# From laboratory observables to intrinsic shapes
# (or stay in the lab?)

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1. The B(E2) sum rule
2. Alhassid's AFMC model
3. Working with moments
3. Can we use lab frame moments directly?

References

1. "Nuclear Theory, A.M. Lane (Benjamin, 1964), p. 80

2. "Transition probability from the ground to the first-excited 2+ state of even-even nuclei", S. Raman et al., Atomic Data and Nuclear Data Tables **78** 1 (2001).

3. "Statistical theory of deformation distributions in nuclear spectra", M.T. Mustonen, et al., Phys Rev. C 97 014315 (2018), Phys. Rev. C **98** 034317 (2018) and PRL **113** 262503.

## Sum Rules

An alternative version of criterion (2) is that the transition exhausts at least a fair fraction (say $\gtrsim$ 5 per cent) of a sum rule. There are two sum rules that are relevant; in obvious notations, these are

$$\sum_n |\langle n|Q_{TL0}|0\rangle|^2 = \langle 0|(Q_{TL0})^2|0\rangle \equiv S_{NEW}{}^{TL}$$

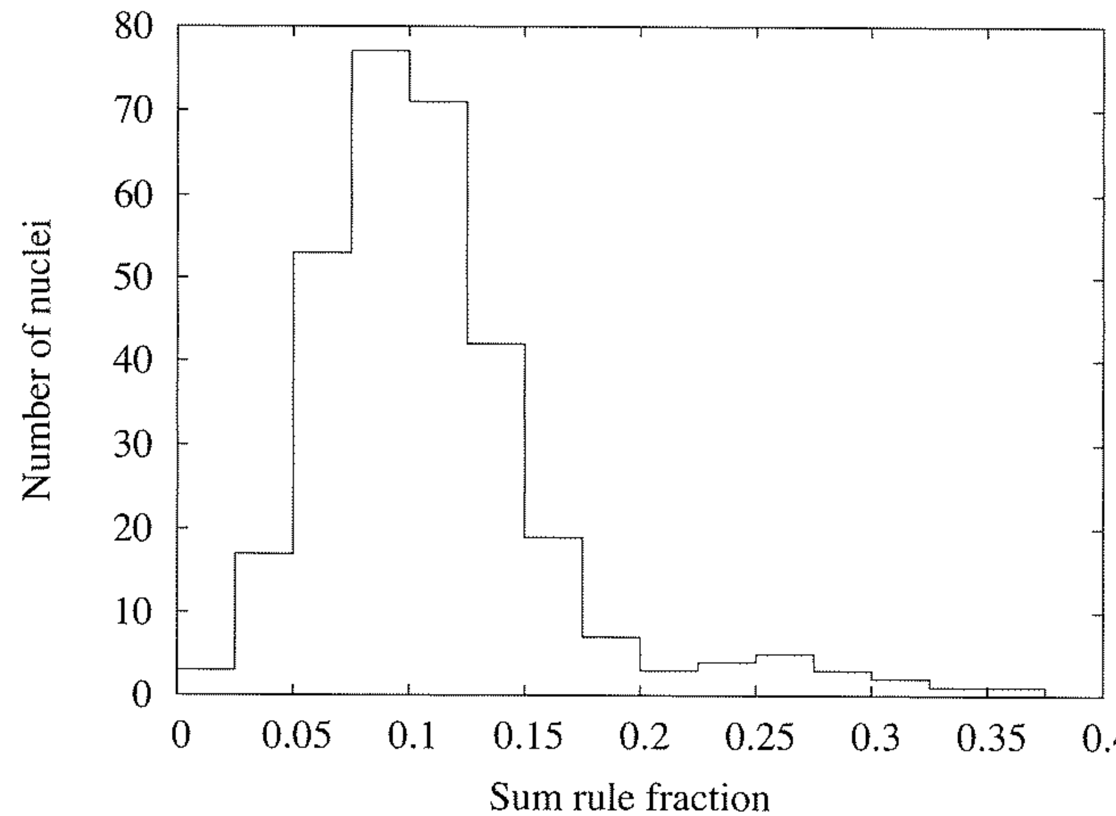
$$\sum_n (E_n - E_0)|\langle n|Q_{TL0}|0\rangle|^2$$

$$= \tfrac{1}{2}\langle 0|[Q_{TL0}, [H, Q_{TL0}]]|0\rangle \equiv S_{EW}{}^{TL}$$

We call these the non-energy-weighted (NEW) and energy-weighted (EW) sum rules. The only sum that can be evaluated exactly (i.e., without reference to a model) is the EWS[1,2] for T = 0:

$$S_{EW}{}^{0L} = \frac{\hbar^2 A}{8\pi M} L(2L+1)\langle r^{2L-2}\rangle$$

The only assumption is that H contains no explicitly velocity-depend ent forces (exchange forces are permitted). Strictly, this value is




$$S = \sum_i (E_i - E_0)| < i|r^2 Y_{20}(\theta)|0 > |^2 \approx \frac{3\hbar^2}{4\pi m_N} A(1.2\,\mathrm{fm}\,A^{1/3})^2$$

Most of sum rule is in the giant quadrupole resonance $E_{GQ} \approx \frac{60}{A^{1/3}}\mathrm{MeV}$

$^{238}$U sum rule fraction in rotational band 0.058

Relative strengths: $\frac{0.054}{0.946}\frac{10\,\mathrm{MeV}}{45\,\mathrm{keV}} \approx 13$

How does that affect the extracted beta?

$$\beta_{GQ}^2 = \frac{1 - S_1/S}{E_{GQ}} \frac{E_1}{S_1/S} \beta_1^2$$

$$\beta_+ = (\beta_1^2 + \beta_{GQ}^2)^{1/2}$$

| Nucleus | $E_1$ (MeV)[a] | $S_1/S$[a] | $\beta_1$[a] | $\beta_{GQ}$ | $\beta_+$ |
|---|---|---|---|---|---|
| $^{238}U$ | 0.045 | 0.058 | 0.286 | 0.079 | 0.295 |
| $^{96}Ru$ | 0.832 | 0.089 | 0.158 | 0.127 | 0.203 |

a) Raman, et al. ADNDT 78 1 (2001)

Significant for $^{96}$Ru but not for $^{238}$U

## The auxiliary-field Monte Carlo (AFMC) calculational framework

- —one-body + two-body Hamiltonian
- —spherical shell-model basis of a full major shell for p,n
- —Woods-Saxon single-particle potential
- —separable residual interaction
- —delivers by MC the full many-body wave function
- —delivers MC expectation values of one-body operators
- —and their exponentials $\langle \exp(i\hat{O}) \rangle$

To calculate instantaneous lab-frame quadrupole distribution:

$$P(q) = \langle \delta(\hat{Q}_{20} - q) \rangle = \int \frac{d\phi}{2\pi} e^{-i\phi q} \langle \exp(i\phi \hat{Q}_{20} \rangle \qquad \int_{-\infty}^{\infty} dq\, P(q) = 1$$

Simple example: lab-frame quadrupole moment of a prolate rigid rotor:

$$P(q) = \frac{1}{q_0(3 + 6q/q_0)^{1/2}} \quad \text{for} \quad -q_0/2 < q < q_0$$

PRL **113**, 262503 (2014) PHYSICAL REVI

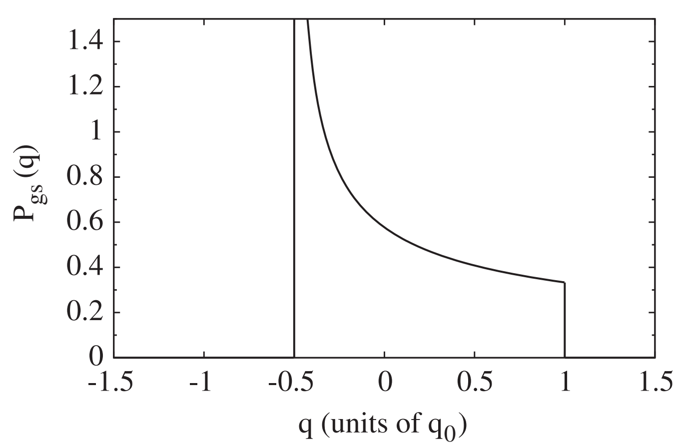
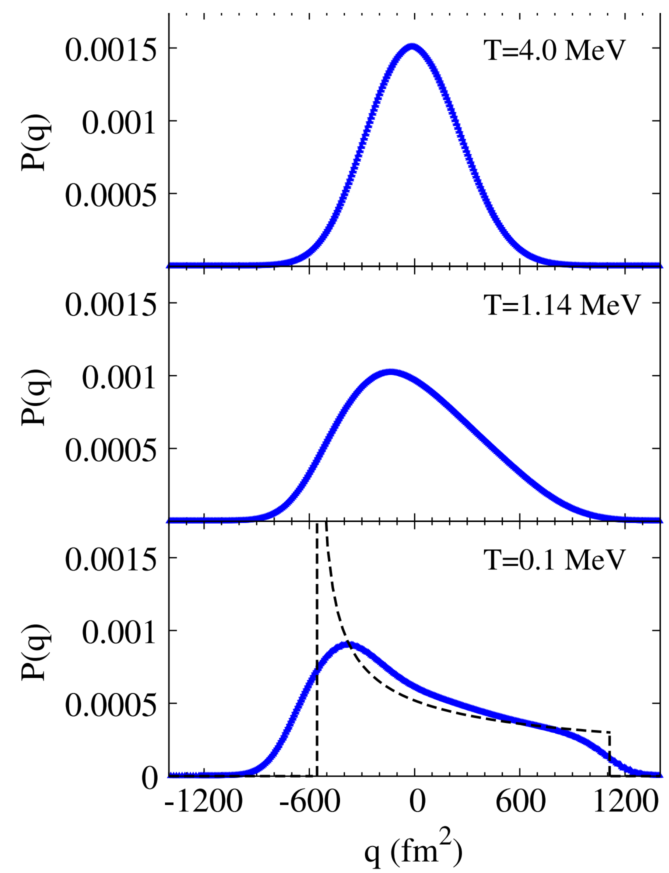



FIG. 1. The ground-state distribution $P_{\rm gs}(q)$ vs $q/q_0$ for a prolate rotor with intrinsic quadrupole moment $q_0$.

# Results for $^{154}Sm$




Blue lines: P(q) in AFMC
Dash line: P(q) for a rigid rotot in its ground state.

## Transformation to Intrinsic frame

Shape parameters are defined by the invariants

$$\beta = \frac{\sqrt{5\pi}}{3r_0^2 A^{5/3}} \langle \hat{Q} \cdot \hat{Q} \rangle^{1/2}; \;\; \cos 3\gamma = -\sqrt{\frac{7}{2}} \frac{\langle (\hat{Q} \times \hat{Q}) \cdot \hat{Q} \rangle}{\langle \hat{Q} \cdot \hat{Q} \rangle^{3/2}}. \tag{41}$$

A fourth-order invariant can be used to define $\Delta\beta$

The invariants are related to powers of the operator

$$\langle \hat{Q} \cdot \hat{Q} \rangle = 5 \big\langle \hat{Q}_{20}^2 \big\rangle,$$

$$\langle (\hat{Q} \times \hat{Q})^{(2)} \cdot \hat{Q} \rangle = -5\sqrt{\frac{7}{2}} \big\langle \hat{Q}_{20}^3 \big\rangle,$$

$$\langle (\hat{Q} \cdot \hat{Q})^2 \rangle = \frac{35}{3} \big\langle \hat{Q}_{20}^4 \big\rangle,$$

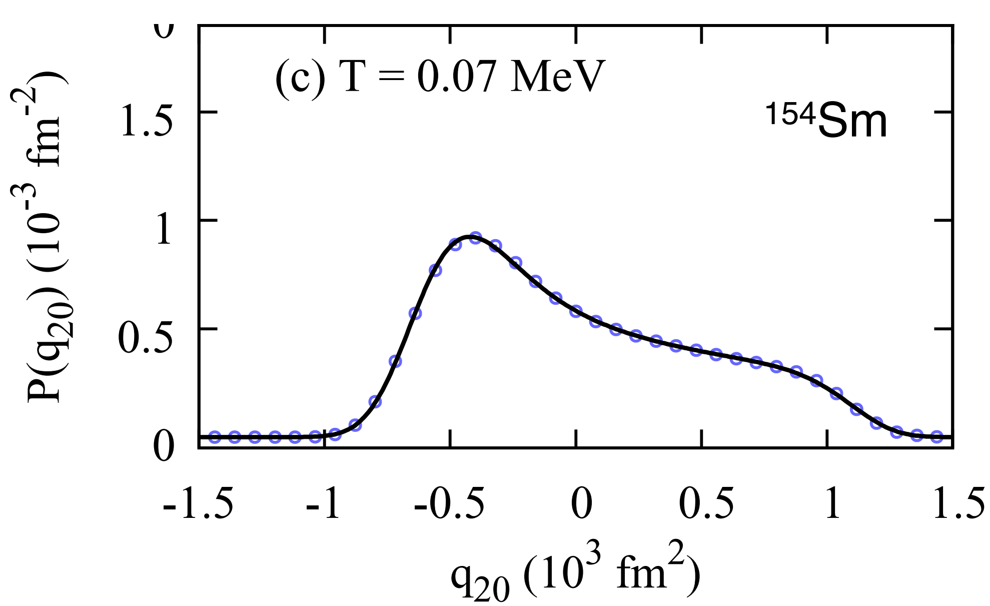
Powers of the operator are extracted from the distribution function

$$< \hat{Q}_{20}^n >= \int_{-\infty}^{\infty} dq \; q^n P(q)$$

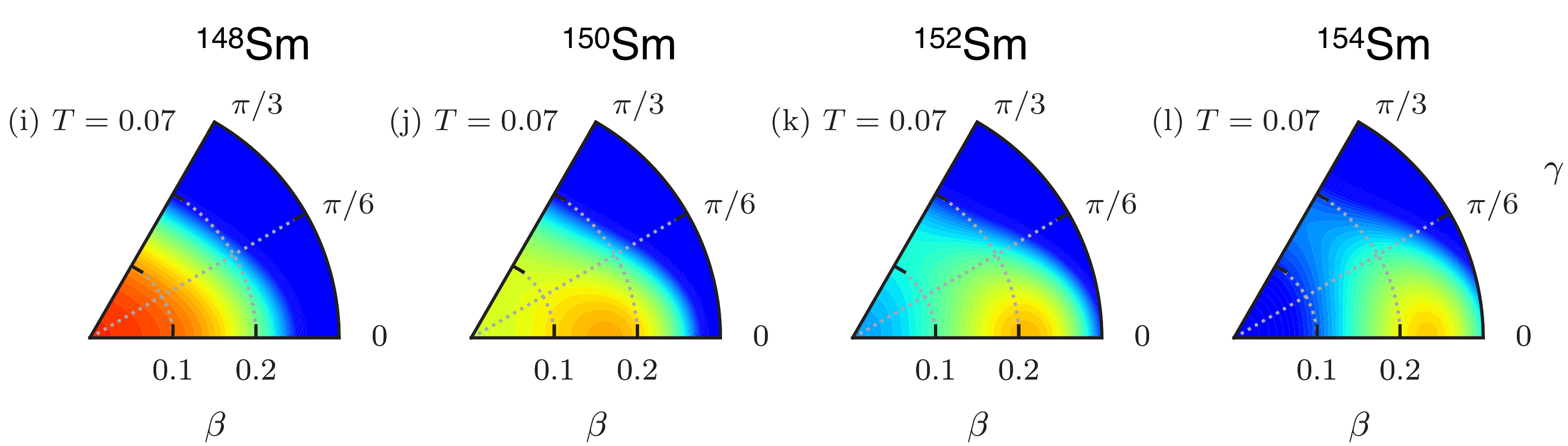
There is a better way to parameterize the fluctuating shape [3].

$$P(\beta, \gamma) = K \exp(-a\beta^2 - b\beta^3 \cos(3\gamma) - c\beta^4)$$

to be used with the integration measure $$\int d\beta\, d\gamma\, \beta^4 |\sin(3\gamma)|$$




Solid line: the distribution P(q) calculated from P(beta,gamma) above.
Circles: P(q) from AFMC



Now here 's a question: would it be better to calculate the distribution of the pair of operators

$$(\hat{Q}_{2,2} + \hat{Q}_{2,-2}), \quad i(\hat{Q}_{2,2} - \hat{Q}_{2,-2})$$

These operators directly measure the v_2 components of the thickness function

$$r_\perp^2 \cos(2\phi), \quad r_\perp^2 \sin(2\phi)$$

avoiding the intrinsic shape entirely.