EM PROBES: “Is it possible to see the EOS softest point from EM probes measured by STAR? How reliable are comparisons to theory?” (Akamatsu, Du, Pisarski, Ruan)

Criticality in emission rate of photons and dileptons

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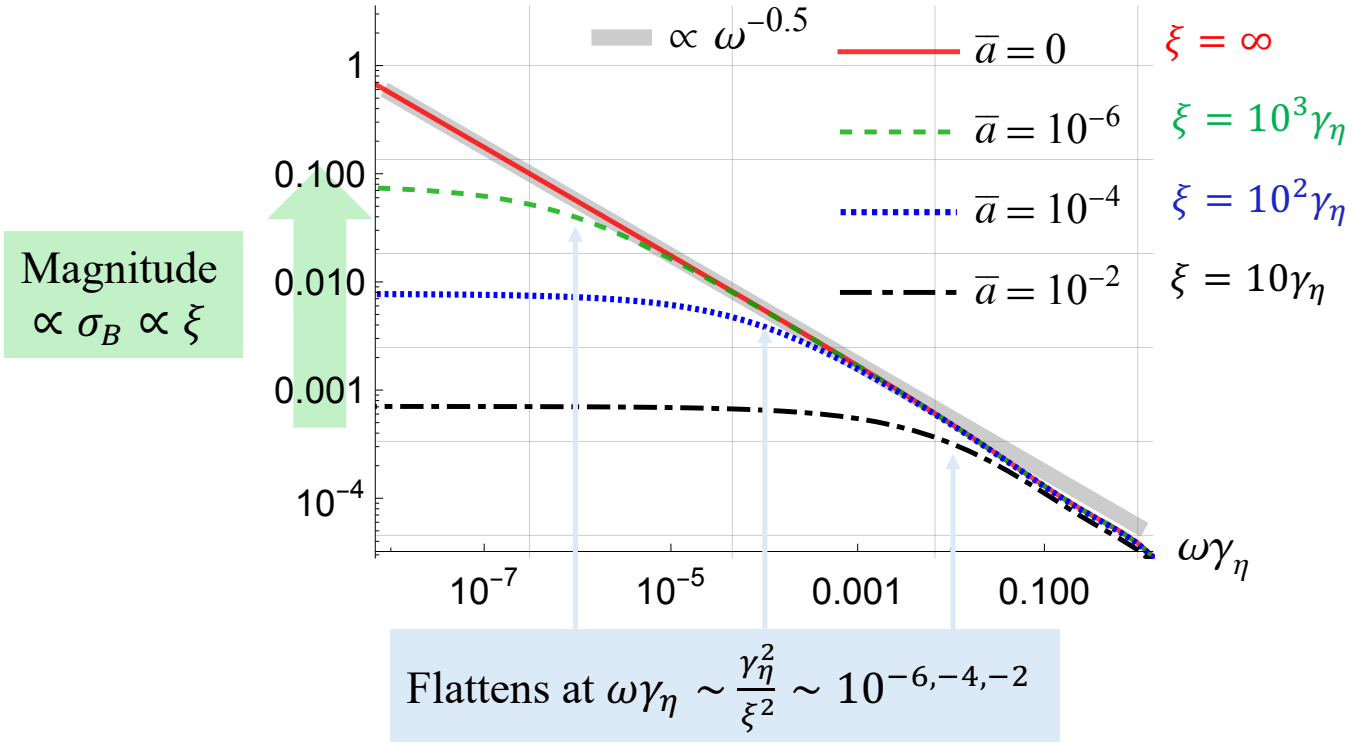
INT workshop: The QCD Critical Point: Are We There Yet? October 31, 2025

Criticality in emission rate of photons and dileptons

Universal photon spectrum at low energy

- Photon spectrum $\omega\, dN_\gamma/d^3k d^4x$ (a.u.)

Akamatsu-Asakawa-Hongo
-Stephanov-Yee 25



Criticality in emission rate of photons and dileptons

Experimental Challenges

- Yield of low energy photons

$$\omega \frac{dN_\gamma}{d^3k}\Big|_{\omega\sim 0} \simeq \frac{\alpha}{12\pi^3}\cdot\frac{T^2}{\bar{\eta}}\cdot\frac{\chi_T}{\xi^2}\cdot\xi V_4 \propto \xi V_4$$

$$\sim 2\times 10^{-5}\cdot\left(\frac{T^2}{\bar{\eta}}[\mathrm{fm}]\right)\cdot\left(\frac{\chi_T}{\xi^2}[\mathrm{fm}^{-4}]\right)\cdot\left(\xi V_4[\mathrm{fm}^5]\right)$$

Roughly, substituting 1fm for all unknown scales, the plateau is around $\omega\frac{dN_\gamma}{d^3k} \sim 2\ \mathrm{fm}^2 \simeq 50\ \mathrm{GeV}^{-2}$ for $\xi \simeq 10$ fm, $V_4 \sim (10\ \mathrm{fm})^4$

- How low is the photon energy?

Typical photon energy is $\omega \sim \frac{\gamma_\eta}{\xi^2} \sim 0.01\ \mathrm{fm}^{-1} \simeq 2\ \mathrm{MeV}$ for $\xi \simeq 10$ fm

Criticality in emission rate of photons and dileptons

Theoretical Directions

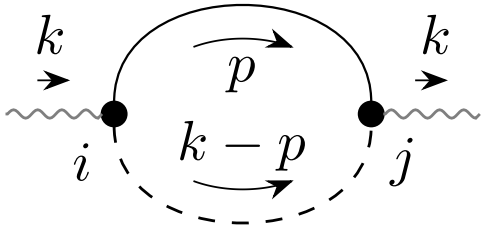
- Dileptons
    - Enhancement is expected from conductivity $\sigma \propto \xi$
    - NJL model calculation Nishimura-Kitazawa-Kunihiro (23,24)
    - Model H calculation to obtain universal scaling

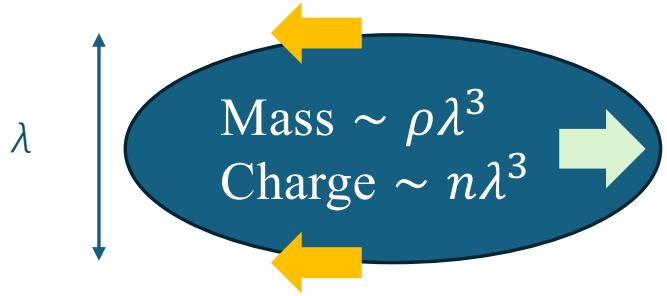
- Nonequilibrium correction to the emission rate ~ similar to Kibble-Zurek scaling

Criticality in emission rate of photons and dileptons

Theoretical Description

$$\Pi_S^{Tii}(k,\boldsymbol{k})\big|_{\text{1-loop}} \simeq$$





Drag $\sim -\lambda^2 \cdot \eta v \lambda^{-1} = -\eta v \lambda$

Field $\sim E = E\,(\omega) e^{-i\omega t}$
Force $\sim n\lambda^3 E$

EoM: $-i\omega\rho v(\omega)\lambda^3 = -\eta v(\omega)\lambda + n\lambda^3 E\,(\omega)$

Current $J(\omega) = nv(\omega) = \frac{n^2}{-i\omega\rho + \eta\lambda^{-2}}$

Charge fluctuation $\langle n^2 \rangle = \frac{1}{\lambda^{-2} + \xi^{-2}}$

Criticality in emission rate of photons and dileptons

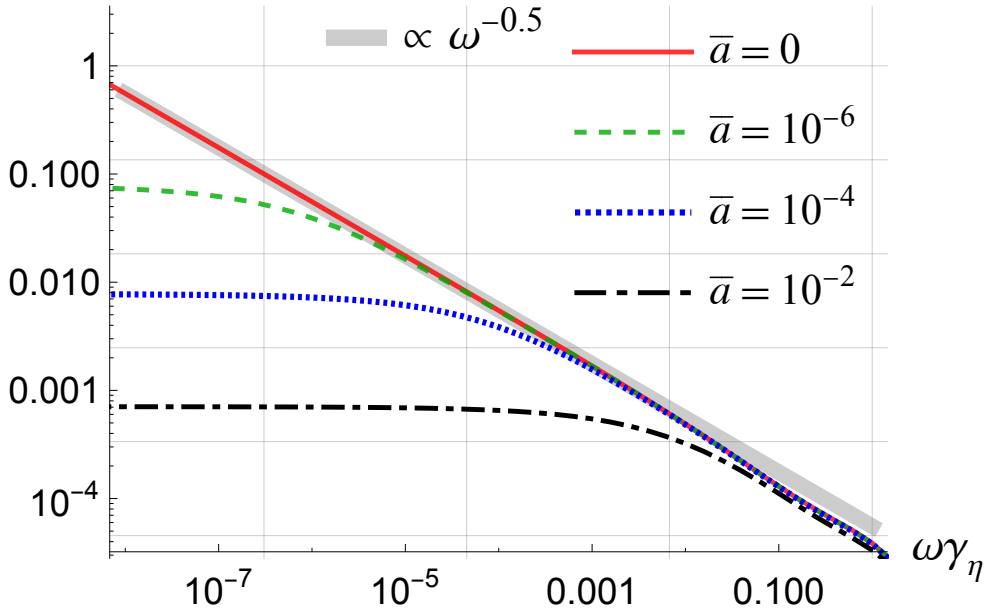
Theoretical Description

Response function $\Pi_R(\omega, k=\omega) \sim \int d^3p \frac{1}{-i\omega\rho+\eta p^2} \frac{1}{p^2+\xi^{-2}} \sim \frac{1}{p_{IR}}$

$$p_{IR} = \left(\frac{\omega\rho}{\eta}\right)^{\frac{1}{2}} = \left(\frac{\omega}{\gamma_\eta}\right)^{\frac{1}{2}} \text{ or } \frac{1}{\xi}$$

Competes at $\omega = \frac{\gamma_\eta}{\xi^2}$

Magnitude $\propto \sigma_B \propto \xi$



Flattens at $\omega\gamma_\eta \sim \frac{\gamma_\eta^2}{\xi^2} \sim 10^{-6,-4,-2}$