Intruder States and the Challenges They Pose

From a χEFT consumer point of view

Anna E. McCoy

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Outline

- Brief review of *ab initio* no-core shell model (NCSM)
- Intruder states in ^{12}Be
- Impacts of intruder states on observables
- Looking for a “LO" picture
  What we know, what we want to know

No-core shell model

Solve many-body Schrodinger equation

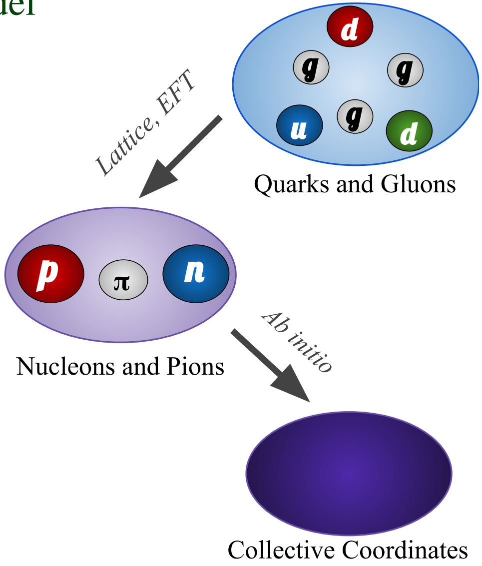
$$\sum_{i}^{A} -\frac{\hbar^2}{2m_i}\nabla_i^2 \Psi + \frac{1}{2}\sum_{i,j=1}^{A} V(|r_i - r_j|)\Psi = E\Psi$$

Expanding wavefunctions in a basis

$$\Psi = \sum_{k=1}^{\infty} a_k \phi_k$$

Reduces to matrix eigenproblem

$$\begin{pmatrix} H_{11} & H_{12} & \dots \\ H_{21} & H_{22} & \dots \\ \vdots & \vdots & \end{pmatrix}\begin{pmatrix} a_1 \\ a_2 \\ \vdots \end{pmatrix} = E\begin{pmatrix} a_1 \\ a_2 \\ \vdots \end{pmatrix}$$



Harmonic oscillator basis

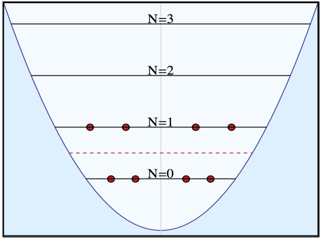
- Basis states are configurations, i.e., distributions of particles over harmonic oscillator shells (*nlj substates*)

- States are organized by total number of oscillator quanta above the lowest Pauli allowed number $N_{\rm ex}$

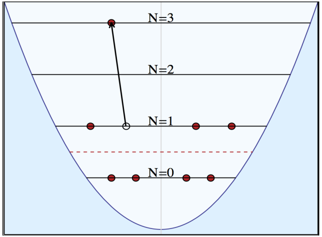
- States with higher $N_{\rm ex}$ contribute less to the wavefunction

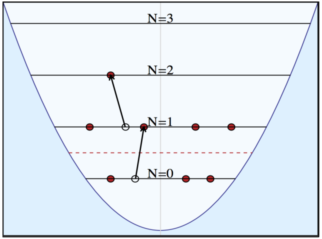
- Basis must be truncated:
  Restrict $N_{\rm ex} \leq N_{\rm max}$

$N = 2n + l$



$N_{\rm ex} = 0$



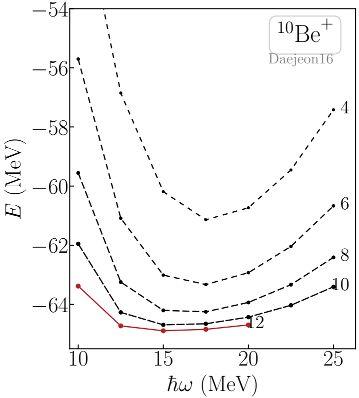


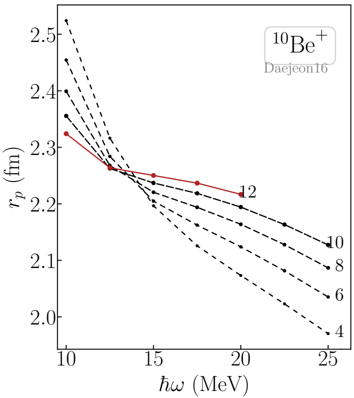
$N_{\rm ex} = 2$

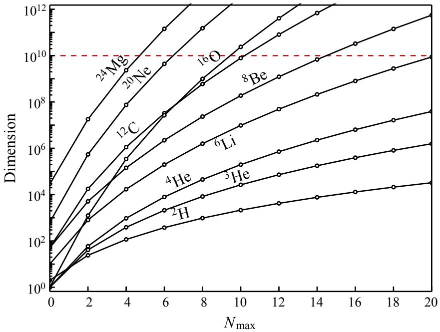
Convergence Challenge

Results for calculations in a finite space depend upon:

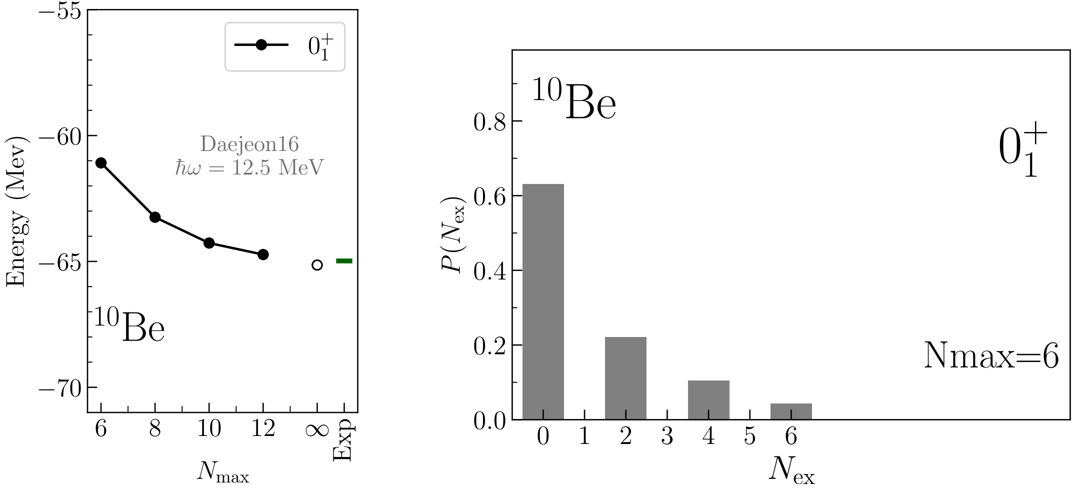
- Many-body truncation $N_{\max}$
- Single-particle basis scale $\hbar\omega$







Normal States vs. Intruder States



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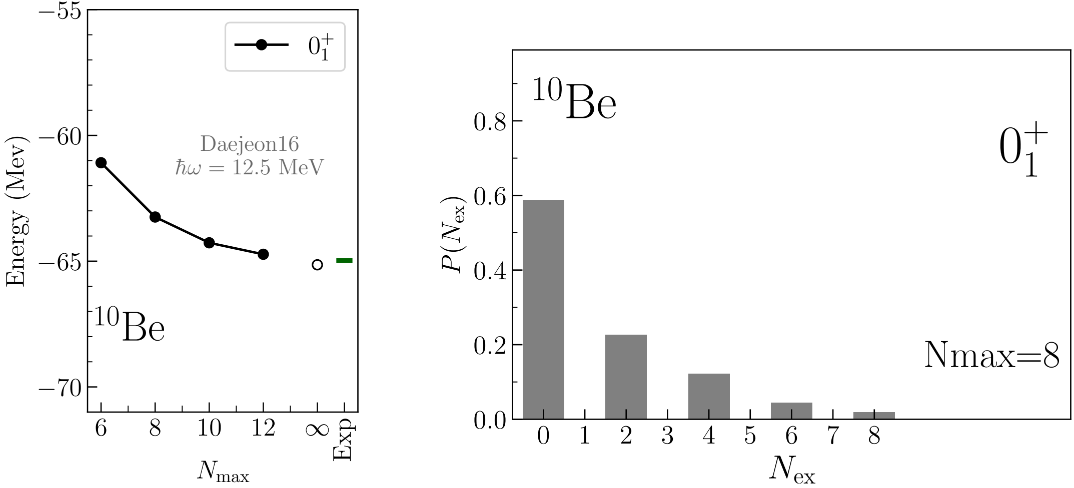

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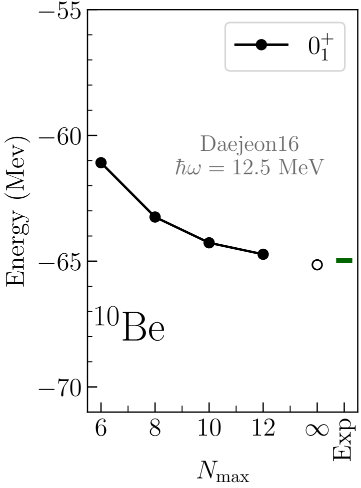

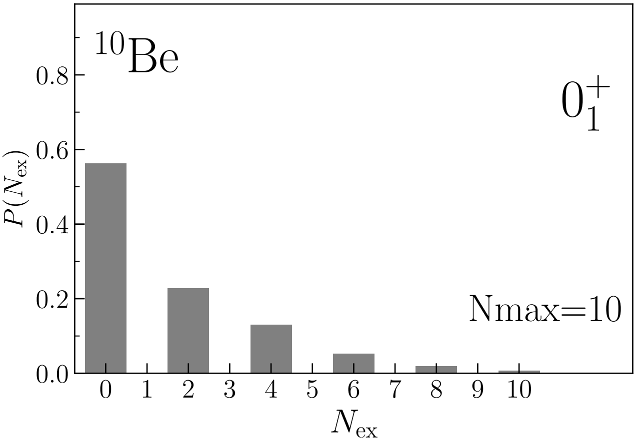

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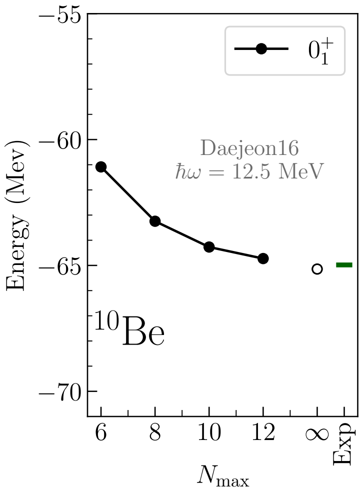

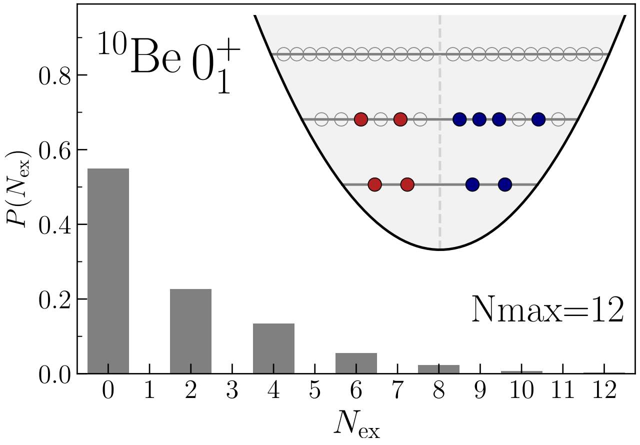

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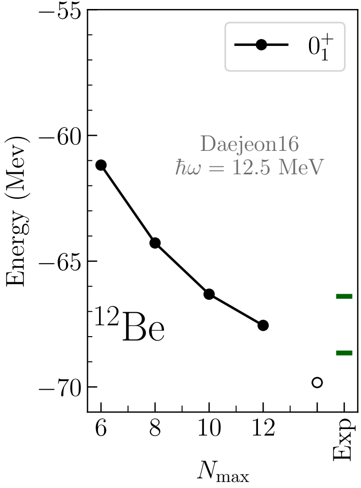

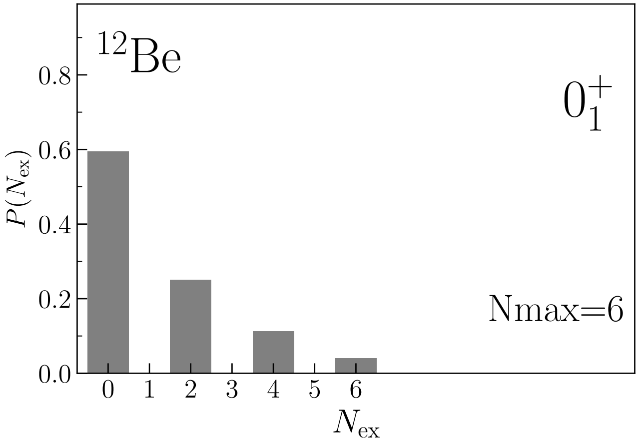

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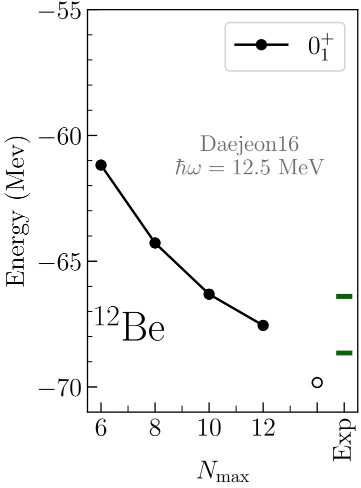

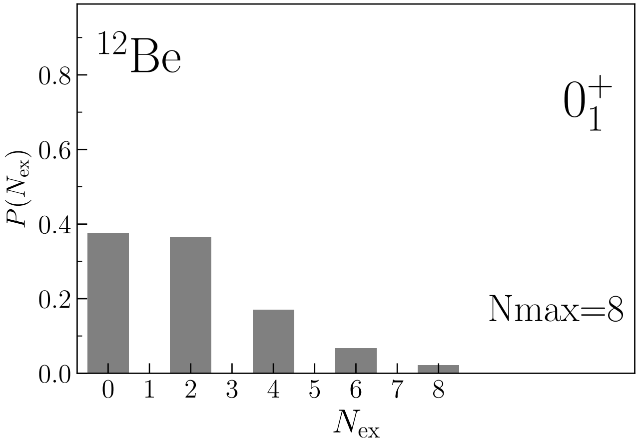

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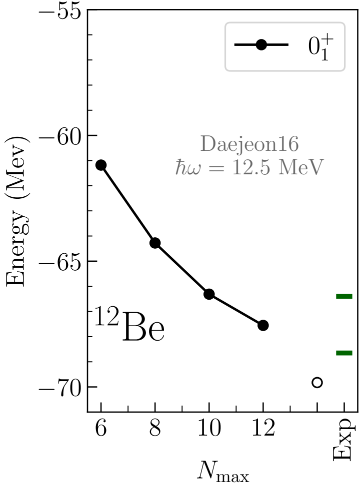

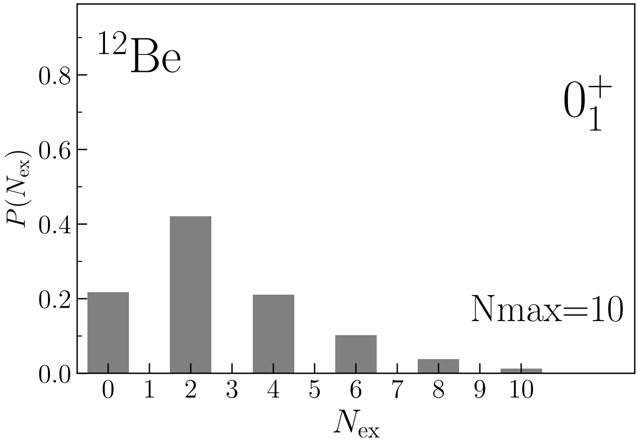

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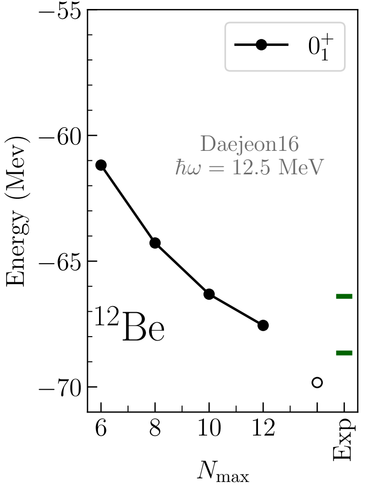
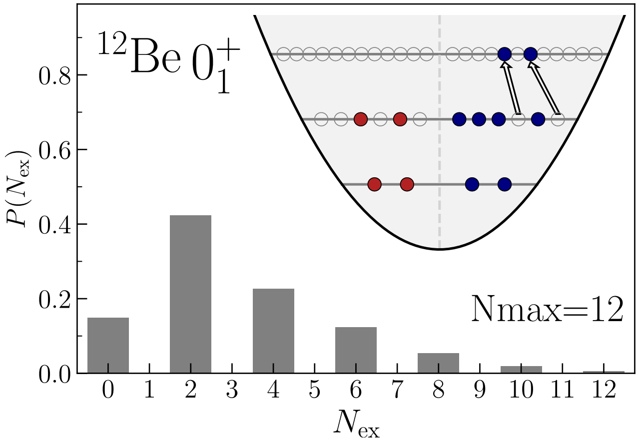


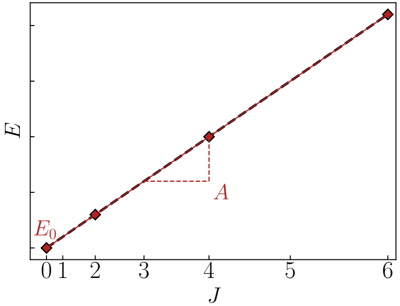


Nuclear rotations

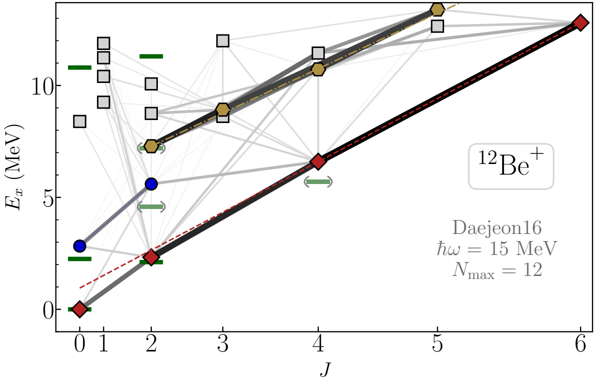
Rotation of intrinsic state $|\phi_K\rangle$ by Euler angles ϑ $\quad (J = K, K+1, \ldots)$

Characteristic energies

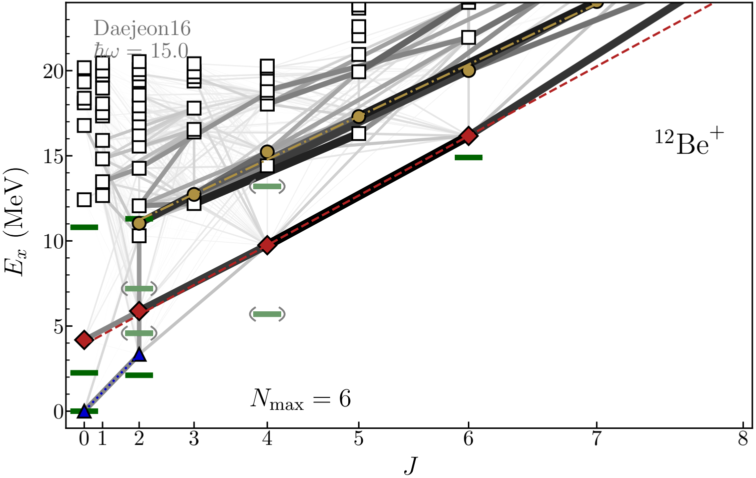
$$E(J) = E_0 + A[J(J+1)]$$

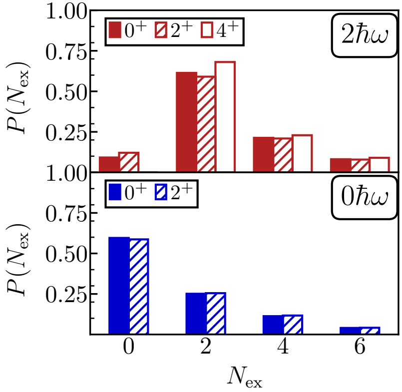


Enhanced $E2$ transitions within band



^{12}Be Band Evolution





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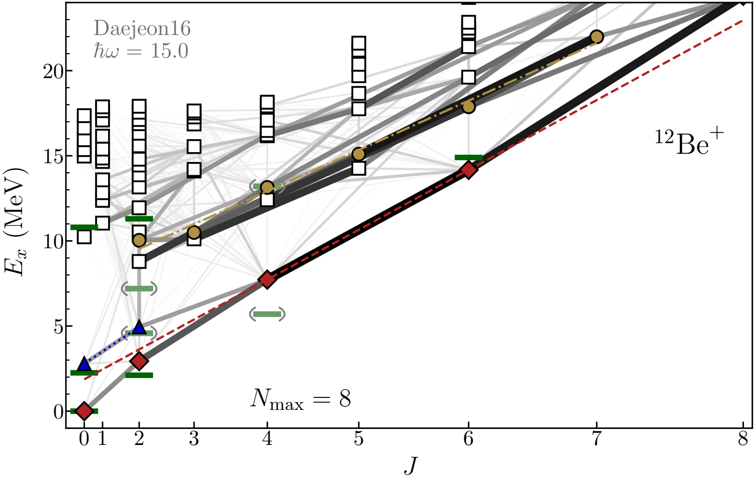

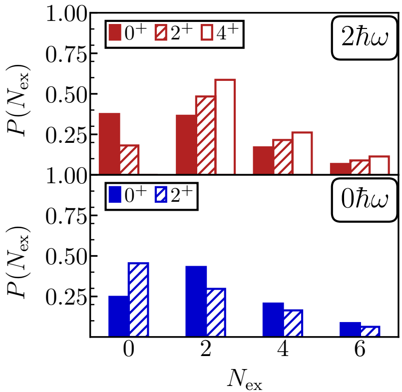

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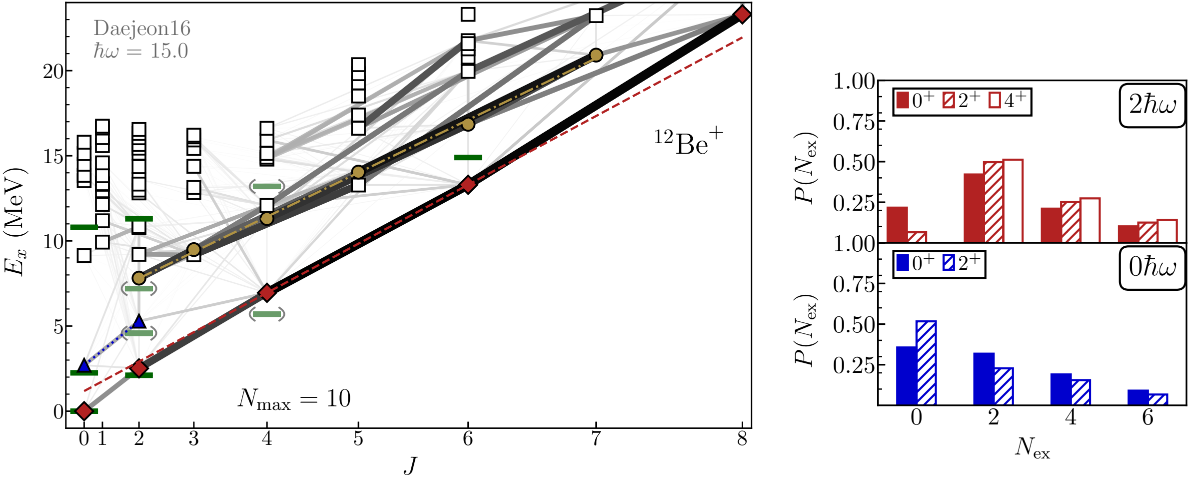

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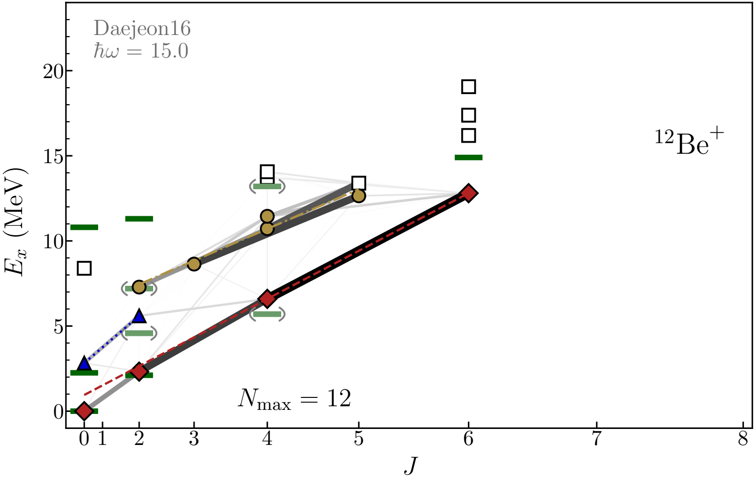
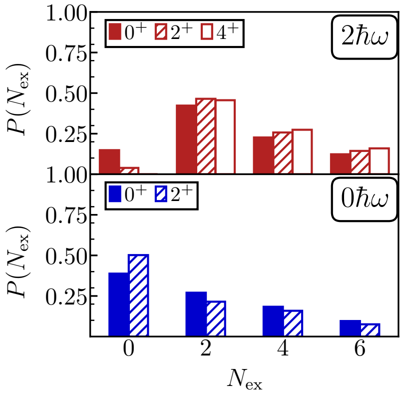
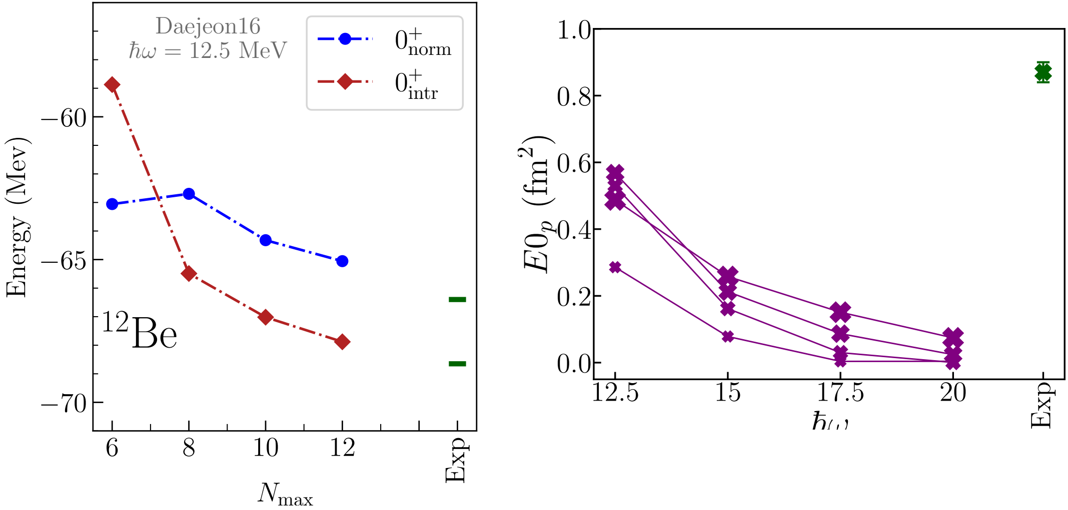


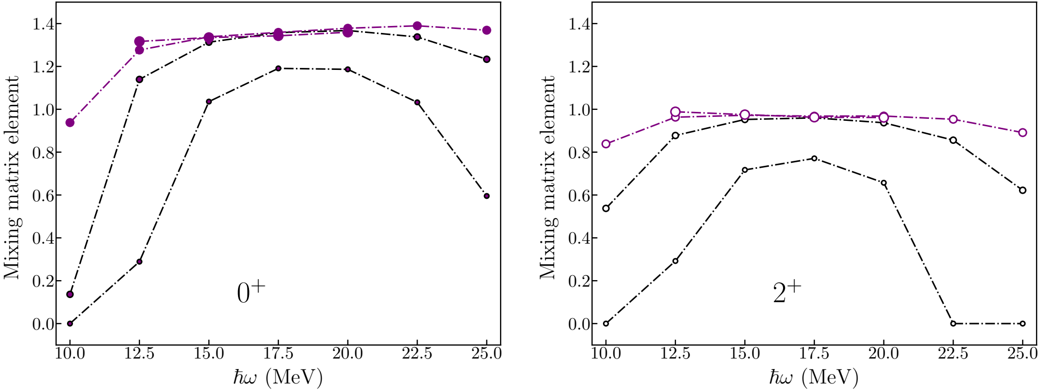


Mixing of intruder and normal states



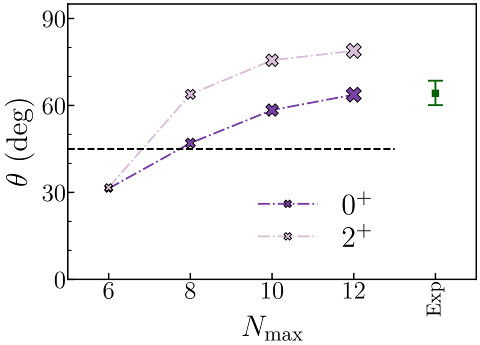
Two state mixing

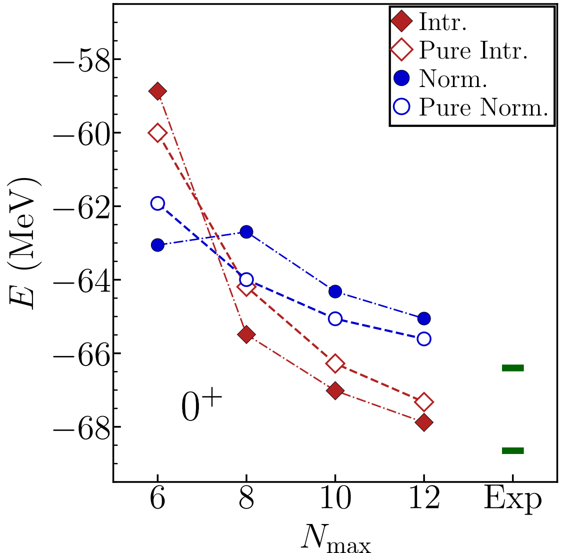
$$H_{\text{mix}} = \underbrace{\begin{pmatrix} E_1 & V \\ V & E_2 \end{pmatrix}}_{\text{mixing Hamiltonian}} \quad \rightarrow \quad \underbrace{\begin{pmatrix} \Psi'_1 \\ \Psi'_2 \end{pmatrix}}_{\text{``mixed"}} = \underbrace{\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}}_{\text{mixing matrix}} \underbrace{\begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix}}_{\text{``pure"}}$$



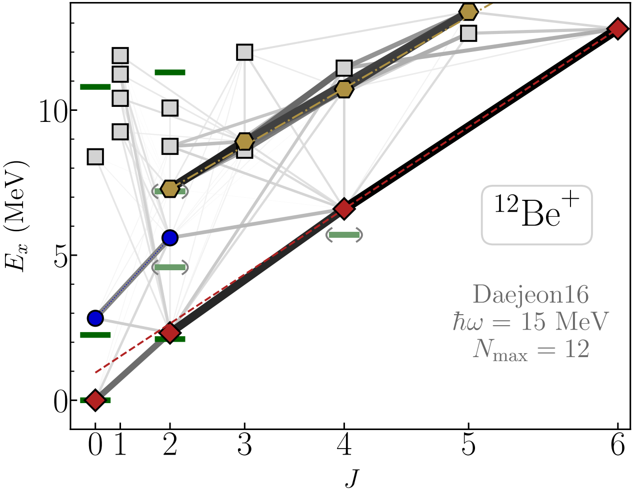
Mixing angle

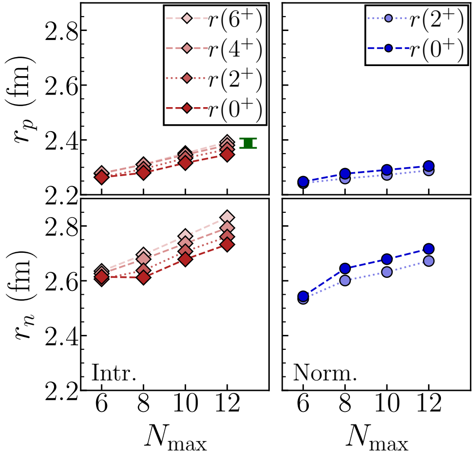
$$\begin{pmatrix} \Psi_1' \\ \Psi_2' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix}$$





Observables with a band





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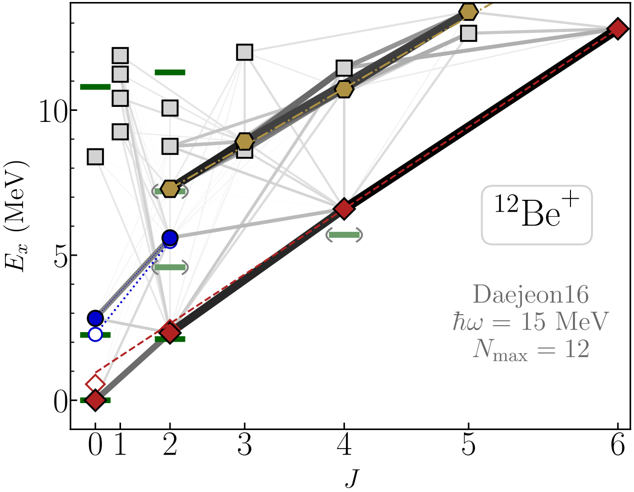

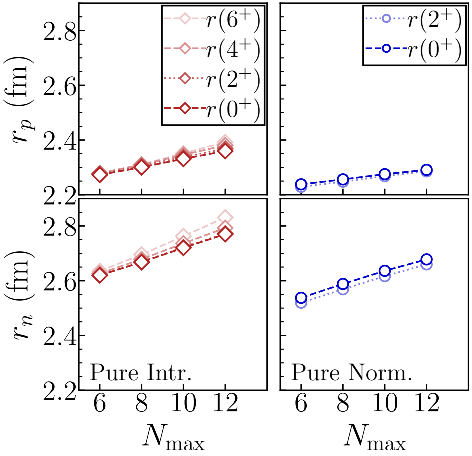

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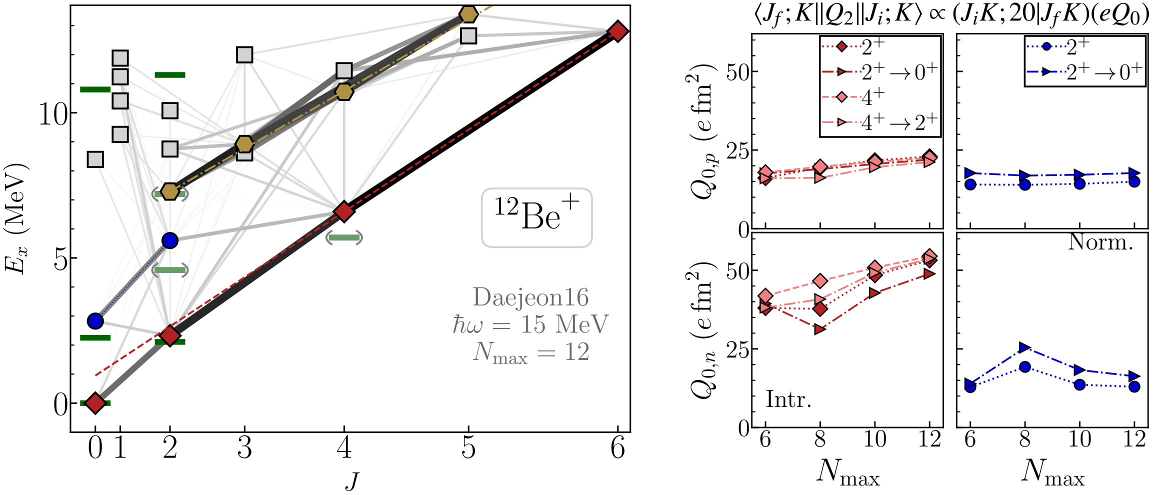

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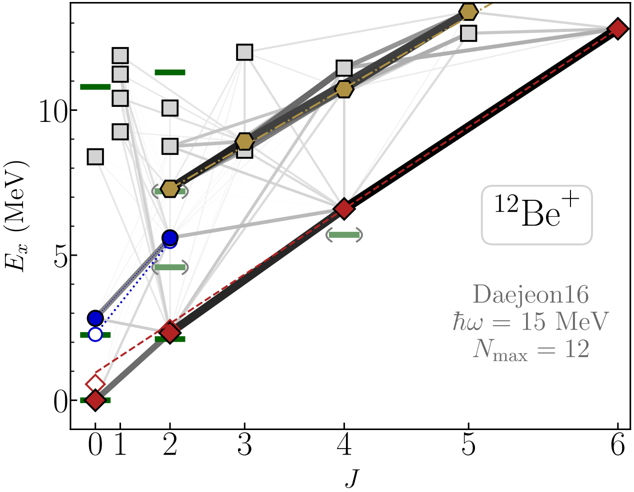
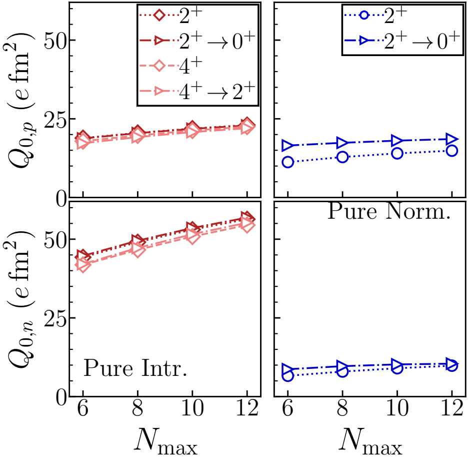


$$\langle J_f; K \| Q_2 \| J_i; K \rangle \propto (J_i K; 20 | J_f K)(eQ_0)$$



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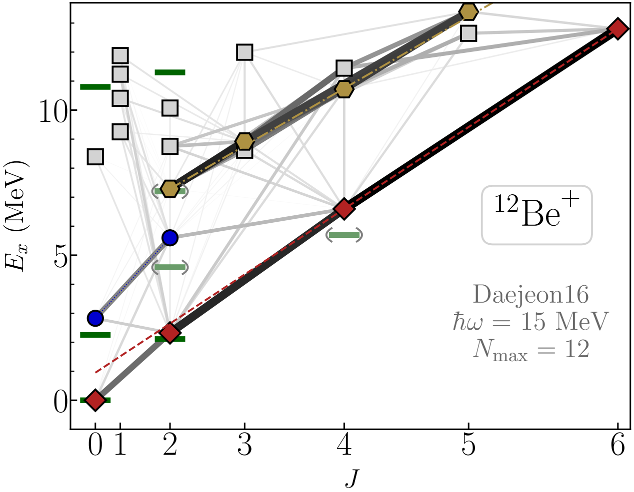

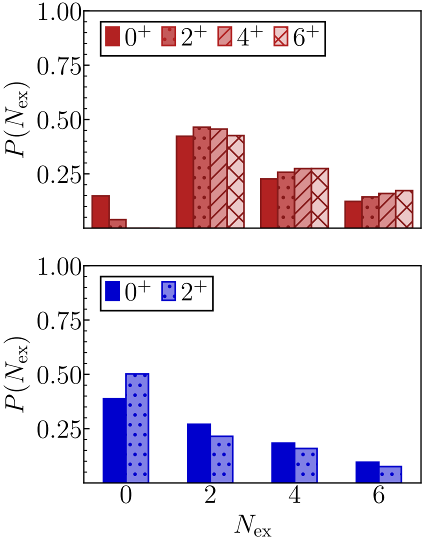

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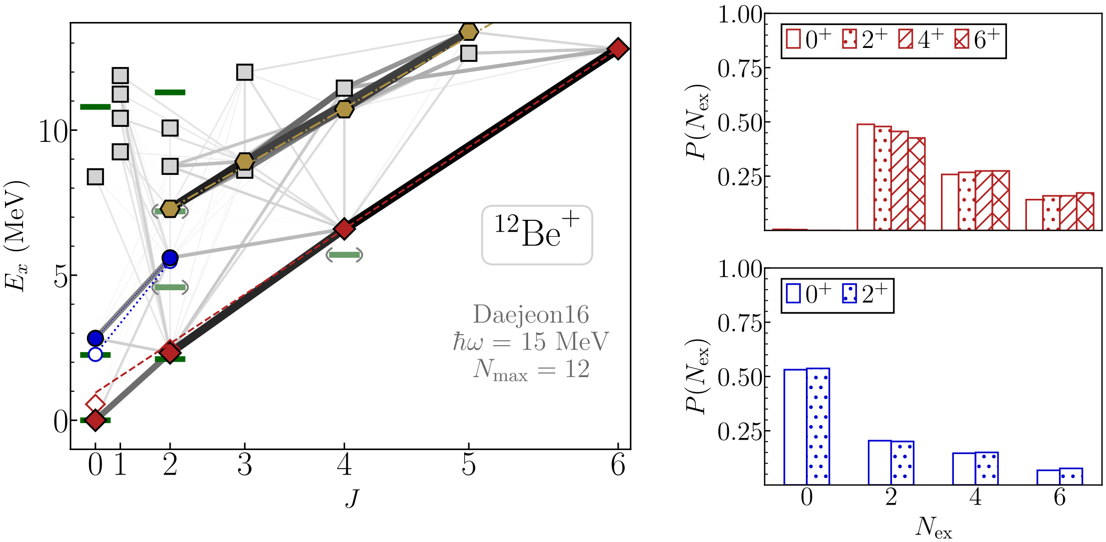
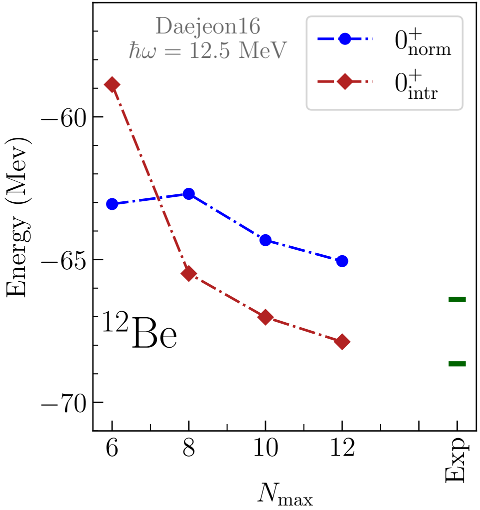


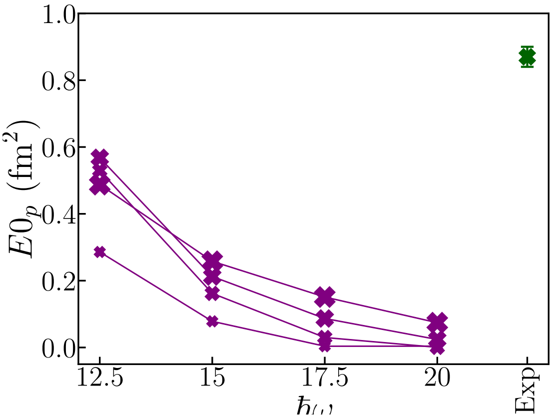
Why do we care about intruder states

– Makes uncertainty quantification hard

  Values depend on degree of mixing

Mixing of intruder and normal states





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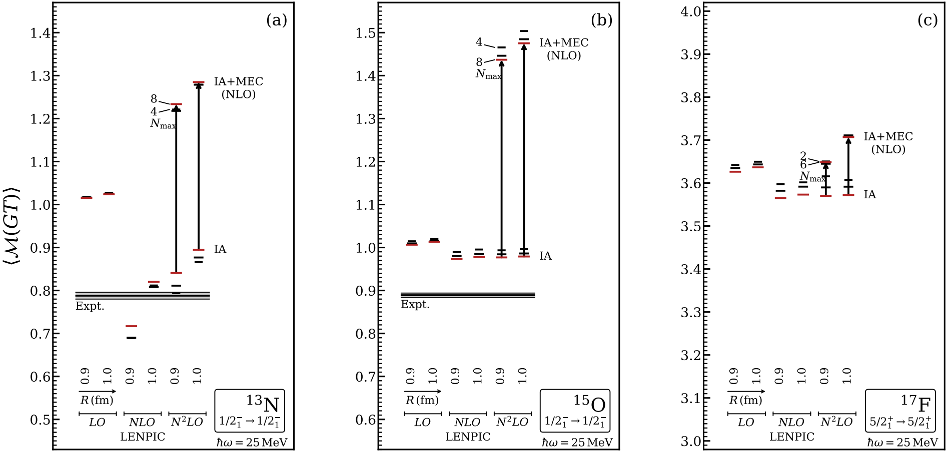
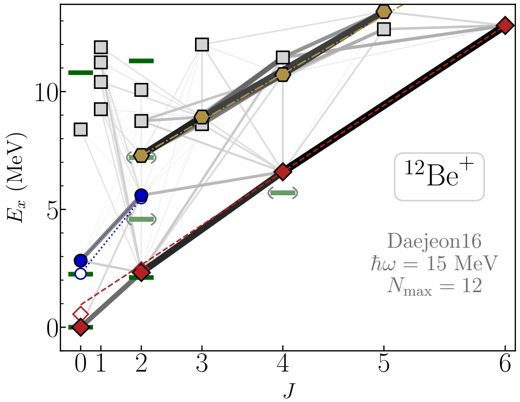
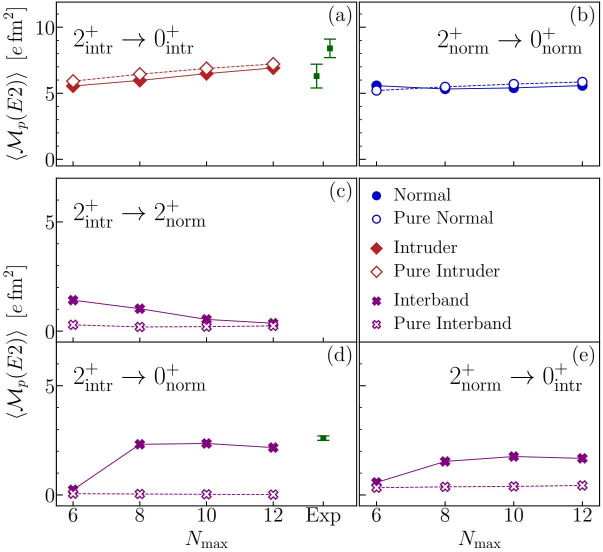
Consistent currents



Patrick Fasano

^{12}Be $E2$ transitions





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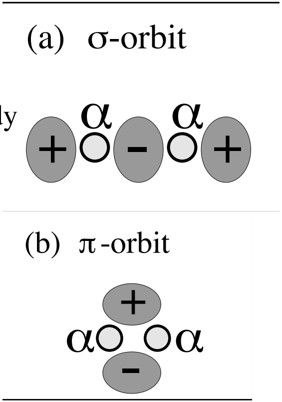
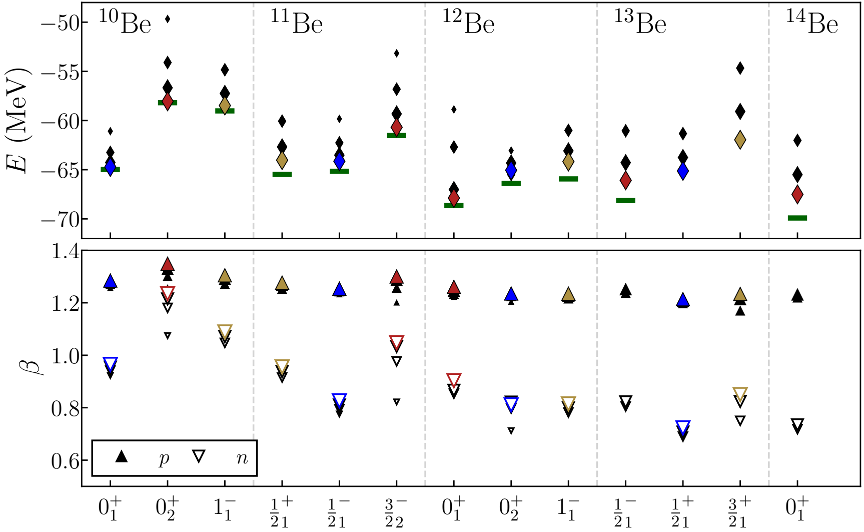
- Want LO description that captures normal and intruder states
  - Molecular orbitals and cluster models
  - Rotational model
  - Nilsson model
  - Algebraic models [Elliott SU(3) and Wigner SU(4)]



Beryllium isotopic chain



Nilsson Model

Wood Saxon potential

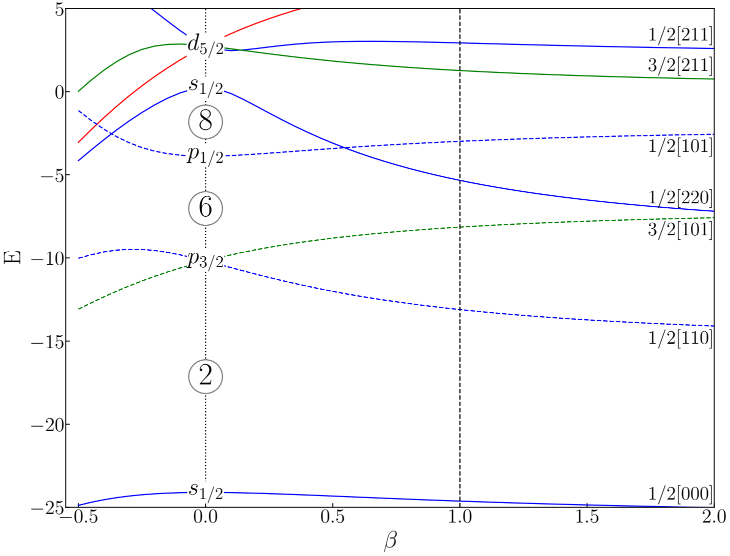
$$V_{\mathrm{WS}} = \frac{-V_0}{1 + e^{(r-R)/a}},$$

$a = 0.67\,\mathrm{fm}$, $V_0 = 57\,\mathrm{MeV}$,
$R = 1.27(A = 8)^{1/3}\,\mathrm{fm}$.

Nilsson Hamiltonian

$$H = V_{\mathrm{WS}} + \beta\hbar\omega r^2 Y_{20}$$

$\hbar\omega = 12.5, \beta = 1$



Wood Saxon parameters: J. Suhonen. From Nucleons to Nuclei
Concepts of Microscopic Nuclear Theory, Chapter 3.

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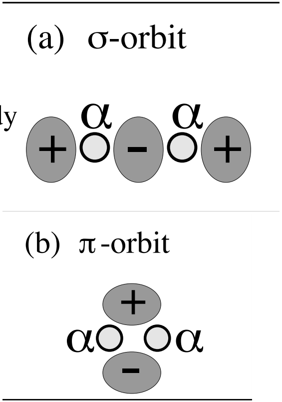
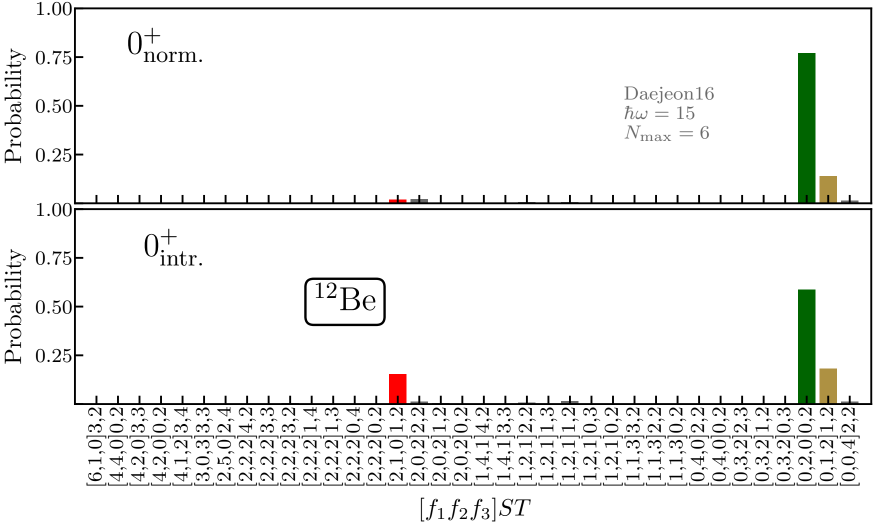
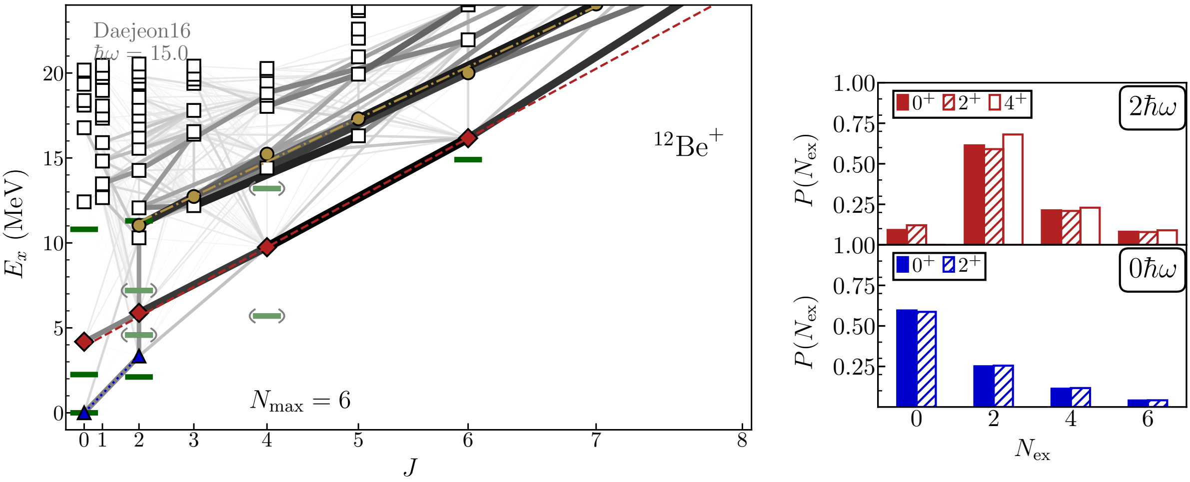


Wigner SU(4) decompositions of ^{12}Be



^{12}Be Band Evolution



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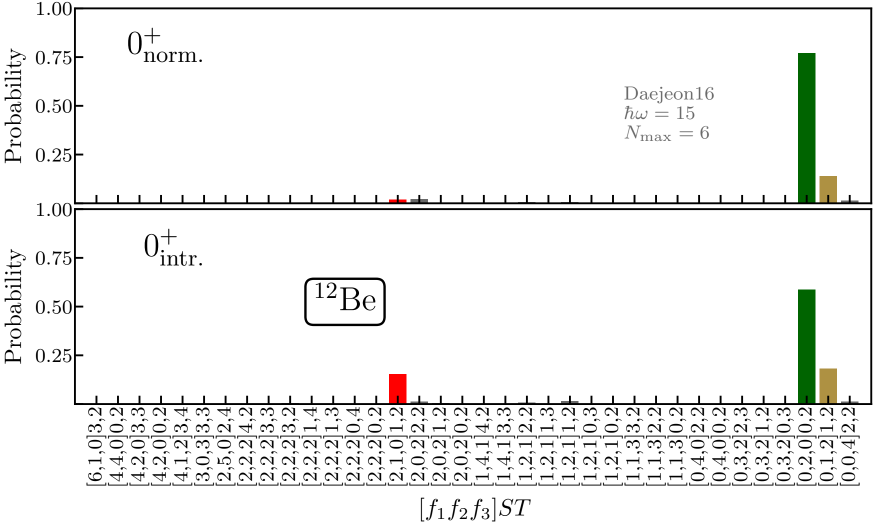


Elliott SU(3)

Labels (λ, μ) associated with deformation parameters β and γ

O. Castanos, J. P. Draayer, Y. Leschber, Z. Phys. A **329** *(1988) 3.*

$$\beta^2 \propto (\lambda^2 + \lambda\mu + \mu^2 + 3\lambda + 3\mu + 3)$$
$$\gamma = \tan^{-1}[\sqrt{3}(\mu + 1)/(2\lambda + \mu + 3)]$$

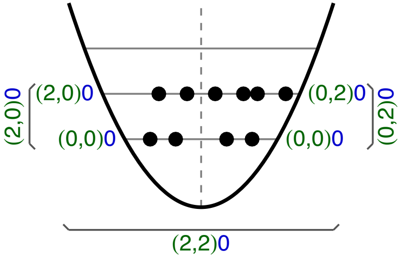
Lowest energies correspond to most deformed state

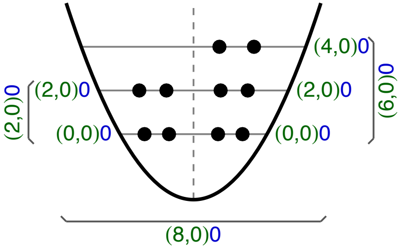
D. J. Rowe, G. Thiamova, and J. L. Wood. Phys. Rev. Lett. **97** *(2006) 202501.*

$$H = H_0 - \underbrace{\kappa Q \cdot Q}_{\propto \beta^2 \langle r^2 \rangle^2} + L \cdot S$$

SU(3) symmetry of a configuration

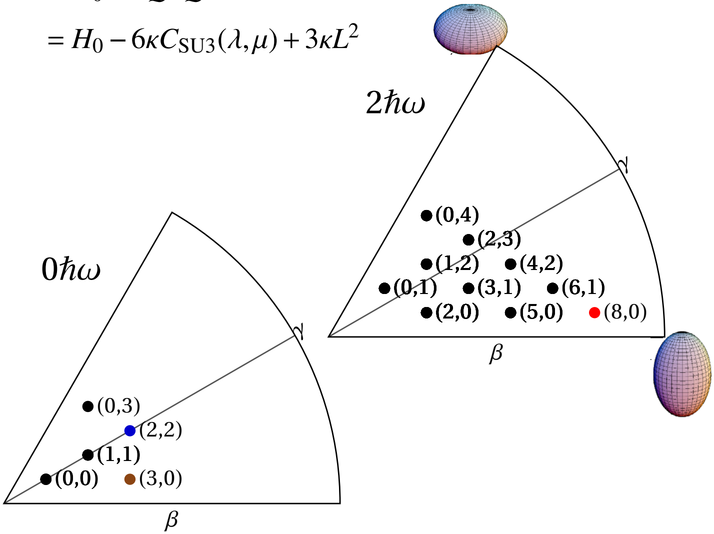
- Each particle has SU(3) symmetry $(N, 0)$, $N = 2n + \ell$
- Allowed spins dictated by antisymmetry constraints
- Final quantum numbers are $N_{\text{ex}}(\lambda\mu)S$.

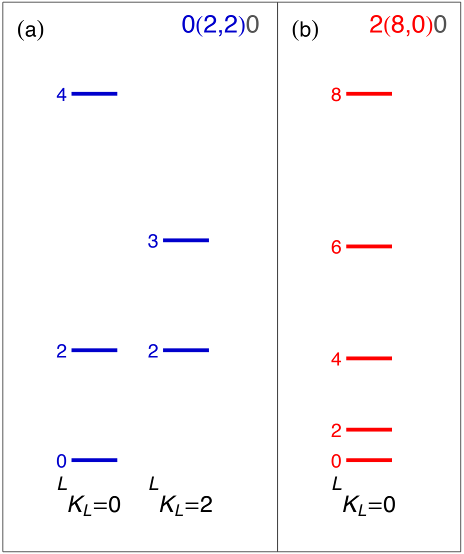




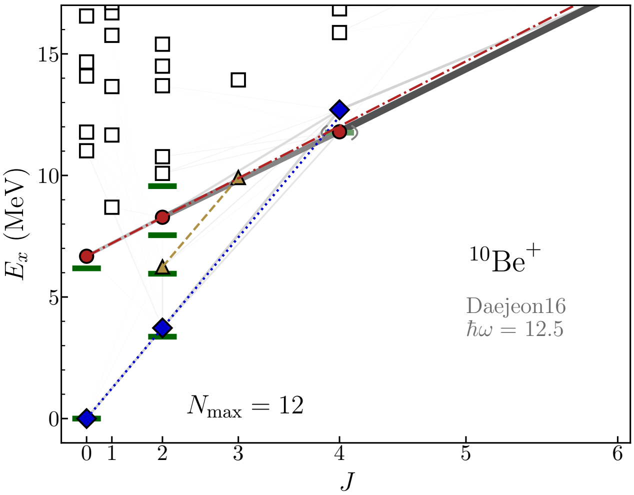
Elliott rotational bands: ^{10}Be

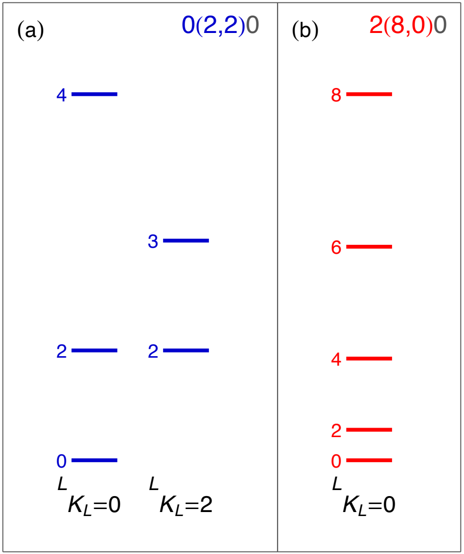
$$H = H_0 - \kappa Q \cdot Q$$
$$= H_0 - 6\kappa C_{SU3}(\lambda,\mu) + 3\kappa L^2$$



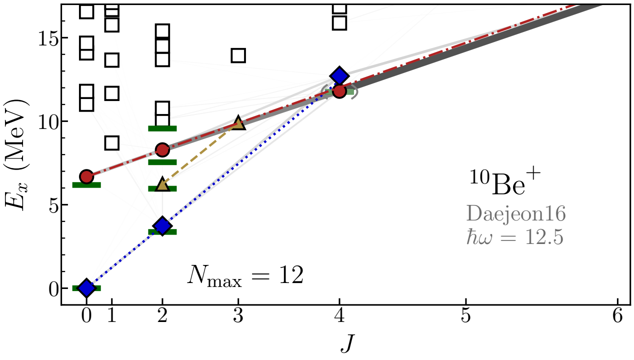


Elliott rotational bands: ^{10}Be

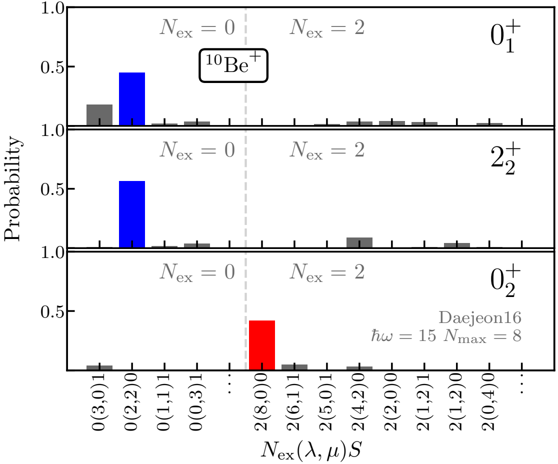




Elliott rotational bands: ^{10}Be



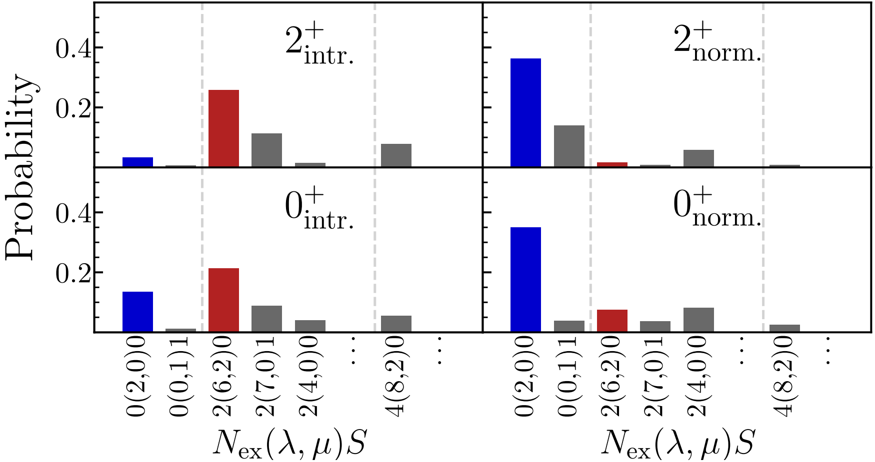
Approximate dynamical symmetry



- Ground state band: $N_{\rm ex}(\lambda\mu)S = 0(2,2)0$
- Intruder band: $N_{\rm ex}(\lambda\mu)S = 2(8,0)0$

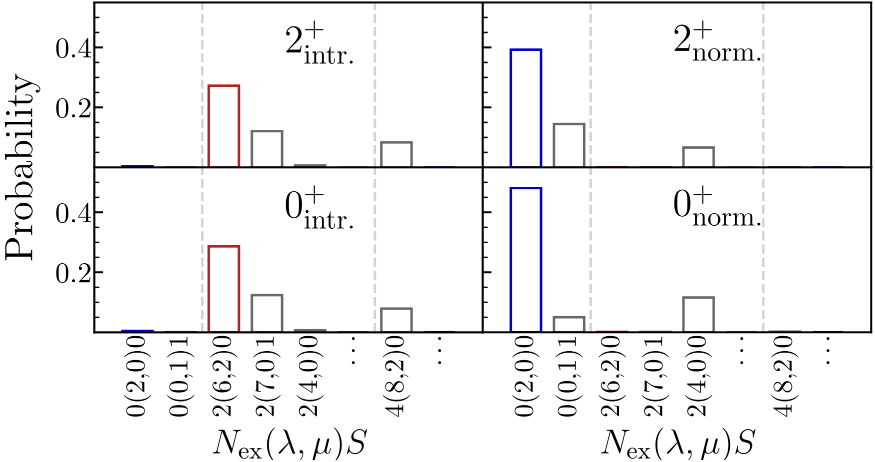
SU(3) decompositions of ^{12}Be

Mixed states

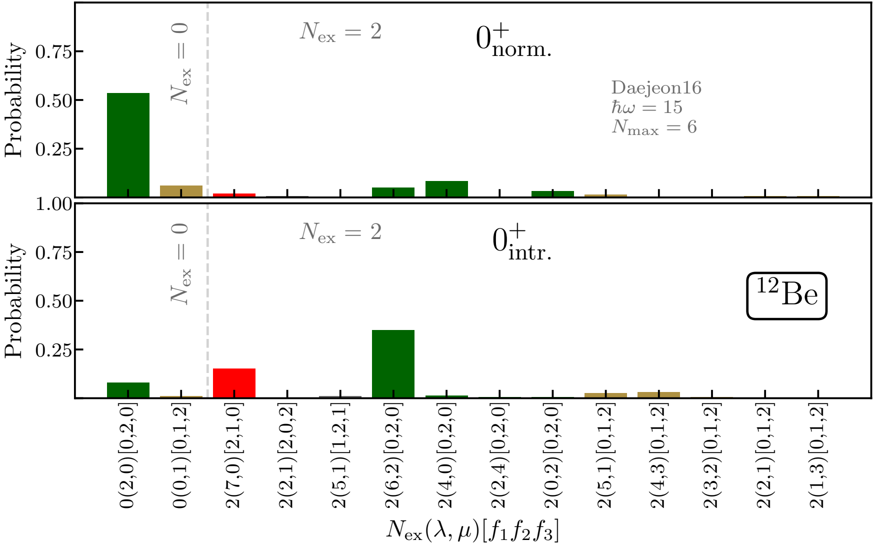


SU(3) decompositions of ^{12}Be

Pure states



Wigner SU(4) and Elliott SU(3)



Why do we care about intruder states

- Makes uncertainty quantification hard
  Values depend on degree of mixing

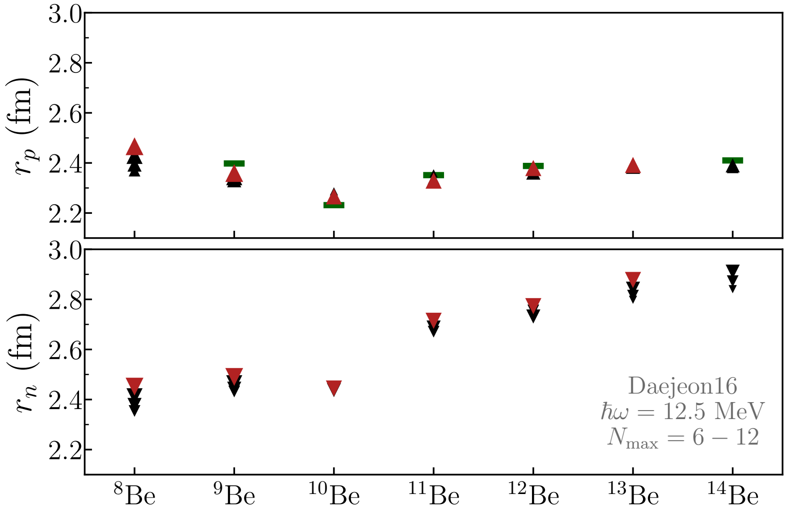
- Makes it harder to detangle error from EFT convergence or many-body convergence

- Want LO description that captures normal and intruder states
  - Molecular orbitals and cluster models
  - Rotational model
  - Nilsson model
  - Algebraic models [Elliott SU(3) and Wigner SU(4)]
- What terms in chiral expansion are important for describing intruder states?
  Would a sensitivity analysis of the normal and intruder states show significant differences?

Conclusions and hopes for the future

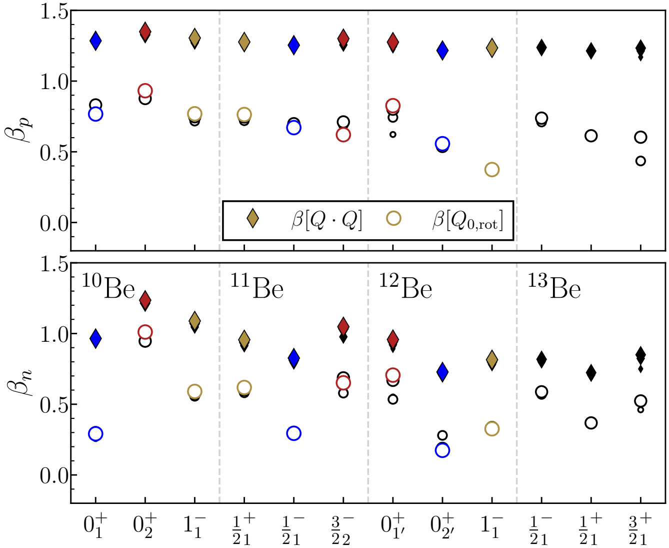
- Intruder states, which appear throughout the nuclear chart, are challenging to describe with current ab initio methods, *e.g.,Hoyle state*.
- Want to be able to provide accurate theoretical predictions with uncertainty quantifications
  Error from chiral truncation, error from many-body method
  Mixing of intruder and normal states can significantly impact structure
- Want to understand from a chiral point of view, how intruder and normal states differ
  What drives deformation?
  Z.H.Sun, A. Ekström, C. Forssén, G. Hagen, G. R. Jansen and T. Papenbrock. Phys. Rev. X **15**, *011028*
- Nuclei exhibit approximate symmetry.
  Want to understand how symmetries are broken from a Chiral EFT perspective
  Guide symmetry adapted approaches

Radii



r_p (fm)
3.0
2.8
2.6
2.4
2.2
r_n (fm)
3.0
2.8
2.6
2.4
2.2
Daejeon16
$\hbar\omega = 12.5$ MeV
$N_{\max} = 6 - 12$
^{8}Be
^{9}Be
^{10}Be
^{11}Be
^{12}Be
^{13}Be
^{14}Be

Quadrupole deformation



$$\beta_p \propto \frac{Q_{0,p}}{\sqrt{Z}\langle r_p^2\rangle} \qquad \beta_p \propto \frac{Q_{0,n}}{\sqrt{N}\langle r_n^2\rangle}$$

From q-invariant (*dynamic deformation*)

$$\langle Q \cdot Q\rangle = \langle Q_0 \cdot Q_0\rangle$$

If symmetric rotor

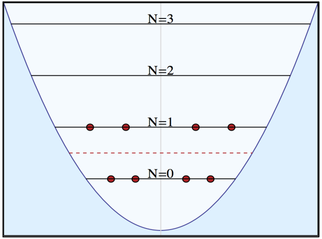
$$Q(J) = \frac{\hat{J}}{(1+\delta_{K,0})}(JK20|JK)Q_0$$

D. J. Rowe. Rep. Prog. Phys. **48**(1985) 1419.
D. J. Rowe, Nuclear Collective Motion: Models and Theory (2010).

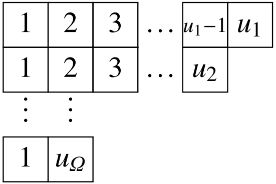
Wigner SU(4) and U(N) symmetries

E.g., Wigner SU(4) associated with spin, isospin and beta decay

- U(Ω) associated with nuclear shells
  Creation and annihilation operators $a_i^\dagger$ *and* a_j *generate* U(Ω)
  Ω *is number of single particle states*



- U(Ω) labeled by young tableau $[u] = [u_1 u_2 \cdots u_\Omega]$
  - Boxes in same column are antisymmetric
  - Boxes in same row are symmetric
  - Fully antisymmeterized slater determinant: $[u] = [1^\Omega]$.

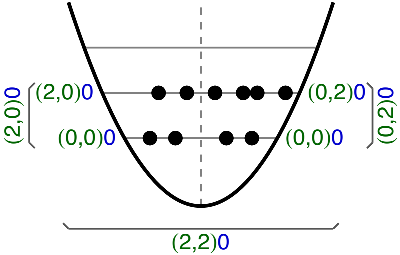


- For SU(Ω), remove columns with Ω blocks
  - $[f_1 f_2 \cdots f_{\Omega-1}] = [u_1 - u_\omega, u_2 - u_\Omega, \cdots, u_{\Omega-1} - u_\Omega]$
  - $[f_1 f_2 \cdots f_{\Omega-1}] = [u_1 - u_2, u_2 - u_3, \cdots u_\Omega - 1 - u_\omega]$

Factorize U(N) into spatial and spin symmetries

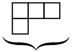
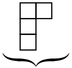
$\mathrm{U}(\Omega) \to \mathrm{U}(N_\mathrm{s}) \times \mathrm{U}(N_\mathrm{x})$

$N_\mathrm{s} = 2$ number of different spin states
$N_\mathrm{x} = \frac{(N+1)(N+2)}{2}$ is number of spatial states:

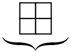
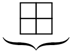


Antisymmetry requires conjugate tableau
Conjugate tableau: exchange rows and columns

$N = 1$: $\mathrm{U}(6) \to \mathrm{U}(3) \times \mathrm{U}(2)$

 $\times$ 

spin: S=1 spatial: $(\lambda\mu)$=(1,1)

 $\times$ 

spin: S=0 spatial: $(\lambda\mu)$=(0,2)

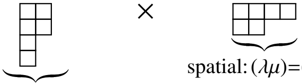
Factorize U(M) into spatial and spin symmetries

$$U(\Omega) \rightarrow U(N_s) \times U(N_x)$$

$N_s = 4$ number of different spin and isospin states
$N_x = \frac{(N+1)(N+2)}{2}$ is number of spatial states

Antisymmetry requires conjugate tableau
Conjugate tableau: exchange rows and columns

$N = 1$: $U(12) \rightarrow U(3) \times U(4)$



spin: $[f]=[0,1,0]$ spatial: $(\lambda\mu)=(2,2)$