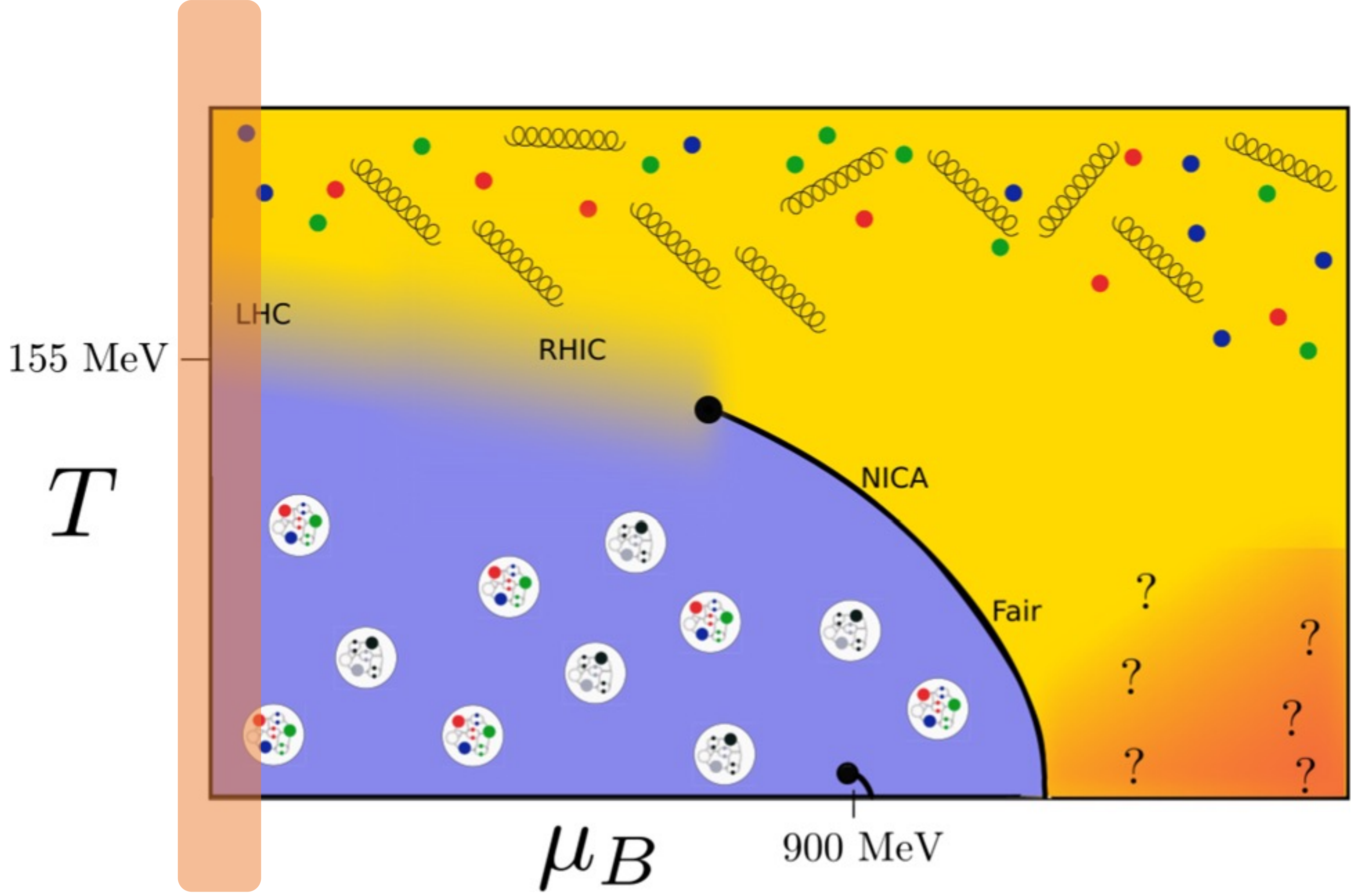
Heavy hadron spectroscopy at T>0

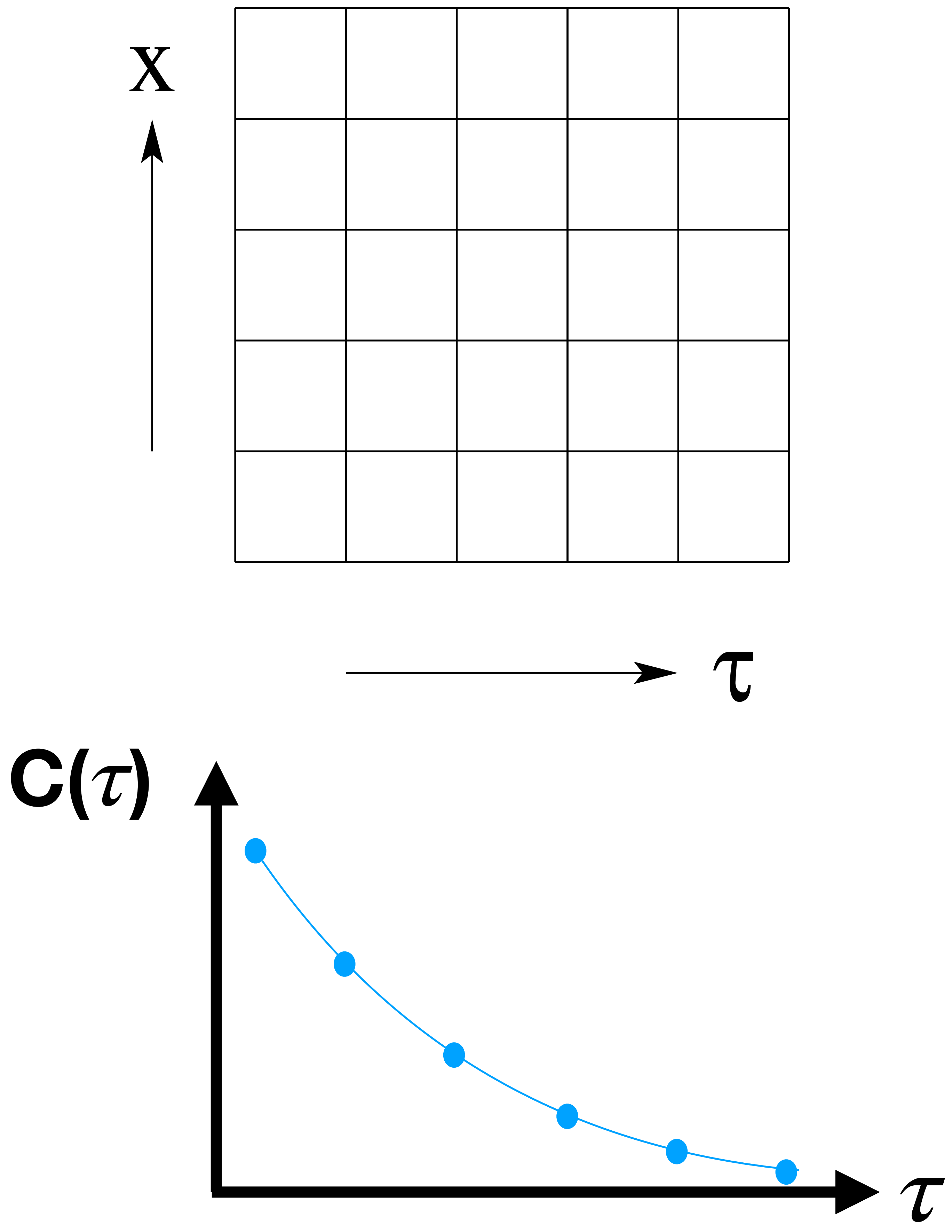
FASTSUM Collaboration

Gert Aarts, Chris Allton, Muhammad Anwar, Ed Bennett, Ryan Bignell, Tim Burns, Sergio Chaves, Alan Kirby, Sam Offler, Ben Page, Thomas Spriggs, Dawid Stasiak	*Swansea University, UK*
Simon Hands	*University of Liverpool, UK*
Benjamin Jäger	*Southern Denmark University, Denmark*
Seyong Kim	*Sejong University, Korea*
Maria Paola Lombardo	*INFN, Florence, Italy*
Sinead Ryan	*Trinity College, Dublin, Ireland*
Jon-Ivar Skullerud, Jesuel Marques, Dale Lawlor	*National University of Ireland, Maynooth, Ireland*
Liang-Kai Wu	*Jiangsu University, China*
Felix Ziegler	*University of Edinburgh, UK*

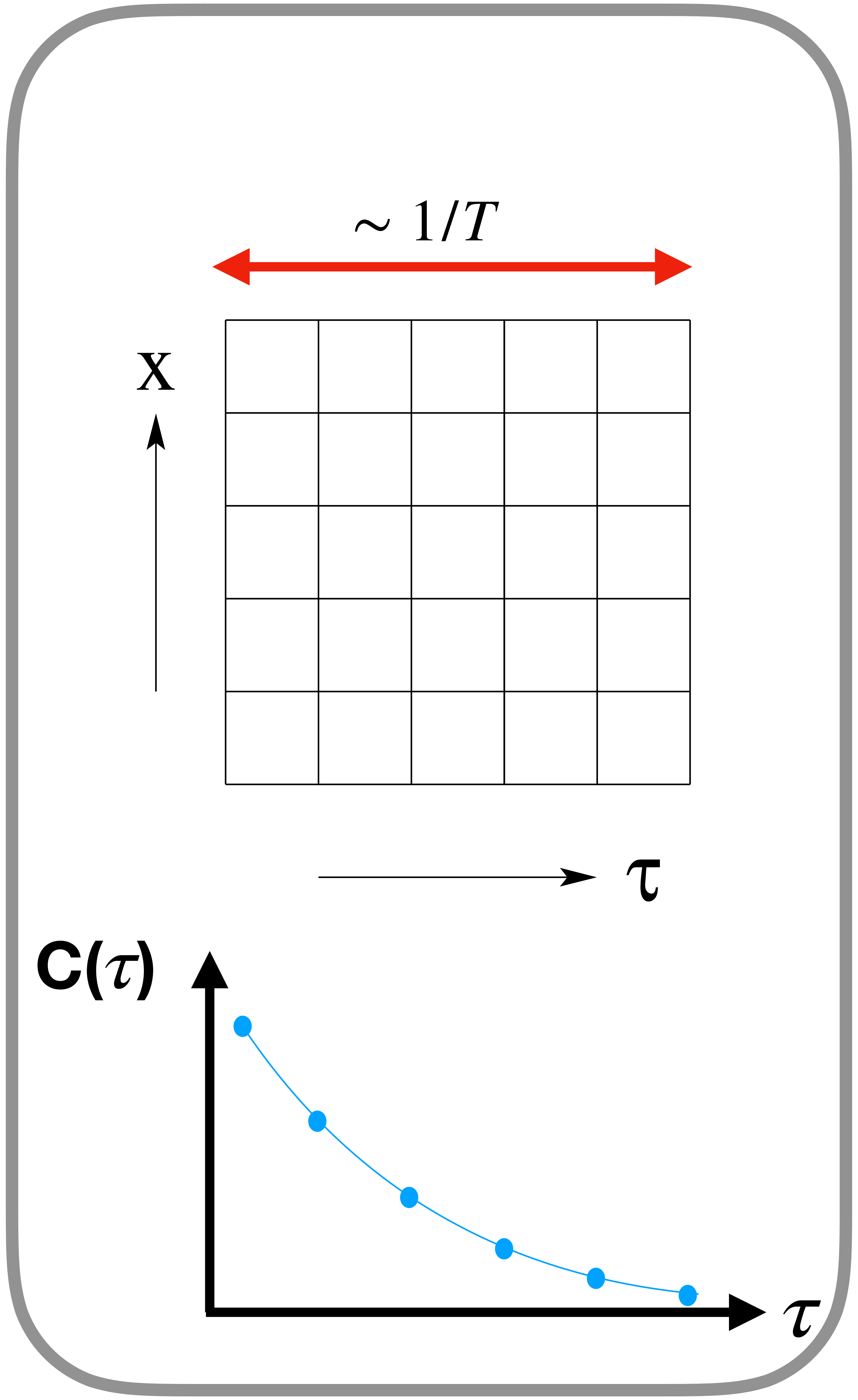
Overview

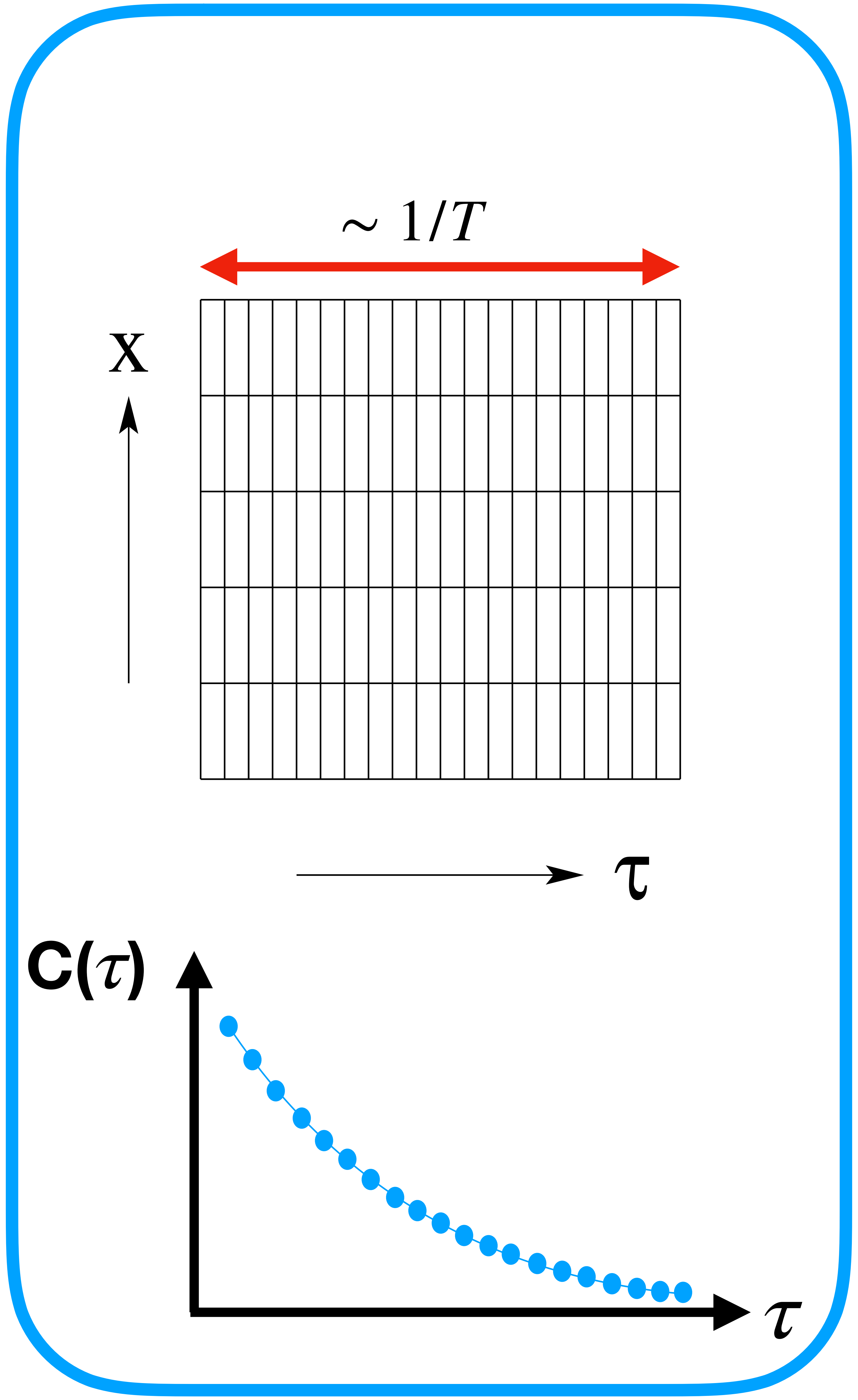
- Overview of FASTSUM approach
- Charmed meson spectrum
- Charmed baryonic spectrum
- Bottomonium (NRQCD) spectrum




x
τ
C(τ)
τ

Our Lattice Setup: *Anisotropic*





$$T = \frac{1}{L_\tau}$$

$$= \frac{1}{a_\tau N_\tau}$$

Spectral Quantities:

Bottomonium

Charmed mesons

Heavy Baryons

Light Hadrons

Interquark potential

Conductivity

...

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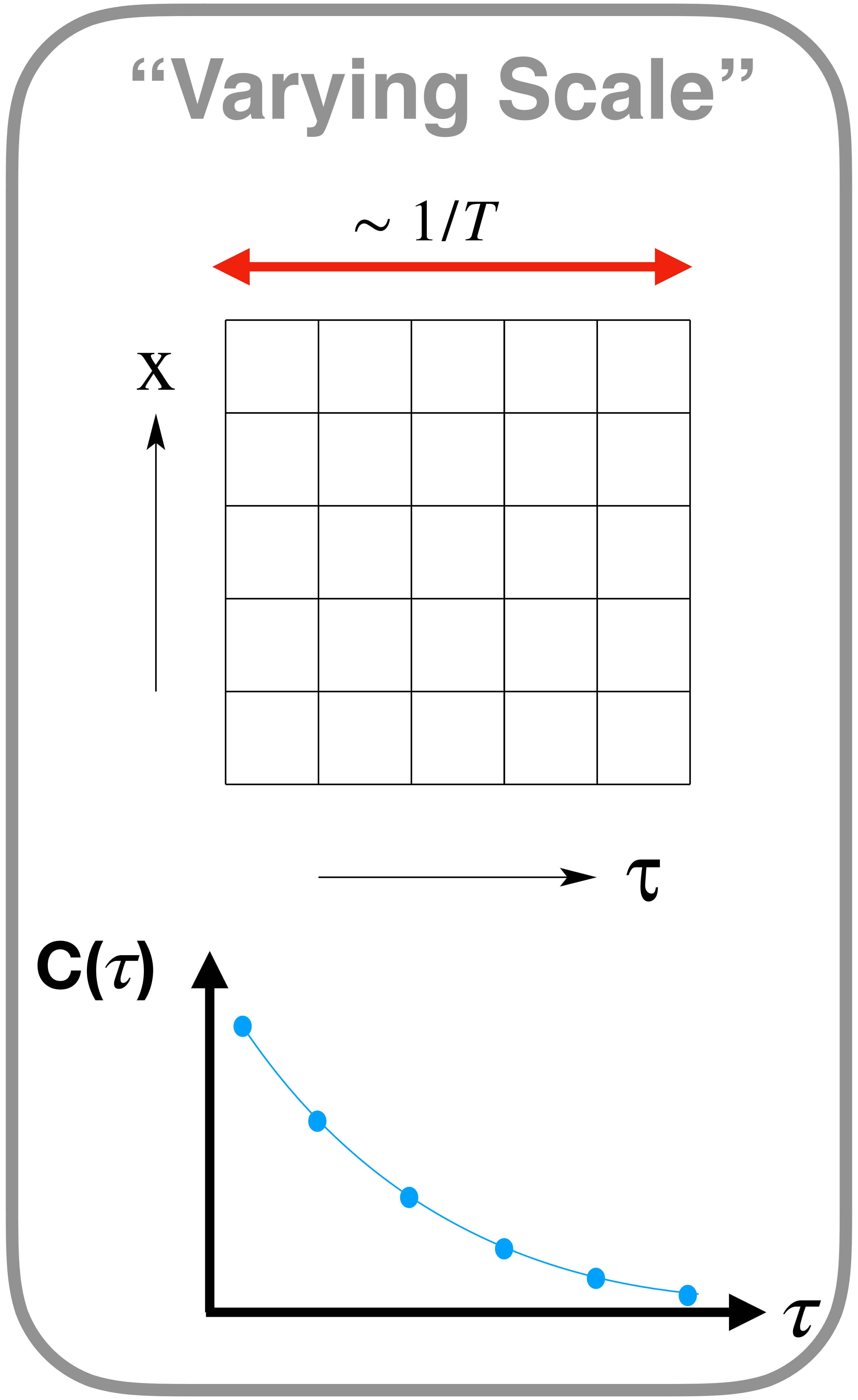

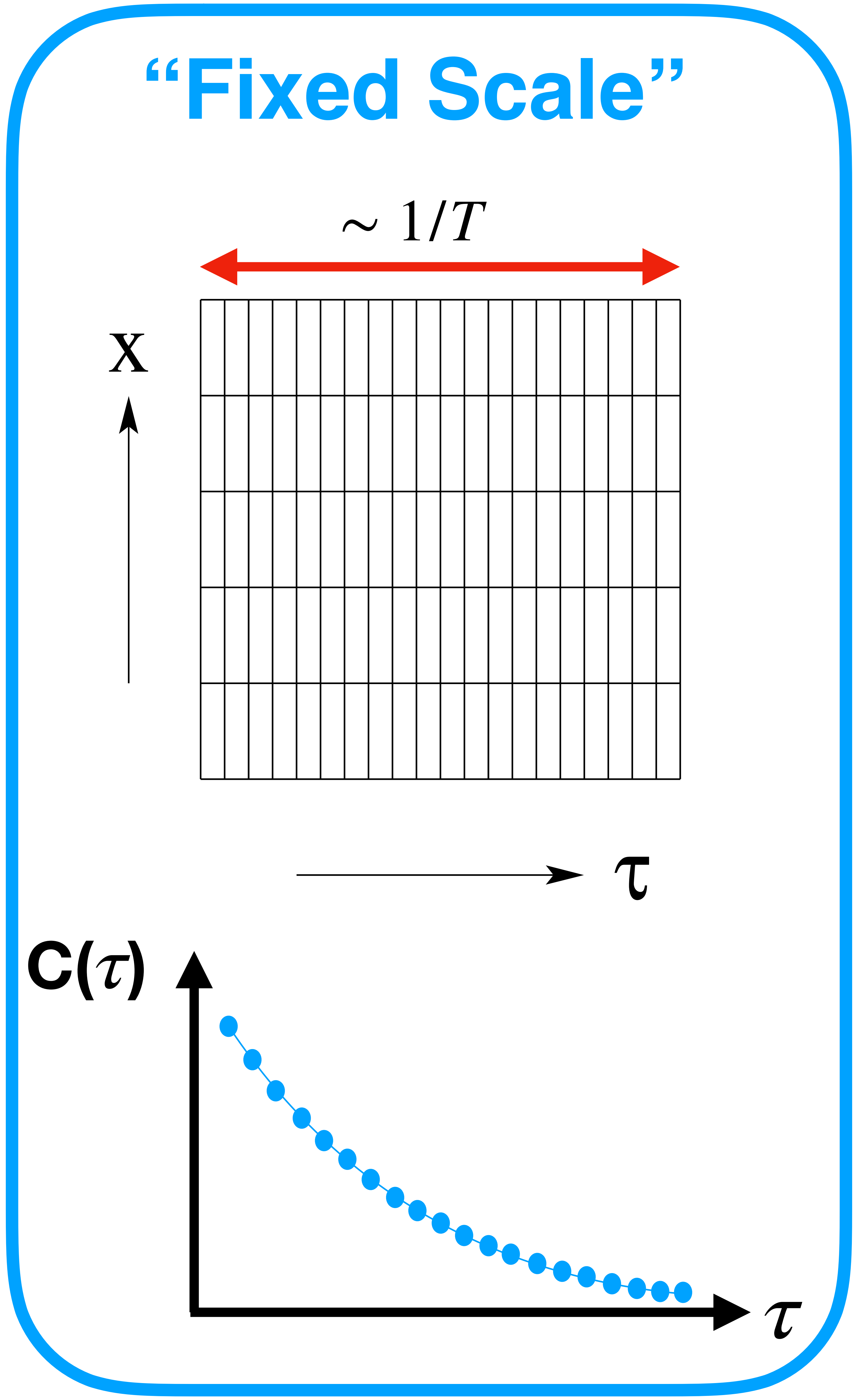

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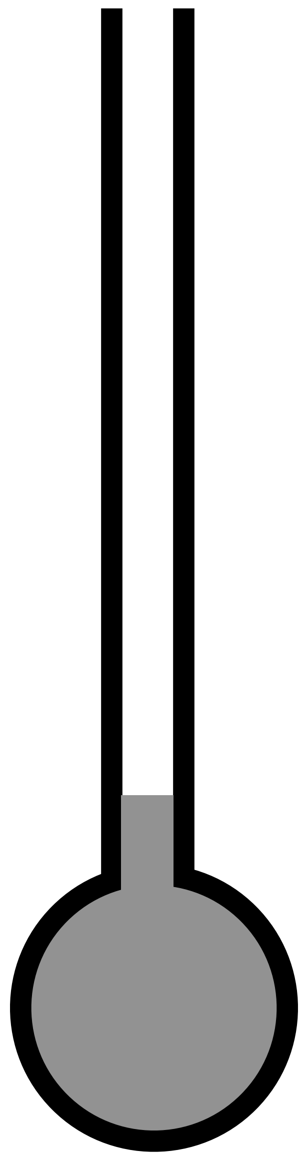

Going hotter…

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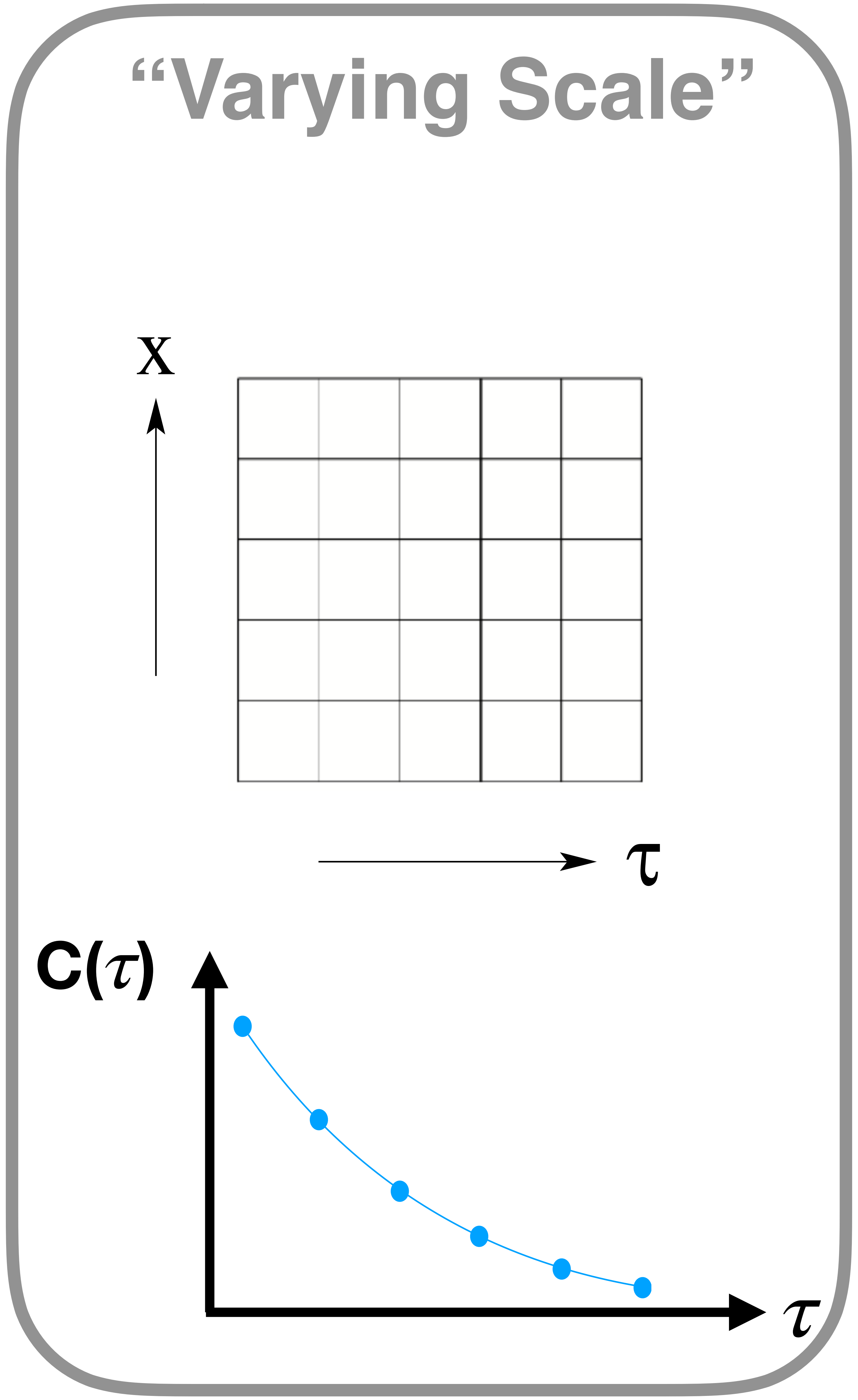

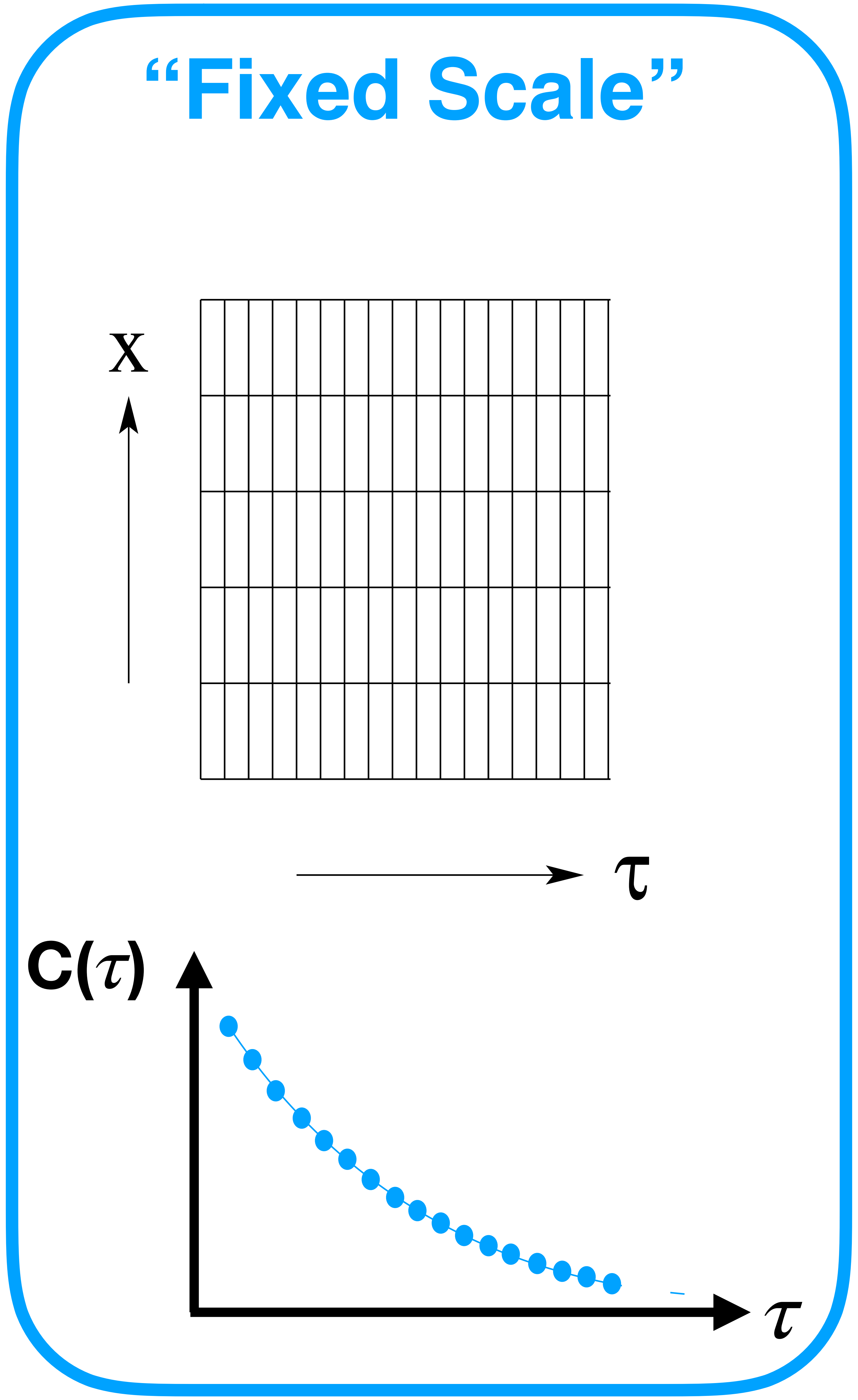

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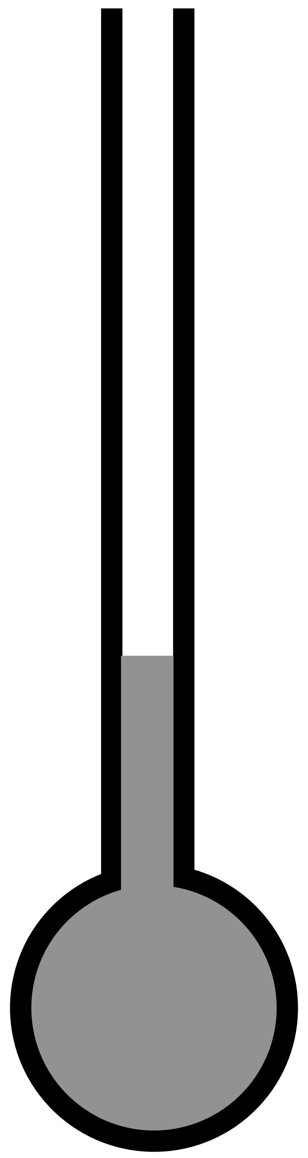
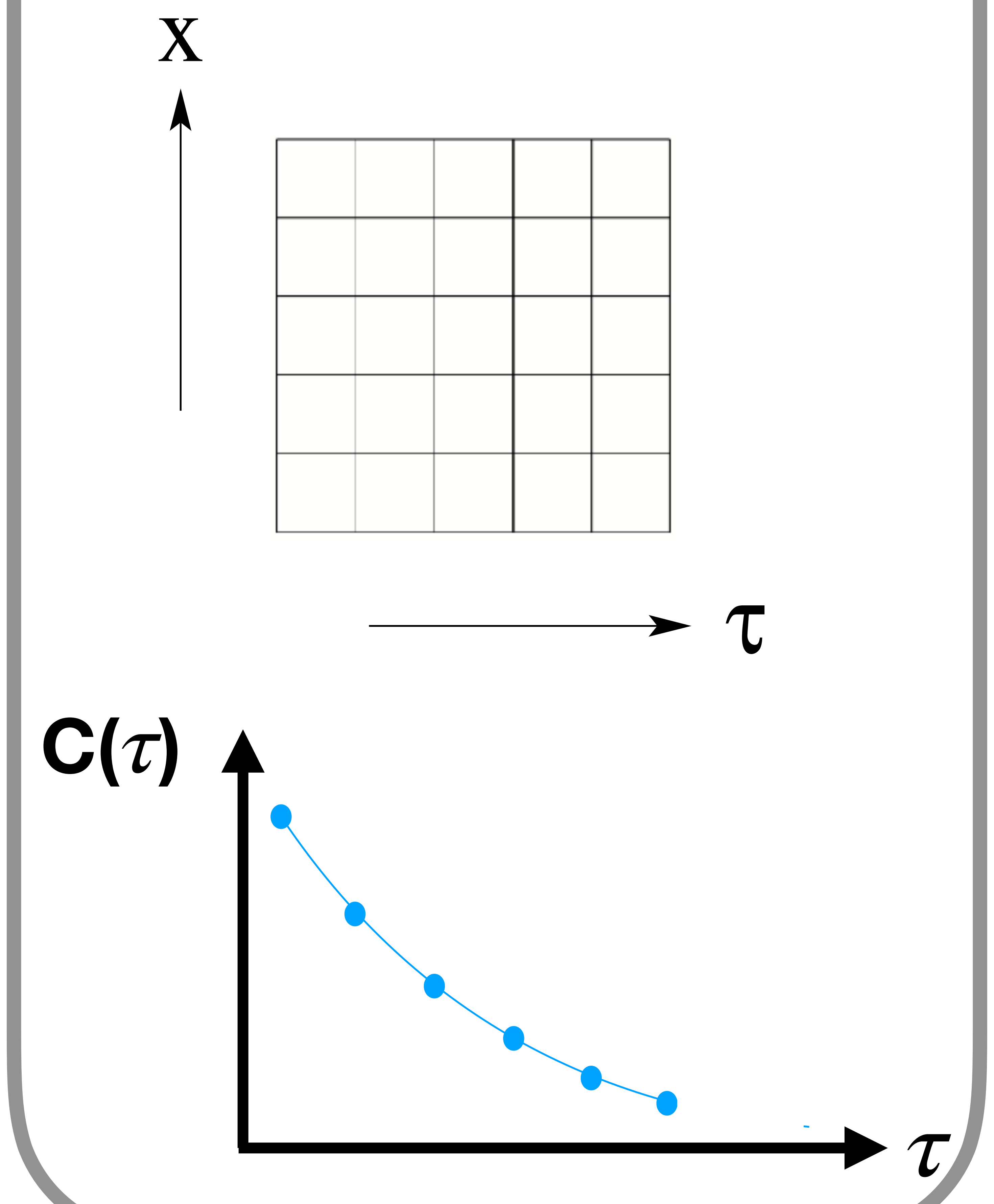


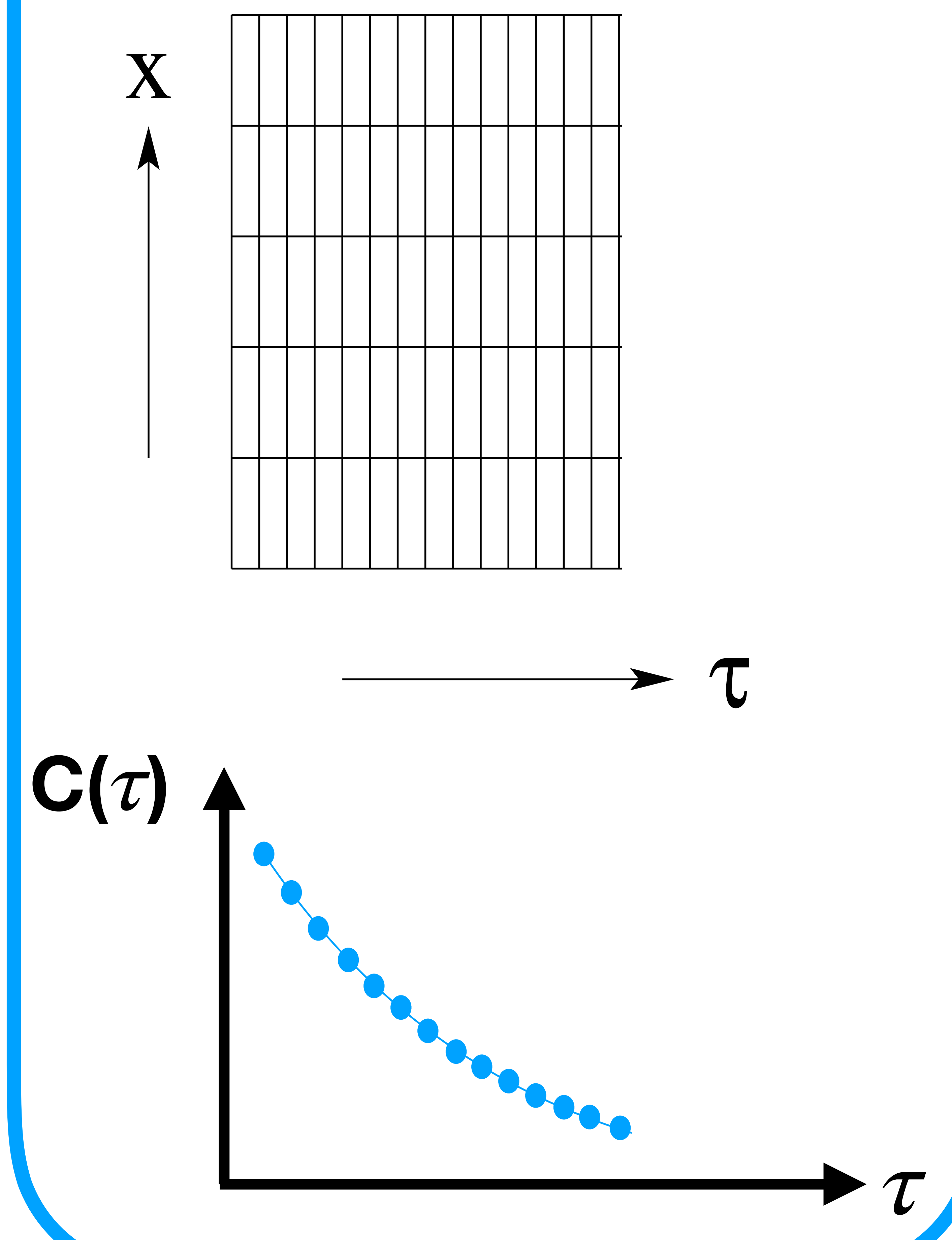
Going hotter…

Our Lattice Setup: *Anisotropic*

"Varying Scale"



"Fixed Scale"



$$T = \frac{1}{L_\tau}$$

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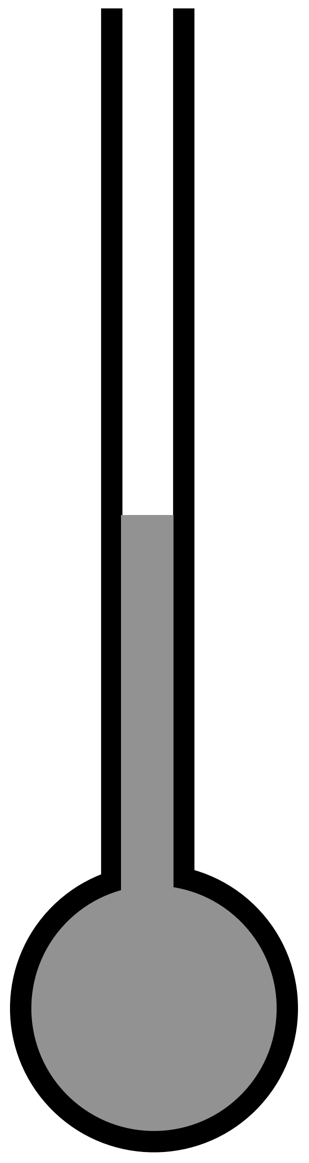
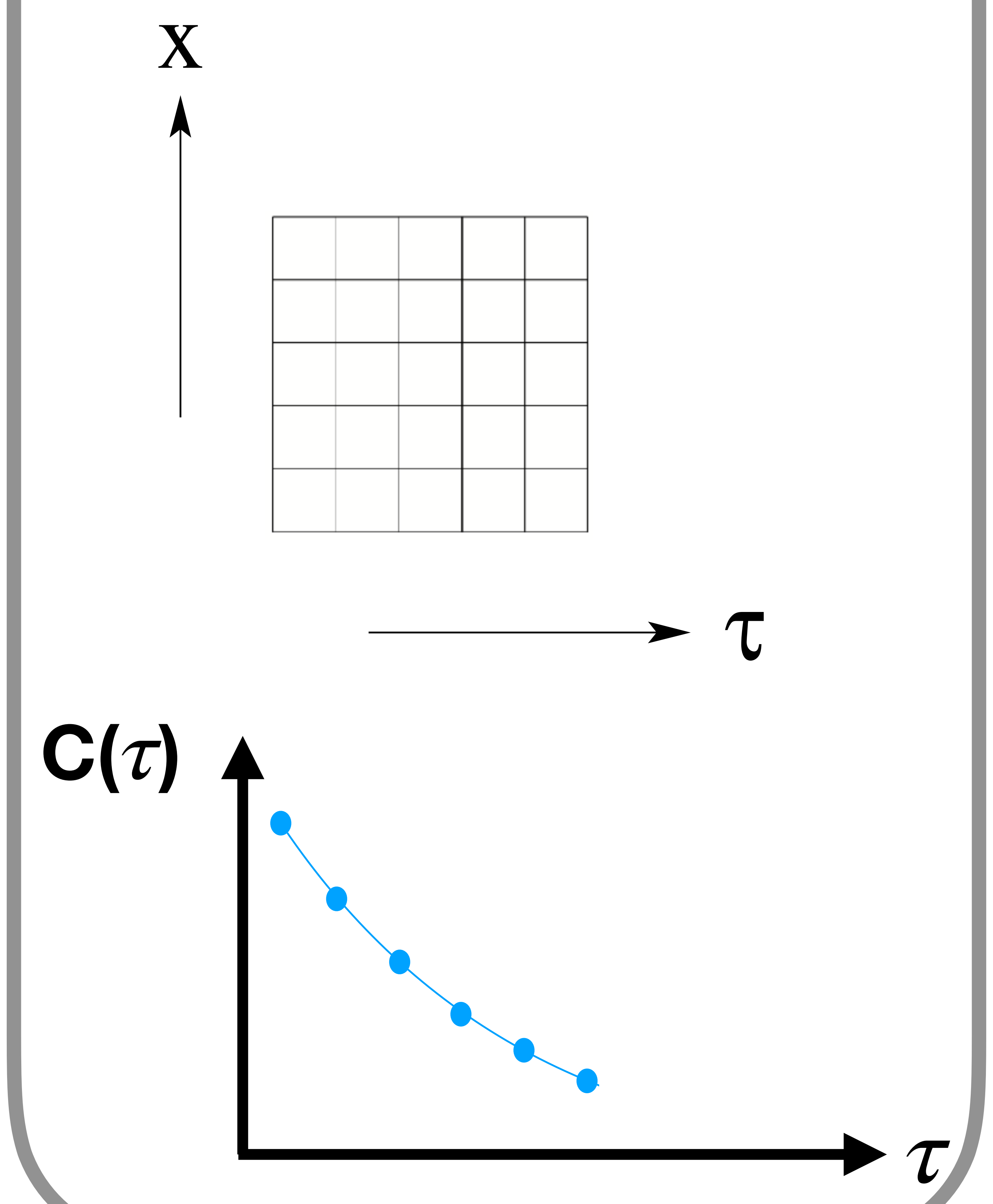


Going hotter…

Our Lattice Setup: *Anisotropic*

“Varying Scale”



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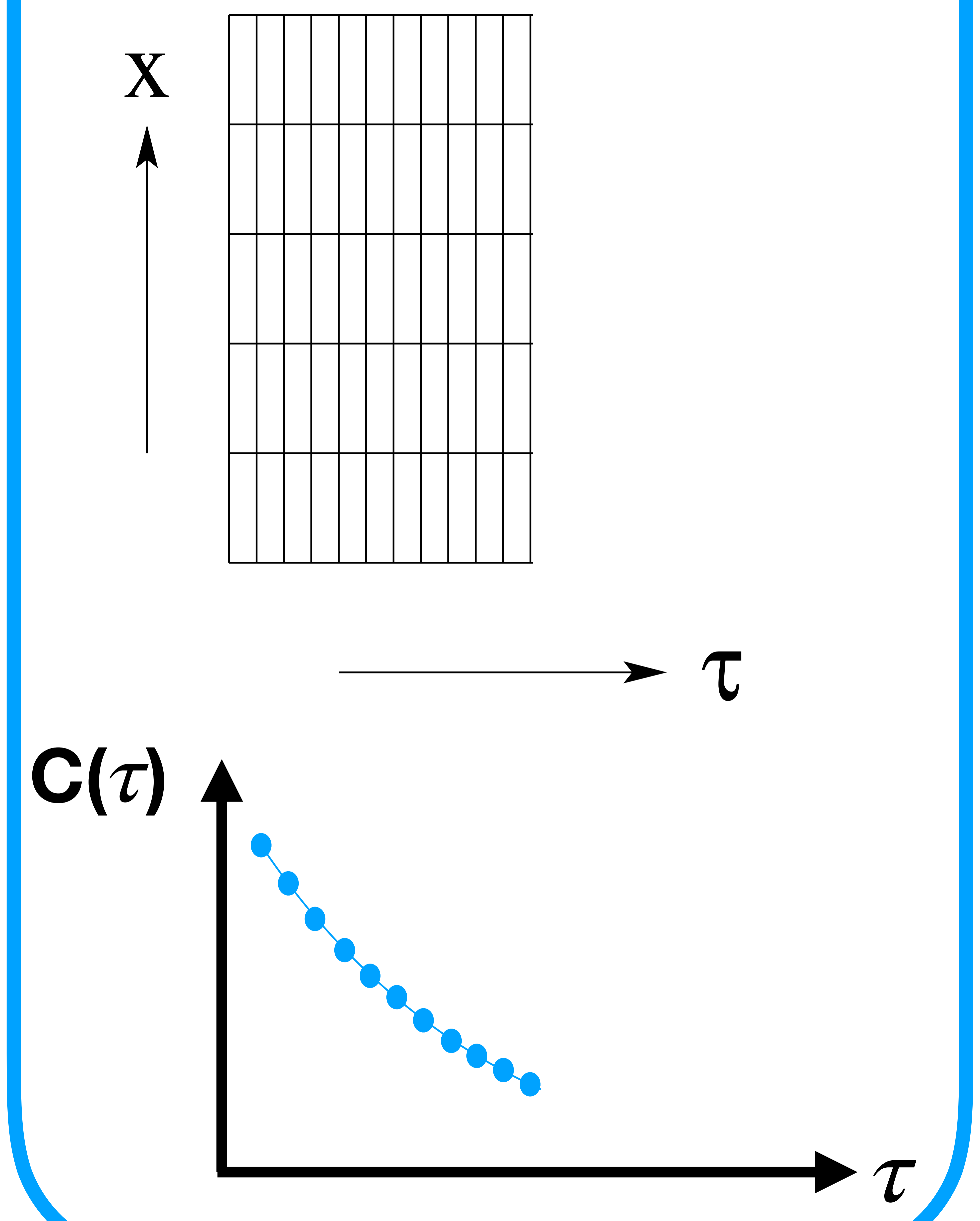

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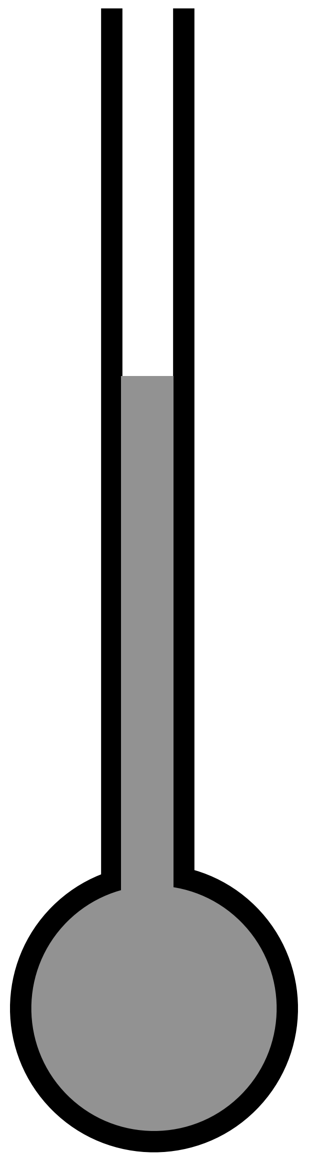

Going hotter…

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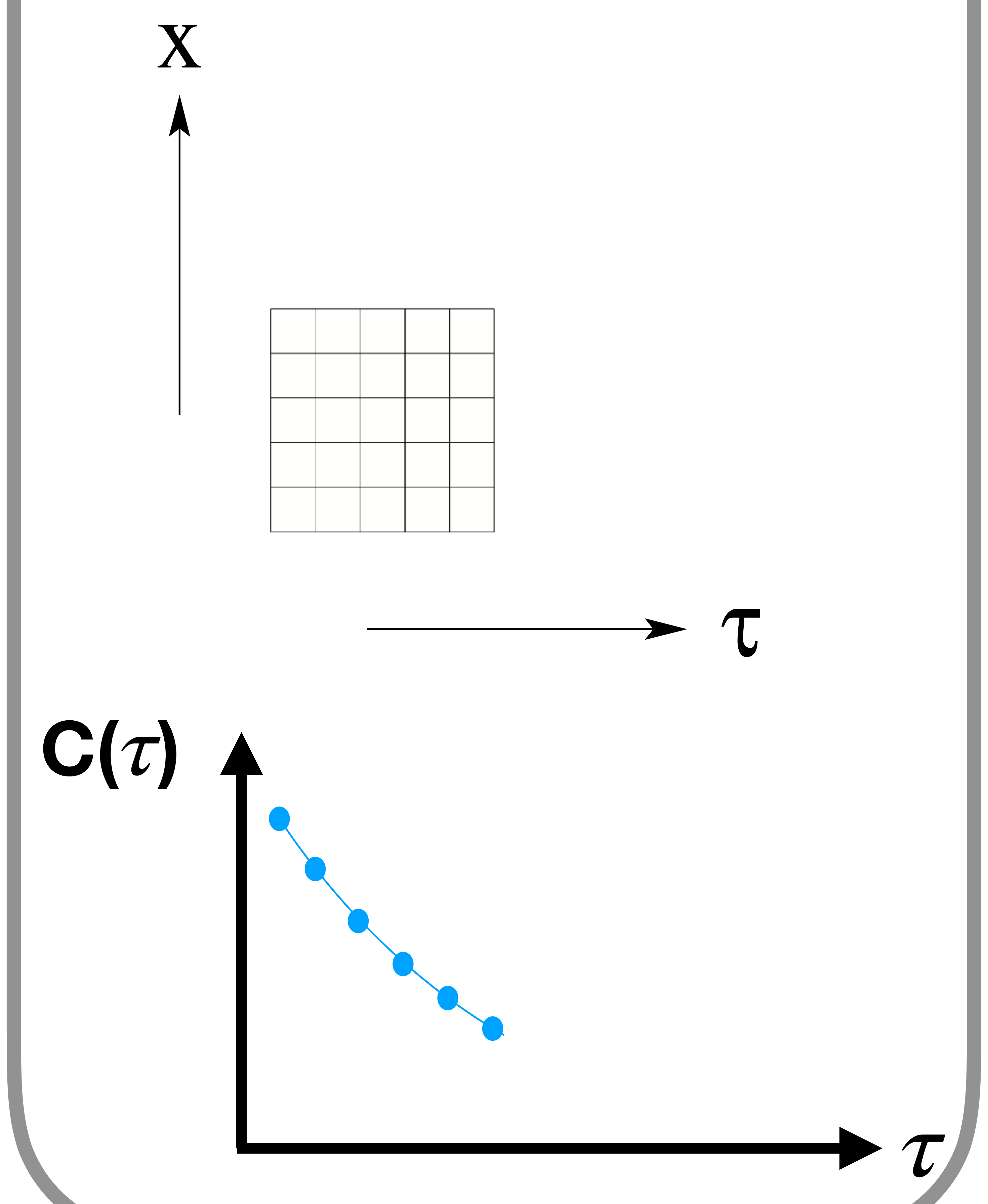

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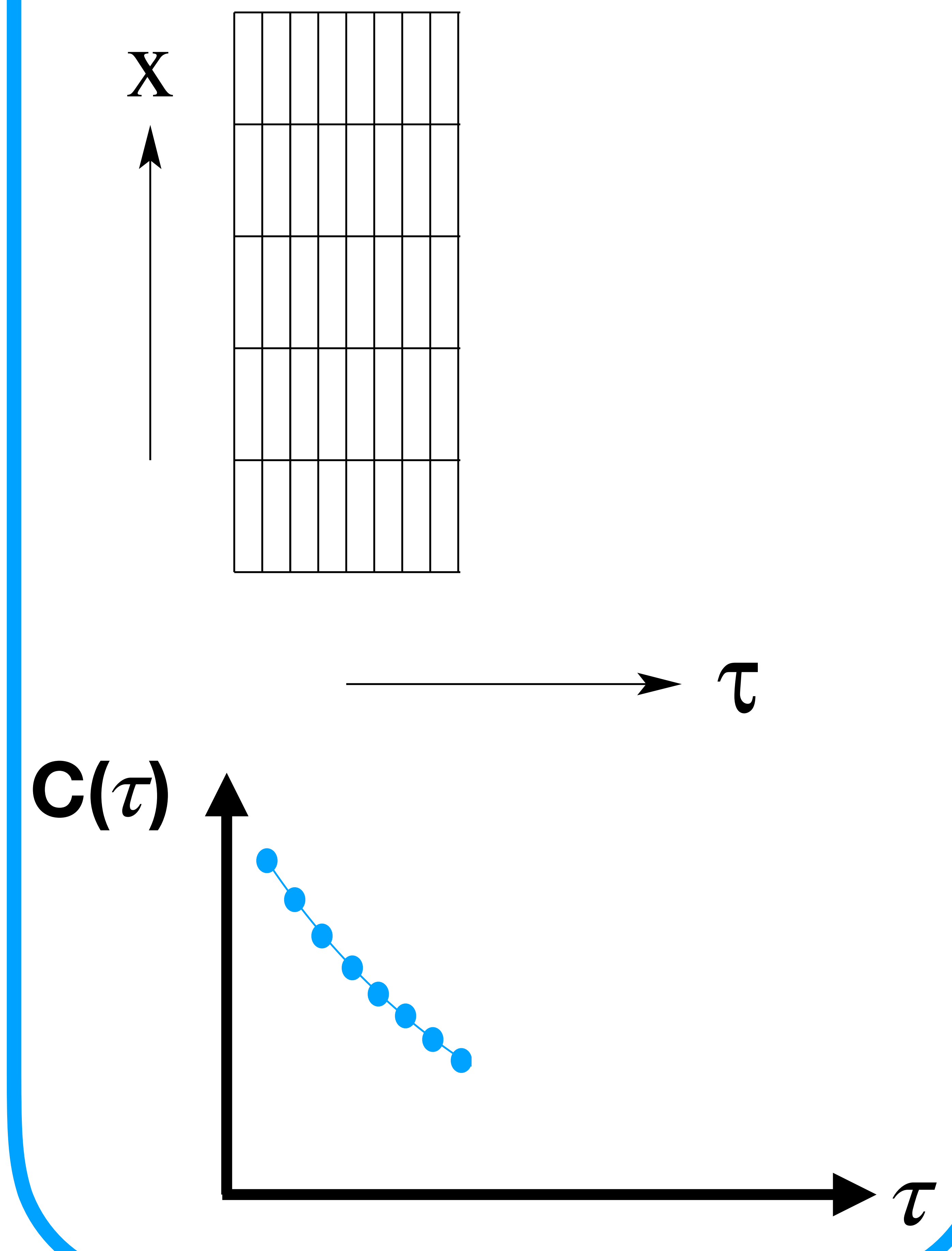

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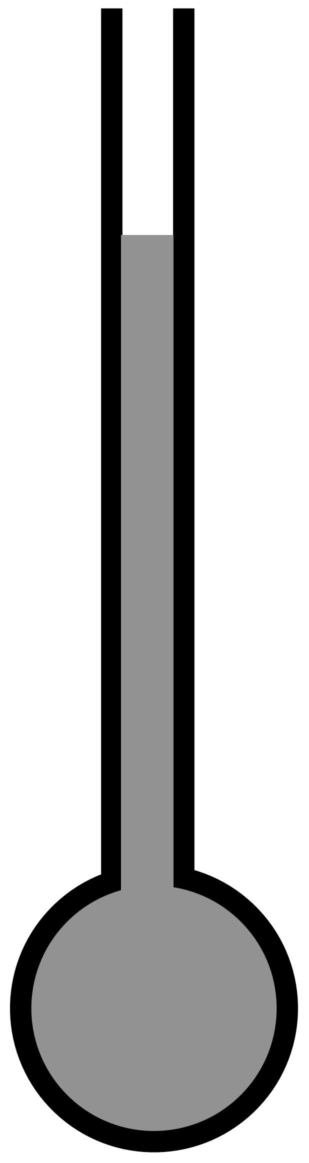

Going hotter…

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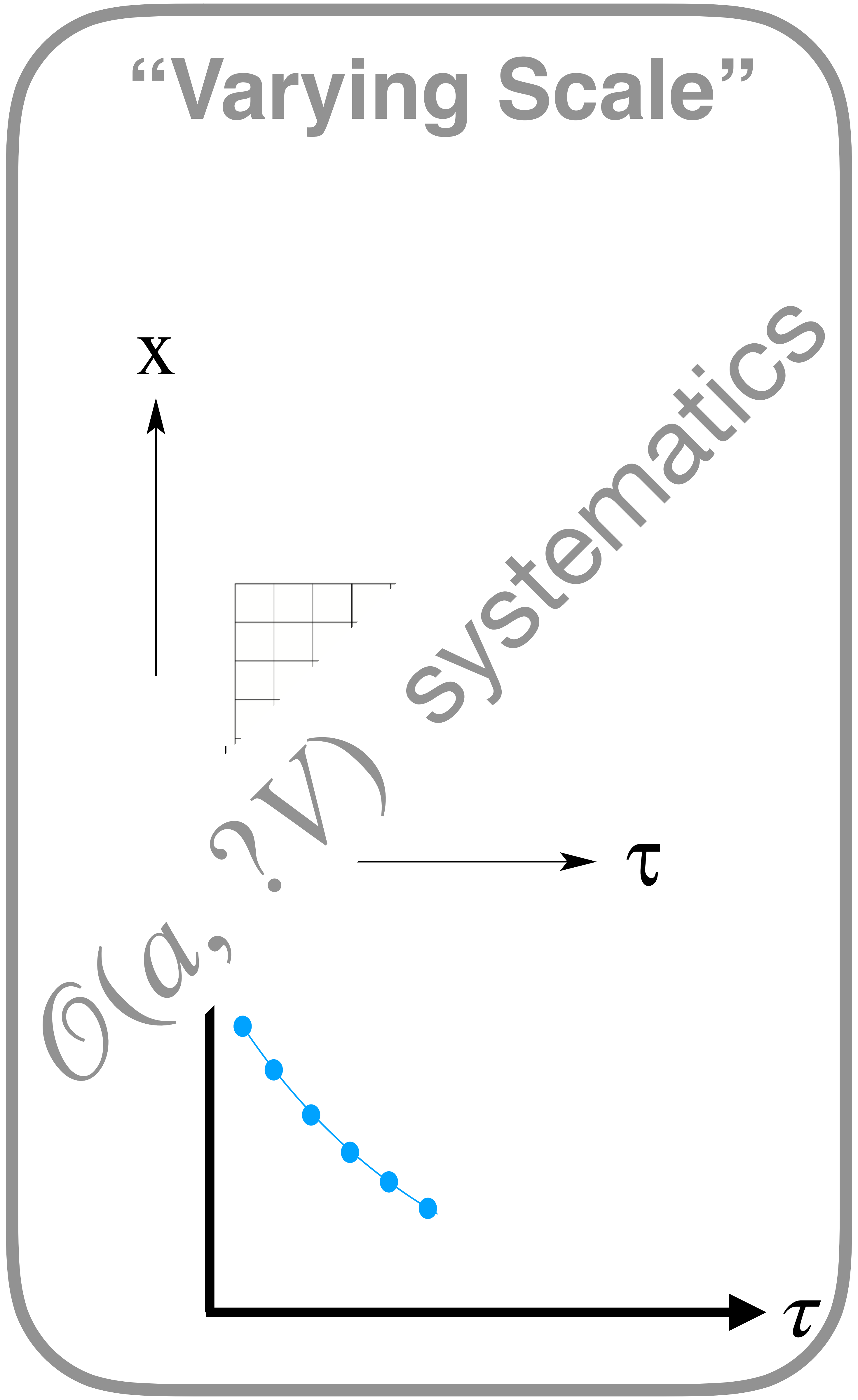

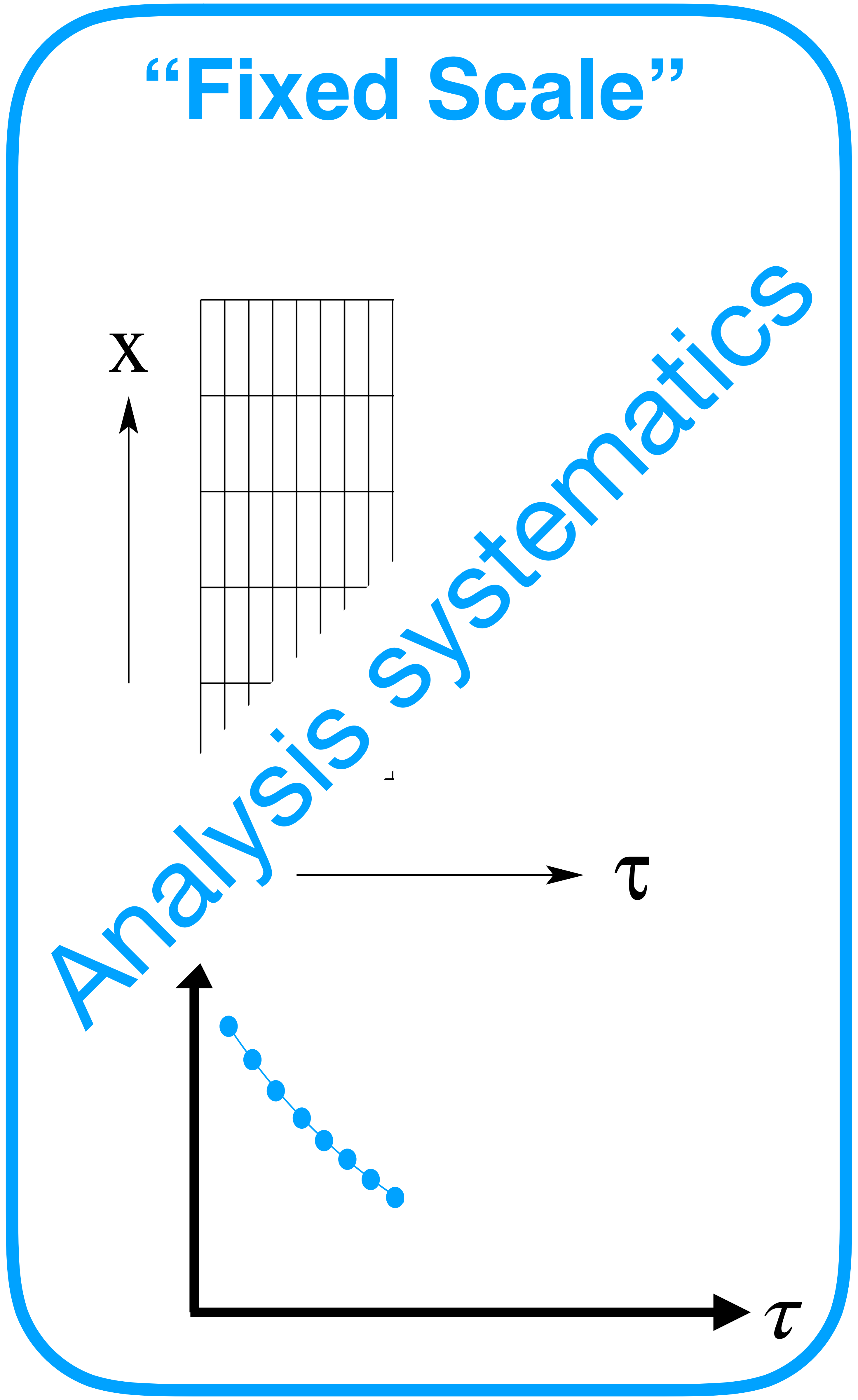

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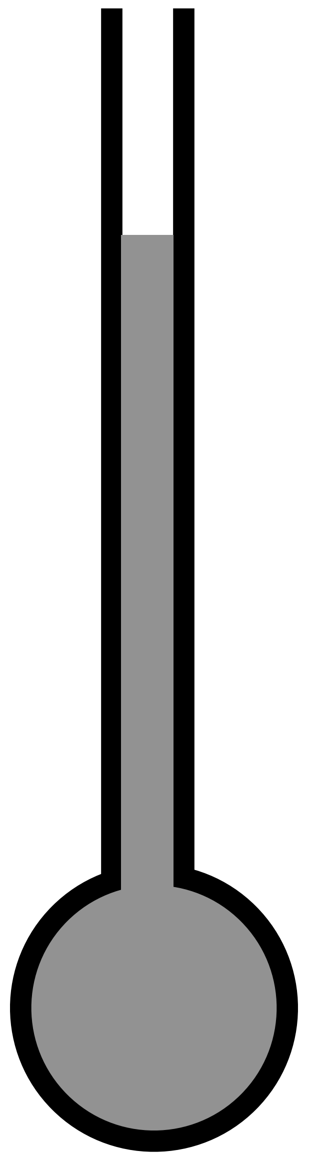
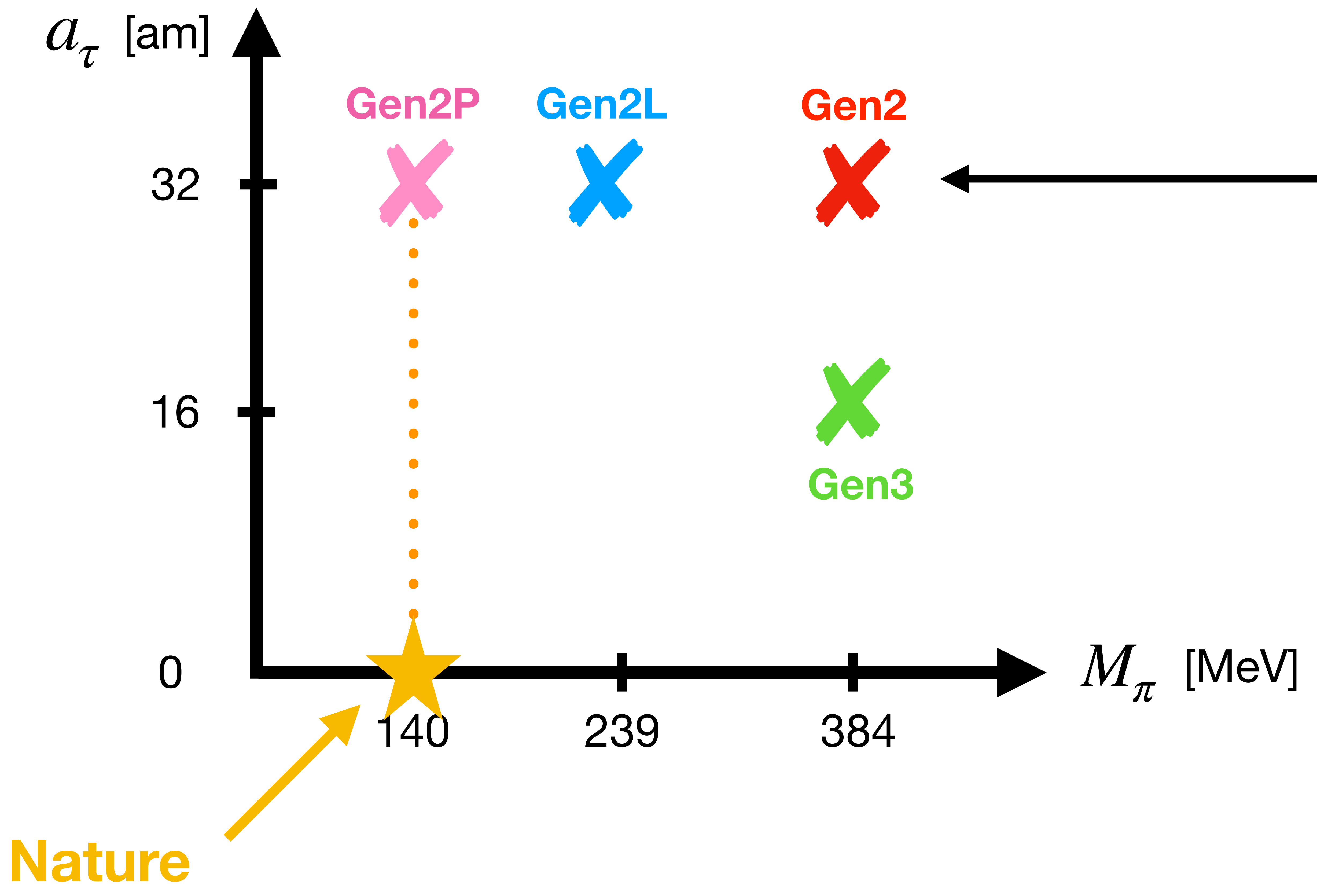


Going hotter…

Lattice Parameters

(2+1) flavour
$a_s \sim 0.112$ fm



Parameters from **HadSpec Collaboration**
R. G. Edwards, B. Joo and H. W. Lin,
Phys. Rev. D 78 (2008) 054501

Gauge Action: Symanzik-improved anisotropic
Fermion Action: Wilson-clover, tree-level tadpole with stout-smeared links

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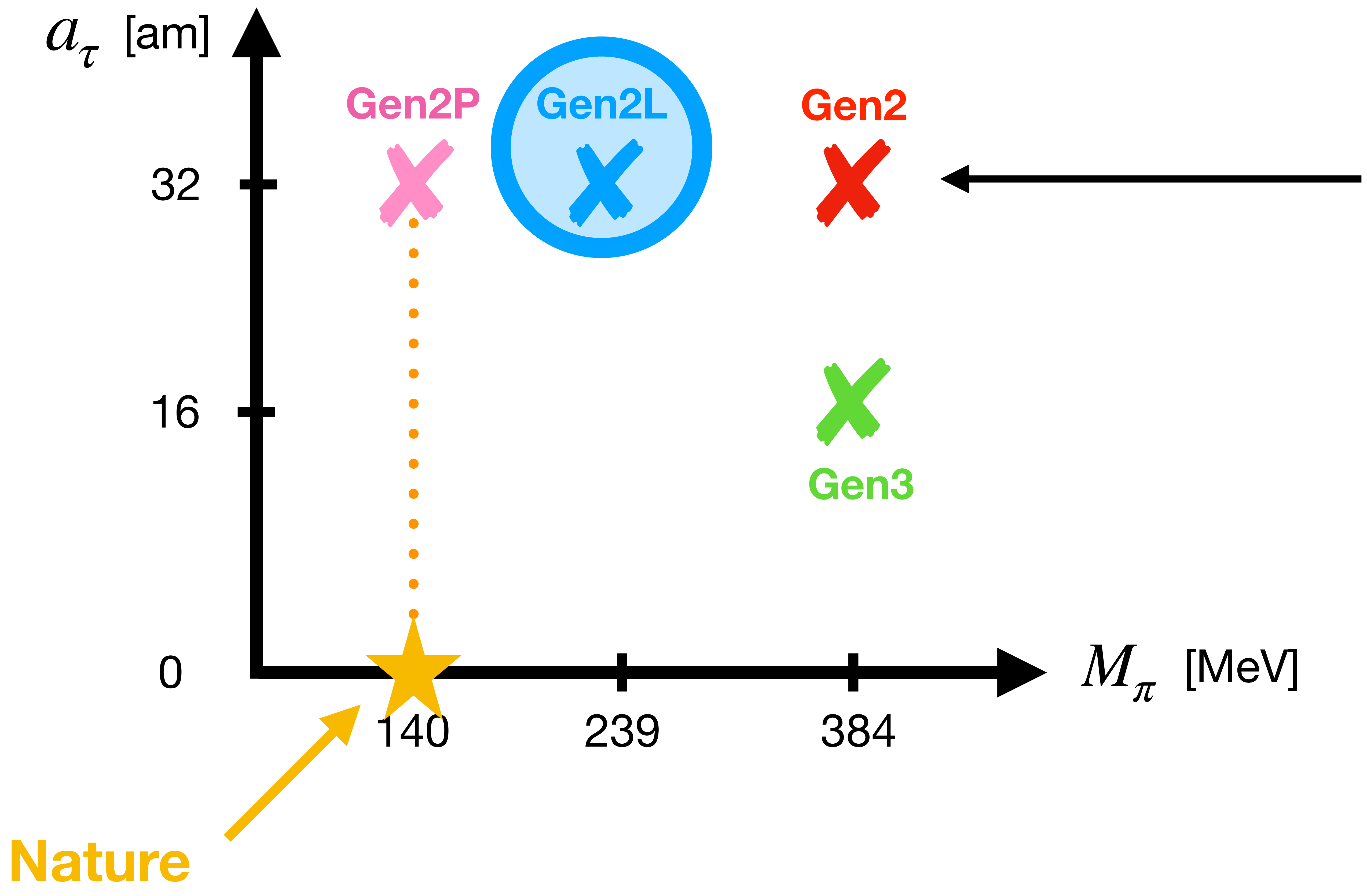

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Generation 2L

a_τ [am]	a_τ^{-1} [GeV]	$\xi = a_s/a_\tau$	a_s [fm]	m_π [MeV]	$T_c^{\bar{\psi}\psi}$ [MeV]
32.46(7)	6.079(13)	3.453(6)	0.1121(3)	239(1)	167(2)(1)

Generation 2L, $32^3 \times N_\tau$										
N_τ	128	64	56	48	40	36	32	28	24	20
T [MeV]	47	95	109	127	152	169	190	217	253	304
N_{cfg}	1024	1041	1042	1123	1102	1119	1090	1031	1016	1030

T_c ~ 167 MeV

a^{-1} = 6.079(13) GeV from HadSpec calculation of **Ω** baryon,

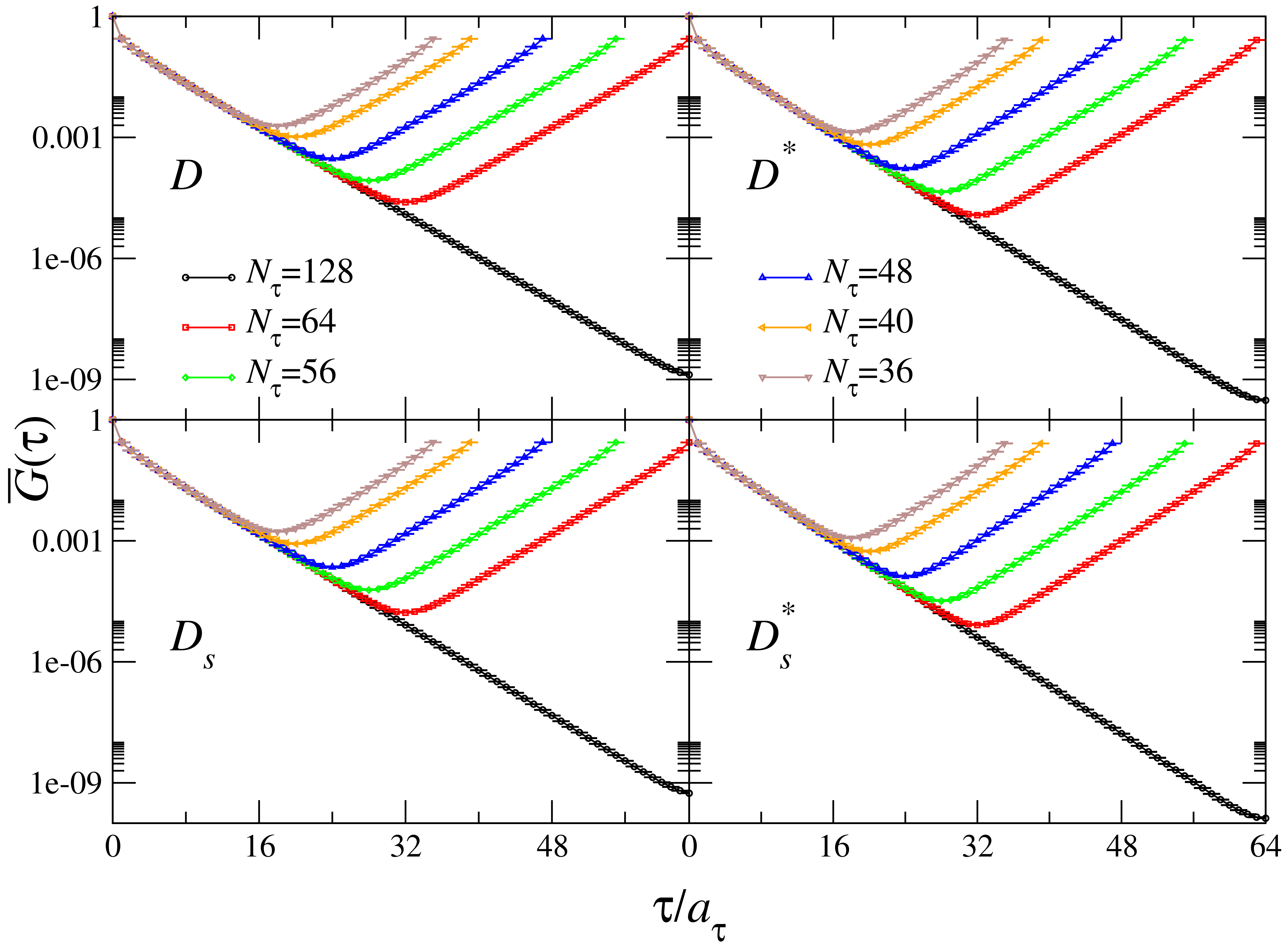
D. J. Wilson, et al., Phys. Rev. Lett. 123 (2019)

Charmed Mesons: $D_{(s)}$ and $D^*_{(s)}$

Sergio Chaves

- Not studied at T ≠ 0 before (Open Charm)
- Confined phase: $G(\tau) \sim e^{-M\tau}$
- Periodic at $T \neq 0$: $G(1/T - \tau) = G(\tau)$

		J^P	PDG [MeV]	M [MeV]
D	pseudoscalar	0^-	1869.65(5)	1876(4)
D^*	vector	1^-	2010.26(5)	2001(4)
D^*_0	scalar	0^+	2300(19)	2222(10)
D_1	axial-vector	1^+	2420.8(5)	2325(43)
D_s	pseudoscalar	0^-	1968.34(7)	1972(5)
D^*_s	vector	1^-	2112.2(4)	2092(4)
D^*_{s0}	scalar	0^+	2317.8(5)	2115(29)
D_{s1}	axial-vector	1^+	2459.5(6)	2512(6)



Studying Thermal Effects

Correlation Function's Spectral Representation:

$$G(\tau; T) = \int_0^\infty \frac{d\omega}{2\pi} K(\tau, \omega; T)\, \rho(\omega; T)$$

Two sources of Thermal Effects:

Kernel (Geometry / Periodicity)

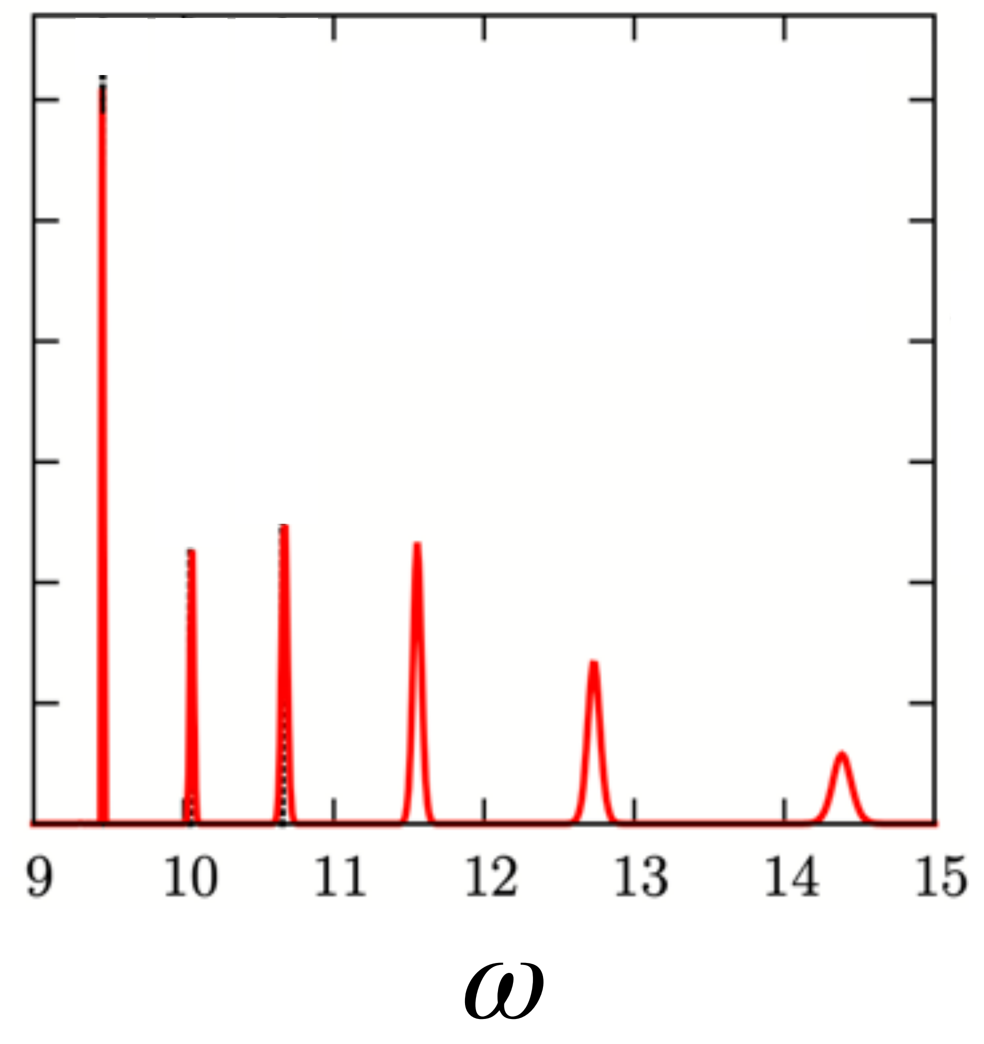
Spectral F'n (Physics)

Kernel:

$$K(\tau, \omega; T) = \frac{\cosh[\omega(\tau - 1/2T)]}{\sinh(\omega/2T)}$$

Spectral F'n:

$\rho(\omega; T)$



Studying Thermal Effects

We use a 2-step procedure

Kernel Spectrum Spectrum Kernel

Dominant behaviour is gnd state (confined phase):

$$G_{\text{model}}(\tau; T, T_0) = Z \frac{\cosh[M(T_0)(\tau - 1/2T)]}{\sinh[M(T_0)/2T]}$$

Divide correlation f'n by this

$$R(\tau; T, T_0) = \frac{G(\tau; T)}{G_{\text{model}}(\tau; T, T_0)}$$

This is a constant as $(\tau \to \infty)$ if ground state has mass $M(T_0)$

Can now compare 2 temps by taking ratio-of-ratios:

$$RoR(\tau; T, T_0) = \frac{R(\tau; T, T_0)}{R(\tau; T_0, T_0)}$$

This is a unity (as $\tau \to \infty$) when T and T_0 have same ground state mass $M(T_0)$

cf. Reconstructed Correlators

Def'n "Reconstructed Correlator": $G_{\text{rec}}(\tau; T, T_0) = \int_0^\infty \frac{d\omega}{2\pi} K(\tau, \omega; T)\ \rho(\omega; T_0)$

Compare this with actual correlation f'n: $\frac{G(\tau, T)}{G_{\text{rec}}(\tau; T, T_0)} \sim$ *constant* if $\rho(\omega; T_0) \neq f(T_0)$

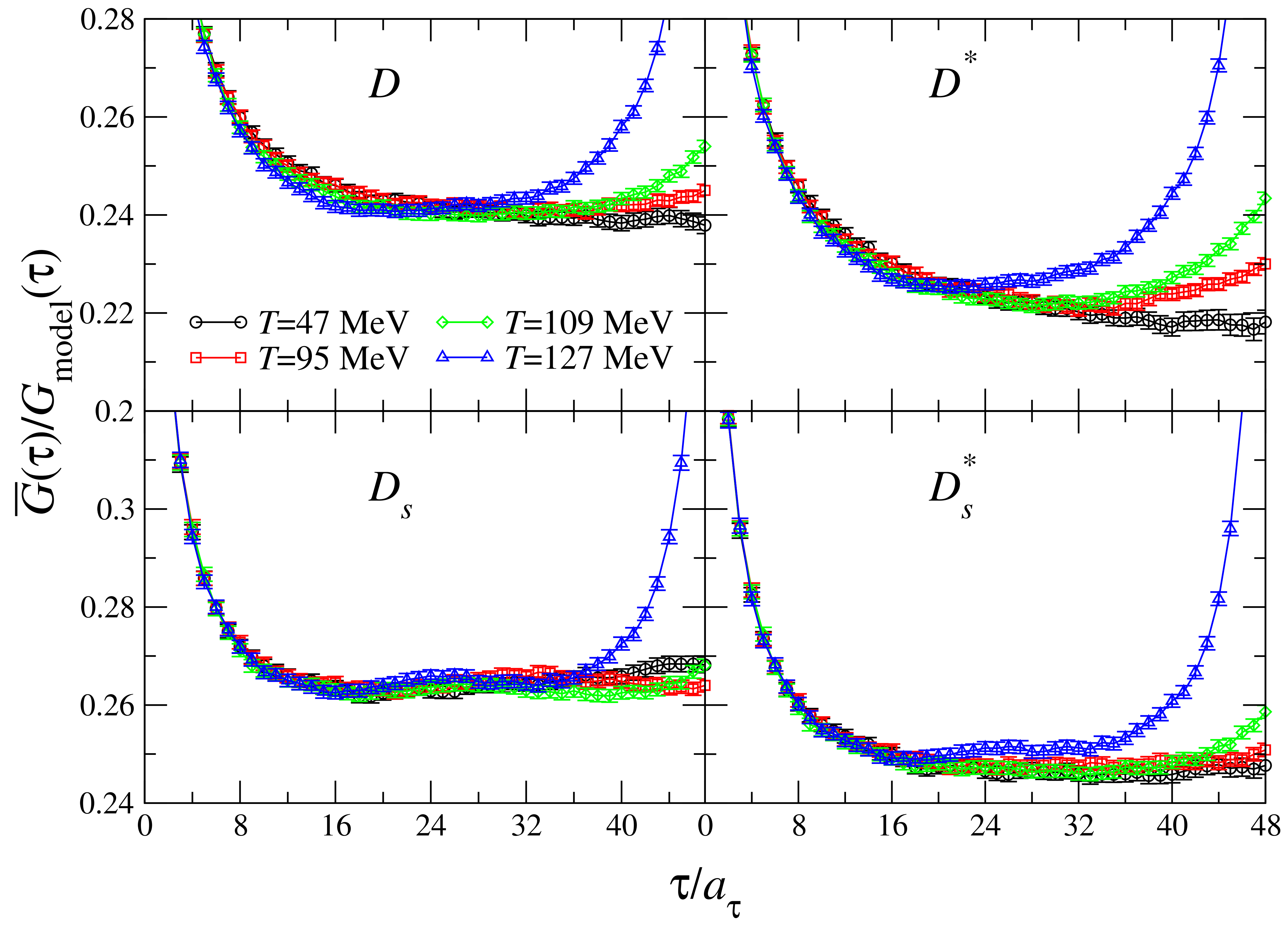
BUT G_{rec} requires knowledge of $\rho(\omega; T_0)$

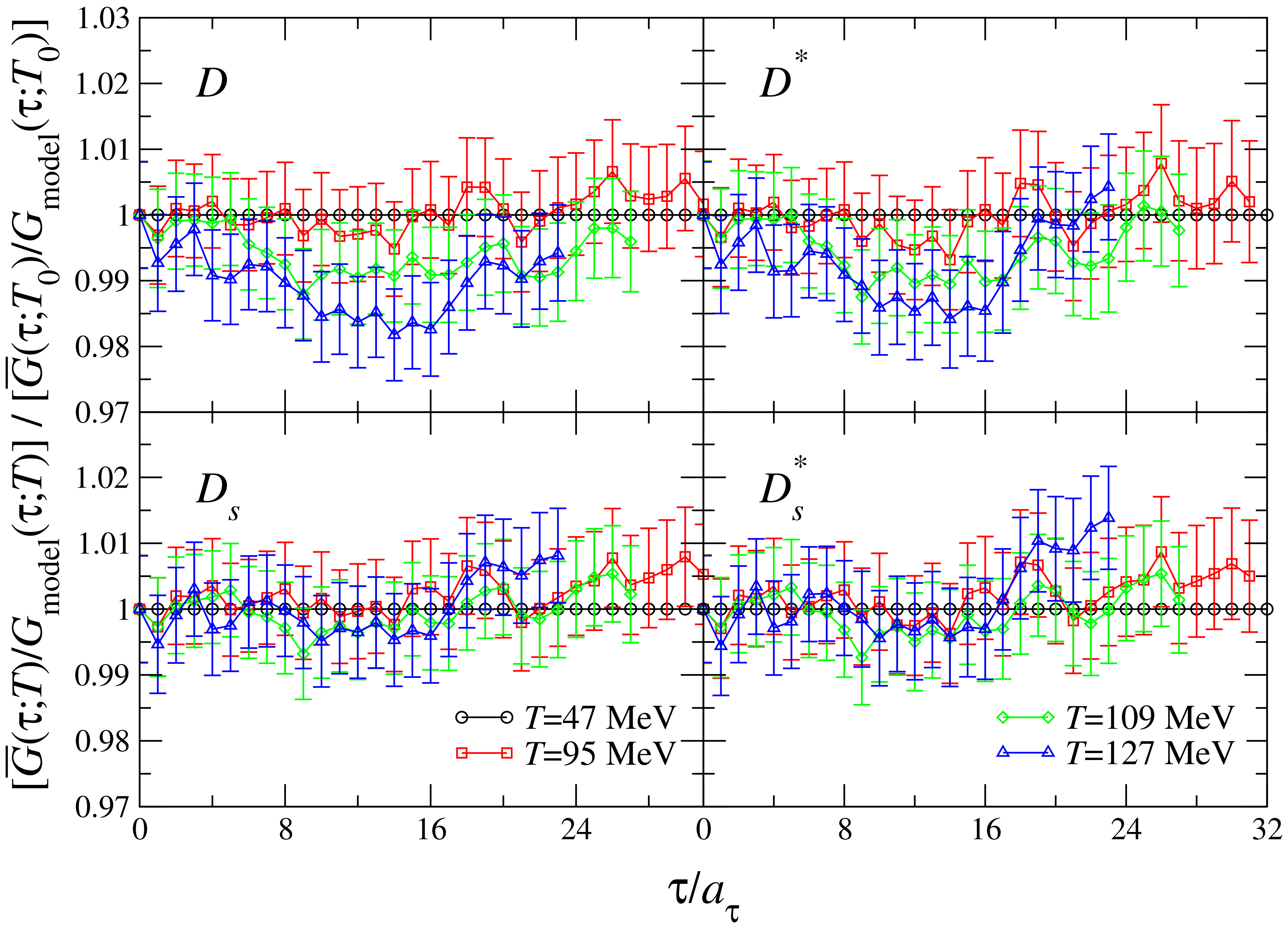
Ratio-of-Ratios $RoR(\tau; T, T_0)$ is a "Poor Man's" Reconstructed Correlator:

- it compares correlation f'n at one T using spectral f'n from T_0
- it requires $M(T_0)$
- but does not require knowledge of $\rho(\omega; T_0)$

$D_{(s)}$ and $D^*_{(s)}$ $T \leq 127$ MeV

$$R(\tau; T, T_0) = \frac{G(\tau; T)}{G_{\text{model}}(\tau; T, T_0)} \qquad (T_0 = 47 \text{ MeV}) \qquad RoR(\tau; T, T_0) = \frac{R(\tau; T, T_0)}{R(\tau; T_0, T_0)}$$



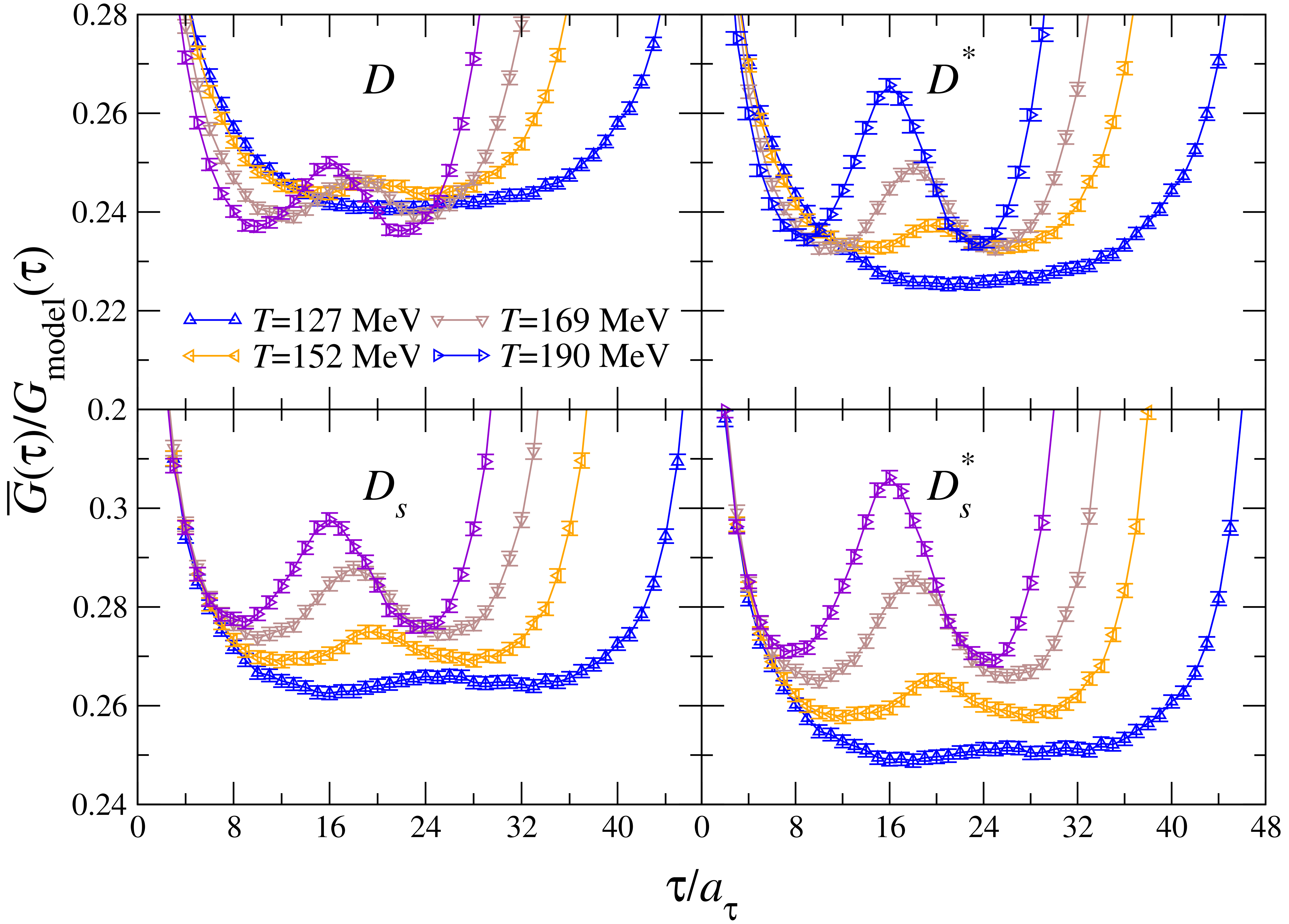


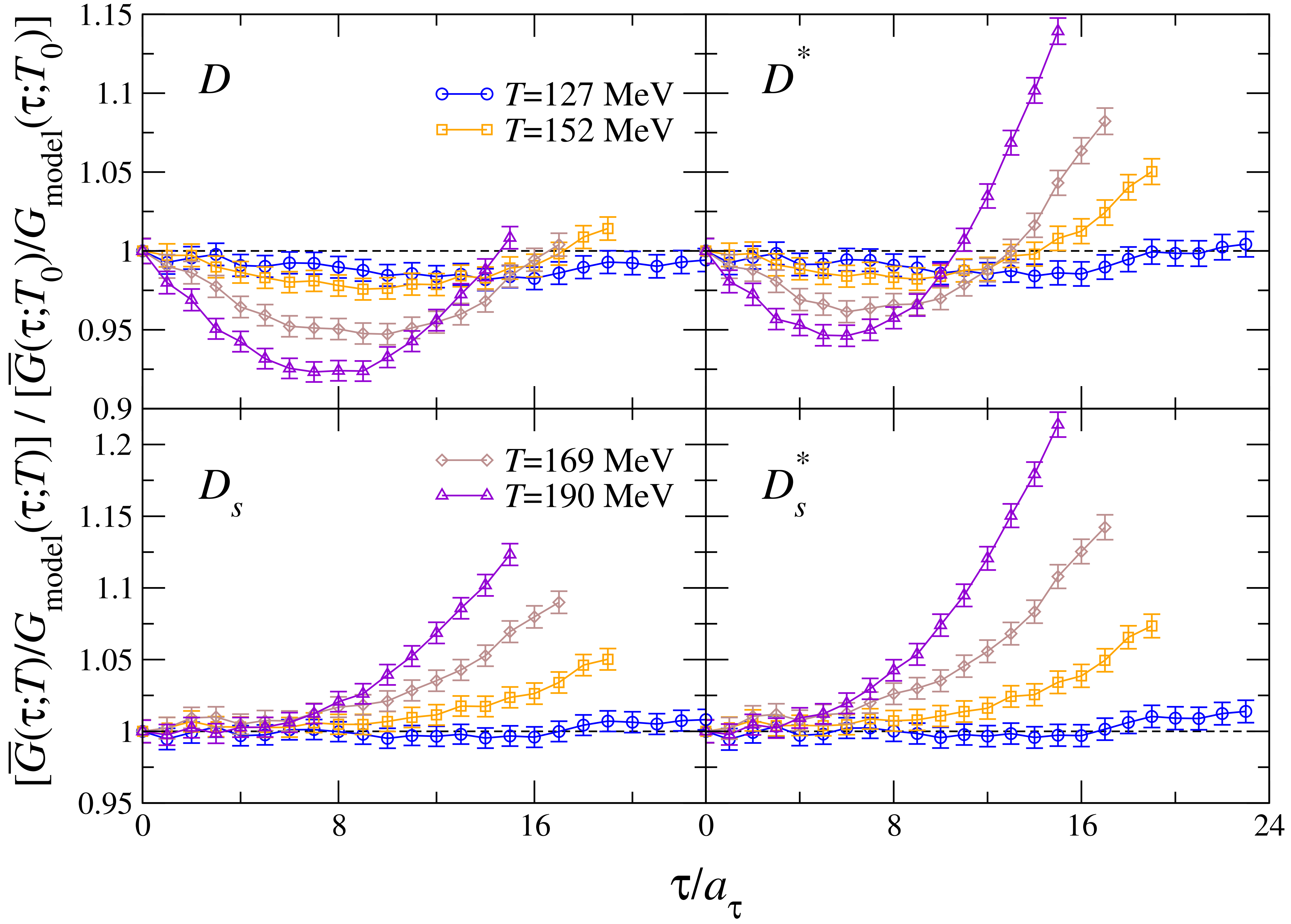
No temperature dependence

$D_{(s)}$ and $D^*_{(s)}$ $127 \leq T \leq 190$ MeV

$$R(\tau; T, T_0) = \frac{G(\tau; T)}{G_{\rm model}(\tau; T, T_0)}$$

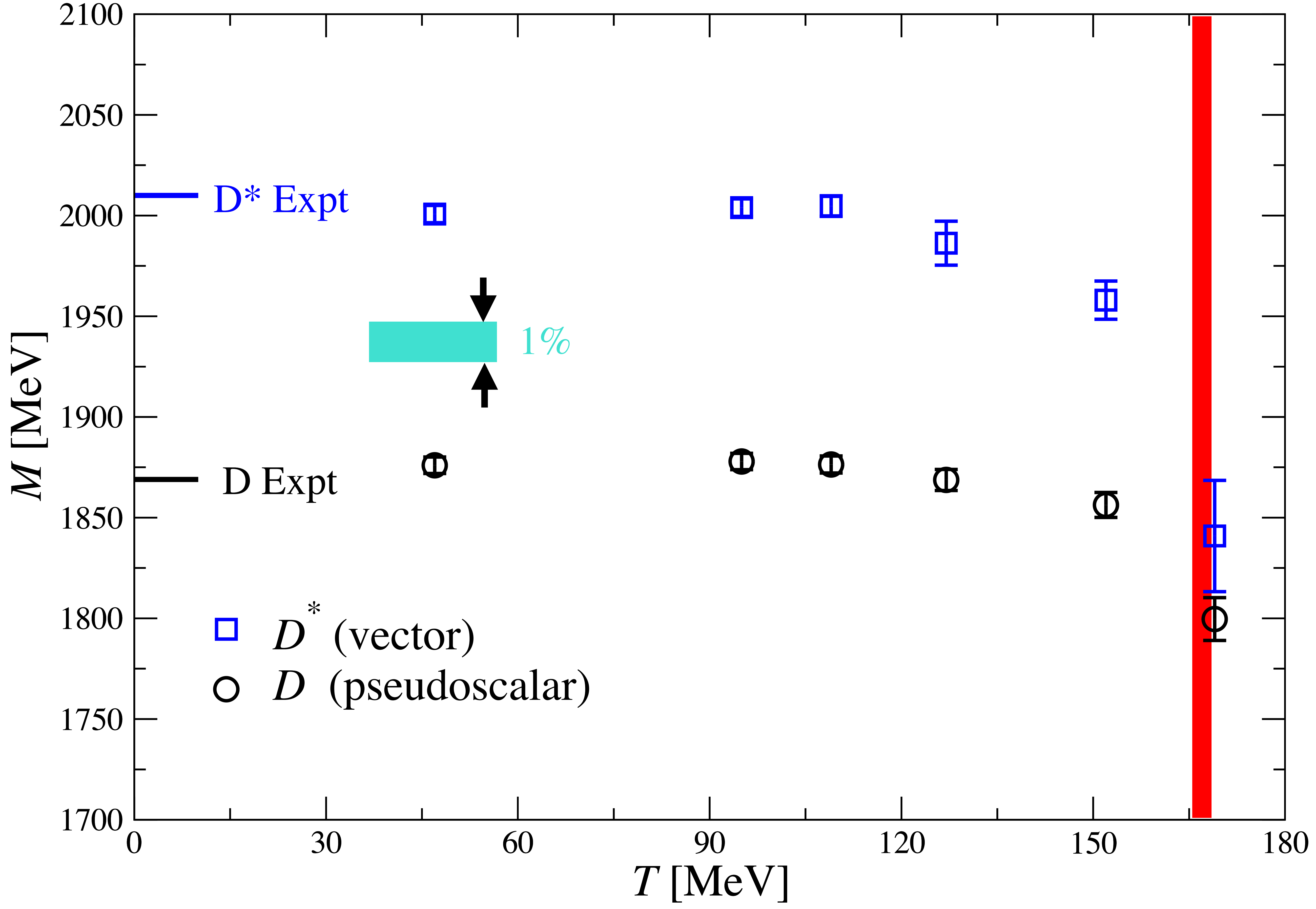
$$RoR(\tau; T, T_0) = \frac{R(\tau; T, T_0)}{R(\tau; T_0, T_0)}$$





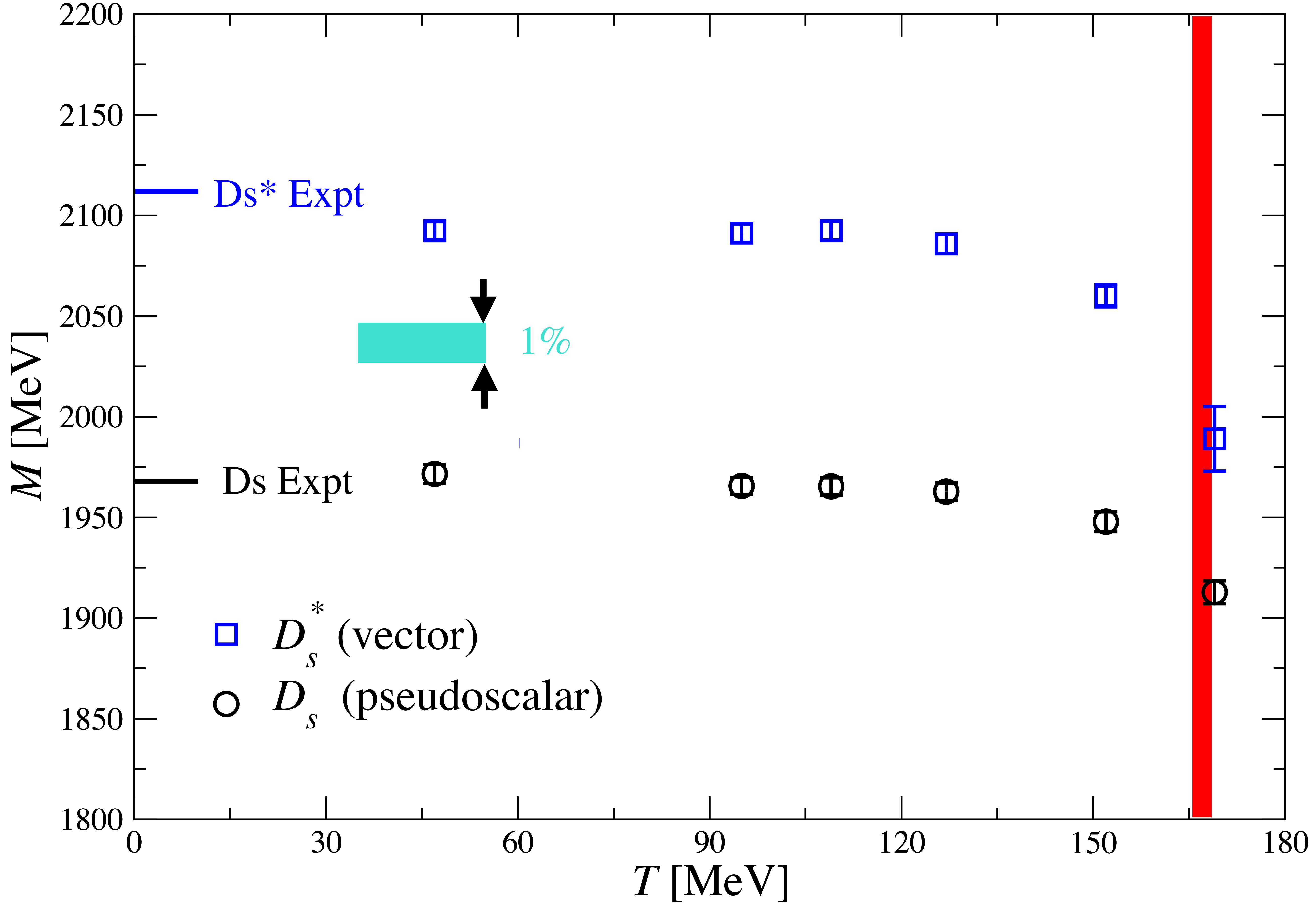
Clear temperature dependence

D and D^* masses



Temperature in confined phase effects O(1%)

D_s and D_s^* masses



Temperature in confined phase effects O(1%)

$D_{(s)}$ and $D^*_{(s)}$ Interpretation

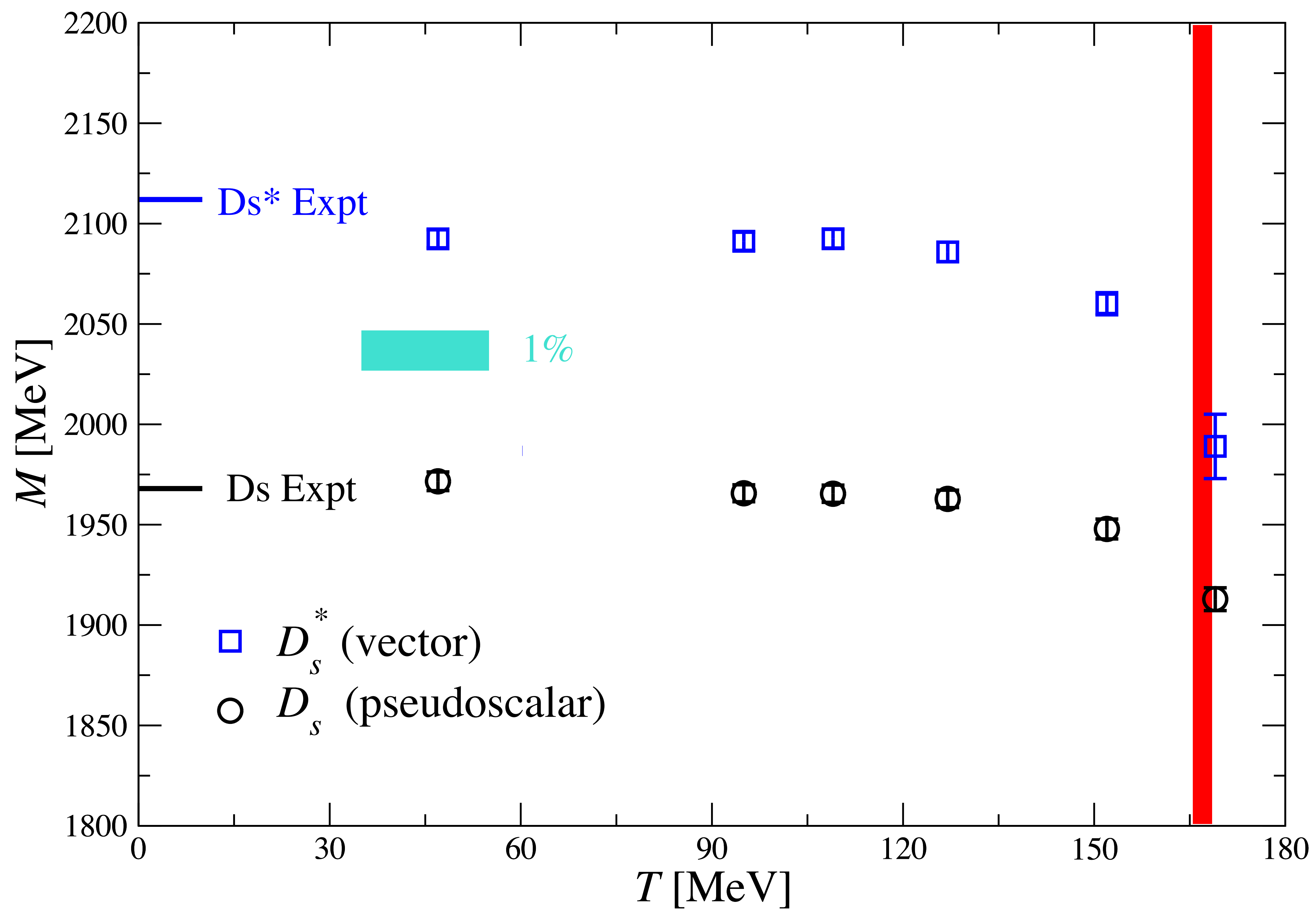
- Ratio-of-ratio shows no temperature dependence up to $T \sim 127$ MeV
- Temperature dependence clearly visible at $T \sim 152$ MeV
- Results for mass have 5MeV accuracy

	J^P	PDG	T[MeV]= 47	95	109	127	152	169
D	0^-	1869.65(5)	1876(4)	1878(4)	1876(4)	1869(5)	1856(6)	1800(11)
D^*	1^-	2010.26(5)	2001(4)	2004(4)	2005(5)	1986(11)	1958(9)	1841(28)
D_s	0^-	1968.34(7)	1972(5)	1966(4)	1965(4)	1963(4)	1948(5)	1913(6)
D_s^*	1^-	2112.2(4)	2092(4)	2091(5)	2092(5)	2086(5)	2060(6)	1989(16)

Comparison with Other Approaches

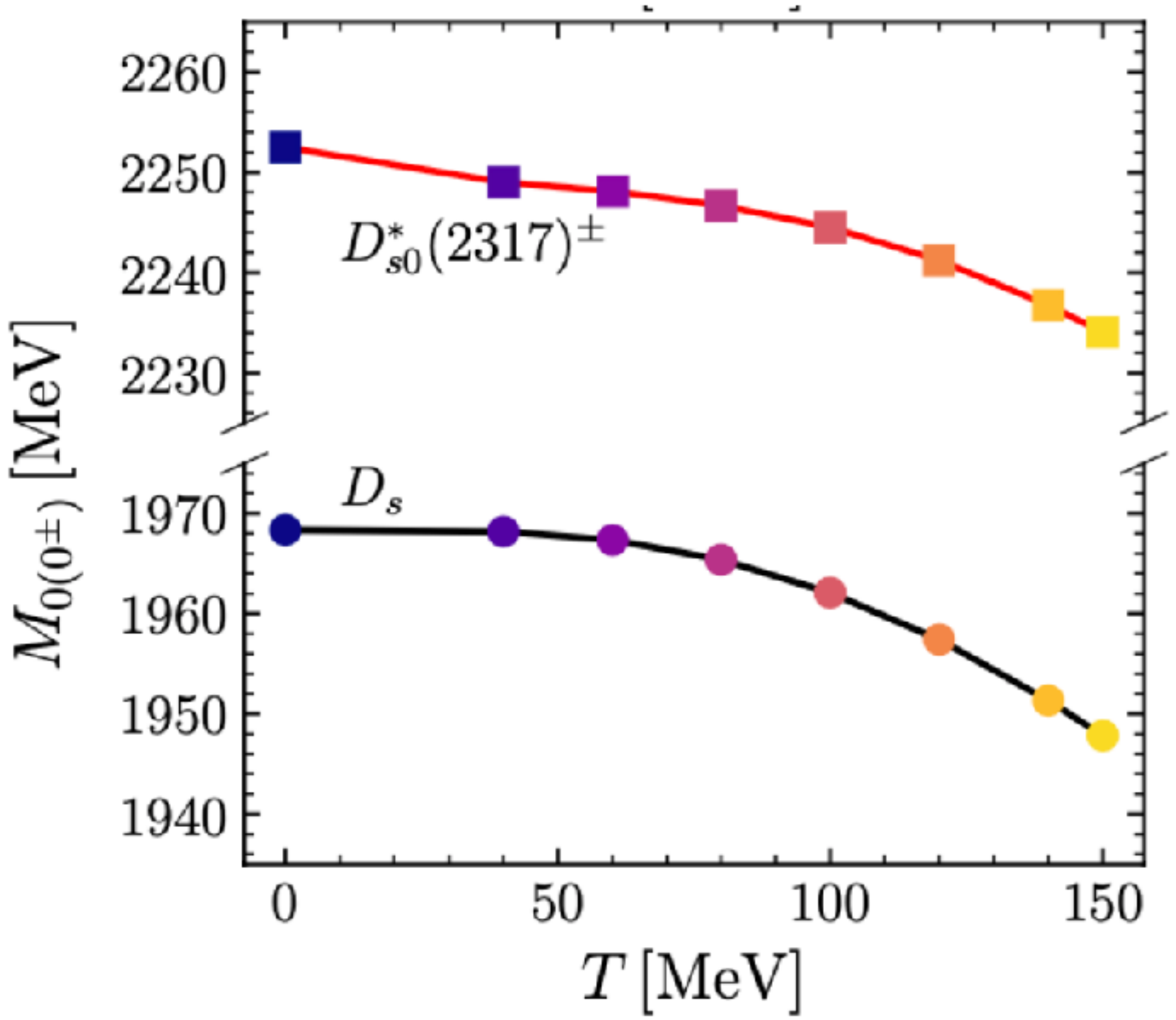
This work

Reduction of D_s mass by ~24(10) MeV



Montaña et al, PLB 806 (2020) 135464

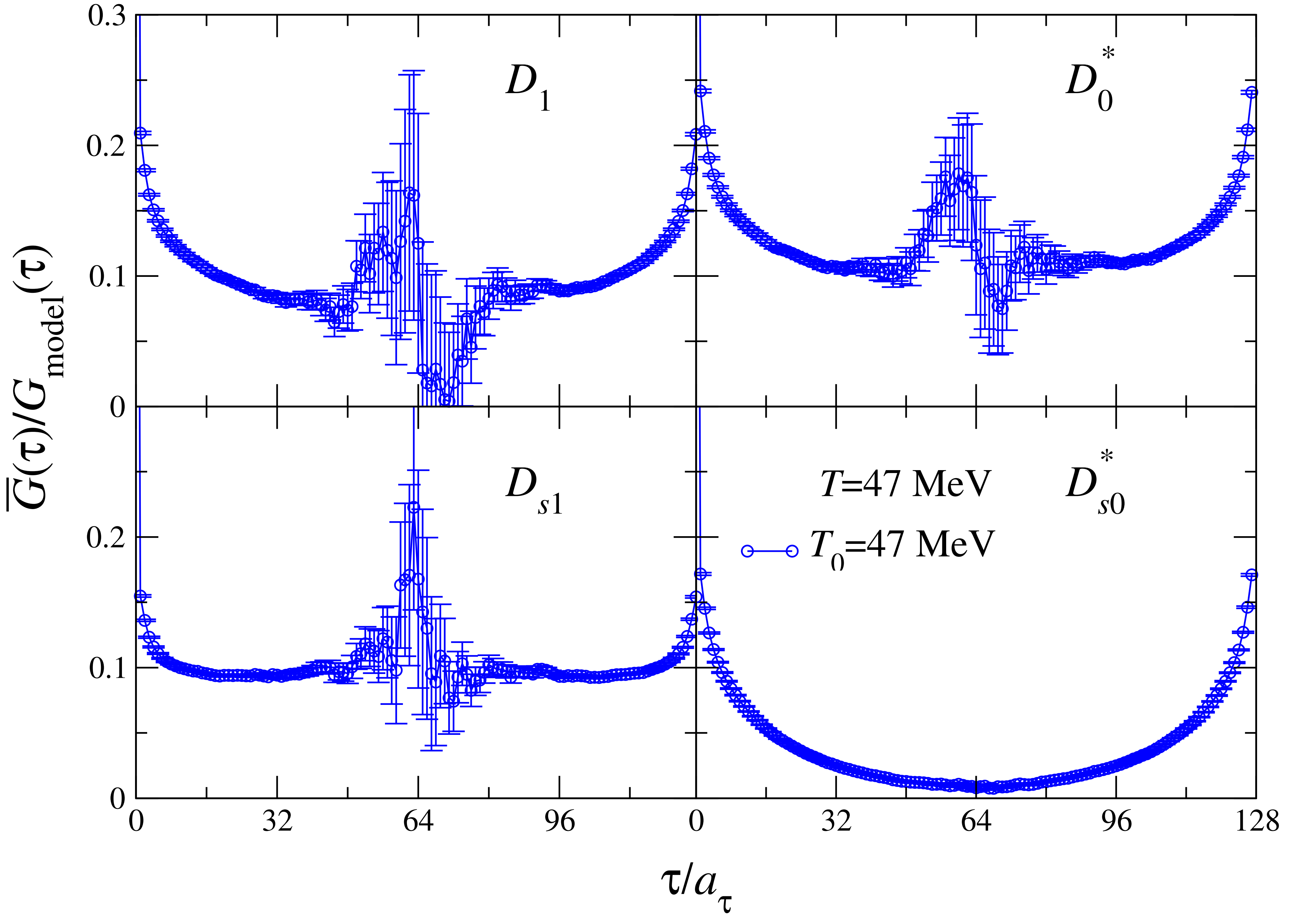
Reduction of D_s mass by ~20 MeV

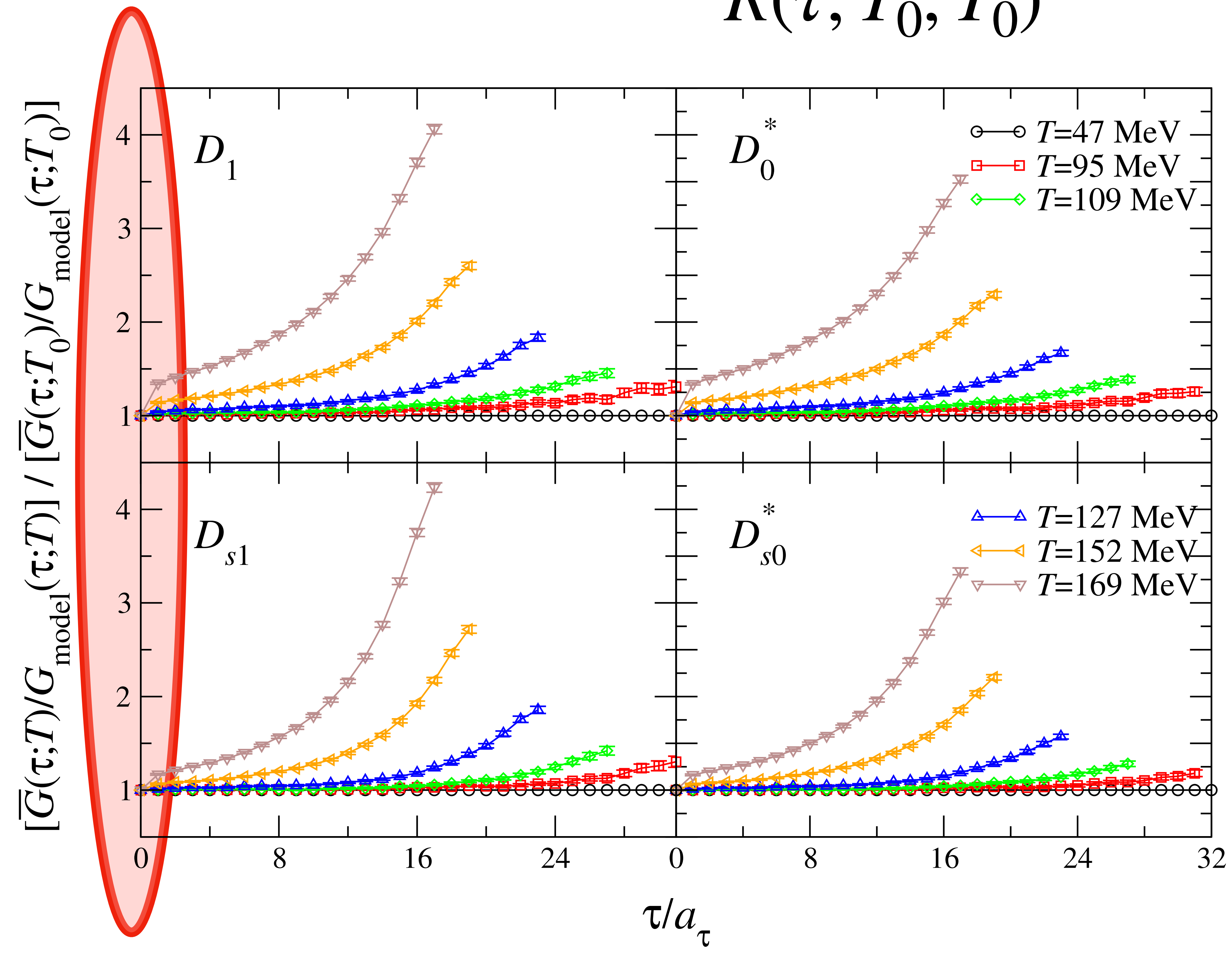


$D_{(s)1}$ and $D^*_{(s)0}$

$$R(\tau; T, T_0) = \frac{G(\tau; T)}{G_{\text{model}}(\tau; T, T_0)}$$

$$RoR(\tau; T, T_0) = \frac{R(\tau; T, T_0)}{R(\tau; T_0, T_0)}$$





Clear temperature dependence

Note threshold effects

Charmed Baryonic Spectrum - Parity

Ryan Bignell

No parity doubling in (T=0) Nature:

+ve parity: $m_+ = m_N = 0.939$ GeV
-ve parity: $m_- = m_{N*} = 1.535$ GeV

Question: What happens as T increases?

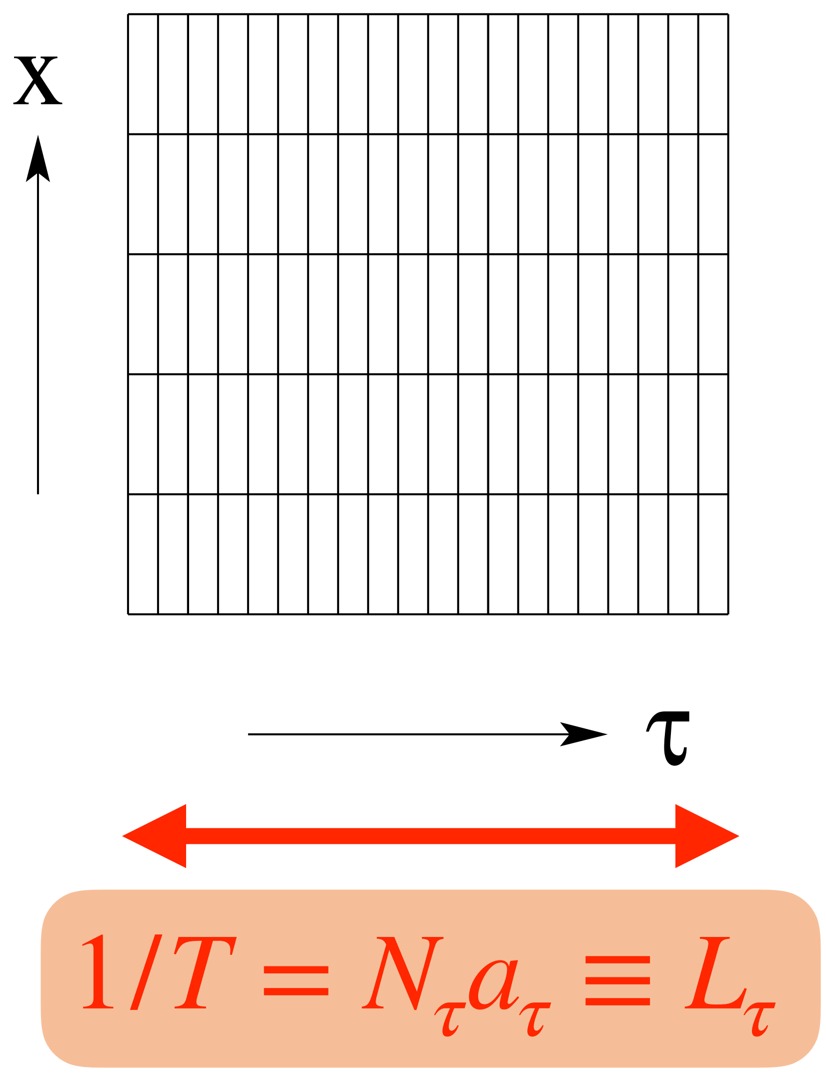
Lattice:

Parity operation: $P\mathcal{O}(\tau, \vec{x})P^{-1} = \gamma_4 \mathcal{O}(\tau, -\vec{x})$

Construct correlation functions:

$$G_\pm(\tau) = \int \mathrm{d}\mathbf{x} \, \langle \mathrm{tr} O(\mathbf{x}, \tau) P_\pm \overline{O}(\mathbf{0}, 0) \rangle \, , \qquad P_\pm = \frac{1}{2}(\mathbb{1} \pm \gamma_4)$$

Symmetries



Charge conjugation (at zero density): $G_{\pm}(\tau) = -G_{\mp}(1/T - \tau)$ (✶)

i.e. positive/negative parity states propagate forward/backward in τ

Eg. for a single state: $G_{+}(\tau) = A_{+}e^{-m_{+}\tau} + A_{-}e^{-m_{-}(1/T-\tau)}$

(Contrasts with meson sector)

Chiral symmetry:

Constrains spinor structure so that $G_{+}(\tau) = -G_{-}(\tau)$

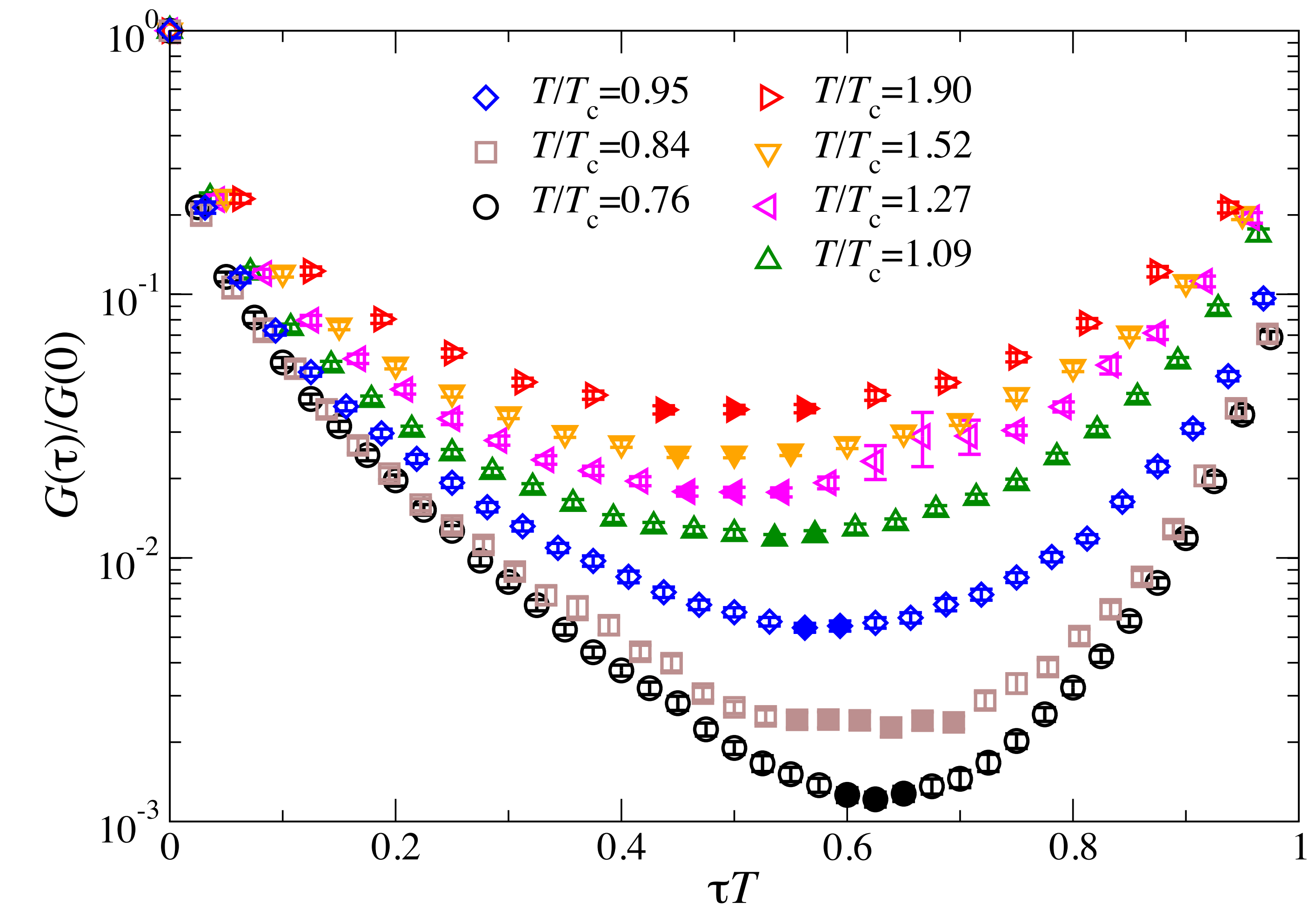
ie. parity doubling: $m_{+} = m_{-}$

Together with (✶) $\longrightarrow G_{+}(\tau) = G_{+}(1/T - \tau)$

i.e. forward/back symmetry

+ve parity

-ve parity



Question: Does this happen in Nature in deconfined phase?

- assuming $m_q \sim 0$
- what about the strange-quark sector

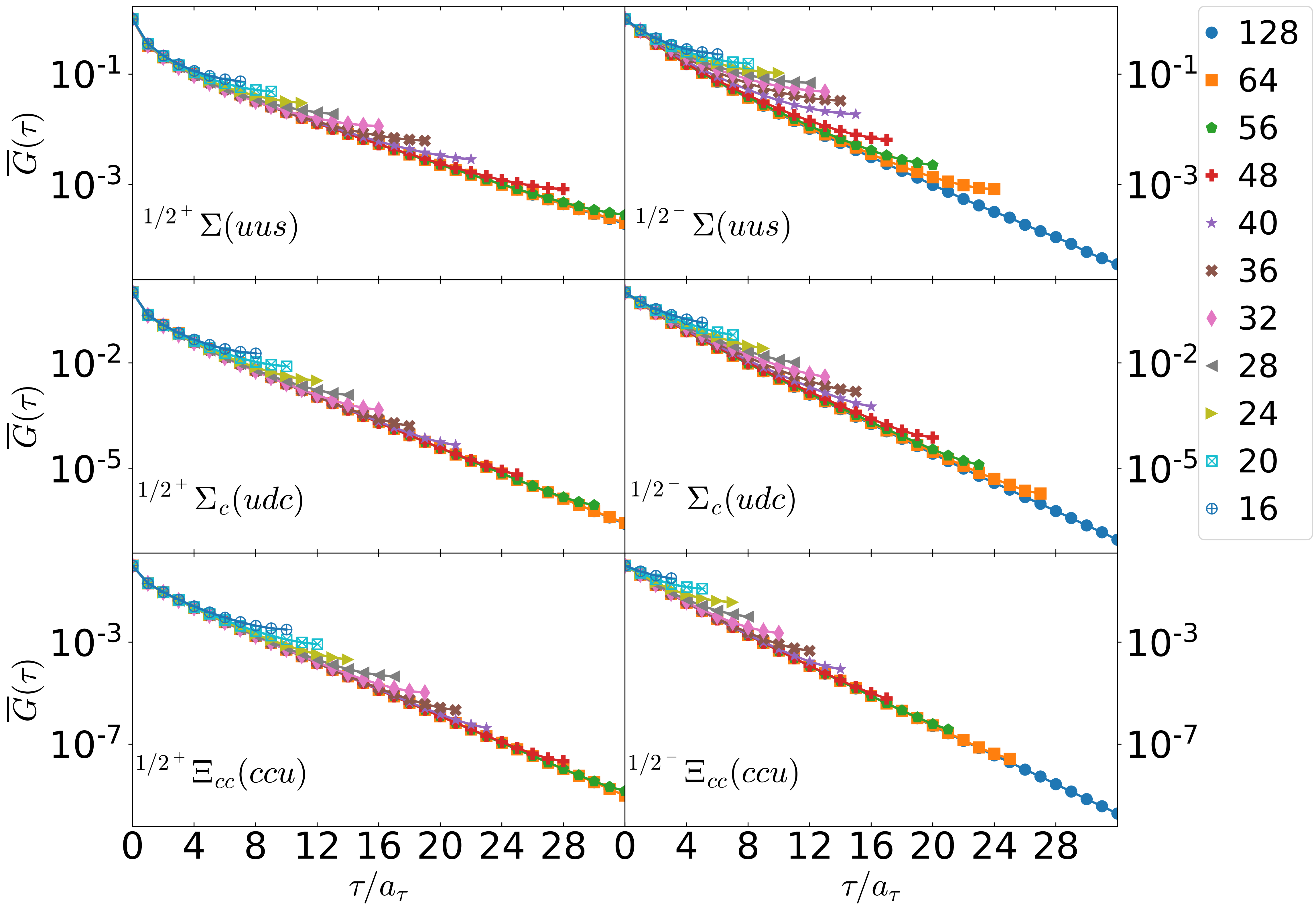
$$R(\tau) = \frac{G_+(\tau) - G_+(1/T - \tau)}{G_+(\tau) + G_+(1/T - \tau)}$$

$R(\tau) \sim 0 \rightarrow$ parity doubling

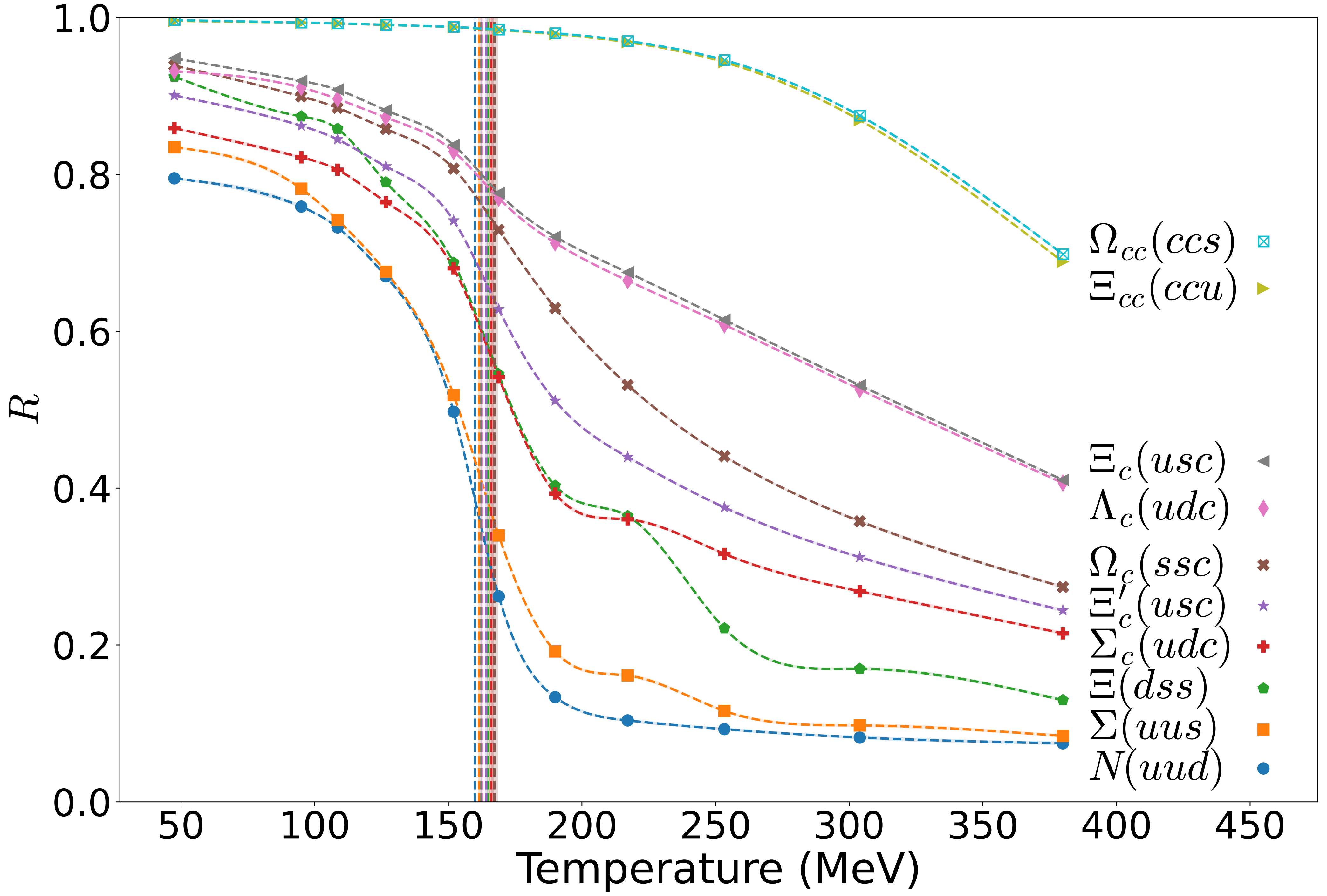
$R(\tau) \sim 1 \rightarrow$ parity max broken

$$R = \frac{\sum_\tau R(\tau)/\sigma^2(\tau)}{\sum_\tau 1/\sigma^2(\tau)}$$

$G(\tau)$

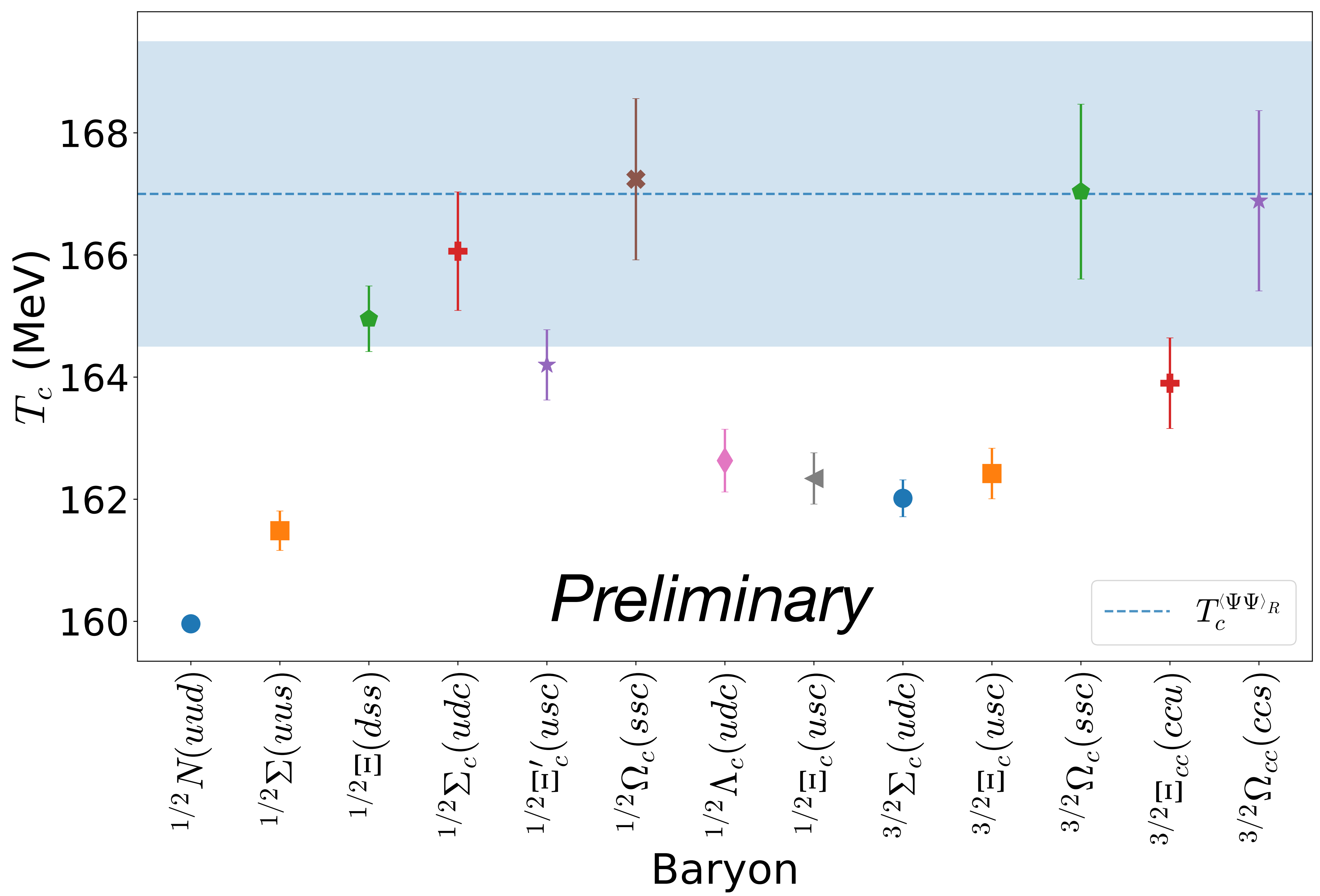


R

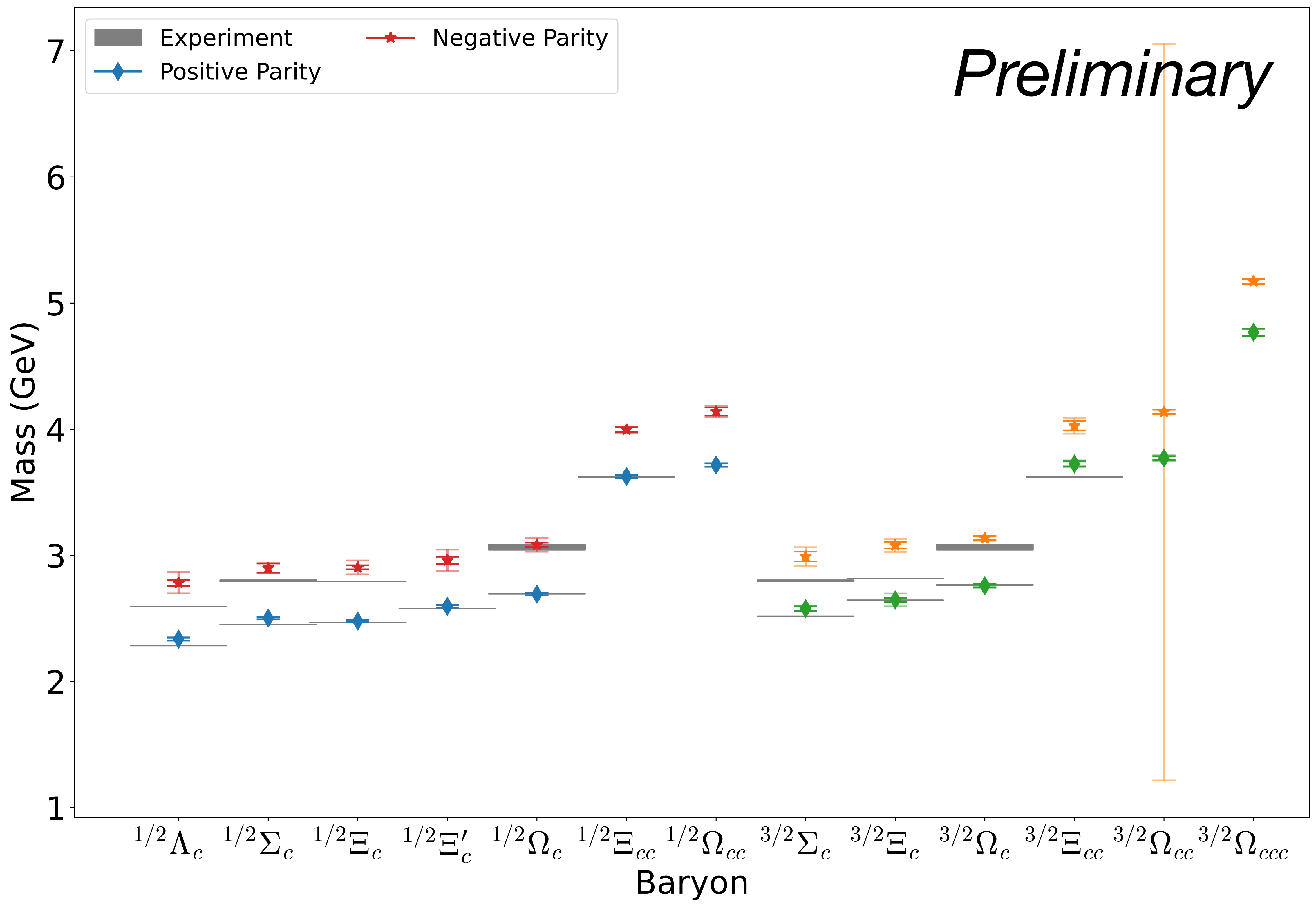


Charmed Baryonic T_c & Spectrum

T_C

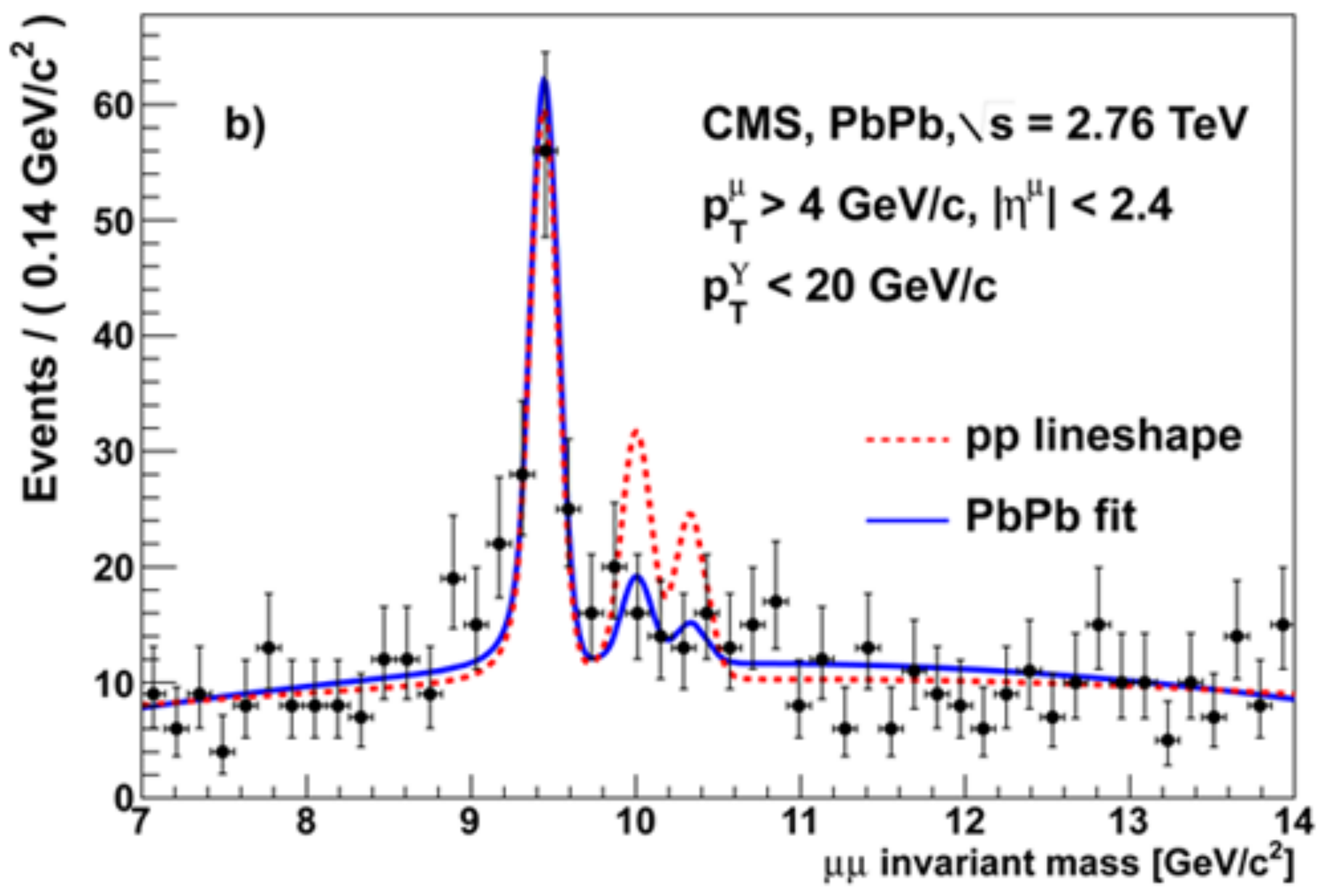


Spectrum



Bottomonium Spectrum & Widths
(via NRQCD)

Great interest to experimentalists and lattice groups



1. Exponential (Conventional δ f'ns)
2. Gaussian Ground State (+ δ f'n excited)

} Maximum Likelihood (Minimise χ^2)

3. Moments of Correlation F'ns — Direct Method - "no" fit

4. BR Method
5. Maximum Entropy Method

} Bayesian Approaches

6. Kernel Ridge Regression — Machine Learning

7. Backus Gilbert — from Geophysics

Bottomonium Spectrum (Backus Gilbert)

Ben Page

Recall $G(\tau;T) = \int_0^\infty \frac{d\omega}{2\pi} K(\tau,\omega;T)\ \rho(\omega;T)$

Use linear combinations of $K(\tau,\omega)$ to make δ f'n:

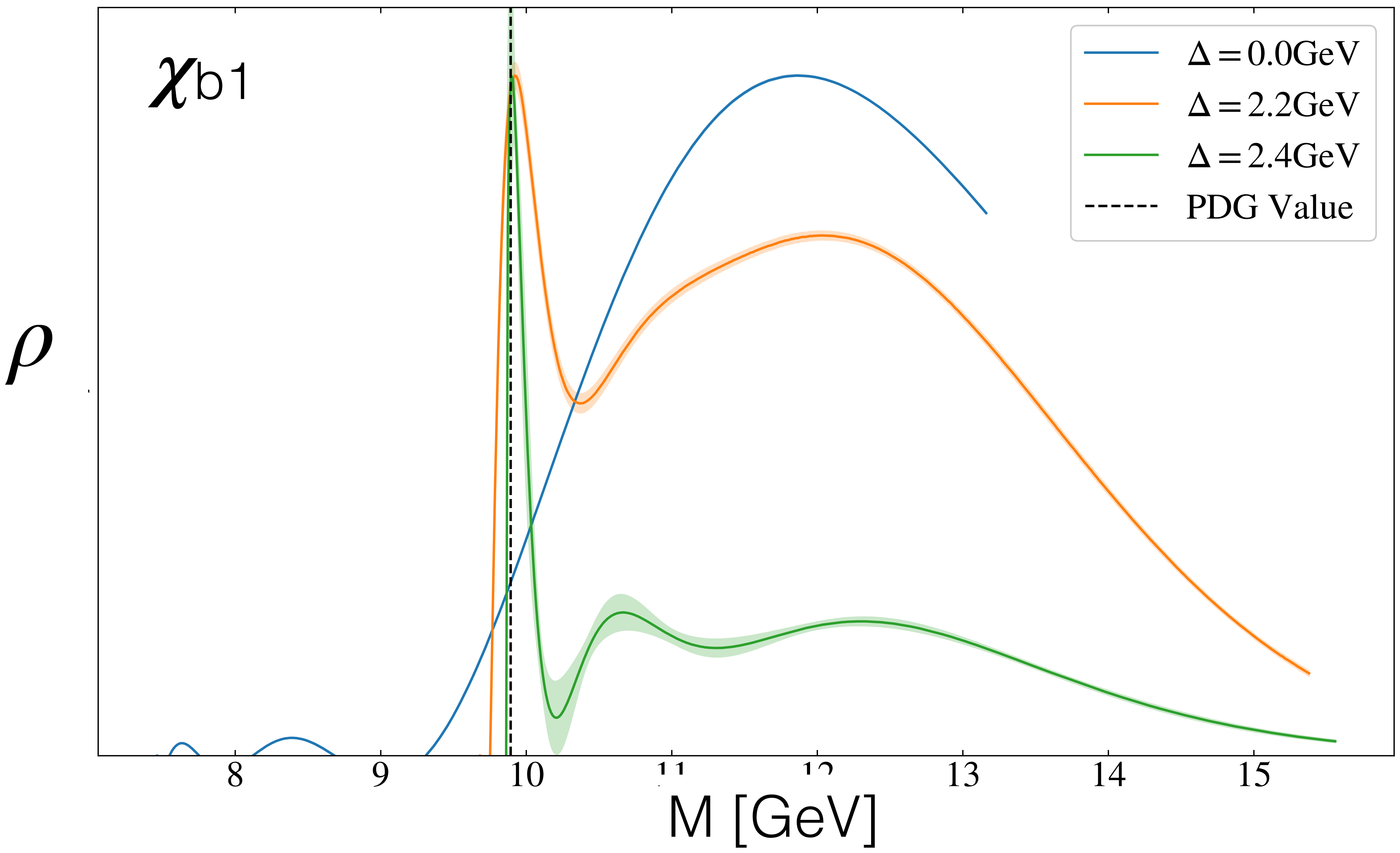
$$\sum_\tau C_{\omega_0}(\tau)\, K(\tau,\omega) \approx \delta(\omega-\omega_0)$$

$$\longrightarrow \sum_\tau C_{\omega_0}(\tau)\, G(\tau;T) \approx \rho(\omega_0)$$

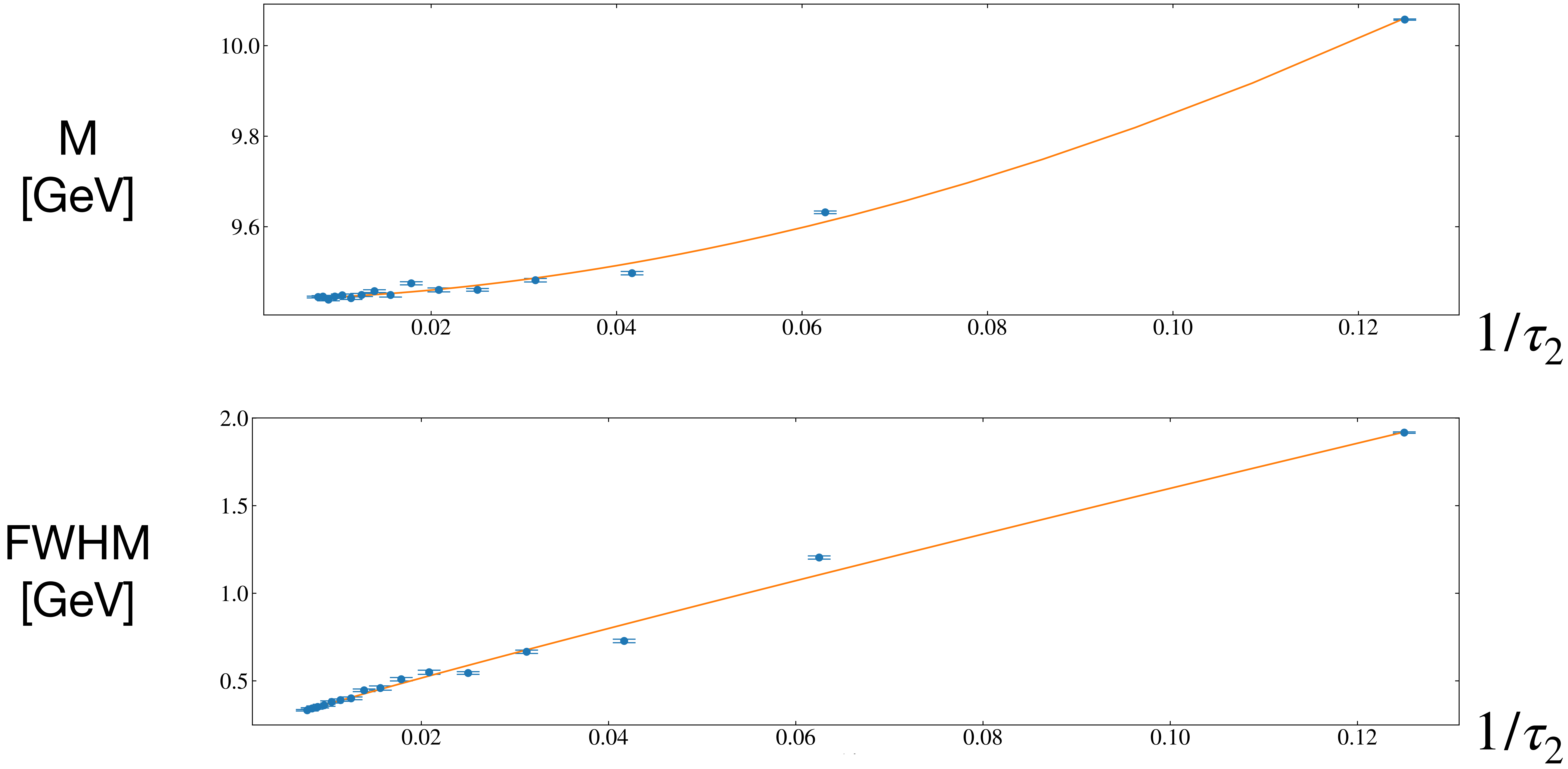
Fitting ("meta") parameters include:

- Δ = energy window shift
- τ_2 time window

Spectral F'n of χ_{b1} using **Backus-Gilbert** with **energy shift** Δ

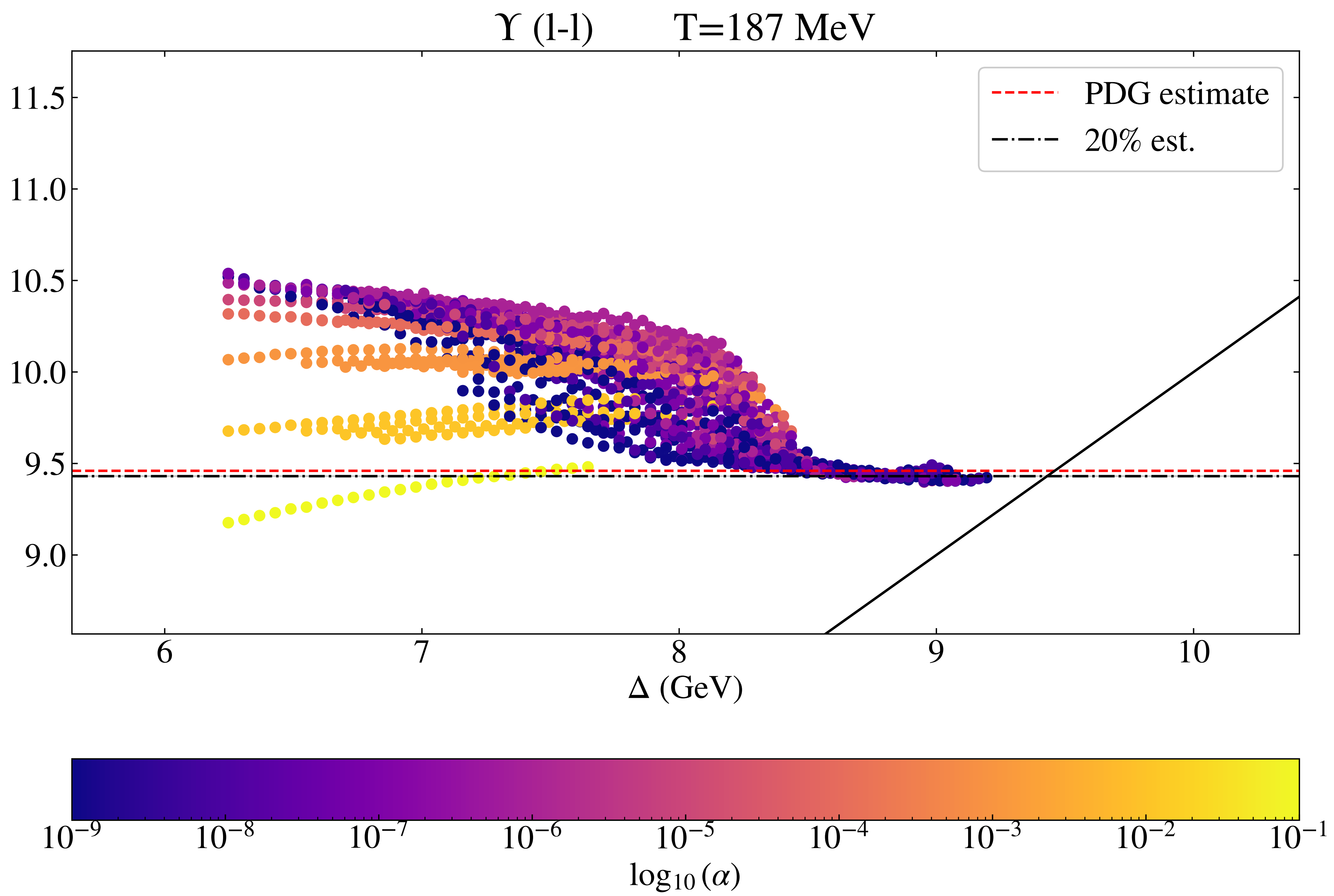


Systematics in time window

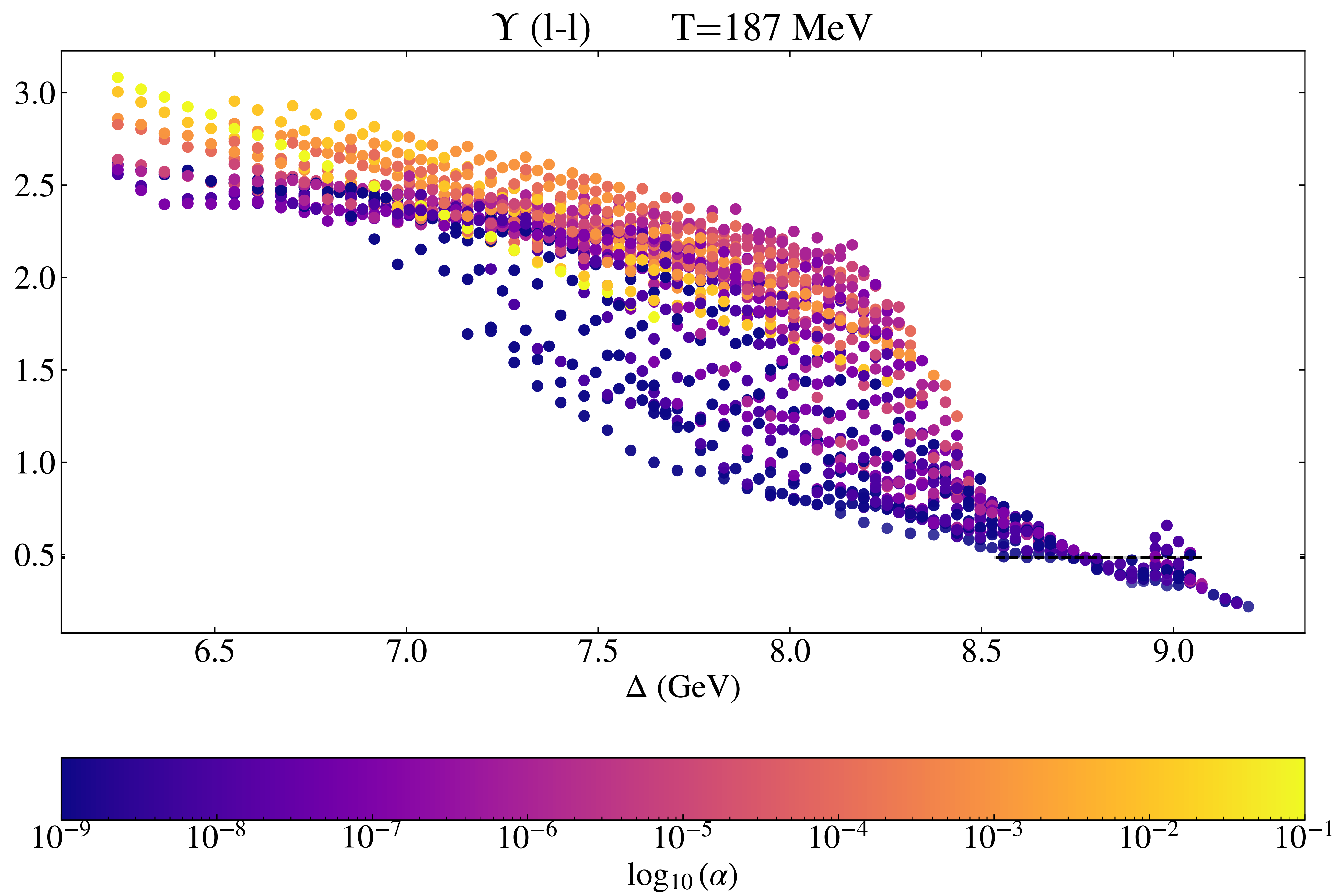


Systematics in energy & time window

M [GeV]



FWHM [GeV]



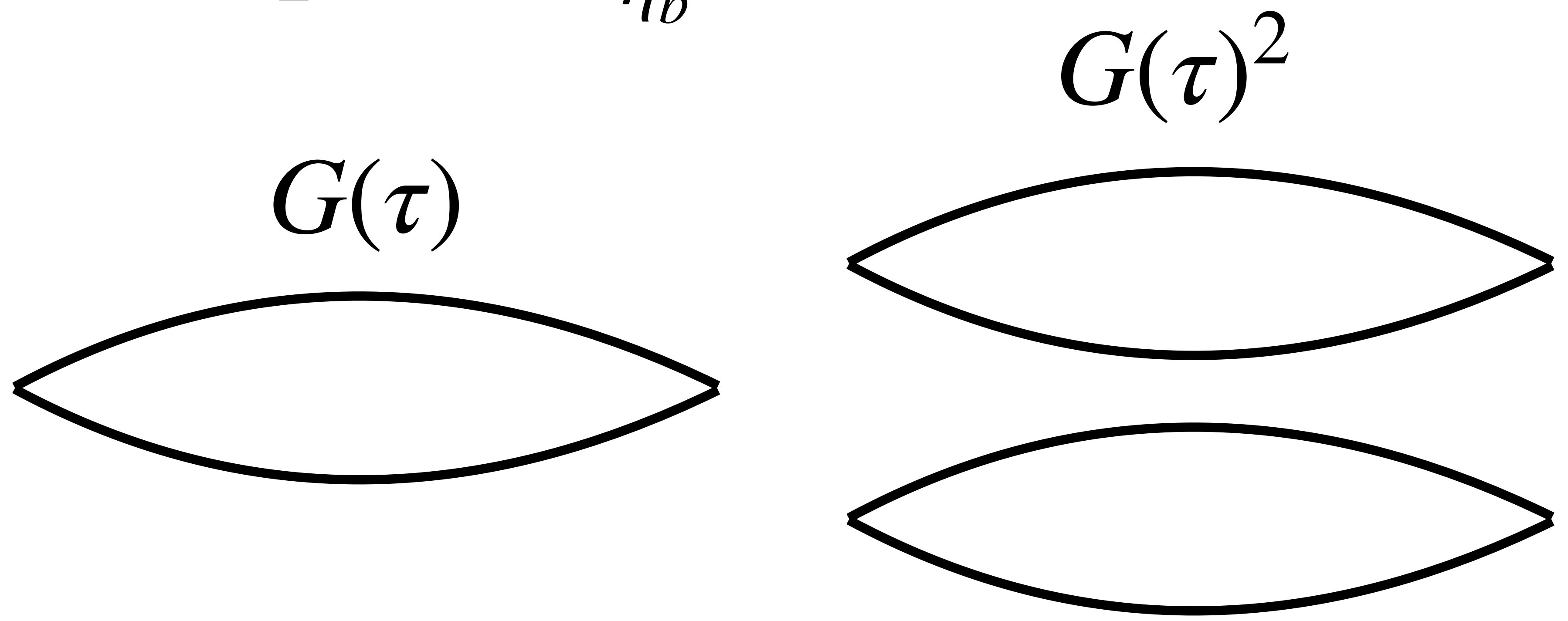
Parisi-Lepage Error Analysis

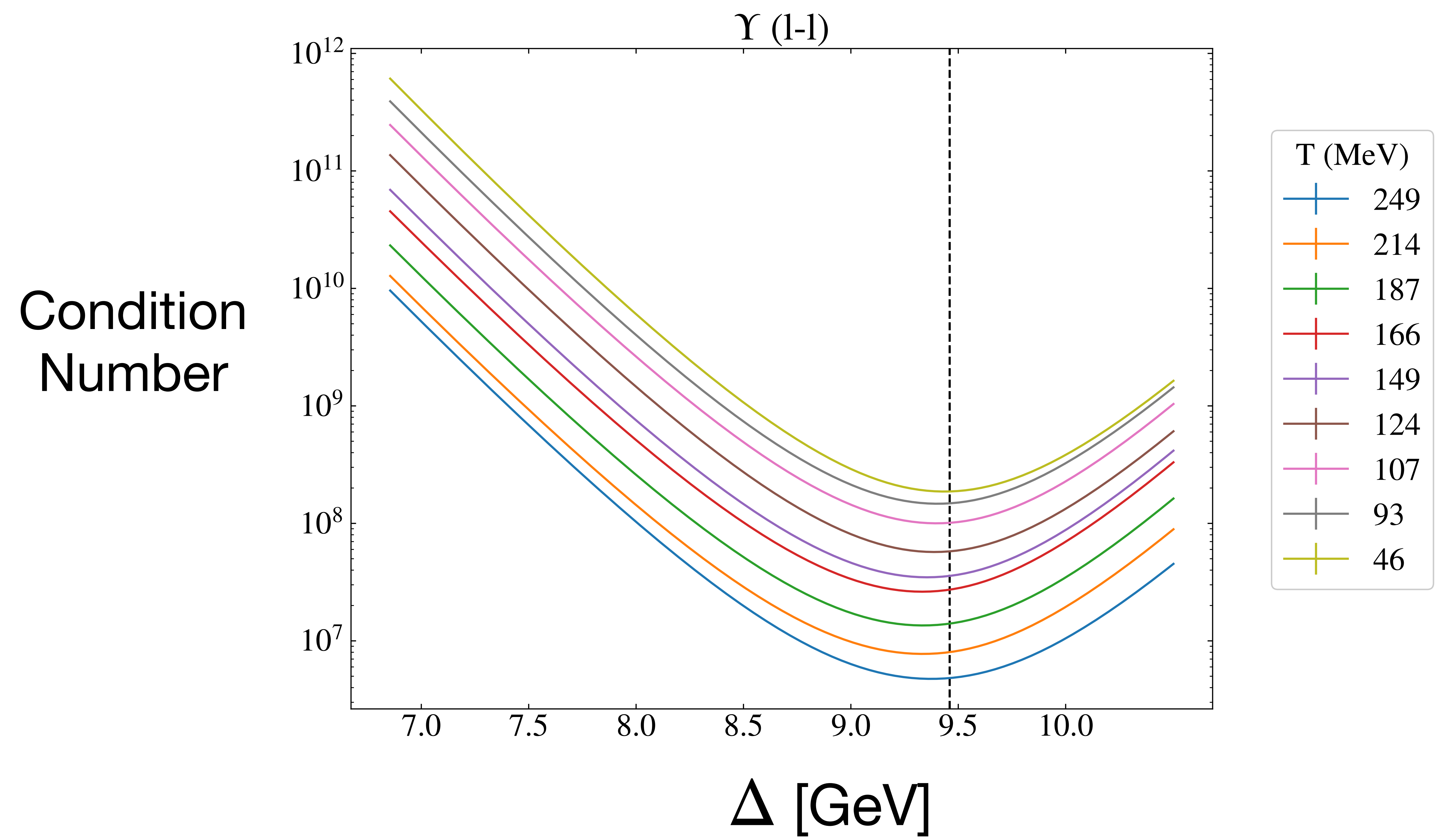
Analysis of η_b, Υ, h_b, χ_{b1} **Covariance Mx** Σ shows it's dominated by $\exp(-2M_{\eta_b}t)$

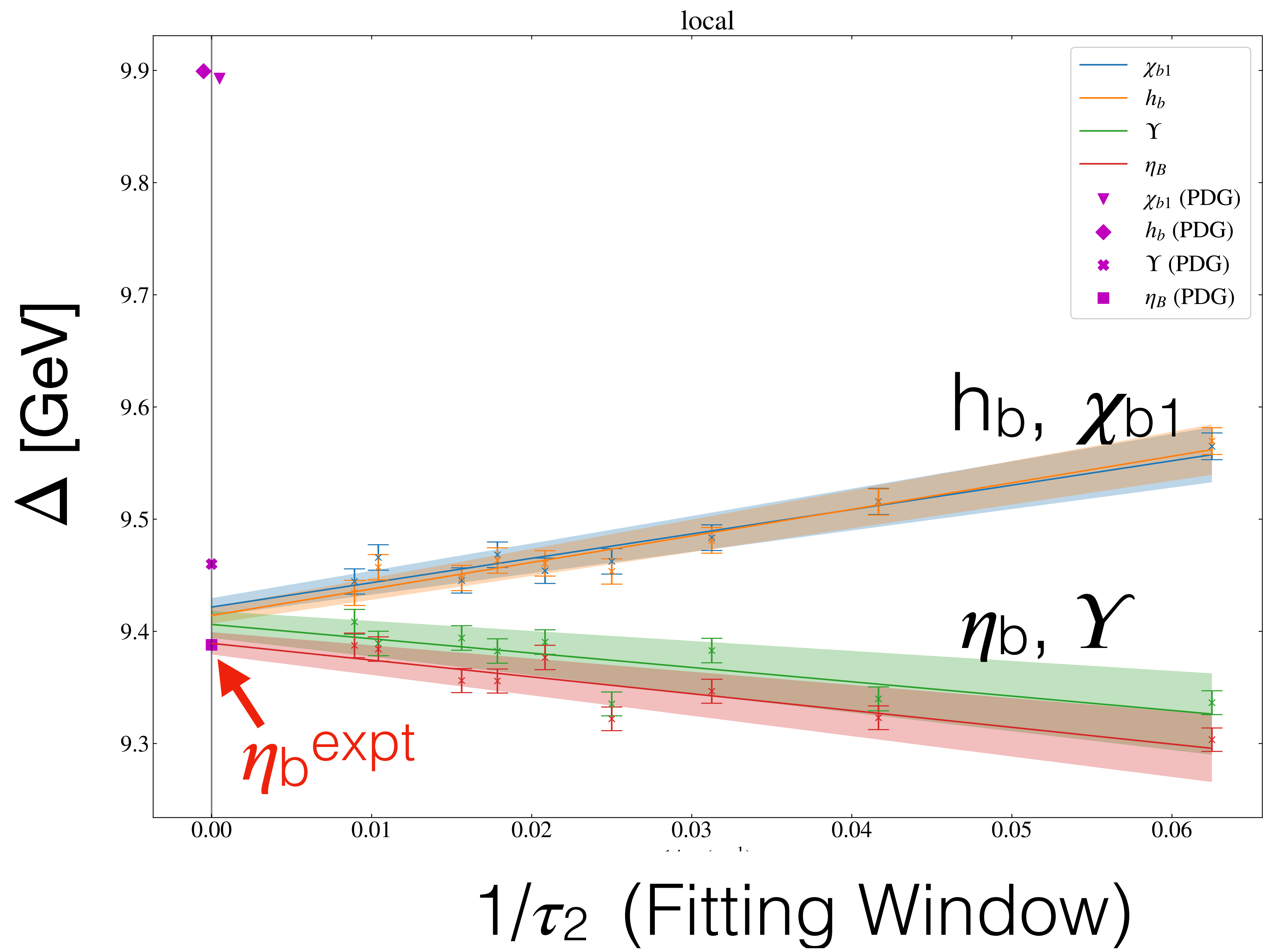
(Parisi-Lepage)

$$\Sigma \sim < x^2 > - < x >^2$$

Accessed via **Condition Number** of $e^{\Delta\tau}\Sigma e^{\Delta\tau}$







Summary

- Overview of FASTSUM approach
  - anisotropic, designed for spectroscopy
- Charmed meson spectrum (Sergio Chaves)
  - PS & V have little thermal effect, Scalar & Axialvector much more
- Charmed baryonic spectrum (Ryan Bignell)
  - Parity doubling at large temperature
- Bottomonium (NRQCD) spectrum (Benjamin Page)
  - Towards quantitative results