Ultraheavy Magnetic Monopole

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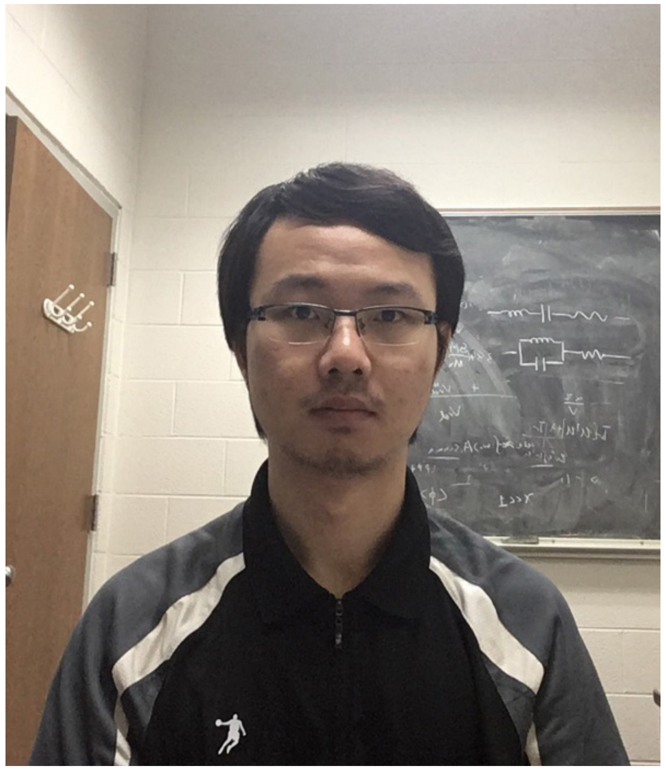
Topological Phases of Matter: From Low to High Energy, March 9

Collaborators



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arXiv: 2007.03703, 2012.15430, 2111.10360, 2103.06286

E

10^{19} GeV

10^{16} GeV

domain walls, cosmic strings, monopoles

difficulty to be produced in a lab, but early universe may generate them

100 GeV

100 MeV

latticed vortices, chiral soliton lattice

…

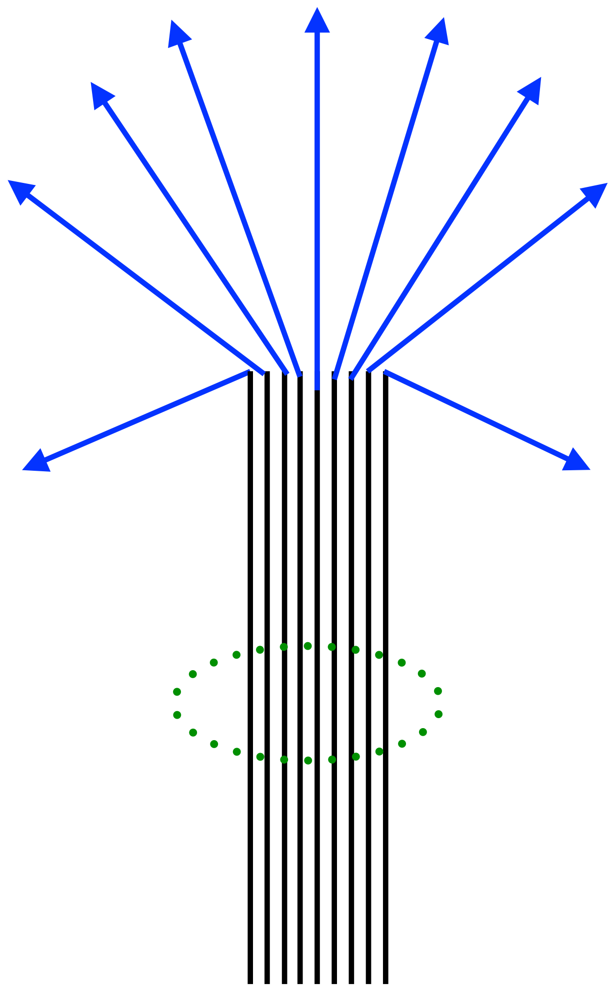
1 eV

toric code model, SPT, fractons

…

Dirac Monopole

- Pierre Curie (1894) proposed the possible existence of monopole
- Dirac (1931) used it explain quantization of electric charge



$$\mathbf{B} = q\,\frac{h\,\hat{\mathbf{r}}}{4\pi r^2}$$

$$q = 1$$

$$h = \frac{2\pi}{e} \approx 68.5\,e$$

t ‘Hooft-Polyakov Monopole

- **Based on spontaneously broken gauge theory: SU(2)/U(1)**

$$\mathscr{L} = \frac{1}{2}(D_\mu \Phi)^2 - \frac{1}{4}\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}) - \frac{\lambda}{4}\left(|\Phi|^2 - f^2\right)^2$$

triplet

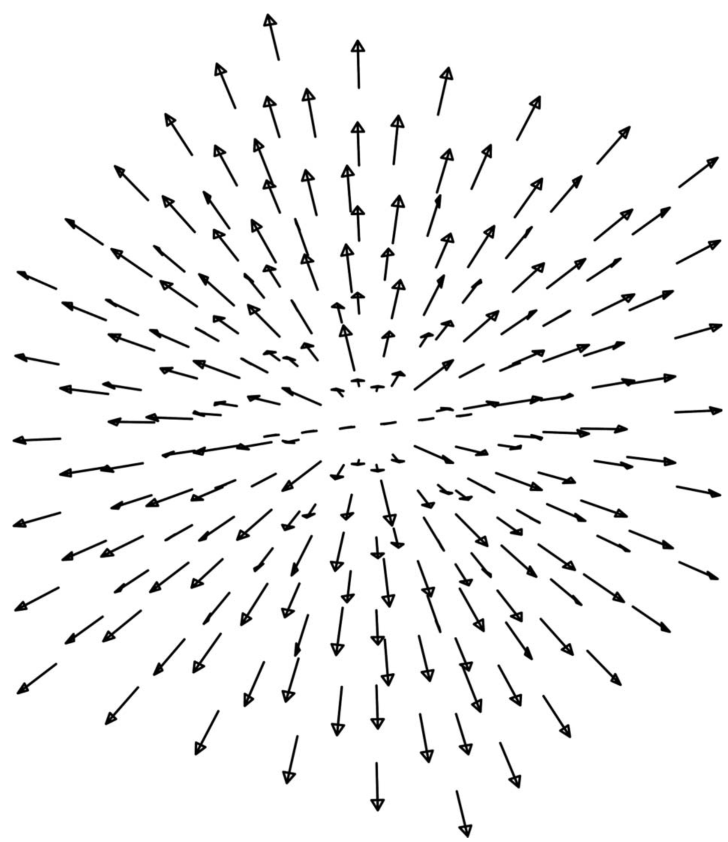
$$D_\mu \Phi^a = \partial_\mu \Phi^a + g\,\epsilon^{abc} A^b_\mu \Phi^c \qquad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\,\epsilon^{abc} A^b_\mu A^c_\nu$$

- **In the “hedgehog gauge” with $A^a_0 = 0$ (spherically symmetric)**

$$\Phi^a = \hat{r}^a f \phi(r)$$

$$A^a_i = \frac{1}{g}\epsilon^{aij}\hat{r}^j\left(\frac{1-u(r)}{r}\right)$$

$q = 2$



‘t Hooft-Polyakov Monopole

- **Total energy or mass (finite)**

$$M_{\mathscr{M}} = \int 4\,\pi\, r^2 \left(\frac{1}{2}\, B_i^a\, B_i^a + \frac{1}{2}\,(D_i\Phi^a)(D_i\Phi^a) + V(\Phi) \right)$$

$$= \frac{4\pi f}{g} \int d\bar{r}\bar{r}^2 \left(\frac{\bar{r}^2\, \phi'^2 + 2\, u^2 \phi^2}{2\, \bar{r}^2} + \frac{(1-u^2)^2 + 2\, \bar{r}^2\, u'^2}{2\, \bar{r}^4} + \frac{\lambda}{4g^2}(\phi^2 - 1)^2 \right)$$

- **Classical equations of motion ($\bar{r} \equiv g\, f\, r = m_W\, r$)**

$$\frac{d^2\phi}{d\bar{r}^2} + \frac{2}{\bar{r}}\frac{d\phi}{d\bar{r}} = \frac{2\, u^2\, \phi}{\bar{r}^2} + \frac{\lambda}{g^2}\phi\,(\phi^2 - 1)$$

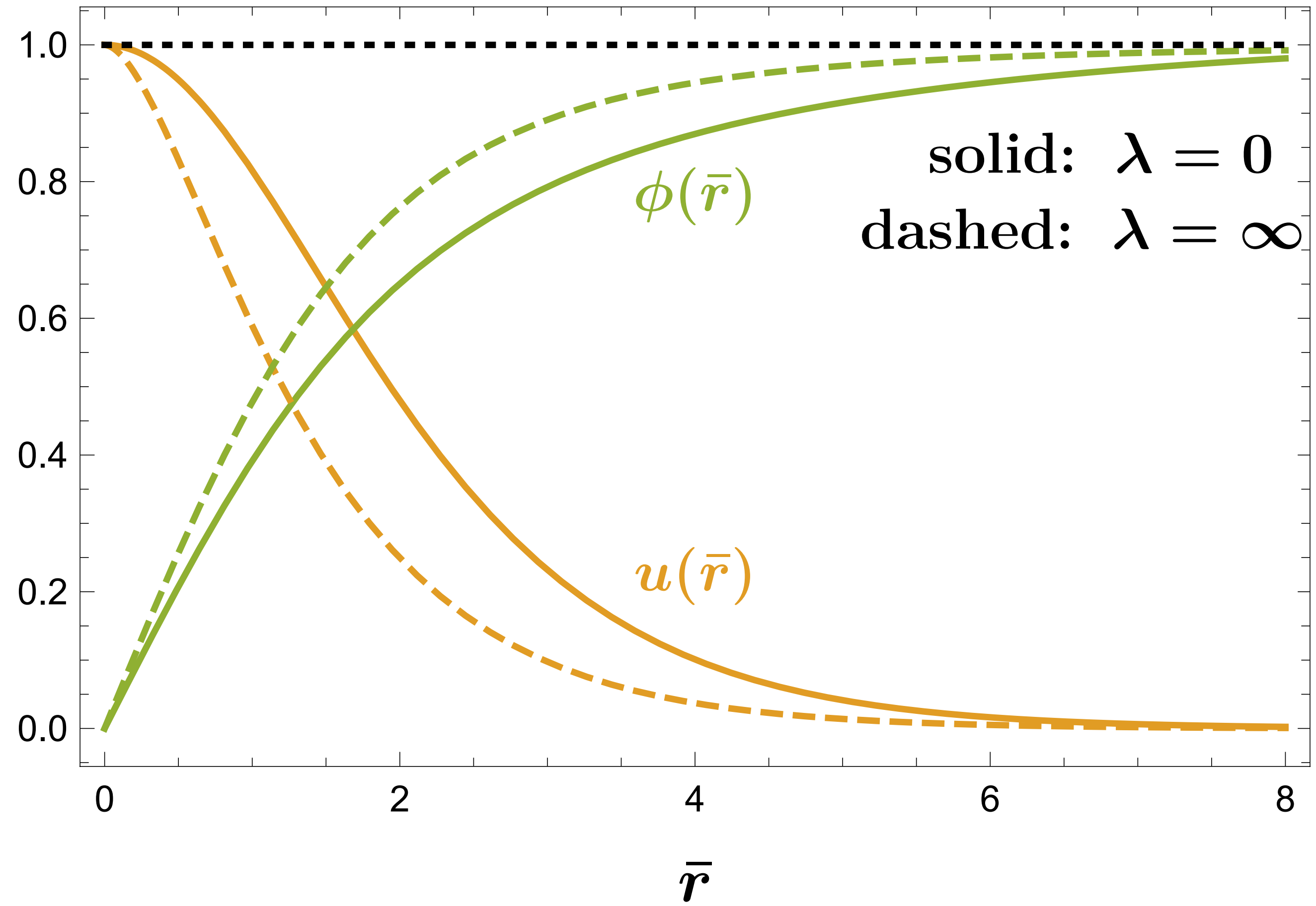
$$\frac{d^2 u}{d\bar{r}^2} = \frac{u\,(u^2-1)}{\bar{r}^2} + u\,\phi^2$$

- **Boundary conditions**

$$\phi(0) = 0\,, \quad \phi(\infty) = 1\,, \quad u(0) = 1\,, \quad u(\infty) = 0$$

‘t Hooft-Polyakov Monopole

$$M_{\mathscr{M}} \equiv \frac{4\pi f}{g} Y(\lambda/g^2) \qquad Y(0) = 1 \qquad Y(\infty) \approx 1.787$$



- **Topological reason:** $\pi_2[G/U(1)] = \pi_1[U(1)] = \mathbb{Z}$
- **GUT monopole:** $SU(5) \to SU(3) \times SU(2) \times U(1)$

$$M_{\mathscr{M}}^{\rm GUT} \sim 10^{17}\,\text{GeV}$$

Monopole in the Standard Model

- **In the SM:** $SU(2)_W \times U(1)_Y \to U(1)_{\rm EM}$ **with a Higgs doublet**

- **Topological reason:** $\pi_2[SU(2)_W \times U(1)_Y / U(1)_{\rm EW}] = 0$ **, no finite-energy EW monopole**

- **In more detail and again making a spherical configuration**

$$H = \frac{v}{\sqrt{2}}\,\phi(r)\,\xi\,, \qquad \xi = i\begin{pmatrix} \sin(\frac{\theta}{2})\,e^{-i\,\phi} \\ -\cos(\frac{\theta}{2}) \end{pmatrix} \qquad H^\dagger\,\vec{\sigma}\,H = -\,\frac{v^2}{2}\,\phi(r)^2\,\hat{r}$$

as the triplet case

$$A_i^a = \frac{1}{g}\,\epsilon^{aij}\,\hat{r}^j\left(\frac{1-u(r)}{r}\right) \longleftarrow SU(2)_W$$

$$B_i = -\,\frac{1}{g_Y}\,(1-\cos\theta)\,\partial_i\phi \longleftarrow U(1)_Y$$

Nambu, NPB130 (1977) 505

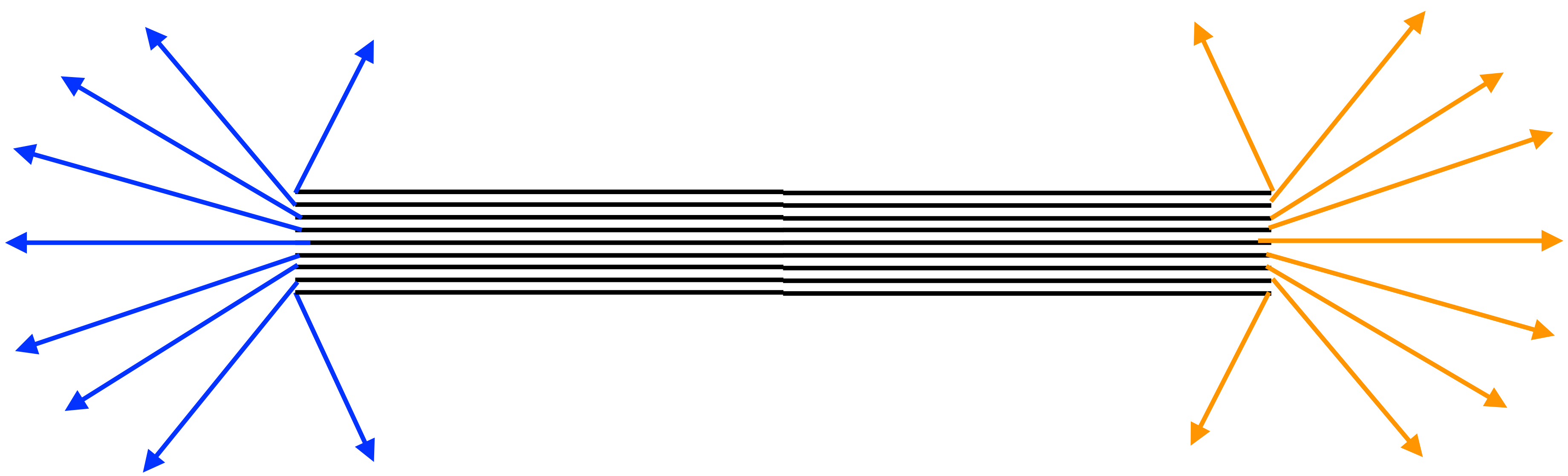
Cho, Maison, hep-th/9601028

Monopole in the Standard Model

$$S = -4\pi \int dt\, dr\, r^2\, (K + U)$$

$$K = \frac{(u')^2}{g^2 r^2} + \frac{1}{2} v^2 (\phi')^2 \qquad U = \frac{(u^2-1)^2}{2 g^2 r^4} + \frac{v^2 u^2 \phi^2}{4 r^2} + \frac{\lambda_h v^4}{8}\left(\phi^2 - 1\right)^2 + \frac{1}{2 g_Y^2 r^4}$$

- The spherical EW monopole has an infinite mass
- Nambu’s monopole-anti-monopole dumbbell configuration



- Unstable! May be produced at a future collider

- **Introduce BSM physics to have a finite-energy monopole**

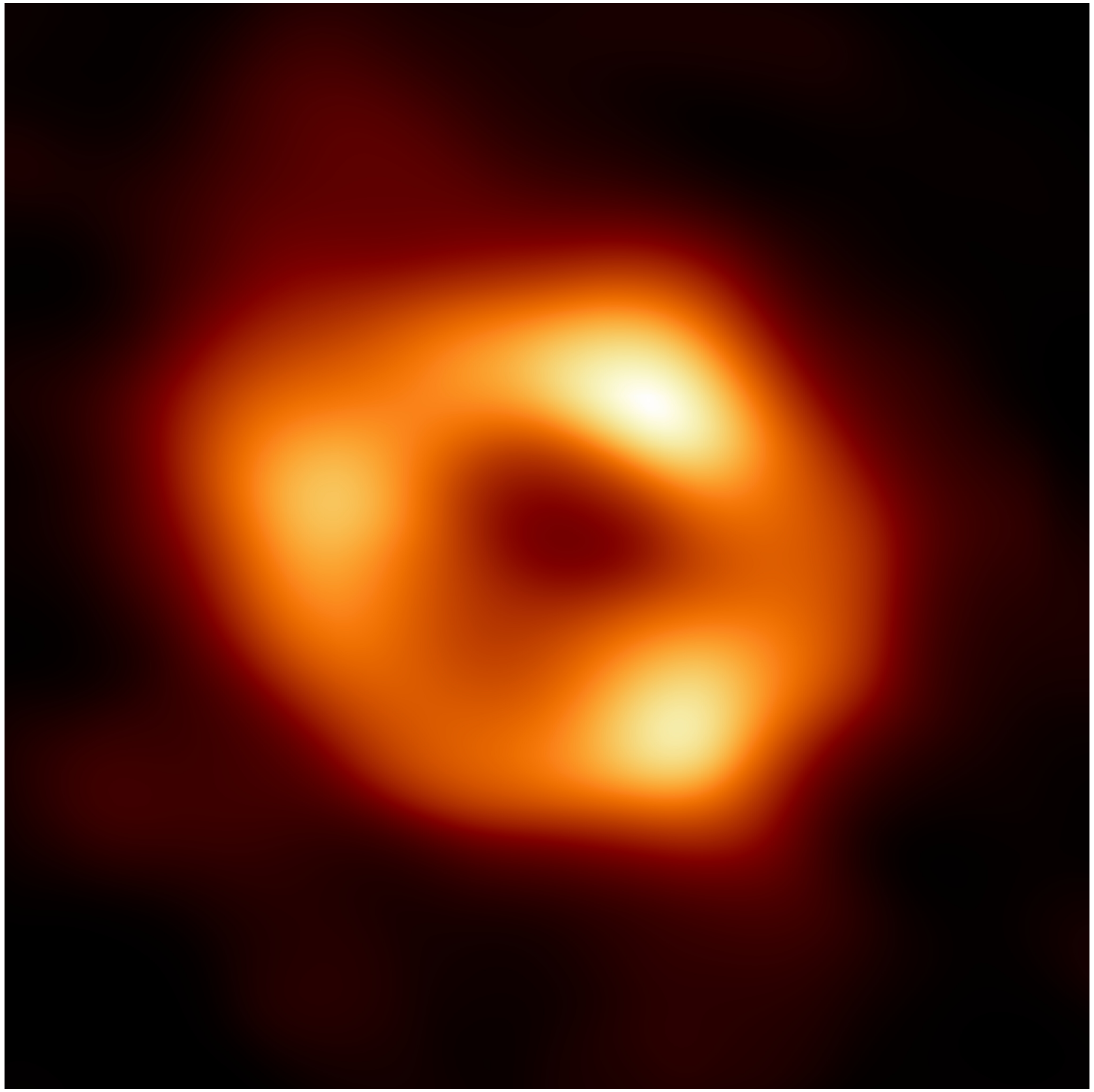
  for instance, $U(1)_Y \subset SU(2)_R$

- **Or hide the divergent part behind the event horizon of a black hole**

- **For the second avenue, no new BSM physics is needed. We just need to study the possible states based on**

Standard Model + General Relativity

Black Hole



Credit: EHT Collaboration

Black Holes

- **Schwarzschild black hole**

r_{H}

$$ds^2 = -\left(1 - \frac{2GM}{r}\right) dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

- **Charged or Reissner-Nordstrom (RN) black hole**

$$ds^2 = -B_{\mathrm{RN}}(r)dt^2 + B_{\mathrm{RN}}(r)^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

$$B_{\mathrm{RN}}(r) = 1 - \frac{2GM}{r} + \frac{G(Q_{\mathrm{E}}^2 e^2 + Q_{\mathrm{M}}^2 h^2)}{4\pi r^2}$$

- **The outer horizon radius is**

$$r_+ = \frac{\left(M_{\mathrm{eBH}} + \sqrt{M_{\mathrm{eBH}}^2 - (Q_{\mathrm{E}}^2 e^2 + Q_{\mathrm{M}}^2 h^2)M_{\mathrm{pl}}^2/4\pi}\right)}{M_{\mathrm{pl}}^2}$$

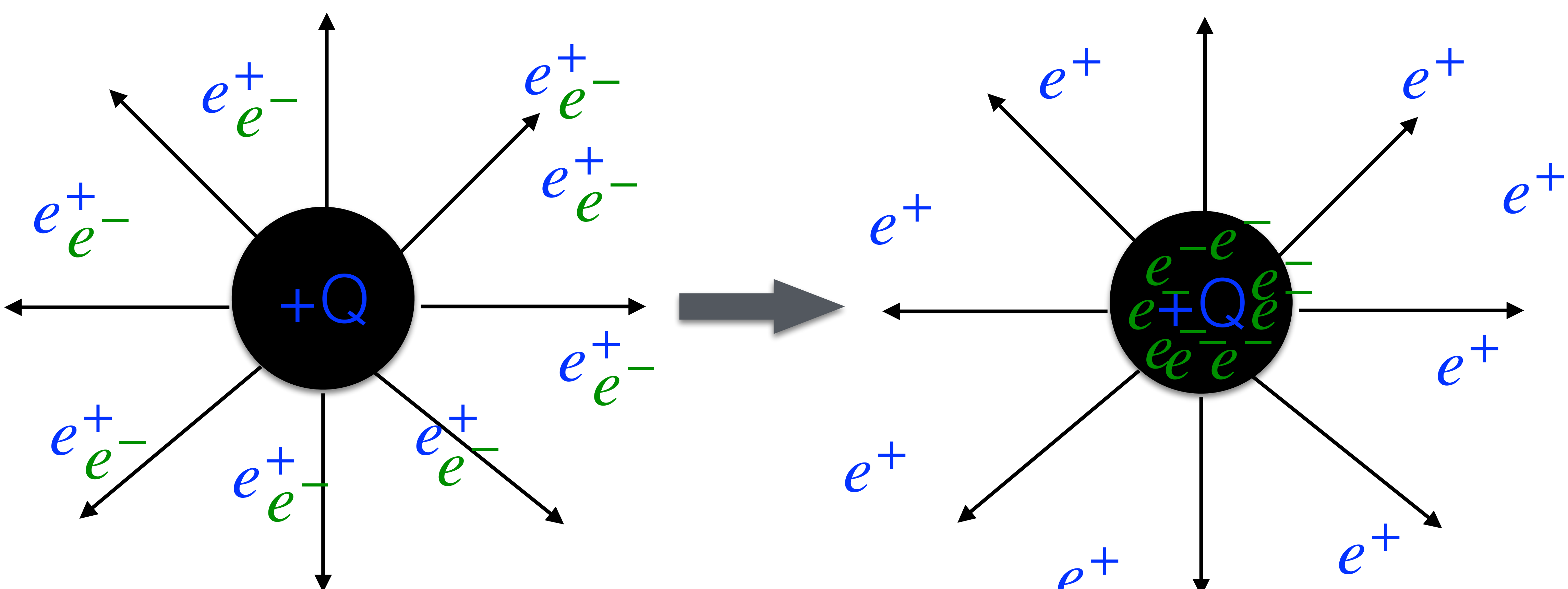
$$M_{\mathrm{eBH}} = \frac{\sqrt{Q_{\mathrm{E}}^2 e^2 + Q_{\mathrm{M}}^2 h^2}}{\sqrt{4\pi}} M_{\mathrm{pl}}$$

Electrically-Charged BH in SM

- **The charged BH has a large electric field close to the event horizon**

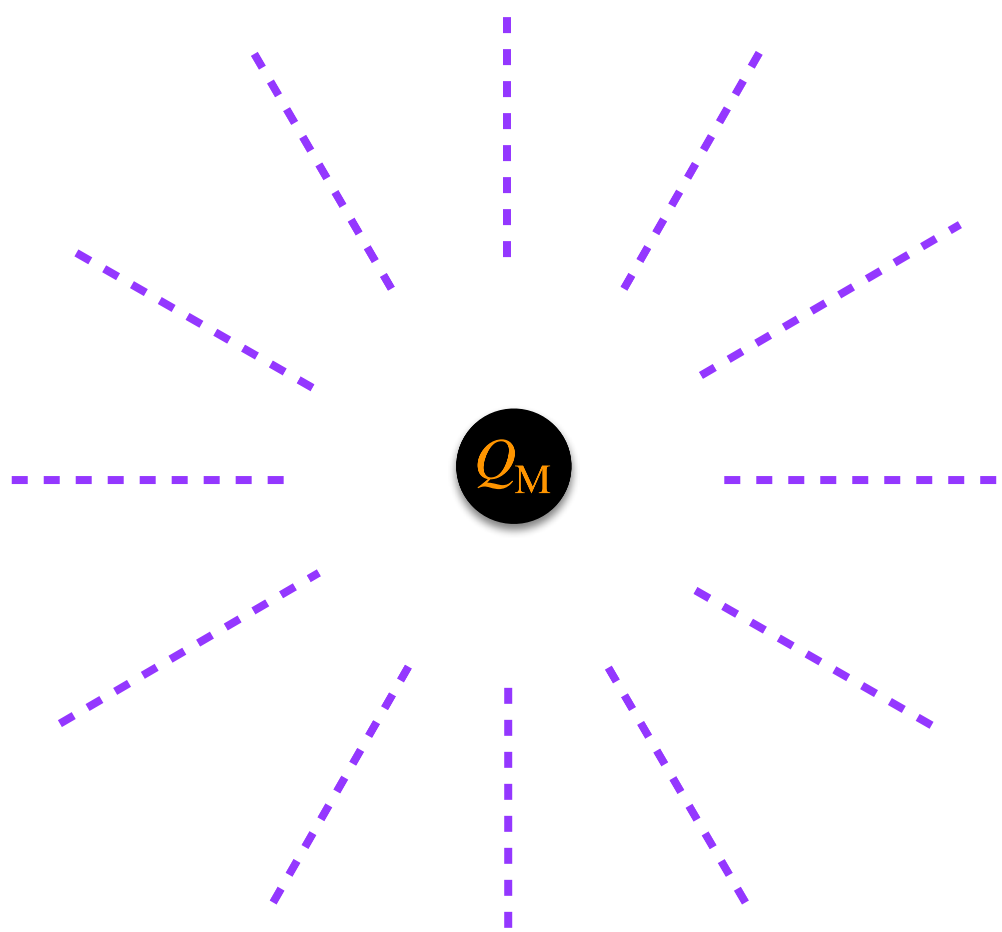
$$E = \frac{M_{\rm pl}^3}{\sqrt{4\pi}\, M_{\rm eBH}} > m_e^2 \qquad \text{for } M_{\rm eBH} < 10^8\, M_\odot$$

- **The Schwinger effects can generate electrons and positrons from vacuum and discharge the eBH**



Magnetically-Charged BH in SM

- Since there is no finite-energy magnetic monopole in the SM, no worry about Schwinger discharge
- If the GUT exists, it may worry its emission of GUT monopole, which is very heavy



$$B(R_{\rm eBH}) = \frac{Q}{2\,e\,R_{\rm eBH}^2} \approx \frac{e\,M_{\rm pl}^2}{2\,\pi\,Q}$$

$$Q \gtrsim 10^6$$

EW Symmetry Restoration in B Field

- **In a large B field background, the electroweak symmetry is restored**

Salam and Strathdee, NPB90 (1975) 203

Ambjorn and Olesen, NPB330 (1990) 193

$$
\begin{aligned}
\mathscr{E} \supset\ & \frac{1}{2}|D_iW_j - D_jW_i|^2 + \frac{1}{4}F_{ij}^2 + \frac{1}{4}Z_{ij}^2 + \frac{1}{2}\,g^2\varphi^2 W_iW_i^\dagger + (g^2\varphi^2/4\cos^2\theta_W)Z_i^2 \\
&+ig(F_{ij}\sin\theta_W + Z_{ij}\cos\theta_W)W_i^\dagger W_j + \frac{1}{2}g^2\left[(W_iW_i^\dagger)^2 - (W_i^\dagger)^2(W_j)^2\right]^2 \\
&+(\partial_i\varphi)^2 + \lambda(\varphi^2 - \varphi_0^2)^2
\end{aligned}
$$

$$
(W_1^\dagger, W_2^\dagger)\begin{pmatrix} \frac{1}{2}g^2\varphi_0^2 & i\,e\,F_{12} \\ -i\,e\,F_{12} & \frac{1}{2}g^2\varphi_0^2 \end{pmatrix}\begin{pmatrix} W_1 \\ W_2 \end{pmatrix}
$$

- **For a large $|F_{12}|$, a negative determinant leads to W-condensation and electroweak restoration. This happens when**

$$
e\,B \gtrsim m_h^2
$$

Electroweak Symmetry Restoration

$$B(R_{\rm eBH}) = \frac{Q}{2\,e\,R_{\rm eBH}^2} \approx \frac{e\,M_{\rm pl}^2}{2\,\pi\,Q} \qquad\qquad e\,B(R_{\rm eBH}) \gtrsim m_h^2$$

- **Electroweak symmetry restoration happens for**

$$Q \lesssim Q_{\rm max} \equiv \frac{e^2\,M_{\rm pl}^2}{2\pi\,m_h^2} \approx 1.4 \times 10^{32}$$

Lee, Nair, Weinberg, PRD45(1992) 2751

Maldacena, arXiv:2004.06084

- **For Q=2, one can obtain the spherically symmetric configuration**

- **For Q > 2, a non-spherically symmetric configuration is anticipated, and requires complicated numerical calculations**

Guth, Weinberg, PRD14(1976) 1660

Q=2: spherical solution

$$ds^2 = P^2(r)\, N(r)\, dt^2 - N(r)^{-1}\, dr^2 - r^2\, d\theta^2 - r^2\, \sin^2\theta\, d\phi^2$$

$$S_{\rm E} = -\frac{1}{16\pi\, G}\int d^4x\sqrt{-g}\, R = -\frac{1}{2\, G}\int dt\, dr\, r\, P'(1-N)$$

$$S_{\rm matter} \supset \int d^4x\sqrt{-g}\, \mathcal{L}_{\rm EW}$$

$$N(r) = 1 - \frac{2\, G\, F(r)}{r} + \frac{4\,\pi\, G}{g_{\rm Y}^2\, r^2}$$

- **The asymptotic mass of the system has**

$$M = F(\infty)$$

Q=2: spherical solution

$$\mathcal{L}_{\rm SM} \supset \mathcal{L}_{\rm EW} = -\frac{1}{4} W^a_{\mu\nu} W^{a\,\mu\nu} - \frac{1}{4} Y_{\mu\nu} Y^{\mu\nu} + |D_\mu H|^2 - \frac{\lambda}{2}\left(H^\dagger H - \frac{v^2}{2}\right)^2$$

$$D_\mu H = \left(\partial_\mu - i\,\frac{g}{2}\,\sigma^a\, W^a_\mu - i\,\frac{g_Y}{2}\, Y_\mu\right) H$$

$$H = \frac{v}{\sqrt{2}}\,\rho(r)\,\xi \qquad \xi = i\begin{pmatrix} \sin\left(\frac{\theta}{2}\right) e^{-i\,\phi} \\ -\cos\left(\frac{\theta}{2}\right) \end{pmatrix}$$

$$W^a_i = \epsilon^{aij}\,\frac{r^j}{r^2}\left(\frac{1 - f(r)}{g}\right) \qquad Y_i = -\frac{1}{g_Y}(1 - \cos\theta)\,\partial_i\phi$$

- **Change from the hedgehog gauge to the unitary gauge**

$$\xi \longrightarrow U\xi = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad \text{with} \qquad U = -i\begin{pmatrix} \cos\left(\frac{\theta}{2}\right) & \sin\left(\frac{\theta}{2}\right) e^{-i\phi} \\ \sin\left(\frac{\theta}{2}\right) e^{i\phi} & -\cos\left(\frac{\theta}{2}\right) \end{pmatrix}$$

$$A_\mu = -\frac{1}{e}(1 - \cos\theta_W)\partial_\mu\phi$$

$$Z_\mu = 0$$

Cho and Maison, hep-th/9601028

Q=2: EOMs and BCs

$$S_{\rm matter} \;=\; -4\pi \int dt\, dr\, r^2\, \left[P(r)\, N(r)\, \mathcal{K} + P(r)\, \mathcal{U}\right]$$

$$\mathcal{K} = \frac{v^2\,\rho'^2}{2} + \frac{f'^2}{g^2\,r^2}\,,$$
$$\mathcal{U} = \frac{v^2\,f^2\,\rho^2}{4\,r^2} + \frac{(1-f^2)^2}{2\,g^2\,r^4} + \frac{\lambda}{8}\,v^4\,(\rho^2-1)^2 + \frac{1}{2\,g_Y^2\,r^4} \equiv \mathcal{U}_1 + \frac{1}{2\,g_Y^2\,r^4}$$

EOM's

$$F' = 4\pi\, r^2\,(\mathcal{U}_1 + N\,\mathcal{K})\,,$$
$$(N\,f')' + 8\pi\,G\,r\,N\,f'\,\mathcal{K} = \frac{f(f^2-1)}{r^2} + \frac{g^2}{4}\,v^2\,f\,\rho^2\,,$$
$$\left(r^2\,N\,\rho'\right)' + 8\pi\,G\,r^3\,N\,\rho'\,\mathcal{K} = \frac{1}{2}\,\rho\,f^2 + \frac{\lambda\,v^2}{2}\,r^2\,\rho(\rho^2-1)\,.$$

BC's

$$N' = \frac{1}{r} - 8\pi\,G\,r\,\mathcal{U}\,, \qquad \text{at } r = r_H$$
$$N'\,f' = \frac{f(f^2-1)}{r^2} + \frac{g^2}{4}\,v^2\,f\,\rho^2\,, \qquad \text{at } r = r_H\,,$$
$$N'\,\rho' = \frac{1}{2}\,\frac{f^2\,\rho}{r^2} + \frac{\lambda\,v^2}{2}\,\rho(\rho^2-1)\,, \qquad \text{at } r = r_H\,.$$

$$f(\infty) = 0$$
$$\rho(\infty) = 1$$

Q=2: solutions

- **Setting $f(r) = 0$ and $\rho(r) = 1$, one has the ordinary RN magnetic black hole solution**

$$M_{\rm BH}^{\rm RN} = \frac{r_H}{2\,G} + \frac{2\pi}{e^2\,r_H} \qquad \geq \qquad M_{\rm eBH}^{\rm RN} = \frac{\sqrt{4\pi}\,M_{\rm pl}}{e}$$

- **For the hairy magnetic black hole solution:**

$$M_{\rm hMBH} = F(\infty) = \int_{r_H}^{\infty} dr' 4\pi\, r'^2 \left[\mathcal{K}(r') + \mathcal{U}_1(r')\right] + \frac{r_H}{2\,G} + \frac{2\pi}{g_Y^2\,r_H}$$

Hair mass $\ll$ **Black hole mass**

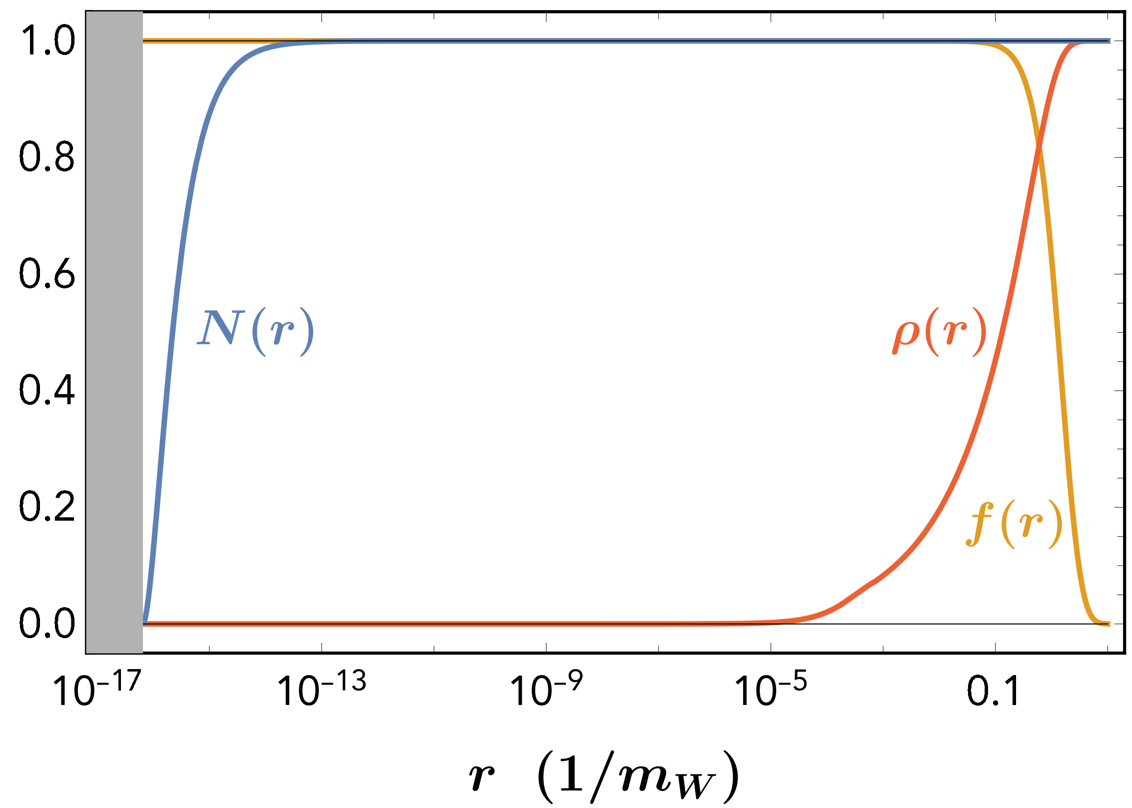
- **Ignoring the hair mass, one has** $e = g_Y \cos\theta_W$

$$M_{\rm hMBH} \approx \frac{r_H}{2\,G} + \frac{2\pi}{g_Y^2\,r_H} \geq M_{\rm ehMBH} = \cos\theta_W\,\frac{\sqrt{4\pi}\,M_{\rm pl}}{e}$$

Hyper-magnetic black hole!

Q=2: profiles

extremal hMBH

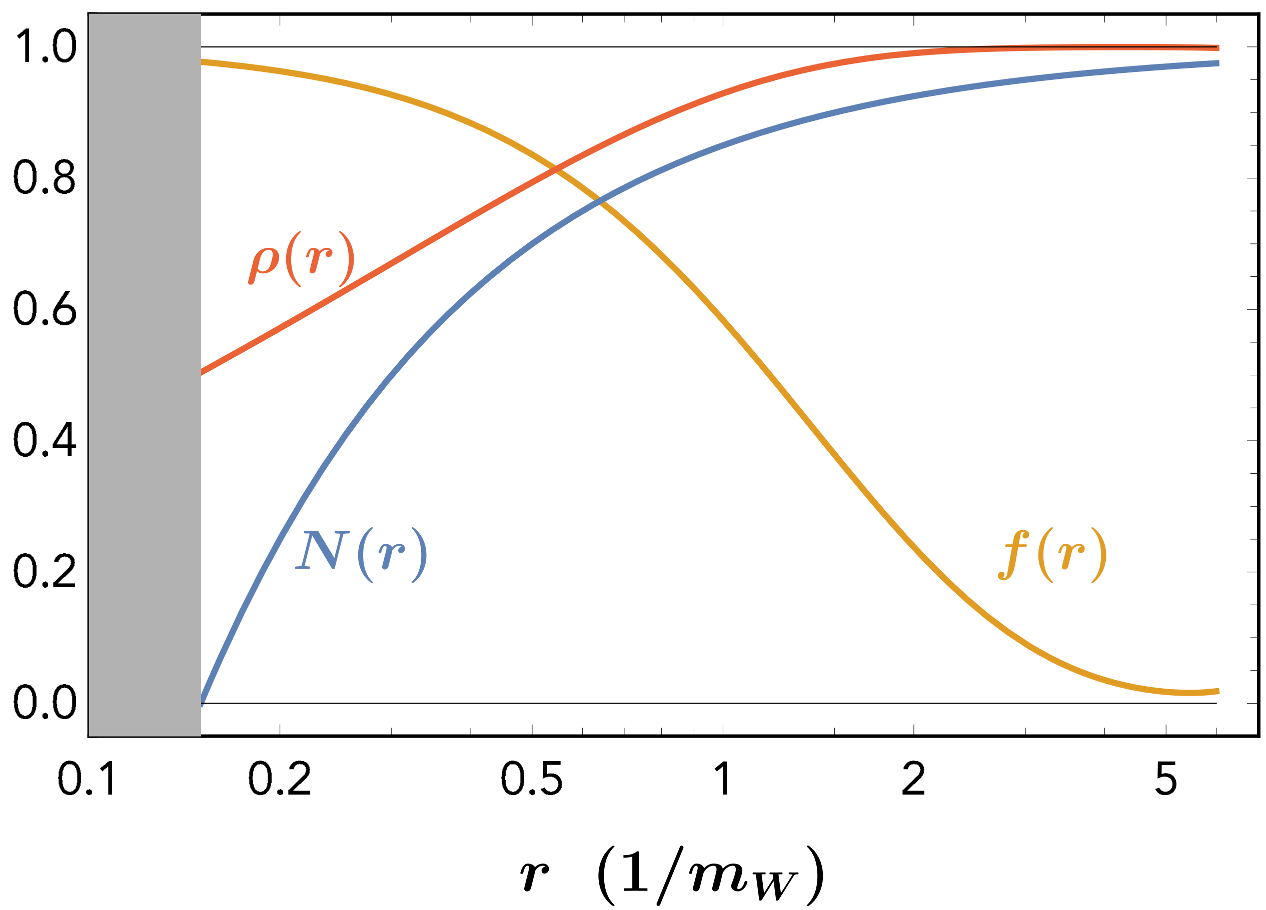


$$M_{\rm ehMBH} \approx \cos\theta_W \frac{\sqrt{4\pi}\, M_{\rm pl}}{e} + 0.75 \times \frac{2\pi\, v^2}{m_W} = (1.2 \times 10^{20} + 3.6 \times 10^3)\,\text{GeV}$$

The electroweak symmetry is restored inside

Q=2: profiles

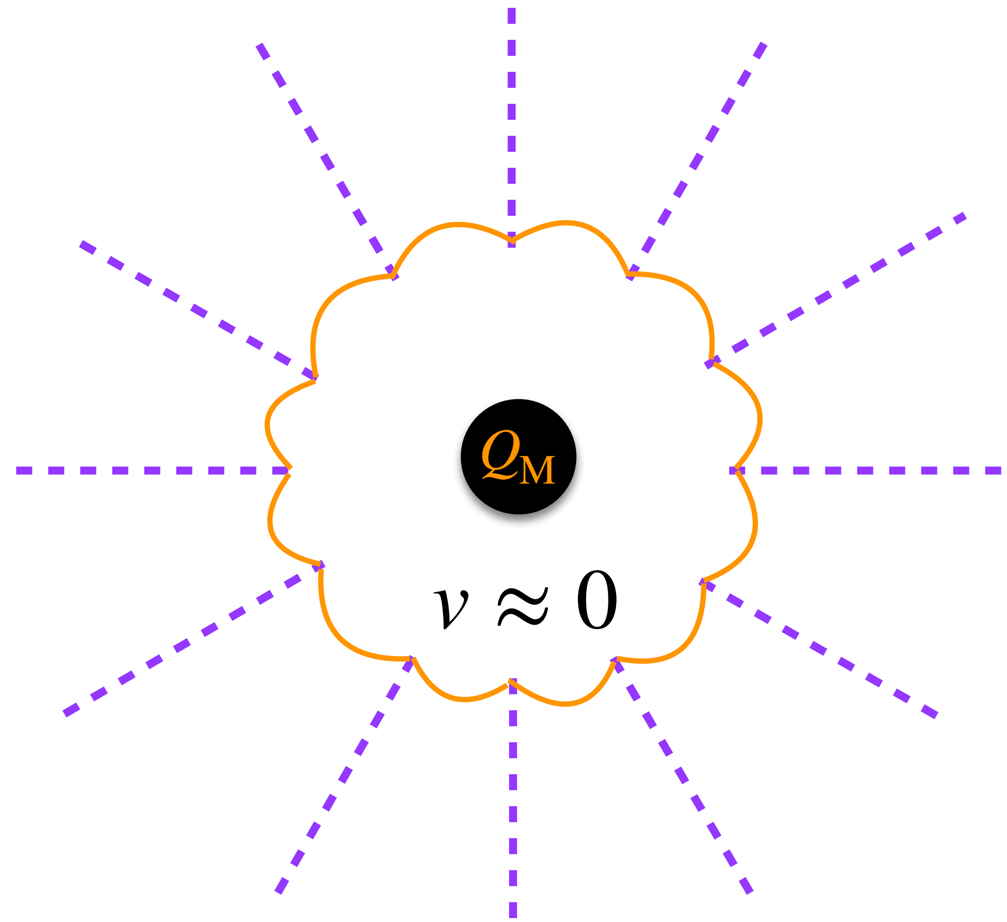
non-extremal hMBH



$r_H = 0.15/m_W$

$$M_{\mathrm{hMBH}} \leq M_{\mathrm{hMBH}}^{\mathrm{max}} = \frac{1}{2\,G\,m_W} + \mathcal{O}(m_W) \approx 9.3 \times 10^{35} \ \mathrm{GeV}$$

Q>2: non-spherical



$v = 246\,\mathrm{GeV}$

$$R_{\mathrm{EW}} \simeq \sqrt{\frac{Q}{2}}\,\frac{1}{m_h}$$

$$M^{\mathrm{tot}}_{\mathrm{MeBH}}(Q) \;\simeq\; c_W\,\frac{\sqrt{\pi}\,Q}{e}\,M_{\mathrm{pl}} + \frac{4\pi}{3}\,R^3_{\mathrm{EW}}\,\frac{m_h^2\,v^2}{8} = c_W\,\frac{\sqrt{\pi}\,Q}{e}\,M_{\mathrm{pl}} + \frac{\pi}{12\sqrt{2}}\,Q^{3/2}\,\frac{v^2}{m_h}$$

$$\equiv\; M_{\ast}(Q) + \frac{\pi}{12\sqrt{2}}\,Q^{3/2}\,\frac{v^2}{m_h}\,, \qquad\qquad M_{\ast}(Q) = c_W\,M^{\mathrm{RN}}_{\mathrm{eBH}}$$

- To have the electroweak hair, $r_H < R_{\mathrm{EW}}$

$$Q < Q_{\mathrm{max}} \simeq 10^{32} \qquad M_{\ast} \lesssim 9\times 10^{51}\,\mathrm{GeV} \sim M_{\oplus} \qquad R^{\mathrm{max}}_{\mathrm{EW}} \sim 1\,\mathrm{cm}$$

2d Modes

- **For non-extremal BH, the Hawking temperature is**

$$T(M_{\rm BH}, M_{\ast}) = \frac{M_{\rm pl}^2}{2\pi} \frac{\sqrt{M_{\rm BH}^2 - M_{\ast}^2}}{\left(M_{\rm BH} + \sqrt{M_{\rm BH}^2 - M_{\ast}^2}\right)^2}$$

- **In the existence of magnetic field, the massless 2d modes exist for a Dirac 4D massless fermion**

$$ds^2 = e^{2\sigma(t,x)}\left(-dt^2 + dx^2\right) + R^2(t,x)\left(d\theta^2 + \sin^2\theta\, d\phi^2\right) \qquad A_\phi = \frac{Q}{2}\cos\theta$$

$$dx = \frac{dr}{f(r)}, \qquad e^{2\sigma(t,x)} = f(r) \equiv (1 - R_{\rm e}/r)^2, \qquad R(t,x) = r$$

$$\not{D}\widetilde{\chi} = m_\chi \widetilde{\chi} \qquad \widetilde{\chi}_{\alpha\beta} = \frac{e^{-\frac{1}{2}\sigma}}{R}\psi_\alpha(t,x)\,\eta_\beta(\theta,\phi)$$

$$\left[\sigma_y \frac{\partial_\phi - iA_\phi}{\sin\theta} + \sigma_x\left(\partial_\theta + \frac{\cot\theta}{2}\right)\right]\eta = 0,$$

$$(i\sigma_x\partial_t + \sigma_y\partial_x)\,\psi = m_\chi\, e^\sigma \psi\,.$$

2d fermion

2d Modes

Kazama, Yang, Goldhaber, ’1977

- **Solutions for Q > 0,**

$$\propto {}_qY_{q,-m}(\theta,\phi)$$

$$\begin{aligned}\eta_1 &= 0\ ,\\ \eta_2 &= \left(\sin\frac{\theta}{2}\right)^{j-m}\left(\cos\frac{\theta}{2}\right)^{j+m} e^{im\phi} = \frac{(1-\cos\theta)^{\frac{q-m}{2}}(1+\cos\theta)^{\frac{q+m}{2}}}{2^{q-\frac{1}{2}}(\sin\theta)^{\frac{1}{2}}}\,e^{im\phi}\end{aligned}$$

$$j = (|Q|-1)/2 \equiv q - 1/2 \text{ and } -j \le m \le j$$

total Q 2d modes

- **There are |Q| massless modes for $m_\chi = 0$**

Field	$SU(3)\times SU(2)\times U(1)$	Number of 2d modes (left - right)
q_L	$(\mathbf{3},\mathbf{2})_{\frac{1}{6}}$	Q
u_R	$(\mathbf{3},\mathbf{1})_{\frac{2}{3}}$	- 2 Q
d_R	$(\mathbf{3},\mathbf{1})_{-\frac{1}{3}}$	Q
l_L	$(\mathbf{1},\mathbf{2})_{-\frac{1}{2}}$	- Q
e_R	$(\mathbf{1},\mathbf{1})_{-1}$	Q

Maldacena, arXiv:2004.06084

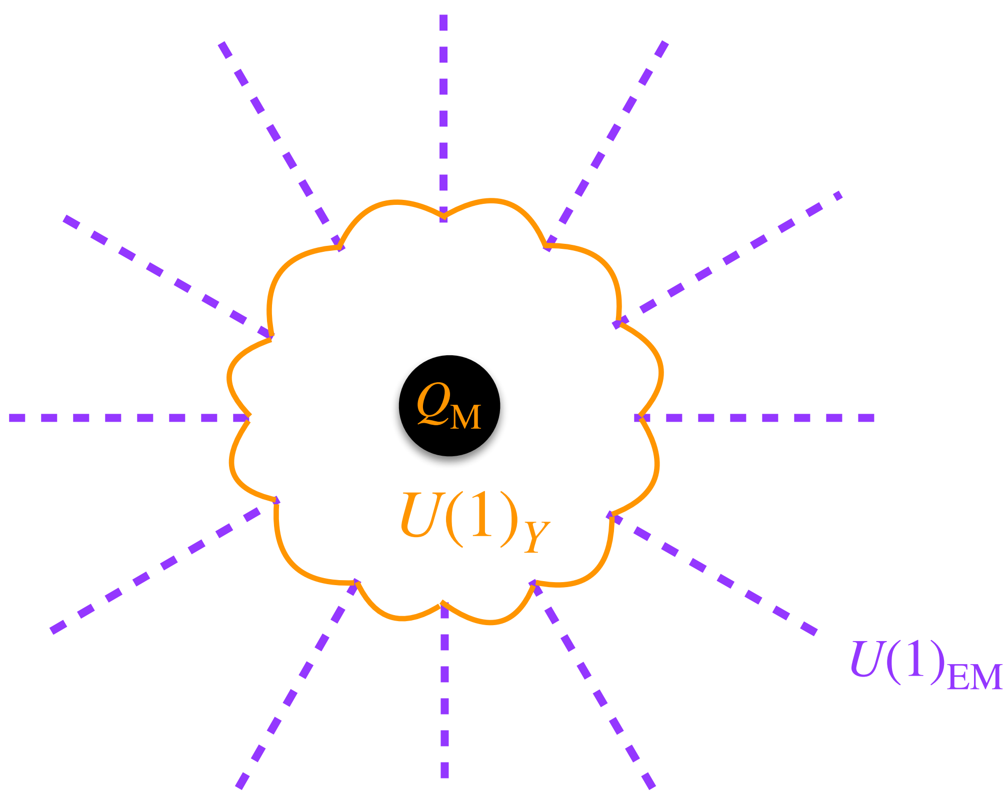
2d Hawking radiation

- **Fermions are massless (ignoring QCD vacuum) inside the EW-corona region**

$$P_2 = \frac{dE}{dt} = \frac{\pi\, g_*}{24} T^2(M_{\rm BH}, M_{\ast})$$

- **For high T, $g_* = 18\,|\,Q\,|$ for three-family fermions**

- **The 2d radiation is very fast; it reaches extremal very quickly**



- 2d neutrino modes can not escape
- EM charged states can travel outside of coronas

2d Hawking radiation

- **For $T < m_e$, the 2d radiation is suppressed. The 4D radiation dominants**

$$P_4 = \frac{dE}{dt} \approx \frac{\pi^2 g_*}{120}(4\pi R_{\rm EW}^2)\, T^4(M_{\rm BH}, M_{\circledast})$$

with $g_* = 2$ for photon and $g_* = 21/4$ for neutrinos

- **For $T > m_e$, the 2d radiation usually dominants over 4D**

$$\tau_{\rm BH} \approx \frac{24\pi^{3/2}\, c_W\, M_{\circledast}^2}{e\, M_{\rm pl}^3} \log\left[\frac{M_{\rm pl}^4\,(M_{\rm BH} - M_{\circledast})}{2\pi^2\, m_e^2\, M_{\circledast}^3}\right]$$

shorter than the 4D time scale by a factor of $M_{\rm pl}/M_{\rm BH}$

Primordial MBHs ?

- There are various ways to form primordial black holes
    - Large primordial fluctuations
    - Phase transitions, boson stars, ……
- Produce large number of monopoles and anti-monopoles (maybe Nambu's dumbbell configurations)
- The formation of black holes eat totally N objects
- Anticipate the net BH magnetic charge: $\sim \sqrt{N}$

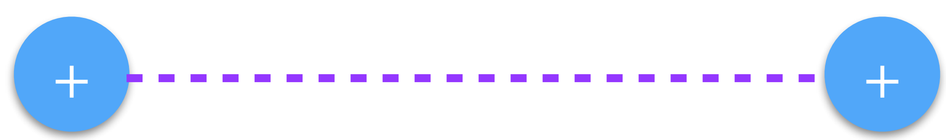
YB, Orlofsky, arXiv: 1906.04858

- To be studied more

- **Gravity could bind monopole charges together:**

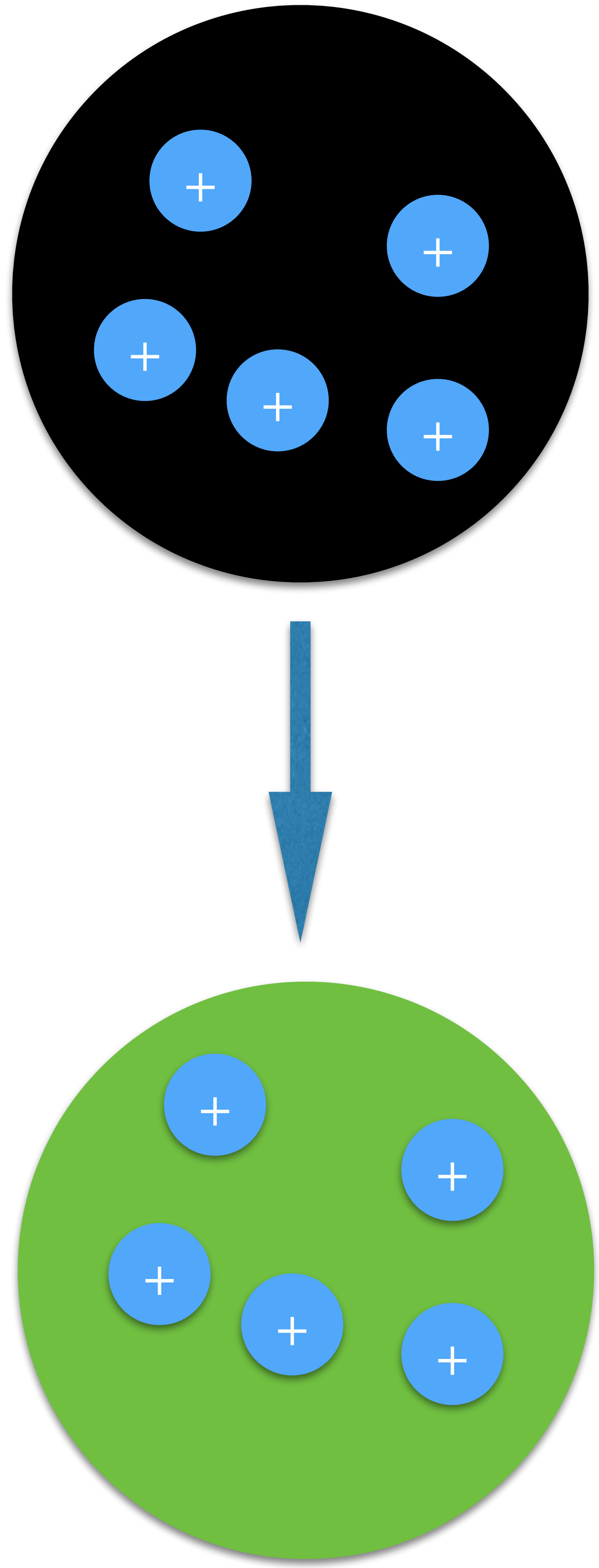
$$\frac{M}{q} \sim M_{\rm pl}$$

- **Can we have a large-charge magnetic monopole object with different relation between M and q?**



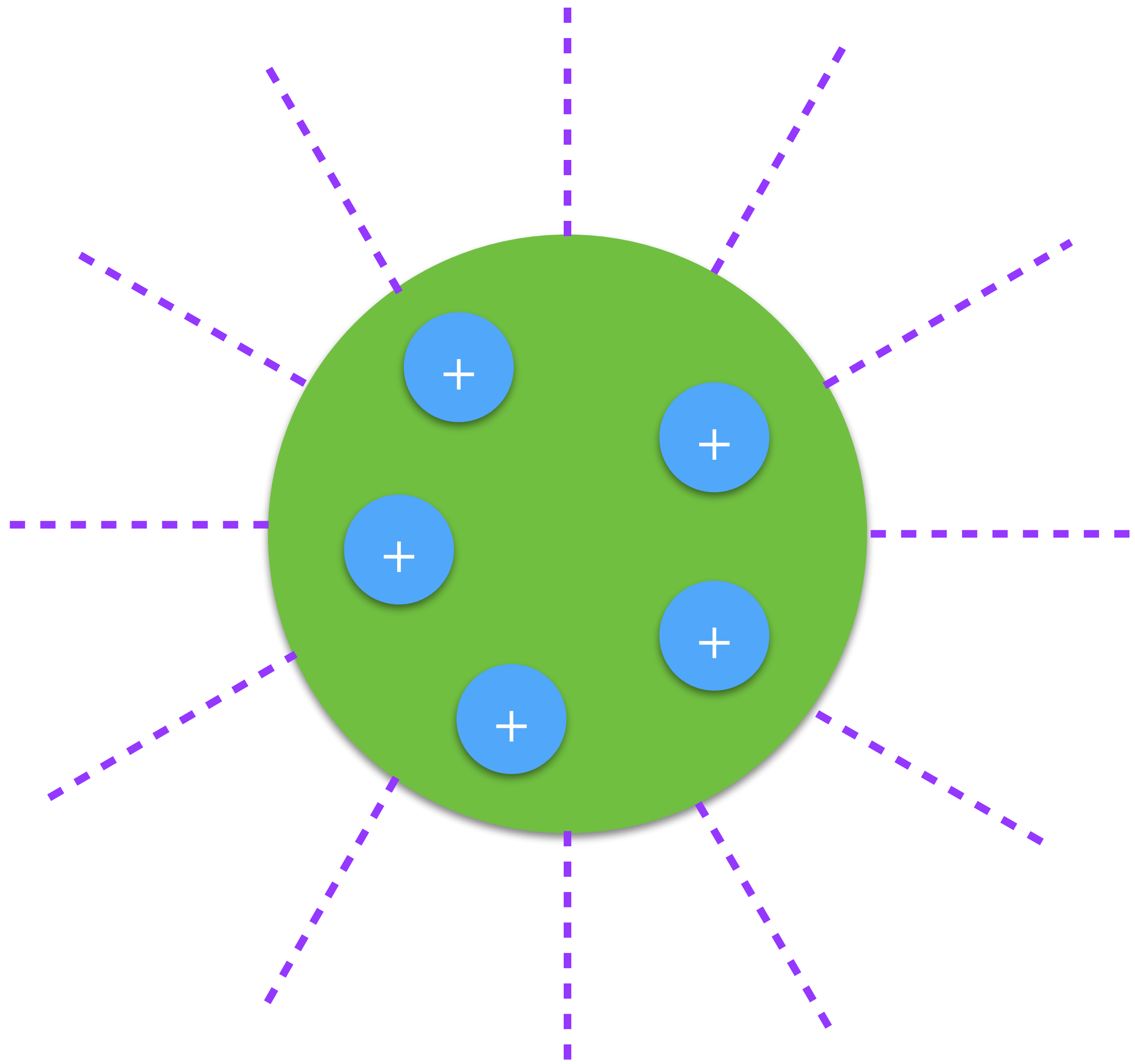
- **The forces from gauge boson and scalar are only cancelled in the BPS limit. Otherwise, they repel each other.**

Black hole — **gravitation pressure**



Q-ball — **vacuum pressure**

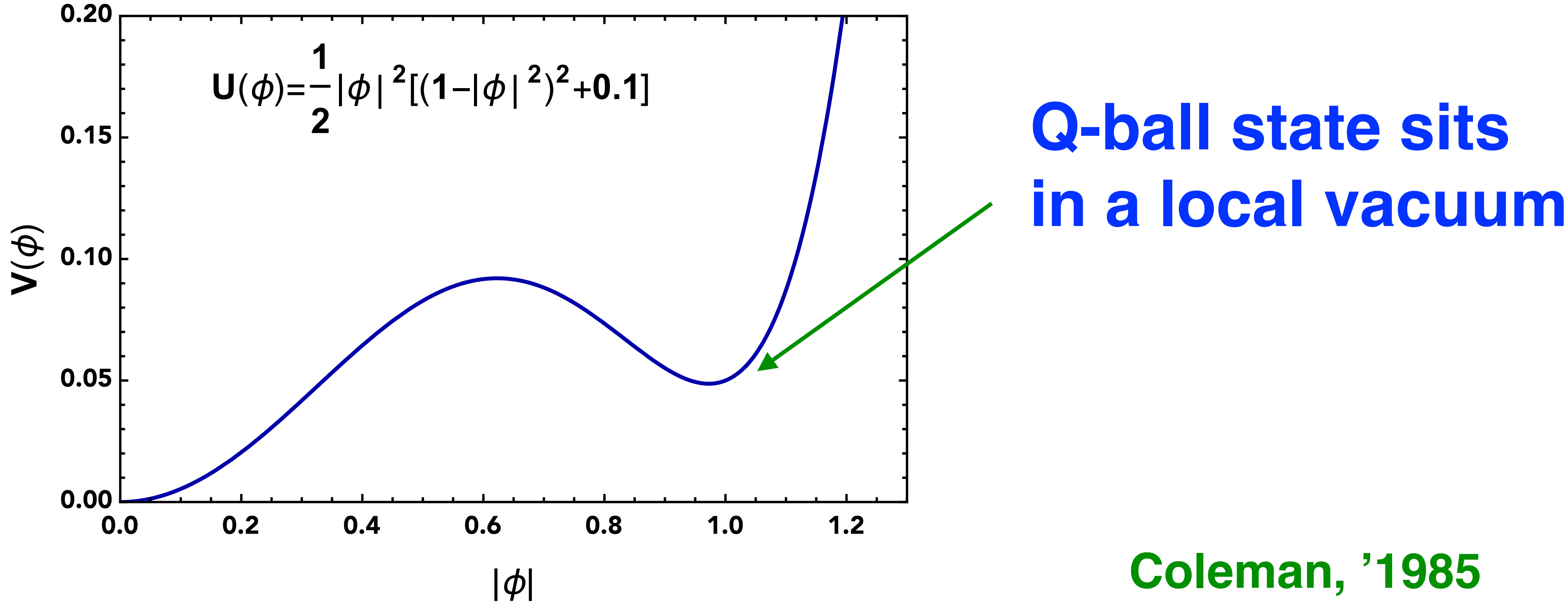
Q-Monopole-Ball



YB, Lu, Orlofsky, arXiv: 2111.10360

Non-topological Soliton

- For a complex scalar field with an unbroken global symmetry, there exist nondissipative solutions of the classical field equations that are absolute minima of the energy for a fixed (sufficiently large) Q.



Coleman, '1985

- This will be a non-renormalizable potential for a single field

Q-Monopole-Ball

- **A Lagrangian provides a soliton state carrying both topological and non-topological charges**

$$\mathscr{L} = |\partial_\mu S|^2 + \frac{1}{2}(D_\mu \phi^a)^2 - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} - V(S,\phi)$$

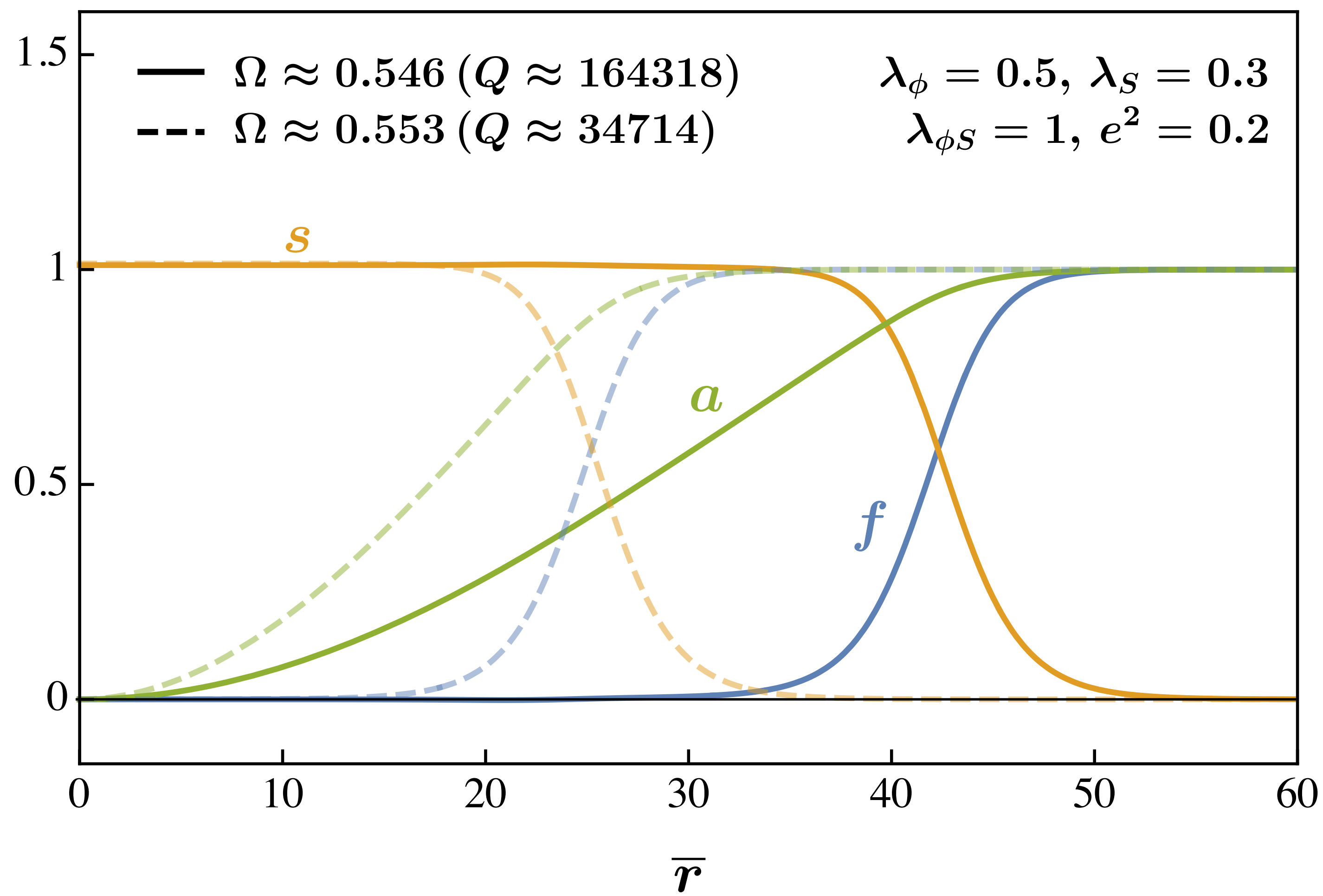
$$V(S,\phi) = \frac{1}{8}\lambda_\phi(\phi^a\phi^a - v^2)^2 + \frac{1}{2}\lambda_{\phi S}|S|^2(\phi^a\phi^a) + \lambda_s|S|^4 + M^2_{S,0}|S|^2$$

- **The symmetry has** $[SU(2)/U(1)] \times U(1)_S$ **. The gauge group is chosen for simple calculation. One could extend it to a more realistic gauge group with the right topology**

- **The state can have charge (q, Q). For q=2, we can perform a detailed calculation for a spherical configuration**

Profiles of Solutions (large Q)

$$q = 2, Q \gg 1$$

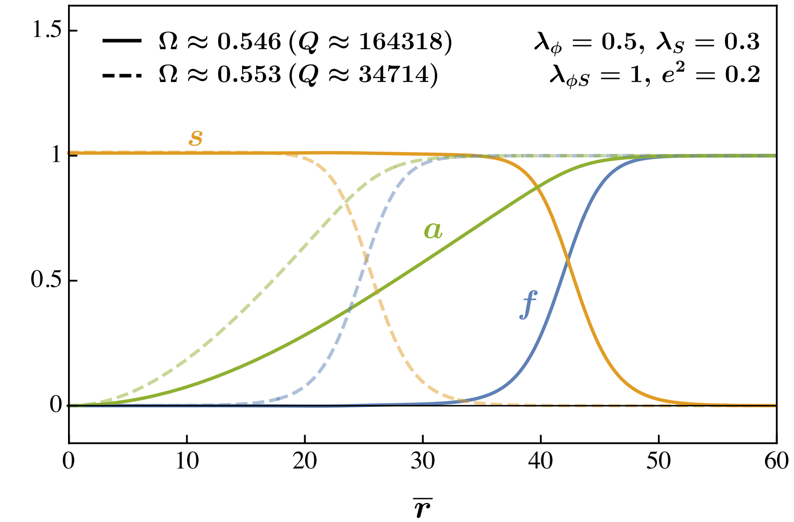


- The total mass and size are dominated by Q-ball properties
- The magnetic field spreads and follows the Q-ball radius, which is a reduction of magnetic energy for this system

Large Magnetic Charge (q >> 1)

- **Again, non-spherically configurations are needed**

- **For a large Q, the Q-ball can contain more monopole charges**



$$M_{(q,Q)} \sim \frac{304\pi v (q/2)^2}{35 e^2 \bar{r}_b} + \frac{4\pi}{3}\bar{r}_b^3 v \left(\frac{1}{4}\lambda_S s_0^4 + \frac{1}{8}\lambda_\phi + \frac{1}{2}\Omega^2 s_0^2\right)$$

energy from magnetic field

- **Requiring it to be stable against the decay of**

$$(q, Q) \to (q-2, Q) + (2, 0)$$

$$q \lesssim e \frac{\lambda_S^{1/12}}{\lambda_\phi^{1/4}} Q^{1/3} \longrightarrow M_{(q,Q)} \gtrsim \frac{\lambda_\phi}{e^3} q^3 v$$

Searching for ultraheavy monopoles

Magnetic Monopoles Inside Earth?

Carl Friedrich Gauss – *General Theory of Terrestrial Magnetism* – a revised translation of the German text

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Abstract. This is a translation of the *Allgemeine Theorie des Erdmagnetismus* published by Carl Friedrich Gauss in 1839 in the *Resultate aus den Beobachtungen des Magnetischen Vereins im Jahre 1838*. The current translation is based on an earlier translation by Elizabeth Juliana Sabine published in 1841. This earlier translation has been revised, corrected, and extended. Numerous biographical comments on the scientists named in the original text have been added as well as further information on the observational material used by Carl Friedrich Gauss. An attempt is made to provide a readable text to a wider scientific community, a text laying the foundation of today's understanding of planetary magnetic fields.

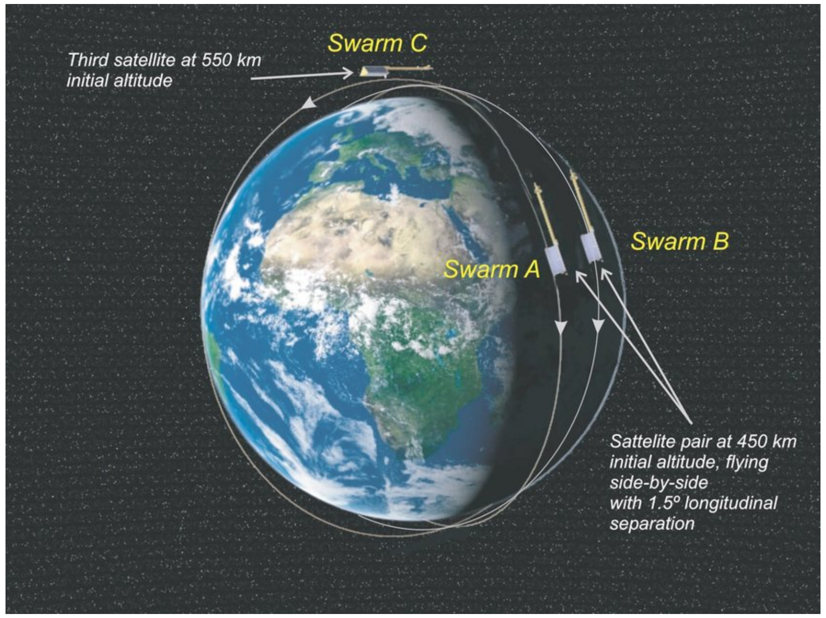
Y components would not at all be affected. Once the future has provided a more extensive opulence of precise observations than currently offered, one might determine whether their precise representation requires a non-vanishing value of P^0 or not[74]. Based on the current state of the data, such an undertaking would be completely unsuccessful.

[74]T: Gauss is discussing here the possible existence of magnetic monopoles. It is remarkable how important experimental results are for this mathematician.

Monopole Moment of Earth Magnetic Field

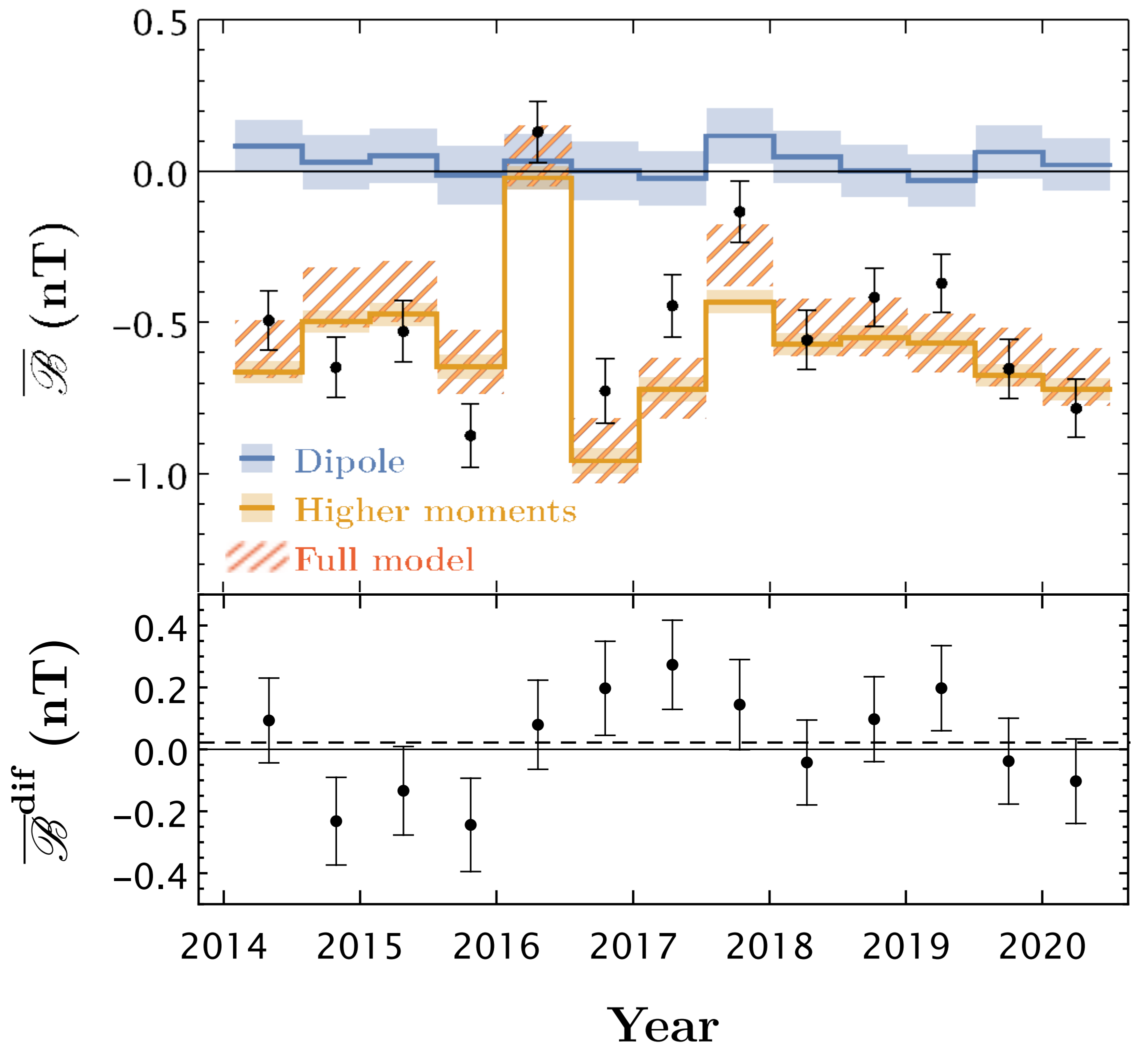
- Using Gauss law to search for monopoles

$$\overline{B}_{\rm m} \;\equiv\; \frac{1}{4\pi}\oint \boldsymbol{B}_{\rm m}(r,\theta,\phi)\cdot\hat{\mathbf{n}}\,d\Omega \;=\; Q\,h\,\frac{1}{4\pi\,R^2}$$



$$\overline{\mathscr{B}} = \frac{1}{4\pi}\int\left[\frac{r(\theta,\phi)}{R_{\rm ref}}\right]^3 \boldsymbol{B}(r,\theta,\phi)\cdot\hat{\mathbf{r}}\,d\Omega$$

Monopole Moment of Earth Magnetic Field



$$|B_m(r = R_\oplus)| < 0.13\,\text{nT}$$

or $$|Q_{\rm net}| < 1.6 \times 10^{19}$$

YB, Lu, Orlofsky, 2103.06286

Conclusions

- Magnetic black holes with $Q < 10^{32}$ have electroweak-symmetric coronas

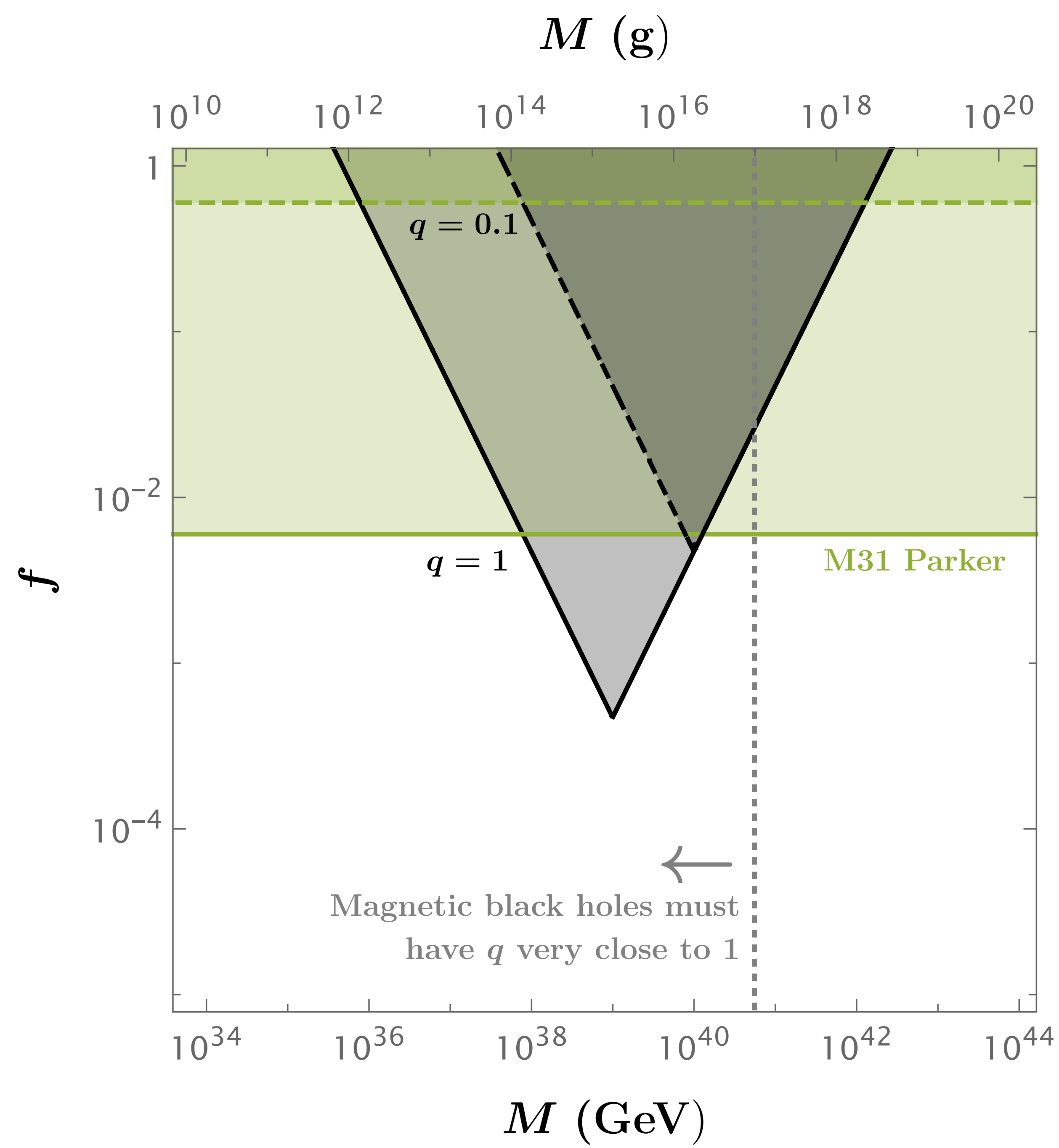
- It has a fast 2d Hawking radiation rate and can reach the extremal state quickly

- The existence of such objects only requires the known physics, SM+GR, and they deserve more studies

- Q-Monopole-Ball serves as another example to have a large magnetically-charged object

Thanks!

Monopole Moment of Earth Magnetic Field



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