The Migdal Effect in Semiconductors from Effective Field Theory

Kim V. Berghaus
Yang Institute for Theoretical Physics
Stony Brook University

Based on upcoming work with Rouven Essig, Angelo Esposito, and Mukul Sholapurkar

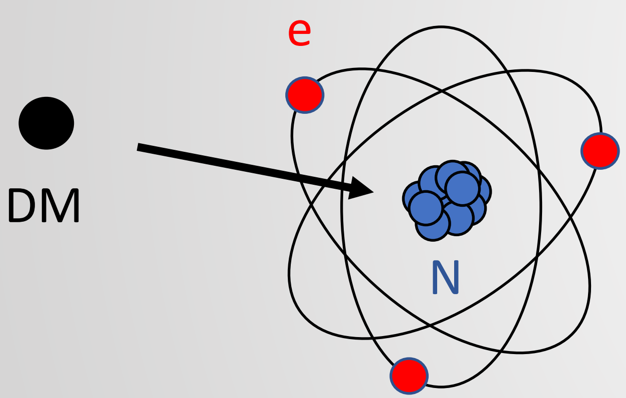
Outline

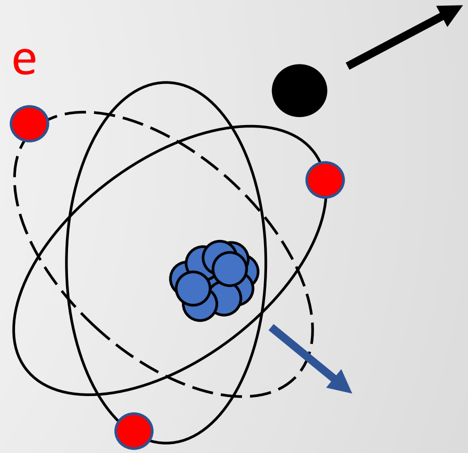
- The Migdal Effect
- The Migdal Effect in Semiconductors
- Results
- Outlook

The Migdal Effect

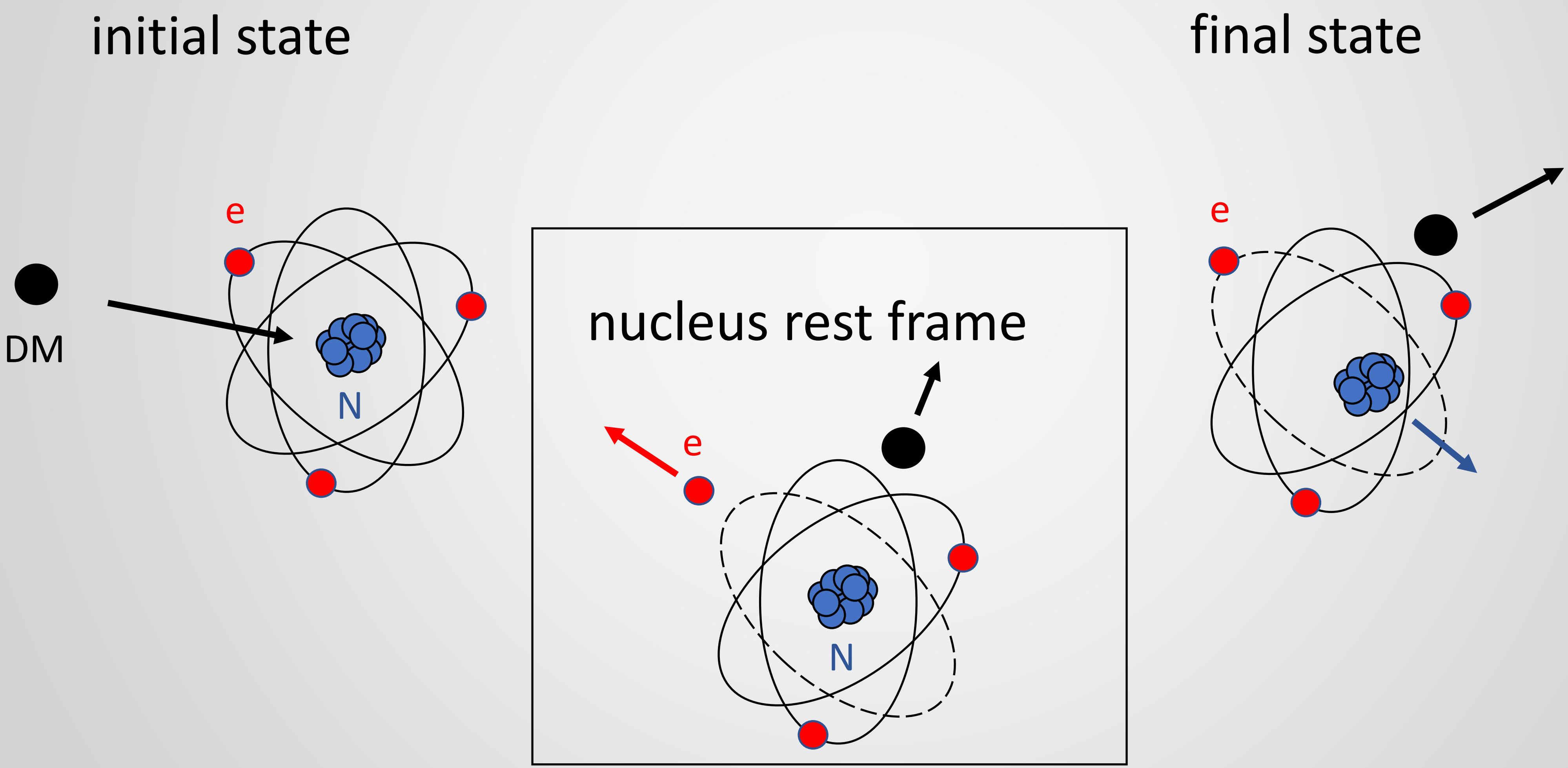
initial state

final state





The Migdal Effect



The Migdal Effect in DM Direct Detection

- Migdal Effect has rate suppression over nuclear recoil rate but more easily detectable signature for light DM
- Process factorizes:

$$\frac{d\sigma}{dE_r\, d\omega} = \frac{d\sigma}{dE_r}\frac{dP}{d\omega}(E_r)$$

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- Electronic wave functions, Ibe et. al., JHEP 2018
- Photoabsorption, Liu, Wu, Chi, Chen, PRD 2020

The Migdal Effect in Semiconductors

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Knapen, Kozaczuk, Lin PRL 2021

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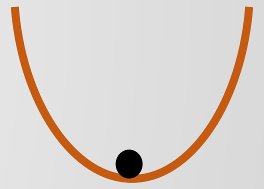
The Migdal Effect in Semiconductors

$$\frac{d\sigma}{dE_r\, d\omega} = \frac{d\sigma}{dE_r}\frac{dP}{d\omega}(E_r)$$

Response of nucleus in semiconductor differs from free nucleus



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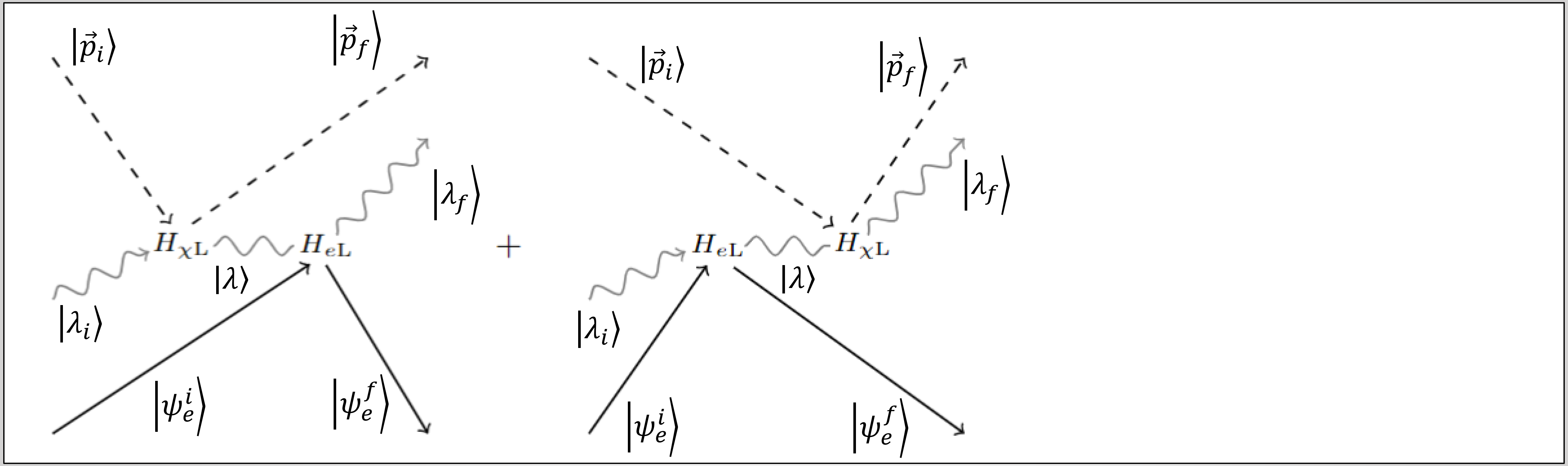

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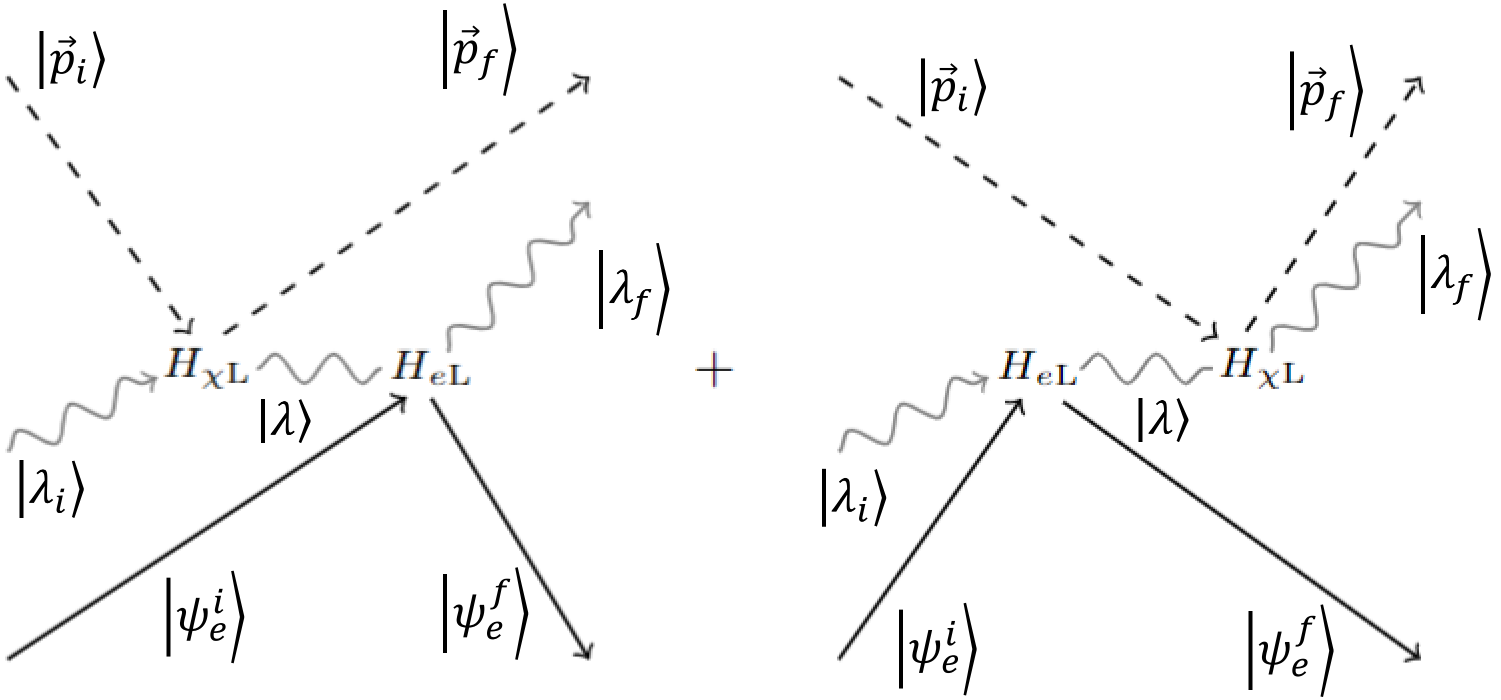


$|\lambda_i\rangle$ ~ harmonic oscillator ground state

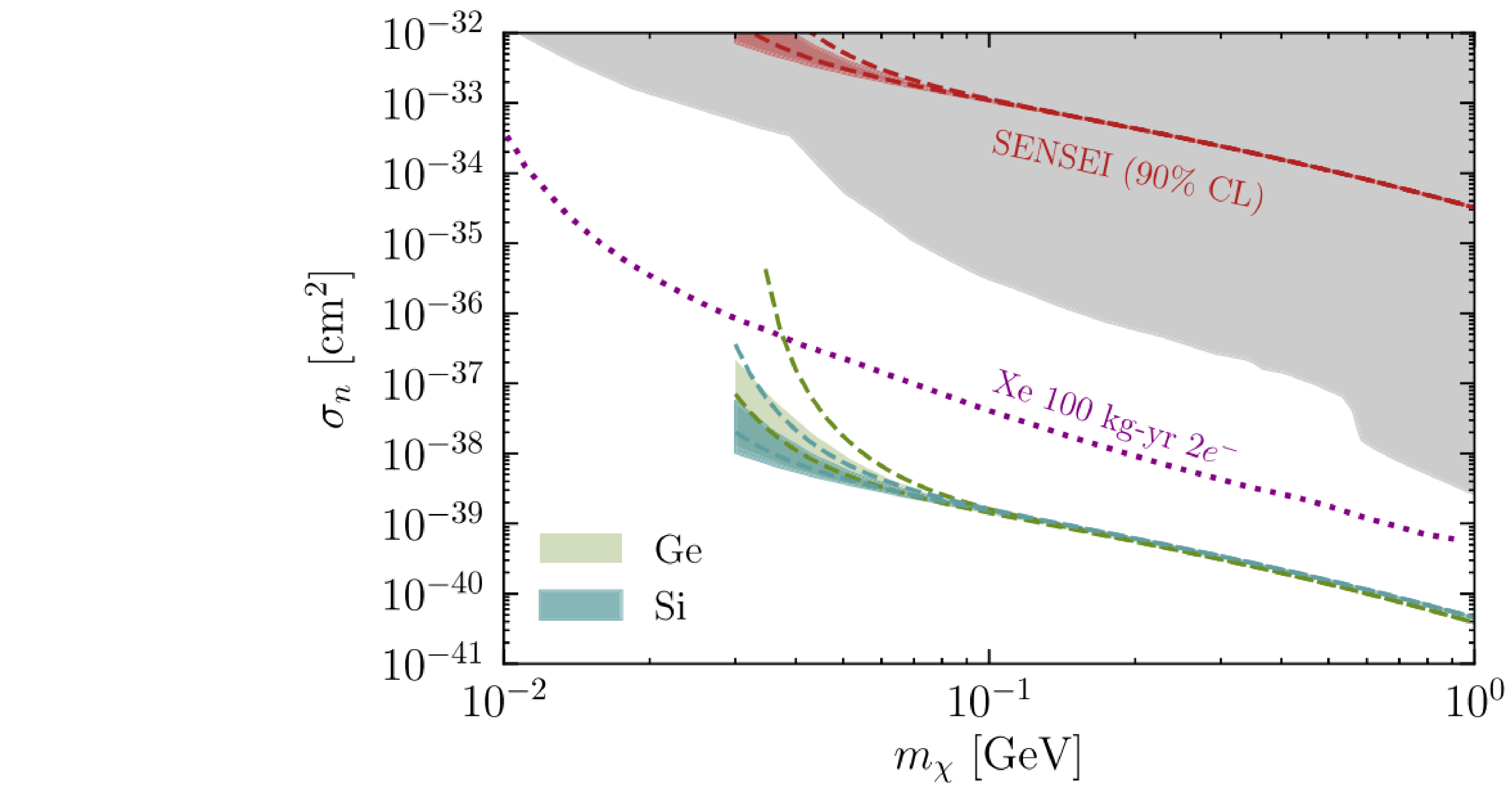
$|\lambda\rangle$ ~ plane wave

$|\lambda_f\rangle$ ~ plane wave

Impulse approximation

Impulse approximation breaks down when $q \sim \sqrt{2m_N E_{ph}}$

The Migdal Effect in Semiconductors



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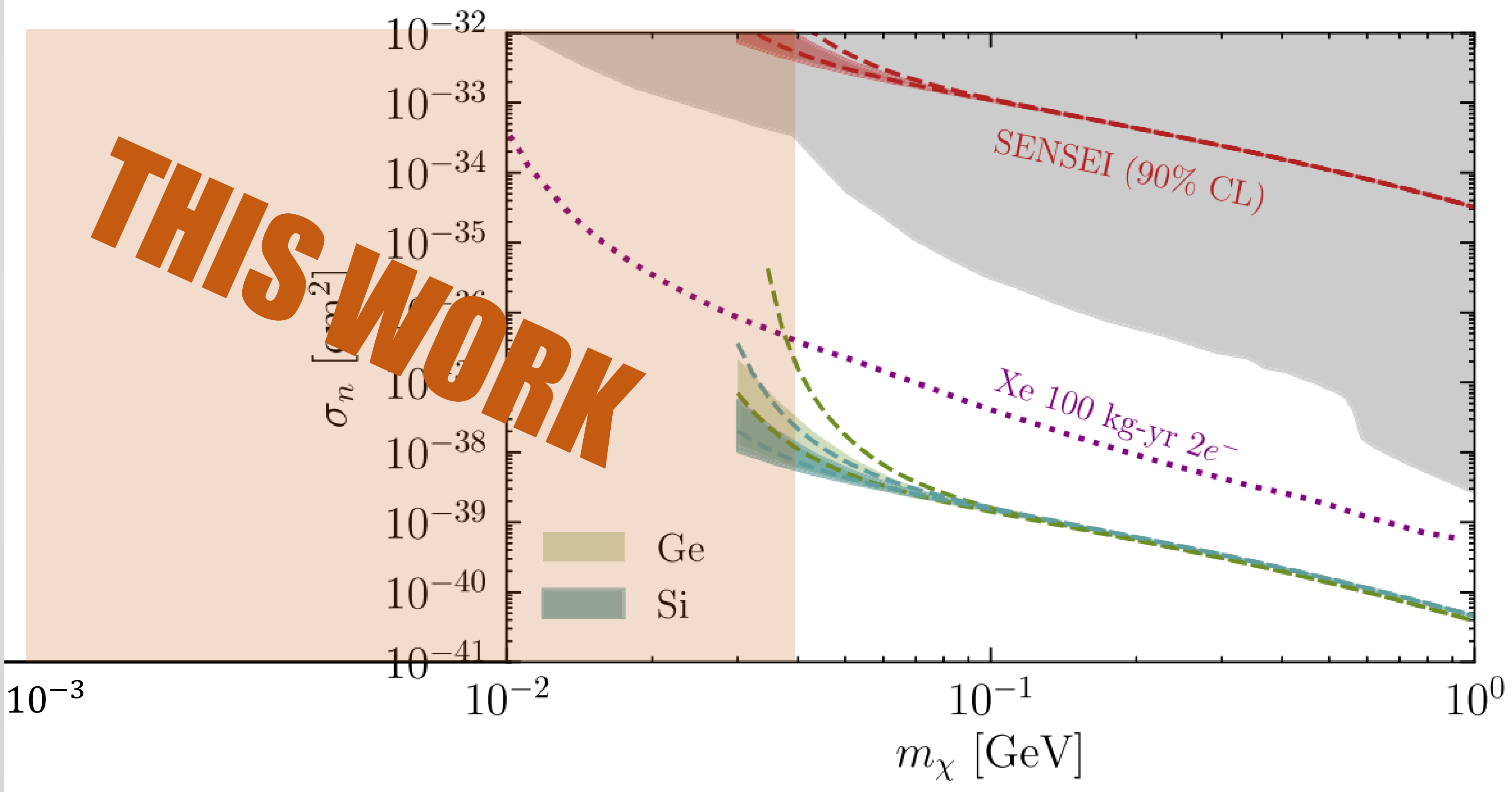

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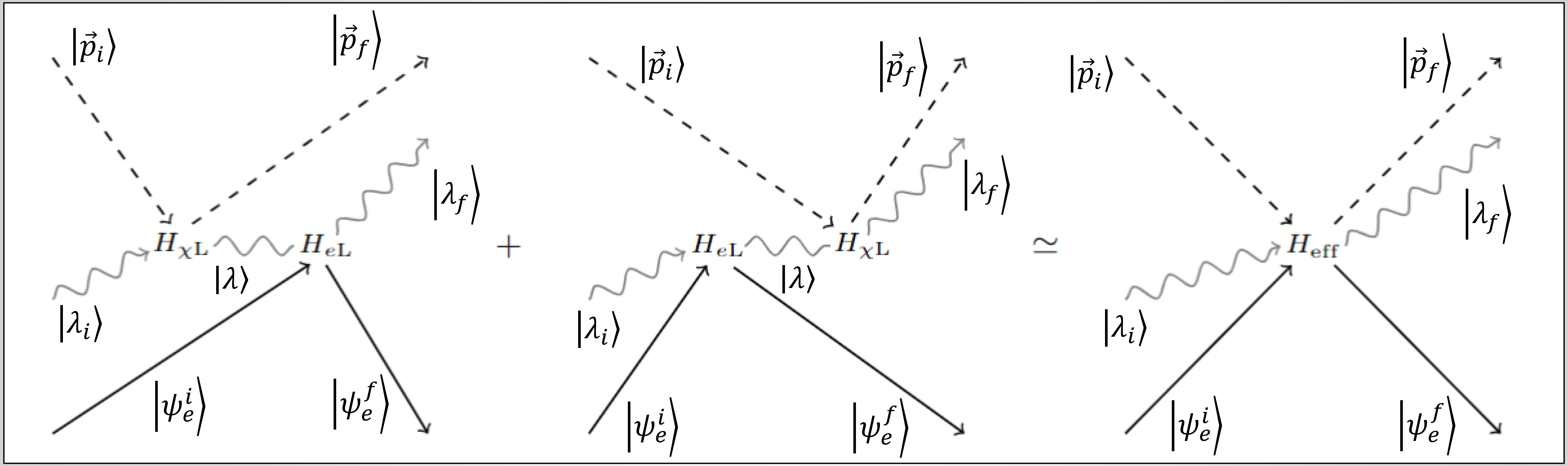
The Migdal Effect in Semiconductors from Effective Field Theory



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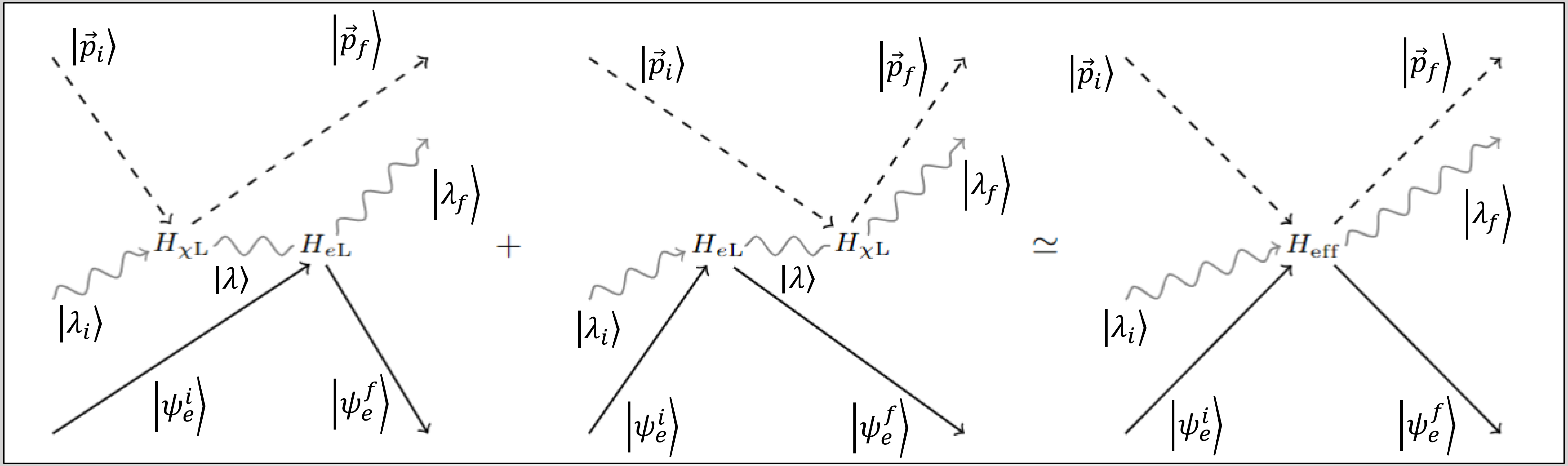

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Separation of scales ($\omega(\text{eV}) \gg \lambda_i, \lambda_f, \lambda$ ($\text{O}(100)$ meV)) allows us to integrate out intermediate lattice modes

$$H_{eff} = \frac{1}{m_N \omega^2} \nabla H_{\chi L} \cdot \nabla H_{eL} + O(\frac{1}{\omega^3})$$

The Migdal Effect in Semiconductors from Effective Field Theory

We derive an expression for the cross-section that depends only on the crystal dynamic structure function of the material

$$\frac{d\sigma_{crystal}}{d\omega} \propto \int d\vec{q}^3\, S(\vec{q}, \vec{q}\cdot\vec{v} - \frac{q^2}{2m_\chi} - \omega)$$

$$S(q, E) = \sum_{\lambda_f} \left|\langle\lambda_f|\, e^{i\,\vec{q}\cdot\vec{x}} |\lambda_i\rangle\right|^2 \delta\left(E_{\lambda_i} - E_{\lambda_f} - E\right)$$

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Can be measured directly!

$$S(q, E) = \sum_{\lambda_f} \left|\langle\lambda_f|\, e^{i\,\vec{q}\cdot\vec{x}} |\lambda_i\rangle\right|^2 \delta\left(E_{\lambda_i} - E_{\lambda_f} - E\right)$$

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$$\frac{d\sigma_{atom}}{d\omega} \propto \int d\vec{q}^3\, \delta\left(\vec{q}\cdot\vec{v} - \frac{q^2}{2m_\chi} - \omega - \frac{q^2}{2m_N}\right)$$

The Dynamic Structure Function $S(q,E)$ in the Harmonic Approximation

$$2W(q) = \frac{q^2}{2\, m_N E_{ph}}$$

$$S(q,E) = \int_{-\infty}^{\infty} \frac{dt}{2\pi} e^{-i\,E\,t} e^{-2W(q)} e^{\langle \vec{q}\cdot\vec{u}(0)\vec{q}\cdot\vec{u}(t)\rangle}$$

Phonon density of states

$$\propto \int_{-\infty}^{\infty} dE' \frac{g(E')}{E'} e^{i\,E'\,t}$$

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$$T_p(E) = \int_{-\infty}^{\infty} dE' \, T_1(E-E') T_{p-1}(E')$$

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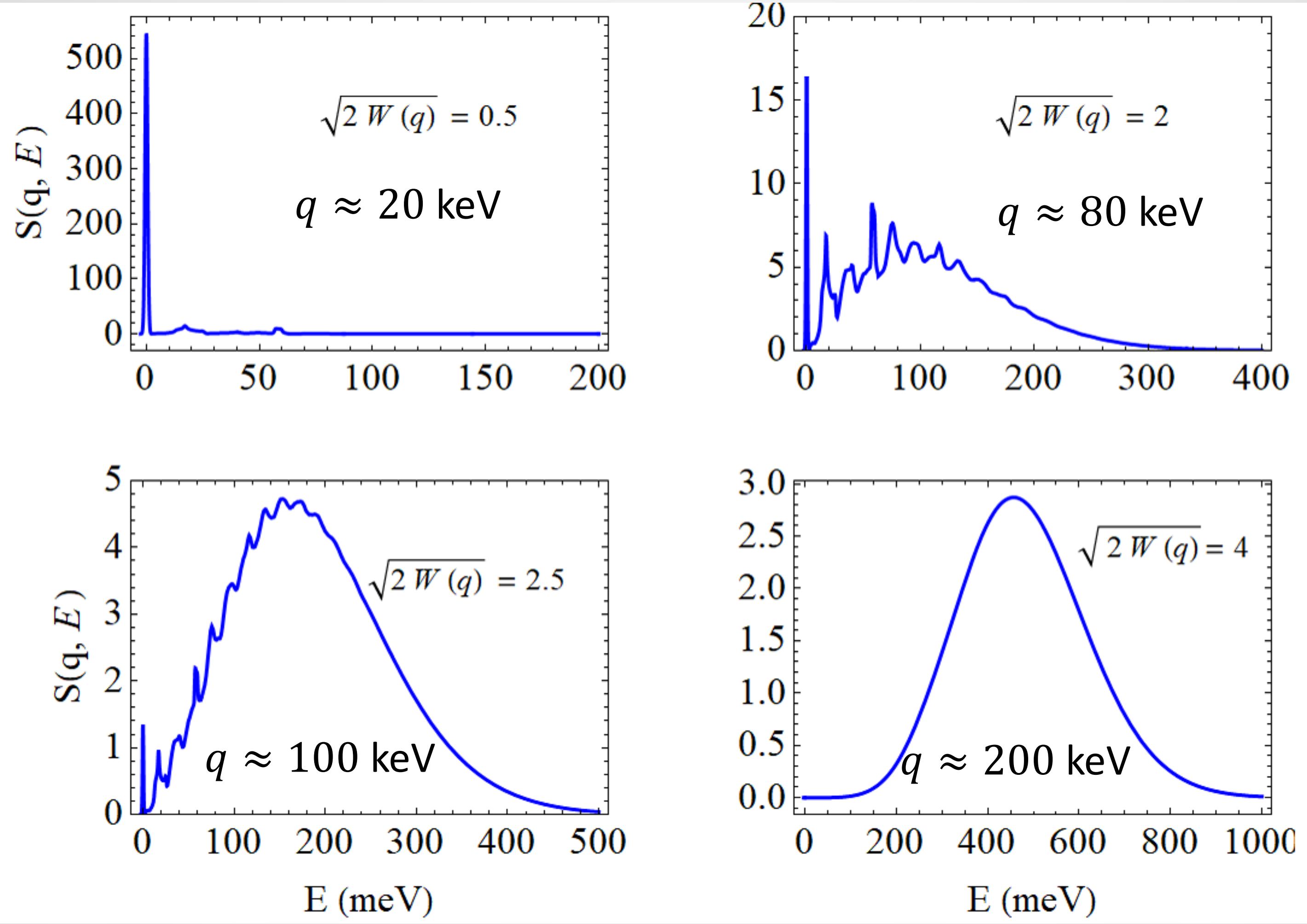

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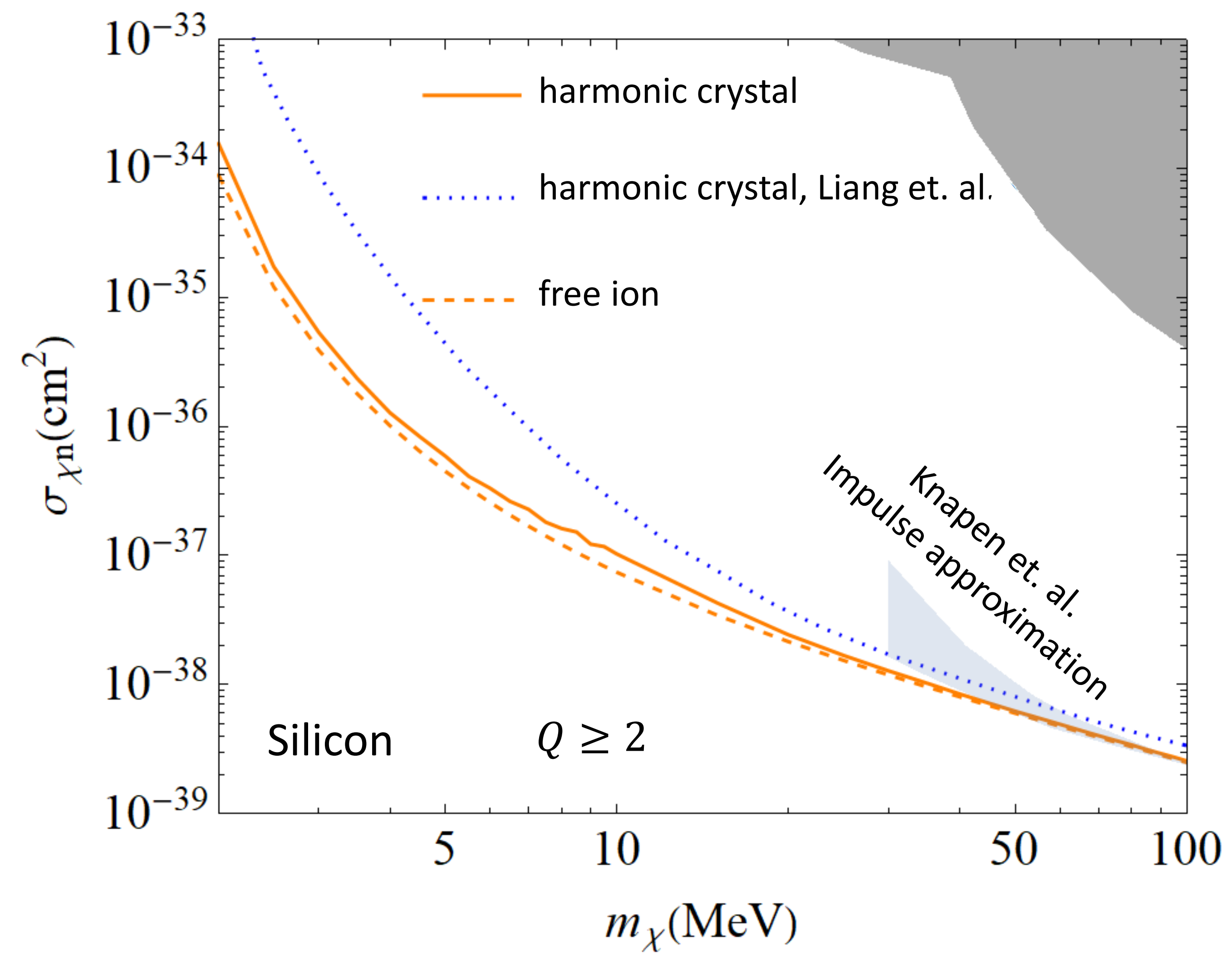
The Migdal Effect in Semiconductors for small DM masses



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Summary and Outlook

- Effective field theory approach connects DM rate with measurable dynamic structure function and electron loss function
- For precise calculations S(q,E) should be obtained from data directly
- Our framework fully quantifies differential cross-section $\frac{d\sigma}{dE\ d\omega}$
- Semiconductors have unmatched sensitivity to light DM down to ~ MeV

Thank you for your attention!