



Finite-Temperature Equations of State for Core-Collapse Supernovae and Binary Neutron Star Mergers

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Finite-Temperature Effects in Multi-Messenger Astrophysics,
19th June, 2026

Finite-Temperature Equations of State for CCSNe and BNS Mergers

Introduction

Main part

- Effective masses & Finite- T effects
- Bayesian setup
- Properties of NS matter at $T = 0$
- Properties of Finite- T matter

Conclusions

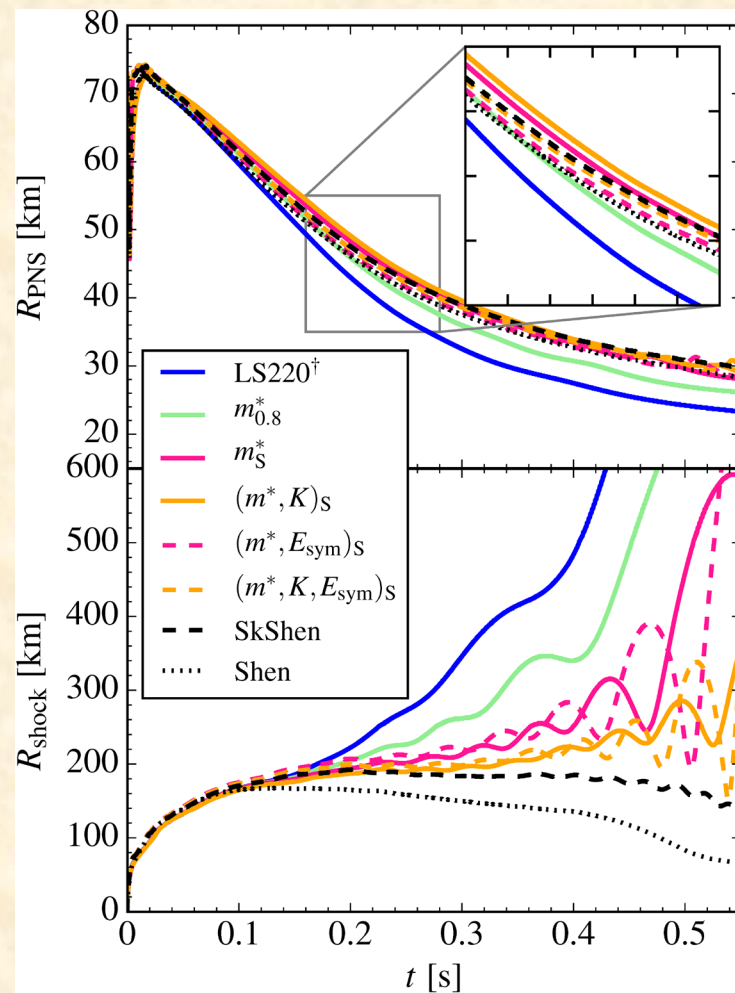
Introduction

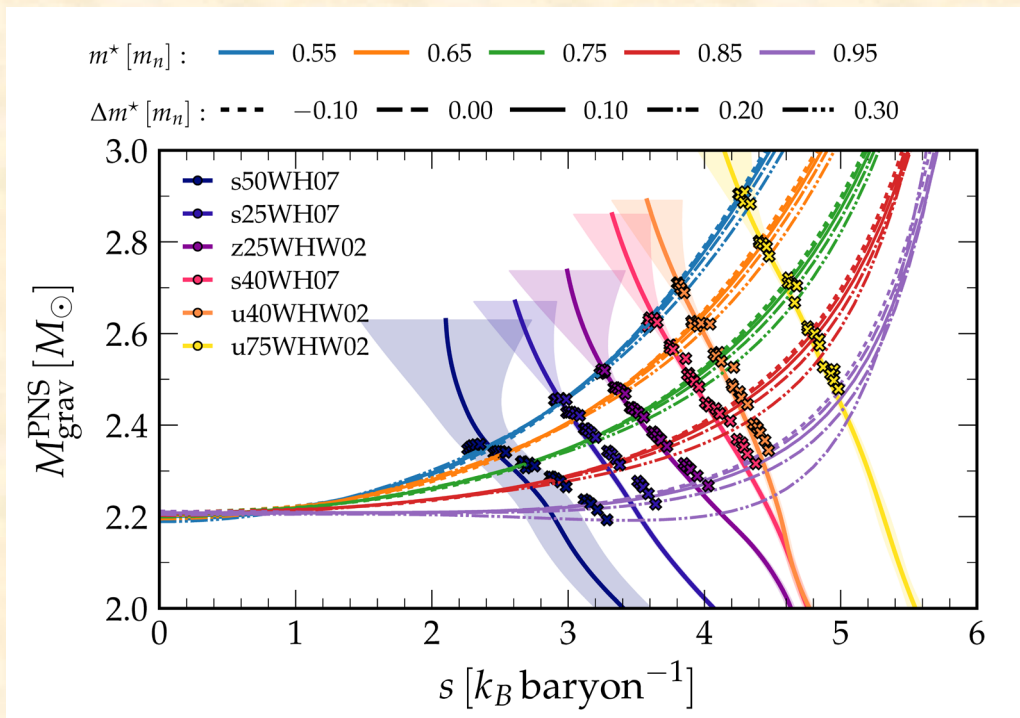
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It is well-known (e.g., [1, 2]) that cataclysmic astrophysical phenomena, such as core-collapse supernovae (CCSNe) and binary neutron star (BNS) mergers, strongly depend on the equation of state (EOS) of dense matter and, **in particular, on the effective masses.**

From simulations: larger m_{eff} \rightarrow faster explosion, larger R_{shock} , faster proto-NS contraction.

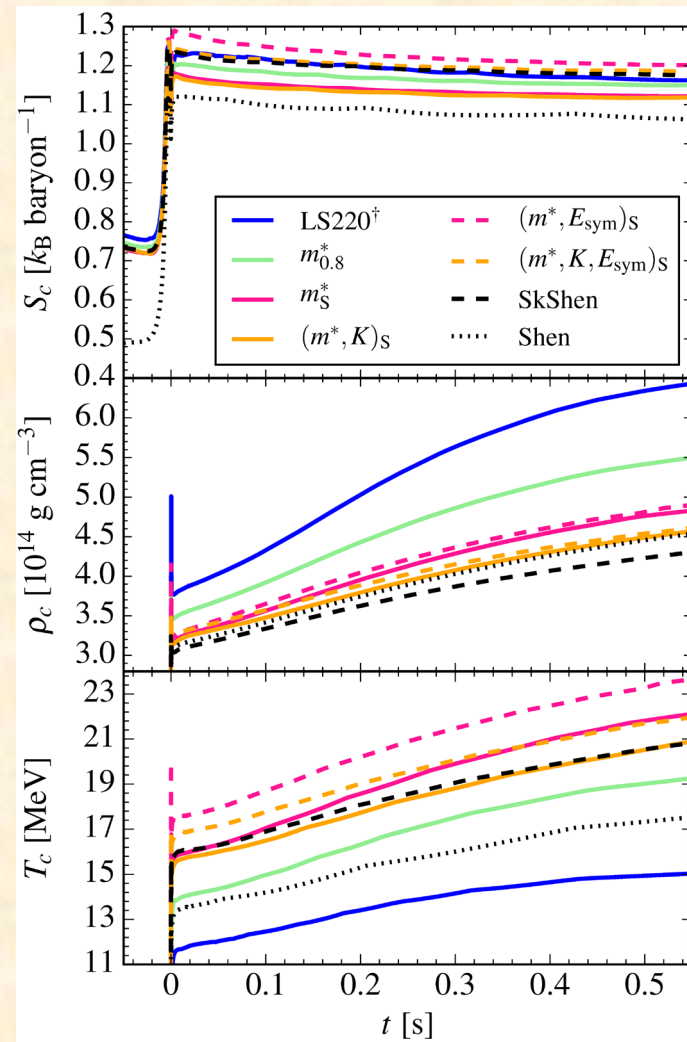
[1, Fig] Yasin et al., PRL **124**, 092701 (2020);
[2] Schneider et al., PRC **100**, 055802 (2019).





Larger $m_{\text{eff}} \rightarrow$ lower T_c , larger $\rho_c [1]$;
 earlier collapse [2].

[1, right] Yasin et al., PRL **124**, 092701 (2020);
 [2, top] Schneider et al., ApJ **894**, 4 (2020);



The simulation results demonstrated above were obtained with Skyrme-like EOSs that have a very simple monotonic dependence of the effective mass on density (or non at all).

However, modern BHF [1,2] and χ EFT [3] calculations with 3-body forces predict that the Landau effective mass should have an U-shaped behavior. More flexible EOSs are required.

Consequently, we performed a massive Bayesian investigation of EOSs build upon extended Skyrme interactions, investigated finite-temperature properties of such EOSs, computed and uploaded on CompOSE general purpose EOSs suitable for simulations.

[1] Baldo et al., PRC **89**, 048801 (2014); [2] Shang et al., PRC **101**, 065801 (2020); [3] Somasundaram et al., PRC **103**, 045803 (2021).

Effective masses & finite- T properties

$$\frac{1}{m_{\text{eff};i}} = \frac{1}{\hbar^2 k_i} \left. \frac{\partial e_i}{\partial k_i} \right|_{k_i=k_{F;i}} = \frac{1}{m_{0;i}} + \frac{2}{\hbar^2} \left(C_{\text{eff}} [n_n + n_p] + D_{\text{eff}} [n_n - n_p] \right), \quad i = n, p;$$

$$e_{\text{th}} = e(T) - e(T=0) = \sum_{i=n,p} \frac{\hbar^2}{2m_{\text{eff};i}} [\tau_i(T) - \tau_i(T=0)] \approx \left(\frac{\pi}{3} \right)^{2/3} \frac{T^2}{2\hbar^2} \sum_{i=n,p} m_{\text{eff};i} n_i^{1/3}, \quad T \ll \mu;$$

$$p_{\text{th}} = p(T) - p(T=0) = \sum_{i=n,p} \frac{\hbar^2}{3m_{\text{eff};i}} \left(1 - \frac{3}{2} \frac{n_B}{m_{\text{eff};i}} \frac{\partial m_{\text{eff};i}}{\partial n_B} \right) [\tau_i(T) - \tau_i(T=0)] \approx$$

$$\approx \left(\frac{\pi}{3} \right)^{2/3} \frac{T^2}{2\hbar^2} \sum_{i=n,p} m_{\text{eff};i} \left(1 - \frac{3}{2} \frac{n_B}{m_{\text{eff};i}} \frac{\partial m_{\text{eff};i}}{\partial n_B} \right) n_i^{1/3}, \quad T \ll \mu;$$

$$\tau_i = \frac{1}{\pi^2} \int k^4 \left[1 + \exp \left(\frac{\frac{\hbar^2 k^2}{2m_{\text{eff};i}} + U_i - \mu_i}{T} \right) \right]^{-1} dk.$$

Bayesian setup

Effective interaction: Brussels-Skyrme (BSk) [1].

Constraints for Bayesian analysis (all at $T = 0$; $i = n, p$):

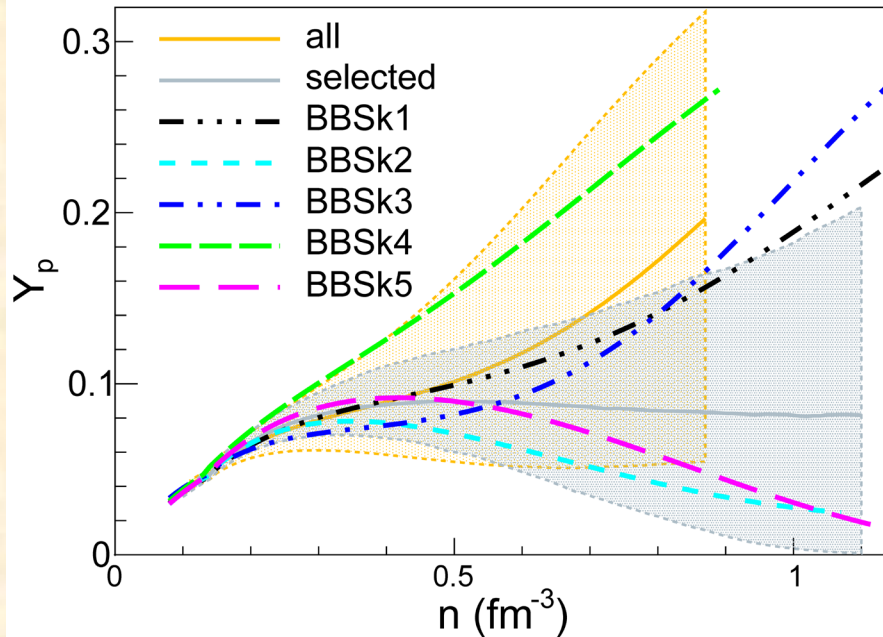
- The best-know nuclear matter (NM) parameters: $n_{\text{sat}}, E_{\text{sat}}, K_{\text{sat}}, J_{\text{sym}}$;
- E/A in PNM for $n_{\text{B}} \leq 0.16 \text{ fm}^{-3}$ as computed by means of χEFT [2];
- For SNM and PNM, $0 < m_{\text{eff};i}/m_{0;i} < 1$ for $n \leq n_{\text{lim}}$; *optionally* with χEFT constraints [2];
- NS EOSs should be thermodynamically stable & causal up to n_{c}^* ;
- For SNM and PNM, $v_{\text{F};i} = \hbar k_{\text{F};i}/m_{\text{eff};i} < c$ [3] for $n \leq n_{\text{lim}}$;
- $n_{\text{lim}} = 0.8 \text{ fm}^{-3}$ [4] or $n_{\text{lim}} = n_{\text{c}}^*$ and PNM is causal up to n_{c}^* [5];
- Maximum NS mass: $M_{\text{G}}^* > 2.0 M_{\odot}$; *optionally* $M_{\text{DU}} \geq 1.5 M_{\odot}$ [6].

[1] Chamel et al., PRC **80**, 065804 (2009); [2] Somasundaram et al., PRC **103**, 045803 (2021); [3] Duan & Urban, PRC **108**, 025813 (2023); [4] MB & AR, PRC **110**, 035805 (2024); [5] AR & MB, A&A **705**, A151 (2026); [6] MB & AR, in prep.

Caveats & things to keep in mind:

- Inference of NM parameters from experiments is model dependent; so the “target” values should be obtained based on “compatible” models (e.g., “target” values for relativistic and non-relativistic models can be different);
- Due to absence of data, we always treat NM parameters as independent constraints; in reality, they are most likely correlated;
- We have also considered correlations between the values that energy per nucleon and/or effective masses in PNM/SNM have at different densities, but we will not discuss this here.

NS matter properties at $T = 0$

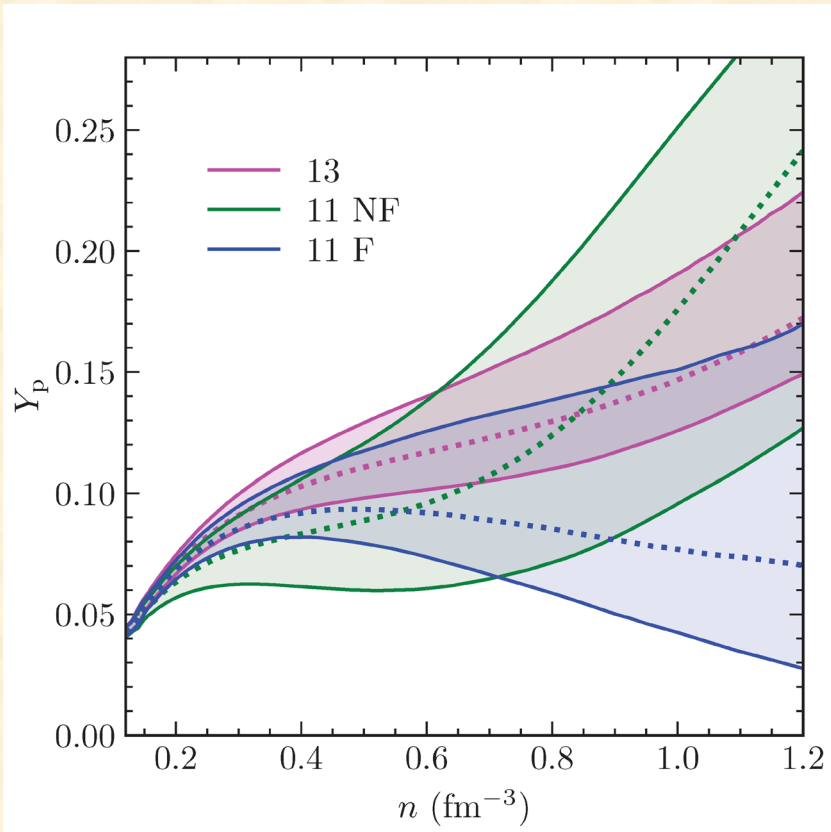


- “All”: run 1 of [1], $n_{\text{lim}} = 0.8 \text{ fm}^{-3}$;
- “Selected”: according to [2], $n_{\text{lim}} = n_c^*$ and PNM is causal up to n_c^* ;
- BBSk1 is the median of $m_{\text{eff}}(n_B)$ from run 2 of [1];
- BBSk2 and BBSk3 lie close to 0.98 and 0.02 quantiles of $m_{\text{eff}}(n_B)$ from “all” models, respectively;
- BBSk4 and BBSk5 provide large negative and positive p_{th} , respectively.

[1] MB & AR, PRC **110**, 035805 (2024);

[2, Fig] AR & MB, A&A **705**, A151 (2026);

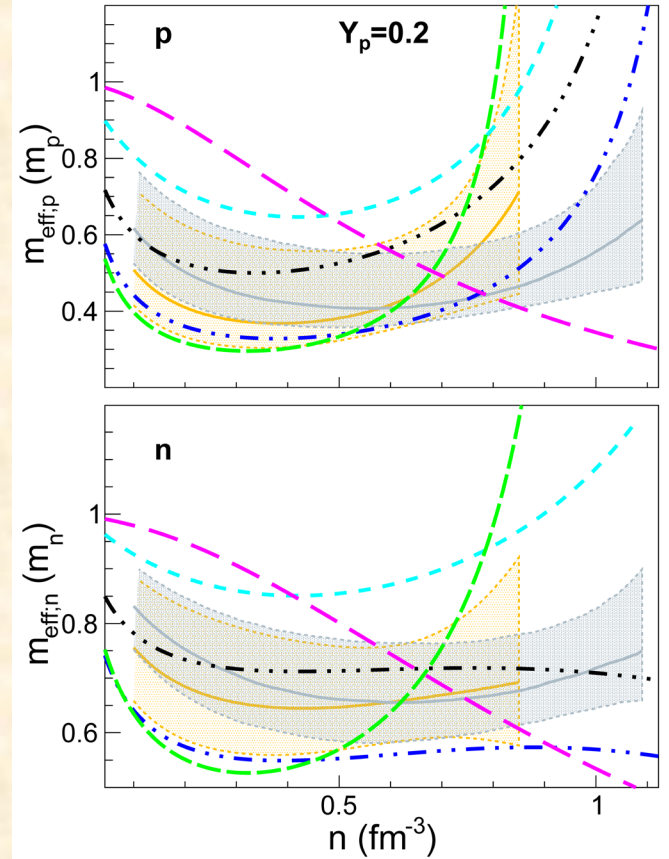
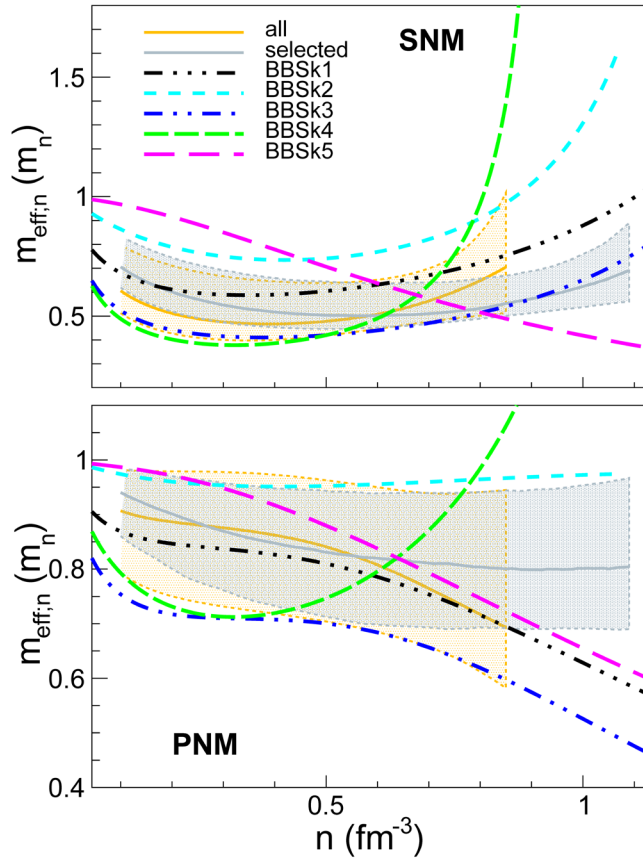
The impact of the direct Urca constraint & extra BSk parameters



- “11 NF” (“not filtered”, green): 11 BSk parameters, 2 fixed, $n_{\text{lim}} = 0.8 \text{ fm}^{-3}$;
- “11 F” (“filtered”, blue): 11 BSk parameters, 2 fixed, $n_{\text{lim}} = n_c^*$ and PNM is causal up to n_c^* ;
- “13” (magenta): all 13 BSk parameters, $n_{\text{lim}} = n_c^*$, PNM is causal up to n_c^* , and $M_{\text{DU}} \geq 1.5 M_{\odot}$.

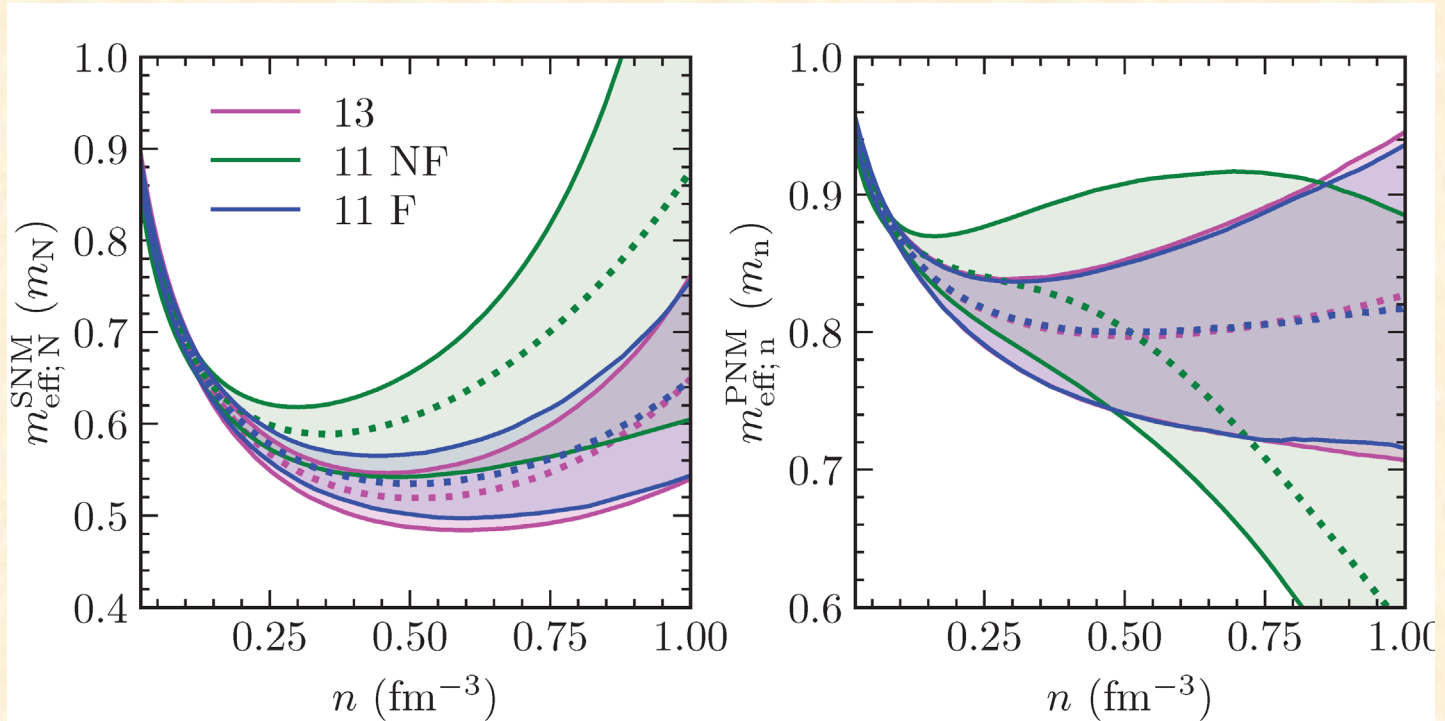
[Fig] AR & MB, in prep. (2026);

Effective masses



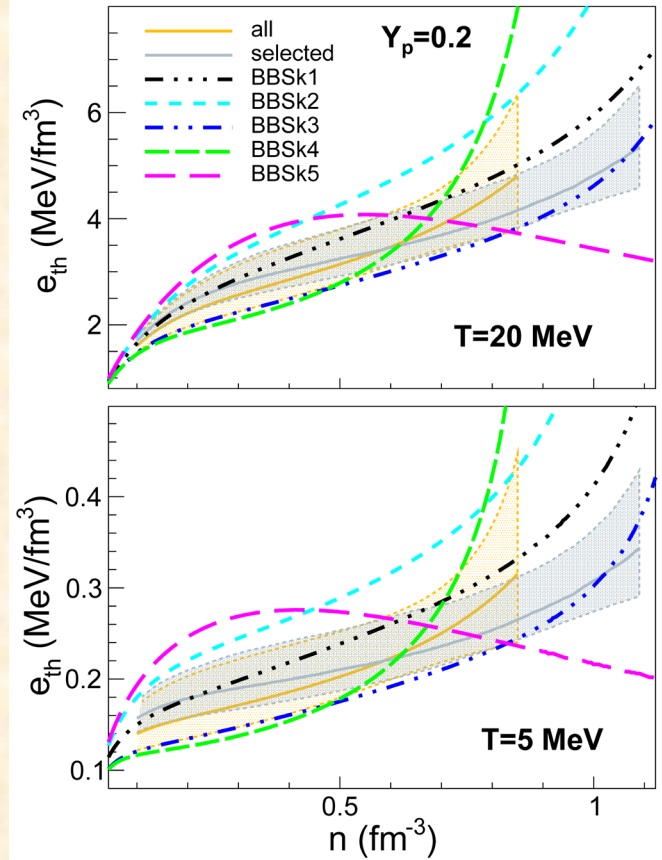
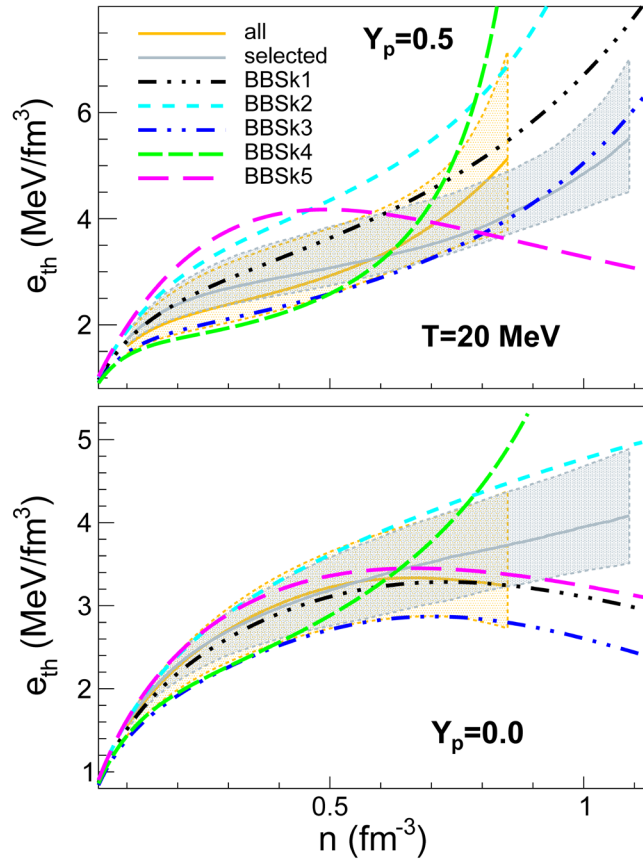
[Figs] AR &
MB, A&A 705,
A151 (2026);

Effective masses



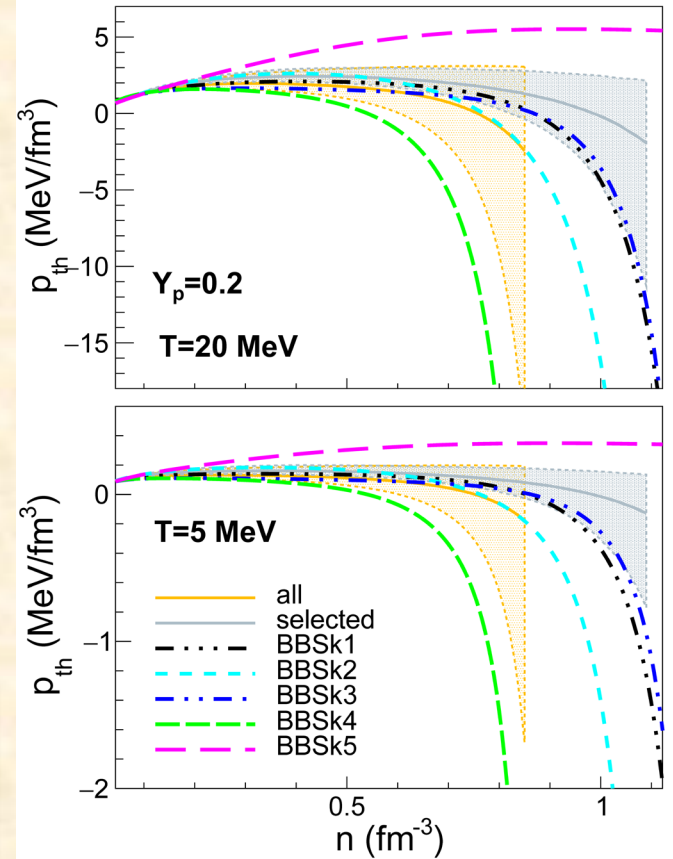
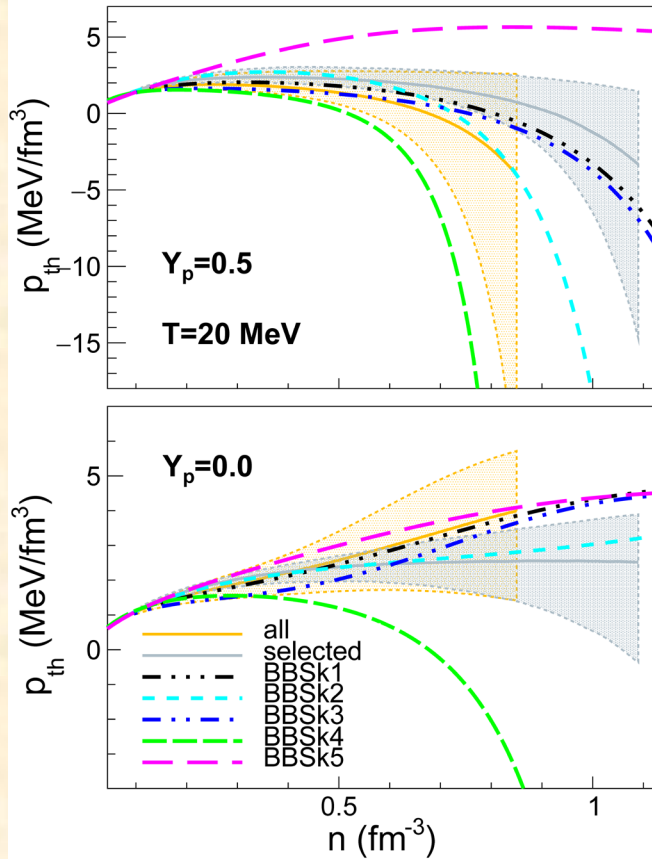
[Fig] AR & MB, in prep. (2026).

Thermal energy



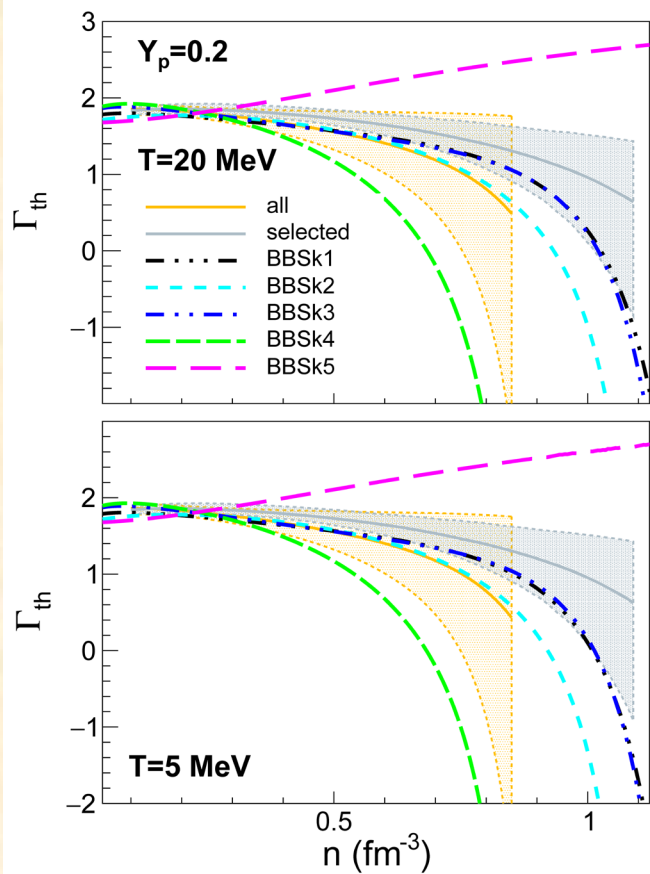
[Figs] AR & MB, A&A 705, A151 (2026);

Thermal pressure



[Figs] AR & MB, A&A 705, A151 (2026);

Thermal index



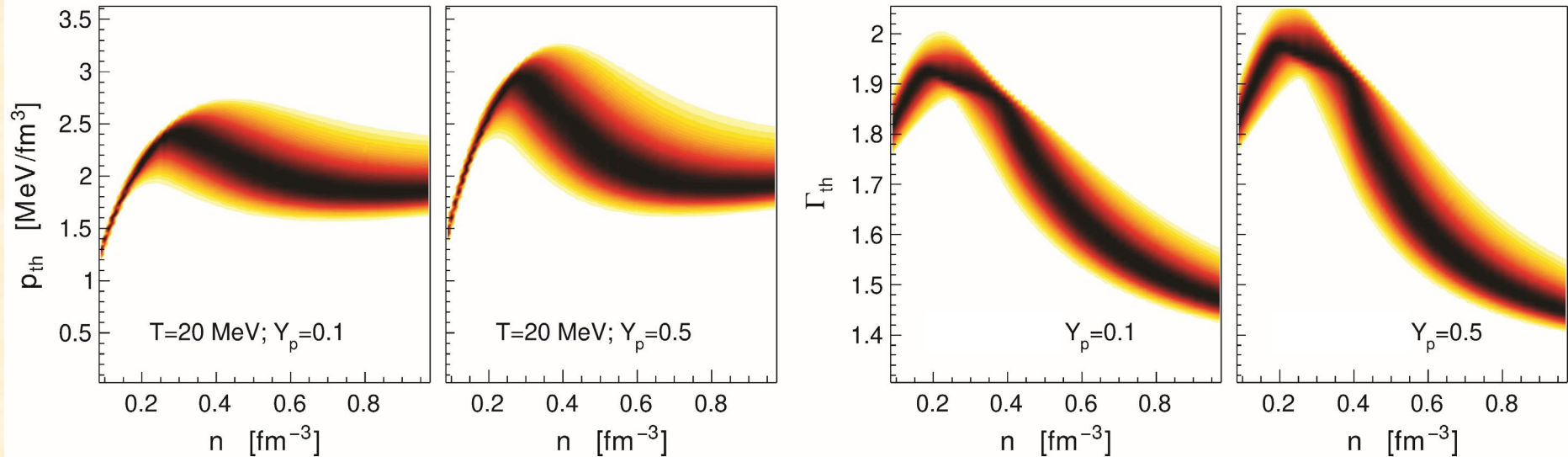
$$\Gamma_{th} = 1 + \frac{p_{th}}{e_{th}}$$

- Thermal index measures the departure from the ideal gas;
- Approximation $\Gamma_{th} = const$ is frequently employed to construct finite- T EOS from a zero-temperature one. Clearly, this approximation is far from being adequate.

[Fig] AR & MB, A&A **705**, A151 (2026);

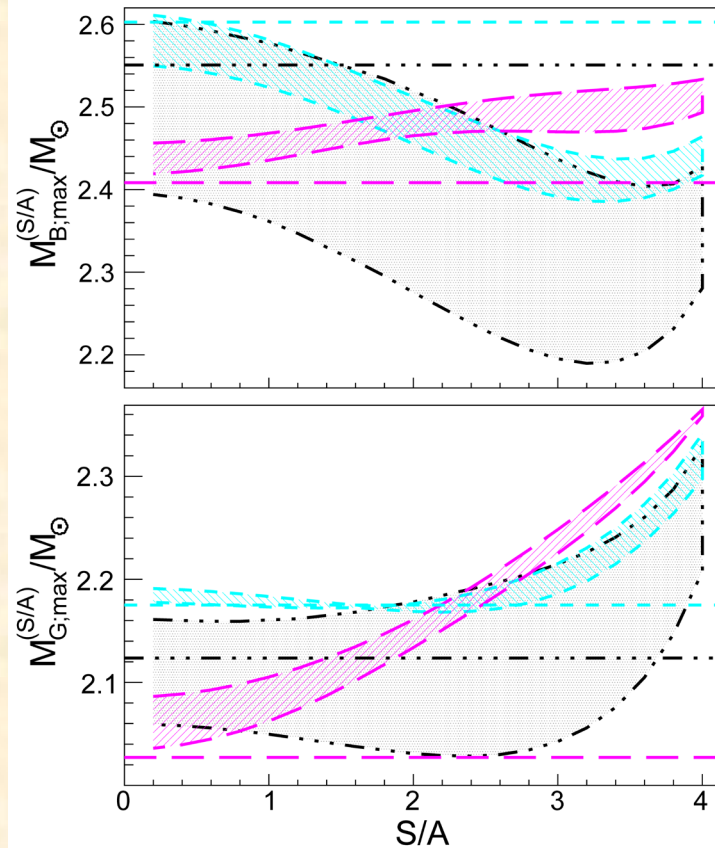
A quick comparison.

Thermal pressure (left) and thermal index (right) for a DD-RMF model [1, 2, 3].



[1] Malik et al., ApJ **930**, 17 (2022); [2] MB & AR, PRC **107**, 045803 (2023);
[3] AR et al., PLB **853**, 138696 (2024).

Stability of finite- T (proto-)NSs



Isentropic PNSs with constant profiles of Y_p , $0.06 \leq Y_p \leq 0.3$ for BBSk1 (black), BBSk2 (cyan), BBSk5 (magenta) EOSs; horizontal lines correspond to zero-temperature beta-equilibrium NSs.

- BBSk1 has strong Y_p -dependence;
- BBSk5: thermal pressure stabilizes the PNS for any value of S/A ;
- BBSk2: thermal pressure *destabilizes* the PNS [1] for any value of S/A .

[Fig] AR & MB, A&A **705**, A151 (2026);

[1] Lu et al., PRC **100**, 054335 (2019).

Conclusions

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- We have performed several Bayesian investigations of dense matter EOSs based on Brussels extended Skyrme effective interactions;
- Many resulting EOSs feature U-shaped $m_{\text{eff}}(n_{\text{B}})$ dependence and, consequently, $p_{\text{th}} < 0$;
- Evolution of CCSNe and BNS mergers depend on finite- T properties and, thus, on m_{eff} .
- The (more)realistic EOSs have a complex $m_{\text{eff}}(n_{\text{B}})$ behavior that makes the mapping between thermal properties and $m_{\text{eff}}(n_{\text{sat}})$ difficult; various parts of $m_{\text{eff}}(n_{\text{B}})$ curve play role during various moments of evolution.
- 8 general purpose EOS tables covering wide range of densities, proton fractions, and temperatures are available on CompOSE: RB(BBSk1) ... RB(BBSk8).

Thank you!



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