Extraction of Transverse Momentum Dependent Distributions

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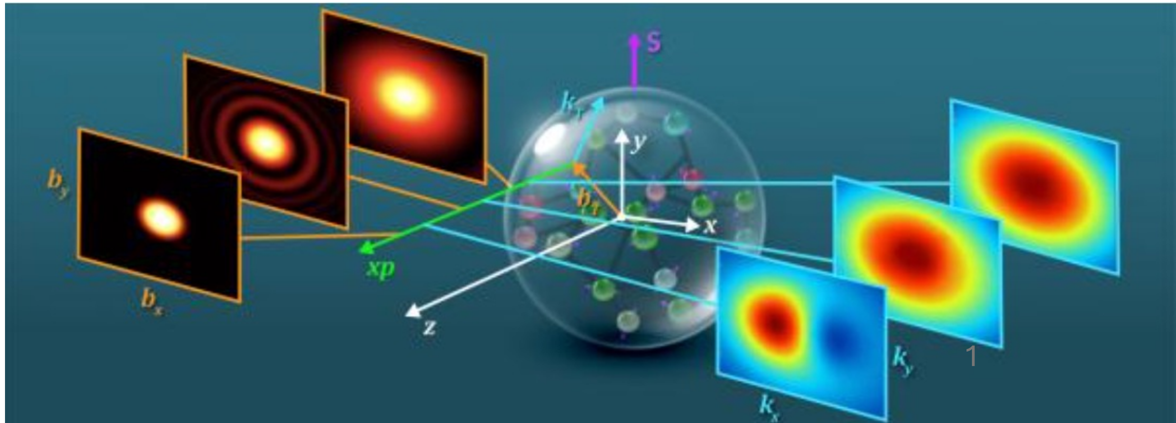
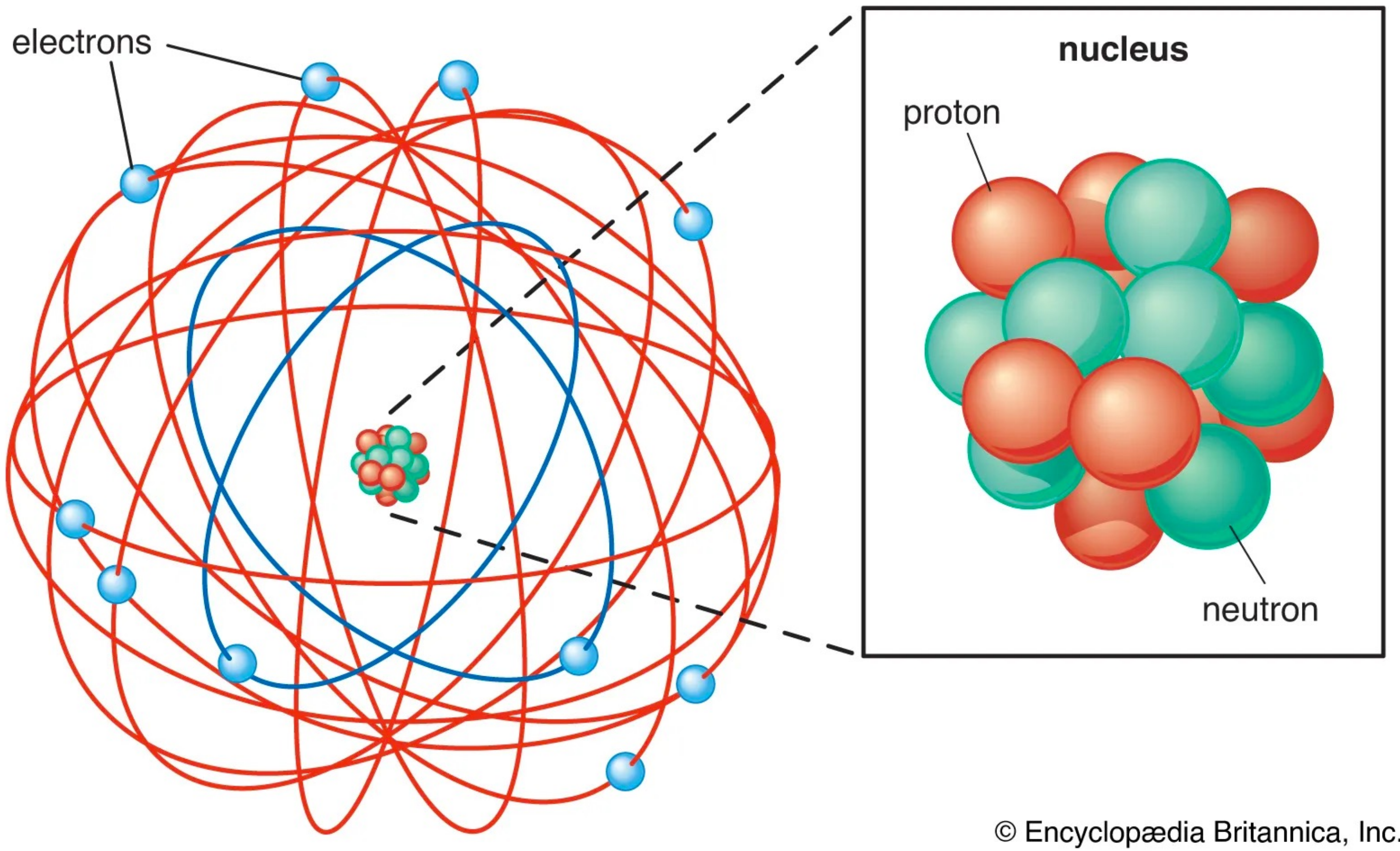


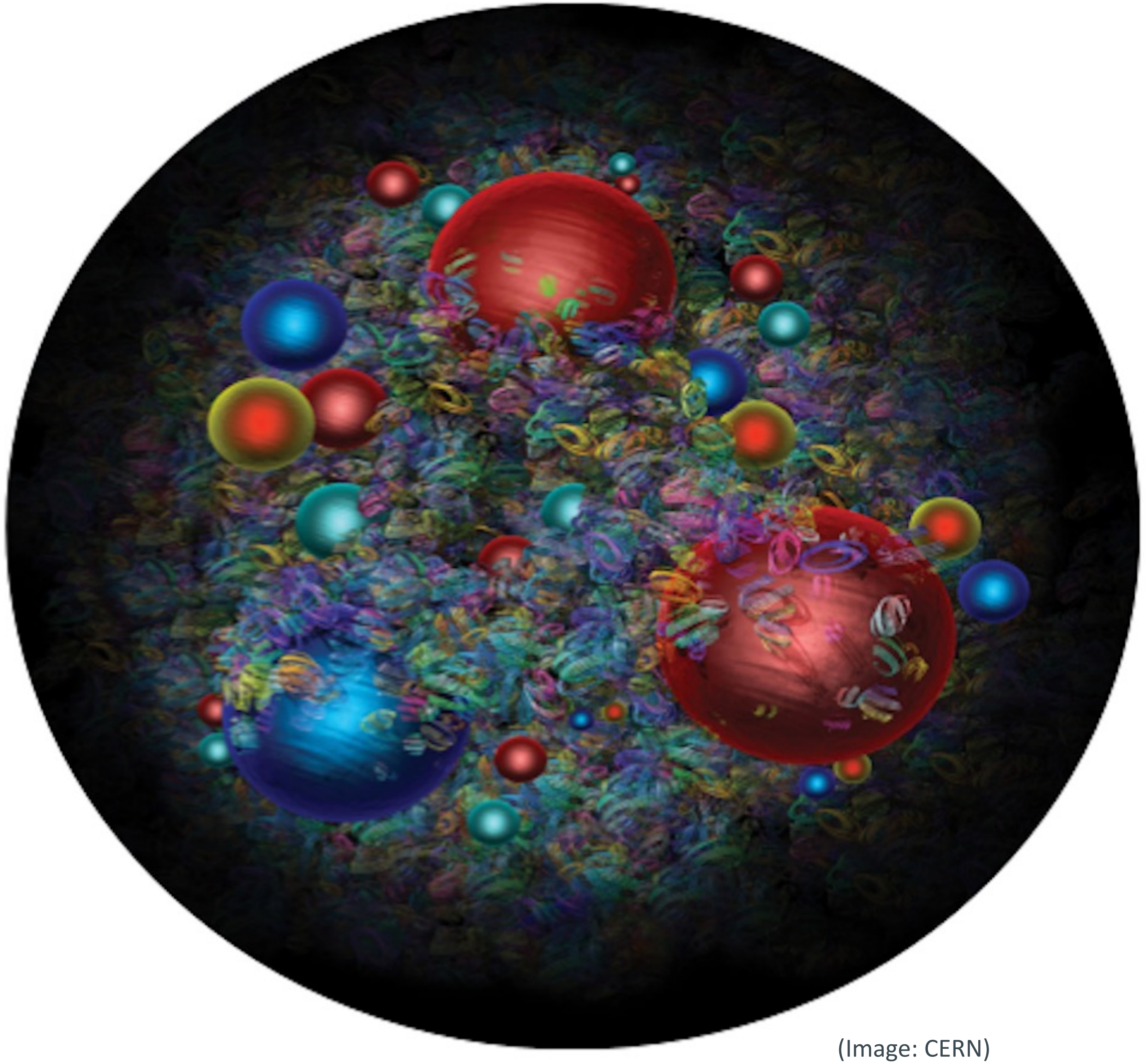
Motivation

- All visible matter is made up of atoms
- The mass of these atoms are largely from the nucleus
- The nucleus is made up of protons and neutrons



Motivation

- In turn, these protons and neutrons are made of quarks and gluons
- We want to study the structure of the nuclear matter



(Image: CERN)

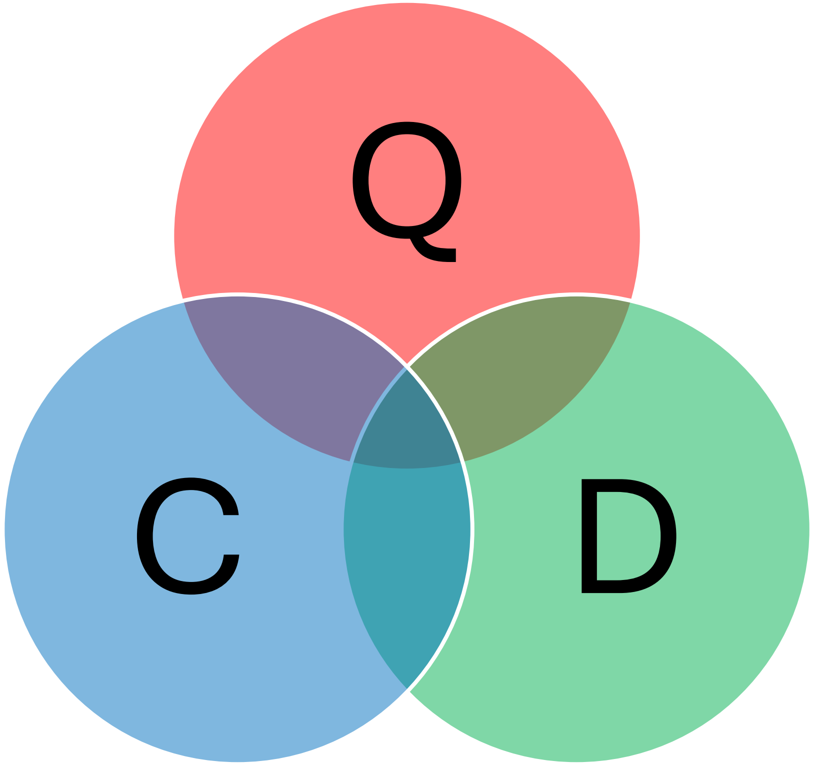
What's the problem?

Quarks and gluons are not directly measurable because of color confinement!

Have to be inferred from experimental data

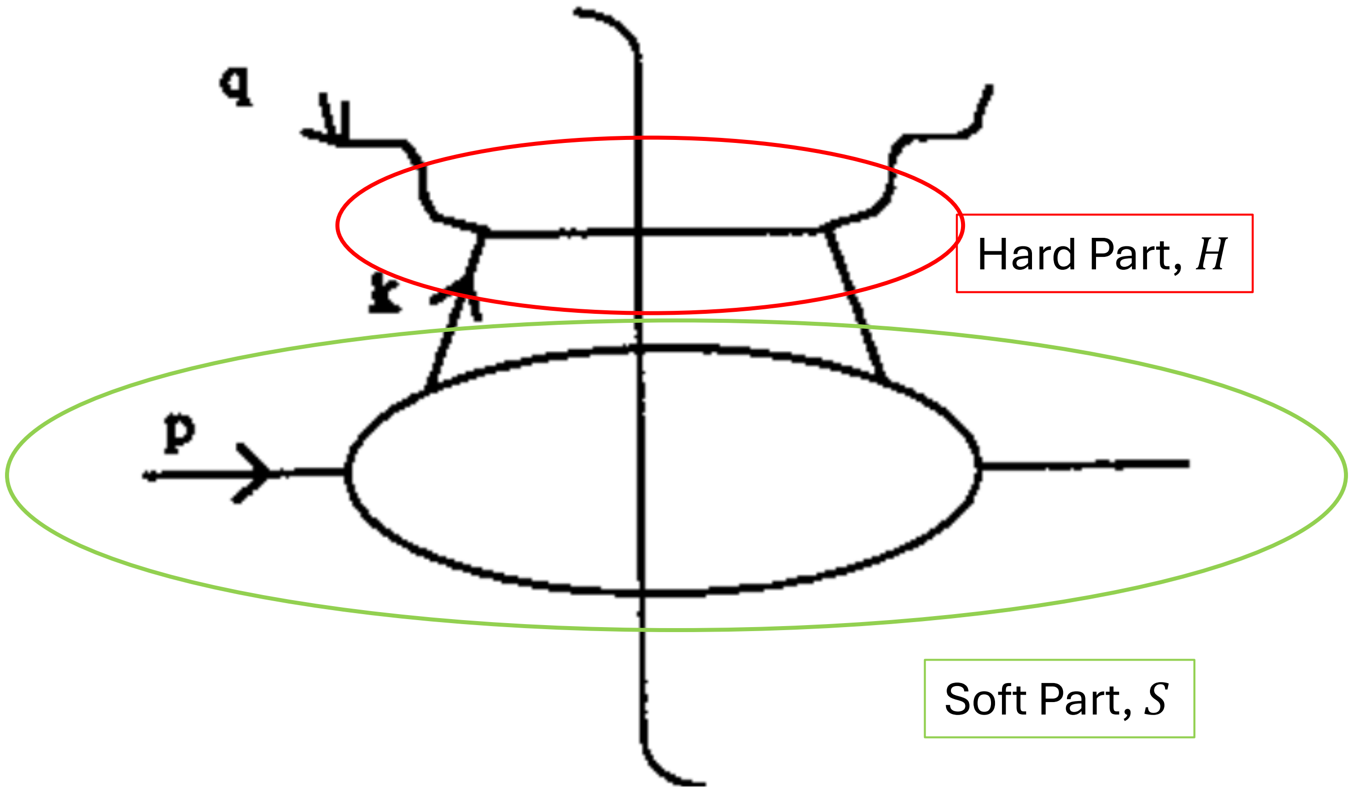
How to handle this

- We make use of QCD, which allows us to study the structure of hadrons in terms of partons (quarks, antiquarks, and gluons)
- Use factorization theorems to separate hard partonic physics out of soft, non-perturbative objects to quantify structure



Factorization Theorems

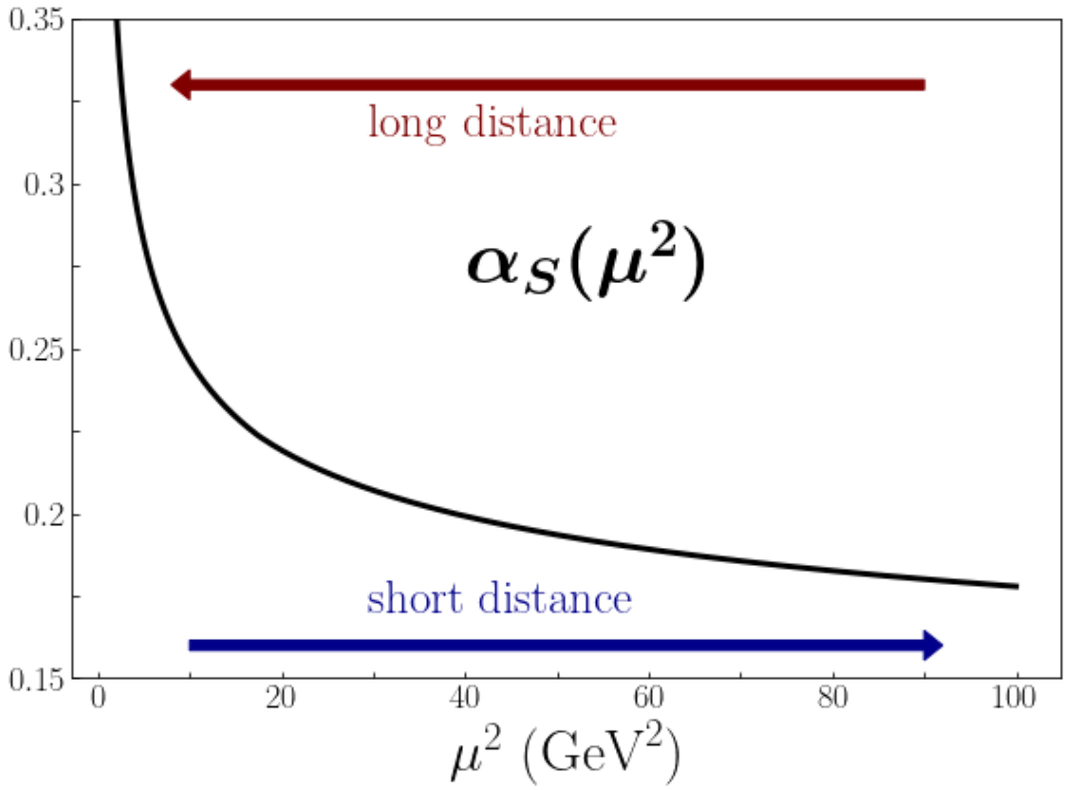
- Deep Inelastic Scattering (DIS) $|\mathcal{M}|^2$ at Leading Order shown to the right
- At large $Q^2 = -q^2$, can decouple the soft part from the hard part
- At short distances, virtual photon picks out individual parton



$$W^{\mu\nu} \propto S \otimes H$$

Strong coupling constant

- Because of large enough energy scales, we can safely compute hard coefficients perturbatively in α_S



Game plan

What to do:

- Define a structure of hadrons in terms of quantum field theories
- Identify physical observables that can be theoretically factorized with controllable approximations, or factorizable lattice QCD observables
- Perform global QCD analysis as structures are universal and are the same in all processes

Complicated Inverse Problem

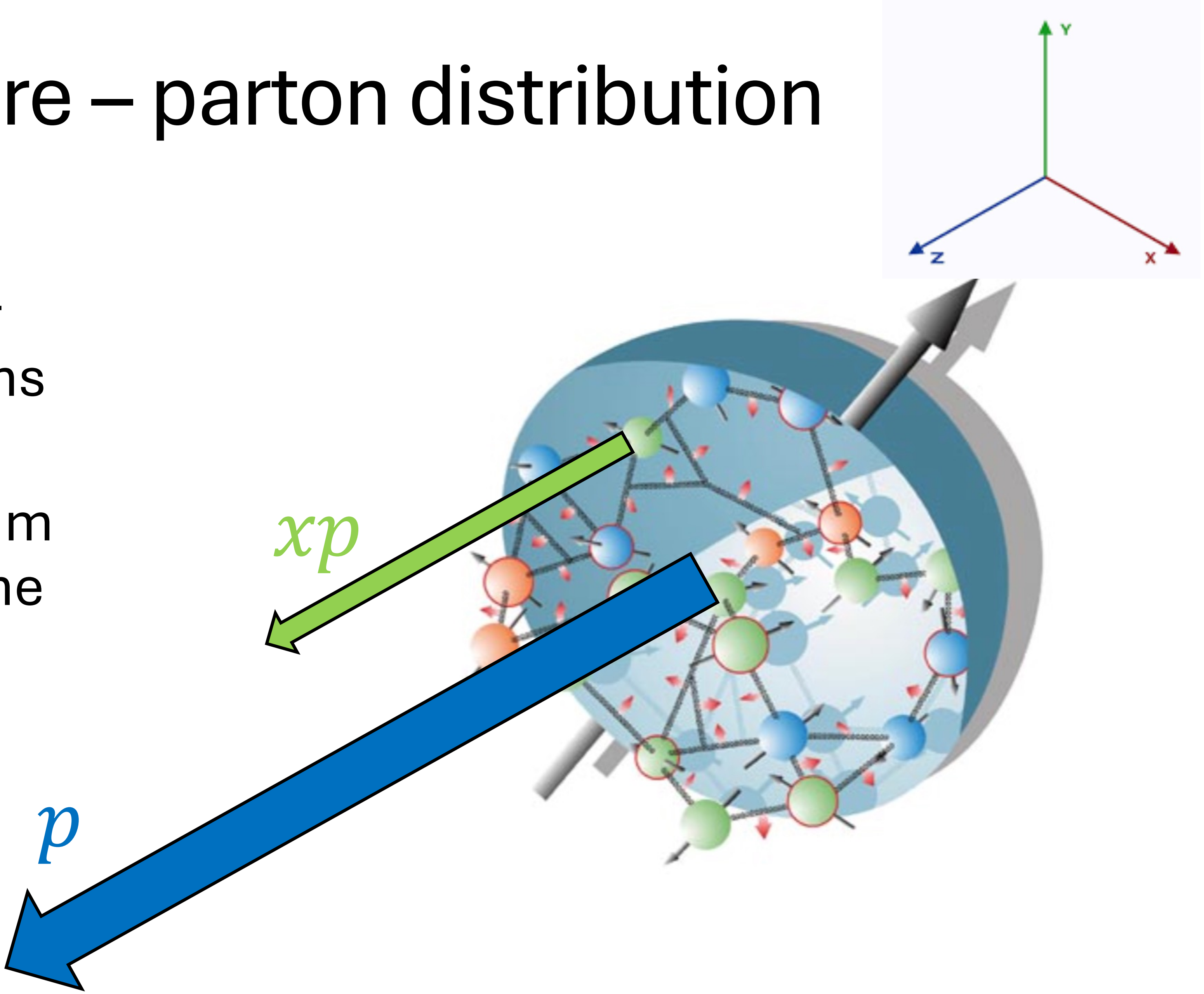
- Factorization theorems involve convolutions of hard perturbatively calculable physics and non-perturbative objects

$$\frac{d\sigma}{d\Omega} \propto \mathcal{H} \otimes f = \int_x^1 \frac{d\xi}{\xi} \mathcal{H}\left(\frac{x}{\xi}\right) f(\xi)$$

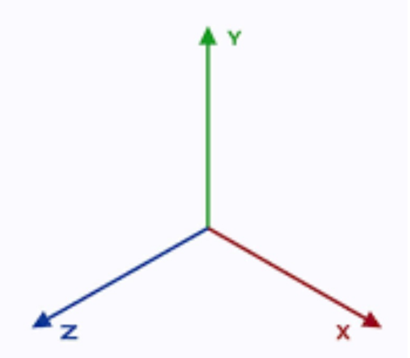
- Parametrize the non-perturbative objects and perform global analysis

Collinear structure – parton distribution function (PDF)

- Describes the collinear momentum distributions of quarks and gluons
- Partons have momentum along the direction of the hadron

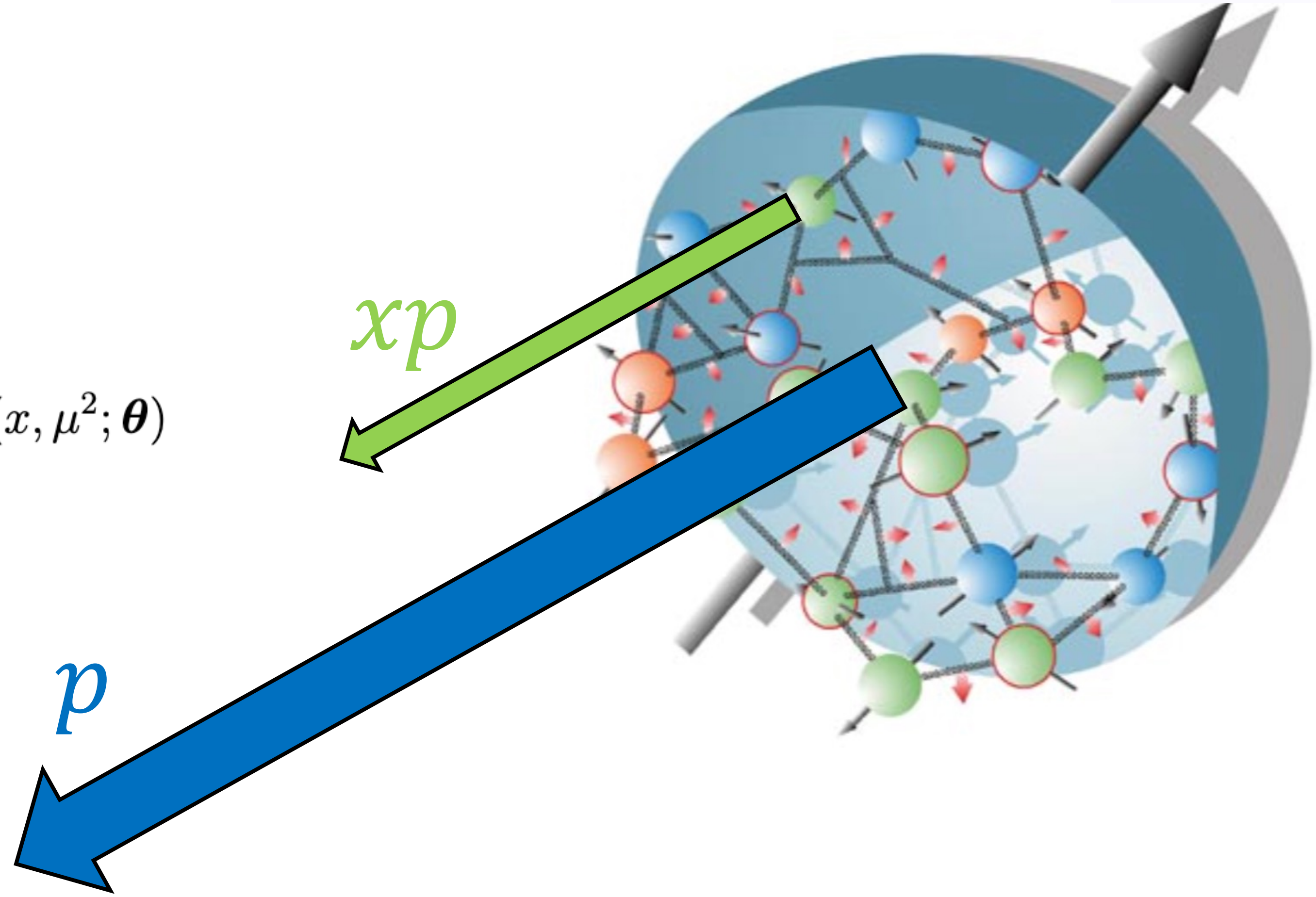


Collinear structure – parton distribution function (PDF)



- Evolution in the renormalization scale according to DGLAP:

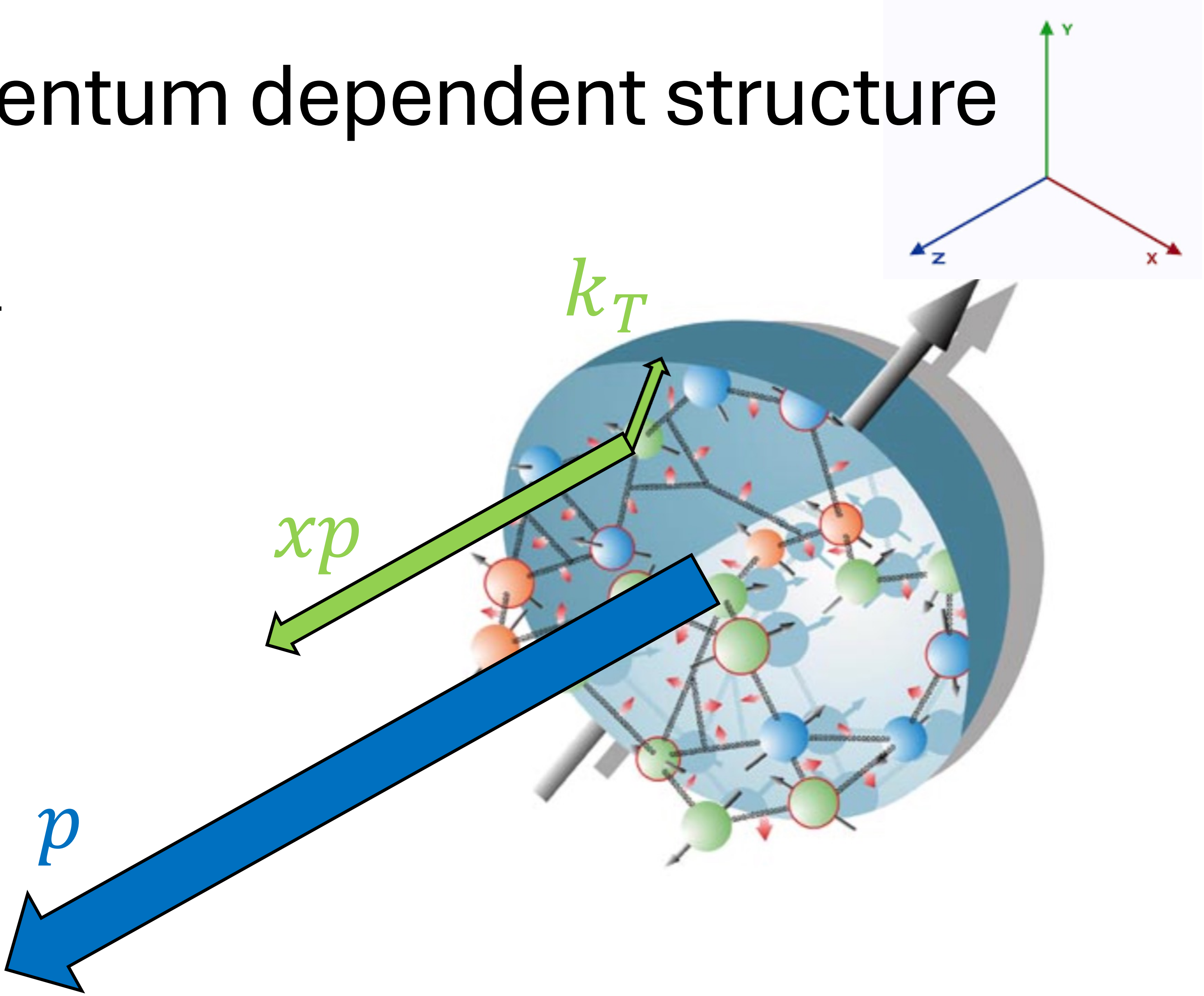
$$\frac{\partial f(x,\mu^2;\boldsymbol{\theta})}{\partial \log \mu^2} = \int_x^1 dz\, \mathcal{P}\left(\frac{x}{z}, \alpha_S(\mu^2)\right) f(x,\mu^2;\boldsymbol{\theta})$$



- In practice, we implement the evolution in Mellin space

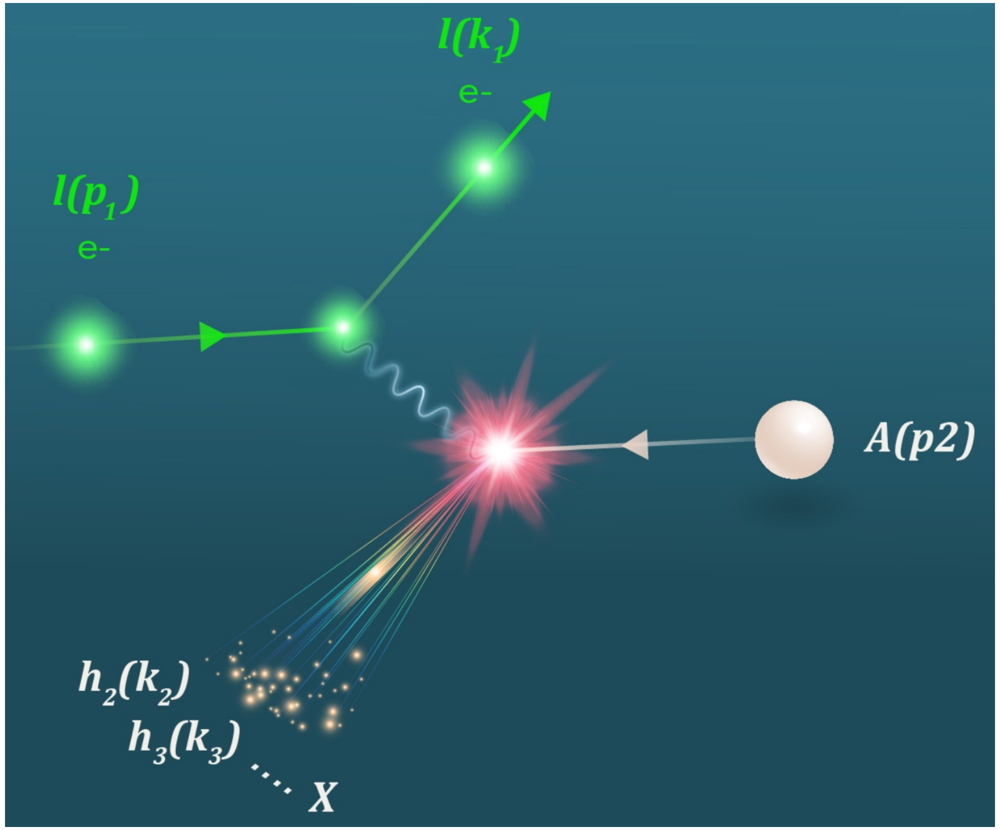
Transverse momentum dependent structure (TMD PDF)

- Describes the collinear momentum and transverse momentum that partons carry

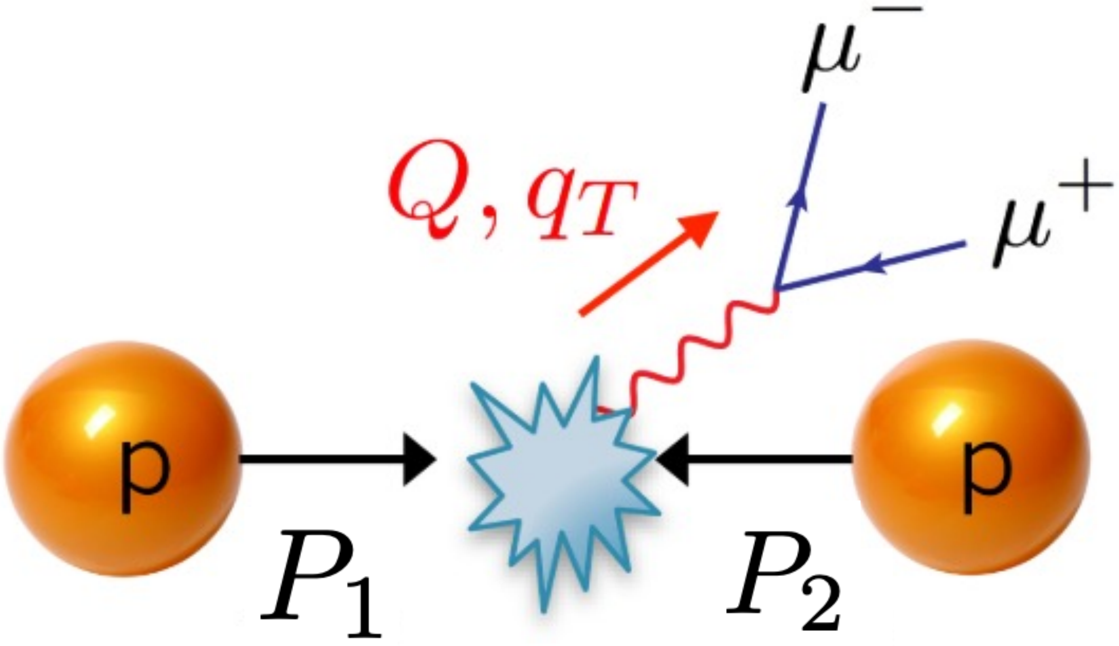


How to access from current experiments?

Semi-inclusive deep inelastic scattering (SIDIS)

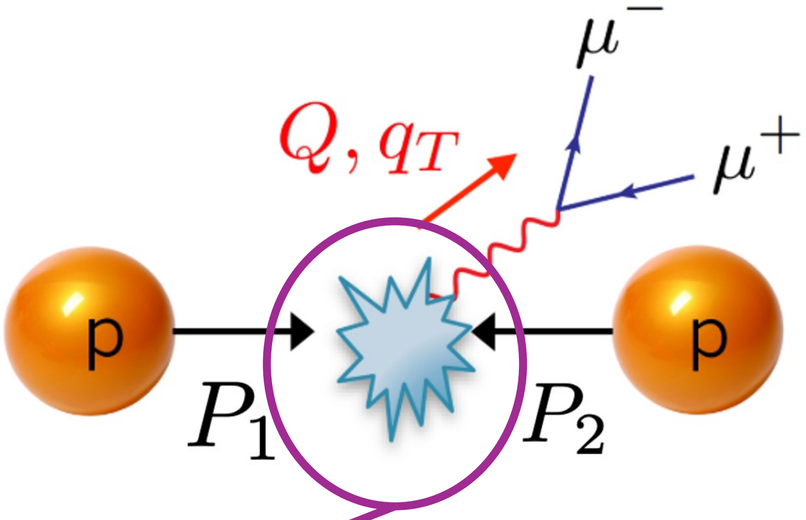


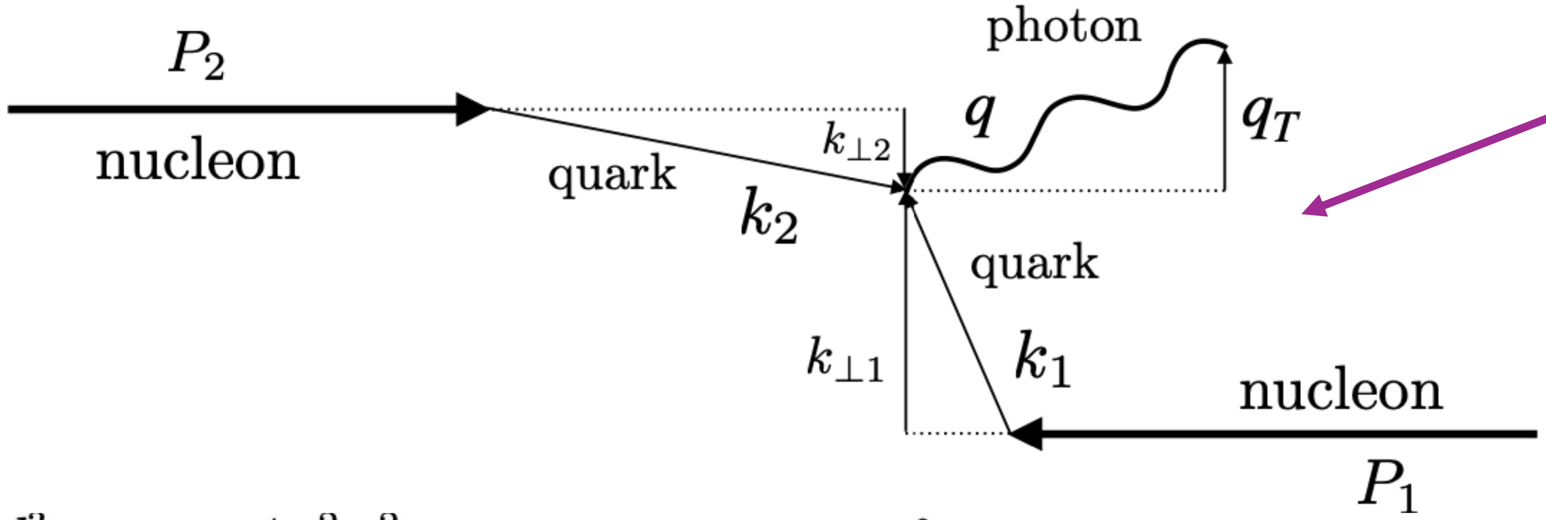
Drell-Yan (DY)



Drell-Yan Observable

- Where does the q_T come from?
  - Intrinsic transverse momenta of quarks!





This framework is optimized for small q_T

$$\frac{d^3\sigma}{dQ^2 dy dq_T^2} = \frac{4\pi^2\alpha^2}{9Q^2 s}\mathcal{P}\sum_q H_{q\bar{q}}^{\mathrm{DY}}(Q,\mu_Q)\int d^2\boldsymbol{k}_{\perp 1} d^2\boldsymbol{k}_{\perp 2}$$

$$f_{q/\mathcal{N}}(x_1, \boldsymbol{k}_{\perp 1}^2; \mu, \zeta_1) f_{\bar{q}/\mathcal{N}}(x_2, \boldsymbol{k}_{\perp 2}^2; \mu, \zeta_2)\delta^{(2)}(\boldsymbol{k}_{\perp 1} + \boldsymbol{k}_{\perp 2} - \boldsymbol{q}_T)$$

How to practically build the cross section

- Practically, the TMD is more convenient to work in the Fourier-conjugate space
- Evolution equations in μ and ζ become much simpler

$$\tilde{f}_{q/\mathcal{N}}(x, b_T; \mu, \zeta) = (2\pi)^2 \int d^2\boldsymbol{k}_T e^{-i\boldsymbol{b}_T \cdot \boldsymbol{k}_T^2} f_{q/\mathcal{N}}(x, \boldsymbol{k}_\perp^2; \mu, \zeta)$$

$$\frac{d^3\sigma}{dQ^2 dy dq_T^2} = \frac{4\pi^2\alpha^2}{9Q^2 s} \mathcal{P} \sum_q H_{q\bar{q}}^{\mathrm{DY}}(Q, \mu_Q) \int \frac{d^2\boldsymbol{b}_T}{(2\pi)^2} e^{i\boldsymbol{b}_T \cdot \boldsymbol{q}_T} \tilde{f}_{q/\mathcal{N}}(x_1, b_T; \mu, \zeta_1) \tilde{f}_{\bar{q}/\mathcal{N}}(x_2, b_T; \mu, \zeta_2)$$

- To connect with measurements, typical choices: $\mu = Q, \zeta = Q^2$

Operator product expansion (OPE)

- At small b_T, the TMD PDF can be described in terms of its OPE:

$$\tilde{f}_{q/\mathcal{N}}(x, b_T; \mu_0, \zeta_0) = \sum_j \int_x^1 \frac{d\xi}{\xi} \tilde{C}_{q/j}(x/\xi, b_T; \mu_0, \zeta_0) f_{j/\mathcal{N}}(\xi; \mu_0)$$

where $\tilde{C}$ are the Wilson coefficients, and $f_{q/\mathcal{N}}$ is the collinear PDF

- Here, sum over all flavors of quarks and gluons with nonzero contributions in $\tilde{C}$ for off-diagonal components at $\mathcal{O}(\alpha_S)$

Operator product expansion (OPE)

$$\tilde{f}_{q/\mathcal{N}}(x, b_T; \mu_0, \zeta_0) = \sum_j \int_x^1 \frac{d\xi}{\xi} \tilde{\mathcal{C}}_{q/j}(x/\xi, b_T; \mu_0, \zeta_0) f_{j/\mathcal{N}}(\xi; \mu_0)$$

- $\tilde{C}$ contains terms proportional to $\log\left(\frac{\mu_0 b_T}{C_1}\right)$ and $\log\left(\frac{\zeta}{\mu^2}\right)$, where $C_1 = 2e^{\gamma_E}$
- To eliminate logarithms, choose initial scale $\mu_0 = C_1/b_T$ and $\zeta_0 = C_1^2/b_T^2$
- For each b_T, there is a new initial scale μ_0
- Evolve $\mu_0 \to \mu$ and $\zeta_0 \to \zeta$ for each b_T

b_* prescription – a large b_T regulator

- When b_T gets too large, the scale μ_0 becomes too small to evaluate the TMD perturbatively
  - Have to regulate the large b_T behavior
- A common approach is the b_*-prescription

$$\mathbf{b}_*(\mathbf{b}_T) \equiv \frac{\mathbf{b}_T}{\sqrt{1 + b_T^2/b_{\max}^2}}.$$

Must choose an appropriate value; a transition from perturbative to non-perturbative physics

- At small b_T, $b_*(b_T) = b_T$
- At large b_T, $b_*(b_T) = b_{\max}$

$$\mu_b = \frac{C_1}{b_*(\mathbf{b}_T)}.$$

Evolution equations for the TMD PDF

$$\frac{\partial \ln \tilde{f}_{q/\mathcal{N}}(x, b_T; \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_T; \mu)$$

Collins-Soper (CS) kernel

Rapidity scale

Has its own renormalization group equation

$$\frac{\mathrm{d}\tilde{K}(b_T; \mu)}{\mathrm{d}\ln \mu} = -\gamma_K\left(\alpha_S(\mu)\right)$$

Anomalous dimension of CS kernel

$$\frac{\mathrm{d}\ln \tilde{f}_{q/\mathcal{N}}(x, b_T; \mu, \zeta)}{\mathrm{d}\ln \mu} = \gamma_f(\alpha_S(\mu)) - \frac{1}{2}\gamma_K(\alpha_S(\mu)) \ln \frac{\zeta}{\mu^2}$$

Renormalization scale

Anomalous dimension of TMDPDF

Introduction of non-perturbative functions

- Because $b_* \neq b_T$, have to non-perturbatively describe large b_T behavior

Completely general – independent of quark, hadron, PDF or FF

$$g_K(b_T; b_{\max}) = -\tilde{K}(b_T, \mu) + \tilde{K}(b_*, \mu)$$

Non-perturbative function dependent in principle on flavor, hadron, etc.

$$e^{-g_{q/\mathcal{N}(A)}(x, b_T)} = \frac{\tilde{f}_{q/\mathcal{N}(A)}(x, b_T; \mu, \zeta)}{\tilde{f}_{q/\mathcal{N}(A)}(x, b_*; \mu, \zeta)}$$

Putting all the pieces together

- A single TMD PDF

$$\tilde{f}_{q/\mathcal{N}}(x, b_T; \mu, \zeta) = \boxed{\sum_j \int_x^1 \frac{d\xi}{\xi} \boxed{f_{j/\mathcal{N}}(\xi; \mu_{b_*})} \tilde{C}_{q/j}\left(\frac{x}{\xi}, b_*; \mu_{b_*}, \mu_{b_*}^2\right)}$$
$$\times \boxed{\exp\left\{\tilde{K}(b_*; \mu_{b_*}) \log\sqrt{\frac{\zeta}{\mu_{b_*}^2}} + \int_{\mu_{b_*}}^{\mu} \frac{d\mu'}{\mu'}\left[\gamma_j(\alpha_S(\mu')) - \log\frac{Q}{\mu'}\gamma_K(\alpha_S(\mu'))\right]\right\}}$$
$$\times \boxed{\exp\left\{-g_{q/\mathcal{N}}(x, b_T) - g_K(b_T)\log\frac{\sqrt{\zeta}}{Q_0}\right\}}$$

$\tilde{C}$ are perturbatively calculable

Explicit dependence on collinear PDF

In practice, integral is done with a Mellin inversion

Perturbatively calculable evolution, can be saved in storage.

If we use ζ-prescription, the integral over μ' is trivial

Nonperturbative objects that are parametrized

TMD factorization in Drell-Yan

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\,\mathrm{d}y\,\mathrm{d}q_\mathrm{T}^2} = \frac{4\pi^2\alpha^2}{9Q^2 s} \sum_{j,j_A,j_B} H^{\mathrm{DY}}_{j\bar{j}}(Q,\mu_Q,a_s(\mu_Q)) \int \frac{\mathrm{d}^2\boldsymbol{b}_\mathrm{T}}{(2\pi)^2} e^{i\boldsymbol{q}_\mathrm{T}\cdot\boldsymbol{b}_\mathrm{T}}$$

$$\times\, e^{-g_{j/A}(x_A,b_\mathrm{T};b_\mathrm{max})} \int_{x_A}^{1} \frac{\mathrm{d}\xi_A}{\xi_A} f_{j_A/A}(\xi_A;\mu_{b_*})\, \tilde{C}^{\mathrm{PDF}}_{j/j_A}\left(\frac{x_A}{\xi_A}, b_*; \mu^2_{b_*}, \mu_{b_*}, a_s(\mu_{b_*})\right)$$

$$\times\, e^{-g_{\bar{j}/B}(x_B,b_\mathrm{T};b_\mathrm{max})} \int_{x_B}^{1} \frac{\mathrm{d}\xi_B}{\xi_B} f_{j_B/B}(\xi_B;\mu_{b_*})\, \tilde{C}^{\mathrm{PDF}}_{\bar{j}/j_B}\left(\frac{x_B}{\xi_B}, b_*; \mu^2_{b_*}, \mu_{b_*}, a_s(\mu_{b_*})\right)$$

$$\times \exp\left\{-\boxed{g_K}(b_\mathrm{T};b_\mathrm{max}) \ln\frac{Q^2}{Q_0^2} + \tilde{K}(b_*;\mu_{b_*}) \ln\frac{Q^2}{\mu^2_{b_*}} + \int_{\mu_{b_*}}^{\mu_Q} \frac{\mathrm{d}\mu'}{\mu'}\left[2\gamma_j(a_s(\mu')) - \ln\frac{Q^2}{(\mu')^2}\gamma_K(a_s(\mu'))\right]\right\}$$

Perturbative pieces

Non-perturbative pieces

Non-perturbative piece of the CS kernel

TMD factorization in Drell-Yan

- We combine the two b_T-space TMD PDFs into $\widetilde{W}$

$$\frac{d^3\sigma}{dQ^2 dy dq_T^2} = \frac{4\pi^2\alpha^2}{9Q^2 s}\mathcal{P}\sum_q H_{q\bar{q}}^{\rm DY}(Q,\mu_Q)\int \frac{d^2\boldsymbol{b}_T}{(2\pi)^2} e^{i\boldsymbol{b}_T\cdot\boldsymbol{q}_T}\widetilde{W}(b_T,Q,s,y;\mu,\zeta)$$

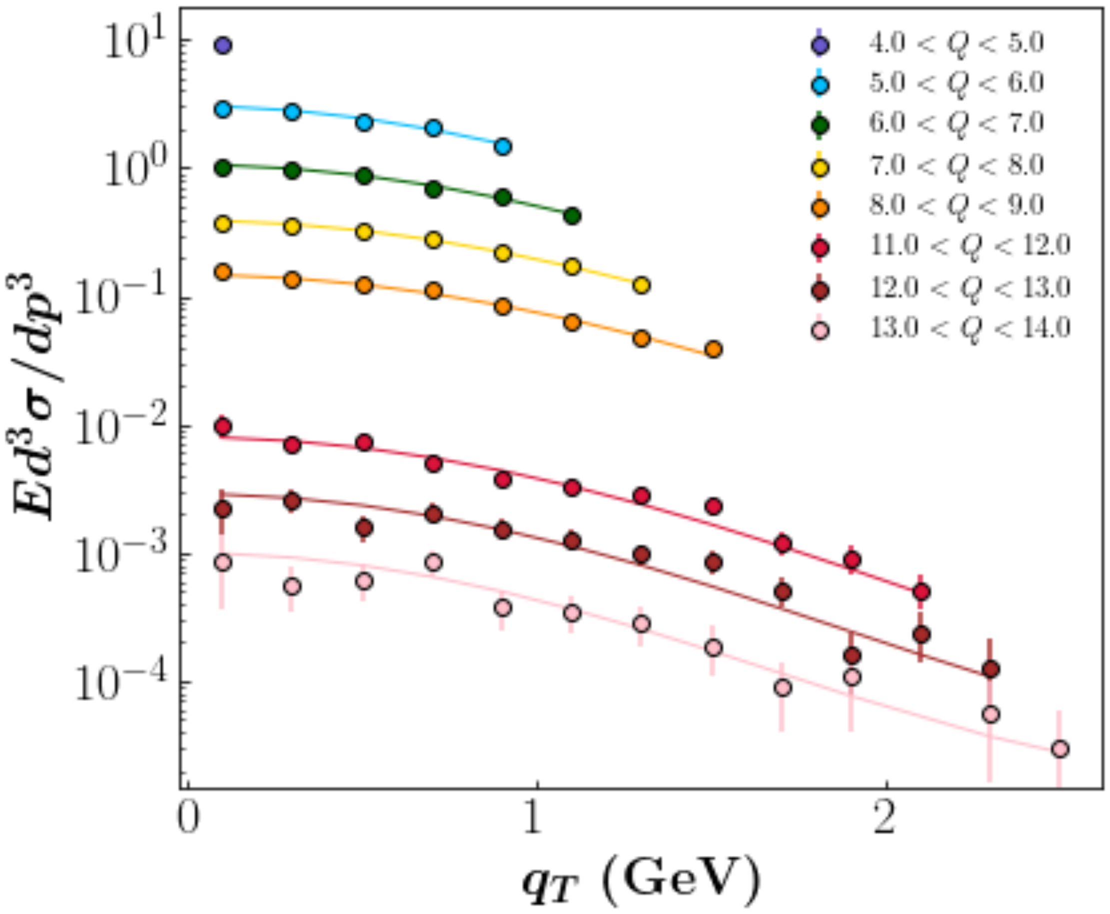
- Notice that $\widetilde{W}$ is independent of q_T

- Simplify according to Hankel transform

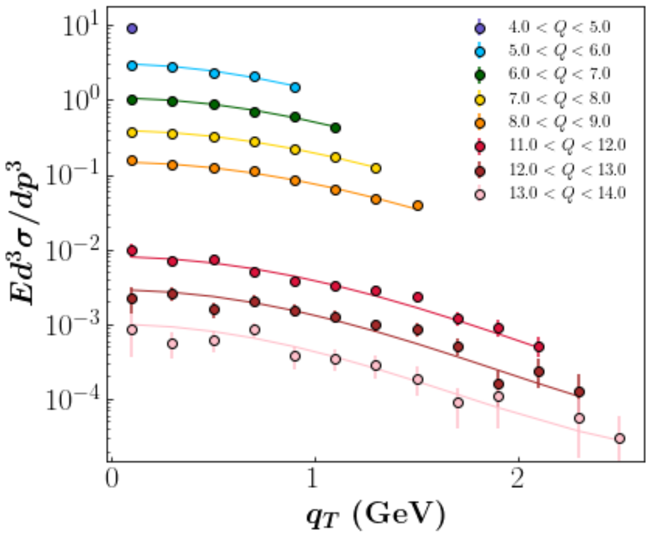
$$\frac{d^3\sigma}{dQ^2 dy dq_T^2} = \frac{4\pi^2\alpha^2}{9Q^2 s}\mathcal{P}\sum_q H_{q\bar{q}}^{\rm DY}(Q,\mu_Q)\int_0^\infty db_T b_T J_0(b_T q_T)\widetilde{W}(b_T,Q,s,y;\mu,\zeta)$$

Fixed target DY data example – E288 400 GeV

- $E\,\frac{d^3\sigma}{dp^3} = \frac{d^2\sigma}{\pi dy dq_T^2}$
  - We must **integrate** over the range in Q^2
- Only fit $q_T < 0.2\,Q$
- In this dataset, we are given a rapidity value $y = 0.03$
- Evaluate the differential cross section at central points in q_T bin
- Uncertainties are relatively large
- 25% normalization uncertainty



Strategy – E288 400 GeV



$$E\frac{d^3\sigma}{dp^3} = \sum_q \frac{4\pi^2\alpha^2}{9s} \int_{Q_{\min}}^{Q_{\max}} 2QdQ \frac{1}{Q^2} H_{q\bar{q}}^{\mathrm{DY}}(Q,\mu_Q) \int_0^\infty db_T b_T J_0(b_T q_T) \widetilde{W}(b_T, Q, s, y; \mu, \zeta)$$

- For each bin of Q, we evaluate at 5 Q interval points
- For each of those Q points, we compute $\widetilde{W}$ as a function of b_T along 100 points from 10^{-4} to 11 GeV^{-1}
- Build a 1d interpolated version of $\widetilde{W}$ over b_T
- Perform b_T integration for the q_T values from the dataset
- Perform Q integration

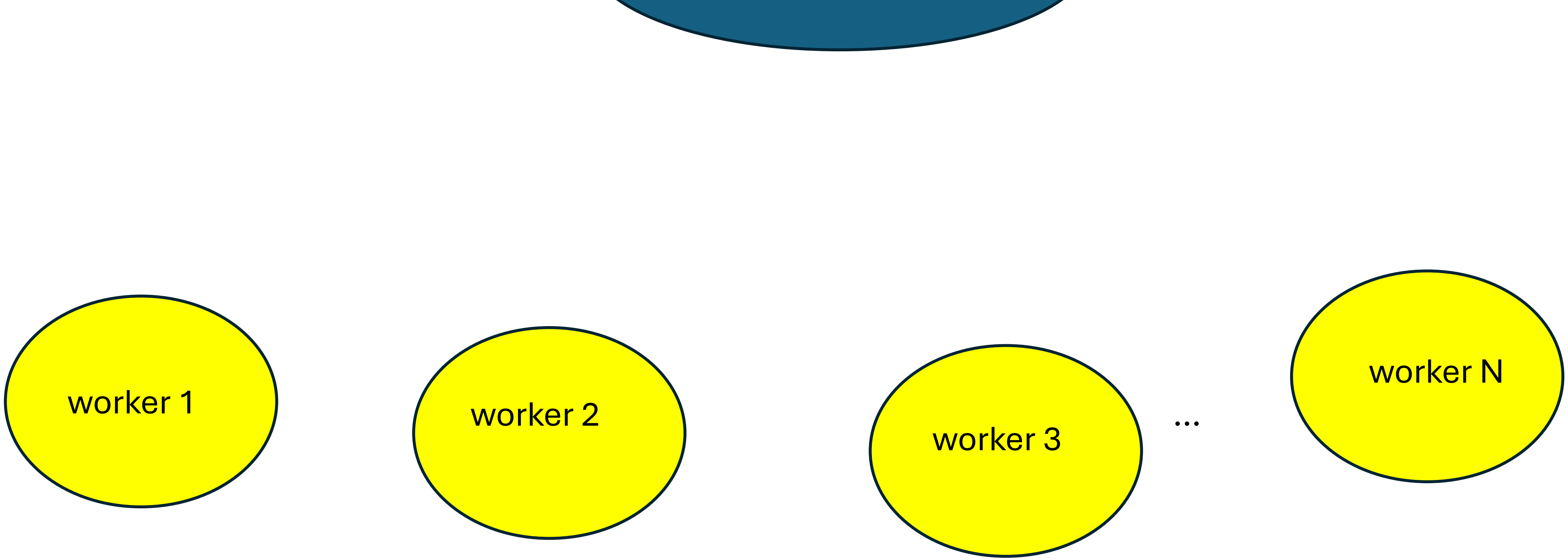
Parallelization details

- Two parallelized tasks:
  1. Computation of $\widetilde{W}$ for all kinematic points - all (b_T, Q, s, y) needed for interpolation
  2. Performing the integral over b_T for all the q_T points
- Upon initialization – run over all kinematics and create an empty storages for each kinematic point

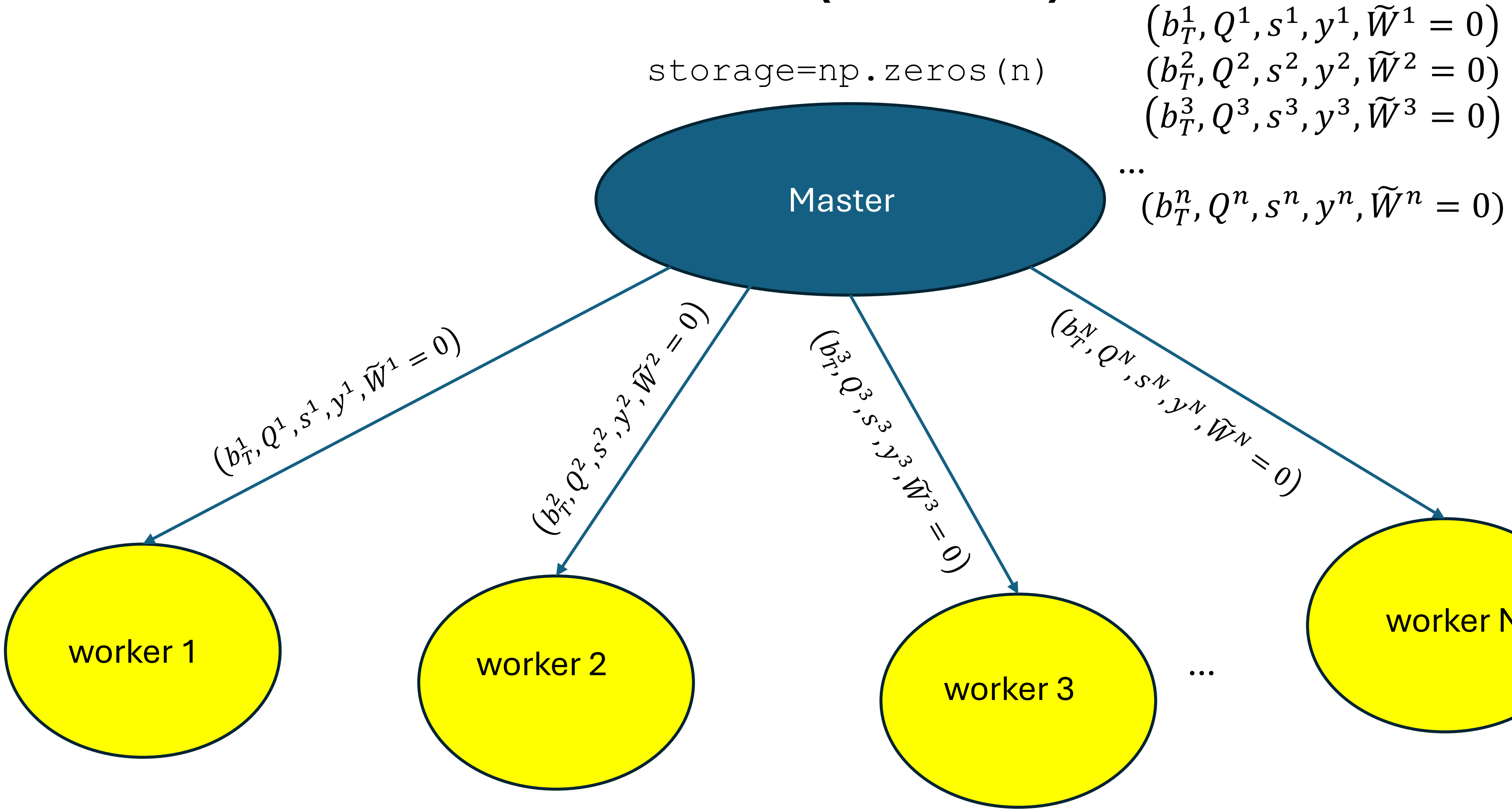
Parallelization details (task 1)

`storage=np.zeros(n)`

$$\left(b_T^1, Q^1, s^1, y^1, \widetilde{W}^1 = 0\right)$$
$$(b_T^2, Q^2, s^2, y^2, \widetilde{W}^2 = 0)$$
$$\left(b_T^3, Q^3, s^3, y^3, \widetilde{W}^3 = 0\right)$$
…
$$(b_T^n, Q^n, s^n, y^n, \widetilde{W}^n = 0)$$



Parallelization details (task 1)



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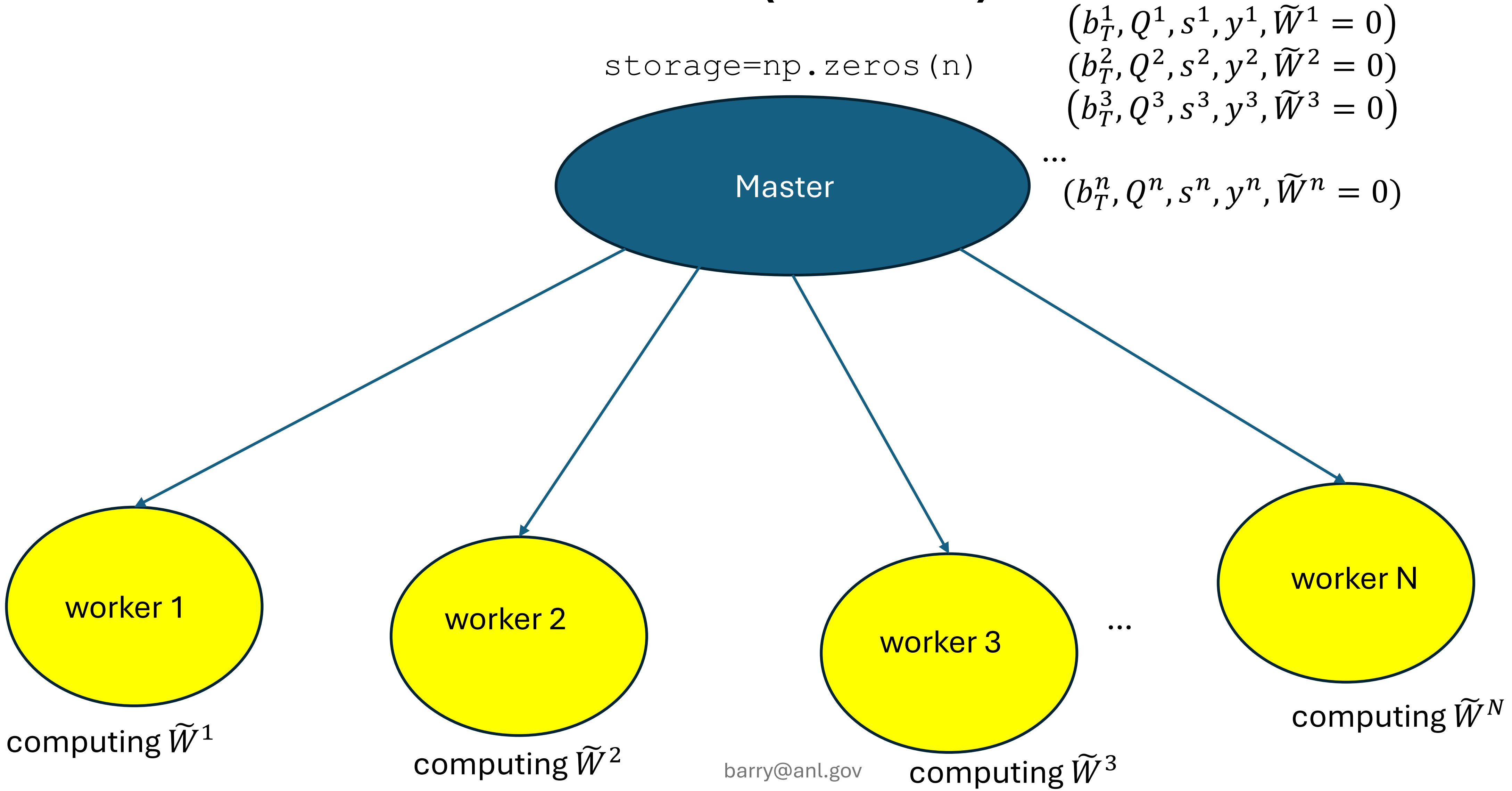

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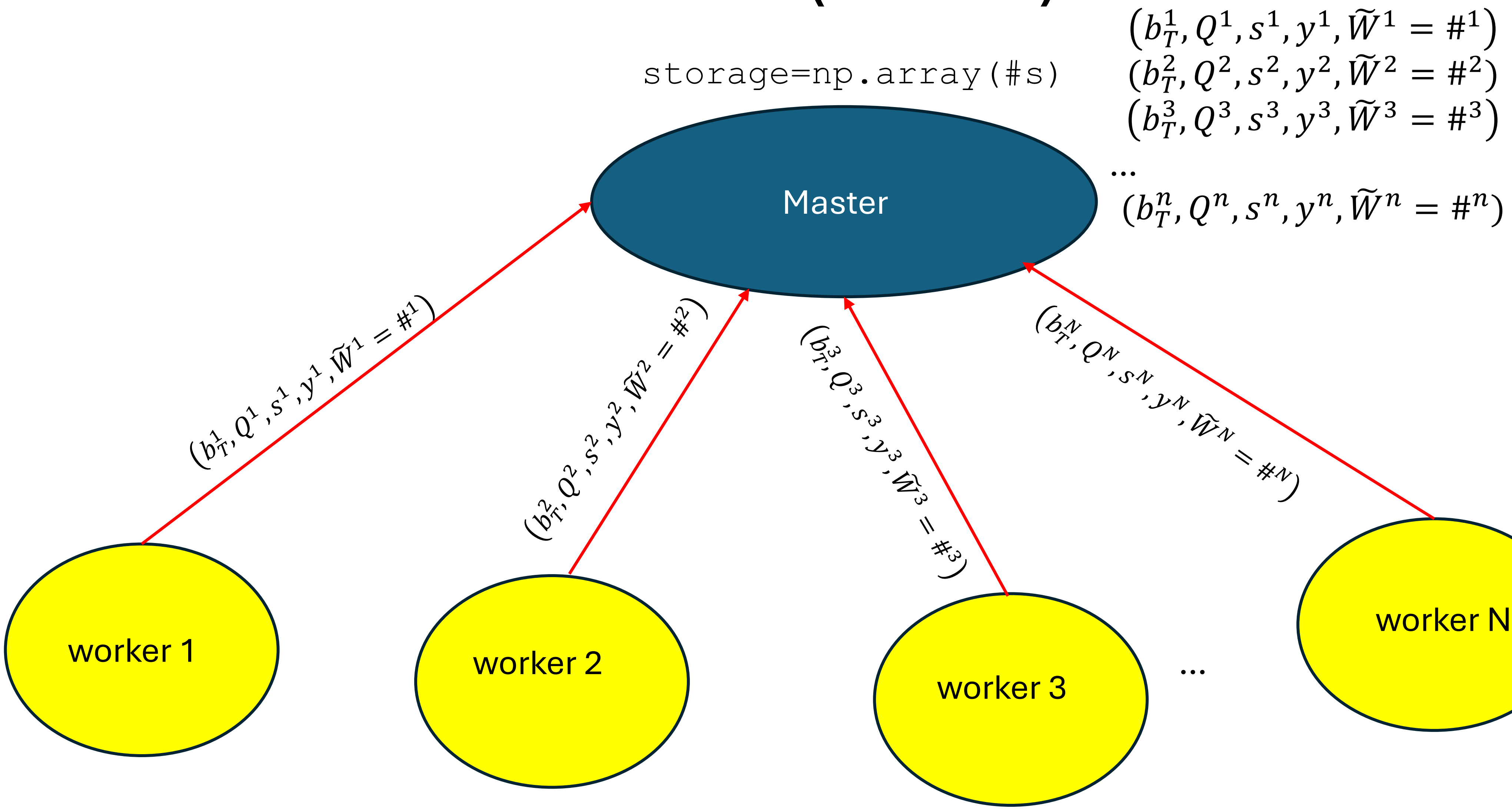
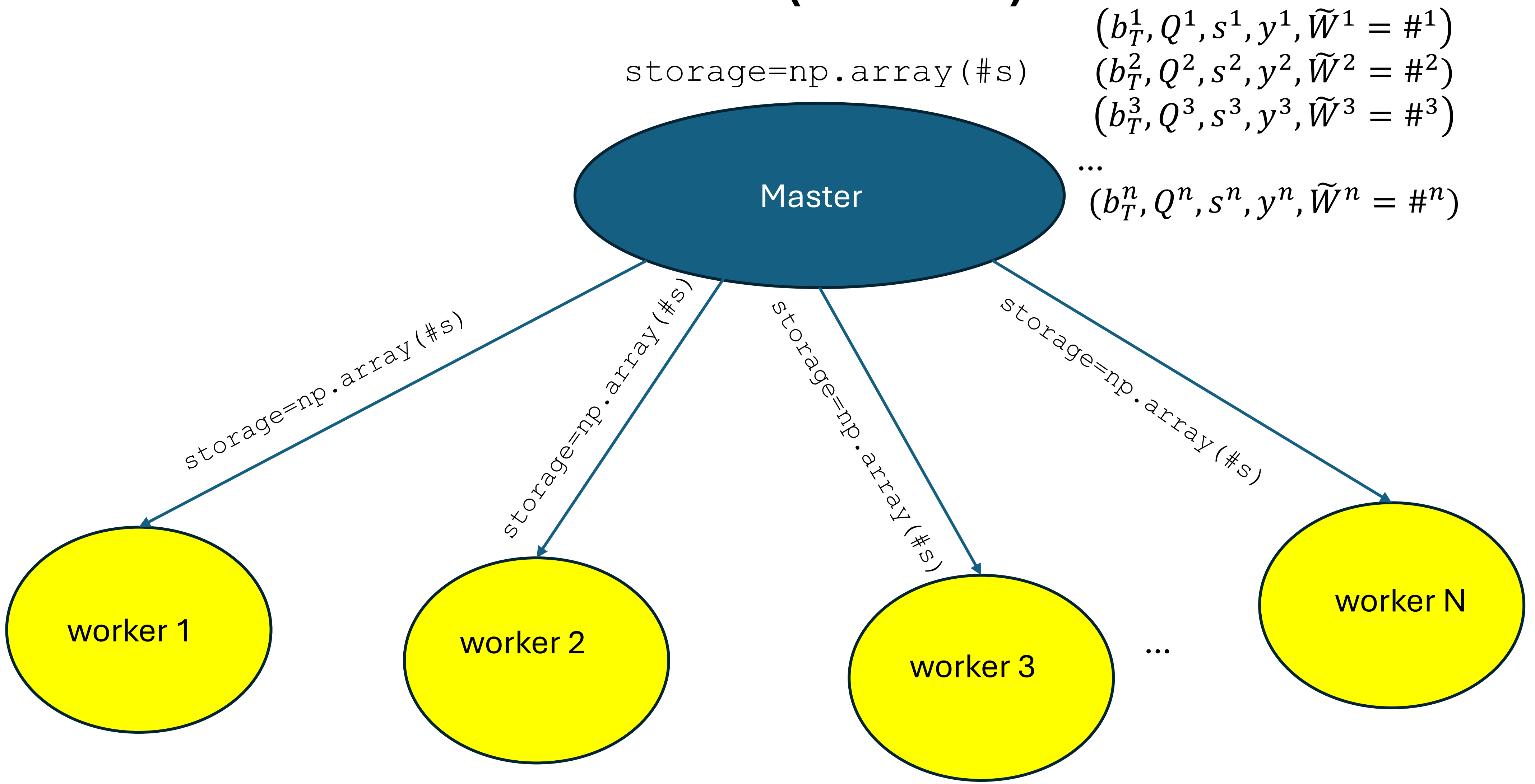
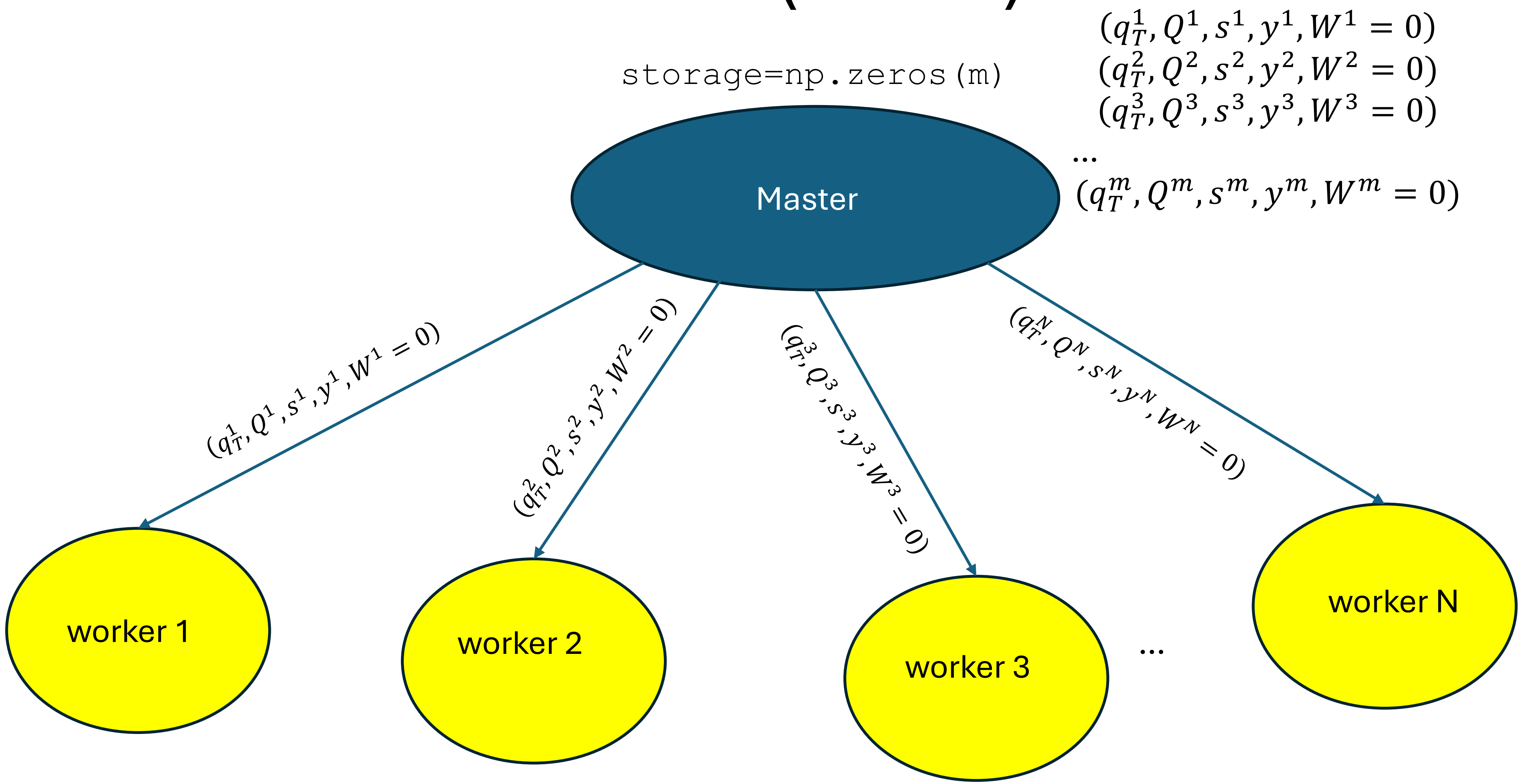


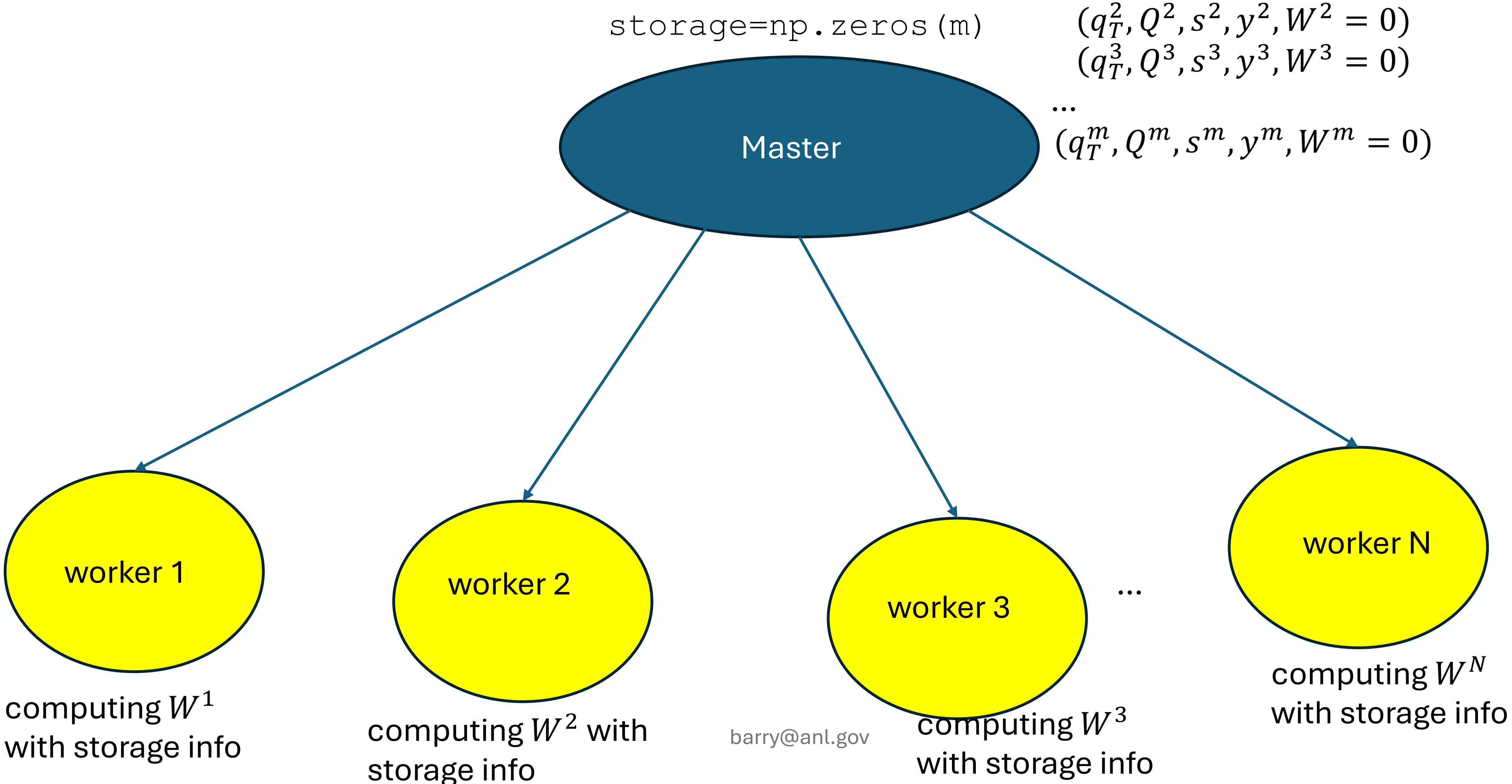
Parallelization details (task 1)



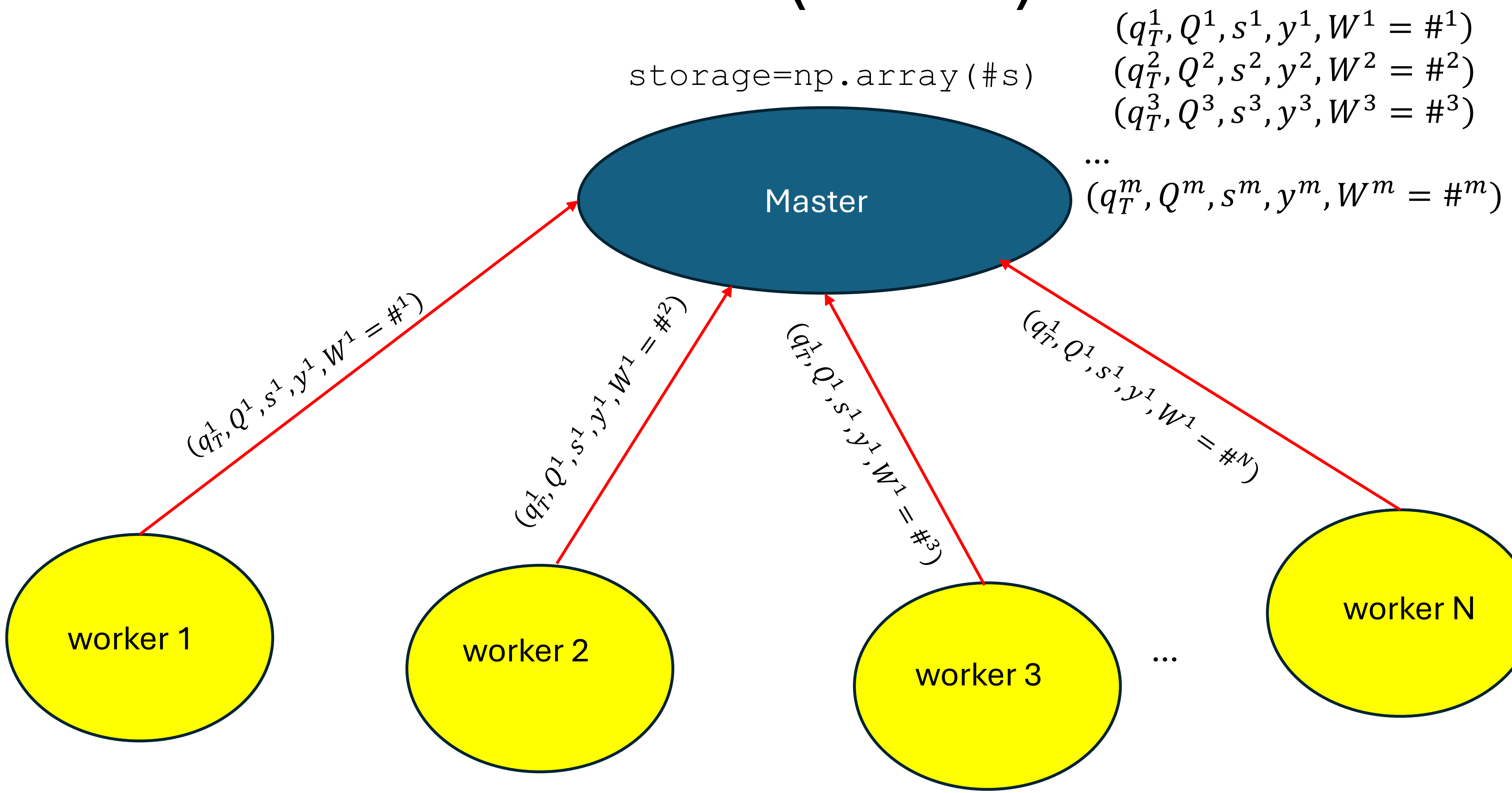
Parallelization details (task 2)



Parallelization details (task 2)



Parallelization details (task 2)



Computation of differential cross sections on master node

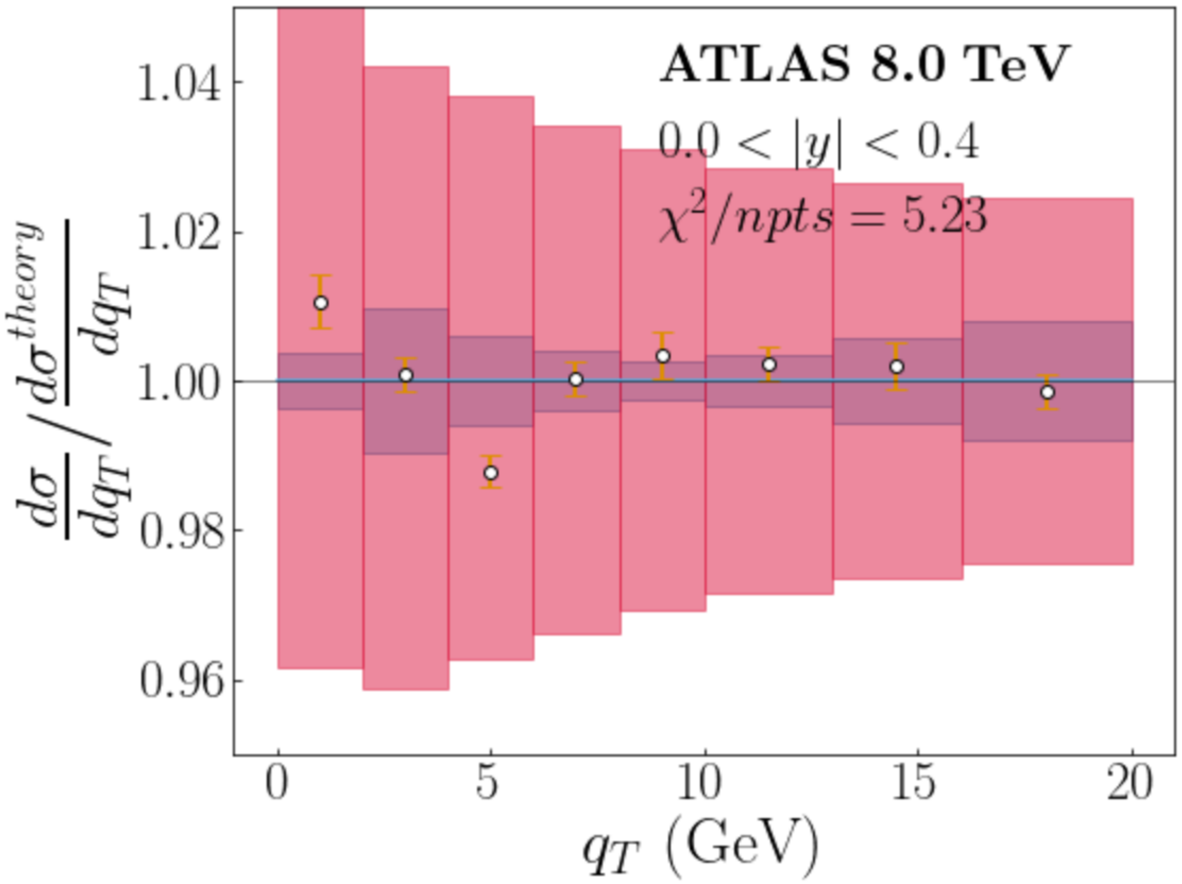
- Takes information from storage from the workers
- Interpolates over Q for each bin
- Integrates over Q
- Sends value to the residuals

Fixed target performance

- These computations are relatively quick and efficient
- 8 workers on 16 cpu allocation
- 224 data points
- 91.2 seconds (CSS prescription)
- 13.1 seconds (ζ-prescription)

Collider data - LHC

- These data are *much* more precise than the fixed-target counterparts
- Spectrum is peaked around the Z-boson, which is a narrow peak near $Q = 91$ GeV
- Optimized TMD region goes to larger q_T - more oscillatory integrand



Collider data - LHC

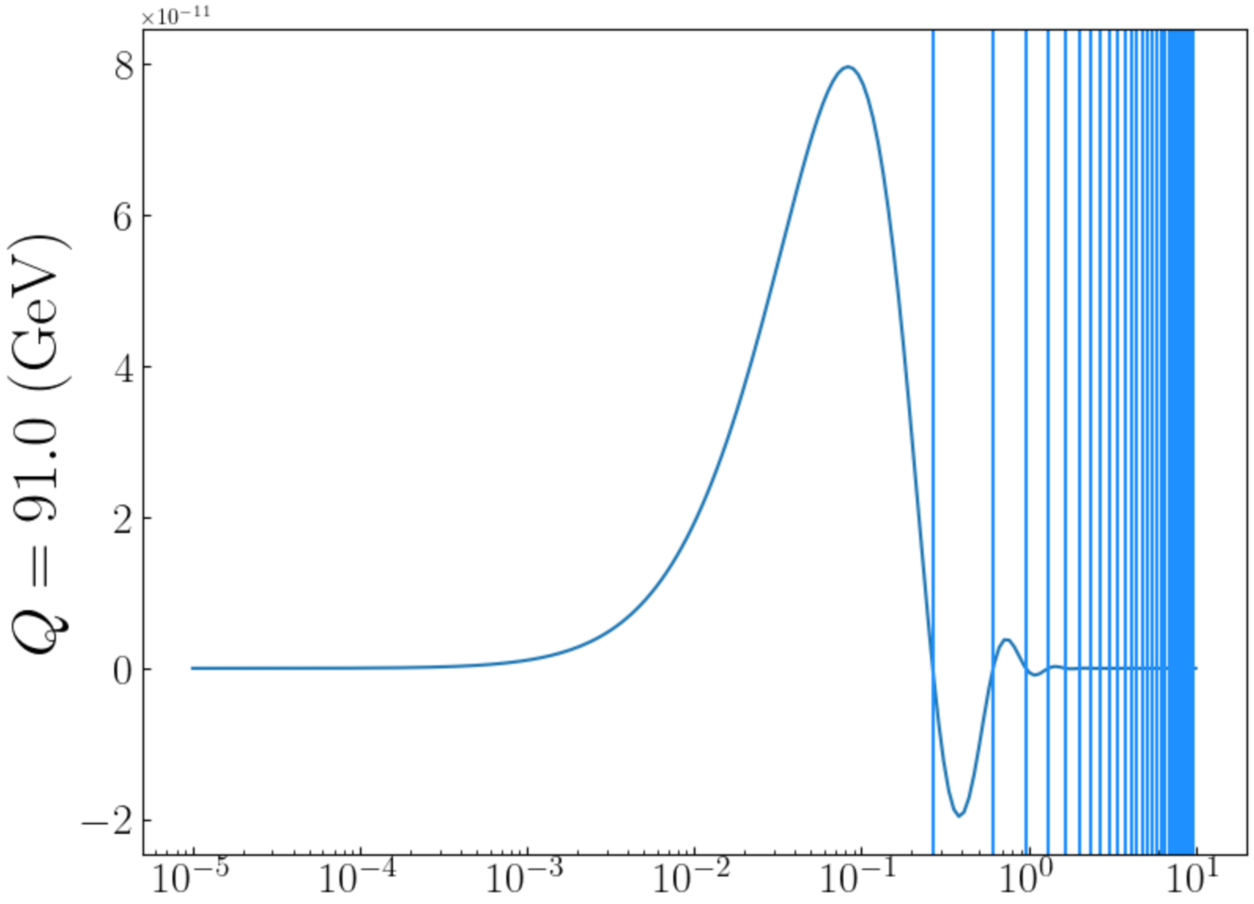
- Reported data are more integrated quantities

$$\frac{d\sigma}{dq_T} = \frac{1}{\Delta q_T} \int dq_T \int dQ \int dy \mathcal{P}(Q, y, q_T) \int db_T b_T J_0(b_T q_T) \widetilde{W}(Q, y, b_T, s)$$

- Instead of a given y value, we have to integrate over the range provided
- Bins in q_T are wider, so performing a bin averaging becomes necessary
- Here, we introduce the fiducial volume $\mathcal{P}$ – this is computable when we know the grid points to use – no parameter dependence

Strategy to compute b_T integral efficiently

- Bessel function is highly oscillatory
- Subdivide integrations over b_T at the nodes of the Bessel function
- Compute integrations for each region and sum



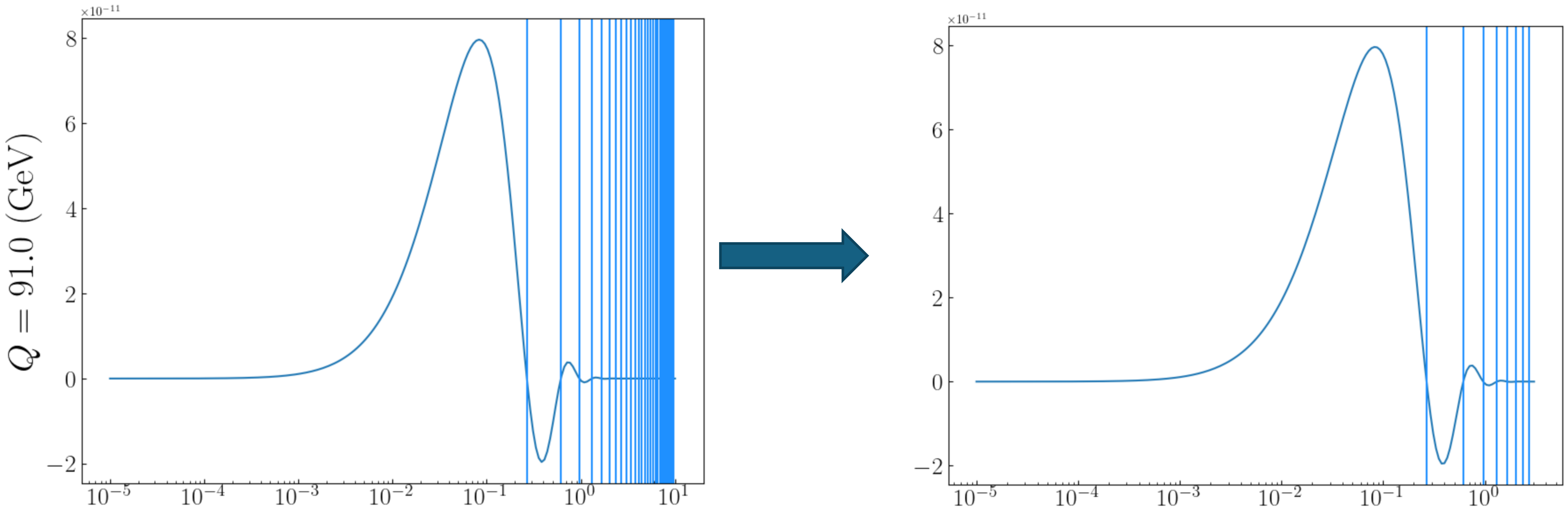
Implementation

- Pass through the `bTnodes` in argument

```
bTmin =1e-5
bTmax =10
if len(bTnodes)==0:
    W=quad(get_WZ_integrand,args,bTmin,bTmax,ngrid=4,eps_rel=1e-10,eps_abs=1e-10)
else:
    flag='calculating'
    W=quad(get_WZ_integrand,args,bTmin,bTnodes[0],ngrid=4,eps_rel=1e-10,eps_abs=1e-10)
    for i in range(len(bTnodes)-1):
        if flag=='finished': continue
        intgrl=quad(get_WZ_integrand,args,bTnodes[i],bTnodes[i+1],ngrid=4,eps_rel=1e-10,eps_abs=1e-10)
        W+=intgrl
        if np.abs(intgrl)/W < 1e-4: flag='finished'
    if flag!='finished':
        W+=quad(get_WZ_integrand,args,bTnodes[-1],bTmax,ngrid=4,eps_rel=1e-10,eps_abs=1e-10)
born=2*pT/2/np.pi
return born*W
```

Implement a truncation if the next integral is relatively small

Truncation



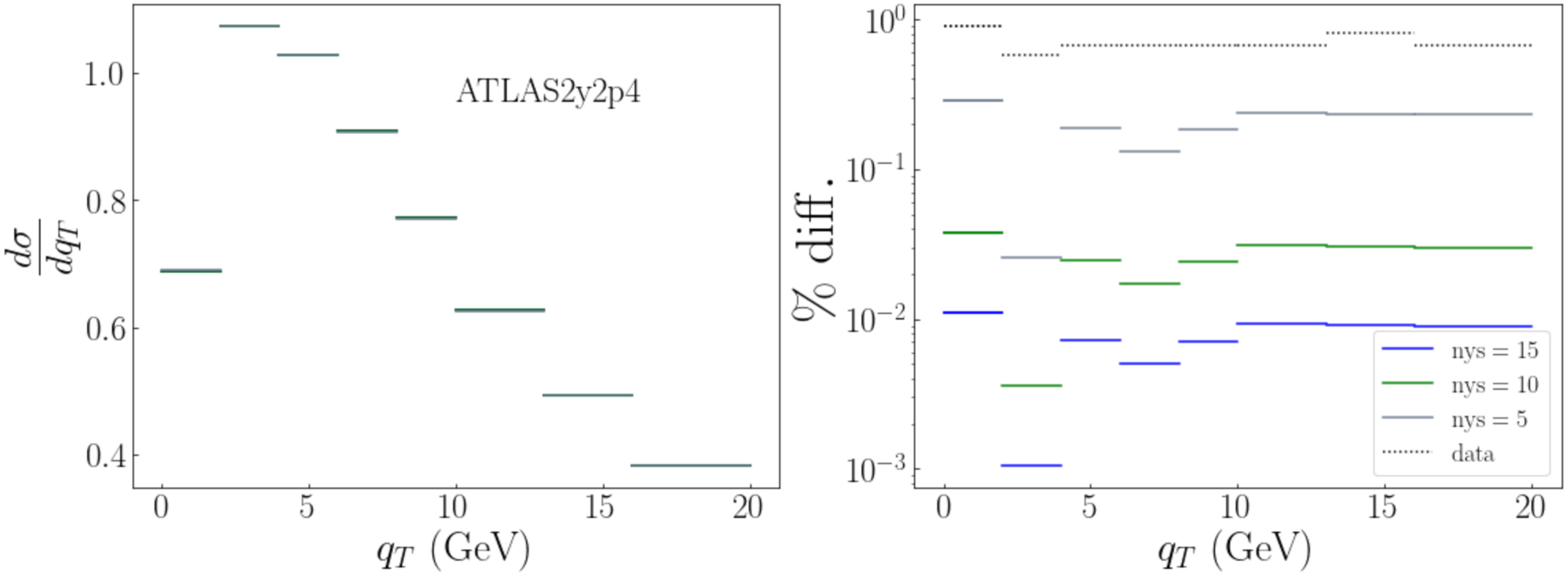
- Stop computing integrations after certain point
- Assume rest of integral = 0

Strategy – collider data

- Parallelization is same task force as the fixed target regime
- We set a predefined grid of (Q, y) to compute $\frac{d^3\sigma}{dQ^2 dy dq_T^2}$, and interpolate over the 2d grid to compute the Q and y integrals using a fixed Gaussian quadrature for a given q_T
- Hard code the q_T integrations with fixed Gaussian quadrature points
- To speed up – test the accuracy of fewer grid points against the uncertainty on the data

Reducing number of grid points

- As a baseline, we use 50 points in y
  - We can see that 10 y points has an order of magnitude less % difference than the uncertainty on the data

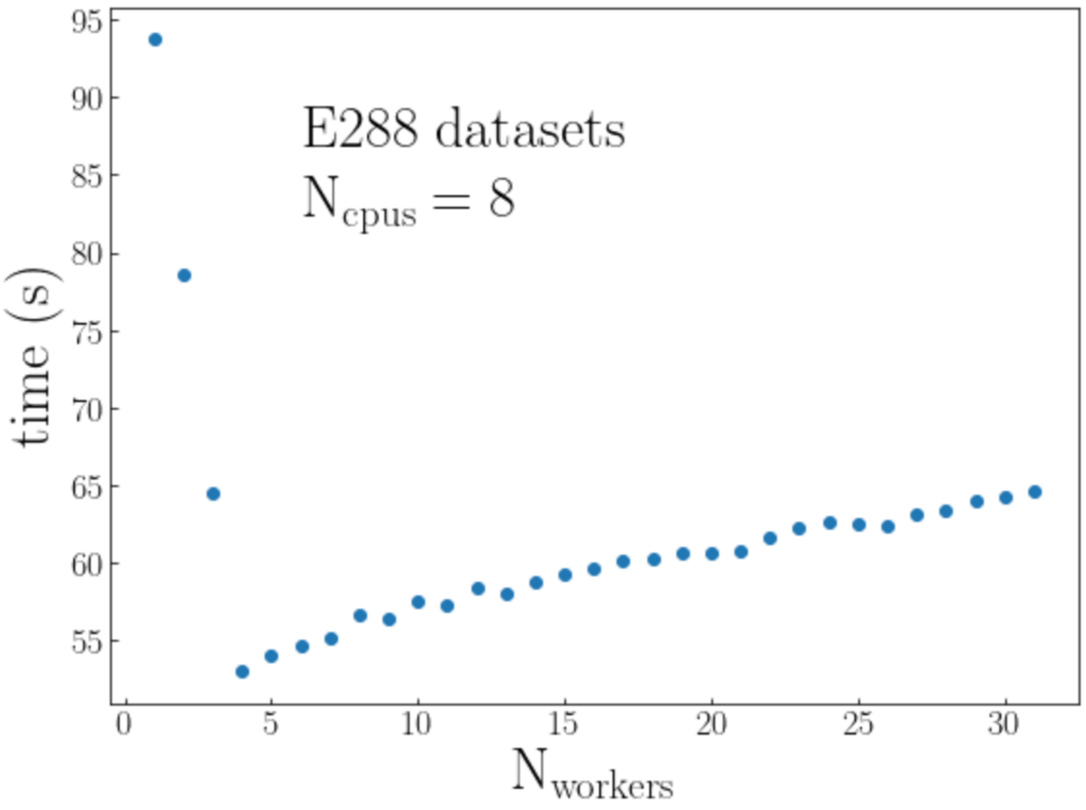


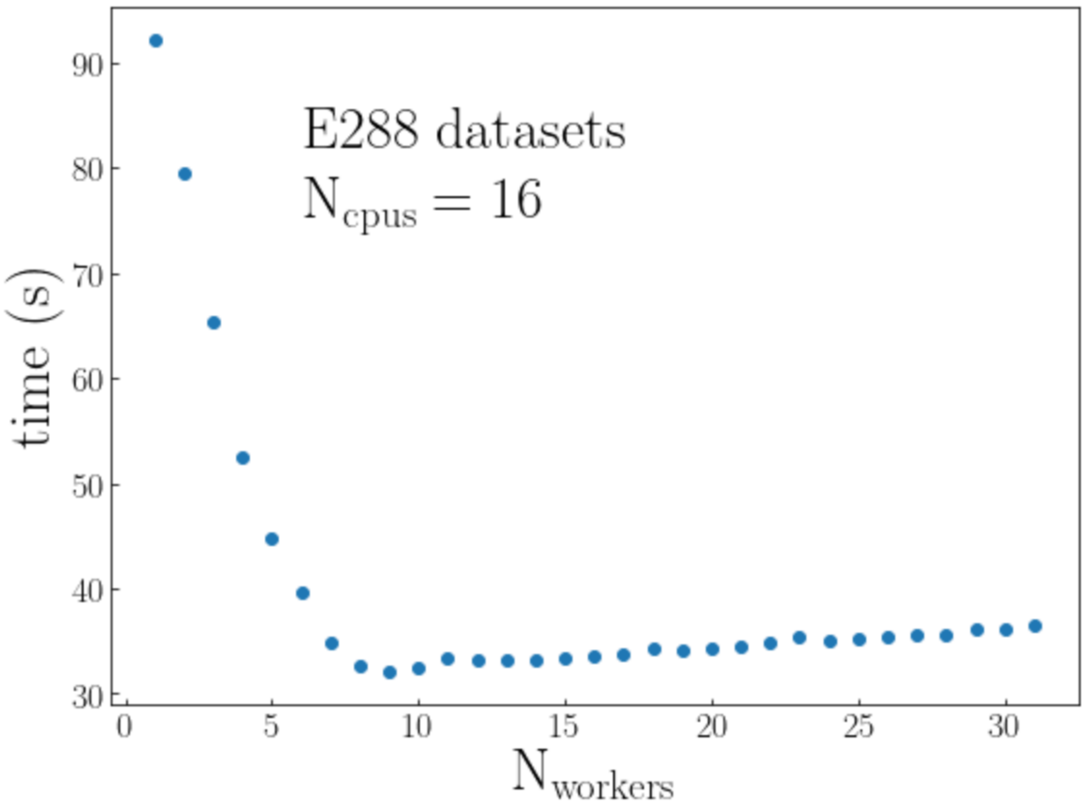
Performance of the collider regime

- Example on the ATLAS 8 TeV dataset (most precise): $0 < |y| < 0.4$
- 8 workers on 16 cpu allocation
- 8 data points
- (CSS): ~180 seconds → ~50 seconds: 50 y points → 5 y points
- (ζ): ~140 seconds → ~15 seconds: 50 y points → 5 y points

Varying number of workers

- Example for low energy datasets – not really sure how to interpret





Preliminary results

- Open both collinear and TMD parameters
- Fit to all TMD data and DIS and DY data – more conservative cuts
- No bootstrap yet on the data – only fits to central values
- We need more results to make any conclusions, but we are making progress

6 replicas

Zeta - NLO+N3LL - fit TMDs and PDFs to collinear and TMD data

Preliminary

Fixed target

	expt	obs	npts	chi2/npts	Z-score	norm_e
dy_qT-12881	E288	Ed3sigma/dp3	31	0.607475	1.724092	0.66197
dy_qT-12882	E288	Ed3sigma/dp3	41	0.983002	0.003593	0.640664
dy_qT-12883	E288	Ed3sigma/dp3	64	0.889252	0.591703	0.71856
dy_qT-10605	E605	Ed3sigma/dp3	43	1.112895	0.576295	0.713713
dy_qT-10772	E772	Ed3sigma/dp3	45	3.291467	7.097962	0.674534

Tevatron

	expt	obs	npts	chi2/npts	Z-score	norm_e
dy_qT-90001	CDF	dsig/dpT	30	0.780858	0.832231	0.992312
dy_qT-90002	CDF	dsig/dpT	36	1.949567	3.259774	1.051179
dy_qT-90003	D0	dsig/dpT	14	0.711364	0.723025	1.079405

LHC

	expt	obs	npts	chi2/npts	Z-score	norm_e
dy_qT-70101	ATLAS	dsig/dpT	8	5.232315	4.679255	0.983501
dy_qT-70102	ATLAS	dsig/dpT	8	1.79634	1.456697	0.983501
dy_qT-70103	ATLAS	dsig/dpT	8	1.402437	0.879483	0.983501
dy_qT-70104	ATLAS	dsig/dpT	8	3.624246	3.415888	0.983501
dy_qT-70105	ATLAS	dsig/dpT	8	1.987644	1.708013	0.983501
dy_qT-70106	ATLAS	dsig/dpT	8	1.295692	0.705334	0.983501
dy_qT-30001	LHCb	dsig/dpT	10	1.062624	0.286185	0.978208
dy_qT-40001	LHCb	sig	10	1.41666	0.971992	0.992076
dy_qT-50001	LHCb	sig	10	1.902854	1.751828	0.986994
dy_qT-20011	CMS	dsig/dpT	15	3.150494	3.986823	0.966343
dy_qT-20012	CMS	dsig/dpT	15	1.284569	0.835131	0.966343
dy_qT-20013	CMS	dsig/dpT	15	0.677952	0.874066	0.966343
dy_qT-20014	CMS	dsig/dpT	15	0.469023	1.713193	0.966343
dy_qT-20015	CMS	dsig/dpT	15	0.570745	1.277703	0.966343

Zeta - NLO+N3LL - fit TMDs and PDFs to collinear and TMD data

Preliminary

DIS

	expt	obs	npts	chi2/npts	Z-score	norm_e
idis-10010	SLAC	F2	218	1.047646	0.521561	1.051022
idis-10016	BCDMS	F2	348	1.184574	2.32412	0.994994
idis-10020	NMC	F2	273	1.752686	7.25825	1.03092
idis-10026	HERA II NC e+ (1)	sig_r	402	1.588192	7.128131	--
idis-10027	HERA II NC e+ (2)	sig_r	75	1.157858	0.973823	--
idis-10028	HERA II NC e+ (3)	sig_r	259	0.979654	0.203777	--
idis-10029	HERA II NC e+ (4)	sig_r	209	1.071991	0.751404	--
idis-10030	HERA II NC e-	sig_r	159	1.641513	4.852066	--
idis-10031	HERA II CC e+	sig_r	39	1.135045	0.646196	--
idis-10032	HERA II CC e-	sig_r	42	1.032389	0.219479	--
idis-10011	SLAC	F2	228	0.828944	1.910629	1.052565
idis-10017	BCDMS	F2	254	1.19829	2.131071	1.023282
idis-10021	NMC	F2d/F2p	174	1.111182	1.036343	1.002163

DY

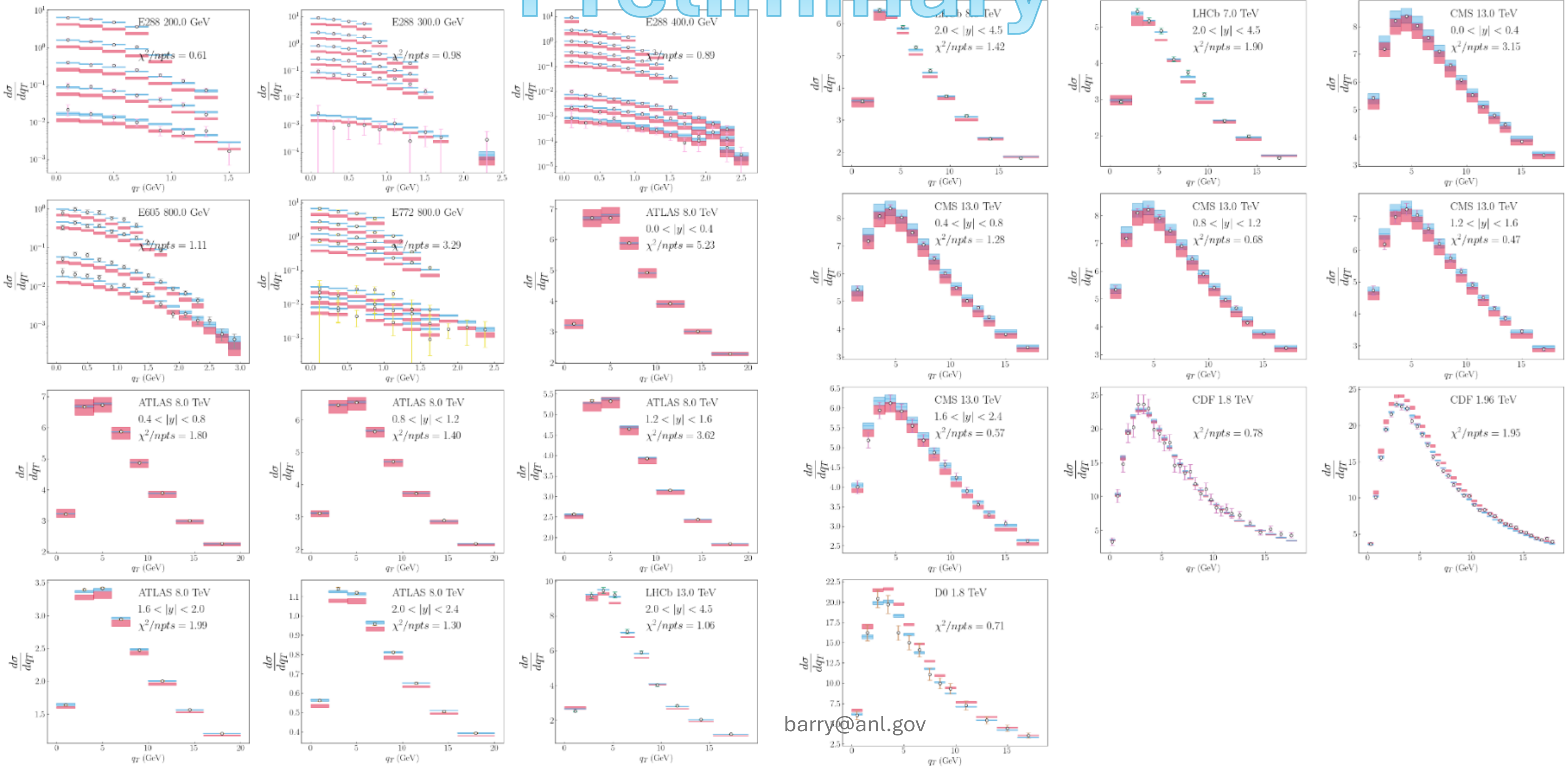
	expt	obs	npts	chi2/npts	Z-score	norm_e
dy-10001	E866	M3 dsig/dM dxF	184	1.405966	3.498452	1.086221
dy-20001	E866	sigpd/2sigpp	15	2.100541	2.432061	0.992887
dy-20002	E906	sigpd/2sigpp	6	1.032193	0.248422	0.999449

Total

	expt	obs	npts	chi2/npts	Z-score	norm_e
total	--	--	3342	1.295128	inf	--

Zeta - NLO+N3LL - fit TMDs and PDFs to collinear and TMD data

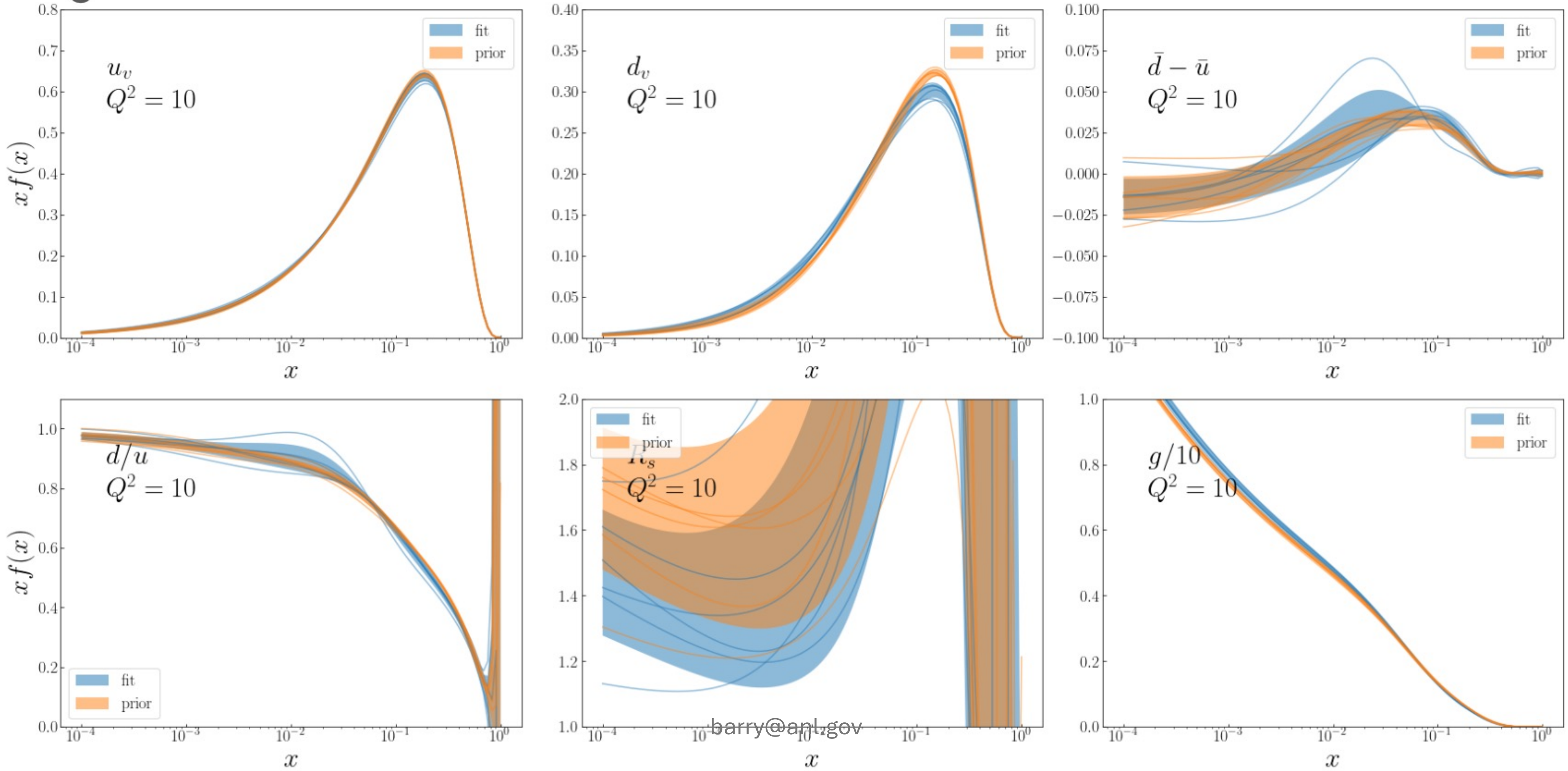
Preliminary



Zeta - NLO+N3LL - fit TMDs and PDFs to collinear and TMD data

Preliminary

Resulting PDFs

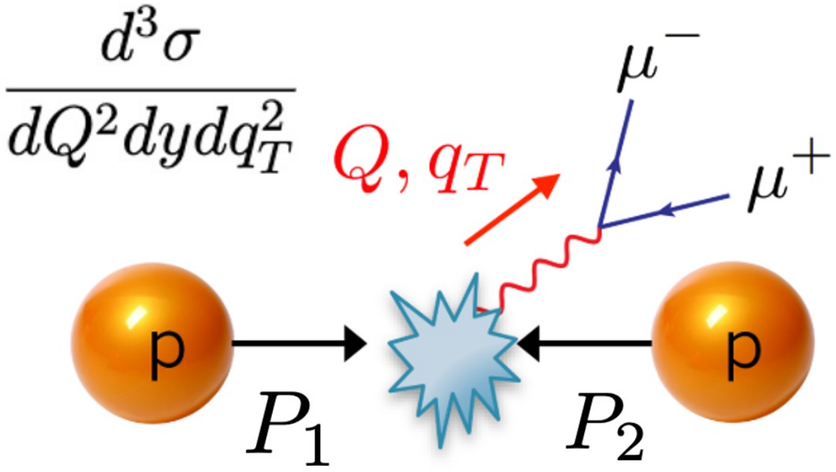


Summary

- Computational time is right now a bottleneck in our simultaneous extractions of TMDs and PDFs
- Precision of the theoretical calculation must match or be better than the uncertainties of the data
- More replicas are needed to draw meaningful conclusion from preliminary results
- We need to more rigorously explore the perturbative accuracies and the way we implement the TMDs

Backup

Drell-Yan kinematics



- Q^2 is the invariant mass of the virtual photon
- y is the rapidity, $y = \frac{1}{2}\log\frac{q^+}{q^-}$ is a measure of how forward/backward the $q\bar{q}$ annihilation occurred relative to the beam line
- q_T is the transverse momentum of the virtual photon, which is inherited by the $\mu^-\mu^+$
- s is the incoming center of mass energy squared of the hadrons
- $x_1 = Q/\sqrt{s}e^y$, $x_2 = Q/\sqrt{s}e^{-y}$ are the partonic momentum fractions relative to the parent hadrons
- $\mathcal{P}$ is a fiducial volume – more on this later

Deriving nonperturbative functions

- Start with the ζ-scale evolution – either for b_T or b_*

$$\tilde{f}_{q/\mathcal{N}}(x, b_T; \mu, \zeta) = \tilde{f}_{q/\mathcal{N}}(x, b_T; \mu, Q_0^2) \exp\left(\tilde{K}(b_T; \mu) \log \frac{\sqrt{\zeta}}{Q_0}\right)$$

$$\tilde{f}_{q/\mathcal{N}}(x, b_*; \mu, \zeta) = \tilde{f}_{q/\mathcal{N}}(x, b_*; \mu, Q_0^2) \exp\left(\tilde{K}(b_*; \mu) \log \frac{\sqrt{\zeta}}{Q_0}\right)$$

- Take the ratio

$$\frac{\tilde{f}_{q/\mathcal{N}}(x, b_T; \mu, \zeta)}{\tilde{f}_{q/\mathcal{N}}(x, b_*; \mu, \zeta)} = \frac{\tilde{f}_{q/\mathcal{N}}(x, b_T; \mu, Q_0^2)}{\tilde{f}_{q/\mathcal{N}}(x, b_*; \mu, Q_0^2)} \exp\left(-\left[\tilde{K}(b_*; \mu) - \tilde{K}(b_T; \mu)\right] \log \frac{\sqrt{\zeta}}{Q_0}\right)$$

Deriving nonperturbative functions

$$\frac{\tilde{f}_{q/\mathcal{N}}(x, b_T; \mu, \zeta)}{\tilde{f}_{q/\mathcal{N}}(x, b_*; \mu, \zeta)} = \frac{\tilde{f}_{q/\mathcal{N}}(x, b_T; \mu, Q_0^2)}{\tilde{f}_{q/\mathcal{N}}(x, b_*; \mu, Q_0^2)} \exp\left(- \left[\tilde{K}(b_*; \mu) - \tilde{K}(b_T; \mu) \right] \log \frac{\sqrt{\zeta}}{Q_0} \right)$$

- Since the evolution of $\widetilde{K}$ with respect to μ is b_T-independent, the μ-dependence of the difference cancels out, and we can write for general μ

$$\frac{\tilde{f}_{q/\mathcal{N}}(x, b_T; \mu, \zeta)}{\tilde{f}_{q/\mathcal{N}}(x, b_*; \mu, \zeta)} = \frac{\tilde{f}_{q/\mathcal{N}}(x, b_T; Q_0, Q_0^2)}{\tilde{f}_{q/\mathcal{N}}(x, b_*; Q_0, Q_0^2)} \exp\left(-g_K(b_T) \log \frac{\sqrt{\zeta}}{Q_0} \right)$$

Deriving nonperturbative functions

- We may also define the ratio of the TMDs themselves as being nonperturbative

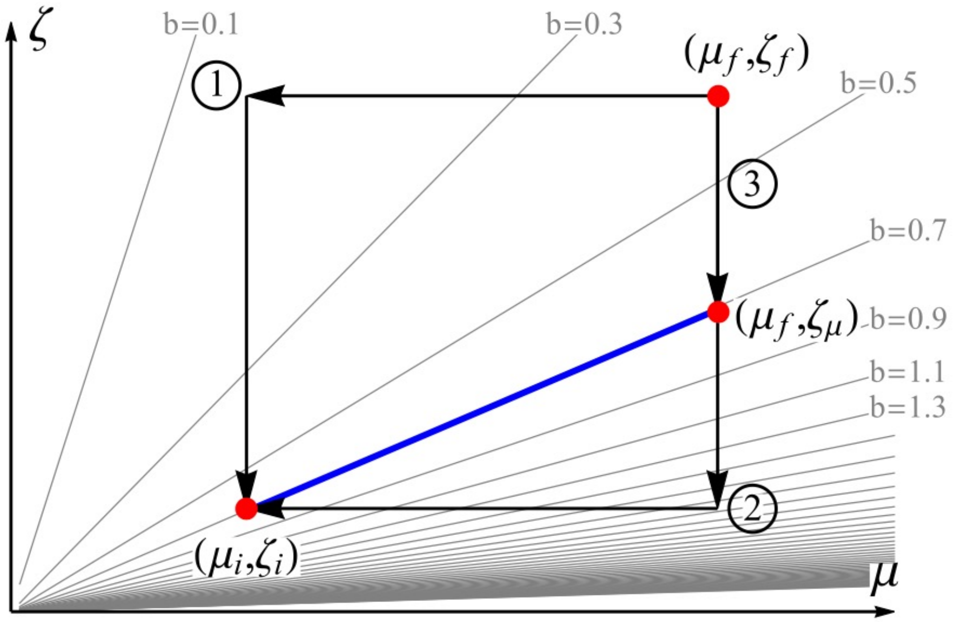
$$-g_{q/\mathcal{N}}(x, b_T) \equiv \log\left(\frac{\tilde{f}_{q/\mathcal{N}}(x, b_T; Q_0, Q_0^2)}{\tilde{f}_{q/\mathcal{N}}(x, b_*; Q_0, Q_0^2)}\right)$$

- And we can write our full b_T-space TMD in terms of the b_* TMD and the nonperturbative functions

$$\tilde{f}_{q/\mathcal{N}}(x, b_T; \mu, \zeta) = \tilde{f}_{q/\mathcal{N}}(x, b_*; \mu, \zeta) \exp\left(-g_{q/\mathcal{N}}(x, b_T) - g_K(b_T) \log\frac{\sqrt{\zeta}}{Q_0}\right)$$

Ways to evolve the TMD

- We can evolve from (μ_i, ζ_i) up to (μ_f, ζ_f) in multiple ways
- (**CSS**): Separately evolve up in μ and ζ
- (**ζ-prescription**): Evaluate the TMD PDF at the scales (μ_f, ζ_μ), which is along the null-evolution line
  - $\tilde{f}_q(x, b_T; \mu_i, \zeta_i) = \tilde{f}_q(x, b_T; \mu_f, \zeta_\mu)$
  - Then evolve simply $\zeta_\mu \to \zeta_f$



MAP parametrization

- The MAP collaboration (JHEP 10 (2022) 127) used the following form for the non-perturbative function

$$f_{1\,NP}(x, \boldsymbol{b}_T^2; \zeta, Q_0) = \frac{g_1(x)\, e^{-g_1(x)\frac{\boldsymbol{b}_T^2}{4}} + \lambda^2\, g_{1B}^2(x)\left[1 - g_{1B}(x)\frac{\boldsymbol{b}_T^2}{4}\right] e^{-g_{1B}(x)\frac{\boldsymbol{b}_T^2}{4}} + \lambda_2^2\, g_{1C}(x)\, e^{-g_{1C}(x)\frac{\boldsymbol{b}_T^2}{4}}}{g_1(x) + \lambda^2\, g_{1B}^2(x) + \lambda_2^2\, g_{1C}(x)} \left[\frac{\zeta}{Q_0^2}\right]^{\boxed{g_K(\boldsymbol{b}_T^2)/2}}, \quad (38)$$

$$g_{\{1,1B,1C\}}(x) = N_{\{1,1B,1C\}} \frac{x^{\sigma_{\{1,2,3\}}}(1-x)^{\alpha^2_{\{1,2,3\}}}}{\hat{x}^{\sigma_{\{1,2,3\}}}(1-\hat{x})^{\alpha^2_{\{1,2,3\}}}},$$

$$\boxed{g_K(\boldsymbol{b}_T^2) = -g_2^2\,\frac{\boldsymbol{b}_T^2}{2}}$$

CS kernel

- 11 free parameters for each hadron (flavor dependence not necessary) (12 if we include the nuclear TMD parameter)

Zeta in JAM

$$f_{NP}^{f}(x,b) = \frac{1}{\cosh\left(\left(\lambda_1^f(1-x)+\lambda_2^f x\right)b\right)},$$

- Parametrize $u, d, \bar{u}, \bar{d}$ and sea quarks ($s = \bar{s} = c = \bar{c} = b = \bar{b}$)
- Evaluates OPE at b_T (not b_*)
- Scale for PDFs in OPE i $\mu_{\text{OPE}} = \frac{2e^{-\gamma_E}}{b} + 2\text{GeV}.$ and non-trivial logs appear
- Non-perturbative piece of the ζ-evolutio $\mathcal{D}_{\text{NP}}(b) = bb^* \left[c_0 + c_1 \ln\left(\frac{b^*}{B_{\text{NP}}}\right)\right],$
- Fit c_0, c_1, B_{NP}

Bayesian Inference

- Minimize the χ^2 for each replica

Normalization parameter

$$\chi^2(\boldsymbol{a}, \text{data}) = \sum_e \left(\sum_i \left[\frac{d_i^e - \sum_k r_k^e \beta_{k,i}^e - t_i^e(\boldsymbol{a})/n_e}{\alpha_i^e} \right]^2 + \left(\frac{1 - n_e}{\delta n_e} \right)^2 + \sum_k \left(r_k^e \right)^2 \right)$$

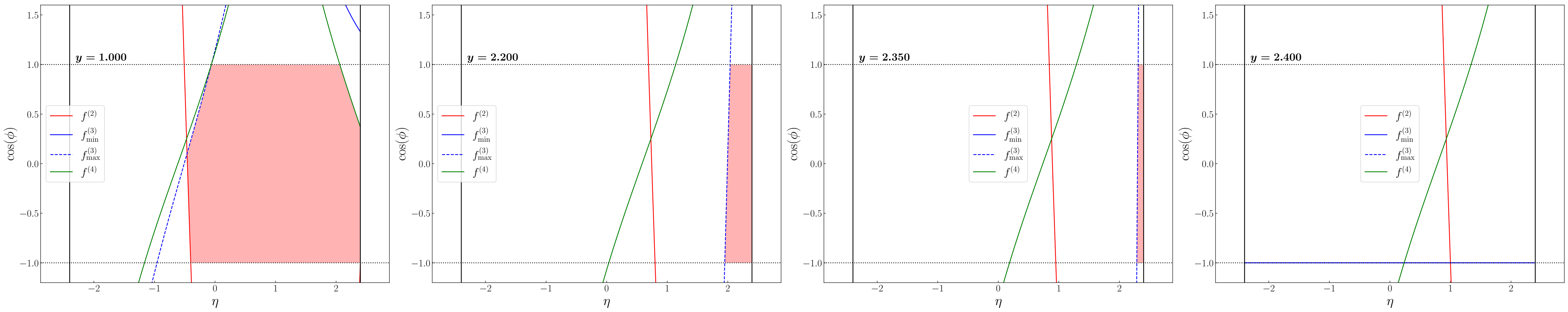
- Perform N total χ^2 minimizations and compute statistical quantities

Expectation value $$\mathrm{E}[\mathcal{O}] = \frac{1}{N} \sum_k \mathcal{O}(\boldsymbol{a}_k),$$

Variance $$\mathrm{V}[\mathcal{O}] = \frac{1}{N} \sum_k \left[\mathcal{O}(\boldsymbol{a}_k) - \mathrm{E}[\mathcal{O}] \right]^2,$$

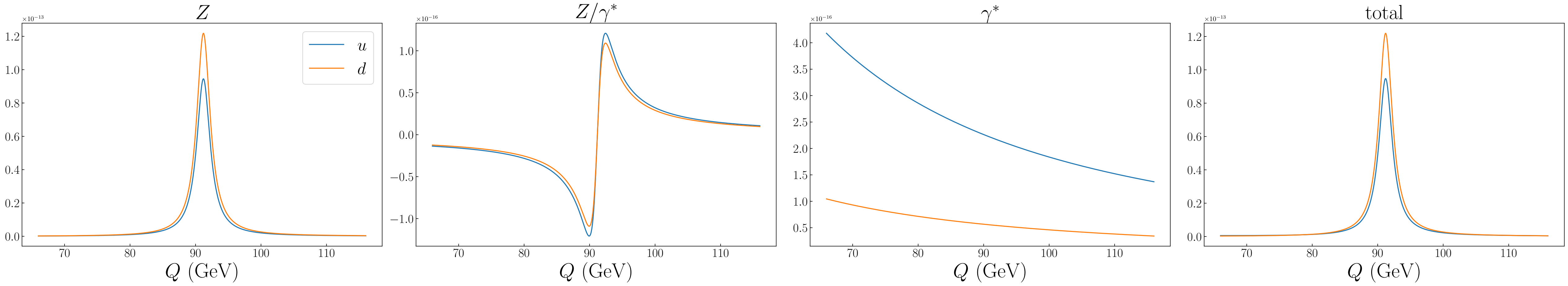
Fiducial volume

- To make systematic uncertainties more uniform within the bins, experiments make fiducial cuts on the phase space of the detected leptons



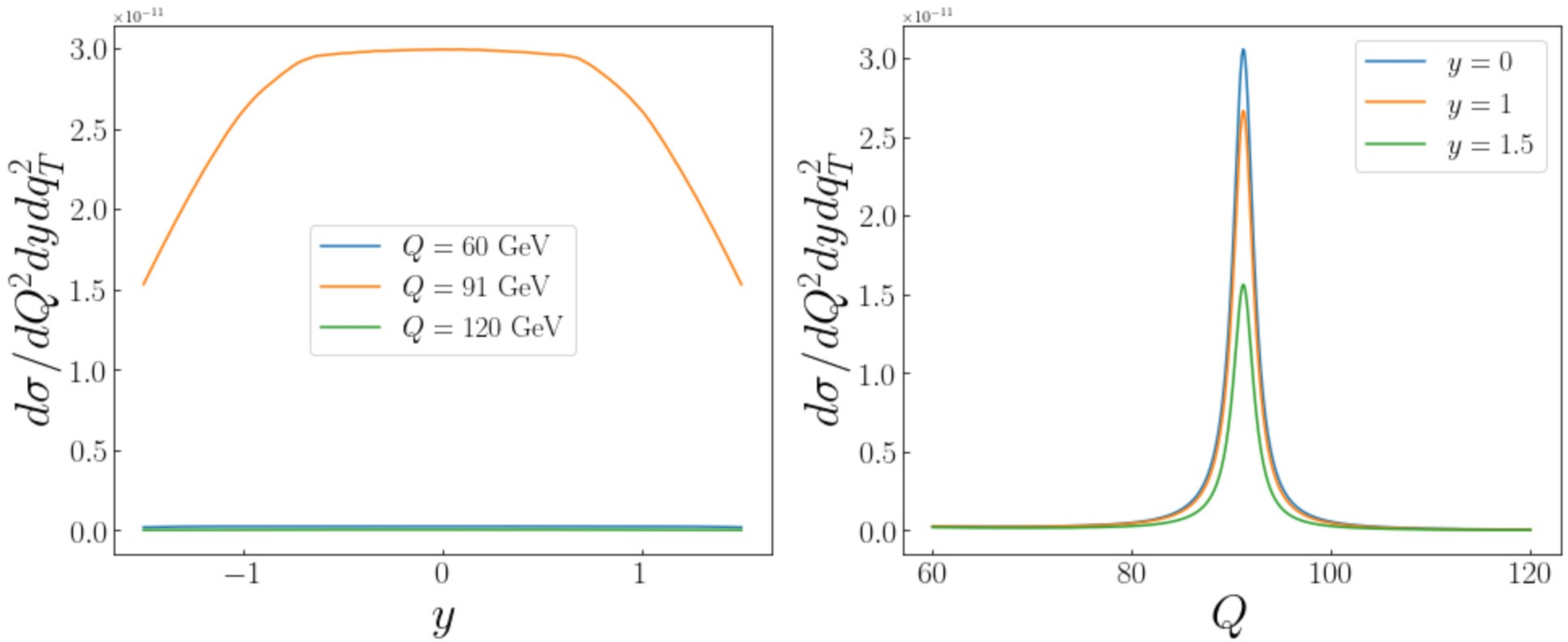
Hard part

- Electroweak charges



Mapping out (Q, y) space

- Shapes are more intricate than in fixed target case
- For Q interpolation, we take out the Z-boson peak, as it appears the same in all calculations



2 replicas

CSS - NLO+N2LL - fit TMDs and PDFs to collinear and qT data

Preliminary

Fixed target

	expt	obs	npts	chi2/npts	Z-score	norm_e
dy_qT-12881	E288	Ed3sigma/dp3	31	1.381872	1.428279	0.780313
dy_qT-12882	E288	Ed3sigma/dp3	41	1.230335	1.044495	0.90703
dy_qT-12883	E288	Ed3sigma/dp3	64	1.376707	1.967496	1.029205
dy_qT-10605	E605	Ed3sigma/dp3	43	1.342743	1.5072	1.047657
dy_qT-10772	E772	Ed3sigma/dp3	45	4.308618	inf	0.726835

TeVatron

	expt	obs	npts	chi2/npts	Z-score	norm_e
dy_qT-90001	CDF	dsig/dpT	30	0.728009	1.079378	1.04394
dy_qT-90002	CDF	dsig/dpT	36	1.61094	2.272648	1.105844
dy_qT-90003	D0	dsig/dpT	14	0.564288	1.250647	1.130121

LHC

	expt	obs	npts	chi2/npts	Z-score	norm_e
dy_qT-70101	ATLAS	dsig/dpT	8	16.758502	inf	0.998635
dy_qT-70102	ATLAS	dsig/dpT	8	11.160682	7.991574	0.998635
dy_qT-70103	ATLAS	dsig/dpT	8	12.523221	inf	0.998635
dy_qT-70104	ATLAS	dsig/dpT	8	8.517499	6.685377	0.998635
dy_qT-70105	ATLAS	dsig/dpT	8	8.562184	6.709281	0.998635
dy_qT-70106	ATLAS	dsig/dpT	8	5.989541	5.194743	0.998635
dy_qT-30001	LHCb	dsig/dpT	10	1.721916	1.47838	0.999438
dy_qT-40001	LHCb	sig	10	2.678211	2.768104	1.013274
dy_qT-50001	LHCb	sig	10	1.665449	1.389321	1.022167
dy_qT-20011	CMS	dsig/dpT	15	3.974359	4.995349	0.992292
dy_qT-20012	CMS	dsig/dpT	15	1.775538	1.853965	0.992292
dy_qT-20013	CMS	dsig/dpT	15	1.205987	0.649604	0.992292
dy_qT-20014	CMS	dsig/dpT	15	1.07154	0.312817	0.992292
dy_qT-20015	CMS	dsig/dpT	15	0.495897	1.592129	0.992292

2 replicas

CSS - NLO+N2LL - fit TMDs and PDFs to collinear and qT data

Preliminary

DIS

	expt	obs	npts	chi2/npts	Z-score	norm_e
idis-10010	SLAC	F2	218	1.798172	6.818831	1.063213
idis-10016	BCDMS	F2	348	1.131218	1.685498	1.002403
idis-10020	NMC	F2	273	1.9903	inf	1.045339
idis-10026	HERA II NC e+ (1)	sig_r	402	1.840115	inf	--
idis-10027	HERA II NC e+ (2)	sig_r	75	1.221055	1.318634	--
idis-10028	HERA II NC e+ (3)	sig_r	259	1.062278	0.723665	--
idis-10029	HERA II NC e+ (4)	sig_r	209	1.12587	1.268656	--
idis-10030	HERA II NC e-	sig_r	159	1.749297	5.533041	--
idis-10031	HERA II CC e+	sig_r	39	1.258311	1.129105	--
idis-10032	HERA II CC e-	sig_r	42	1.279029	1.247263	--
idis-10011	SLAC	F2	228	1.646766	5.83784	1.062495
idis-10017	BCDMS	F2	254	1.063362	0.728933	1.031765
idis-10021	NMC	F2d/F2p	174	1.864843	6.51827	1.004052

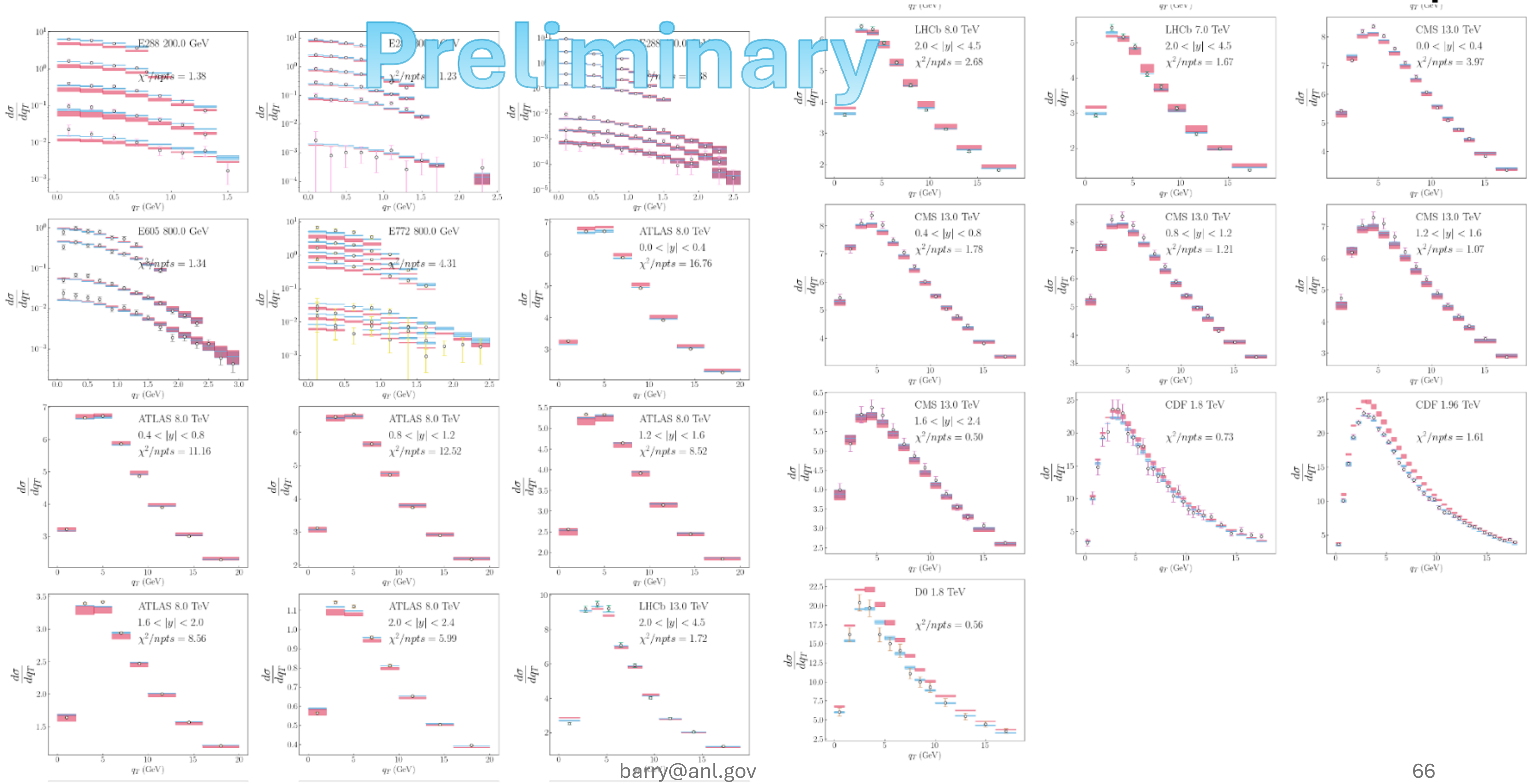
DY

	expt	obs	npts	chi2/npts	Z-score	norm_e
dy-10001	E866	M3 dsig/dM dxF	184	2.31974	inf	1.071864
dy-20001	E866	sigpd/2sigpp	15	4.1107	5.149757	0.990762
dy-20002	E906	sigpd/2sigpp	6	1.317787	0.690254	0.995907

Total

	expt	obs	npts	chi2/npts	Z-score	norm_e
total	--	--	3342	1.716182	inf	--

CSS - NLO+N2LL - fit TMDs and PDFs to collinear and qT data



CSS - NLO+N2LL - fit TMDs and PDFs to collinear and qT data

Preliminary

PDFs