^2 P_{avg,a}^0 P_{avg,a}^3}\right)\boldsymbol{A_8}$$

Main results

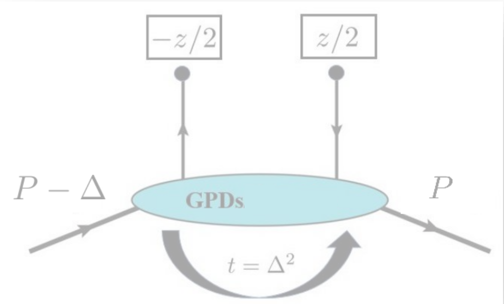
Exploring historical definitions of quasi-GPDs

Frame-dependent expressions: Explicit non-invariance from kinematics factors



Symmetric frame:

$$H_{\mathrm{Q}(0)}(z,P_s,\Delta_s)\big|_s = A_1 + \frac{\Delta_s^0}{P_s^0}A_3 - \frac{\Delta_s^0 z^3}{2P_s^0P_s^3}A_4 + \left(\frac{(\Delta_s^0)^2 z^3}{2M^2P_s^3} - \frac{\Delta_s^0\Delta_s^3 z^3 P_s^0}{2M^2(P_s^3)^2} - \frac{z^3\Delta_\perp^2}{2M^2P_s^3}\right)A_6 + \left(\frac{(\Delta_s^0)^3 z^3}{2M^2P_s^0P_s^3} - \frac{(\Delta_s^0)^2\Delta_s^3 z^3}{2M^2(P_s^3)^2} - \frac{\Delta_s^0 z^3\Delta_\perp^2}{2M^2P_s^0P_s^3}\right)A_8$$



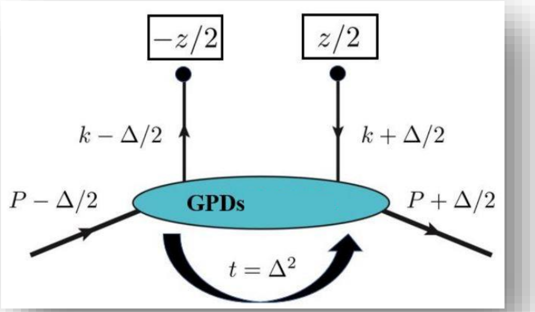
Asymmetric frame:

$$\begin{aligned} H_{\mathrm{Q}(0)}\big|_a(z,P_a,\Delta_a) &= A_1 + \frac{\Delta_a^0}{P^0_{avg,a}}A_3 - \left(\frac{\Delta_a^0 z^3}{2P^0_{avg,a}P^3_{avg,a}} - \frac{1}{(1+\frac{\Delta_a^3}{2P^3_{avg,a}})}\frac{\Delta_a^0\Delta_a^3 z^3}{4P^0_{avg,a}(P^3_{avg,a})^2}\right)A_4 \\ &+ \left(\frac{(\Delta_a^0)^2 z^3}{2M^2P^3_{avg,a}} - \frac{1}{(1+\frac{\Delta_a^3}{2P^3_{avg,a}})}\frac{(\Delta_a^0)^2\Delta_a^3 z^3}{4M^2(P^3_{avg,a})^2} - \frac{1}{(1+\frac{\Delta_a^3}{2P^3_{avg,a}})}\frac{P^0_{avg,a}\Delta_a^0\Delta_a^3 z^3}{2M^2(P^3_{avg,a})^2} - \frac{z^3\Delta_\perp^2}{2M^2P^3_{avg,a}}\right)A_6 \\ &+ \left(\frac{(\Delta_a^0)^3 z^3}{2M^2P^0_{avg,a}P^3_{avg,a}} - \frac{1}{(1+\frac{\Delta_a^3}{2P^3_{avg,a}})}\frac{(\Delta_a^0)^3\Delta_a^3 z^3}{4M^2P^0_{avg,a}(P^3_{avg,a})^2} - \frac{1}{(1+\frac{\Delta_a^3}{2P^3_{avg,a}})}\frac{(\Delta_a^0)^2\Delta_a^3 z^3}{2M^2(P^3_{avg,a})^2} - \frac{z^3\Delta_\perp^2\Delta_a^0}{2M^2P^0_{avg,a}P^3_{avg,a}}\right)A_8 \end{aligned}$$

Main results

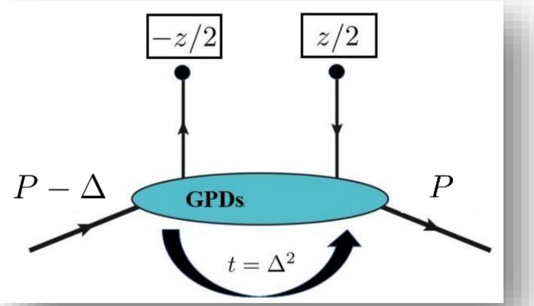
Light-cone GPDs

Mapping Form Factors to the light-cone GPDs: (Sample results)



Lorentz-invariant definition:

$$H(x,\xi,t) \to \int \frac{d(P\cdot z)}{4\pi} e^{ixP\cdot z} \frac{1}{P\cdot z} \langle p'|\bar{q}\,\not{z}\,q|p\rangle$$



Relation between light-cone GPD H & Form Factors:

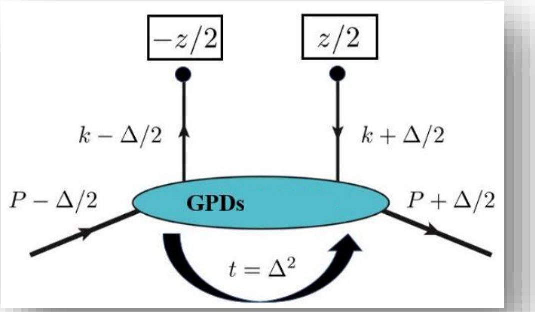
$$H(z\cdot P, z\cdot\Delta, t=\Delta^2, z^2) = \mathbf{A_1} + \frac{\Delta_{s/a}\cdot z}{P_{avg,s/a}\cdot z}\mathbf{A_3}$$

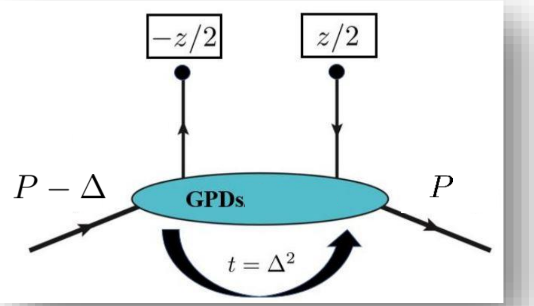
Lorentz-invariant expression

Main results

Sketch of the essence of a Lorentz-invariant definition of quasi-GPDs

Lorentz covariant formalism





Main results

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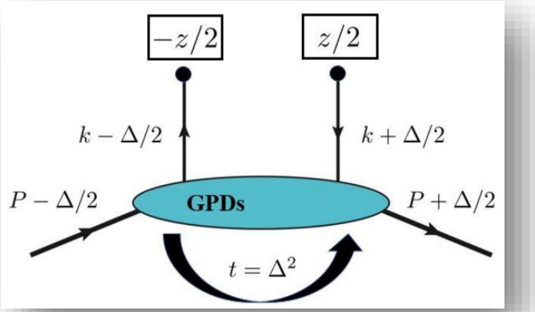
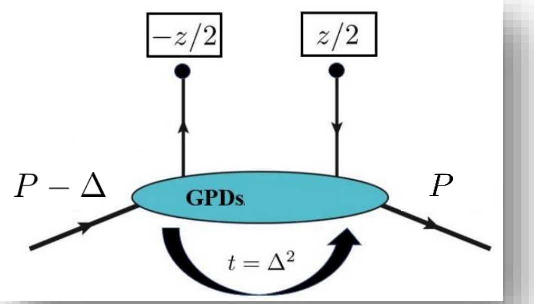
si-GPDs & Form Factors: (Sample results)

Symmetric frame:



$$H_{Q(0)}(z,P_s,\Delta_s)\big|_s = A_1 + \frac{\Delta_s^0}{P_s^0}A_3 - \frac{\Delta_s^0 z^3}{2P_s^0 P_s^3}A_4 + \left(\frac{(\Delta_s^0)^2 z^3}{2M^2 P_s^3} - \frac{\Delta_s^0\Delta_s^3 z^3 P_s^0}{2M^2(P_s^3)^2} - \frac{z^3\Delta_\perp^2}{2M^2 P_s^3}\right)A_6$$
$$+ \left(\frac{(\Delta_s^0)^3 z^3}{2M^2 P_s^0 P_s^3} - \frac{(\Delta_s^0)^2\Delta_s^3 z^3}{2M^2(P_s^3)^2} - \frac{\Delta_s^0 z^3\Delta_\perp^2}{2M^2 P_s^0 P_s^3}\right)A_8$$

Asymmetric frame:



$$H_{Q(0)}\big|_a(z,P_a,\Delta_a) = A_1 + \frac{\Delta_a^0}{P^0_{avg,a}}A_3 - \left(\frac{\Delta_a^0 z^3}{2P^0_{avg,a}P^3_{avg,a}} - \frac{1}{\left(1+\frac{\Delta_a^3}{2P^3_{avg,a}}\right)}\frac{\Delta_a^0\Delta_a^3 z^3}{4P^0_{avg,a}(P^3_{avg,a})^2}\right)A_4$$
$$+ \left(\frac{(\Delta_a^0)^2 z^3}{2M^2 P^3_{avg,a}} - \frac{1}{\left(1+\frac{\Delta_a^3}{2P^3_{avg,a}}\right)}\frac{(\Delta_a^0)^2\Delta_a^3 z^3}{4M^2(P^3_{avg,a})^2} - \frac{1}{\left(1+\frac{\Delta_a^3}{2P^3_{avg,a}}\right)}\frac{P^0_{avg,a}\Delta_a^0\Delta_a^3 z^3}{2M^2(P^3_{avg,a})^2} - \frac{z^3\Delta_\perp^2}{2M^2 P^3_{avg,a}}\right)A_6$$
$$+ \left(\frac{(\Delta_a^0)^3 z^3}{2M^2 P^0_{avg,a}P^3_{avg,a}} - \frac{1}{\left(1+\frac{\Delta_a^3}{2P^3_{avg,a}}\right)}\frac{(\Delta_a^0)^3\Delta_a^3 z^3}{4M^2 P^0_{avg,a}(P^3_{avg,a})^2} - \frac{1}{\left(1+\frac{\Delta_a^3}{2P^3_{avg,a}}\right)}\frac{(\Delta_a^0)^2\Delta_a^3 z^3}{2M^2(P^3_{avg,a})^2} - \frac{z^3\Delta_\perp^2\Delta_a^0}{2M^2 P^0_{avg,a}P^3_{avg,a}}\right)A_8$$

Main results

Sketch of the essence of a Lorentz-invariant definition of quasi-GPDs

Lorentz covariant formalism

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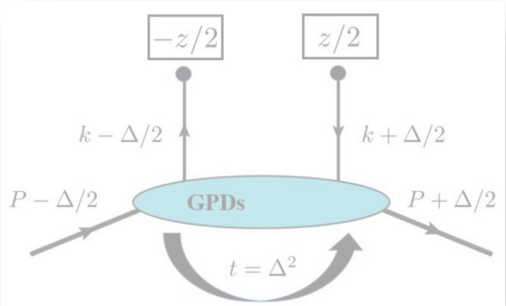
...si-GPDs & Form Factors: (Sample results)

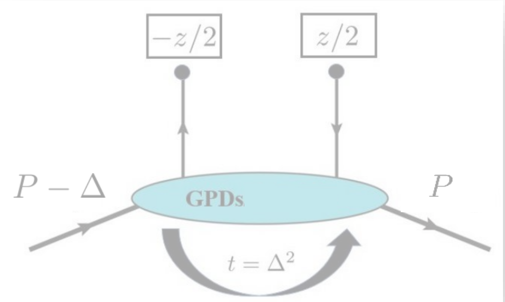
Symmetric frame:



$$H_{Q(0)}(z,P_s,\Delta_s)\big|_s = A_1 + \frac{\Delta_s^0}{P_s^0}A_3 - \frac{\Delta_s^0 z^3}{2P_s^0 P_s^3}A_4 + \left(\frac{(\Delta_s^0)^2 z^3}{2M^2 P_s^3} - \frac{\Delta_s^0\Delta_s^3 z^3 P_s^0}{2M^2 (P_s^3)^2} - \frac{z^3\Delta_\perp^2}{2M^2 P_s^3}\right)A_6 + \left(\frac{(\Delta_s^0)^3 z^3}{2M^2 P_s^0 P_s^3} - \frac{(\Delta_s^0)^2\Delta_s^3 z^3}{2M^2 (P_s^3)^2} - \frac{\Delta_s^0 z^3 \Delta_\perp^2}{2M^2 P_s^0 P_s^3}\right)A_8$$

Contamination from additional amplitudes or explicit power corrections

Asymmetric frame:



$$H_{Q(0)}\big|_a(z,P_a,\Delta_a) = A_1 + \frac{\Delta_a^0}{P_{avg,a}^0}A_3 - \left(\frac{\Delta_a^0 z^3}{2P_{avg,a}^0 P_{avg,a}^3} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{\Delta_a^0\Delta_a^3 z^3}{4P_{avg,a}^0 (P_{avg,a}^3)^2}\right)A_4$$
$$+ \left(\frac{(\Delta_a^0)^2 z^3}{2M^2 P_{avg,a}^3} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{(\Delta_a^0)^2\Delta_a^3 z^3}{4M^2 (P_{avg,a}^3)^2} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{P_{avg,a}^0\Delta_a^0\Delta_a^3 z^3}{2M^2 (P_{avg,a}^3)^2} - \frac{z^3\Delta_\perp^2}{2M^2 P_{avg,a}^3}\right)A_6$$
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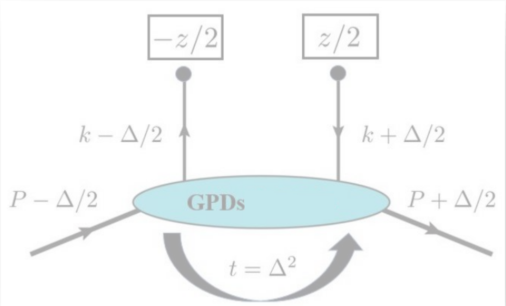

$$H_{Q(0)}(z, P_s, \Delta_s)\big|_s = A_1 + \frac{\Delta_s^0}{P_s^0} A_3 - \frac{\Delta_s^0 z^3}{2P_s^0 P_s^3} A_4 + \left(\frac{(\Delta_s^0)^2 z^3}{2M^2 P_s^3} - \frac{\Delta_s^0 \Delta_s^3 z^3 P_s^0}{2M^2 (P_s^3)^2} - \frac{z^3 \Delta_\perp^2}{2M^2 P_s^3}\right) A_6 + \left(\frac{(\Delta_s^0)^3 z^3}{2M^2 P_s^0 P_s^3} - \frac{(\Delta_s^0)^2 \Delta_s^3 z^3}{2M^2 (P_s^3)^2} - \frac{\Delta_s^0 z^3 \Delta_\perp^2}{2M^2 P_s^0 P_s^3}\right) A_8$$

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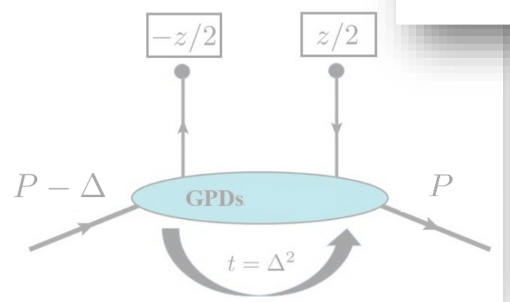
In the large-momentum limit, these expressions reduce to light-cone results



$$\dots + \left(\frac{(\Delta_a^0)^2 z^3}{2M^2 P_{avg,a}^3} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{(\Delta_a^0)^2 \Delta_a^3 z^3}{4M^2 (P_{avg,a}^3)^2} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{P_{avg,a}^0 \Delta_a^0 \Delta_a^3 z^3}{2M^2 (P_{avg,a}^3)^2} - \frac{z^3 \Delta_\perp^2}{2M^2 P_{avg,a}^3}\right) A_6$$
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Sketch of the essence of a … tion of quasi-GPDs

Lore…

Interlude:

Lorentz covariant formalism

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Let's go back to PDFs

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Contamination from additional amplitudes or explicit power corrections

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$$+\left(\frac{(\Delta_a^0)^2z^3}{2M^2P_{avg,a}^3} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{(\Delta_a^0)^2\Delta_a^3z^3}{4M^2(P_{avg,a}^3)^2} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{P_{avg,a}^0\Delta_a^0\Delta_a^3z^3}{2M^2(P_{avg,a}^3)^2} - \frac{z^3\Delta_\perp^2}{2M^2P_{avg,a}^3}\right)A_6$$

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arXiv: 1705.01488

Quasi-PDFs, momentum distributions and pseudo-PDFs

A. V. Radyushkin

Old Dominion University, Norfolk, VA 23529, USA and
Thomas Jefferson National Accelerator Facility, Newport News, VA 23606, USA

...r explicit power corrections

...m limit, these expressions reduce to light-cone results

$$\mathcal{M}^\alpha(z,p) \equiv \langle p|\bar\psi(0)\,\gamma^\alpha\,\hat E(0,z;A)\psi(z)|p\rangle \qquad (12)$$

type, where $\hat E(0,z;A)$ is the standard $0\to z$ straight-line gauge link in the quark (fundamental) representation. These matrix elements may be decomposed into p^α and z^α parts:

$$\mathcal{M}^\alpha(z,p) = 2p^\alpha \mathcal{M}_p(-(zp),-z^2) + z^\alpha \mathcal{M}_z(-(zp),-z^2)\,. \qquad (13)$$

2 Amplitudes

The $\mathcal{M}_p(-(zp),-z^2)$ part gives the twist-2 distribution when $z^2\to 0$, while $\mathcal{M}_z((zp),-z^2)$ is a purely higher-twist contamination, and it is better to get rid of it.

$$\ldots \frac{z^3\Delta_\perp^2}{2M^2P_s^3}\Big) A_6$$

$$\ldots \frac{1}{\left(1+\frac{\Delta_a^3}{2P_{avg,a}^3}\right)}\frac{P^0_{avg,a}\Delta^0_a\Delta^3_a z^3}{2M^2(P^3_{avg,a})^2} - \frac{z^3\Delta_\perp^2}{2M^2P^3_{avg,a}}\Big) A_6$$

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Main results

Let's go back to PDFs

arXiv: 1705.01488

Quasi-PDFs, momentum distributions and pseudo-PDFs

A. V. Radyushkin
Old Dominion University, Norfolk, VA 23529, USA and Thomas Jefferson National Accelerator Facility, Newport News, VA 23606, USA

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Lorentz covariant formalism

Interlude:

...GPDs & Form Factors: (Sample results)

Let's go back to PDFs

arXiv: 1705.01488

Quasi-PDFs, momentum distributions and pseudo-PDFs

A. V. Radyushkin

Old Dominion University, Norfolk, VA 23529, USA and
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...explicit power corrections

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...cone results

formula (6). For quasi-distributions, the easiest way to remove the z^α contamination is to take the time component of $\mathcal{M}^\alpha(z=z_3,p)$ and define

$$\mathcal{M}^0(z_3,p) = 2p^0\int_{-1}^{1} dy\, Q(y,P)\, e^{iyPz_3}\,. \qquad (14)$$

Therefore, γ^0 is better behaved than γ^3 with respect to power corrections

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Interlude:

Quasi-GPDs & Form Factors: (Sample results)

Let's go back to PDFs

arXiv: 1705.01488

Quasi-PDFs, momentum distributions and pseudo-PDFs

Old Domini...
Thomas Jefferson Natio...

Statement needs a qualifier: Situation more complicated for quasi-GPDs

(See next slide)

$$\mathcal{M}^\alpha(z,p) \equiv \langle p|\bar\psi(0)\,\gamma^\alpha\,\hat E(0,z;A)\psi(z)|p\rangle \qquad (12)$$

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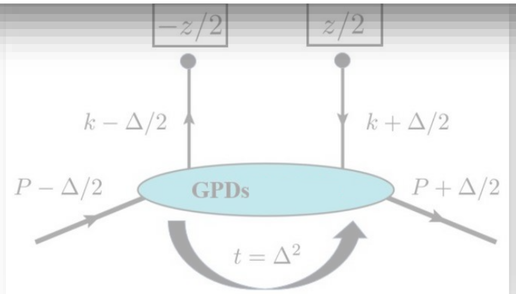

$$H_{Q(0)}(z,P_s,\Delta_s)\Big|_s = A_1 + \frac{\Delta_s^0}{P_s^0}A_3 - \frac{\Delta_s^0 z^3}{2P_s^0 P_s^3}A_4 + \left(\frac{(\Delta_s^0)^2 z^3}{2M^2 P_s^3} - \frac{\Delta_s^0\Delta_s^3 z^3 P_s^0}{2M^2(P_s^3)^2} - \frac{z^3\Delta_\perp^2}{2M^2P_s^3}\right)A_6 + \left(\frac{(\Delta_s^0)^3 z^3}{2M^2P_s^0P_s^3} - \frac{(\Delta_s^0)^2\Delta_s^3 z^3}{2M^2(P_s^3)^2} - \frac{\Delta_s^0 z^3\Delta_\perp^2}{2M^2P_s^0P_s^3}\right)A_8$$

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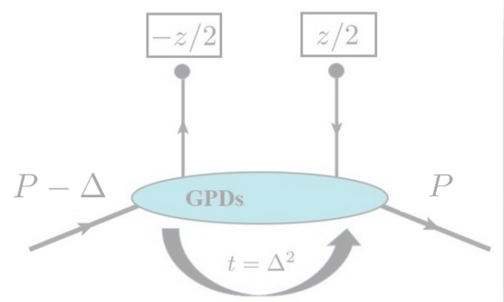

$$H_{Q(0)}\Big|_a(z,P_a,\Delta_a) = A_1 + \frac{\Delta_a^0}{P^0_{avg,a}}A_3 - \left(\frac{\Delta_a^0 z^3}{2P^0_{avg,a}P^3_{avg,a}} - \frac{1}{(1+\frac{\Delta_a^3}{2P^3_{avg,a}})}\frac{\Delta_a^0\Delta_a^3 z^3}{4P^0_{avg,a}(P^3_{avg,a})^2}\right)A_4$$
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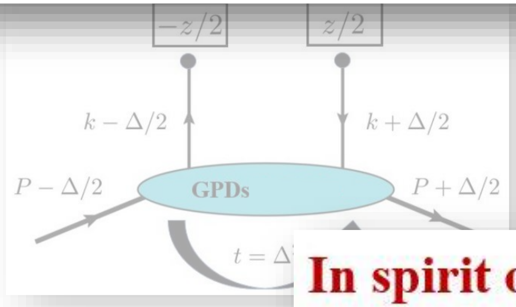

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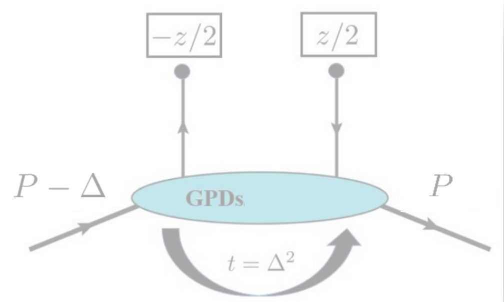
In spirit of what's done for PDFs:

You can think of eliminating additional amplitudes by the addition of other operators:

$$(\gamma^1, \gamma^2)$$

Asymmetric frame:



$$H_{Q(0)}\big|_a(z, P_a, \Delta_a) = A_1 + \frac{\Delta_a^0}{P^0_{avg,a}} A_3 - \left(\frac{\Delta_a^0 z^3}{2P^0_{avg,a} P^3_{avg,a}} - \frac{1}{\left(1+\frac{\Delta_a^3}{2P^3_{avg,a}}\right)} \frac{\Delta_a^0 \Delta_a^3 z^3}{4P^0_{avg,a}(P^3_{avg,a})^2}\right) A_4$$
$$+ \left(\frac{(\Delta_a^0)^2 z^3}{2M^2 P^3_{avg,a}} - \frac{1}{\left(1+\frac{\Delta_a^3}{2P^3_{avg,a}}\right)} \frac{(\Delta_a^0)^2 \Delta_a^3 z^3}{4M^2 (P^3_{avg,a})^2} - \frac{1}{\left(1+\frac{\Delta_a^3}{2P^3_{avg,a}}\right)} \frac{P^0_{avg,a} \Delta_a^0 \Delta_a^3 z^3}{2M^2 (P^3_{avg,a})^2} - \frac{z^3 \Delta_\perp^2}{2M^2 P^3_{avg,a}}\right) A_6$$
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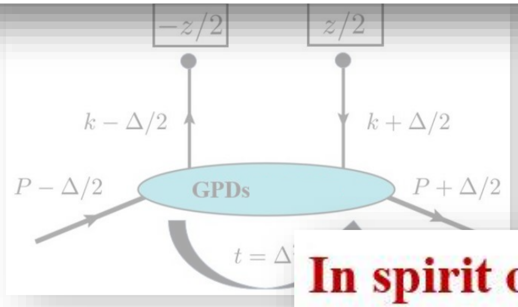

$$H_{Q(0)}(z, P_s, \Delta_s)\big|_s = A_1 + \frac{\Delta_s^0}{P_s^0} A_3 - \frac{\Delta_s^0 z^3}{2P_s^0 P_s^3} A_4 + \left(\frac{(\Delta_s^0)^2 z^3}{2M^2 P_s^3} - \frac{\Delta_s^0 \Delta_s^3 z^3 P_s^0}{2M^2 (P_s^3)^2} - \frac{z^3 \Delta_\perp^2}{2M^2 P_s^3}\right) A_6$$

You can think of eliminating additional amplitudes by the addition of other operators:

$$(\gamma^1, \gamma^2)$$

In spirit of what's done for PDFs:

Asymmetric frame:

Lorentz-invariant definition of quasi-GPDs:

Main finding:

Schematic structure: $H_Q \to c_0\langle\bar\psi\gamma^0\psi\rangle + c_1\langle\bar\psi\gamma^1\psi\rangle + c_2\langle\bar\psi\gamma^2\psi\rangle$

Note: Here c's are frame-dependent kinematic factors that cancel additional amplitudes to project quasi-GPD potentially faster (vs historic def.) onto light-cone GPD

Main results

Sketch of the essence of a Lorentz-invariant definition of quasi-GPDs

Lorentz covariant formalism

Relation between light-cone GPD H & Form Factors:

$$H(z\cdot P, z\cdot\Delta, t=\Delta^2, z^2) = \boxed{A_1} + \frac{\Delta_{s/a}\cdot z}{P_{avg,s/a}\cdot z}\boxed{A_3}$$

Contrary to quasi-PDFs, γ^0 operator for quasi-GPDs is
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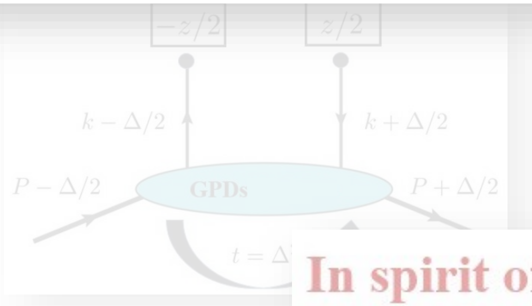

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Krzysztof's talk

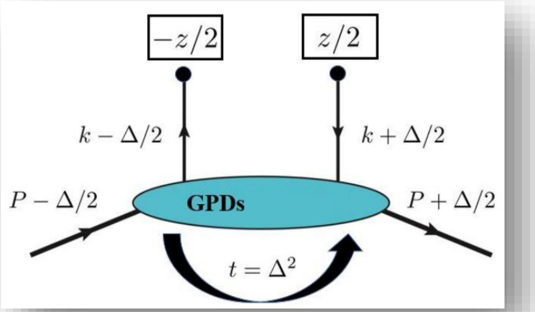
Note: Here c's are frame-de… project quasi-GPD pot…

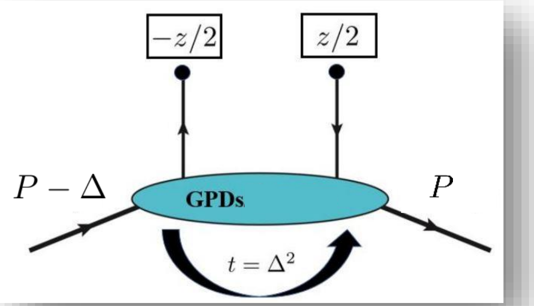
Agreement of results for H & E between frames confirmed by Lattice results

Main results

Lorentz covariant formalism

Sketch of the essence of a Lorentz-invariant definition of quasi-GPDs

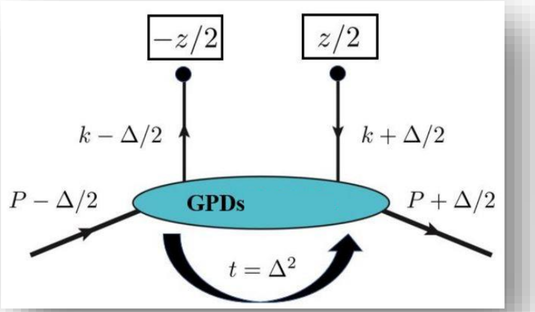




Main results

Lorentz covariant formalism

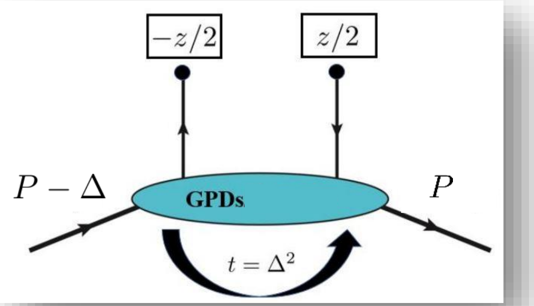
Sketch of the essence of a Lorentz-invariant definition of quasi-GPDs



Matching equation:

$$H_{\mathrm{Q}}(z\cdot P, z\cdot\Delta, t, z^2, \mu) = \int_{-1}^{1} du\, \overline{C}(u, z\cdot P, z\cdot\Delta, z^2, \mu^2)\, H(u, z\cdot P, z\cdot\Delta, t, \mu)$$

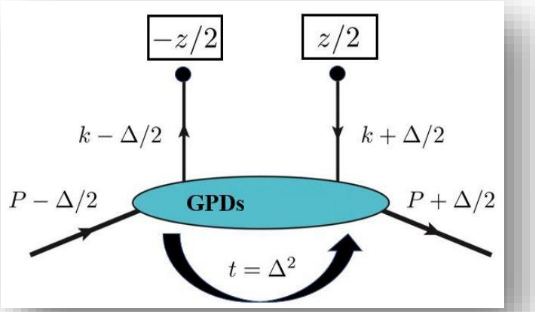
Schematic structure



Main results

Lorentz covariant formalism

Sketch of the essence of a Lorentz-invariant definition of quasi-GPDs



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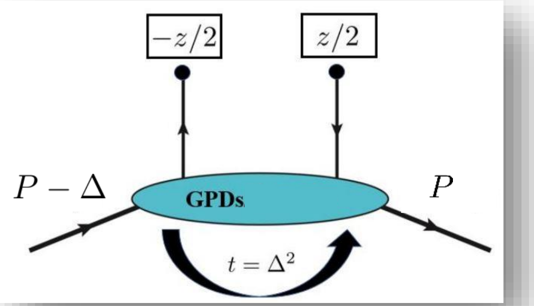
Schematic structure

Essence of matching: Equivalence of light-cone & quasi-GPDs at LO

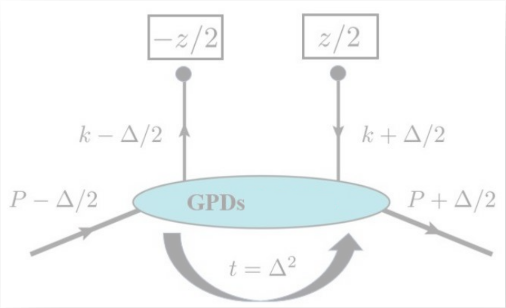
$$\lim_{z^2\to 0} H_{\mathrm{Q}}(z\cdot P, z\cdot\Delta, \Delta^2, z^2) = H(z\cdot P, z\cdot\Delta, \Delta^2)$$



Main results

Lorentz covariant formalism

Sketch of the essence of a Lorentz-invariant definition of quasi-GPDs



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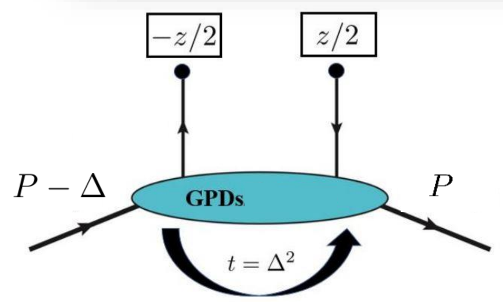
Schematic structure

Essence of matching: Equivalence of light-cone & quasi-GPDs at LO

$$\lim_{z^2\to 0} H_{\rm Q}(z\cdot P, z\cdot\Delta, \Delta^2, z^2) = H(z\cdot P, z\cdot\Delta, \Delta^2)$$

Same functional forms

$$A_i \equiv A_i(z^2 \neq 0)$$



Relation between light-cone GPD H & Form Factors:

$$H(z\cdot P, z\cdot\Delta, t=\Delta^2, z^2) = A_1 + \frac{\Delta_{s/a}\cdot z}{P_{avg,s/a}\cdot z} A_3$$

$$A_i \equiv A_i(z^2 = 0)$$

Natural candidate:

Lorentz-invariant generalization of LC definition to $z^2 \neq 0$

$$H_{\rm Q}(z\cdot P, z\cdot\Delta, t=\Delta^2, z^2) = A_1 + \frac{\Delta_{s/a}\cdot z}{P_{avg,s/a}\cdot z} A_3$$

Main results

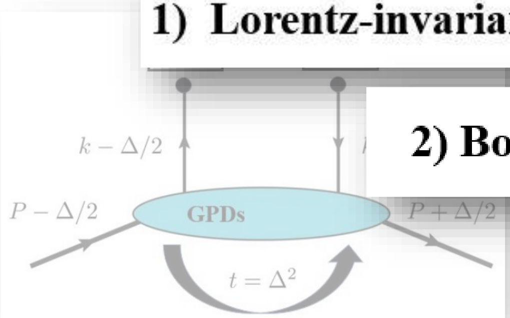
Lorentz covariant formalism

Sketch of the essence of a L... t definition of quasi-GPDs

Key points:

Matching equation:

1) **Lorentz-invariant generalization of LC definition to $z^2 \neq 0$ might converge faster at LO**



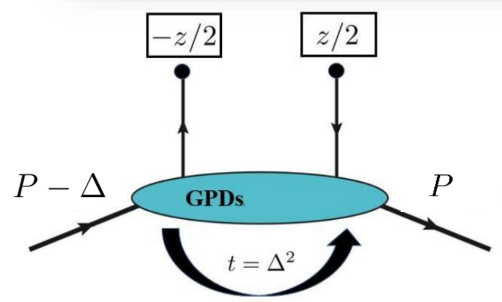
2) **Both sides Lorentz invariant → Differences suppressed by frame-independent power corrections**

Essence of matching: Equivalence of light-cone & quasi-GPDs at LO

$$\lim_{z^2\to 0} H_Q(z\cdot P, z\cdot\Delta, \Delta^2, z^2) = H(z\cdot P, z\cdot\Delta, \Delta^2)$$

Same functional forms

$A_i \equiv A_i(z^2 \neq 0)$



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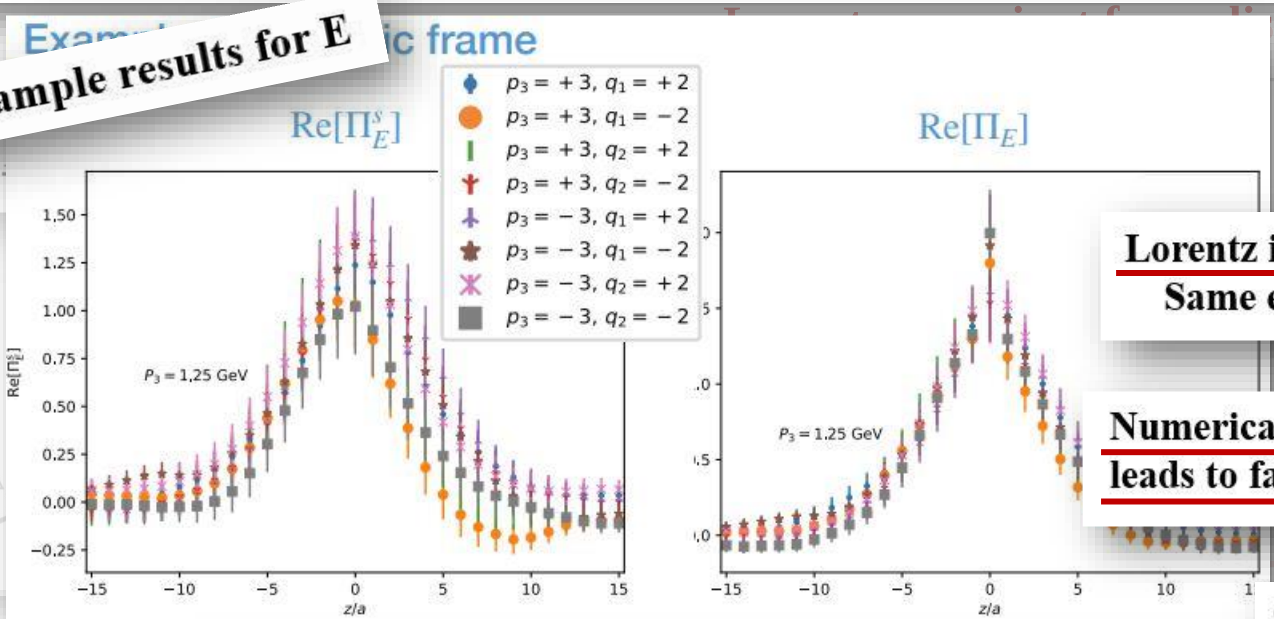
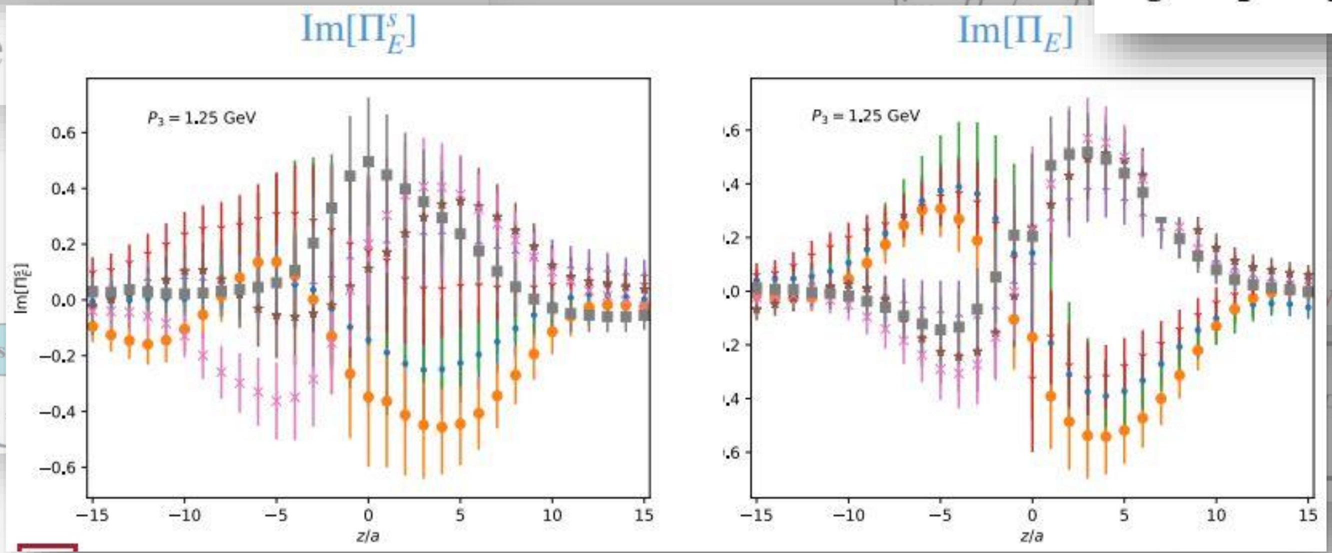
$A_i \equiv A_i(z^2 = 0)$

Main results

Numerical comparison between Lorentz invariant and historical definitions of quasi-GPDs:

Sample results for E





Krzysztof's talk

Lorentz invariant definition leads to more precise results for E;
Same effect of improvement for asymmetric frame

Numerical indications that using Lorentz invariant definition for E leads to faster convergence to LC GPD with respect to P^3

Signal quality for H same for all cases (not shown)

$A_i \equiv A_i(z^2 \neq 0)$

Natural candidate:

Summary

Connecting dots: Ending with what I started with

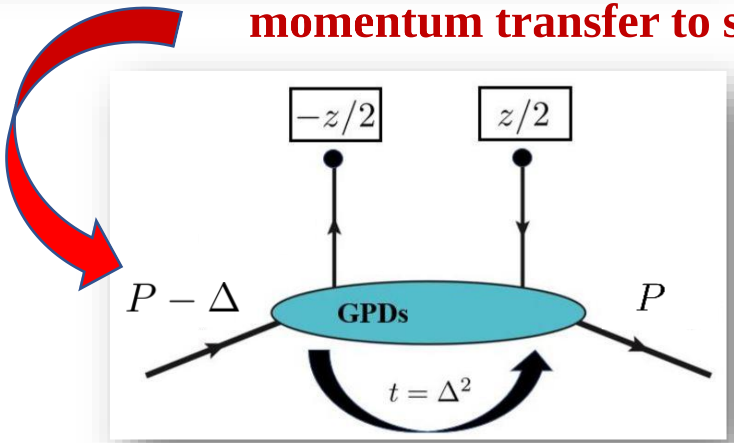
Summary

Goal:

Connecting dots: Ending with what I started with

Perform Lattice QCD calculations of GPDs in asymmetric frames

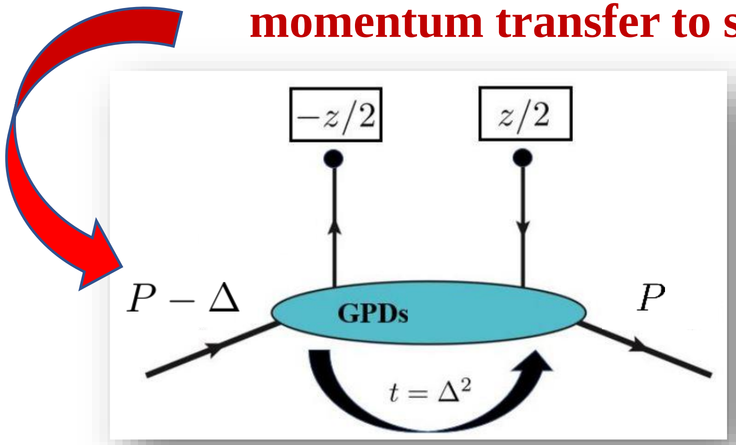
All momentum transfer to source



Summary

Connecting dots: Ending with what I started with

All
momentum transfer to source



Approach 1: Can we calculate a quasi-GPD in symmetric frame through an asymmetric frame?

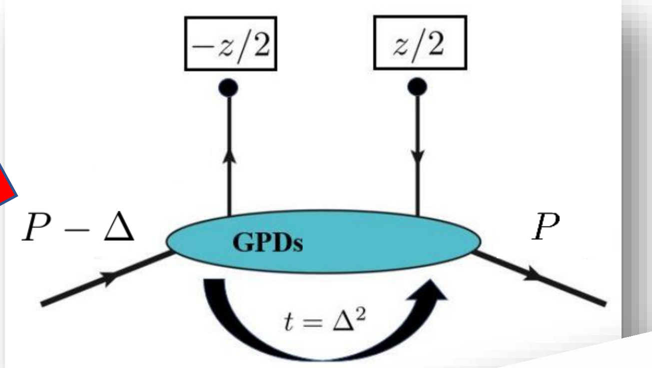
Transverse boost: This Lorentz transformation allows for an exact calculation of quasi-GPDs in symmetric frame through matrix elements of asymmetric frame

Summary

Connecting dots: Ending with what I started with

Approach 2: Why does it matter in which frame quasi-GPDs are calculated?

All momentum transfer to source



Key findings: QCD calc... asymmetric fram...

1) **Historic definitions of H & E quasi-GPDs are not manifestly Lorentz invariant**

$$H_Q \to c\langle\bar{\psi}\gamma^0\psi\rangle$$

Symmetric frame:

$$H_{Q(0)}(z,P_s,\Delta_s)\big|_s = A_1 + \frac{\Delta_s^0}{P_s^0}A_3 - \frac{\Delta_s^0 z^3}{2P_s^0 P_s^3}A_4 + \left(\frac{(\Delta_s^0)^2 z^3}{2M^2 P_s^3} - \frac{\Delta_s^0\Delta_s^3 z^3 P_s^0}{2M^2 (P_s^3)^2} - \frac{z^3\Delta_\perp^2}{2M^2 P_s^3}\right)A_6 + \left(\frac{(\Delta_s^0)^3 z^3}{2M^2 P_s^0 P_s^3} - \frac{(\Delta_s^0)^2\Delta_s^3 z^3}{2M^2 (P_s^3)^2} - \frac{\Delta_s^0 z^3\Delta_\perp^2}{2M^2 P_s^0 P_s^3}\right)A_8$$

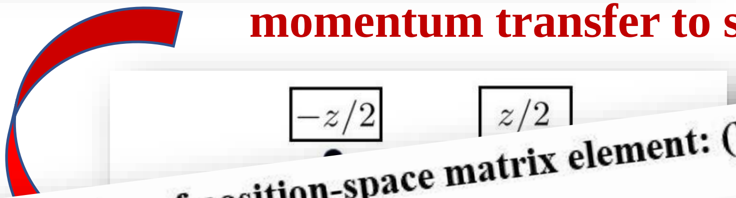
Contamination from additional amplitudes or explicit power corrections

Summary

Connecting dots: Ending with what I started with

Approach 2: Why does it matter in which frame quasi-GPDs are calculated?

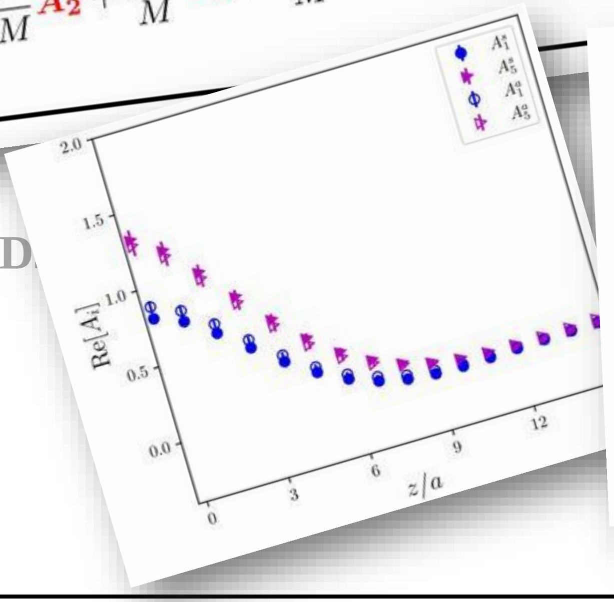
All momentum transfer to source

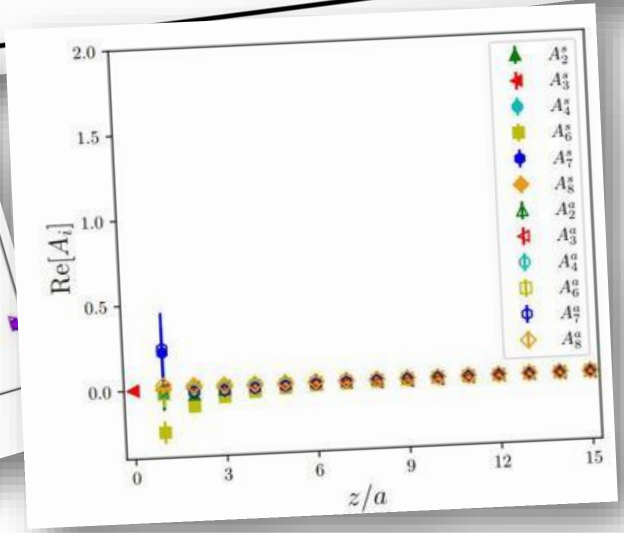


2) Novel parameterization of position-space matrix element: (Vector operator)

$$F^{\mu}_{\lambda,\lambda'} = \bar{u}(p',\lambda')\left[\frac{P^{\mu}}{M}A_1 + \frac{z^{\mu}}{M}A_2 + \frac{\Delta^{\mu}}{M}A_3 + \frac{i\sigma^{\mu z}}{M}A_4 + \frac{i\sigma^{\mu\Delta}}{M}A_5 + \frac{P^{\mu}i\sigma^{z\Delta}}{M^3}A_6 + \frac{z^{\mu}i\sigma^{z\Delta}}{M^3}A_7 + \frac{\Delta^{\mu}i\sigma^{z\Delta}}{M^3}A_8\right]u(p,\lambda)$$

Key findings: QCD calculations of GPD...



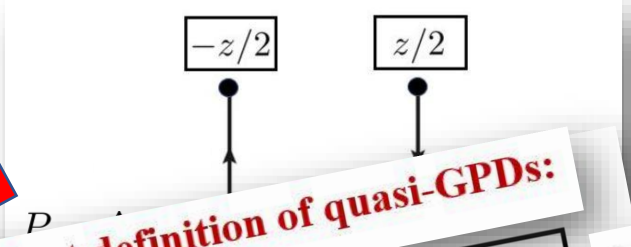


Summary

Connecting dots: Ending with what I started with

Approach 2: Why does it matter in which frame quasi-GPDs are calculated?

All momentum transfer to source



3)

Lorentz-invariant definition of quasi-GPDs:

$$H_Q(z\cdot P, z\cdot\Delta, t=\Delta^2, z^2) = A_1 + \frac{\Delta_{s/a}\cdot z}{P_{avg,s/a}\cdot z} A_3$$

$$H_Q \to c_0 \langle\bar{\psi}\gamma^0\psi\rangle + c_1\langle\bar{\psi}\gamma^1\psi\rangle + c_2\langle\bar{\psi}\gamma^2\psi\rangle$$

QCD calculations of GPDs in asymmetric frames

Key findings:

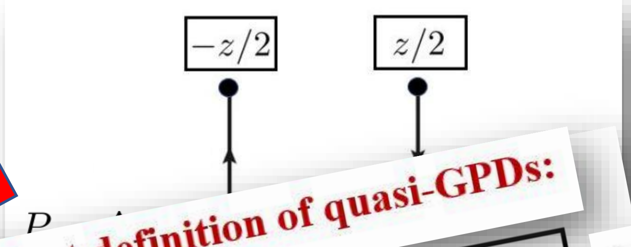
- **Lorentz covariant formalism for calculating quasi-GPDs in any frame**
- **Elimination of power corrections potentially allowing faster convergence to light-cone GPDs at LO**

Summary

Connecting dots: Ending with what I started with

Approach 2: Why does it matter in which frame quasi-GPDs are calculated?

All momentum transfer to source



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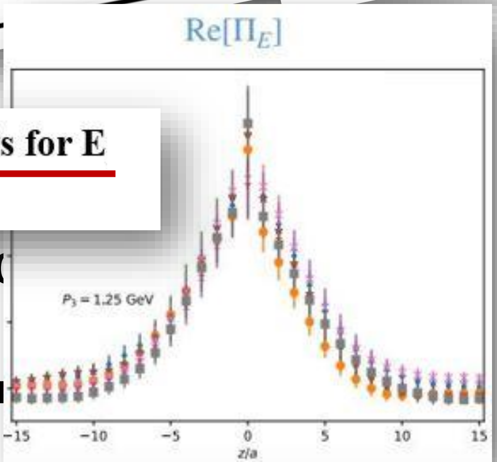

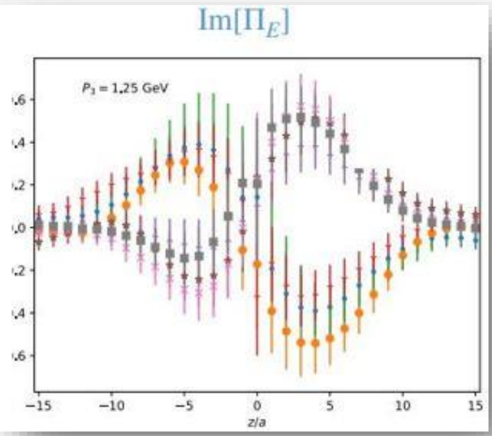

Key fi

Lorentz invariant definition leads to more precise results for E

- **Lorentz covariant formalism for calculating quasi-(**
- **Elimination of power corrections potentially allowi lig**

Backup slides

Main results

Renormalization: Sketch

Few words on operators:

- **Schematic structure of Lorentz non-invariant quasi-GPD:** $H_{\rm Q} \to c\langle\bar{\psi}\gamma^0\psi\rangle$

- **Schematic structure of Lorentz invariant quasi-GPD:** $H_{\rm Q} \to c_0\langle\bar{\psi}\gamma^0\psi\rangle + c_1\langle\bar{\psi}\gamma^1\psi\rangle + c_2\langle\bar{\psi}\gamma^2\psi\rangle$

How to renormalize?

Main results

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Few words on renormalization:

RI-MOM

- **Renormalization factors are different for** $\langle\bar{\psi}\gamma^0\psi\rangle$, $\langle\bar{\psi}\gamma^1\psi\rangle$, $\langle\bar{\psi}\gamma^2\psi\rangle$
  --- UV-divergent terms same
  --- Finite terms different
- **Matching:**
  --- Frame-independent
  --- Available for only γ^0
  --- Takes care of finite terms for γ^0
- **Strategy to renormalize: Use Renormalization factor for operator whose matching is known**