

# Quarkonium and open heavy flavor in the statistical hadronization model

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- The statistical model and the thermal fits
  - The charm quarks
  - The beauty quarks

Andronic, Braun-Munzinger, Redlich, Stachel, [Nature 561 \(2018\) 321](#)

...+ Köhler, Mazeliauskas, Vislavicius, [JHEP 07 \(2021\) 035](#)

# The statistical (thermal) model

grand canonical partition function for specie (hadron)  $i$  :

$$\ln Z_i = \frac{V g_i}{2\pi^2} \int_0^\infty \pm p^2 dp \ln[1 \pm \exp(-(E_i - \mu_i)/T)]$$

$g_i = (2J_i + 1)$  spin degeneracy factor;  $T$  temperature;

$E_i = \sqrt{p^2 + m_i^2}$  total energy; (+) for fermions (-) for bosons

$\mu_i = \mu_B B_i + \mu_{I_3} I_{3i} + \mu_S S_i + \mu_C C_i$  chemical potentials

$\mu$  ensure conservation (on average) of quantum numbers, fixed by “initial conditions”

i) isospin:  $\sum_i n_i I_{3i} / \sum_i n_i B_i = I_3^{tot} / N_B^{tot}$ ,  $N_B^{tot} \sim \mu_B$

$I_3^{tot}$ ,  $N_B^{tot}$  isospin and baryon number of the system (=0 at high energies)

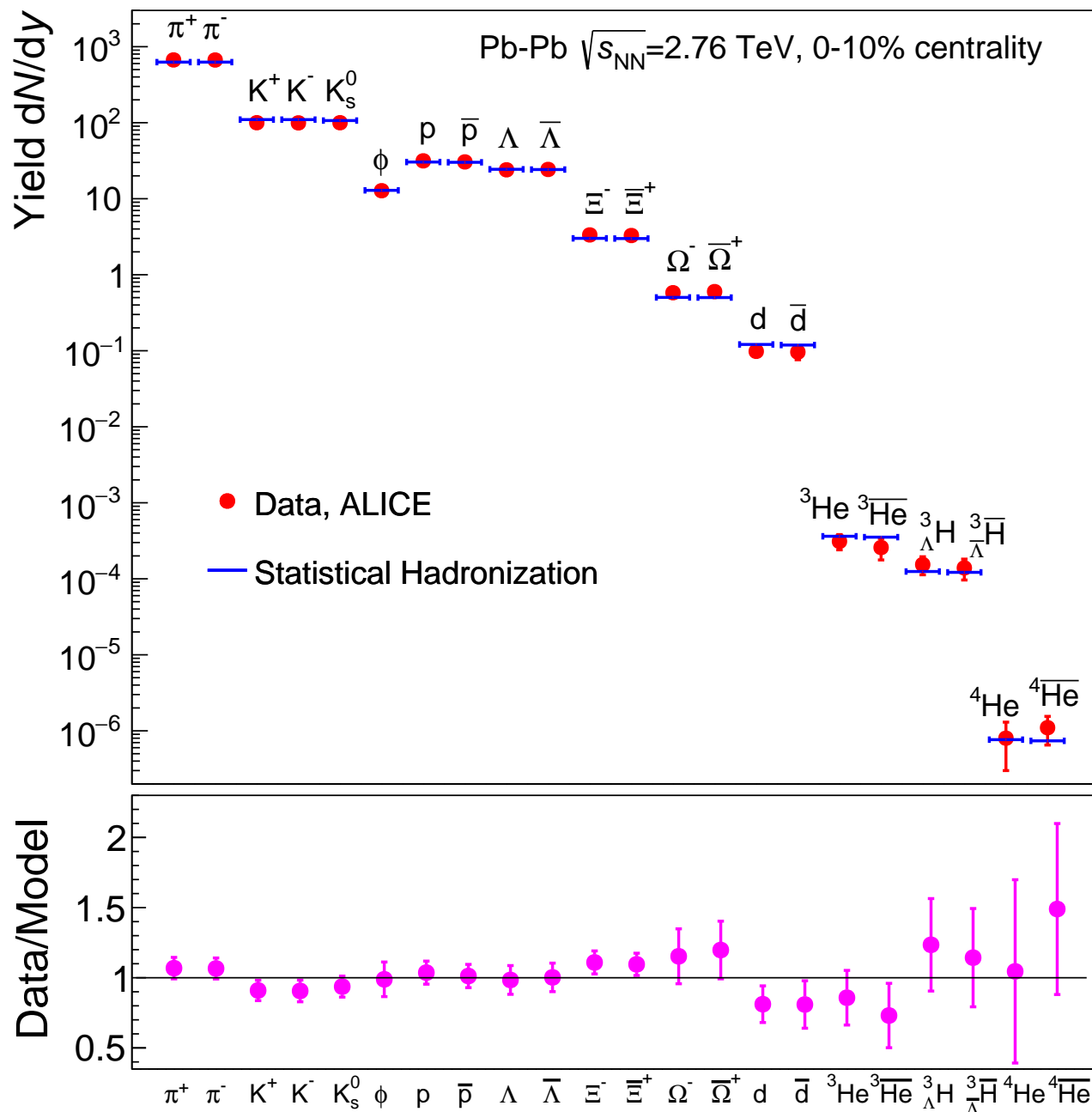
ii) strangeness:  $\sum_i n_i S_i = 0$

iii) charm:  $\sum_i n_i C_i = 0$ .

# Thermal fit – LHC, Pb–Pb, 0-10%

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matter and antimatter produced in equal amounts

$$T_{CF} = 156.6 \pm 1.7 \text{ MeV}$$

$$\mu_B = 0.7 \pm 3.8 \text{ MeV}$$

$$V_{\Delta y=1} = 4175 \pm 380 \text{ fm}^3$$

$$\chi^2/N_{df} = 16.7/19$$

*S-matrix treatment*

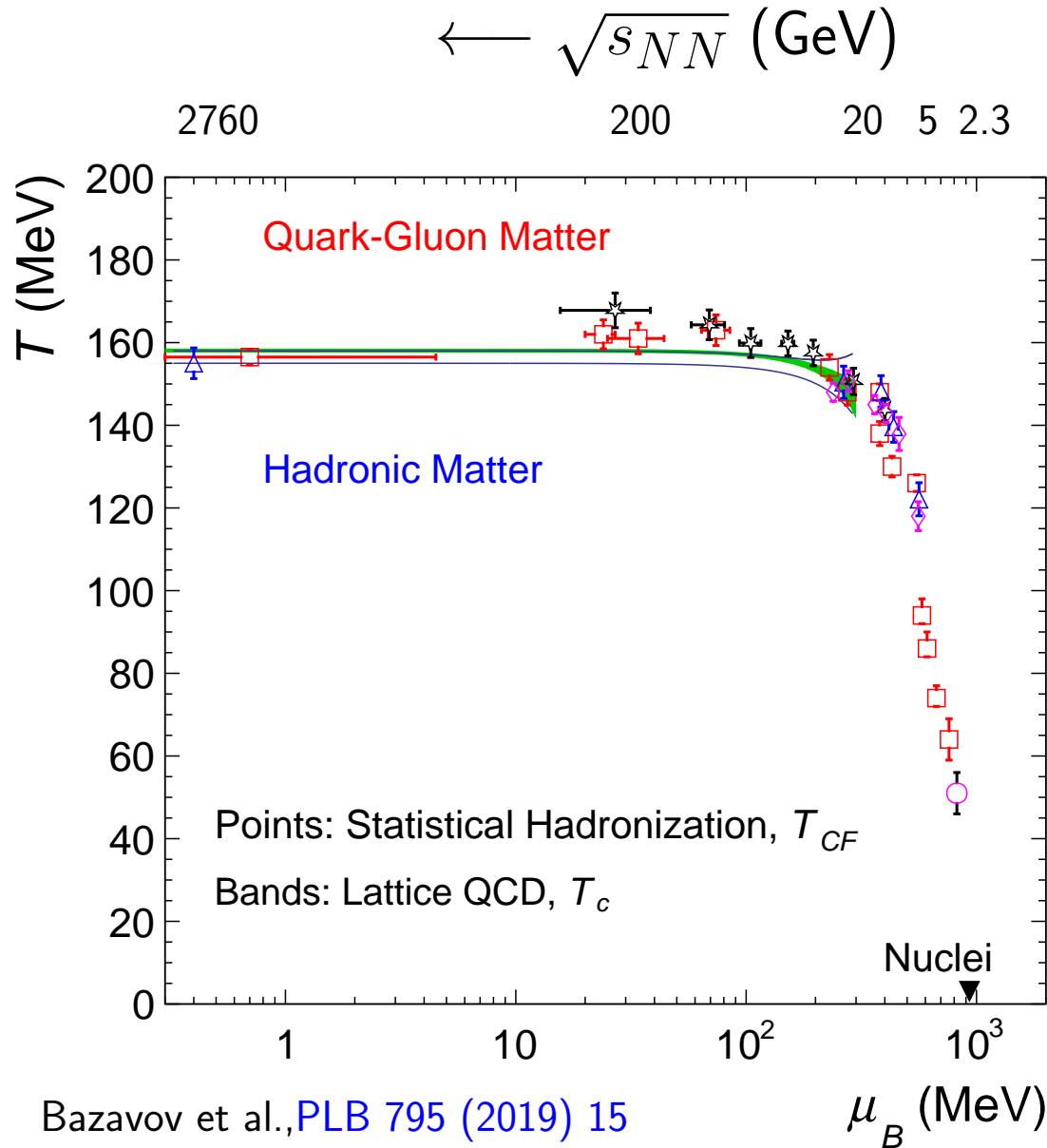
remarkably, loosely-bound objects are also well described ( ${}^3_{\Lambda}\text{H}$  with 25% B.R.)

hadronization as bags of quarks and gluons?

# The phase diagram of QCD

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at LHC, remarkable “coincidence” with Lattice QCD results

at LHC ( $\mu_B \simeq 0$ ): purely-produced (anti)matter ( $m = E/c^2$ ), as in the Early Universe

$\mu_B > 0$ : more matter, from “remnants” of the colliding nuclei

$\mu_B \gtrsim 400$  MeV: *the critical point awaiting discovery*  
(RHIC BES / FAIR)

Bazavov et al., [PLB 795 \(2019\) 15](#)

Borsanyi et al., [PLB 370 \(2014\) 99](#)

see refs. in [Nature 561 \(2018\) 321](#)

points: independent analyses of same data → “model/code uncert.” are small

# SHM for charm (SHMc)

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pQCD production, "throw in":  $N_{c\bar{c}} = 9.6 \rightarrow g_c = 30.1$  ( $I_1/I_0 = 0.974$ )

LHC, central collisions

assume:

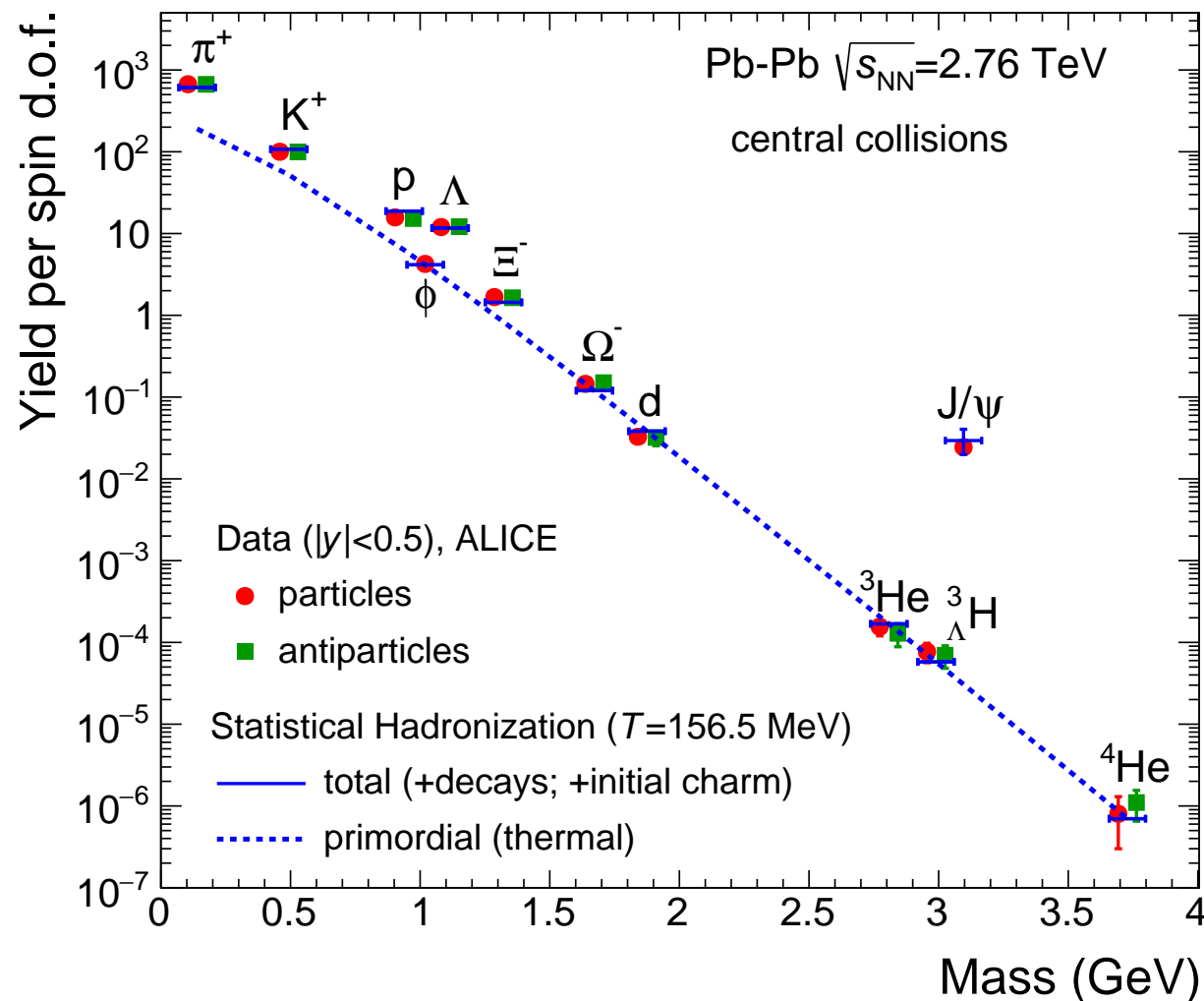
- full thermalization of  $c, \bar{c}$   
("mobility" in  $V \simeq 4000 \text{ fm}^3$ )

- full color screening  
(Matsui-Satz)

Braun-Munzinger, Stachel, [PLB 490 \(2000\) 196](#)

Model predicts all charm  
chemistry ( $\psi(2S), X(3872)$ )

$\pi, K^\pm, K^0$  from charm included in the thermal fit  
(0.7%, 2.9%, 3.1% for  $T=156.5 \text{ MeV}$ )



[PLB 797 \(2019\) 134836](#)

# SHMc: method and inputs

Braun-Munzinger, Stachel, [PLB 490 \(2000\) 196](#), [NPA 690 \(2001\) 119](#)

- Thermal model calculation (grand canonical)  $T, \mu_B$ :  $\rightarrow n_X^{th}$
- $N_{c\bar{c}}^{dir} = \frac{1}{2}g_c V (\sum_i n_{D_i}^{th} + n_{\Lambda_i}^{th}) + g_c^2 V (\sum_i n_{\psi_i}^{th} + n_{\chi_i}^{th})$
- $N_{c\bar{c}} \ll 1 \rightarrow$  Canonical (Cleymans, Redlich, Suhonen, Z. Phys. C51 (1991) 137):

Gorenstein, Kostyuk, Stöcker, Greiner, [PLB 509 \(2001\) 277](#)

$$N_{c\bar{c}}^{dir} = \frac{1}{2}g_c N_{oc}^{th} \frac{I_1(g_c N_{oc}^{th})}{I_0(g_c N_{oc}^{th})} + g_c^2 N_{c\bar{c}}^{th} \rightarrow g_c(N_{part}) \text{ (charm fugacity)}$$

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$$\text{Outcome: } N_D = g_c V n_D^{th} I_1/I_0 \quad N_{J/\psi} = g_c^2 V n_{J/\psi}^{th}$$

$$\text{Inputs: } T, \mu_B, \quad V_{\Delta y=1} (= (dN_{ch}^{exp}/dy)/n_{ch}^{th}), \quad N_{c\bar{c}}^{dir} \text{ (exp. or pQCD)}$$

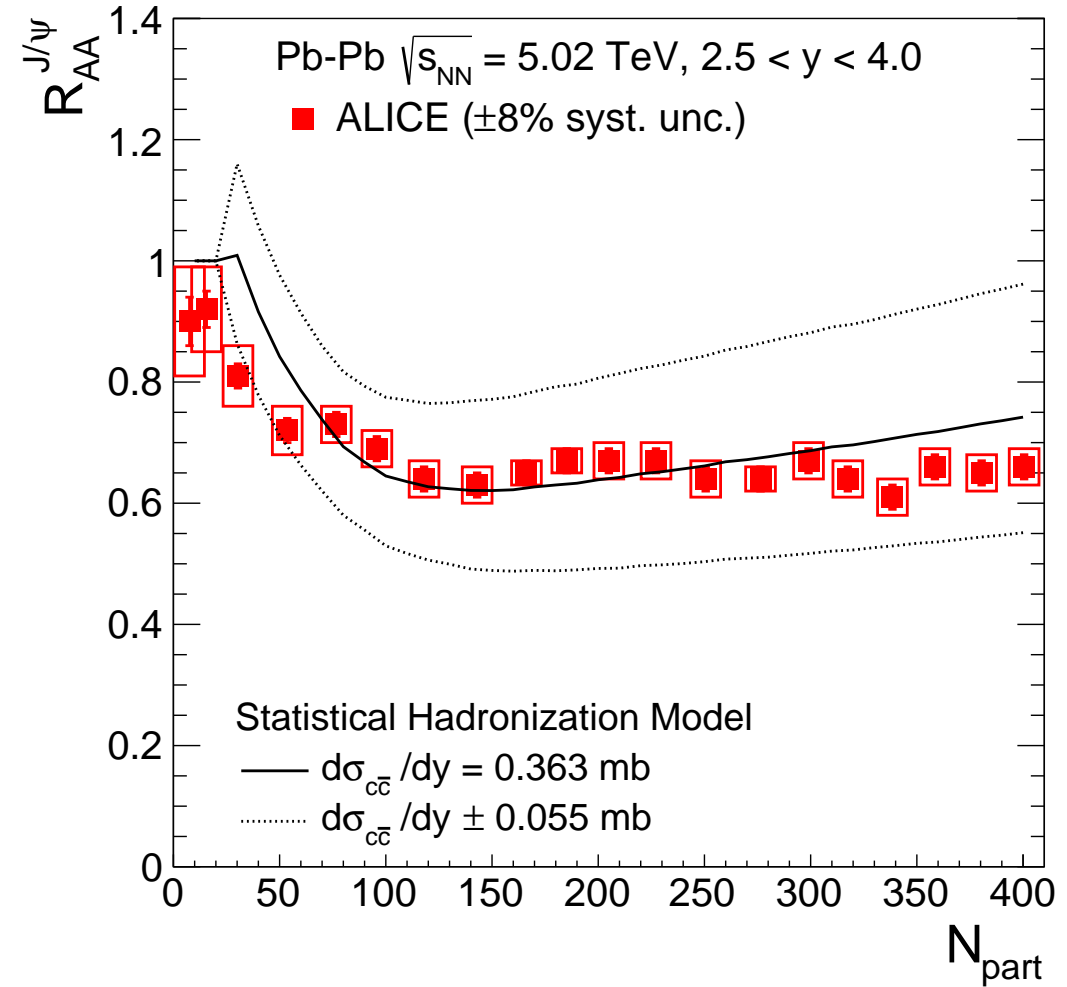
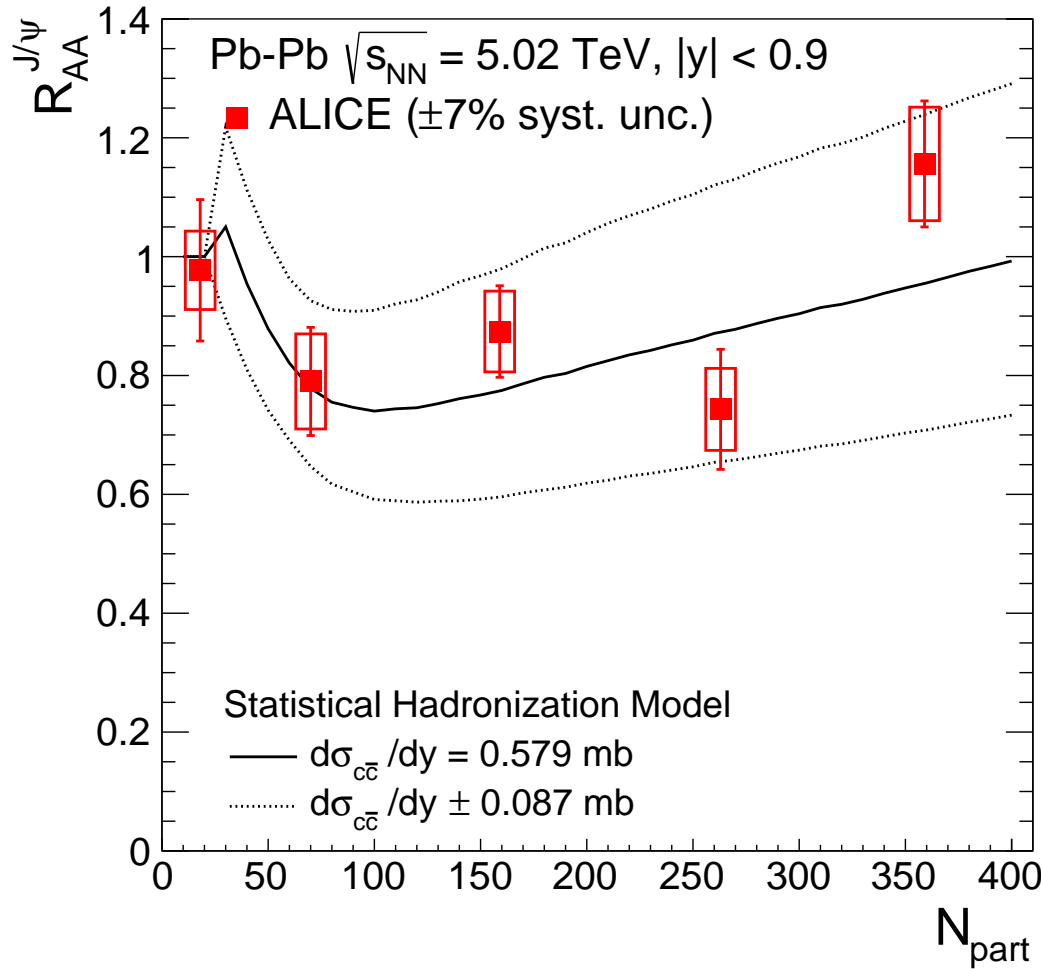
Assumed minimal volume for QGP:  $V_{QGP}^{min} = 200 \text{ fm}^3$

# SHMc and charmonium data at the LHC

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*full thermalization of  $c$  quarks in QGP, hadronization at chemical freeze-out*



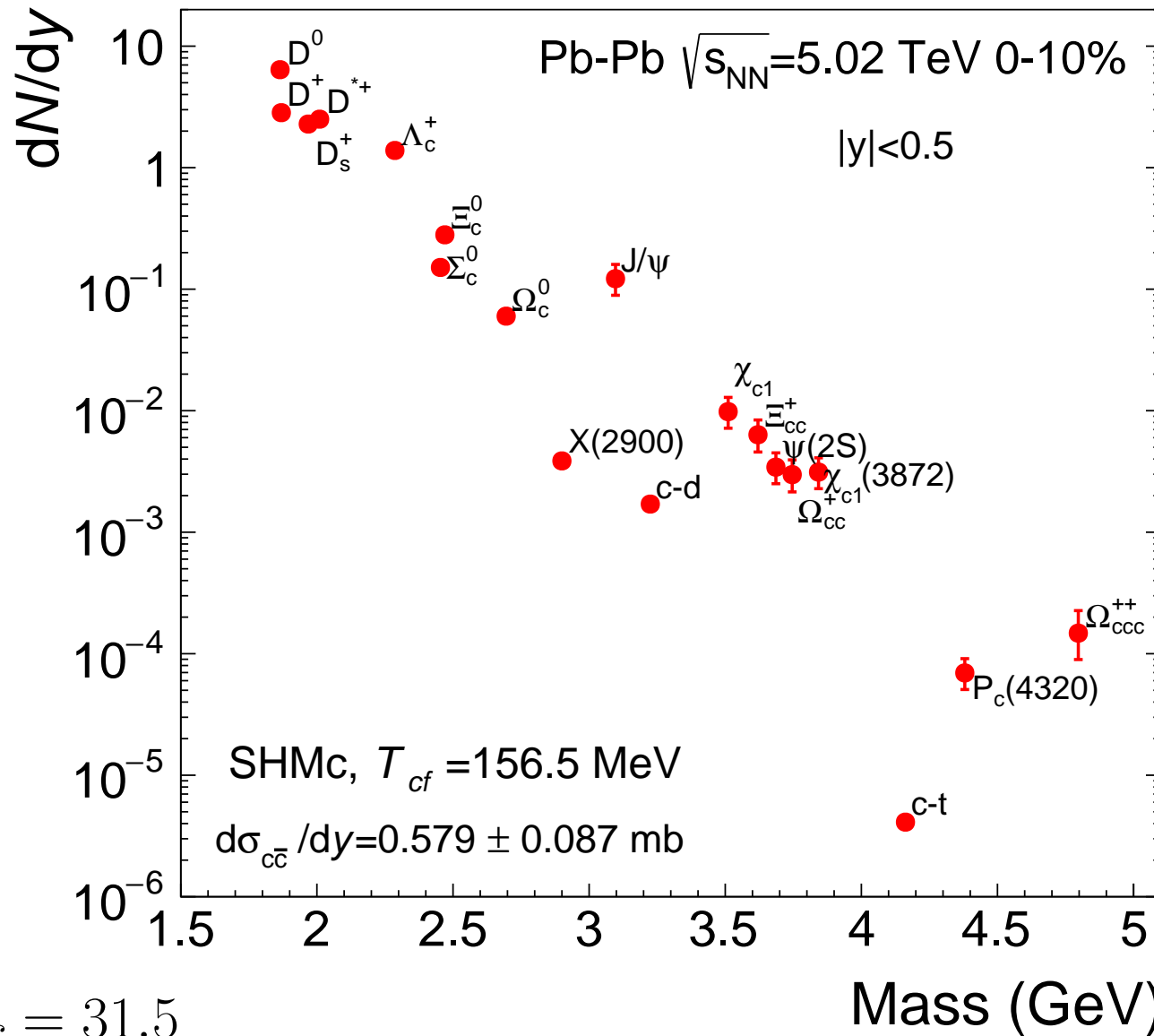
$d\sigma_{c\bar{c}}/dy$  via normalization to  $D^0$  in Pb-Pb 0-10%, ALICE, [arXiv:2110.09420](https://arxiv.org/abs/2110.09420)

$dN/dy = 6.82 \pm 1.03$  ( $|y| < 0.5$ ; FONLL for  $y=2.5-4$ ; assuming hadronization fractions in data as in SHMc)

# SHMc: the full charm zoo

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$$\frac{dN_{c\bar{c}}}{dy} = 13.8$$

$$\rightarrow g_c = 31.5$$

$$T_{cc}^+ \simeq 0.9 \cdot \chi_{c1}(3872)$$

$$X(6900) \sim 10^{-8}$$

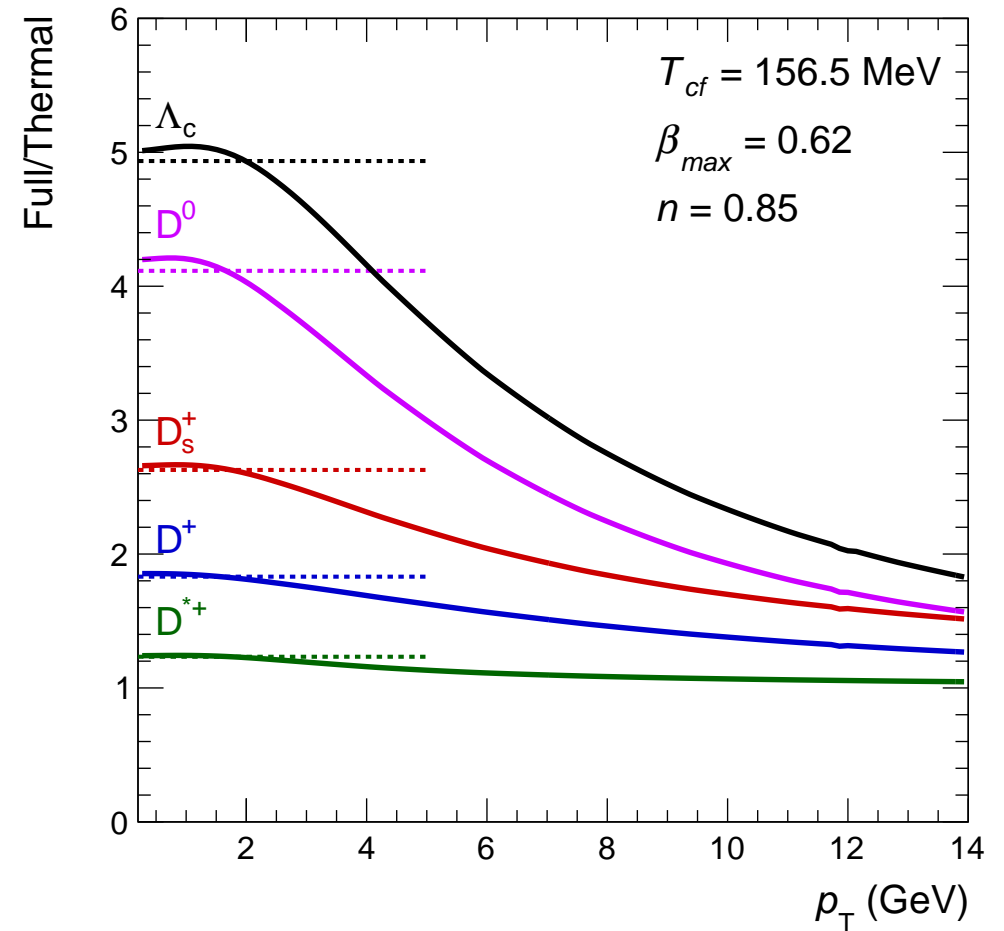
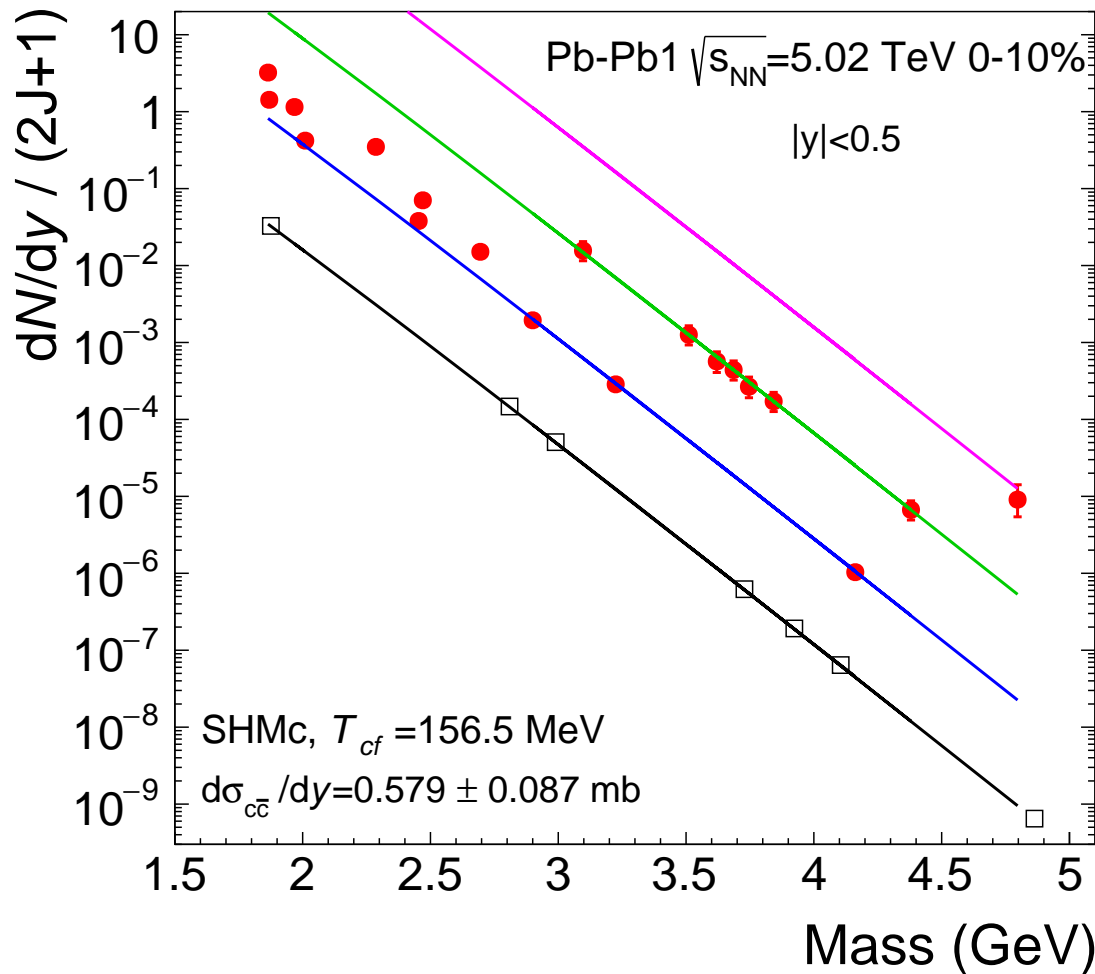
The power of the model: predicting the full suite of charmed hadrons



# Full charm predictions for the LHC

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Charm-hadron spectrum as in PDG: 55 c-mesons, 74 c-baryons (part.+antipart.)

...large, but may not be complete

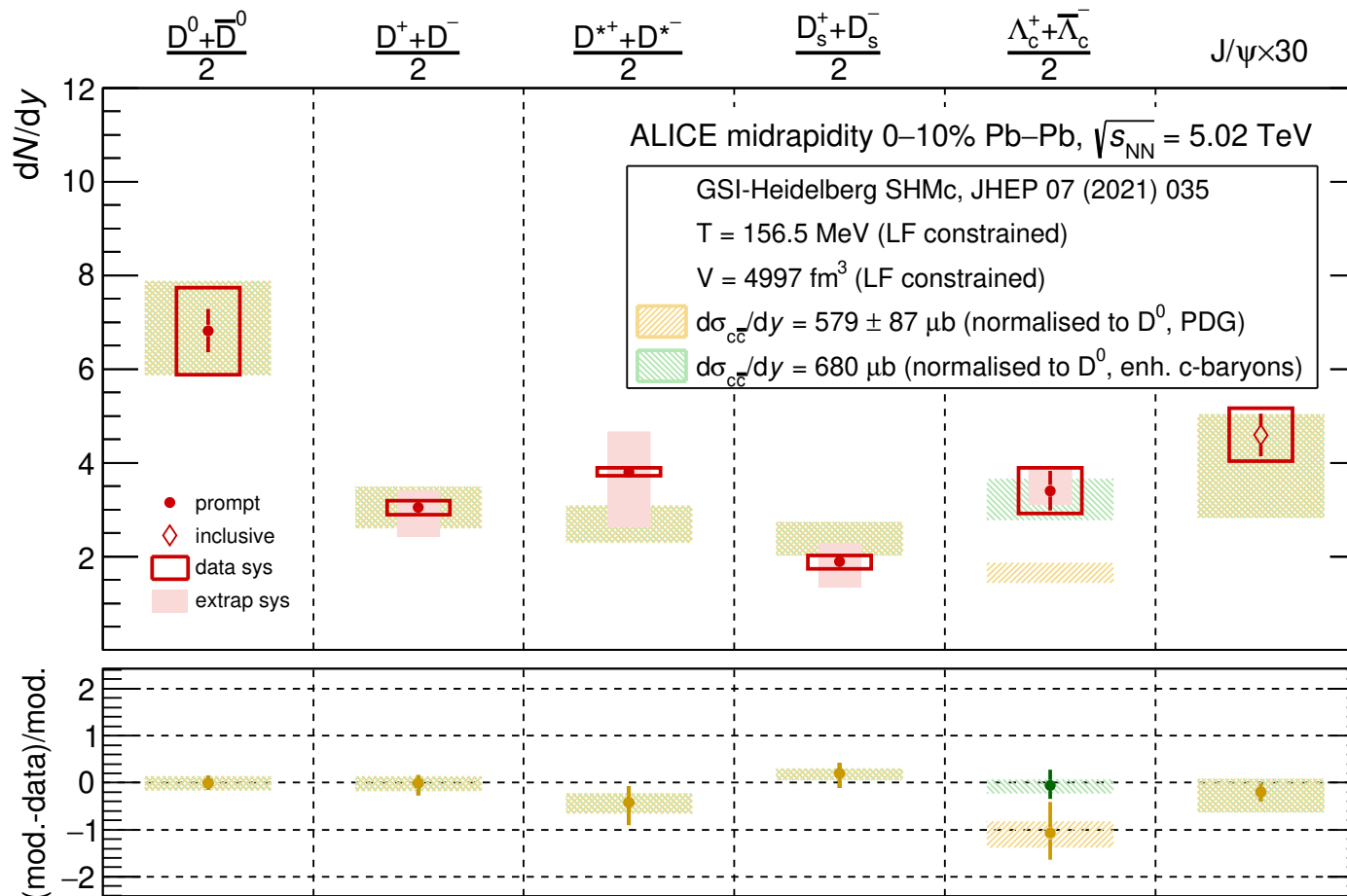
# Modified charm-hadron spectrum

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...ad-hoc: *tripled* the excited charm-baryon states, enhanced  $d\sigma_{c\bar{c}}/dy$  by 19%

RQM: He,Rapp, [PLB 795 \(2019\) 117](#); LQCD, Bazavov et al., [PLB 737 \(2014\) 210](#)



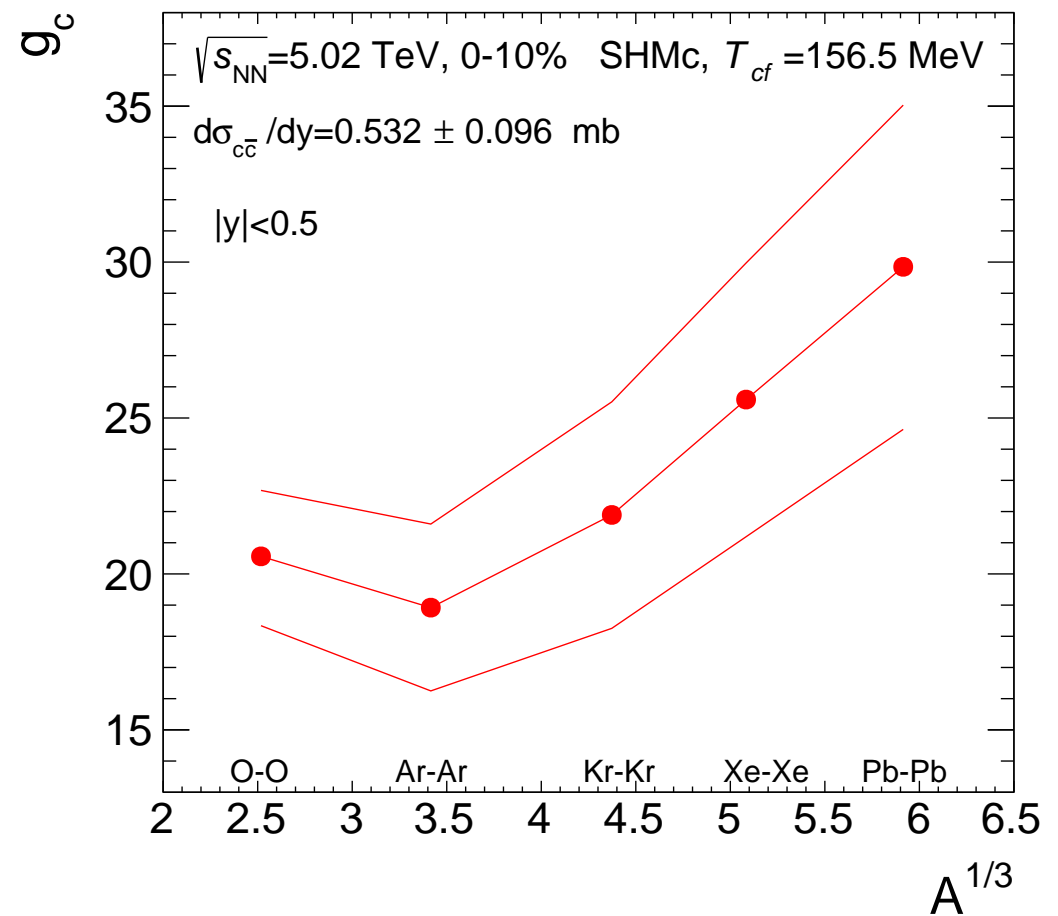
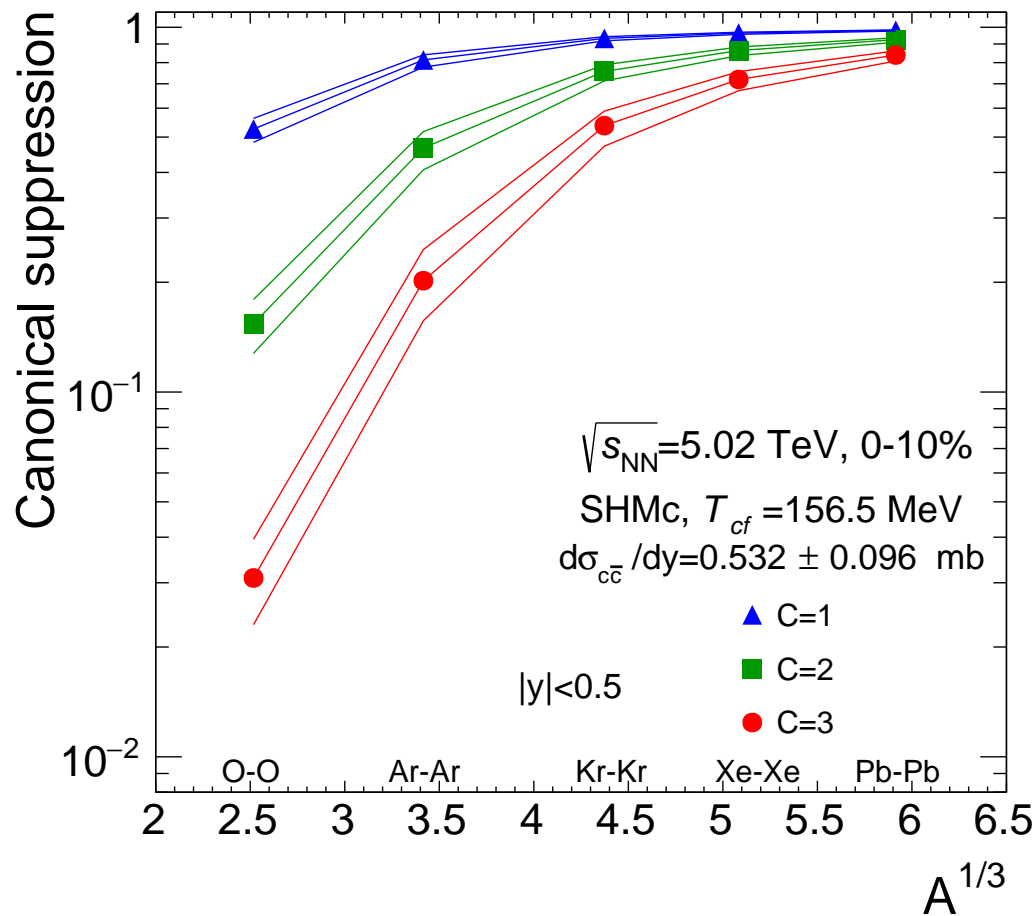
ALICE (QM'22 talk, L. Vermunt)

leaves the mesonic sector unaffected, for the commensurately larger  $\sigma_{c\bar{c}}$

# SHMc: system dependence (central, 0-10%)

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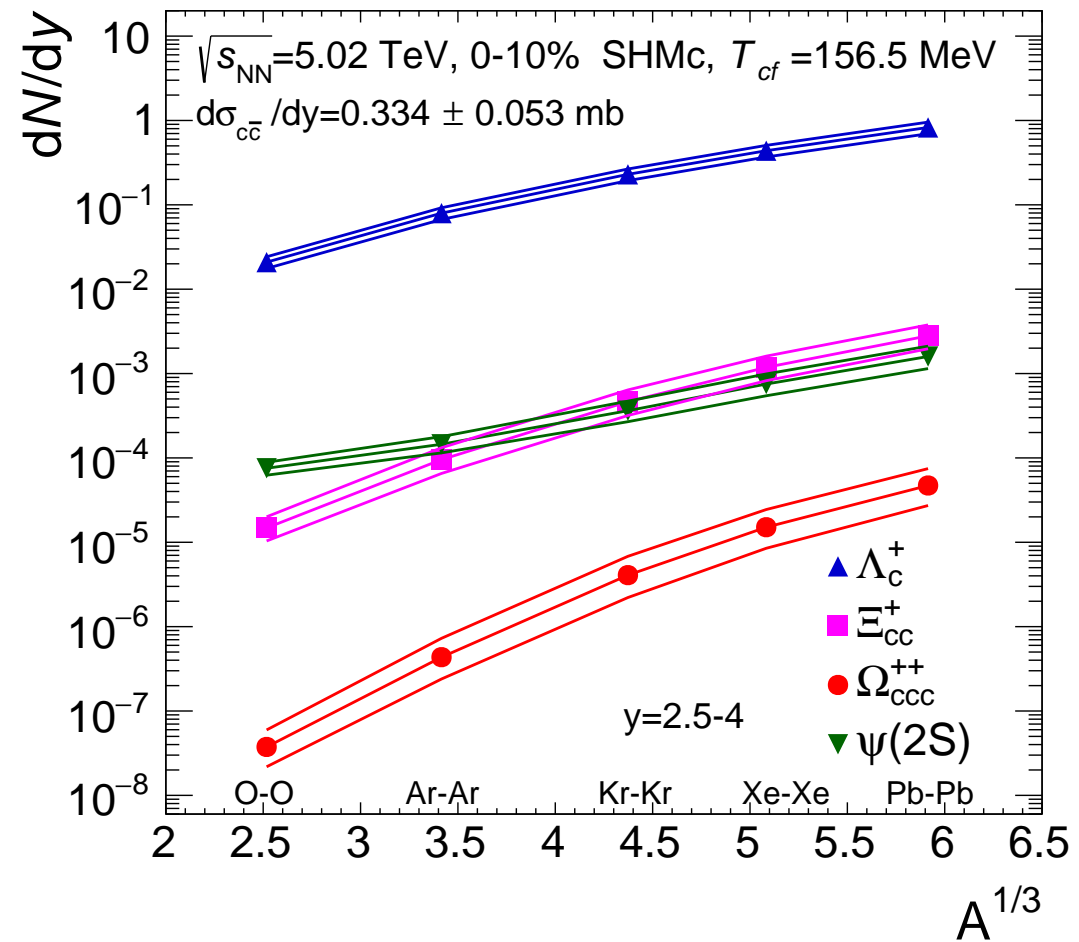
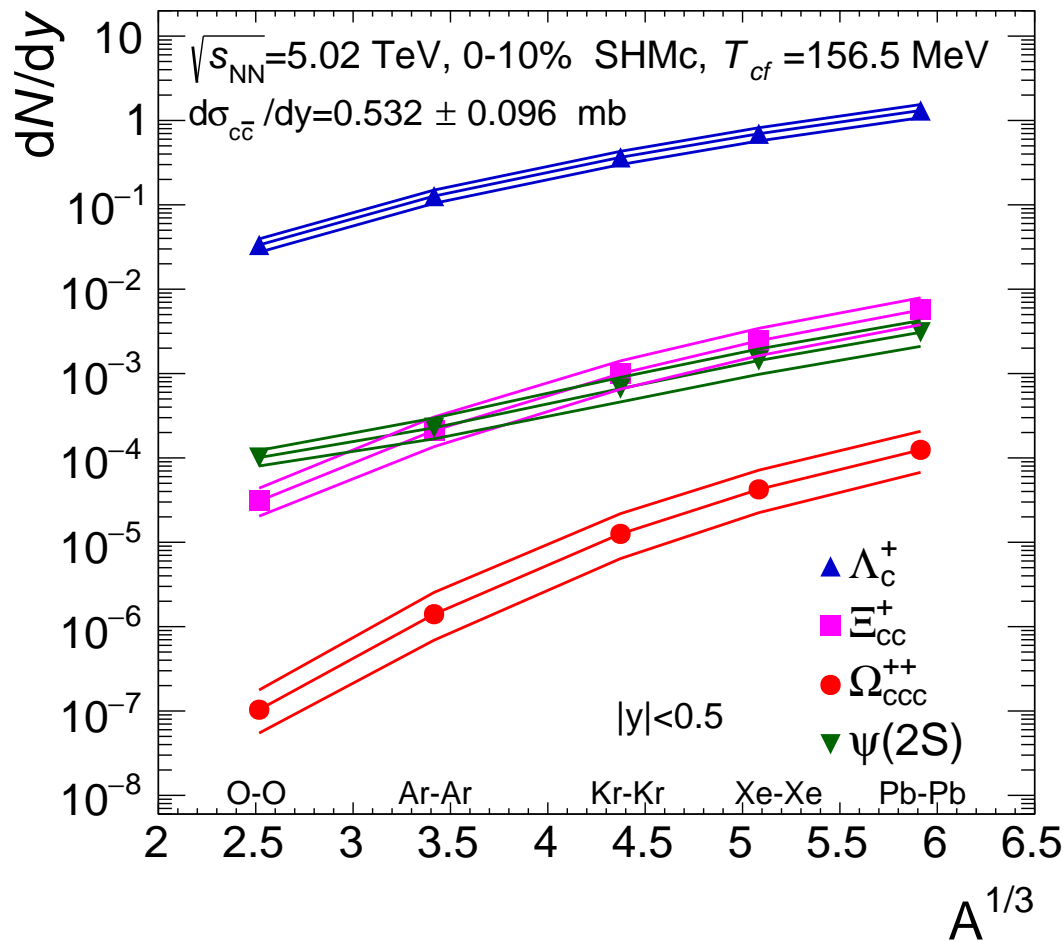


Strong canonical suppression for light systems (for multi-charm hadrons)

# SHMc: system dependence (central, 0-10%)

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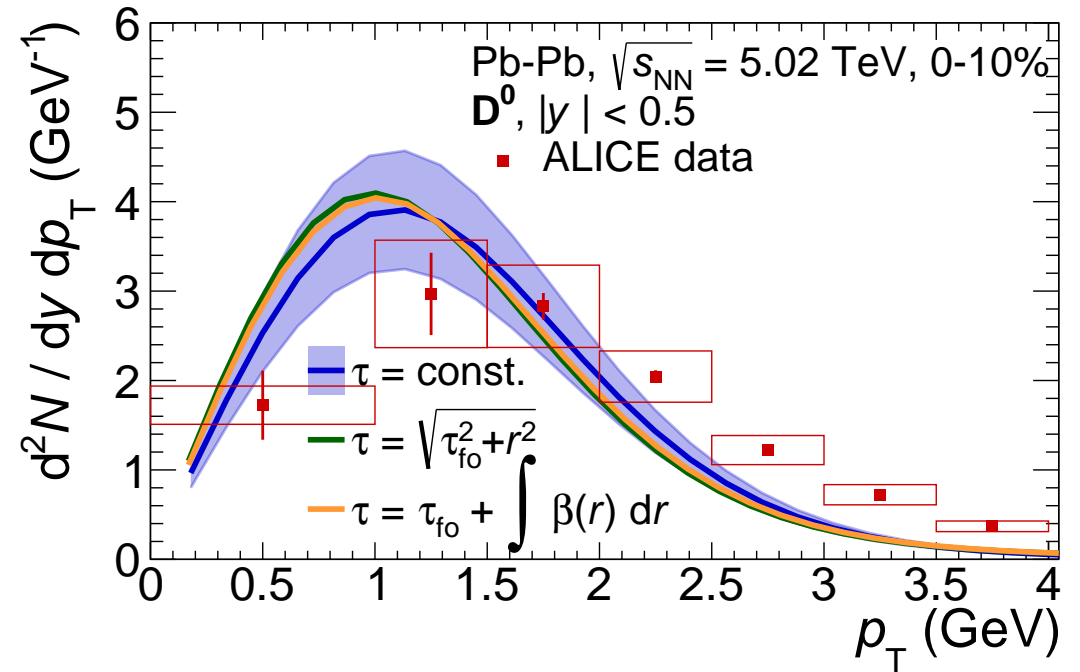
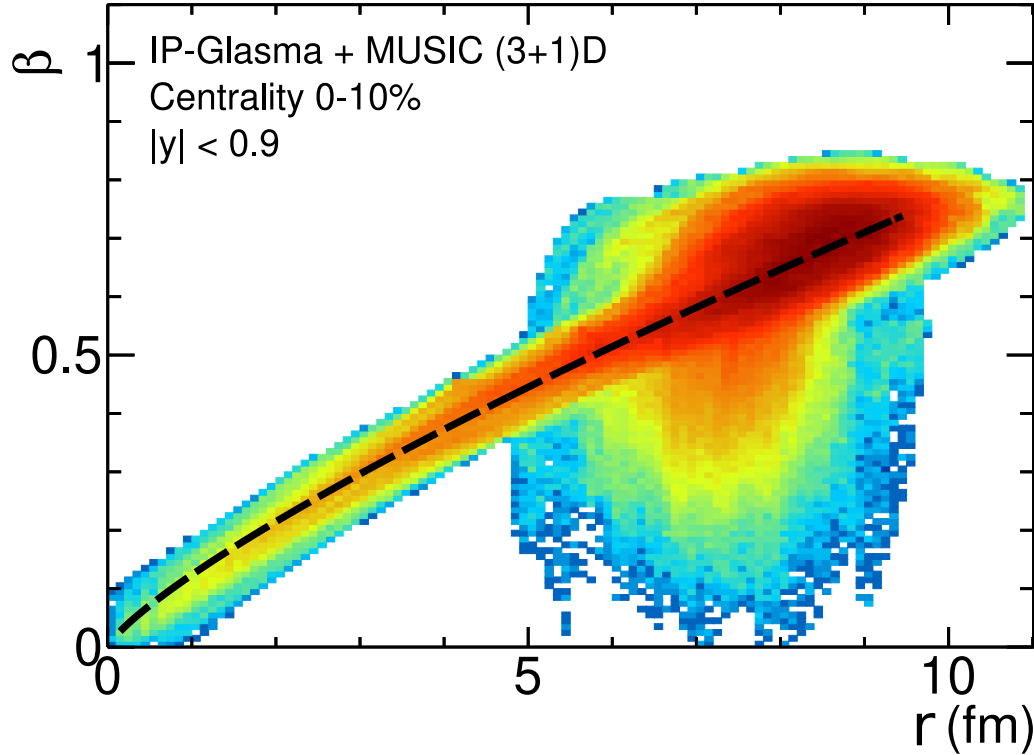


# SHMc: $p_T$ dependence

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full hydrodynamic flow (MUSIC(3+1)D, IP-Glasma; parametrized via blast-wave



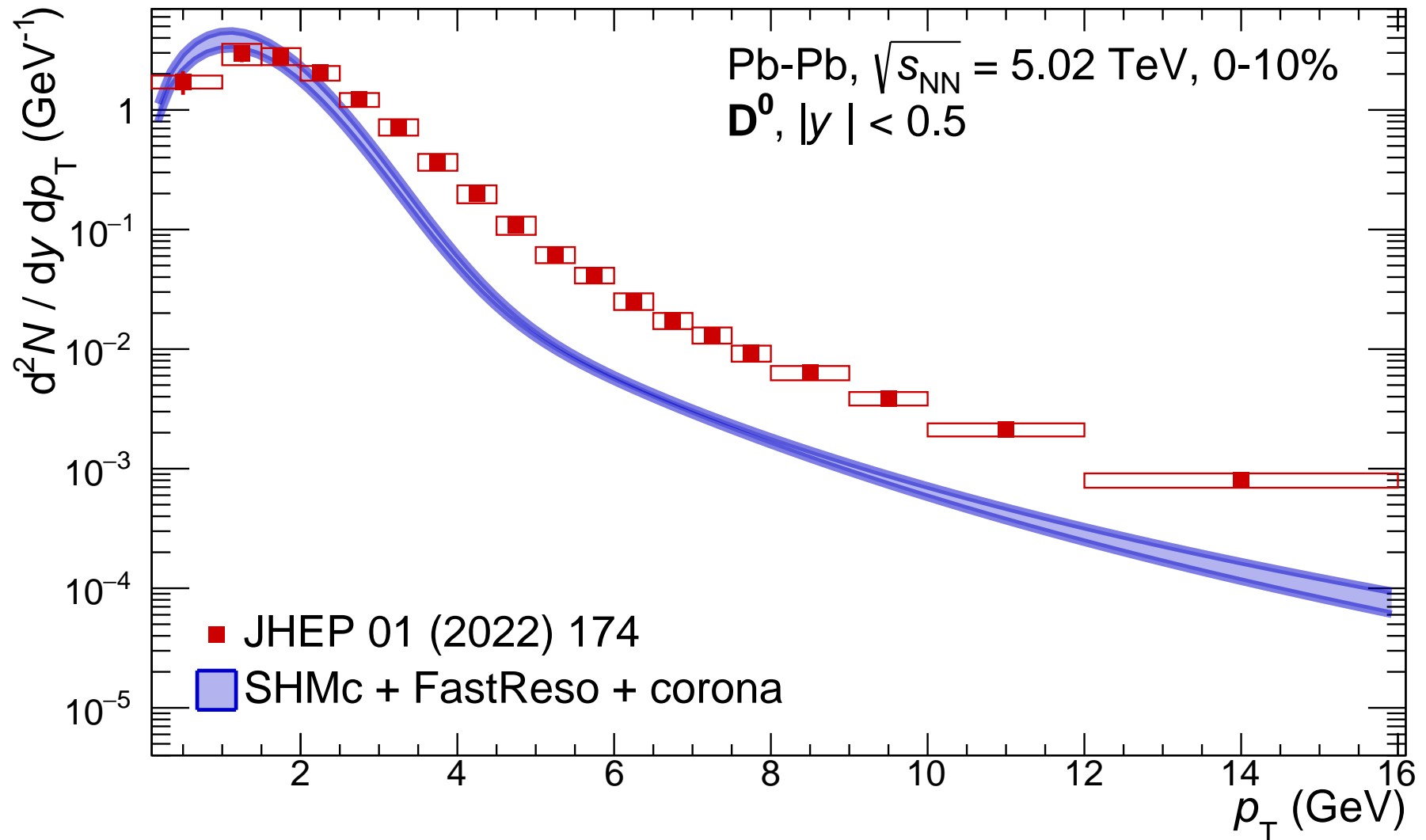
$$\frac{d^2N}{2\pi p_T dp_T dy} = \frac{2J+1}{(2\pi)^3} \int d\sigma_\mu p^\mu f(p) = \frac{2J+1}{(2\pi)^3} \int_0^{r_m} dr \tau(r) r \left[ K_1^{\text{eq}} - \frac{\partial \tau}{\partial r} K_2^{\text{eq}} \right]$$

$$K_1^{\text{eq}}(p_T, u^r) = 4\pi m_T I_0 \left( \frac{p_T u^r}{T} \right) K_1 \left( \frac{m_T u^r}{T} \right), \quad K_2^{\text{eq}}(p_T, u^r) = 4\pi p_T I_1 \left( \frac{p_T u^r}{T} \right) K_0 \left( \frac{m_T u^r}{T} \right)$$

# SHMc: $p_T$ dependence

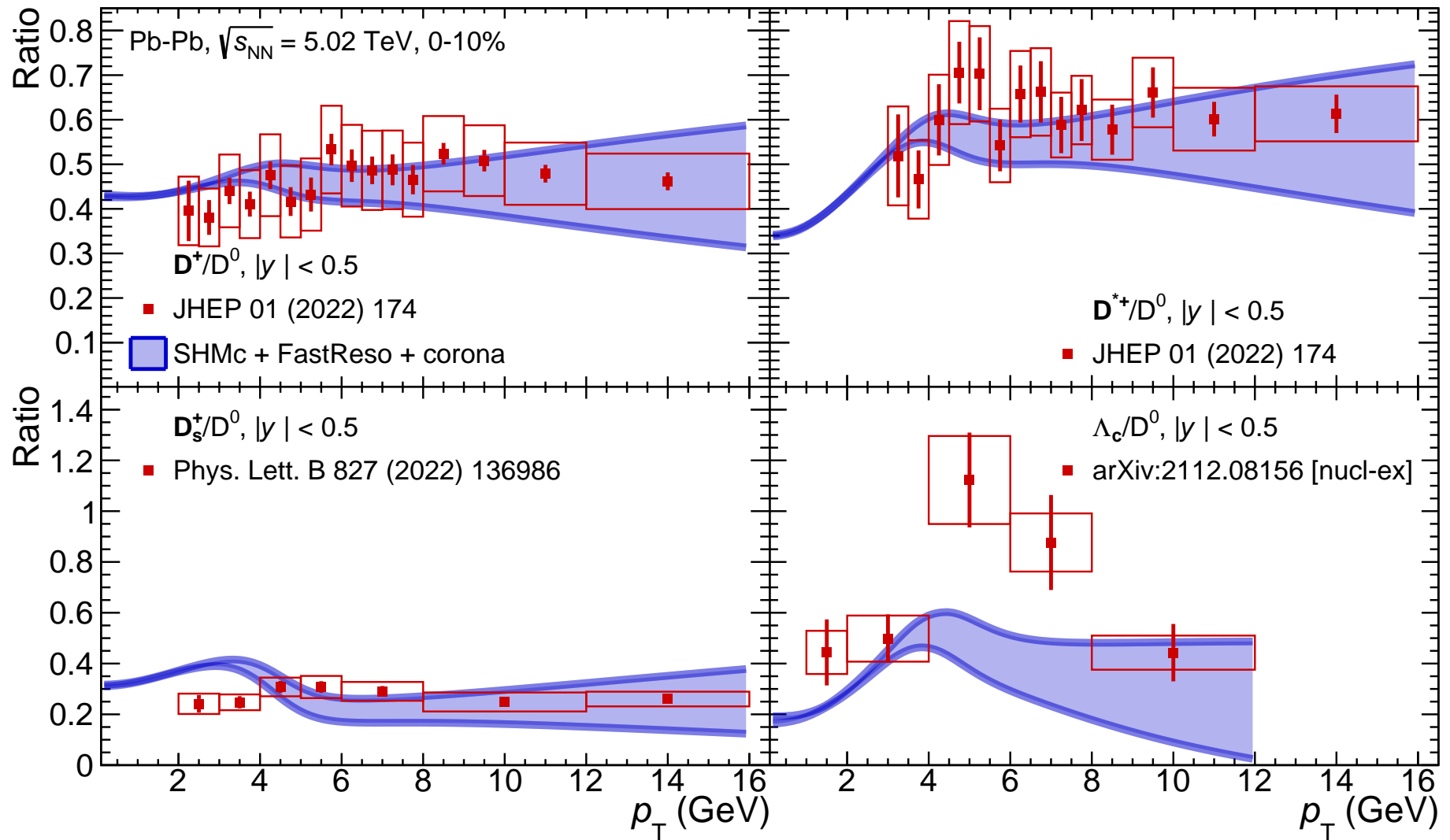
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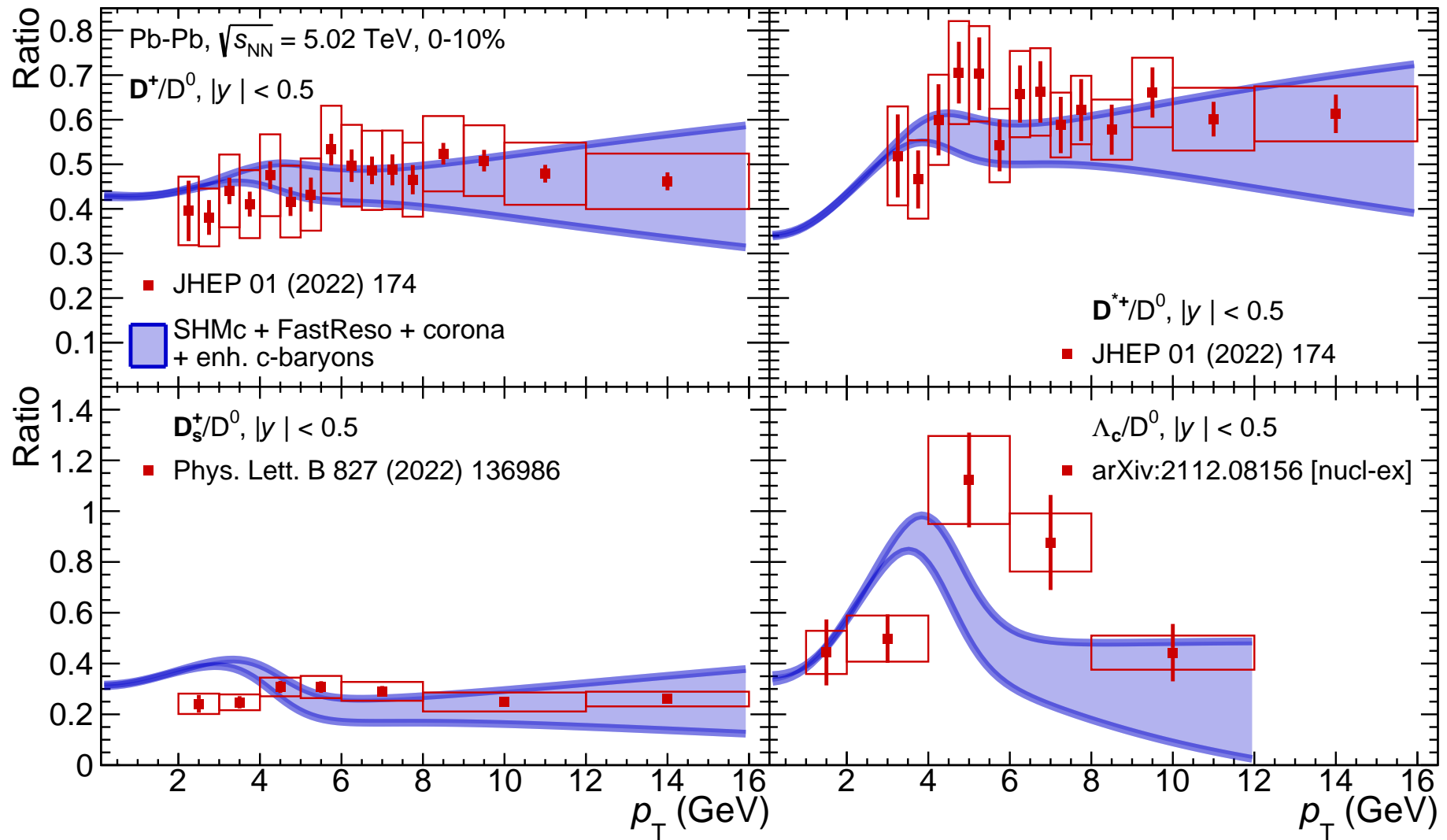
**SHMc:** low  $p_T$ ; high  $p_T$ : only nuclear-corona contribution (incl. uncert.)

# Ratios to $D^0$



Charm-hadron spectrum: PDG

# Ratios to $D^0$



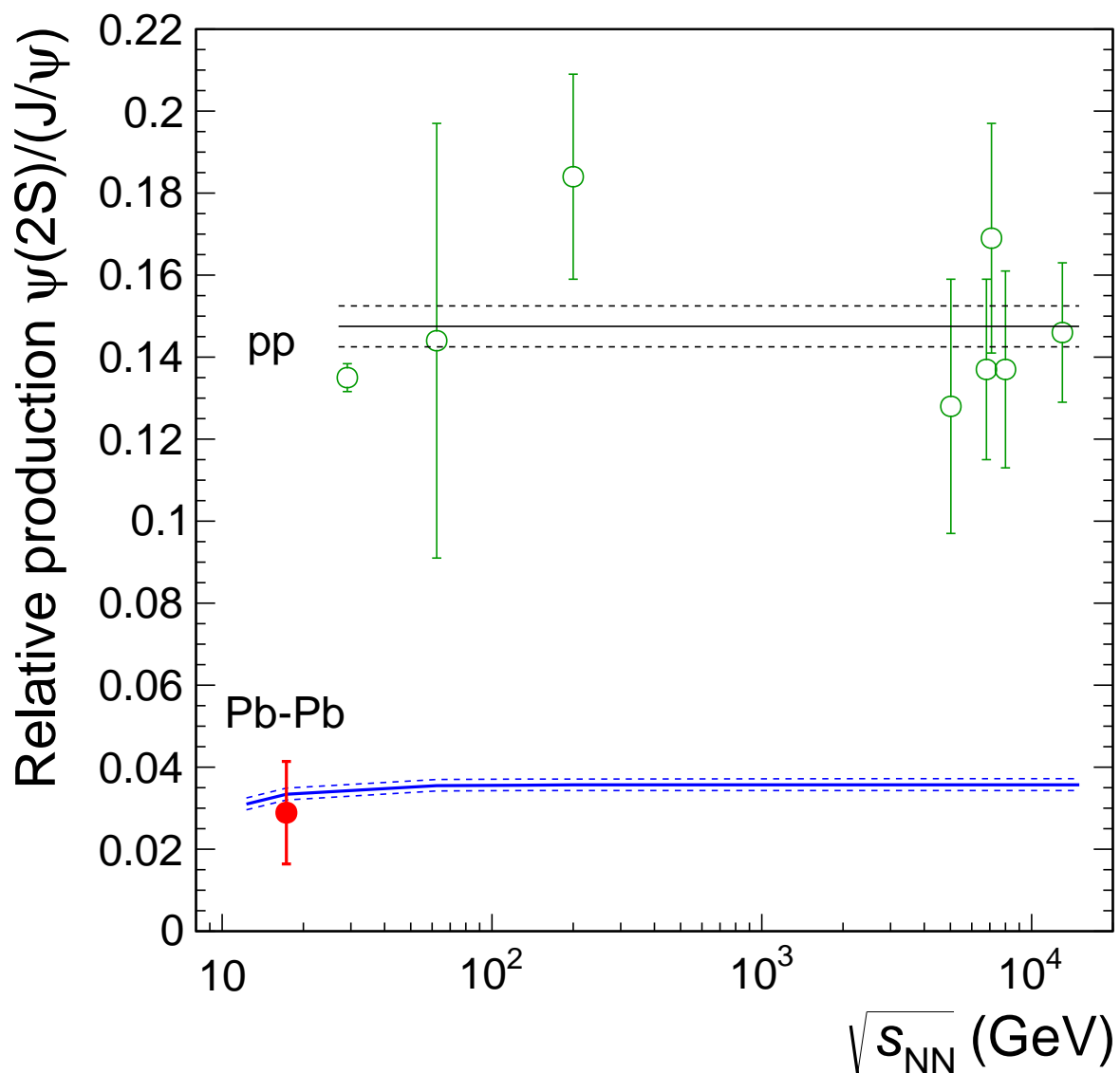
Charm-hadron spectrum: enhanced c-baryons (tripled excited states)



# IF charm thermalizes (fully) also at lower energies

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...SHMc is easy to be extended to lower energies

AA, Braun-Munzinger, Redlich, Stachel,  
[PLB 659 \(2008\) 149](#)

..litmus test:  $\psi(2S)$  ( $+v_2$ ,  $R_{AA}$ )

SHMc works (was proposed) at SPS

Braun-Munzinger, Stachel, [PLB 490 \(2000\) 196](#)

for D, stat. hadronization is a simpler act, may be at work in pp and in  $e^+e^-$

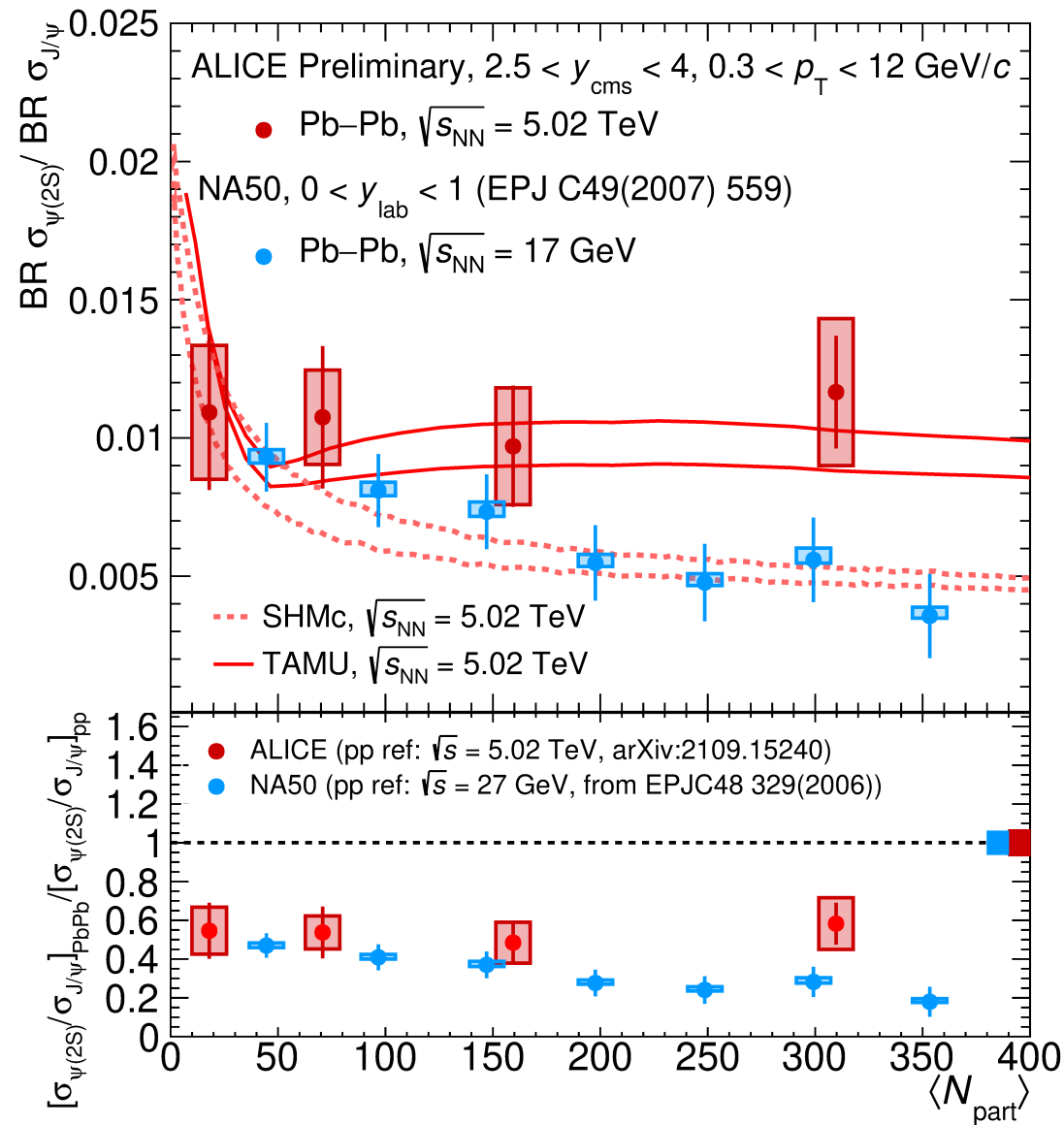
[PLB 678 \(2009\) 350](#)

The measurement in Pb-Pb at LHC is a central goal for Run 3,4 ([YR, WG5 HL-LHC](#))

# $\psi(2S)/J/\psi$ at the LHC

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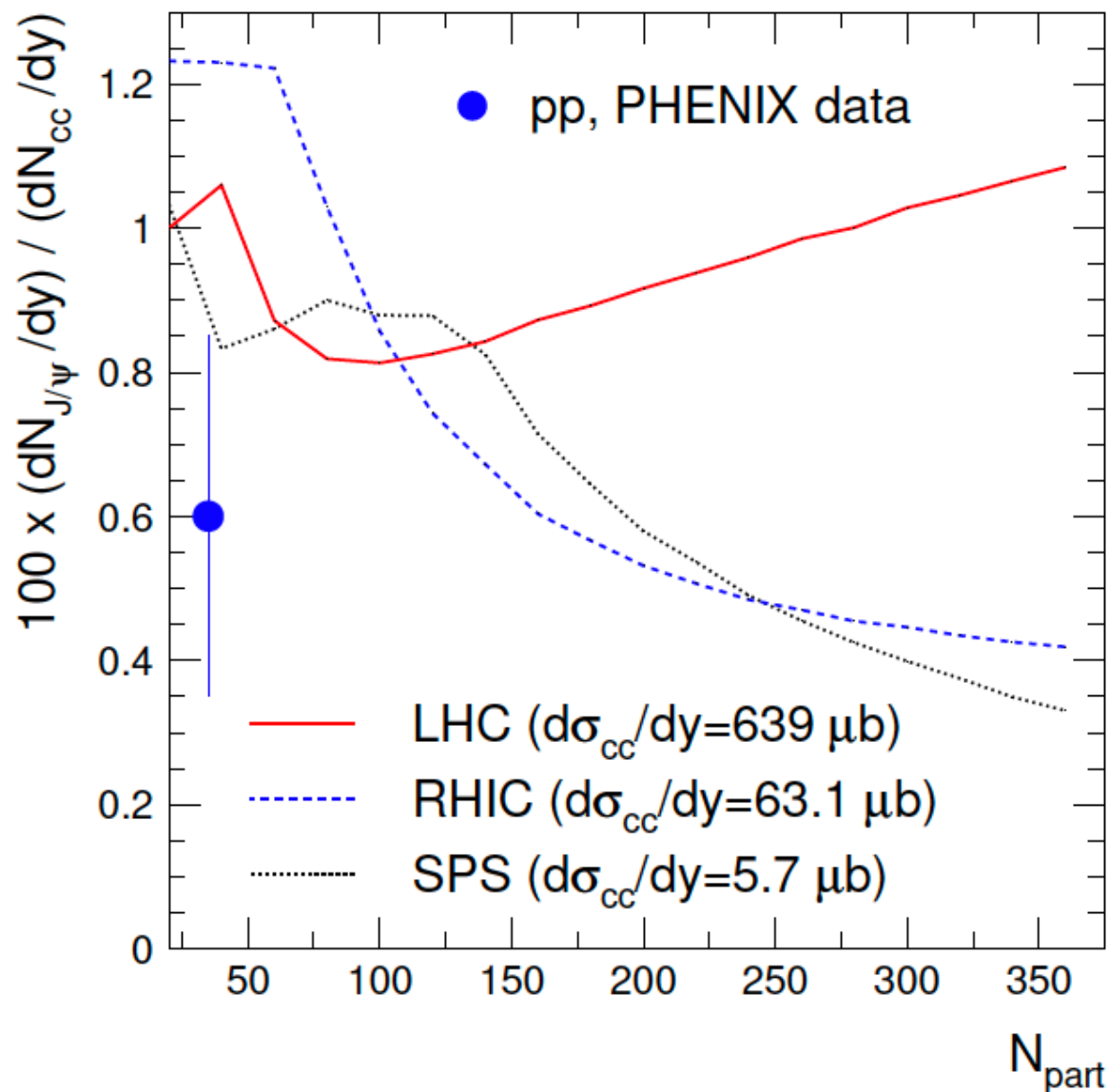
In SHMc uncertainty only due to nuclear-corona  
 ( $\sigma_{c\bar{c}}$  cancels out completely)

# $J/\psi$ production relative to charm

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...an observable with similar features as  $R_{AA}$



NPA 789 (2007) 334

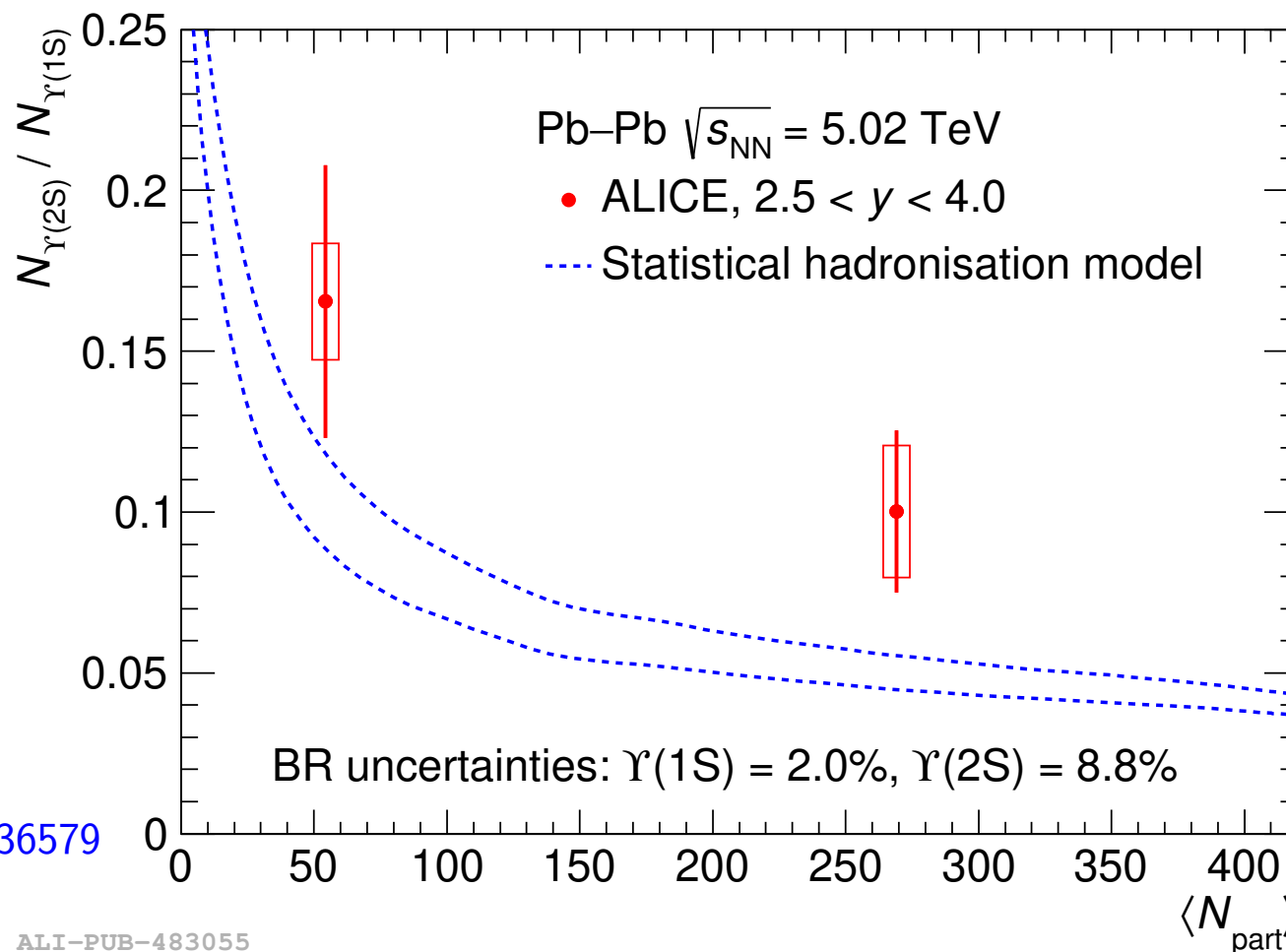
(see also: Satz, *Adv.HEP* 2013 (2013) 242918)

- similar values at RHIC and SPS  
...with differences in fine details  
...determined by canonical suppression of open charm  
same features in RHIC BES data
- enhancement-like at LHC  
can. suppr. lifted, quadratic term dominant

# ...and on to beauty (at the LHC)

Beauty is expected ( $R_{AA}$ ,  $v_2$  data) to be less thermalized compared to charm

The beauty-hadron spectrum is less well known (PDG: 48 b-mes, 46 b-bar total)



ALICE,

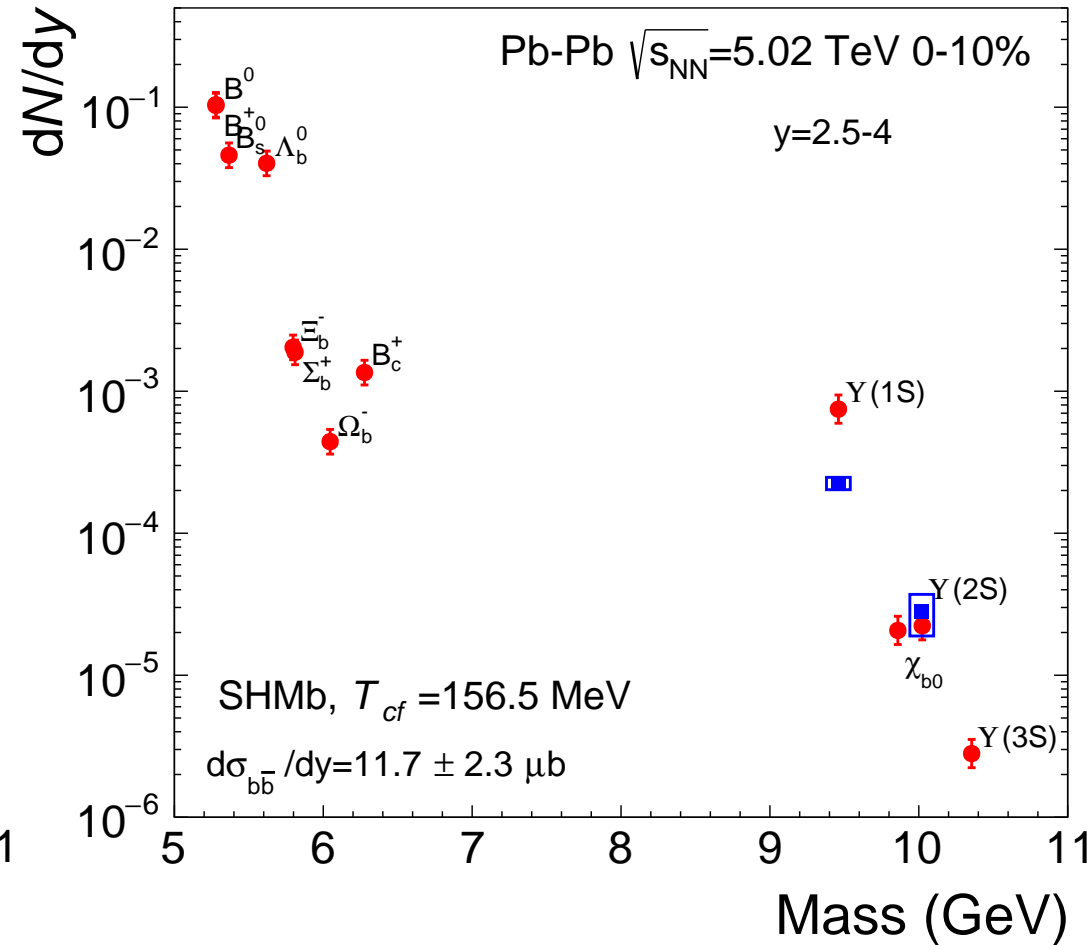
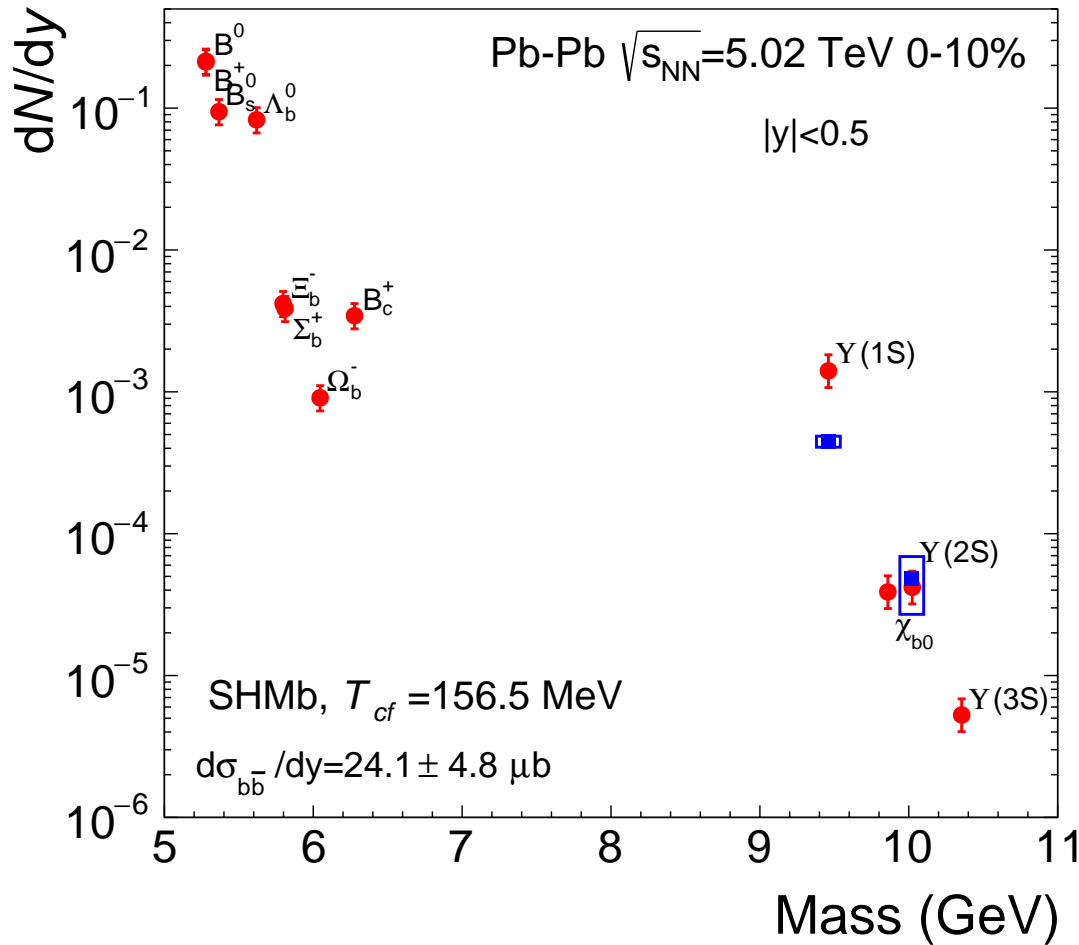
PLB 822 (2021) 136579

ALI-PUB-483055

Color screening may not destroy all  $\Upsilon$  mesons in QGP

Uncertainty band determined by nuclear-corona

# The limiting case: full beauty thermalization



$$g_b = 1.05 \cdot 10^9 \quad \left( \frac{dN_{bb^-}}{dy} = 0.57 \right)$$

$$B_c : 3.44 \cdot 10^{-3}$$

$$g_b = 0.86 \cdot 10^9 \quad \left( \frac{dN_{bb^-}}{dy} = 0.28 \right)$$

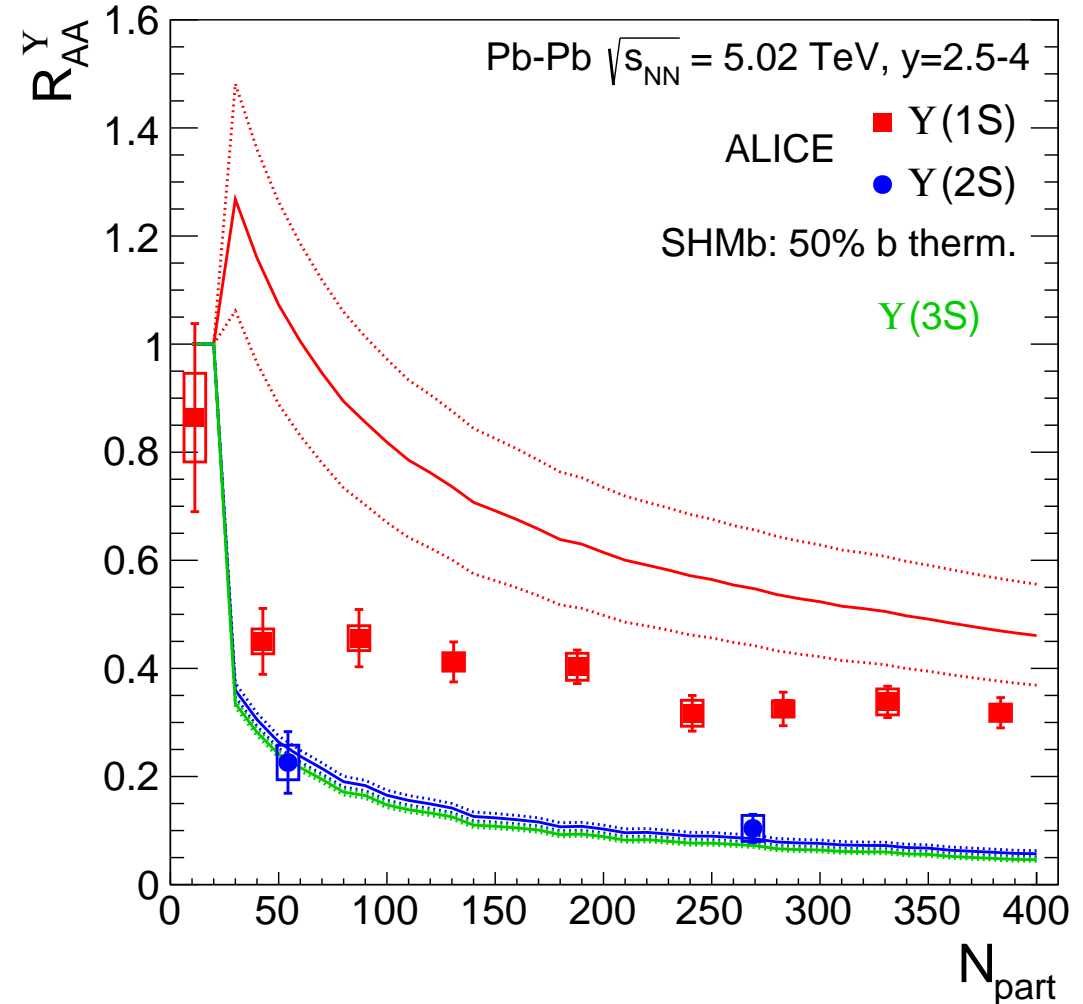
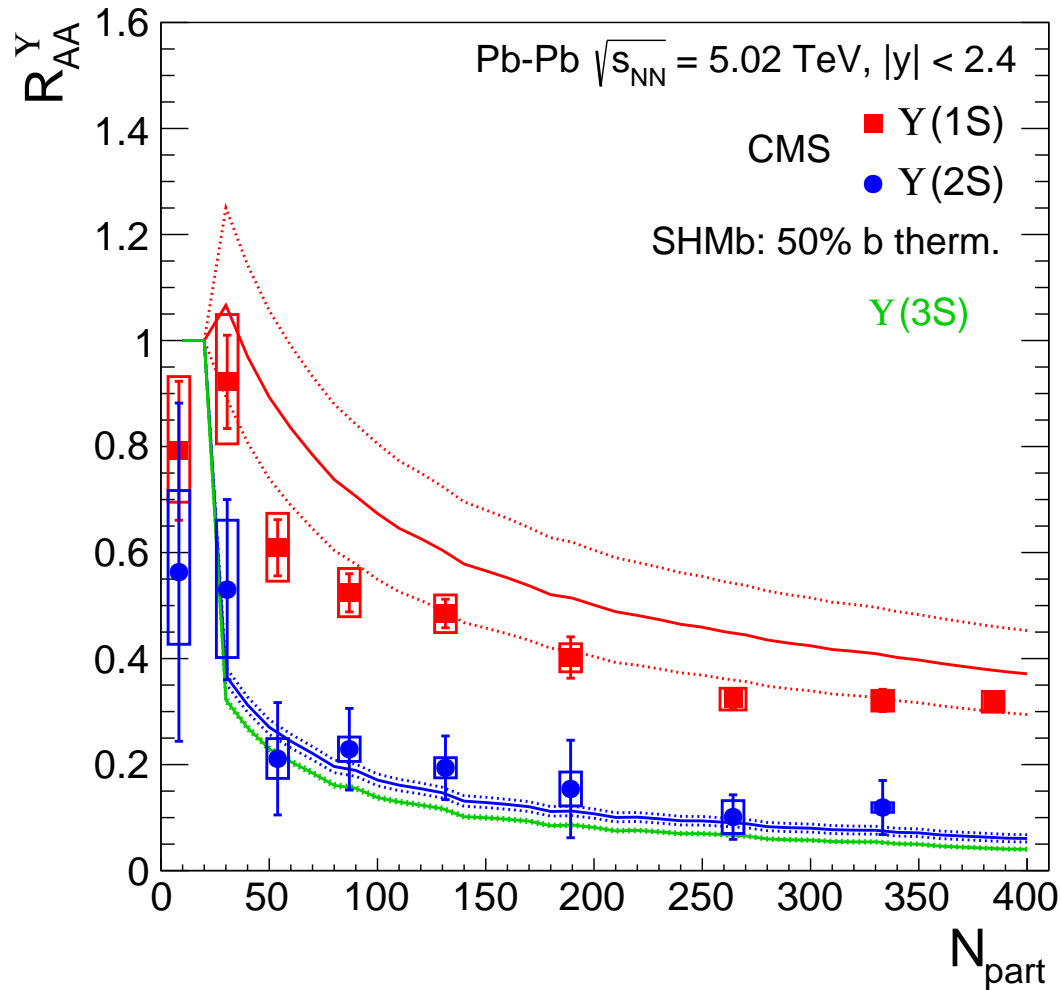
$$B_c : 1.36 \cdot 10^{-3}$$

Blue:  $\Upsilon$  data (CMS, ALICE): calc. based on  $R_{AA}$  and pp (would be nice to include in publications  $dN/dy$ )

# $R_{AA}$ , 50% $b\bar{b}$ thermalized

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CMS, PRL 120 (2018) 142301

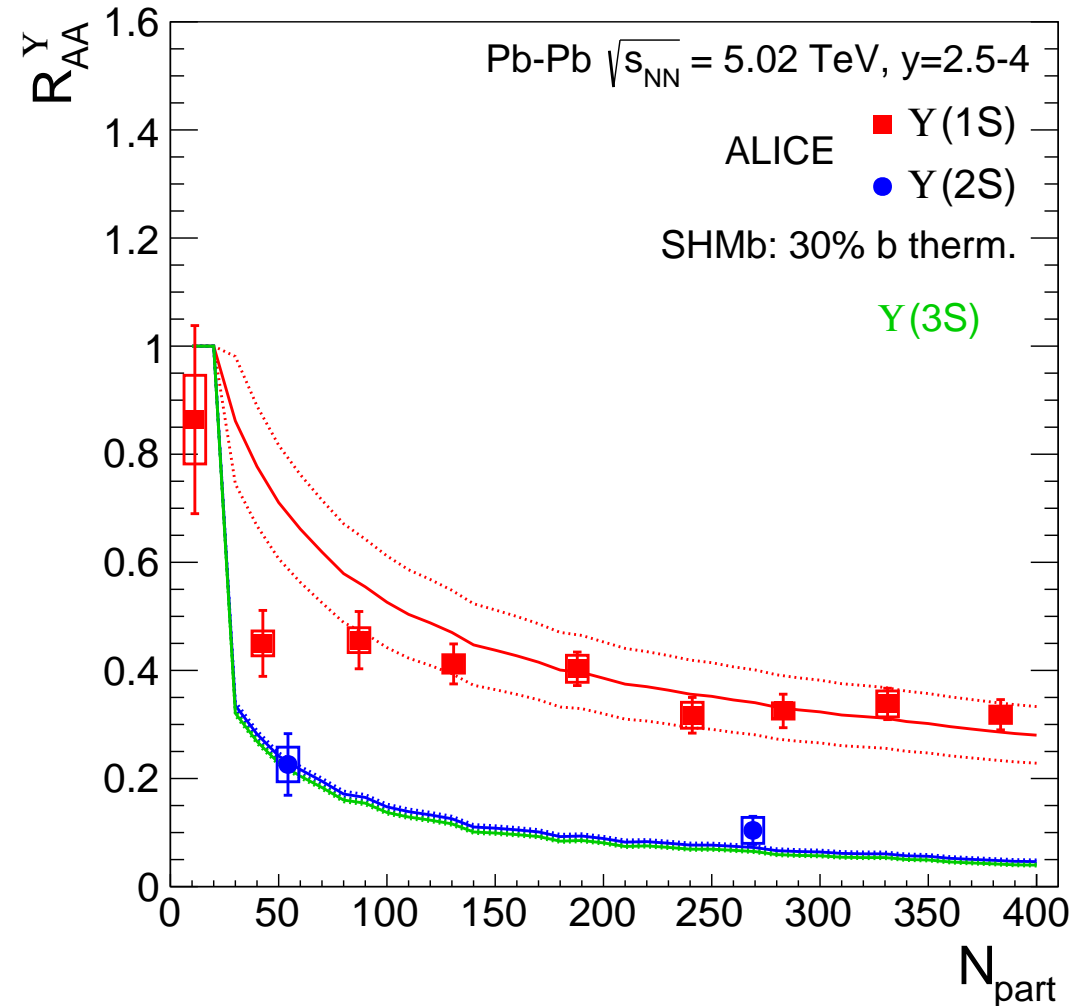
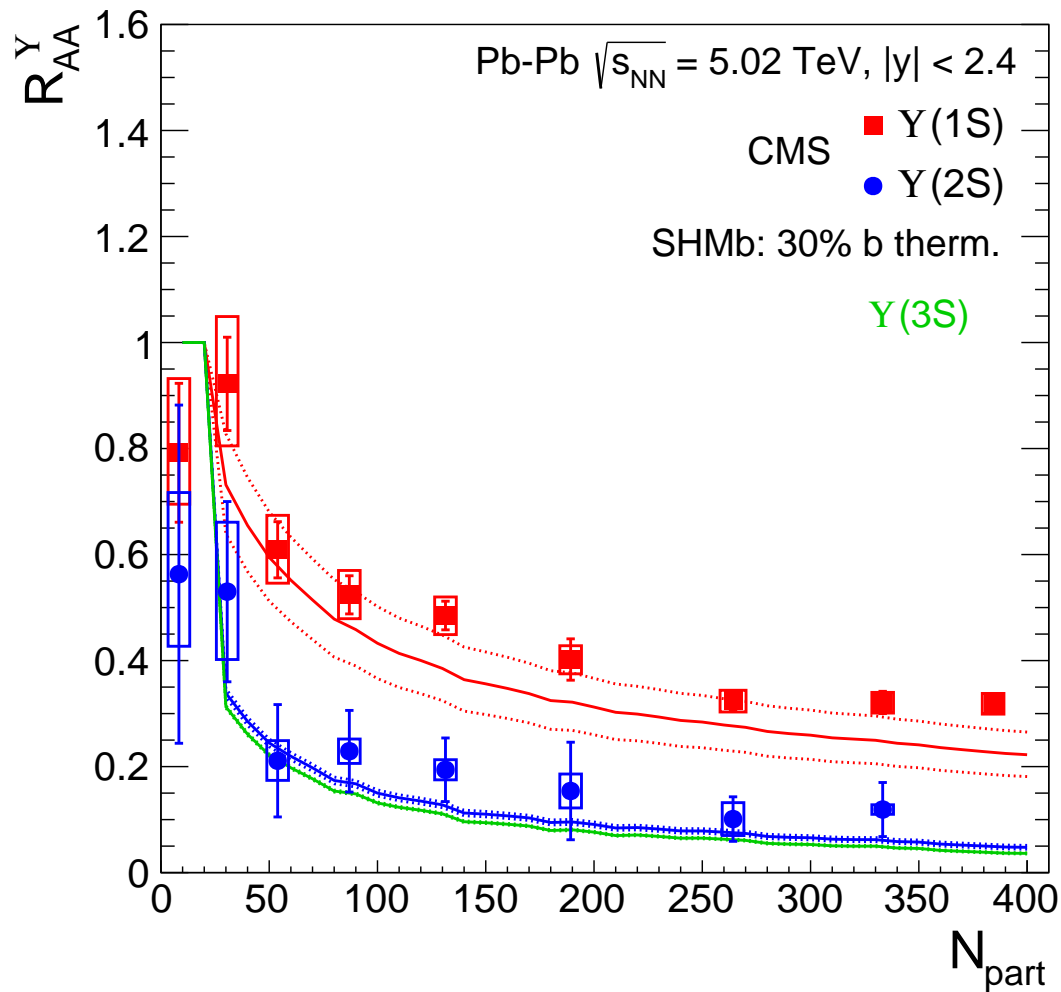
ALICE, PLB 822 (2021) 136579

*What does non-thermalized beauty produce? (no room for it in SHMb)*

# $R_{AA}$ , 30% $b\bar{b}$ thermalized

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CMS, PRL 120 (2018) 142301

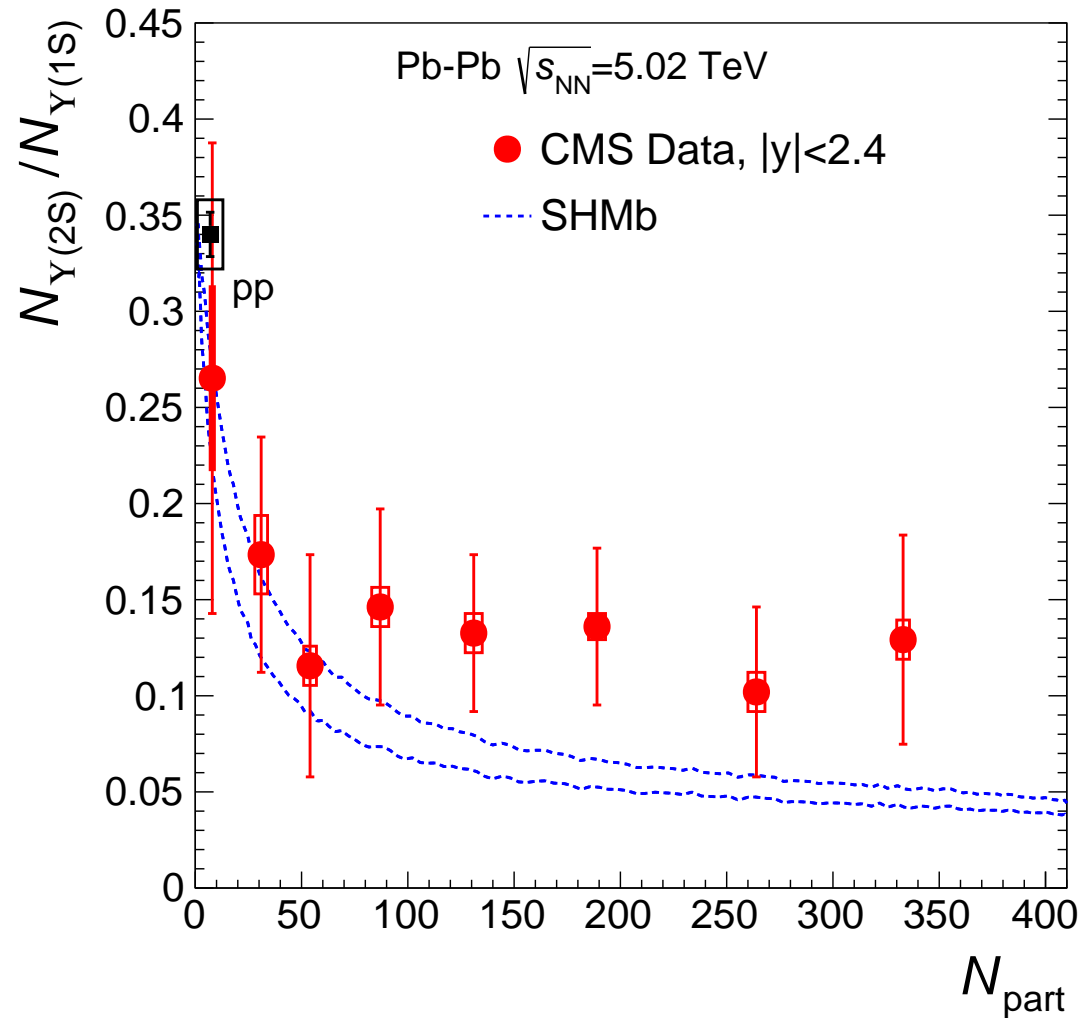
ALICE, PLB 822 (2021) 136579

*What does non-thermalized beauty produce? (no room for it in SHMb)*

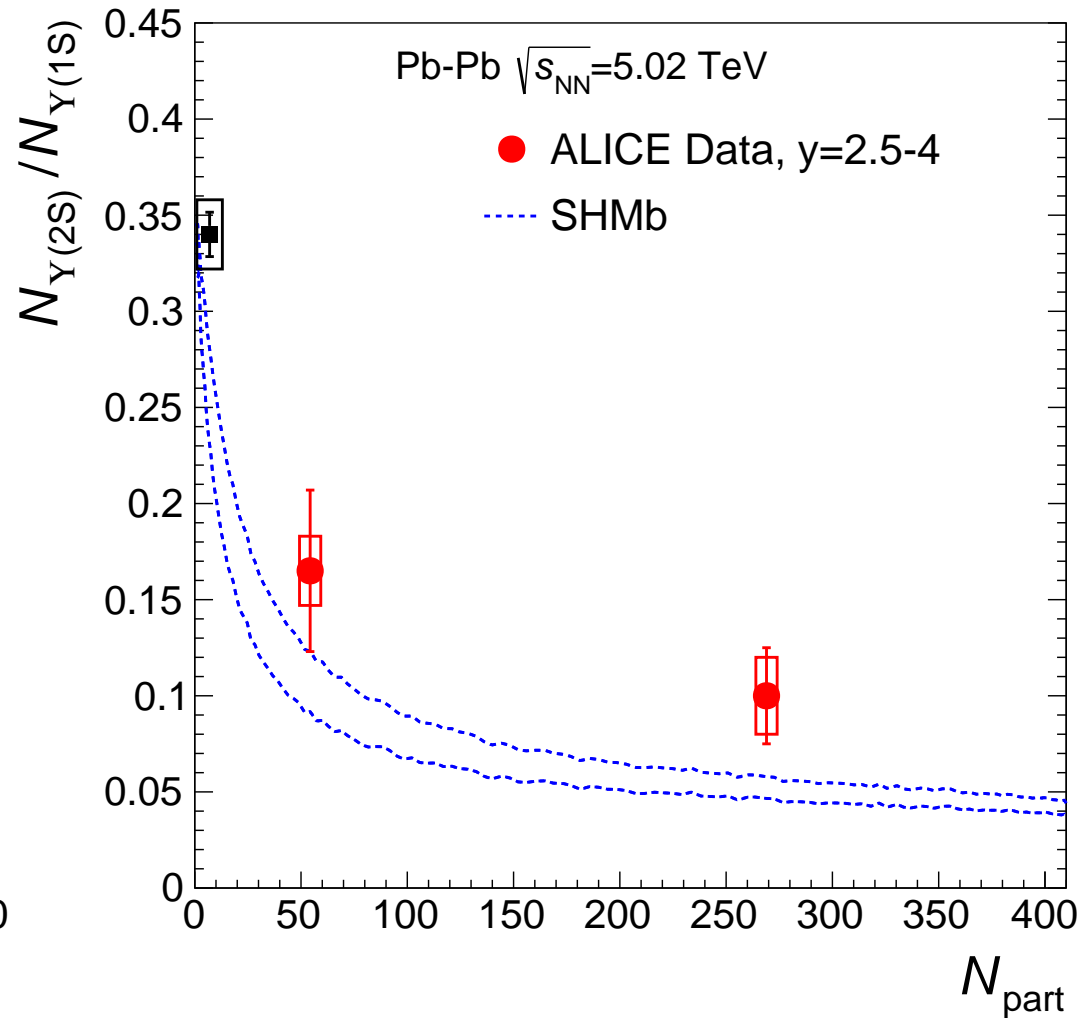
# $\Upsilon(2S)/\Upsilon(1S)$ ratio (100% b thermalization)

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CMS, [PRL 120 \(2018\) 142301](#)



ALICE, [PLB 822 \(2021\) 136579](#)

ALICE pp:  $\Upsilon(2S)/\Upsilon(1S) = 0.5 \pm 0.1$ , [arXiv:2109.15240](#)

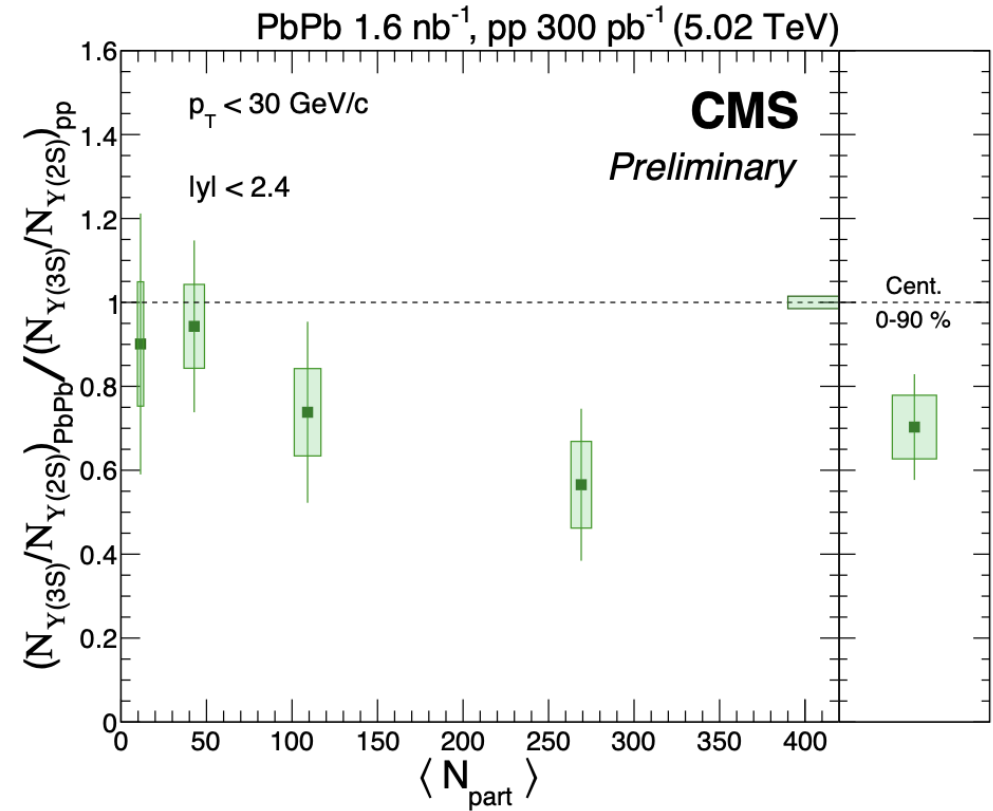
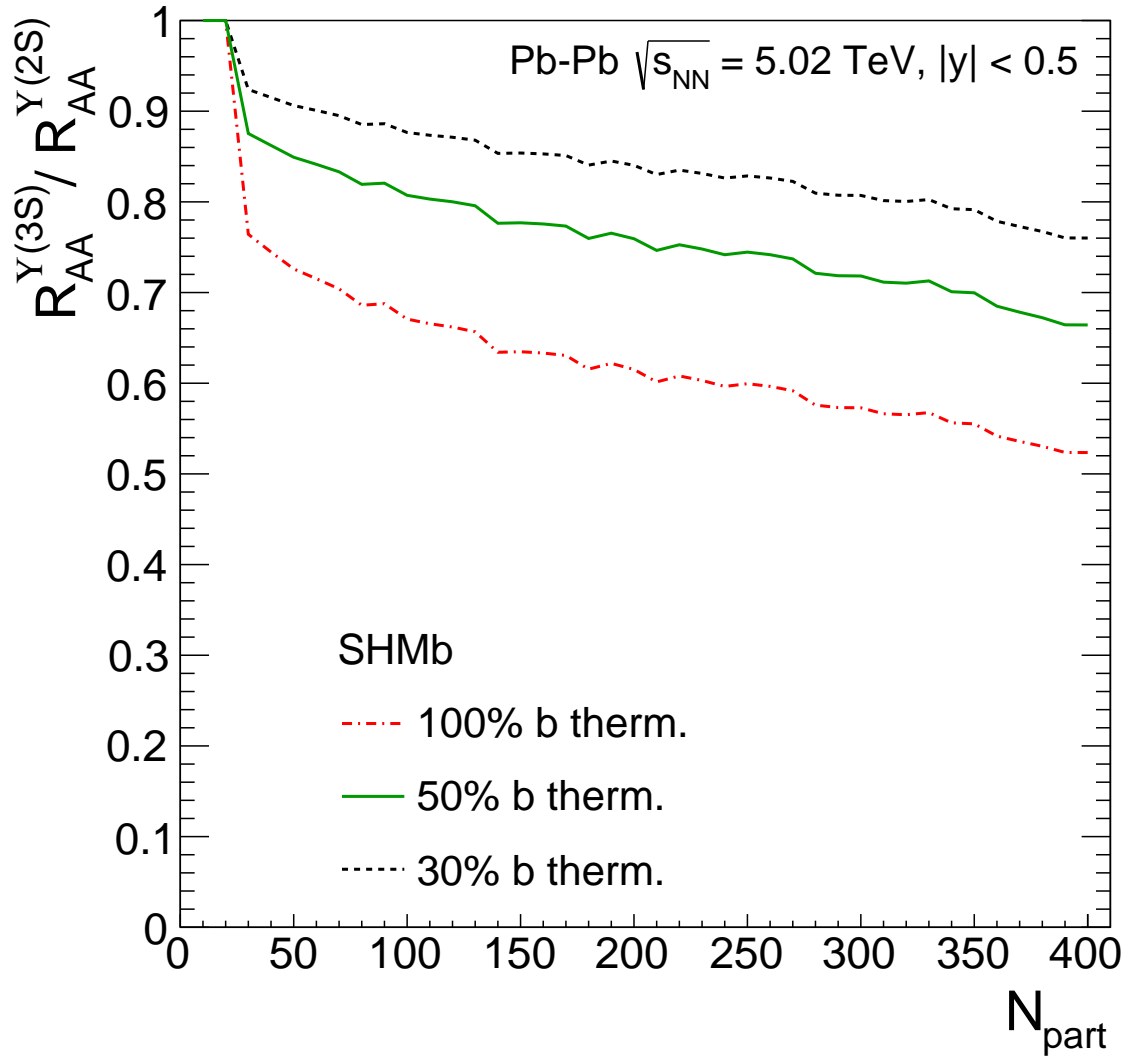
SHMb uncert.: nuclear-corona (fraction)



# $\Upsilon(3S)/\Upsilon(2S) R_{AA}$ ratio

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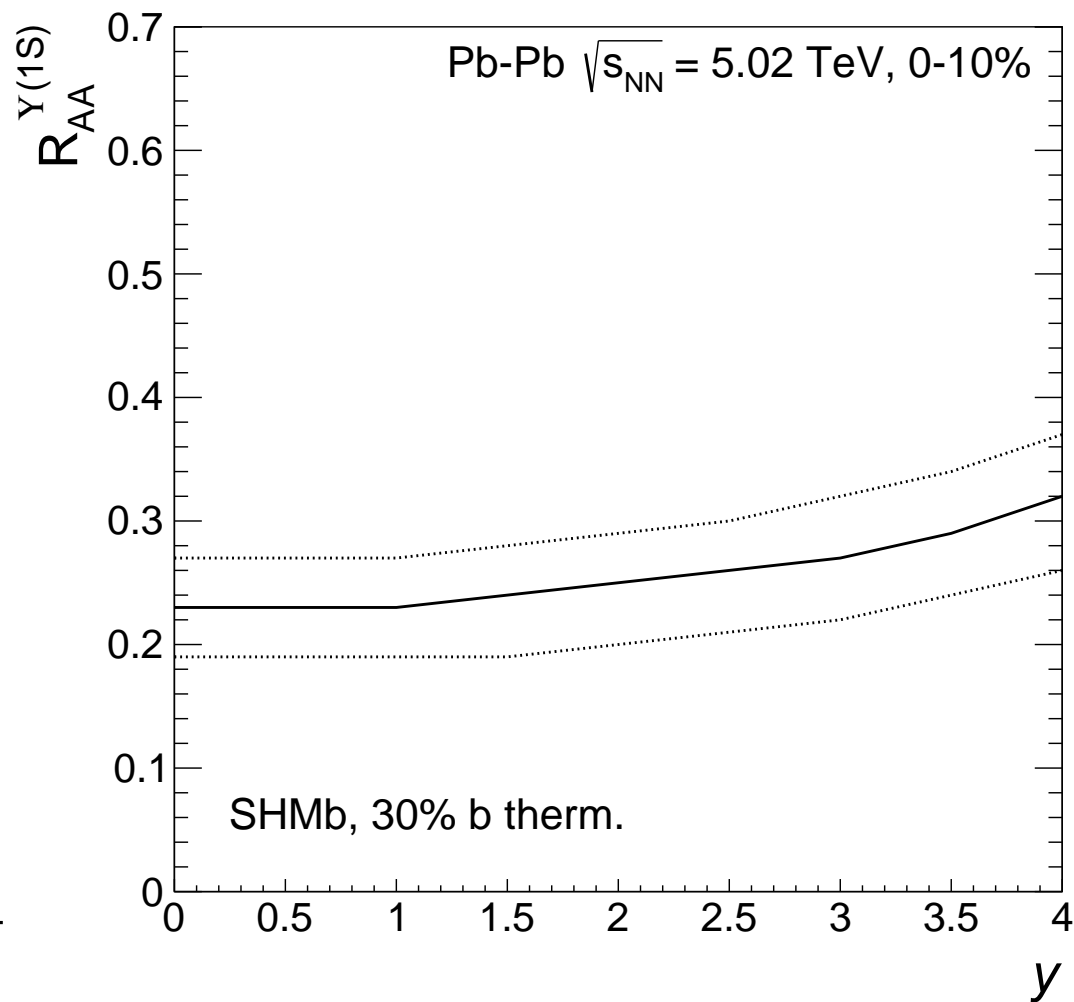
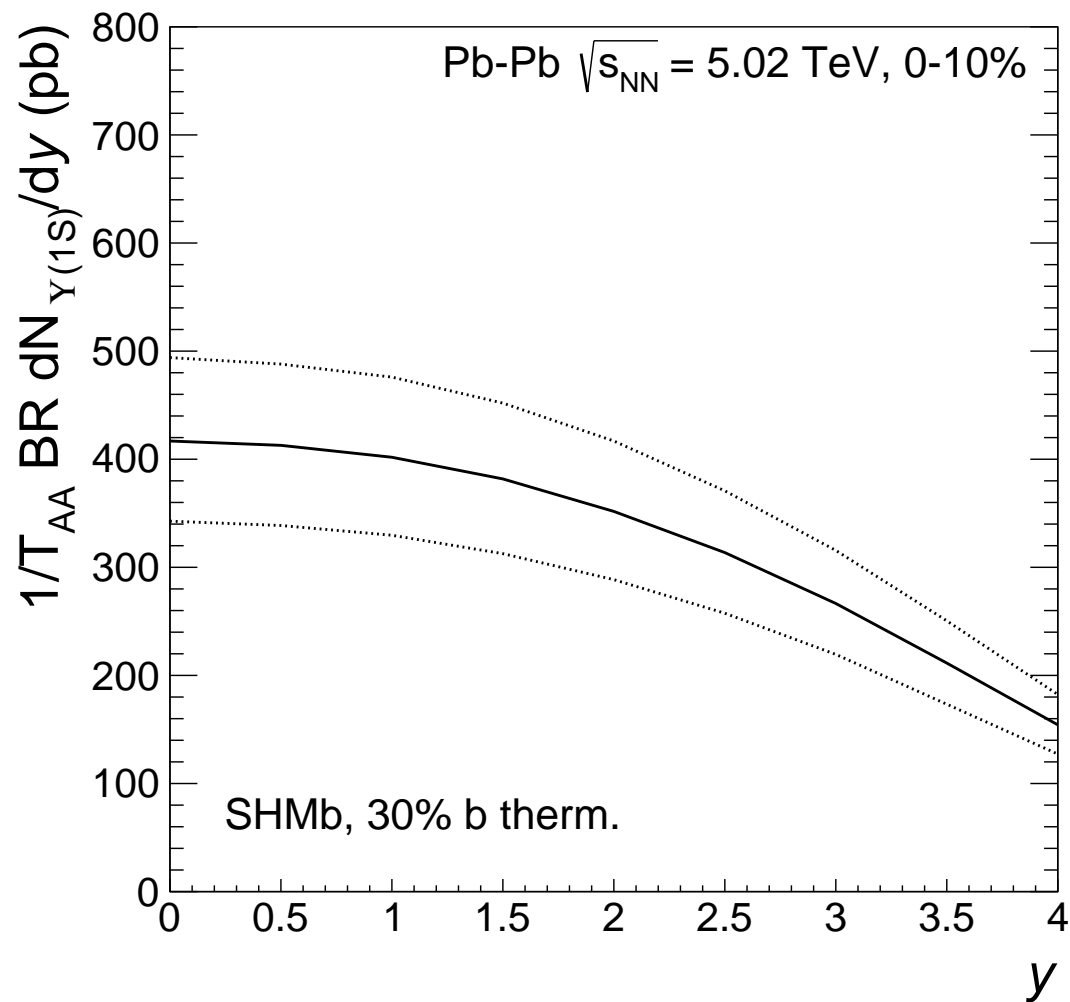
CMS, QM'22, [HIN-21-007](#)

The  $\Upsilon(3S)/\Upsilon(2S) R_{AA}$  ratio is quite sensitive to the degree of b thermalization

# Rapidity dependence $\Upsilon(1S)$ , 30% $b\bar{b}$ thermalized

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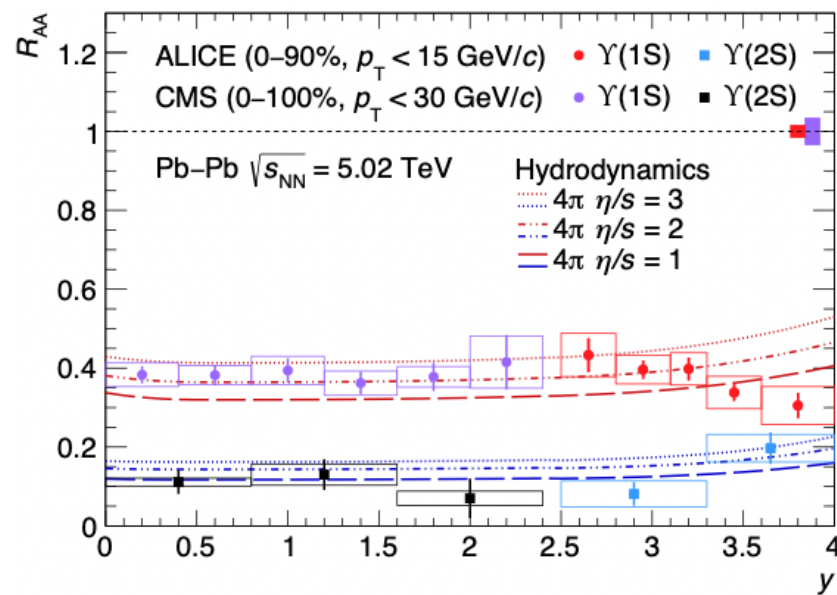
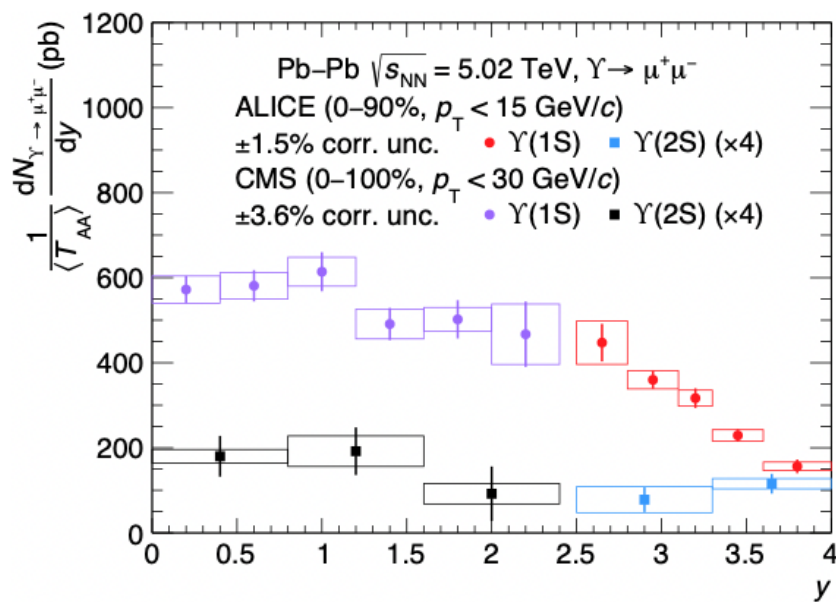
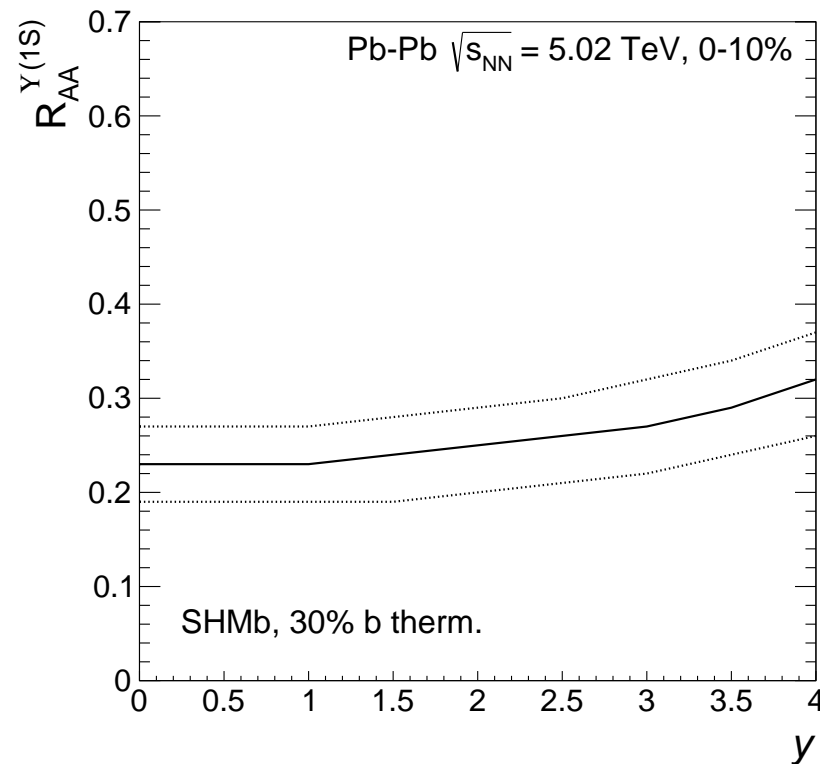
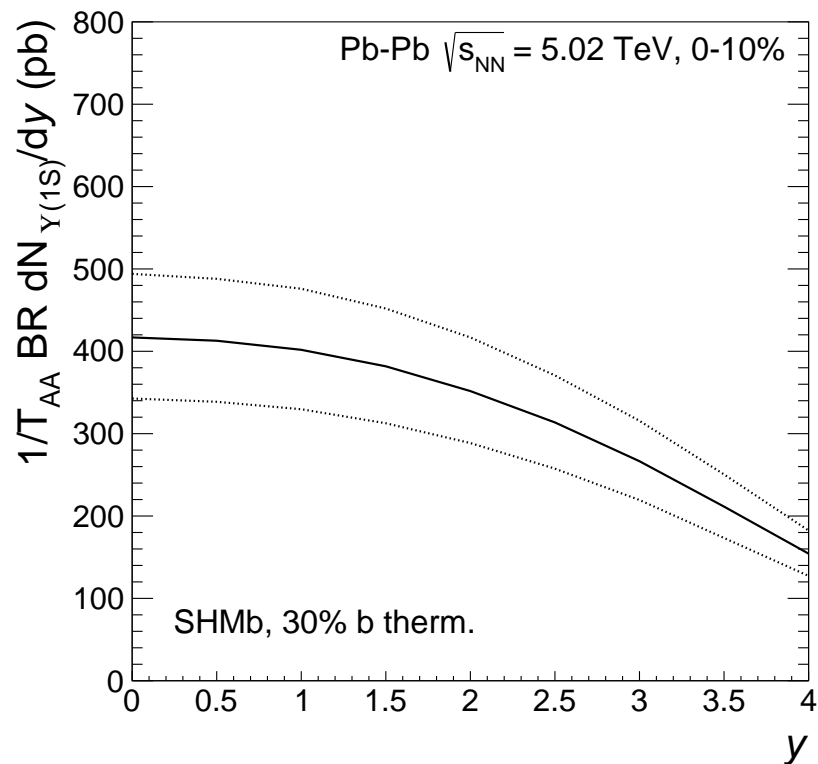


Data available only centrality-integrated ...is 0-10% (or 0-20%) doable?

# Rapidity dependence $\Upsilon(1S)$ , 30% $b\bar{b}$ thermalized

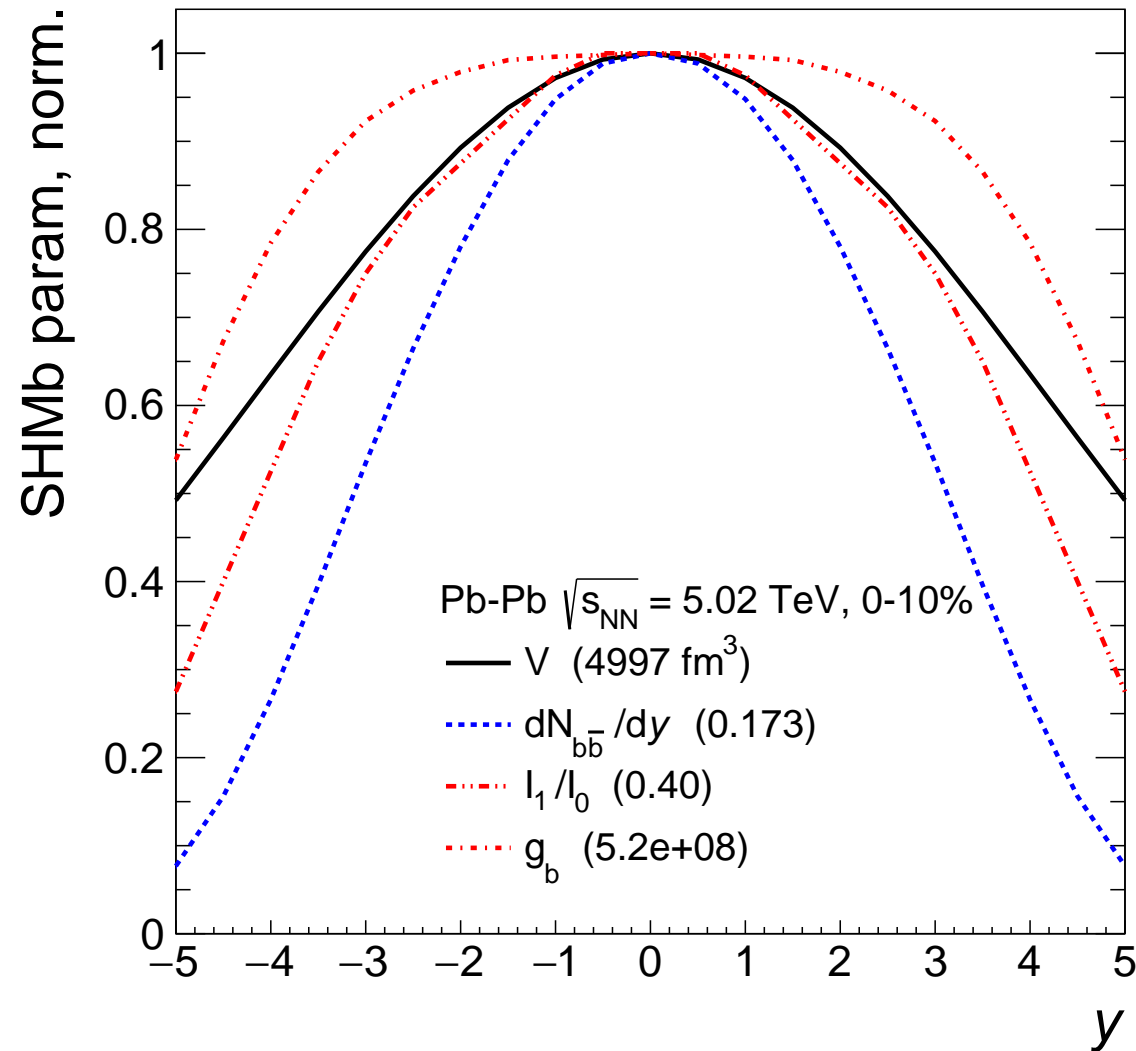
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# SHMb parameters

NB: none is a free parameter, except the 30% b thermalization fraction



# Summary / Conclusions: charm

In the (our) statistical hadronization model:

- The hadronization is a rapid process in which all quark flavors take part concurrently
- All charmonium and open charm states are generated exclusively at hadronization (chemical freeze-out) ...full color screening

The model is very successful in reproducing the  $J/\psi$  and open charm data  
*A handle for hadronization  $T$  with a mass scale well above  $T$*

"The competition":

the kinetic model, continuous  $J/\psi$  destruction and (re)generation in QGP

(only up to 2/3 of the  $J/\psi$  yield (LHC, central collisions) originates from deconfined  $c$  and  $\bar{c}$  quarks)

*Discriminating the two pictures implies providing an answer to fundamental questions related to the fate of hadrons in a hot deconfined medium.*

A precision ( $\pm 10\%$ ) measurement of  $d\sigma_{c\bar{c}}/dy$  in Pb-Pb (Au-Au) collisions needed for a stringent test  
(within reach with the upgraded detectors at the LHC and RHIC)

# Summary / Conclusions: beauty

- Full beauty thermalization seems not realized in nature
  - ...with 30-50% of beauty quarks fully thermalized we can explain the  $\Upsilon$  data
  - ...but this fraction is (significantly) dependent on the b-hadron spectrum
- What does non/partially-thermalized beauty produce?
  - no  $\Upsilon$  because strong coupling with the medium destroys the  $b\bar{b}$  correlation?
  - ...related: is there non-screened bottomonium at all? (...or maybe just  $\Upsilon(1S)$ ?)
- Another difficulty:  $R_{AA}^{Y(1S)}(p_T)$  is flat (we would predict a bump),  $v_2$  is small

similar to Reygers et al., [PRC 101 \(2020\) 064905](#)

forthcoming LHC data will (hopefully) clarify these questions (...in a while)

*inspite (because:) of its simplicity SHM provides for all more sophisticated models a meaningful (powerful) limit to check (worth even if not fully realized in nature:)*