Lecture M2a: Chiral EFT 1

based on chiral symmetry of QCD / connects nuclear physics to QCD

$$\begin{array}{l} 0 \\ \vdash\!\!-\!\!\!-\!\!\!-\!\!\!-\!\!\!-\!\!\!-\!\!\!-\!\!\!-\!\!\!-\!\!\!\rightarrow Q \\ \underbrace{\left|\tfrac{1}{a}\right| \quad m_\pi}\; < m_\Delta - m_N \qquad\qquad m_{heavy} \end{array}$$

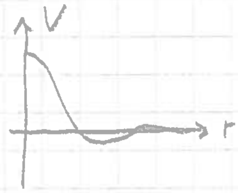
$Q \sim m_\pi$ + nonrelativistic $\frac{1}{m}$ expansion

chiral EFT: Weinberg '90, '91; degrees of freedom N, π without explicit Δ → Δ-full chiral EFT see tomorrow

dimensionless exp. parameter $\frac{Q}{\Lambda_b \sim 500\,\text{MeV}}$

references: Epelbaum, Prog. Nucl. Part. Phys. (2006)
Entem + Machleidt, Phys. Rept. (2011)
Epelbaum + Meißner, Annu. Rev. Nucl. Part. Sci (2012)

LO pionless EFT $\quad V_{NN}^{(0)} = C_S + C_T\, \vec{\sigma}_1 \cdot \vec{\sigma}_2$

→ 

smeared out δ functions
→ nonlocal at least at Q^4

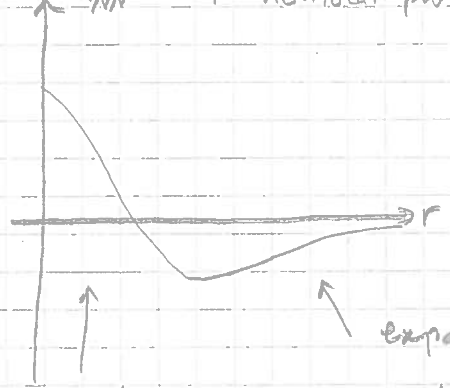
include pion exchange

$$V_{OPE} = -\left(\frac{g_A}{2f_\pi}\right)^2 \frac{\vec{\sigma}_1 \cdot \vec{q}\ \ \vec{\sigma}_2 \cdot \vec{q}}{q^2 + m_\pi^2}\, \vec{\tau}_1 \cdot \vec{\tau}_2 \qquad f_\pi = 92.4\ \text{MeV}$$

order of OPE $\sim \frac{Q \cdot Q}{Q^2} \sim 1 \implies V_{OPE} = V_{NN}^{(0)}$

combined LO NN potential

$V_{NN}^{(0)}$ + nonlocal part at least at order Q^4



expand long-range part systematically in pion exchanges

and short-range part in contact interactions pions are Goldstone bosons $\Rightarrow$ derivatively coupled

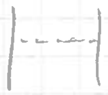
so that pion self-interactions remain massless

 $\sim Q^2$

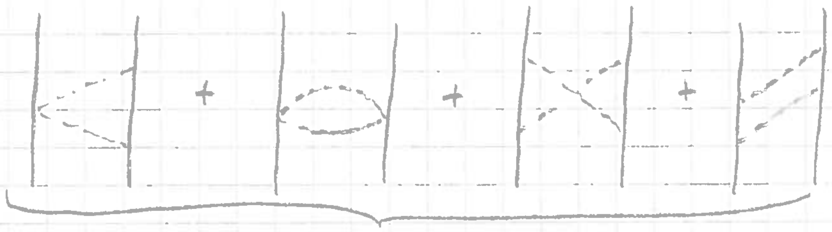
$\rightarrow$ see pion nucleon coupling  $\sim \vec{\sigma}\cdot\vec{q}$

pion interactions are weak at low energies

but why is V_{OPE} then Q^0

$\rightarrow$ from  intermediate state $\frac{1}{q^2+m_\pi^2}$ intermediate pion $E \sim m_\pi$

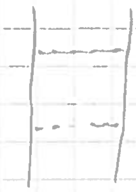
How does this work for two-pion exchange (TPE)?



two-pion intermediate state

$E \sim 2m_\pi \sim Q$

general mom. flowing through $\sim Q$



(*)

intermediate state

$E \sim \frac{q^2}{m_N} \sim \frac{Q^2}{m_N}$ $\rightarrow$ LS eqn

intermediate state from the LS eqn. is infrared enhanced $\rightarrow$ formally count $m_N \Rightarrow \Lambda_b$

$\Rightarrow$ Weinberg power counting

for nuclear forces power count the potential, then iterate to all orders solving the Schrödinger / LS equation.

Connected diagrams contribute at Q^ν with

$$\nu = -4 + 2N + 2L + \sum_{\text{vertices } i} \Delta_i \geq 0$$

$$\Delta_i = d_i + \frac{n_i}{2} - 2$$

\# nucleon lines

\# loops

\# derivatives or m_π insertions

\# nucleon field operators at vertex

chiral EFT connects (perturbative) $\pi\pi$, πN systems with $NN, \dots$ interactions

leading chiral Lagrangian $\Delta_i = 0$

$$\mathcal{L}^{(0)} = \frac{1}{2}\partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi} - \frac{1}{2}m_\pi^2\vec{\pi}^2 + N^\dagger\left(i\partial_0 + \frac{g_A}{2f_\pi}\vec{\tau}\,\vec{\sigma}\cdot\vec{\nabla}\vec{\pi} - \frac{1}{4f_\pi^2}\vec{\tau}\,(\vec{\pi}\times\dot{\vec{\pi}})\right)N$$

$$- \frac{1}{2}C_S\,(N^\dagger N)^2 - \frac{1}{2}C_T\left(N^\dagger\vec{\sigma}N\right)\cdot\left(N^\dagger\vec{\sigma}N\right) \quad + \text{terms with additional } \vec{\pi} \text{ fields}$$

next-to-leading chiral Lagrangian $\Delta_i = 1$

$$\mathcal{L}^{(1)} = N^\dagger\Big(4c_1 m_\pi^2 - \frac{2c_1}{f_\pi^2}m_\pi^2\vec{\pi}^2 + \frac{c_2}{f_\pi^2}\dot{\vec{\pi}}^2 + \frac{c_3}{f_\pi^2}\partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi}$$

$$- \frac{c_4}{2f_\pi^2}\varepsilon_{ijk}\varepsilon_{abc}\,\sigma_i\tau_a\left(\nabla_j\pi_b\right)\left(\nabla_k\pi_c\right)\Big)N$$

$$- \frac{D}{4f_\pi}(N^\dagger N)\left(N^\dagger\vec{\sigma}\vec{\tau}N\right)\cdot\vec{\nabla}\vec{\pi} - \frac{E}{2}(N^\dagger N)\left(N^\dagger\vec{\tau}N\right)\left(N^\dagger\vec{\tau}N\right) + \cdots$$

before applying Weinberg power counting, discuss two aspects

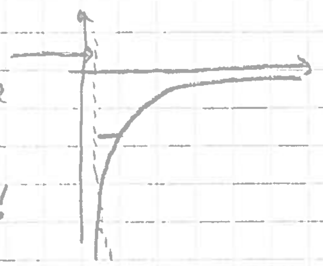
renormalization issue

iterating the leading order $V_{NN}^{(0)} = C_S + C_T\,\vec{\sigma}_1\cdot\vec{\sigma}_2 + V_{OPE}^{(0)}$ in the LS equation

generates cutoff dependence, with divergences in spin $S=1$ channels where $V_{OPE}^{(0)}$ is attractive

$\rightarrow$ due to very singular potentials $S=1 \rightarrow$ tensor forces $V_{OPE,\,tensor}^{(0)} \sim \frac{1}{r^3}$

large Λ
$\rightarrow$ short distance cutoff
$\rightarrow$ spurious bound states!



(and $\frac{1}{r^{3n}}$ for n-π exchange)

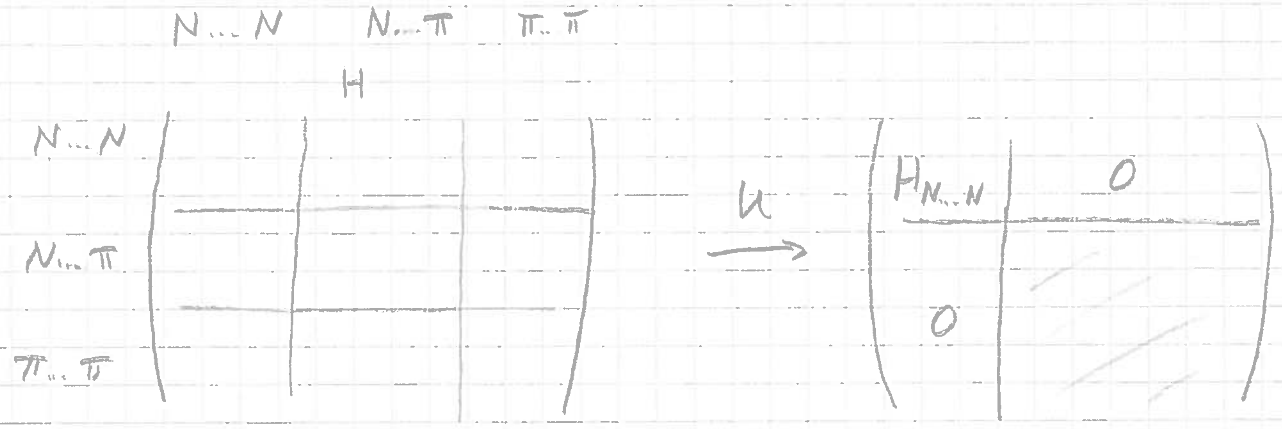
$\rightarrow$ to subtract these requires higher order contact interactions, already at LO!

practical scheme to avoid these is Weinberg power counting with $\Lambda \sim \Lambda_b$

ongoing developments to remedy this

in addition, need to decouple $\pi \dots N$ sectors from nuclear forces

→ achieved by means of a unitary transformation



leads to energy-independent $V_{NN}, V_{3N}, V_{4N}, \dots$ with cutoff $f(\frac{k}{\Lambda}), f(\frac{k'}{\Lambda})$
and spectral fn cutoff $\tilde{\Lambda}$ or dim. reg to calculate pion loop integrals

apply Weinberg power counting to V_{NN}

$\nu = \underbrace{\Delta_1}_{=1+\frac{2}{2}-2} + \Delta_2 = 0$

$\nu = \Delta_1 = \frac{4}{2} - 2 = 0$

NLO $\sim Q^2$ due to parity conservation

$\sim Q^2$ + TPE

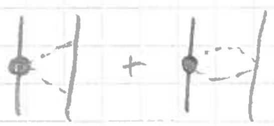
$\nu = 2 + \underbrace{\Delta_1}_{=0} + \underbrace{\Delta_2}_{=0} + \underbrace{\Delta_3}_{=0} = 2$

see $\mathcal{L}^{(0)}$

similarly for  and other diagrams

note: from (*) need to subtract iterated OPE!

$N^2LO \sim Q^3$ by replacing one of the leading  vertices from $\mathcal{L}^{(0)}$ by one from $\mathcal{L}^{(1)}$

→ subleading TPE  no new contact interactions at N^2LO!

→ both NLO and N^2LO have similar cutoff regularization error $\sim \left(\frac{Q}{\Lambda}\right)^4$

(5)

→ slide NN expansion in chiral EFT

→ NN, 3N, 4N, ... slide all worked out completely up to N^3LO.

available NN potentials from Entem + Machleidt (EM) N^3LO $\Lambda = 500, 600$ MeV dim reg. for TPE

Epelbaum, Glöckle, Meißner (EGM) NLO, N^2LO, N^3LO

$\Lambda = 450 - 600$ MeV

$\hat{\Lambda} = 500 - 700$ MeV

POUNDERS N^2LO $\Lambda = 500$ MeV

local LO, NLO, N^2LO potentials → Alex Gezerlis' lecture

include isospin-symmetry-breaking corrections

counting $\varepsilon = \frac{m_u - m_d}{m_u + m_d} \approx -\frac{1}{3}$ and $e = \sqrt{4\pi\alpha} \sim \frac{Q}{\Lambda_b}$

dominant strong ISB effects from pion mass difference in OPE, TPE

nucleon mass " in TPE

and two momentum-ind. contact interactions