TALENT/INT Course on Nuclear Forces
Exercises and Discussion Questions F1

[Last revised on July 5, 2013 at 10:48:33.]

Friday 1: QMC, local chiral EFT; 3N forces and halo nuclei

We have again grouped all of the two-minute and discussion questions toward the beginning. But remember to spend only about an hour working on questions and then try some of the other problems as well. When you need a break, go back and try another question!

1. Student project: "Nuclear Forces and their impact on my research". Your task in the coming days is to prepare a short presentation that relates some of the topics discussed in this course to the research you are doing now or are preparing to do. Some of you who are more advanced in your research will present next week but most of you will present in the third week.

   (a) The outcome will be a ten-minute talk including a few minutes for questions and discussion.

   (b) Prepare no more than 3 slides that give an overview of your research area with the focus on how nuclear forces are relevant. Any further explanations will be on the blackboard.

   (c) Within your talk identify and present a discussion question about nuclear forces (new or from the exercises) that pertains to some aspect of your research.

   (d) You are encouraged to discuss your ideas with your fellow participants and the instructors.

2. If you haven't had a chance and have time, it would be great to go over the following problems of the first week: Monday 5), Tuesday 13), Wednesday 6), and Thursday 5). Also, don't forget to try the Mathematica (or iPython) notebook on scattering.

3. Two-minute exercises and discussion questions:

   (a) The factor
   $$2^A \frac{A!}{Z!(A-Z)!}$$
   describes the scaling with nucleon number A and proton number Z of the spin and isospin part of the state vector in a GFMC calculation of a nucleus. Let's dissect it.

      i. How many spin states are there?

      ii. How many isospin states are there? Why isn't this the same as for the spin?

      iii. How much larger is the factor for ^{12}C than for ^{4}He? How about ^{16}O? How about ^{40}Ca? Why is this a problem?

(b) The "AF" in AFDMC stands for "auxiliary field". What is an auxiliary field?

(c) Why do QMC methods like GFMC or AFDMC need local potentials? Why has this been a problem for chiral EFT?

(d) Why can we apply pionless EFT to atomic gases?

(e) Why is there only one Q^0 short-range three-body force for nucleons? (In general, we could construct several possible combinations of spin and isospin.) Is there a Q^0 short-range three-body force for neutrons only?

(f) Show that the operator

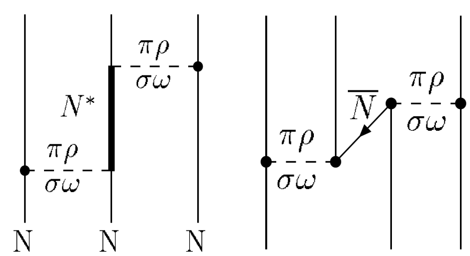
$$\begin{aligned} \mathcal{A}_{123} &= (1 + P_{12}P_{23} + P_{13}P_{23})(1 - P_{23}) \\ &= 1 - P_{12} - P_{13} - P_{23} + P_{12}P_{23} + P_{13}P_{23} \end{aligned} \quad (1)$$

is the antisymmetrizer in the three-body system. Why are the two ways of writing $\mathcal{A}_{123}$ equal?

(g) What is a limit cycle? What are the consequences for observables?

(h) Find all halo nuclei with proton number $Z < 8$. What is their evidence for a halo? Are they Borromean?

4. More two-minute exercises: Consider these figures in the context of three-body forces.



(a) If our low-energy pionful theory does not include nucleon resonances (an excited state of the nucleon represented on the left by a heavy line and N^*), why do we get a three-body force?

(b) The right Feynman diagram includes an anti-nucleon ($\overline{N}$) state. (This is called a "Z-graph".) Why does this diagram lead to a three-body force in our low-energy EFT? (Hint: what is the energy of the intermediate state?)

(c) The exchanged mesons can be either pions or heavier mesons (e.g., ρ or ω). For each combination, what are the ranges of the three-body forces (choosing among short-range, mid-range, and long-range)? Draw the corresponding diagrams in the chiral EFT that has pions but no heavy mesons or nucleon resonances or anti-nucleons.

5. Applying the exponential of a matrix $e^{-(H-E_T)\tau}$ to a trial ground-state vector. Consider a vector $|\Psi_{\text{var.}}\rangle$ and its expansion in eigenstates of the Hamiltonian matrix H:

$$|\Psi_{\text{var.}}\rangle = \sum_k C_k|\psi_k\rangle \quad \text{where} \quad H|\psi_k\rangle = E_k|\psi_k\rangle \ ,$$

where (for example) $|\Psi_{\text{var.}}\rangle$ is a variational guess for the ground-state wave function.

(a) Show in general that $f(H)|\psi_k\rangle = f(E_k)|\psi_k\rangle$ (where f is specified by a power series).

(b) Apply imaginary time propagation e^{-iHt} with $\tau = it$ to show

$$|\Psi(\tau \to \infty)\rangle = \lim_{\tau\to\infty} e^{-(H-E_T)\tau}|\Psi_{\text{var.}}\rangle \stackrel{\tau\to\infty}{\longrightarrow} C_0 e^{-(E_0-E_T)\tau}|\psi_0\rangle$$

That is, we project out the ground state.

(c) Note the use of the trial energy E_T. Why put that in? If I change E_T, how can I extract E_0?

(d) Why do we, in practice, break up the imaginary time into small intervals to be able to calculate: $e^{-(H-E_T)\tau} = \Pi_{\Delta\tau} e^{-(H-E_T)\Delta\tau}$ rather than just apply the exponential?

6. When discussing the AFDMC method we started from a two-particle operator:

$$V_{\text{SD}} = \frac{1}{2} \sum_{j,\alpha,k,\beta} \sigma_{j,\alpha}\ A_{j,\alpha;k,\beta}\ \sigma_{k,\beta}\ , \tag{2}$$

where roman indices are particle labels and greek indices are cartesian components. We then jumped to the result that this can be expressed as the square of a one-body operator

$$V_{\text{SD}} = \frac{1}{2}\sum_{n=1}^{3N}(O_n)^2\lambda_n\ , \tag{3}$$

where n runs over the distinct one-body operators. Here we provide more details.

(a) Write down the equation defining the real eigenvalues and eigenvectors of the $3N$ by $3N$ symmetric matrix A,

$$\sum_{k,\beta} A_{j,\alpha;k,\beta}\ \psi_n^{k,\beta} = \lambda_n \psi_n^{j,\alpha}\ . \tag{4}$$

(b) The operators O are a combination of the spin matrix and the eigenvectors. Write them out explicitly as a sum over one particle label and one cartesian component.

(c) Using the eigendecomposition $A = Q\Lambda Q^{-1}$ reach the following result:

$$V_2 \ = \ V_{\text{SI}} + \frac{1}{2}\sum_{j,\alpha,k,\beta,n} \sigma_{j,\alpha}\,\psi_n^{j,\alpha}\,\lambda_n\,\sigma_{k,\beta}\,\psi_n^{k,\beta} \tag{5}$$

and from there arrive at the result shown in the slide (and above) in terms of the square of a one-body operator.