Lecture QCD 1

will develop and work with EFT of QCD

connection to QCD and its symmetries important throughout the lectures

$\Rightarrow$ brief introduction to the theory of strong interactions

Quantum Chromodynamics

theory of quarks $(s=\frac{1}{2})$ and gluons $(s=1)$ $\longrightarrow$ A_μ^a fields

$$\mathcal{L}_{QCD} = \underset{\text{flavor}}{\bar{\Psi}_i}\left((i\gamma^\mu D_\mu)_{ij} - m_i\delta_{ij}\right)\Psi_j - \frac{1}{4}F_{\mu\nu}^a F^{\mu\nu a}$$

$$D_\mu = \partial_\mu - igA_\mu^a \qquad F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c$$

input: quark masses and g

	charge	mass
u (up)	2/3	2–3 MeV
d (down)	−1/3	4–6 MeV
c (charm)	2/3	≈1.3 GeV
s (strange)	−1/3	≈100 MeV
t (top)	2/3	≈170 GeV
b (bottom)	−1/3	≈ 4.5 GeV

with color charge (a) = red, green, blue

focus

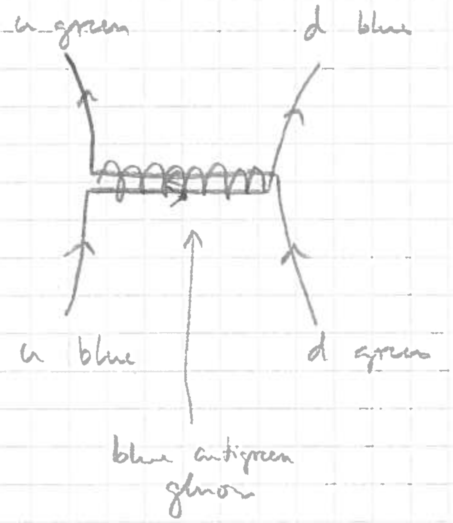
2 light quarks

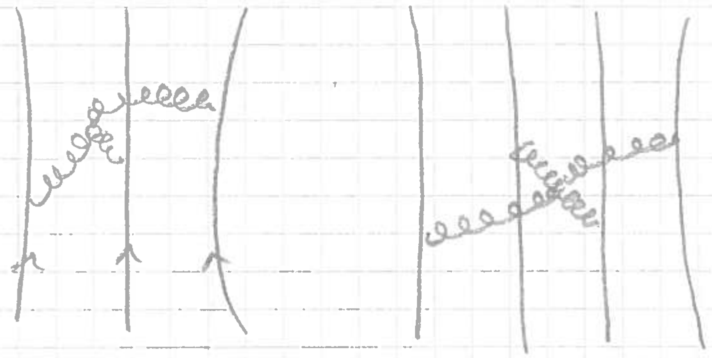
$N_f = 2+1$

Comparison with QED

QED		QCD
e^+, e^- electric charge e	$\longrightarrow$	quarks color charge g
1 photon	$\longrightarrow$	8 gluons (massless)
gauge group U(1)	$\longrightarrow$	SU(3)

Forces between quarks mediated by massless gluons





3- and 4-gluon vertex lead to 3- and 4-quark interactions (→ 3N and 4N forces)

only 8 gluons = 9 − color-blind singlet $\frac{1}{\sqrt{3}}\left(|r\bar{r}\rangle + |b\bar{b}\rangle + |g\bar{g}\rangle\right)$

$\sim \frac{g^2}{4\pi} = \alpha_s$ strong coupling

compared to $\frac{e^2}{4\pi} = \frac{1}{137}$ in QED

Running coupling

QED: electric charge: screening increases effective charge at short distances = high momentum scales Q

⇒ Coupling strength changes with scale "running coupling" (→ key concept for nuclear forces)

QCD: color antiscreening leads to

$$\frac{1}{\alpha_s(Q)} = \frac{33 - 2N_F}{6\pi} \log \frac{Q}{\Lambda_{QCD}} \qquad N_f = \# \text{ of flavors}$$

⇒ α_s becomes weaker for high momenta Q for $N_f \leq 16$ ⇒ asymptotic freedom

Gross, Politzer, Wilczek, Nobel 2004

→ $\alpha_s(Q)$ figure

Λ_{QCD} is the scale of QCD: $\Lambda_{QCD} \sim 200-400$ MeV

⇒ input to QCD: m_q, Λ_{QCD} instead of g

for chiral limit $m_{light} \to 0$, $m_{heavy} \to \infty$, Λ_{QCD} is only scale

$\Rightarrow$ QCD is perturbative at high energies, verified in exp., e.g. 3 vs 2 jet events
$\rightarrow$ jet figure

QCD is nonperturbative at low energies $\longrightarrow$ EFT for nuclear forces

leads to 1) confinement and 2) chiral symmetry breaking

$\downarrow$

quarks cannot be isolated, confined to color singlet (colorless) hadrons

energy to separate $q\bar{q}$: $E = \sigma r$

$\uparrow$
string tension

E is sufficient to
string / gluon flux breaks as r increases, new $q\bar{q}$ pair created

$\rightarrow$ $V_{q\bar{q}}$ figure

$\Longrightarrow$ degrees of freedom at low energies are hadrons

masses of hadrons bosons: mesons $\pi, \rho, \ldots$ ($q\bar{q}$) focus on hadrons of u,d quarks

fermions: baryons $N, \Delta, \ldots$ (qqq)

$m_{hadron} \sim 1$ GeV, except for light π, K

$\sim \Lambda_{QCD} \gg m_u, m_d$ $\Rightarrow$ can think of Λ_{QCD} as standard QCD kilogram

QCD symmetries of quarks $\longrightarrow$ symmetries in hadron spectrum

$m_u \approx m_d$ $\Rightarrow$ u,d quarks form isospin multiplet $|u\rangle = |\text{isospin}\uparrow\rangle = |T=\tfrac{1}{2}, M_T=\tfrac{1}{2}\rangle$
$|d\rangle = |\text{isospin}\downarrow\rangle = |\text{''}, =-\tfrac{1}{2}\rangle$

isospin operator $\vec{T} = \hbar \frac{\vec{\tau}}{2}$ with Pauli matrices τ_i

isospin symmetry (approximate symmetry because $m_u \neq m_d$)

clearly seen in hadron spectrum

baryons: nucleon $N\left(\frac{1}{2}^+\right)$ 940 MeV

$|n\rangle = |T=\tfrac{1}{2}, M_T=-\tfrac{1}{2}\rangle$ d(ud) (spin S=0)

$|p\rangle = |T=\tfrac{1}{2}, M_T=+\tfrac{1}{2}\rangle$ u(ud)

isospin doublet is nonstrange part ($S=0$ strangeness) of baryon octet

Delta isobars $\Delta\left(\frac{3}{2}^+\right)$ 1232 MeV

$|\Delta^-\rangle, |\Delta^0\rangle, |\Delta^+\rangle, |\Delta^{++}\rangle$

ddd, udd, uud, uuu

$|T=3/2, M_T=-3/2, -1/2, 1/2, 3/2\rangle$

isospin quartet of baryon decouplet

Simple constituent quark mass model: $m_{\text{constituent}} = \frac{m_{\Delta^{++}}}{3} \simeq 400\,\text{MeV} \sim \Lambda_{QCD}$

$m_N = 3 \cdot m_{\text{constituent}}$ − diquark ud $S=0$ binding

$B_{\text{diquark}} \simeq 300\,\text{MeV}$

mesons: pseudoscalar pions $\pi(0^-)$ 140 MeV

$|\pi^-\rangle, |\pi^0\rangle, |\pi^+\rangle$

$|T=1, M_T=-1,0,1\rangle$

vector mesons $\rho(1^-)$ 770 MeV

$|\rho^-\rangle, |\rho^0\rangle, |\rho^+\rangle$

meson masses $m_\rho \approx 2\, m_{\text{constituent}} \simeq 800\,\text{MeV}$ o.k.

but $m_\pi \simeq 140\,\text{MeV} \ll 2 m_{\text{constituent}} - B_{\text{diquark}} = 500\,\text{MeV}$

$\mathcal{L}_{QCD}$ symmetries with massless quarks

$$\mathcal{L}_q = \bar{u}\, i\not{D}\, u + \bar{d}\, i\not{D}\, d = \bar{u}_L\, i\not{D}\, u_L + \bar{u}_R\, i\not{D}\, u_R + \bar{d}_L\, i\not{D}\, d_L + \bar{d}_R\, i\not{D}\, d_R$$

spinors decomposed into left- and right-handed quarks

$\Rightarrow$ $\mathcal{L}_{QCD}$ is symmetric under independent rotations in u,d space of L- an R-handed quarks

symmetry $\quad SU(2)_L \times SU(2)_R \times U(1)_L \times U(1)_R$

$$= \underbrace{SU(2)_{L+R}}_{=\text{ vector }=\text{ isospin}} \times \underbrace{SU(2)_{L-R}}_{=\text{ axial }=\text{ chiral}} \times \underbrace{U(1)_V}_{\text{baryon number symmetry}} \times \underbrace{U(1)_A}_{\text{broken by quantum effects (anomaly)}}$$

$SU(2)_{isospin}$ is present in hadron spectrum

$SU(2)_{axial}$ implies degenerate parity partners

e.g. for the nucleon $\quad N(\tfrac{1}{2}^+)$ and $N(\tfrac{1}{2}^-)$

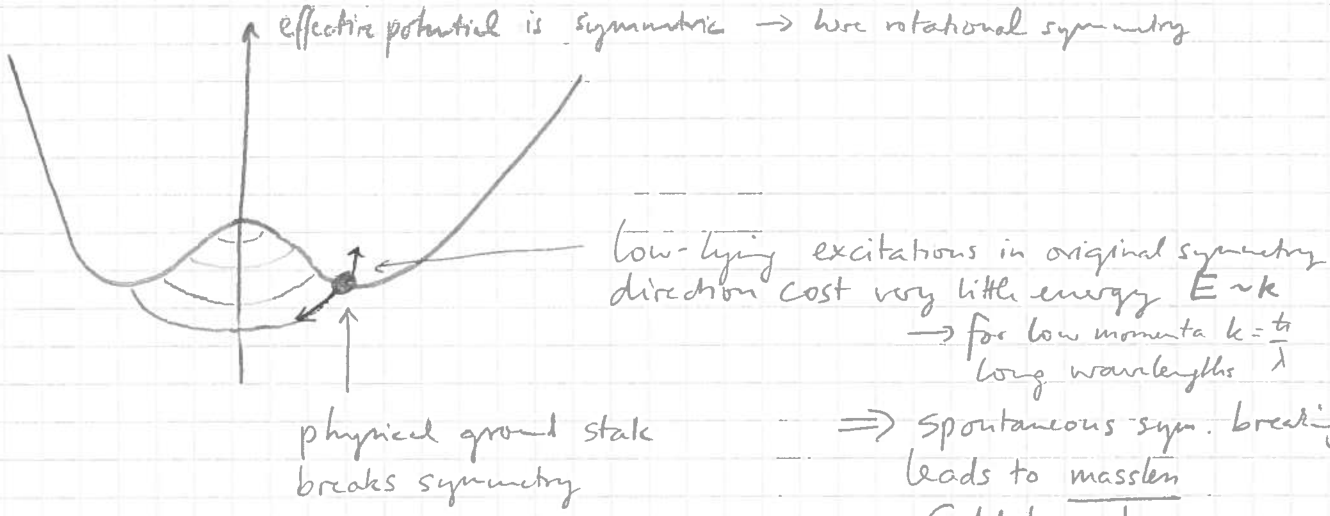
$$m_N^{\frac{1}{2}^+} \approx 940\,\text{MeV} \qquad \approx 1535\,\text{MeV}$$

$\Rightarrow$ <u>chiral symmetry is spontaneously broken</u> in the QCD ground state / vacuum

in addition $SU(2)_A$ is <u>explicitly broken by $m_u, m_d \neq 0$</u> mixes L, R

$$\mathcal{L}_{q,m} = -\bar{u}_R m_u u_L - \bar{u}_L m_u u_R - \bar{d}_R m_d d_L - \bar{d}_L m_d d_R$$

Spontaneous symmetry breaking



⇒ spontaneous sym. breaking leads to massless **Goldstone bosons**

Light pions are Goldstone bosons of chiral symmetry breaking

Gell-Mann-Oakes-Renner relation $\quad m_\pi^2 \sim m_q$

finite pion mass due to explicit chiral symmetry breaking

} → consequences for nuclear forces: π's interact weakly $\sim Q$ and can self-interact

Other examples of SSB and Goldstone bosons

phase	broken symmetry	Goldstone boson
crystal	translations	phonon = lattice vibrations
magnet ↑↑↑↑↑↑	rotations	magnon

In addition to the light pions, chiral symmetry breaking is responsible for the dynamical mass generation of $m_{constituent} \approx 300\,\text{MeV} \gg m_{u,d}$

QCD phase diagram

at high temperatures and densities = high momenta → asymptotic freedom

transition to deconfinement and chiral symmetry restoration

$m_{constituent} \to m_{u,d}$

↓ quarks and gluons become free of their confinement into hadrons

lattice QCD at zero chemical potential for $T \gtrsim 170\,\text{MeV} \sim 10^{12}\,\text{K}$

We will focus on the low T, low baryon density region of the QCD phase diagrams

⇒ degrees of freedom: nucleons and pions (and Δ's)

Units

We will work in units with $\hbar = c = 1$

use $\hbar c = 197.327$ MeV fm to convert MeV $\leftrightarrow$ fm^{-1}
fm $\leftrightarrow$ MeV^{-1}

e.g. pion mass $m_\pi = 140\,\text{MeV} = \frac{140\,\text{MeV}}{\hbar c} \approx 0.7\,\text{fm}^{-1}$ (inverse de-Broglie wavelength)

also useful to remember $\frac{\hbar^2}{m_N} = \frac{\hbar^2 c^2}{m_N c^2} = 41.4\,\text{MeV fm}^2$

Naive dimensional analysis and naturalness

$-e \; \bullet \; m_e$

$+Ze \; \circ \; m_{Nucleus}$

Example: Radius r and energy E of hydrogen-like atoms

reduced mass $\mu = \frac{m_e m_{Nucleus}}{m_e + m_{Nucleus}}$

What can r and E depend on? $\Rightarrow$ relevant quantities $\approx m_e$

		dimensions
reduced mass	m_e	$[M]$
Coulomb potential	$V(r) = -\frac{kZe^2}{r} \to kZe^2$ (depends on conventions)	$[M][L]^3[T]^{-2}$
quantization	$\hbar$	$[M][L]^2[T]^{-1}$

$$\Rightarrow \quad r \sim \frac{\hbar^2}{kZe^2 \cdot m_e} \quad \text{and} \quad E \sim \frac{kZe^2}{r} = \frac{(kZe^2)^2 m_e}{\hbar^2}$$

$$= \frac{a_0}{Z} \quad \text{Bohr radius}$$

QM: $r = \frac{a_0}{Z}$ QM: constant $\frac{1}{2}$

so constant $\sim 1 = 1$

NDA often allows one to estimate the answer and scaling law up to an overall factor that is usually of $O(1)$ $\Rightarrow$ naturalness