Summary of weeks 1-3: Uncertainty Quantification

INT26-2a

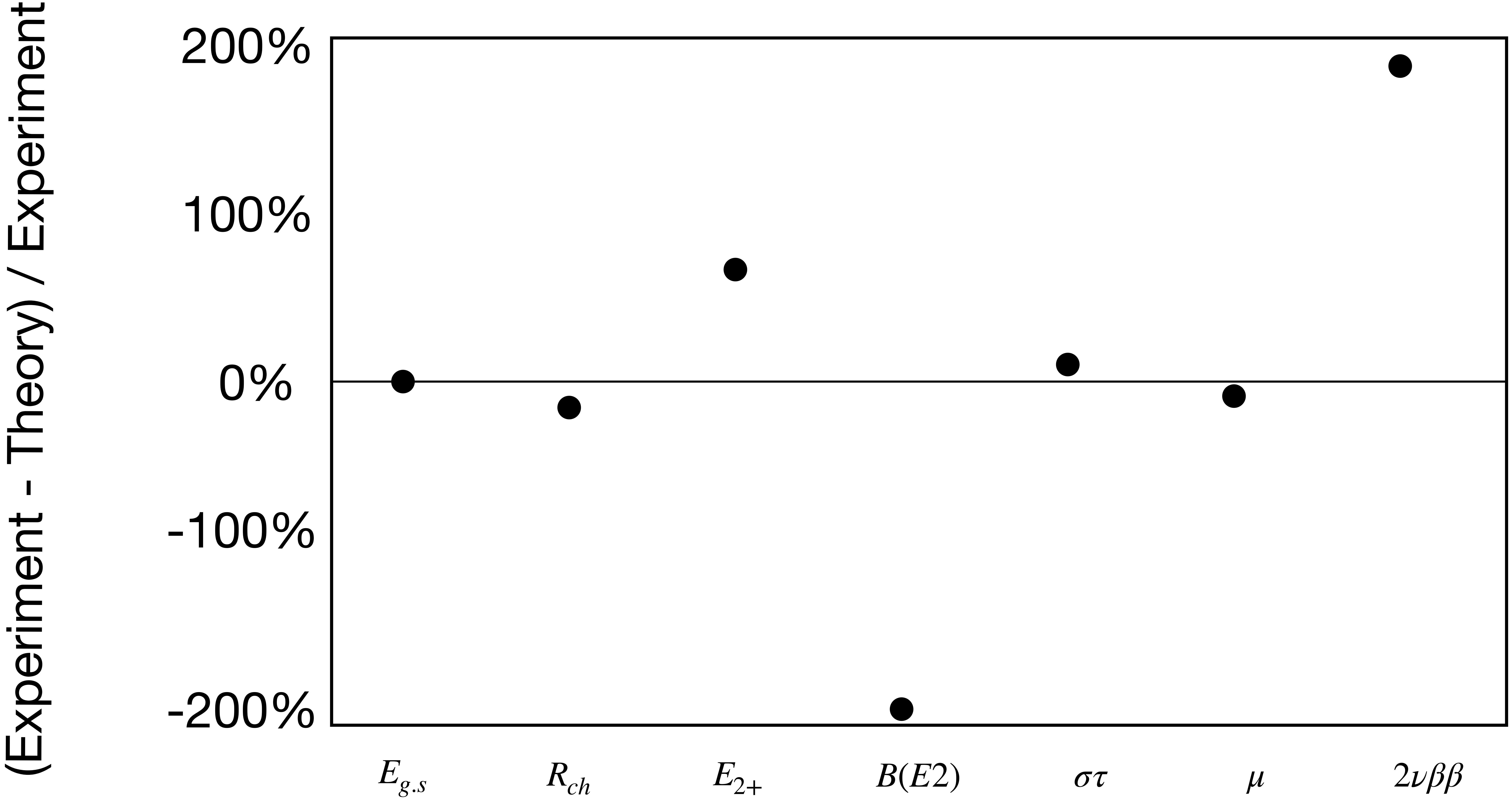
Antoine Belley

20 July 2026

Outline

1. Sources of uncertainties in NME calculations.
2. Status of uncertainty evaluations.
3. Framework to assess uncertainties.
4. How to improve uncertainty models?

The Current Picture



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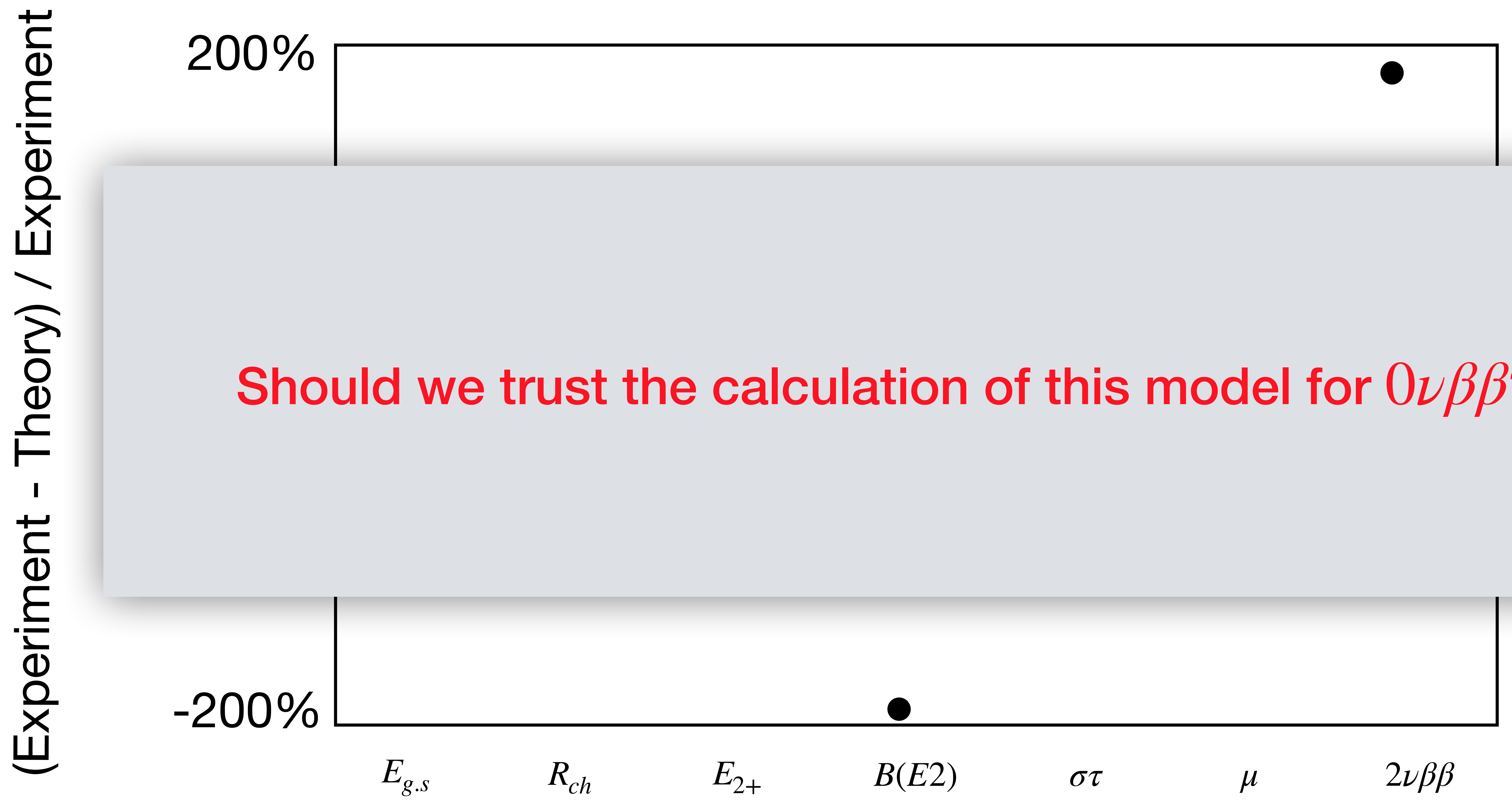


Should we trust the calculation of this model for $0\nu\beta\beta$?

Sources of uncertainties in calculations of NME

Parametric uncertainties

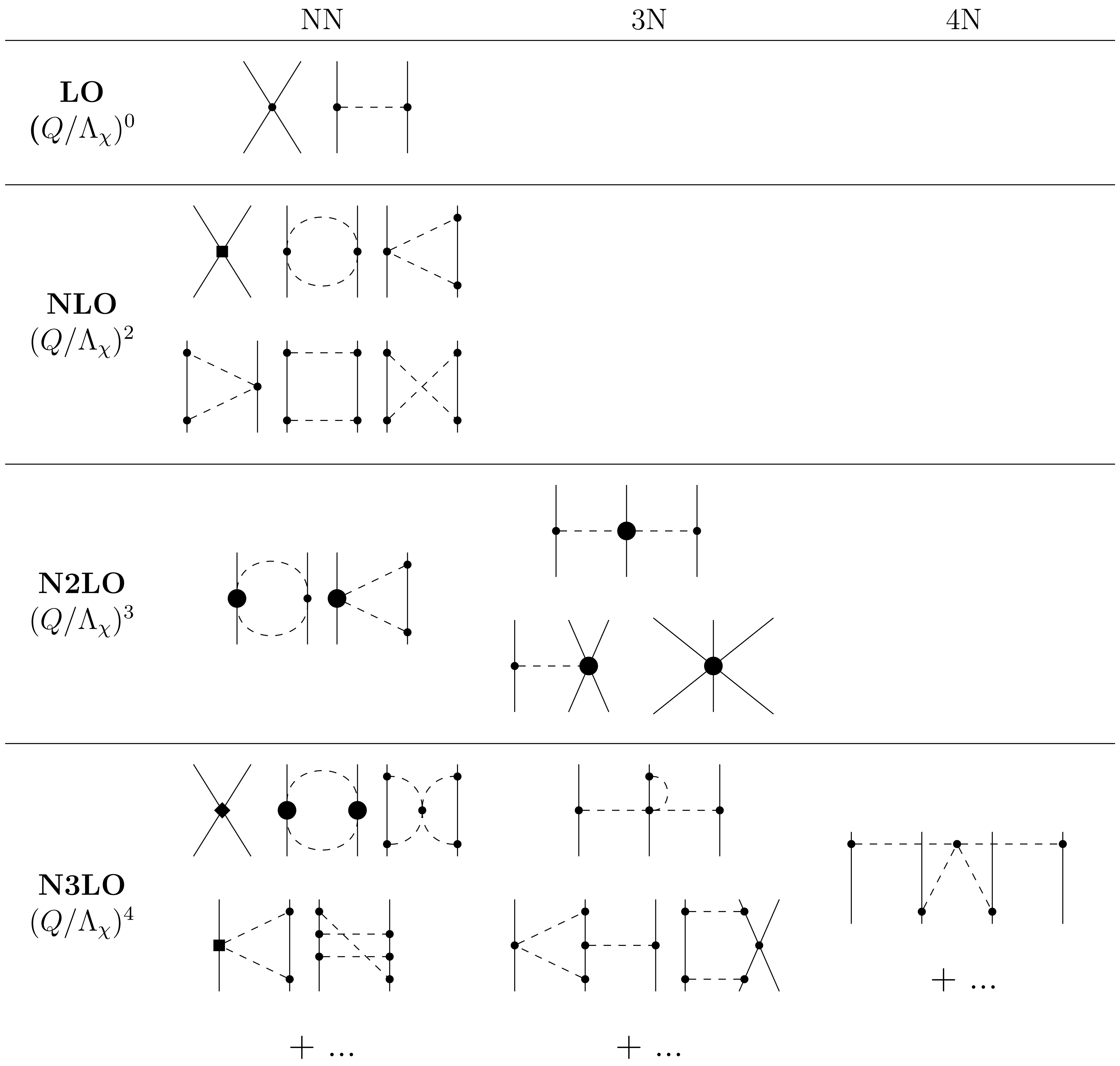
Uncertainty associated with the variation of parameters in the nuclear models, e.g.:

- **Shell-Model:** Matrix elements of effective shell-model interactions.
- **Ab initio methods:** Low-energy constants of nuclear interactions.

Closest thing to a statistical uncertainty.

Nuclear Interaction Uncertainties

Systematic uncertainties coming from the nuclear interaction:



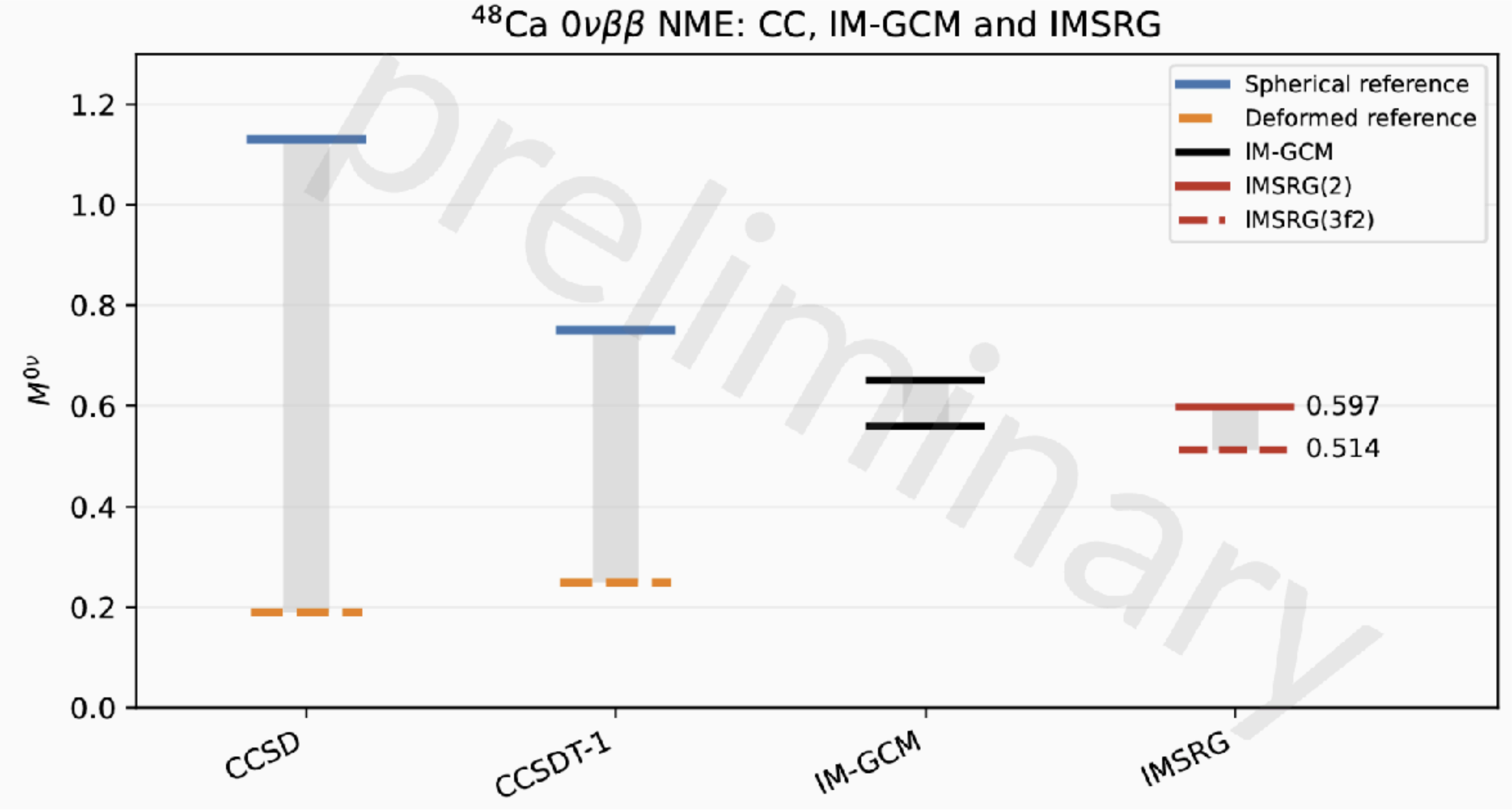
Ab initio: Interaction truncated at a fixed order in chiral expansion.

Power counting issues remain to be clarified/investigated.

Phenomenological: Challenging to evaluate.

Nuclear Model Uncertainties

Systematic error coming from the model used to solve the many-body Schrödinger equation and obtain the wave functions.



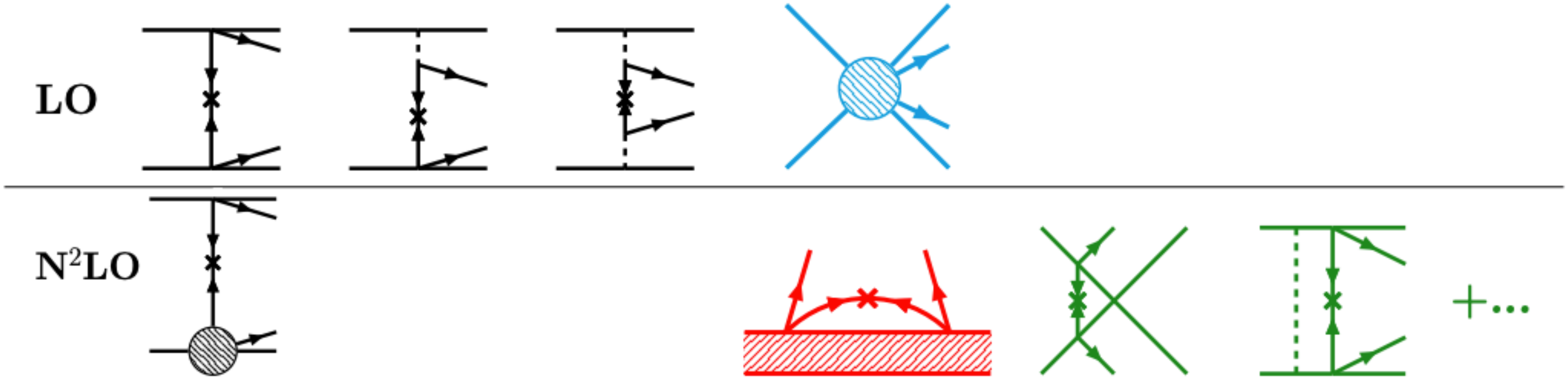
See Bincheng He's and Ragnar Stroberg's talks from week 1.

Decay Operator Uncertainties

Complete approach based on EFT allows to find corrections to operators:

$$[T_{1/2}^{0\nu}]^{-1} = g_A^4 G^{0\nu} \,|\, M_{LR}^{0\nu} + M_{SR}^{0\nu} + M_{\text{usoft}}^{0\nu} + M_{\text{loops}}^{0\nu} \,|^2 \left(\frac{m_{\beta\beta}}{m_e} \right)^2$$

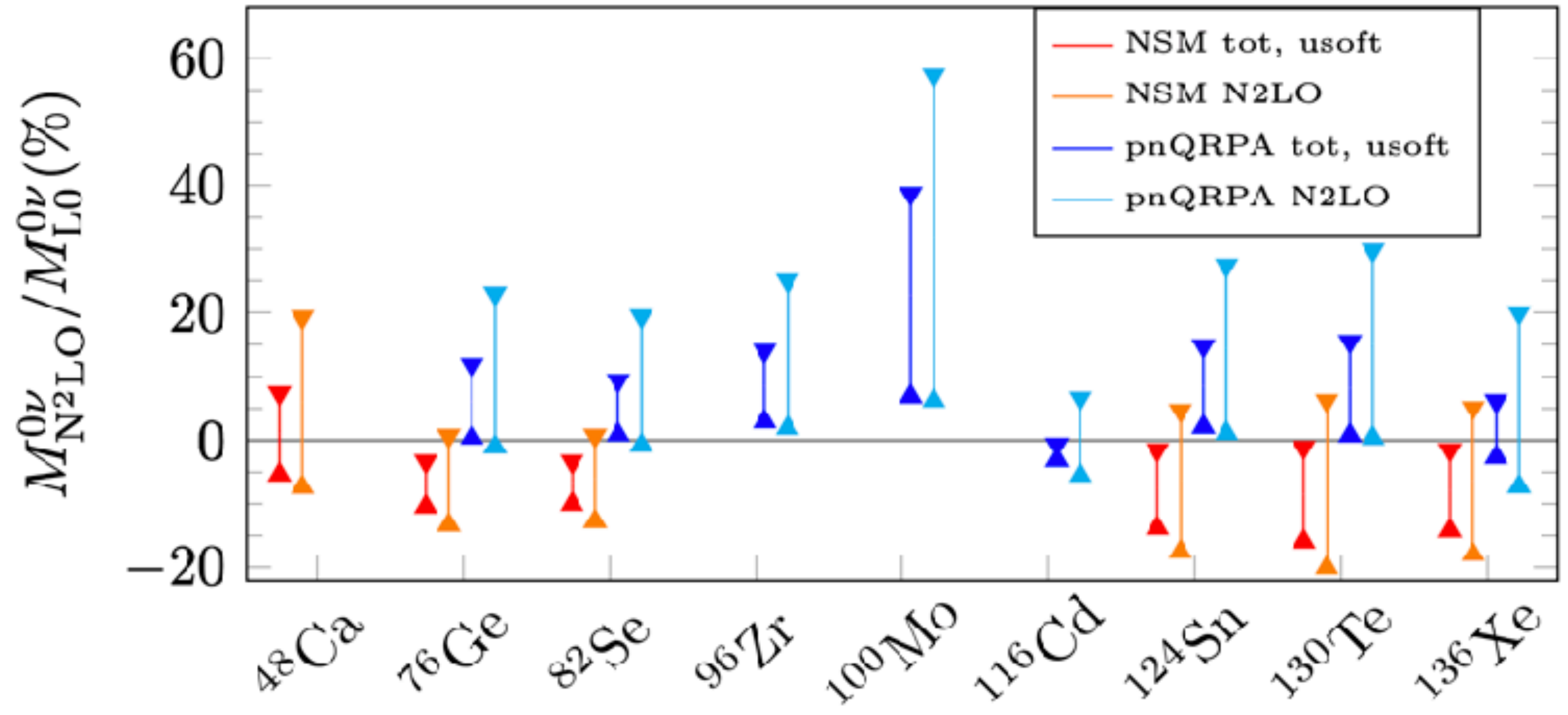
V. Cirigliano et al., Phys. Rev. C 97, 065501 (2018), Phys. Rev. Lett. 120, 202001 (2018), Phys. Rev. C 100, 055504 (2019)



See EFT summary.

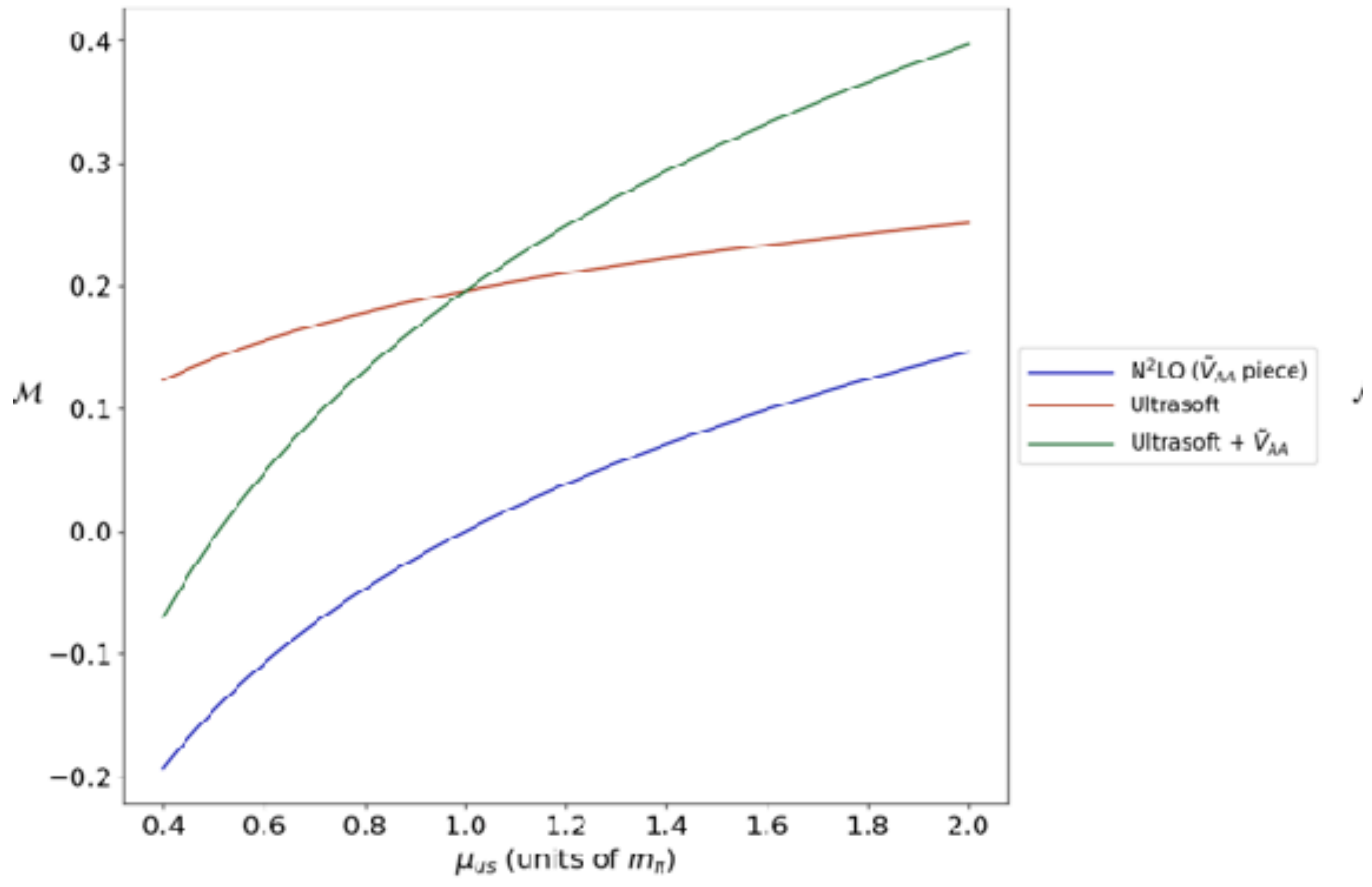
Figure courtesy of L. Jokiniemi

Decay Operator Uncertainties



DC, Jokiniemi, Menéndez, Soriano, Phys. Lett. B, 860, 2025

See Daniel Castillo Garcia's talk from week 2 for evaluation in phenomenological methods.



^{48}Ca → ^{48}Ti (Magic)

See Amelia Remnant's talk from week 1 for evaluation in ab initio method.

Current status of UQ

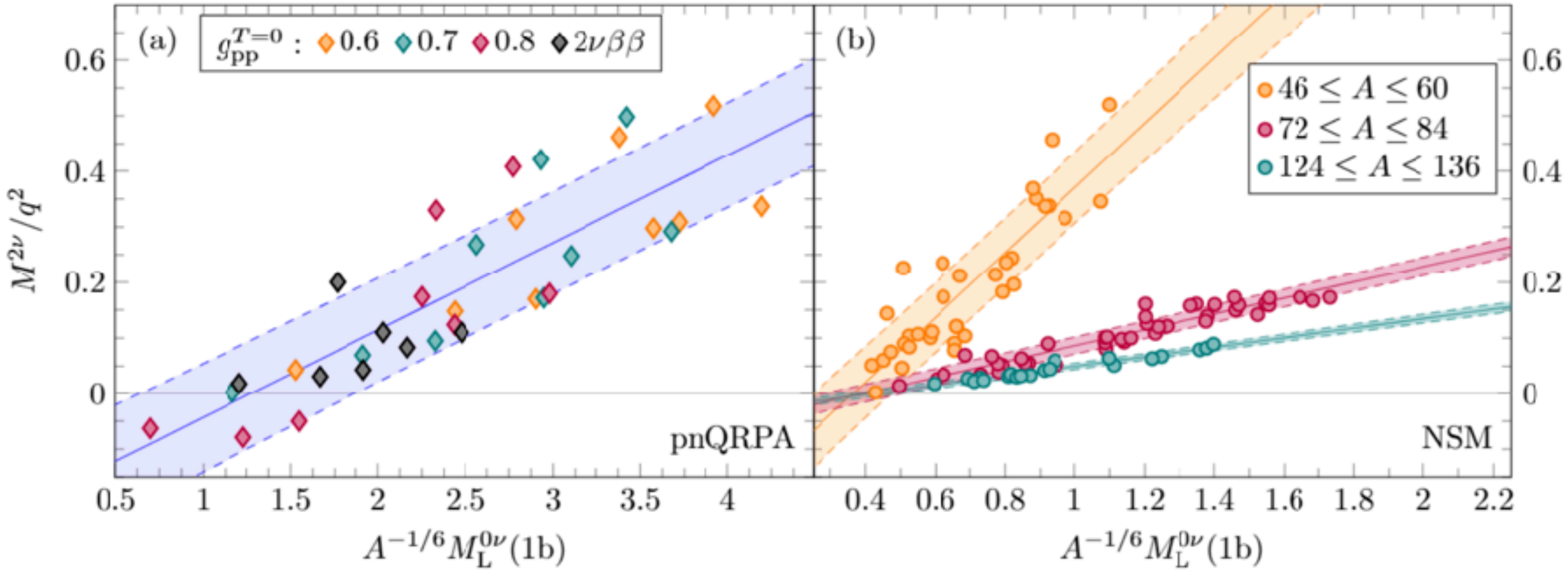
Using Correlations with Other Observables

$\underline{2\nu\beta\beta}$

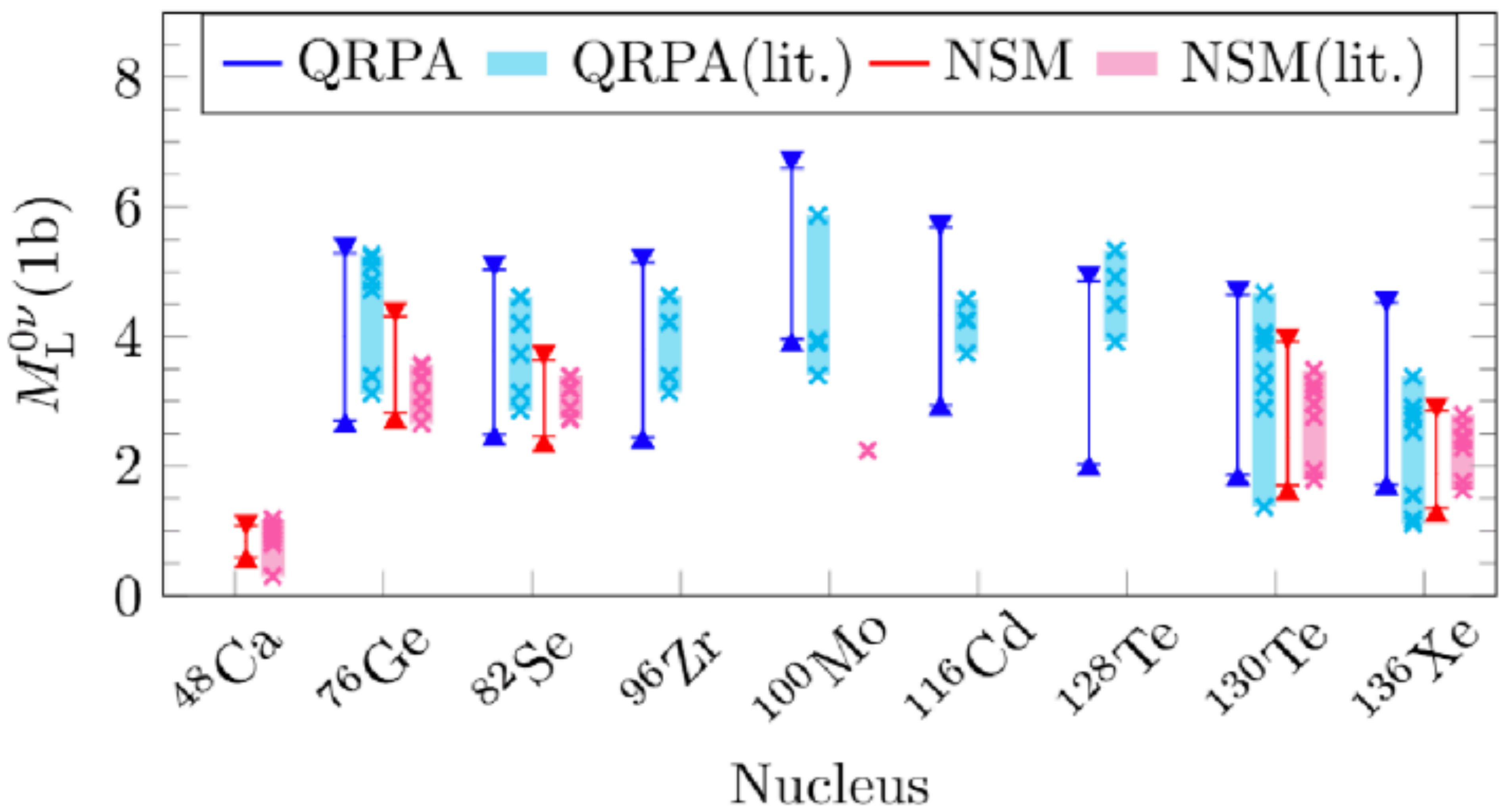
Using Correlations with Other Observables

$2\nu\beta\beta$

See Lotta Jokiniemi's talk from week 3.



Jokiniemi, Romeo, Soriano, Menéndez, Phys Rev. C 107, 044305 (2023)



Jokiniemi, Romeo, Soriano, Menéndez, Phys Rev. C 107, 044305 (2023)

Using Correlations with Other Observables

$\underline{2\nu\beta\beta}$

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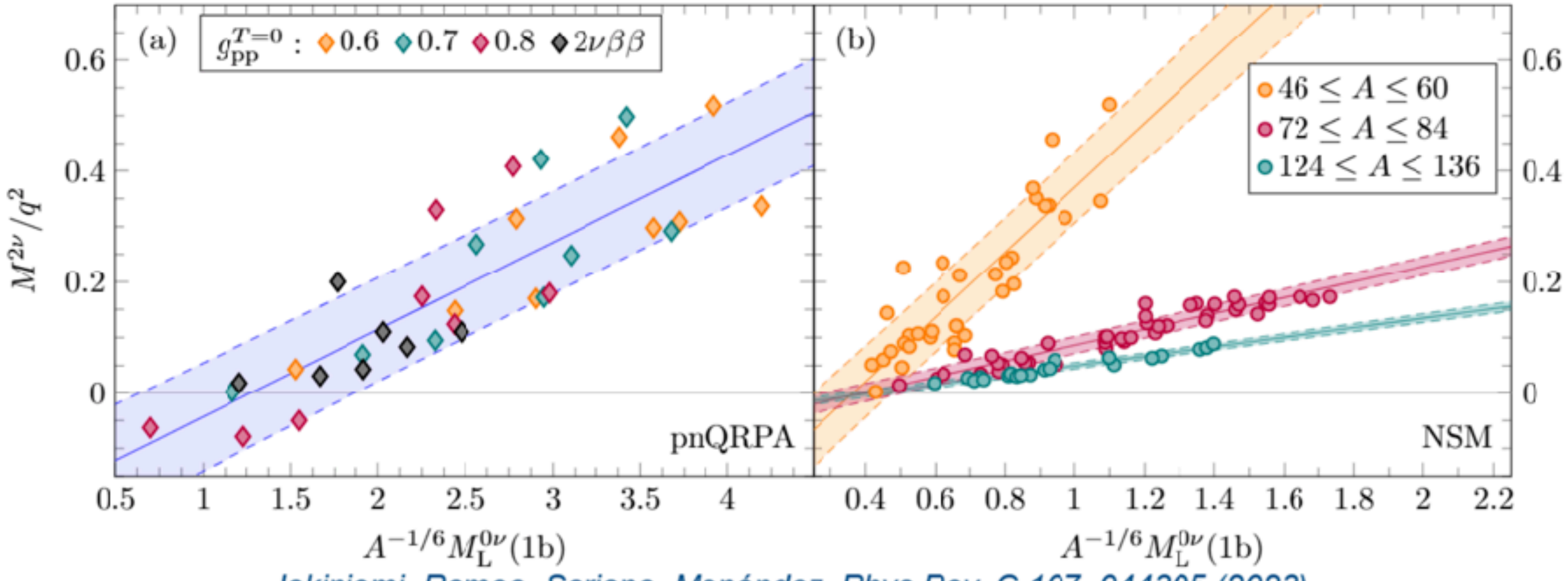
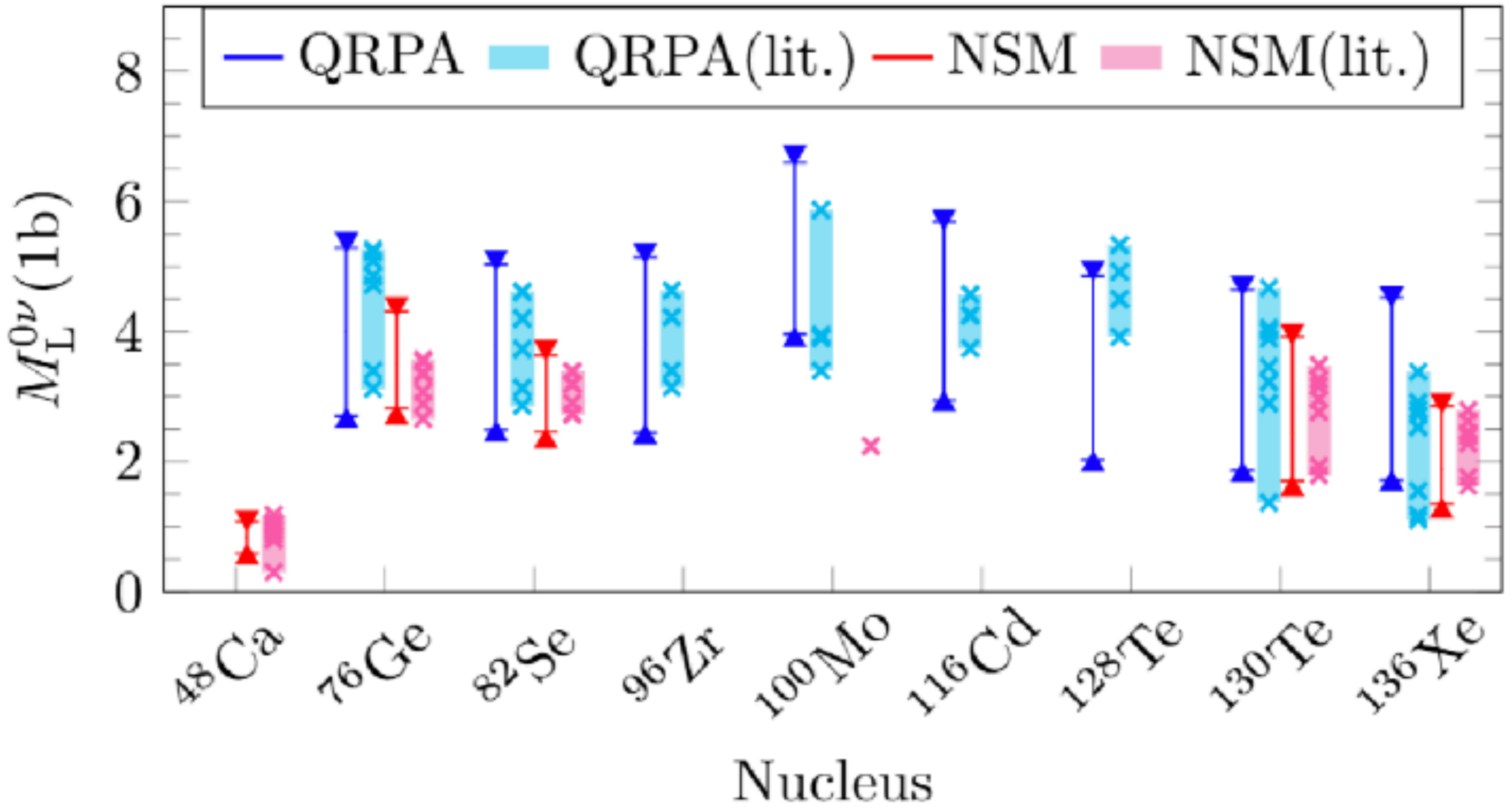
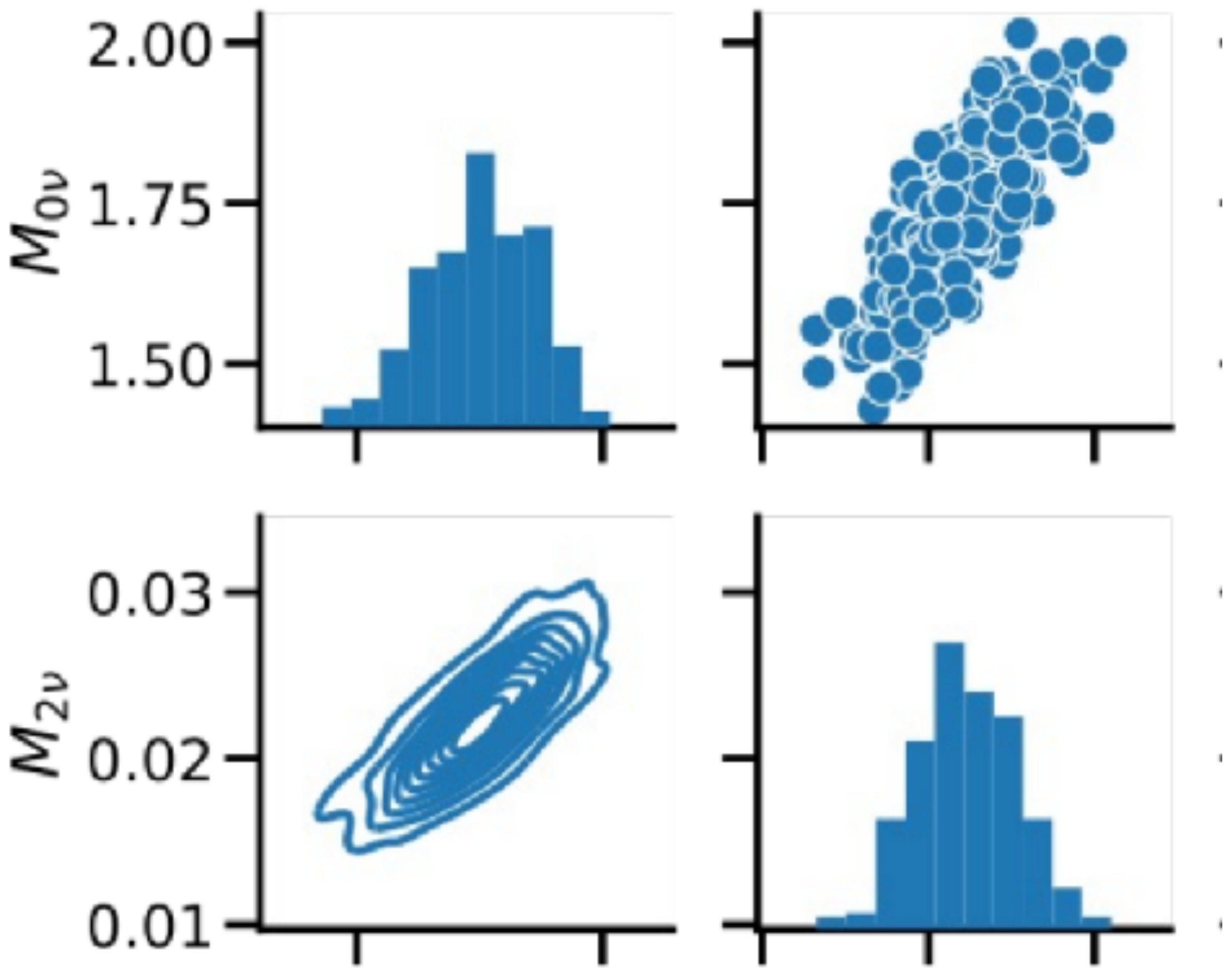


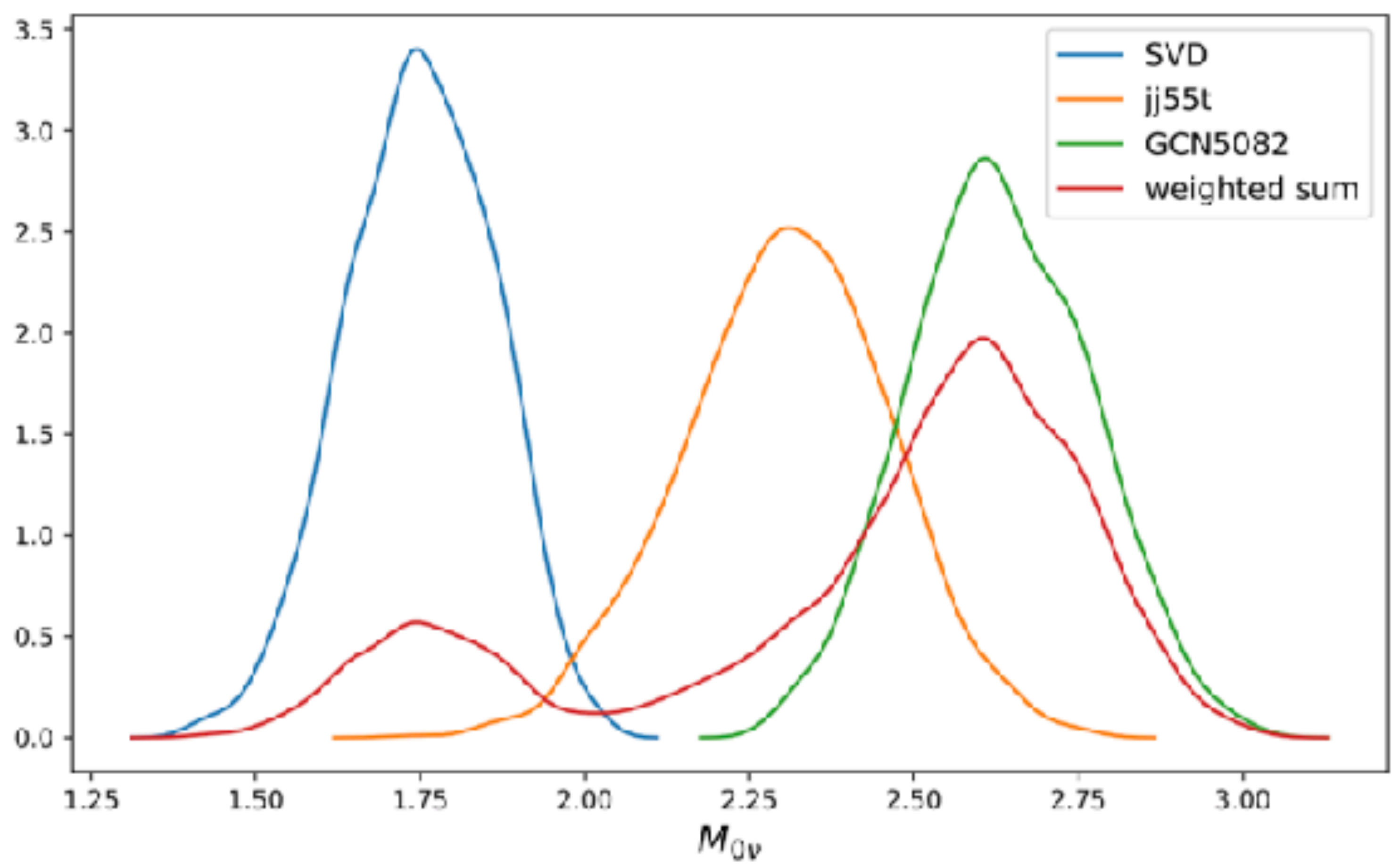
Jokiniemi, Romeo, Soriano, Menéndez, Phys Rev. C 107, 044305 (2023)



Jokiniemi, Romeo, Soriano, Menéndez, Phys Rev. C 107, 044305 (2023)



M. Horoi, Universe 10, 252 (2024)



See Mihai Horoi's talk from week 3.

Using Correlations with Other Observables

$\underline{2\nu\beta\beta}$

Using Correlations with Other Observables

$2\nu\beta\beta$

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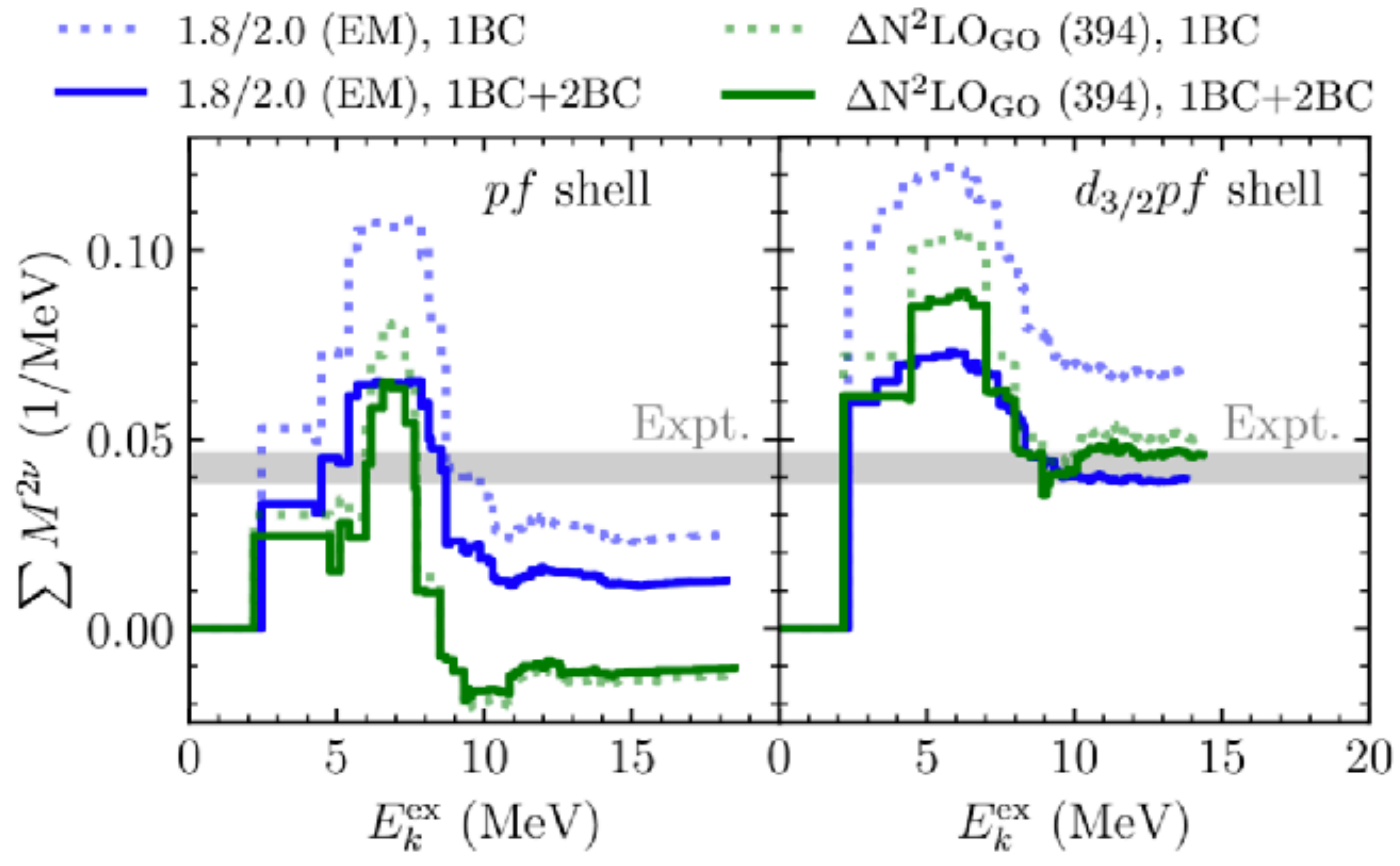
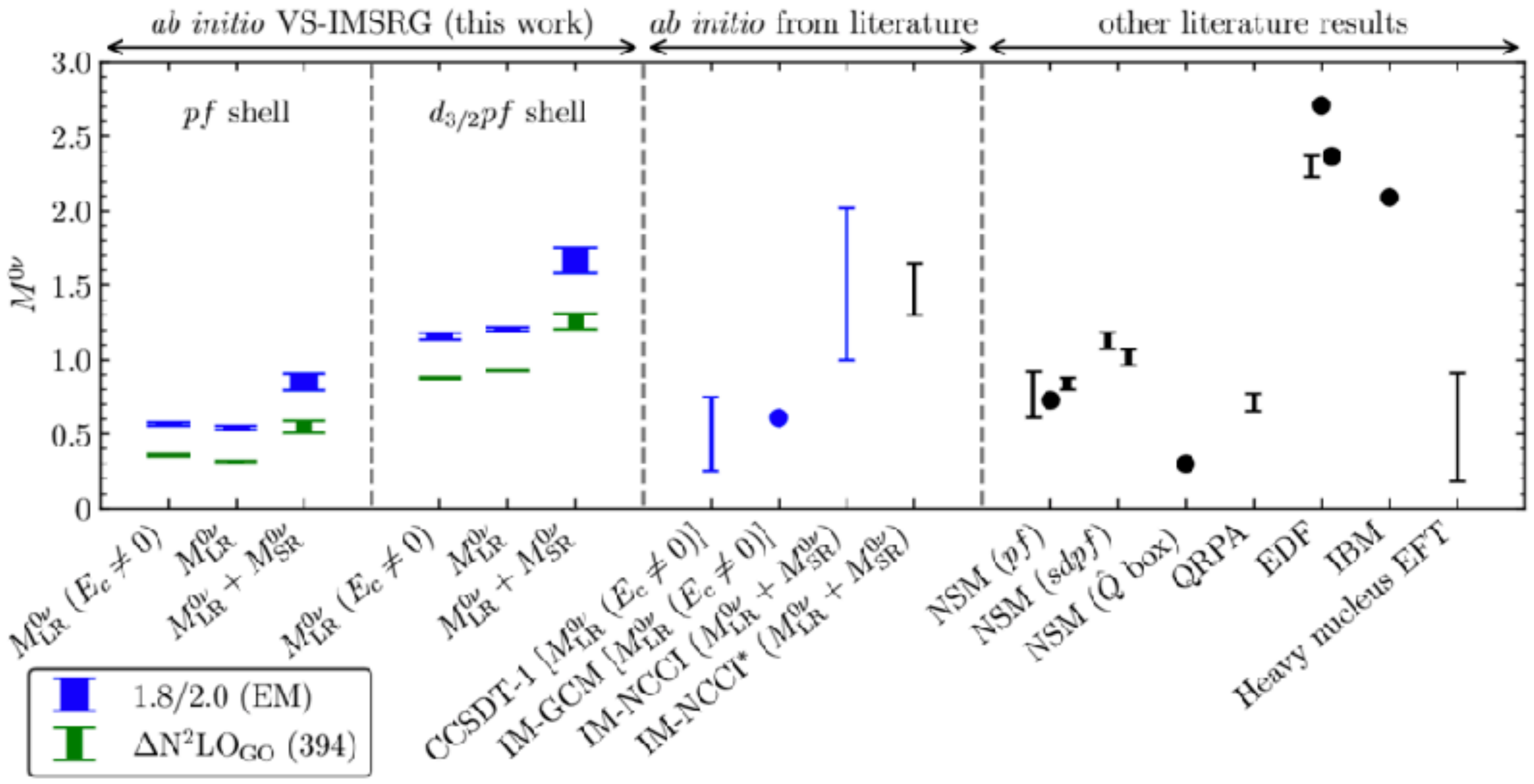


Li, Jokiniemi, Schwenk, arXiv:2607.11733



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Using Correlations with Other Observables

$2\nu\beta\beta$

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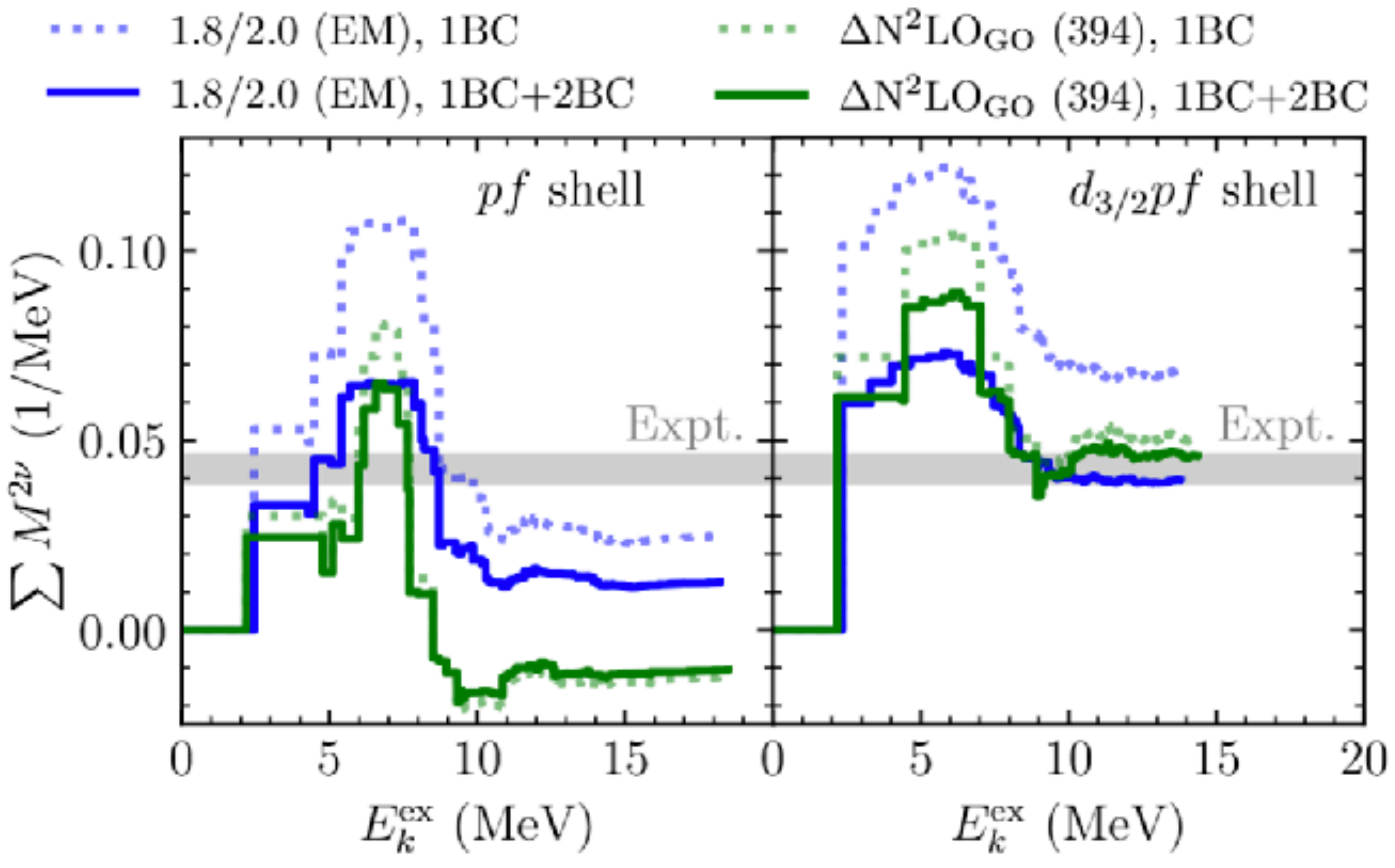
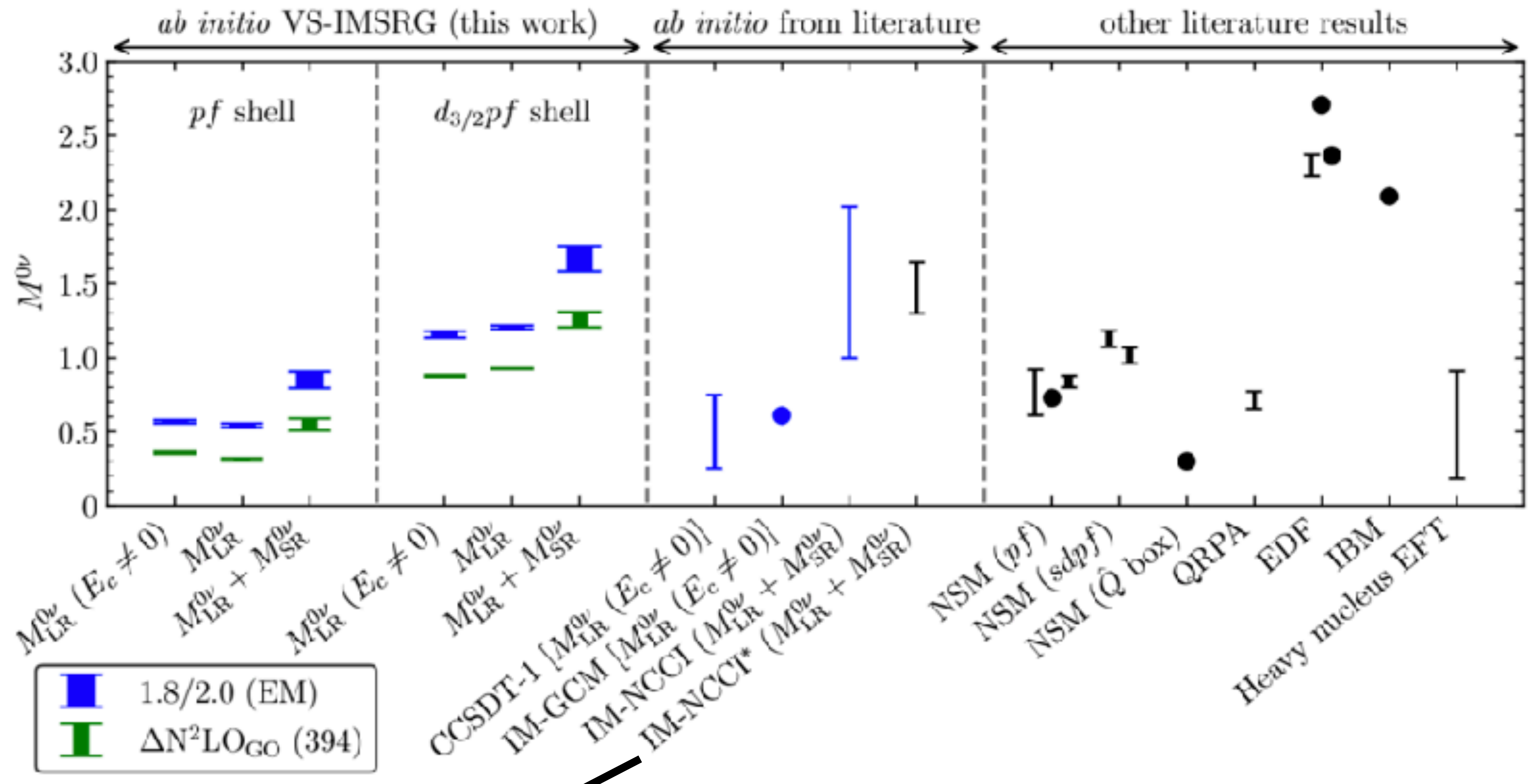
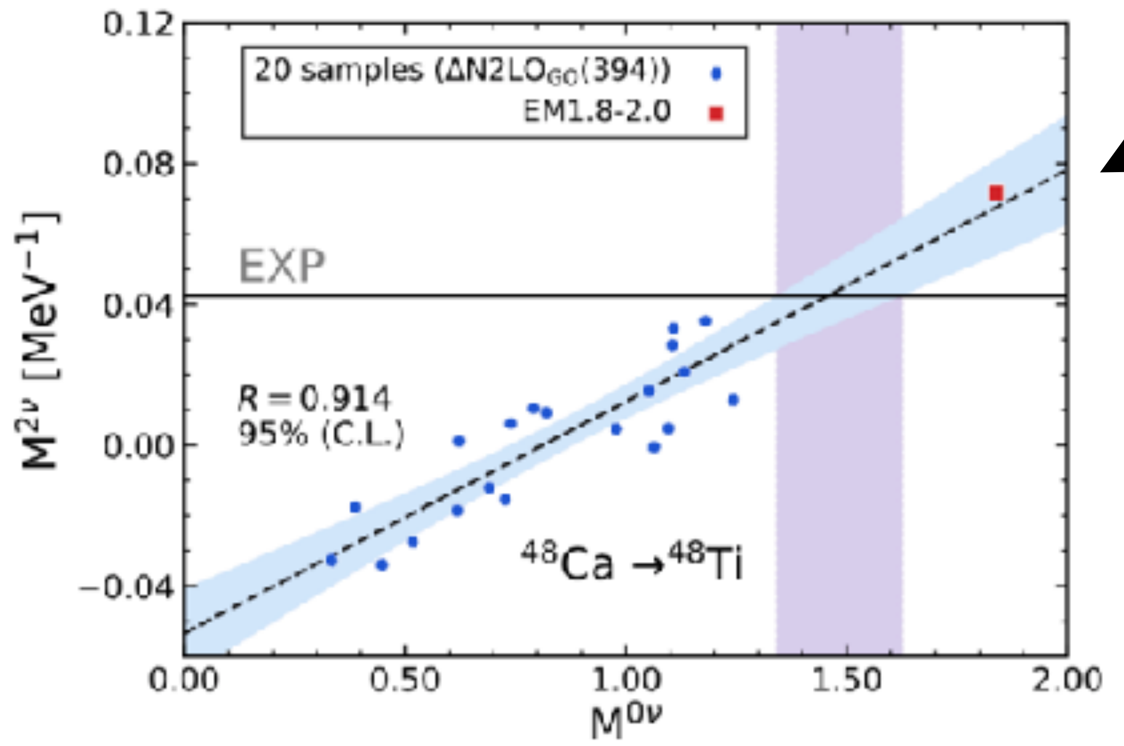


Li, Jokiniemi, Schwenk, arXiv:2607.11733



Li, Jokiniemi, Schwenk, arXiv:2607.11733

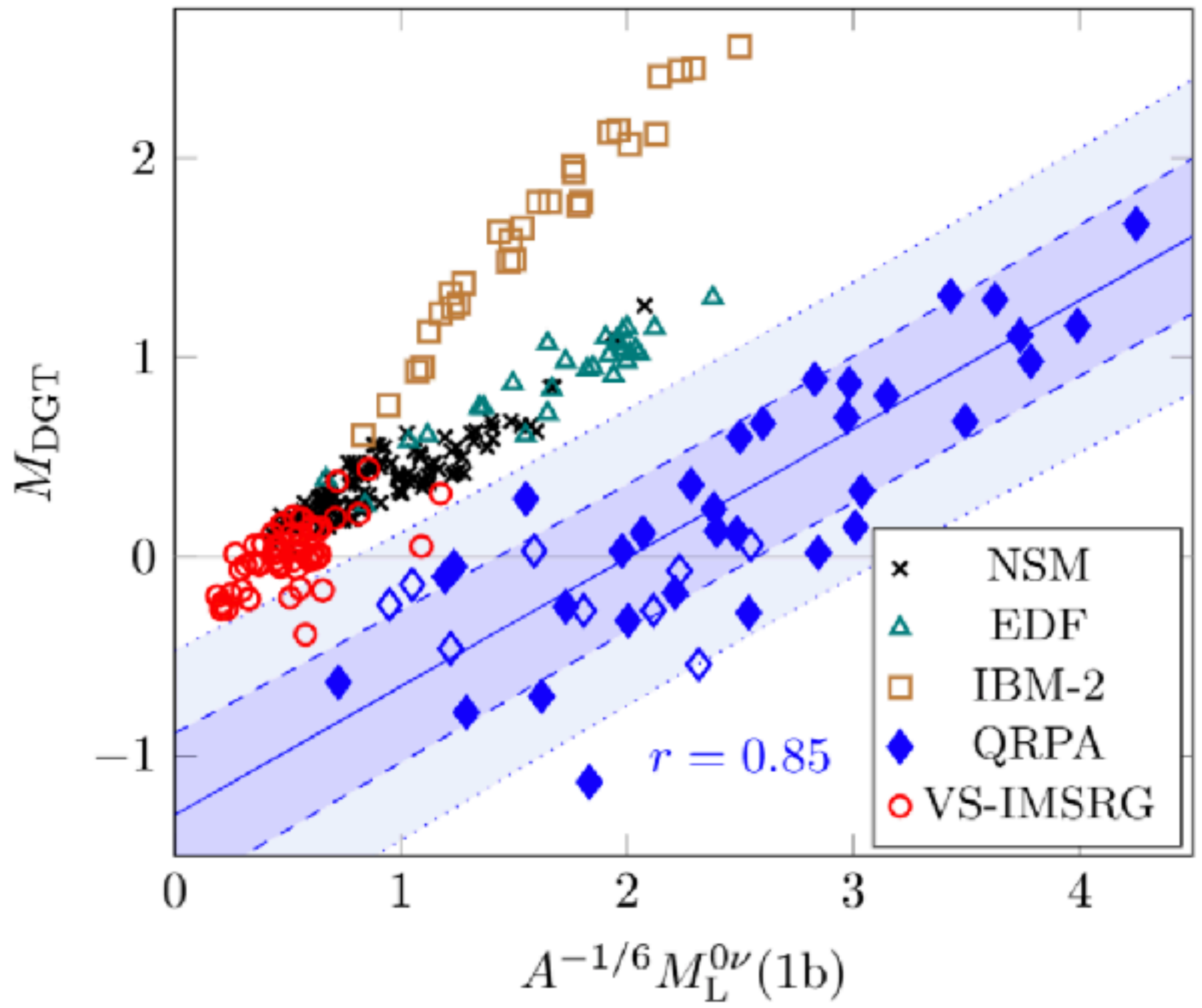


Lian, et al., arxiv:2605.19479

Using Correlations with Other Observables

Double Gamow-Teller

$$M^{DGT} = \langle f || [\sum_{ik} \sigma_j \tau_j^- \times \sigma_k \tau_k^-]^{(0)} || i \rangle$$



Jokiniemi, Menéndez, Phys. Rev. C 107, 044316 (2024)

Correlation observed both in phenomenological and ab initio models.

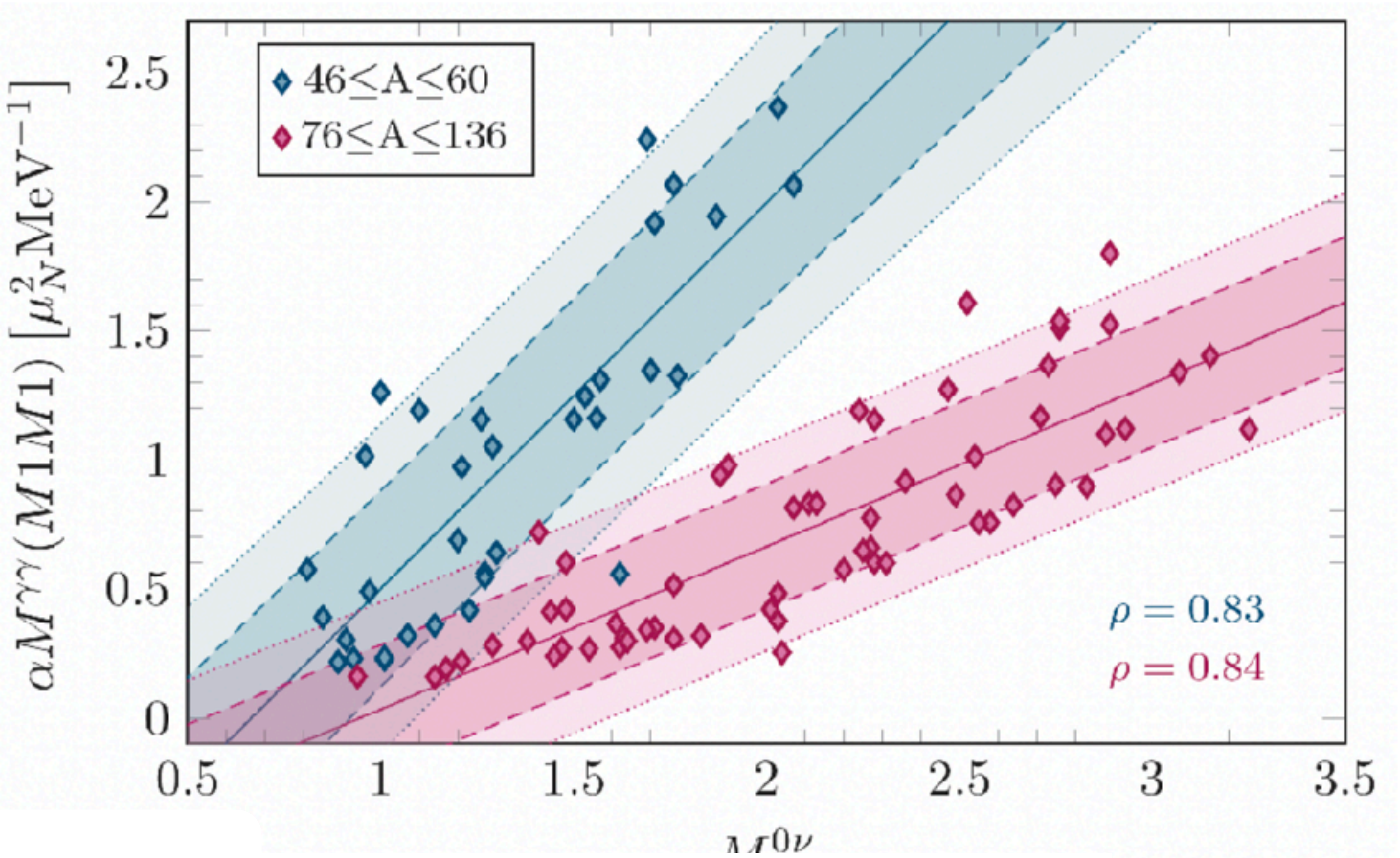
Process has not been measured.

See Lotta Jokiniemi's talk from week 3.

Using Correlations with Other Observables

Double Gamma-Decay

$$M^{\gamma\gamma}(M1M1) = \sum_n \frac{\langle 0^+_{gs}||\mathbf{M1}||1^+_n\rangle\langle 1^+_n||\mathbf{M1}||0^+_i(DIAS)\rangle}{E_n - \frac{1}{2}(E_i + E_f)}$$



BR, J.Menéndez and C. Peña-Garay, Phys. Lett. B 827, 136965 (2022)
$M_L^{0\nu}$ from N. Shimizu *et. al.* Phys. Rev. Lett 120, 142502 (2018)

Correlation observed in phenomenological models.

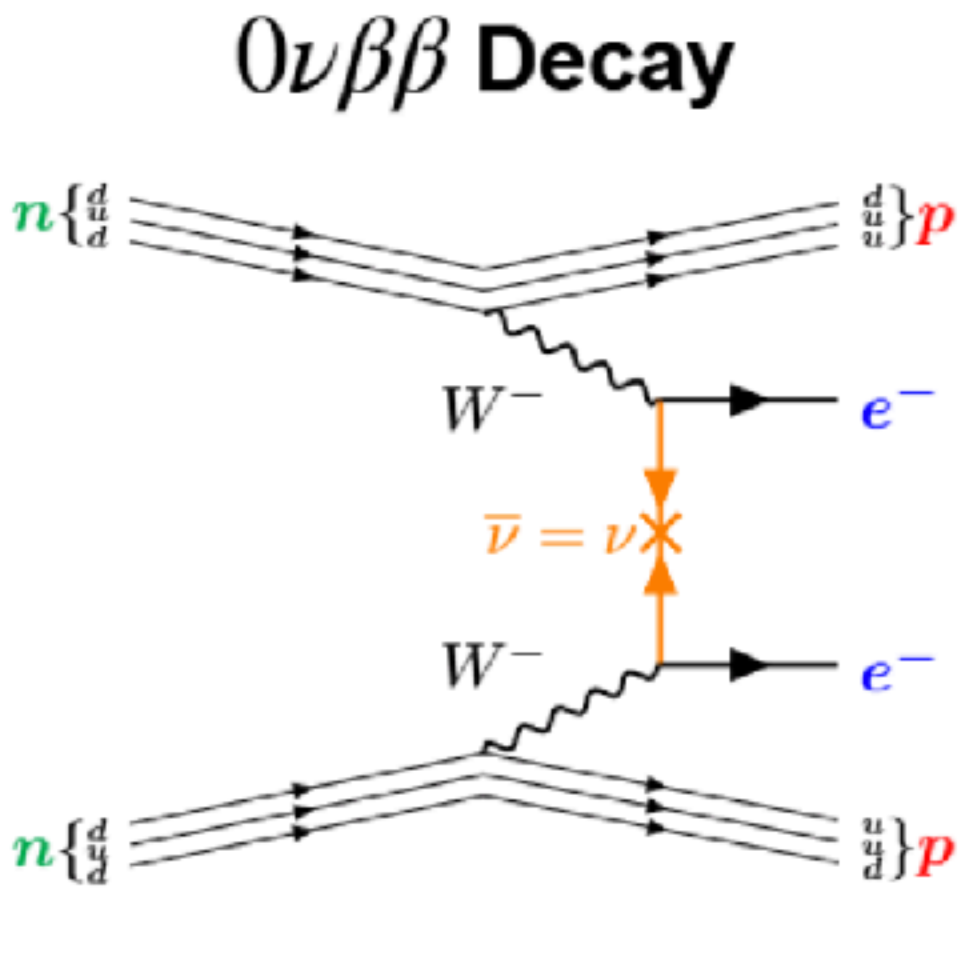
Process has not been measured but possibility of measurement was discussed.

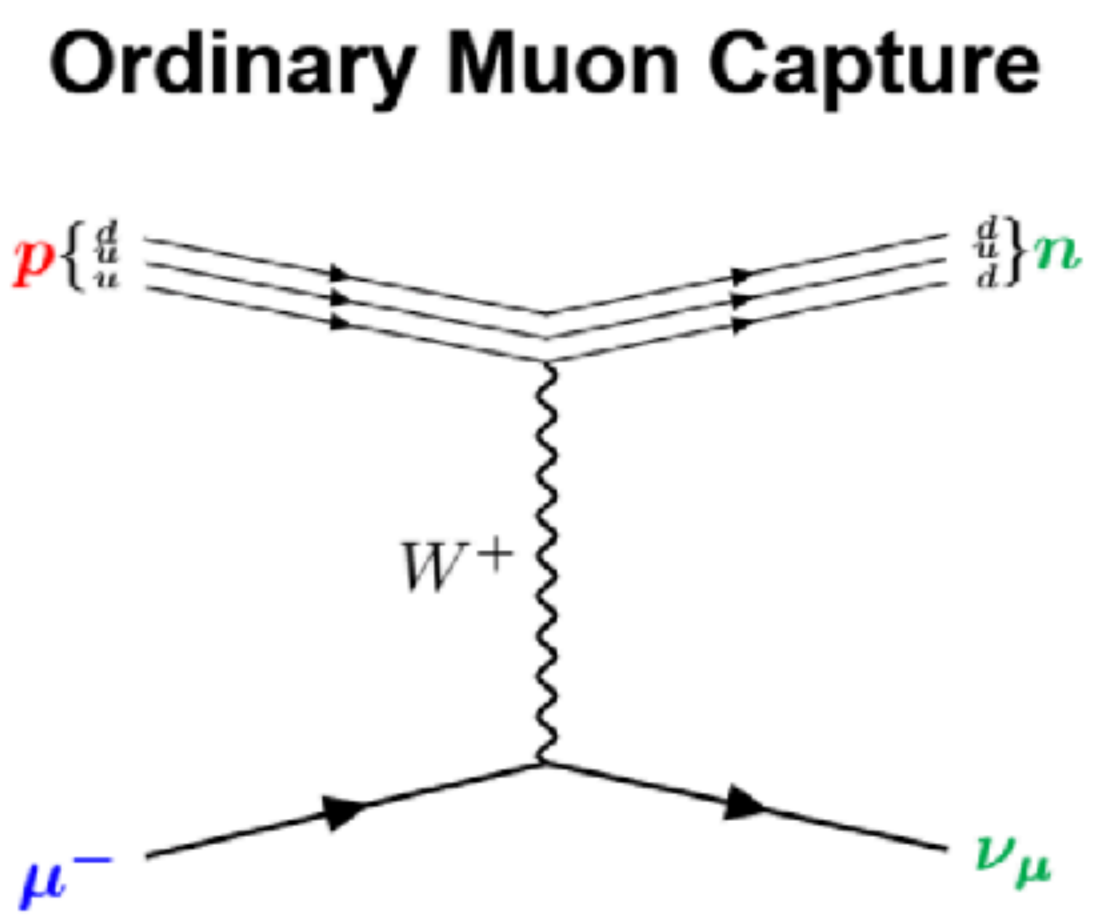
See Beatriz Romeo's talk from week 3.

Using Correlations with Other Observables

Ordinary Muon Capture

Figure courtesy of L. Jokiniemi





- $q \approx 1/|\mathbf{r}_1 - \mathbf{r}_2| \approx 100 - 200$ MeV
- No experimental signal so far

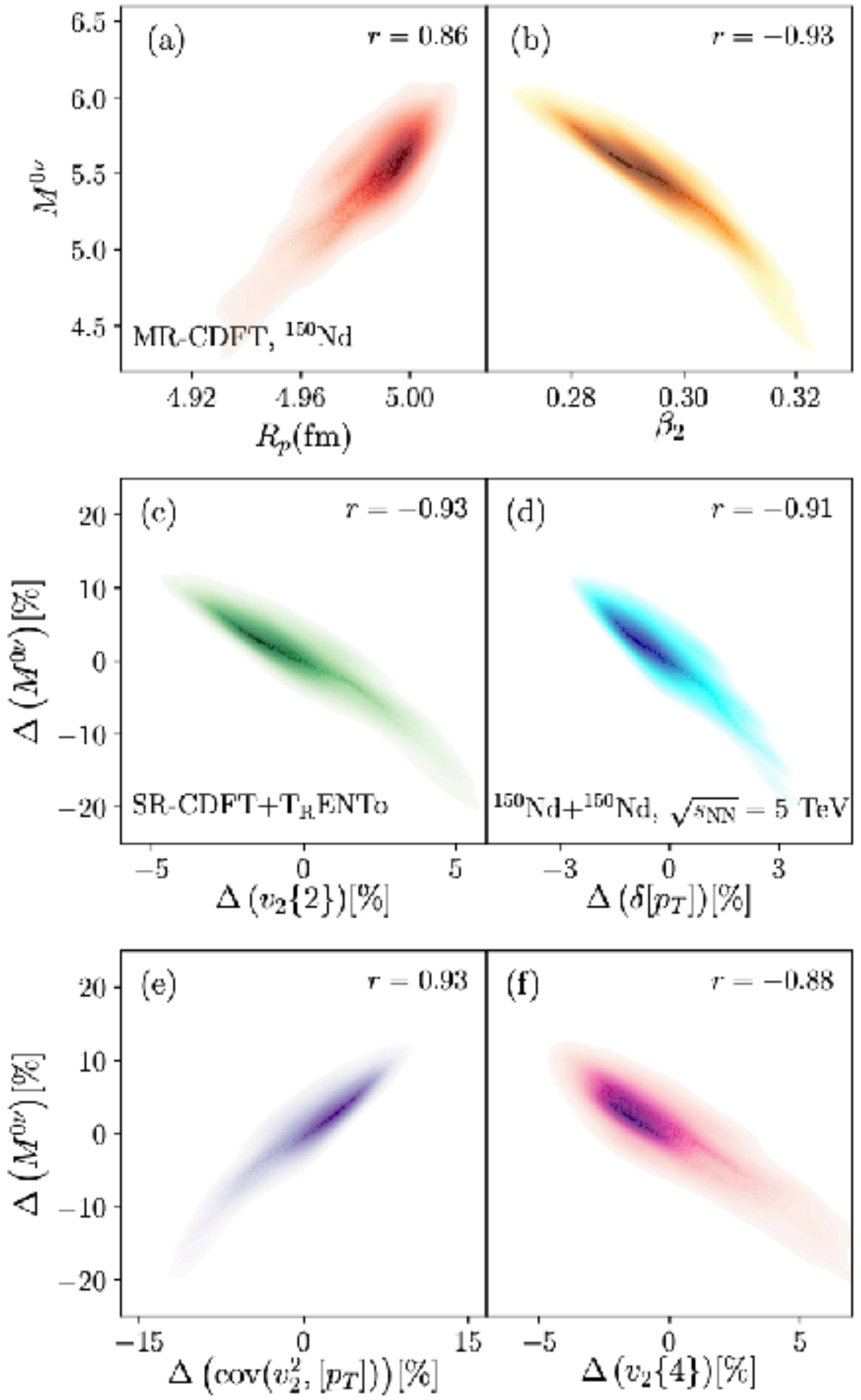
- $q \approx m_\mu + E_i - E_f \approx 100$ MeV
- Has been measured

$$j^{\alpha\dagger} = \bar{\Psi}[g_V(\mathbf{q}^2)\gamma^\alpha + i g_M(\mathbf{q}^2)\frac{\sigma^{\alpha\beta}}{2M}q_\beta - g_A(\mathbf{q}^2)\gamma^\alpha\gamma_5 - g_P(\mathbf{q}^2)q^\alpha\gamma_5]\Psi$$

See Lotta Jokiniemi's talk from week 3.

Using Correlations with Other Observables

Ultrarelativistic collisions

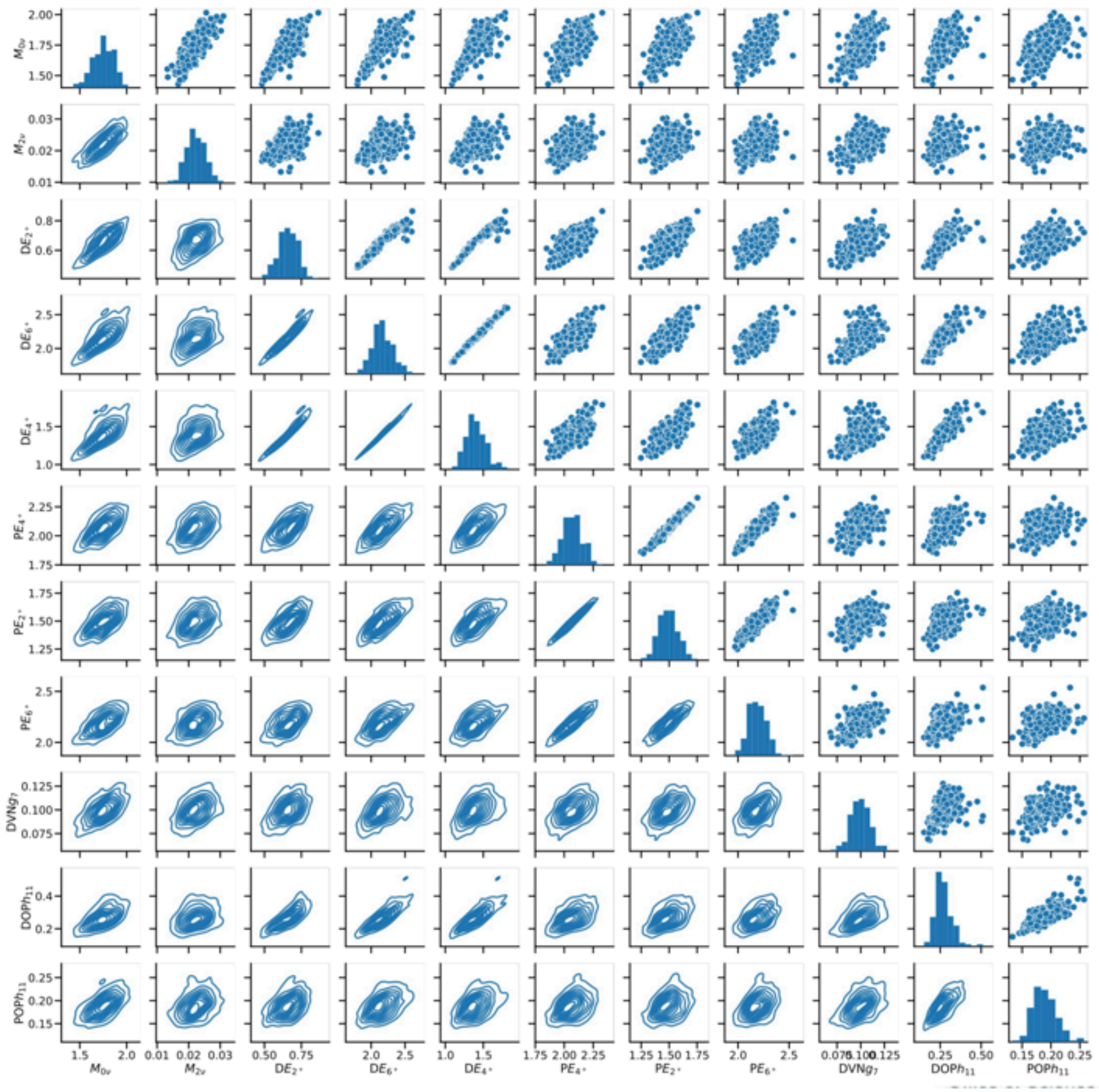


Li, Zhang, Giacalone, Yao; Phys. Rev. Lett. 135, 022301 (2025)

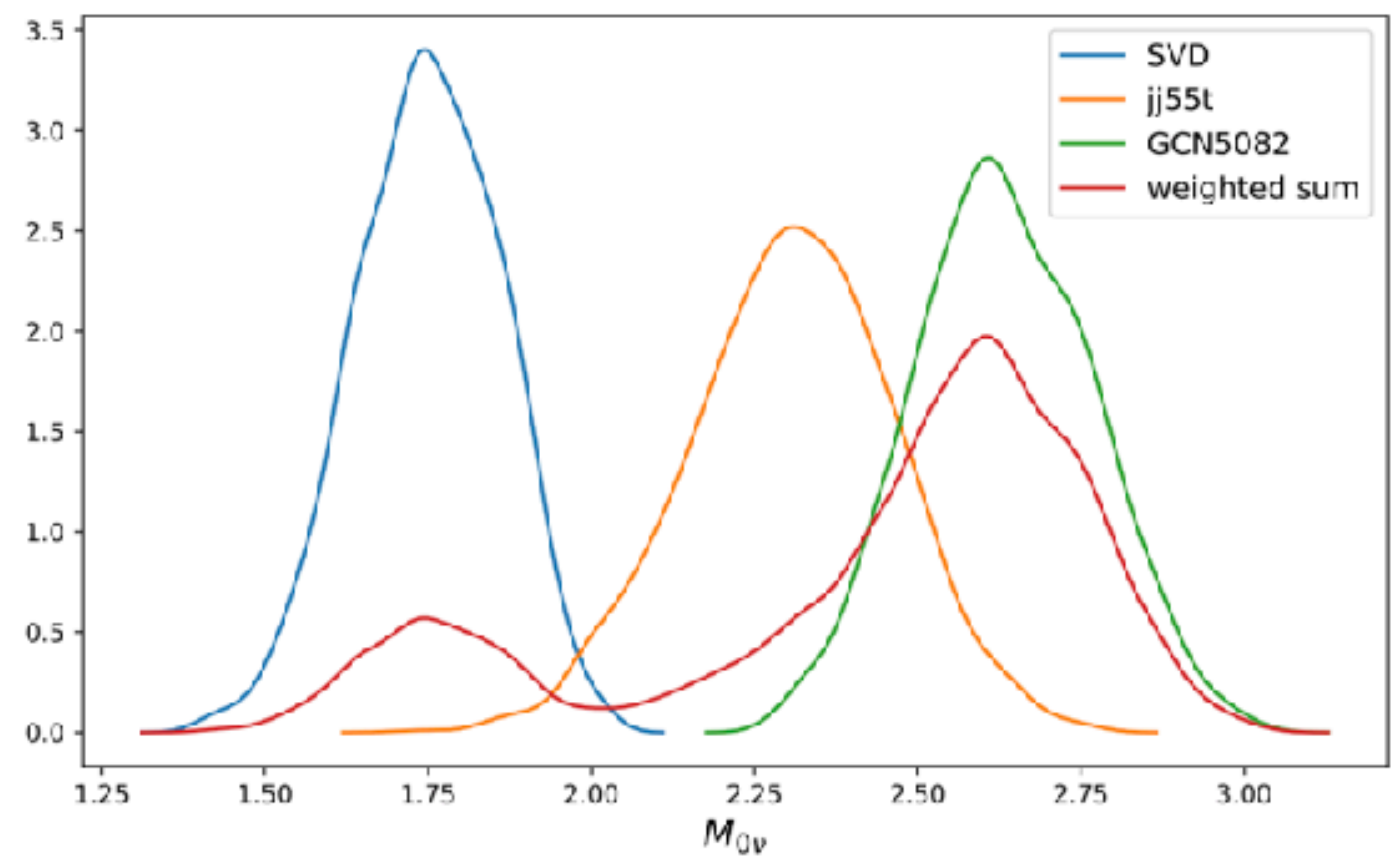
See Matthew Luzum's talk from week 3.

Study in the Shell-Model

Assessment of parametric uncertainty of shell-model calculations by varying parameters of shell-model calculations and doing Bayesian averaging.



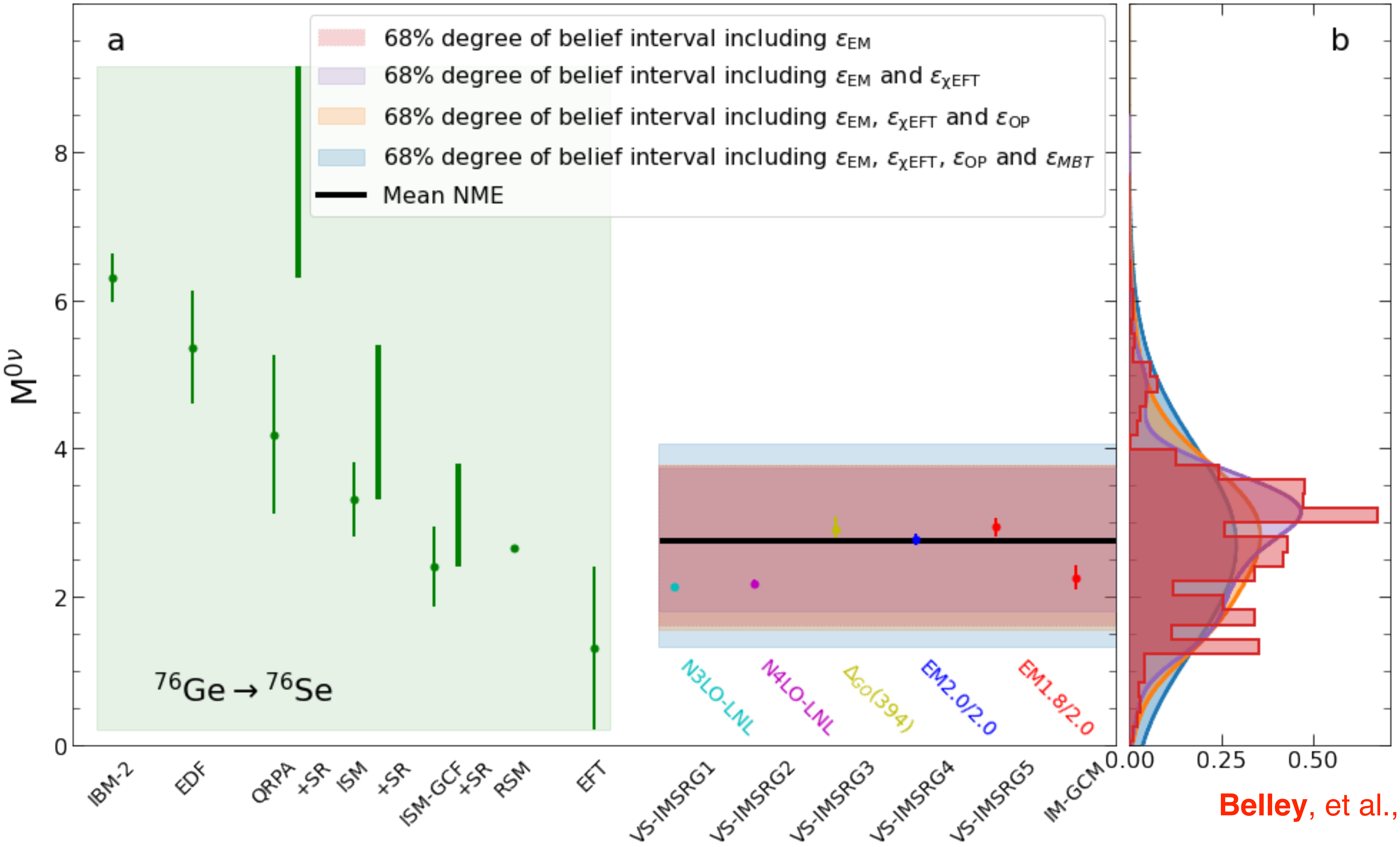
M. Horoi, Universe 10, 252 (2024)



$$p(x = M_{0\nu}|y_e, \sigma_e) = \sum_{k=svd}^{j5t} p(x = M_{0\nu}|y_e, \sigma_e, \mathcal{M}_k)p(\mathcal{M}_k|y_e, \sigma_e)$$

See Mihai Horoi's talk from week 3.

Ab initio Evaluation



$$M^{0\nu\beta\beta} = 2.60^{+1.28}_{-1.36}$$

Belley, et al., Phys. Rev. Lett. 132, 182502

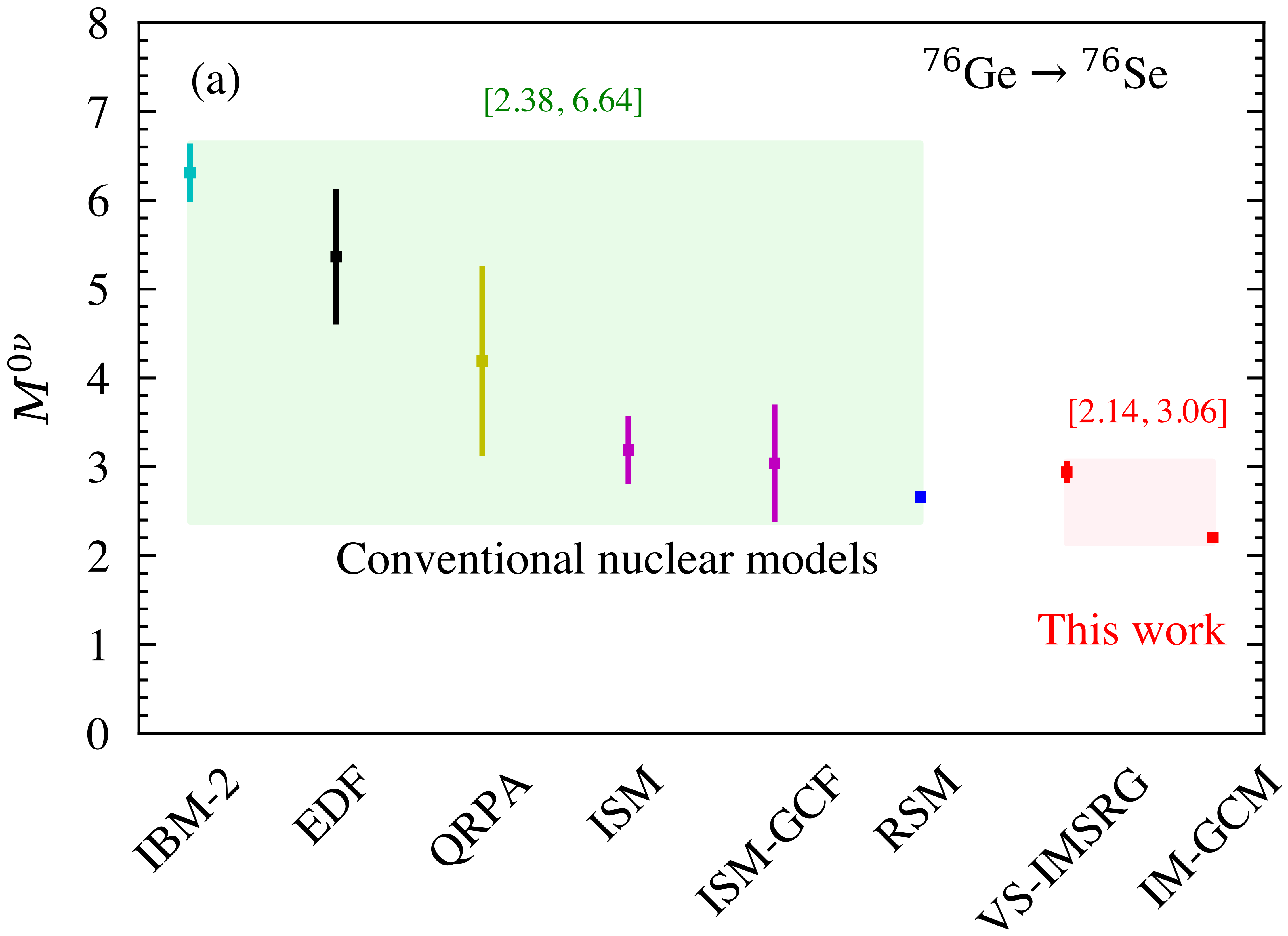
Many-Body Truncation Error

$$y = y_{MM-DGP} + \epsilon_{emulator} + \epsilon_{EFT} + \epsilon_{many-body} + \epsilon_{operator}$$

Error due to the truncation of the many-body method. This is studied by comparing the results of the IM-GCM and VS-IMSRG using the magic interaction.

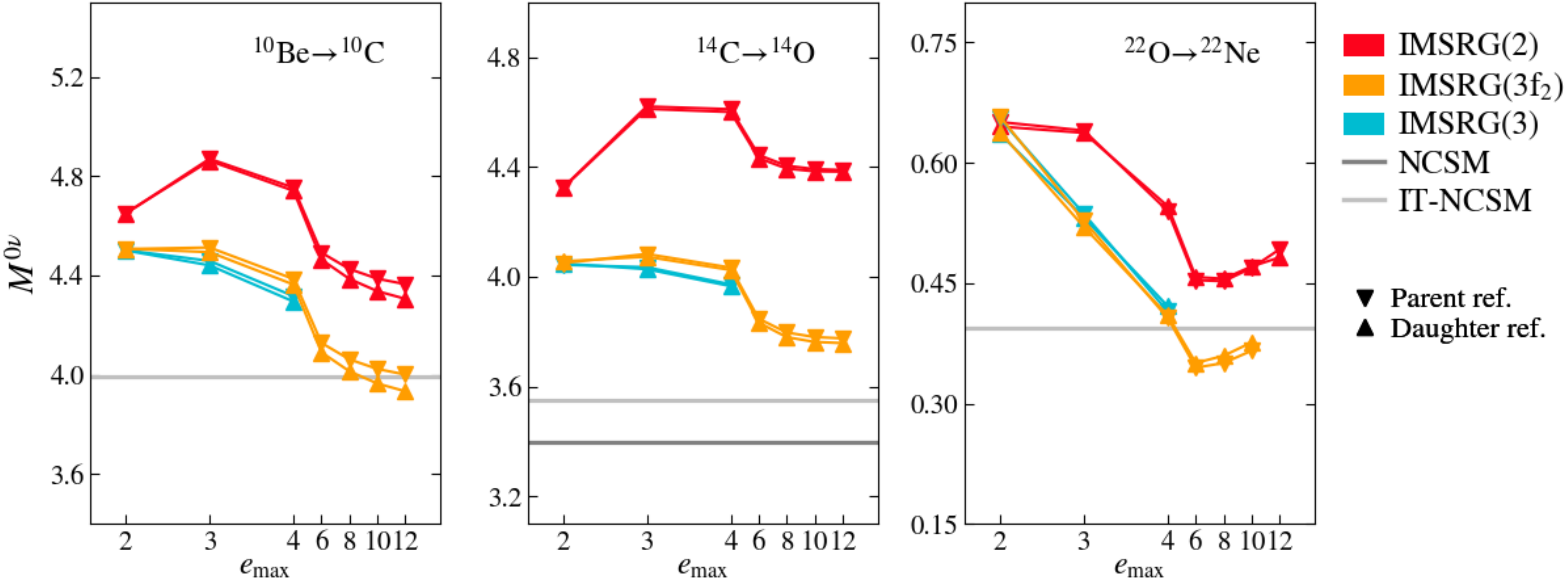
This error is surprisingly large as we find $\epsilon_{many-body}$ = 0.88.

By far the weakest point of the analysis!!!



Many-Body Truncation Error

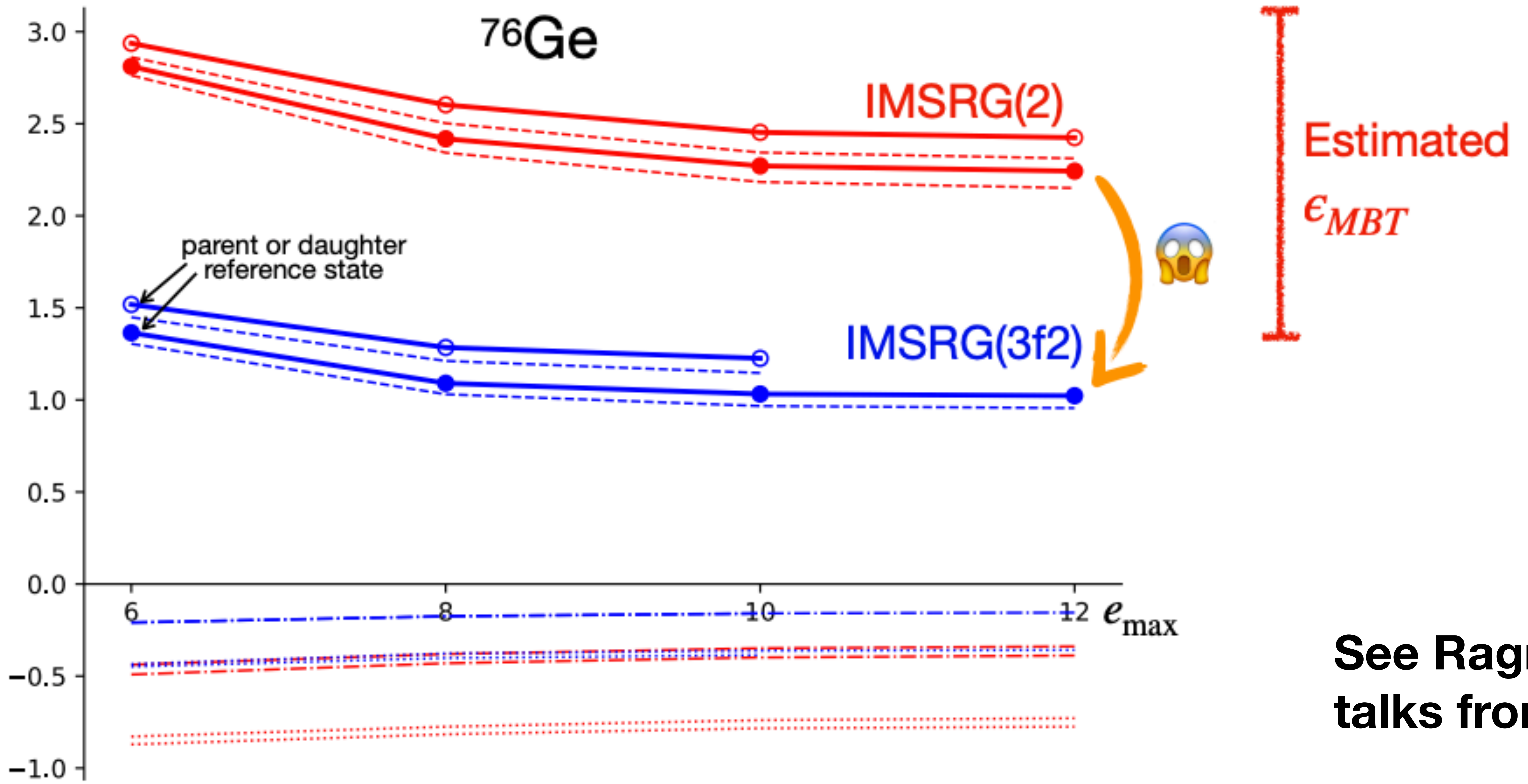
Preliminary results of IMSRG(3f2) show expected improvements.



Many-Body Truncation Error

$$y = y_{MM-DGP} + \epsilon_{emulator} + \epsilon_{EFT} + \epsilon_{many-body} + \epsilon_{operator}$$

Effect from IMSRG(3f2) is larger than the error we originally estimated…



See Ragnar Stroberg's talks from week 1.

Uncertainty quantification framework

Getting Posterior Distributions

See AB's talk from week 3.

Getting Posterior Distributions

Prior: 8188 "non-implausible" samples (Phys. Rev. C **109**, 064314)

See AB's talk from week 3.

Getting Posterior Distributions

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“Solve” many-body with emulator for all samples

See AB’s talk from week 3.

Getting Posterior Distributions

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Weight the samples by a likelihood for some calibration observables (correlation with target observable)

See AB’s talk from week 3.

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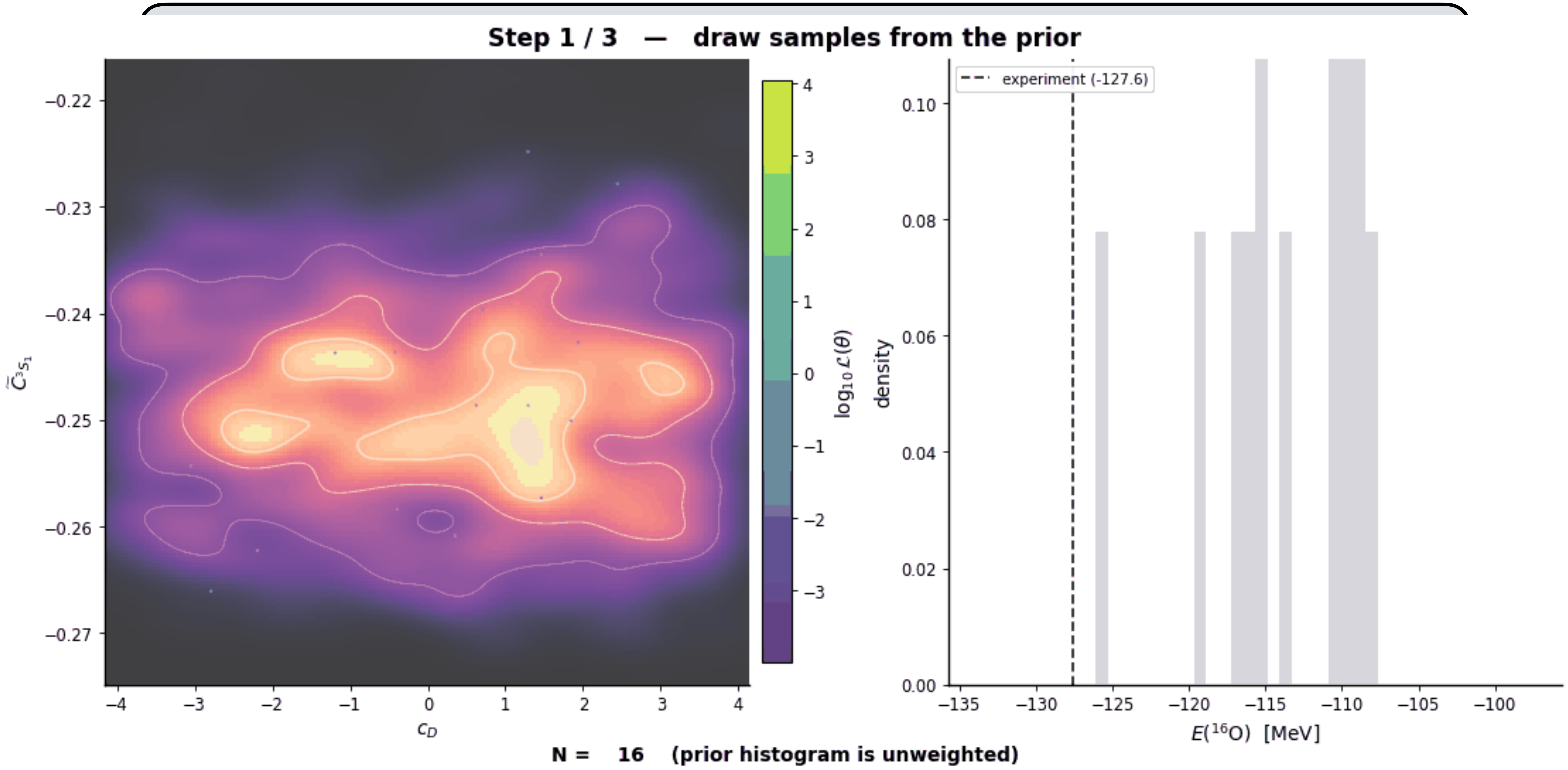

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Final results is given by

$$y = y_{LECs} + \epsilon_{emulator} + \epsilon_{EFT} + \epsilon_{many-body} + \epsilon_{operator}$$

where y_{LECs} is sampled from the weighted interactions

See AB’s talk from week 3.

Getting Posterior Distributions

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The different sources of errors ϵ are assumed to be normally distributed and independent

See AB’s talk from week 3.

Getting Posterior Distributions

ϵ's do not necessarily correlate between isotopes and observables!!!

Prior: 8188 "non-implausible" samples (Phys. Rev. C **109**, 064314)

"Solve" many-body with emulator for all samples

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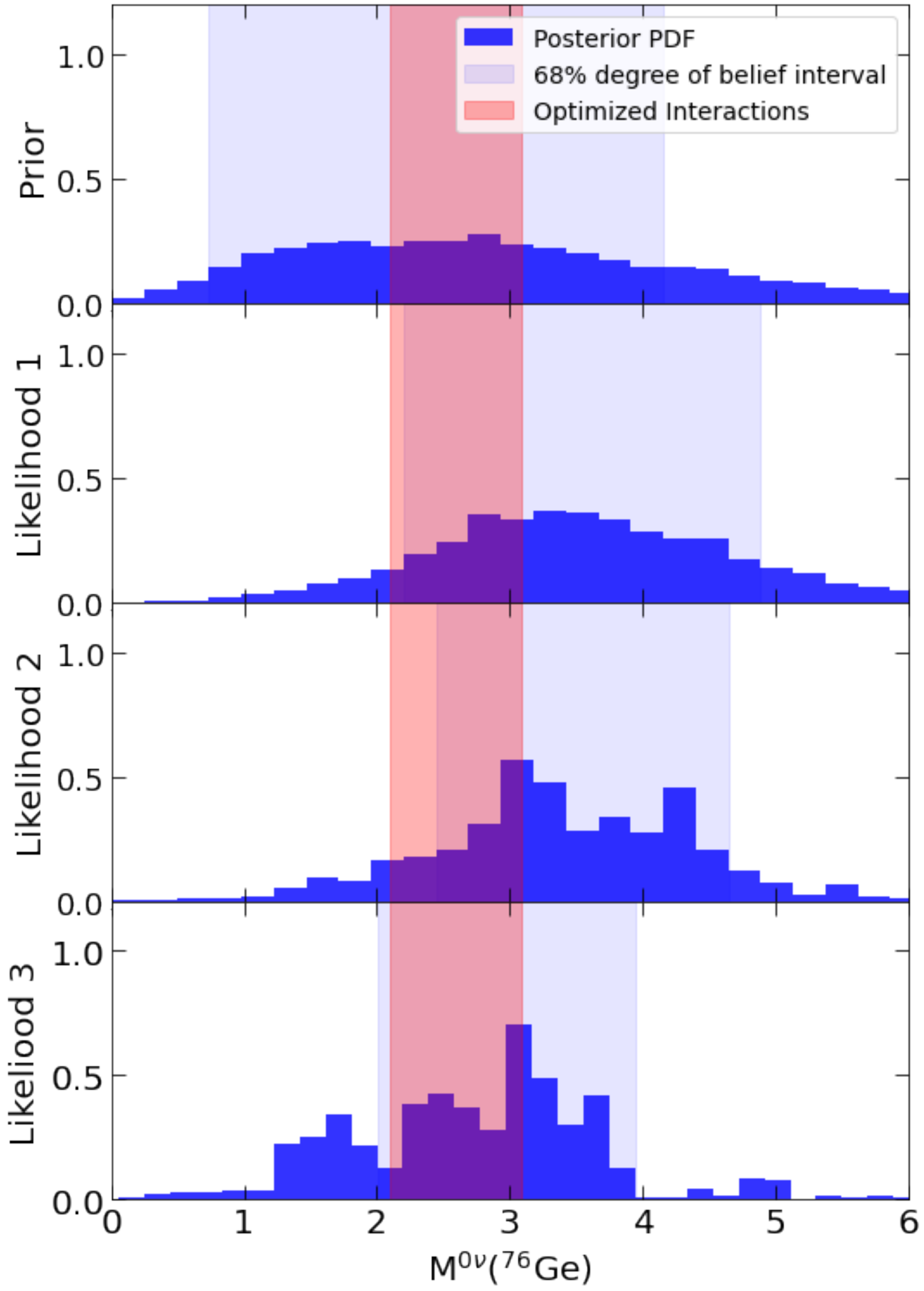
See AB's talk from week 3.

Choosing a Likelihood



Likelihood 1: Only contains $^1S_0$ neutron-proton phase shifts at 50 MeV.

Likelihood 2: Contains $^1S_0$ neutron-proton phase shifts at 50 MeV and observables for A=2-4.

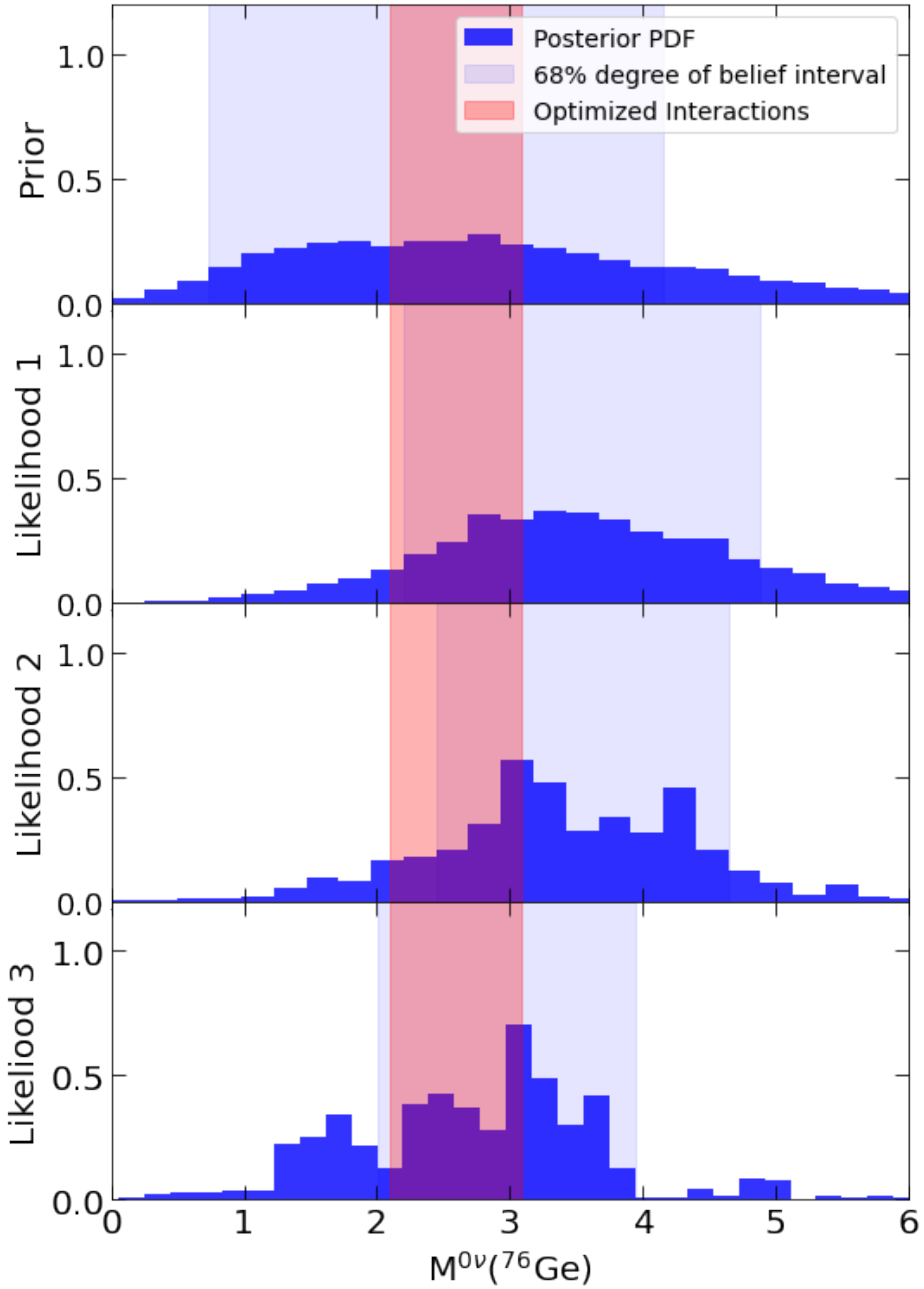
Likelihood 3: Contains $^1S_0$ neutron-proton phase shifts at 50 MeV and observables for A=2-4,16.

$$L \propto \exp(-1/2 \sum_n \frac{(O_n^{exp.} - O_n^{th.})^2}{\sigma_n^2})$$

Include both theoretical and experimental errors

A2-4: $E(^2H)$, $R_p(^2H)$, $Q(^2H)$, $E(^3H)$, $E(^4He)$, $R_p(^4He)$ A16: $E(^{16}O)$, $R_p(^{16}O)$

Choosing a Likelihood



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Likelihood 3: Contains $^1S_0$ neutron-proton phase shifts at 50 MeV and observables for A=2-4,16.

Correlations with other observables should be used here!

$$L \propto \exp(-1/2 \sum_n \frac{(O_n^{exp.} - O_n^{th.})^2}{\sigma_n^2})$$

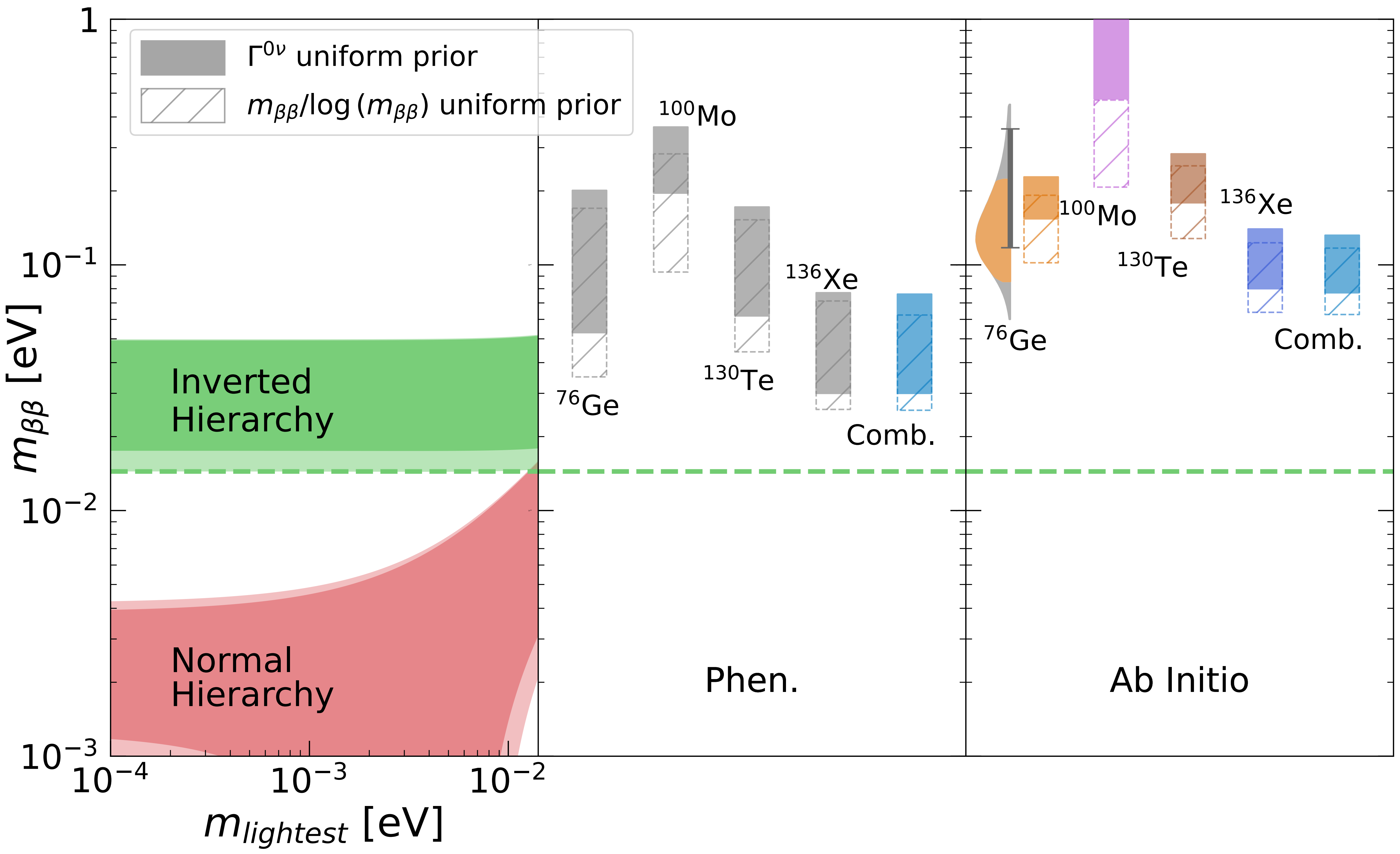
Include both theoretical and experimental errors

A2-4: $E(^2H)$, $R_p(^2H)$, $Q(^2H)$, $E(^3H)$, $E(^4He)$, $R_p(^4He)$

A16: $E(^{16}O)$, $R_p(^{16}O)$

Combining Limits of Different Isotopes

Taiki Shickele

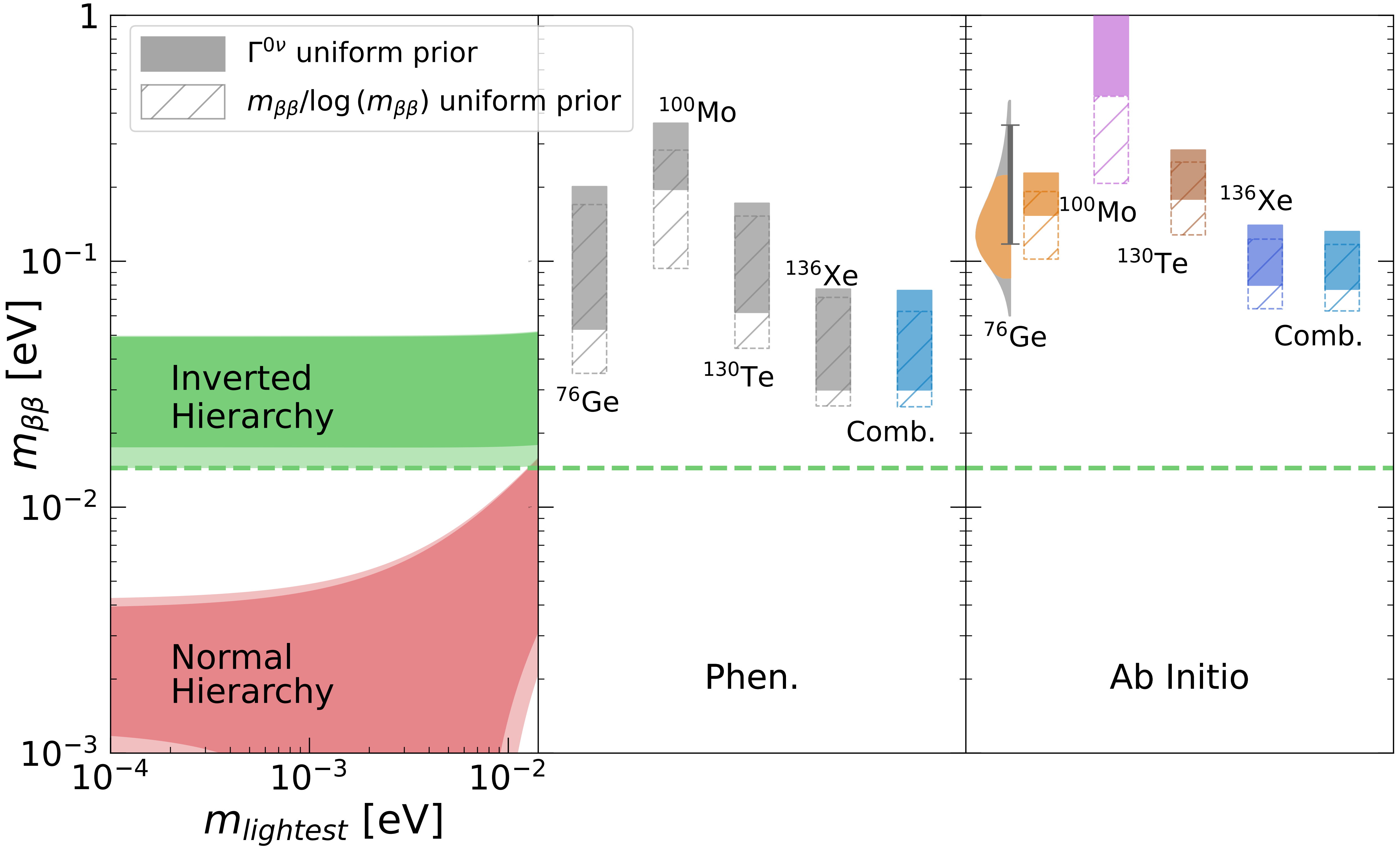


Current

Experimental limits: **GERDA (^{76}Ge)** Phys. Rev. Lett. 125, 252502, **CUPID-Mo (^{100}100)** Eur. Phys. J. C 82 11, 1033, **CUORE(^{130}Te)** arXiv:2404.04453, **EXO(^{136}Xe)** Phys. Rev. Lett. 123, 161802 and **Kamland Zen (^{136}Xe)** arXiv:2406.11438

Combining Limits of Different Isotopes

Taiki Shickele



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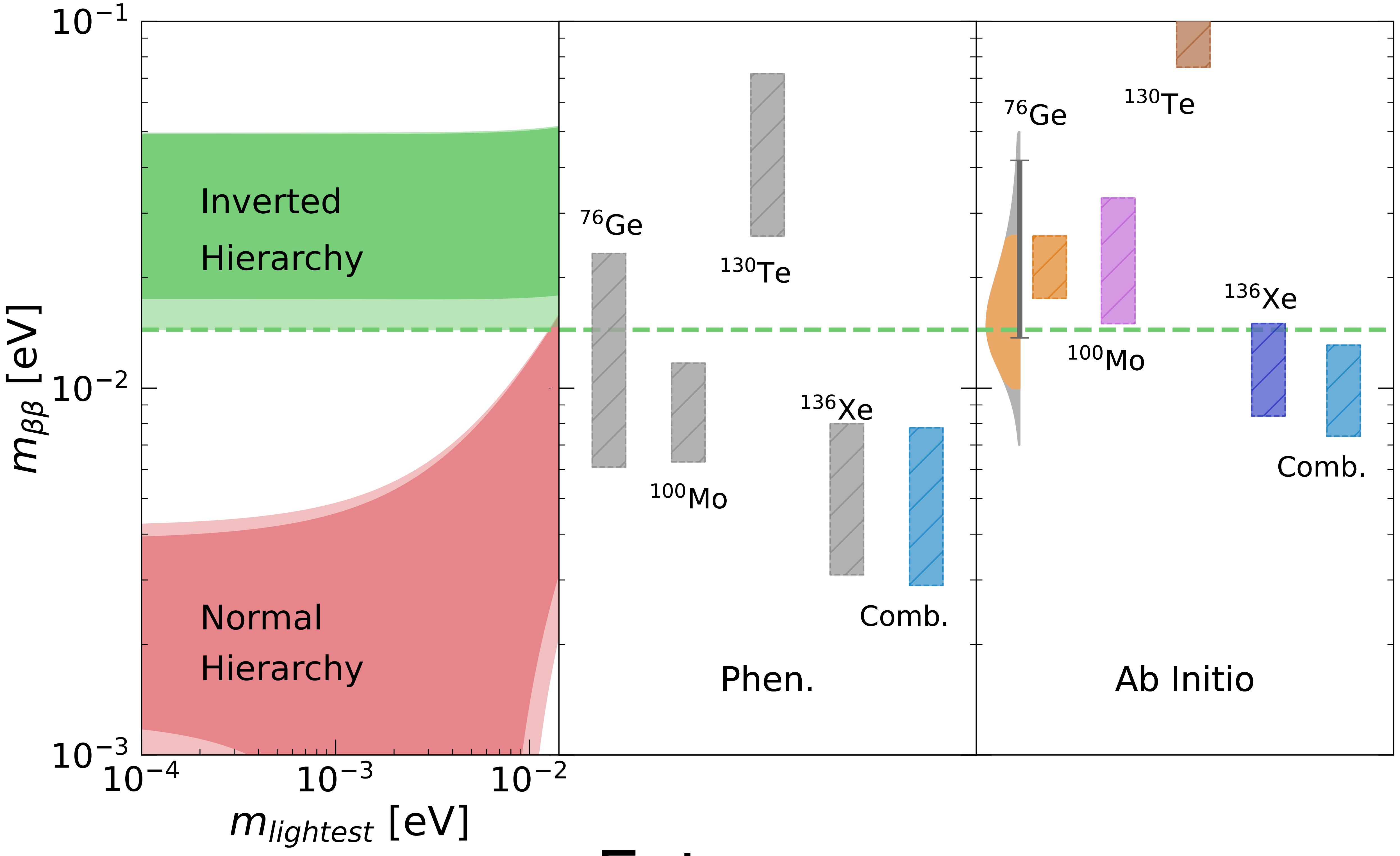
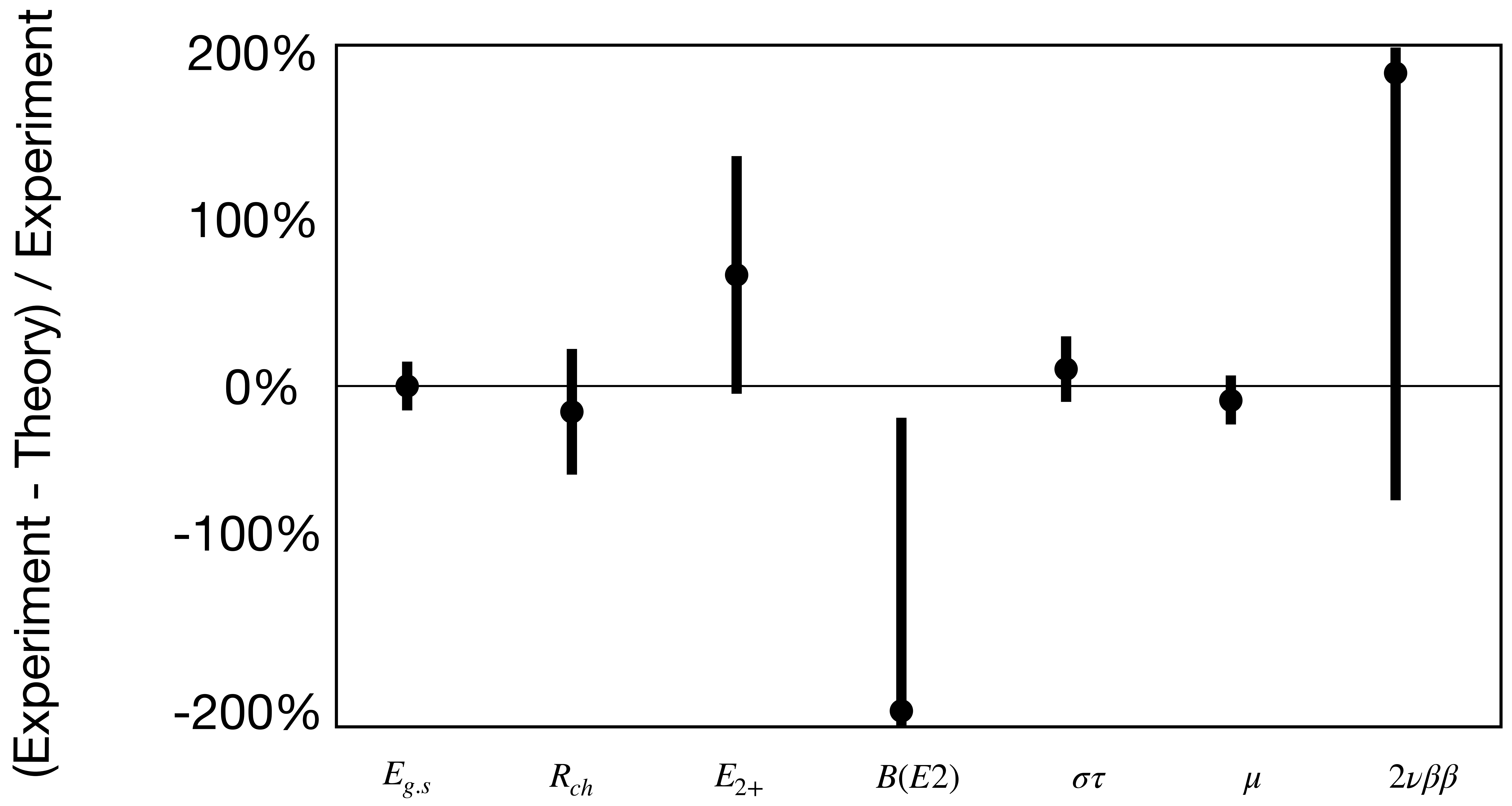


Future

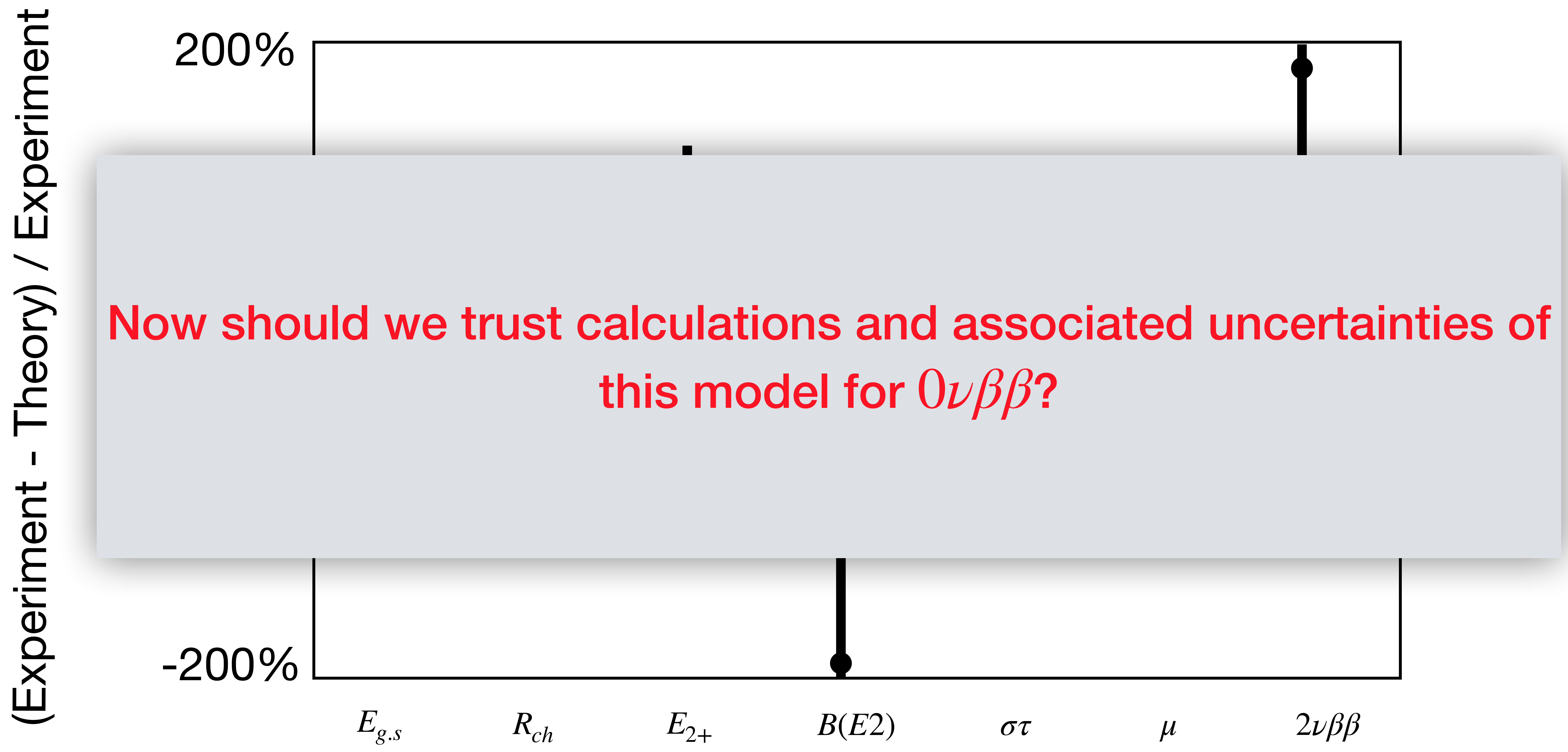
Experimental limits: **GERDA (^{76}Ge)** Phys. Rev. Lett. 125, 252502, **CUPID-Mo (^{100}100)** Eur. Phys. J. C 82 11, 1033, **CUORE(^{130}Te)** arXiv:2404.04453, **EXO(^{136}Xe)** Phys. Rev. Lett. 123, 161802 and **Kamland Zen (^{136}Xe)** arXiv:2406.11438

Expected limits: **LEGEND (^{76}Ge)** arXiv:2107.11462, **CUPID-1T (^{100}100)** arXiv:2203.08386, **AMoRE (^{100}100)** arXiv:2406.09698, **SNO+(^{130}Te)** arXiv:2104.11687, **NEXT (^{136}Xe)** JHEP09(2023)190 and **nEXO (^{136}Xe)** J. Phys .G 49 1, 015104.

Using other observables



Using other observables



How to improve uncertainty models?

Weakness of uncertainty model

1. Theoretical uncertainty are somewhat double counted since they are included in the likelihood.

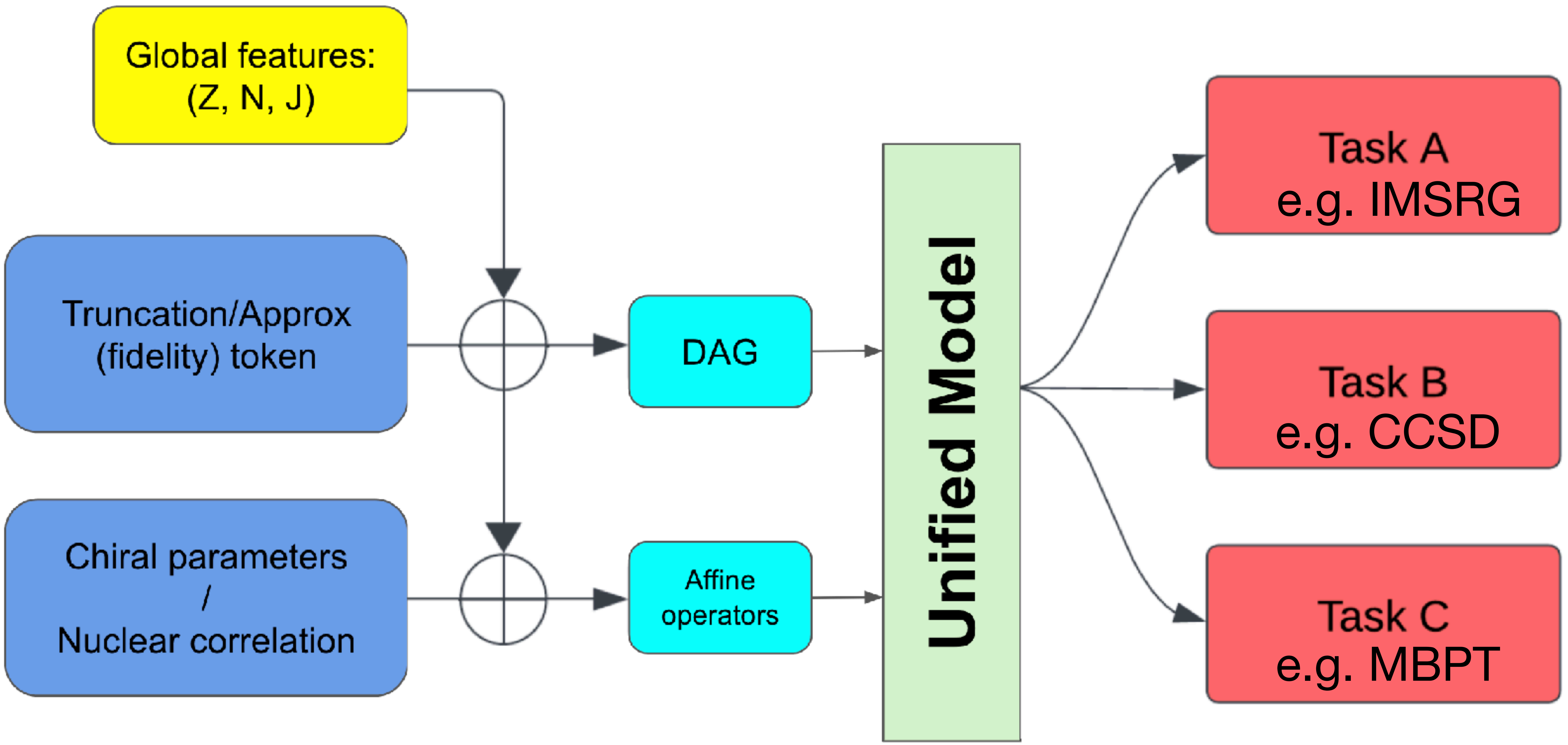
2. Correlations between the different sources of uncertainty are not included.

3. Uncertainty limited to one many-body method.

4. Choice of observables used in the likelihood might not constrain the LECs optimally for $0\nu\beta\beta$.

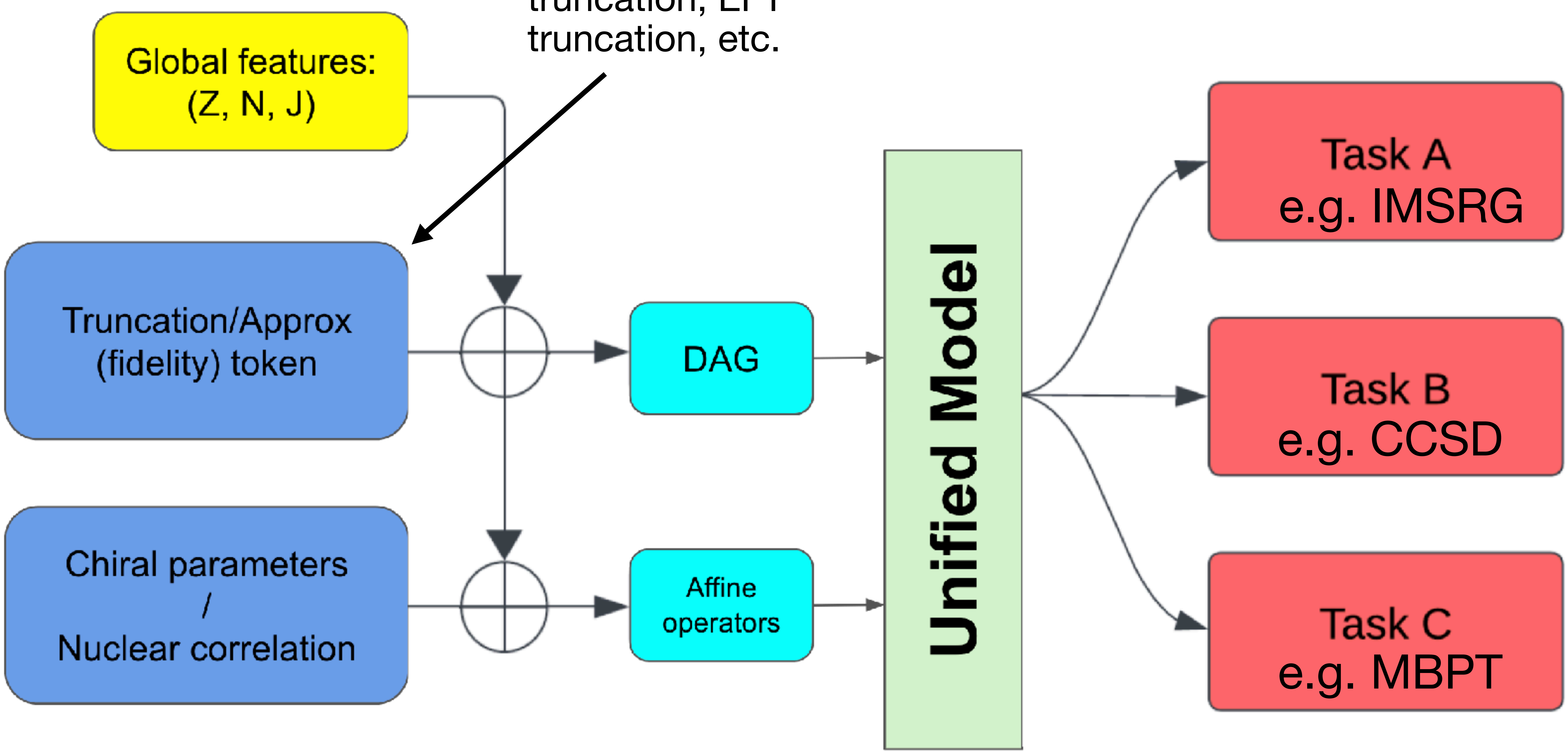
5. Not considering correlation between the different isotopes.

Foundation Model For UQ

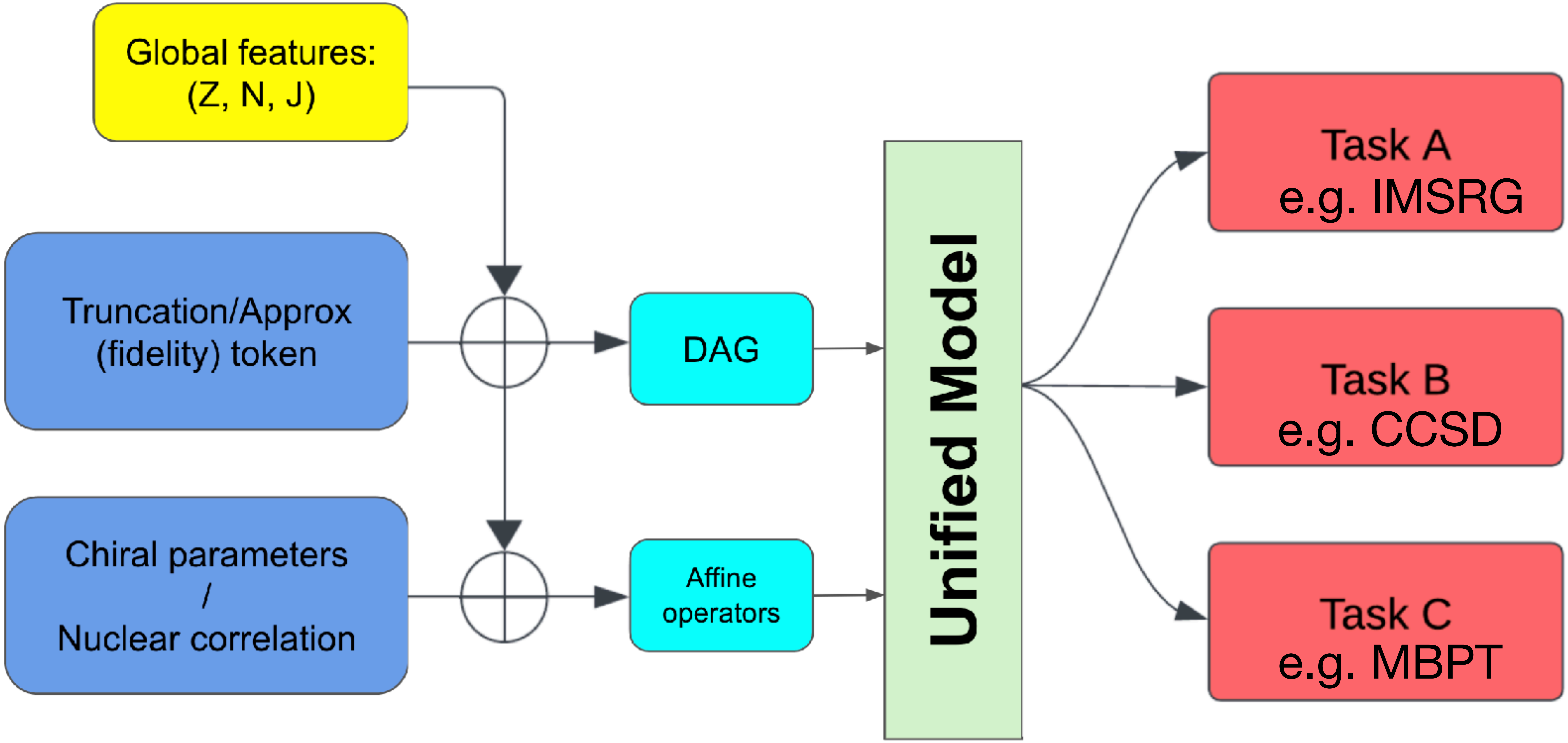


Foundation Model For UQ

Includes model-space truncation, many-body truncation, EFT truncation, etc.

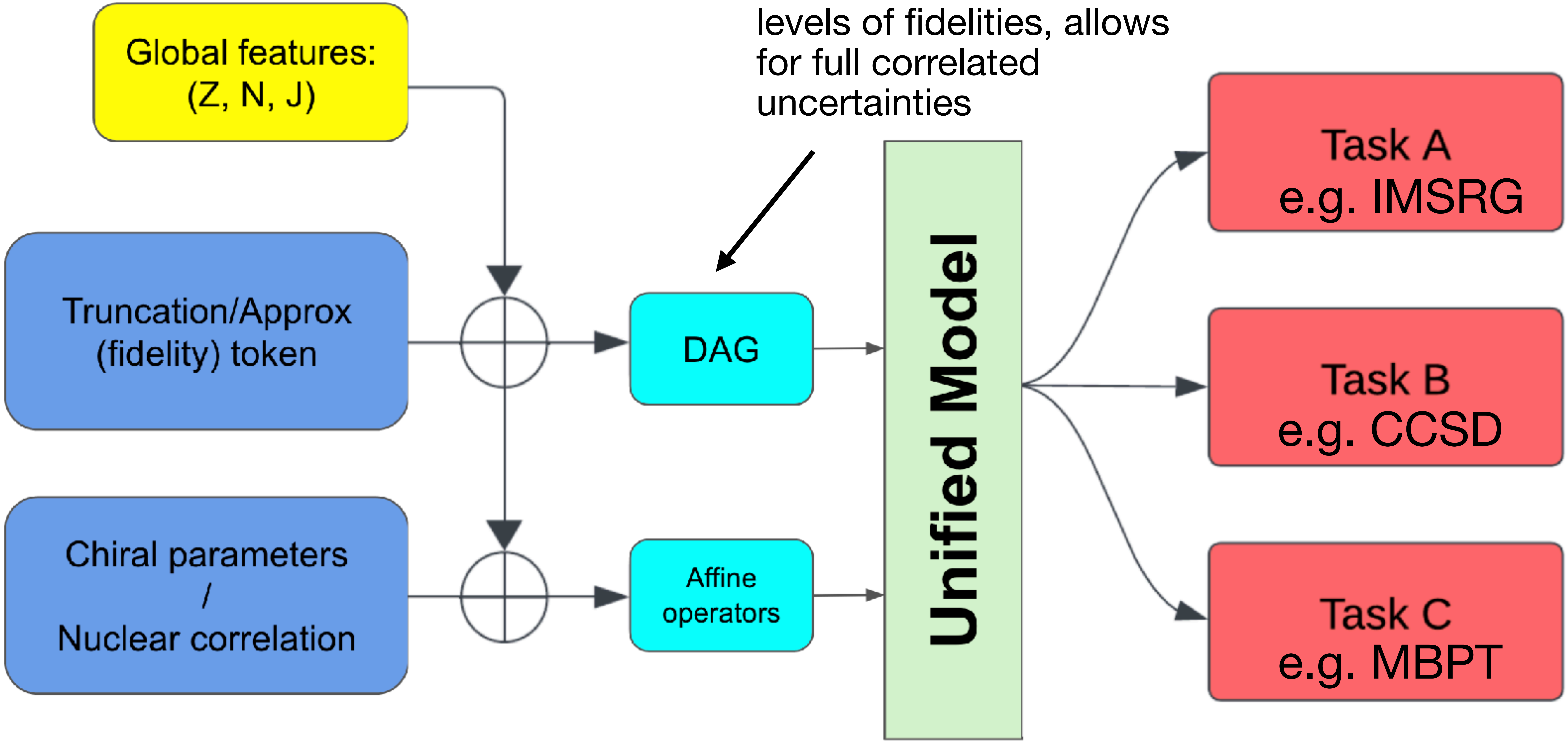


Foundation Model For UQ

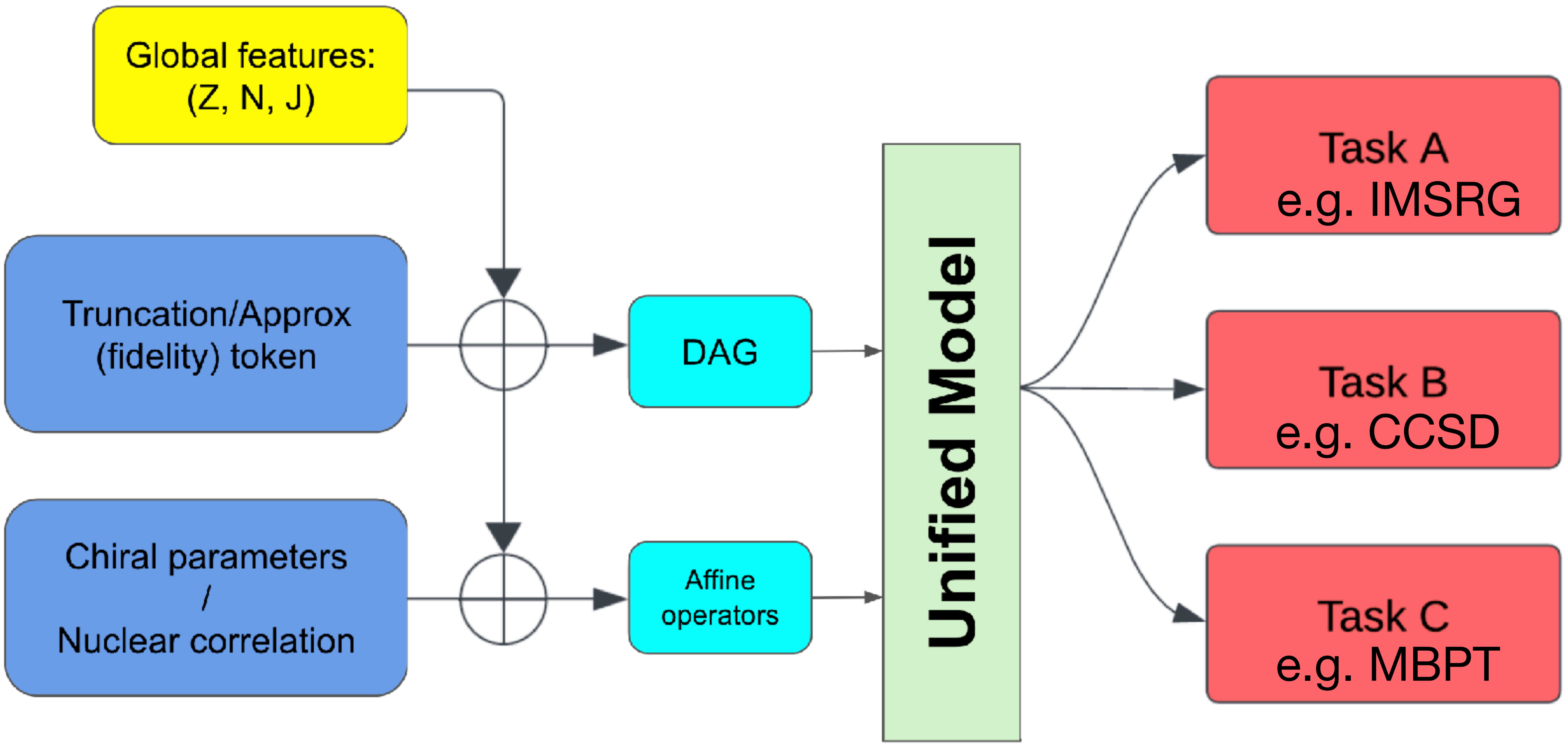


Foundation Model For UQ

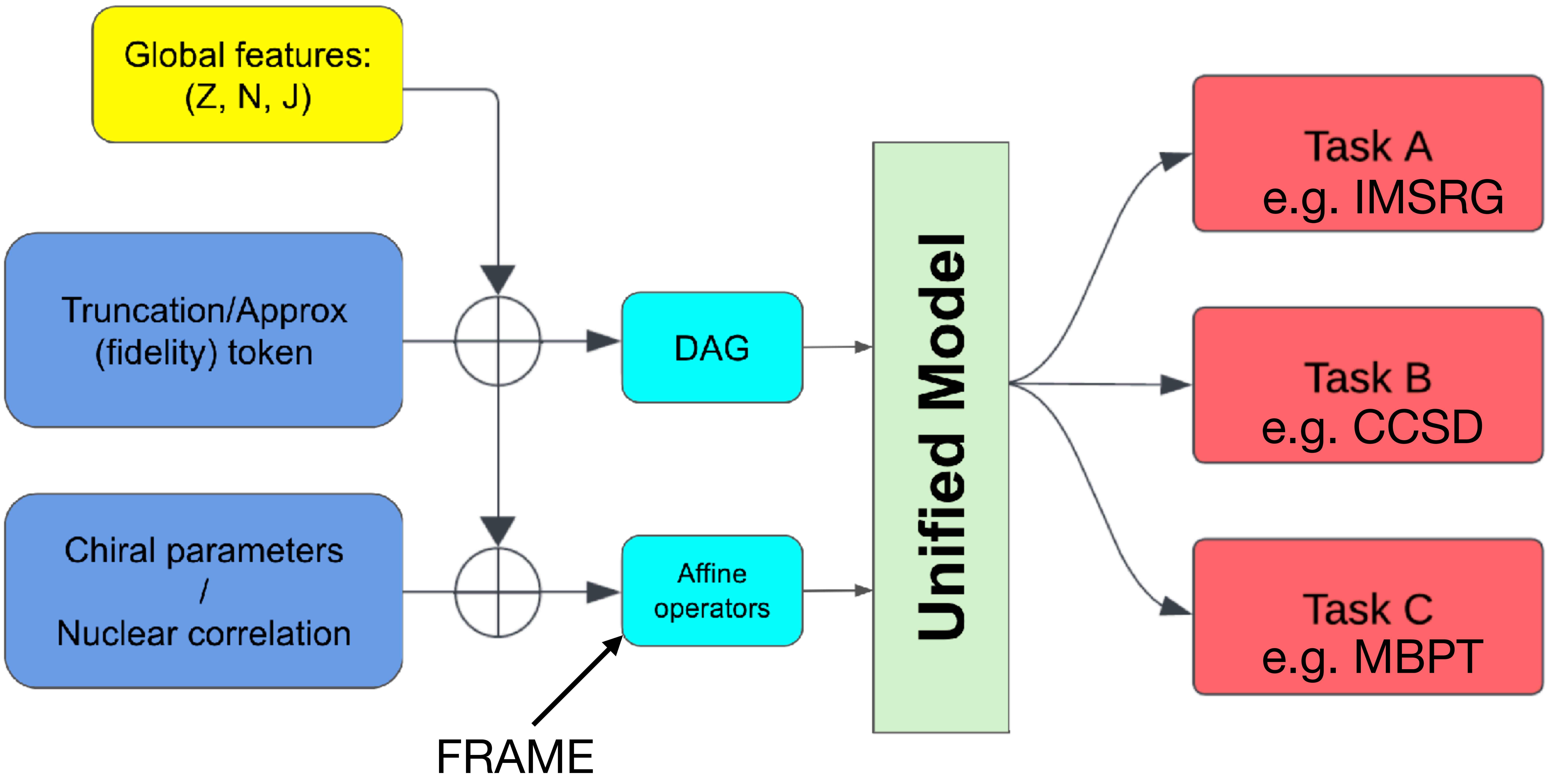
Directed Acyclic Graph is used to order the different levels of fidelities, allows for full correlated uncertainties



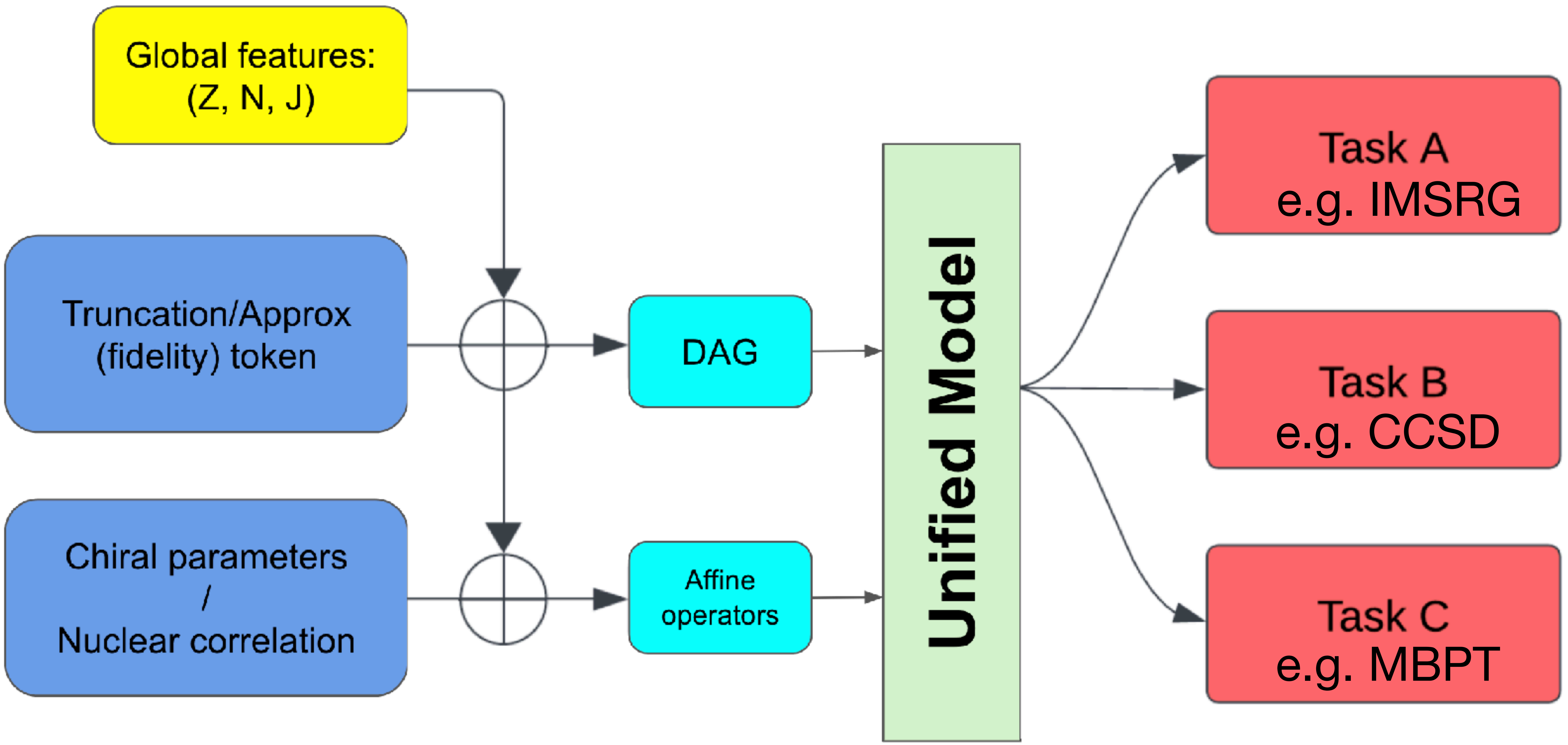
Foundation Model For UQ



Foundation Model For UQ



Foundation Model For UQ



Summary…

- Uncertainty quantification is required for calculations to be trusted.
- All sources of uncertainties should be evaluated.
- Correlation with other observables can be useful to reduce parametric uncertainties of the NMEs.
- Parametric uncertainties of NMEs have been evaluated in Shell-Model.
- Complete uncertainties have been evaluated in the VS-ISMRG but newer evaluation of the model uncertainties bring the result lower.

… and Homework

- Assessment of correlation between uncertainties.
- Combining results of different models.
- Improving model uncertainties..
- Find best calibration observables to constrain likelihoods.