Flux Tube Entanglement Entropy

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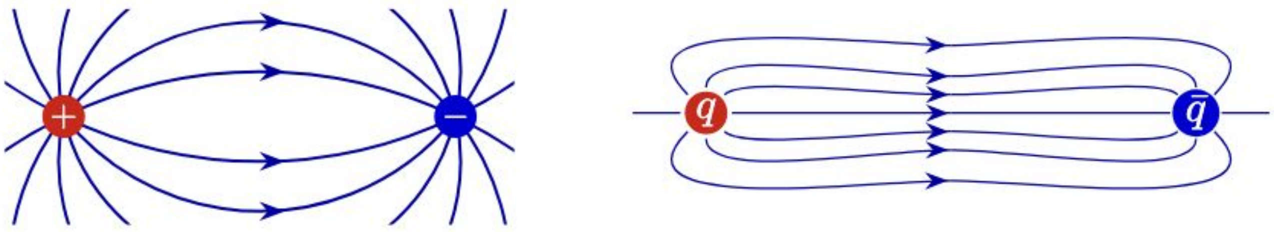
Why study the color flux tube?

- QCD string much easier to study on the lattice than in experiment
- Color flux tube is highly relevant to the confinement problem
- Incomplete understanding of its effective theory
    - “Critical radius” R_c below which EST not applicable
    - Non-Gaussian profile is unexplained
    - Unclear how intrinsic width contributes



Entanglement Entropy (Renyi Entropy)

$$\hat{\rho}_{\mathcal{A}} = \mathrm{tr}_{\bar{\mathcal{A}}}\left(\hat{\rho}\right)$$

Example:

$$\psi = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle)$$

$$\rho = \begin{array}{c} \begin{array}{cccc} |\uparrow\uparrow\rangle & |\uparrow\downarrow\rangle & |\downarrow\uparrow\rangle & |\downarrow\downarrow\rangle \end{array} \\ \begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix} \end{array}$$

$$\hat{\rho}_{\mathcal{A}} = \mathrm{Tr}_{\bar{\mathcal{A}}}\rho = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$$

$$S_{\mathrm{EE}} = -\,\mathrm{tr}_{\mathcal{A}}\left(\hat{\rho}_{\mathcal{A}} \log\left(\hat{\rho}_{\mathcal{A}}\right)\right)$$

$$S^{(q)} = \frac{1}{1-q} \log\left(\mathrm{tr}_{\mathcal{A}}\left(\hat{\rho}^{q}_{\mathcal{A}}\right)\right) \forall\ q \in \mathbb{N},\ \ q \geq 2.$$

$$S_{\mathrm{EE}} = \lim_{q \to 1} S^{(q)}.$$

UV-finite Entanglement Entropy

$$\frac{1}{|\partial A|} S = \frac{1}{|\partial A|} S_{UV} + \frac{1}{|\partial A|} S_f$$

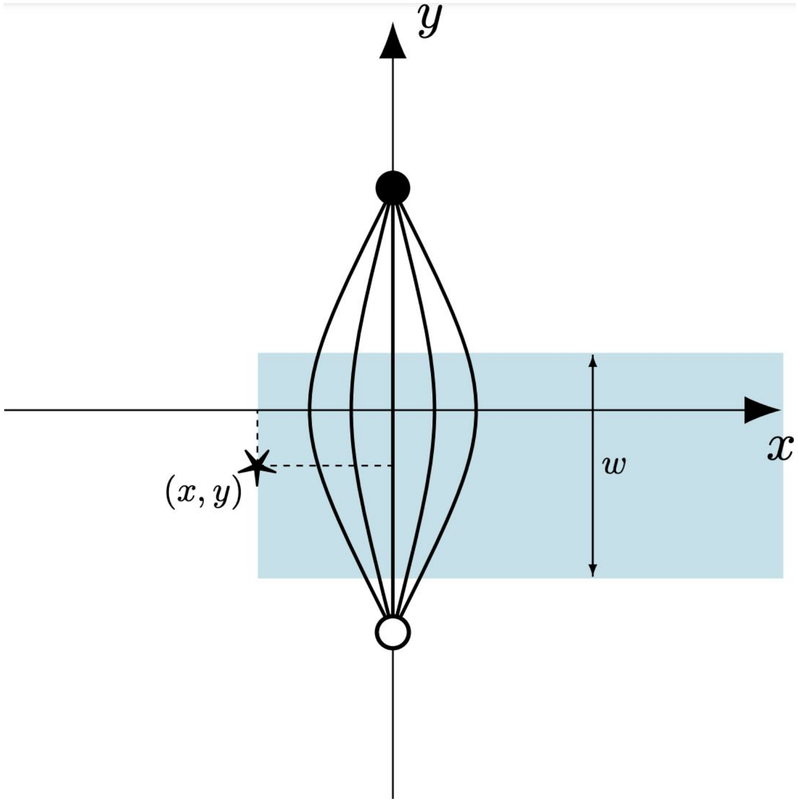
(P. V. Buividovich, M. I. Polikarpov arXiv:0802.4247)
(S. Ryu et al arXiv:hep-th/0603001)
(T. Nishioka et al arXiv:hep-th/0611035)
(I. Klebanov et al arXiv:0709.2140)

$$\tilde{S}^{(q)}_{|Q\bar{Q}} \equiv S^{(q)}_{|Q\bar{Q}} - S^{(q)}$$

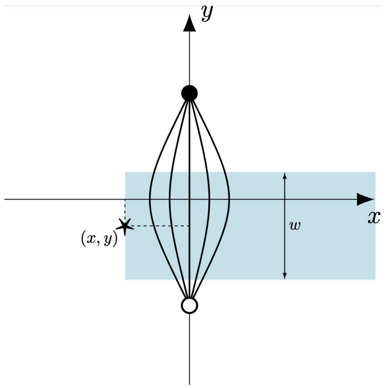
(Flux Tube Entanglement Entropy)

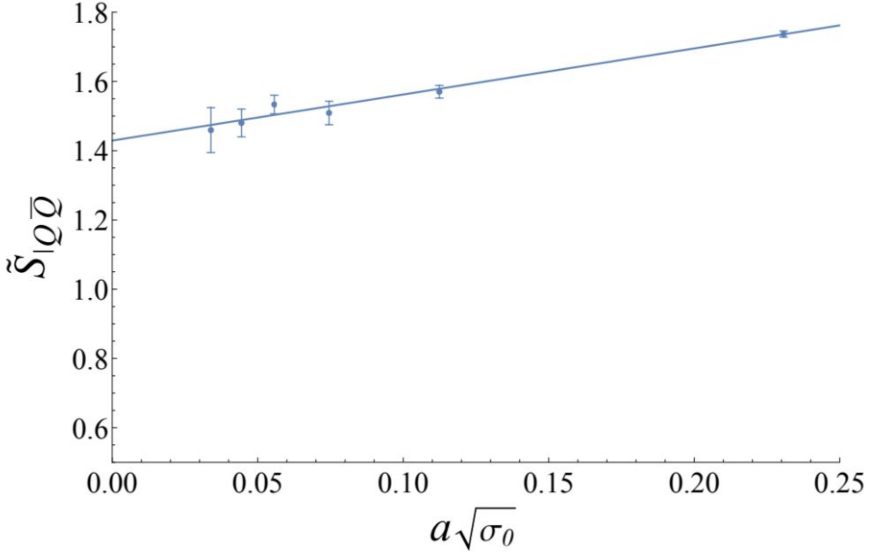
Can be realized on the lattice using Polyakov lines and the replica method

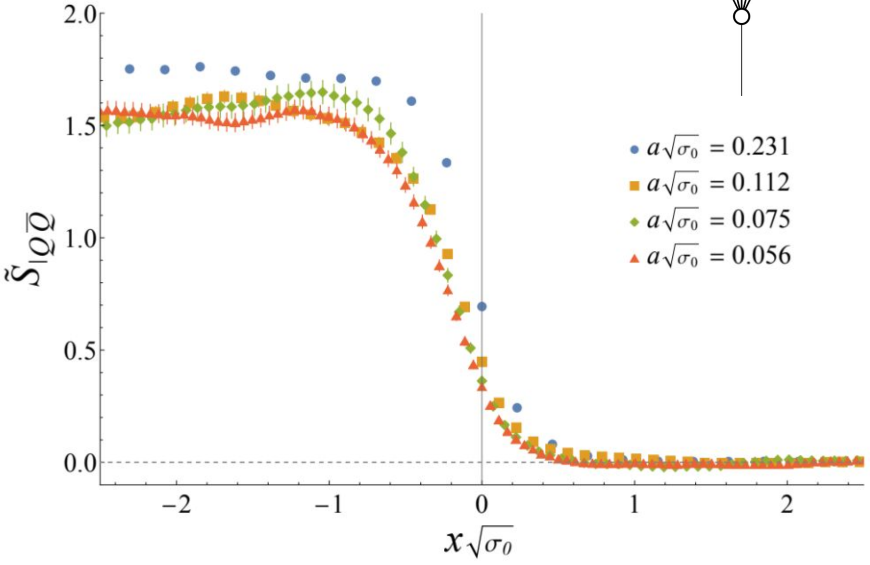
Lattice Setup: A = Half-slab



Finite Entanglement Entropy (Scaling Study)



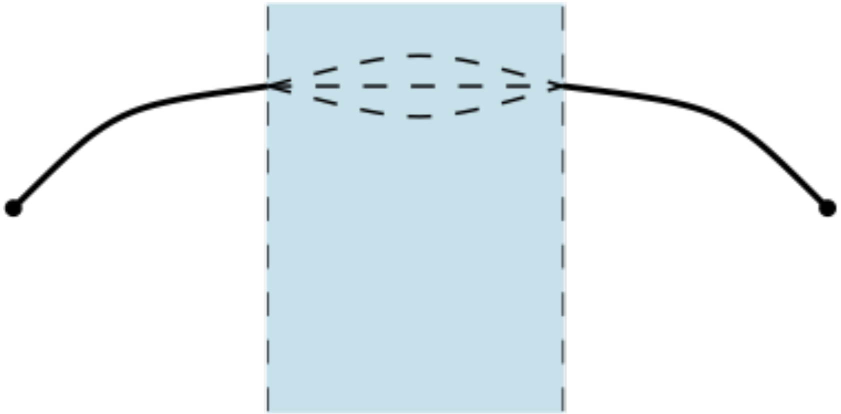


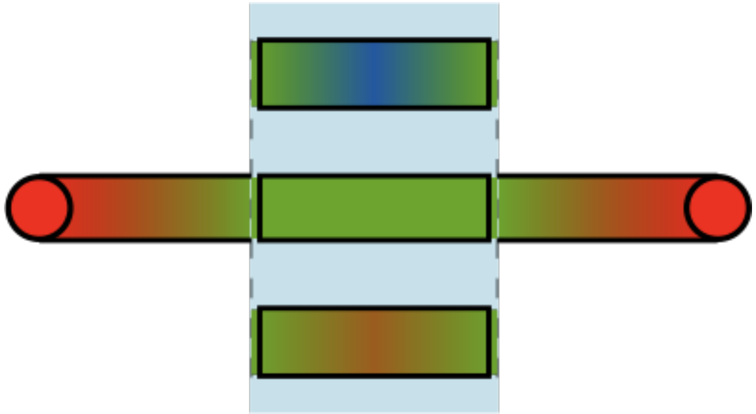


Takeaway: FTE^2 has finite non-zero continuum limit for $x \lesssim 0$

String Model Predictions

- FTE^2 splits into internal and vibrational components
- Internal: Due to colorful degrees of freedom inside flux tube
- Vibrational: Due to mechanical vibrations of flux tube

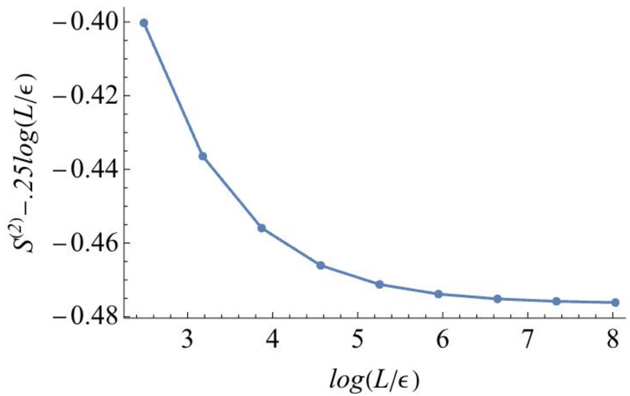


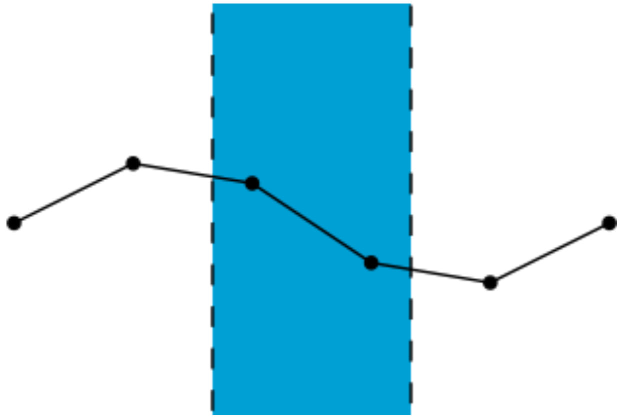


String Model Predictions

- Thin relativistic string in long string limit (1 transverse dimension)
- Follow procedure of (L. Bombelli et al PhysRevD.34.373)
- Numerically calculate entropy taking L/ε to infinity

$$S^{(2)} = \frac{1}{4}\log(L/\epsilon) + \text{finite contributions}$$

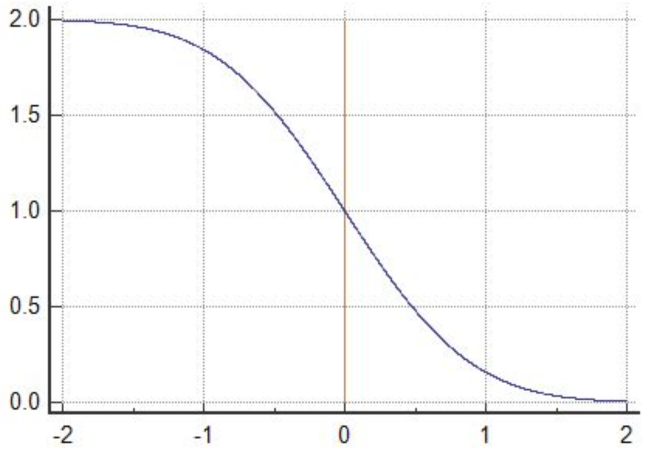


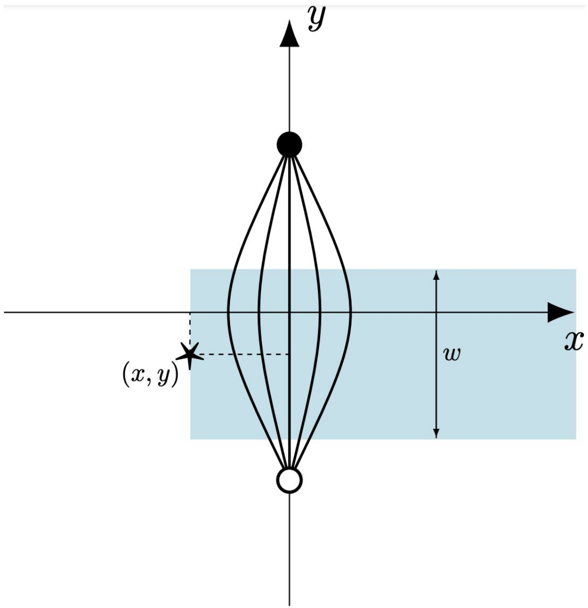


$$H = M^2 L + \frac{\pi}{2M^2 L}\int_0^\pi ds\left(p^2 + M^4 x'(s)^2\right)$$

String Model Predictions (Internal)

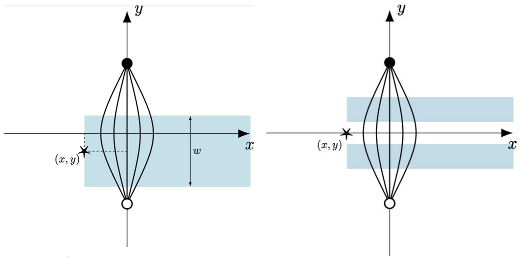
- 1+1-D Yang Mills predicts F*log(Nc) (2411:12818)
- 2+1-D <F>*log(Nc), Displacement of string is Gaussian, Naively expect error function shape

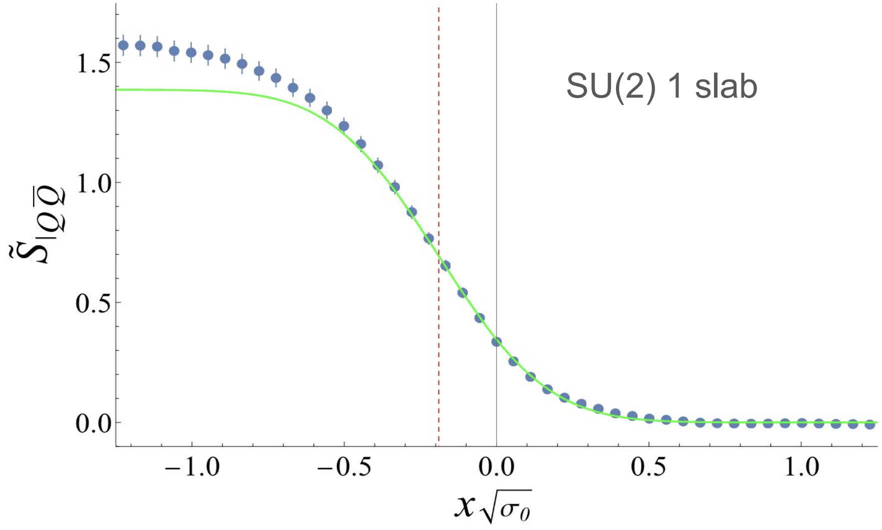


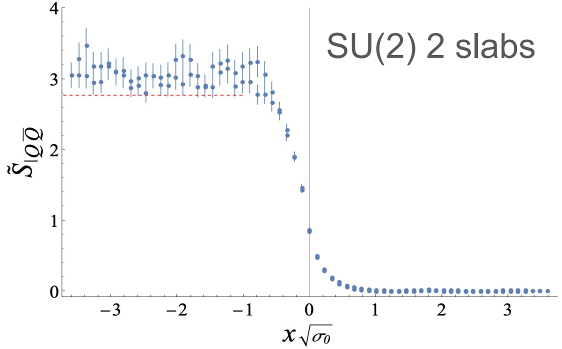


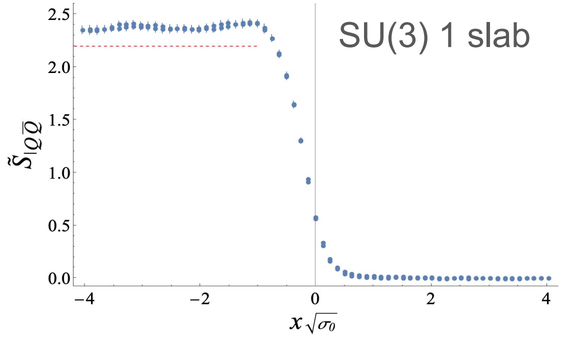
Results

General Properties (Internal Entropy)



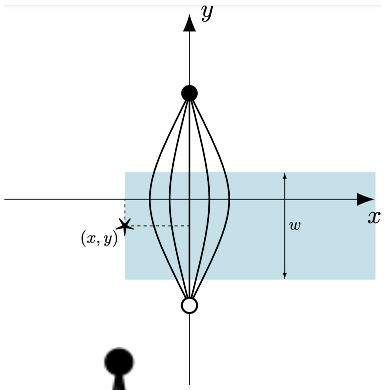


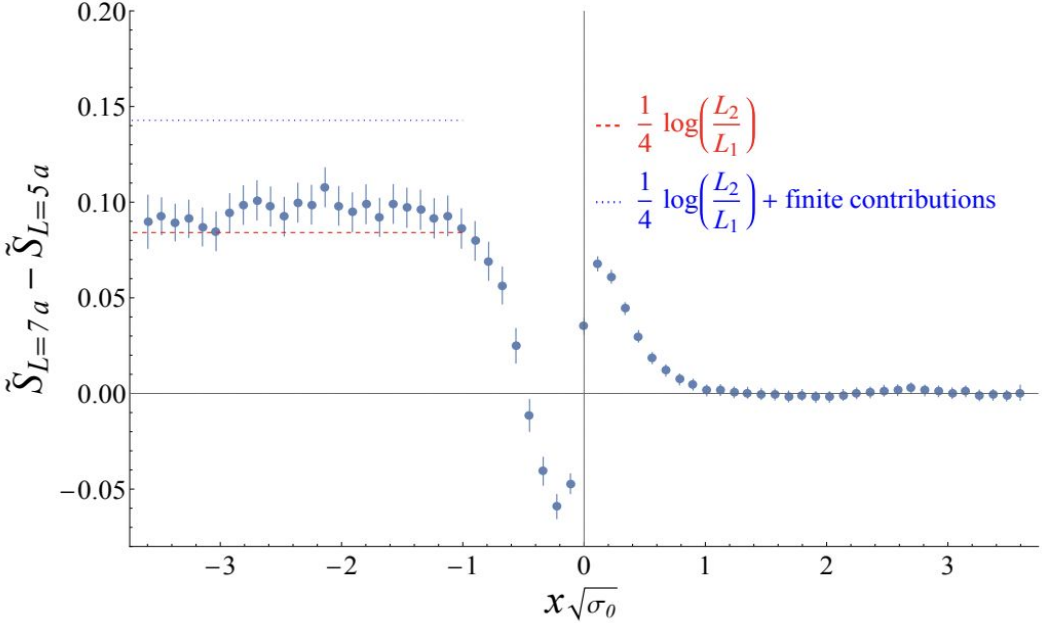


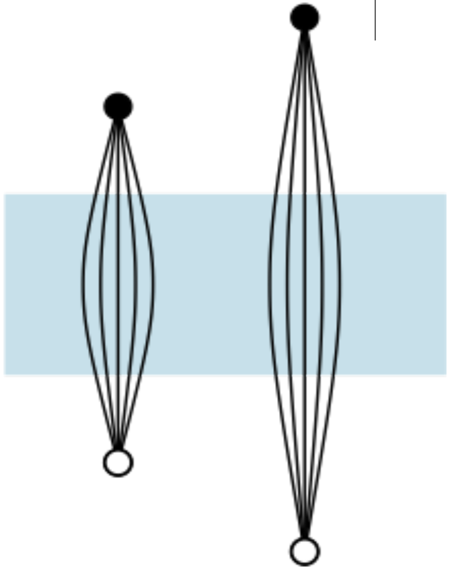


Takeaway: FTE^2 is dominated by the internal entropy contribution.

Entropy String Length Dependence

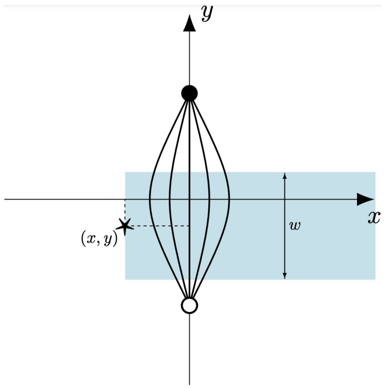


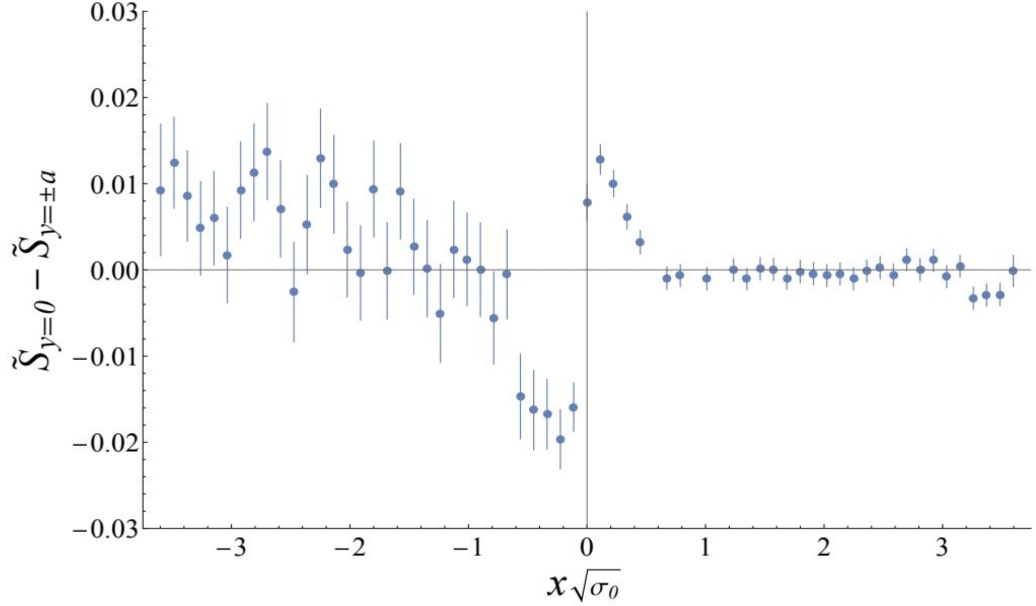


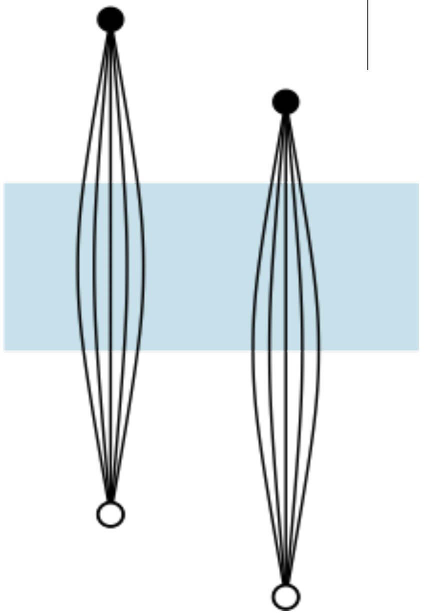


Takeaway: Present length scaling term, other finite contributions small/absent.

Entropy Location along the String Dependence

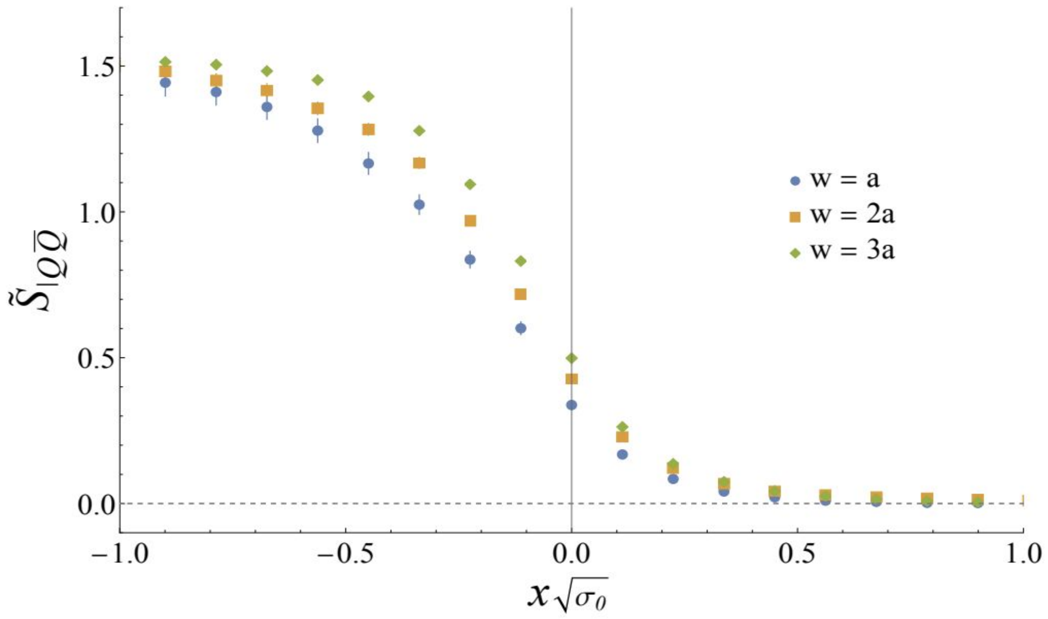


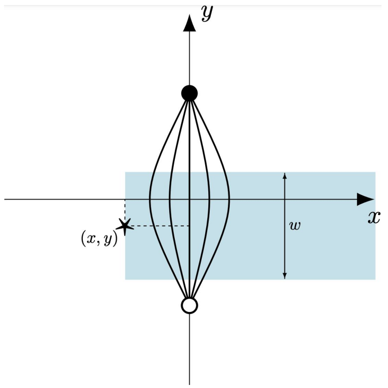


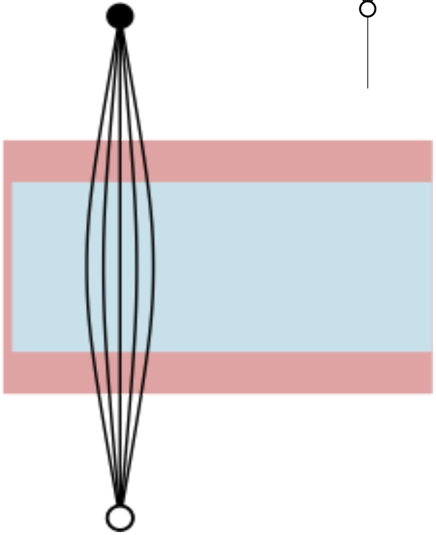


Takeaway: Finite contributions small/absent from vibrational entropy.

Entropy Region Width Dependence







Takeaway: Thin string model has success explaining FTE^2 dynamics.

Results Recap

- FTE^2 is dominated by the internal entropy contribution for quark separations probed.
- Logarithmic length scaling term is present for vibrational entropy, other finite contributions small/absent.
- Limited y-dependence
- Thin string model does a good job of explaining our results*