

Cabibbo angle anomalies and vector-like quarks

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F. Albergaria, J. F. Bastos, B. B., G. C. Branco, J. T. Penedo, A. Rodríguez-Sánchez, J. I. Silva-Marcos, ``Vector-like quark doublets, weak-basis invariants and CP violation," JHEP 08, 061 (2025)

B. B. and S. Trifinopoulos, 2302.14097 *Phys.Rev.D* 108 (2023) 3, 035022

B. B. and Z. Berezhiani JHEP 10, 079 (2021), 2103.05549

B. B., R. Beradze and Z. Berezhiani, Eur. Phys. J. C 80, no.2, 149 (2020), 1906.02714

The Standard Model

$$\mathcal{L}_{\text{Yuk}} = Y_u^{ij} \tilde{\phi} \overline{q_{Li}} u_{Rj} + Y_d^{ij} \phi \overline{q_{Li}} d_{Rj} + Y_e^{ij} \phi \overline{\ell_{Li}} e_{Rj} + \text{h.c.}$$

- Three fermion families in identical representations of gauge symmetry; a single Higgs doublet
- Yukawa matrices are not diagonal in weak basis:

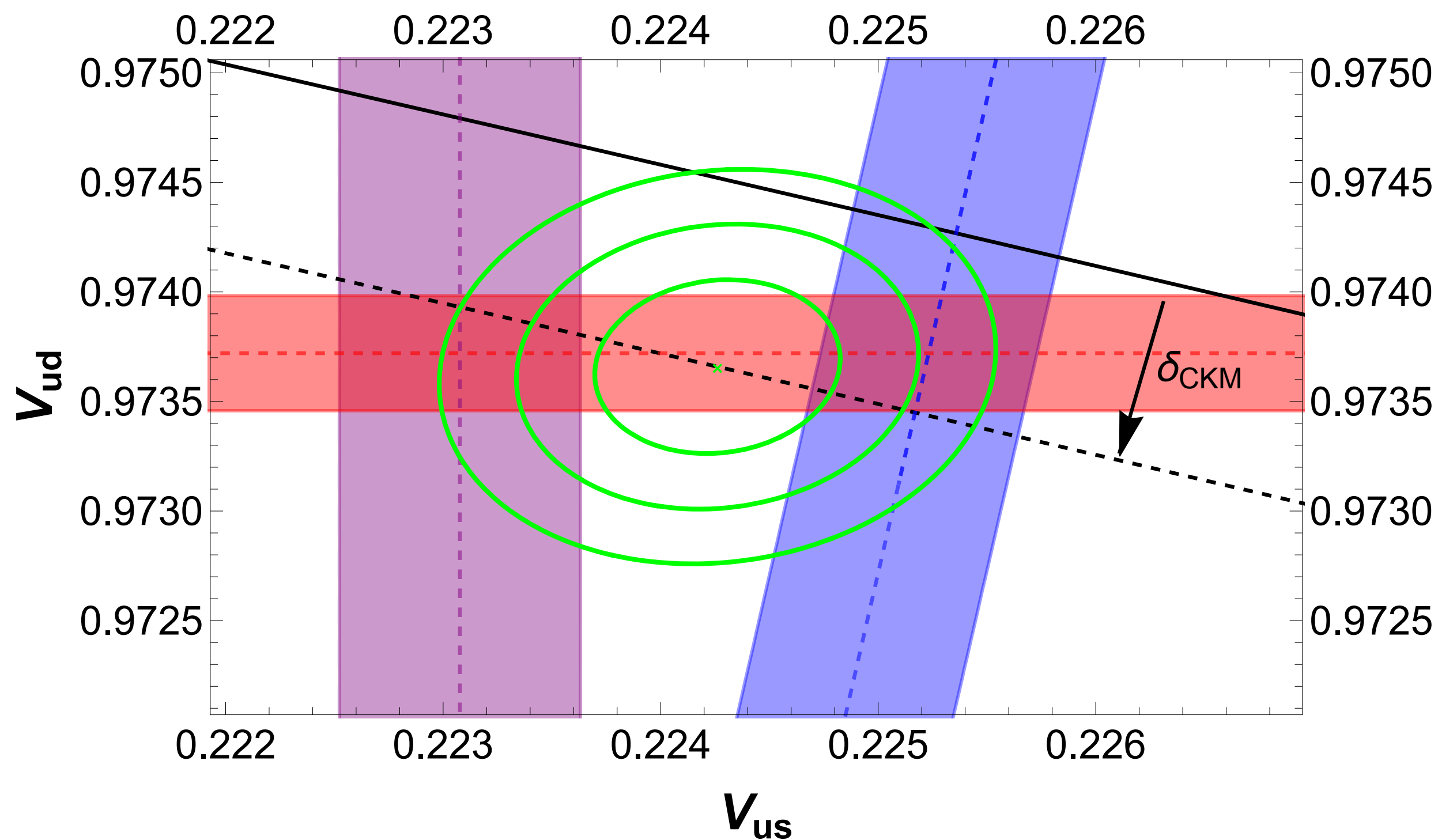
$$m^{(u,d,e)ij} = Y_{u,d,e}^{ij} v_w, \quad \mathbf{m}_{\text{diag}}^{u,d,e} = V_L^{(u,d,e)\dagger} \mathbf{m}^{(u,d,e)} V_R^{(u,d,e)}$$

- Quark charged currents in mass basis: $\frac{g}{\sqrt{2}} \overline{\begin{pmatrix} u & c & t \end{pmatrix}_L} \gamma^\mu V_L^{(u)\dagger} V_L^{(d)} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L W_\mu^\dagger$
- $V_{\text{CKM}} = V_L^{(u)\dagger} V_L^{(d)} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}.$
- **V_{CKM} is unitary.**
- The unitarity condition for the first row is: $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$
- Yukawa couplings and photon/Z couplings (for **unitarity** of $V_L^{(u)}$ and $V_L^{(d)}$) are diagonal in mass basis.
- **NO flavour changing neutral currents** at tree level.

Cabibbo angle anomalies

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 \quad \longrightarrow \quad \cos^2 \theta_C + \sin^2 \theta_C = 1$$

Three determinations: $|V_{us}|_A = \sin \theta_C$, $|V_{us}/V_{ud}|_B = \tan \theta_C$, $|V_{ud}|_C = \cos \theta_C$



C: $|V_{ud}| = 0.97372(26)$ β decays ($\Delta_R^V = 0.02467(22)$)

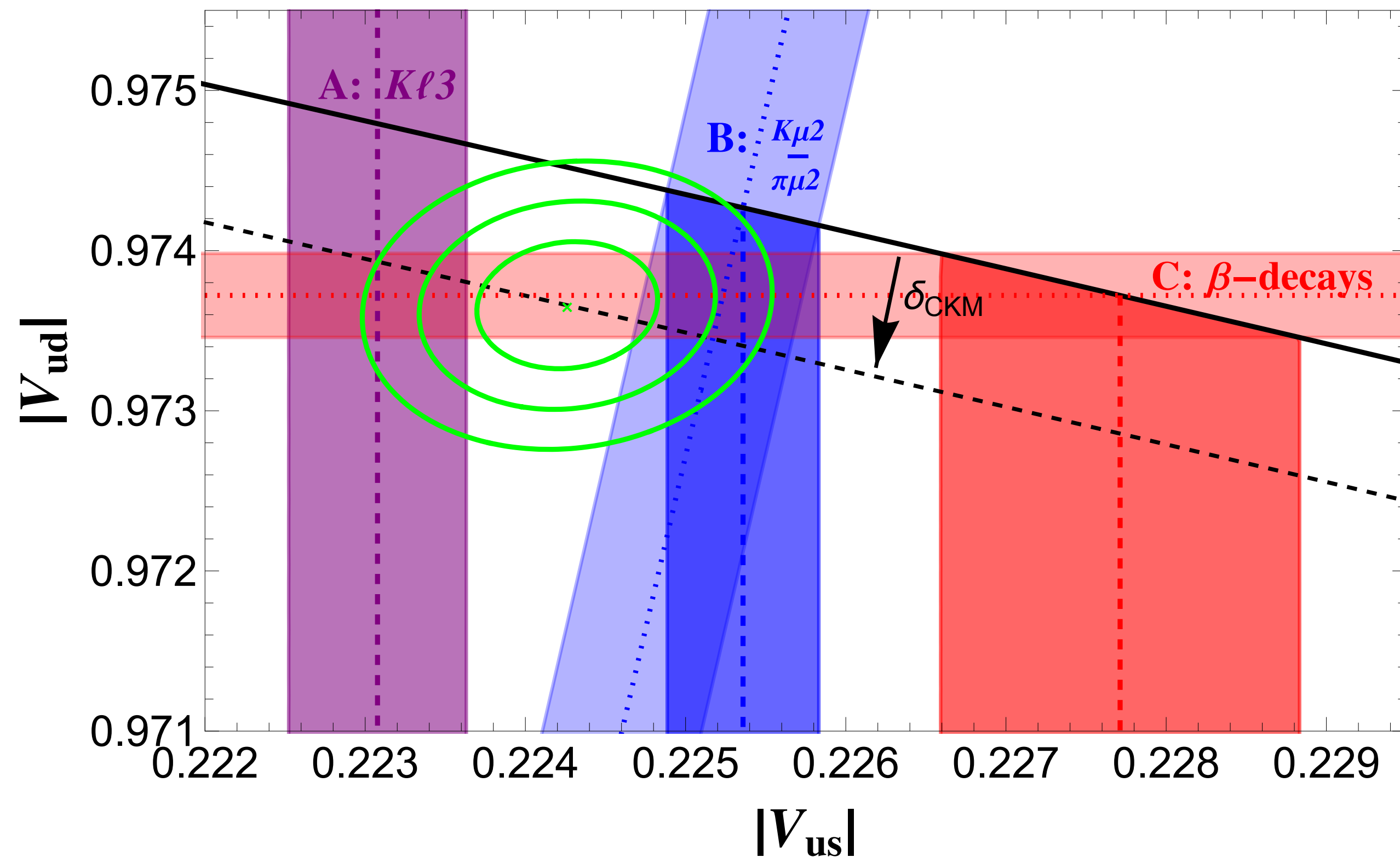
A: $|V_{us}| = 0.22308(55)$ semileptonic $K \rightarrow \pi \ell \nu$ decays
 $f_+(0) = 0.9698(17)$ FLAG24(21)

B: $\frac{|V_{us}|}{|V_{ud}|} = 0.23126(48)$ ratio of leptonic $K\mu 2/\pi\mu 2$
 $f_K/f_\pi = 1.1934(19)$ FLAG21

— Unitarity

- CKM unitarity deficit: $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 - \delta_{\text{CKM}} \rightarrow \delta_{\text{CKM}} \approx 2 \times 10^{-3}$
- Discrepancy between determinations A and B.

Cabibbo angle anomalies



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— Unitarity

- CKM unitarity problem: $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 - \delta_{\text{CKM}} \rightarrow \delta_{\text{CKM}} \approx 2 \times 10^{-3}$
- Discrepancy between determinations A and B.

Cabibbo angle anomalies

A: $|V_{us}| = 0.22308(55)$

B: $\frac{|V_{us}|}{|V_{ud}|} = 0.23131(51)$

C: $|V_{ud}| = 0.97372(26)$

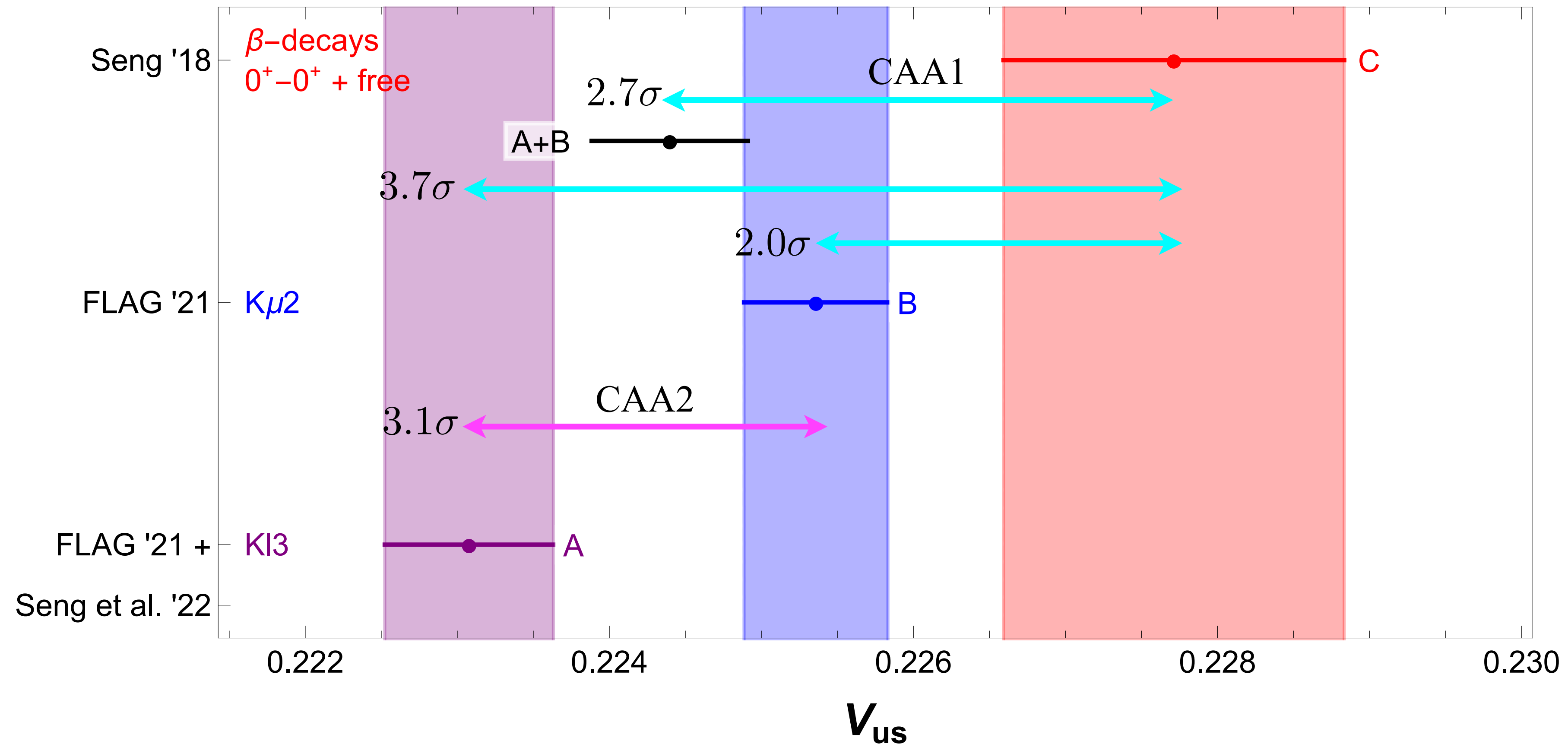
$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$



$|V_{us}|_A = 0.22308(55)$

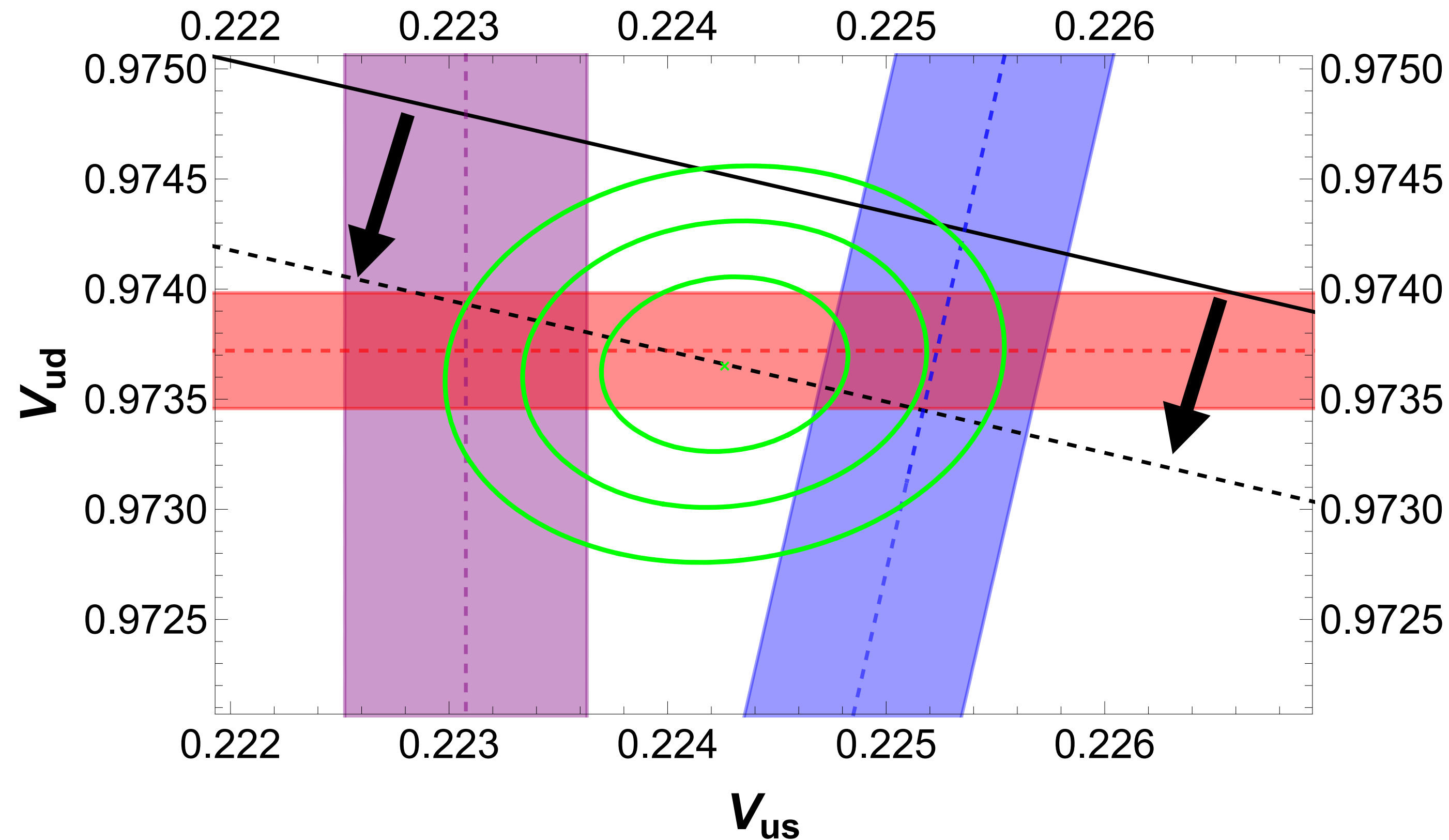
$|V_{us}|_B = 0.22536(47)$

$|V_{us}|_C = 0.2277(11)$



New quark-mixing

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 - |V_{ub'}|^2, \quad |V_{ub'}| \approx 0.041 \quad (|V_{ub'}| \gg |V_{ub}|)$$



By combining electroweak precision data, Higgs production rate via gluon fusion and LHC mass limits, 4th sequential family is excluded. However additional vector-like fermions can be introduced.

Vector-like quarks

- LH and RH in the same representation of the SM
- predicted in some extensions of the SM
- Their mass terms are not protected by SM gauge symmetries
- Are they a solution for anomalies in the first row of CKM?
- Fermion species in the same representation as the standard quarks
 - Extra down-type weak singlet
 - Extra up-type weak singlet
 - Extra weak doublet ($Y=1/6$)

Down-type weak singlet

Down-type vector-like quark $D_{L,R}$, with left and right components both $SU(2)$ singlets, mixing with SM quarks:

$$\dots + h_{Dj} \overline{q_{Lj}} \phi D_R + M_D \overline{D_L} D_R + \text{h.c.}$$

- $\overline{d_{Li}} \mathbf{m}_{ij}^{(d)} d_{Rj} + \text{h.c.} = (\overline{d_{L1}}, \overline{d_{L2}}, \overline{d_{L3}}, \overline{D_L}) \begin{pmatrix} \mathbf{Y}^{(d)} v_w & h_1 v_w \\ h_2 v_w \\ h_3 v_w \\ 0 & 0 & 0 & M_D \end{pmatrix} \begin{pmatrix} d_{R1} \\ d_{R2} \\ d_{R3} \\ D_R \end{pmatrix} + \text{h.c.}$

- $V_L^{(d)\dagger} \mathbf{m}^{(d)} V_R^{(d)} = \mathbf{m}_{\text{diag}}^{(d)} = \text{diag}(m_d, m_s, m_b, M_{b'})$

$$h = (h_d, h_s, h_b)^T = U_{Ld}^\dagger (h_1, h_2, h_3)^T$$

$$\begin{pmatrix} d_{L1} \\ d_{L2} \\ d_{L3} \\ D_L \end{pmatrix} = V_L^{(d)} \begin{pmatrix} d_L \\ s_L \\ b_L \\ b'_L \end{pmatrix}, \quad V_L^{(d)} = \begin{pmatrix} V_{L1d} & V_{L1s} & V_{L1b} & V_{L1b'} \\ V_{L2d} & V_{L2s} & V_{L2b} & V_{L2b'} \\ V_{L3d} & V_{L3s} & V_{L3b} & V_{L3b'} \\ V_{LDd} & V_{LDs} & V_{LDb} & V_{LDb'} \end{pmatrix} \approx \begin{pmatrix} & 0 \\ U_{Ld} & 0 \\ & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & \frac{h_d v_w}{M_D} \\ 0 & 1 & 0 & \frac{h_s v_w}{M_D} \\ 0 & 0 & 1 & \frac{h_b v_w}{M_D} \\ -\frac{h_d^* v_w}{M_D} & -\frac{h_s^* v_w}{M_D} & -\frac{h_b^* v_w}{M_D} & 1 \end{pmatrix}$$

$$|V_{LD\alpha}| = s_{L\alpha}^d \approx |h_\alpha| v_w / M_D \quad |s_{Ri}^d| \approx \frac{y_i |h_i| v_w^2}{M_D^2}$$

Down-type weak singlet

- Charged weak interactions:

$$\mathcal{L}_{cc} = \frac{g}{\sqrt{2}} \begin{pmatrix} \overline{u_{L1}} & \overline{u_{L2}} & \overline{u_{L3}} \end{pmatrix} \gamma^\mu \begin{pmatrix} d_{L1} \\ d_{L2} \\ d_{L3} \end{pmatrix} W_\mu^+ + \text{h.c.} = \frac{g}{\sqrt{2}} \begin{pmatrix} \overline{u_L} & \overline{c_L} & \overline{t_L} \end{pmatrix} \gamma^\mu V_L \begin{pmatrix} d_L \\ s_L \\ b_L \\ b'_L \end{pmatrix} W_\mu^+ + \text{h.c.}$$

- V_L is a 3×4 matrix, $V_L^\dagger V_L \neq \mathbf{1}$, $V_L V_L^\dagger = \mathbf{1}$

$$V_L = U_{Lu}^\dagger \tilde{V}_L^{(d)} = \tilde{V}_L^{(d)} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} & V_{ub'} \\ V_{cd} & V_{cs} & V_{cb} & V_{cb'} \\ V_{td} & V_{ts} & V_{tb} & V_{tb'} \end{pmatrix}$$

$$|\mathbf{V}_{ud}|^2 + |\mathbf{V}_{us}|^2 + |\mathbf{V}_{ub}|^2 = 1 - |\mathbf{V}_{ub'}|^2$$

$$\begin{pmatrix} d_{L1} \\ d_{L2} \\ d_{L3} \\ D_L \end{pmatrix} = \begin{pmatrix} V_{L1d} & V_{L1s} & V_{L1b} & V_{L1b'} \\ V_{L2d} & V_{L2s} & V_{L2b} & V_{L2b'} \\ V_{L3d} & V_{L3s} & V_{L3b} & V_{L3b'} \\ V_{LDd} & V_{LDs} & V_{LDb} & V_{LDb'} \end{pmatrix} \begin{pmatrix} d_L \\ s_L \\ b_L \\ b'_L \end{pmatrix} \approx \begin{pmatrix} U_{Ld} \left(1 - hh^\dagger \frac{v_w^2}{2M_D^2}\right) & U_{Ld} h \frac{v_w}{M_D} \\ -h^\dagger \frac{v_w}{M_D} & 1 - h^\dagger h \end{pmatrix} \begin{pmatrix} d_L \\ s_L \\ b_L \\ b'_L \end{pmatrix}$$

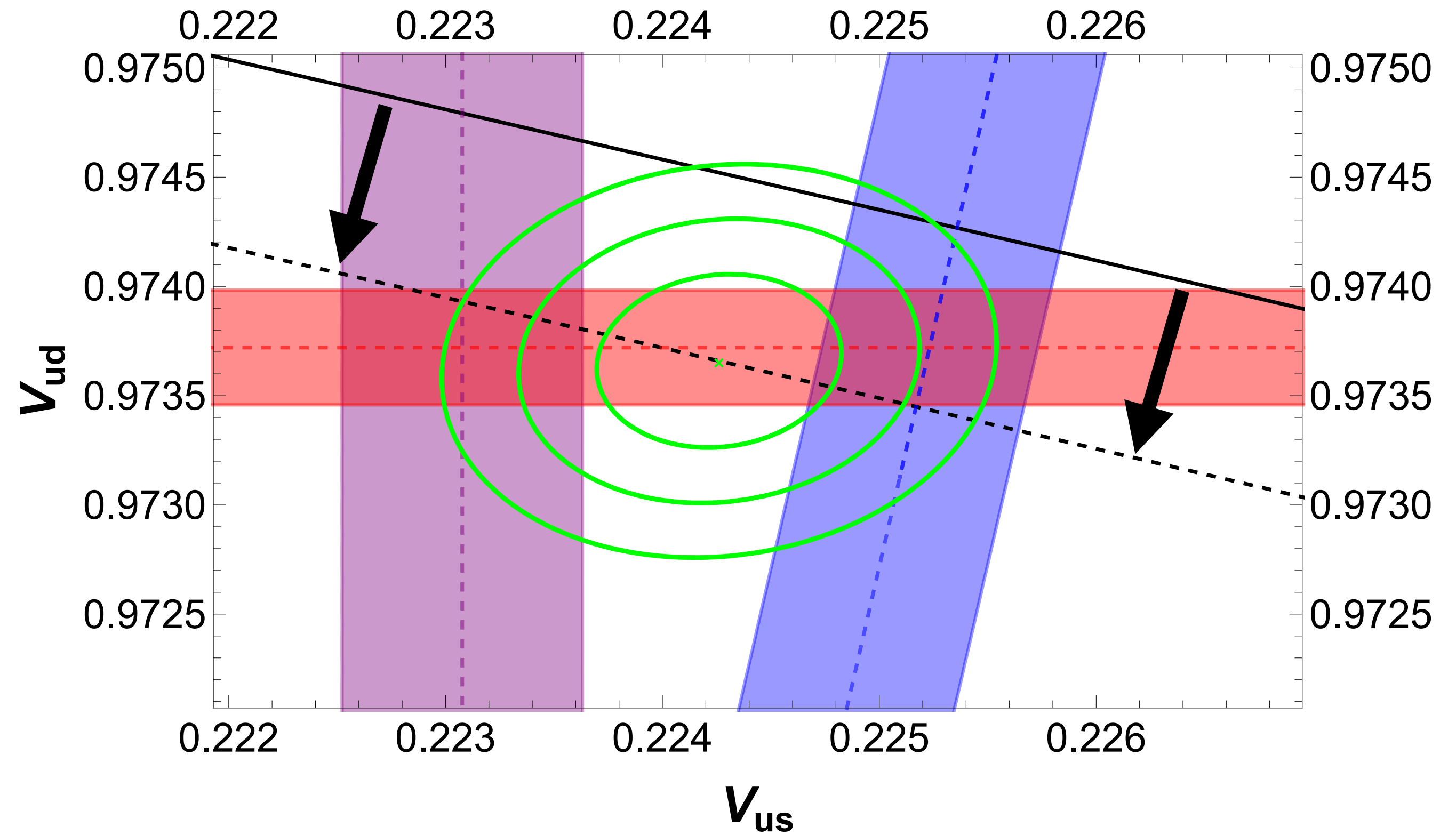
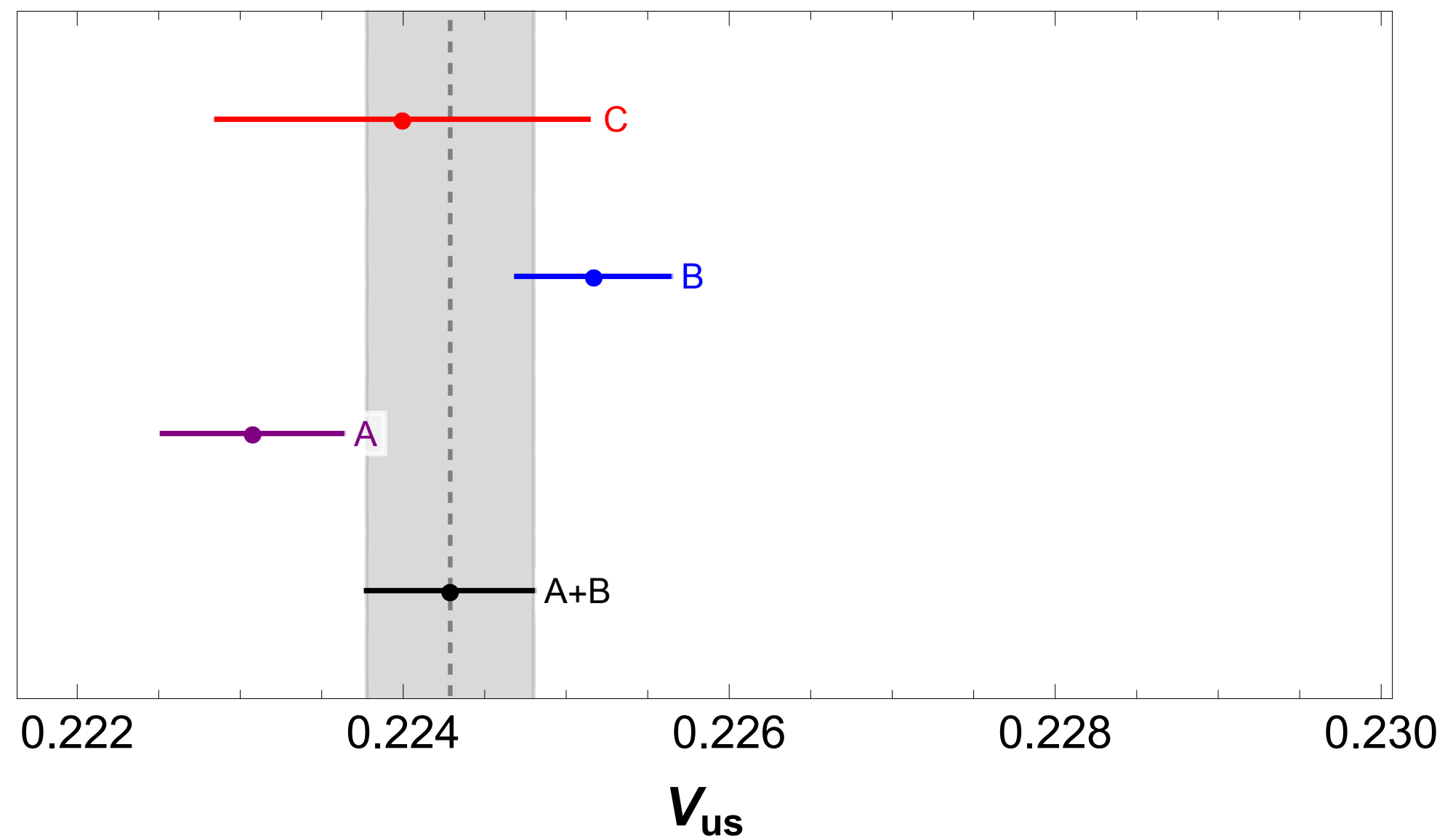
$$|V_{ub'}| \approx |V_{LDd}| \approx |h_d| v_w / M_D$$

Vector-like weak singlets

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 - |V_{ub'}|^2 ,$$

$$|V_{ub'}| \approx 0.041 \quad (|V_{ub'}| \gg |V_{ub}|)$$

- Extra down-type weak singlets
- Extra up-type weak singlets



Down-type weak singlet

- Weak neutral currents:

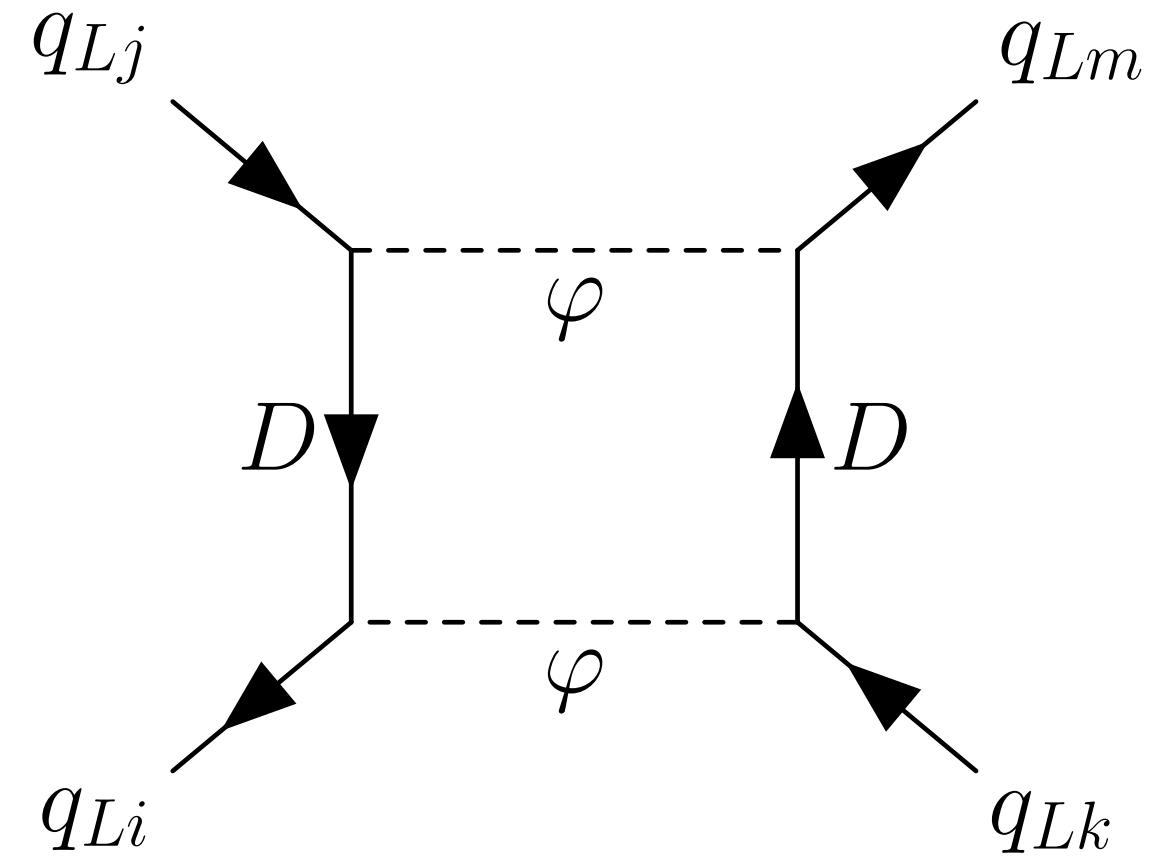
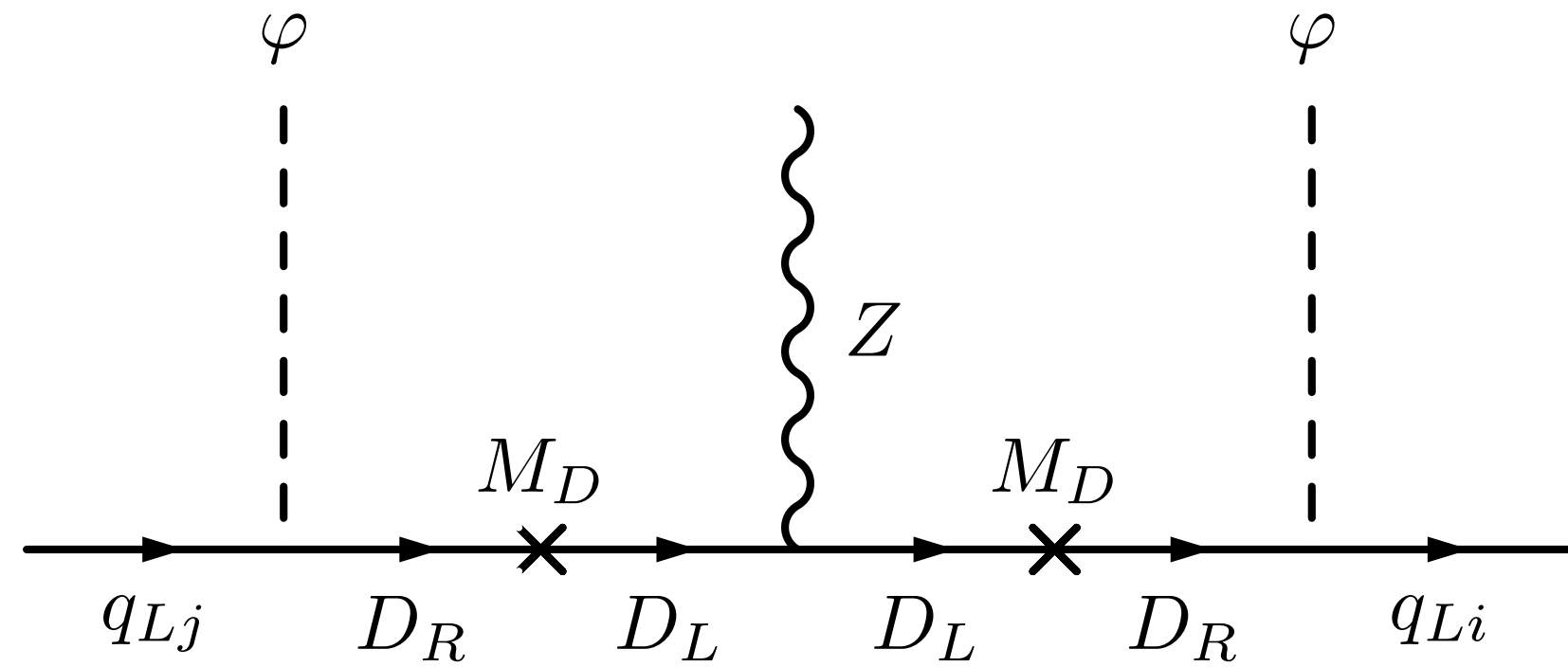
$$\mathcal{L}_{\text{nc}} \supset -\frac{1}{2} \frac{g}{\cos \theta_W} \left(\overline{d_L} \quad \overline{s_L} \quad \overline{b_L} \quad \overline{b'_L} \right) \gamma^\mu V_L^{(d)\dagger} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} V_L^{(d)} \begin{pmatrix} d_L \\ s_L \\ b_L \\ b'_L \end{pmatrix} Z_\mu$$

$$V_{\text{nc}}^{(d)} = V_L^{(d)\dagger} \text{diag}(1, 1, 1, 0) V_L^{(d)} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} -|V_{L4d}|^2 & -V_{L4d}^* V_{L4s} & -V_{L4d}^* V_{L4b} & -V_{L4d}^* V_{L4b'} \\ -V_{L4s}^* V_{L4d} & -|V_{L4s}|^2 & -V_{L4s}^* V_{L4b} & -V_{L4s}^* V_{L4b'} \\ -V_{L4b}^* V_{L4d} & -V_{L4b}^* V_{L4s} & -|V_{L4b}|^2 & -V_{L4b}^* V_{L4b'} \\ -V_{L4b'}^* V_{L4d} & -V_{L4b'}^* V_{L4s} & -V_{L4b'}^* V_{L4b} & |V_{L1b'}|^2 + |V_{L2b'}|^2 + |V_{L3b'}|^2 \end{pmatrix}$$

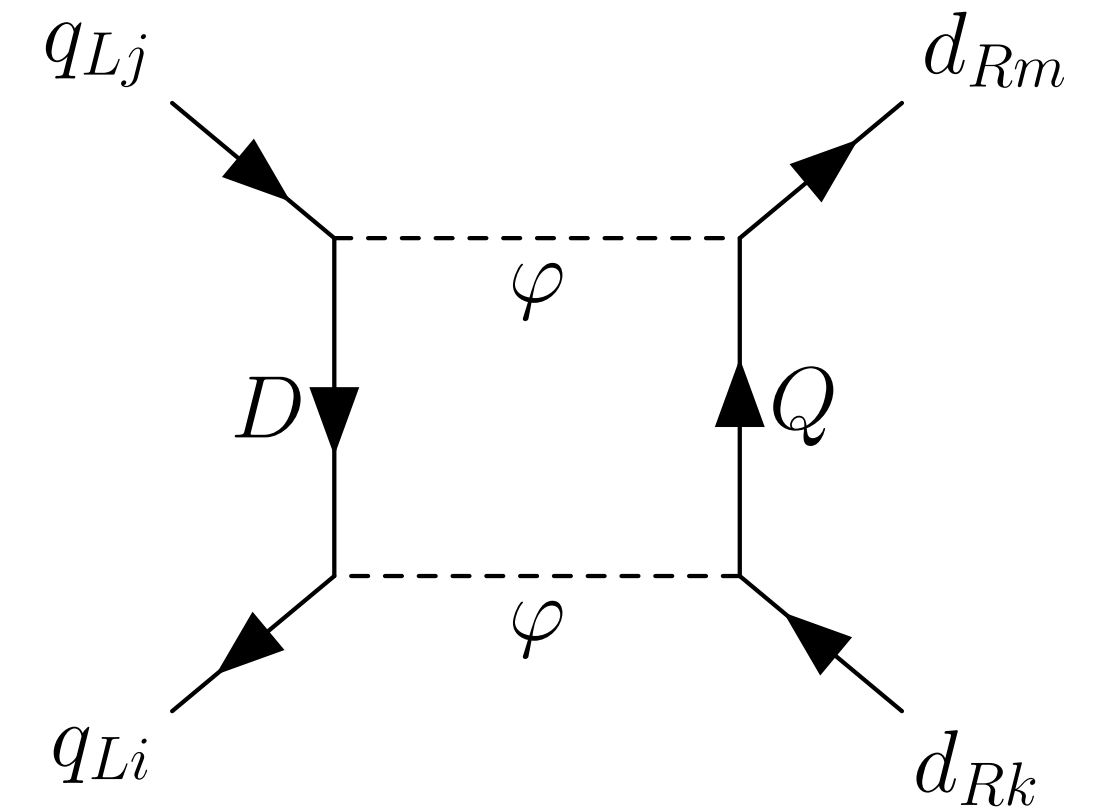
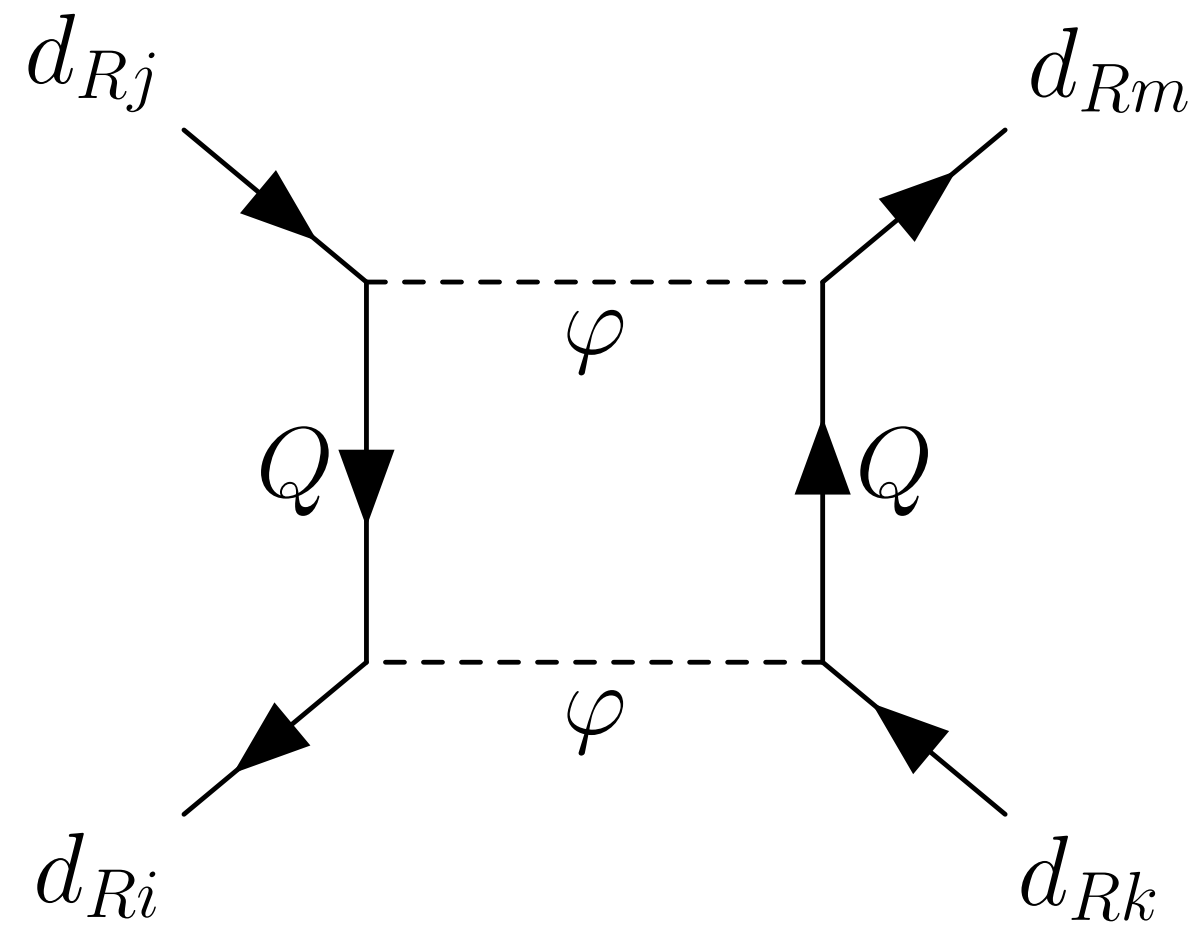
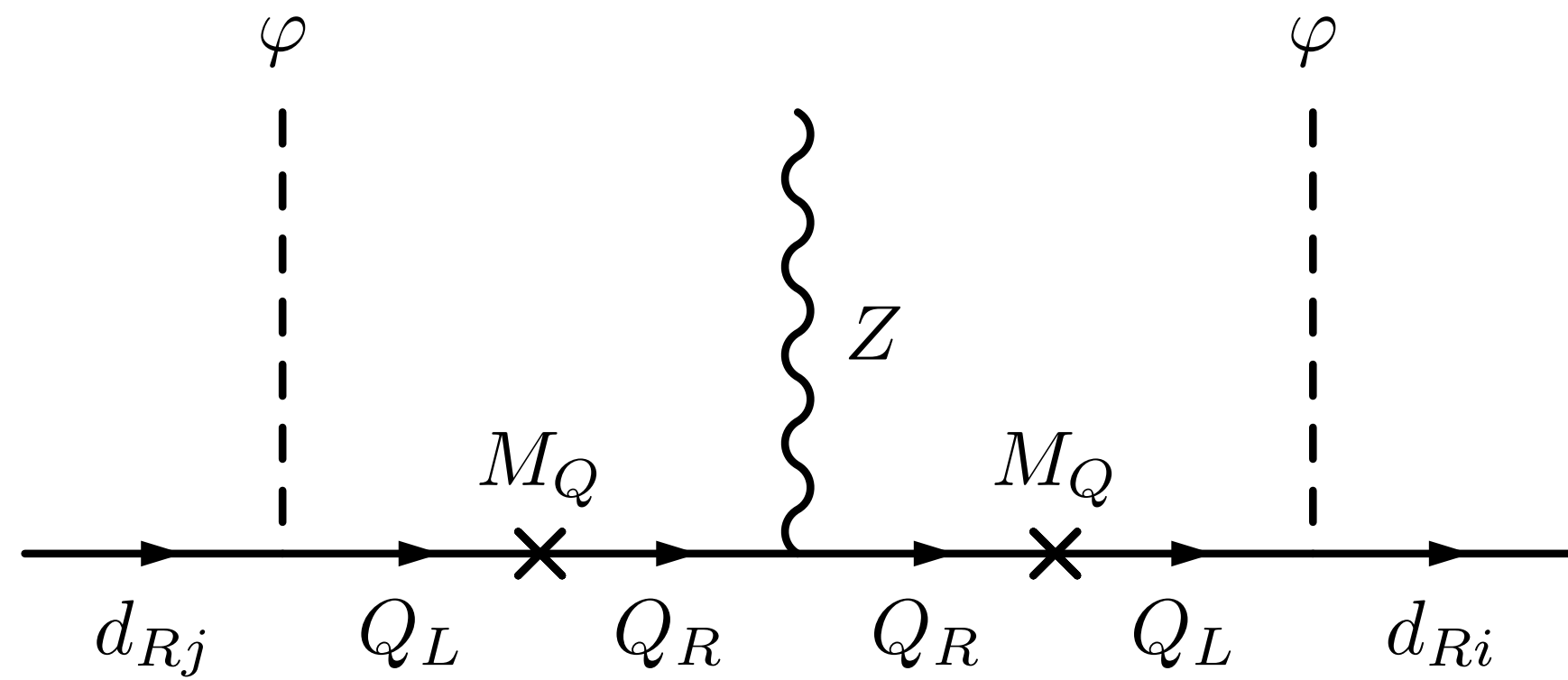
- $\mathcal{L}_{\text{nc}} = \frac{g}{\cos \theta_W} Z_\mu \left[T_3(f_{L,R}) - Q(f) \sin^2 \theta_W \right] \overline{f_{L,R}} \gamma^\mu f_{L,R}$ (not the same quantum numbers)
- Yukawa and mass matrices are not diagonalized by the same transformation;
- Tree and loop level **flavour changing** couplings with the Higgs boson and with Z-boson.
- Also **flavour diagonal** processes change.
- $\Gamma(Z \rightarrow \text{had}) - \Gamma(Z \rightarrow \text{had})_{\text{SM}} \approx \frac{G_F M_Z^3}{\sqrt{2}\pi} \left(-\frac{1}{2} + \frac{1}{3} \sin^2 \theta_W \right) (|V_{LDd}|^2 + |V_{LDs}|^2 + |V_{LDb}|^2) < 0$

$$|V_{LDd}|^2 + |V_{LDs}|^2 + |V_{LDb}|^2 \lesssim 2 \times 10^{-3}$$

Flavour changing



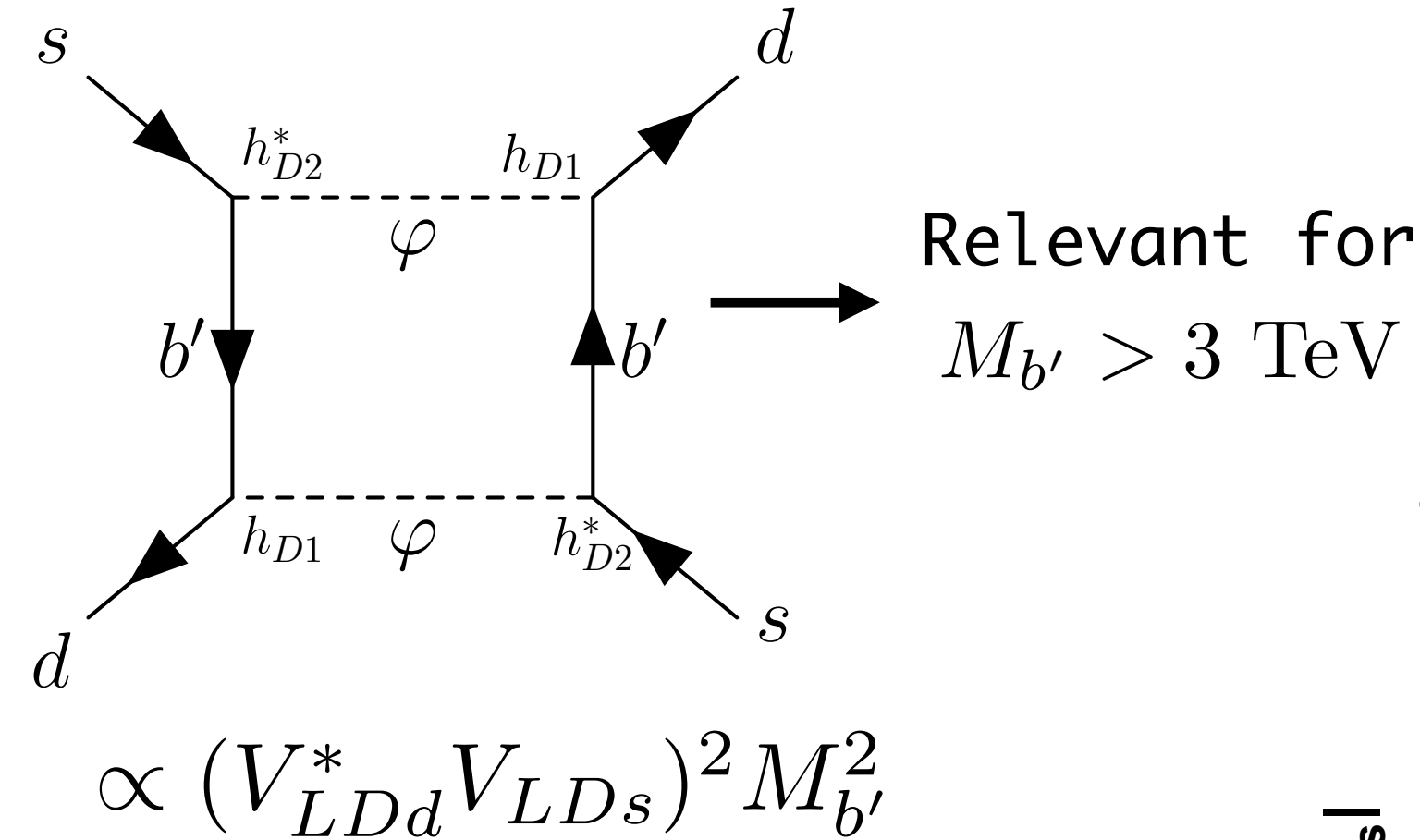
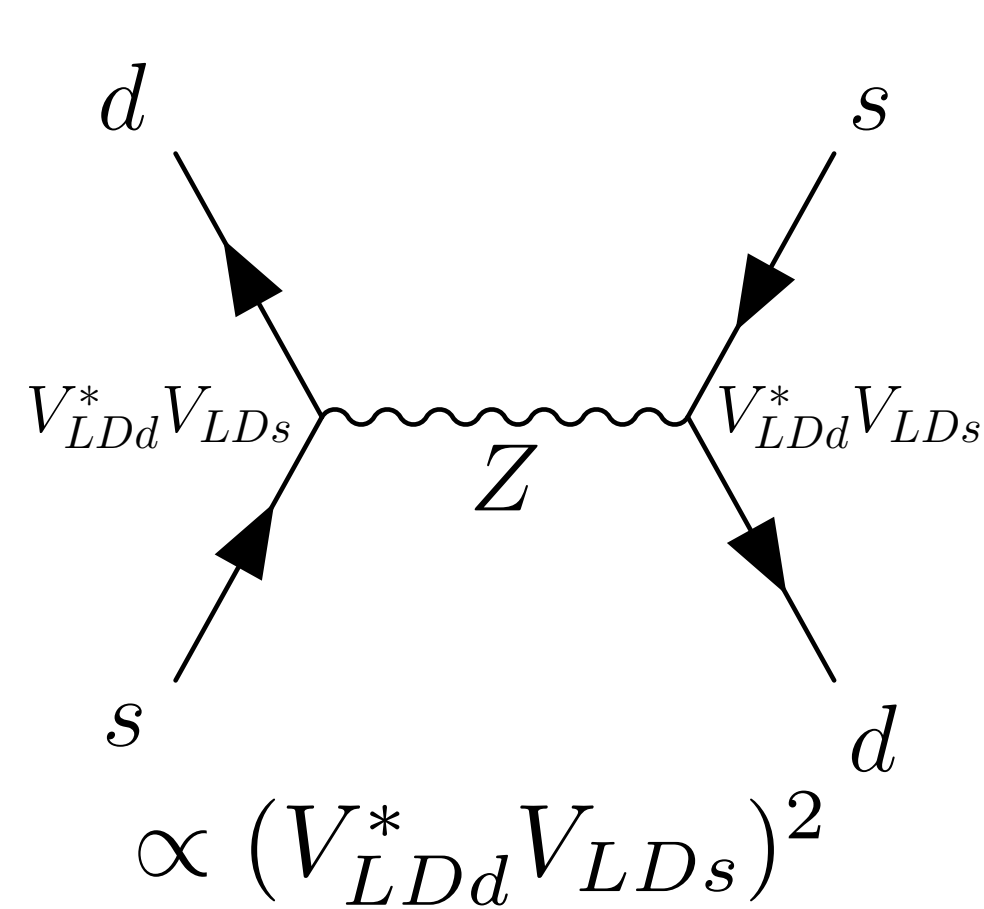
(similarly for up)



Down-type weak singlet

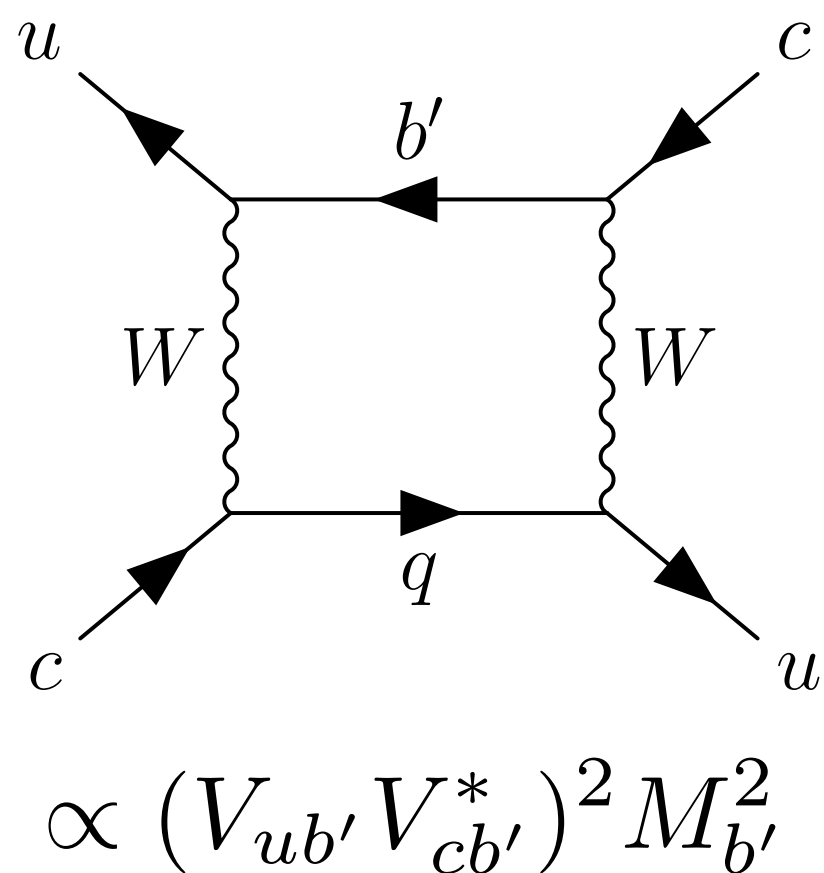
- Constraints become stronger for larger mass of the vector-like quarks

Neutral K mesons mixing:

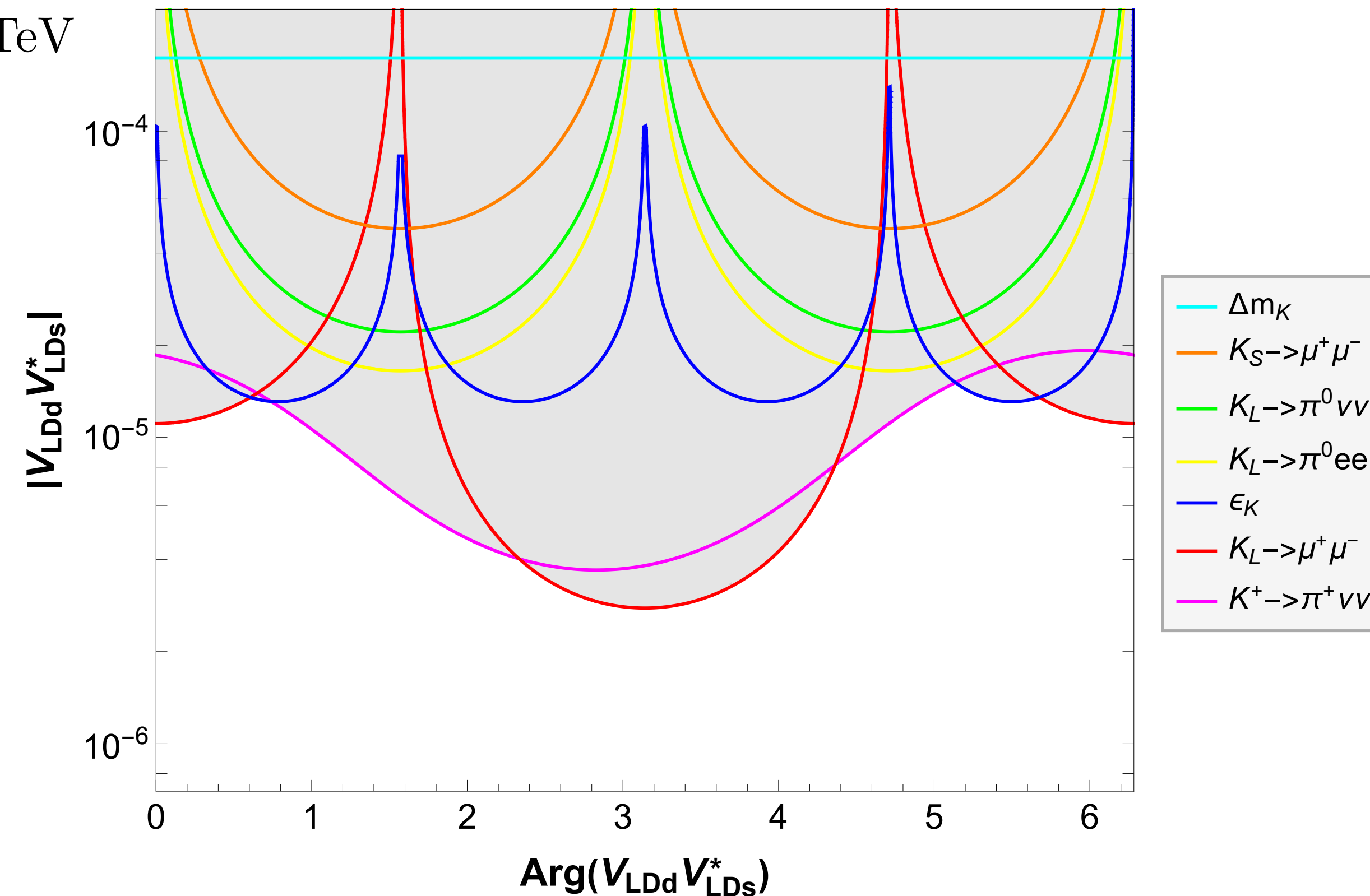
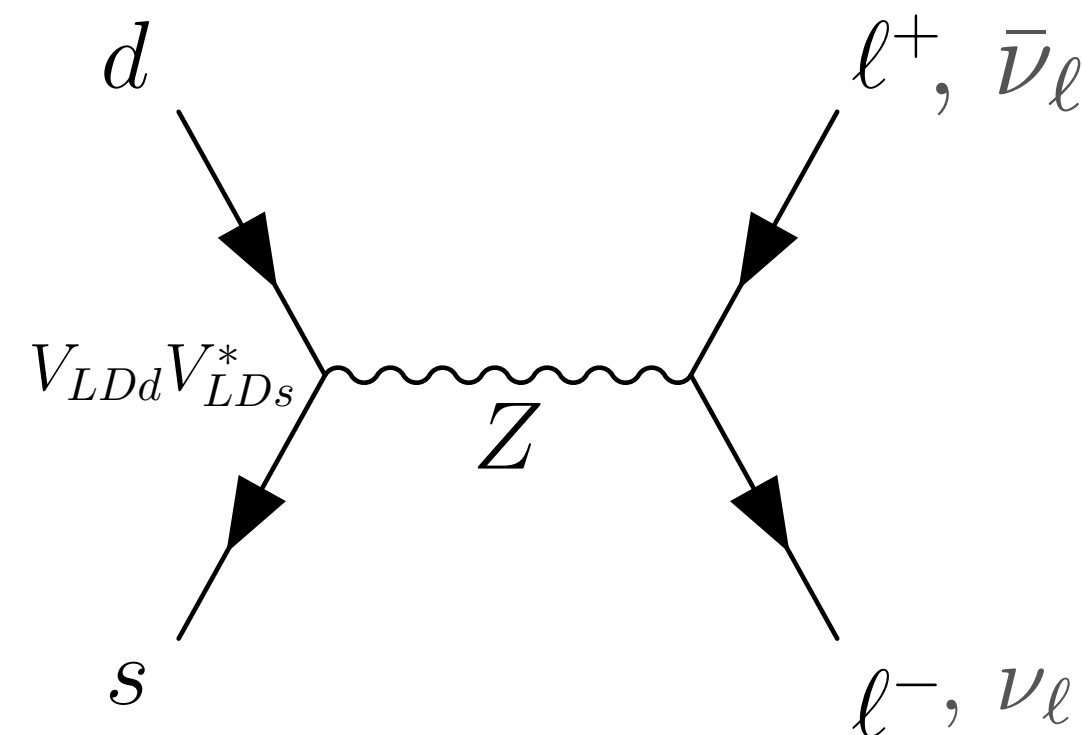


$M_{b'} = 1 \text{ TeV}$

Neutral D mesons mixing:



FC kaon decays:



Down-type weak singlet

• $|V_{LDs}^* V_{LDd}| < 5 \times 10^{-6}$ from K-mesons fc processes.

• $|V_{LDb}^* V_{LDd}| < 2 \times 10^{-4}$ from B-mesons fc processes.

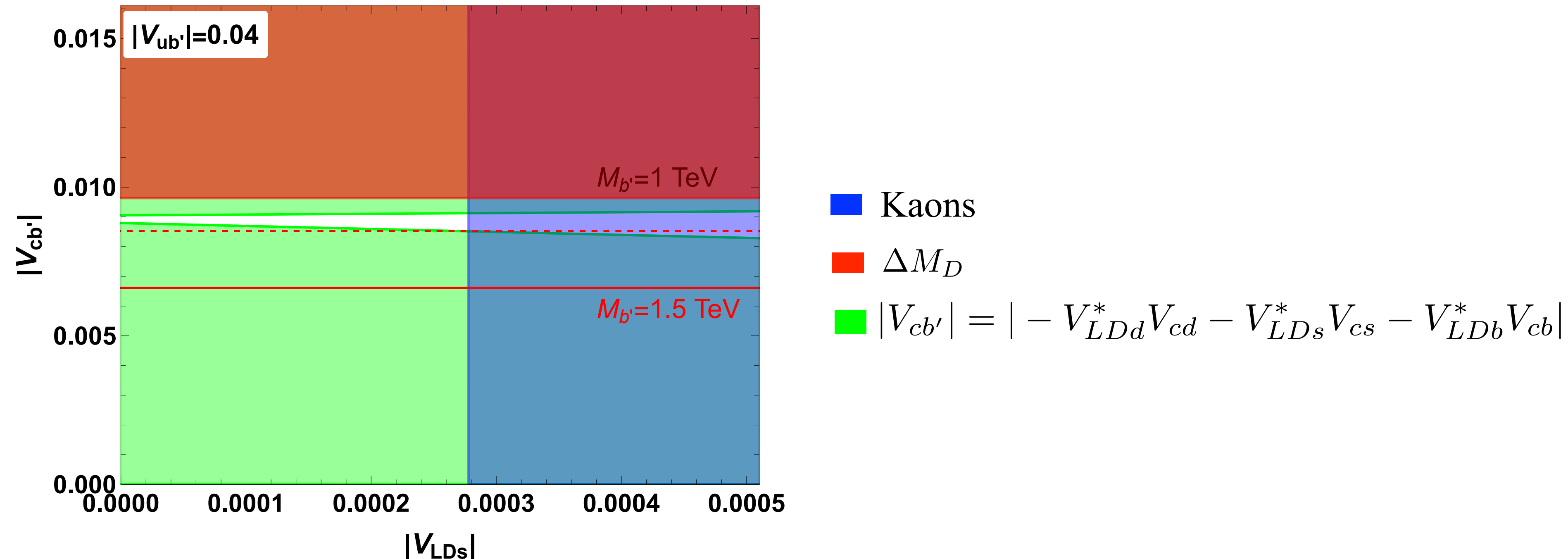
• $|V_{cb'}| \approx | -V_{LDd}^* V_{cd} - V_{LDs}^* V_{cs} - V_{LDb}^* V_{cb} | \approx |V_{cd} V_{ub'}|$

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 - |V_{ub'}|^2$$

$$|V_{ub'}| \approx 0.041$$

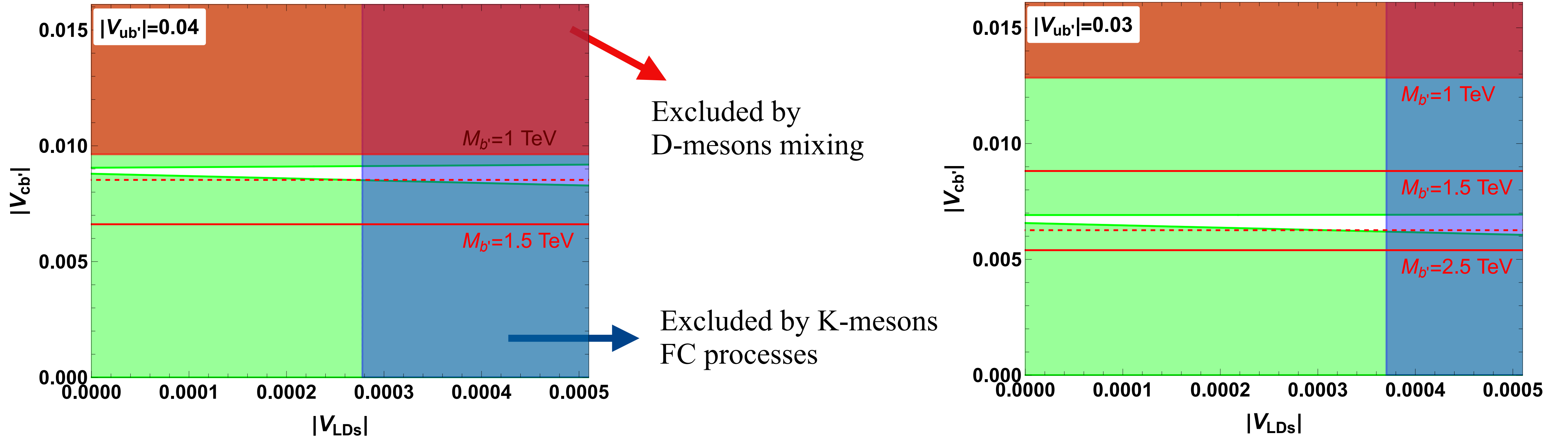
$$|V_{ub'}| \approx |V_{LDd}| \approx |h_d| v_w / M_D$$

D mixing: $|V_{ub'} V_{cb'}^*| \approx |V_{cd}| |V_{ub'}| \lesssim 4 \times 10^{-4} \left[\frac{1 \text{ TeV}}{M_{b'}} \right] \longrightarrow |V_{ub'}| < 0.042 \left[\frac{1 \text{ TeV}}{M_{b'}} \right]^{1/2}$



Down-type weak singlet

$$\blacksquare |V_{cb'}| = | -V_{LDd}^* V_{cd} - V_{LDs}^* V_{cs} - V_{LDb}^* V_{cb} |$$



- Mass of the extra quark cannot exceed **few TeV**, also in the most conservative case.
- $|V_{ub'}| \approx |V_{LDd}| \approx |h_d|v_w/M_D < 0.042$ with $M_{b'} = 1$ TeV (D^0 -mesons mixing).
- $|V_{ub'}| < 0.050$ from Z boson decay rate.
- $|V_{cb'}| \lesssim 10^{-2}$. From B-mesons physics: $|V_{tb'}| = |V_{LDb}| = |h_b|v_w/M_D \lesssim 6 \cdot 10^{-3}$.
- Yukawa couplings h_s, h_b should be respectively 50 and 4 times smaller than h_d .

Up-type weak singlet

Vector-like up-type quark $U_{L,R}$ with left and right components both $SU(2)$ singlets, mixing with SM quarks:

$$\dots + z_j \overline{q_{Lj}} \tilde{\phi} U_R + M_U \overline{U_L} U_R + \text{h.c.}$$

$$\overline{u_{Li}} \mathbf{m}_{ij}^{(u)} u_{Rj} + \text{h.c.} = (\overline{u_{L1}}, \overline{u_{L2}}, \overline{u_{L3}}, \overline{U_L}) \begin{pmatrix} \mathbf{Y}^{(u)} v_w & z_1 v_w \\ 0 & z_2 v_w \\ 0 & z_3 v_w \\ 0 & M_U \end{pmatrix} \begin{pmatrix} u_{R1} \\ u_{R2} \\ u_{R3} \\ U_R \end{pmatrix} + \text{h.c.}$$

- $V_L^{(u)\dagger} \mathbf{m}^{(u)} V_R^{(u)} = \mathbf{m}_{\text{diag}}^{(u)} = \text{diag}(m_u, m_c, m_t, M_{t'})$

$$(z_u, z_c, z_t)^T = U_{Lu}^\dagger (z_1, z_2, z_3)^T$$

$$\begin{pmatrix} u_L \\ c_L \\ t_L \\ t'_L \end{pmatrix} = V_L^{(u)\dagger} \begin{pmatrix} u_{L1} \\ u_{L2} \\ u_{L3} \\ U_L \end{pmatrix}, \quad V_L^{(u)} = \begin{pmatrix} U_{L1u} & U_{L1c} & U_{L1t} & U_{L1t'} \\ U_{L2u} & U_{L2c} & U_{L2t} & U_{L2t'} \\ U_{L3u} & U_{L3c} & U_{L3t} & U_{L3t'} \\ U_{LUu} & U_{LUc} & U_{LUt} & U_{LUt'} \end{pmatrix} \xrightarrow{V_{\text{CKM}}^\dagger} \approx \begin{pmatrix} & 0 \\ U_{Lu} & 0 \\ & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & \frac{z_u v_w}{M_U} \\ 0 & 1 & 0 & \frac{z_c v_w}{M_U} \\ 0 & 0 & 1 & \frac{z_t v_w}{M_U} \\ -\frac{z_u^* v_w}{M_U} & -\frac{z_c^* v_w}{M_U} & -\frac{z_t^* v_w}{M_U} & 1 \end{pmatrix}$$

$$|U_{LU\alpha}| = s_{L\alpha}^u \approx |z_\alpha| v_w / M_U$$

$$|s_{Ri}^u| \approx \frac{y_i |z_i| v_w^2}{M_U^2}$$

Up-type weak singlet

$$\mathcal{L}_{cc} \supset -\frac{g}{\sqrt{2}} \left(\overline{u}_L \quad \overline{c}_L \quad \overline{t}_L \quad \overline{t}'_L \right) \gamma^\mu \tilde{V}_{\text{CKM}} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} W_\mu^+ + \text{h.c.}$$

- $\tilde{V}_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \\ V_{t'd} & V_{t's} & V_{t'b} \end{pmatrix} = \tilde{V}_L^{(u)\dagger} V_L^{(d)}$, not unitary $\tilde{V}_{\text{CKM}} \tilde{V}_{\text{CKM}}^\dagger = \tilde{V}_L^{(u)\dagger} \tilde{V}_L^{(u)} \neq \mathbf{1}$.

- First row: $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 - |U_{LUu}|^2$, $|U_{LUu}| \approx |h_u|v_w/M_{t'} \approx 0.041$.

- **Flavour changing** couplings of quarks with Z boson and Higgs at tree level.

$$-\mathcal{L}_{\text{nc}} \supset \frac{g}{\cos \theta_W} \left[\frac{1}{2} \left(\overline{u}_L \quad \overline{c}_L \quad \overline{t}_L \quad \overline{t}'_L \right) \gamma^\mu V_L^{(u)\dagger} \text{diag}(1, 1, 1, 0) V_L^{(u)} \begin{pmatrix} u_L \\ c_L \\ t_L \\ t'_L \end{pmatrix} - \frac{2}{3} \sin^2 \theta_W (\overline{\mathbf{u}}_L \gamma^\mu \mathbf{u}_L + \overline{\mathbf{u}}_R \gamma^\mu \mathbf{u}_R) \right] Z_\mu$$

$$\mathcal{L}_H \supset (\overline{u}_L, \overline{c}_L, \overline{t}_L, \overline{t}'_L) V_L^{(u)\dagger} \begin{pmatrix} & & z_1 \\ Y_{3 \times 3}^{(u)} & & z_2 \\ & & z_3 \\ 0 & 0 & 0 & 0 \end{pmatrix} V_R^{(u)} \begin{pmatrix} u_R \\ c_R \\ t_R \\ t'_R \end{pmatrix} \frac{H^0}{\sqrt{2}} + \text{h.c.}$$

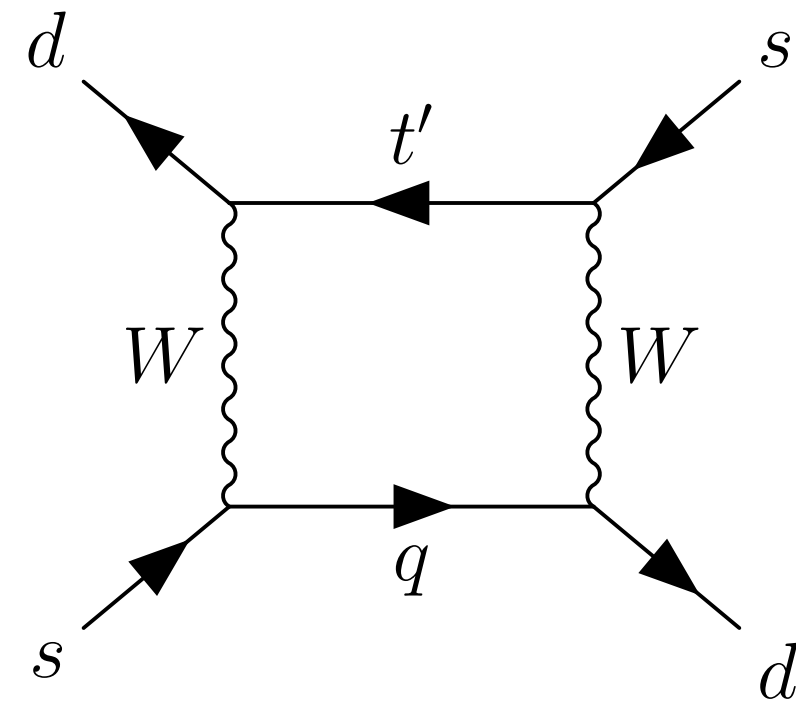
Up-type weak singlet

- Constraints from K and B mesons mixing

$$\longrightarrow (V_{t'd}V_{t's}), \quad (V_{t'd}V_{t'b}^*)$$

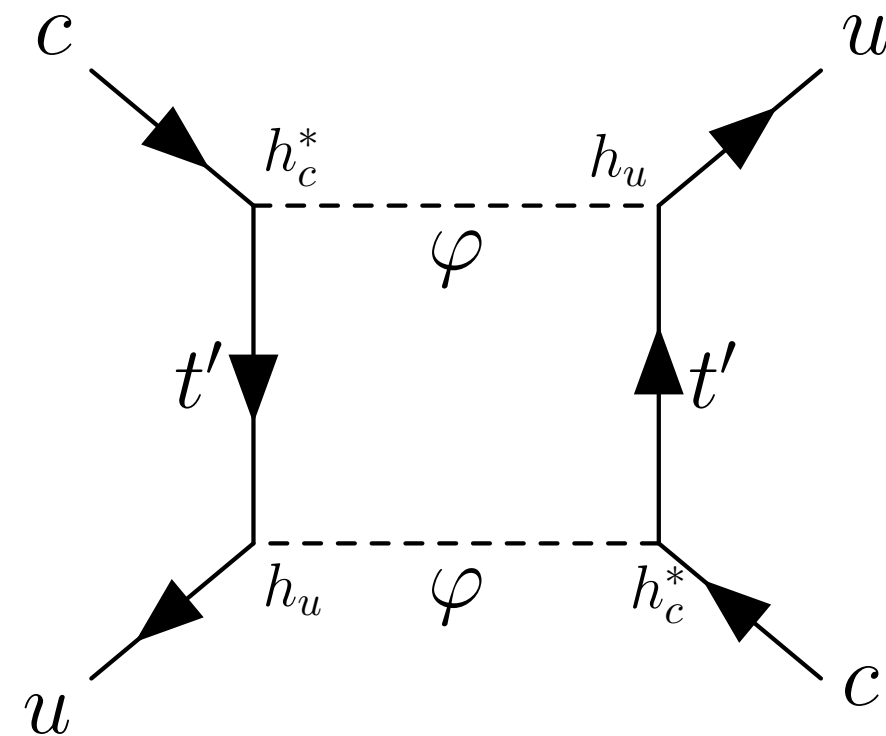
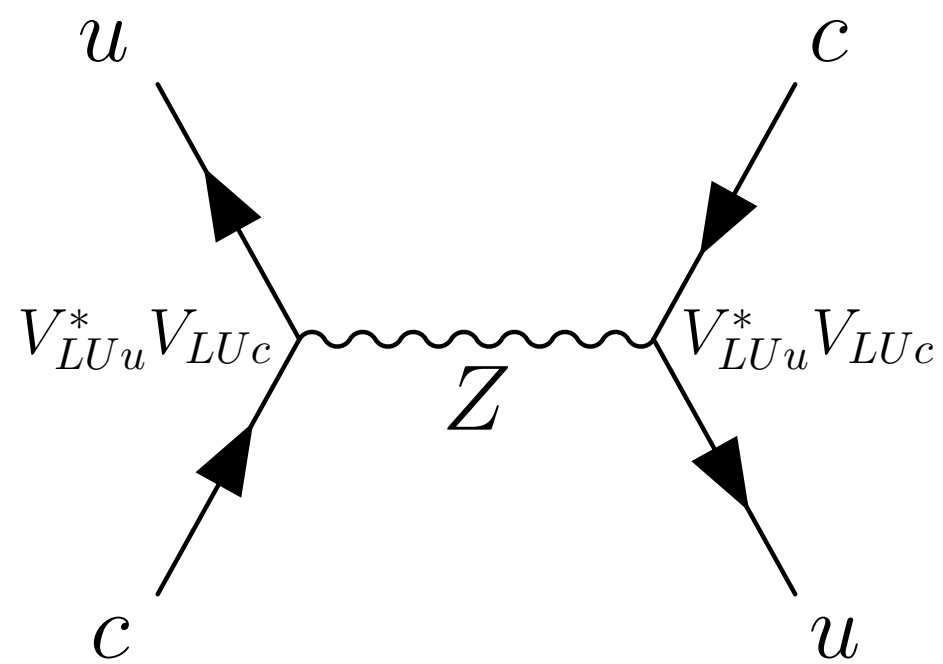
$$V_{t'd} \approx -U_{LUu} \approx h_u^* v_w / M_{t'}, \quad V_{t'b} \approx -U_{LUt} \approx z_t^* v_w / M_U$$

$$V_{t's} = -U_{LUu}V_{us} - U_{LUc}V_{cs} - U_{LUt}V_{ts}$$

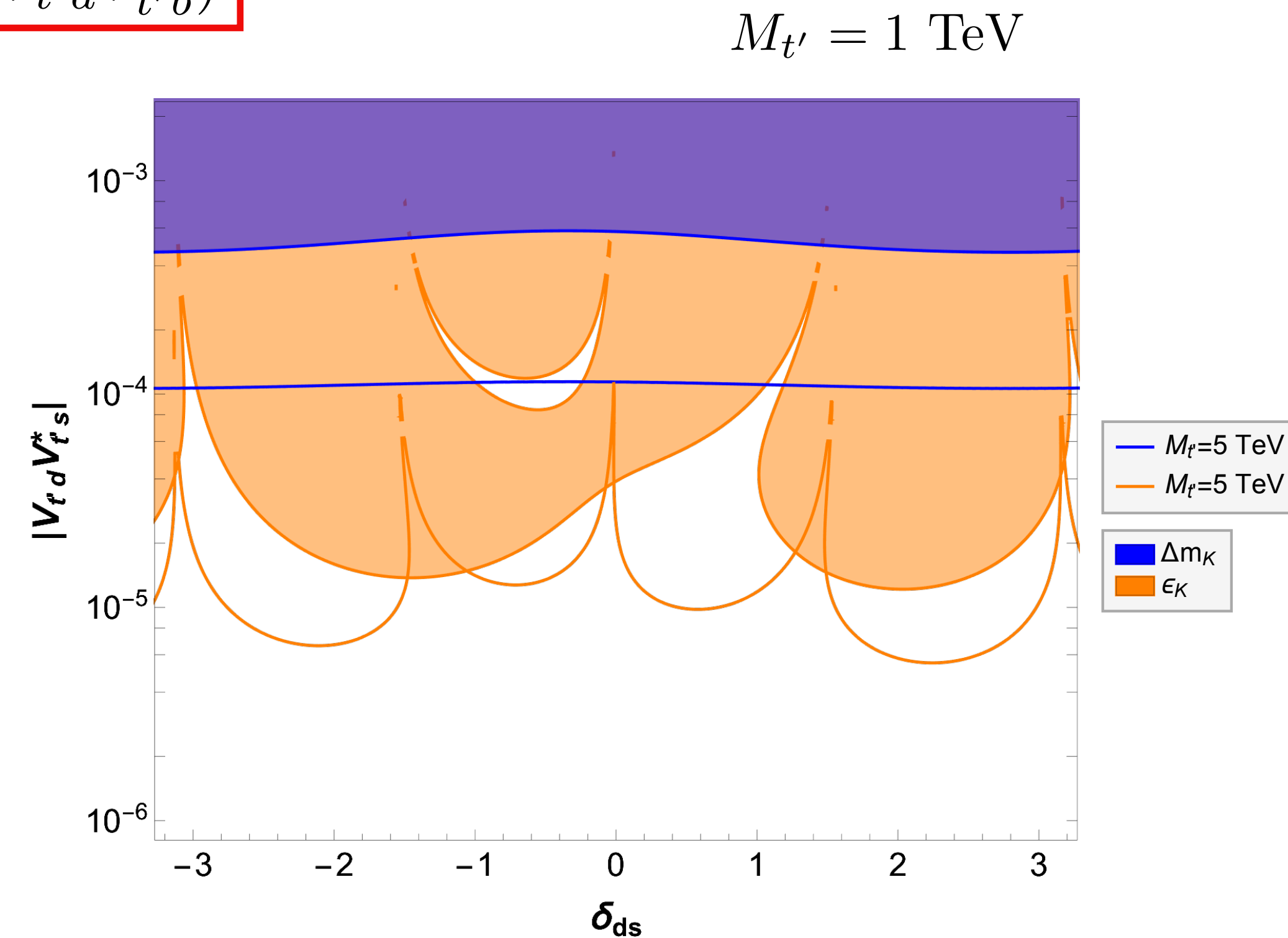


$$\sim (V_{t'd}V_{t's}^*)^2 M_{t'}^2 + \dots$$

- Constraints from D mesons mixing



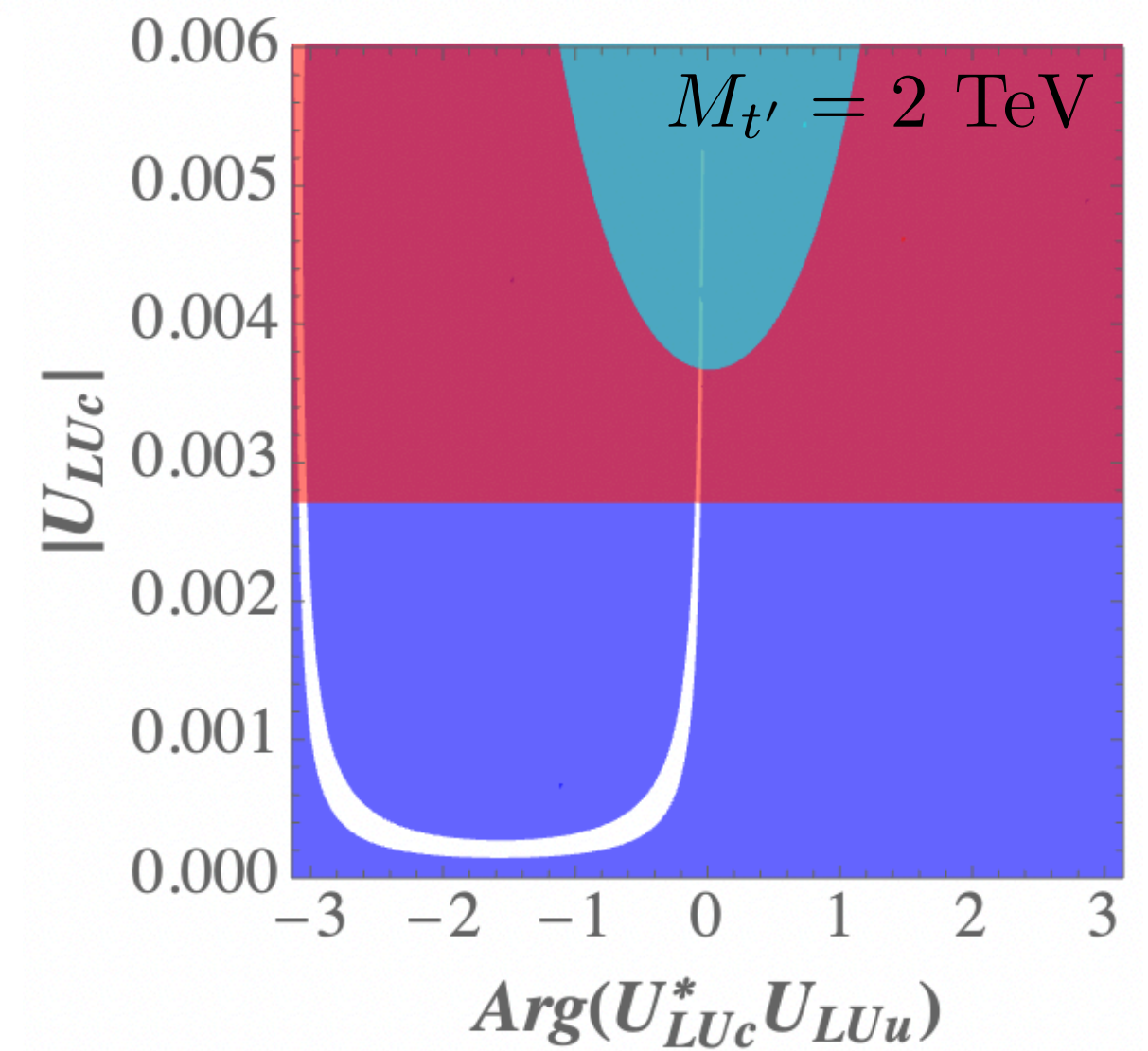
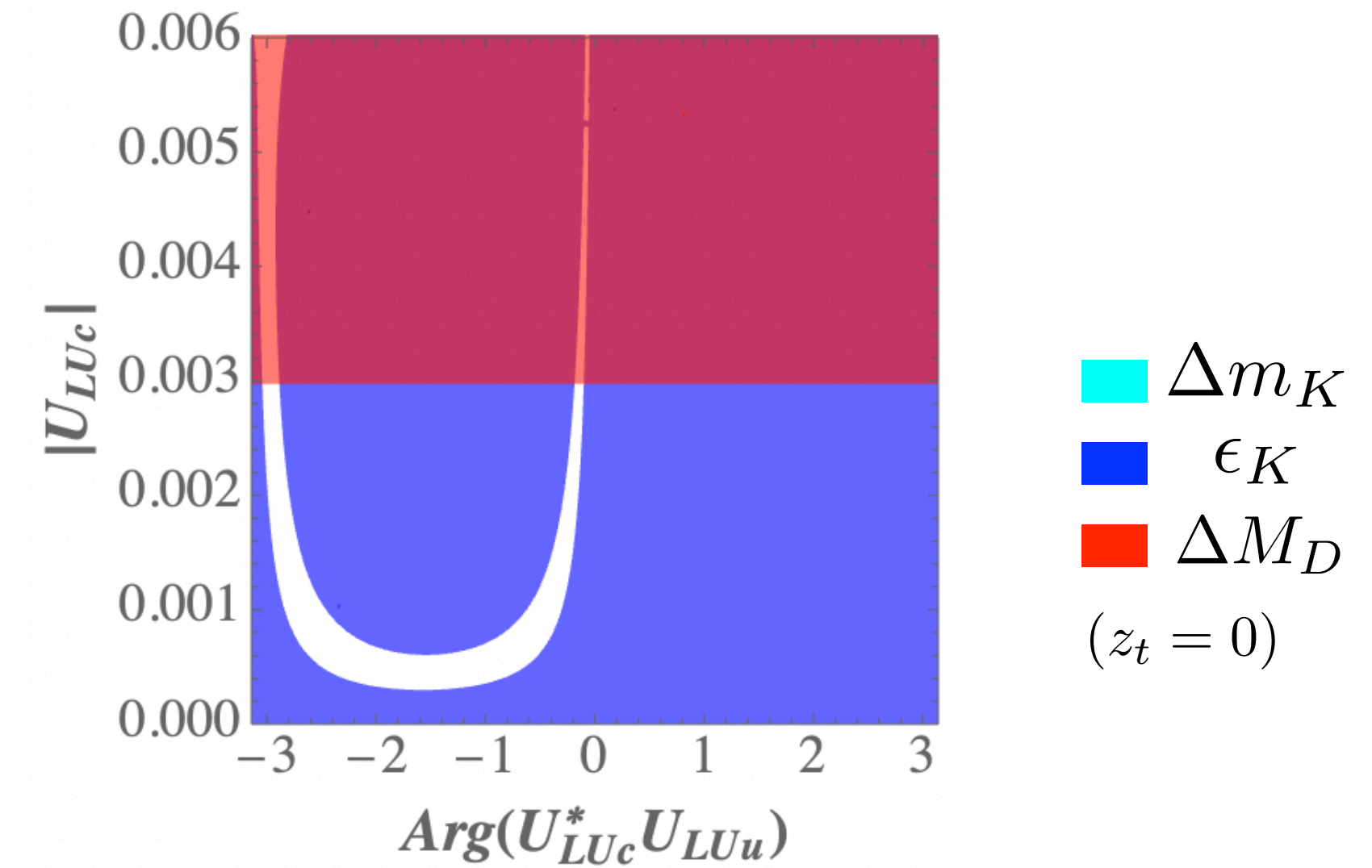
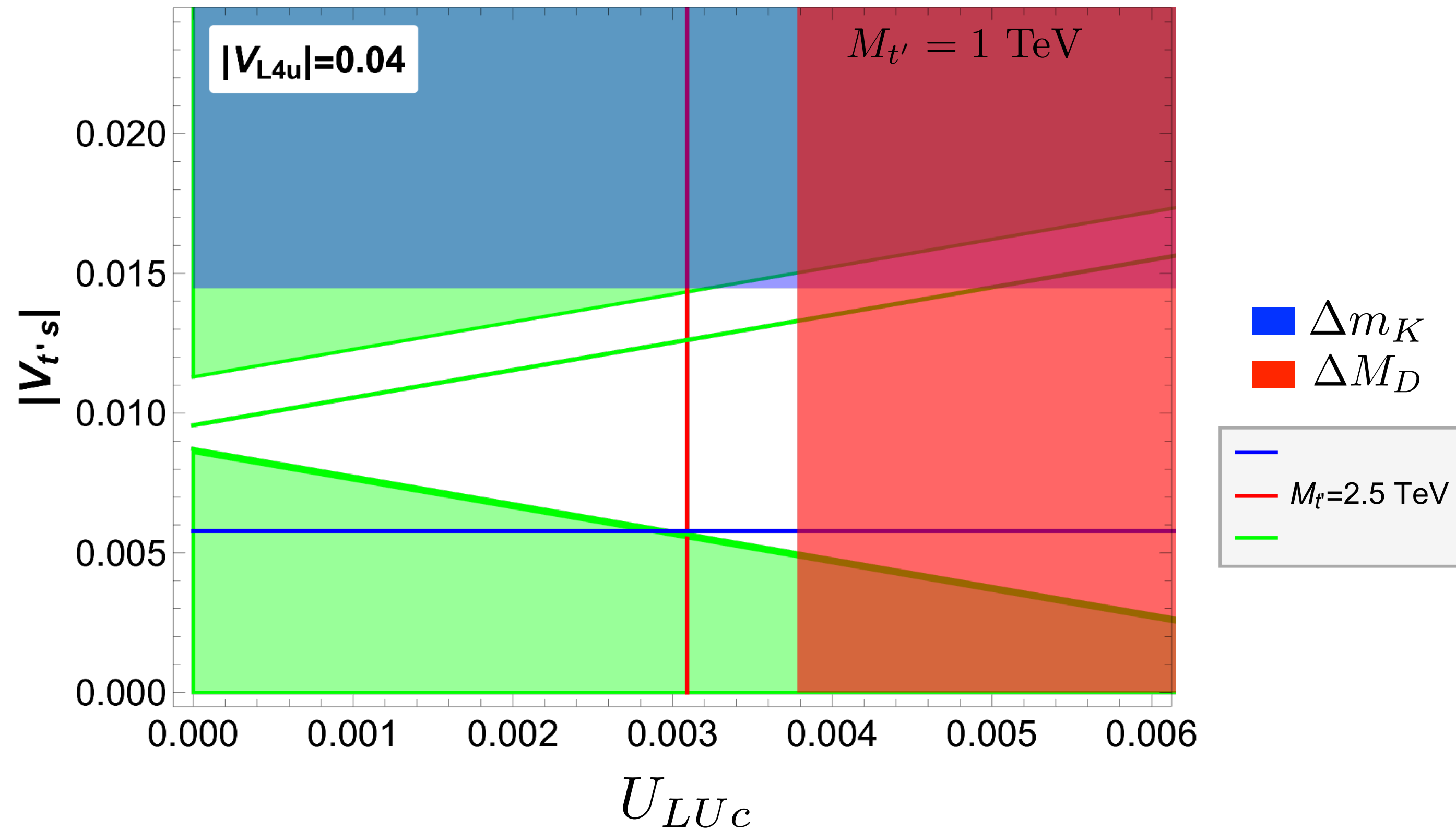
$$M_{12, \text{NP}}^D \approx \frac{1}{3} f_D^2 M_{D^0} \frac{G_F}{\sqrt{2}} (U_{LUu}^* U_{LUc})^2 \left[1 + \left(\frac{M_{t'}}{3.1 \text{ TeV}} \right)^2 \right]$$



Up-type weak singlet

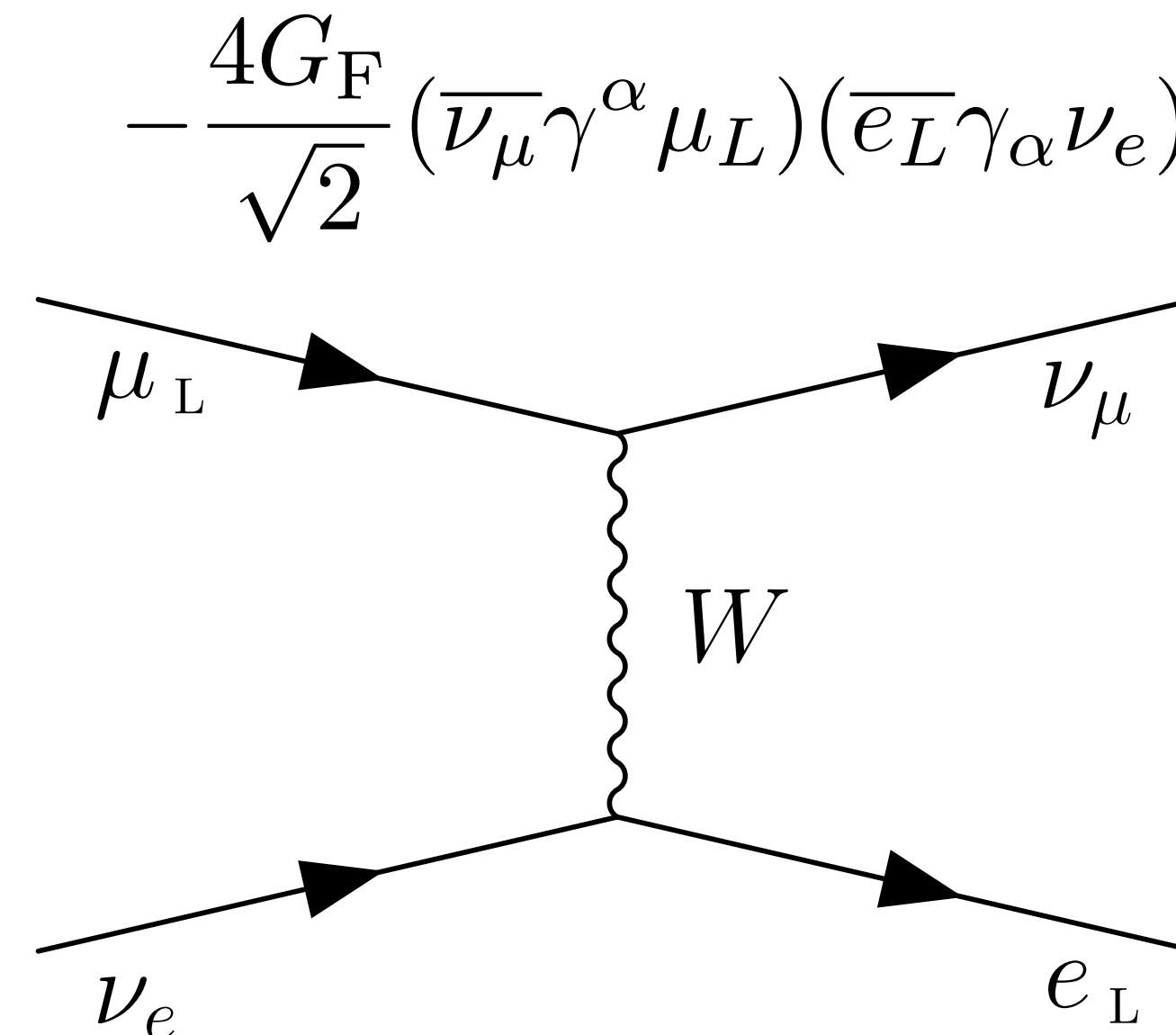
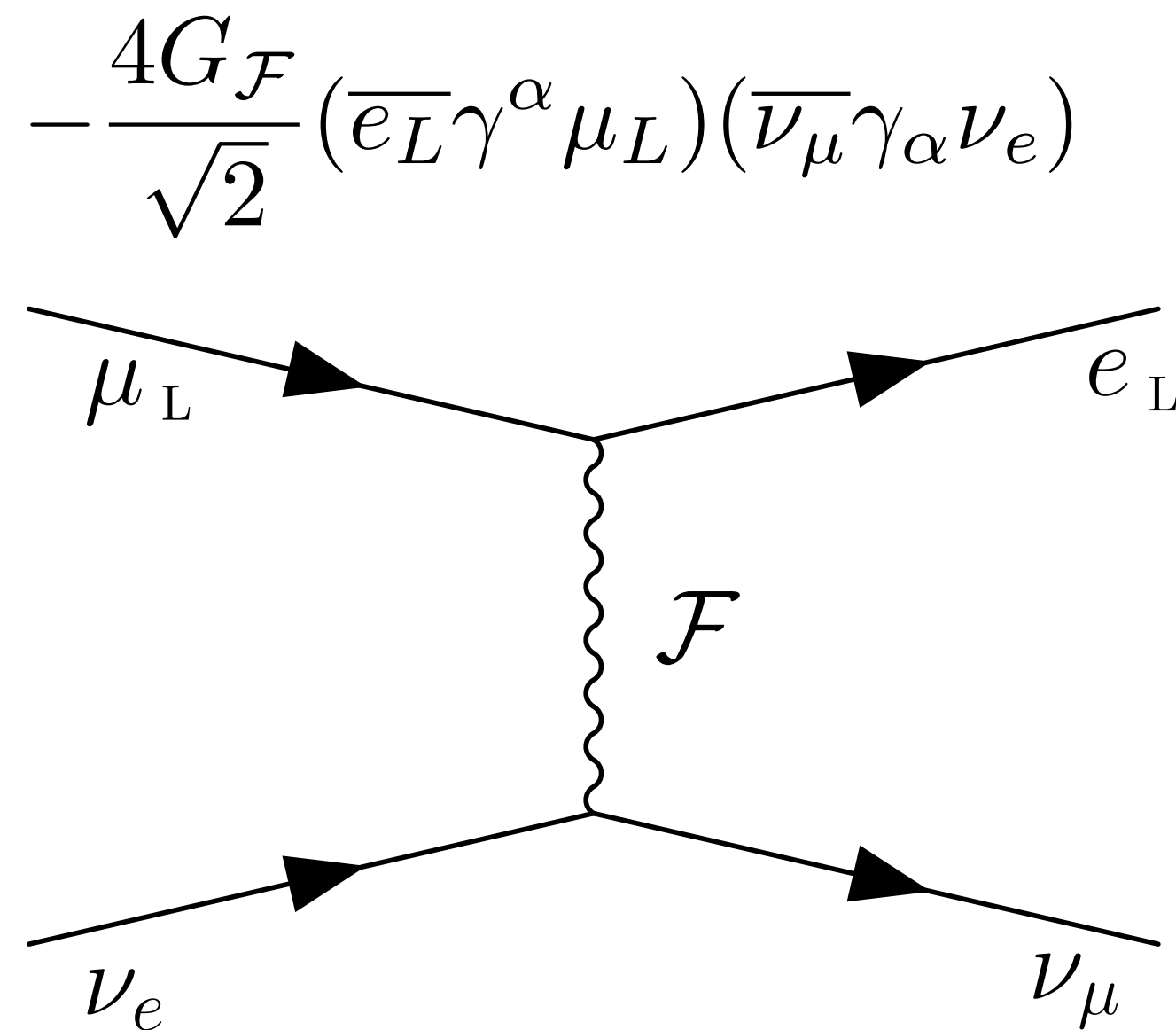
$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 - |U_{LUu}|^2, \quad |V_{t'd}| \approx |U_{LUu}| \approx |h_u|v_w/M_{t'} \approx 0.041$$

■ $|V_{t's}| = | -U_{LUu}V_{us} - U_{LUc}V_{cs} - U_{LUt}V_{ts} |$



Flavour bosons for CKM unitarity

- Suppose the existence of **flavor changing bosons**.



- Horizontal interactions have positive interference with SM;
- After Fierz transformation, the sum of the diagrams gives the operator:

$$-\frac{4G_\mu}{\sqrt{2}}(\bar{\nu}_\mu\gamma^\alpha\mu_L)(\bar{e}_L\gamma_\alpha\nu_e)$$

$$G_\mu = G_F + G_{\mathcal{F}} = G_F(1 + \delta_\mu)$$

$$G_\mu \neq G_F$$

G_F shift

- Different $\mathbf{G}_\mu = \mathbf{G}_F + \mathbf{G}_\mathcal{F} = \mathbf{G}_F(1 + \delta_\mu) = 1 + \frac{v_w^2}{v_\mathcal{F}^2}$
- The values of V_{us} , V_{ud} (and corresponding errorbars) should be rescaled:

$$|V_{us}| = 0.22333(60) \times (1 + \delta_\mu), \quad |V_{ud}| = 0.97370(14) \times (1 + \delta_\mu)$$

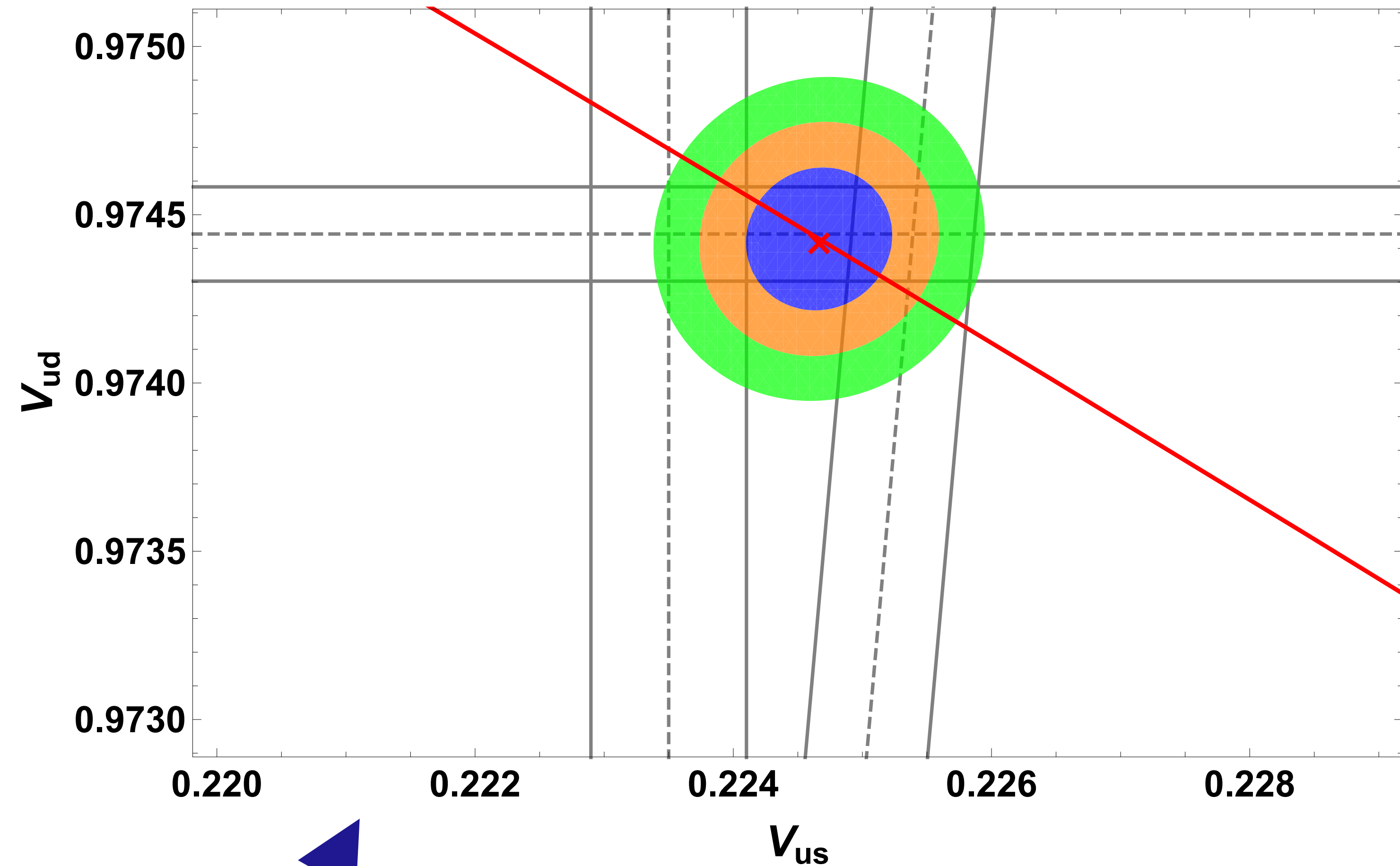
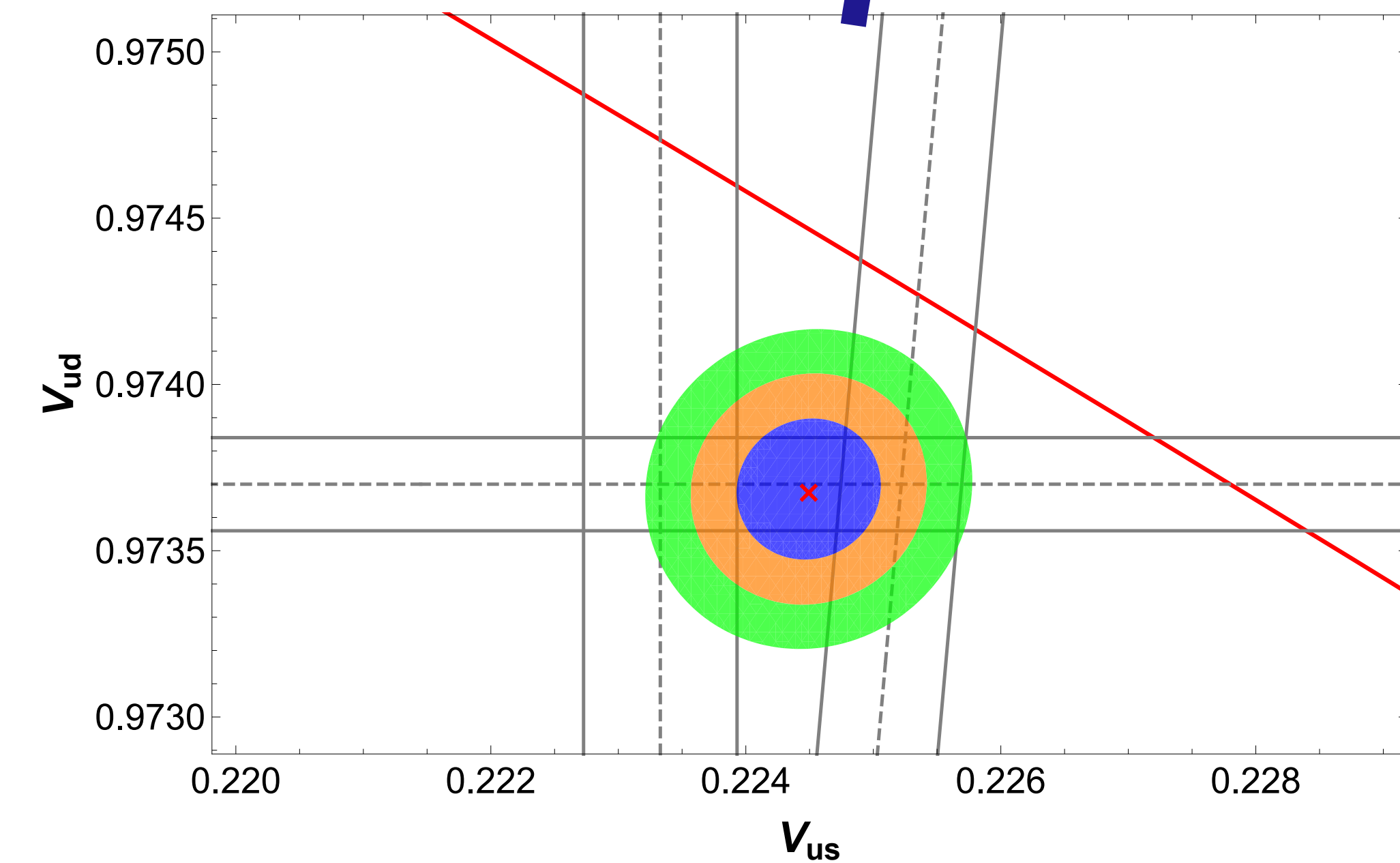
while the ratio is not affected.

- Unitarity recovered: $\left(\frac{G_F}{G_\mu}\right)^2 (|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2) = 1 - \frac{2G_\mathcal{F}}{G_F}$
- CKM unitarity is explained with $\delta_\mu \sim \mathbf{7.6 \times 10^{-4}}$, or

$$v_\mathcal{F} = 6\text{-}7 \text{ TeV}$$

Flavour bosons for CKM unitarity

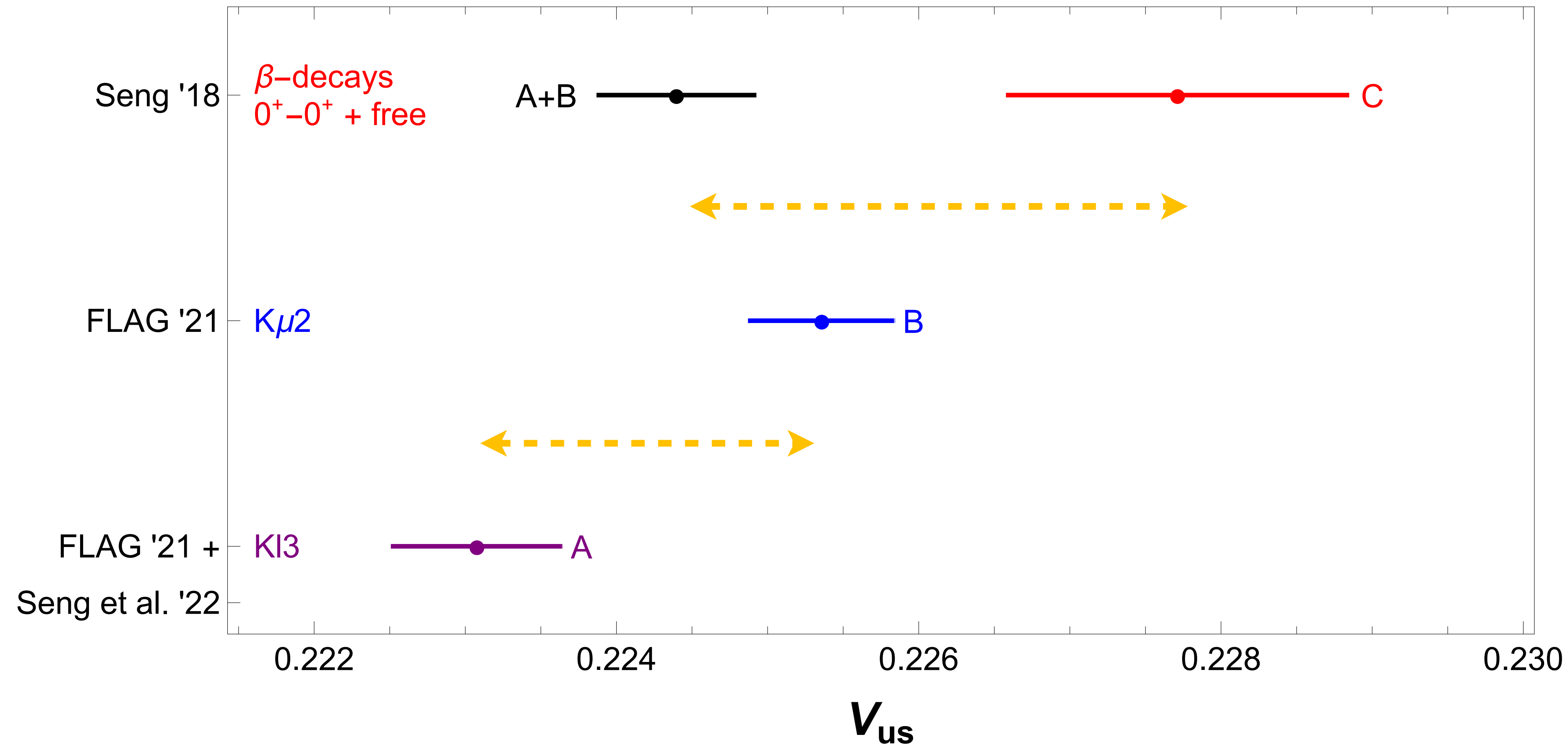
$$\delta_\mu = 7.6 \cdot 10^{-4}$$
$$v_{\mathcal{F}} = 6.3 \text{ TeV}$$



Flavour gauge bosons can be as light as few TeV !

Vector-like weak doublet

- Vector-like $SU(2)$ -doublet: $Q_{L,R} = \begin{pmatrix} T \\ B \end{pmatrix}_{L,R}$



trying to explain both the gaps...

Vector-like weak doublet

- Vector-like $SU(2)$ -doublet: $Q_{L,R} = \begin{pmatrix} T \\ B \end{pmatrix}_{L,R}$ $\mathcal{L}_Y \supset \dots + h_{ui} \bar{Q}_L \tilde{\varphi} u_{Ri} + h_{di} \bar{Q}_L \varphi d_{Ri} + M_Q \bar{Q}_L Q_R + \text{h.c.}$

$$\overline{d_{Li}} \mathbf{m}_{ij}^{(d)} d_{Rj} + \text{h.c.} = \begin{pmatrix} \overline{d_{L1}} & \overline{d_{L2}} & \overline{d_{L3}} & \overline{B_L} \end{pmatrix} \begin{pmatrix} & & 0 \\ & \mathbf{Y}_d v_w & 0 \\ & & 0 \\ h_{d1} v_w & h_{d2} v_w & h_{d3} v_w & M_Q \end{pmatrix} \begin{pmatrix} d_{R1} \\ d_{R2} \\ d_{R3} \\ B_R \end{pmatrix} + \text{h.c.}$$

$$U_{dL}^\dagger \mathbf{m}^{(d)} U_{dR} = \mathbf{m}_{\text{diag}}^{(d)}, \quad U_{uL}^\dagger \mathbf{m}^{(u)} U_{uR} = \mathbf{m}_{\text{diag}}^{(u)}$$

- $U_{d,uL,R}$ are unitary 4×4 matrices: $\begin{pmatrix} d_{R1} \\ d_{R2} \\ d_{R3} \\ B_R \end{pmatrix} = U_{dR} \begin{pmatrix} d_R \\ s_R \\ b_R \\ B'_R \end{pmatrix}, \quad \begin{pmatrix} u_{R1} \\ u_{R2} \\ u_{R3} \\ T_R \end{pmatrix} = U_{uR} \begin{pmatrix} u_R \\ c_R \\ t_R \\ T'_R \end{pmatrix}$

Basis in which the up Yukawa submatrix $\hat{U}_{uL}^\dagger Y_u \hat{U}_{uR}$ is diagonal $(h_u, h_c, h_t) = (h_{u1}, h_{u2}, h_{u3}) \hat{U}_{uR}$

$$U_{dL} = \begin{pmatrix} U_{L1d} & U_{L1s} & U_{L1b} & U_{L1B'} \\ U_{L2d} & U_{L2s} & U_{L2b} & U_{L2B'} \\ U_{L3d} & U_{L3s} & U_{L3b} & U_{L3B'} \\ U_{LBd} & U_{LBs} & U_{LBb} & U_{LBB'} \end{pmatrix} \approx \begin{pmatrix} & & 0 \\ & \hat{U}_{dL} & 0 \\ & & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad U_{uL} \approx \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{y_t h_t^* v_W^2}{M_Q^2} \\ 0 & 0 & -\frac{y_t h_t v_W^2}{M_Q^2} & 1 \end{pmatrix}$$

$$s_{Li}^{u,d} \approx \frac{y_i |h_i| v_w^2}{M_Q^2}$$

Vector-like weak doublet

- Vector-like $SU(2)$ -doublet: $Q_{L,R} = \begin{pmatrix} T \\ B \end{pmatrix}_{L,R}$ $\mathcal{L}_Y \supset \dots + h_{ui} \bar{Q}_L \tilde{\varphi} u_{Ri} + h_{di} \bar{Q}_L \varphi d_{Ri} + M_Q \bar{Q}_L Q_R + \text{h.c.}$

$$\begin{pmatrix} d_{R1} \\ d_{R2} \\ d_{R3} \\ \textcolor{red}{B_R} \end{pmatrix} = U_{dR} \begin{pmatrix} d_R \\ s_R \\ b_R \\ B'_R \end{pmatrix}, \quad \begin{pmatrix} u_{R1} \\ u_{R2} \\ u_{R3} \\ \textcolor{red}{T_R} \end{pmatrix} = U_{uR} \begin{pmatrix} u_R \\ c_R \\ t_R \\ T'_R \end{pmatrix} \quad U_{dL}^\dagger \mathbf{m}^{(d)} U_{dR} = \mathbf{m}_{\text{diag}}^{(d)}, \quad U_{uL}^\dagger \mathbf{m}^{(u)} U_{uR} = \mathbf{m}_{\text{diag}}^{(u)}$$

Basis in which $\hat{U}_{dR}^\dagger Y_d^\dagger Y_d \hat{U}_{dR}$ is diagonal $(h_d, h_s, h_b) = (h_{d1}, h_{d2}, h_{d3}) \hat{U}_{dR}$

$$U_{dR} = \begin{pmatrix} U_{R1d} & U_{R1s} & U_{R1b} & U_{R1B'} \\ U_{R2d} & U_{R2s} & U_{R2b} & U_{R2B'} \\ U_{R3d} & U_{R3s} & U_{R3b} & U_{R3B'} \\ \textcolor{red}{U_{RBd}} & \textcolor{red}{U_{RBs}} & \textcolor{red}{U_{RBb}} & U_{RBB'} \end{pmatrix} \approx \begin{pmatrix} 1 & 0 & 0 & \frac{h_d^* v_w}{M_Q} \\ 0 & 1 & 0 & \frac{h_s^* v_w}{M_Q} \\ 0 & 0 & 1 & \frac{h_b^* v_w}{M_Q} \\ \textcolor{red}{-\frac{h_d v_w}{M_Q}} & \textcolor{red}{-\frac{h_s v_w}{M_Q}} & \textcolor{red}{-\frac{h_b v_w}{M_Q}} & 1 \end{pmatrix} + \mathcal{O}\left(\frac{v_w^2}{M_Q^2}\right) \quad \begin{matrix} U_{RB\alpha} \approx -h_\alpha v_w / M_Q \\ U_{RT\alpha} \approx -h_\alpha v_w / M_Q \end{matrix}$$

- Mass splitting: $\frac{M_{T'}^2}{M_Q^2} = 1 + (h_t^2 + h_c^2 + h_u^2) \frac{v_w^2}{M_Q^2} + \mathcal{O}\left(\frac{v_w^4}{M_Q^4}\right), \quad \frac{M_{B'}^2}{M_Q^2} = 1 + (h_d^2 + h_s^2 + h_b^2) \frac{v_w^2}{M_Q^2} + \mathcal{O}\left(\frac{v_w^4}{M_Q^4}\right)$

Vector-like weak doublet

- The charged-current Lagrangian is:

$$\begin{aligned}
 -\mathcal{L}_{cc} &\supset \frac{g}{\sqrt{2}} \sum_{i=1}^4 (\overline{u_{Li}} \gamma^\mu d_{Li}) W_\mu^+ + \frac{g}{\sqrt{2}} \overline{T_R} \gamma^\mu B_R W_\mu^+ + \text{h.c.} = \\
 &= \frac{g}{\sqrt{2}} \left(\overline{u_L} \quad \overline{c_L} \quad \overline{t_L} \quad \overline{T'_L} \right) \gamma^\mu V_L \begin{pmatrix} d_L \\ s_L \\ b_L \\ B'_L \end{pmatrix} W_\mu^+ + \frac{g}{\sqrt{2}} \left(\overline{u_R} \quad \overline{c_R} \quad \overline{t_R} \quad \overline{T'_R} \right) \gamma^\mu V_R \begin{pmatrix} d_R \\ s_R \\ b_R \\ b'_R \end{pmatrix} W_\mu^+ + \text{h.c.}
 \end{aligned}$$

- $\mathbf{V}_L = \mathbf{U}_{uL}^\dagger \mathbf{U}_{dL}$ is a 4×4 unitary matrix.
- Weak **charged RH currents** with mixing $\mathbf{V}_R$.

$$V_R = U_{uR}^\dagger \text{diag}(0, 0, 0, 1) U_{dR} = \begin{pmatrix} U_{RTu}^* U_{RBd} & U_{RTu}^* U_{RBs} & U_{RTu}^* U_{RBb} & U_{RTu}^* U_{RBB'} \\ U_{RTc}^* U_{RBd} & U_{RTc}^* U_{RBs} & U_{RTc}^* U_{RBb} & U_{RTc}^* U_{RBB'} \\ U_{RTt}^* U_{RBd} & U_{RTt}^* U_{RBs} & U_{RTt}^* U_{RBb} & U_{RTt}^* U_{RBB'} \\ U_{RTT'}^* U_{RBd} & U_{RTT'}^* U_{RBs} & U_{RTT'}^* U_{RBb} & U_{RTT'}^* U_{RBB'} \end{pmatrix}$$

$$U_{RB\alpha}, U_{RT\alpha} \approx -h_\alpha v_w / M_Q$$

Vector-like weak doublet

- Weak interaction of SM quarks with the W -boson loses its pure $V - A$ character

$$\mathcal{L}_{cc} \supset -\frac{g}{2\sqrt{2}} W_{\mu}^{+} \overline{(u \ c \ t)} \left[\gamma^{\mu}(\hat{V}_L + \hat{V}_R) - \gamma^{\mu}\gamma^5(\hat{V}_L - \hat{V}_R) \right] \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

- In this scenario, we are determining vector and axial-vector couplings:

$$\text{semileptonic } K\ell 3 \text{ decay} \quad A : \quad |V_{L\ us} + V_{R\ us}| = 0.22308(55)$$

$$\text{leptonic } K\mu 2 \text{ decay} \quad B : \quad \frac{|V_{L\ us} - V_{R\ us}|}{|V_{L\ ud} - V_{R\ ud}|} = 0.23131(51)$$

$$\beta\text{-decays} \quad C : \quad |V_{L\ ud} + V_{R\ ud}| = 0.97372(26)$$

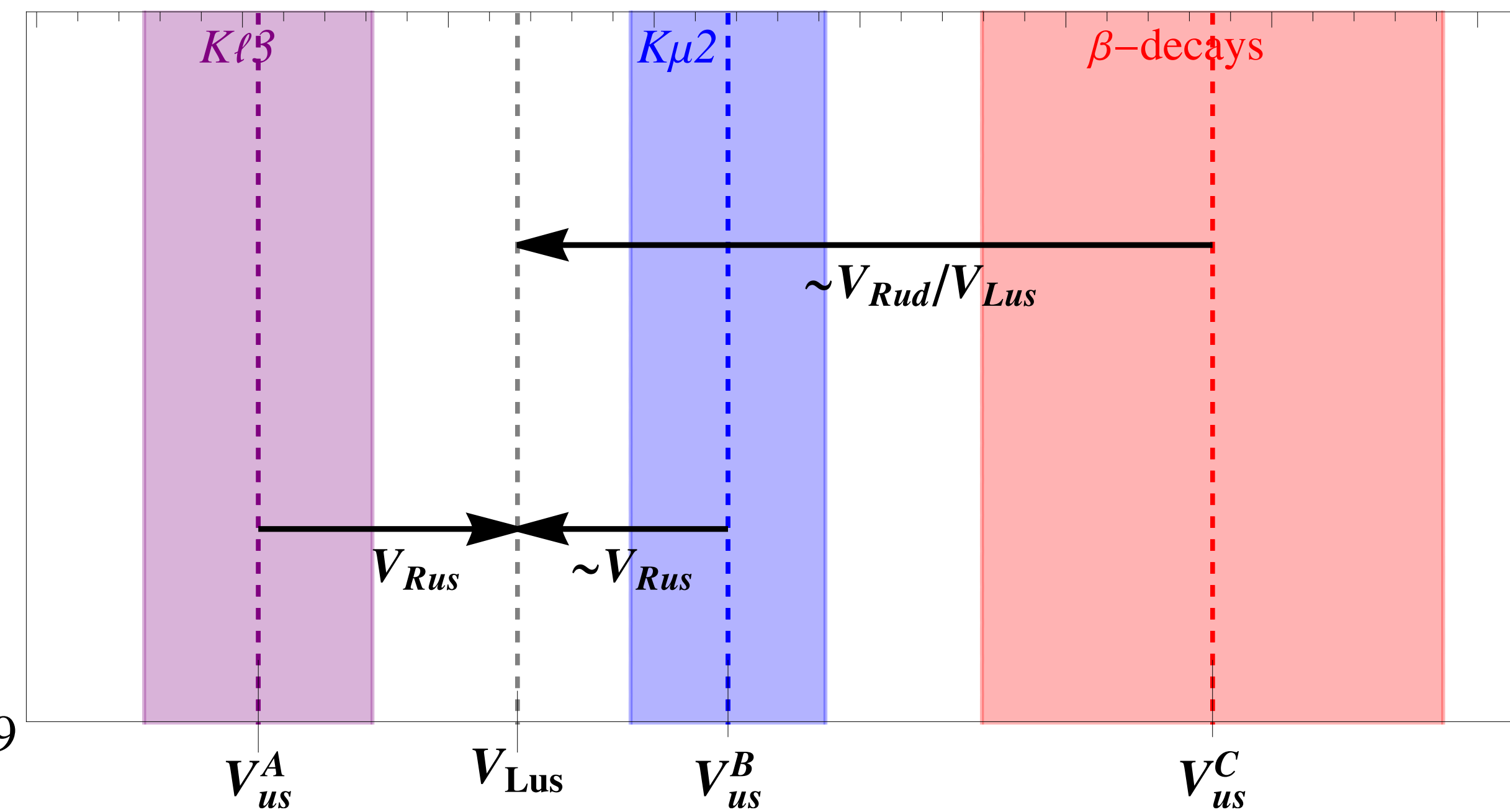
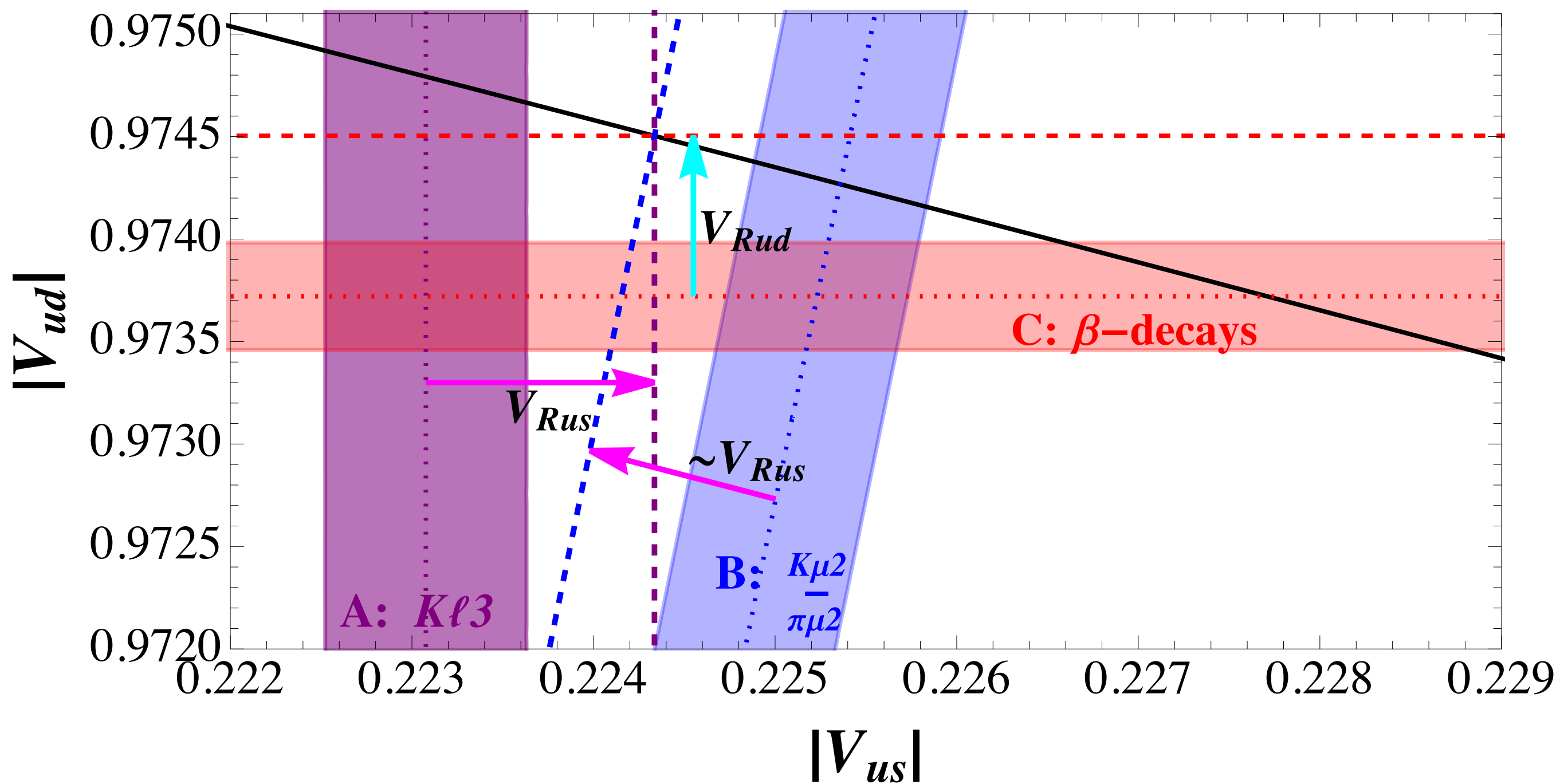
- $V_{Rud} = U_{RTu}^{*} U_{RBd}, \quad V_{Rus} = U_{RTu}^{*} U_{RBs}$
- $|V_{Lud}|^2 + |V_{Lus}|^2 + |V_{Lub}|^2 = 1 - |V_{Lub'}|^2 = 1$

Vector-like weak doublet

- Needed values:

$$V_{Rud} = U_{RTu}^* U_{RBd} = -0.78(27) \times 10^{-3}, \quad V_{Rus} = U_{RTu}^* U_{RBs} = -1.26(38) \times 10^{-3}$$

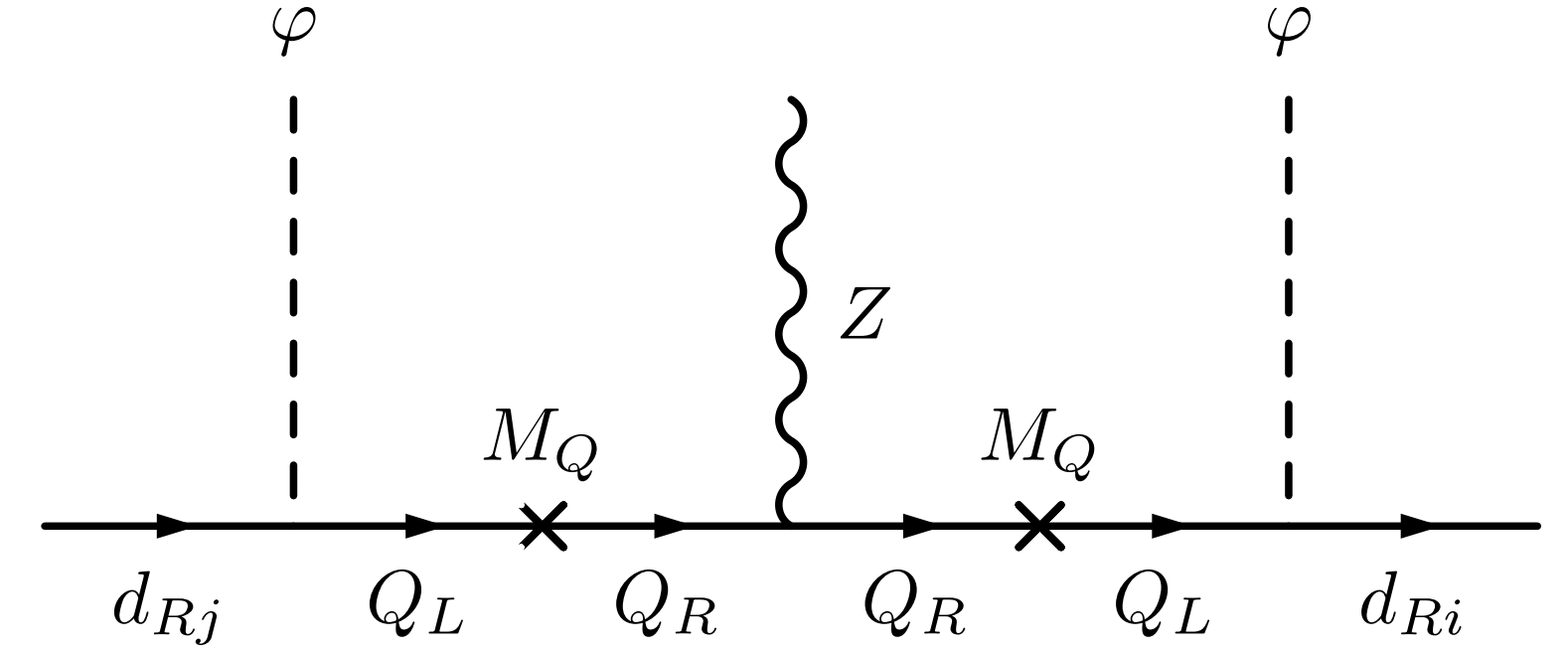
with $V_{Lus} = 0.22434(36)$



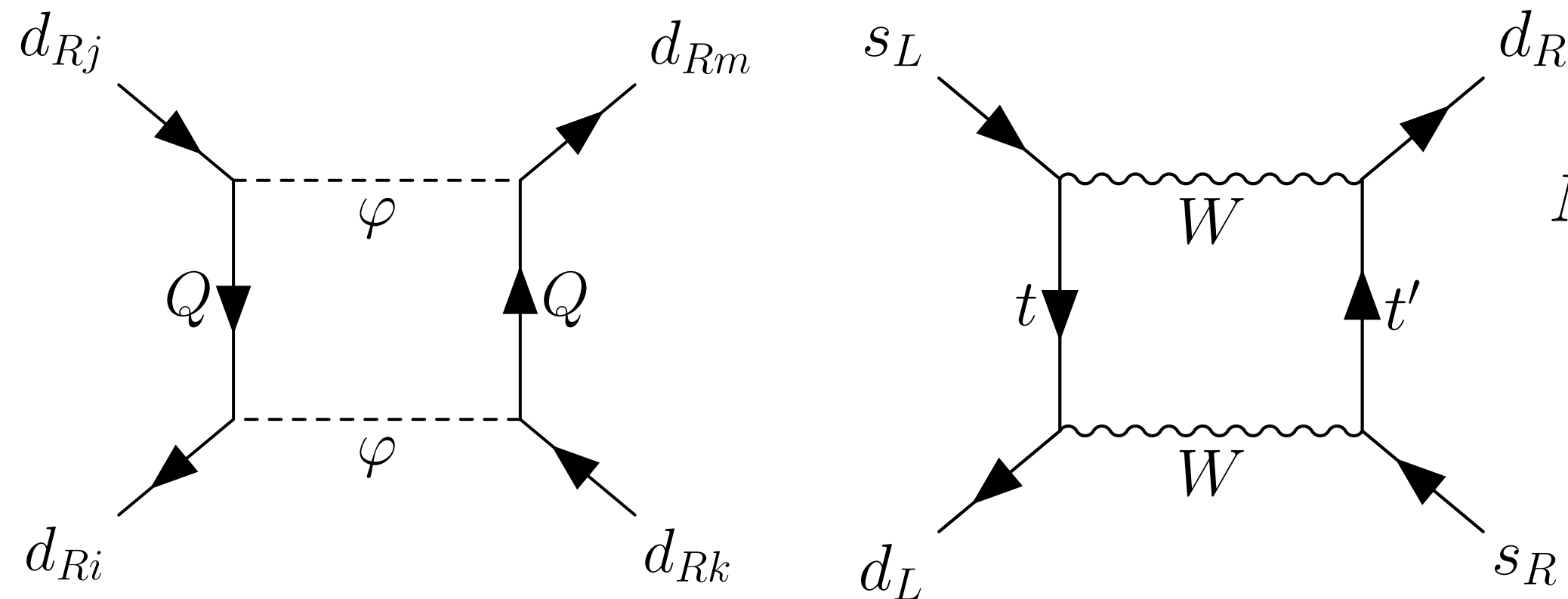
Vector-like weak doublet

- However also in this scenario FCNC at tree (and loop) level and constraints from Z -boson physics and EW observables.

$$\begin{aligned}
 -\mathcal{L}_{\text{fcnc}} = & \frac{1}{2} \frac{g}{\cos \theta_W} Z^\mu \left(\overline{u_R} \overline{c_R} \overline{t_R} \overline{T'_R} \right) \gamma^\mu U_{uR}^\dagger \text{diag}(0, 0, 0, 1) U_{uR} \begin{pmatrix} u_R \\ c_R \\ t_R \\ T'_R \end{pmatrix} + \\
 & - \frac{1}{2} \frac{g}{\cos \theta_W} Z^\mu \left(\overline{d_R} \overline{s_R} \overline{b_R} \overline{B'_R} \right) \gamma^\mu U_{dR}^\dagger \text{diag}(0, 0, 0, 1) U_{dR} \begin{pmatrix} d_R \\ s_R \\ b_R \\ B'_R \end{pmatrix} \\
 \mathcal{L}_H \supset & \frac{1}{\sqrt{2}} H \left(\overline{d_L} \quad \overline{s_L} \quad \overline{b_L} \quad \overline{B'_L} \right) U_{dL}^\dagger \begin{pmatrix} Y_d & 0 \\ h_d & 0 \end{pmatrix} U_{dR} \begin{pmatrix} d_R \\ s_R \\ b_R \\ B'_R \end{pmatrix}
 \end{aligned}$$



and loop level:

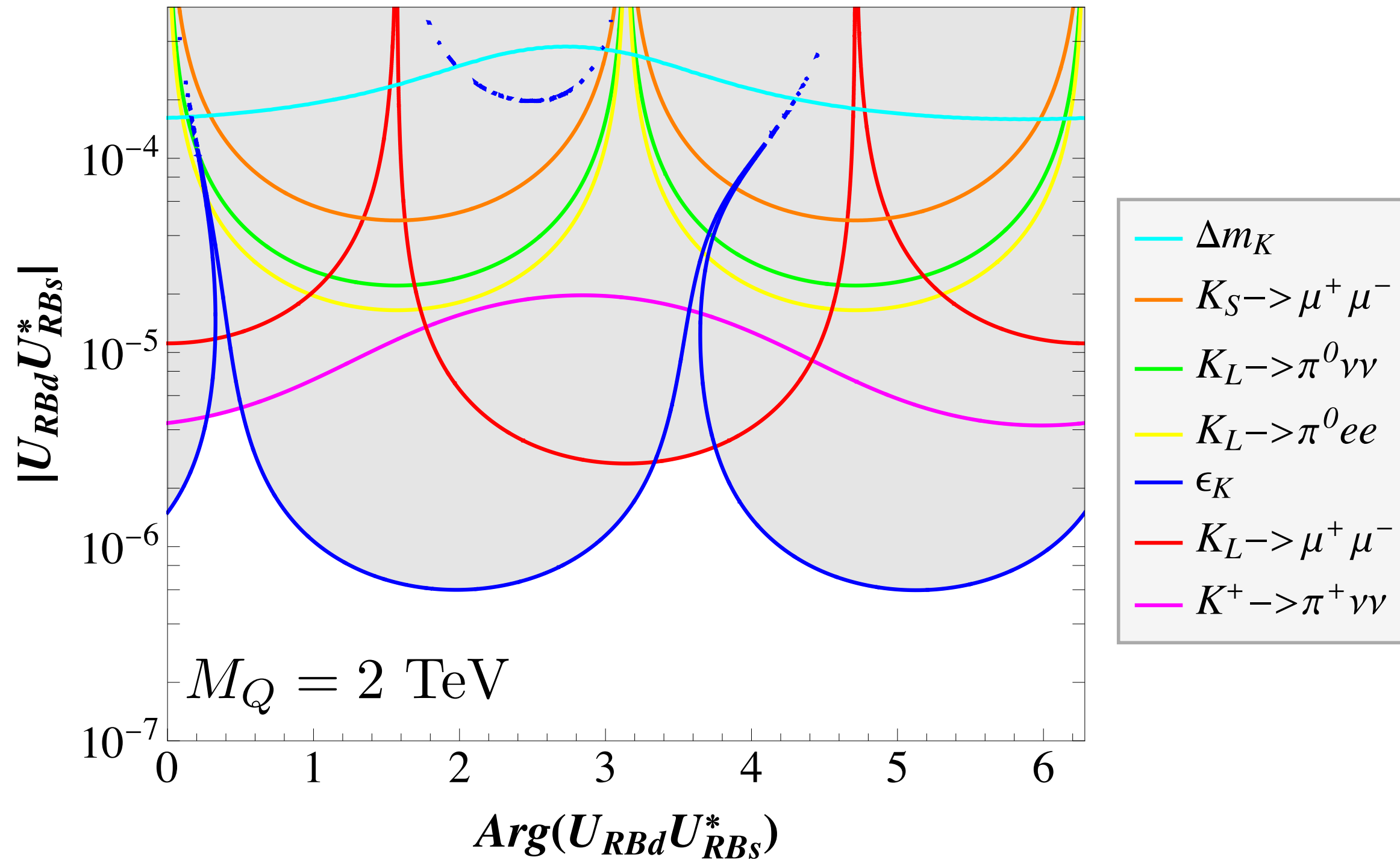


$$\propto (U_{RBd}^* U_{RBs})^2 M_Q^2 \quad (\text{in kaons mixing})$$

$$\begin{aligned}
 M_{12, \text{NP}}^K \approx & \frac{1}{3} m_{K^0} f_K^2 0.43 \left\{ \frac{G_F}{\sqrt{2}} (U_{RBd}^* U_{RBs})^2 + \frac{G_F^2}{4\pi^2} \left[\frac{1}{2} M_Q^2 (U_{RBd}^* U_{RBs})^2 + \right. \right. \\
 & \left. \left. - 3.1 \frac{m_{K^0}^2}{(m_d + m_s)^2} (U_{RBd}^* U_{RBs}) (V_{Ltd}^* V_{Lts}) \frac{m_t^2}{4} \log \left(\frac{M_Q^2}{m_W^2} \right) \right] \right\}
 \end{aligned}$$

Vector-like weak doublet

- Constraints become more stringent for larger values of the mass of the extra doublet. Assuming $M_Q = 2$ TeV:



K mixing and fc decays:

$$|U_{RBs}^* U_{RBd}| \approx |h_s^* h_d| v_w^2 / M_Q^2 \lesssim (0.6 - 5.2) \times 10^{-6}$$

B mixing and fc decays:

$$|U_{RBd}^* U_{RBb}| \approx |h_d^* h_b| v_w^2 / M_Q^2 < (0.4 - 1.7) \times 10^{-4}$$

$$|U_{RBs}^* U_{RBb}| \approx |h_s^* h_b| v_w^2 / M_Q^2 < 8 \times 10^{-7} - 6.4 \times 10^{-4}$$

D mixing:

$$|U_{RTu}^* U_{RTc}| \approx |h_u^* h_c| v_w^2 / M_Q^2 < 1.0 \times 10^{-4}$$

Vector-like weak doublet

Explanation of CAA1 and CAA2:

$$V_{Rus} = U_{RTu}^* U_{RBs} = -1.26(38) \times 10^{-3}$$

$$V_{Rud} = U_{RTu}^* U_{RBd} = -0.78(27) \times 10^{-3}$$

$$|U_{RTu}|^2 U_{RBs}^* U_{RBd} \approx 1.0 \times 10^{-6}$$

Constraints from Z-boson physics and EW observables, e.g.

$$\Gamma(Z) - \Gamma(Z)_{\text{SM}} \approx \frac{G_F M_Z^3}{\sqrt{2}\pi} \left[-\frac{2}{3} \sin^2 \theta_W (|U_{RTu}|^2 + |U_{RTc}|^2) - \frac{1}{3} \sin^2 \theta_W (|U_{RBd}|^2 + |U_{RBs}|^2 + |U_{RBb}|^2) \right]$$

$$\longrightarrow |U_{RTu}|^2 + |U_{RTc}|^2 + \frac{1}{2} (|U_{RBd}|^2 + |U_{RBs}|^2 + |U_{RBb}|^2) \lesssim 5 \times 10^{-3}$$

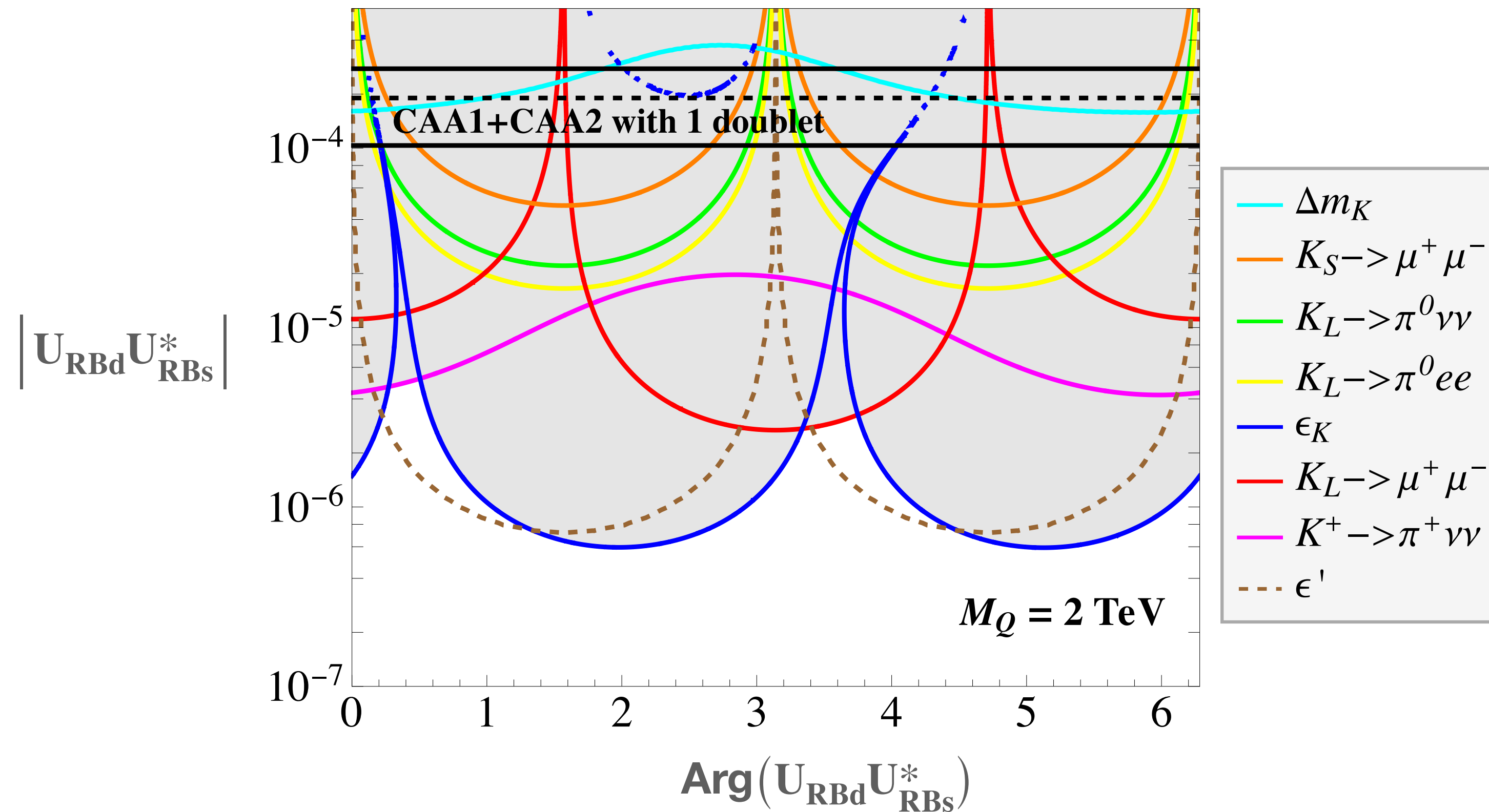
kaon flavor changing processes: $|U_{RBs}^* U_{RBd}| \lesssim 5.2 \times 10^{-6}$

$$|V_{Rud}| = |U_{RTu}^* U_{RBd}| = \frac{|U_{RTu}|^2 |U_{RBs}^* U_{RBd}|}{|U_{RTu}^* U_{RBs}|} \lesssim \frac{5 \times 10^{-3} \times 5 \times 10^{-6}}{10^{-3}} \approx 2.5 \times 10^{-5} \ll 10^{-3}$$

Vector-like weak doublet

- One weak doublet of quarks cannot explain both the Cabibbo angle anomalies (CAA1 and CAA2) alone.

$$V_{Rus} = U_{RTu}^* U_{RBs} = -1.26(38) \times 10^{-3}, \quad V_{Rud} = U_{RTu}^* U_{RBd} = -0.78(27) \times 10^{-3}$$



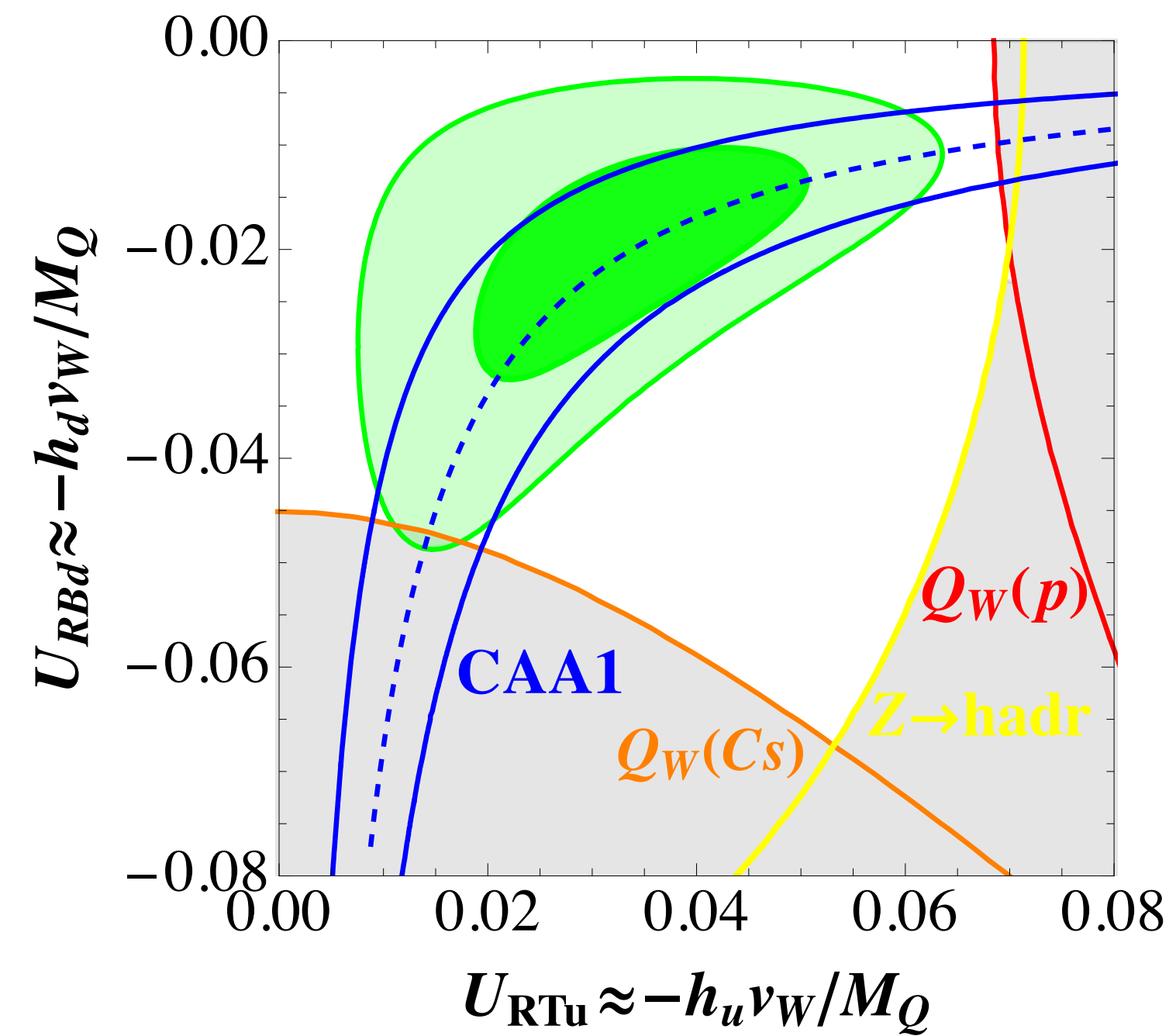
$$U_{RB\alpha}, U_{RT\alpha} \approx -h_\alpha v_w / M_Q$$

CAA: vector-like weak doublet

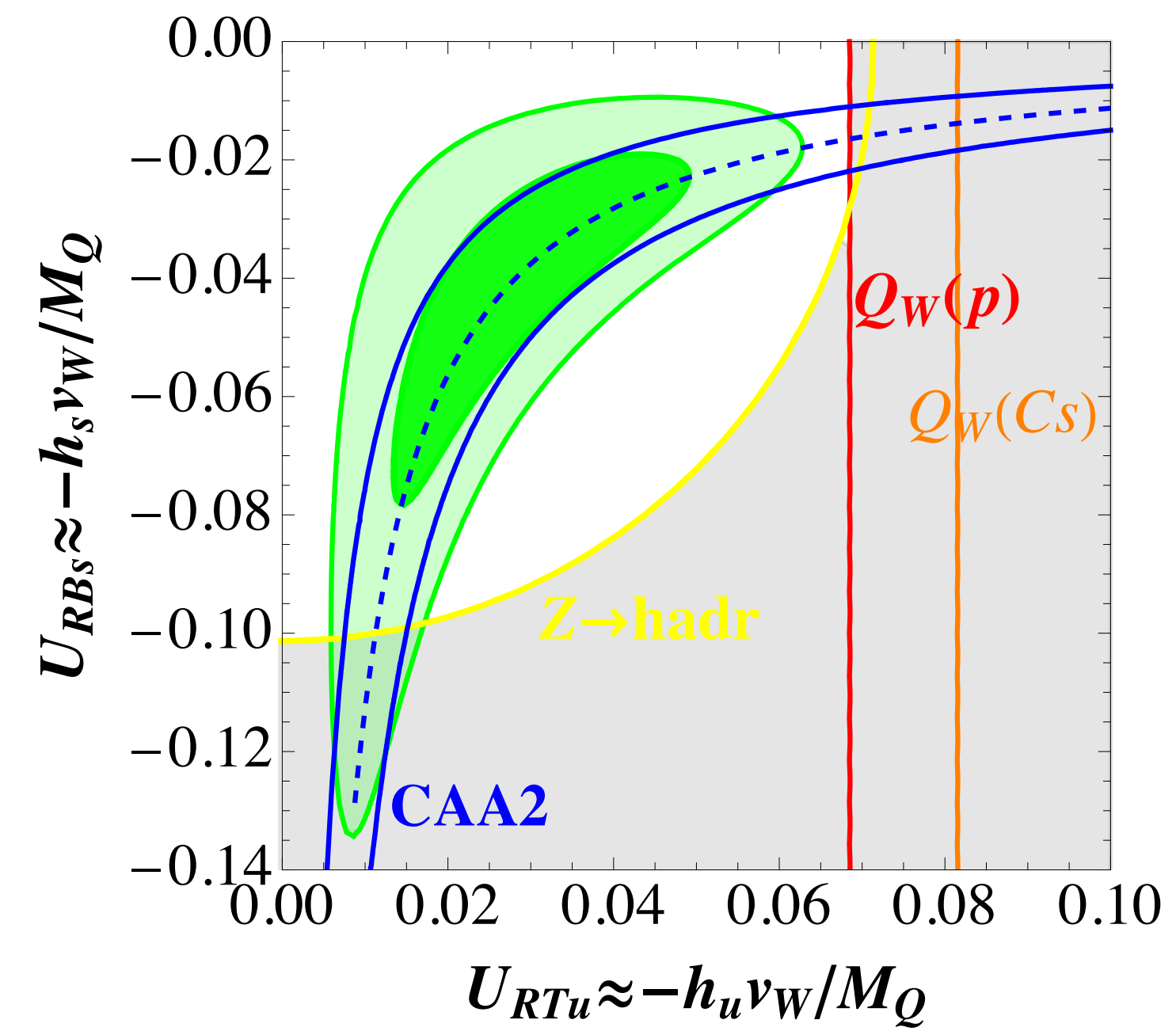
$$V_{Rud} = U_{RTu}^* U_{RBd} = -0.68(27) \times 10^{-3}$$

$$V_{Rus} = U_{RTu}^* U_{RBs} = -1.13(37) \times 10^{-3}$$

1. CAA1 (h_u , h_d)



2. CAA2 (h_u , h_s)



Possible extensions

- Two weak doublets or one isodoublet with isosinglet (up or down type or both as a "complete vector-like family") (or one isodoublet and family symmetries...) can alleviate FC phenomena and explain all discrepancies.

- Two or more doublets:
$$\sum_{i=1}^5 \sum_{j=1}^3 [y_{ij}^u \tilde{\varphi} \overline{q_{Li}} u_{Rj} + y_{ij}^d \varphi \overline{q_{Li}} d_{Rj}] + M_4 \overline{q_{L4}} q_{R4} + M_5 \overline{q_{L5}} q_{R5} + \text{h.c.}$$

$$\begin{pmatrix} \overline{d_{L1}} & \overline{d_{L2}} & \overline{d_{L3}} & \overline{d_{L4}} & \overline{d_{L5}} \end{pmatrix} \begin{pmatrix} & \mathbf{y}_{3 \times 3}^{(d)} v_w & & 0 & 0 \\ & & & 0 & 0 \\ & & & 0 & 0 \\ y_{41}^d v_w & 0 & 0 & M_4 & 0 \\ 0 & y_{52}^d v_w & 0 & 0 & M_5 \end{pmatrix} \begin{pmatrix} d_{R1} \\ d_{R2} \\ d_{R3} \\ d_{R4} \\ d_{R5} \end{pmatrix}$$

$$\begin{pmatrix} \overline{u_{L1}} & \overline{u_{L2}} & \overline{u_{L3}} & \overline{u_{L4}} & \overline{u_{L5}} \end{pmatrix} \begin{pmatrix} & \mathbf{y}_{3 \times 3}^{(u)} v_w & & 0 & 0 \\ & & & 0 & 0 \\ & & & 0 & 0 \\ y_{41}^u v_w & 0 & 0 & M_4 & 0 \\ y_{51}^u v_w & 0 & 0 & 0 & M_5 \end{pmatrix} \begin{pmatrix} u_{R1} \\ u_{R2} \\ u_{R3} \\ u_{R4} \\ u_{R5} \end{pmatrix}$$

$$U_{R5u}^* U_{R5s} = -1.26(38) \times 10^{-3}, \quad U_{R4u}^* U_{R4d} = -0.78(27) \times 10^{-3}$$

- Z decay: $|U_{R4u}|^2 + |U_{R5u}|^2 + 0.50(|U_{R4d}|^2 + |U_{R5s}|^2) \lesssim 5 \times 10^{-3}$

Vector-like doublets of quarks and CP violation

Vector-like quark isodoublets

- We extend the SM by adding N vector-like quarks isodoublets

$$Q_{L\alpha}^0 = \begin{pmatrix} U_{L\alpha}^0 \\ D_{L\alpha}^0 \end{pmatrix}, \quad Q_{R\alpha}^0 = \begin{pmatrix} U_{R\alpha}^0 \\ D_{R\alpha}^0 \end{pmatrix} \quad (\alpha = 1, \dots, N)$$

- Prior to electroweak symmetry breaking

$$\begin{aligned} -\mathcal{L}_Y &= (Y_u)_{ij} \overline{q_{Li}^0} \tilde{\Phi} u_{Rj}^0 + (Y_d)_{ij} \overline{q_{Li}^0} \Phi d_{Rj}^0 + \text{h.c.} & \} & \xrightarrow{\text{red}} \text{SM} \\ &+ (Z_u)_{\alpha j} \overline{Q_{L\alpha}^0} \tilde{\Phi} u_{Rj}^0 + (Z_d)_{\alpha j} \overline{Q_{L\alpha}^0} \Phi d_{Rj}^0 + \text{h.c.} & \} & \xrightarrow{\text{blue}} \text{NP} \\ &+ (\bar{M})_{i\beta} \overline{q_{Li}^0} Q_{R\beta}^0 + (M_Q)_{\alpha\beta} \overline{Q_{L\alpha}^0} Q_{R\beta}^0 + \text{h.c.} \end{aligned}$$

- Collecting all LH SU(2) doublets in a single 3+N-dimensional flavour vector

$$\mathcal{Q}_L^0 = \begin{pmatrix} q_L^0 \\ Q_L^0 \end{pmatrix} \quad -\mathcal{L}_Y = \overline{\mathcal{Q}_L^0} \tilde{\Phi} \mathcal{Y}_u u_R^0 + \overline{\mathcal{Q}_L^0} \Phi \mathcal{Y}_d d_R^0 + \overline{\mathcal{Q}_L^0} M Q_R^0 + \text{h.c.}$$

Vector-like quark isodoublets

- We extend the SM by adding N vector-like quarks isodoublets

$$Q_{L\alpha}^0 = \begin{pmatrix} U_{L\alpha}^0 \\ D_{L\alpha}^0 \end{pmatrix}, \quad Q_{R\alpha}^0 = \begin{pmatrix} U_{R\alpha}^0 \\ D_{R\alpha}^0 \end{pmatrix} \quad (\alpha = 1, \dots, N)$$

- After electroweak symmetry breaking one obtains the tree level mass terms

$$\begin{aligned} -\mathcal{L}_m &= \begin{pmatrix} \overline{u_L^0} & \overline{U_L^0} \end{pmatrix} \begin{pmatrix} v_w Y_u & \bar{M} \\ v_w Z_u & M_Q \end{pmatrix} \begin{pmatrix} u_R^0 \\ U_R^0 \end{pmatrix} + \begin{pmatrix} \overline{d_L^0} & \overline{D_L^0} \end{pmatrix} \begin{pmatrix} v_w Y_d & \bar{M} \\ v_w Z_d & M_Q \end{pmatrix} \begin{pmatrix} d_R^0 \\ D_R^0 \end{pmatrix} + \text{h.c.} \\ &\equiv \overline{\mathcal{U}}_L^0 M_u \mathcal{U}_R^0 + \overline{\mathcal{D}}_L^0 M_d \mathcal{D}_R^0 + \text{h.c.} \end{aligned}$$

- Given that the $3+N$ species of LH doublets have identical quantum numbers, one can consider a basis in field space where $\bar{M}$ vanishes and M_Q is real and diagonal $M_q \rightarrow D_Q$

- Mass basis

$$\mathcal{U}_{L,R}^0 = U_{uL,R} \mathcal{U}_{L,R} = U_{uL,R} \begin{pmatrix} u_{L,R} \\ U_{L,R} \end{pmatrix}, \quad \mathcal{D}_{L,R}^0 = U_{dL,R} \mathcal{D}_{L,R} = U_{dL,R} \begin{pmatrix} d_{L,R} \\ D_{L,R} \end{pmatrix}$$

EW interactions with vector-like quark isodoublets

- Charged current weak interactions

$$\mathcal{L}_W = -\frac{g}{\sqrt{2}} W_\mu^+ \left[\overline{\mathcal{U}}_L^0 \gamma^\mu \mathcal{D}_L^0 + \overline{U}_R^0 \gamma^\mu D_R^0 \right] + \text{h.c.} \quad \} \text{ Flavor basis}$$

$$= -\frac{g}{\sqrt{2}} W_\mu^+ \left[\overline{\mathcal{U}}_L \gamma^\mu V_L \mathcal{D}_L + \overline{U}_R \gamma^\mu V_R D_R \right] + \text{h.c.} \quad \} \text{ Mass basis}$$

- (3+N)-dimensional unitary matrix V_L , and (3+N)-dimensional RH current matrix V_R

$$\boxed{V_L = U_{uL}^\dagger U_{dL}} = \begin{pmatrix} \begin{matrix} & \text{V}_{\text{CKM}} \\ V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \\ \vdots & \vdots & \vdots \end{matrix} & \begin{matrix} \cdots \\ \cdots \\ \cdots \\ \ddots \end{matrix} \end{pmatrix}$$

LH charged current UNITARY mixing matrix

$$\boxed{V_R = U_{uR}^\dagger \text{diag}(0, 0, 0, 1, \dots, 1) U_{dR}}$$

RH charged current mixing matrix

EW interactions with vector-like quark isodoublets

- Neutral current weak interactions

$$\begin{aligned}
 \mathcal{L}_Z &= -\frac{g}{2c_W} Z_\mu \left[\overline{\mathcal{U}}_L^0 \gamma^\mu \mathcal{U}_L^0 - \overline{\mathcal{D}}_L^0 \gamma^\mu \mathcal{D}_L^0 + \overline{U}_R^0 \gamma^\mu U_R^0 - \overline{D}_R^0 \gamma^\mu D_R^0 - 2s_W^2 J_{\text{em}}^\mu \right] & \} \text{ Flavor basis} \\
 &= -\frac{g}{2c_W} Z_\mu \left[\overline{\mathcal{U}}_L \gamma^\mu \mathcal{U}_L - \overline{\mathcal{D}}_L \gamma^\mu \mathcal{D}_L + \overline{\mathcal{U}}_R \gamma^\mu F_u \mathcal{U}_R - \overline{\mathcal{D}}_R \gamma^\mu F_d \mathcal{D}_R - 2s_W^2 J_{\text{em}}^\mu \right] & \} \text{ Mass basis}
 \end{aligned}$$

- (3+N)-dimensional hermitian matrices

$$F_u = V_R V_R^\dagger = U_{uR}^\dagger \text{diag}(0, 0, 0, 1, \dots, 1) U_{uR}$$

$$F_d = V_R^\dagger V_R = U_{dR}^\dagger \text{diag}(0, 0, 0, 1, \dots, 1) U_{dR}$$

- Higgs interactions

$$\begin{aligned}
 \mathcal{L}_h &= -\frac{h}{\sqrt{2}} \left[\overline{\mathcal{U}}_L^0 \begin{pmatrix} Y_u & 0 \\ Z_U & 0 \end{pmatrix} \mathcal{U}_R^0 + \overline{\mathcal{D}}_L^0 \begin{pmatrix} Y_d & 0 \\ Z_d & 0 \end{pmatrix} \mathcal{D}_R^0 \right] + \text{h.c.} \\
 &= -\frac{h}{\sqrt{2}} \left[\overline{\mathcal{U}}_L \frac{\mathcal{D}_u}{v_w} (\mathbf{1} - F_u) \mathcal{U}_R + \overline{\mathcal{D}}_L \frac{\mathcal{D}_d}{v_w} (\mathbf{1} - F_d) \mathcal{D}_R \right] + \text{h.c.}
 \end{aligned}$$

non-negligible couplings between RH light states and LH heavy states, while heavy-light transitions of the opposite chirality are generically suppressed by small SM Yukawas

Weak basis transformations

- After electroweak symmetry breaking one obtains the tree level mass terms

$$\begin{aligned} -\mathcal{L}_m &= \begin{pmatrix} \overline{u_L^0} & \overline{U_L^0} \end{pmatrix} \begin{pmatrix} v_w Y_u & \bar{M} \\ v_w Z_u & M_Q \end{pmatrix} \begin{pmatrix} u_R^0 \\ U_R^0 \end{pmatrix} + \begin{pmatrix} \overline{d_L^0} & \overline{D_L^0} \end{pmatrix} \begin{pmatrix} v_w Y_d & \bar{M} \\ v_w Z_d & M_Q \end{pmatrix} \begin{pmatrix} d_R^0 \\ D_R^0 \end{pmatrix} + \text{h.c.} \\ &\equiv \overline{\mathcal{U}}_L^0 M_u \mathcal{U}_R^0 + \overline{\mathcal{D}}_L^0 M_d \mathcal{D}_R^0 + \text{h.c.} \end{aligned}$$

- Underlying symmetries give choices of **weak basis** and **redundancy of parameters**
- Weak basis are connected by **weak basis transformations** which keep the kinetic terms invariant, i.e. do not modify the form of the EW gauge interactions.
- Any two sets of Yukawa and mass matrices related by a WBT represent the **same physical system**.

$$\begin{aligned} \mathcal{U}_L^0 &\rightarrow \mathcal{W}_L \mathcal{U}_L^0, & u_R^0 &\rightarrow W_R^u u_R^0, & U_R^0 &\rightarrow W_R U_R^0, \\ \mathcal{D}_L^0 &\rightarrow \mathcal{W}_L \mathcal{D}_L^0, & d_R^0 &\rightarrow W_R^d d_R^0, & D_R^0 &\rightarrow W_R D_R^0 \end{aligned}$$

Weak basis transformations

$$\begin{aligned}
 -\mathcal{L}_m &= \begin{pmatrix} \overline{u_L^0} & \overline{U_L^0} \end{pmatrix} \begin{pmatrix} \boxed{v_w Y_u} & \boxed{\bar{M}} \\ v_w Z_u & M_Q \end{pmatrix} \begin{pmatrix} u_R^0 \\ U_R^0 \end{pmatrix} + \begin{pmatrix} \overline{d_L^0} & \overline{D_L^0} \end{pmatrix} \begin{pmatrix} \boxed{v_w Y_d} & \boxed{\bar{M}} \\ v_w Z_d & M_Q \end{pmatrix} \begin{pmatrix} d_R^0 \\ D_R^0 \end{pmatrix} + \text{h.c.} \\
 &\equiv \overline{\mathcal{U}_L^0} M_u \mathcal{U}_R^0 + \overline{\mathcal{D}_L^0} M_d \mathcal{D}_R^0 + \text{h.c.}
 \end{aligned}$$

- weak basis transformations**

$$\begin{aligned}
 \mathcal{U}_L^0 &\rightarrow \mathcal{W}_L \mathcal{U}_L^0, & u_R^0 &\rightarrow W_R^u u_R^0, & U_R^0 &\rightarrow W_R U_R^0, \\
 \mathcal{D}_L^0 &\rightarrow \mathcal{W}_L \mathcal{D}_L^0, & d_R^0 &\rightarrow W_R^d d_R^0, & D_R^0 &\rightarrow W_R D_R^0
 \end{aligned}$$

- Under a WBT the Yukawa and mass matrices are not left invariant, but instead transform as

$$\boxed{\mathcal{Y}_q} \rightarrow \mathcal{W}_L^\dagger \mathcal{Y}_q W_R^q, \quad \boxed{M} \rightarrow \mathcal{W}_L^\dagger M W_R,$$

- CP is violated only when there is no possible choice of CP transformation which leaves $\mathcal{L}$ invariant
- Construct **weak basis invariants**, i.e. **physical quantities** which do not depend on the WB choice

CP violation in the SM

- **CP violation** is related to **CP-odd** weak basis invariants and imaginary part of **rephasing invariants**
- In the **SM**, the presence of CP violation is connected to the non-vanishing of **one CP-odd weak-basis invariant**

$$\text{tr}[M_U M_U^\dagger, M_d M_d^\dagger]^3 = -6i (m_t^2 - m_c^2)(m_t^2 - m_u^2)(m_c^2 - m_u^2)(m_b^2 - m_s^2)(m_b^2 - m_d^2)(m_s^2 - m_d^2) J$$

- The vanishing of the trace would be a necessary and sufficient condition for CP conservation in the SM.
- This trace is the non-real WBI with lowest **mass dimension (=12)** which can appear in the SM, as J is the simplest complex rephasing invariant that can be built.
- **J** is the rephasing invariant

$$J \sum_{\gamma, k} \epsilon_{\alpha\beta\gamma} \epsilon_{ijk} = \text{Im}(V_{\alpha i} V_{\beta j} V_{\alpha j}^* V_{\beta i}^*) = \text{Im} Q_{\alpha i \beta j} = \text{Im}(V_{ud} V_{cs} V_{us}^* V_{cd}^*)$$

- **Quartets** of the form $Q_{\alpha i \beta j}$ involving **four different quarks** are the simplest generically-complex rephasing invariants.
- The non-vanishing of the imaginary parts of the quartets is directly connected to the existence of CP violation in the SM.
- From unitarity of CKM, all nine $\text{Im} Q_{\alpha i \beta j}$ are equal to **J** up to a sign and
- **all CP-violating effects in the SM are proportional to J.**

Vector-like doublets of quarks and CP violation

- In presence of vector-like quark doublets one expects new weak basis invariants.
- To construct **CP-odd weak basis invariants** one has to look into combinations of at least three distinct Hermitian matrices.
- Then, the simplest CP-odd WBI are of the form

$$\text{Im}[\text{tr}[(M_u M_u^\dagger)^n (M_d M_d^\dagger)^k M M^\dagger]] = \sum_{\alpha, i=1}^{3+N} m_\alpha^{2n+1} m_i^{2k+1} \text{Im}(V_{L\alpha i} V_{R\alpha i}^*)$$

- the first CP-odd weak-basis invariant appears at **mass dimension 6** with **three "blocks"** of Hermitian matrices
- Lowest mass dimension CP-odd WBI brings with it the rephasing invariant **bilinears**

$$\mathcal{B}_{\alpha i}^{LR} = V_{L\alpha i} V_{R\alpha i}^*$$

- CP-violating effects involving only **two quarks**.
- Rephasing transformations:

$$\begin{aligned} V_{\alpha i}^L &\rightarrow e^{-i(\varphi_\alpha - \varphi_i)} V_{\alpha i}^L, & V_{\alpha i}^R &\rightarrow e^{-i(\varphi_\alpha - \varphi_i)} V_{\alpha i}^R, \\ F_{\alpha\beta}^u &\rightarrow e^{-i(\varphi_\alpha - \varphi_\beta)} F_{\alpha\beta}^u, & F_{ij}^d &\rightarrow e^{-i(\varphi_i - \varphi_j)} F_{ij}^d, \end{aligned}$$

Vector-like doublets of quarks and CP violation

- The simplest CP-odd WBI are of the form

$$\text{Im}[\text{tr}[(M_u M_u^\dagger)^n (M_d M_d^\dagger)^k M M^\dagger]] = \sum_{\alpha, i=1}^{3+N} m_\alpha^{2n+1} m_i^{2k+1} \text{Im}(V_{L\alpha i} V_{R\alpha i}^*)$$

- We can also add other hermitian blocks

$$\text{Im}\left[\text{tr}\left[(M_d M_d^\dagger)^m (M_u M_u^\dagger)^\ell (M_d M_d^\dagger)^n M M^\dagger\right]\right] = \sum_{\alpha, i=1}^{3+N} m_i^{2m+1} m_\alpha^{2\ell} m_j^{2n+1} \text{Im}(V_{L\alpha i}^* V_{L\alpha j} F_{ji})$$

$$\text{Im}\left[\text{tr}\left[(M_u M_u^\dagger)^m (M_d M_d^\dagger)^\ell (M_u M_u^\dagger)^n M M^\dagger\right]\right] = \sum_{\alpha, i=1}^{3+N} m_\alpha^{2m+1} m_i^{2\ell} m_\beta^{2n+1} \text{Im}(V_{L\alpha i} V_{L\beta i}^* F_{\beta\alpha})$$

- After adding blocks, larger rephasing invariants appear involving more quarks:

bilinears $\mathcal{B}_{\alpha i}^{LR} = V_{L\alpha i} V_{R\alpha i}^*$

trilinears $\mathcal{T}_{\alpha, ij} = V_{L\alpha i}^* V_{L\alpha j} F_{ji}$

$$\mathcal{T}_{i, \alpha\beta} = V_{L\alpha i} V_{L\beta i}^* F_{\beta\alpha}$$

quartets $Q_{\alpha i \beta j}^{LLLR} = V_{L\alpha i} V_{L\beta i}^* V_{L\beta j} V_{R\alpha j}^*$

$$Q_{\alpha i \beta j}^{LLLL} = V_{L\alpha i} V_{L\beta i}^* V_{L\beta j} V_{L\alpha j}^*, \dots$$

Effective CP-odd weak basis invariants (N=1)

- By using weak basis transformations one can always write

$$\mathcal{M}_u = \begin{pmatrix} v_w Y_u & 0 \\ v_w Z_u & M_Q \end{pmatrix} \quad \mathcal{M}_d = \begin{pmatrix} v_w Y_d & 0 \\ v_w Z_d & M_Q \end{pmatrix}$$

- Using

$$\hat{Y}_u = \hat{U}_{uL}^\dagger Y_u \hat{U}_{uR} = \text{diag}(\hat{y}_u, \hat{y}_c, \hat{y}_t), \quad \hat{Y}_d = \hat{U}_{dL}^\dagger Y_d \hat{U}_{dR} = \text{diag}(\hat{y}_d, \hat{y}_s, \hat{y}_b)$$

$$\mathbf{z}_u = Z_u \hat{U}_{uR} = (z_u, z_c, z_t), \quad \mathbf{z}_d = Z_d \hat{U}_{dR} = (z_d, z_s, z_b)$$

- we can write

$$\mathcal{M}_{u,d} = \begin{pmatrix} \hat{U}_{u,dL} & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v_w \hat{Y}_{u,d} & 0 \\ v_w \mathbf{z}_{u,d} & M_Q \end{pmatrix} \begin{pmatrix} \hat{U}_{u,dR}^\dagger & 0 \\ 0 & 1 \end{pmatrix}$$

$$U_{u,dL,R} = \begin{pmatrix} \hat{U}_{u,dL,R} & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \delta U_{u,dL,R} & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} R^{u,dL,R} \end{pmatrix}$$

Effective CP-odd weak basis invariants (N=1)

- Then the mixing matrices in weak currents are

$$V_R = \begin{pmatrix} \boxed{z_u^* z_d \frac{v^2}{M_Q^2}} & z_u^* z_s \frac{v^2}{M_Q^2} & z_u^* z_b \frac{v^2}{M_Q^2} & -z_u^* \frac{v}{M_Q} \\ z_c^* z_d \frac{v^2}{M_Q^2} & \boxed{z_c^* z_s \frac{v^2}{M_Q^2}} & z_c^* z_b \frac{v^2}{M_Q^2} & -z_c^* \frac{v}{M_Q} \\ z_t^* z_d \frac{v^2}{M_Q^2} & z_t^* z_s \frac{v^2}{M_Q^2} & \boxed{z_t^* z_b \frac{v^2}{M_Q^2}} & -z_t^* \frac{v}{M_Q} \\ -z_d \frac{v}{M_Q} & -z_s \frac{v}{M_Q} & -z_b \frac{v}{M_Q} & 1 - \frac{1}{2} (|z_u|^2 + |z_d|^2) \frac{v^2}{M_Q^2} \end{pmatrix} = \hat{V}_R + \mathcal{O}\left(\frac{v^3}{M_Q^3}\right)$$

$$F_u = \begin{pmatrix} \boxed{|z_u|^2 \frac{v^2}{M_Q^2}} & z_u^* z_c \frac{v^2}{M_Q^2} & z_u^* z_t \frac{v^2}{M_Q^2} & -z_u^* \frac{v}{M_Q} \\ z_c^* z_u \frac{v^2}{M_Q^2} & \boxed{|z_c|^2 \frac{v^2}{M_Q^2}} & z_c^* z_t \frac{v^2}{M_Q^2} & -z_c^* \frac{v}{M_Q} \\ z_t^* z_u \frac{v^2}{M_Q^2} & z_t^* z_c \frac{v^2}{M_Q^2} & \boxed{|z_t|^2 \frac{v^2}{M_Q^2}} & -z_t^* \frac{v}{M_Q} \\ -z_u \frac{v}{M_Q} & -z_c \frac{v}{M_Q} & -z_t \frac{v}{M_Q} & 1 - |z_u|^2 \frac{v^2}{M_Q^2} \end{pmatrix} = \hat{F}^u + \mathcal{O}\left(\frac{v^3}{M_Q^3}\right)$$

$$V_L \approx \begin{pmatrix} \boxed{\hat{U}_{uL}^\dagger \hat{U}_{dL}} = \hat{V}_L & \mathcal{O}\left(yz \frac{v^2}{M_Q^2}\right) \\ \hline \mathcal{O}\left(yz \frac{v^2}{M_Q^2}\right) & 1 \end{pmatrix}$$

Effective rephasing invariants

$$\hat{\mathcal{B}}_{\alpha i} \equiv \frac{v^2}{M_Q^2} z_\alpha^* z_i \hat{V}_{\alpha i}^{L*} \equiv \hat{V}_{\alpha i}^R \hat{V}_{\alpha i}^{L*},$$

$$\hat{\mathcal{T}}_{i, \alpha \beta} \equiv \frac{v^2}{M_Q^2} z_\alpha^* z_\beta \hat{V}_{\alpha i}^{L*} \hat{V}_{\beta i}^L \equiv \hat{F}_{\alpha \beta}^u \hat{V}_{\alpha i}^{L*} \hat{V}_{\beta i}^L,$$

$$\hat{\mathcal{T}}_{\alpha, ij} \equiv \frac{v^2}{M_Q^2} z_i^* z_j \hat{V}_{\alpha i}^L \hat{V}_{\alpha j}^{L*} \equiv \hat{F}_{ij}^d \hat{V}_{\alpha i}^L \hat{V}_{\alpha j}^{L*},$$

$$\hat{\mathcal{Q}}_{\alpha i \beta j} \equiv \frac{v^2}{M_Q^2} z_\alpha^* z_i \hat{V}_{\beta j}^L \hat{V}_{\alpha j}^{L*} \hat{V}_{\beta i}^{L*} \equiv \hat{V}_{\alpha i}^R \hat{V}_{\beta j}^L \hat{V}_{\alpha j}^{L*} \hat{V}_{\beta i}^{L*}$$

$$\hat{\mathcal{Q}}_{\alpha i \beta j} \equiv \hat{V}_{\alpha i}^L \hat{V}_{\beta j}^L \hat{V}_{\alpha j}^{L*} \hat{V}_{\beta i}^{L*}$$

Effective CP-odd weak basis invariants (N=1)

- **Effective CP-odd weak basis invariants** for **one** vector-like doublet ($M_Q \gg v_w$)

$$\frac{1}{M_Q^{2m+2n+2}} \text{Im}[\text{tr}[(M_u M_u^\dagger)^n (M_d M_d^\dagger)^k M M^\dagger]] = \frac{v_w^4}{M_Q^4} \sum_{i,\alpha=1}^3 \text{Im} \left(\hat{V}_{L\alpha i} z_\alpha z_i^* \right) y_\alpha y_i (1 + \delta k_{mn\alpha i})$$

$$\text{Im} \left[\text{tr} \left[(M_u M_u^\dagger)^m (M_d M_d^\dagger)^\ell (M_u M_u^\dagger)^n M M^\dagger \right] \right] = \frac{v_w^{4+2m+2\ell}}{M_Q^{4+2m+2\ell}} \sum_{i,j,\alpha,\beta=1}^3 \text{Im} \left(\hat{V}_{L\alpha i} \hat{V}_{L\beta i}^* z_\alpha z_\beta^* \right) y_i^{2\ell} y_\alpha^{2m+1} y_\beta \left(1 + \delta k_{\alpha\beta i}^{nm\ell} \right) + \text{bilinears}$$

$$\text{Im} \left[\text{tr} \left[(M_d M_d^\dagger)^m (M_u M_u^\dagger)^\ell (M_d M_d^\dagger)^n M M^\dagger \right] \right] = \frac{v_w^{4+2m+2\ell}}{M_Q^{4+2m+2\ell}} \sum_{i,j,\alpha,\beta=1}^3 \text{Im} \left(\hat{V}_{L\alpha i} \hat{V}_{L\alpha j}^* z_j z_i^* \right) y_\alpha^{2\ell} y_i^{2m+1} y_j \left(1 + \delta k_{\alpha ij}^{nm\ell} \right) + \text{bilinears}$$

$$\text{Im} \left[\text{tr} \left[(M_u M_u^\dagger)^m (M_d M_d^\dagger)^\ell (M_u M_u^\dagger)^n (M_d M_d^\dagger)^k M M^\dagger \right] \right] \longrightarrow \frac{v_w^2}{M_Q^2} \left(z_\beta z_j^* \hat{V}_{L\alpha i}^* \hat{V}_{L\beta i} \hat{V}_{L\alpha j} \right) = \hat{V}_{R\beta j}^* \hat{V}_{L\alpha i}^* \hat{V}_{L\beta i} \hat{V}_{L\alpha j} + \text{bilinears} + \text{trilinears}$$

- direct connection between weak basis invariants and **“effective” rephasing invariants**

$$\text{Im} \left(\hat{V}_{L\alpha i} z_\alpha z_i^* \frac{v_w^2}{M_Q^2} \right) = \text{Im} \left(\hat{V}_{L\alpha i} \hat{V}_{R\alpha i}^* \right) = \text{Im} \hat{\mathcal{B}}_{\alpha i}^* \approx \text{Im} \left(V_{L\alpha i} V_{R\alpha i}^* \right), \quad \frac{v_w^2}{M_Q^2} h_i^* h_j \hat{V}_{L\alpha i} \hat{V}_{L\alpha j}^* = \hat{F}_{ij}^d \hat{V}_{L\alpha i} \hat{V}_{L\alpha j}^* = \hat{\mathcal{J}}_{\alpha,ij} \approx V_{L\alpha i} V_{L\alpha j}^* F_{ij}^d \dots$$

- **conditions** for CP conservation

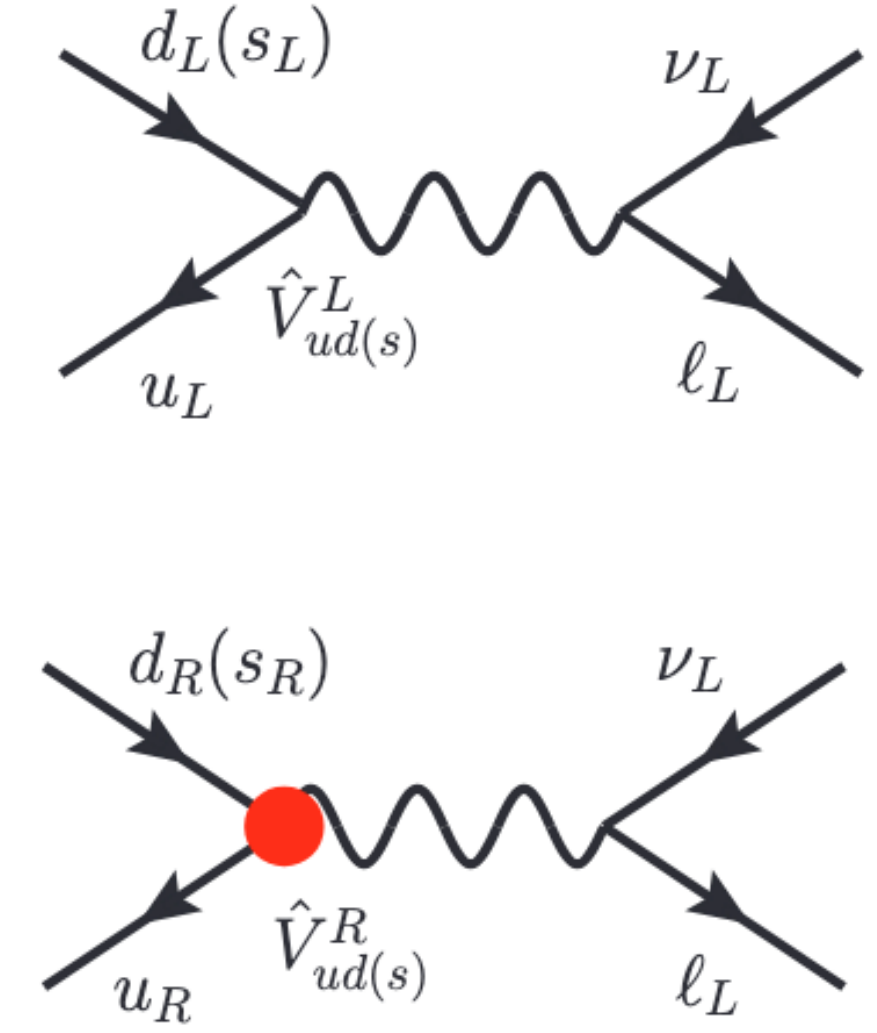
Bilinears and the Cabibbo sector

- Cabibbo angle anomalies

$$|V_{ud}|_{\beta} = |\hat{V}_{ud}^L + \hat{V}_{ud}^R| \simeq |\hat{V}_{ud}^L| + \frac{\text{Re}(\hat{V}_{ud}^L \hat{V}_{ud}^{R*})}{|\hat{V}_{ud}^L|},$$

$$|V_{us}|_{K\ell 3} = |\hat{V}_{us}^L + \hat{V}_{us}^R| \simeq |\hat{V}_{us}^L| + \frac{\text{Re}(\hat{V}_{us}^L \hat{V}_{us}^{R*})}{|\hat{V}_{us}^L|},$$

$$\left| \frac{V_{us}}{V_{ud}} \right|_{\frac{K\mu 2}{\pi\mu 2}} = \frac{|\hat{V}_{us}^L - \hat{V}_{us}^R|}{|\hat{V}_{ud}^L - \hat{V}_{ud}^R|} \simeq \frac{|\hat{V}_{us}^L|}{|\hat{V}_{ud}^L|} \left(1 - \frac{\text{Re}(\hat{V}_{us}^L \hat{V}_{us}^{R*})}{|\hat{V}_{us}^L|^2} + \frac{\text{Re}(\hat{V}_{ud}^L \hat{V}_{ud}^{R*})}{|\hat{V}_{ud}^L|^2} \right)$$

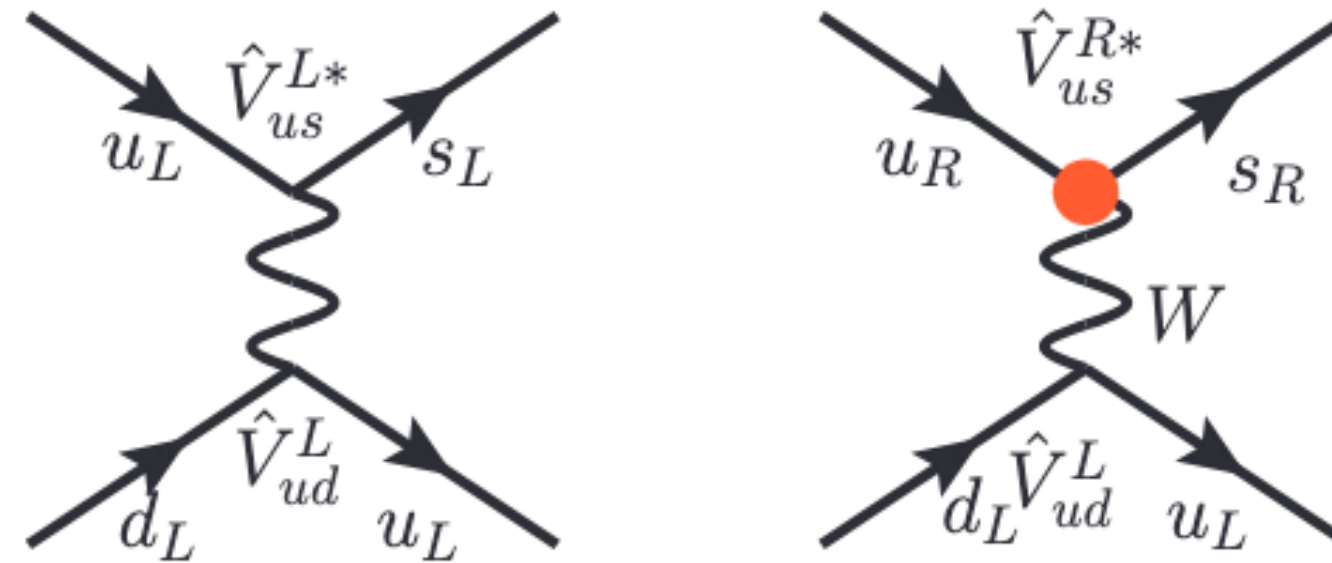


$$|\hat{V}_{ud}^L| = 0.97450(8), \quad \frac{\text{Re}(\hat{\mathcal{B}}_{ud})}{|\hat{V}_{ud}^L|} = \frac{\text{Re}(\hat{V}_{ud}^{L*} \hat{V}_{ud}^R)}{|\hat{V}_{ud}^L|} = -0.78(27) \times 10^{-3},$$

$$|\hat{V}_{us}^L| = 0.22434(36), \quad \frac{\text{Re}(\hat{\mathcal{B}}_{us})}{|\hat{V}_{us}^L|} = \frac{\text{Re}(\hat{V}_{us}^{L*} \hat{V}_{us}^R)}{|\hat{V}_{us}^L|} = -1.26(38) \times 10^{-3}.$$

Bilinears and the Cabibbo sector

- As noted in 2311.00021 (Cirigliano et al.), significant bounds on right-handed currents can come from the theoretical and experimental results on the decay $K \rightarrow \pi\pi$



- useful parameters

$$\omega = \frac{\langle 2 | H_{\Delta S=1} | K_S \rangle}{\langle 0 | H_{\Delta S=1} | K_S \rangle}, \quad \epsilon = \frac{\langle 0 | H_{\Delta S=1} | K_L \rangle}{\langle 0 | H_{\Delta S=1} | K_S \rangle},$$

$$\epsilon' = \frac{\langle 2 | H_{\Delta S=1} | K_L \rangle \langle 0 | H_{\Delta S=1} | K_S \rangle - \langle 0 | H_{\Delta S=1} | K_L \rangle \langle 2 | H_{\Delta S=1} | K_S \rangle}{\sqrt{2} \langle 0 | H_{\Delta S=1} | K_S \rangle^2}$$

- In presence of vector-like weak doublets of quarks the amplitudes of the decays $K \rightarrow \pi\pi$ receive additional contribution:

$$A_I e^{i\delta_I} = A_I^{\text{SM}} e^{i\delta_I} + A_I^{\text{NP}} e^{i\delta_I} \quad \text{where}$$

$$\langle I | H_{\Delta S=1} | K^0 \rangle = A_I e^{i\delta_I} \sqrt{2}, \quad \langle I | H_{\Delta S=1} | \bar{K}^0 \rangle = \bar{A}_I e^{i\delta_I} \sqrt{2}$$

for $I=0$ and $I=2$ and the phases δ_I are the final-state-interaction phase shifts of the two pion states.

Bilinears and the Cabibbo sector

$\Delta = 1/2$ isospin rule:

- In a **rephasing invariant** description we have (with $\lambda_u = \hat{V}_{Lus}^* \hat{V}_{Lud}$): $|A_2^{\text{SM}} + A_2^{\text{NP}}| \approx |A_2^{\text{SM}}| \left(1 + \frac{\text{Re}(\lambda_u^* A_2^{\text{NP}})}{\text{Re}(\lambda_u^* A_2^{\text{SM}})} \right)$

• with

$$\text{Re}(\lambda_u^* A_2^{\text{NP}}) = \text{Re} \left[|\hat{V}_{us}^L|^2 \hat{V}_{ud}^{L*} \hat{V}_{ud}^R - |\hat{V}_{ud}^L|^2 \hat{V}_{us}^L \hat{V}_{us}^{R*} \right] \times 3.76(16) \times 10^{-6} \text{ GeV}$$

• at 2σ we get

$$-1.1 \times 10^{-3} \lesssim |\hat{V}_{us}^L| \frac{\text{Re}(\hat{V}_{ud}^{L*} \hat{V}_{ud}^R)}{|\hat{V}_{ud}^L|} - |\hat{V}_{ud}^L| \frac{\text{Re}(\hat{V}_{us}^L \hat{V}_{us}^{R*})}{|\hat{V}_{us}^L|} \lesssim 1.0 \times 10^{-3}$$

• by inserting the values for Cabibbo angle anomalies we have

$$|\hat{V}_{us}^L| \frac{\text{Re}(\hat{V}_{ud}^{L*} \hat{V}_{ud}^R)}{|\hat{V}_{ud}^L|} - |\hat{V}_{ud}^L| \frac{\text{Re}(\hat{V}_{us}^L \hat{V}_{us}^{R*})}{|\hat{V}_{us}^L|} = 1.05(37) \times 10^{-3}$$

Bilinears and the Cabibbo sector

Direct CP violation

$$\frac{(\epsilon'/\epsilon)_{\text{NP}}}{(\epsilon'/\epsilon)_{\text{SM}}} \approx \frac{(1 + 2\sqrt{2} |\omega|_{\text{exp}}) \text{Im}(A_2^{\text{NP}} \hat{V}_{Lus} \hat{V}_{Lud}^*)}{\text{Im}(A_2^{\text{SM}} \hat{V}_{Lus} \hat{V}_{Lud}^*) - |\omega|_{\text{exp}} \text{Im}(A_0^{\text{SM}} \hat{V}_{Lus} \hat{V}_{Lud}^*)} \approx \frac{\hat{V}_{Lus} \hat{V}_{Lud}^* (\hat{V}_{Lus}^* \hat{V}_{Rud} - \hat{V}_{Rus}^* \hat{V}_{Lud}) \times 1.9(6) \times 10^6}{|\hat{V}_{Lus} \hat{V}_{Lud}^*|}$$

- the imaginary part of that combination of the bilinears implied by CAA should be 3 orders of magnitude smaller than the CP conserving part.

$$\frac{\text{Im} \left[|\hat{V}_{Lus}|^2 \hat{V}_{Lud}^* \hat{V}_{Rud} - |\hat{V}_{Lud}|^2 \hat{V}_{Lus} \hat{V}_{Rus}^* \right]}{\text{Re} \left[|\hat{V}_{Lus}|^2 \hat{V}_{Lud}^* \hat{V}_{Rud} - |\hat{V}_{Lud}|^2 \hat{V}_{Lus} \hat{V}_{Rus}^* \right]} = \frac{\text{Im} \left[|\hat{V}_{Lus}|^2 \hat{\mathcal{B}}_{ud} - |\hat{V}_{Lud}|^2 \hat{\mathcal{B}}_{us} \right]}{\text{Re} \left[|\hat{V}_{Lus}|^2 \hat{\mathcal{B}}_{ud} - |\hat{V}_{Lud}|^2 \hat{\mathcal{B}}_{us} \right]} \approx 10^{-3}, \quad \text{Arg} \left(\frac{0.23 \hat{V}_{Lud} \hat{V}_{Rud}^*}{|\hat{V}_{Lud}|} - \frac{\hat{V}_{Lus} \hat{V}_{Rus}^*}{|\hat{V}_{Lus}|} \right) \sim 10^{-3}$$

- neutron EDM $d_n < 1.8 \times 10^{-13}$ e fm $d_n \approx \text{Im} \left[\hat{V}_{Lud}^* \hat{V}_{Rud} + 1.9 \hat{V}_{Lus}^* \hat{V}_{Rus} \right] \times 1.4 \times 10^{-7}$ e fm

$$\left| (2.3 \pm 1.1) \frac{\text{Im} [\hat{V}_{Lud}^* \hat{V}_{Rud}]}{|\hat{V}_{Lud}|} + (1.0 \pm 0.5) \frac{\text{Im} [\hat{V}_{Lus}^* \hat{V}_{Rus}]}{|\hat{V}_{Lus}|} \right| \lesssim 3 \times 10^{-6}$$

Trilinears and the Cabibbo sector

Kaon mixing

- In the SM we have

$$\epsilon \approx -\frac{e^{i\pi/4}}{\sqrt{2} \Delta m_K} \frac{\text{Im}\left[M_{12}^{\text{SM}} \lambda_u^2\right]}{|\lambda_u|^2} = \frac{e^{i\pi/4} f_K^2 m_K B_K}{\sqrt{2} \Delta m_K} \frac{G_F^2 m_W^2}{12\pi^2} \frac{1}{|\lambda_u|^2} \text{Im}\left[(Q_{csud})^2 S(x_c) + (Q_{tsud})^2 S(x_t) + 2 Q_{tsud} Q_{csud} S(x_c, x_t)\right]$$

- In presence of vector-like quark doublets we have $M_{12} = M_{12}^{\text{SM}} + M_{12}^{\text{NP}}$

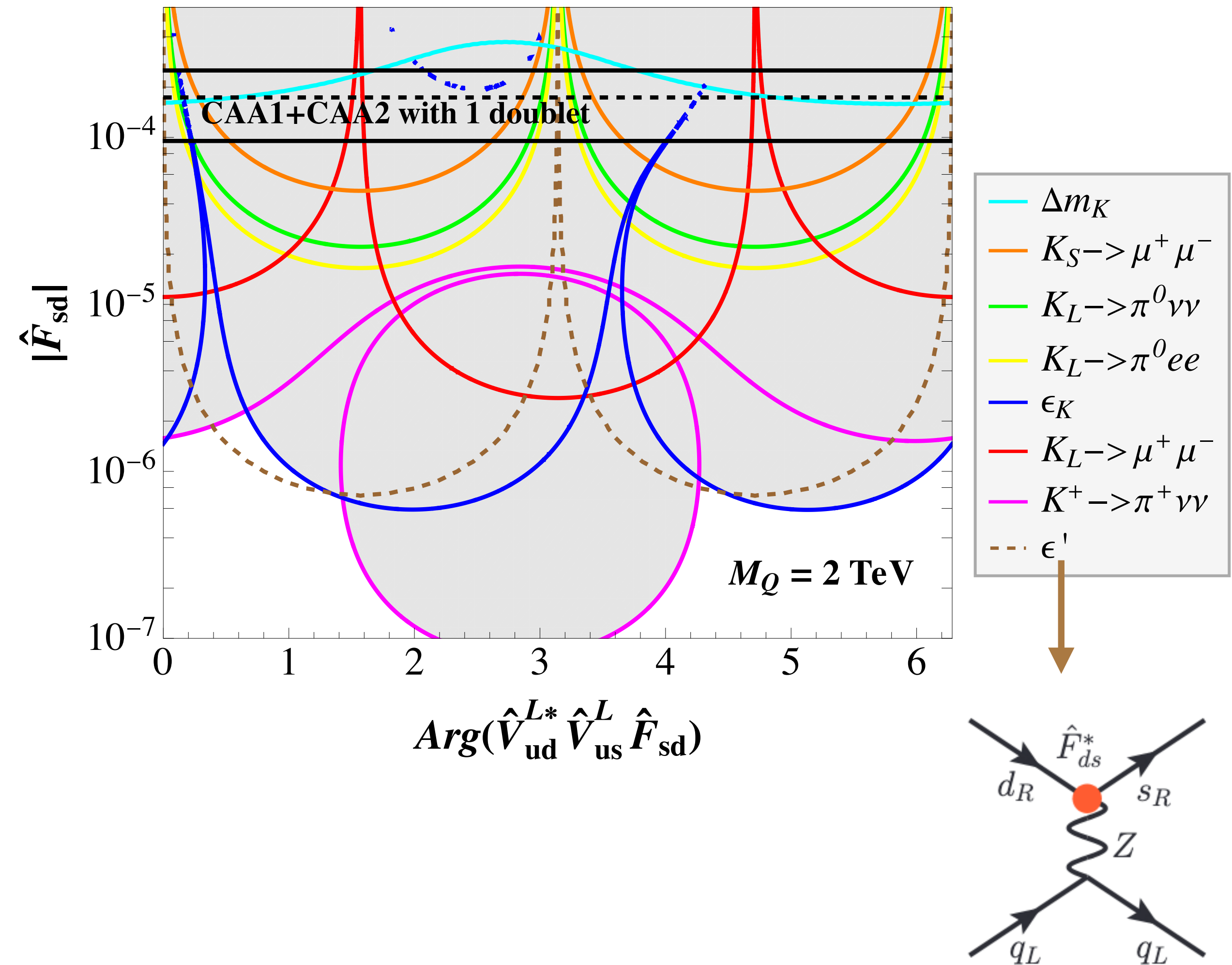
$$\frac{\text{Im}\left[M_{12}^{\text{NP}} (\lambda_u)^2\right]}{|\lambda_u|^2} = \frac{1}{3} m_K f_K^2 0.43 \text{Im}\left\{ \frac{G_F}{\sqrt{2}} \frac{\left(\hat{\mathcal{T}}_{u,ds}\right)^2}{|\lambda_u|^2} + \frac{G_F^2}{4\pi^2} \left[\frac{1}{2} M_Q^2 \frac{\left(\hat{\mathcal{T}}_{u,ds}\right)^2}{|\lambda_u|^2} - 3.1 \frac{m_K^2}{(m_d + m_s)^2} \frac{\hat{\mathcal{T}}_{u,ds} \hat{Q}_{tsud}}{|\lambda_u|^2} m_W^2 f(x_Q, x_t) \right] \right\}$$

Trilinears and the Cabibbo sector

Kaon decays and mixing

- after considering flavour-changing processes together with the constraint from Z decays in the one-doublet case we would have

$$|\hat{V}_{ud}^R| = \frac{\hat{F}_{uu} |\hat{F}_{ds}|}{|\hat{V}_{us}^R|} \lesssim \frac{5 \times 10^{-3} \times 2 \times 10^{-6}}{10^{-3}} \approx 10^{-5} \ll 10^{-3}$$



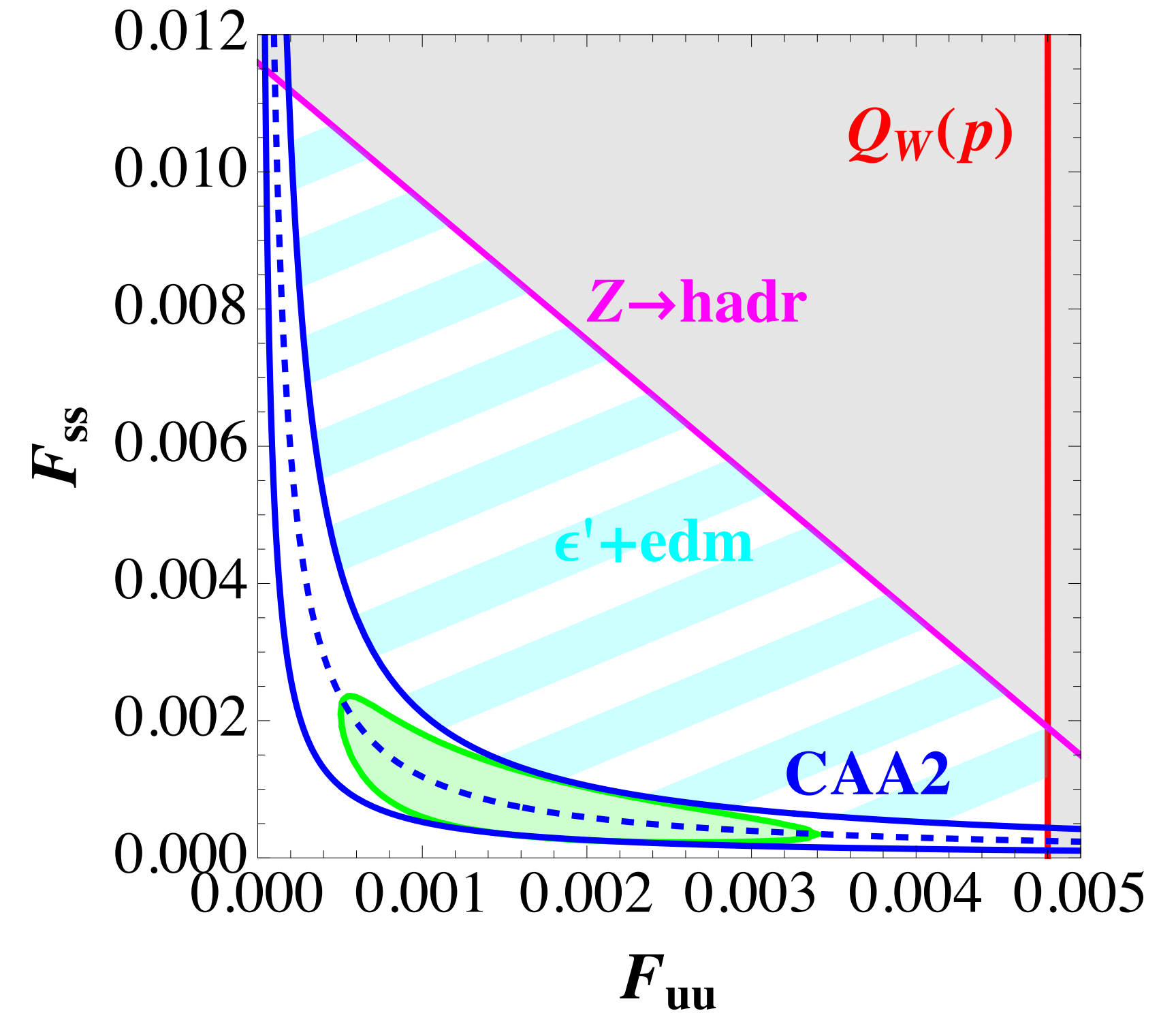
Rephasing invariants and CAA2 (N=1)

$$\hat{V}_{us}^R = \frac{v^2}{M_Q^2} z_u^* z_s, \quad \hat{F}_{ss} = \frac{v^2}{M_Q^2} |z_s|^2 > 0, \quad \hat{F}_{uu} = \frac{v^2}{M_Q^2} |z_u|^2 > 0.$$

$$\frac{\text{Re}(\hat{V}_{us}^{L*} \hat{V}_{us}^R)}{|\hat{V}_{us}^L|} = -1.13(37) \times 10^{-3}, \quad |\hat{V}_{us}^L| = 0.22457(35),$$

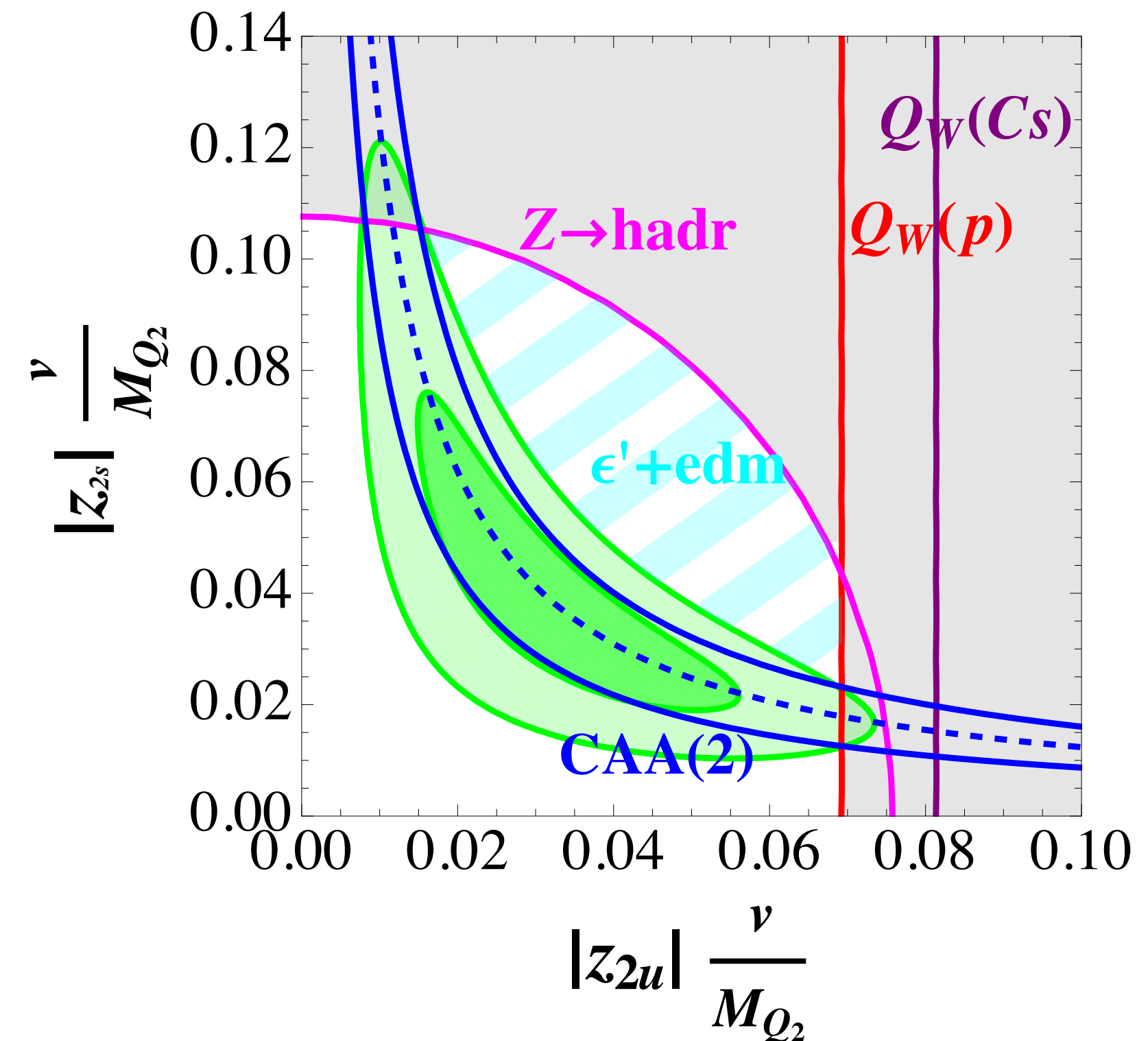
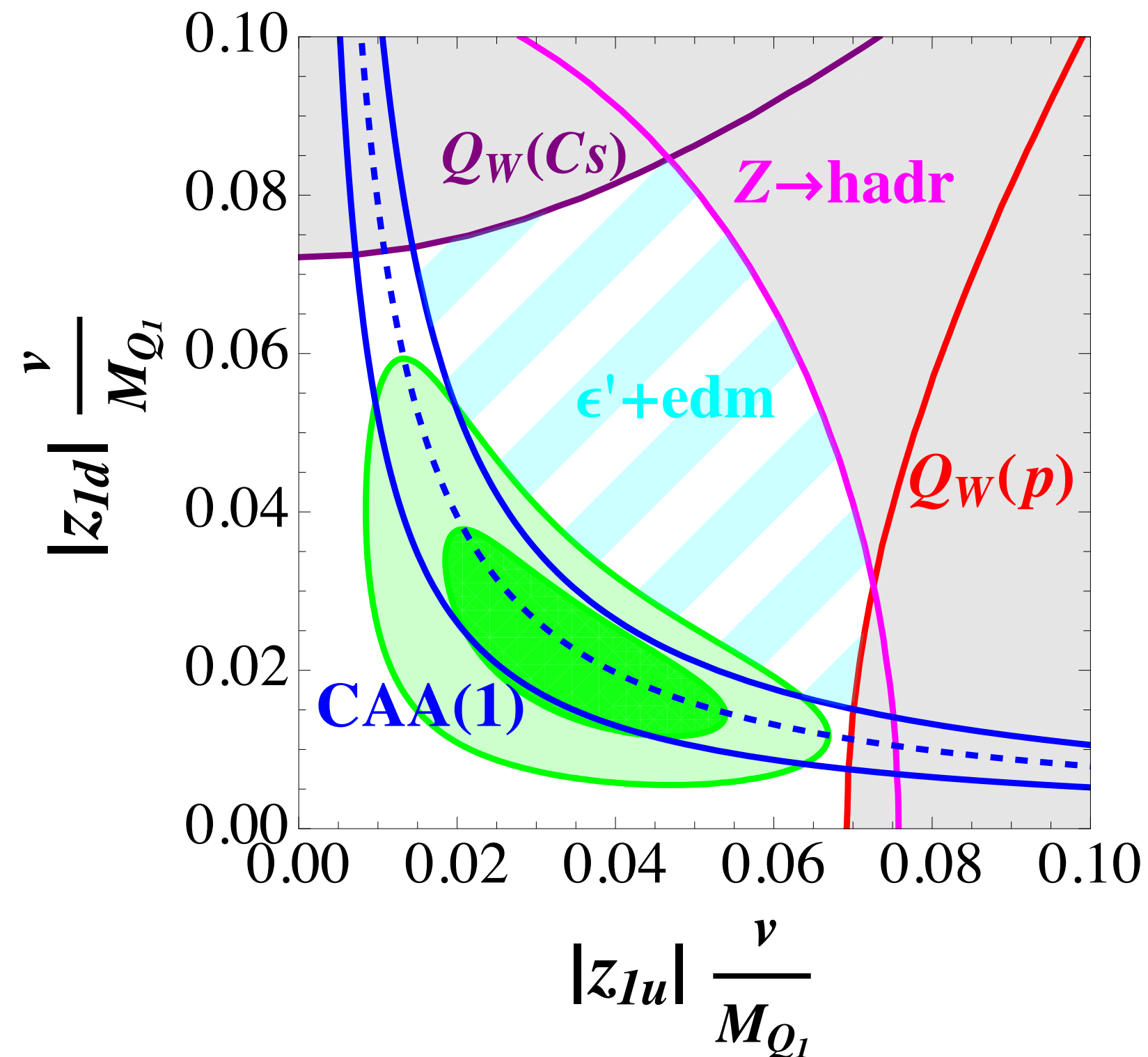
$$\hat{F}_{ss} \hat{F}_{uu} = |\hat{V}_{us}^R|^2 = \frac{[\text{Re}(\hat{\mathcal{B}}_{us})]^2 + [\text{Im}(\hat{\mathcal{B}}_{us})]^2}{|\hat{V}_{us}^L|^2}$$

$$\hat{F}_{ss} \hat{F}_{uu} \geq \frac{(\text{Re}(\hat{\mathcal{B}}_{us}))^2}{|\hat{V}_{us}^L|^2} = (1.13(37) \cdot 10^{-3})^2$$



Rephasing invariants and CAA1+2 (N=2)

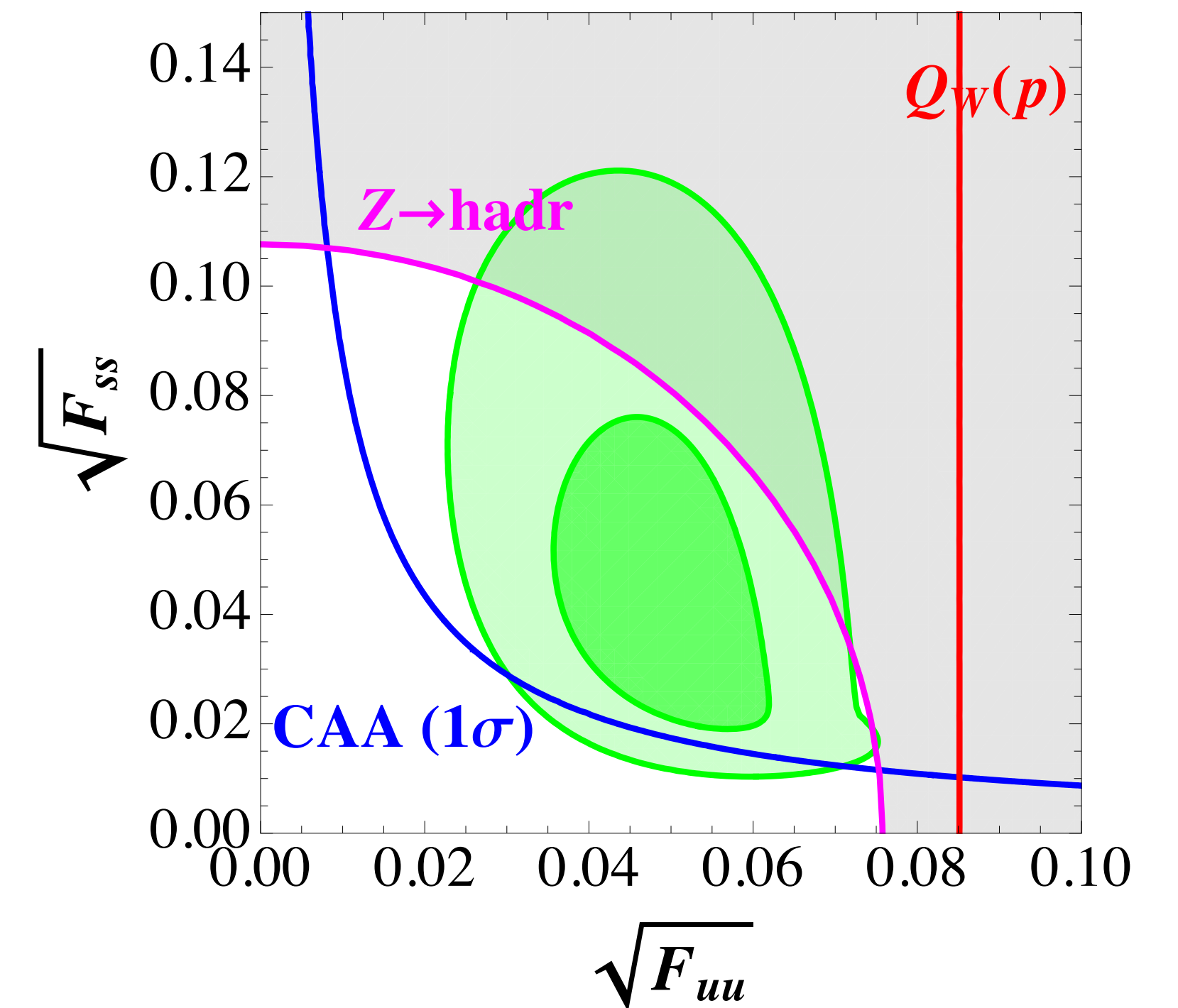
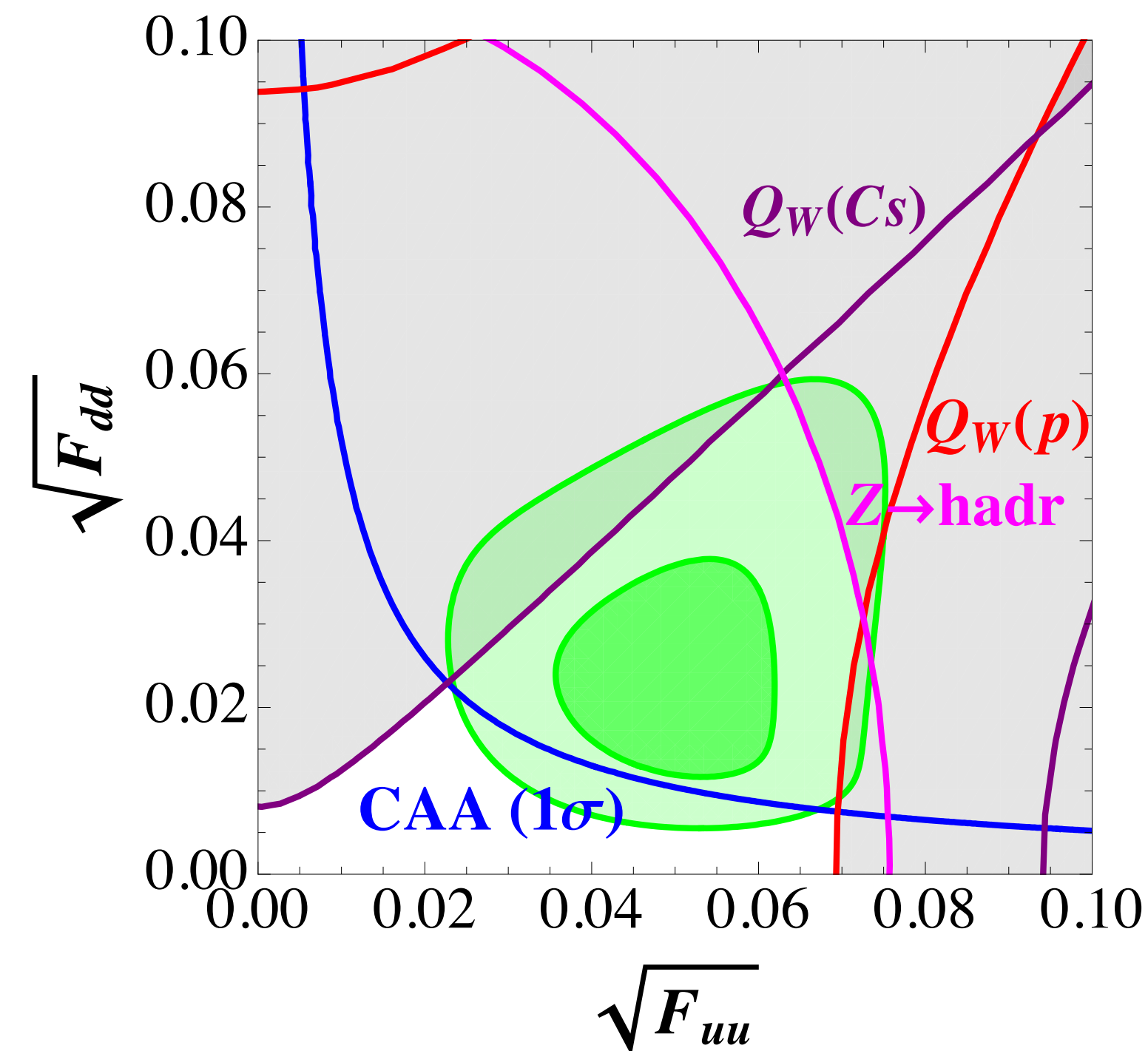
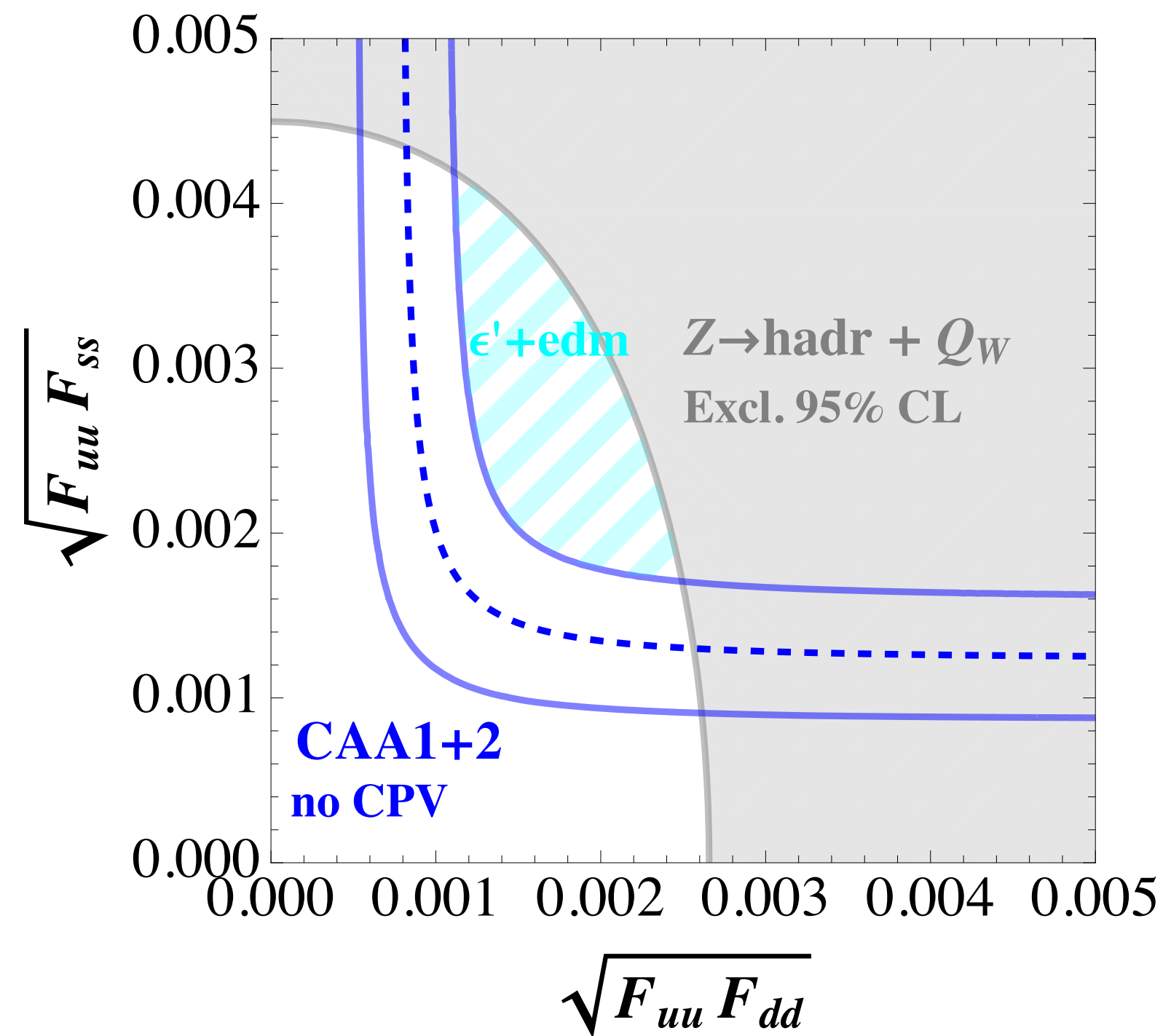
$$\begin{aligned}\hat{V}_{ud}^R &= \frac{v^2}{M_Q^2} z_{1u}^* z_{1d}, & \hat{V}_{us}^R &= \frac{v^2}{a^2 M_Q^2} z_{2u}^* z_{2s}, \\ \hat{F}_{dd} &= \frac{v^2}{M_Q^2} |z_{1d}|^2, & \hat{F}_{ss} &= \frac{v^2}{a^2 M_Q^2} |z_{2s}|^2, & \hat{F}_{uu} &= \frac{v^2}{M_Q^2} \left(|z_{1u}|^2 + \frac{|z_{2u}|^2}{a^2} \right)\end{aligned}$$



Rephasing invariants and CAA1+2 (N=2)

- Predictions in neutral currents

$$1 = \frac{|\hat{V}_{ud}^R|^2}{\hat{F}_{uu} \hat{F}_{dd}} + \frac{|\hat{V}_{us}^R|^2}{\hat{F}_{uu} \hat{F}_{ss}} \geq \frac{(\text{Re } \hat{\mathcal{B}}_{ud})^2}{|\hat{V}_{ud}^L|^2} \frac{1}{\hat{F}_{uu} \hat{F}_{dd}} + \frac{(\text{Re } \hat{\mathcal{B}}_{us})^2}{|\hat{V}_{us}^L|^2} \frac{1}{\hat{F}_{uu} \hat{F}_{ss}} \longrightarrow \hat{F}_{uu} = \frac{(0.79(27) \times 10^{-3})^2}{\hat{F}_{dd}} + \frac{(1.24(37) \times 10^{-3})^2}{\hat{F}_{ss}}$$



Conclusions

- There are **tensions** between independent determinations of the **Cabibbo angle**: unitarity deficit (CAA1) and determination of V_{us} from leptonic and semileptonic kaon decays (CAA2).
- Extra vector-like quarks can be possible explanation for CKM anomalies. A quite large mixing with SM fermions is needed.
- **Vector-like weak doublets of quarks** can be an explanation of both the anomalies.
- Their mass should be no more than few **TeV**, since experimental constraints on flavor changing phenomena become more stringent with larger masses.
- Only one type of extra multiplet cannot entirely explain all the discrepancies, and some their combination is required, e.g. two species of isodoublet, or one isodoublet and up and/or down type isosinglet.
- Constraints from CP violating phenomena imply predictions on neutral currents flavor conserving couplings.
- **Vector-like weak doublets of quarks** can provide an explanation of Cabibbo anomalies without conflict with electroweak precision observables and emerges as a favourite candidate.

Backup

A minimal weak basis

- By using weak basis transformations one can always write

$$\mathcal{M}_u = \begin{pmatrix} v_w Y_u & 0 \\ v_w Z_u & D_Q \end{pmatrix} \quad \mathcal{M}_d = \begin{pmatrix} v_w Y_d & 0 \\ v_w Z_d & D_Q \end{pmatrix}$$

- where the 3×3 block can be diagonalized by a biunitary transformation

$$Y_u = \hat{U}_{uL} \hat{Y}_u \hat{U}_{uR}^\dagger, \quad Y_d = \hat{U}_{dL} \hat{Y}_d \hat{U}_{dR}^\dagger \quad \hat{Y}_u \equiv \text{diag}(\hat{y}_u, \hat{y}_c, \hat{y}_t), \quad \hat{Y}_d \equiv \text{diag}(\hat{y}_d, \hat{y}_s, \hat{y}_b)$$

and we can write

$$\hat{V} = \hat{U}_{uL}^\dagger \hat{U}_{dL}$$

- then we can further use the weak basis transformations

$$\mathcal{M}_u = \begin{pmatrix} v_w \hat{Y}_u & 0 \\ v_w \mathbf{z}_u & D_Q \end{pmatrix} \quad \mathcal{M}_d = \begin{pmatrix} v_w \hat{V} \hat{Y}_d & 0 \\ v_w \mathbf{z}_d & D_Q \end{pmatrix}$$

where $Z_u \rightarrow \mathbf{z}_u = Z_u \hat{U}_{uR}, \quad Z_d \rightarrow \mathbf{z}_d = Z_d \hat{U}_{dR}$

- We can further use weak basis transformations to remove phases

Physical parameters counting

- 5 phases can be removed from $\hat{V}$ by rephasing standard quark fields
- Then, $\hat{V}$ can be parameterized as the standard CKM with three mixing angles and a phase.
- With these choices, $\hat{V}$ becomes the SM CKM in the limit of VLQ doublet decoupling ($D_Q \rightarrow \infty$).
- One can also remove N of the $6N$ phases contained in $\mathbf{z}_u$ and $\mathbf{z}_d$

	$\hat{Y}_u$	$\hat{\mathbf{z}}_u$	$\hat{V}$	$\hat{Y}_d$	$\mathbf{z}_d$	D_Q	Total
Moduli	3	$3N$	3	3	$3N$	N	$9 + 7N$
Phases	0	$2N$	1	0	$3N$	0	$1 + 5N$
Total	3	$5N$	4	3	$6N$	N	$10 + 12N$

Rotating to the mass basis (N=1)

- By using weak basis transformations one can always write

$$\mathcal{M}_u = \begin{pmatrix} v_w Y_u & 0 \\ v_w Z_u & M_Q \end{pmatrix} \quad \mathcal{M}_d = \begin{pmatrix} v_w Y_d & 0 \\ v_w Z_d & M_Q \end{pmatrix}$$

- Diagonalization by bi-unitary transformations

$$U_{uL}^\dagger \mathcal{M}_u U_{uR} = \mathcal{D}_u = \text{diag}(y_u v_w, y_c v_w, y_t v_w, M'_T),$$

$$U_{dL}^\dagger \mathcal{M}_d U_{dR} = \mathcal{D}_d = \text{diag}(y_d v_w, y_s v_w, y_b v_w, M'_B)$$

$$\begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ T \end{pmatrix}_{L,R} = U_{uL,R} \begin{pmatrix} u \\ c \\ t \\ T' \end{pmatrix}_{L,R}, \quad \begin{pmatrix} d_1 \\ d_2 \\ d_3 \\ B \end{pmatrix}_{L,R} = U_{dL,R} \begin{pmatrix} d \\ s \\ b \\ B' \end{pmatrix}_{L,R}$$

- then we can write

$$\mathcal{M}_{u,d} = \begin{pmatrix} \hat{U}_{u,dL} & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v_w \hat{Y}_{u,d} & 0 \\ v_w \mathbf{Z}_{u,d} & M_Q \end{pmatrix} \begin{pmatrix} \hat{U}_{u,dR}^\dagger & 0 \\ 0 & 1 \end{pmatrix}$$

where

$$\hat{Y}_u = \hat{U}_{uL}^\dagger Y_u \hat{U}_{uR} = \text{diag}(\hat{y}_u, \hat{y}_c, \hat{y}_t), \quad \hat{Y}_d = \hat{U}_{dL}^\dagger Y_d \hat{U}_{dR} = \text{diag}(\hat{y}_d, \hat{y}_s, \hat{y}_b)$$

$$\mathbf{Z}_u = Z_u \hat{U}_{uR} = (z_u, z_c, z_t), \quad \mathbf{Z}_d = Z_d \hat{U}_{dR} = (z_d, z_s, z_b)$$

- then

$$U_{u,dL,R} = \begin{pmatrix} \hat{U}_{u,dL,R} & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \delta U_{u,dL,R} & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} R^{u,dL,R} \end{pmatrix}$$

CAA and family symmetry

Vector-like fermions and family symmetry

General Lagrangian includes the mixed Yukawa terms:

$$\mathcal{L}_{\text{Yuk}}^{\text{mix}} = h_U \tilde{\varphi} \bar{q}_L U_R + h_D \varphi \bar{q}_L D_R + h_{\mathcal{U}} \tilde{\varphi} \bar{Q}_L u_R + h_{\mathcal{D}} \varphi \bar{Q}_L d_R + \text{h.c.}$$

and the mass terms

$$\mathcal{L}_{\text{mass}} = M_U \bar{U}_L U_R + M_D \bar{D}_L D_R + M_Q \bar{Q}_L Q_R + \mu_u \bar{U}_L u_R + \mu_d \bar{D}_L d_R + \mu_q \bar{q}_L Q_R + \text{h.c.}$$

- Symmetries may forbid the direct Yukawa terms but allow the mixed ones.
- Mass terms μ and M can be originated from some physical scales.
- SM Yukawa terms for normal fermions will be induced after integrating out the heavy states.

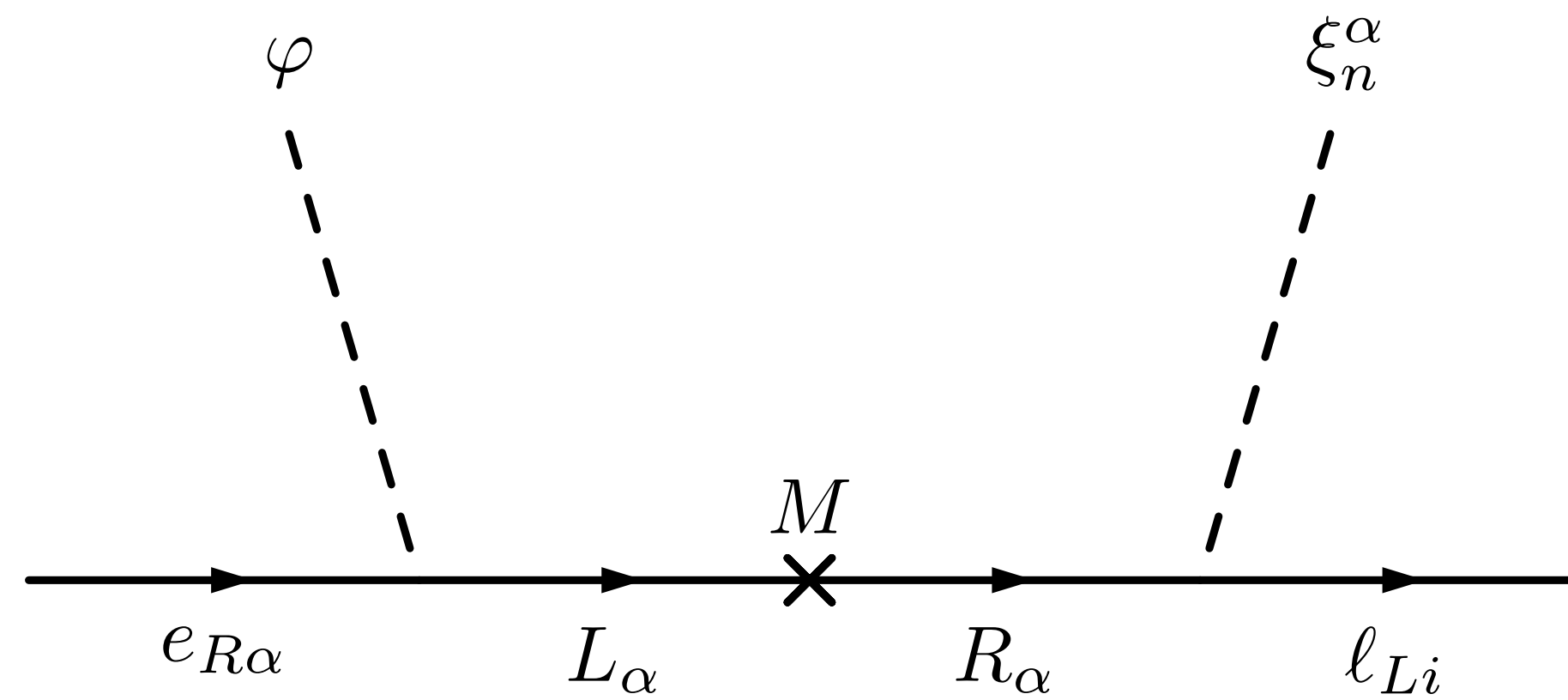
With mixing mass terms $\mu_{u,d,q}$ smaller than $M_{U,D,Q}$:

$$Y_u \simeq h_U M_U^{-1} \mu_u + \mu_q M_Q^{-1} h_{\mathcal{U}}, \quad Y_d \simeq h_D M_D^{-1} \mu_d + \mu_q M_Q^{-1} h_{\mathcal{D}}$$

- Non-zero quark masses are induced via the mixings with the extra vector-like species.
- Predictive model building for fermion masses and mixings.

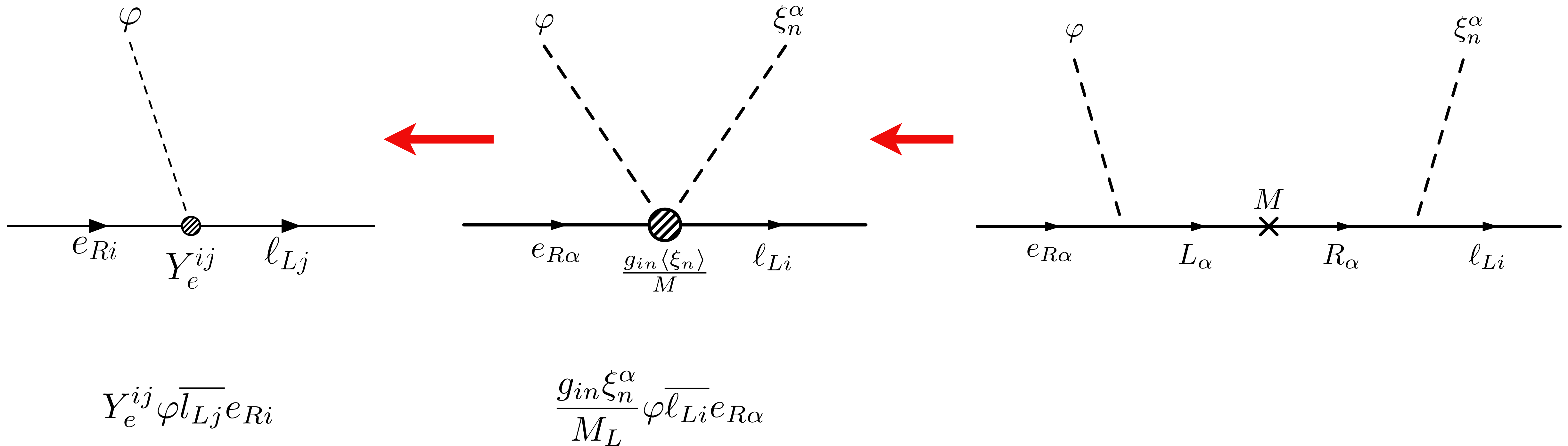
Vector-like fermions and family symmetry

“Universal” seesaw mechanism for fermion masses



Vector-like fermions and family symmetry

“Universal” seesaw mechanism for fermion masses



Family symmetries

- In the limit of vanishing Yukawa couplings, $Y_f \rightarrow 0$, the SM acquires maximal global symmetry:

$$U(3)_\ell \times U(3)_e \times U(3)_q \times U(3)_u \times U(3)_d$$

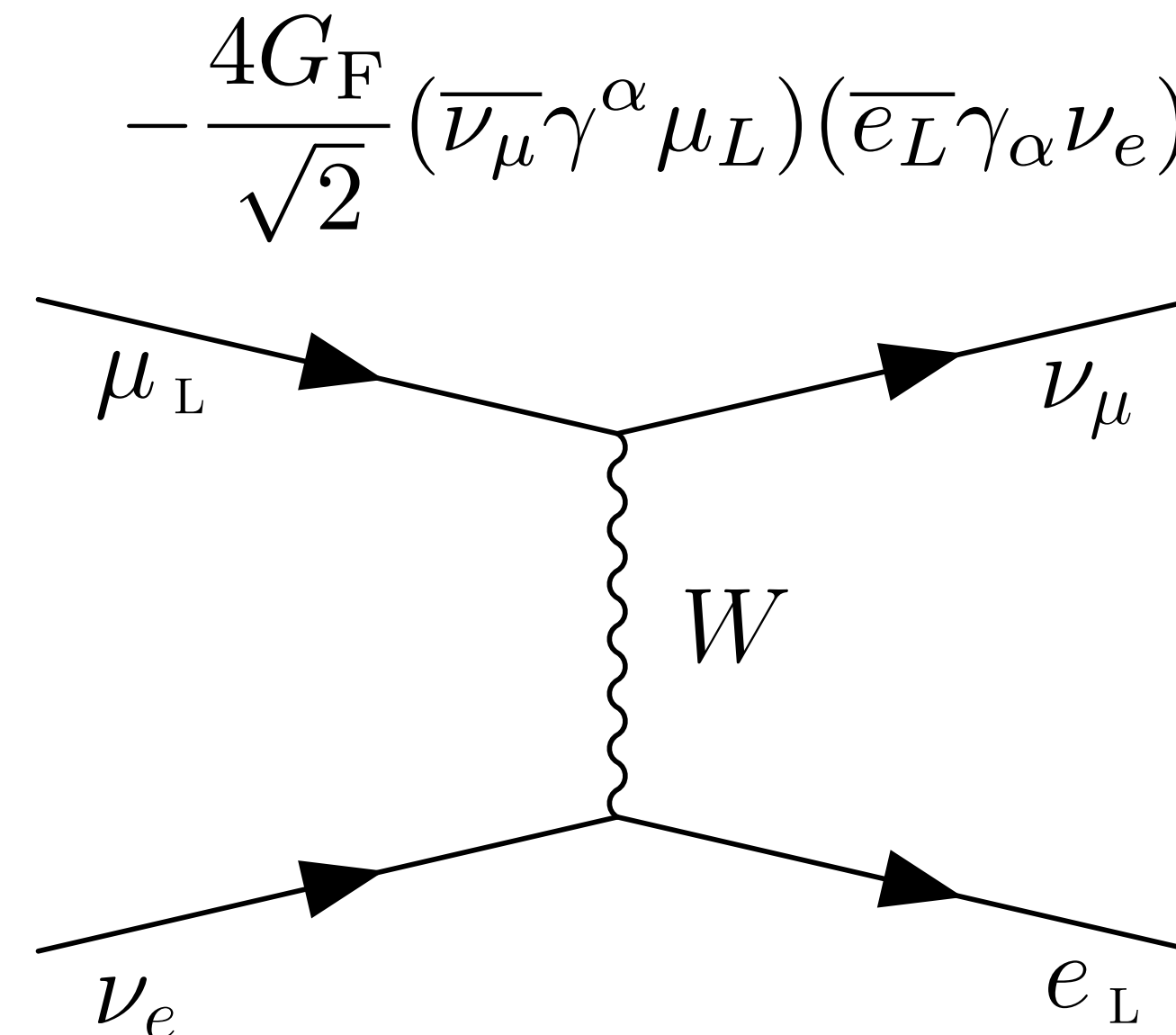
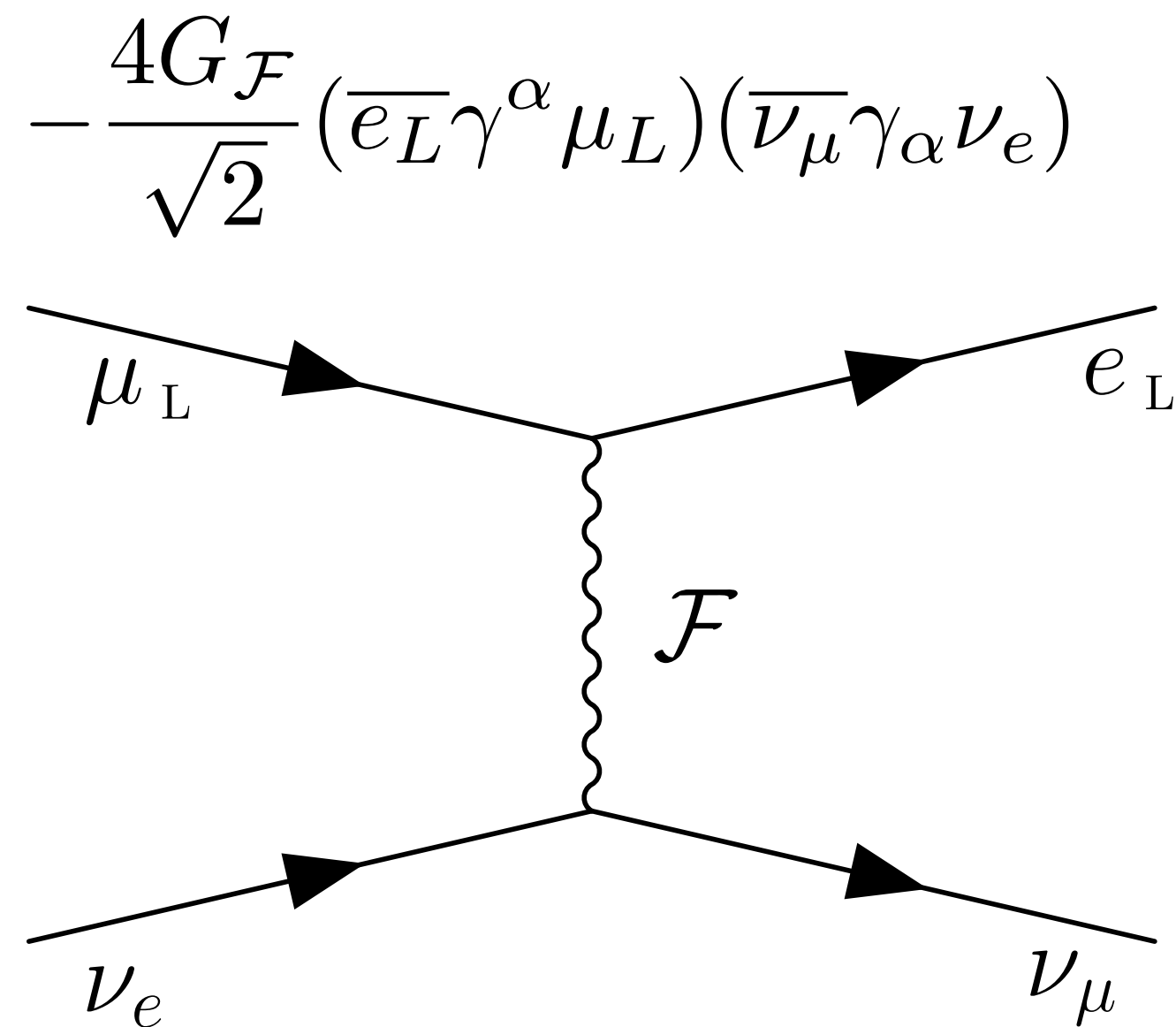
- Fermions transform as triplets, Yukawa interactions break the $SU(3)^5$ symmetry.
- Gauge the symmetry $SU(3)^5$ (the $U(1)$ factors remain as global symmetries).
- Fermions cannot get mass if the symmetry is unbroken: LH and RH particles transform in different representations.
- Yukawa couplings are induced by non-zero VEVs of scalars. The fermion mass hierarchy can be related to the breaking pattern.

$$\frac{y_{ij} \bar{\xi}_j^\gamma \eta_{i\alpha}}{M_L^2} \varphi \overline{\ell_{L\alpha}} e_{Ri} + \frac{h_{ij} \bar{\eta}_i^\alpha \bar{\eta}_j^\beta}{M_\nu^3} \varphi \varphi \ell_{L\alpha}^T C \ell_\beta + \text{h.c.}$$

- Three $SU(3)_e$ triplets $\xi_i \sim 3_e$
- Three $SU(3)_\ell$ triplets $\eta_i \sim 3_\ell$

Flavour bosons for CKM unitarity

- Suppose the existence of **flavor changing bosons**.



- Horizontal interactions have positive interference with SM;
- After Fierz transformation, the sum of the diagrams gives the operator:

$$-\frac{4G_\mu}{\sqrt{2}}(\bar{\nu}_\mu\gamma^\alpha\mu_L)(\bar{e}_L\gamma_\alpha\nu_e)$$

$$G_\mu = G_F + G_{\mathcal{F}} = G_F(1 + \delta_\mu)$$

$$G_\mu \neq G_F$$

Flavour bosons for CKM unitarity

- Different $\mathbf{G}_\mu = \mathbf{G}_F + \mathbf{G}_\mathcal{F} = \mathbf{G}_F(1 + \delta_\mu) = 1 + \frac{v_w^2}{v_\mathcal{F}^2}$
- The values of V_{us} , V_{ud} (and corresponding errorbars) should be rescaled:

$$|V_{us}| = 0.22333(60) \times (1 + \delta_\mu), \quad |V_{ud}| = 0.97370(14) \times (1 + \delta_\mu)$$

while the ratio is not affected.

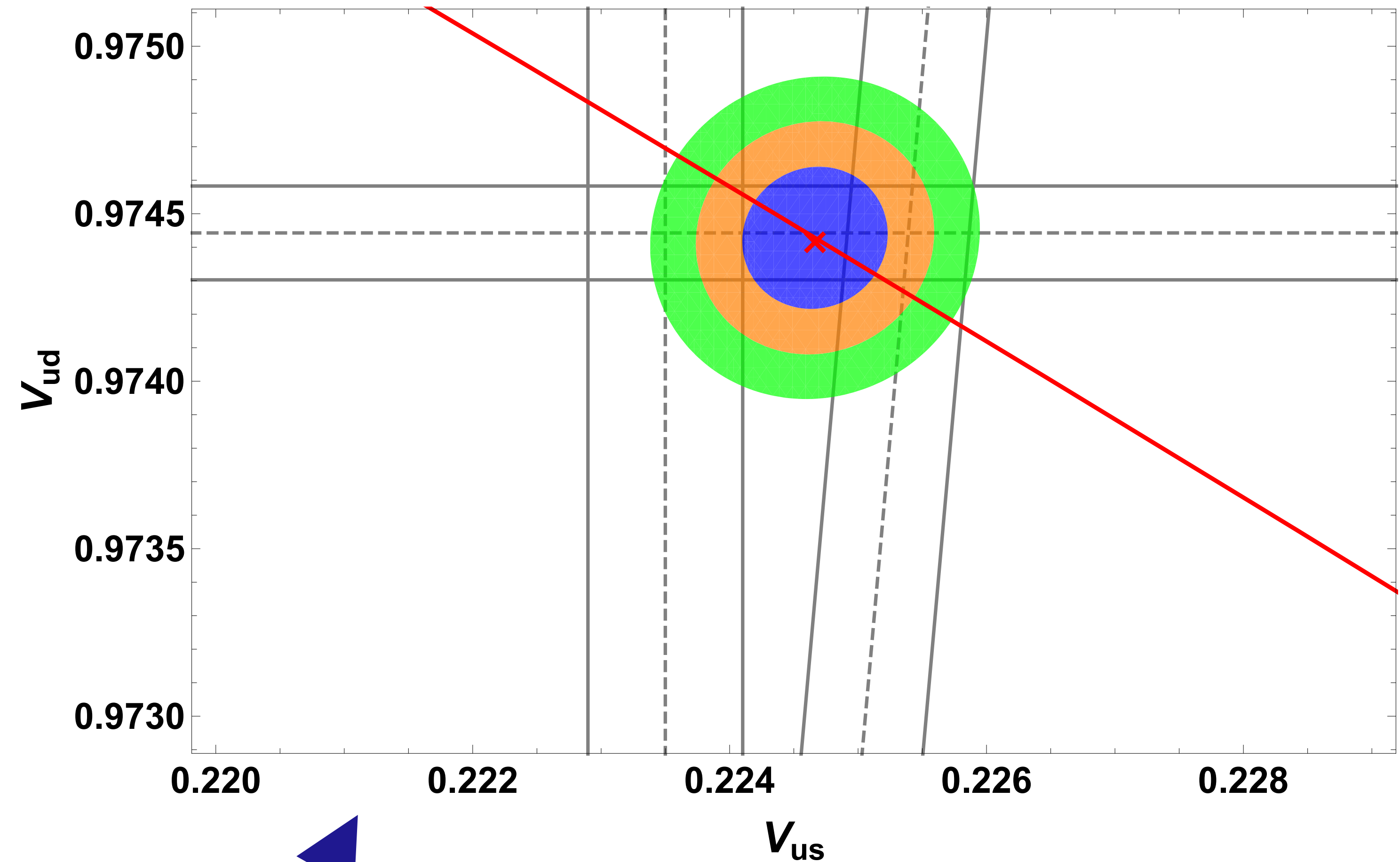
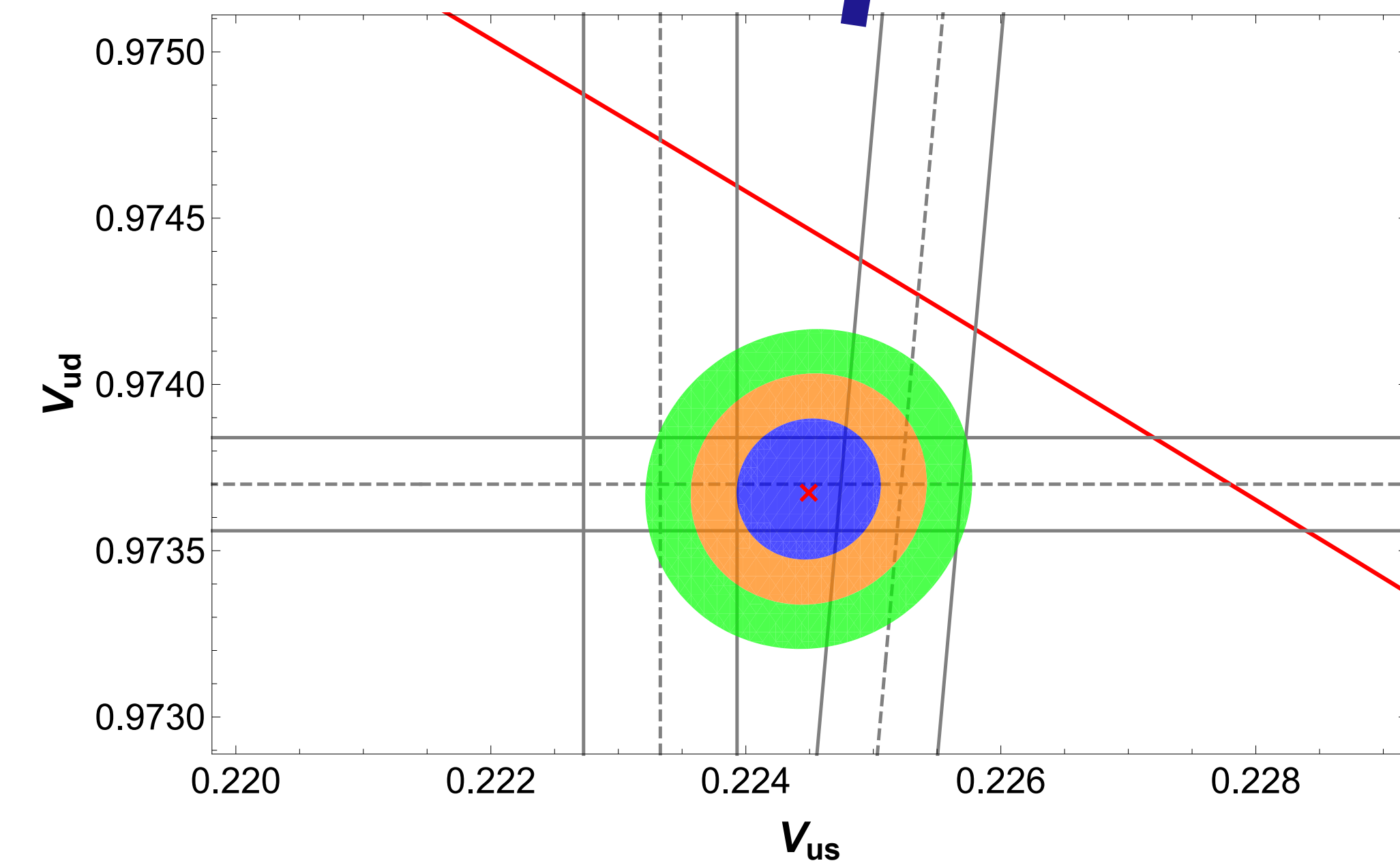
- Unitarity recovered: $\left(\frac{G_F}{G_\mu}\right)^2 (|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2) = 1 - \frac{2G_\mathcal{F}}{G_F}$
- CKM unitarity is explained with $\delta_\mu \sim \mathbf{7.6} \times \mathbf{10^{-4}}$, or

$$v_\mathcal{F} = 6\text{-}7 \text{ TeV}$$

Flavour bosons for CKM

$$\delta_\mu = 7.6 \cdot 10^{-4}$$

$$v_{\mathcal{F}} = 6.3 \text{ TeV}$$



How light flavour can changing gauge bosons be?

FCNC in left-handed sector

$$\mathcal{L}_{\text{eff}}^{ee} = -\frac{G_{\mathcal{F}}}{\sqrt{2}} \sum_{a=1}^8 \left(\overline{\mathbf{e}}_L \gamma_\mu \frac{\lambda_a}{x_a} \mathbf{e}_L \right)^2$$

$$SU(3)_\ell \xrightarrow{w_3} SU(2)_\ell \xrightarrow{w_2} \text{nothing} \quad w_2^2 + w_1^2 = v_{\mathcal{F}}^2 \quad (\delta_\mu = \frac{v_w^2}{v_{\mathcal{F}}^2} \simeq 7 \cdot 10^{-4})$$

- Masses of gauge bosons $M_{\ell 1,2}^2 = \frac{g^2}{2}(w_2^2 + w_1^2) = \frac{g^2}{2}v_{\mathcal{F}}^2$.
- If $w_3 = w_2 = w_1$ (e. g. symmetry between η s), gauge bosons have equal masses, $\lambda_a \rightarrow V^\dagger \lambda_a V$ is simply a basis redetermination of the Gell-Mann matrices. From Fierz identities for λ matrices:

$$\mathcal{L}_{eff} = -\frac{1}{4v_\ell^2} (\overline{\mathbf{e}}_L \lambda^a \gamma^\mu \mathbf{e}_L) (\overline{\mathbf{e}}_L \lambda^a \gamma_\mu \mathbf{e}_L) = -\frac{1}{3v_\ell^2} (\overline{\mathbf{e}}_L \mathbb{I} \gamma_\mu \mathbf{e}_L)^2$$

No FCNC, the global $SO(8)_\ell$ symmetry acts as a custodial symmetry.

- In the general case, FC ($\mu \rightarrow 3e$, $\tau \rightarrow 3\mu, \dots$) under control.
- $v_{\mathcal{F}} \simeq \mathbf{6 \ TeV}$ is not contradicting experimental constraints.

Gauge bosons in left-handed sector, $SU(3)_\ell$

$$SU(3)_\ell \xrightarrow{w_3} SU(2)_\ell \xrightarrow{w_2} \text{nothing}$$

$$\frac{1}{2} \begin{pmatrix} \mathcal{F}_3 + \frac{1}{\sqrt{3}}\mathcal{F}_8 & \mathcal{F}_1 - i\mathcal{F}_2 & \mathcal{F}_4 - i\mathcal{F}_5 \\ \mathcal{F}_1 + i\mathcal{F}_2 & -\mathcal{F}_3 + \frac{1}{\sqrt{3}}\mathcal{F}_8 & \mathcal{F}_6 - i\mathcal{F}_7 \\ \mathcal{F}_4 + i\mathcal{F}_5 & \mathcal{F}_6 + i\mathcal{F}_7 & -\frac{2}{\sqrt{3}}\mathcal{F}_8 \end{pmatrix}$$

$$M_{\ell 4,5}^2 = \frac{g^2}{2}(w_3^2 + w_1^2) \quad M_{\ell 6,7}^2 = \frac{g^2}{2}(w_3^2 + w_2^2) \quad M_{\ell 38}^2 = \frac{g^2}{2} \begin{pmatrix} w_2^2 + w_1^2 & \frac{1}{\sqrt{3}}(w_1^2 - w_2^2) \\ \frac{1}{\sqrt{3}}(w_1^2 - w_2^2) & \frac{1}{3}(4w_3^2 + w_1^2 + w_2^2) \end{pmatrix}$$

$$M_{\ell 1,2}^2 = \frac{g^2}{2}(w_2^2 + w_1^2) = \frac{g^2}{2}v_{\mathcal{F}}^2$$

New interactions in left-handed sector

$$\mathcal{L}_{\text{eff}}^{ee} = -\frac{G_{\mathcal{F}}}{\sqrt{2}} \sum_{a=1}^8 \left(\overline{e}_L \gamma_{\mu} \frac{\lambda_a}{x_a} e_L \right)^2$$

Charged leptons flavour conserving interactions,
lepton flavour violating interactions

$$\mathcal{L}_{\text{eff}}^{e\nu} = -\frac{2G_{\mathcal{F}}}{\sqrt{2}} \sum_{a=1}^8 \left(\overline{e}_L \gamma_{\mu} \frac{\lambda_a}{x_a} e_L \right) \left(\overline{\nu}_L \gamma_{\mu} \frac{\lambda_a}{x_a} \nu_L \right)$$

muon decay, tau decays,
non-standard neutrino interactions
with leptons

$$\mathcal{L}_{\text{eff}}^{\nu\nu} = -\frac{G_{\mathcal{F}}}{\sqrt{2}} \sum_{a=1}^8 \left(\overline{\nu}_L \gamma_{\mu} \frac{\lambda_a}{x_a} \nu_L \right)^2$$

Non-standard interactions between
neutrinos

New interactions in left-handed sector

$$\mathcal{L}_{\text{eff}}^{ee} = -\frac{G_{\mathcal{F}}}{\sqrt{2}} \sum_{a=1}^8 \left(\bar{\mathbf{e}}_L \gamma_\mu \frac{\lambda_a}{x_a} \mathbf{e}_L \right)^2$$

Flavour conserving interactions
(compositeness limits):

$$v_{\mathcal{F}} > 3 \text{ TeV}$$

FCNC in left-handed sector

$$\mathcal{L}_{\text{eff}}^{ee} = -\frac{G_{\mathcal{F}}}{\sqrt{2}} \sum_{a=1}^8 \left(\overline{\mathbf{e}}_L \gamma_{\mu} \frac{\lambda_a}{x_a} \mathbf{e}_L \right)^2$$

If $u_3 = u_2 = u_1$ (e. g. symmetry between η s) then

- Gauge bosons have equal masses and do not mix.
- $\lambda_a \rightarrow V^{\dagger} \lambda_a V$ is simply a basis redetermination of the Gell-Mann matrices
- From Fierz identities for λ matrices:

$$\mathcal{L}_{eff} = -\frac{1}{4v_{\ell}^2} (\overline{\mathbf{e}}_L \lambda^a \gamma^{\mu} \mathbf{e}_L) (\overline{\mathbf{e}}_L \lambda^a \gamma_{\mu} \mathbf{e}_L) = -\frac{1}{3v_{\ell}^2} (\overline{\mathbf{e}}_L \mathbb{I} \gamma_{\mu} \mathbf{e}_L)^2$$

- **no FCNC**, the global $SO(8)_{\ell}$ symmetry acts as a custodial symmetry.

FCNC in left-handed sector

$$\mathcal{L}_{\text{eff}}^{ee} = -\frac{G_{\mathcal{F}}}{\sqrt{2}} \sum_{a=1}^8 \left(\bar{\mathbf{e}}_L \gamma_\mu \frac{\lambda_a}{x_a} \mathbf{e}_L \right)^2$$

- In general case e.g. $\mu \rightarrow 3e$ decay: $(\delta_\mu = \frac{v_w^2}{v_{\mathcal{F}}^2} \simeq 7 \cdot 10^{-4})$

$$\frac{\Gamma(\mu \rightarrow ee\bar{e})}{\Gamma(\mu \rightarrow e\nu_\mu\bar{\nu}_e)} \simeq \frac{1}{8} (C(r)|U_{3e}^*U_{3\mu}|)^2 \delta_\mu^2$$

$r = 2u_3^2/v_\ell^2$, $|C(r)| < 1$. $|U_{3\mu}|$ and $|U_{3e}|$ can be as large as $\sin\theta_C = V_{us}$.

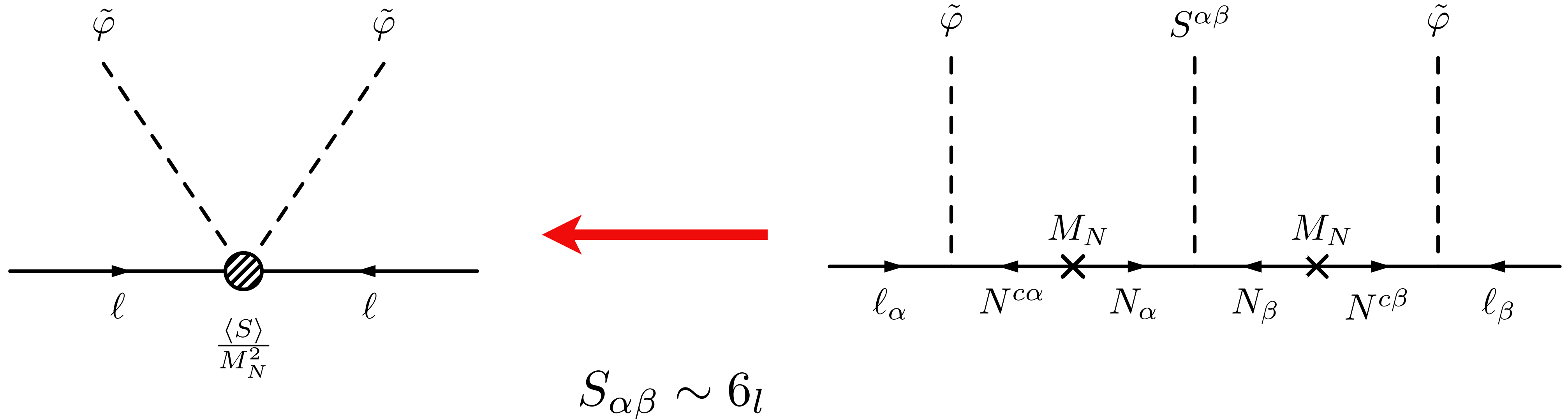
- The experimental limits on other LFV effects as $\tau \rightarrow 3\mu$ are much weaker.
- $\mathbf{v}_\ell \simeq \mathbf{6\ TeV}$ is not contradicting experimental constraints.
- For $r = 1$ all LFV effects are vanishing owing to custodial symmetry.

Effective operators for fermion masses

$$U(3)_\ell \times U(3)_e \times U(3)_q \times U(3)_u \times U(3)_d$$

$$\ell_L \sim 3_\ell, \quad e_R \sim 3_e, \quad q_L \sim 3_q, \quad u_R \sim 3_u, \quad d_R \sim 3_d$$

- Fermions cannot get mass if the symmetry is unbroken.
- Yukawa couplings are induced by non-zero VEVs of scalars.

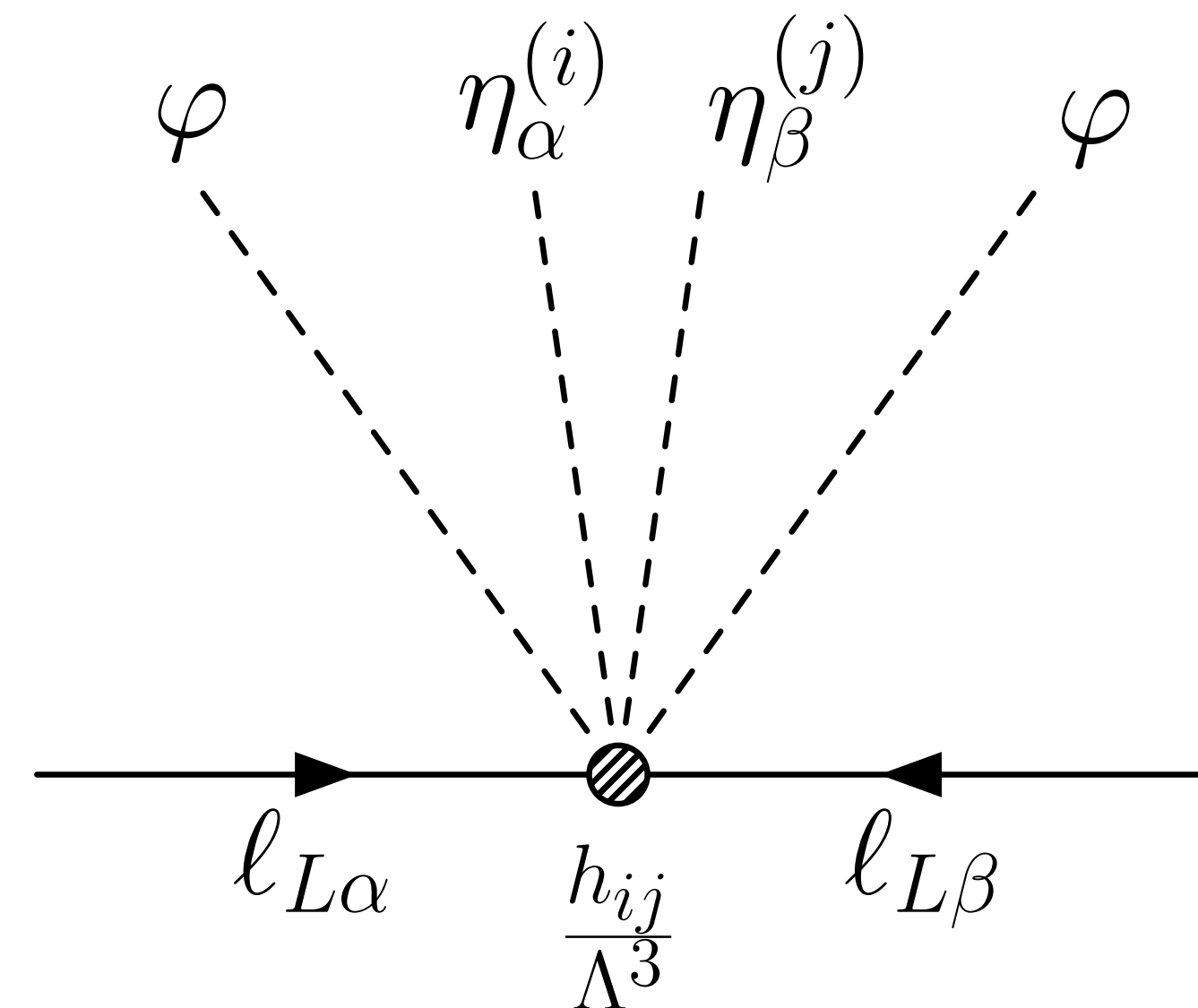
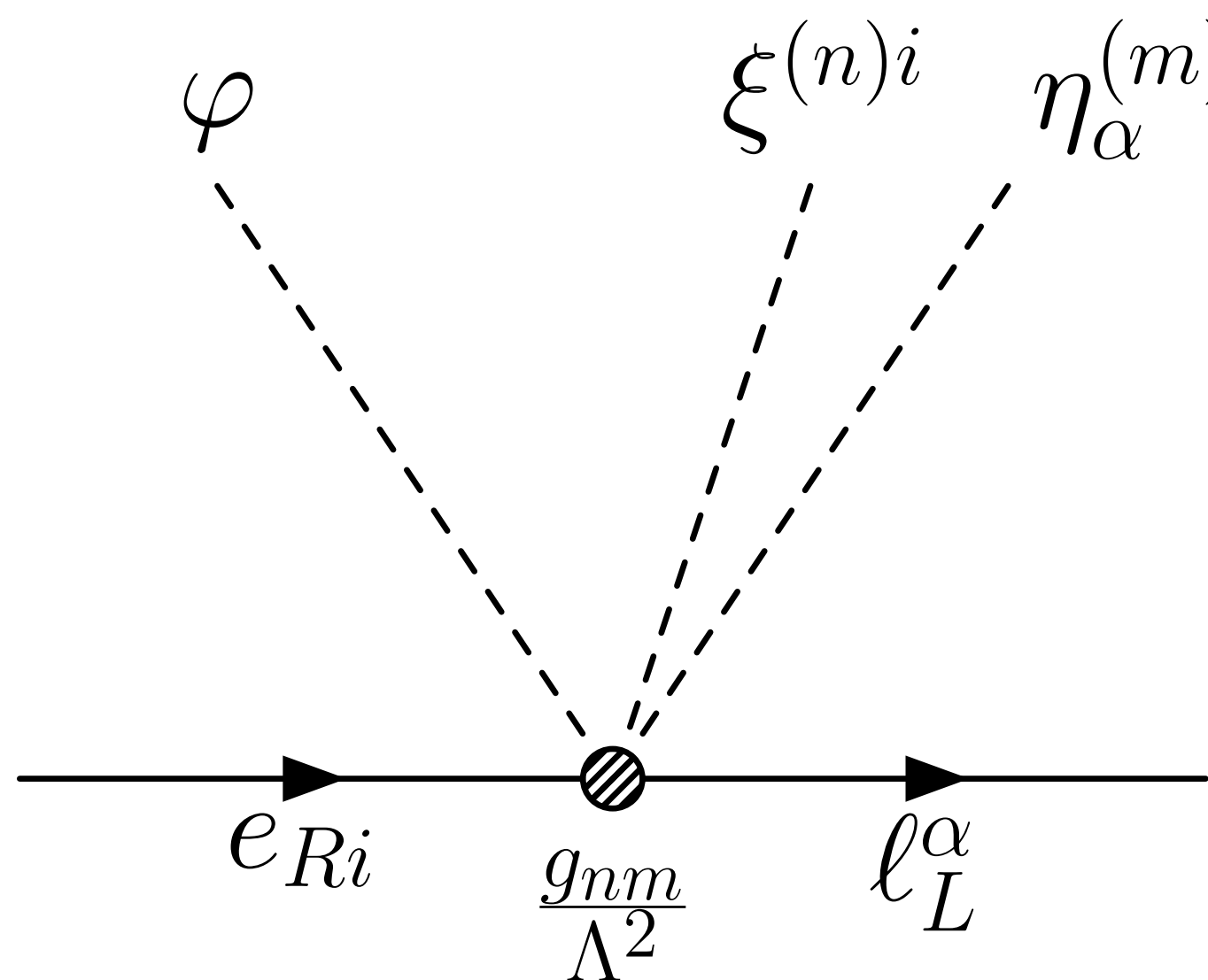


Leptons and family symmetry

$$SU(3)_\ell \times SU(3)_e$$

- Three $SU(3)_e$ triplets $\xi_i \sim 3_e$
- Three $SU(3)_\ell$ triplets $\eta_i \sim 3_\ell$

$$\frac{y_{ij} \bar{\xi}_j^\gamma \eta_{i\alpha}}{M_L^2} \varphi \overline{\ell_{L\alpha}} e_{Ri} + \frac{h_{ij} \bar{\eta}_i^\alpha \bar{\eta}_j^\beta}{M_\nu^3} \varphi \varphi \ell_{L\alpha}^T C \ell_\beta + h.c.$$



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$$SU(3)_e \xrightarrow{v_3} SU(2)_e \xrightarrow{v_2} \text{nothing}$$

$$SU(3)_\ell \xrightarrow{w_3} SU(2)_\ell \xrightarrow{w_2} \text{nothing}$$

$$\langle \eta_1 \rangle = \begin{pmatrix} w_1 \\ 0 \\ 0 \end{pmatrix}, \quad \langle \eta_2 \rangle = \begin{pmatrix} 0 \\ w_2 \\ 0 \end{pmatrix}, \quad \langle \eta_3 \rangle = \begin{pmatrix} 0 \\ 0 \\ w_3 \end{pmatrix}$$

$$m_\nu^{ij} = \frac{h_{ij} w_i w_j v_w^2}{M_\nu^3}, \quad U_{\text{PMNS}}$$

Leptons and family symmetry

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- Three $SU(3)_\ell$ triplets $\eta_i \sim 3_\ell$

$$\frac{y_{ij} \bar{\xi}_j^\gamma \eta_{i\alpha}}{M_L^2} \varphi \overline{\ell_{L\alpha}} e_{Ri} + \frac{h_{ij} \bar{\eta}_i^\alpha \bar{\eta}_j^\beta}{M_\nu^3} \varphi \varphi \ell_{L\alpha}^T C \ell_\beta + h.c.$$

$$SU(3)_e \xrightarrow{v_3} SU(2)_e \xrightarrow{v_2} \text{nothing}$$

$$SU(3)_\ell \xrightarrow{w_3} SU(2)_\ell \xrightarrow{w_2} \text{nothing}$$

$$\langle \xi_1 \rangle = \begin{pmatrix} v_1 \\ 0 \\ 0 \end{pmatrix}, \quad \langle \xi_2 \rangle = \begin{pmatrix} 0 \\ v_2 \\ 0 \end{pmatrix}, \quad \langle \xi_3 \rangle = \begin{pmatrix} 0 \\ 0 \\ v_3 \end{pmatrix}$$

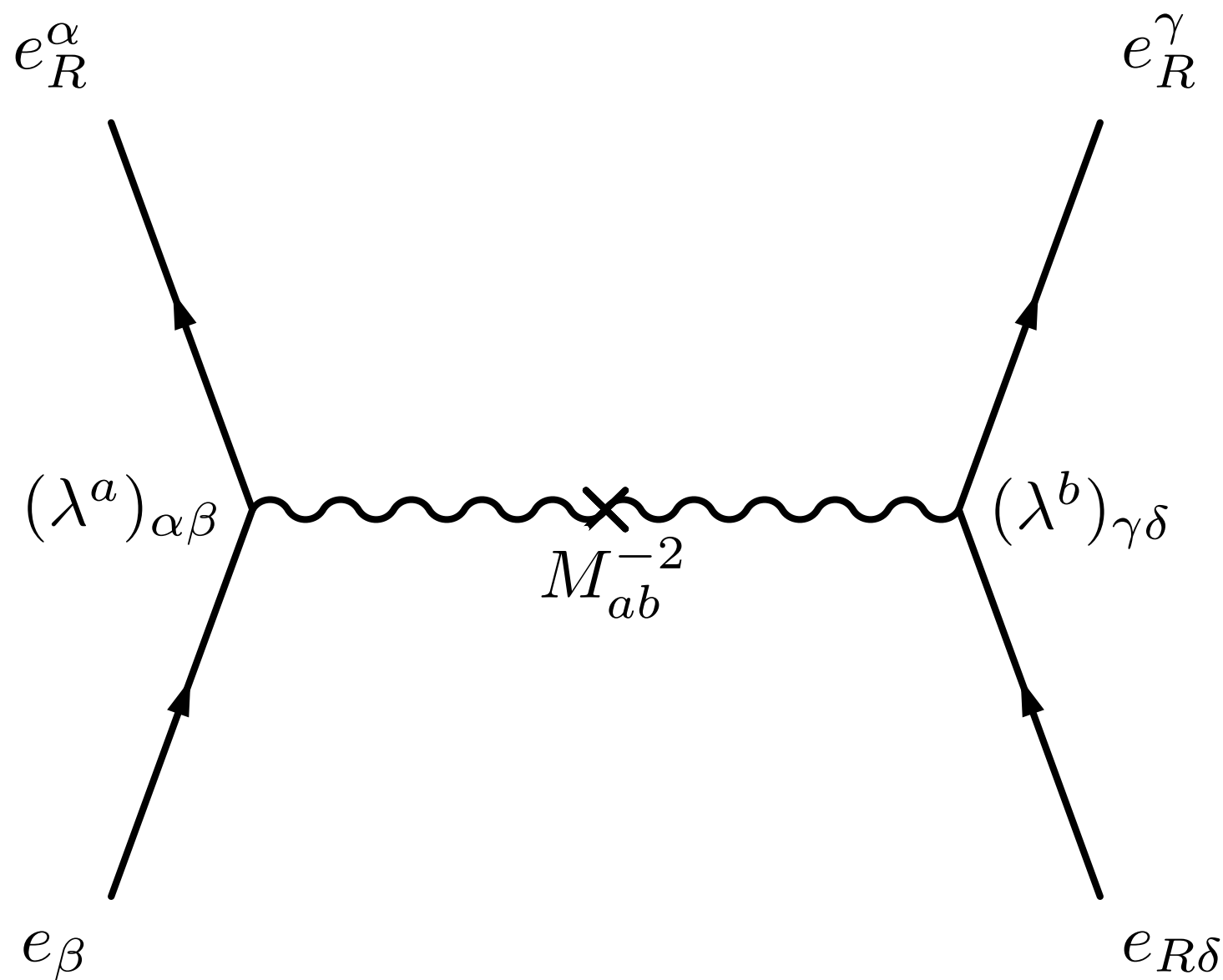
$$v_1 : v_2 : v_3 = \tilde{\epsilon} \epsilon : \epsilon : 1 \longrightarrow Y_e = y_\tau \begin{pmatrix} \sim \tilde{\epsilon} \epsilon & \sim \epsilon & \sim 1 \\ \sim \tilde{\epsilon} \epsilon & \sim \epsilon & \sim 1 \\ \sim \tilde{\epsilon} \epsilon & \sim \epsilon & \sim 1 \end{pmatrix} \longrightarrow m_e, m_\mu, m_\tau$$

$$Y_e^{ij} = \frac{y_{ij} w_i v_j}{M_L^2}$$

Gauge bosons & RH charged leptons

- Generically, FCNC:

$$SU(3)_e$$



$$e_{Ri} = V_{R1e}^{(e)} e_R + V_{R1\mu}^{(e)} \mu_R + V_{R1\tau}^{(e)} \tau_R$$

$$SU(3)_e \xrightarrow{v_3} SU(2)_e \xrightarrow{v_2} \text{nothing} \quad m_\tau : m_\mu : m_e \approx v_3 : v_2 : v_1$$

$$v_1 : v_2 : v_3 = \tilde{\epsilon} \epsilon : \epsilon : 1$$

$$-\frac{1}{4v_2^2} \sum_{a=1}^3 (\overline{\mathbf{e}}_R \lambda_a \gamma^\mu \mathbf{e}_R)^2 =$$

$$= -\frac{1}{4v_2^2} \left[\left(\overline{e}_R \quad \overline{\mu}_R \quad \overline{\tau}_R \right) \gamma_\mu V_R^{(e)\dagger} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} V_R^{(e)} \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix} \right]^2$$

Gauge bosons, $SU(3)_e$

$$SU(3)_e \xrightarrow{v_3} SU(2)_e \xrightarrow{v_2} \text{nothing} \qquad v_1 : v_2 : v_3 = \tilde{\epsilon} \epsilon : \epsilon : 1$$

$$\frac{1}{2} \begin{pmatrix} \boxed{\mathcal{F}_3 + \frac{1}{\sqrt{3}}\mathcal{F}_8} & \boxed{\mathcal{F}_1 - i\mathcal{F}_2} & \mathcal{F}_4 - i\mathcal{F}_5 \\ \boxed{\mathcal{F}_1 + i\mathcal{F}_2} & \boxed{-\mathcal{F}_3 + \frac{1}{\sqrt{3}}\mathcal{F}_8} & \mathcal{F}_6 - i\mathcal{F}_7 \\ \mathcal{F}_4 + i\mathcal{F}_5 & \mathcal{F}_6 + i\mathcal{F}_7 & -\frac{2}{\sqrt{3}}\mathcal{F}_8 \end{pmatrix}$$

$$M_{4,5,6,7}^2 = \frac{g^2 v_3^2}{2}$$

$$M_{1,2}^2 = \frac{g^2 v_2^2}{2}$$

$$M_{38}^2 = \frac{g^2}{2} \begin{pmatrix} \boxed{v_2^2} & -\frac{1}{\sqrt{3}}v_2^2 \\ -\frac{1}{\sqrt{3}}v_2^2 & \frac{1}{3}(4v_3^2 + v_2^2) \end{pmatrix}$$

$$v_2 \gtrsim ?$$

Flavour changing neutral currents

In $\text{SU}(2)_e$ gauge symmetry limit ($v_3 \gg v_2$):

- $SU(2)_e$ gauge bosons have equal masses;
- **no FCNC** for CUSTODIAL SYMMETRY, no matter if two families are mixed:

$$\begin{aligned} -\frac{1}{v_2^2} \sum_{a=1}^3 (J_a^\mu)^2 &= -\frac{1}{4v_2^2} \sum_{a=1}^3 (\bar{\mathbf{e}}_R \lambda_a \gamma^\mu \mathbf{e}_R)^2 = \\ &= -\frac{1}{4v_2^2} \left[\left(\begin{array}{ccc} \bar{e}_R & \bar{\mu}_R & \bar{\tau}_R \end{array} \right) \gamma_\mu V_R^{(e)\dagger} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} V_R^{(e)} \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix} \right]^2 \end{aligned}$$

- no mixing with 3rd family $\rightarrow$ NO FCNC.
- Constraints on masses are proportional to violation of custodial symmetry $(\epsilon, \tilde{\epsilon})$.

Flavour changing neutral currents

- Constraints on masses are proportional to violation of custodial symmetry:

$$\begin{aligned}
 \mathcal{L}_{\text{fcnc}} = & -\frac{1}{4v_2^2} \left[\left(\overline{e_R} \quad \overline{\mu_R} \quad \overline{\tau_R} \right) \gamma_\mu V_R^{(e)\dagger} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} V_R^{(e)} \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix} \right]^2 + \\
 & -\frac{\epsilon^2}{4v_2^2} \left[\left(\overline{e_R} \quad \overline{\mu_R} \quad \overline{\tau_R} \right) \gamma_\mu V_R^{(e)\dagger} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} V_R^{(e)} \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix} \right]^2 = \\
 & -\frac{1}{4v_2^2} \left[\left(\overline{e_R} \quad \overline{\mu_R} \quad \overline{\tau_R} \right) \gamma_\mu \begin{pmatrix} 1 & \sim \tilde{\epsilon}\epsilon^2 & \sim \tilde{\epsilon}\epsilon \\ \sim \tilde{\epsilon}\epsilon^2 & 1 & \sim \epsilon \\ \sim \tilde{\epsilon}\epsilon & \sim \epsilon & \sim \epsilon^2 \end{pmatrix} \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix} \right]^2 + \\
 & -\frac{\epsilon^2}{4v_2^2} \left[\left(\overline{e_R} \quad \overline{\mu_R} \quad \overline{\tau_R} \right) \gamma_\mu \begin{pmatrix} 1 & \sim \tilde{\epsilon} & \sim \tilde{\epsilon}\epsilon \\ \sim \tilde{\epsilon} & \sim \epsilon^2 & \sim \epsilon \\ \sim \tilde{\epsilon}\epsilon & \sim \epsilon & 1 \end{pmatrix} \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix} \right]^2
 \end{aligned}$$

Experimental constraint

- **Flavour conserving operators** constrained by compositeness limits:

$$-\frac{1}{4v_2^2} (\overline{e_R} \gamma^\nu e_R)^2 - \frac{1}{2v_2^2} (\overline{e_R} \gamma^\nu e_R) (\overline{\mu_R} \gamma_\nu \mu_R) \quad \boxed{v_2 > 2 \text{ TeV}} \rightarrow v_3 > 10 \text{ TeV}$$

- From FCNC:

LFV mode	Exp. $\Gamma_i/\Gamma_\mu(\Gamma_\tau)$	Main contribution to $\frac{\Gamma_i}{\Gamma_{\mu/\tau}}$	Predicted value of $\frac{\Gamma_i}{\Gamma_{\mu/\tau}}$
$\mu \rightarrow eee$	$< 1.0 \cdot 10^{-12}$	$\frac{1}{8} \left(\frac{v_{EW}}{v_2} \right)^4 V_{3e}^* V_{3\mu} + V_{2e}^* V_{2\mu} \epsilon^2 ^2$	$\leq 1.1 \cdot 10^{-13} \left(\frac{2 \text{ TeV}}{v_2} \right)^4 \epsilon_{20}^4 \tilde{\epsilon}_{20}^2$
$\tau^- \rightarrow \mu^- e^+ e^-$	$< 1.8 \cdot 10^{-8}$	$\frac{1}{4} \left(\frac{v_{EW}}{v_2} \right)^4 V_{3\mu}^* V_{3\tau} ^2 \frac{\Gamma_w}{\Gamma_\tau}$	$= 6.2 \cdot 10^{-9} \left(\frac{2 \text{ TeV}}{v_2} \right)^4 \epsilon_{20}^2$
$\tau \rightarrow \mu\mu\mu$	$< 2.1 \cdot 10^{-8}$	$\frac{1}{8} \left(\frac{v_{EW}}{v_2} \right)^4 V_{3\mu}^* V_{3\tau} ^2 \frac{\Gamma_w}{\Gamma_\tau}$	$= 3.1 \cdot 10^{-9} \left(\frac{2 \text{ TeV}}{v_2} \right)^4 \epsilon_{20}^2$
$\mu \rightarrow e\gamma$	$< 4.2 \cdot 10^{-13}$	$\frac{3\alpha}{2\pi} \left(\frac{v_{EW}}{v_2} \right)^4 V_{3e}^* V_{3\mu} ^2$	$= 3.1 \cdot 10^{-15} \left(\frac{2 \text{ TeV}}{v_2} \right)^4 \epsilon_{20}^4 \tilde{\epsilon}_{20}^2$
$\tau \rightarrow \mu\gamma$	$< 4.4 \cdot 10^{-8}$	$\frac{3\alpha}{2\pi} \left(\frac{v_{EW}}{v_2} \right)^4 V_{3\mu}^* V_{3\tau} ^2 \frac{\Gamma_w}{\Gamma_\tau}$	$= 8.7 \cdot 10^{-11} \left(\frac{2 \text{ TeV}}{v_2} \right)^4 \epsilon_{20}^2$

Quarks and family symmetries

$U(3)_q \times U(3)_d \times U(3)_u$ with gauge factors $SU(3)_q \times SU(3)_d \times SU(3)_u$

- Quark masses:

$$\frac{y_{ij}^d}{M_d^2} \eta_{i\alpha}^q \bar{\xi}_j^{d\gamma} \phi \overline{q_{L\alpha}} d_{R\gamma} + \frac{y_{ij}^u}{M_u^2} \eta_{i\alpha}^q \bar{\xi}_j^{u\gamma} \tilde{\phi} \overline{q_{L\alpha}} u_{R\gamma} + \text{h.c.}$$

- mass hierarchy is related with hierarchies in breaking of $SU(3)_q \times SU(3)_d \times SU(3)_u$ gauge symmetry:

$$m_b : m_s : m_d = 1 : \epsilon_d \epsilon_q : \epsilon_d \tilde{\epsilon}_d \epsilon_q \tilde{\epsilon}_q$$

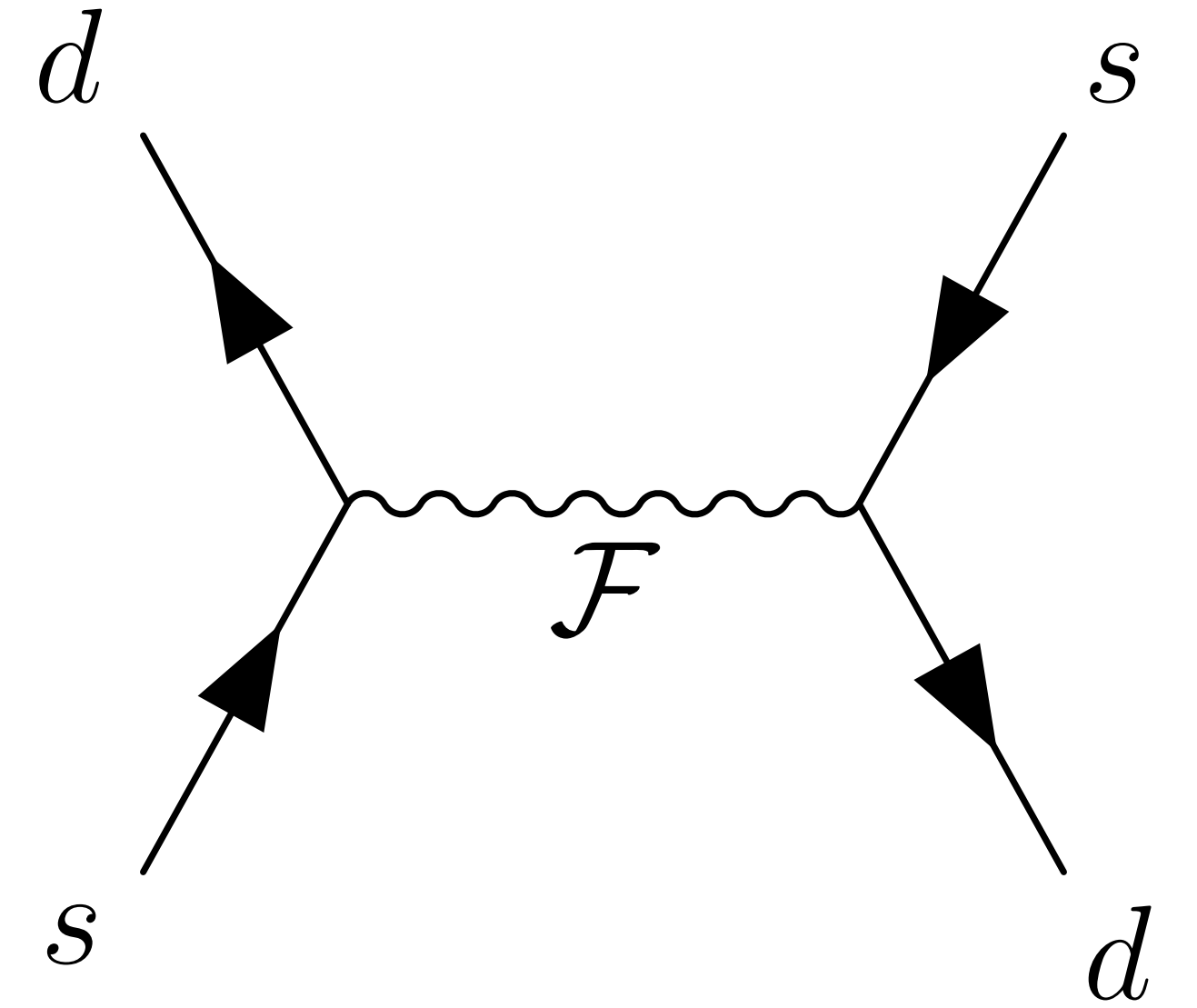
$$m_t : m_c : m_u = 1 : \epsilon_u \epsilon_q : \epsilon_u \tilde{\epsilon}_u \epsilon_q \tilde{\epsilon}_q$$

Quarks and family symmetries

- $K^0 - \bar{K}^0$ oscillation is induced by:

$$- \frac{1}{4v_{d2}^2} \left[(V_{3d}V_{3s}^*)^2 + \epsilon_d^2 (V_{2d}V_{2s}^*)^2 \right] (\bar{s}_R \gamma^\mu d_R)^2$$

- Since $|V_{2d}V_{2s}^*| \sim \tilde{\epsilon}_d$, $|V_{3d}V_{3s}^*| \sim \epsilon_d^2 \tilde{\epsilon}_d$, K^0 mixing is suppressed by $\epsilon_d^2 \tilde{\epsilon}_d^2 \ll 1$.
- New contribution can be constrained to be less than the SM contribution. By taking $\epsilon_d \tilde{\epsilon}_d \sim 10^{-2}$, the mass scale $v_{d2} \sim 7 \text{ TeV}$ is compatible with the constraint from the neutral kaons mass difference.
- As regards the imaginary part contributing to ϵ_K , with the same choice $\epsilon_d \tilde{\epsilon}_d \sim 10^{-2}$, $v_{d2} \sim 7 \text{ TeV}$ is still allowed if the phase of $V_{2d}V_{2s}^*$ is $O(0.1)$.



VEVS

Can the hierarchy of the VEVs of ξ s be natural? The generic potential is:

$$V(\xi) = \lambda_n (|\xi_n|^2 - \frac{\mu_n^2}{2\lambda_n})^2 + \lambda_{klm} \xi_k^\dagger \xi_l \xi_m^\dagger \xi_n + (\mu \xi_1 \xi_2 \xi_3 + \text{h.c.})$$

- The dimensional constant μ can be arbitrarily small since if $\mu \rightarrow 0$ the Lagrangian acquires global $U(1)_e$ symmetry.
- $v_2/v_3 \sim m_\mu/m_\tau$ (one order of magnitude) can emerge from a natural fluctuation of mass terms μ_n^2 and coupling constants λ .
- Small v_1 is naturally obtained when the third flavon ξ_1 has positive mass squared. Then for $\mu \neq 0$ non-zero VEV $\langle \xi_1 \rangle$ is induced:

$$v_1 = \frac{\mu v_2 v_3}{\mu_1^2}$$

- Taking μ small enough, say $\mu < v_2$, one can naturally get $v_1 \ll v_2$. **The hierarchy of the VEVs of ξ s can be natural.**

Flavons and LFV

- Also flavons can mediate the LFV processes.
- Lepton Yukawa couplings with the flavon fields ξ_n :

$$h_{in} \xi_n^\alpha \overline{\ell_{Li}} e_{R\alpha} \quad h_{in} = \frac{g_{in} v_w}{M}$$

which are generically flavor-changing.

- ξ_2 , with mass $\mu_2 \sim v_2$, induces the effective operator:

$$- \frac{h_{32} h_{22}}{\mu_2^2} (\bar{\tau} \mu) (\bar{\mu} \mu), \quad \frac{h_{32} h_{22}}{\mu_2^2} \simeq \frac{m_\mu^2}{v_2^4}$$

For $v_2 > 2$ TeV, the width of $\tau \rightarrow 3\mu$ decay induced by this operator is more than 12 orders of magnitude below the experimental limit.

- The width of $\mu \rightarrow 3e$ decay induced by analogous operator mediated by flavon ξ_1 is also suppressed by orders of magnitude.

Triangle anomalies

$$SU(3)_\ell \times SU(3)_e \times SU(3)_Q \times SU(3)_u \times SU(3)_d$$

$$\ell_L \sim 3_\ell, e_R \sim 3_e, Q_L \sim 3_Q, u_R \sim 3_u, d_R \sim 3_d$$

- In order to cancel $SU(3)^3$ anomalies for each triplet another triplet (SM singlet) with opposite chirality is needed.
- An interesting possibility is to introduce the mirror twins with opposite chirality and analogous representation of mirror SM gauge symmetry $SU(3)' \times SU(2)' \times U(1)'$:

$$\ell'_R \sim 3_\ell, e'_L \sim 3_e, Q'_R \sim 3_Q, u'_L \sim 3_u, d'_L \sim 3_d$$

- Couplings with flavons:

$$\frac{g_{in} \xi_n^\alpha}{M} (\phi \overline{\ell_{Li}} e_{R\alpha} + \phi' \overline{\ell'_{Ri}} e'_{L\alpha}) + h.c.$$

Triangle anomalies 2

- As an example, for $SU(3)_e$, mixed triangle anomaly $U(1) \times SU(3)_e^2$ must be cancelled. New leptons

$$\mathcal{E}_{L\alpha} \sim (1, -2, 3_e; X), \quad \mathcal{E}_{Ri} \sim (1, -2, 1; X)$$

and for mirror parity

$$\mathcal{E}'_{R\alpha} \sim (1, -2', 3_e; X), \quad \mathcal{E}'_{Li} \sim (1, -2', 1; X)$$

cancel the mixed triangle

$$U(1) \times SU(3)_e^2, \quad U(1)_X \times SU(3)_e^2, \quad U(1) \times U(1)_X^2, \quad U(1)_X \times U(1)^2$$

- Masses from Yukawa couplings

$$y_{in} \xi_n^\alpha \overline{\mathcal{E}_{Ri}} \mathcal{E}_{L\alpha} + y_{in} \xi_n^\alpha \overline{\mathcal{E}'_{Li}} \mathcal{E}'_{R\alpha} + h.c.$$

- The lightest has mass $O(100)$ GeV. If $U(1)_X$ is unbroken, then it is stable. Current experimental lower limit on charged new leptons is 102.6 GeV.

Cosmological implications

- Mirror matter is a viable candidate for light dark matter dominantly consisting of mirror helium and hydrogen atoms.
- The flavor gauge bosons are messengers between the two sectors and so a portal for direct detection.
- $T'/T < 0.2 \div 0.3$ from CMB and large scale structures.
- Freeze-out temperature of horizontal interactions between the two sectors should not exceed $T_d \simeq (v_2/2)^{\frac{4}{3}} \times 130$ MeV. Or $v'_{EW} \gg v_{EW}$.
- For neutrinos

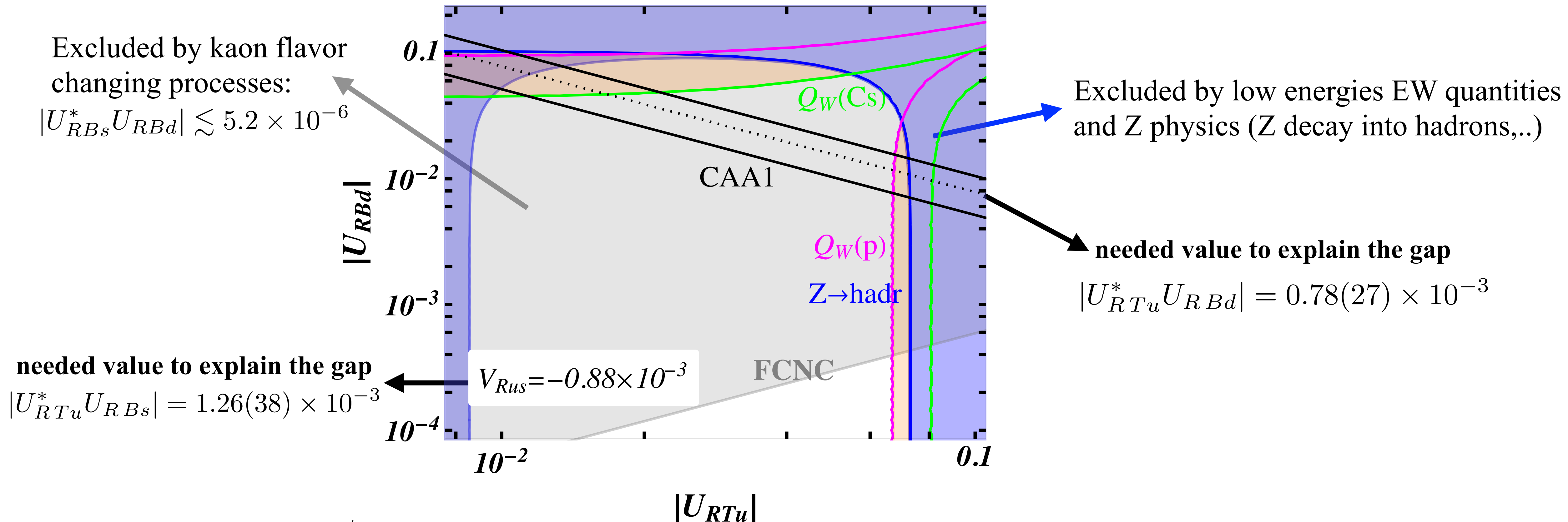
$$\frac{Y_\nu^{ij}}{\mathcal{M}} (\phi \phi l_{Li}^T C l_{Lj} + \phi' \phi' l_{Ri}'^T C l_{Rj}') + \frac{\tilde{Y}_\nu^{ij}}{\mathcal{M}} \phi \phi' \overline{l_{Li}} l_{Rj}' + h.c.$$

the last operator gives COLEPTOGENESIS.

Vector-like weak doublet

- One weak doublet of quarks cannot explain both the Cabibbo angle anomalies (CAA1 and CAA2) alone.

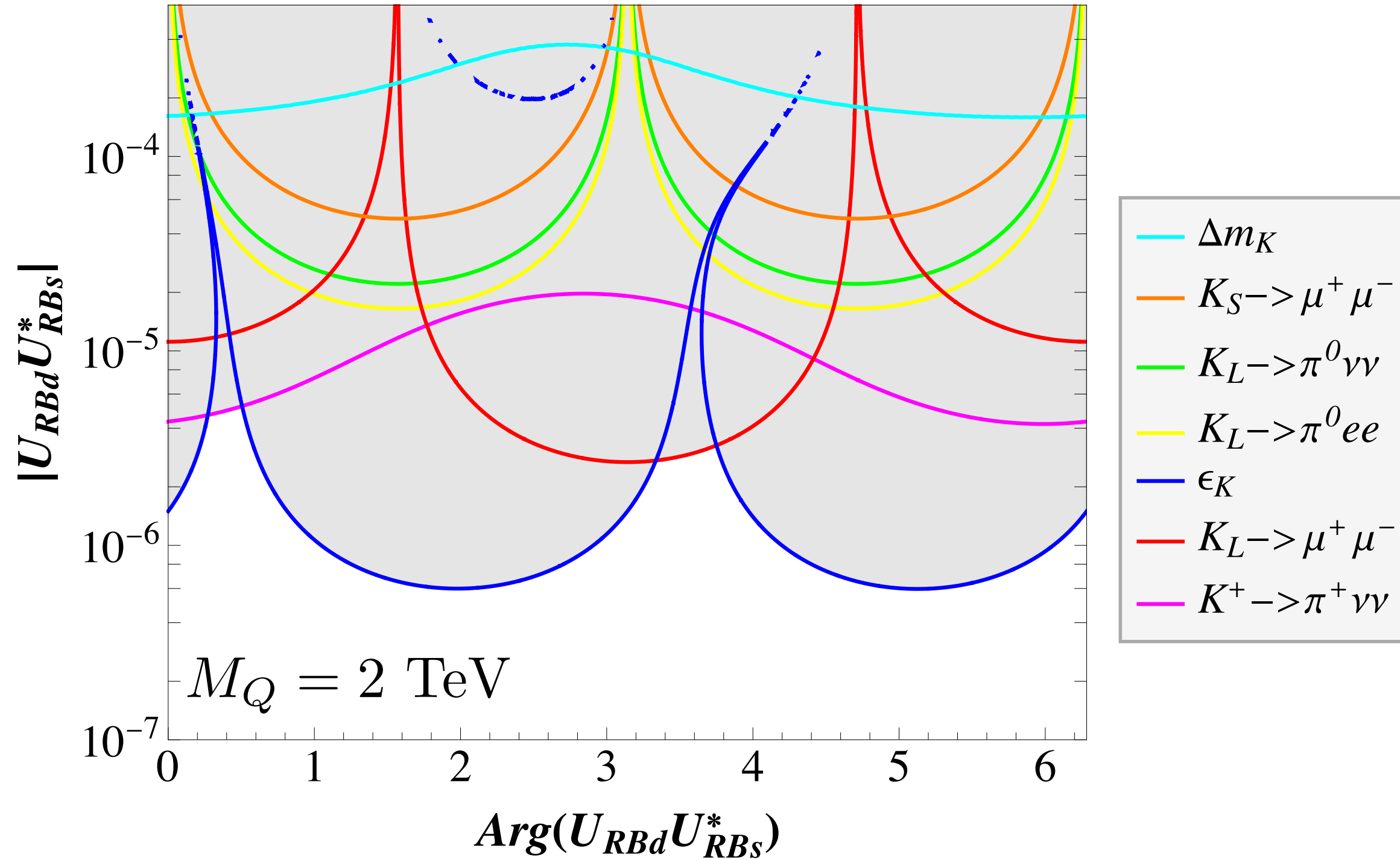
$$V_{Rus} = U_{RTu}^* U_{RBs} = -1.26(38) \times 10^{-3}, \quad V_{Rud} = U_{RTu}^* U_{RBd} = -0.78(27) \times 10^{-3}$$



$$U_{RB\alpha}, U_{RT\alpha} \approx -h_\alpha v_w / M_Q$$

Vector-like weak doublet

- Constraints become more stringent for larger values of the mass of the extra doublet. Assuming $M_Q = 2$ TeV:



K mixing and fc decays:

$$|U_{RBs}^* U_{RBd}| \approx |h_s^* h_d| v_w^2 / M_Q^2 \lesssim (0.6 - 5.2) \times 10^{-6}$$

B mixing and fc decays:

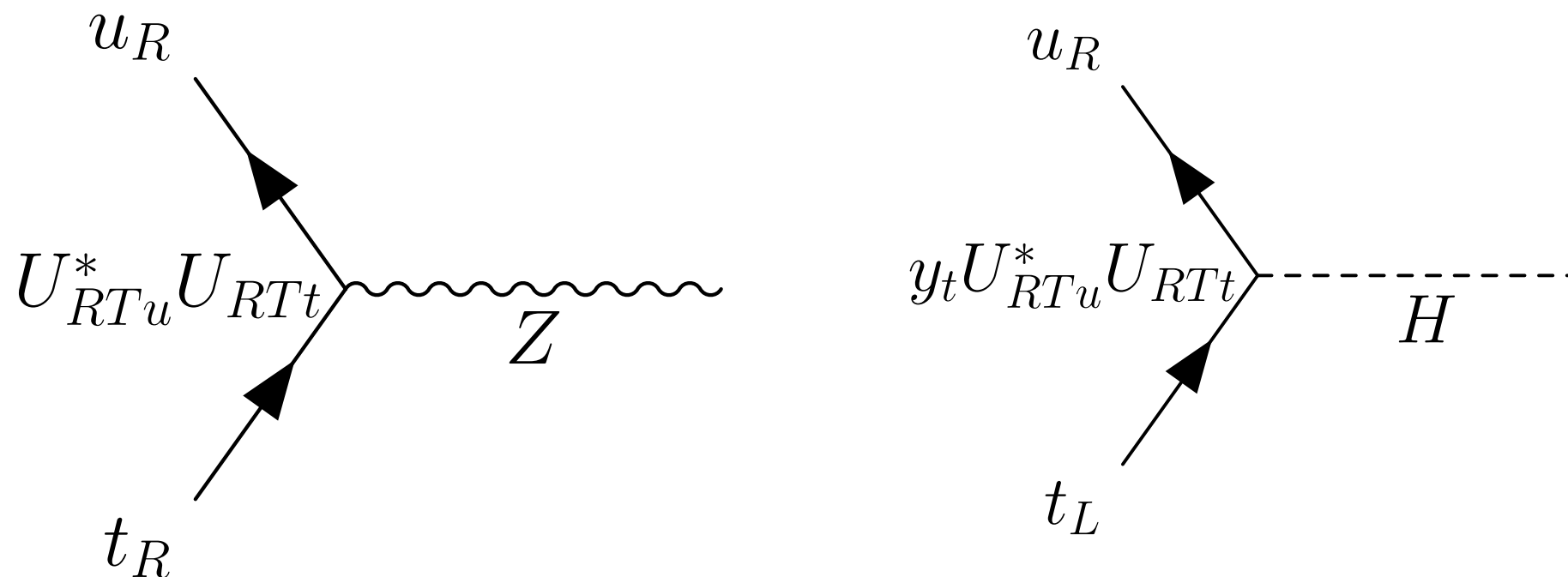
$$|U_{RBd}^* U_{RBb}| \approx |h_d^* h_b| v_w^2 / M_Q^2 < (0.4 - 1.7) \times 10^{-4}$$

$$|U_{RBs}^* U_{RBb}| \approx |h_s^* h_b| v_w^2 / M_Q^2 < 8 \times 10^{-7} - 6.4 \times 10^{-4}$$

D mixing:

$$|U_{RTu}^* U_{RTc}| \approx |h_u^* h_c| v_w^2 / M_Q^2 < 1.0 \times 10^{-4}$$

- Top decays:



$$|U_{RTu}^* U_{RTt}| \approx |h_u^* h_t| v_w^2 / M_Q^2 \lesssim 0.01$$

$$|U_{LTt} h_u^*| \approx y_t |h_u^* h_t| v_w^2 / M_Q^2 \lesssim 0.08$$

Vector-like weak doublet

- However also in this scenario flavour changing neutral currents appear at tree level.

$$\mathcal{L}_{\text{fcnc}} = \frac{1}{2} \frac{g}{\cos \theta_W} Z^\mu \begin{pmatrix} \overline{u_R} & \overline{c_R} & \overline{t_R} & \overline{t'_R} \end{pmatrix} \gamma^\mu V_R^{(u)\dagger} \text{diag}(0, 0, 0, 1) V_R^{(u)} \begin{pmatrix} u_R \\ c_R \\ t_R \\ t'_R \end{pmatrix} +$$

$$- \frac{1}{2} \frac{g}{\cos \theta_W} Z^\mu \begin{pmatrix} \overline{d_R} & \overline{s_R} & \overline{b_R} & \overline{b'_R} \end{pmatrix} \gamma^\mu V_R^{(d)\dagger} \text{diag}(0, 0, 0, 1) V_R^{(d)} \begin{pmatrix} d_R \\ s_R \\ b_R \\ b'_R \end{pmatrix}$$

where

$$V_R^{(u)\dagger} \text{diag}(0, 0, 0, 1) V_R^{(u)} = \begin{pmatrix} |V_{R4u}|^2 & V_{R4u}^* V_{R4c} & V_{R4u}^* V_{R4t} & V_{R4u}^* V_{R4t'} \\ V_{R4c}^* V_{R4u} & |V_{R4c}|^2 & V_{R4c}^* V_{R4t} & V_{R4c}^* V_{R4t'} \\ V_{R4t}^* V_{R4u} & V_{R4t}^* V_{R4c} & |V_{R4t}|^2 & V_{R4t}^* V_{R4t'} \\ V_{R4t'}^* V_{R4u} & V_{R4t'}^* V_{R4c} & V_{R4t'}^* V_{R4t} & |V_{R4t'}|^2 \end{pmatrix}$$

$$V_R^{(d)\dagger} \text{diag}(0, 0, 0, 1) V_R^{(d)} = \begin{pmatrix} |V_{R4d}|^2 & V_{R4d}^* V_{R4s} & V_{R4d}^* V_{R4b} & V_{R4d}^* V_{R4b'} \\ V_{R4s}^* V_{R4d} & |V_{R4s}|^2 & V_{R4s}^* V_{R4b} & V_{R4s}^* V_{R4b'} \\ V_{R4b}^* V_{R4d} & V_{R4b}^* V_{R4s} & |V_{R4b}|^2 & V_{R4b}^* V_{R4b'} \\ V_{R4b'}^* V_{R4d} & V_{R4b'}^* V_{R4s} & V_{R4b'}^* V_{R4b} & |V_{R4b'}|^2 \end{pmatrix}$$

Down-type isosinglet

- Weak neutral currents:

$$\mathcal{L}_{\text{nc}} = \frac{g}{\cos \theta_W} \left[-\frac{1}{2} \begin{pmatrix} \overline{d}_L & \overline{s}_L & \overline{b}_L & \overline{b}'_L \end{pmatrix} \gamma^\mu \tilde{V}_L^{(d)\dagger} \tilde{V}_L^{(d)} \begin{pmatrix} d \\ s \\ b \\ b' \end{pmatrix} + \frac{1}{3} \sin^2 \theta_W (\overline{\mathbf{d}}_L \gamma^\mu \mathbf{d}_L + \overline{\mathbf{d}}_R \gamma^\mu \mathbf{d}_R) \right] Z_\mu$$

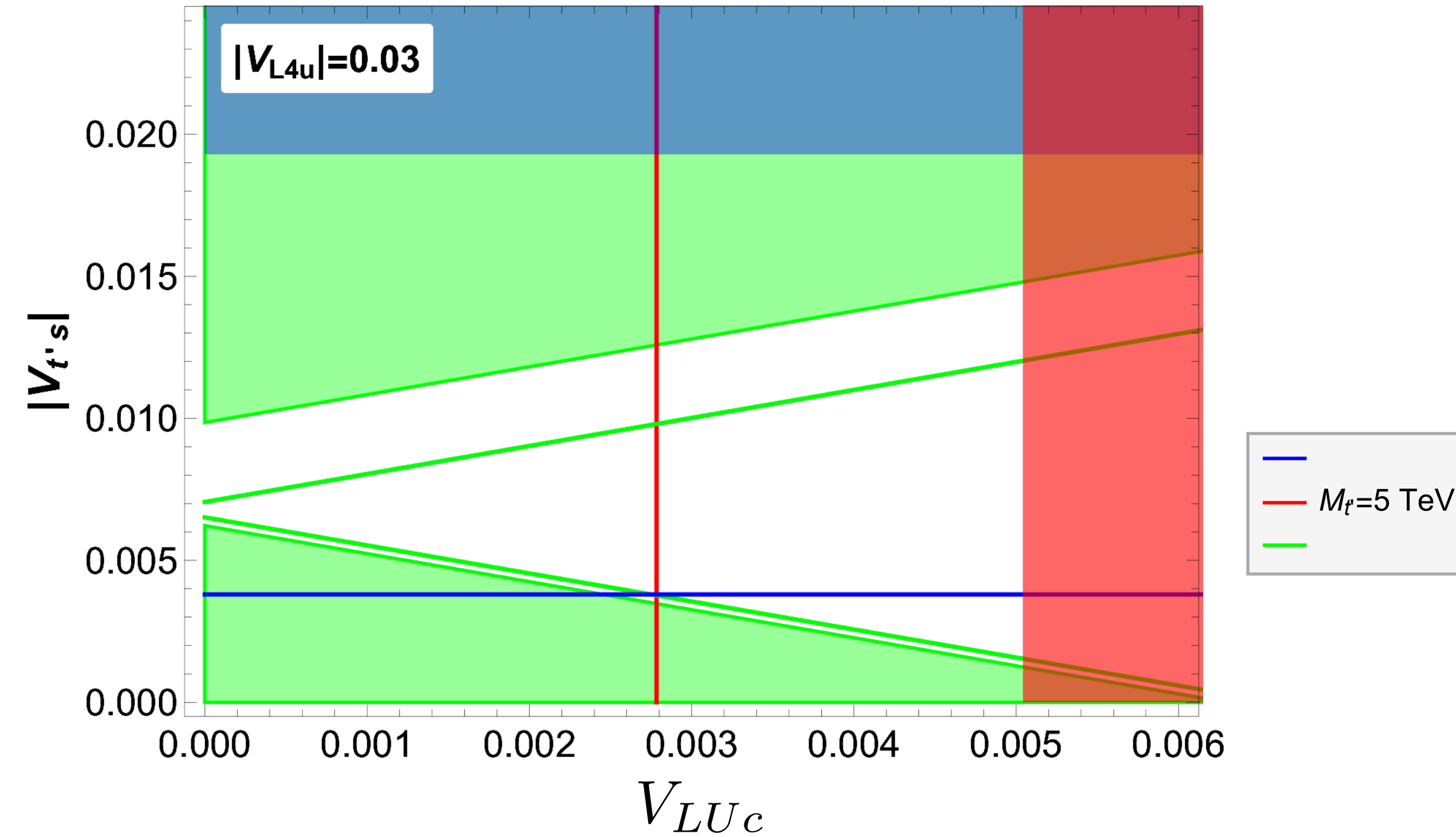
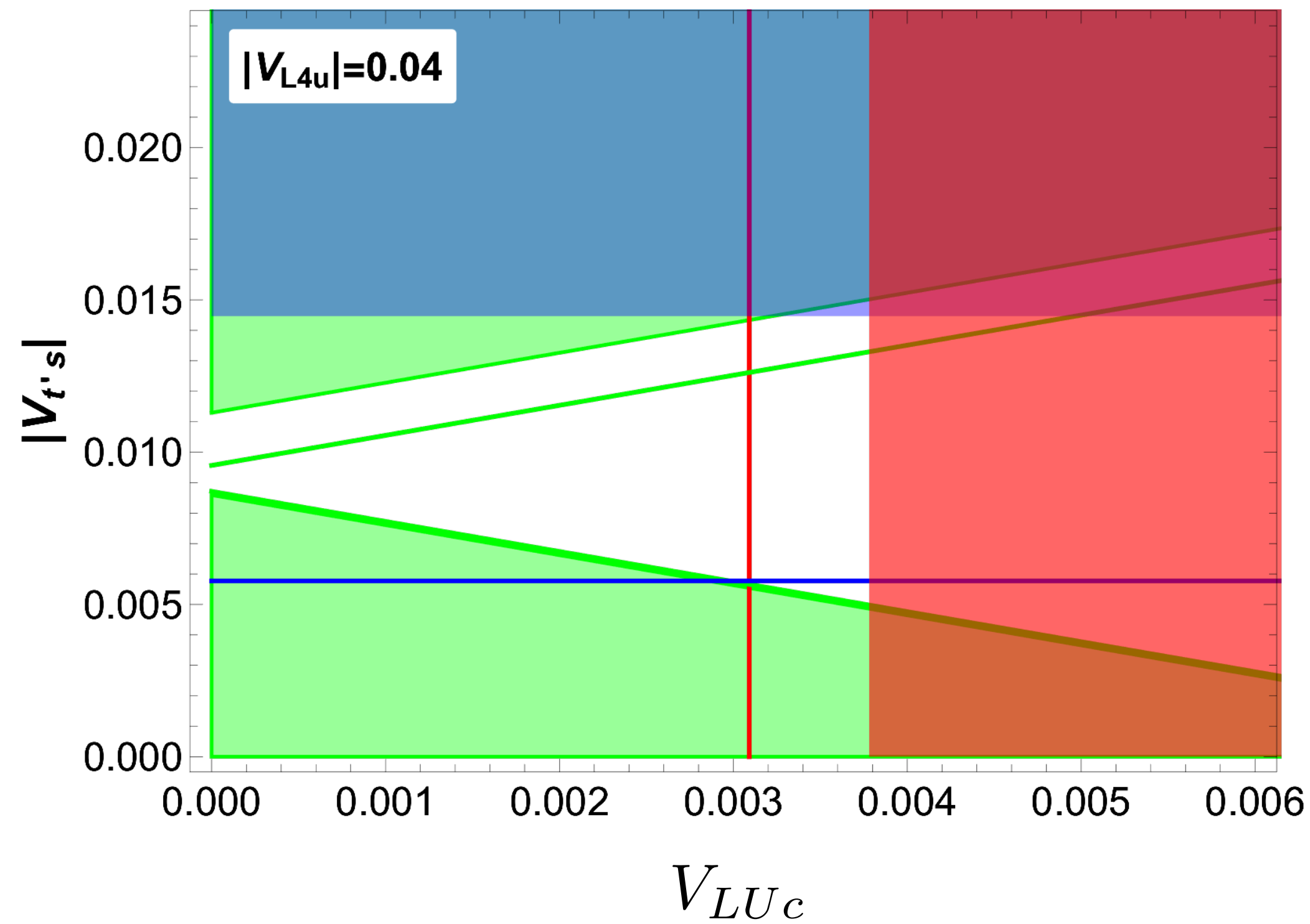
- The non-unitarity of $\tilde{V}_L^{(d)}$ is at the origin of **tree level flavour changing** couplings with Z boson (and Higgs boson), determined by the matrix:

$$\tilde{V}_L^{(d)\dagger} \tilde{V}_L^{(d)} = \begin{pmatrix} 1 - |V_{4d}|^2 & -V_{4d}^* V_{4s} & -V_{4d}^* V_{4b} & -V_{4d}^* V_{4b'} \\ -V_{4s}^* V_{4d} & 1 - |V_{4s}|^2 & -V_{4s}^* V_{4b} & -V_{4s}^* V_{4b'} \\ -V_{4b}^* V_{4d} & -V_{4b}^* V_{4s} & 1 - |V_{4b}|^2 & -V_{4b}^* V_{4b'} \\ -V_{4b'}^* V_{4d} & -V_{4b'}^* V_{4s} & -V_{4b'}^* V_{4b} & |V_{1b'}|^2 + |V_{2b'}|^2 + |V_{3b'}|^2 \end{pmatrix}_L$$

- If the 2nd and 3rd families are not mixed with the fourth, $V_{L4s} = V_{L4b} = 0$, there are not FCNC at tree level between the first three families.

Up-type weak singlets

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 - |V_{LUu}|^2, \quad |V_{LUu}| \approx |h_u|v_w/M_{t'} \approx 0.044$$



W-boson mass

$$\delta m_W^2 = m_W^2 - m_{W,\text{SM}}^2 = c^2 m_Z^2 \alpha \left[\frac{c^2}{c^2 - s^2} T - \frac{1}{2(c^2 - s^2)} S + \frac{1}{4s^2} U \right]$$

- Oblique parameters:

$$\alpha S \equiv 4e^2 [\Pi'_{33}(0) - \Pi'_{3Q}(0)] ,$$

$$\alpha T \equiv \frac{e^2}{s^2 c^2 m_Z^2} [\Pi_{11}(0) - \Pi_{33}(0)] ,$$

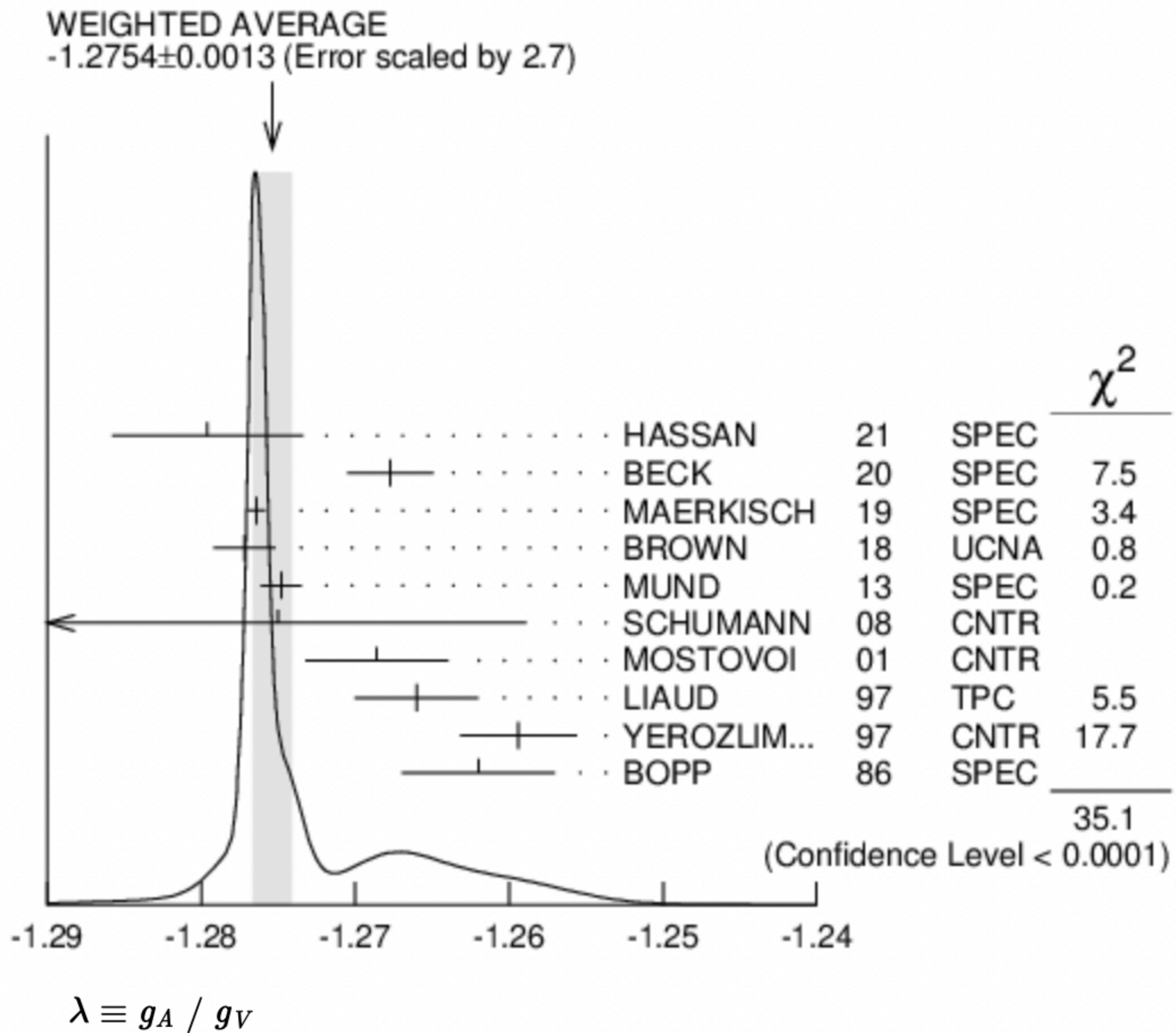
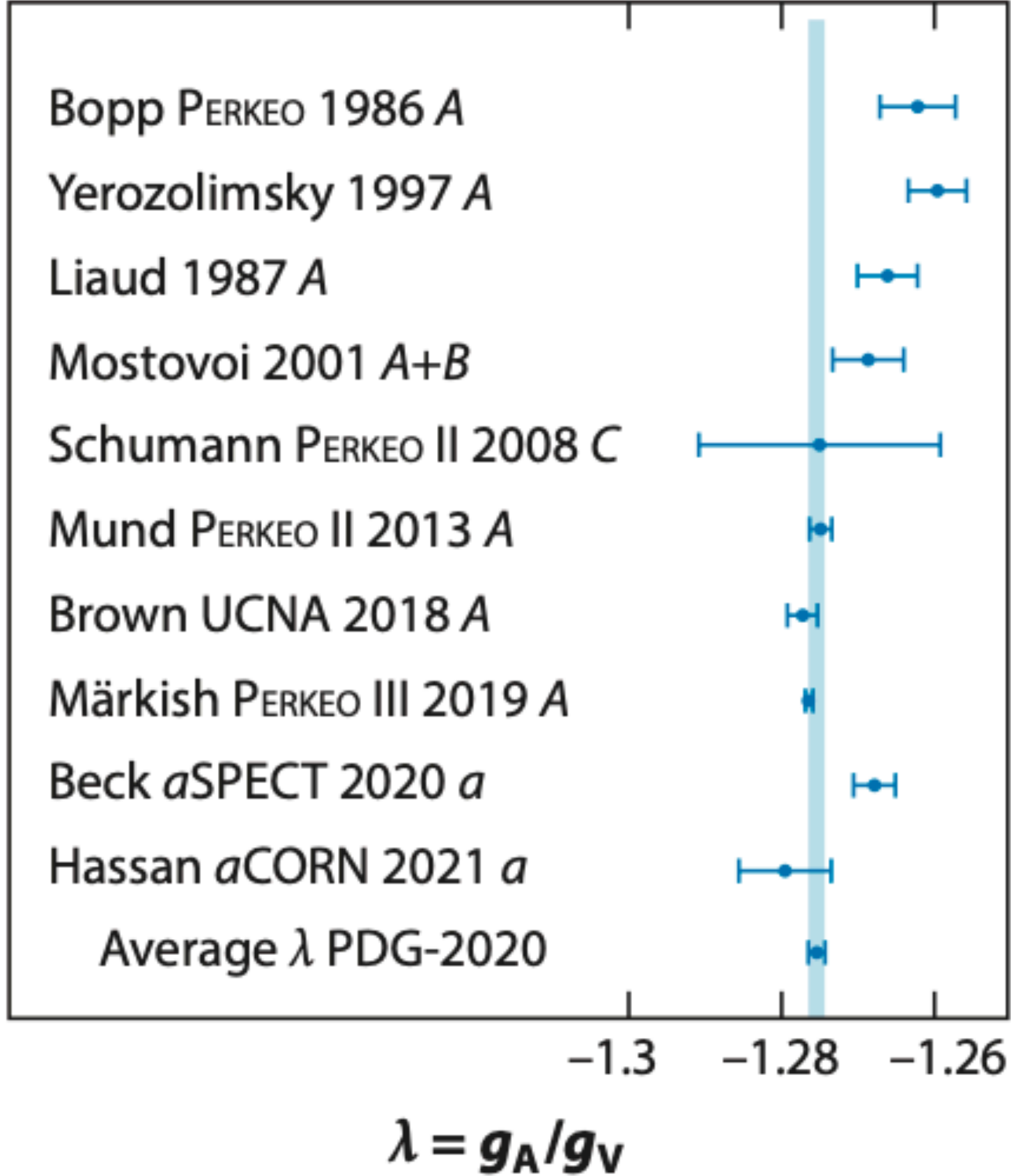
$$\alpha U \equiv 4e^2 [\Pi'_{11}(0) - \Pi'_{33}(0)] .$$

$$T \equiv \frac{1}{\alpha_e} \left(\frac{\Pi_{WW}^{\text{new}}(0)}{m_W^2} - \frac{\Pi_{ZZ}^{\text{new}}(0)}{m_Z^2} \right) = \frac{\rho - 1}{\alpha_e} ,$$

$$S \equiv \frac{4c^2 s^2}{\alpha_e} \left[\frac{\Pi_{ZZ}^{\text{new}}(m_Z^2) - \Pi_{ZZ}^{\text{new}}(0)}{m_Z^2} - \frac{c^2 - s^2}{cs} \frac{\Pi_{Z\gamma}^{\text{new}}(m_Z^2)}{m_Z^2} - \frac{\Pi_{\gamma\gamma}^{\text{new}}(m_Z^2)}{m_Z^2} \right] ,$$

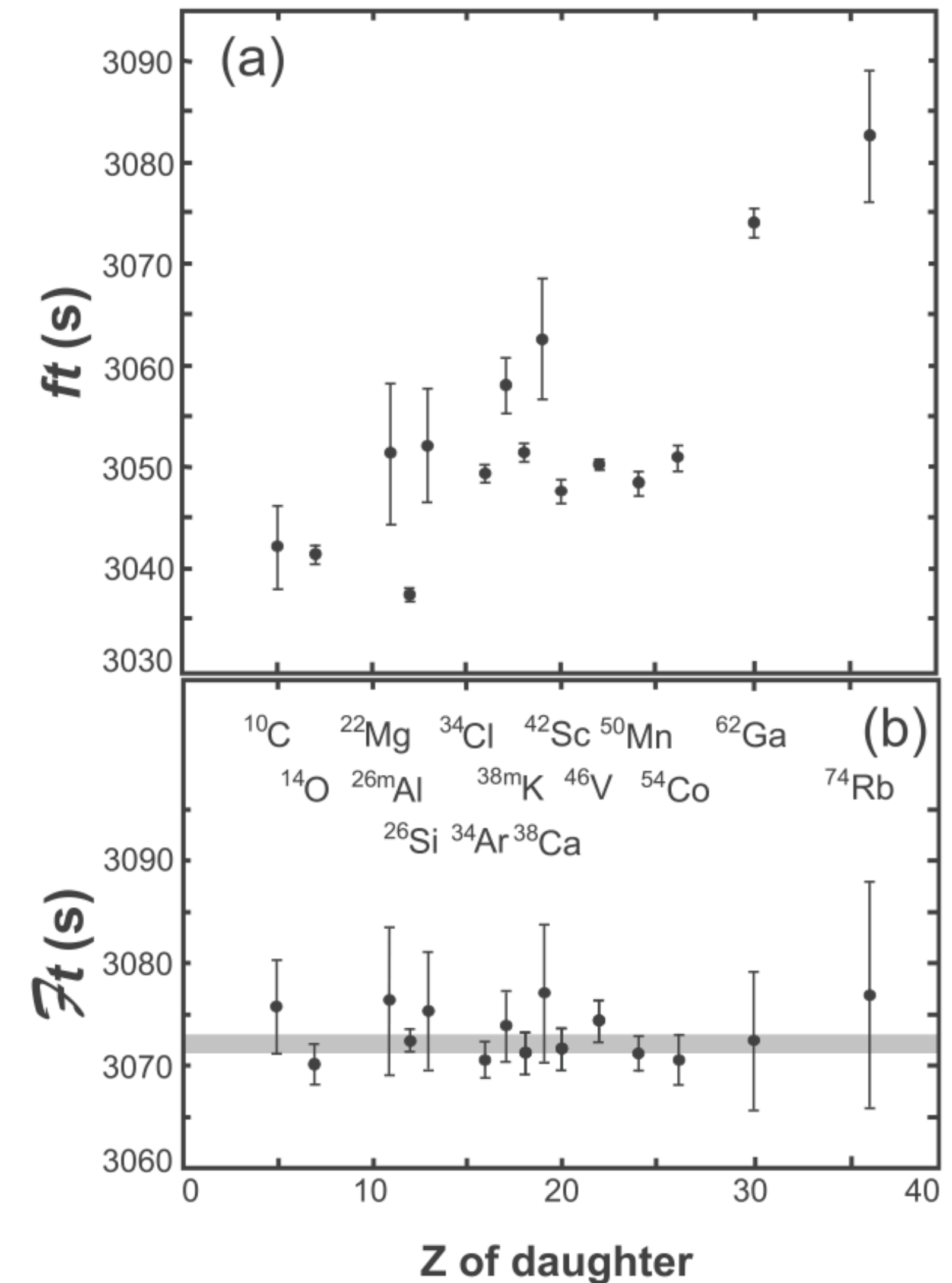
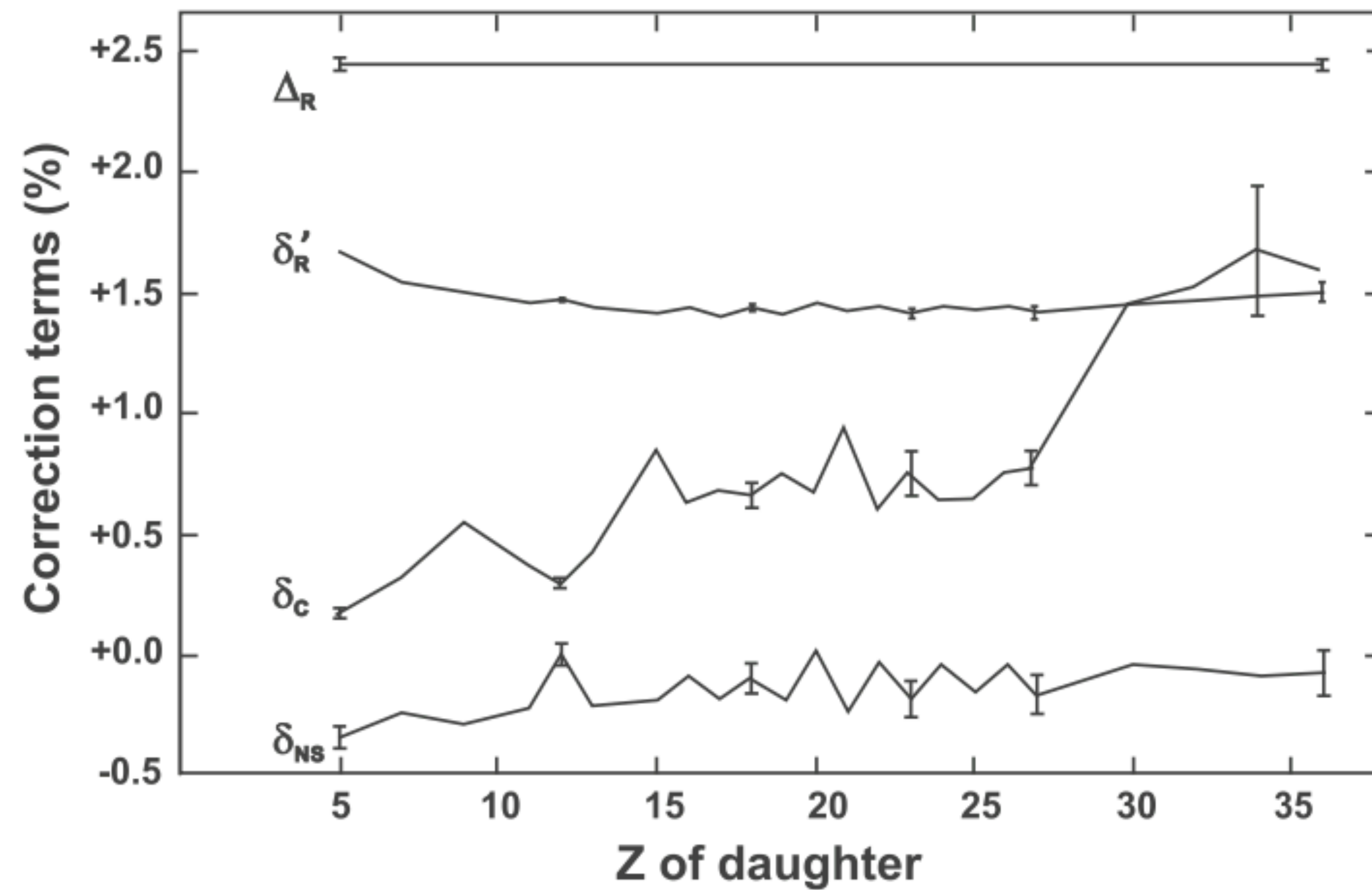
$$U \equiv \frac{4s^2}{\alpha_e} \left[\frac{\Pi_{WW}^{\text{new}}(m_W^2) - \Pi_{WW}^{\text{new}}(0)}{m_W^2} - \frac{c}{s} \frac{\Pi_{Z\gamma}^{\text{new}}(m_Z^2)}{m_Z^2} - \frac{\Pi_{\gamma\gamma}^{\text{new}}(m_Z^2)}{m_Z^2} \right] - S ,$$

Free neutron decay (g_A)

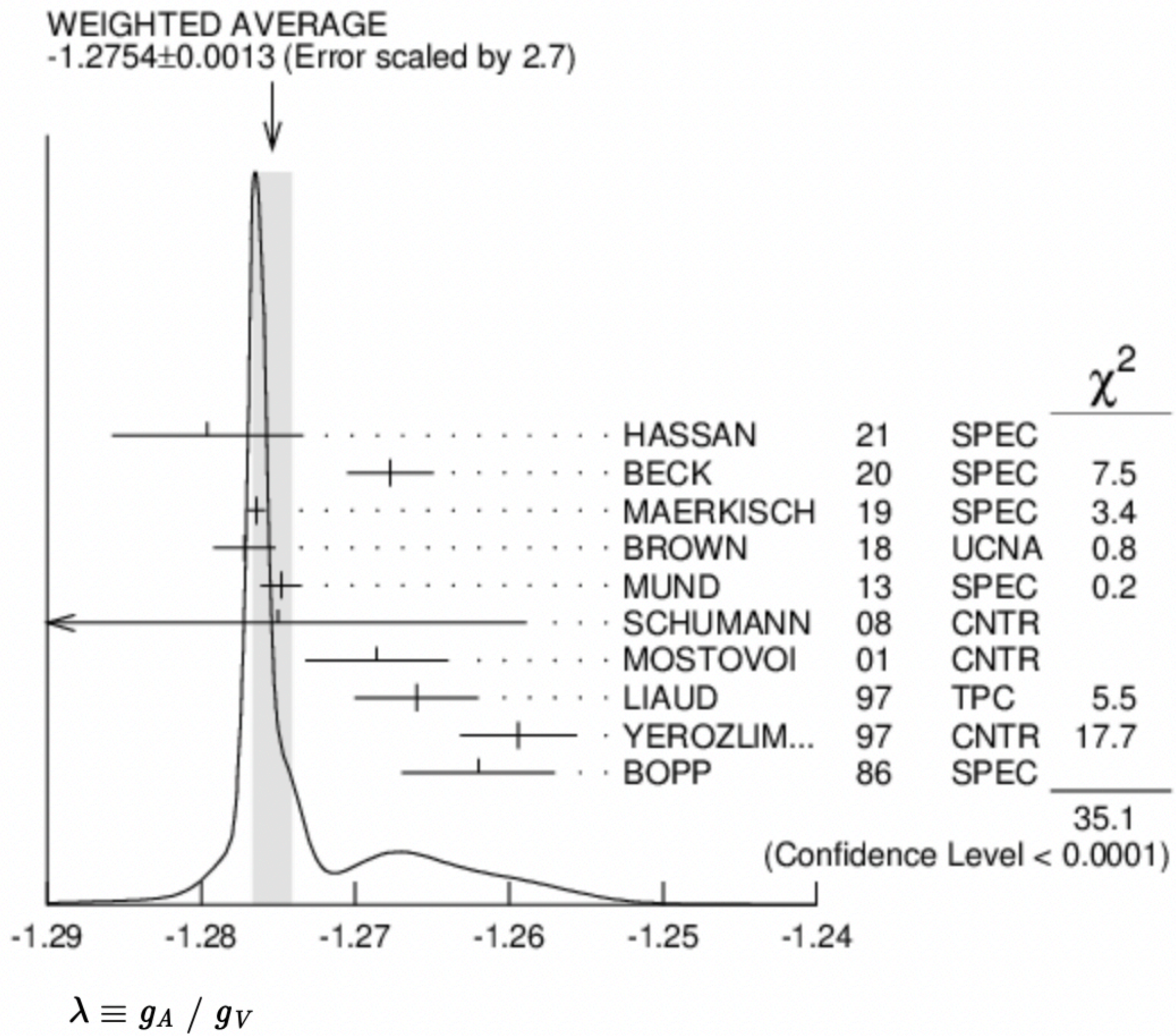
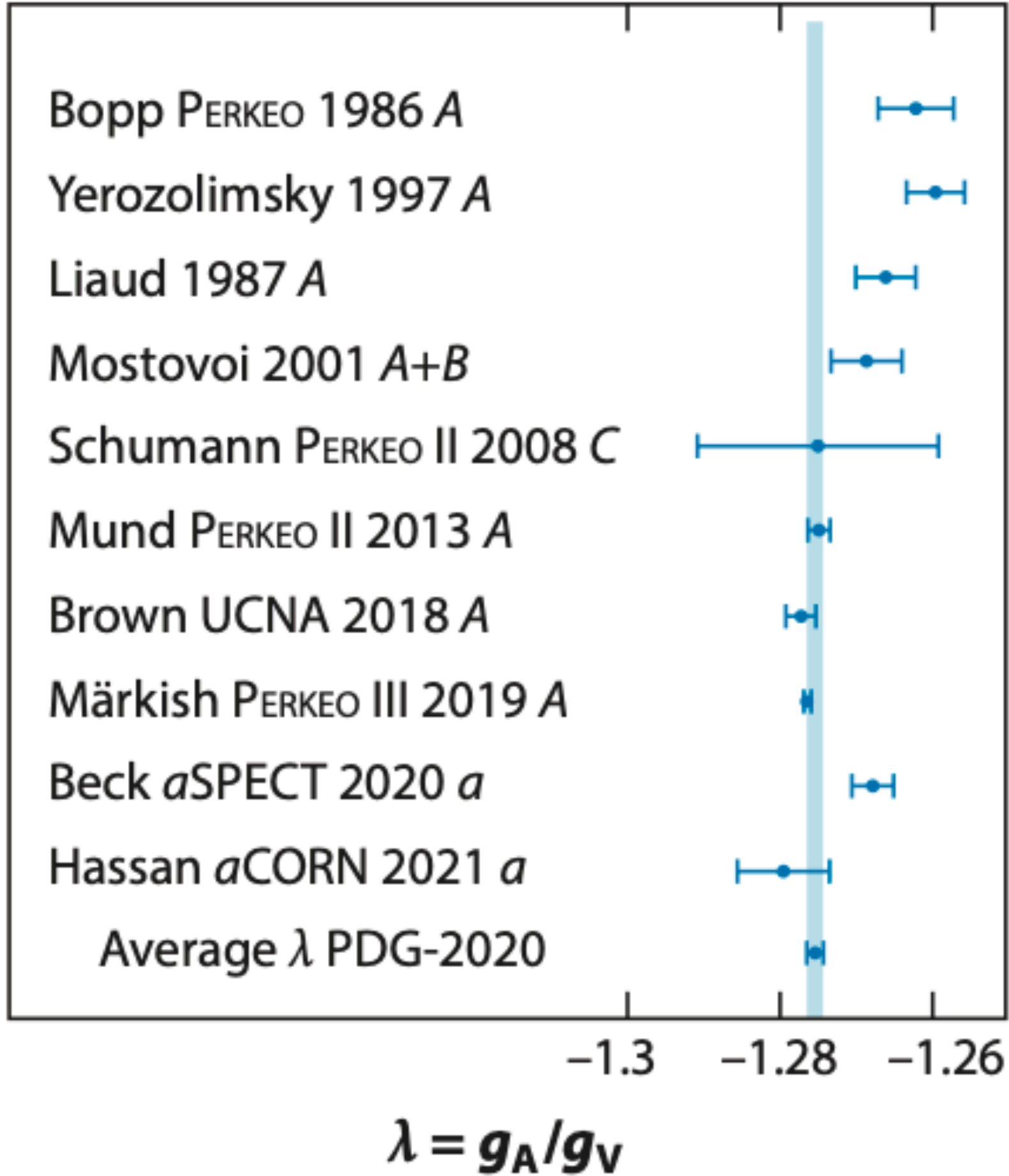


Cabibbo angle anomalies

J.~C.~Hardy and I.~S.~Towner, ``Superaligned  $0^+ \rightarrow 0^+$  nuclear  $\beta$  decays: 2020 critical survey, with implications for  $V_{ud}$  and CKM unitarity,`` Phys. Rev. C 102, no.4, 045501 (2020)



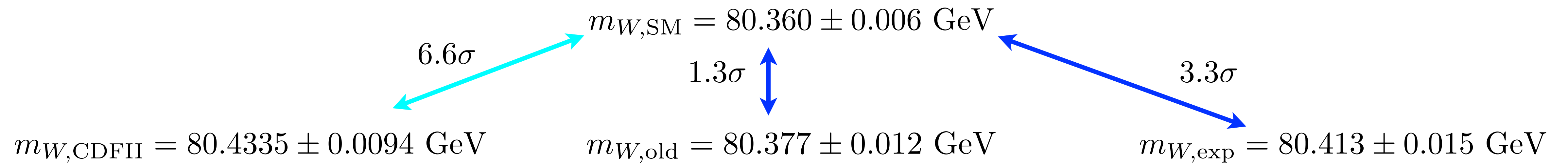
Cabibbo angle anomalies



CAA and EW precision observables

W-boson mass

- CDFII (2022) new physics?



W-boson mass

LEP combination

Phys. Rep. 532 (2013) 119

D0

PRL 108 (2012) 151804

CDF

Science 376 (2022) 6589

LHCb

JHEP 01 (2022) 036

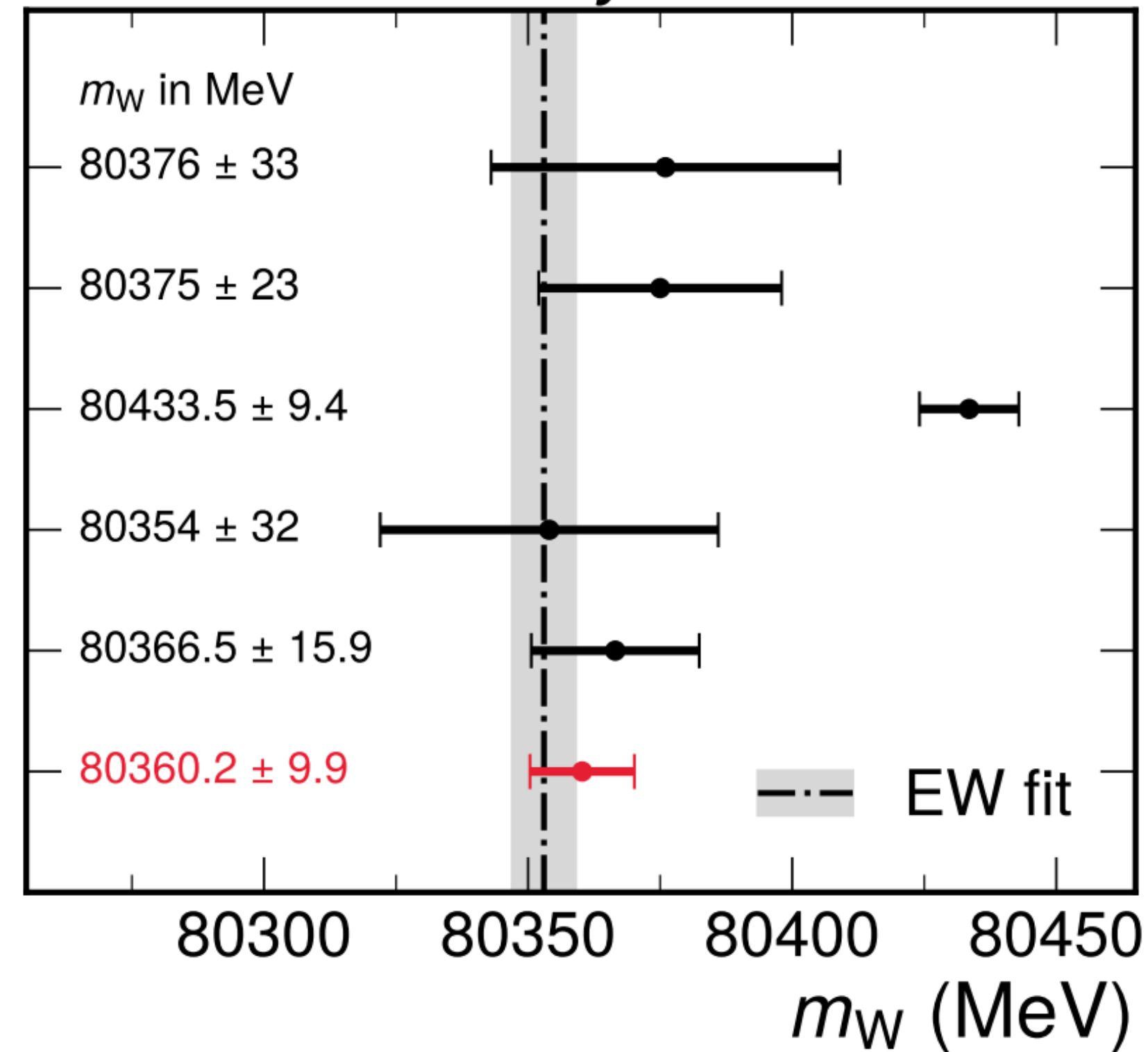
ATLAS

arxiv:2403.15085, subm. to EPJC

CMS

This Work

CMS *Preliminary*



$$m_{W,PDG} = 80.360 \pm 0.012$$

$$m_{W,SM} = 80.356 \pm 0.005$$

CAA and W-boson mass

- Tree level: $m_Z^2 = \frac{e^2 v^2}{2s_W^2 c_W^2}$, $m_W = \frac{e^2 v^2}{2s_W^2}$, $\rho = \frac{m_W^2}{m_Z^2 \cos^2 \theta_W} = 1$
- Loop level is required for electroweak precision tests.
- The most precise measured electroweak quantities are α , G_F , m_Z , from which $\sin \theta_W$ can be defined:

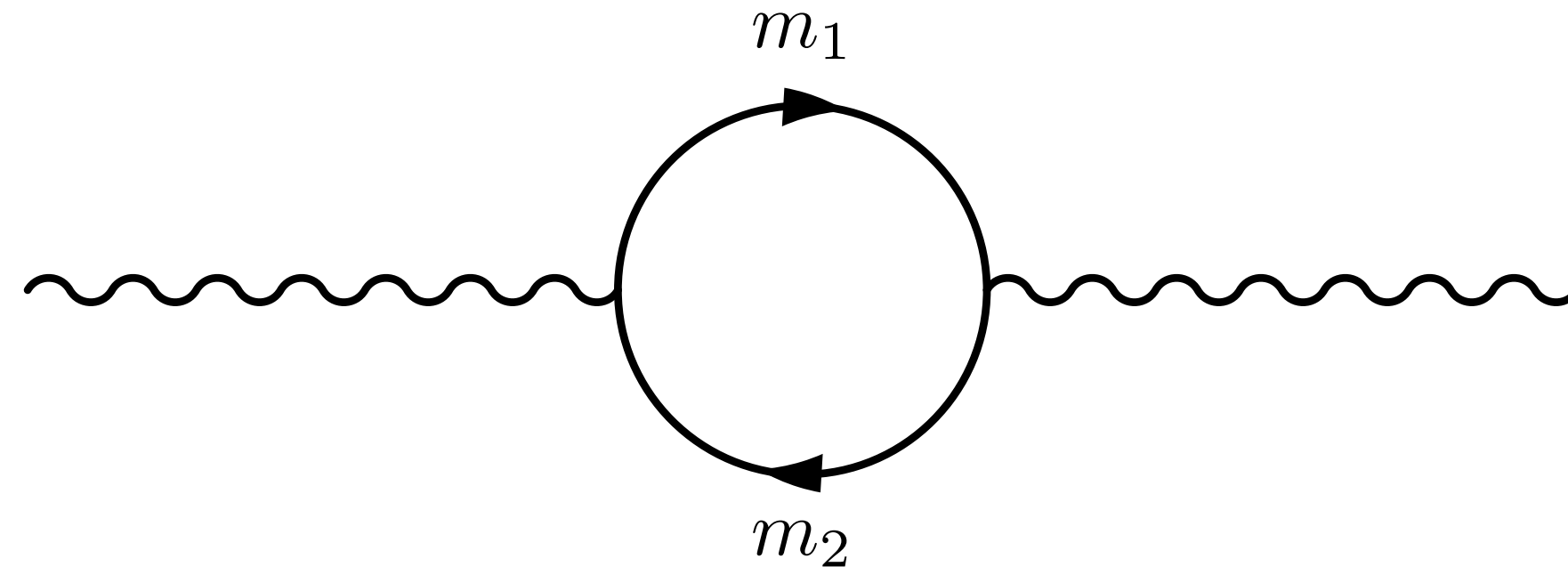
$$\hat{\alpha}, \quad \hat{G}_F, \quad \hat{m}_Z \quad \longrightarrow \quad \sin^2 \hat{\theta}_W \cos^2 \hat{\theta}_W = \frac{\pi \hat{\alpha}(m_Z)}{\sqrt{2} \hat{G}_F \hat{m}_Z^2}, \quad m_{W,\text{SM}}^2 = \hat{c}_W^2 \hat{m}_Z^2 \rho$$

$$G_F = (1 + \delta_\mu)^{-1} G_\mu \quad \longrightarrow \quad s_W^2 \rightarrow s_W^2 + \Delta s_W^2 \quad \longrightarrow \quad m_{W,\text{new}} = m_{W,\text{SM}} - \Delta m_w$$

$$m_{W,\text{SM}} = 80.360 \pm 0.006 \text{ GeV} \quad \xleftarrow{1.3\sigma} \quad m_{W,\text{w/oCDFII}} = 80.377 \pm 0.012 \text{ GeV} \quad \xleftarrow{2.4\sigma} \quad m_{W,\delta_\mu} = 80.344 \pm 0.007 \text{ GeV}$$

- New horizontal gauge bosons go to the opposite direction in m_W shift. If the increased tension is confirmed, some other step would be needed, like mixing with some extra gauge bosons like Z' at the TeV scale or perhaps also with the flavor gauge bosons or a scalar triplet.

Vector-like weak doublet and W-boson mass



$$\delta m_W^2 = m_W^2 - m_{W,\text{SM}}^2 = c^2 m_Z^2 \alpha \left[\frac{c^2}{c^2 - s^2} T - \frac{1}{2(c^2 - s^2)} S + \frac{1}{4s^2} U \right]$$

- Oblique parameters (radiative corrections)

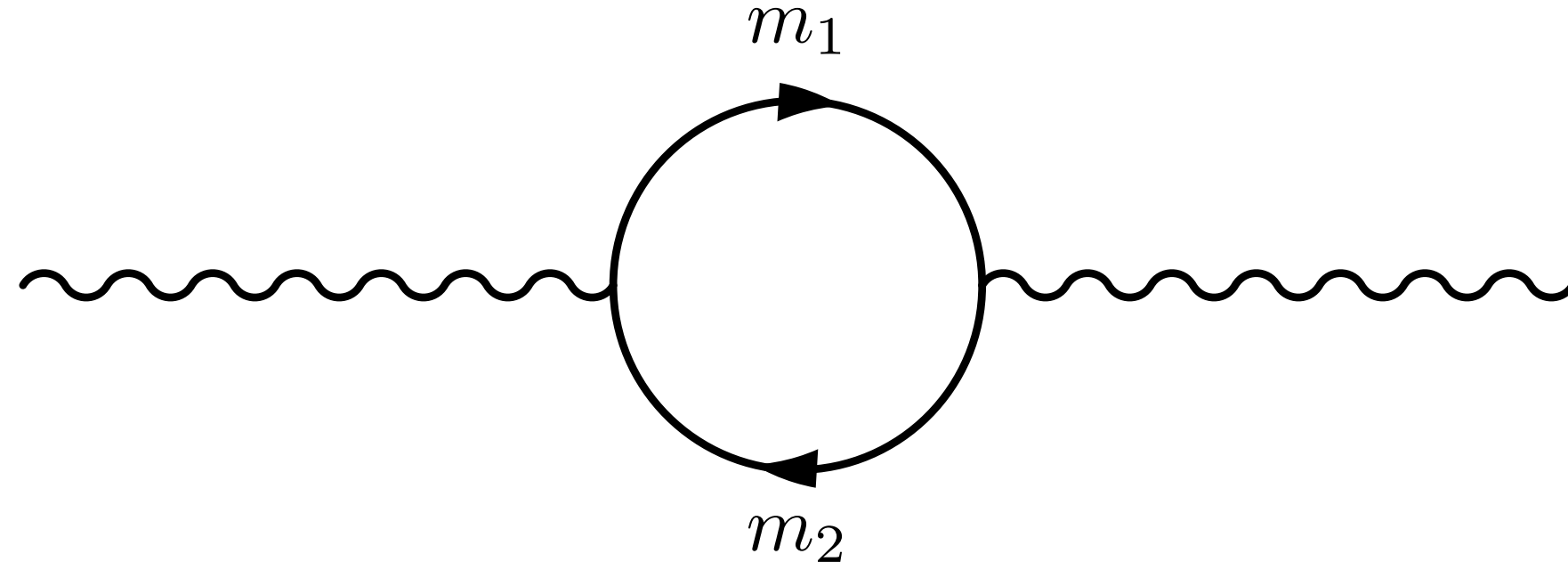
$$T = \frac{4\pi}{s_W^2 c_W^2 m_Z^2} (\Pi_{11}(0) - \Pi_{33}(0)) = \frac{\rho - 1}{\alpha(m_Z)}$$

$$S = \frac{16\pi}{m_Z^2} (\Pi_{33}(m_Z^2) - \Pi_{33}(0) - \Pi_{3Q}(m_Z^2))$$

$$U = \frac{16\pi}{m_W^2} (\Pi_{11}(m_W^2) - \Pi_{11}(0)) - \frac{16\pi}{m_Z^2} (\Pi_{33}(m_Z^2) - \Pi_{33}(0))$$

$$m_{W,\text{SM}}^2 = \hat{c}_W^2 \hat{m}_Z^2 \rho_{\text{SM}} \approx \hat{c}_W^2 \hat{m}_Z^2 \left(1 + \frac{3\alpha(m_Z)}{16\pi s_W^2 c_W^2} \frac{m_t^2}{m_Z^2} \right)$$

Vector-like weak doublet and W-boson mass



$$T \approx \frac{3}{16\pi s_W^2 c_W^2} \left[\frac{4}{3} \frac{M_Q^2}{m_Z^2} \left(\frac{M_{U'}^2 - M_{D'}^2}{M_Q^2} \right)^2 + 2|V_{Ltb'}|^2 \frac{M_Q^2}{m_Z^2} + 2\text{Re}(V_{Ltb'} V_{Rtb'}^*) \frac{2M_Q y_t v_w}{m_Z^2} \left(-2 + \ln \frac{M_Q^2}{m_t^2} \right) + O\left(\frac{v_w^4}{M_Q^4}\right) \right] \approx$$

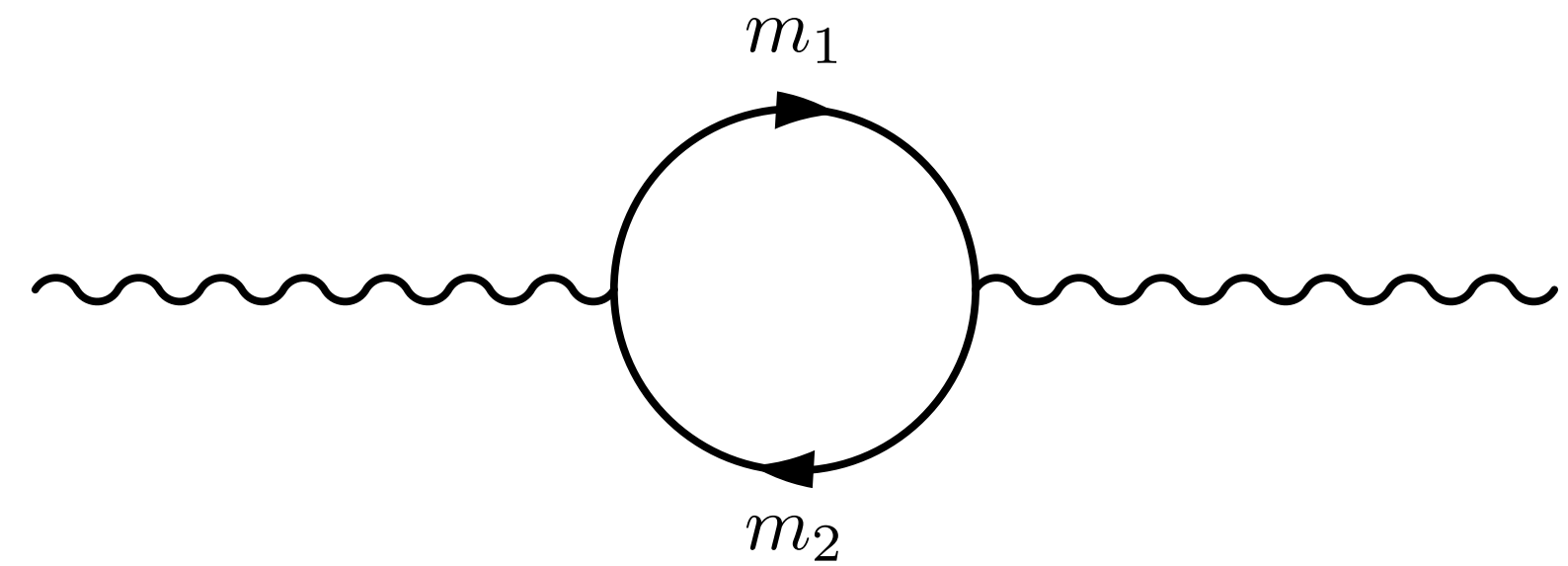
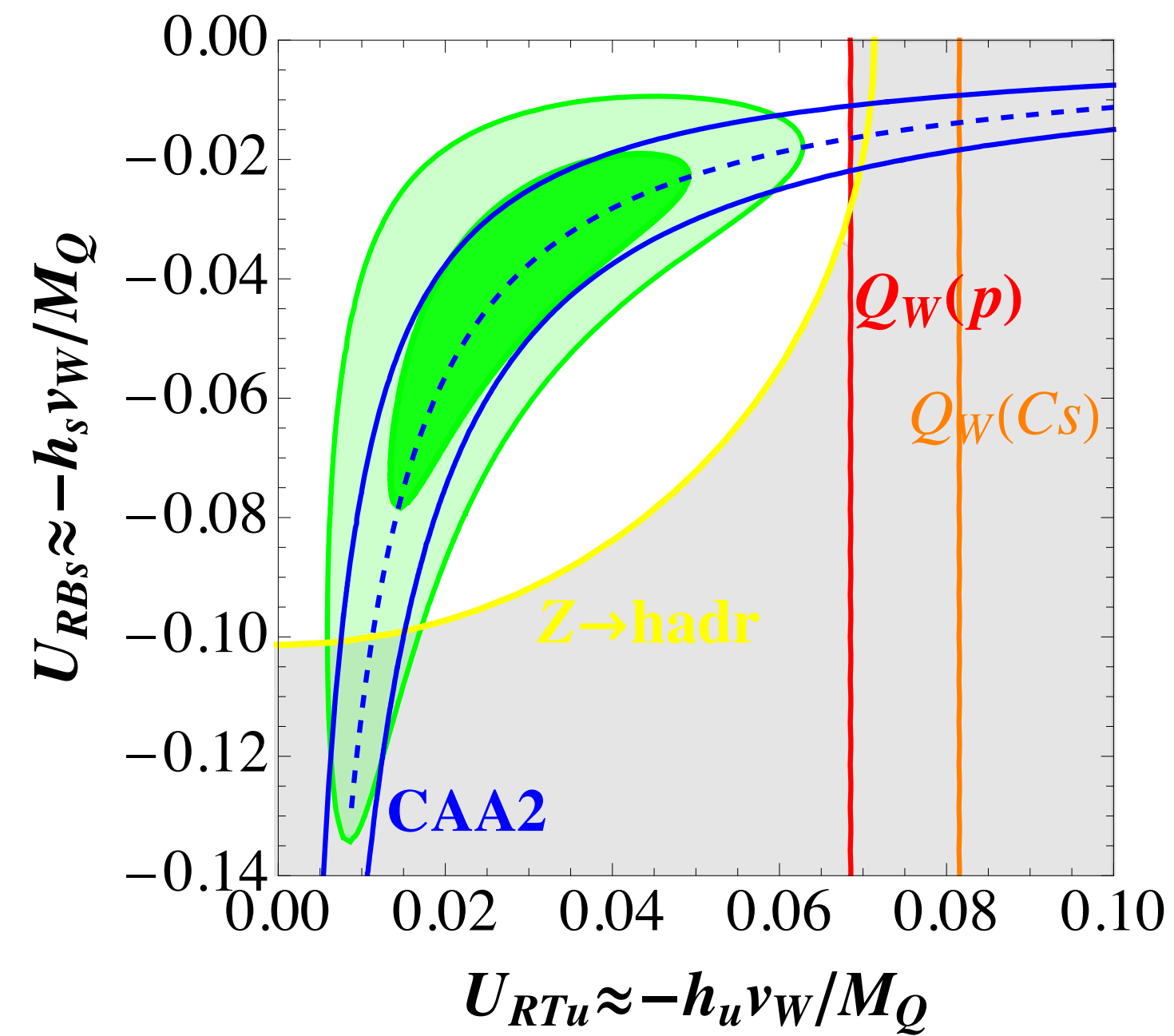
$$\approx \frac{3}{16\pi^2} \frac{2\pi v_w^2}{s_W^2 c_W^2 m_Z^2} \left[\frac{2}{3} \frac{v_w^2}{M_Q^2} \left(\sum_{\alpha=u,c,t} |h_\alpha|^2 - \sum_{\beta=d,s,b} |h_\beta|^2 \right)^2 + y_t^2 |h_t|^2 \frac{v_w^2}{M_Q^2} \left(-3 + 2 \ln \frac{M_Q^2}{m_t^2} \right) + O\left(\frac{v_w^4}{M_Q^4}\right) \right]$$

$$\frac{\delta m_W^2}{c_W^2 m_Z^2} \approx \frac{c_W^2}{c_W^2 - s_W^2} \frac{3}{16\pi^2} \frac{v_w^2}{M_Q^2} \left[\frac{2}{3} \left(|h_d|^2 + |h_s|^2 + |h_b|^2 - |h_u|^2 - |h_c|^2 - |h_t|^2 \right)^2 + y_t^2 |h_t|^2 \left(-3 + 2 \ln \frac{M_Q^2}{m_t^2} \right) \right]$$

- One vector-like doublet with mass $M_Q = 2$ TeV coupling only to the top, the CDFII m_W value would be explained with a Yukawa coupling of $h_t \approx 1.0 \pm 0.1$, corresponding to a mass splitting of 7 GeV.

Vector-like weak doublet and W-boson mass

$$V_{Rus} = U_{RTu}^* U_{RBs} = -1.13(37) \times 10^{-3}$$



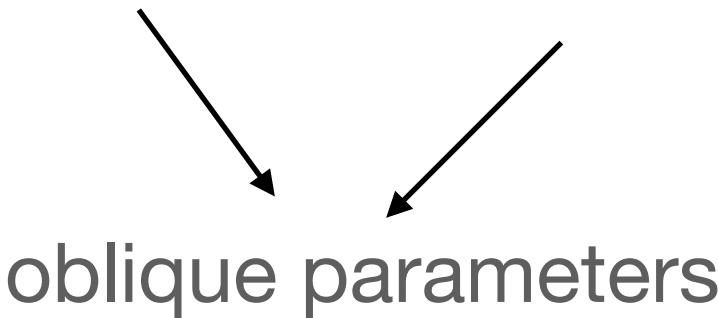
Vector-like doublet of quarks emerges as a favorite explanation of Cabibbo angle anomalies

CAA and EW precision observables

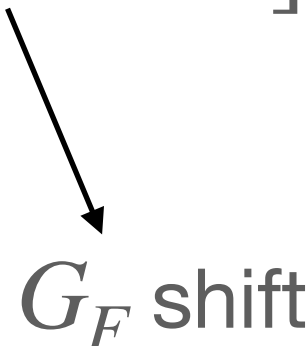
Cirigliano et al 2204.08440

$\mathcal{O}_{HWB}$	$H^\dagger \tau^I H W_{\mu\nu}^I B^{\mu\nu}$
$\mathcal{O}_{HD}$	$ H^\dagger D_\mu H ^2$
$\mathcal{O}_{Hl}^{(3)}$	$(H^\dagger i \overleftrightarrow{D}_\mu^I H)(\bar{l}_p \tau^I \gamma^\mu l_r)$
$\mathcal{O}_{Hq}^{(3)}$	$(H^\dagger i \overleftrightarrow{D}_\mu^I H)(\bar{q}_p \tau^I \gamma^\mu q_r)$
$\mathcal{O}_{ll}$	$(\bar{l}_p \gamma_\mu l_r)(\bar{l}_s \gamma^\mu l_t)$
$\mathcal{O}_{lq}^{(3)}$	$(\bar{l}_p \tau^I \gamma_\mu l_r)(\bar{q}_s \tau^I \gamma^\mu q_t)$

$$\frac{\delta m_W^2}{m_W^2} = 2v_w^2 \frac{s_W c_W}{s_W^2 - c_W^2} \left[2C_{HWB} + \frac{c_W}{2s_W} C_{HD} + \frac{s_W}{c_W} (2C_{H\ell}^{(3)} - C_{\ell\ell}) \right]$$



oblique parameters



G_F shift

Assuming $U(3)^5$ flavour symmetry

$$\Delta_{\text{CKM}} = 4v^2 \left[C_{Hq}^{(3)} - C_{H\ell}^{(3)} + (C_{\ell\ell})_{1221} - C_{\ell q}^{(3)} \right]$$

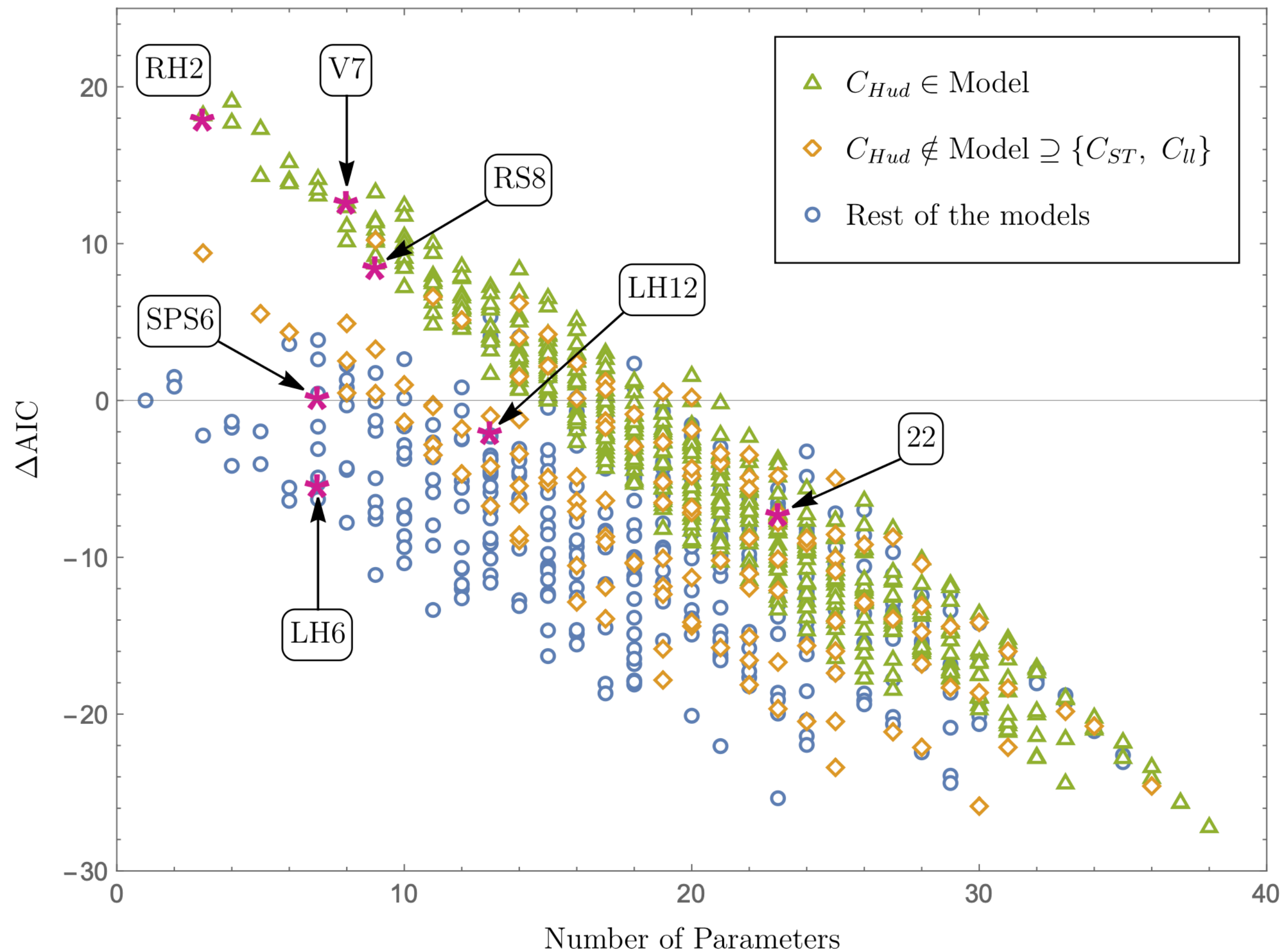


strongly constrained by LHC

Global fit of electroweak precision observables gives a too large value of Δ_{CKM}

Fit of CAA, EW precision observables, LHC

Cirigliano et al. 2311.00021



$$Q_{Hud} + \text{h.c.}$$

$$i(\tilde{H}^\dagger D_\mu H)(\bar{u}_p \gamma^\mu d_r)$$

$$C_{Hud11}, C_{Hud12}$$

vector-like doublets
at tree level

$$C_{ST} \rightarrow c_W C_{HD} + 4s_W C_{HWB}$$

vector-like doublets
at loop level

Present situation of the CKM first row

Present situation of CKM first row

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 \quad \longrightarrow \quad \cos^2 \theta_C + \sin^2 \theta_C = 1$$

Three determinations: $|V_{us}|_A = \sin \theta_C$, $|V_{us}/V_{ud}|_B = \tan \theta_C$, $|V_{ud}|_C = \cos \theta_C$

- $|V_{us}|$ can be directly determined from **semileptonic** $K \rightarrow \pi \ell \nu$ ($K\ell 3$) decays
($K_L e 3$, $K_S e 3$, $K^\pm e 3$, $K^\pm \mu 3$, $K_L \mu 3$, $K_S \mu 3$)

Experimental inputs: KLOE, KLOE-2, KTeV, NA48 (Moulson 1704.04104, only small updates since 2012)

$$\Gamma_{K\ell 3} = \frac{G_F^2 M_K^5 C_K^2}{192\pi^3} |f_+(0) V_{us}|^2 I_{K\ell}^{(0)} S_{EW} (1 + \delta_{EM}^{K\ell} + \delta_{SU(2)}^{K\pi})$$

$K^0 \rightarrow \pi^-$ transition **form factor** at zero momentum transfer
 FLAG average of 4 active quark flavour lattice QCD calculations:
 $f_+(0) = 0.9698(17)$

Isospin factor (1 or $1/\sqrt{2}$)
 phase space factor
 channel independent short distance EW radiative corrections
 long distance EM radiative corrections
 isospin breaking corrections

Present situation of CKM first row

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$K^0 \rightarrow \pi^-$ transition **form factor** at zero momentum transfer
 FLAG average of 4 active quark flavour lattice QCD calculations:
 $f_+(0) = 0.9698(17)$

Isospin factor (1 or $1/\sqrt{2}$)
 phase space factor
 channel independent short distance EW radiative corrections
 long distance EM radiative corrections
 isospin breaking corrections
 Recent re-analysis
 Seng et al. 1910.13208, 2009.00459, 2103.00975, 2103.04843, 2107.14708, 2203.05217 Ma et al. 2102.12048,
 (also with lattice QCD) consistent with ChPT (Cirigliano et al. 08074507) **with much smaller uncertainties**

Present situation of CKM first row

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 \quad \longrightarrow \quad \cos^2 \theta_C + \sin^2 \theta_C = 1$$

Three determinations: $|V_{us}|_A = \sin \theta_C$, $|V_{us}/V_{ud}|_B = \tan \theta_C$, $|V_{ud}|_C = \cos \theta_C$

- $|V_{us}|$ can be directly determined from **semileptonic** $K \rightarrow \pi \ell \nu$ ($K\ell 3$) decays
($K_L e 3$, $K_S e 3$, $K^\pm e 3$, $K^\pm \mu 3$, $K_L \mu 3$, $K_S \mu 3$)

Experimental results + radiative corrections 2203.05217:

$$f_+(0)|V_{us}| = 0.21634 \pm 0.00038$$

- Average of 4-flavour lattice QCD calculations for the vector form factor $\mathbf{f}_+(\mathbf{0}) = \mathbf{0.9698(17)}$
(FLAG 2024)

$$|\mathbf{V}_{us}|_A = \mathbf{0.22308(55)}$$

Present situation of CKM first row

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 \quad \longrightarrow \quad \cos^2 \theta_C + \sin^2 \theta_C = 1$$

Three determinations: $|V_{us}|_A = \sin \theta_C$, $|V_{us}/V_{ud}|_B = \tan \theta_C$, $|V_{ud}|_C = \cos \theta_C$

- The ratio of **leptonic** decay rates $K \rightarrow \mu\nu(\gamma)$ and $\pi \rightarrow \mu\nu(\gamma)$ determines:

Experimental input dominated by KLOE, KLOE-2 (Moulson 1704.04104, not changed since 2014)

$$\frac{|V_{us}|}{|V_{ud}|} \frac{f_{K^\pm}}{f_{\pi^\pm}} = \left(\frac{\Gamma_{K \rightarrow \mu\nu(\gamma)} m_{\pi^\pm}}{\Gamma_{\pi \rightarrow \mu\nu(\gamma)} m_{K^\pm}} \right)^{1/2} \frac{1 - m_\mu^2/m_{\pi^\pm}^2}{1 - m_\mu^2/m_{K^\pm}^2} \left(1 - \frac{\Delta_{RC+IB}^{K\pi}}{2} \right)$$

charged kaon and charged pion **decay constants**
from lattice QCD calculations

residual EM radiative corrections:
Chiral Perturbation Theory -1.12(21)%
Confirmed by **lattice QCD calculation** of radiative
and isospin-breaking corrections
-1.26(14)% (1904.08731), -0.86(40)% (2211.12865)

Present situation of CKM first row

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 \quad \longrightarrow \quad \cos^2 \theta_C + \sin^2 \theta_C = 1$$

Three determinations: $|V_{us}|_A = \sin \theta_C$, $|V_{us}/V_{ud}|_B = \tan \theta_C$, $|V_{ud}|_C = \cos \theta_C$

- The ratio of **leptonic** decay rates $K \rightarrow \mu\nu(\gamma)$ and $\pi \rightarrow \mu\nu(\gamma)$ determines:

$$\frac{f_{K^\pm}}{f_{\pi^\pm}} \frac{|V_{us}|}{|V_{ud}|} = 0.27599 \pm 0.00037$$

- 4-flavour lattice QCD for the decay constants ratio $\mathbf{f_K/f_\pi} = \mathbf{1.1934(19)}$ FLAG 2024 ($f_K/f_\pi = 1.1932(21)$ FLAG21)

$$\frac{|\mathbf{V_{us}}|}{|\mathbf{V_{ud}}|_B} = \mathbf{0.23126(48)}$$

Present situation of CKM first row

$$\boxed{|V_{ud}|^2} + |V_{us}|^2 + |V_{ub}|^2 = 1 \quad \longrightarrow \quad \cos^2 \theta_C + \sin^2 \theta_C = 1$$

Three determinations: $|V_{us}|_A = \sin \theta_C$, $|V_{us}/V_{ud}|_B = \tan \theta_C$, $\boxed{|V_{ud}|_C = \cos \theta_C}$

- Superaligned nuclear beta-decays and free neutron decay:

$$|V_{ud}|_{0^+ \rightarrow 0^+}^2 = \frac{K}{2G_F^2 \mathcal{F} t (1 + \Delta_R^V)}, \quad |V_{ud}|_n^2 = \frac{K / \ln 2}{G_F^2 \mathcal{F}_n \tau_n (1 + 3g_A^2)(1 + \Delta_R^V)}$$

- $\mathbf{K} = 2\pi^3 \log 2 / m_e^5$, $\mathbf{G_F} = \mathbf{G_\mu} = 1.1663787(6) \times 10^{-5} \text{ GeV}^{-2}$;

Present situation of CKM first row

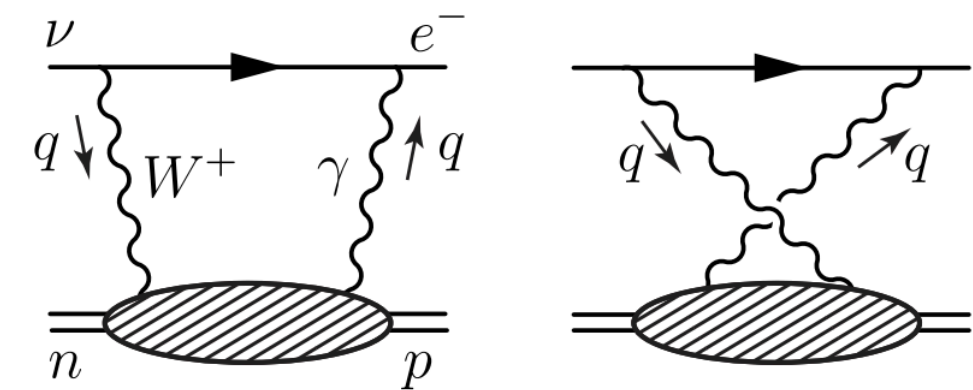
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- $\mathbf{K} = 2\pi^3 \log 2 / m_e^5$, $\mathbf{G_F} = \mathbf{G_\mu} = 1.1663787(6) \times 10^{-5} \text{ GeV}^{-2}$;
- Δ_R^V are short-distance (transition independent, single-nucleon) radiative corrections (γW box diagram).



- Marciano & Sirlin 2006 $\Delta_R^V = 0.02361(38)$ adopted by PDG 2018.
- New calculation with reduced theory uncertainty Seng et al. 2018 $\Delta_R^V = \mathbf{0.02467(22)}$. Other more recent studies confirmed the shift of Δ_R^V .

Present situation of CKM first row

- The most precise determination is from **superaligned nuclear beta-decays** ($0^+ \rightarrow 0^+$ Fermi transitions) determining vector coupling $\mathbf{G}_V = \mathbf{G}_F |\mathbf{V}_{ud}|$:

Hardy and Towner, Phys. Rev. C 102, no.4, 045501 (2020)
(new survey after 2015)

transition dependent radiative correction
function of electron energy and Z

$$\mathcal{F}t = ft (1 + \delta'_R)(1 + \delta_{NS} - \delta_C) = \frac{K}{2 G_F^2 |V_{ud}|^2 (1 + \Delta_R^V)}$$

transition dependent radiative correction,
depends on **nuclear structure** (neutron in nucleus)

$$\delta_{NS} = \delta_{NS,A} + \delta_{NS,B} + \delta_{NS,E}$$

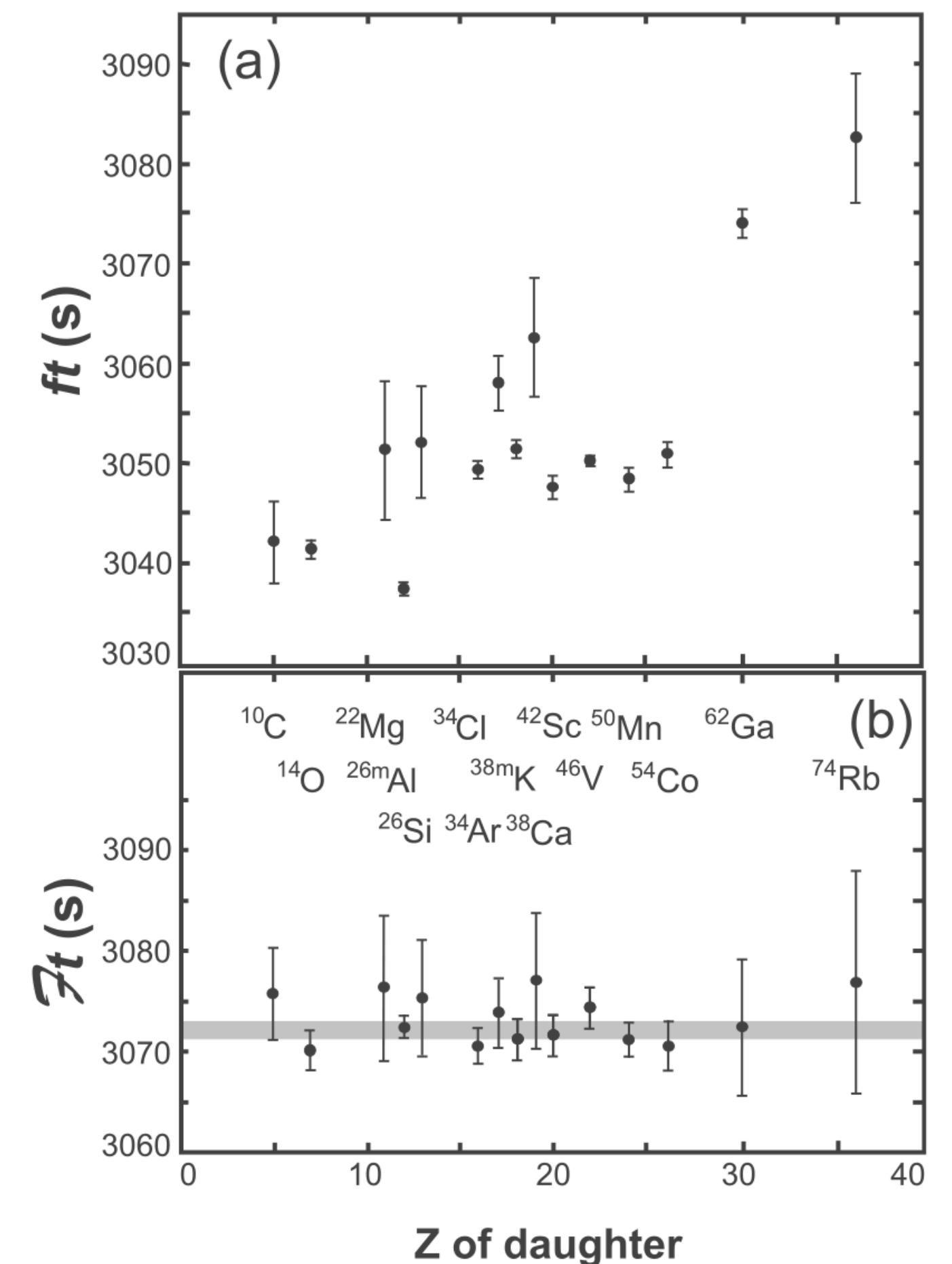
$$\delta_{NS,A} \text{ Seng et al.1812.03352}$$

$$\delta_{NS,E} \text{ Gorchtein 1812.04229}$$

uncertainty increased by 2.6 factor

isospin breaking corrections,
depends on nuclear structure
(aligns ft the most)

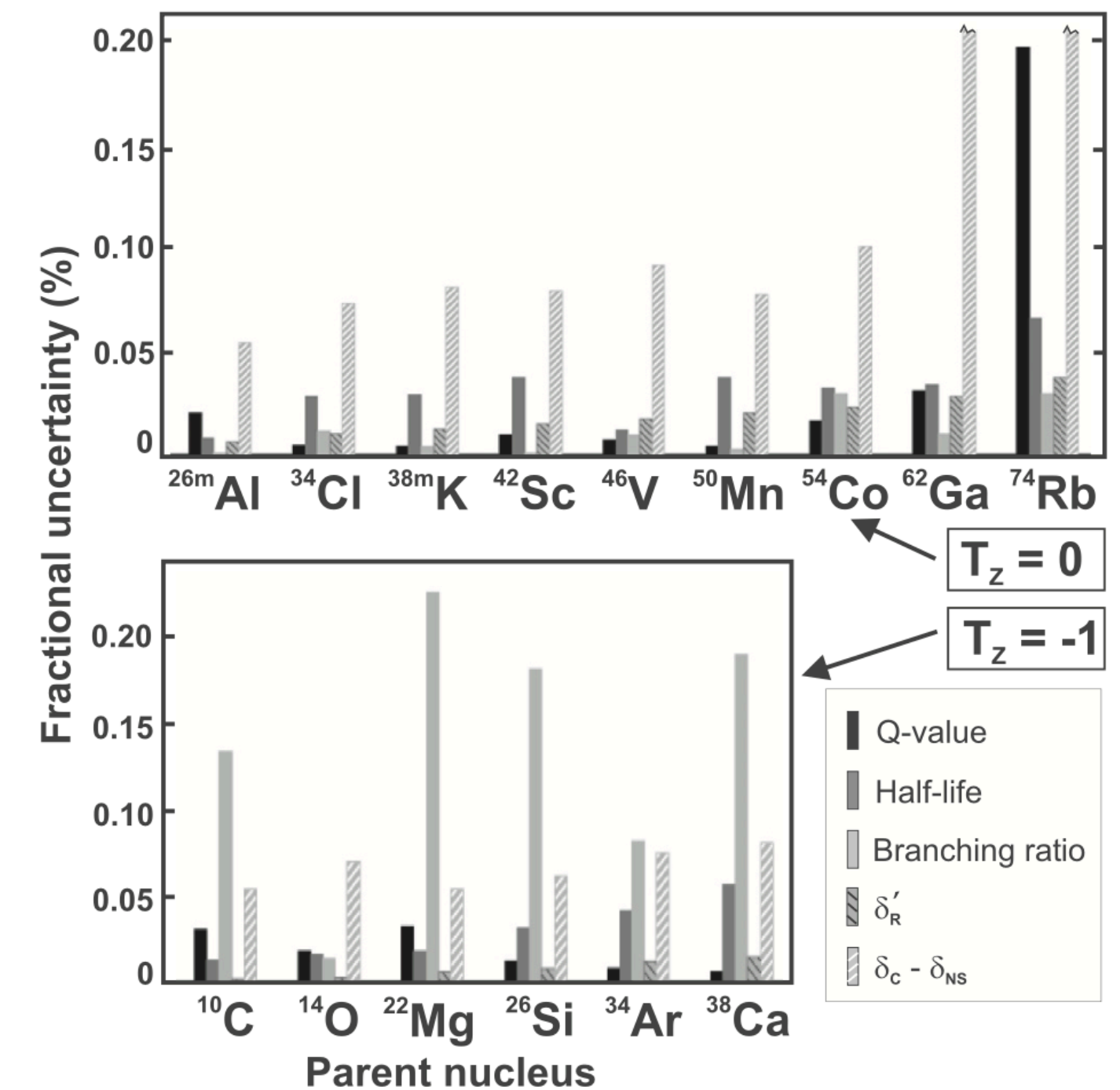
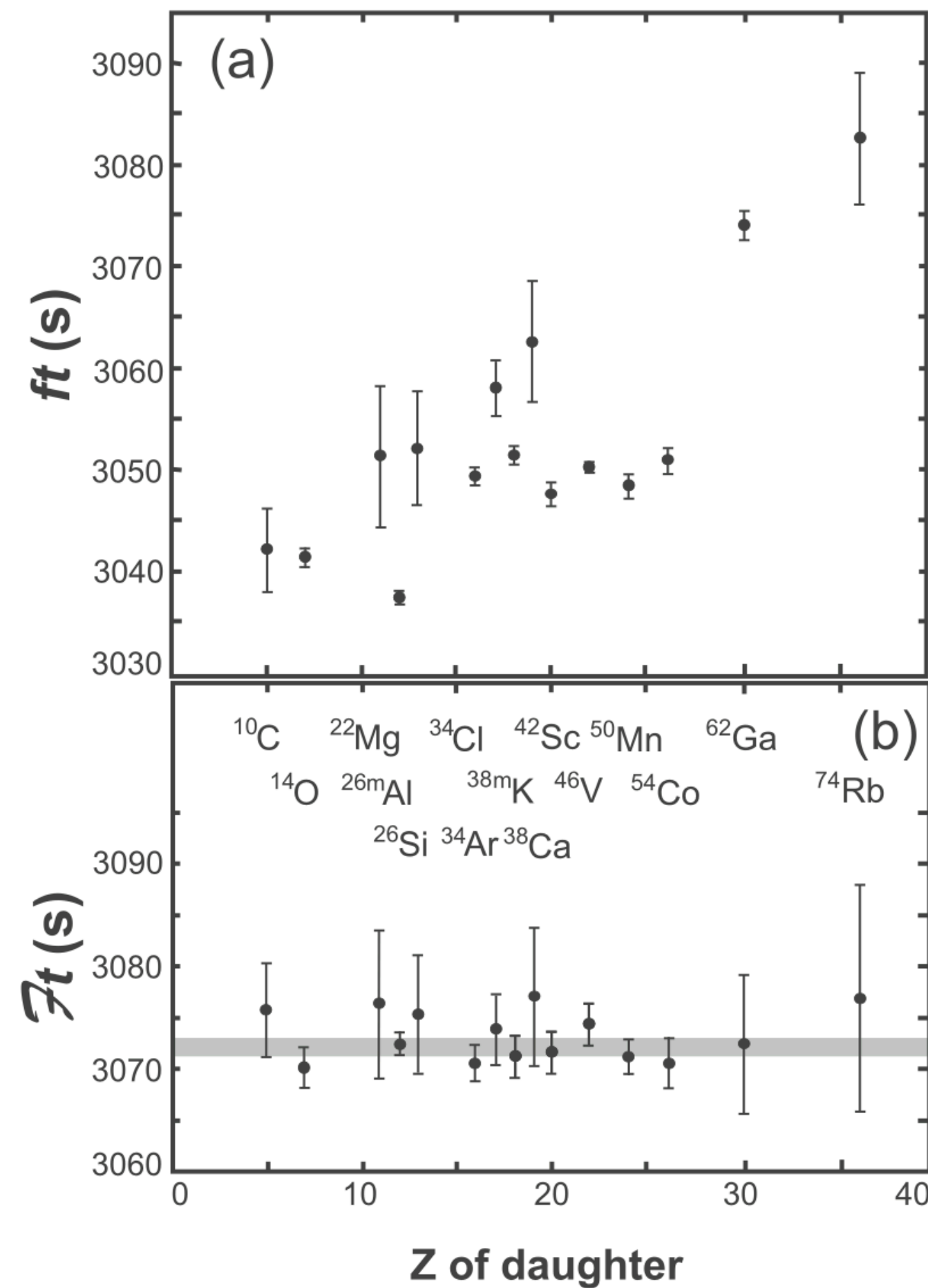
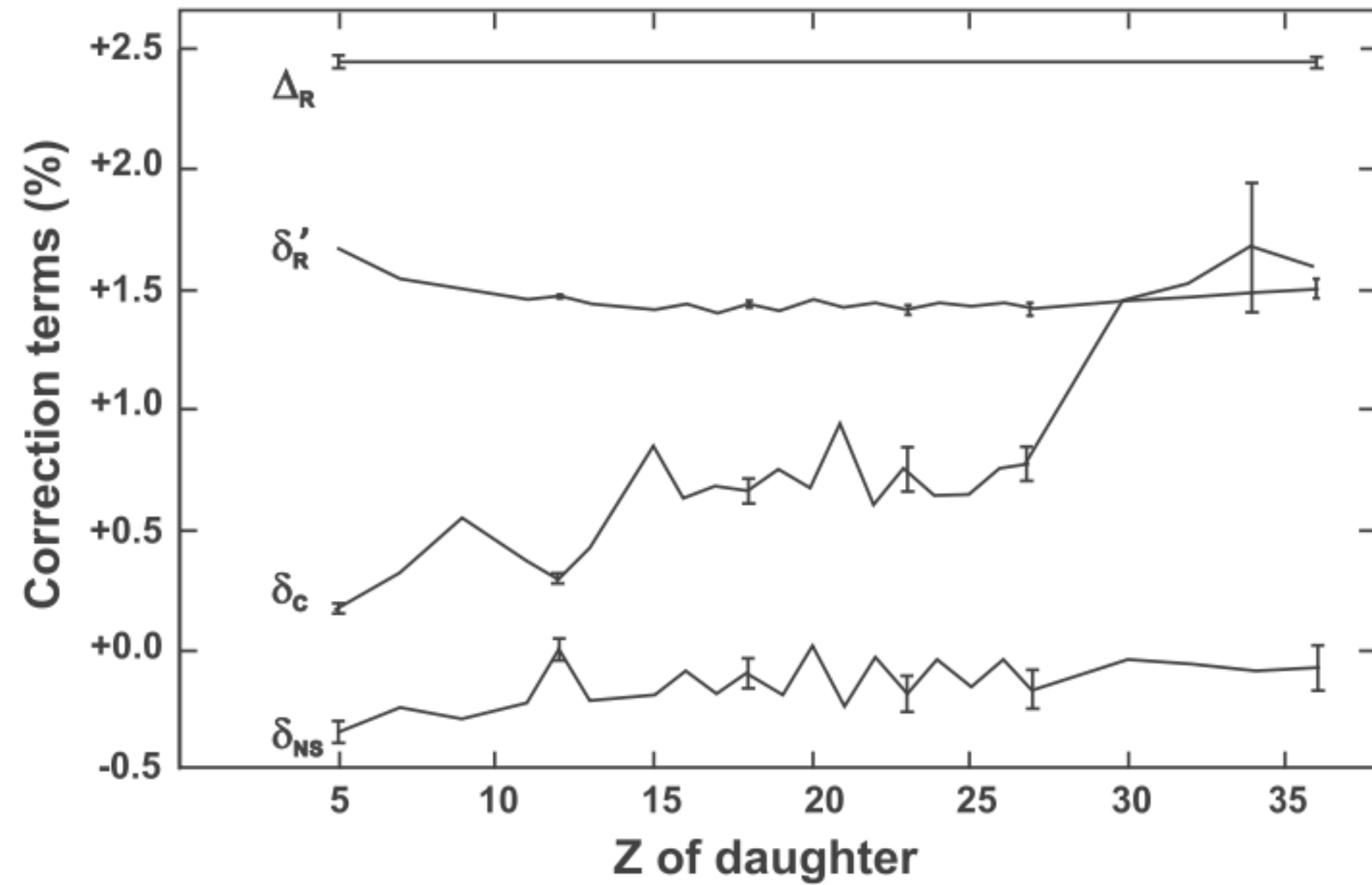
- $K = 2\pi^3 \log 2 / m_e^5$, $G_F = G_\mu = 1.1663787(6) \times 10^{-5} \text{ GeV}^{-2}$, f statistical rate function (depends on the total transition energy), t partial half-life
- $\mathcal{F}t = ft(1 + \delta'_R)(1 + \delta_{NS} - \delta_C)$ nucleus-independent value obtained from ft-values averaging **15 transitions**



Present situation of CKM first row

Hardy and Towner, Phys. Rev. C 102, no.4, 045501 (2020) (new survey after 2015)

$$\mathcal{F}t = ft(1 + \delta'_R)(1 + \delta_{NS} - \delta_C) = \frac{K}{2 G_F^2 |V_{ud}|^2 (1 + \Delta_R^V)}$$



Present situation of CKM first row

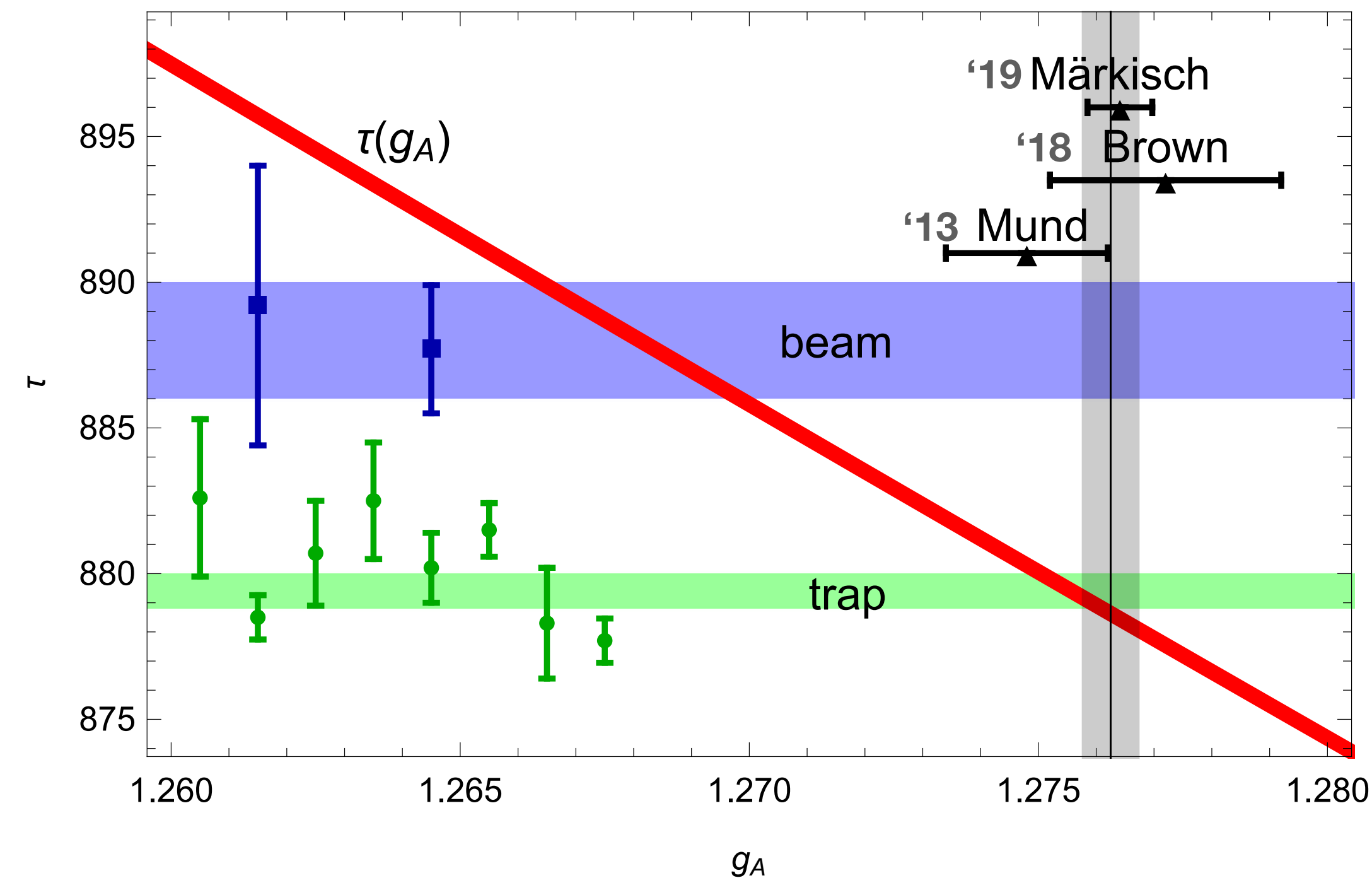
- The most precise determination is from superallowed nuclear beta-decays ($0^+ \rightarrow 0^+$ Fermi transitions) determining vector coupling $\mathbf{G}_V = \mathbf{G}_F |\mathbf{V}_{ud}|$:

$$|V_{ud}|_{0^+ \rightarrow 0^+}^2 = \frac{K}{2 G_F^2 \mathcal{F}t (1 + \Delta_R^V)}$$

- $K = 2\pi^3 \log 2 / m_e^5$, $G_F = G_\mu = 1.1663787(6) \times 10^{-5} \text{ GeV}^{-2}$, f statistical rate function (depends on the total transition energy), t partial half-life;
- $\mathcal{F}t = \mathbf{ft}(1 + \delta'_R)(1 + \delta_{NS} - \delta_C)$ nucleus-independent value obtained from ft -values after including the nucleus-dependent radiative (δ'_R , δ_{NS}) and isospin-breaking correction δ_C , averaging **15 transitions**, recently updated (**uncertainty increased by 2.6 factor** due to nuclear structure corrections δ_{NS} (1812.03352, 1812.04229), which **now dominate uncertainty**): $\mathcal{F}t = \mathbf{3072.24(1.85)}$ s (Hardy & Towner 2020);

$$|V_{ud}|_{0^+ \rightarrow 0^+} = 0.97367(31)$$

Cabibbo angle and free neutron



$$G_V^2 = \frac{K / \ln 2}{\mathcal{F}_n \tau_n (1 + 3g_A^2)(1 + \Delta_R^V)}$$

$$G_V^2 = \frac{K}{2\mathcal{F}t(1 + \Delta_R^V)}$$



$$\tau_n = \frac{2\mathcal{F}t}{\ln 2 \mathcal{F}_n (1 + 3g_A^2)} = \frac{5172.3(3.2) \text{ s}}{(1 + 3g_A^2)}$$

- $|V_{ud}|_n^2 = \frac{K / \ln 2}{G_F^2 \mathcal{F}_n \tau_n (1 + 3g_A^2)(1 + \Delta_R^V)} = \frac{5024.5(6) \text{ s}}{\tau_n (1 + 3g_A^2)(1 + \Delta_R^V)}$
- $\mathcal{F}_n = f_n(1 + \delta'_R)$ f -value corrected by LD QED correction.
- $\tau_n^{\text{bottle}} = 878.4(5) \text{ s}$.
- $\tau_n^{\text{beam}} = 888.0(2.0) \text{ s}$ (4σ tension with SM and bottle experiments).
- $g_A = 1.27624(50)$ axial-vector coupling from parity-violating β -asymmetry parameter A from polarized neutrons
- (some tension with aSPECT (a coefficient) from the recoil spectrum of protons, but agreement with the other recent measurement of the a coefficient by aCORN).
- $\tau_n^{\text{bottle}} \& \Delta_R^V \& g_A \longrightarrow |V_{ud}|_n = 0.97383(44)$
- G_V and Δ_R^V cancel out even in BSM $G_V \neq G_F |V_{ud}|$, $g_A = -G_A/G_V$
- new Δ_R^V calculations have no influence on τ_n determination.
- $g_A = 1.27624(50) \rightarrow \tau_n^{\text{SM}} = 878.7(8) \text{ s} \approx \tau_{\text{bottle}}$

Present situation of CKM first row

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 \longrightarrow \cos^2 \theta_C + \sin^2 \theta_C = 1$$

Three determinations: $|V_{us}|_A = \sin \theta_C$, $|V_{us}/V_{ud}|_B = \tan \theta_C$, $|V_{ud}|_C = \cos \theta_C$

- $|V_{us}|$ can be directly determined from **semileptonic** $K \rightarrow \pi \ell \nu$ ($K\ell 3$) decays
($K_L e 3$, $K_S e 3$, $K^\pm e 3$, $K^\pm \mu 3$, $K_L \mu 3$, $K_S \mu 3$)

Recent re-analysis of long distance EM radiative corrections Seng et al. 2203.05217:

	$ V_{us} f_+^{K^0 \pi^-}(0) $
$K_L e$	$0.21617(46)_{\text{exp}}(10)_{I_K}(4)_{\delta_{\text{EM}}}$
$K_S e$	$0.21530(122)_{\text{exp}}(10)_{I_K}(4)_{\delta_{\text{EM}}}$
$K^+ e$	$0.21714(88)_{\text{exp}}(10)_{I_K}(21)_{\delta_{\text{SU}(2)}}(5)_{\delta_{\text{EM}}}$
$K_L \mu$	$0.21649(50)_{\text{exp}}(16)_{I_K}(4)_{\delta_{\text{EM}}}$
$K_S \mu$	$0.21251(466)_{\text{exp}}(16)_{I_K}(4)_{\delta_{\text{EM}}}$
$K^+ \mu$	$0.21699(108)_{\text{exp}}(16)_{I_K}(21)_{\delta_{\text{SU}(2)}}(6)_{\delta_{\text{EM}}}$
Average: Ke	$0.21626(40)_K(3)_{\text{HO}}$
Average: $K\mu$	$0.21654(48)_K(3)_{\text{HO}}$
Average: tot	$0.21634(38)_K(3)_{\text{HO}}$

Short distance single-nucleon radiative corrections

Reduced Hadronic Uncertainty in the Determination of V_{ud}

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We analyze the universal radiative correction Δ_R^V to neutron and superallowed nuclear β decay by expressing the hadronic γW -box contribution in terms of a dispersion relation, which we identify as an integral over the first Nachtmann moment of the γW interference structure function $F_3^{(0)}$. By connecting the needed input to existing data on neutrino and antineutrino scattering, we obtain an updated value of $\Delta_R^V = 0.02467(22)$, wherein the hadronic uncertainty is reduced. Assuming other standard model theoretical calculations and experimental measurements remain unchanged, we obtain an updated value of $|V_{ud}| = 0.97370(14)$, raising tension with the first row Cabibbo-Kobayashi-Maskawa unitarity constraint. We comment on ways current and future experiments can provide input to our dispersive analysis.

DOI: [10.1103/PhysRevLett.121.241804](https://doi.org/10.1103/PhysRevLett.121.241804)

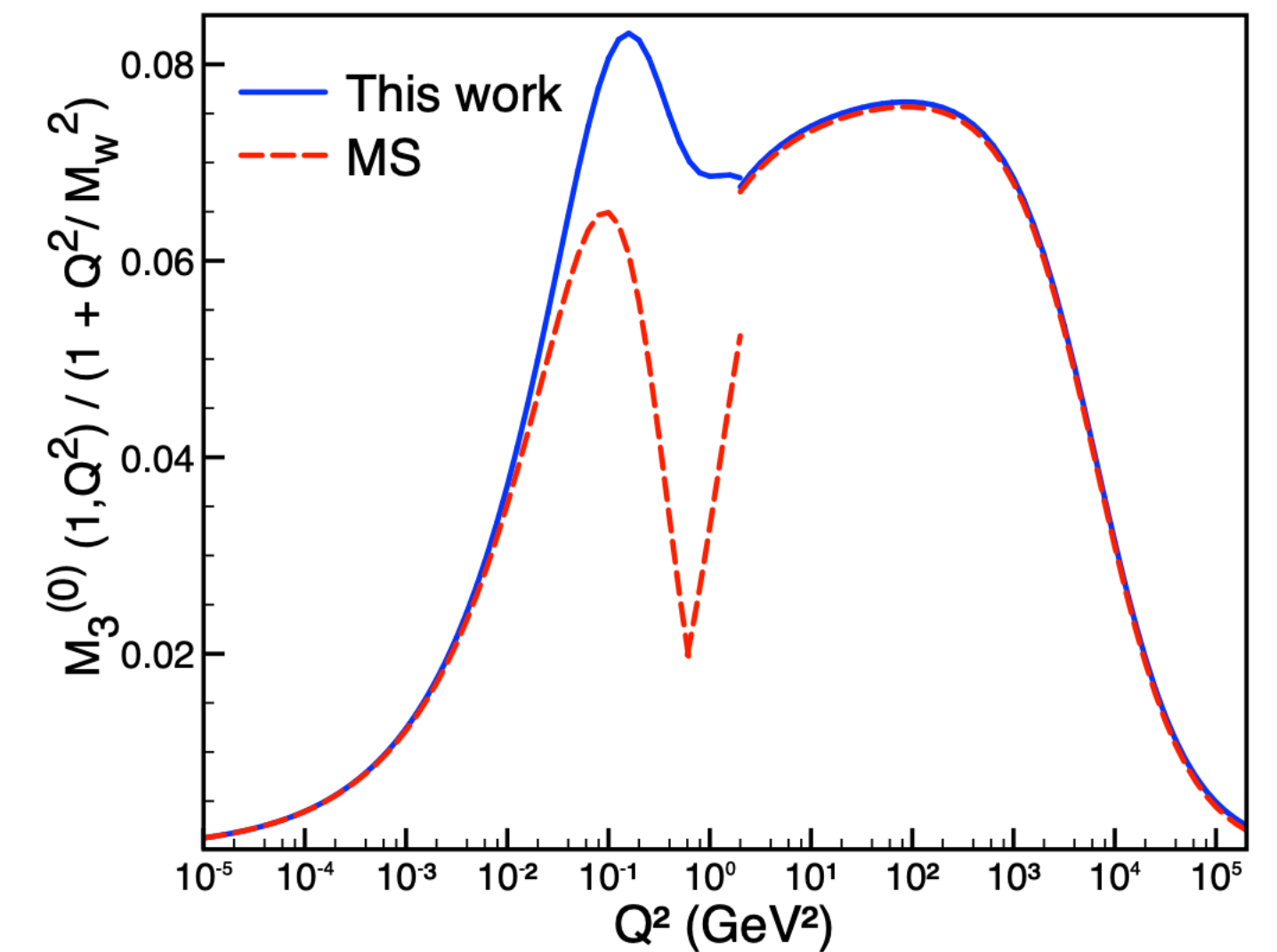
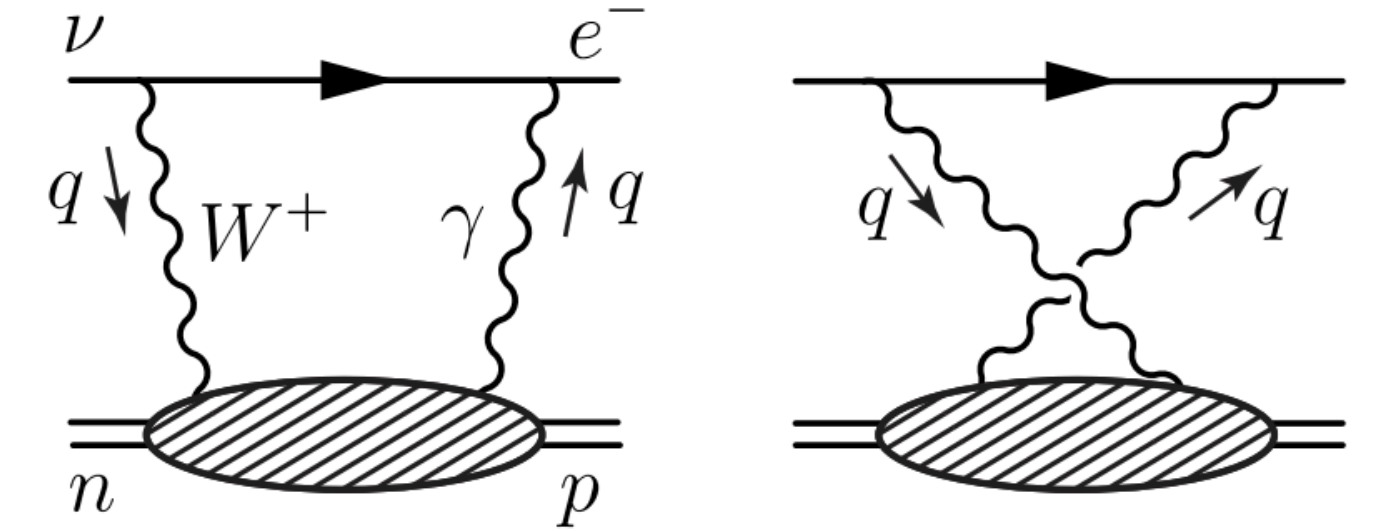
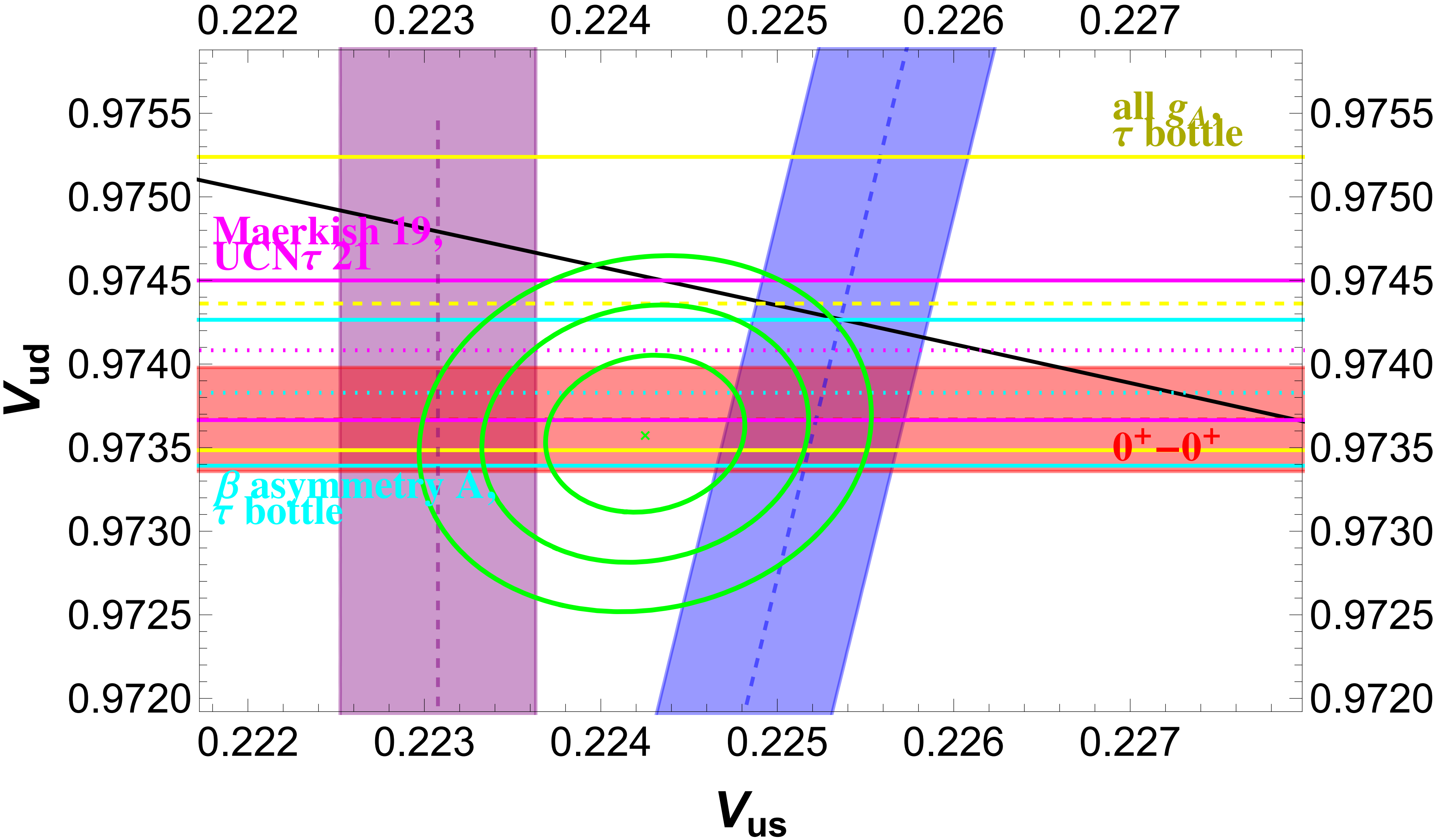


Figure 5: Log-linear plot of $\frac{M_W^2}{M_W^2 + Q^2} M_3^{(0)}(1, Q^2)$ as a function of Q^2 . The blue curve is the result of our parameterization in (12), and the red curve is the piecewise parametrization used by [MS]. For a given parametrization, the contribution to $\square_{\gamma W}^{VA}$ is proportional to the area under the curve, see (9).

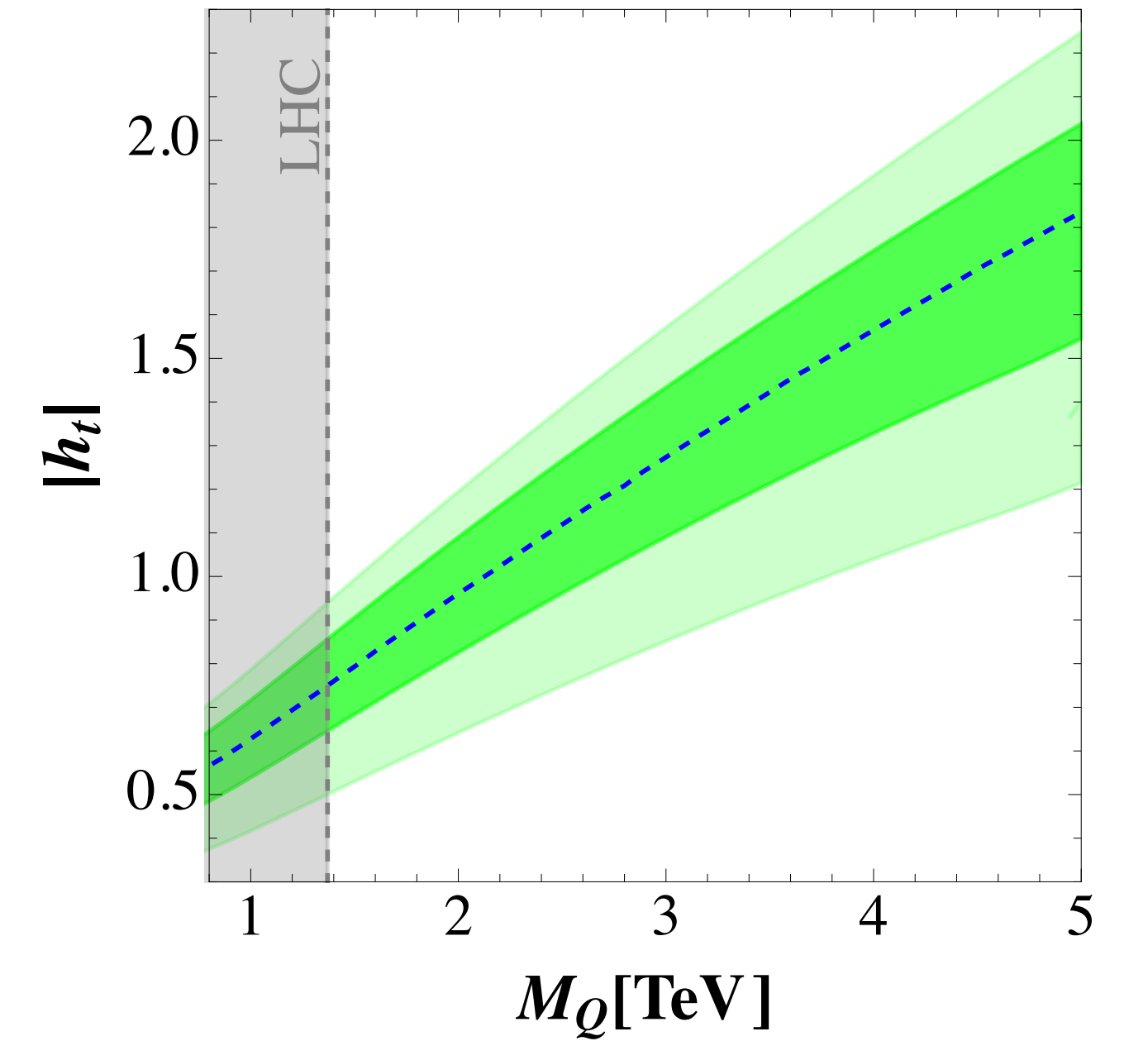
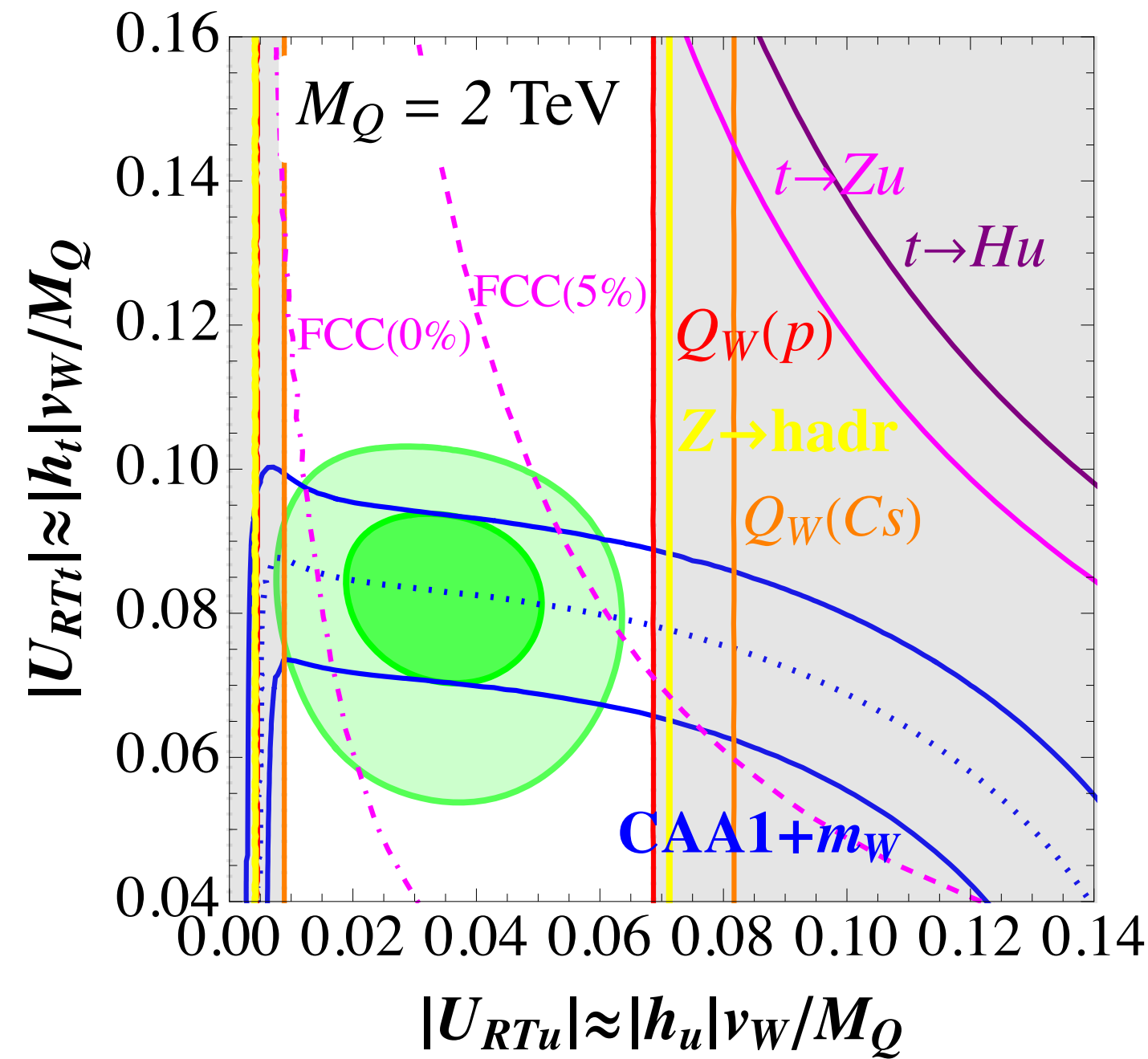
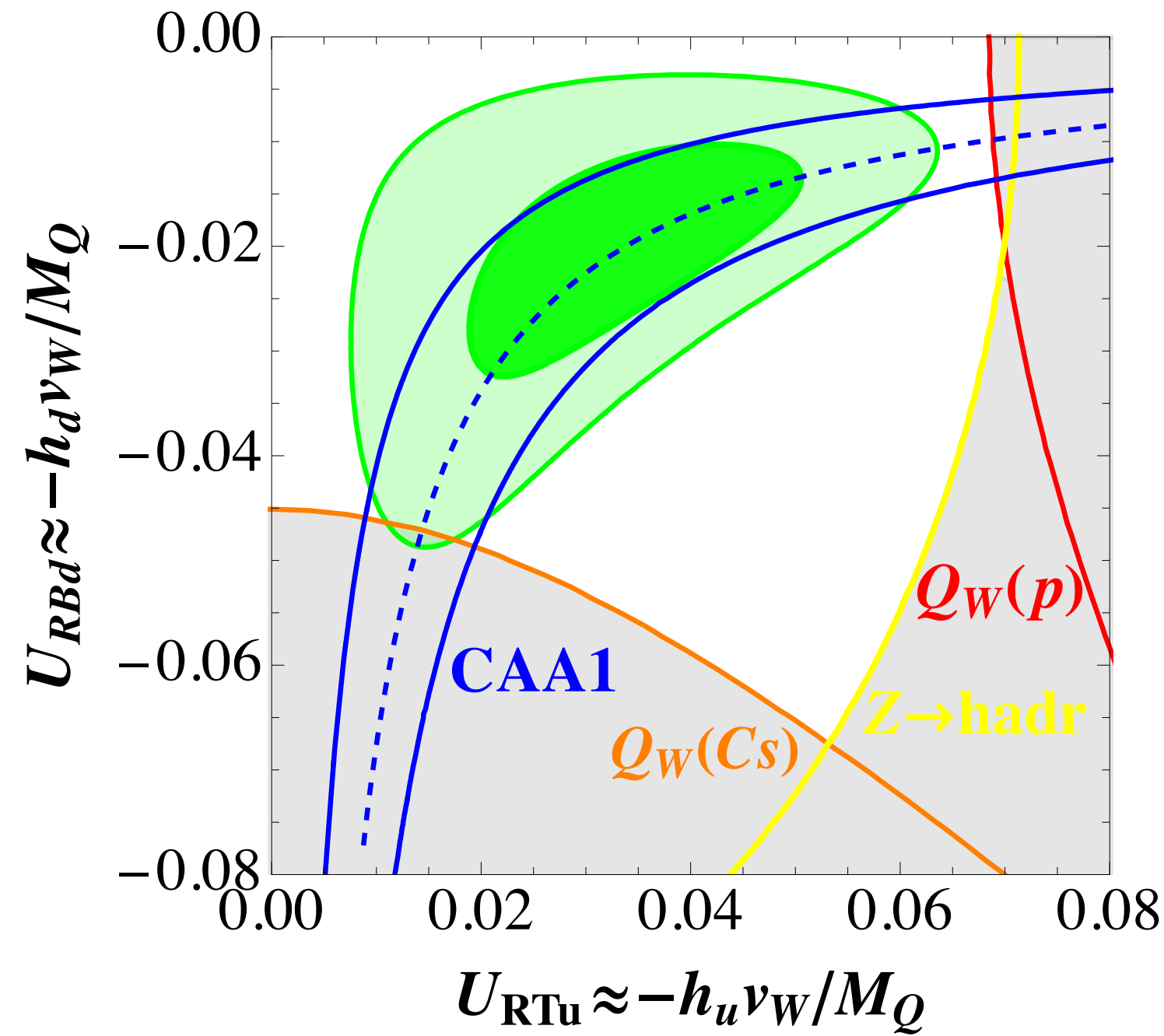
Cabibbo angle anomalies



CAA and W-boson mass: vector-like weak doublet

- Assume the vector-like doublet couples predominantly to the down or the strange quark:
two scenarios to explain CAA1 and m_W or CAA2 and m_W .

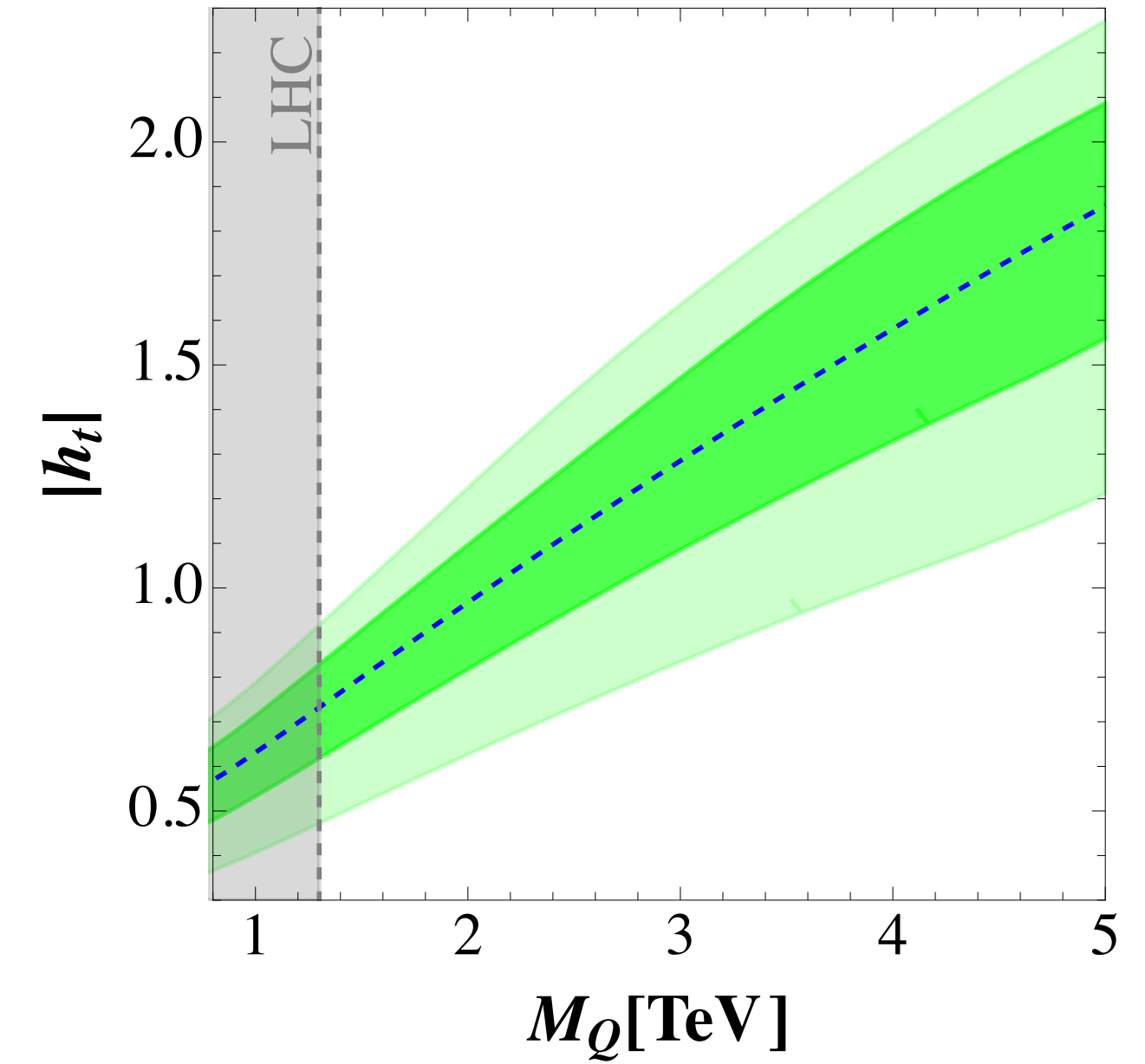
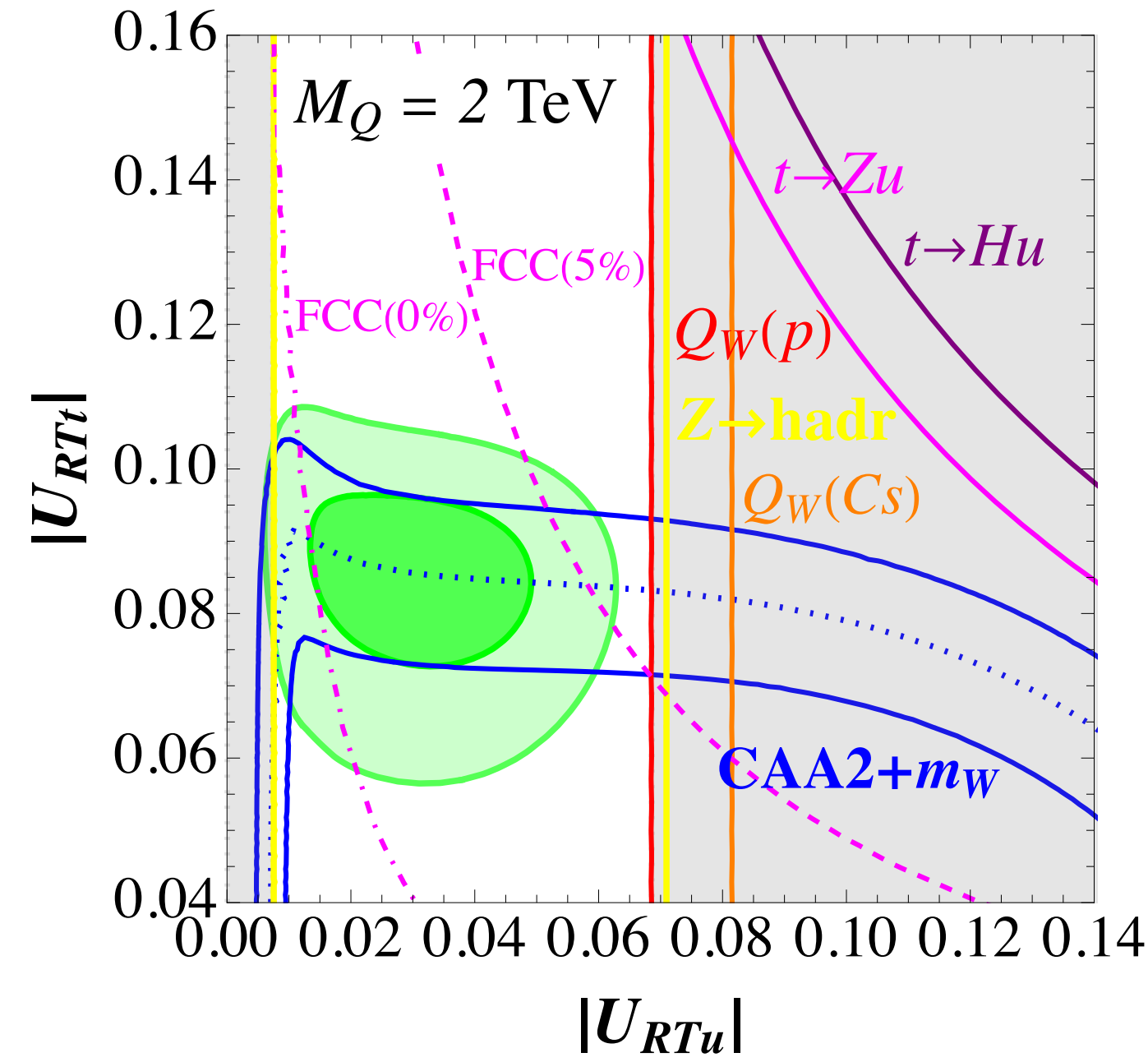
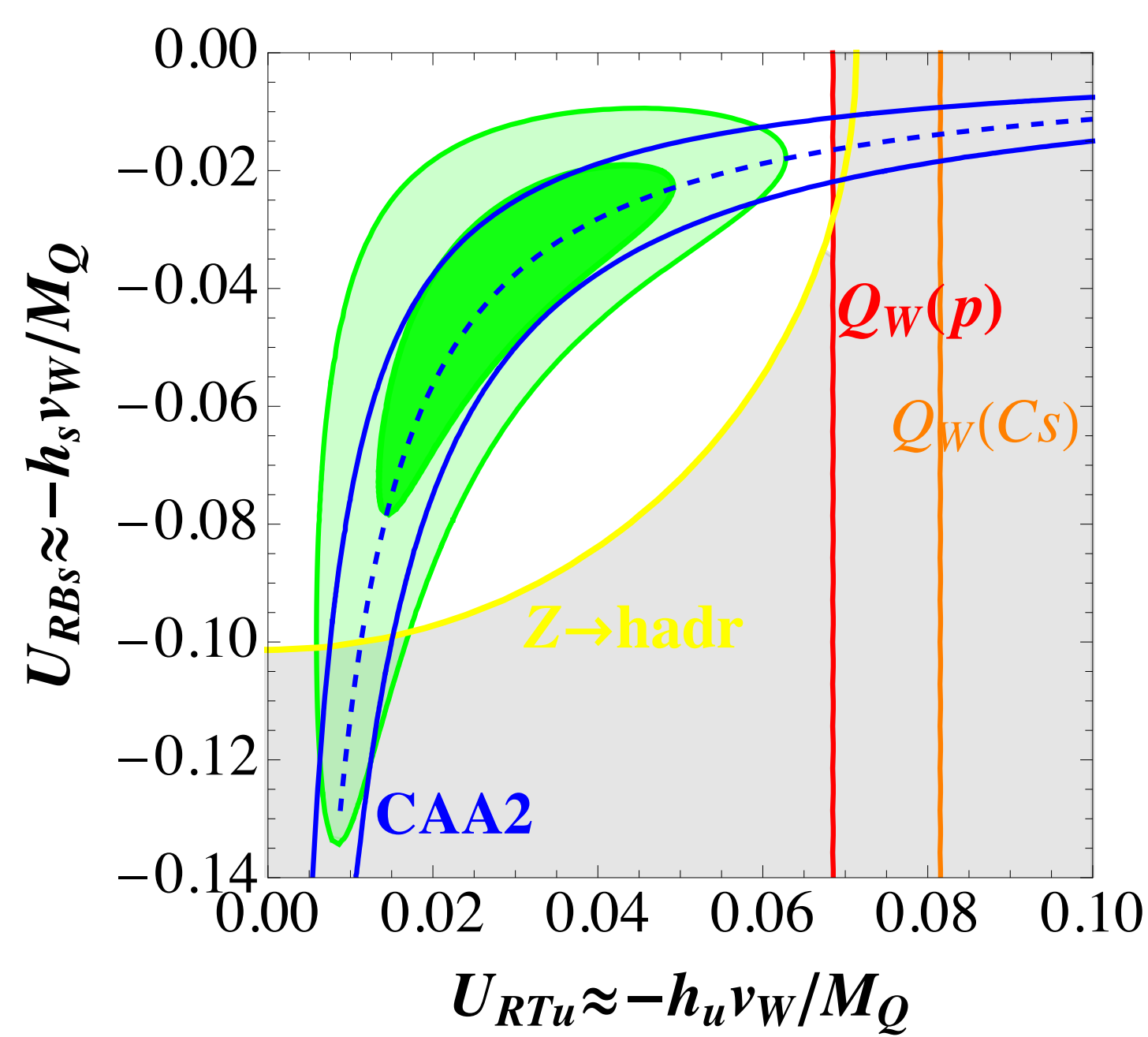
1. CAA1 + m_W (h_u , h_d , h_t)



- $\chi_{\text{SM}}^2 = 31.4$, improvement of $\chi_{\text{SM}}^2 - \chi_{\text{min,new}}^2 = 17.2$. The left discrepancy is due to CAA2.
- If CAA2 is resolved by the presence of another vector-like doublet, then $\chi_{\text{SM}}^2 - \chi_{\text{min}}^2 = 28.1$.
- $h_t \lesssim 1$ for $M_Q \lesssim 3 \text{ TeV}$.

CAA and W-boson mass: vector-like weak doublet

2. CAA2 + m_W (h_u , h_s , h_t)



- $\chi^2_{\text{SM}} = 31.4$, improvement of $\chi^2_{\text{SM}} - \chi^2_{\text{min,new}} = 20.0$. The left discrepancy is due to CAA1.
- $V_{Rus} = U_{RTu}^* U_{RBs} \approx h_u^* h_s v_w^2 / M_Q^2 = -1.13(38) \times 10^{-3}$ induces the difference between the vector coupling in $K\ell 3$ decays and the axial-vector coupling in $K\mu 2/\pi\mu 2$ decays.
- $h_t \lesssim 1$ for $M_Q \lesssim 3$ TeV.

CAA and W-boson mass: vector-like weak doublet

1. CAA1 + m_W (h_u, h_d, h_t)

$$h_u \sim h_d \approx 0.3, \quad h_t \approx 1, \quad h_s \lesssim \mathcal{O}(10^{-3}), \quad h_c \lesssim \mathcal{O}(10^{-2}) \quad h_b \lesssim \mathcal{O}(10^{-1})$$

$$|h_t| \lesssim 1 \text{ for } M_Q \lesssim 3 \text{ TeV}$$

2. CAA2 + m_W (h_u, h_s, h_t)

$$h_u \sim h_s \approx 0.3, \quad h_t \approx 1, \quad h_d \lesssim \mathcal{O}(10^{-3}), \quad h_c \lesssim \mathcal{O}(10^{-2}) \quad h_b \lesssim \mathcal{O}(10^{-1})$$

$$|h_t| \lesssim 1 \text{ for } M_Q \lesssim 3 \text{ TeV}$$

CAA and W-boson mass: vector-like weak singlets

Down-type singlet

$$T_D \approx \frac{3}{16\pi} \frac{v_w^2}{s^2 c^2 m_Z^2} \frac{v_w^2}{M_D^2} \left[-y_t^2 |w_b|^2 \ln \frac{M_D^2}{m_t^2} + \frac{1}{2} (|w_d|^2 + |w_s|^2 + |w_b|^2)^2 \right]$$

- Considering constraints on the mixing elements, which also give $M_D \lesssim 1.5$ TeV, the upper limit is $|w_b| \lesssim 0.05$. It cannot induce the positive shift of m_W .

Up-type singlet

$$T_U \approx \frac{3}{16\pi} \frac{v_w^2}{s^2 c^2 m_Z^2} \frac{v_w^2}{M_U^2} \left[y_t^2 |z_t|^2 \left(\ln \frac{M_U^2}{m_t^2} - 1 \right) + \frac{1}{2} (|z_u|^2 + |z_c|^2 + |z_t|^2)^2 \right]$$

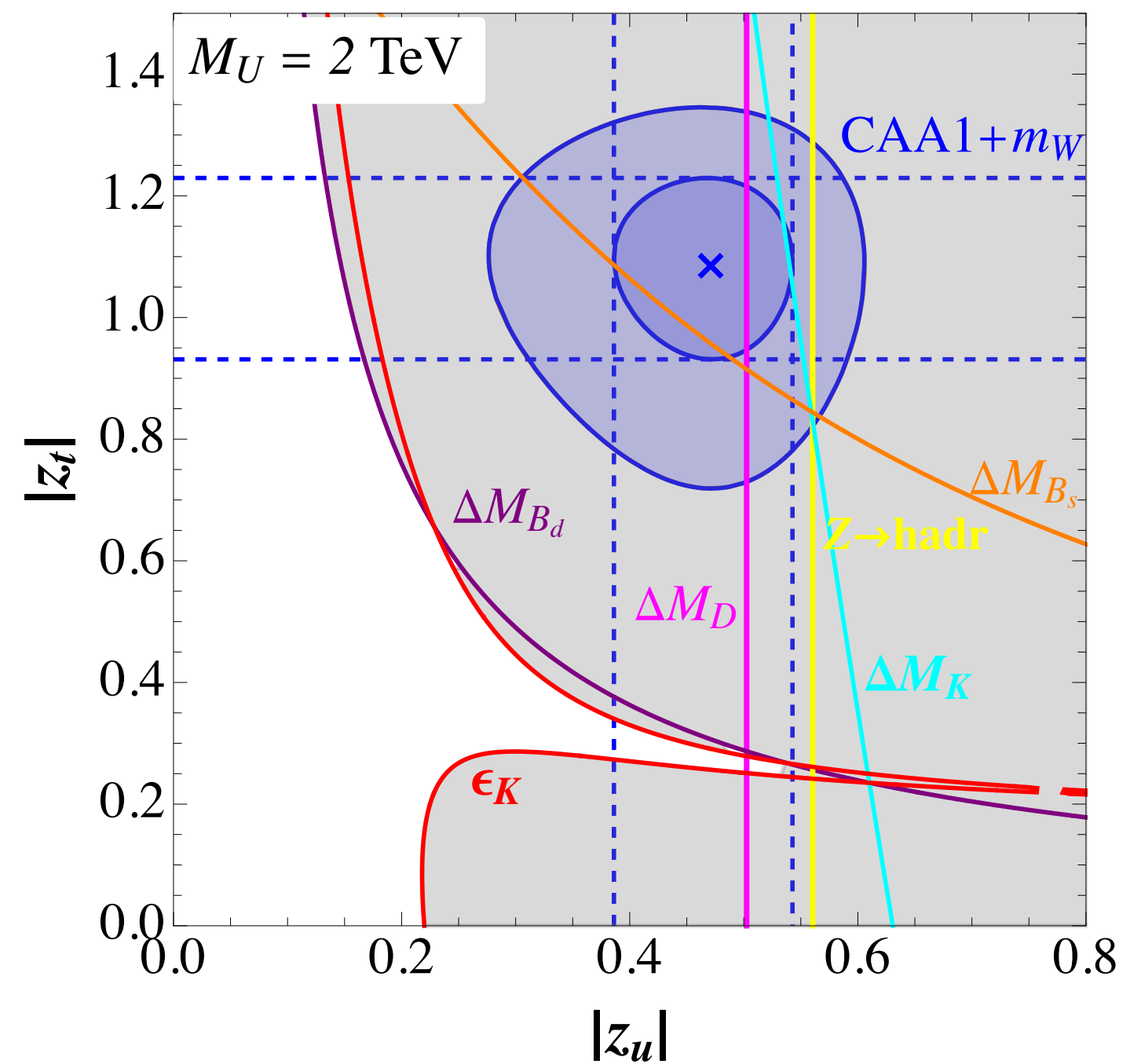
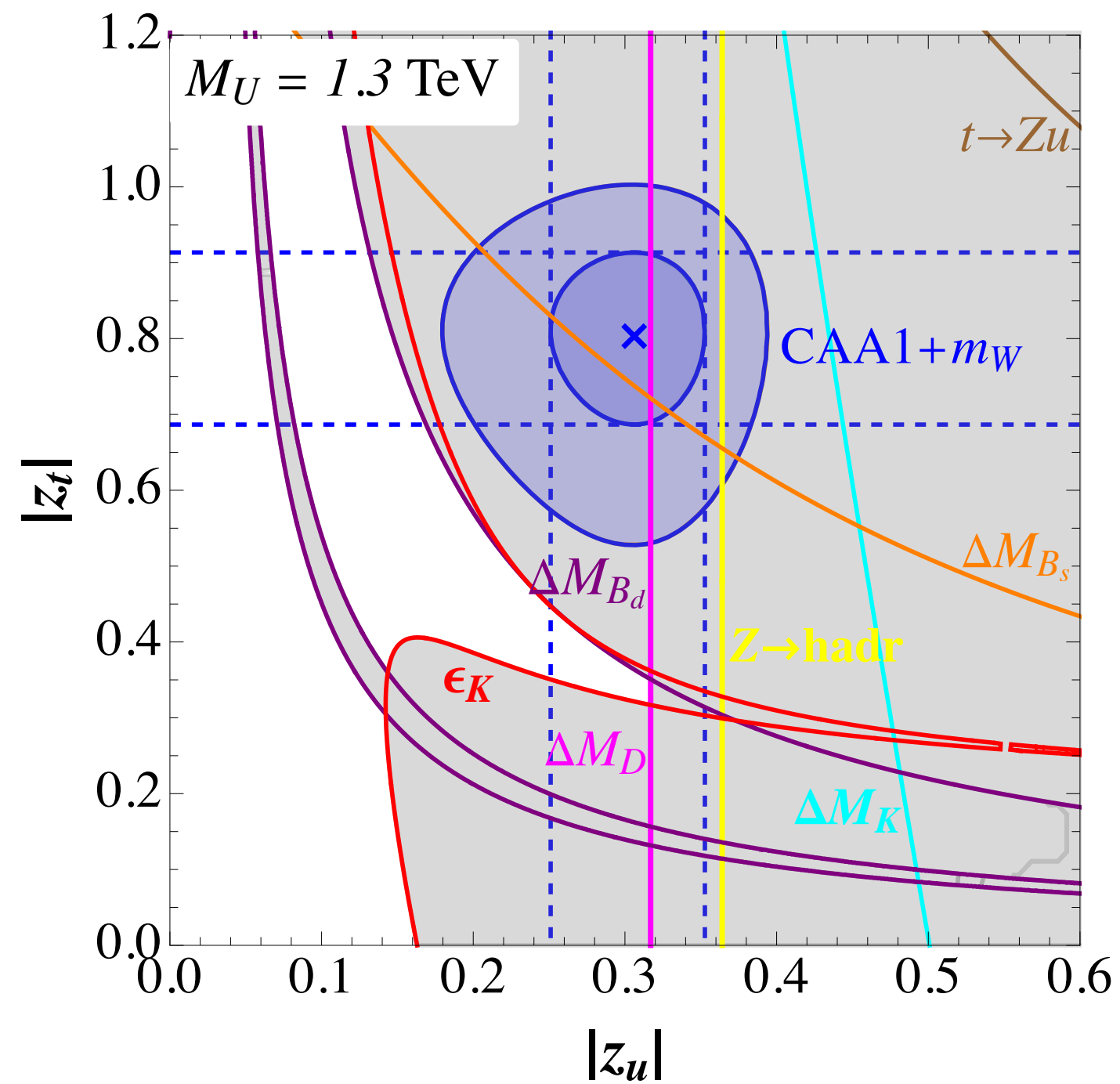
- Taking into account the bound on z_c from the D -mesons mass difference

$$|U_{LUu}^* U_{LUc}| \approx |z_u^* z_c| v_w^2 / M_U^2 \lesssim 1.3 \times 10^{-4} \left[1 + (M_U / 3.1 \text{ TeV})^2 \right]^{-1/2}$$

- $B^0 - \bar{B}^0$ mixing gives a bound which is approximately $|V_{t'b}^* V_{t'd}| \approx |z_u^* z_t| v_w^2 / M_U^2 \lesssim 1.1 \times 10^{-3} \left[\frac{1 \text{ TeV}}{M_U} \right]$
- $|U_{LUu}| = 0.04$ for CAA1. Assuming z_c is chosen to compensate in ϵ_K , the limit gives $|z_t| \lesssim 0.15$.
- However, m_W requires $z_t = 0.67(10)$ for $M_U = 1$ TeV ($z_t = 1.08(15)$ for $M_U = 2$ TeV).

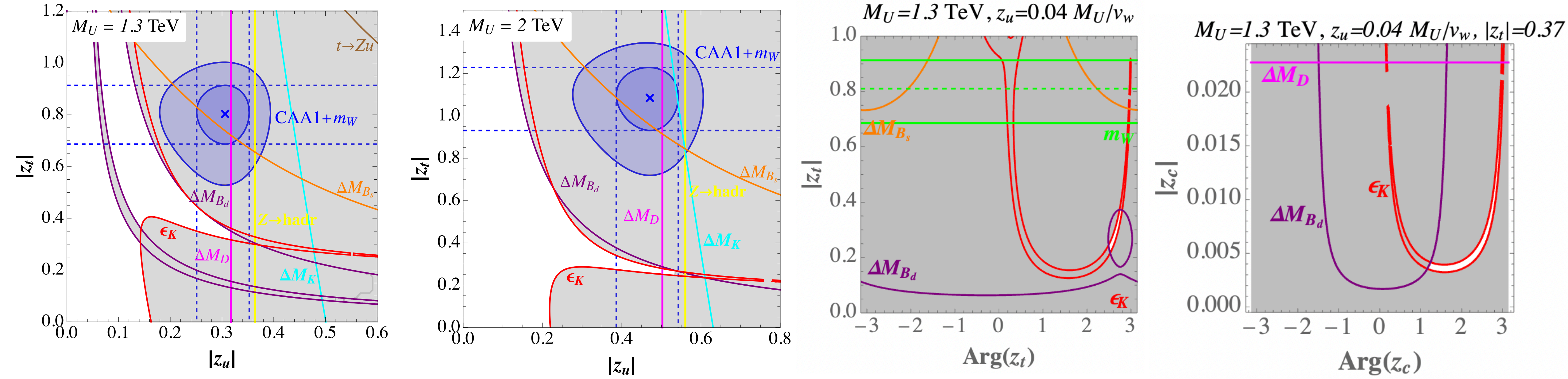
CAA and W-boson mass: vector-like up-type singlet

- $\text{Arg}(z_t)$ can be selected to reduce the contribution in B_d^0 system, z_c set to compensate the CP-violating effect in $K^0-\bar{K}^0$.
- Limit from $B_d^0-\bar{B}_d^0$ still excludes the preferred region for the combined explanation.



- Up-type singlet can be an explanation of CAA1 or m_W , separately.

CAA and W-boson mass



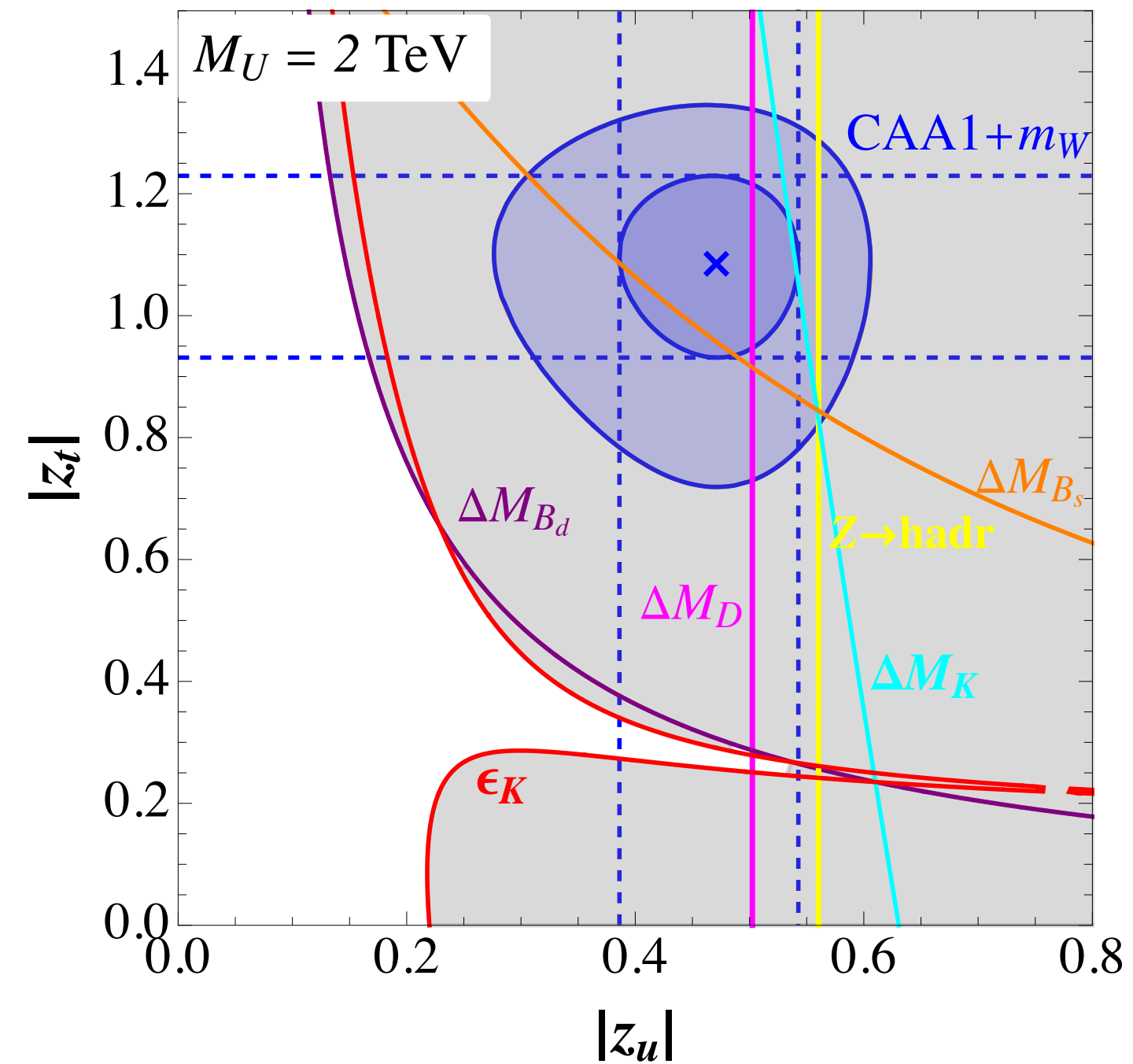
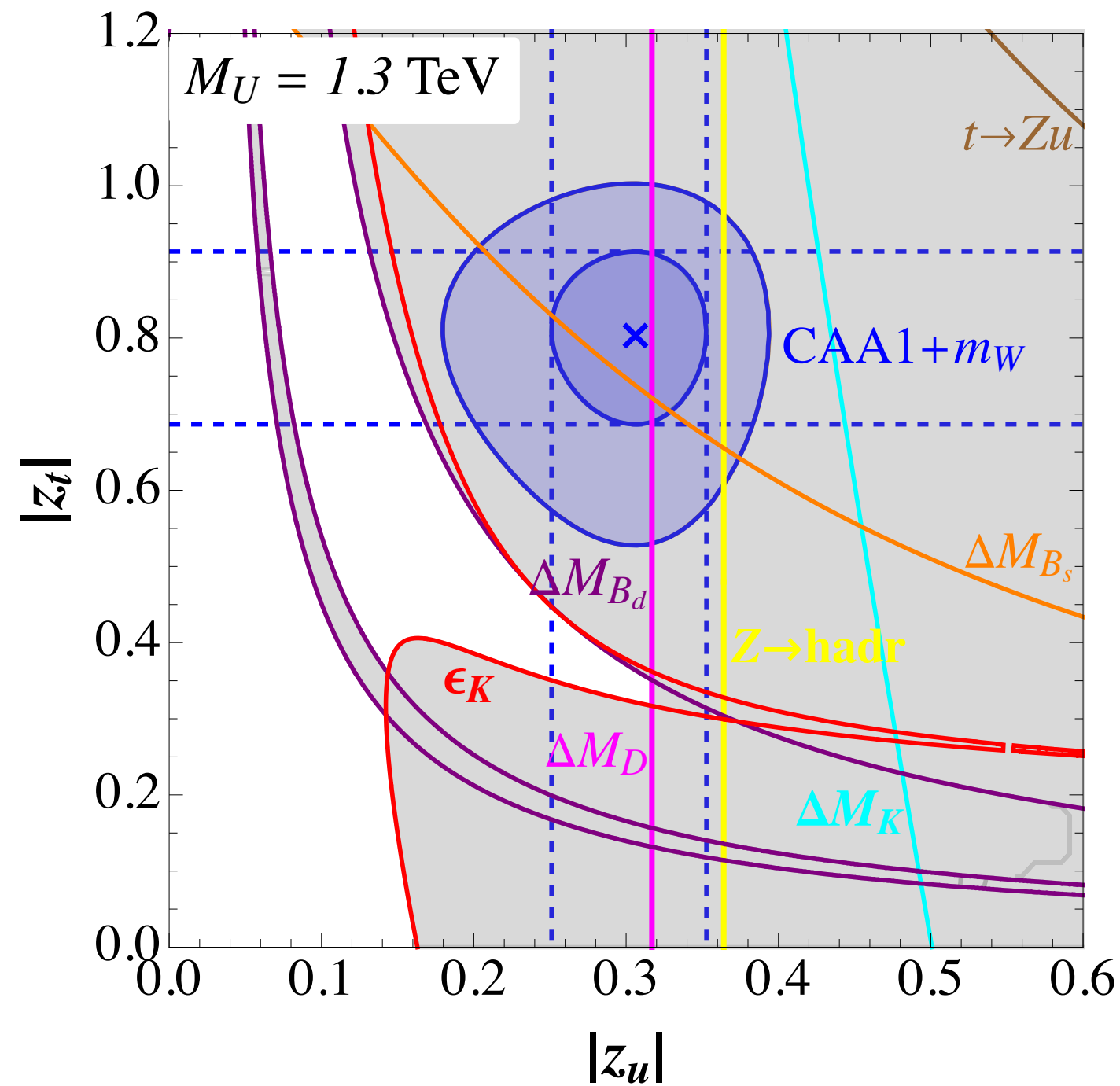
- $\text{Arg}(z_t)$ selected to reduce the contribution in the B_d^0 system, z_c set to compensate in ϵ_K .
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CAA and W-boson mass: vector-like up-type singlet

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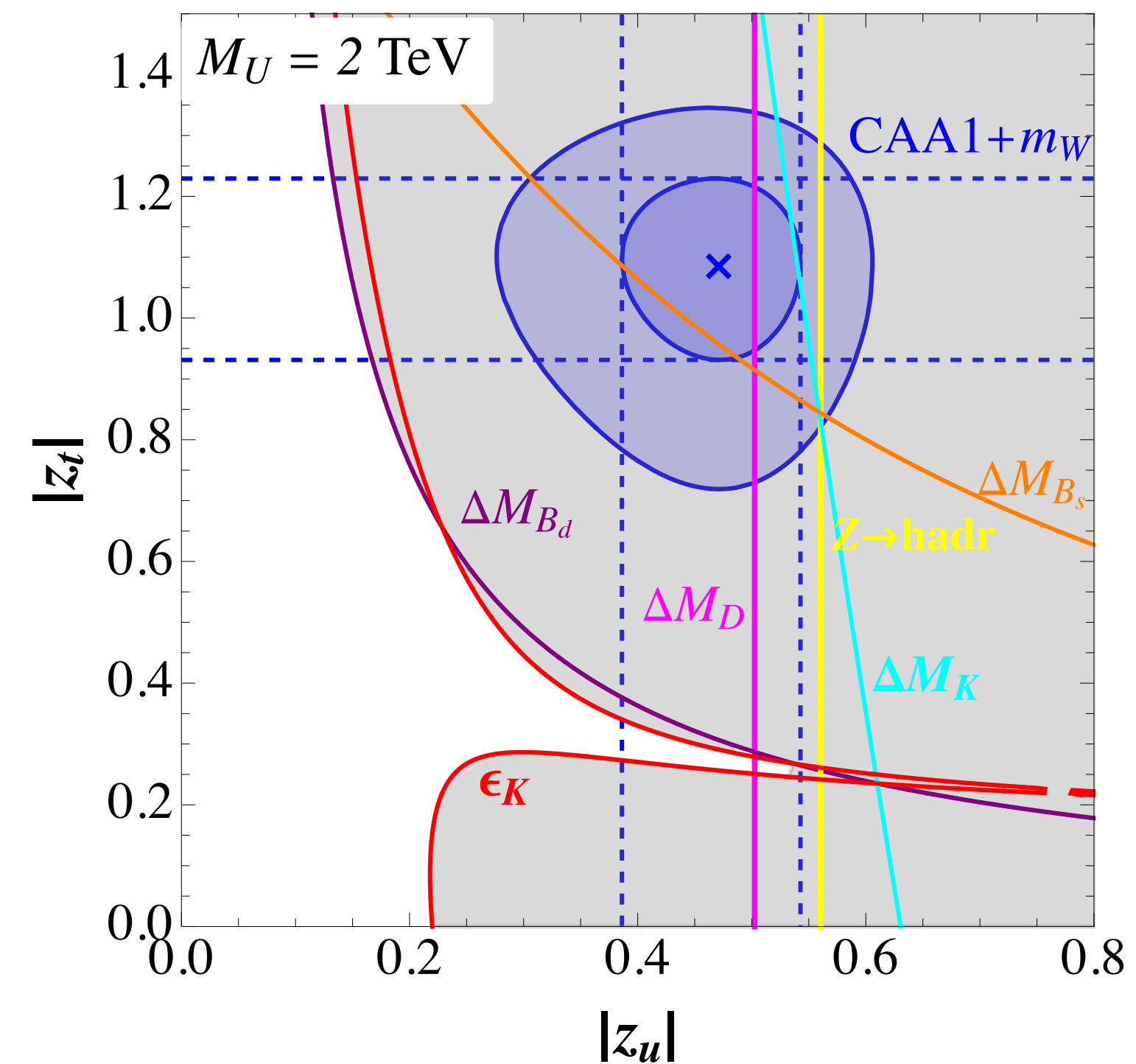
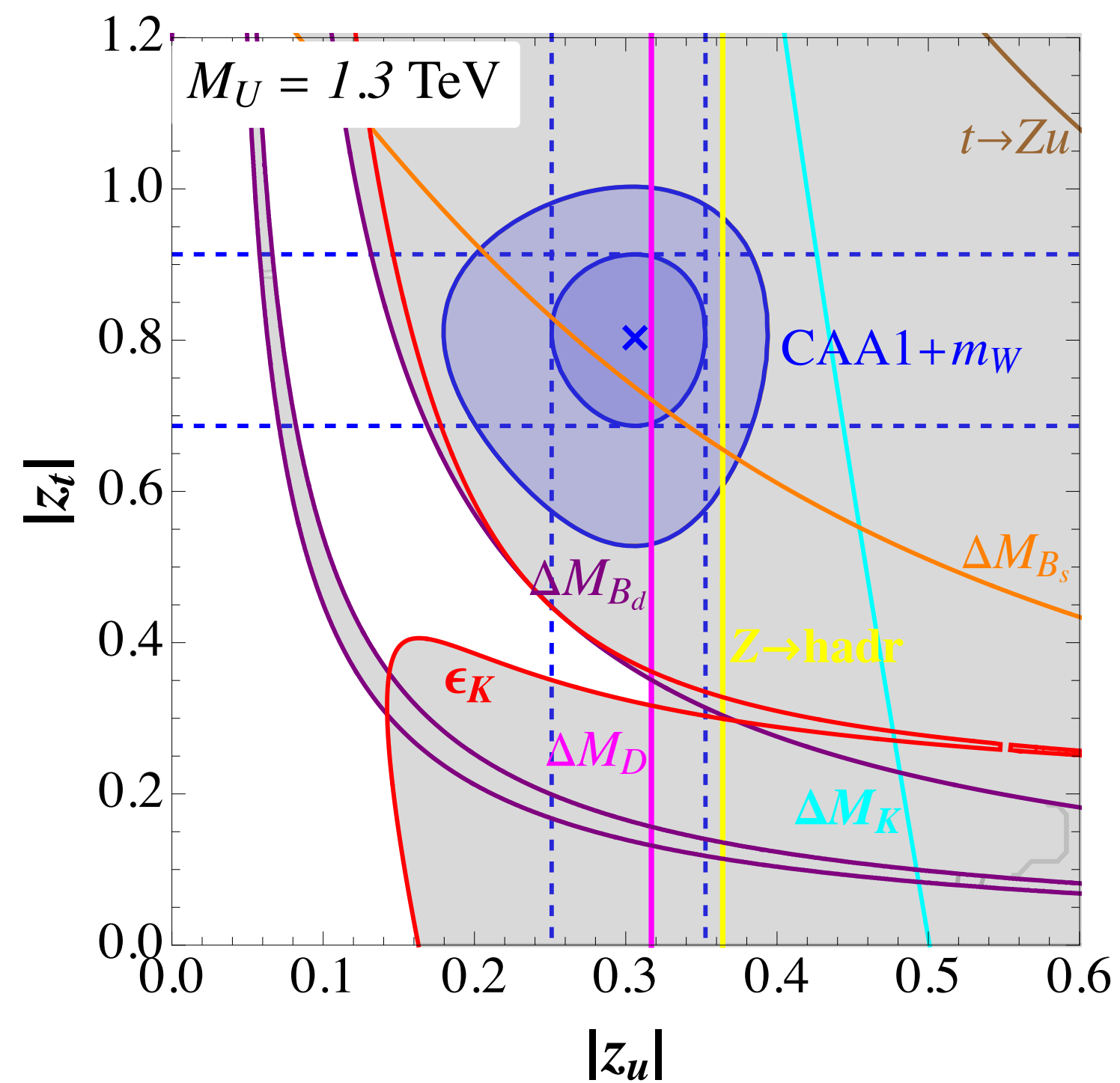


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CAA and W-boson mass

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$$T_U \approx \frac{3}{16\pi} \frac{v_w^2}{s^2 c^2 m_Z^2} \frac{v_w^2}{M_U^2} \left[y_t^2 |z_t|^2 \left(\ln \frac{M_U^2}{m_t^2} - 1 \right) + \frac{1}{2} (|z_u|^2 + |z_c|^2 + |z_t|^2)^2 \right]$$



Present situation of CKM first row

- The most precise determination is from **superallowed nuclear beta-decays** ($0^+ \rightarrow 0^+$ Fermi transitions) determining vector coupling $\mathbf{G}_V = \mathbf{G}_F |\mathbf{V}_{ud}|$:

Hardy and Towner, Phys. Rev. C 102, no.4, 045501 (2020)
(new survey after 2015)

transition dependent QED radiative correction
function of electron energy and Z

$$\mathcal{F}t = ft (1 + \delta'_R)(1 + \delta_{NS} - \delta_C) = \frac{K}{2 G_F^2 |V_{ud}|^2 (1 + \Delta_R^V)}$$

transition dependent radiative correction,
depends on **nuclear structure** (neutron in nucleus)

$$\delta_{NS} = \delta_{NS,A} + \delta_{NS,B} + \delta_{NS,E}$$

$$\delta_{NS,A} \text{ Seng et al.1812.03352}$$

$$\delta_{NS,E} \text{ Gorchtein 1812.04229}$$

uncertainty increased by 2.6 factor

isospin breaking corrections,
depends on nuclear structure
(aligns ft the most)

- $K = 2\pi^3 \log 2 / m_e^5$, $G_F = G_\mu = 1.1663787(6) \times 10^{-5} \text{ GeV}^{-2}$, f statistical rate function (depends on the total transition energy), t partial half-life
- $\mathcal{F}t = \mathbf{ft}(1 + \delta'_R)(1 + \delta_{NS} - \delta_C)$ nucleus-independent value obtained from ft -values averaging **15 transitions**