Using Fermi Liquid Theory for Nuclear Response

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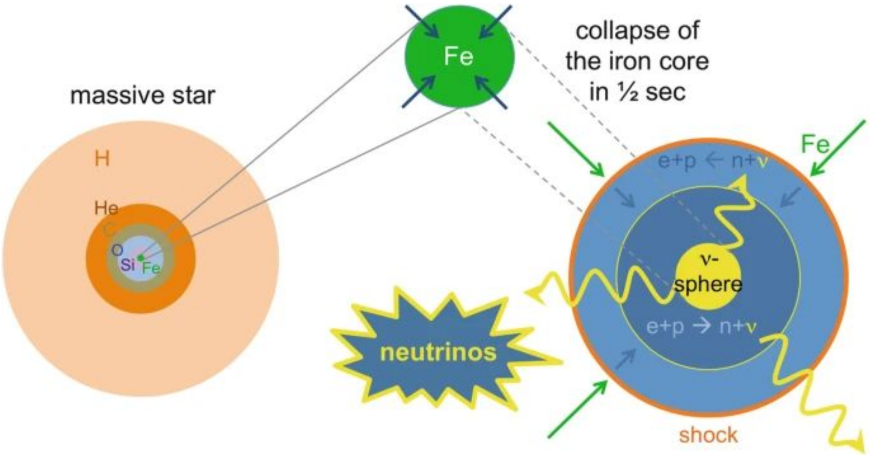
Motivation

- core-collapse supernovae: production of neutrinos
- neutrinos trapped in proto-neutron core
  - neutrino opacity
  - neutrino transport, absorption rates
  - cooling and evolution of the star



Beta processes:	$e^- + p \rightleftharpoons n + \nu_e$
	$e^+ + n \rightleftharpoons p + \bar{\nu}_e$
	$e^- + A \rightleftharpoons \nu_e + A^*$
Neutrino scattering:	$\nu + n, p \rightleftharpoons \nu + n, p$
	$\nu + A \rightleftharpoons \nu + A$
	$\nu + e^{\pm} \rightleftharpoons \nu + e^{\pm}$
Thermal pair processes:	$N + N \rightleftharpoons N + N + \nu + \bar{\nu}$
	$e^+ + e^- \rightleftharpoons \nu + \bar{\nu}$
Neutrino-neutrino reactions:	$\nu_x + \nu_e, \bar{\nu}_e \rightleftharpoons \nu_x + \nu_e, \bar{\nu}_e$ $(\nu_x = \nu_\mu, \bar{\nu}_\mu, \nu_\tau, \text{or } \bar{\nu}_\tau)$
	$\nu_e + \bar{\nu}_e \rightleftharpoons \nu_{\mu,\tau} + \bar{\nu}_{\mu,\tau}$

Janka et. al (2012)

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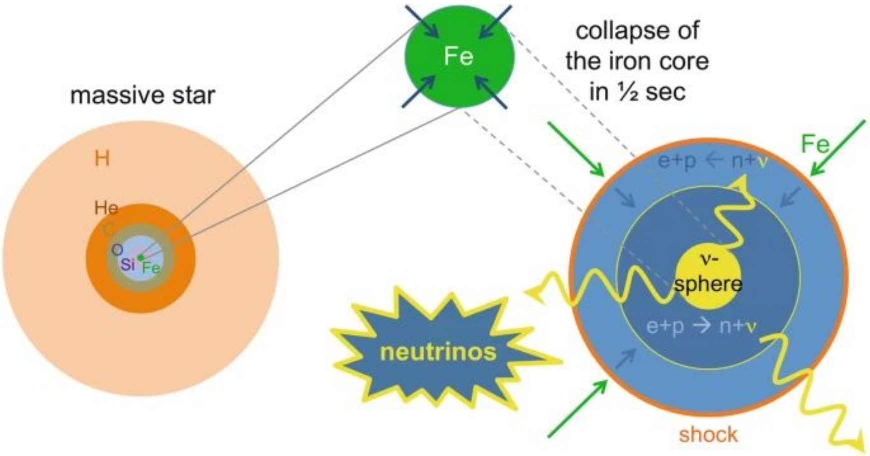
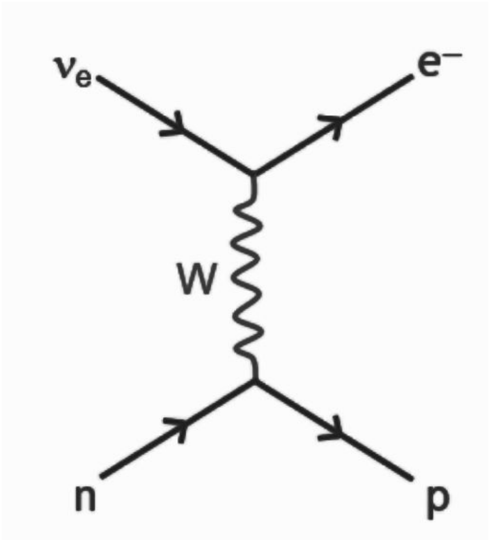




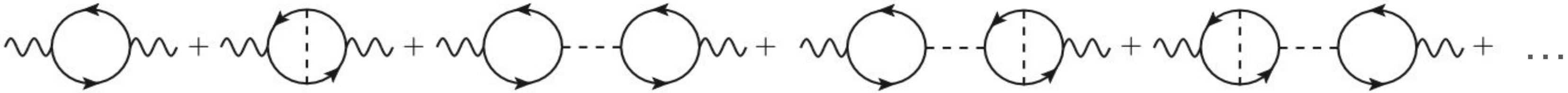
Response functions

- Quantification of system reaction to external perturbation
  - e.g. neutrino scattering, absorptions
  - function of *q*, *ω*
- Imaginary part: dynamical structure functions

$$S_\tau(q,\omega) = -\frac{2\,\mathrm{Im}\chi(q,\omega)}{1 - e^{-(\omega+\mu_n-\mu_p)/T}},$$

Response functions

- **Random Phase Approximation:**
  - many-body, conserving approximation:
    - collective excitations in interacting system
    - derived from **linear response theory**
  - Hartree-Fock ground state - lowest energy state by assuming mean field
    - small oscillations of HF Slater determinant
    - particle-hole excitations



Shin et. al (2023)

$$\chi^{RPA}(q,\omega) = \chi_0(q,\omega) + v_q[\chi_0(q,\omega)]^2 + v_q^2[\chi_0(q,\omega)]^3 + \cdots$$
$$= \frac{\chi_0(q,\omega)}{1 - v_q\chi_0(q,\omega)}.$$

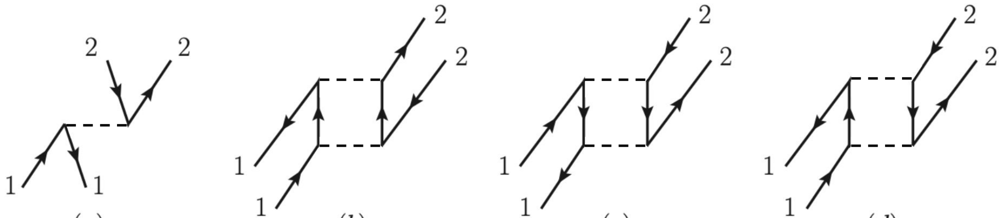
χ_0: non-interacting response function
V_q: interaction potential

Response functions

- **Fermi Liquid Theory:**
  - framed around interacting fermions
    - low T, near Fermi-surface
  - quasiparticles: weakly-interacting, collective excitations
    - effective mass, interaction strength ~ Landau parameters
- **Beyond RPA**: higher order perturbations in interactions between *n, p*
  - More effective at low *q*, low *ω* scales

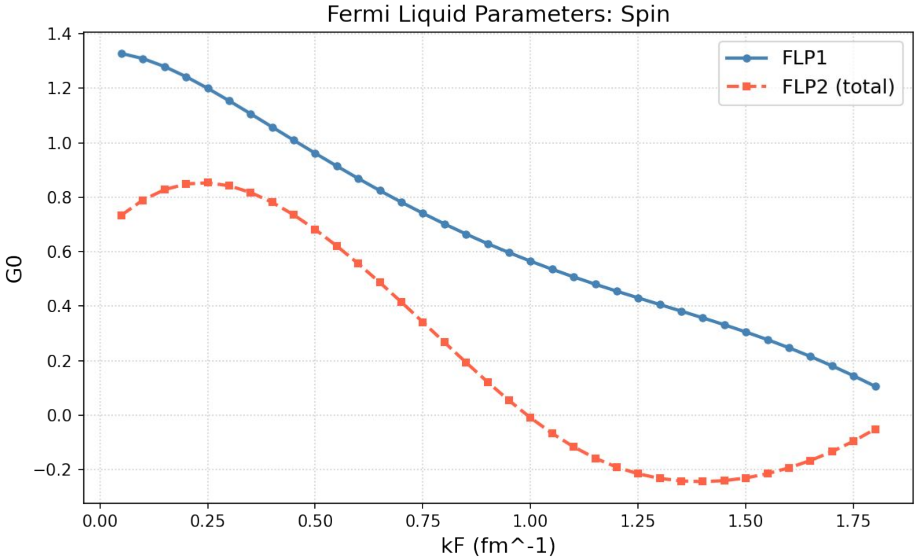
$$\chi^R_{nn}(\mathbf{q},\omega) = \frac{\chi^{0R}_{nn}(\mathbf{q},\omega)}{1 + f^s_0 \chi^{0R}_{nn}(\mathbf{q},\omega)}.$$

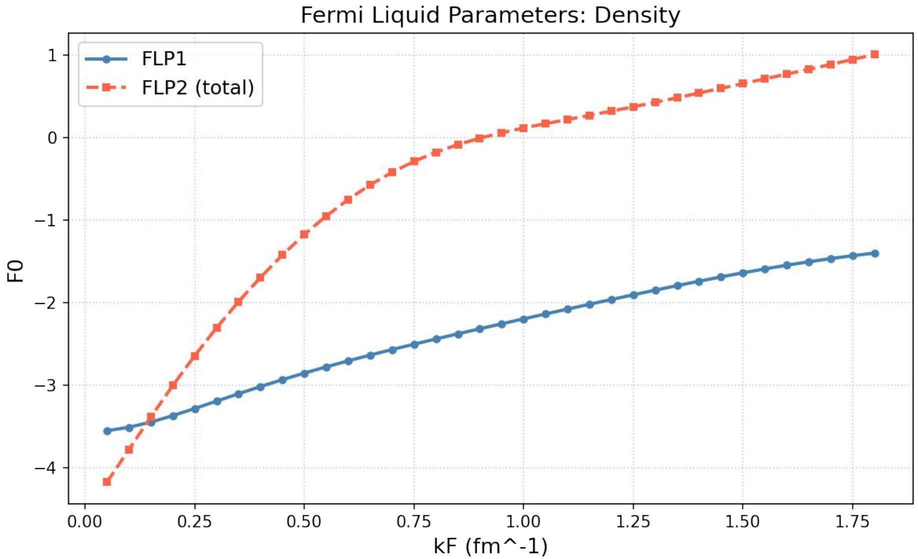
$$\chi_\sigma(\vec{\mathbf{k}},\omega) = \frac{N(0)}{V}\frac{g(\lambda)}{1+[F^a_0+\lambda^2 F^a_1/(1+\frac{1}{3}F^a_1)]g(\lambda)}$$



(a) (b) (c) (d)

Results

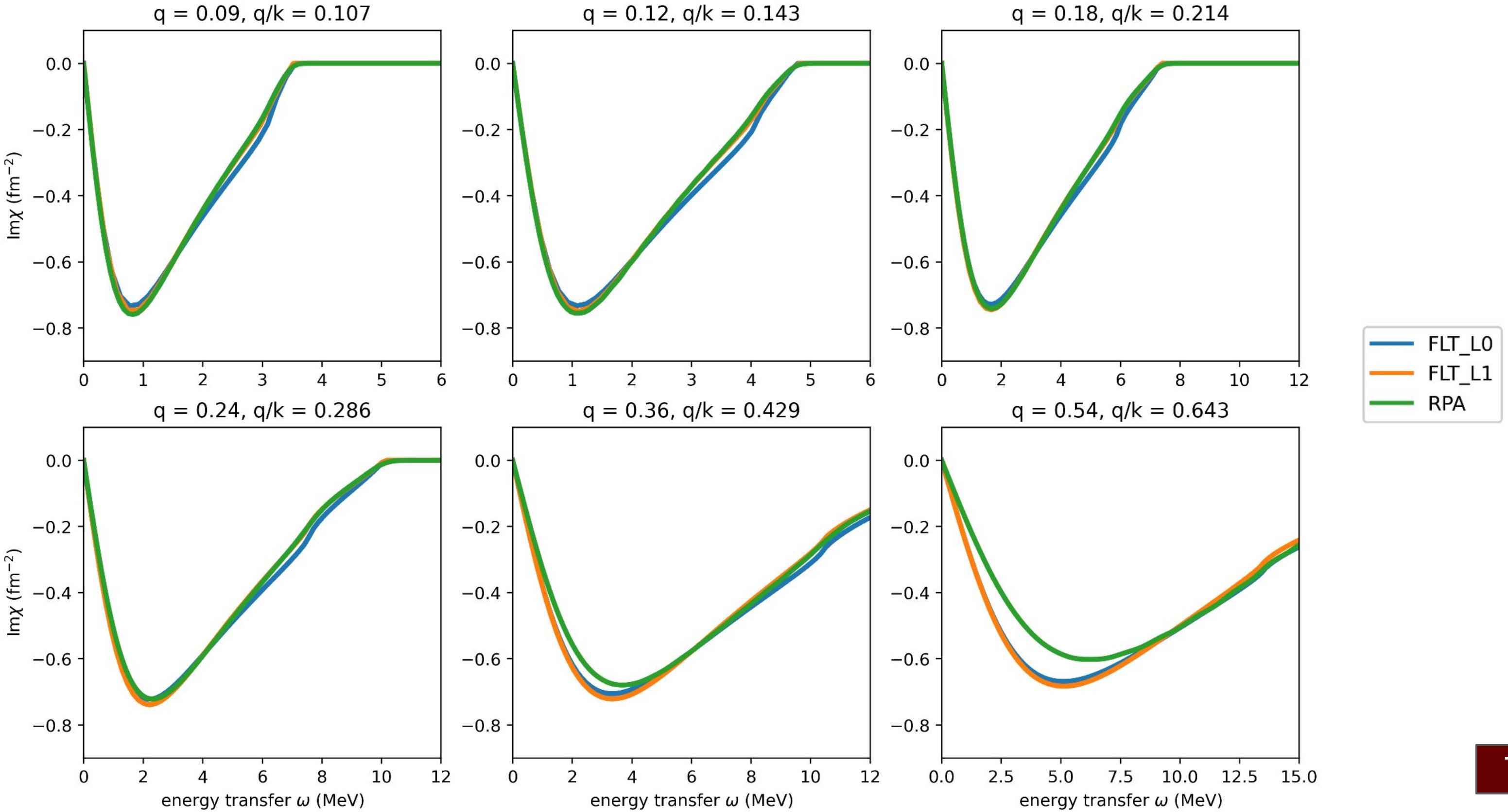

Fermi Liquid Parameters: Spin
FLP1
FLP2 (total)
G0
kF (fm^-1)
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75


Fermi Liquid Parameters: Density
FLP1
FLP2 (total)
F0
kF (fm^-1)
1
0
−1
−2
−3
−4
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75

Results

Density response functions:

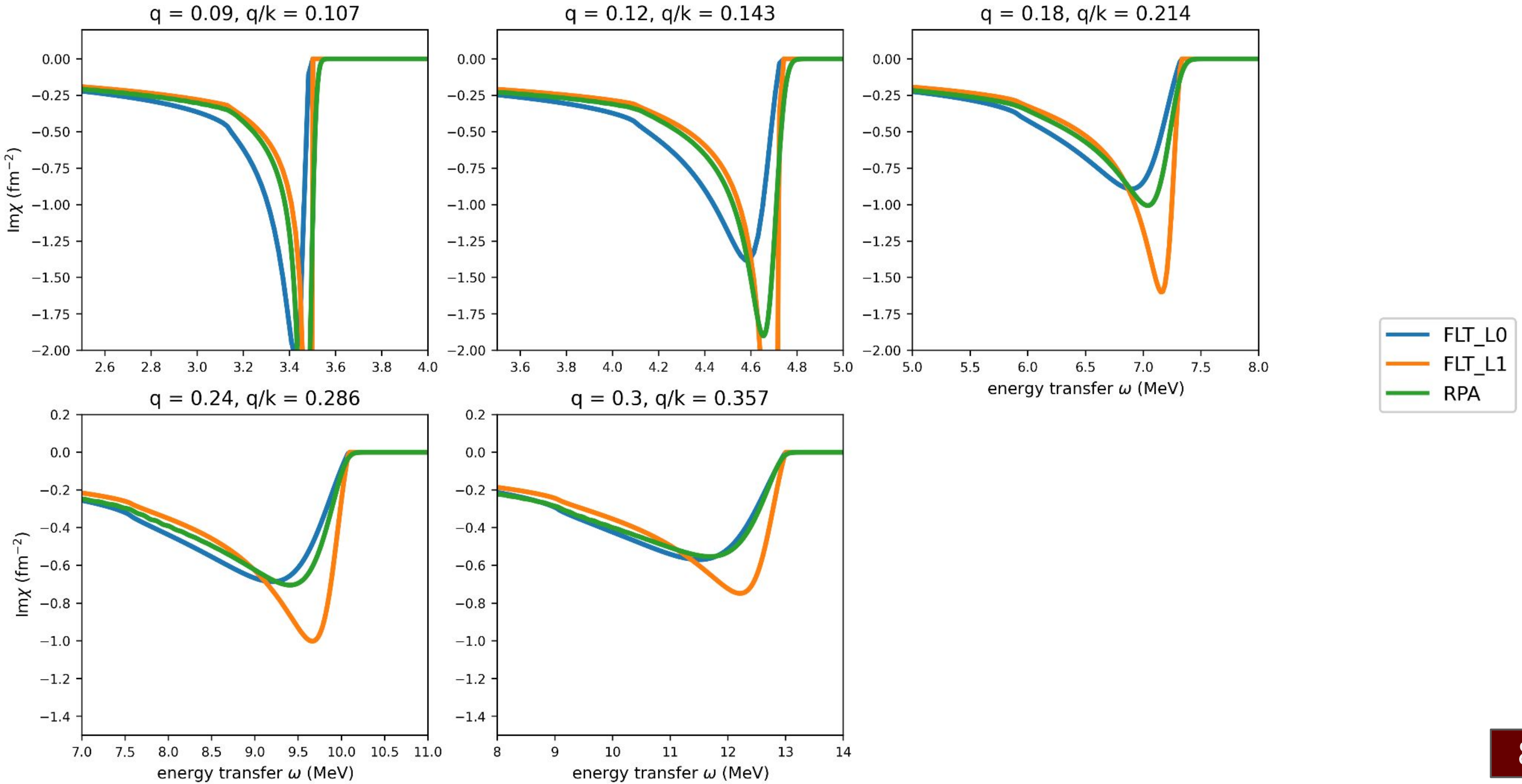
- D = 0.01, 0.02, 0.05 fm^{-3}
- q = 0.06 - 0.54 fm^{-1}



Results

Spin response functions:

- $D = 0.02 - 0.12\ fm^{-3}$
- $q = 0.09 - 0.3\ fm^{-1}$



Future

Next steps:

- Convergence of finite-T to T=0 response functions
- Symmetric nuclear matter vs. pure neutron matter
- Incorporation of second-order Fermi liquid theory diagrams
  - Neutrino opacities
  - Cross sections

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Thank You!