Metasurface-based spin-selective optical cavity

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Outline

▶ Motivation
▶ Introduction to the problem
▶ Proposed cavity design
▶ Metasurface optics
▶ Conclusion
Goals and motivation

We seek a cavity which **differentiates between left- and right-handed light** within the cavity volume.

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\begin{pmatrix}
|H\rangle \\
|V\rangle \\
|L\rangle \\
|R\rangle
\end{pmatrix}
\]

Defined photon spin may facilitate:

- **spintronics**: exciton polariton with known spin
- **quantum information processing**
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\left( \frac{E_{0R}}{E_{0I}} \right)_N = \frac{Z_2 \cos \theta_I - Z_1 \cos \theta_T}{Z_2 \cos \theta_I + Z_1 \cos \theta_T} \approx -1
\]

where \(Z_1, Z_2\) are the impedences of air and the conductor respectively, and \(Z_1 \gg |Z_2|\).

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where \( Z_1, Z_2 \) are the impedences of air and the conductor respectively, and \( Z_1 \gg |Z_2| \).

Hence, \( E_R \) gains a uniform \( \pi \) phase shift and is “reflected” with no preferred transverse axis.

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Quantities with handedness are not invariant under reflections.

In particular, for circularly polarized incident light,

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It is useful to preserve one handedness in our cavity: hence, we may use a quarter wave plate preceding the mirror to “preserve” spin after reflection.
Proposed cavity design

We use birefringent materials to impose polarization-dependent path lengths.
Proposed cavity design

Some nice symmetries

Rotation:

180°
Proposed cavity design

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Reflection:
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(iv) $\hat{U}(z_1)\hat{U}(z_2) = \hat{U}(z_1 + z_2)$. 
Proposed cavity design

Defining transverse rotations

If two transverse polarizations \( \hat{\kappa}, \hat{\nu} \) are non-parallel, then some state \( |u_1(z)\hat{\kappa}\rangle + |u_2(z)\hat{\nu}\rangle \) effectively comprises a vector field.

For an orthonormal polarization basis \( \hat{\imath}, \hat{\jmath} \), denote

\[
|u(z)\rangle = (|u_1(z)\rangle \hat{\imath} + |u_2(z)\rangle \hat{\jmath}).
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Then we may define the expected local rotation operator,

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R(\theta) := \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}.
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Hence we define propagation in some birefringent region aligned with our polarization basis

\[ \hat{Q}(z_i, z_j) = \begin{pmatrix} \hat{U}(z_i) & 0 \\ 0 & \hat{U}(z_j) \end{pmatrix} \]
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and the cavity roundtrip operator follows:

\[
\hat{T} = \hat{Q}(\alpha + \delta, \alpha) R \left( \frac{\pi}{4} \right) \hat{Q}(2\beta, 0) R^\dagger \left( \frac{\pi}{4} \right) \ldots \\
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Proposed cavity design

\[ \hat{T} = \frac{1}{2} \left( \hat{U}(4\alpha + 2\delta) + \hat{U}(4\alpha + 4\beta + 2\delta) \right) l_2 + \]

\[ \frac{1}{2} \hat{U}(4\alpha) \left( \hat{U}(4\beta) - 1 \right) \left( \begin{array}{cc} 0 & \hat{U}(3\delta) \\ \hat{U}(\delta) & 0 \end{array} \right) \]
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Hence, we find (normalized) eigenvectors of

\[ |u_{\pm}\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} \pm \hat{U}(\delta) \\ 1 \end{pmatrix} |u(0)\rangle \]

with eigenvalues of

\[ \frac{1}{2} \hat{U}(4\alpha + 2\delta) \left( 1 + \hat{U}(4\beta) \pm \left( \hat{U}(4\beta) - 1 \right) \right) , \]
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that is,

\[ \hat{U}(4\alpha + 4\beta + 2\delta), \quad \hat{U}(4\alpha + 2\delta) \]
Designing metasurface-based optics

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A phase picture of optical elements
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Phase profile for a thin lens with focal length $f$:

$$\phi(r) = k \left( \sqrt{r^2 + f^2} - f \right)$$
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A phase picture of optical elements
Phase profile for a thin lens with focal length $f$:

$$\phi(r) = k \left( \sqrt{r^2 + f^2} - f \right)$$

If we allow birefringence:
Half wave plate:

$$\phi_x = \pi; \quad \phi_y = 0$$

Quarter wave plate:

$$\phi_x = \frac{\pi}{2}; \quad \phi_y = 0$$
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Arbabi et al. implement arrays of **elliptical, subwavelength high-contrast posts to exhibit birefringence.**

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- lattice constant
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RCWA is used to determine phase and amplitude for a given parameter set.

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Silicon nitride-based metasurfaces

Figure: low-contrast metasurface optics (SEM).\(^3\) (a) lens, (b) vortex beam generator.

Further work

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  - Transverse modes (cavity as system of coupled harmonic oscillators)
  - Explicit definition of propagation operator and mode functions
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  - Transverse modes (cavity as system of coupled harmonic oscillators)
  - Explicit definition of propagation operator and mode functions
- Simulate elements, cavity with FDTD
Acknowledgements

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