Applicability of hydrodynamics in high energy collisions

Illinois Center for Advanced Studies of the Universe

Dekrayat Almaalol

University of Illinois at Urbana-Champaign

Intersection of nuclear structure and high energy nuclear collisions

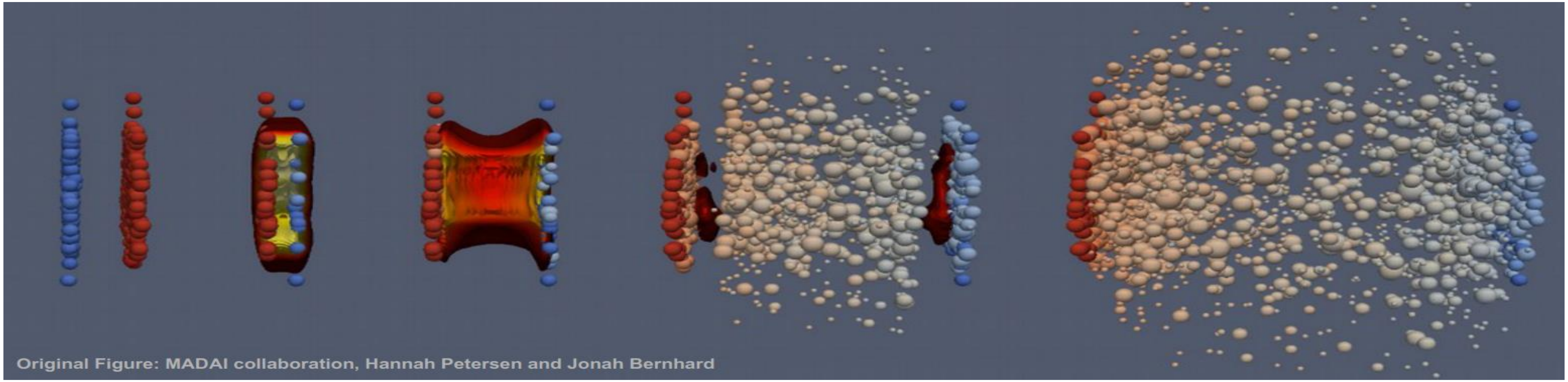
Institute for nuclear theory, Seattle

(Jan 23- Feb 24, 2023)

Dynamical evolution of Heavy-ion collisions

- Hydrodynamics is the workhorse in heavy ion collisions model simulations

Heinz, Snellings, Ann. Rev. Nucl. Part. Sci. 63, 123 (2013)



initial state + pre-equilibrium $\tau = 0.1\,\mathrm{fm/c}$

Hydrodynamics $\tau = 1\,\mathrm{fm/c}$

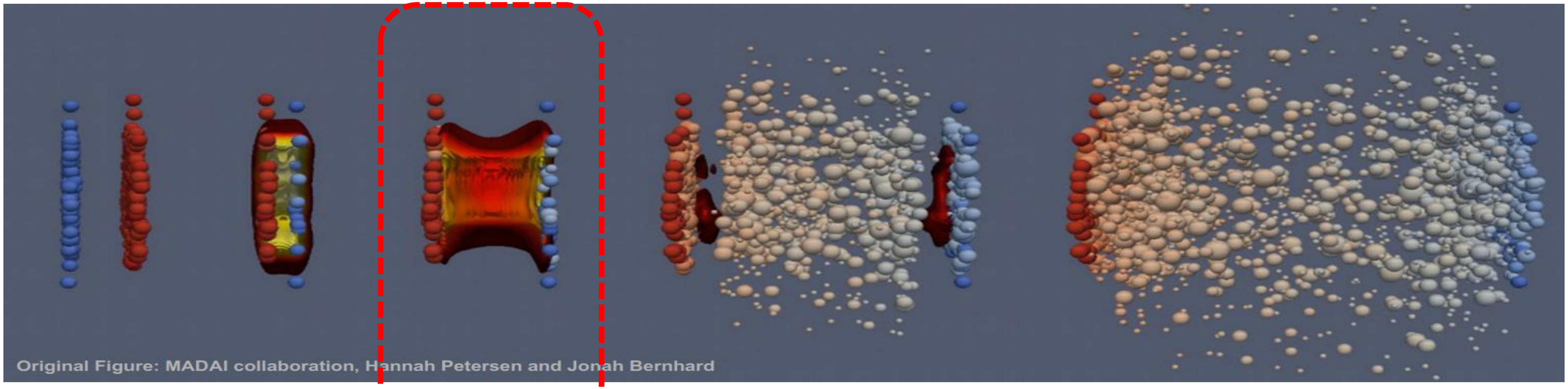
Freezeout/ Particalization $\tau = 10\,\mathrm{fm/c}$

Hadronic afterburner $\tau = 20\,\mathrm{fm/c}$

Dynamical evolution of Heavy-ion collisions

- Hydrodynamics is the workhorse in heavy ion collisions model simulations

Heinz, Snellings, Ann. Rev. Nucl. Part. Sci. 63, 123 (2013)



initial state + pre-equilibrium
$\tau = 0.1\,\mathrm{fm/c}$

Hydrodynamics
$\tau = 1\,\mathrm{fm/c}$

Freezeout/ Particalization
$\tau = 10\,\mathrm{fm/c}$

Hadronic afterburner
$\tau = 20\,\mathrm{fm/c}$

where do we use hydrodynamics?

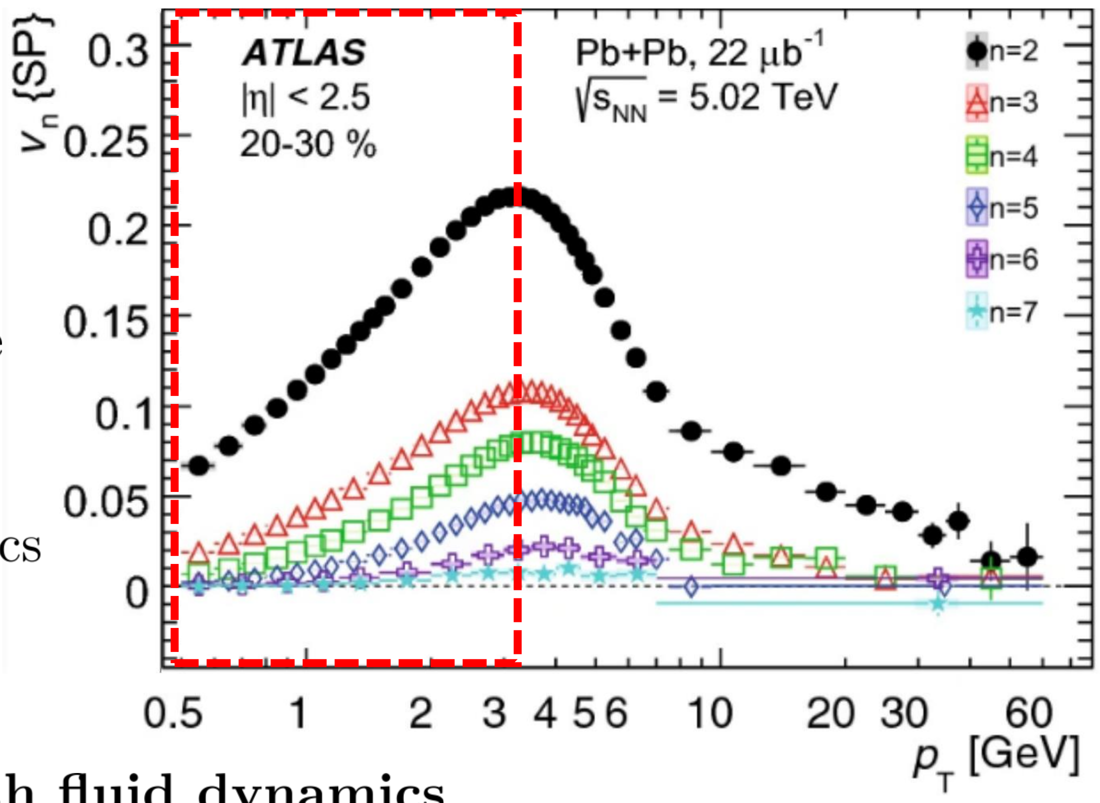
- Hydrodynamics is an effective theory which describes the evolution of long-wavelength modes in the dynamical system.

Aaboud, Aad, Abbott,et al, Eur. Phys.J.C78,(2018).

- Soft probes (low-pT ≤ 3 GeV hadrons): collective behavior of the medium

- Hard probes (high-pT particles): produced in hard pQCD processes in the initial stage

- the exact value of momentum for which fluid dynamics is no longer applicable is not exactly known.

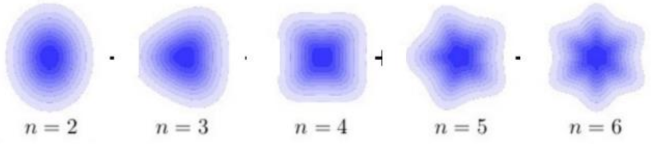


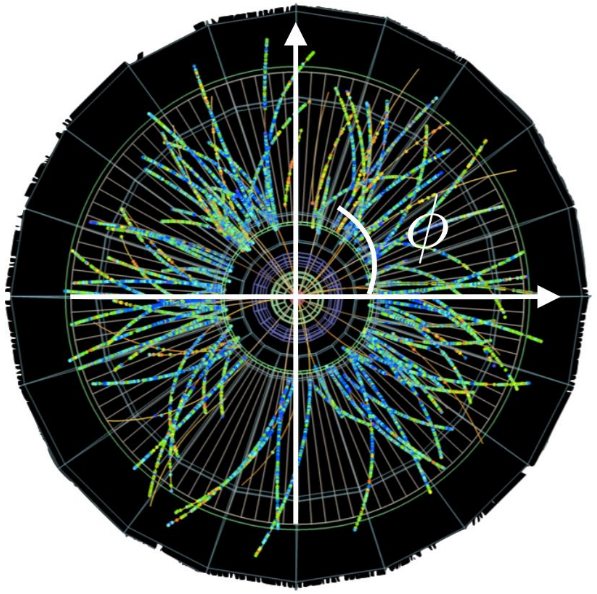
- **Discovery of QGP is a result of agreement with fluid dynamics**

Collectivity in high energy collisions

- Nonlinear medium response to initial state geometry

$$E\frac{dN}{d^3p} \propto \frac{dN}{p_T dp_T dy}\left[1 + 2\sum_{n}^{\infty} v_n(p_T, y)\cos\left[n(\phi - \psi_{RP})\right]\right]$$

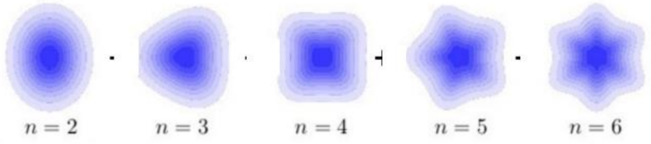


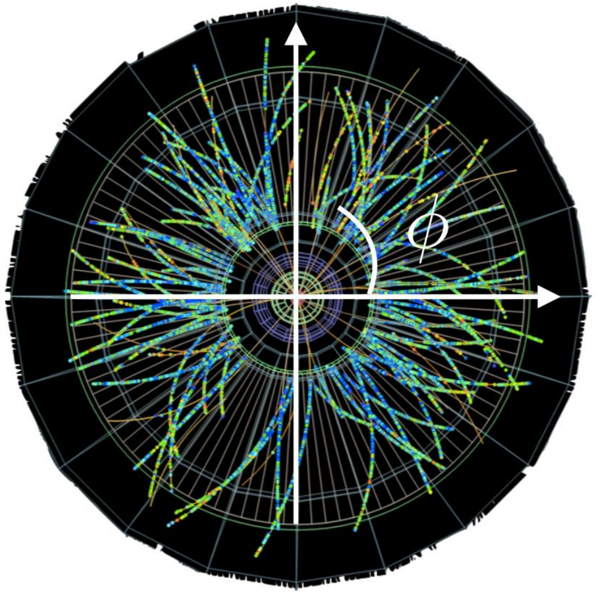


Collectivity in high energy collisions

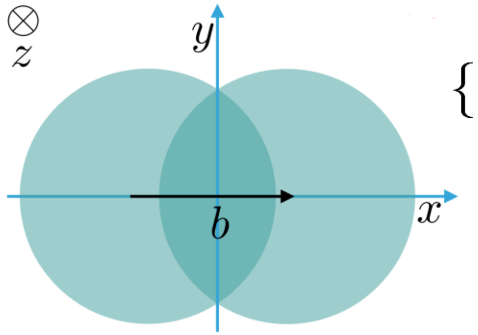
- Nonlinear medium response to initial state geometry

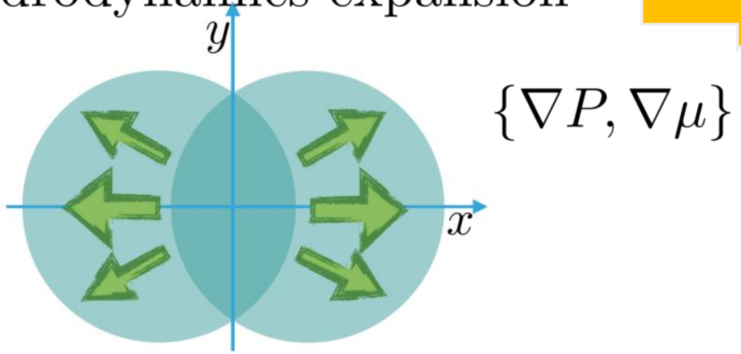
$$E\frac{dN}{d^3p} \propto \frac{dN}{p_T dp_T dy}\left[1 + 2\sum_{n}^{\infty} v_n(p_T, y)\cos\left[n(\phi - \psi_{RP})\right]\right]$$

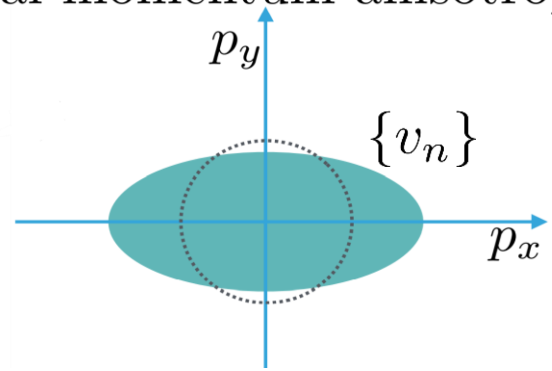




Initial spatial anisotropy → Hydrodynamics expansion → Final momentum anisotropy







$$\mathcal{E}_2 = \varepsilon_2 e^{i2\psi_2} = \frac{\langle x^2 - y^2\rangle + i\langle 2xy\rangle}{\langle x^2 + y^2\rangle},$$

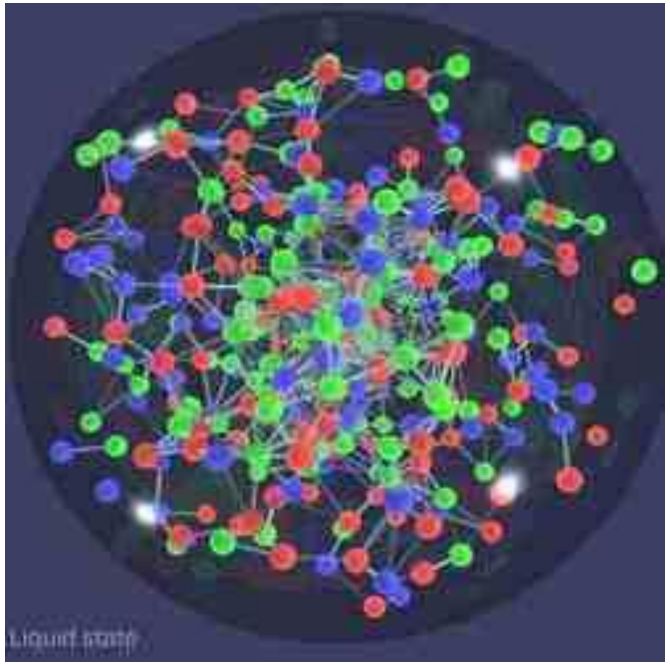
$$\mathcal{E}_p \equiv \varepsilon_p e^{i2\psi_2^p} \equiv \frac{\langle T^{xx} - T^{yy}\rangle + i\langle 2T^{xy}\rangle}{\langle T^{xx} + T^{yy}\rangle},$$

Universality of hydrodynamics

- Universal behaviour in strongly interacting quantum systems

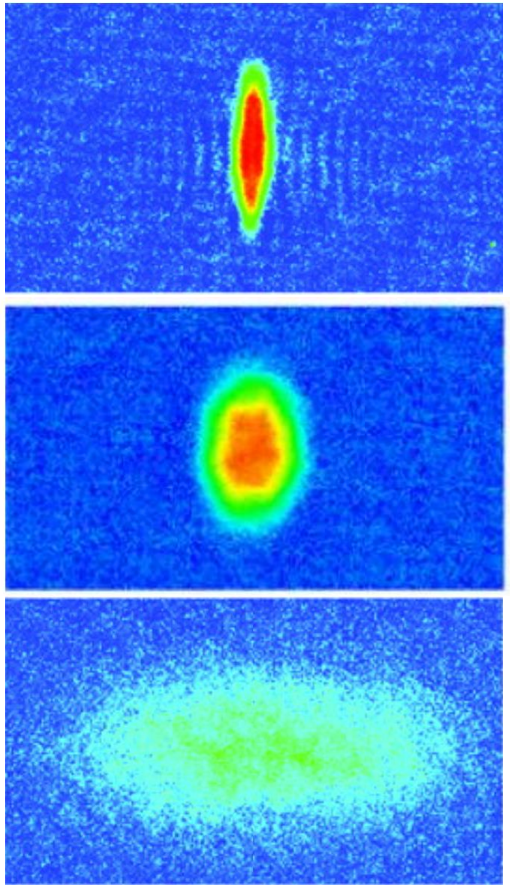
Quark gluon plasma $(T \sim 10^{12} K)$

K M O'Hara et al. 2002 Science 298 2179



Ultra cold atoms $(T \sim 10^{-9} K)$

K M O'Hara et al. 2002 Science 298 2179



- Is Flow an intrinsic feature at all scales?

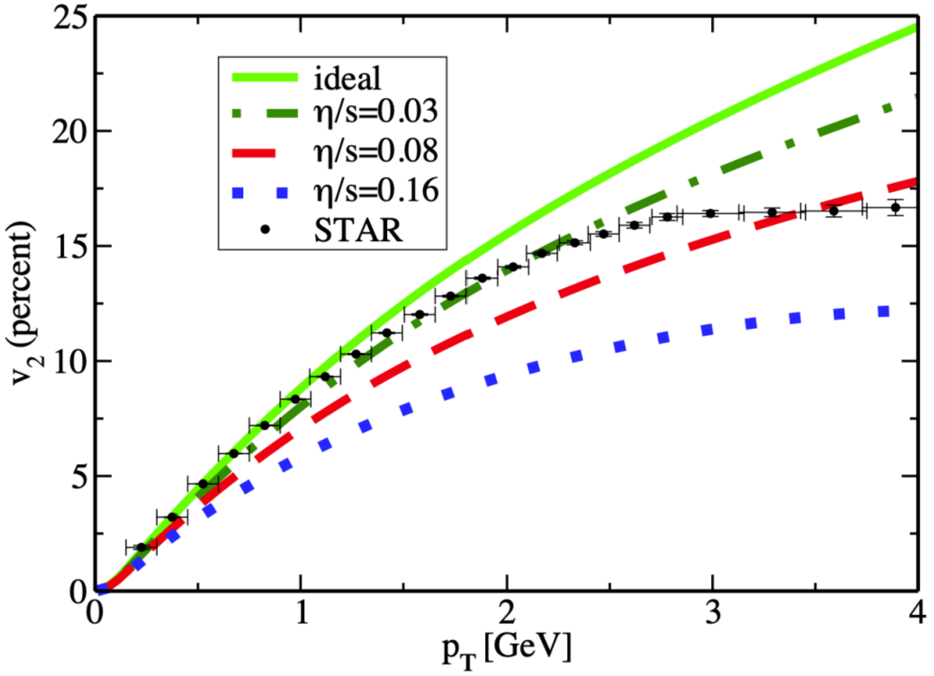
Phenomenological success of hydrodynamics

- QGP discovered in ultra relativistic heavy ion collisions.

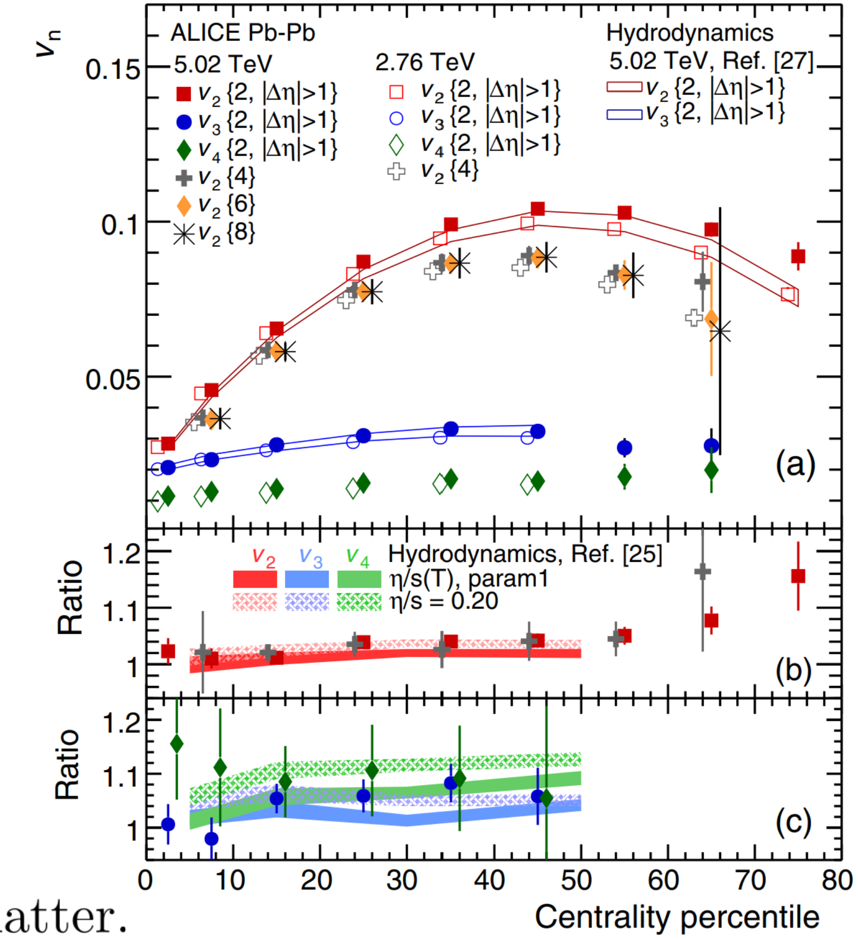
  Huovinen, Kolb, Heinz, Ruuskanen ,Voloshin, Physics Letters B 99,(2001)

- Precise predictive power of URHIC observables.

  P.Romatschke, U.Romatschke, PRL 99,(2007)



The ALICE Collaboration PRL 116, (2016)



- Extraction of the detailed properties of the QGP state of matter.

Hydrodynamics as an effective field theory

I. Macroscopic description

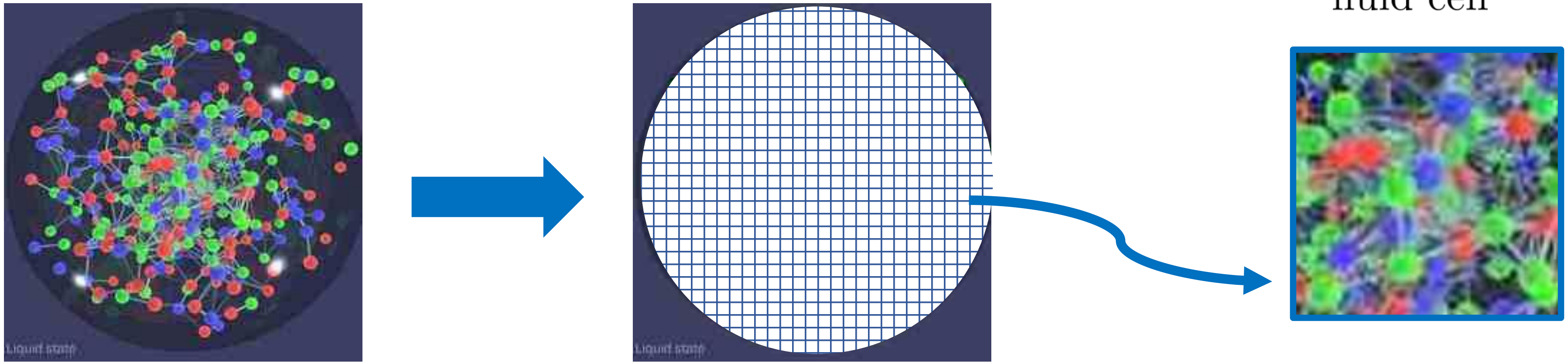
The state of the system is given by the fields $\varphi_i = \{u^\mu(\mathbf{x}), \varepsilon(\mathbf{x}), \rho_q(\mathbf{x}), \Pi(\mathbf{x}), \pi^{\mu\nu}(\mathbf{x}), n_q^\mu(\mathbf{x})\}$

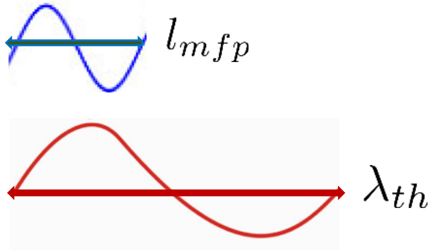
Hydrodynamics as an effective field theory

I. Macroscopic description

The state of the system is given by the fields $\varphi_i = \{u^\mu(\mathbf{x}), \varepsilon(\mathbf{x}), \rho_q(\mathbf{x}), \Pi(\mathbf{x}), \pi^{\mu\nu}(\mathbf{x}), n_q^\mu(\mathbf{x})\}$

fluid cell





$$\mathrm{Kn} = \frac{\text{microscopic scale}}{\text{macroscopic scale}} \leq 0.5 \qquad \mathrm{R}_\pi^{-1} = \frac{\sqrt{\pi^{\mu\nu}\pi_{\mu\nu}}}{P} \leq 0.5$$

An effective description at sufficiently long distance and time scales $L \gg l_{mfp}$ $\quad t \gg \tau_{mfp}$

Hydrodynamics as an effective field theory

I. Macroscopic description

The state of the system is given by the fields $\varphi_i = \{u^\mu(\mathbf{x}), \varepsilon(\mathbf{x}), \rho_q(\mathbf{x}), \Pi(\mathbf{x}), \pi^{\mu\nu}(\mathbf{x}), n_q^\mu(\mathbf{x})\}$

II. Conservation laws

$$D_\mu T^{\mu\nu} = 0\,,$$
$$D_\mu N_q^\mu = 0$$

(Energy-momentum conservation)

(Charge conservation)

III. Constitutive relations

$$T^{\mu\nu} = \varepsilon u^\mu u^\nu - p\Delta^{\mu\nu} + \Pi^{\mu\nu}$$
$$N_q^\mu = \rho_q u^\mu + n_q^\mu$$

$(u^\mu u_\mu = -1\,, \Delta_{\mu\nu} = g_{\mu\nu} + u_\mu u_\nu)$

Hydrodynamics as an effective field theory

I. Macroscopic description

The state of the system is given by the fields $\varphi_i = \{u^\mu(\mathbf{x}), \varepsilon(\mathbf{x}), \rho_q(\mathbf{x}), \Pi(\mathbf{x}), \pi^{\mu\nu}(\mathbf{x}), n_q^\mu(\mathbf{x})\}$

II. Conservation laws

$$D_\mu T^{\mu\nu} = 0\,,$$
$$D_\mu N_q^\mu = 0$$

(Energy-momentum conservation)

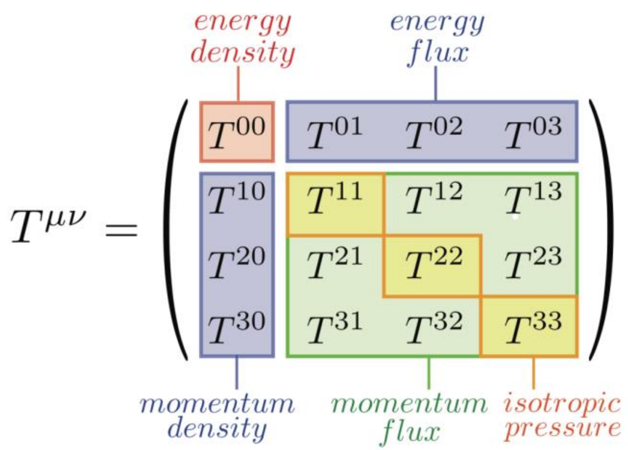
(Charge conservation)

$$u_\mu T^{\mu\nu} = \varepsilon u_\nu\,,$$

- How do we evolve the dissipative fields $(\Pi, \pi^{\mu\nu}, n^\mu)$?

III. Constitutive relations

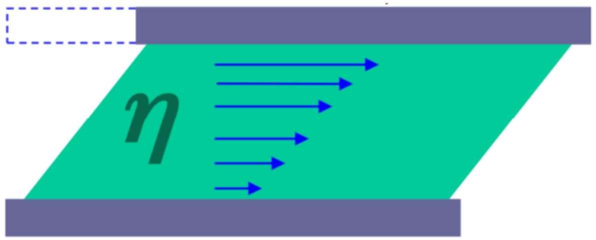
$$T^{\mu\nu} = \varepsilon u^\mu u^\nu - p\Delta^{\mu\nu} + \Pi^{\mu\nu}$$
$$N_q^\mu = \rho_q u^\mu + n_q^\mu$$



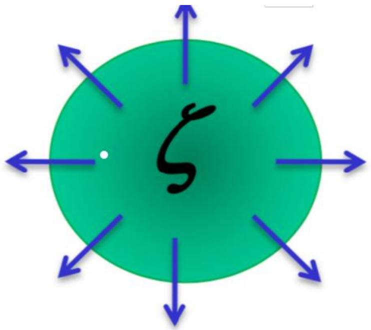
Hydrodynamics as an effective field theory

Navier Stokes

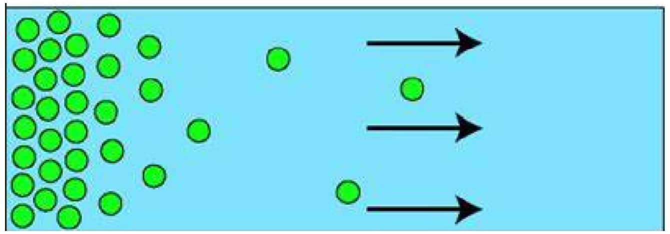
$$\pi^{\mu\nu} = 2\eta\sigma^{\mu\nu}$$



$$\Pi = -\zeta\theta$$



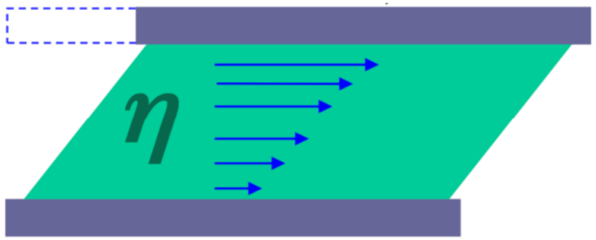
$$n_q^{\mu} = -\kappa_{qq'}\nabla^{\mu}\alpha_{q'}$$



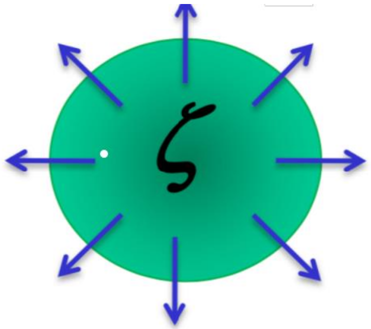
Hydrodynamics as an effective field theory

Navier Stokes

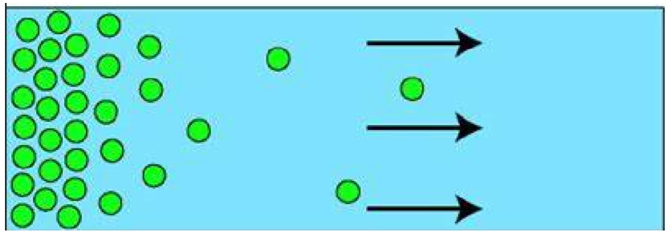
$$\pi^{\mu\nu} = 2\eta\sigma^{\mu\nu}$$



$$\Pi = -\zeta\theta$$



$$n_q^{\mu} = -\kappa_{qq'}\nabla^{\mu}\alpha_{q'}$$



The dissipative currents $(\pi^{\mu\nu}, \Pi)$ are independent dynamical DoF

2nd law of thermodynamics

Israel, Stewart, Ann. Phys. 118 (1979)

$$\nabla_{\mu} S^{\mu} \geq 0$$

- Higher-order terms are suppressed by powers of the cutoff l_{mfp}.

- **Relaxation type equations:**

$$\tau_{\Pi}\dot{\Pi} + \Pi = -\zeta\theta + \mathcal{F}_{\Pi}[\varphi_i]$$

$$\tau_{\pi}\dot{\pi}^{\mu\nu} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} + \mathcal{F}_{\pi^{\mu\nu}}[\varphi_i]$$

$$\tau_{qq'}\dot{n}_{q'}^{\mu} + n_q^{\mu} = -\kappa_{qq'}\nabla^{\mu}\alpha_{q'} + \mathcal{F}_{n_{\mu}^{q}}[\varphi_i]$$

Hydrodynamics as an effective field theory

The dissipative currents $(\pi^{\mu\nu}, \Pi)$ are independent dynamical DoF

Moment method (DNMR)

$$P_\mu \partial^\mu f = 0$$

systematic expansion of $\{Kn, Re_n^{-1}\}$

Denicol,Niemi, Molnar,Rischke PRD85(2012)

- **Relaxation type equations:**

$$\tau_\Pi \dot{\Pi} + \Pi = -\zeta\theta + \mathcal{J} + \mathcal{K} + \mathcal{R}$$

$$\tau_\pi \dot{\pi}^{\mu\nu} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} + \mathcal{J} + \mathcal{K} + \mathcal{R}$$

$$\tau_{qq'} \dot{n}^\mu_{q'} + n^\mu_q = -\kappa_{qq'} \nabla^\mu \alpha_{q'} + \mathcal{J} + \mathcal{K} + \mathcal{R}$$

Hydrodynamics as an effective field theory

The dissipative currents $(\pi^{\mu\nu}, \Pi)$ are independent dynamical DoF

Moment method (DNMR)

$$P_\mu \partial^\mu f = 0$$

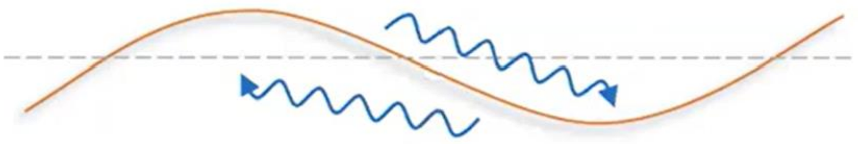
systematic expansion of $\{Kn, Re_n^{-1}\}$

Denicol,Niemi, Molnar,Rischke PRD85(2012)

- **Relaxation type equations:**

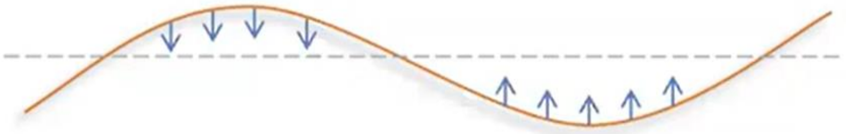
$$\tau_\Pi \dot{\Pi} + \Pi = -\zeta\theta + \mathcal{J} + \mathcal{K} + \mathcal{R}$$
$$\tau_\pi \dot{\pi}^{\mu\nu} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} + \mathcal{J} + \mathcal{K} + \mathcal{R}$$
$$\tau_{qq'} \dot{n}^\mu_{q'} + n^\mu_q = -\kappa_{qq'} \nabla^\mu \alpha_{q'} + \mathcal{J} + \mathcal{K} + \mathcal{R}$$

- conserved quantities



$\tau_{rel} \to \infty$ as $\lambda_{th} \to \infty$

- non-conserved quantities



$\tau_{rel} \sim \tau_{mfp}$

Hydro simulations: input parameters

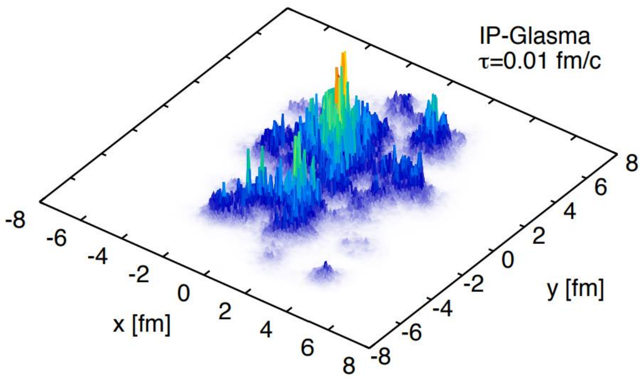
Conservation laws $D_\mu T^{\mu\nu} = 0\,, \quad D_\mu N_q^\mu = 0$

Relaxation type equations:

$$\tau_\Pi \dot{\Pi} + \Pi = -\zeta\theta + \mathcal{J} + \mathcal{K} + \mathcal{R}$$

initial state

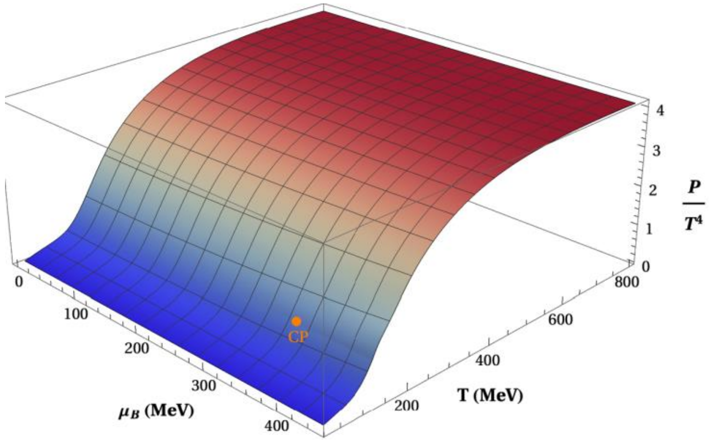
- $[\varepsilon_0(\tau_0), \rho_0^q(\tau_0), \Pi, \pi^{\mu\nu}, n_q^\mu]$



B. Schenke et al, PRL 108, (2012)

thermodynamics

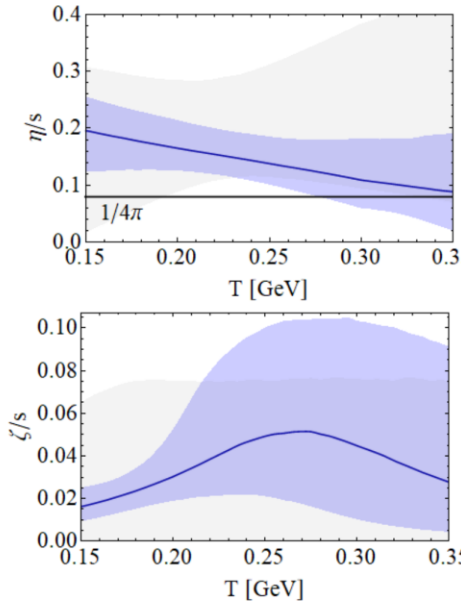
- $P_0(T, \mu_q)$



Karthein, Mroczek et al, arxiv.2211.04566

Transport

- $\{\zeta, \eta, \tau_\pi\}$
- $\{\tau_\Pi, \tau_{\pi\pi}, \delta_{\pi\pi}, \delta_{\Pi\Pi}, \lambda_{\pi\Pi}, \lambda_{\Pi\pi}\}$



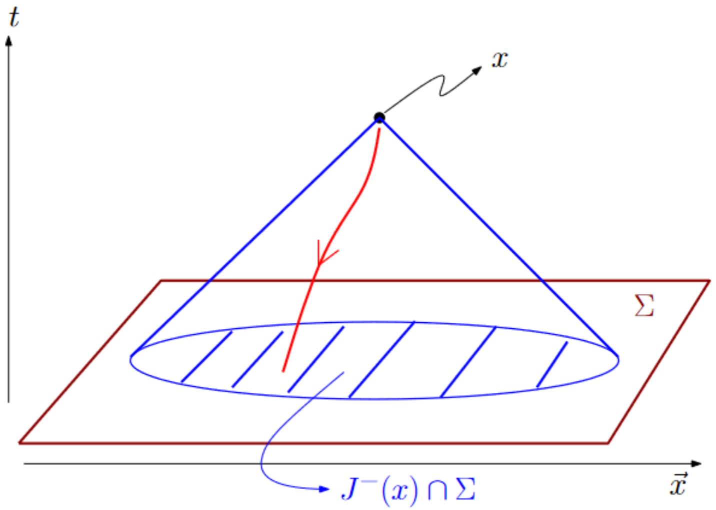
Bernhard, Moreland, Bass, Nat. Phys. 15, (2019).

Theoretical benchmarks

- A relativistic hydrodynamics theory must be

  I. Causal: no signal propagates faster than the speed of light.

  II. Stable: Perturbations around the state of global equilibrium states must decay.

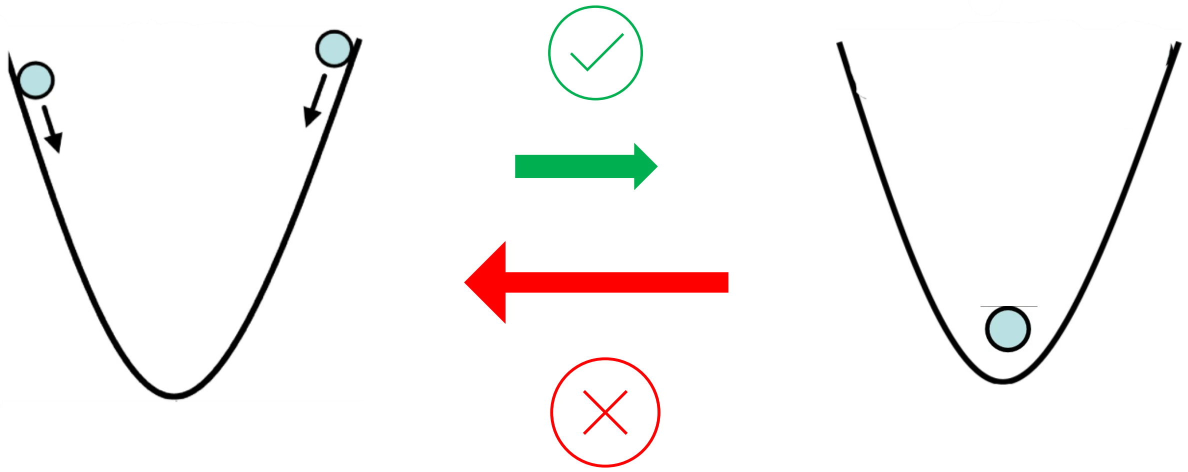


- linear statbility-causality analysis

  Hiscock, Lindblom, Annals of Physics (1983);

- The two concepts are related

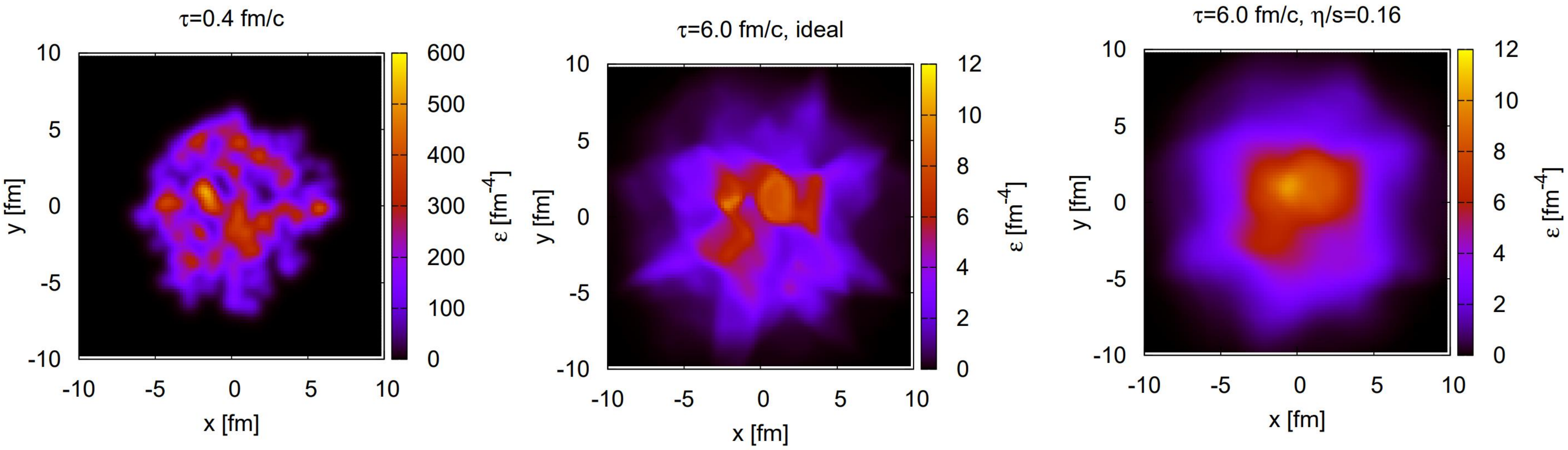
L. Gavassino, M. Antonelli, and B. Haskell PRL. 128, (2022)



Effect of viscosity on the dynamics

Hydrodynamic evolution of Au-Au at top RHIC energy Schenke, Jeon, and Gale, PRL106(2011)

- Large density gradients at initial times!
- viscosity smoothes out the initial condition faster → directly modifies flow



Dissipation effects in model-to-data comparison

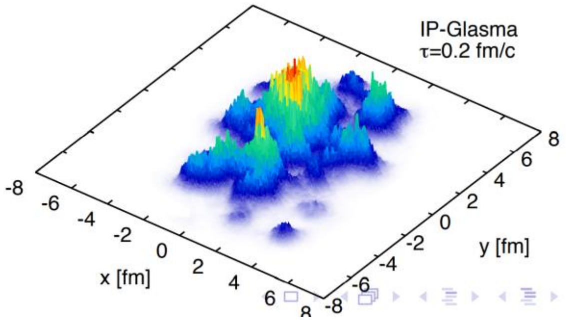
- Dissipation effects extend the agreement with data to higher p_T

  P.Romatschke, U.Romatschke, PRL 99,(2007)

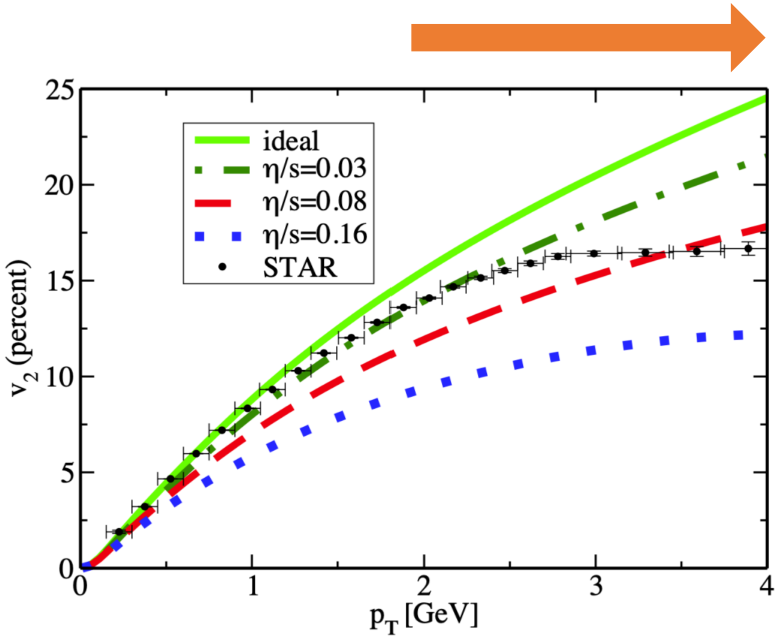
- non-equilibrium effects are a source of uncertainty at high p_T range

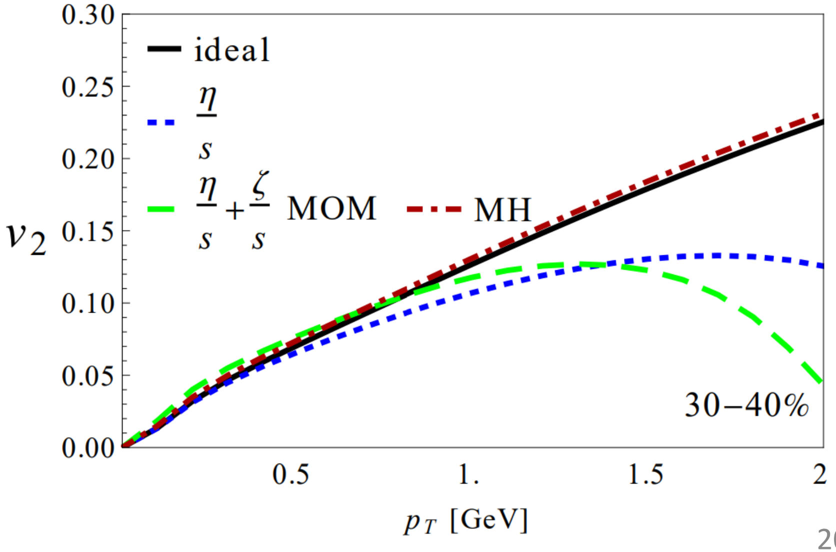
  Noronha-Hostler, Noronha, Grassi PRC90(2014)

- Include initial state fluctuations



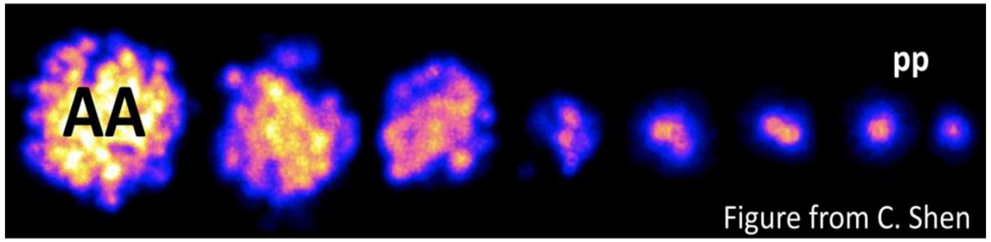
- Earlier initialization time of hydrodynamics $(0.6 - 1\, fm/c)$

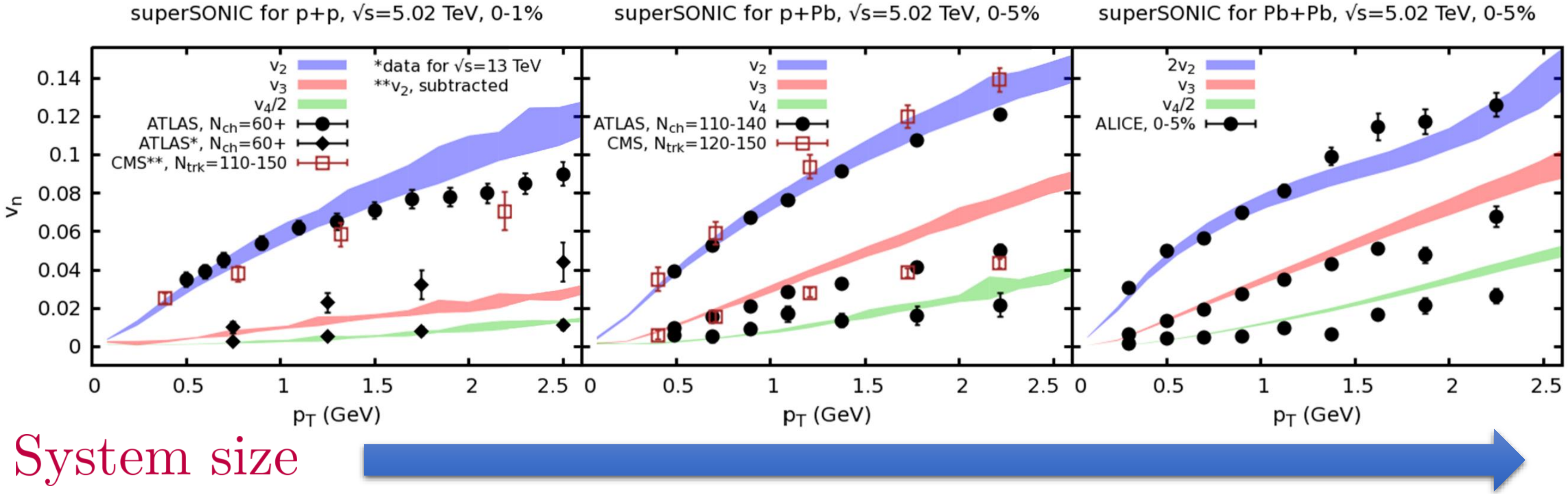




To the extreme: flow in small systems

- No separation of scale/ No statistical limit
- Hydro quantitatively describes small systems flow data





Weller R D and Romatschke P 2017 Phys. Lett. B774

Ulrich Heinz, J. Scott Moreland 2019 J.Phys.Conf.Ser.1271

So far so good!!..

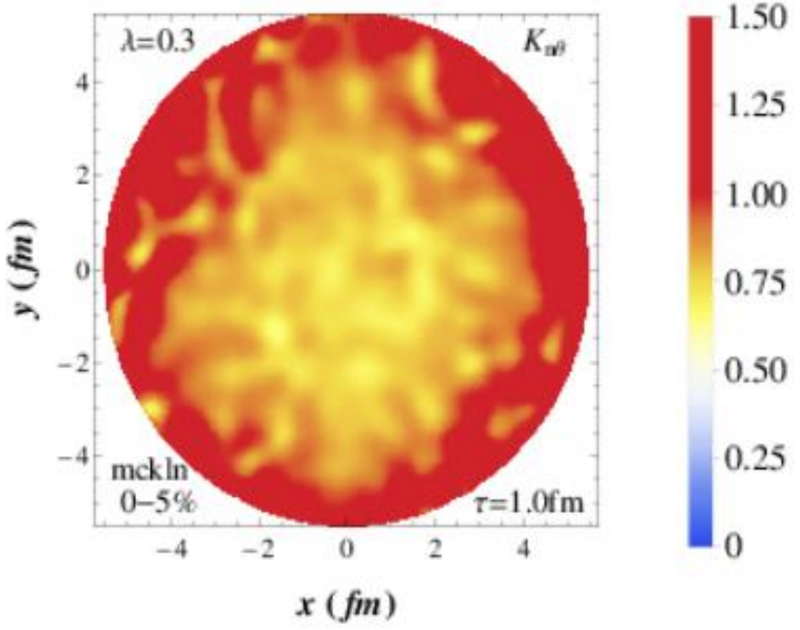
So far so good!!..

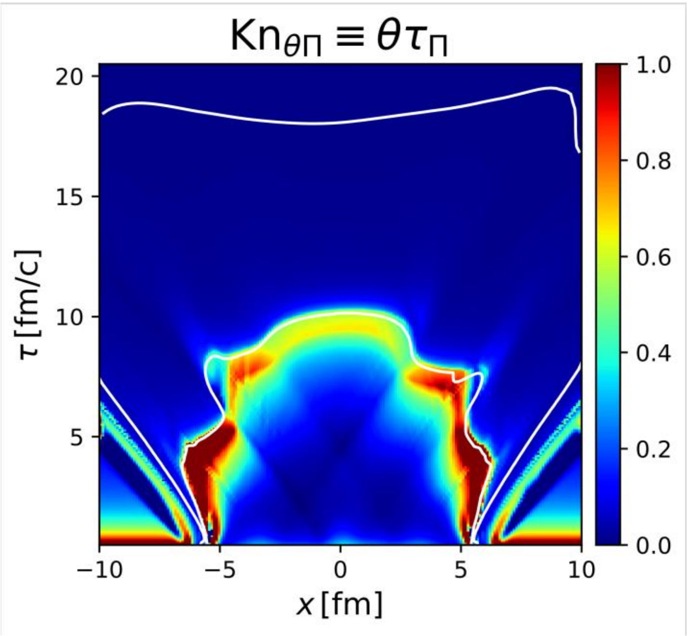
The paradigm: Hydrodynamics far from equilibrium

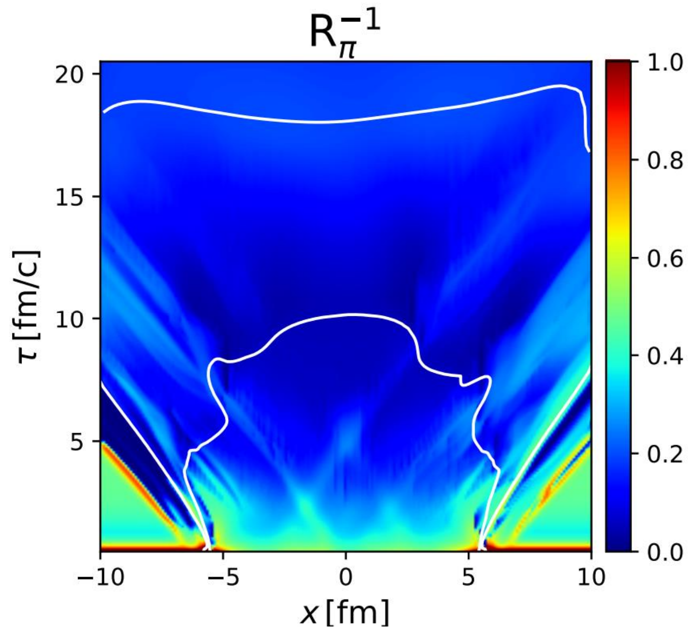
- Large out of equilibrium corrections near the edges and at early times!

$$\mathrm{Kn} = \frac{\text{microscopic scale}}{\text{macroscopic scale}} \leq 0.5$$

$$\mathrm{R}_{\pi}^{-1} = \frac{\sqrt{\pi^{\mu\nu}\pi_{\mu\nu}}}{P} \leq 0.5$$







$\sqrt{s} = 2.76$ TeV Pb+Pb

Noronha-Hostler, Noronha, Gyulassy PRC 93(2016)

$\sqrt{s} = 5.02$ TeV Pb+Pb

Bazow, Heinz, Strickland Comp. Phys. Comm. 225 (2018)

Effort to extend applicability of hydro out-of-equilibrium

Great ongoing theory efforts on the hydrodynamics framework side

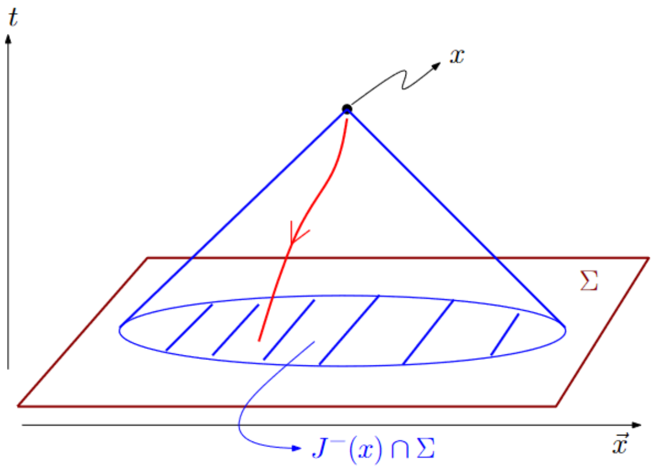
- Resummation of higher order terms into the transport coefficients
- Development of Pre-equilibrium models as an intermediate stage
- Hydro-dynamization and attractors, trajectories
- Comparison to microscopic theory to quantify the limits of hydrodynamics
- Origins of flow in small systems.

Let's go back and review the basics

Nonlinear causality constraints

- **Nonlinear** constraints for DNMR equations of motion Noronha et al [PRL 126 (2021), 222301]
- Nonlinear constraint all $2nd$ order transport:
  $\{\tau_\Pi, \tau_{\pi\pi}, \delta_{\pi\pi}, \delta_{\Pi\Pi}, \lambda_{\pi\Pi}, \lambda_{\Pi\pi}\}$

  Note: linear causality analysis constraint only $\{\zeta, \eta, \tau_\pi\}$.
- Causality implies $0 \leq v^2 \leq c^2$, so evolution equations **must**:
  (i) be hyperbolic $(v^2 \geq 0)$
  (ii) have no superluminal propagation $(v^2 \leq c^2)$



- can be investigated by determining the characteristic manifolds associated with a system of PDEs
- Inequalities which constrain the allowed dynamical configuration for the fluid. For example:

$$\varepsilon + P + \Pi + \Lambda_a - \frac{1}{2\tau_\pi}(2\eta + \lambda_{\pi\Pi}\Pi) - \frac{\tau_{\pi\pi}}{4\tau_\pi}(\Lambda_d + \Lambda_a) \geq 0, \quad a \neq d$$

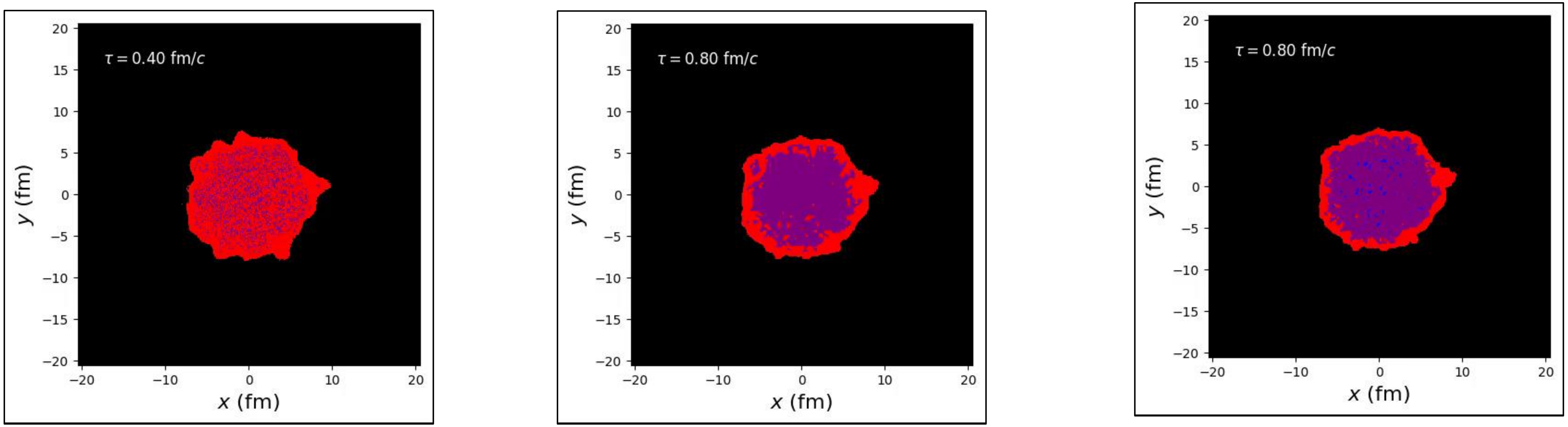
- **6 necessary and 8 sufficient can be checked for each fluid cell**

Non-linear causality analysis: explicit test

IPGlasma+MUSIC Pb+Pb 2.76 TeV

Plumberg, **DA**, Dore, Noronha,Noronha-Hostler, PRC105,(2021)

Chiu and Shen, PRC103,(2021)



Pure Hydro ($\tau_i = 0.4$ fm/c) FS + Hydro ($\tau_i = 0.8$ fm/c) KøMPøST + Hydro ($\tau_i = 0.8$ fm/c)

- At early times, 70% of fluid cells are **acausal**!
- Acausal regions on the edge persist until later times in the evolution
- Acausal regions are consistent with large $\{Kn, Re_n^{-1}\}$ criteria.
- pre equilibrium EKT reduces the acausal regions to $\sim 30\%$

Red: Acausal

Purple: Unknown

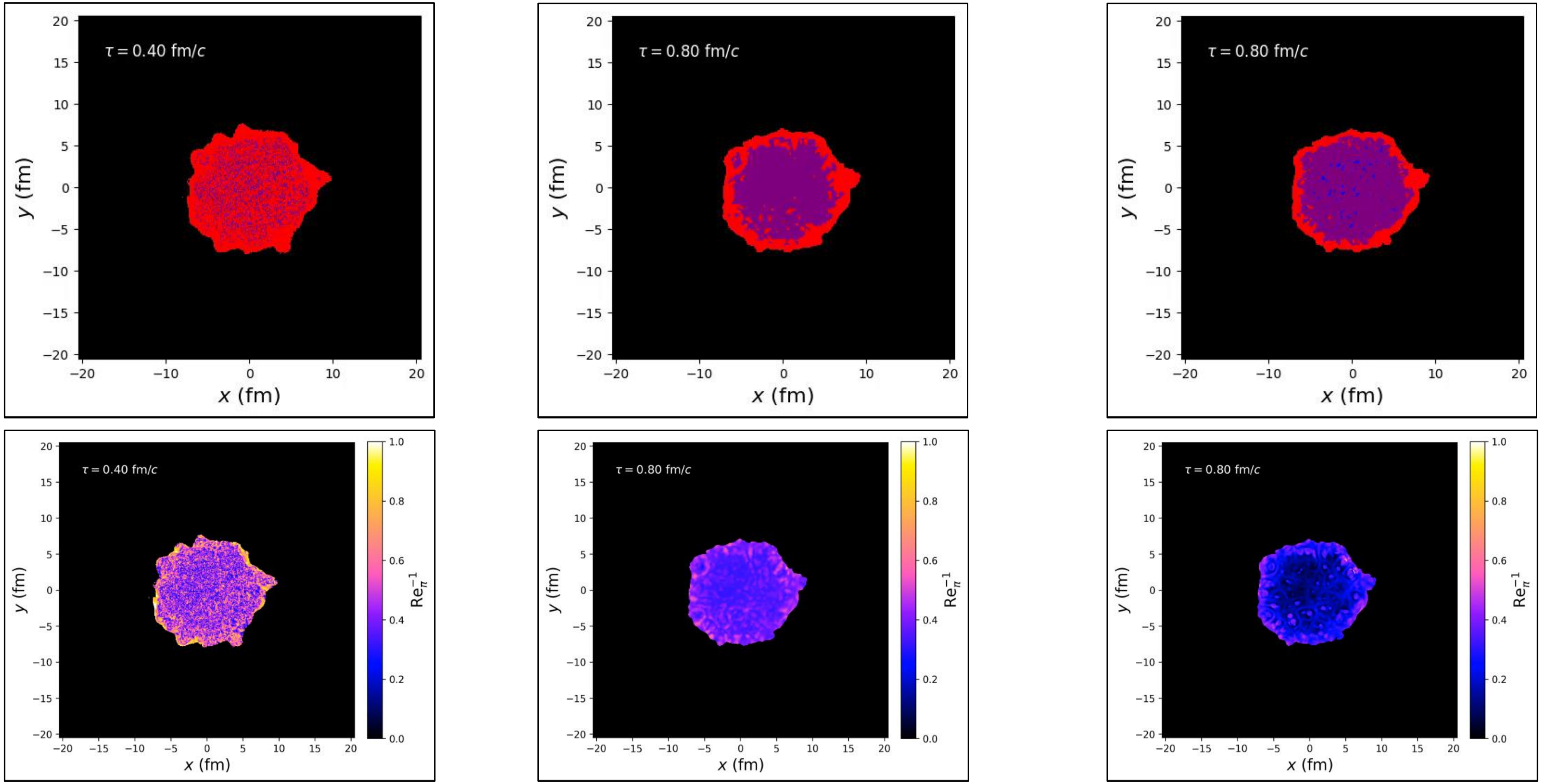
Blue: Causal

Non-linear causality analysis

IPGlasma+
MUSIC

Pb+Pb
@2.76 TeV

Re_π^{-1}



Hydrodyanmics | FS + hydrodynamics | EKT KøMPøST

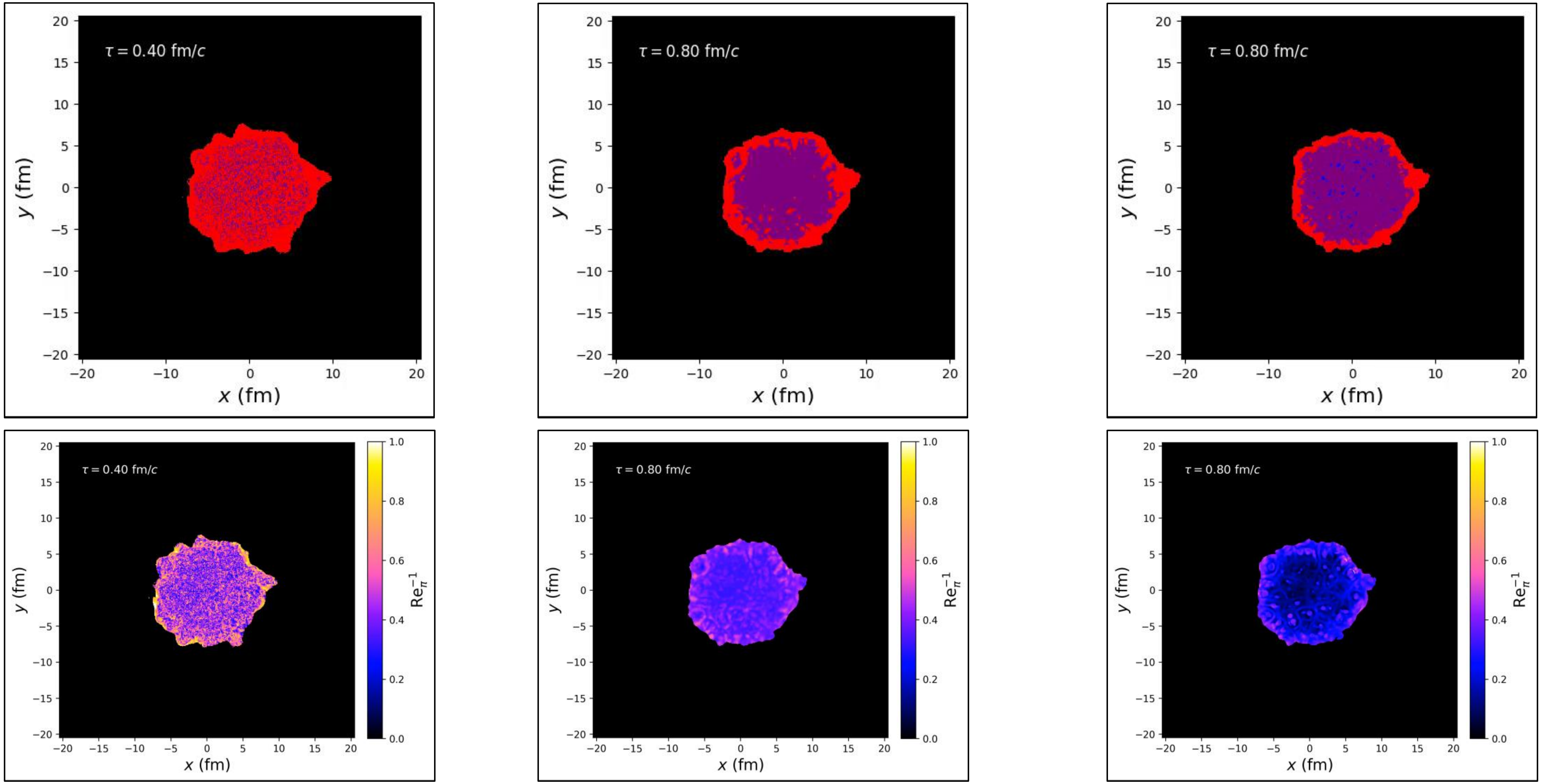
Non-linear causality analysis

We have a problem!

IPGlasma+
MUSIC

Pb+Pb
@2.76 TeV

Re_{π}^{-1}



Hydrodyanmics

FS + hydrodynamics

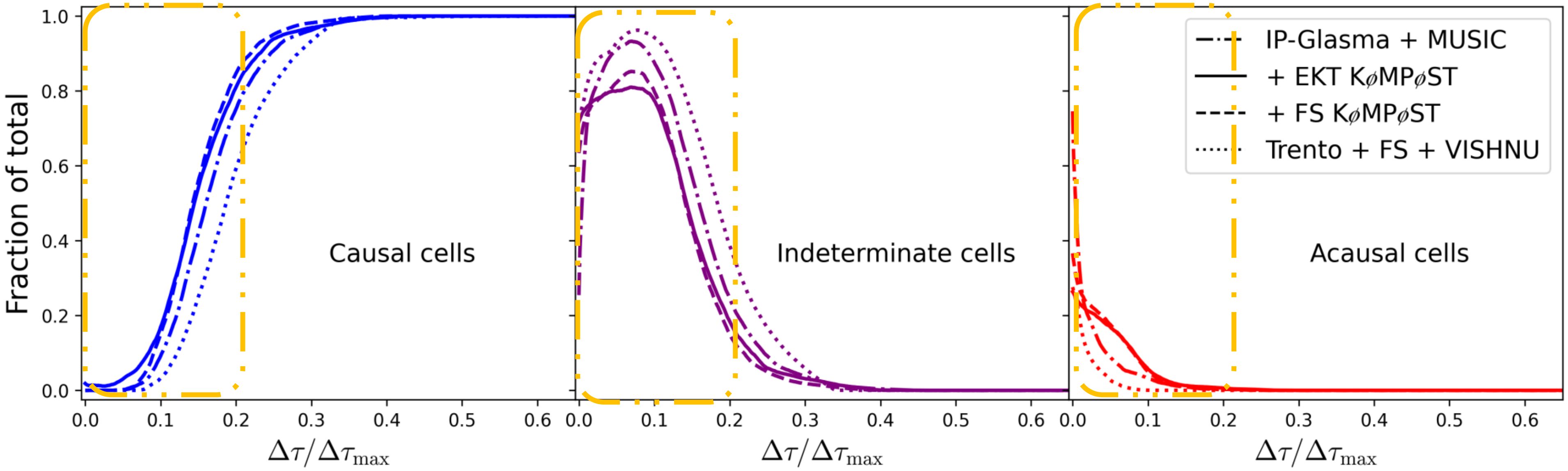
EKT KøMPøST

Quantification of the causality violations

Plumberg, **DA**, Dore, Noronha, Noronha-Hostler, PRC105,(2021)

- cell fraction

$$\Delta\tau \equiv \tau - \tau_0$$



- Most definite causality violations resolved in first 15% of evolution
- 50% of cells definitely causal after 20% of evolution ($2 - 3$ fm)
- System complete causal after 40% of evolution ($4 - 5$ fm)

Impact on observables

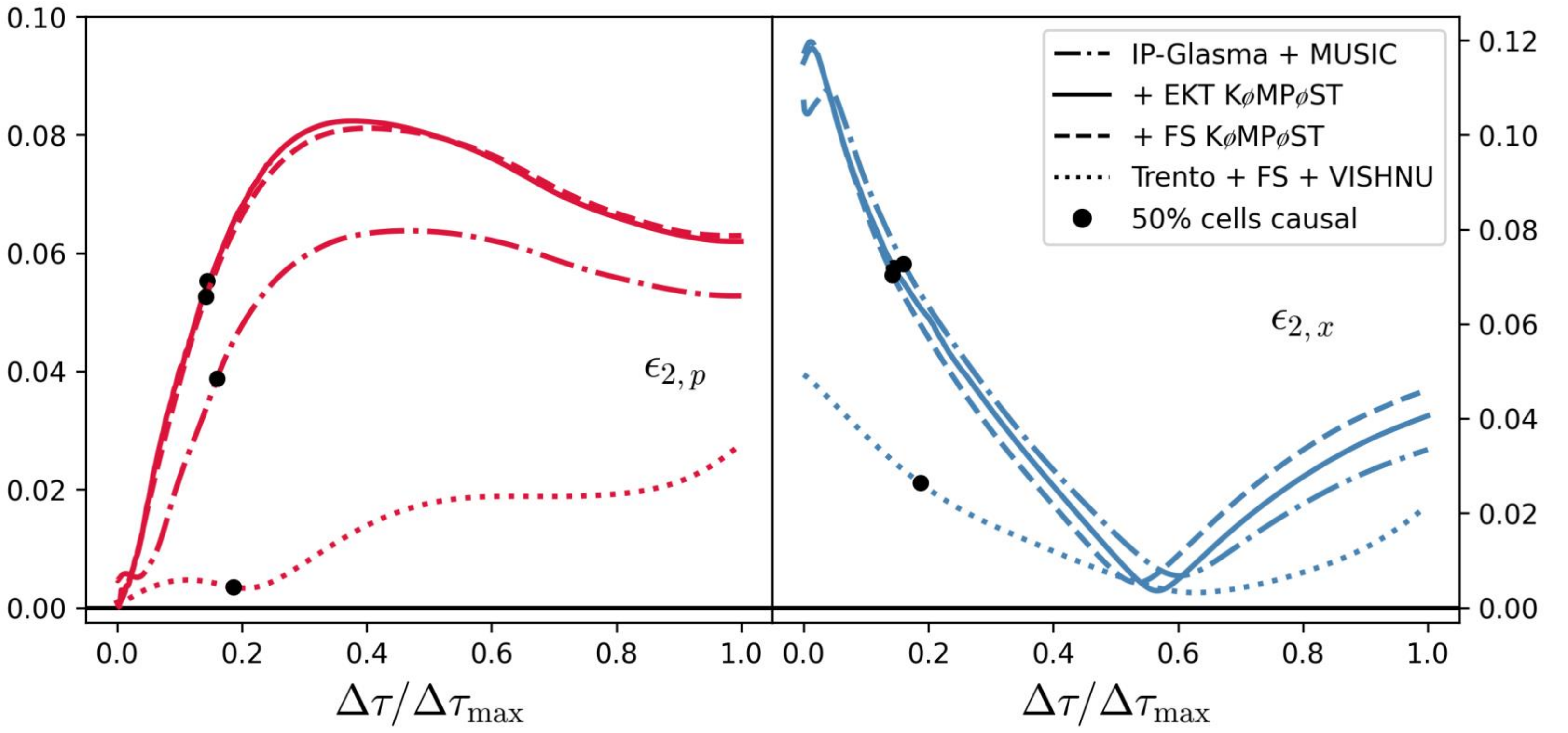
Plumberg, **DA**, Dore, Noronha, Noronha-Hostler, PRC105, (2021)

Signifcant potential development of flow when < 50% of fluid cells are causal

$$\epsilon_p = \left[\frac{\langle T^{xx} \rangle^2 - \langle T^{YY} \rangle^2 + 2\langle T^{xy} \rangle}{\langle T^{xx} \rangle^2 + \langle T^{yy} \rangle^2} \right]^{1/2}$$

$$\epsilon_x = \left[\frac{\langle x \rangle^2 - \langle y \rangle^2 + 2\langle xy \rangle^2}{\langle x \rangle^2 + \langle y \rangle^2} \right]^{1/2}$$

- Where do we go from here?



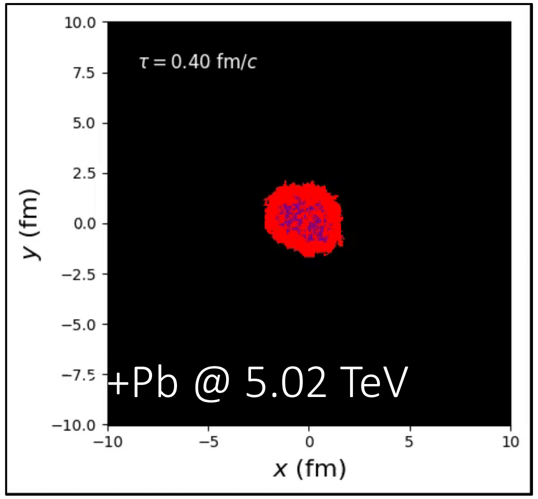
	Initial		Final		Initial		Final	
	$\epsilon_{2,x}$ [all]	$\epsilon_{2,x}$ [causal]	$\epsilon_{2,x}$ [all]	$\epsilon_{2,x}$ [causal]	$\epsilon_{2,p}$ [all]	$\epsilon_{2,p}$ [causal]	$\epsilon_{2,p}$ [all]	$\epsilon_{2,p}$ [causal]
VISHNU	0.0396	0.0515	0.0218	0.0347	0.00281	0.0238	0.0277	0.0348
MUSIC (EKT pre-eq.)	0.101	0.119	0.406	0.141	0.0177	0.0372	0.0620	0.0731
MUSIC (FS pre-eq.)	0.101	0.141	0.0461	0.161	0.0156	0.0253	0.0630	0.0918
MUSIC (no pre-eq.)	0.0997	0.120	0.0335	0.156	0.0074	0.0233	0.0528	0.0882

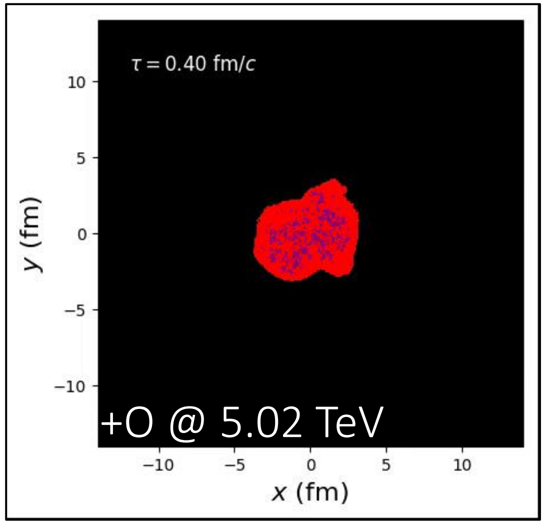
Non-linear causality in Small Systems

Plumberg, **DA**, Dore, Noronha,Noronha-Hostler, PRC105,(2021)

- The question in small systems goes beyond out of equilibrium corrections
- Criteria of separation of scale is no longer applicable
- Understanding the origin of flow could help us determine the region of applicability of hydrodynamics

IPGlasma+MUSIC





Chiu and Shen, PRC103,(2021)

Collision system	Transport coefficients	Violate necessary conditions	Violate sufficient conditions
30–40% Au+Au	Restricted DNMR with $\tau_{\Pi,1}$	1.8%	33%
	DNMR with $\tau_{\Pi,1}$	3.8%	22%
0–5% p+Au	Restricted DNMR with $\tau_{\Pi,1}$	9%	66%
	DNMR with $\tau_{\Pi,1}$	17%	48%
30–40% Au+Au	Restricted DNMR with $\tau_{\Pi,2}$	0.1%	14%
	DNMR with $\tau_{\Pi,2}$	1.7%	16%
0–5% p+Au	Restricted DNMR with $\tau_{\Pi,2}$	0.2%	25%
	DNMR with $\tau_{\Pi,2}$	7%	40%

Non-linear Causality analysis

T$_R$ENTo
free-streaming + VISHNU

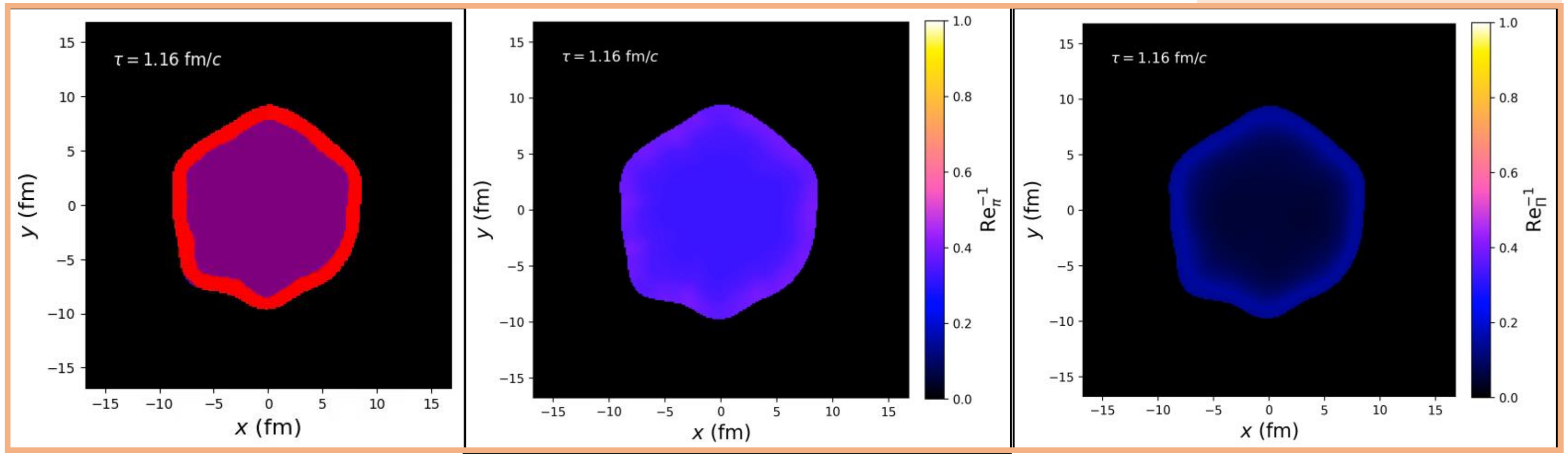
- The issue is not resolved in other initial state models
- It worsens for small systems

- Hints for particular issues?

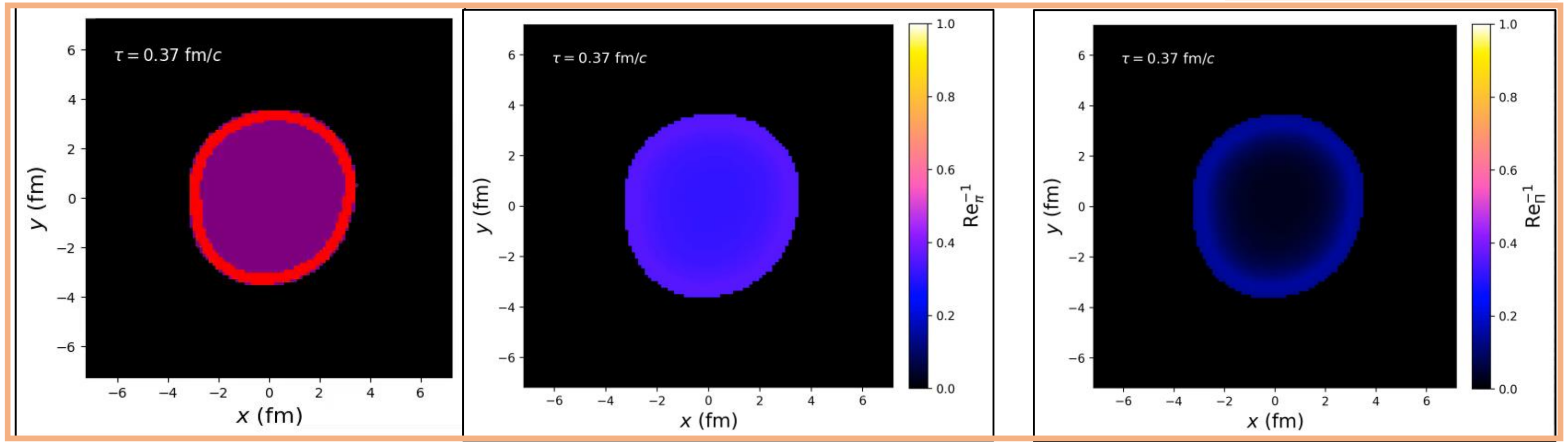
Conformality,

far from equilibrium?

Pb+Pb 2.76 TeV



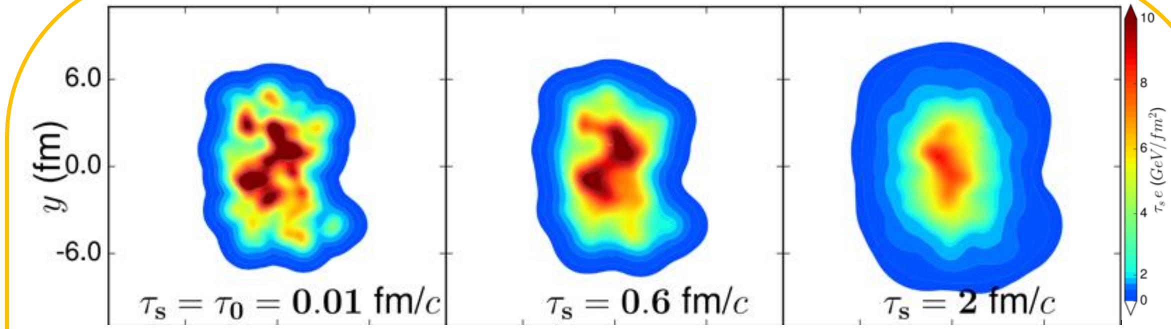
p+Pb 5.02 TeV

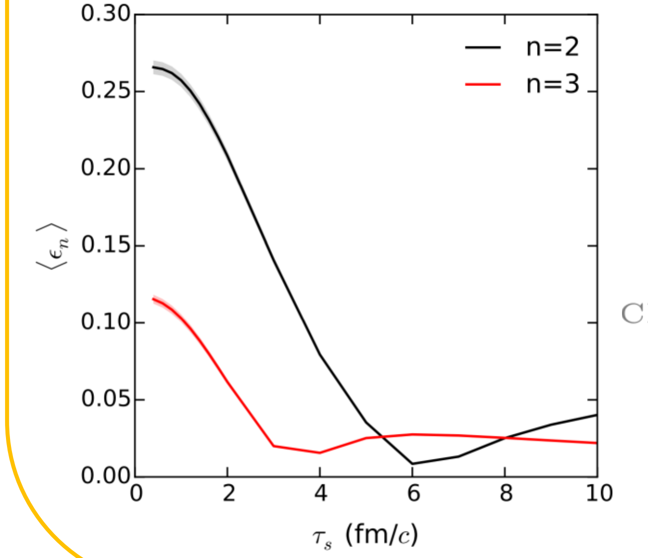


Re_π^{-1} Re_Π^{-1}

Status of Pre-equilibrium evolution

- A pre-equilibrium needed to smooth out gradients

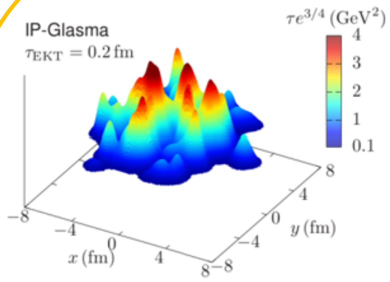


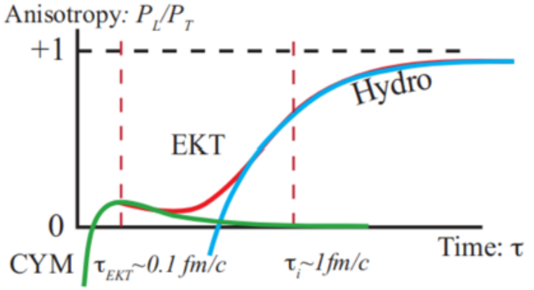


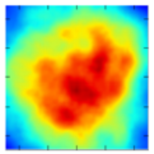
Free stream

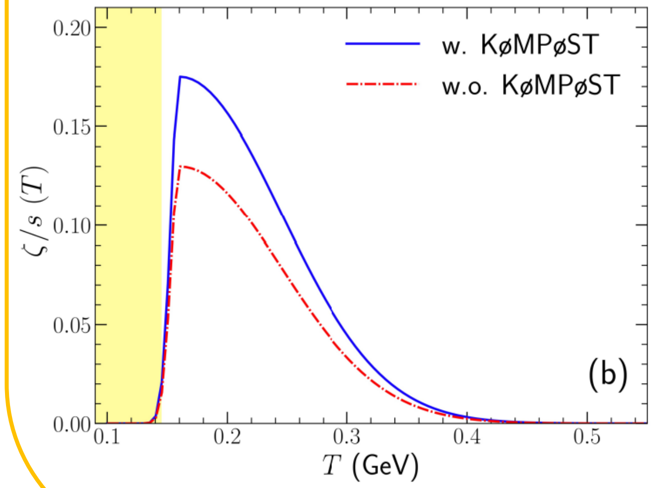
$$P_\mu \partial^\mu f = 0$$

Chun, Heinz, PRC92,(2015)









KoMPoST

$$P_\mu \partial^\mu f = \mathcal{C}$$

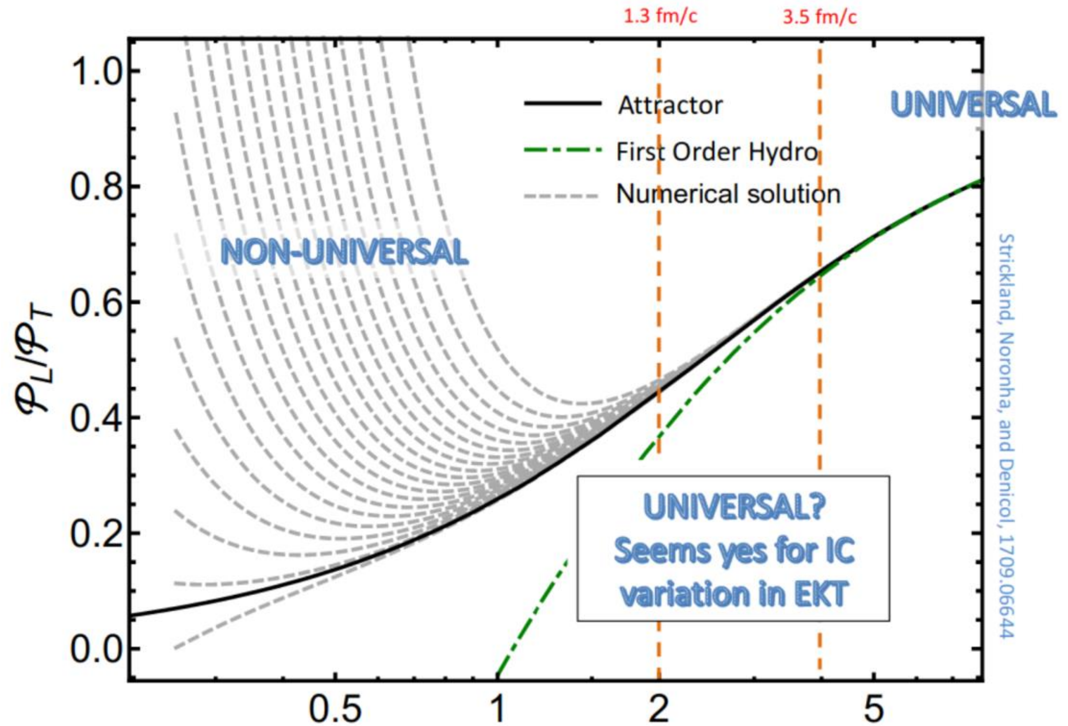
. Kurkela, Mazeliauskas, Paquet

S. Schlichting, Teaney, PRL. 122 (2019)

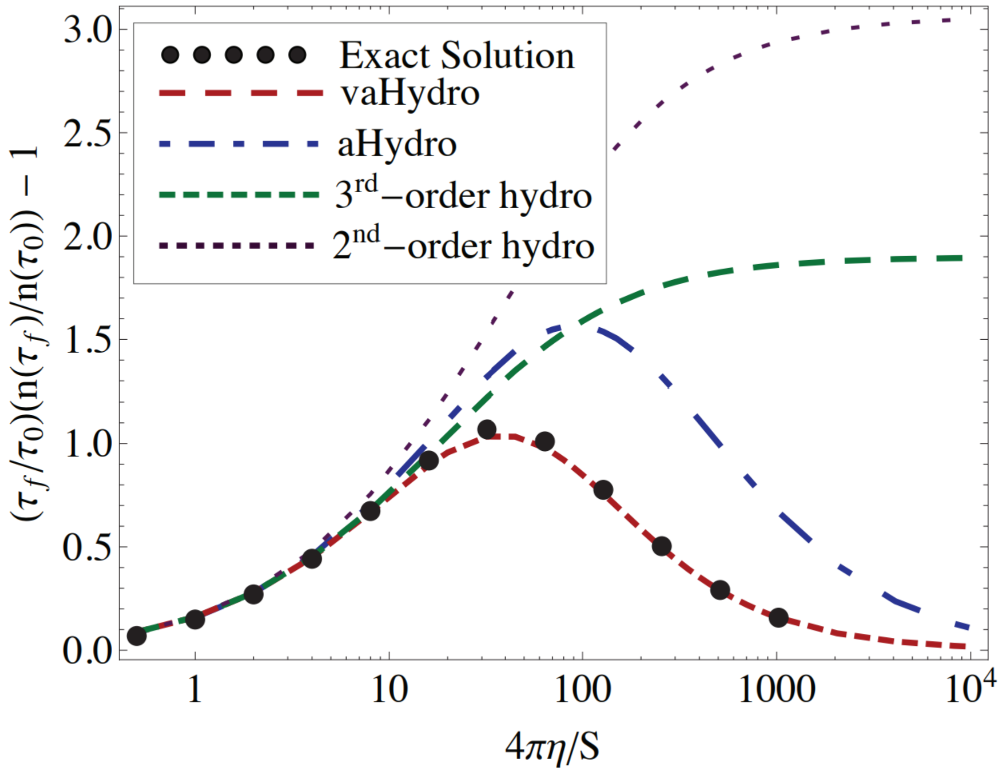
- Late initialization time impacts flow and transport

Emergence of hydrodynamics from microscopic theory

- Emergence of universal hydrodynamic behavior even far from equilibrium Heller, Spalinski, PRL 115,(2015)
- Towrads a non-perturbative description? Bazow, Heinz, and Strickland PRC90,(2014)]



$$\overline{\mathrm{W}} = \frac{\tau}{\tau_R} = \frac{\tau \mathrm{T}}{4\pi\,\bar{\eta}} = \mathrm{K_n}^{-1}$$



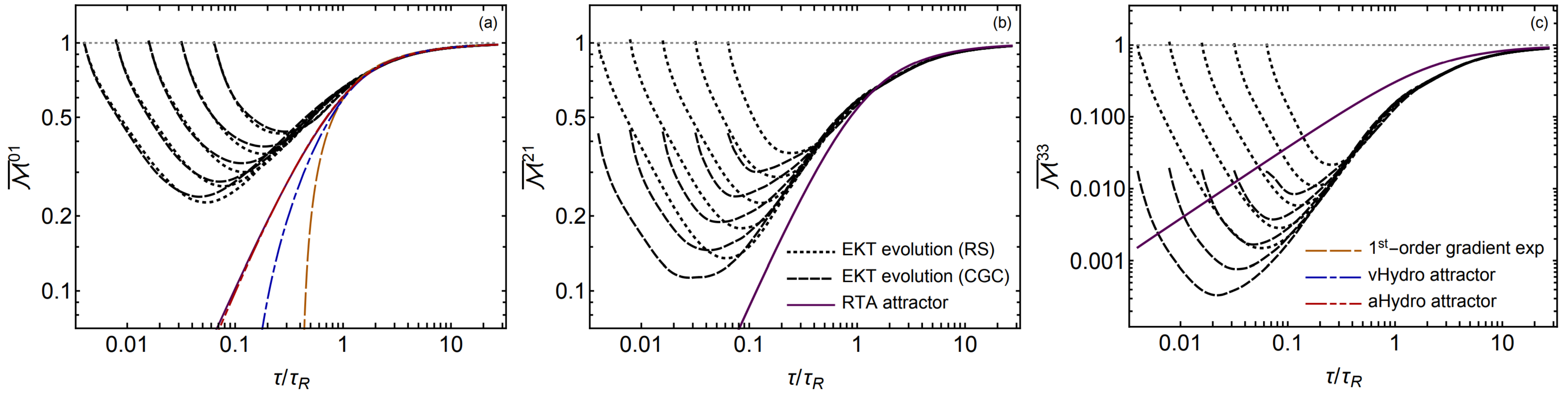
Emergence of hydrodynamics from microscopic theory

- Emergence of universal hydrodynamic behavior even far from equilibrium Heller, Spalinski, PRL 115,(2015)
- Towrads a non-perturbative description? Bazow, Heinz, and Strickland PRC90,(2014)]
- A non equilibrium attractor for higher moments of the distribution function? DA, Kurkela,Strickland PRL. 125, (2020)

τ/τ_R	τ
0.2	0.32 fm/c
0.5	0.86 fm/c
1	1.88 fm/c
2	4.23 fm/c
5	14.1 fm/c
10	38.5 fm/c

Pressure anisotropy

$\mathcal{M}^{01}[f] =< p^{-1}p_z^2 >$ $\mathcal{M}^{21}[f] =< p^{1}p_z^2 >$ $\mathcal{M}^{33}[f] =< p^{2}p_z^6 >$



Conclusions and outlook

- Hydrodynamics is a powerful tool in heavy ion collisions simulations
- Causality provides a fundamental constraint on the physics of heavy ion collisions.
- pre equilibrium is crucial to reduce uncertainties due to out-of-equilibrium corrections in the initial state
- Small system opens a new windows into investigating the domain of hydrodynamics behaviour

Challenges ..

- How well engineered are the current initial state models?. What physics are we possibly missing?
- Is there a path towards uncertainty quantification on hydrodynamics (theory and in URHIC simulations)?
- What could causality convey us on the non-thermal sector of the URHIC initial state?

Thank you for your attention

Checking causality: procedure

Step 1: Enforce preconditions for causality analysis

$$\zeta, \eta, \tau_\pi, \tau_\Pi, \tau_{\pi\pi}, \delta_{\pi\pi}, \delta_{\Pi\Pi}, \lambda_{\pi\Pi}, \lambda_{\Pi\pi}, \ldots \text{ are all positive}$$

Step 2: Get eigenvalues of shear stress tensor π^μ_ν, Λ_i:

$$\Lambda_0 = 0, \quad \Lambda_1 \leq \Lambda_2 \leq \Lambda_3 \text{ and } \Lambda_1 + \Lambda_2 + \Lambda_3 = 0$$

(follows from $\pi^\mu_\nu u^\nu = 0$ and $\mathrm{Tr}\,\pi = 0$)

Step 3: Evaluate necessary and sufficient conditions for causality in DNMR

Step 4: Assess hydrodynamic validity using

$$\mathrm{Re}_\pi^{-1} = \sqrt{\pi_{\mu\nu}\pi^{\mu\nu}}/(\varepsilon + P), \qquad \mathrm{Re}_\Pi^{-1} = |\Pi|/(\varepsilon + P)$$

DNMR: necessary conditions for causality

$$(2\eta + \lambda_{\pi\Pi}\Pi) - \frac{1}{2}\tau_{\pi\pi}|\Lambda_1| \geq 0$$

$$\varepsilon + P + \Pi - \frac{1}{2\tau_\pi}(2\eta + \lambda_{\pi\Pi}\Pi) - \frac{\tau_{\pi\pi}}{4\tau_\pi}\Lambda_3 \geq 0,$$

$$\frac{1}{2\tau_\pi}(2\eta + \lambda_{\pi\Pi}\Pi) + \frac{\tau_{\pi\pi}}{4\tau_\pi}(\Lambda_a + \Lambda_d) \geq 0, \quad a \neq d,$$

$$\varepsilon + P + \Pi + \Lambda_a - \frac{1}{2\tau_\pi}(2\eta + \lambda_{\pi\Pi}\Pi) - \frac{\tau_{\pi\pi}}{4\tau_\pi}(\Lambda_d + \Lambda_a) \geq 0, \quad a \neq d$$

$$\frac{1}{2\tau_\pi}(2\eta + \lambda_{\pi\Pi}\Pi) + \frac{\tau_{\pi\pi}}{2\tau_\pi}\Lambda_d + \frac{1}{6\tau_\pi}[2\eta + \lambda_{\pi\Pi}\Pi + (6\delta_{\pi\pi} - \tau_{\pi\pi})\Lambda_d] + \frac{\zeta + \delta_{\Pi\Pi}\Pi + \lambda_{\Pi\pi}\Lambda_d}{\tau_\Pi} + (\varepsilon + P + \Pi + \Lambda_d)c_s^2 \geq 0,$$

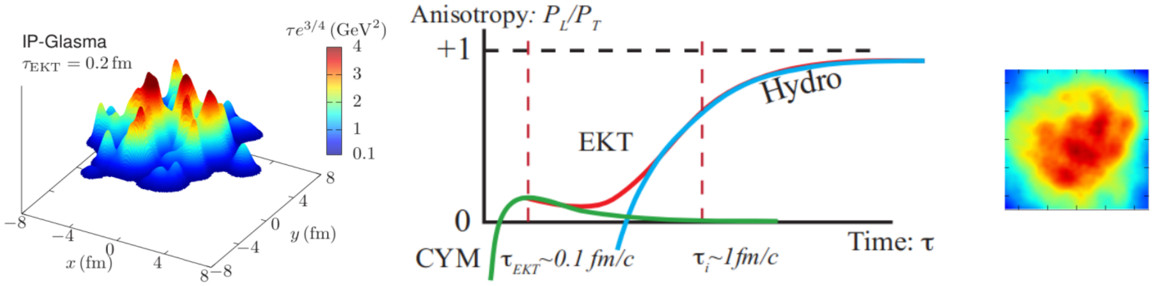
$$\varepsilon + P + \Pi + \Lambda_d - \frac{1}{2\tau_\pi}(2\eta + \lambda_{\pi\Pi}\Pi) - \frac{\tau_{\pi\pi}}{2\tau_\pi}\Lambda_d - \frac{1}{6\tau_\pi}[2\eta + \lambda_{\pi\Pi}\Pi + (6\delta_{\pi\pi} - \tau_{\pi\pi})\Lambda_d] - \frac{\zeta + \delta_{\Pi\Pi}\Pi + \lambda_{\Pi\pi}\Lambda_d}{\tau_\Pi} - (\varepsilon + P + \Pi + \Lambda_d)c_s^2 \geq 0,$$

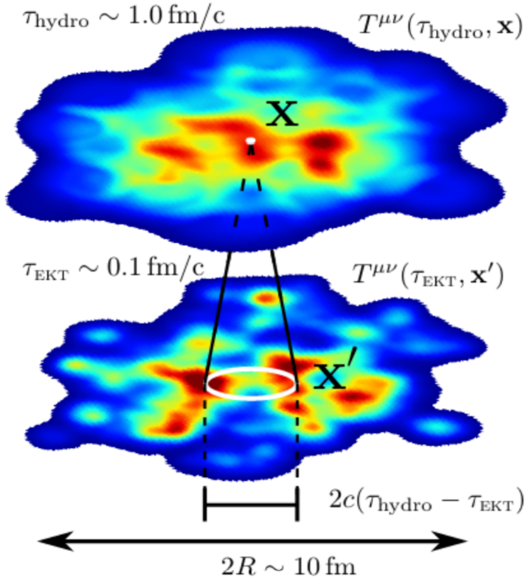
Total of six necessary conditions: if any conditions are violated, fluid cell is *guaranteed* to be **acausal**

KoMPoST

A. Kurkela, A. Mazeliauskas, J.F. Paquet, S. Schlichting, D. Teaney, Phys.Rev.Lett. 122 (2019) 12, 122302

- hydrodynamic model results are dependent on initialization time, and different hydrodynamic codes regulate these extreme initial conditions in different ad hoc ways





$$T^{\mu\nu}(\tau_{\text{EKT}}, \mathbf{x}') = \overline{T}^{\mu\nu}_{\mathbf{x}}(\tau_{\text{EKT}}) + \delta T^{\mu\nu}_{\mathbf{x}}(\tau_{\text{EKT}}, \mathbf{x}').$$

$$f_{\mathbf{x},\mathbf{p}} = \bar{f}_{\mathbf{p}} + \int \frac{d^2\mathbf{k}}{(2\pi)^2}\, \delta f_{\mathbf{k},\mathbf{p}}\, e^{i\mathbf{k}\cdot\mathbf{x}}.$$

$$T^{\mu\nu}(\tau_{\text{hydro}}, \mathbf{x}) = \overline{T}^{\mu\nu}_{\mathbf{x}}(\tau_{\text{hydro}}) + \frac{\overline{T}^{\tau\tau}_{\mathbf{x}}(\tau_{\text{hydro}})}{\overline{T}^{\tau\tau}_{\mathbf{x}}(\tau_{\text{EKT}})} \times \times \int d^2\mathbf{x}'\, G^{\mu\nu}_{\alpha\beta}(\mathbf{x}, \mathbf{x}', \tau_{\text{hydro}}, \tau_{\text{EKT}})\, \delta T^{\alpha\beta}_{\mathbf{x}}(\tau_{\text{EKT}}, \mathbf{x}').$$

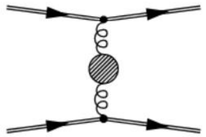
the subsequent hydrodynamic evolution becomes independent of the hydrodynamic initialization time !!

QCD medium at high temperatures: Effective kinetic theory AMY JHEP0301 (2003) 030

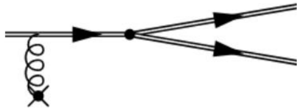
$$P^{\mu}\partial_{\mu} f_{q,g}(\mathbf{p}) = -C[f(\mathbf{p})], \qquad f_{q,g} \propto \frac{dN_{g,q}}{d^3x d^3p} \qquad g, q(u, d, s, \bar{u}, \bar{d}, \bar{s}))$$

$$\frac{df_{q,g}(\mathbf{p})}{d\tau} - \frac{p_z}{\tau}\partial_{p_z} f_{q,g}(\mathbf{p}) = -\mathcal{C}_{\mathbf{2\leftrightarrow 2}}[f_{q,g}(\mathbf{p})] - \mathcal{C}_{\mathbf{1\leftrightarrow 2}}[f_{q,g}(\mathbf{p})] \qquad \underline{0 + 1\text{d Bjorken}}$$

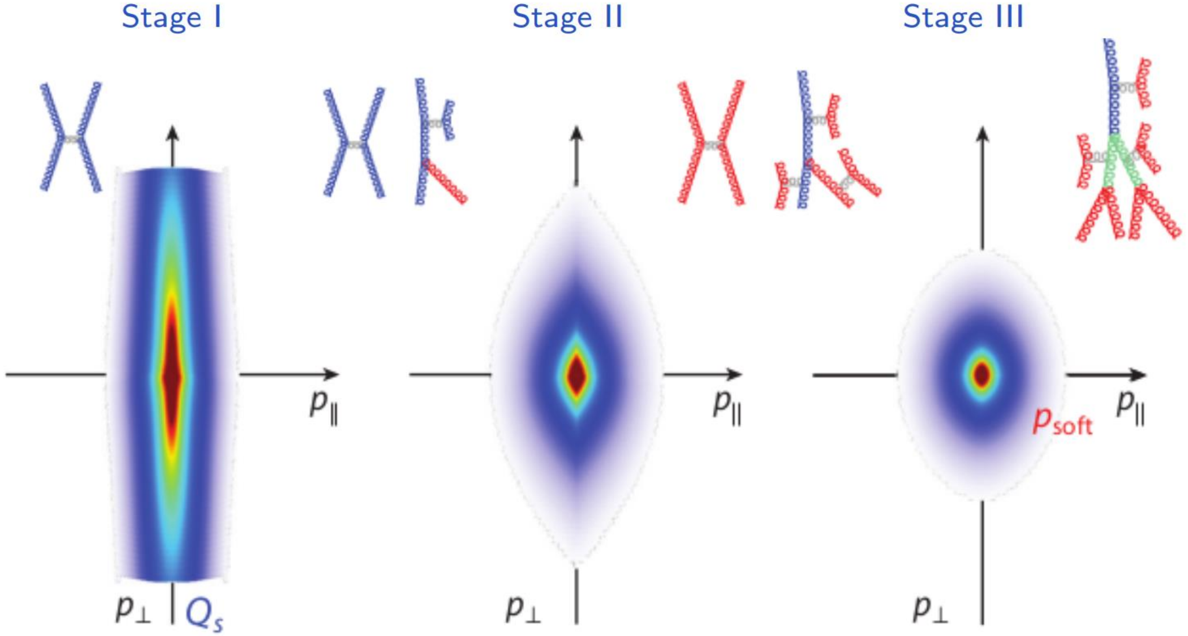
At Leading order, transport at different momentum scales in C[f]



regulated by HTL



LPM included



"Bottom up thermalisation"

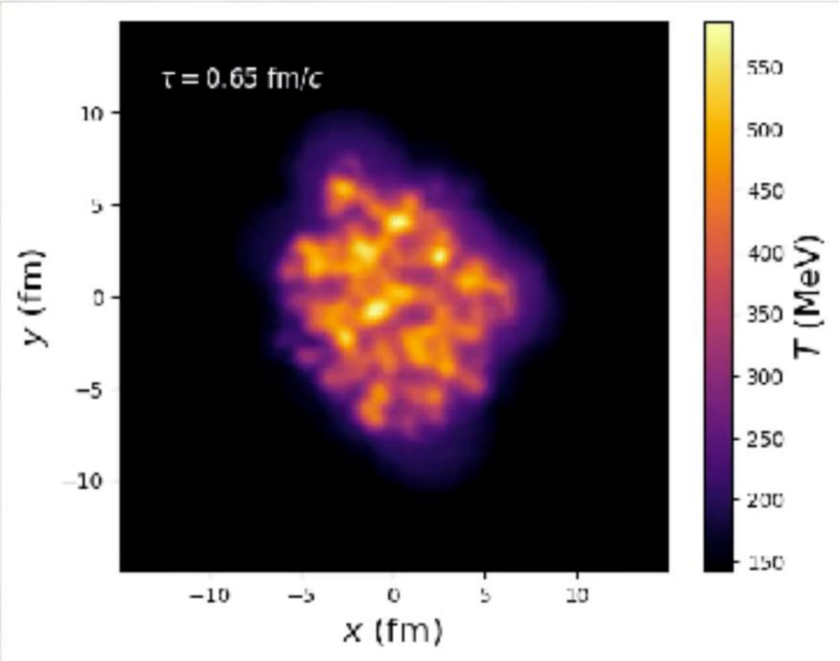
Baier, Mueller, Schiff, and Son (2001); J.Berges, M.Heller, A.Mazeliauskas and R.Venugopalan arxiv.2005.12299 (2020); Schlichting, Teaney, Ann. Rev. of Nuc Part. Sci.(2019); Arnold, P. Gorda, T. Iqbal, S. JHEP. 2020, 53

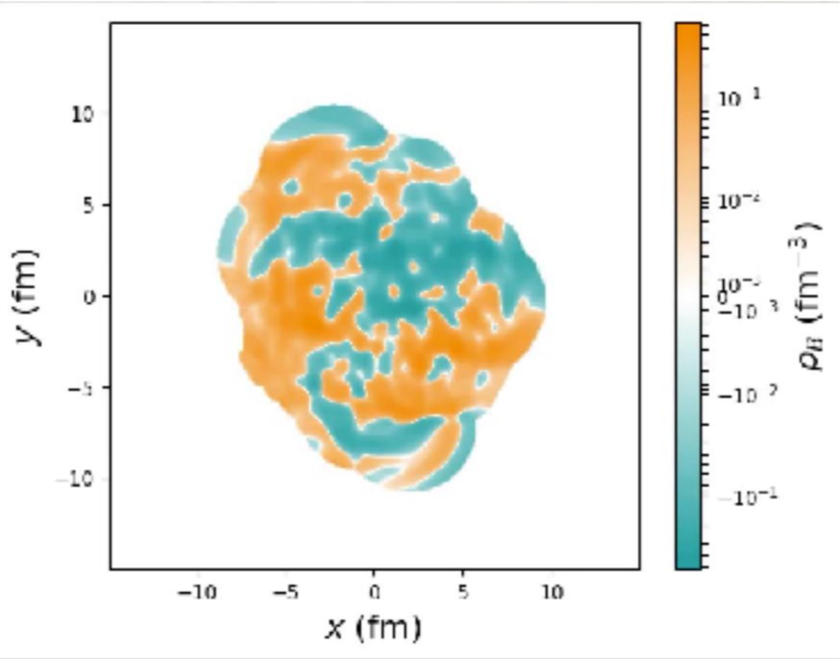
INITIAL STATE FLUCTUATIONS

- 4D Initial state input $[\varepsilon_0(\tau_0), \rho_0^B(\tau_0), \rho_0^S(\tau_0), \rho_0^Q(\tau_0)]$
- large shear stress at early time
- Gradients in 4D

$$\{\nabla P, \nabla \frac{\mu_B}{T}, \nabla \frac{\mu_S}{T}, \nabla \frac{\mu_Q}{T}\}$$

- A pre-equilibrium evolution for the charge densities





- Larger out of equilibrium corrections are expected at finite densities!

BSQ multi-component transient hydrodynamics

- How build a dissipative theory to be stable and causal? DA, Dore, Noronha-Hostler arxiv.2209-11210

$$S^\mu = \underbrace{s u^\mu}_{\text{ideal}} - \underbrace{\sum_q^{B,S,Q} \alpha_q n_q^\mu}_{\text{NS}} - \frac{1}{2}u^\mu \underbrace{\left(\beta_\Pi \Pi^2 + \beta_\pi \pi^{\mu\nu}\pi_{\mu\nu} + \sum_q^{B,S,Q} \beta_n^{qq'} n_q^\mu n_\mu^{q'}\right)}_{2^{\text{nd}}\text{-order}} - \underbrace{\sum_q^{B,S,Q} \left(\gamma_{n\Pi}^q n_q^\mu \Pi + \gamma_{n\pi}^q n_q^\nu \pi_\nu^\mu\right) - \frac{1}{2}\left(u^\nu \beta_{\Pi\pi} \Pi \pi_{\mu\nu}\right)}_{2^{\text{nd}}\text{-order coupling}}$$

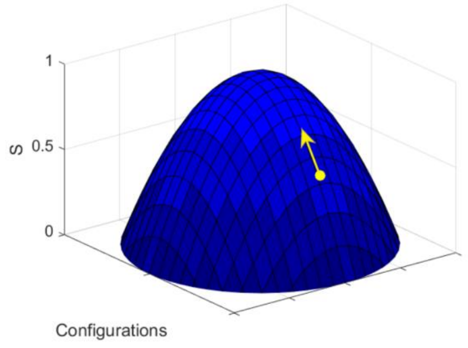
$$\nabla_\mu T^{\mu\nu} = 0$$
$$\nabla_\mu N_q^\mu = 0$$
$$+$$
$$\nabla_\mu S^\mu = \frac{\beta_0}{2\eta}\pi_{\mu\nu}\pi^{\mu\nu} + \frac{\beta_0}{\zeta}\Pi^2 + \frac{1}{\kappa_{qq'}} n_\mu^q n_{q'}^\mu \geq 0$$

linear Stability analysis

DA, Dore, Noronha-Hostler arxiv.2209-11210

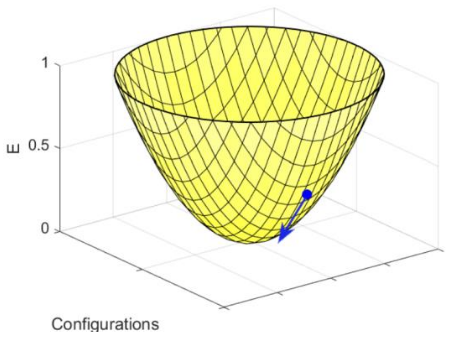
Key: Dissipation drives the system to equilibrium as $\tau \to \infty$

The entropy difference between the $S(\varphi + \delta\varphi) - S(\varphi)$ can be quantified as an information current E^μ



E is the net information carried by the perturbation

- $E^\mu n_\mu \geq 0$ E^μ is time/light like (connection to causality!)
- $E^\mu = 0; \ \forall$ $\delta\varphi = 0$
- $\nabla_\mu E^\mu \leq 0$ Information are lost in time



Task: Determine the form of the Lyapunov energy functional

See: Gavassino Class. Quant.Grav. 38 (2021)

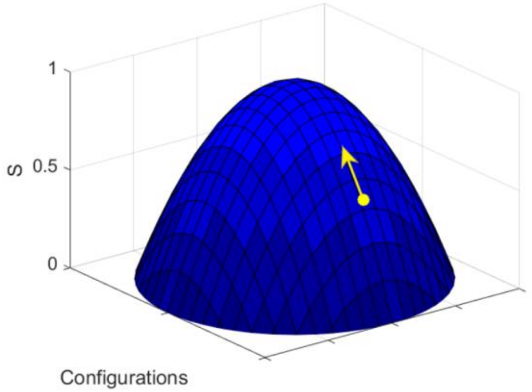
Applicability||Stability analysis of BSQ Isreal-Stewart

DA, Dore, Noronha-Hostler (In preparation)

- Dissipation leads thermodynamic systems to converge to the equilibrium state as $\tau \to \infty$

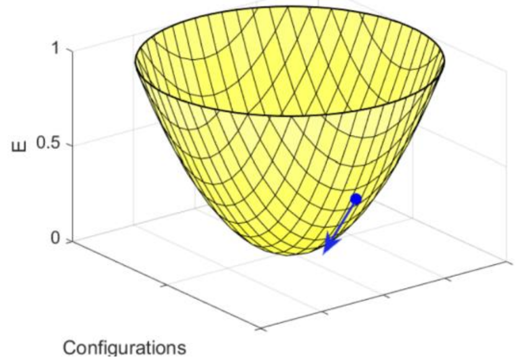
Lyapunov Functional

- $\partial_\mu s^\mu \geq 0$
- $s(\Omega_i) \leq s(\Omega_{eq}) \qquad \forall \qquad \Omega_i$
- s is max only when $\Omega_i = \Omega_{eq} \Rightarrow \; s_{eq}$ is unique



- For an isolated system, this total entropy is a positive definite function of time $\to$ if we evaluate it on any later surface its value must increase

$$S(\Sigma') - S(\Sigma) = \int_\Sigma \nabla_\mu S^\mu dV$$



Task: Determine the form of the Lyabunov energy functional

See also: Gavassino Class. Quant. Grav. 38 (2021) 21, for single charge analysis in Ekart frame

linear Stability analysis

- Determine the form of the Lyabunov energy functional

  - $E^{\mu} n_{\mu} \geq 0$ $\quad$ E^{μ} is time/light like (connection to causality!)
  - $E^{\mu} = 0; \ \forall$ $\quad$ $\delta\varphi = 0$
  - $\nabla_{\mu} E^{\mu} \leq 0$ $\quad$ Information are lost in time

$$(\mathrm{I}) \quad \frac{1}{\varepsilon + p} \frac{\partial \varepsilon}{\partial p}\bigg|_s \geq 0 \qquad \frac{1}{\varepsilon + p} \frac{\partial \varepsilon}{\partial s}\bigg|_p \frac{\partial p}{\partial s}\bigg|_{\alpha_B} \geq 0$$

Thermodynamics constraints

$$(\mathrm{II}) \quad \beta_{\Pi} \geq 0 \qquad \beta_{\pi} \geq 0$$

Transport constraints

$$(\mathrm{IV}) \quad \frac{\kappa_n^{BB}}{2\lambda^2} - \frac{(\gamma_{n\Pi}^B)^2}{\beta_{\Pi}} - \frac{(\gamma_{n\pi}^B)^2}{\beta_{\pi}} \geq (\varepsilon + p)\left(T^2 \frac{\partial s}{\partial \varepsilon}\bigg|_p \frac{\partial s}{\partial p}\bigg|_{\alpha_B} \left(\frac{\partial \alpha_B}{\partial s}\bigg|_p\right)^2 - \frac{1}{4} \frac{\partial p}{\partial \mathcal{E}}\bigg|_s \left(\frac{\partial \alpha_B}{\partial p}\bigg|_s\right)^2\right)$$

+ 3 constraints!

Linear stability and causality constraints on BSQ hysrodynamics