Multi-messenger constraints on Equation of State of hot and dense matter and astrophysical implications

- N. Barman, B. K. Pradhan and D. Chatterjee [Phys.Rev.D 111 (2025) 2, 023017]
- N. Barman and D. Chatterjee [arXiv:2506.03288]

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INT-2b program
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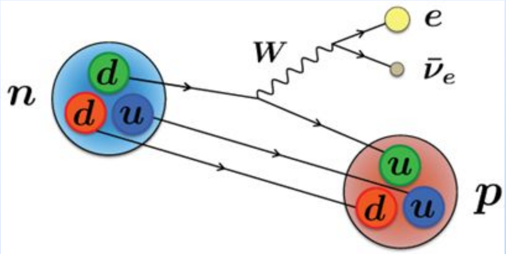
Outline

- Motivation
  - Cold Neutron Star matter
  - Nuclear Physics Input
  - Quasi Normal Mode (f-mode)
  - Hot and Dense Matter
  - General Purpose Equation of State (EOS)
  - Multi-messenger constraints on EOS of hot and dense matter

- Hot and dense matter EOS modelling and macroscopic structure and oscillation modes in hot Neutron Stars
  - EOS modelling
  - Nucleonic matter (N-matter)
  - Neutrino trapping
  - Hyperonic matter (NY-matter)

Motivation: Cold Neutron Star (NSs) matter



Cold (T < 1 MeV)

β-equilibrium

($\mu_p + \mu_e - \mu_n = 0$)

Cold Asymmetric Nuclear Matter (ANM)

Cold NS

Non-zero temperature and out-of-β-equilibrium effects?

Motivation: Nuclear physics input

Energy per particle (T=0),

$$\frac{E}{A} = \text{Isoscalar contribution} + \text{Isovector Contribution}$$

$$= E_{sat} + \frac{K_{sat}}{2!}x^2 + O(x^3, \dots) + \delta^2 \left[J_{sym} + \frac{L_{sym}}{1!}x + O(x^2, \dots) \right]$$

$+$ Higher order terms

	n_{sat} (fm^{-3})	E_{sat} (MeV)	K_{sat} (MeV)	J_{sym} (MeV)	L_{sym} (MeV)	m^*/m
Fixed	0.15	−16.0	240	32	60	0.65
Varied	0.14–0.17	-16 ± 0.2	200–300	28–34	40–70	0.55–0.75

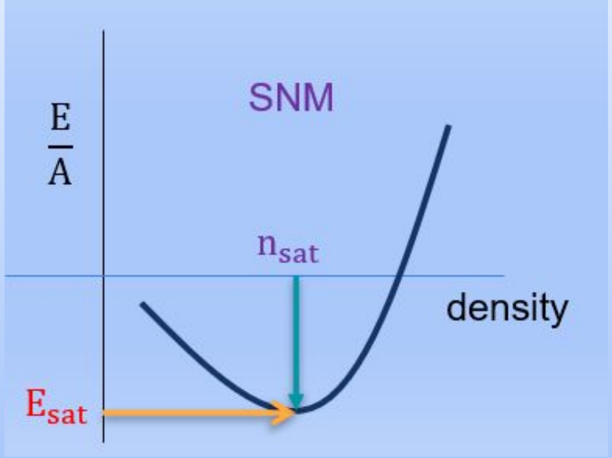
Normalized density, $x = \frac{n - n_{sat}}{3n_{sat}}$

Asymmetry parameter, $\delta = \frac{n_n - n_p}{n_n + n_p}$

Symmetric Nuclear matter (SNM), $\delta = 0$

Pure Neutron matter (PNM), $\delta = 1$

Asymmetric Nuclear matter (ANM), $\delta \neq 0$



Uncertainties in nuclear parameters manifest themselves as uncertainties in Nuclear Equation of State [p(ε)]

Motivation: Unstable oscillation modes and Gravitational wave (GW) emission

- Fundamental mode: f-mode (1-3 kHz)
- The frequencies strongly depend on the particular cold EOS i.e. nuclear parameters

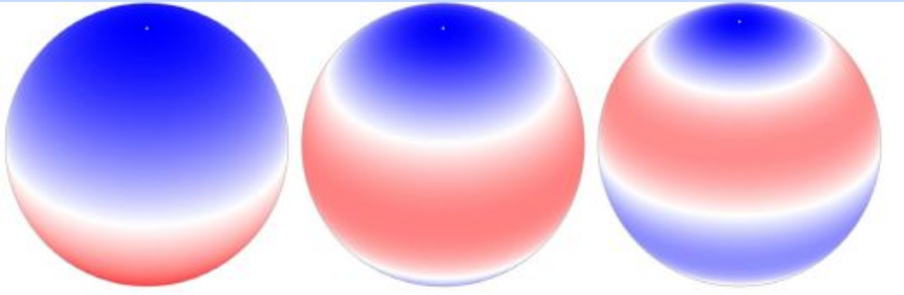
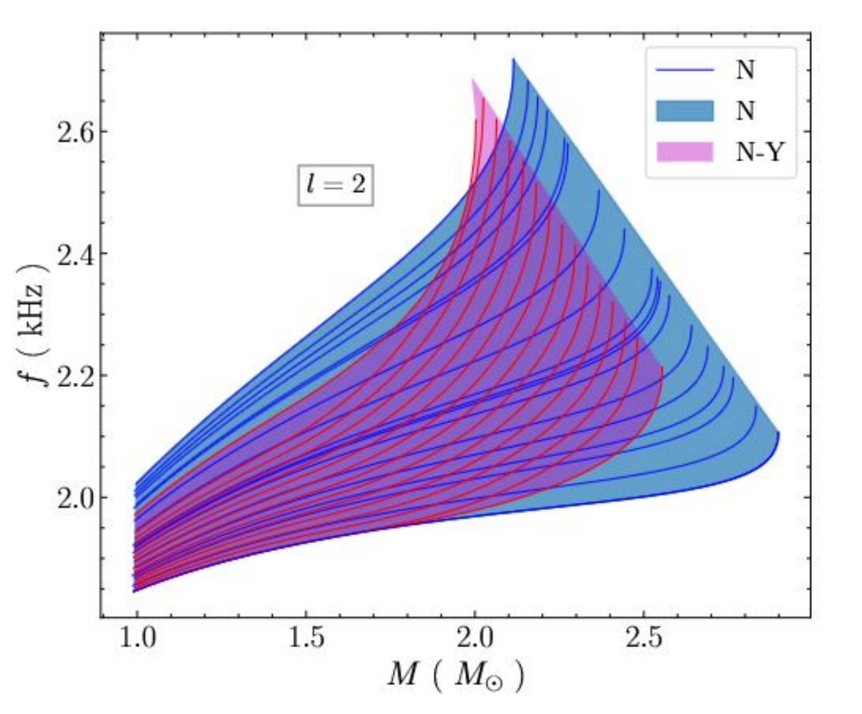


Fig: *l* = 1, 2, 3 modes [credit: C. Aerts]

Band corresponds to uncertainty in nuclear EOS



f-mode frequency vs mass for cold NSs (Pradhan+ 2021)

Motivation: Hot and Dense environment in Astrophysics

- Neutron Stars (NSs): cold beta equilibrated matter
- Finite Temperature effects:
  - Proto-Neutron Stars (PNSs)
  - Binary Neutron Star (BNS) merger remnants
- Out-of-β-equilibrium ($\mu_p+\mu_e-\mu_n\neq0$)
- Neutrino trapping
- T ~ 0-100 MeV, Y_Q ~ 0.01-0.60, n_B/n_{sat} ~ 0.5-6.0
  - ⇒ Hot NS Configuration

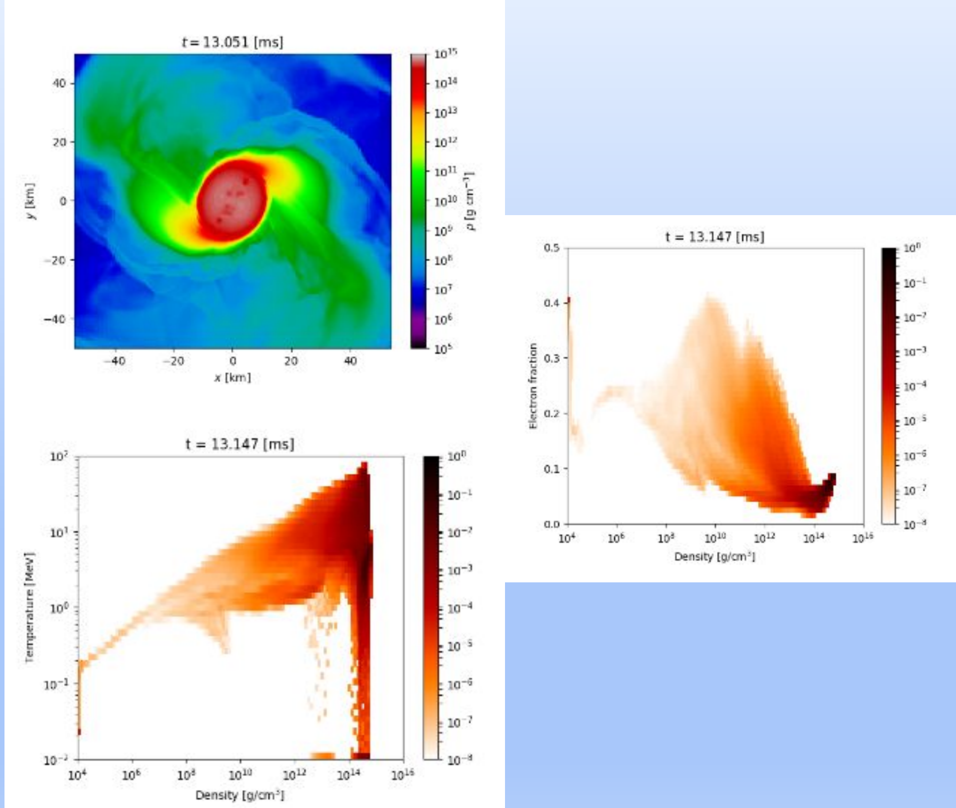


Fig: BNS post-merger [Credit: Perego et al. (2019)]

General Purpose Equations of State (EOSs)

- Hot EOSs are completely specified by - (T, n_B, Y_Q)
- State of the art EOSs available in CompOSE database ⇒ Tabulated EOSs
- These tabulated EOSs use fixed parameterization of nuclear saturation properties
  - might not capture full ranges of uncertainty of nuclear properties based on current multi-messenger data

Nuclear Saturation Properties:

- ❑ $\{n_{sat}, E_{sat}, K_{sat}, J_{sym}, L_{sym}, m^*/m\}$+Higher order terms→Cold EoS
- ❑ n_{sat} = nuclear saturation density
- ❑ E_{sat} = Binding energy at saturation for SNM
- ❑ K_{sat} = Curvature of Binding energy at saturation for SNM
- ❑ J_{sym} = Symmetry energy at saturation
- ❑ L_{sym} = Slope of Symmetry energy at saturation
- ❑ m^*/m = effective mass

Gamma Law / hybrid prescription

$$\epsilon_{th}(T, n_B, Y_Q) = \epsilon(T, n_B, Y_Q) - \epsilon(T=0, n_B, Y_Q) ,$$
$$p_{th}(T, n_B, Y_Q) = p(T, n_B, Y_Q) - p(T=0, n_B, Y_Q)$$

$$\Gamma_{th}(T, n_B, Y_Q) = 1 + \frac{p_{th}(T, n_B, Y_Q)}{\epsilon_{th}(T, n_B, Y_Q)}$$

Γ-law (cold EOS + Thermal contribution of ideal gas) substitution commonly used in simulations is found to be inconsistent for non-nucleonic degrees of freedom
(Raduta+ 2022, Kochankovski+ 2022)

- Typical range: $1.3 < \Gamma_{th} < 1.75$

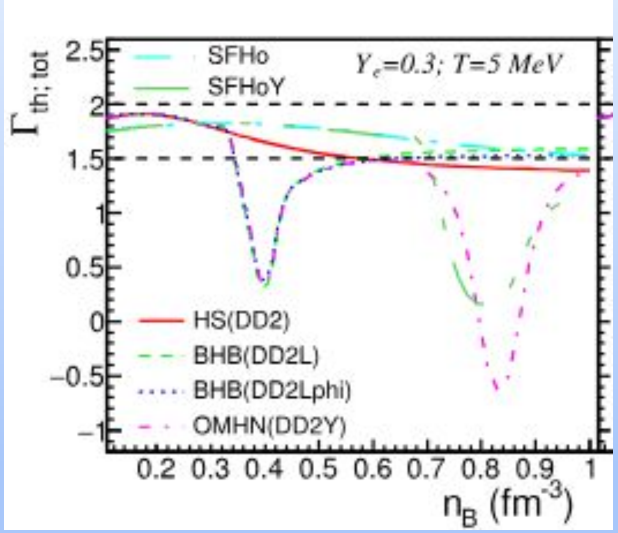
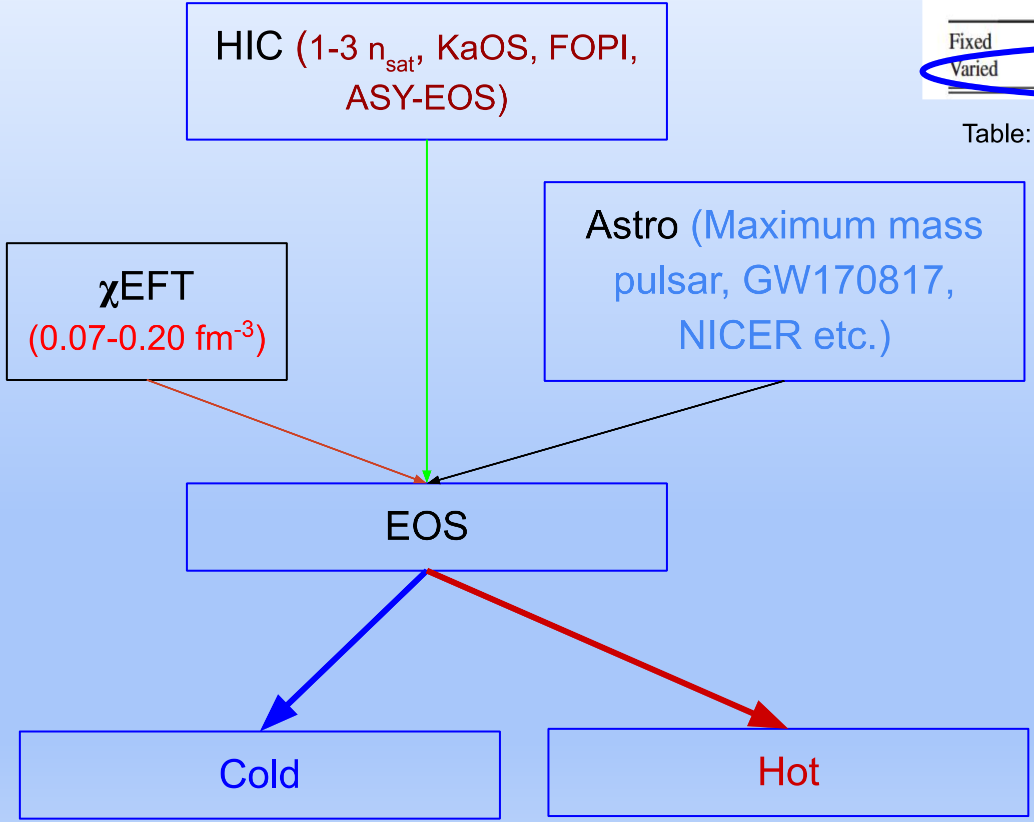


Image credit: Raduta+ 2022

Multi-messenger constraints on EOS



	n_{sat} (fm^{-3})	E_{sat} (MeV)	K_{sat} (MeV)	J_{sym} (MeV)	L_{sym} (MeV)	m^*/m
Fixed	0.15	−16.0	240	32	60	0.65
Varied	0.14–0.17	-16 ± 0.2	200–300	28–34	40–70	0.55–0.75

Table: Uncertainty ranges or priors of nuclear parameters (Ghosh+ 2022)

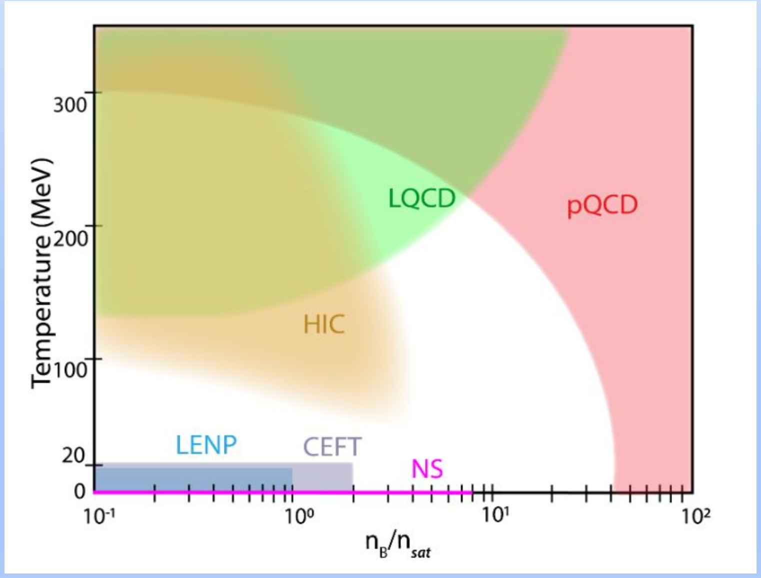


Fig: QCD phase diagram

[MUSES Collab.]

Hot NS configuration

Cold and β-equilibrated NSs

Entropy per baryon (S/A): Proxy for finite T

Charge/lepton fraction (Y_Q/Y_L): denotes out-of-β-equilibrium state

Hot NSs [N/NY(S/A,Y_Q/Y_L)]

$Y_Q=Y_e$

$Y_L=Y_e+Y_\nu$

PNS evolution path

N/NY(1, 0.4) → N/NY(2, 0.2) → N/NY(T=0)

Pons+ 1999

Objectives

- Systematically include finite temperature effects and nuclear physics inputs in hot and dense matter EOS
- Imposition of multi-messenger constraints
- Study the macroscopic properties and oscillation modes of hot NS
- Inclusion of neutrino trapping in hot NS configurations
- Establish connection between nuclear/hyper-nuclear properties with hot NS observables

EOS Modelling

- Non-linear Relativistic Mean Field (NL-RMF) model fitted to reproduce nuclear saturation properties (Chen and Piekarewicz 2014, Hornick+2018)

$$\mathcal{L} = \sum_B \bar{\psi}_B (i\gamma_\mu \partial^\mu - m_B + g_{\sigma B}\sigma - g_{\omega B}\gamma^\mu \omega_\mu - g_{\phi B}\gamma^\mu \phi_\mu - g_{\rho B}\gamma^\mu \rho^a_\mu \tau^a)\psi_B + \frac{1}{2}\partial^\mu \sigma \partial_\mu \sigma - \frac{1}{2}m_\sigma^2 \sigma^2 - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega^2 \omega^2 - \frac{1}{2}\vec{\rho}_{\mu\nu}\vec{\rho}^{\mu\nu} + \frac{1}{2}m_\rho^2 \vec{\rho}^2 - \frac{1}{4}\phi_{\mu\nu}\phi^{\mu\nu} + \frac{1}{2}m_\phi^2 \phi^2 + U_\sigma + \Lambda_\omega (g_{\rho N}^2 \vec{\rho}_\mu . \vec{\rho}^\mu)(g_{\omega N}^2 \omega_\mu \omega^\mu) + \sum_L \bar{\psi}_L (i\gamma_\mu \partial^\mu - m_L)\psi_L + \sum_{i=B,L} \bar{\psi}_i (-q_i \gamma_\mu A^\mu)\psi_i - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

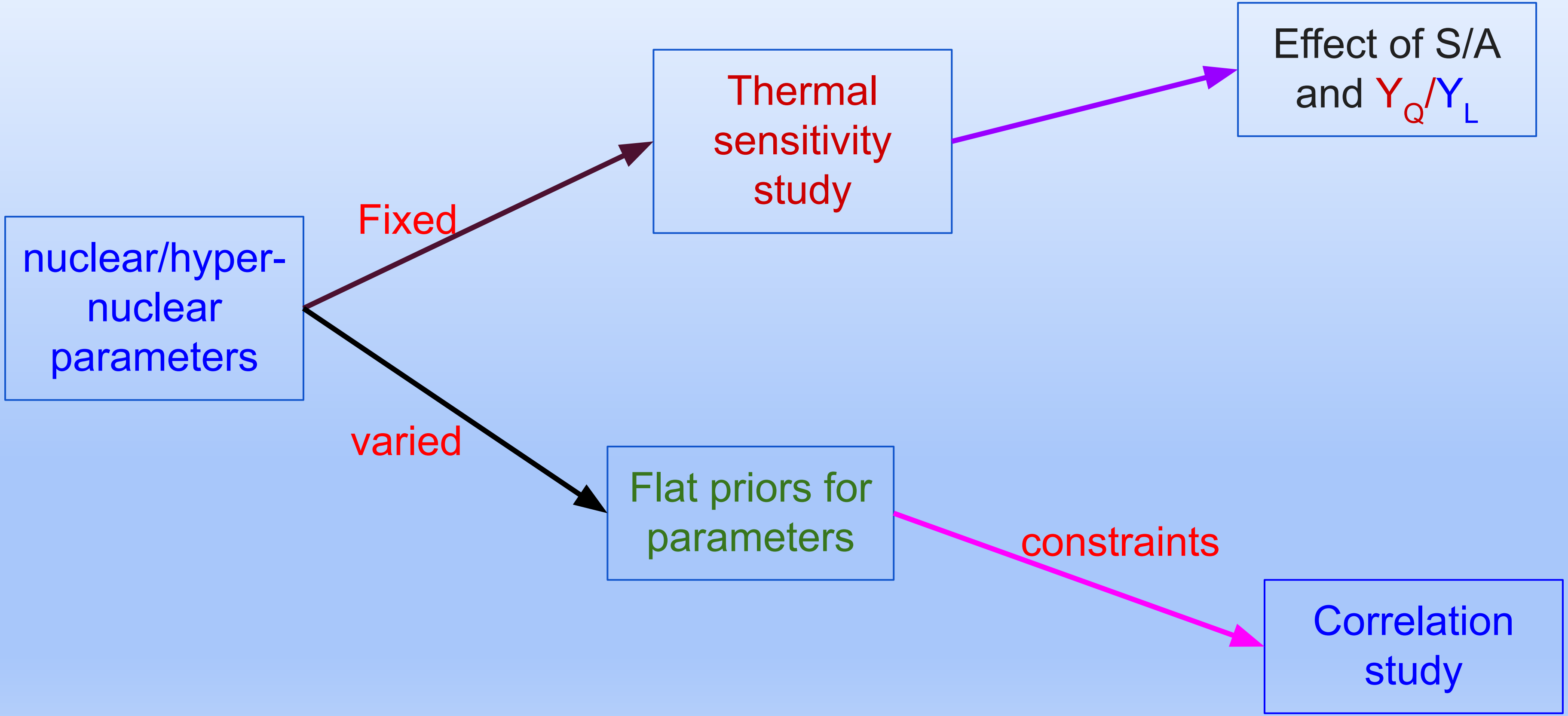
- $U_\sigma = \frac{1}{3} b m_N \sigma^3 + \frac{1}{4} c\sigma^4$
- Isoscalar channel: $\{g_\sigma, g_\omega, b, c\} \rightarrow \{n_{sat}, E_{sat}, K_{sat}, m^*/m\}$
- Isovector channel: $\{g_\rho, \Lambda_\omega\} \rightarrow \{J_{sym}, L_{sym}\}$

	n_{sat} (fm^{-3})	E_{sat} (MeV)	K_{sat} (MeV)	J_{sym} (MeV)	L_{sym} (MeV)	m^*/m
Fixed	0.15	−16.0	240	32	60	0.65
Varied	0.14–0.17	-16 ± 0.2	200–300	28–34	40–70	0.55–0.75

Table: Uncertainty ranges or priors of nuclear parameters (Ghosh+ 2022)

$$U_i^N = -g_{\sigma i}\bar{\sigma}_0 + g_{\omega i}\bar{\omega}_0$$

Thermal effects v/s nuclear effects



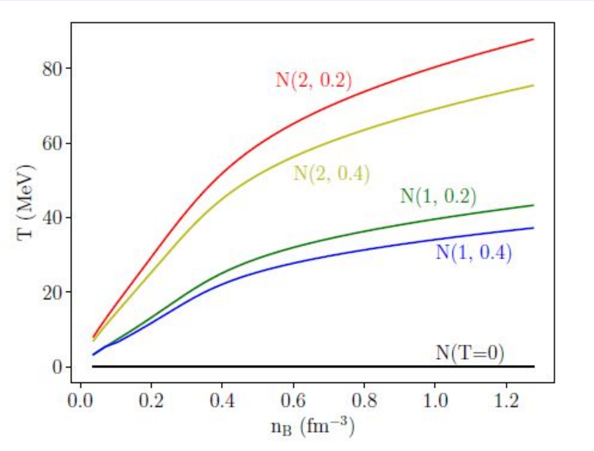
N-matter

- Nonlinear Relativistic Mean Field (NL-RMF) model
- npe-matter
- Neutrino free
- Thermodynamic conditions:
  - $N(S/A = 1, Y_Q = 0.4)$
  - $N(S/A = 2, Y_Q = 0.2)$

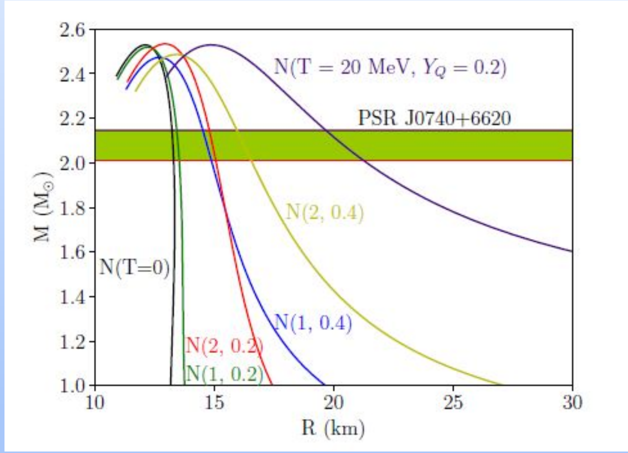
	n_{sat} (fm^{-3})	E_{sat} (MeV)	K_{sat} (MeV)	J_{sym} (MeV)	L_{sym} (MeV)	m^*/m
Fixed	0.15	−16.0	240	32	60	0.65
Varied	0.14–0.17	-16 ± 0.2	200–300	28–34	40–70	0.55–0.75

Table: Uncertainty ranges or priors of nuclear parameters (Ghosh+ 2022)

Sensitivity of thermal configurations for a fixed nuclear model

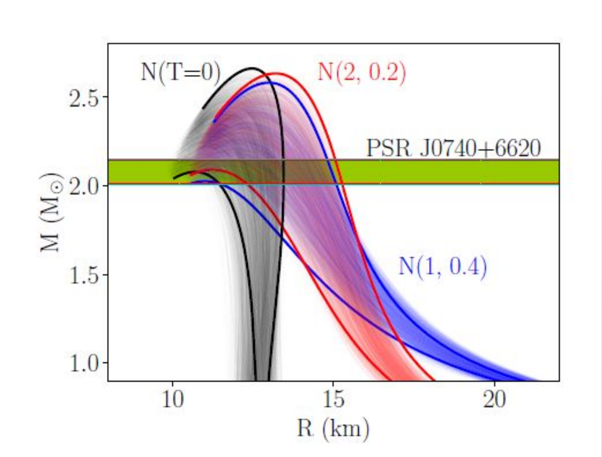
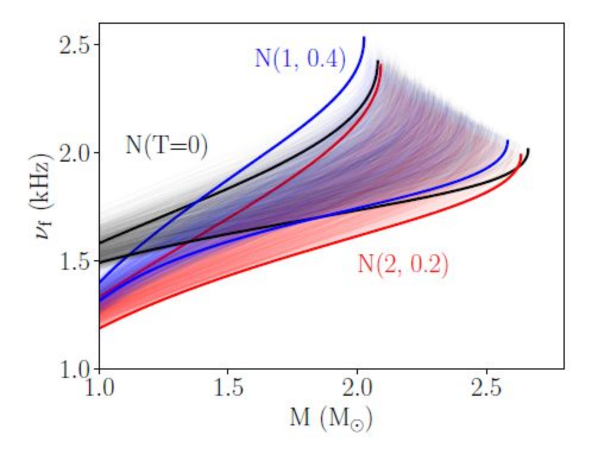
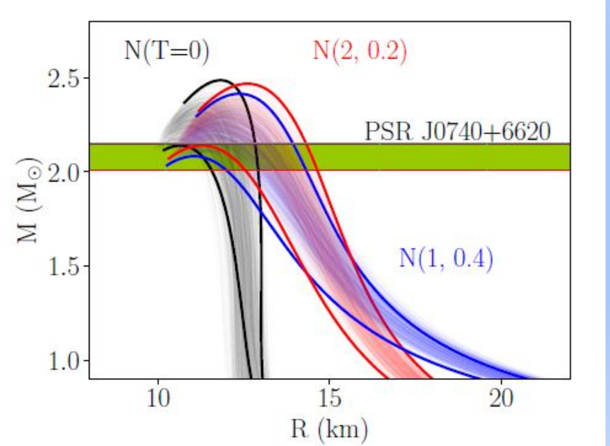
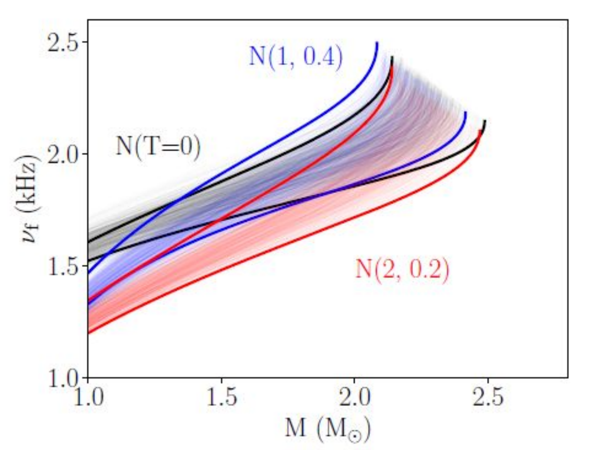


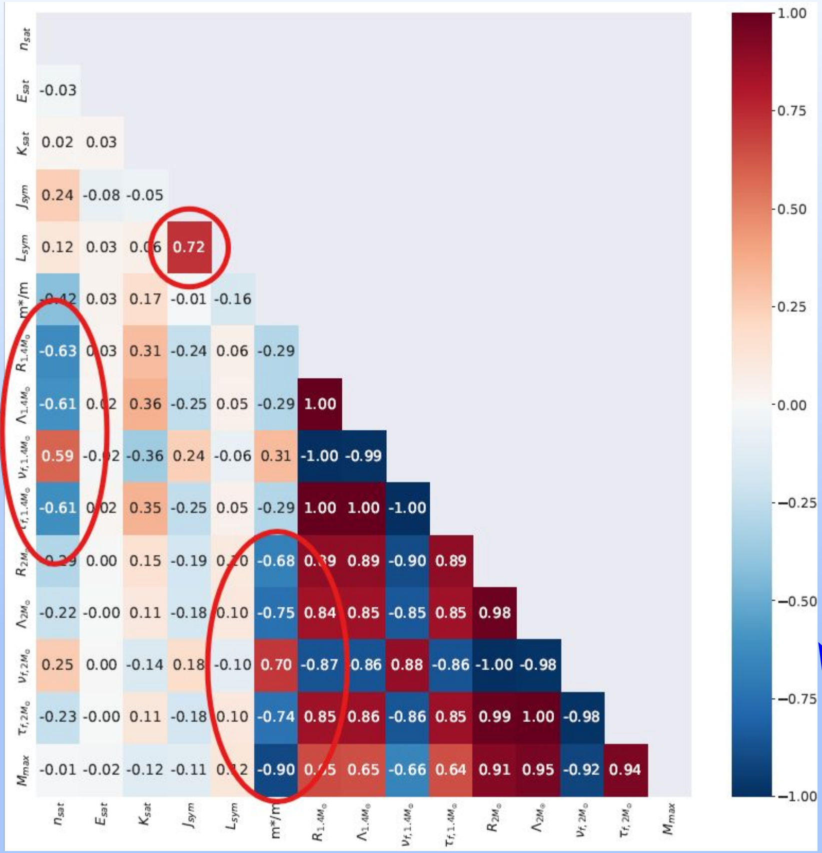
a. T vs n_B profile inside core



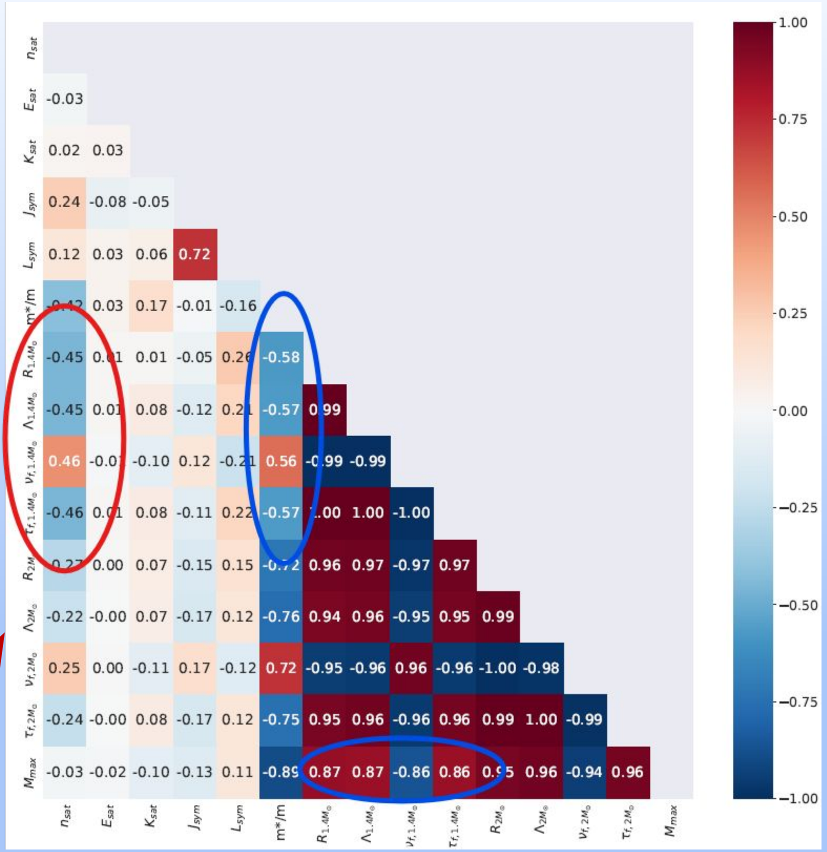
b. Mass radius relations

Nuclear saturation properties: n_{sat}= 0.15 fm^{-3}, E_{sat} = -16 MeV, K_{sat} = 240 MeV, J_{sym} = 32 MeV, L_{sym} = 60 MeV, m*/m = 0.65 (satisfies χEFT + Astro)

	Mass-Radius relations	f-mode frequencies
χEFT + Astro		
χEFT + Astro + HIC		



N(S/A = 1, Y_Q= 0.4)

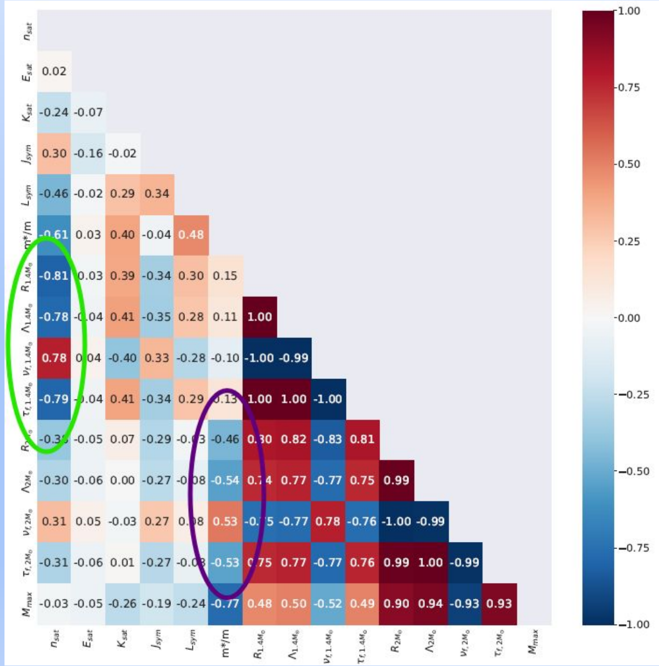


N(S/A = 2, Y_Q= 0.2)

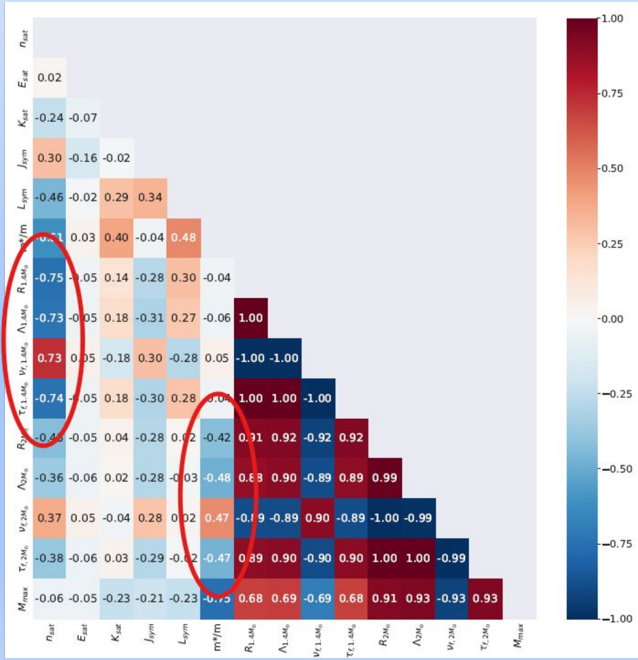
χEFT + Astro

Correlations (χEFT + Astro + HIC)

- Increase of n_{sat} correlations with properties of canonical mass of $1.4M_{\odot}$ hot NS
- reduced effective mass correlations
- similar behavior for both cases
- Model dependency of HIC: IQMD transport models was used for analysis!!!!!



N(S/A = 1, Y_Q= 0.4)



N(S/A = 2, Y_Q= 0.2)

UR (C-Love relations)

- $C = \sum_{i=0}^{2} a_i [\ln(\Lambda)]^i$

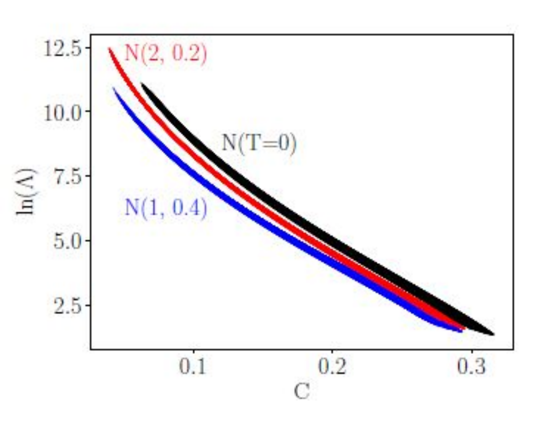


Fig.: C-Love relations (χEFT + Astro)

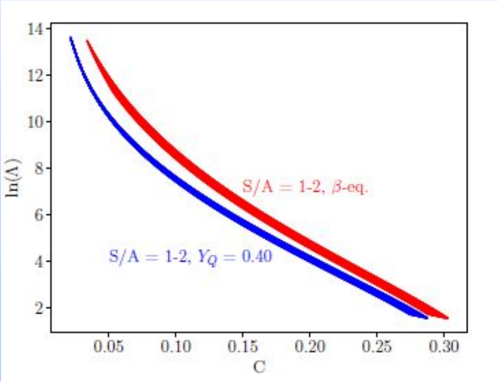


Fig.: C-Love relations (S/A variation from 1-2)

➢ Out-of-beta-equilibrium → deviations from universality

Also shown in Simulation [Phys. Rev. D **109** (2024), 083040 (V. Guedes et al.)]

N-matter (Summary)

- Dominant parameters affecting astro observables: effective nucleon mass (m*/m), nuclear saturation density (n_{sat})
- Correlations change with **thermal conditions**
- **HIC constraints** reduce m*/m sensitivity
- Universal Relation:
  - Robust vs nuclear inputs
  - Sensitive to thermal effects
- **Charge fraction** → drives universality breaking

Neutrino trapping

- Neutrinos can be in thermal equilibrium for T > 5 MeV (Alford+ 2018, Alford+ 2021)
- Modifications of composition and macroscopic structure
- Hot NS: $N(S/A,Y_L)$ or $NY(S/A,Y_L)$

$$n \rightarrow p + e^{-} \text{ or } \mu^{-} + \bar{\nu}_e \text{ or } \bar{\nu}_\mu$$
$$p + e^{-} \text{ or } \mu^{-} \rightarrow n + \nu_e \text{ or } \nu_\mu$$

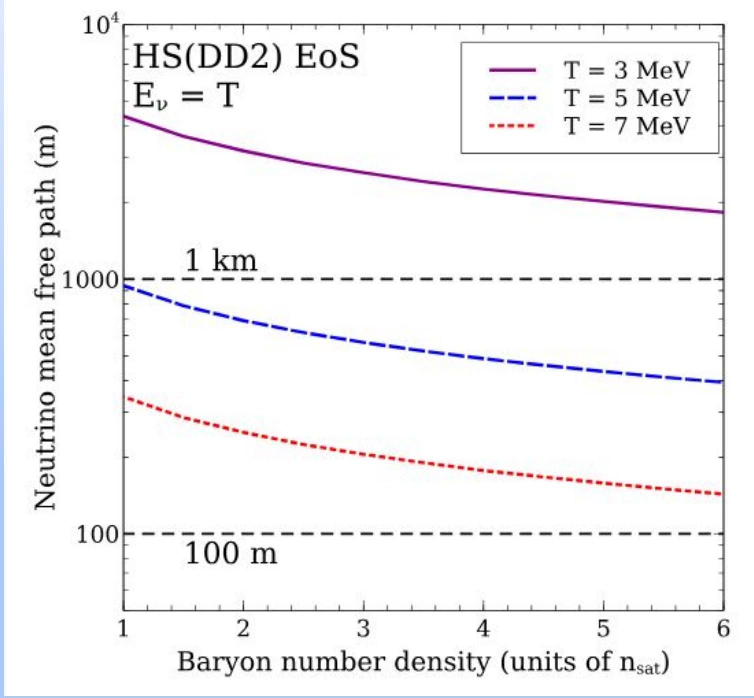


Image credit: Alford+ 2018

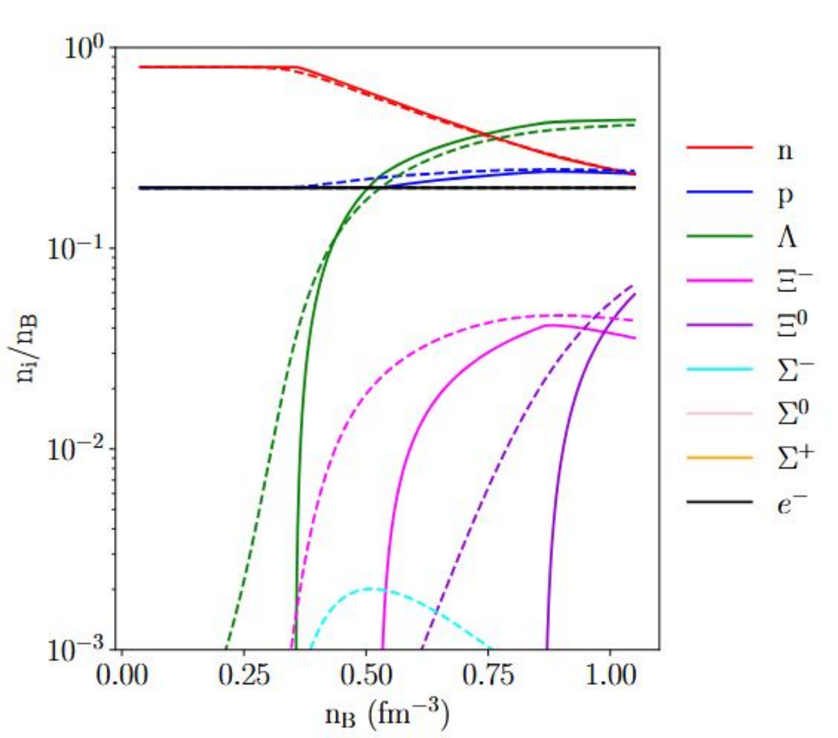
Hyperonic matter

- NL-RMF extended to include neutrino contributions
- NY-matter (npe + Λ + Σ + Ξ)
- Neutrino trapped
- Thermodynamic conditions:
  - N(S/A = 1, Y_L = 0.4)
  - N(S/A = 2, Y_L = 0.2)

	n_{sat} (fm^{-3})	E_{sat} (MeV)	K_{sat} (MeV)	J_{sym} (MeV)	L_{sym} (MeV)	m^*/m	U_Λ^N (MeV)	U_Σ^N (MeV)	U_Ξ^N (MeV)
Fixed	0.15	-16.0	240	32	60	0.65	-30	30	-18
Varied	0.14 to 0.17	-16±0.2	200 to 300	28 to 34	40 to 70	0.55 to 0.75	-30	0 to 30	-30 to 0

Table: Nuclear and hypernuclear uncertainty

Composition (fixed params)



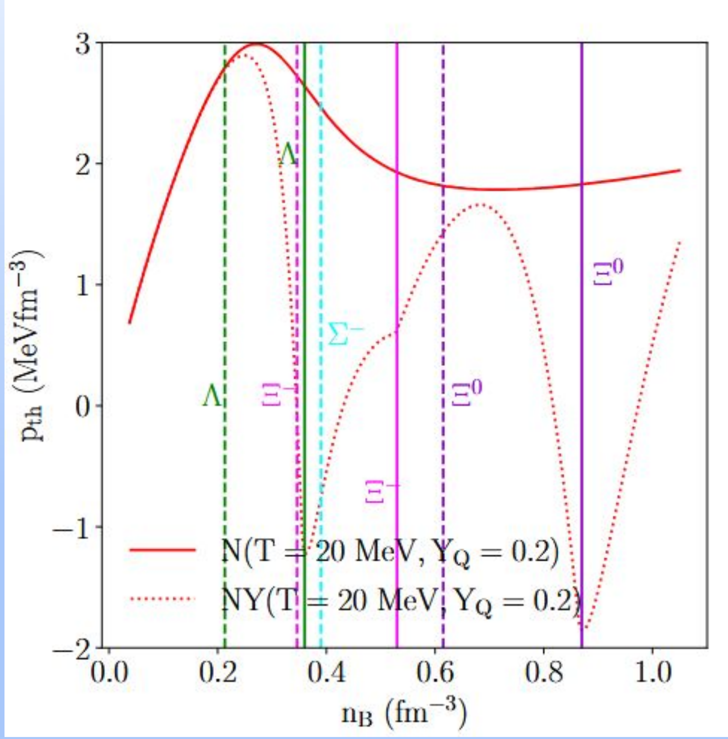
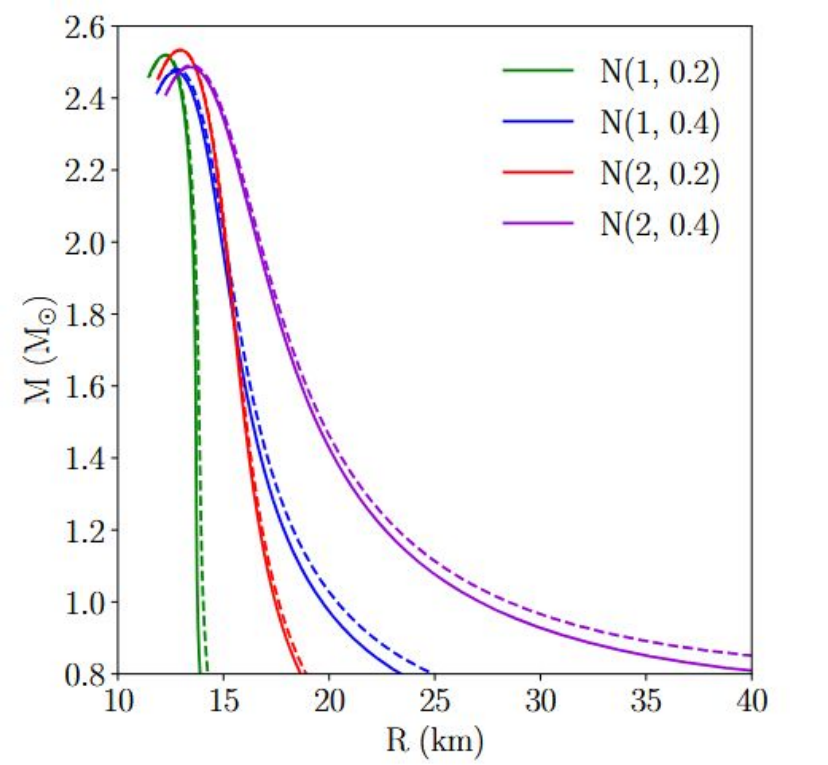


Fig: Microphysics of NY(T=20 MeV, Y_Q=0.2) [dashed] and NY(T=0 MeV, Y_Q=0.2) [solid]



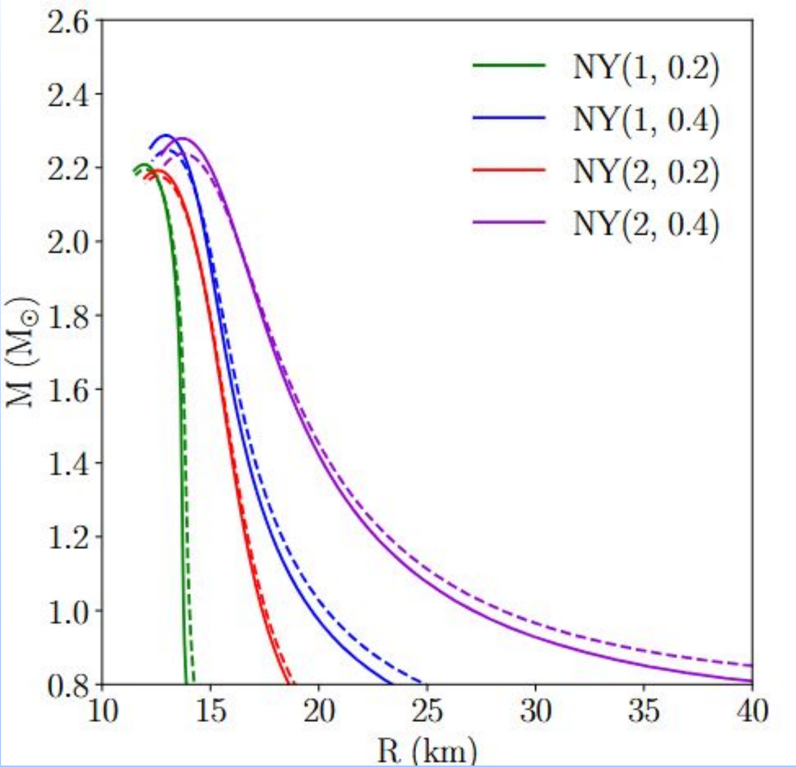
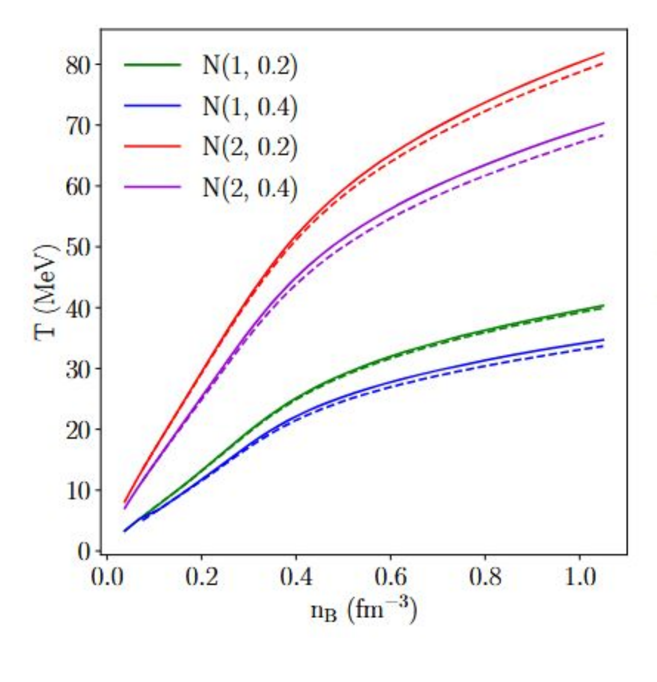


Figure: Mass-Radius relations for hot NS configuration. Solid (dashed) curves indicate neutrino-free (neutrino-trapped) cases.

Temperature profile



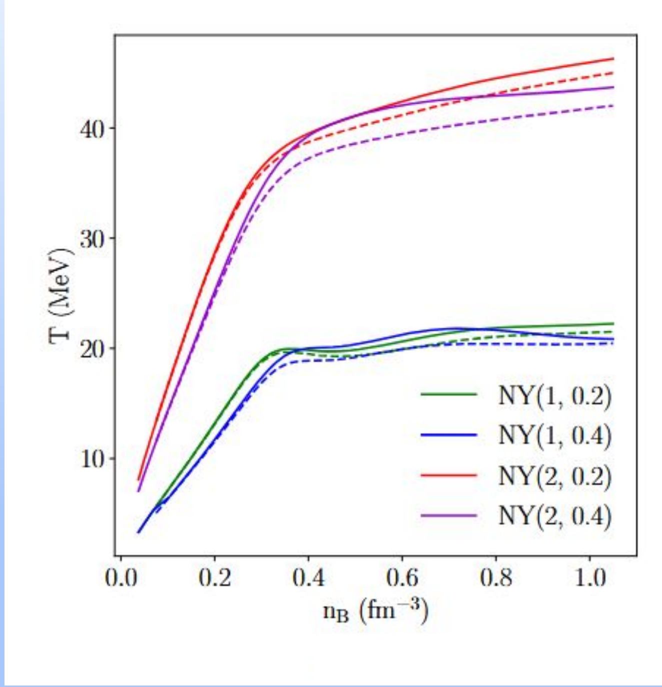


Figure: Temperature vs baryon density. Solid (dashed) curves indicate neutrino-free (neutrino-trapped) cases.

NY(S/A = 1,Y_L=0.4)

NY(S/A = 2,Y_L=0.2)

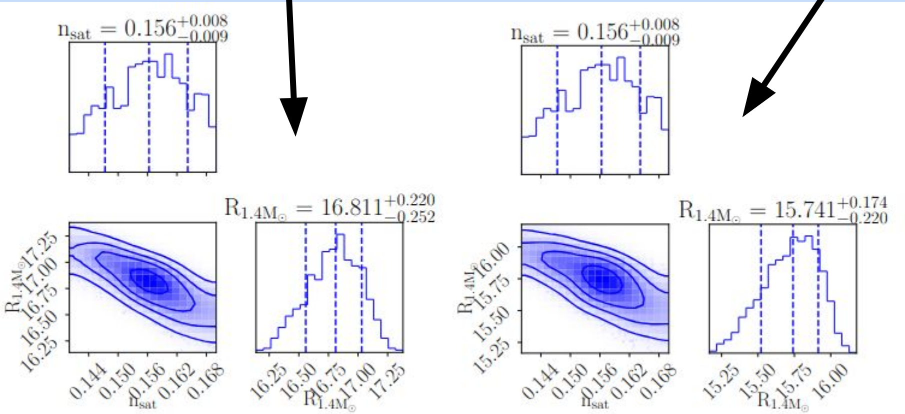
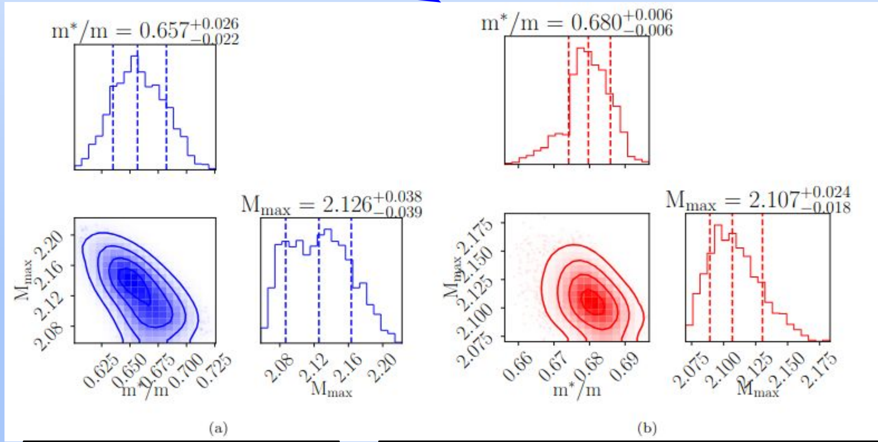


Figure: saturation density vs radius of intermediate mass config., [χEFT + Astro]



χEFT + Astro

χEFT + Astro + HIC

Figure: m*/m vs maximum mass

NY-matter (Summary)

- Inclusion of neutrinos alters radii and this effect increases with increasing lepton fraction

- Maximum mass supported:
    - N(1, 0.4) < N(2, 0.4) < N(1, 0.2) < N(2, 0.2)
    - NY(2, 0.2) < NY(1, 0.2) < NY(2, 0.4) < NY(1, 0.4)

- Nuclear saturation density shows moderate to strong correlation with NS astrophysical observables like radius and f-mode frequency

Conclusions

- We systematically investigate the the role of nuclear parameters and thermal effects in hot NSs
- We extended the formalism to include effects of neutrinos in thermal equilibrium

- Nuclear parameters playing dominant roles:
  - N-matter: nucleon effective mass and nuclear saturation density
  - NY-matter: nuclear saturation density

- Effect of Neutrino trapping:
  - suppresses temperature profiles
  - alters macroscopic structure

- Universal relations change with thermal conditions
- Which nuclear physics inputs are important to study PNS oscillations and BNS merger remnants?

Extra Slides

$$\mu_i^* = B_i\mu_B + Q_i\mu_Q + L_i\mu_L - g_{\omega i}\bar{\omega}_0 - g_{\rho i}\bar{\rho}_{03}I_{3i}/2 - g_{\phi i}\bar{\phi}_0$$

$$s = -\sum_{i=B,L}\frac{2J_i+1}{2\pi^2}\int dkk^2\left[f_{FD}\left(\frac{E_i(k)-\mu_i^*}{T}\right)\right.$$
$$\ln f_{FD}\left(\frac{E_i(k)-\mu_i^*}{T}\right) + \bar{f}_{FD}\left(\frac{E_i(k)-\mu_i^*}{T}\right)$$
$$\left.\ln \bar{f}_{FD}\left(\frac{E_i(k)-\mu_i^*}{T}\right)\right]$$
$$+\, anti - particle\ contribution\ (\mu_i^* \to -\mu_i^*) + s_\gamma\ ,$$

$$\epsilon = \frac{1}{2}m_\sigma^2\bar{\sigma}_0^2 + \frac{1}{2}m_\omega^2\bar{\omega}_0^2 + \frac{1}{2}m_\rho^2\bar{\rho}_{03}^2 + \frac{1}{2}m_\phi^2\bar{\phi}_0^2$$
$$+ U_\sigma + 3\Lambda_\omega(g_{\rho N}g_{\omega N}\bar{\rho}_{03}.\bar{\omega}_0)^2$$
$$+ \sum_{i=B,L}\frac{2J_i+1}{2\pi^2}\int dkk^2E_i(k)\left[f_{FD}\left(\frac{E_i(k)-\mu_i^*}{T}\right)\right.$$
$$\left.+\, f_{FD}\left(\frac{E_i(k)+\mu_i^*}{T}\right)\right] + \epsilon_\gamma\ ,$$
$$p = \sum_{i=B,L}\mu_i n_i + Ts - \epsilon\ .$$

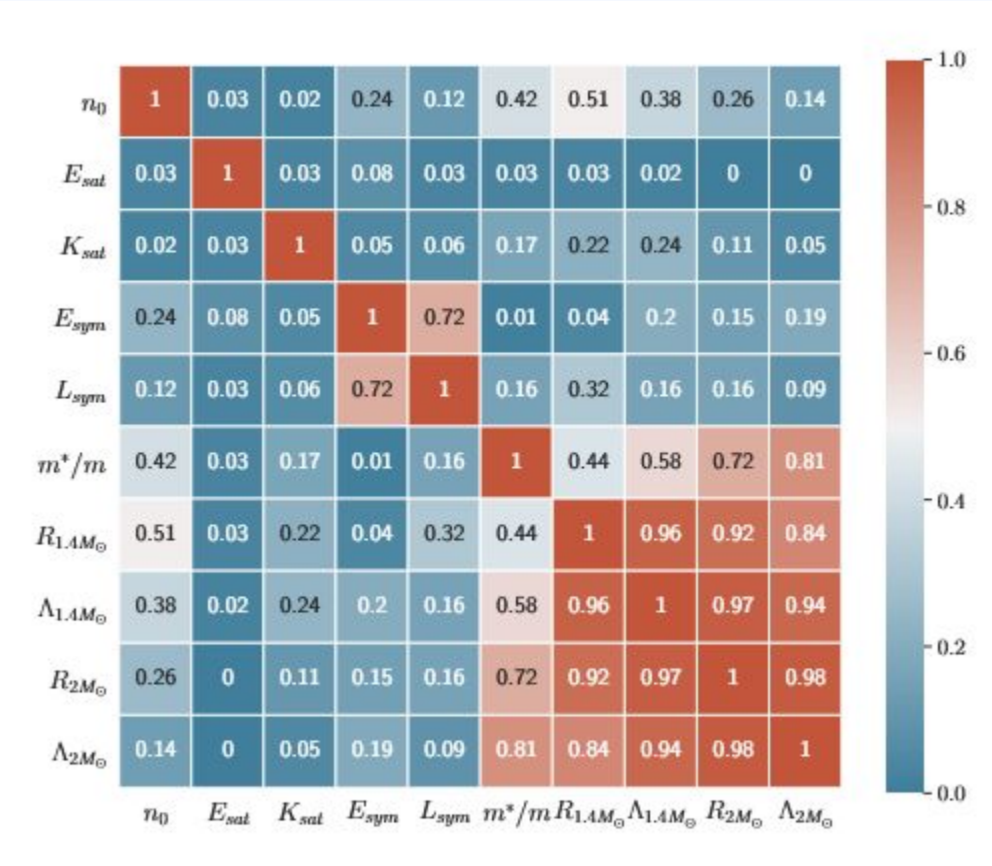


Image credit: Ghosh+ 2022

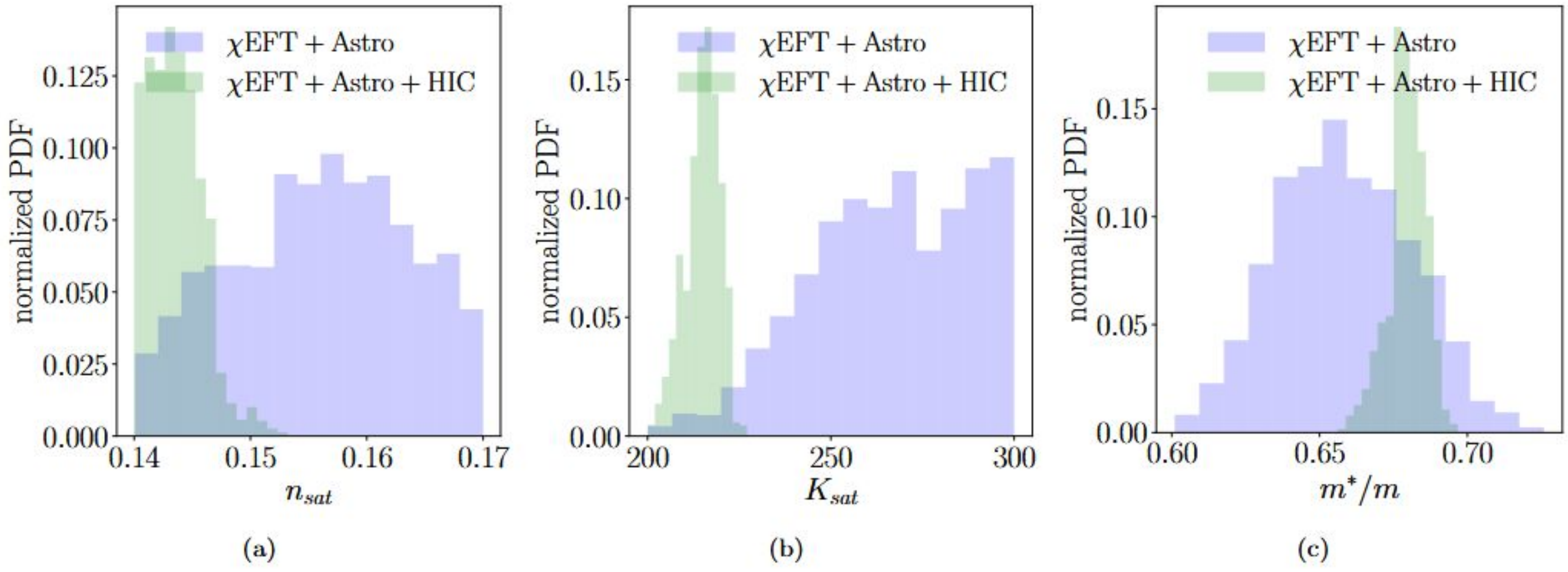


Figure 1. Posteriors of isoscalar nuclear parameters with different constraints for NY-matter. (a) n_{sat} posteriors, (b) K_{sat} posteriors, (c) m^*/m posteriors

2.2 Thermal contribution and Γ-law

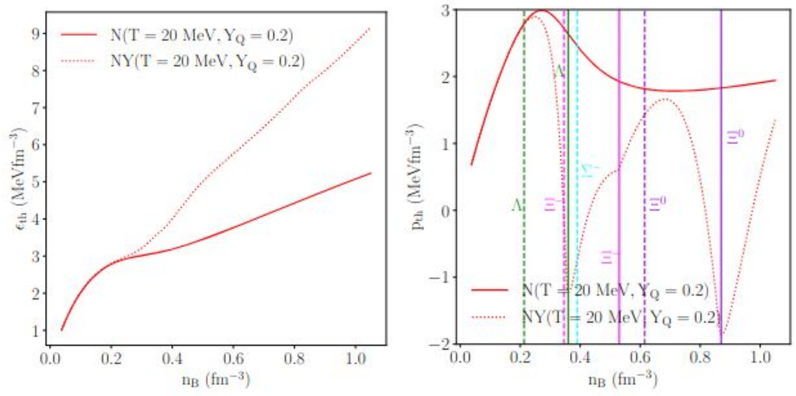
The Γ-law is a simplified model for hot nuclear EOS that connects it to the corresponding cold EOS. In this prescription, it is assumed that the gas is ideal and that the effects of degeneracy are negligible. We denote thermal contributions to energy density and pressure by ϵ_{th} and p_{th} respectively.

$$\epsilon_{th}(T, n_B, Y_Q) = \epsilon(T, n_B, Y_Q) - \epsilon(T = 0, n_B, Y_Q) \,, \tag{7}$$

$$p_{th}(T, n_B, Y_Q) = p(T, n_B, Y_Q) - p(T = 0, n_B, Y_Q) \tag{8}$$

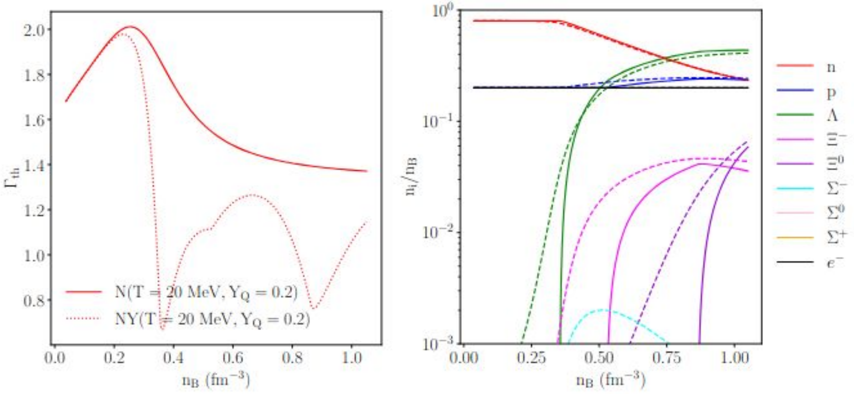
From these definitions we can define the so-called Γ-law as,

$$\Gamma_{th}(T, n_B, Y_Q) = 1 + \frac{p_{th}(T, n_B, Y_Q)}{\epsilon_{th}(T, n_B, Y_Q)} \tag{9}$$



(a) (b)

Figure 3. Thermal contributions to energy density and pressure. Nuclear and hypernuclear properties are set to fixed values mentioned in Table 1 (a) ϵ_{th} vs n_B, (b) p_{th} vs n_B [Hyperon thresholds are shown with vertical lines: dashed(solid) lines correspond to appearance in case of hot(cold) matter]



(a) (b)

Figure 4. Γ-law and particle fractions. Nuclear and hypernuclear properties are set to fixed values mentioned in Table 1 (a) Γ_{th} vs n_B, (b) particle fractions vs n_B [Solid (dashed) curves are for $T = 0$ ($T = 20$ MeV), $Y_Q = 0.2$ case(s)]

Composition

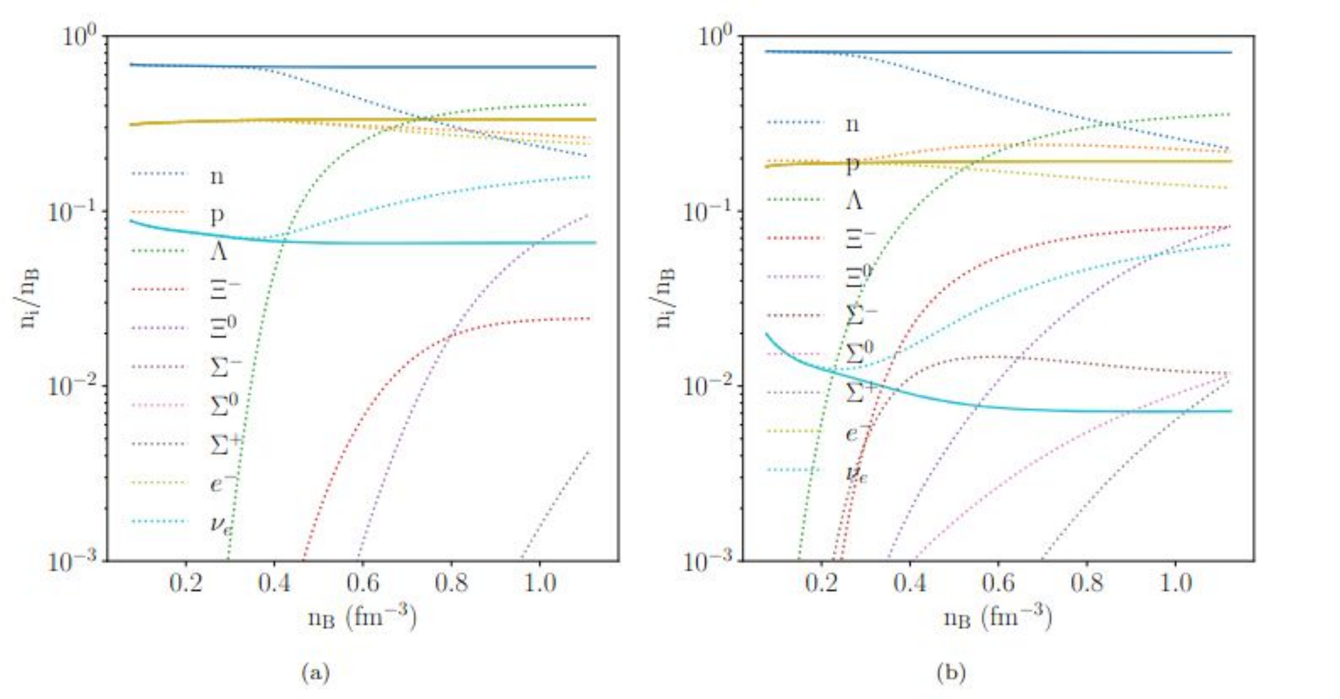


Fig: particle fractions vs baryon density, N-matter (solid lines) and NY-matter (dotted lines).
(a) N/NY(S/A = 1, Y_L=0.4), (b) N/NY(S/A = 2, Y_L=0.2)

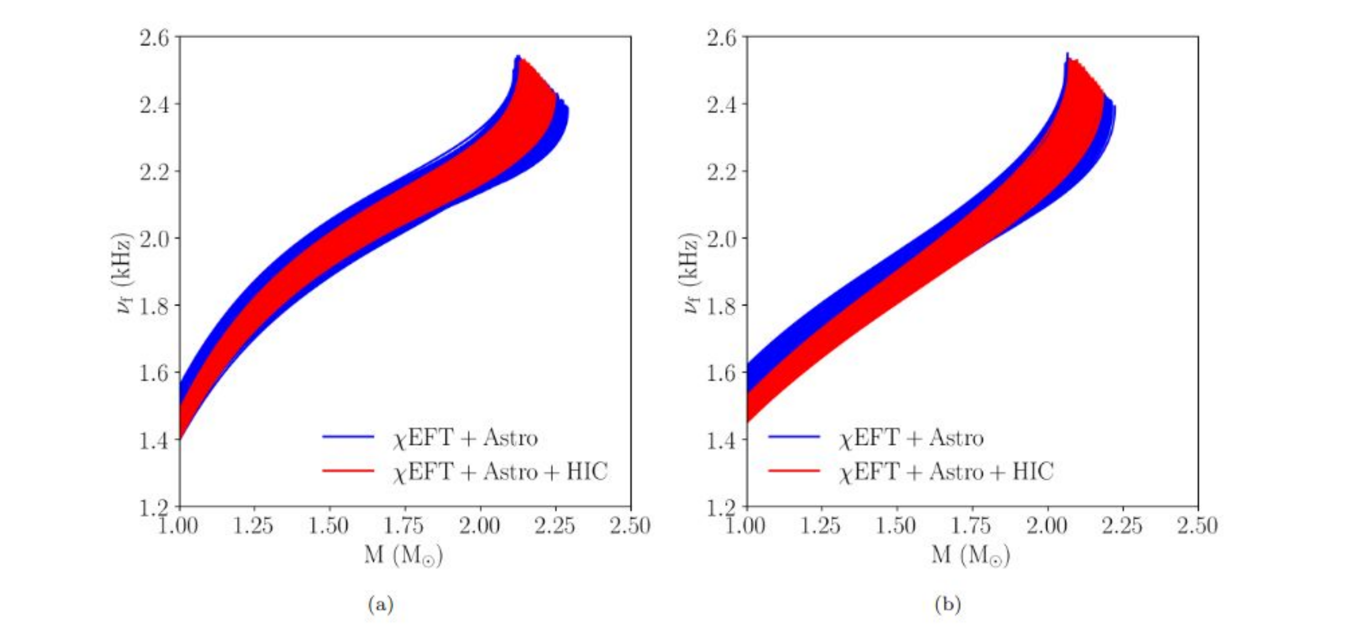
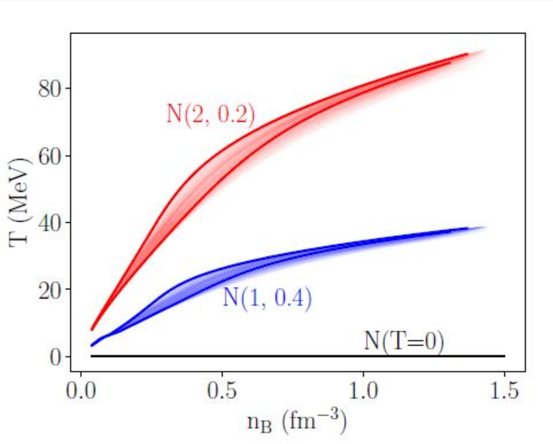
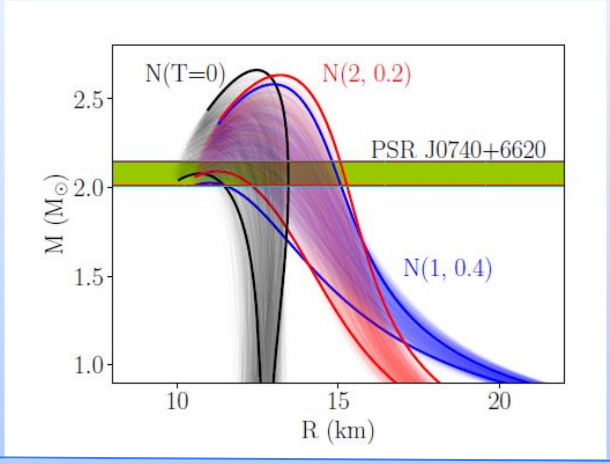
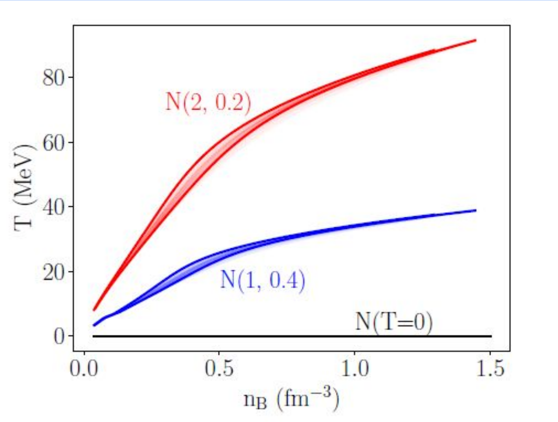
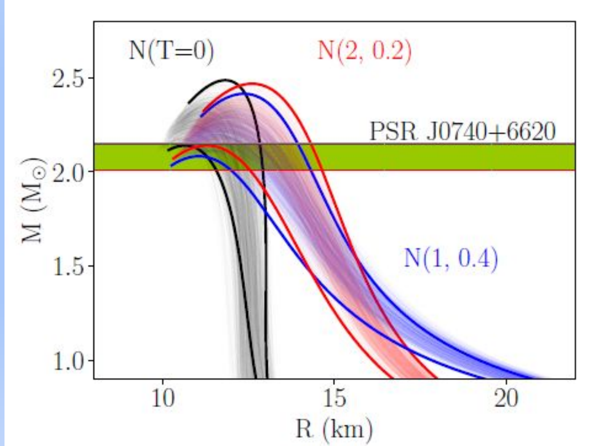
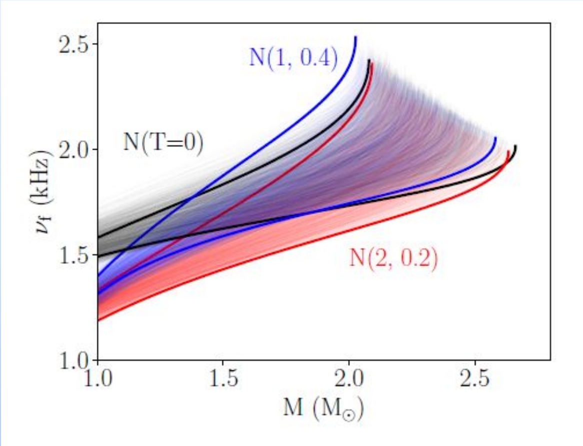
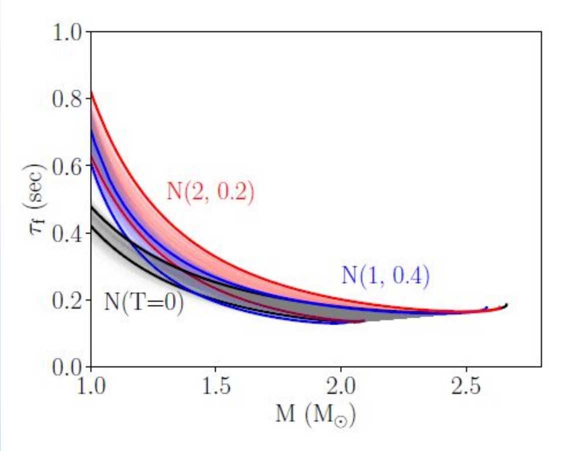
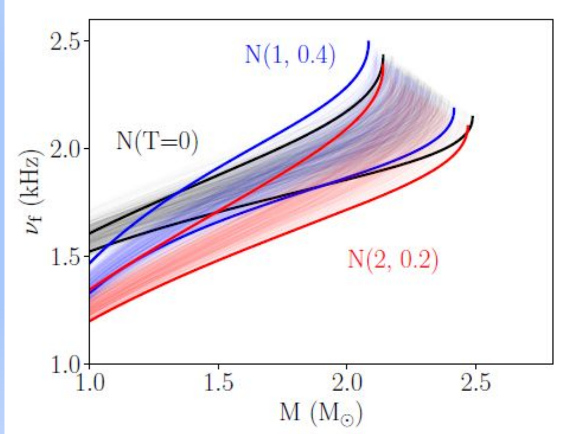
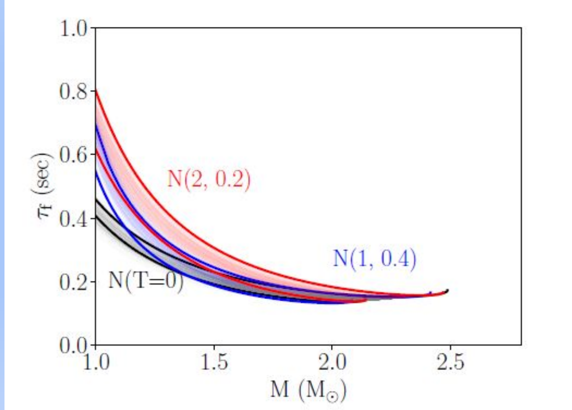


Figure: f-mode frequencies (Cowling) vs mass. (a) NY(S/A = 1,Y_L=0.4), (b) NY(S/A = 2,Y_L=0.2)

	Temperature profiles	Mass-Radius relations
χEFT + Astro		
χEFT + Astro +HIC		

	f-mode frequencies	Damping times
χEFT + Astro		
χEFT + Astro + HIC		

Early evolution of PNS

- Evolution from low entropy to high entropy state.
- Simultaneous deleptonization.

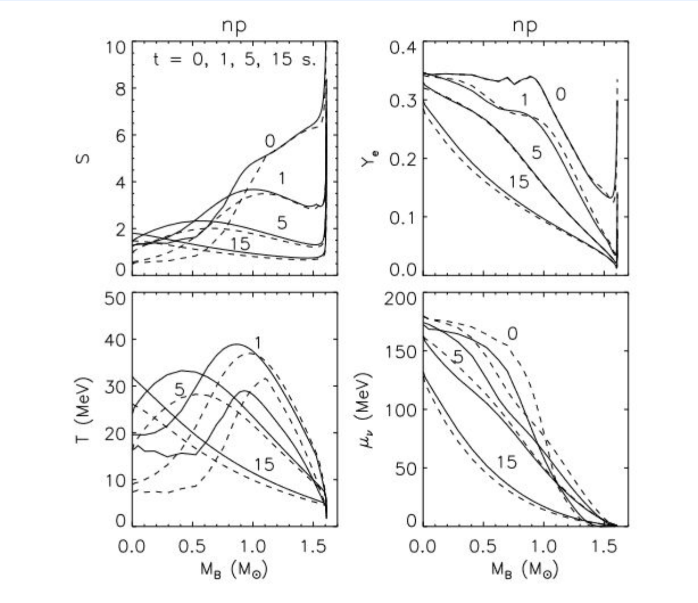


Fig.: Thermal evolution (Pons+1999)

UR (C-Love relations)

- $C = \sum_{i=0}^{2} a_i [\ln(\Lambda)]^i$

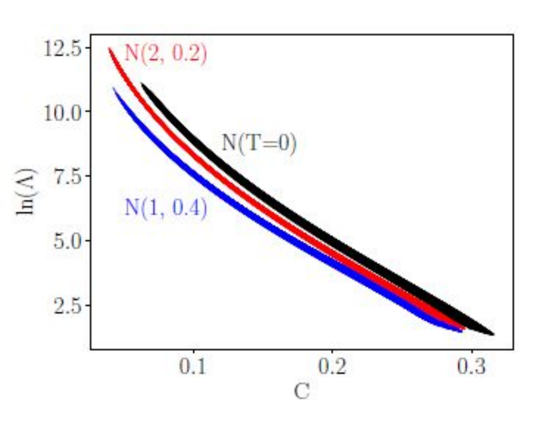


Fig.: C-Love relations (χEFT + Astro)

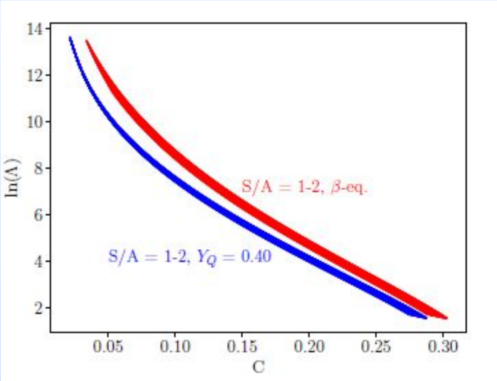


Fig.: C-Love relations (S/A variation from 1-2)

	a_0	a_1	a_2
$N(1,0.4)$	3.556×10^{-1}	-4.209×10^{-2}	1.173×10^{-3}
$N(2,0.2)$	3.598×10^{-1}	-4.070×10^{-2}	1.182×10^{-2}

Table: Polynomial fitting

➢ Out-of-beta-equilibrium → deviations from universality